Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.2-Quartic-trinomial/119-1.2.2.5

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**108**]. This is test number [119].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (108)	0.00 (0)
Mathematica	100.00 (108)	0.00 (0)
Maple	96.30 (104)	3.70 (4)
Fricas	85.19 (92)	14.81 (16)
Mupad	75.93 (82)	24.07 (26)
Giac	75.00 (81)	25.00 (27)
Reduce	75.00 (81)	25.00 (27)
Maxima	58.33 (63)	41.67 (45)
Sympy	37.96 (41)	62.04 (67)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

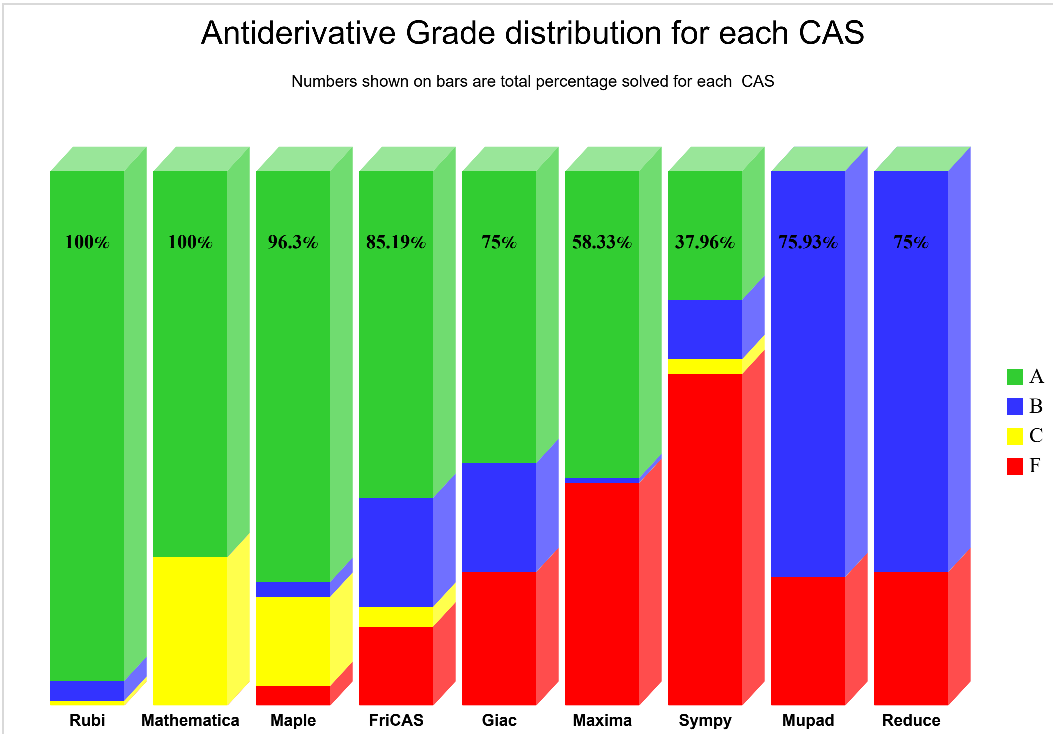
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

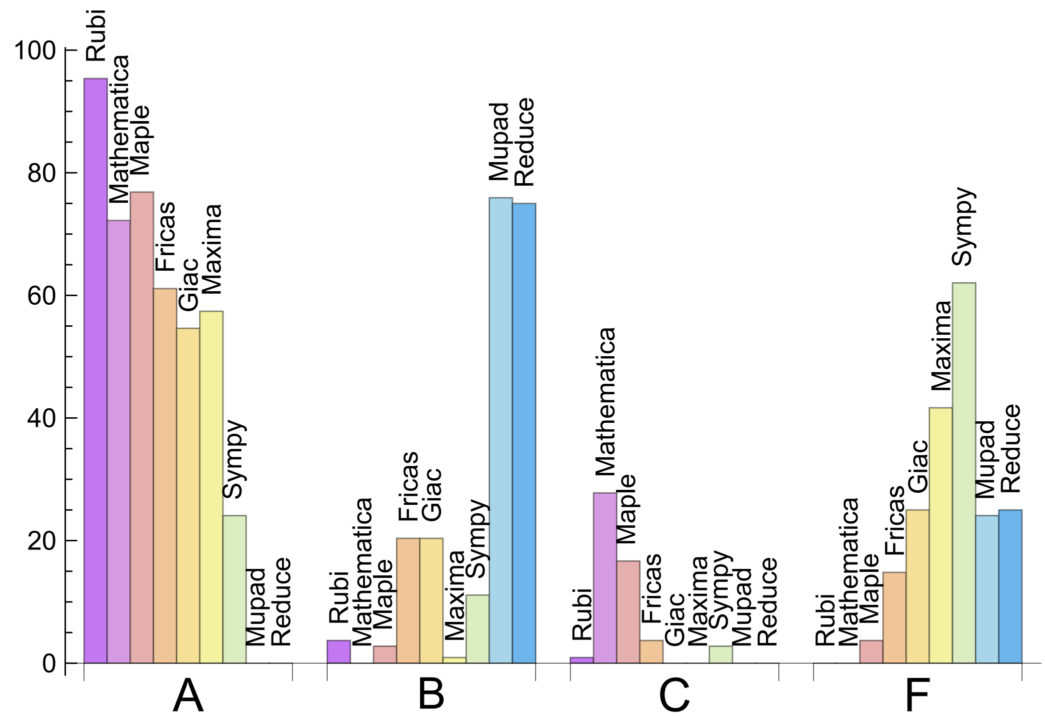
System	% A grade	% B grade	% C grade	% F grade
Rubi	95.370	3.704	0.926	0.000
Maple	76.852	2.778	16.667	3.704
Mathematica	72.222	0.000	27.778	0.000
Fricas	61.111	20.370	3.704	14.815
Maxima	57.407	0.926	0.000	41.667
Giac	54.630	20.370	0.000	25.000
Sympy	24.074	11.111	2.778	62.037
Mupad	0.000	75.926	0.000	24.074
Reduce	0.000	75.000	0.000	25.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	4	100.00	0.00	0.00
Fricas	16	25.00	75.00	0.00
Mupad	26	0.00	100.00	0.00
Giac	27	100.00	0.00	0.00
Reduce	27	100.00	0.00	0.00
Maxima	45	100.00	0.00	0.00
Sympy	67	35.82	64.18	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.05
Giac	0.46
Rubi	0.65
Reduce	1.12
Maple	1.88
Fricas	2.73
Mathematica	2.84
Mupad	14.49
Sympy	17.38

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	92.76	0.93	82.00	0.89
Maple	266.49	1.01	115.50	0.98
Rubi	269.40	1.25	154.00	1.00
Mathematica	271.43	1.05	157.50	1.00
Sympy	724.59	7.75	121.00	1.21
Giac	1916.93	3.89	111.00	1.00
Reduce	2264.63	4.74	231.00	2.22
Mupad	6308.49	9.84	108.00	0.98
Fricas	28162.41	132.10	177.50	1.12

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

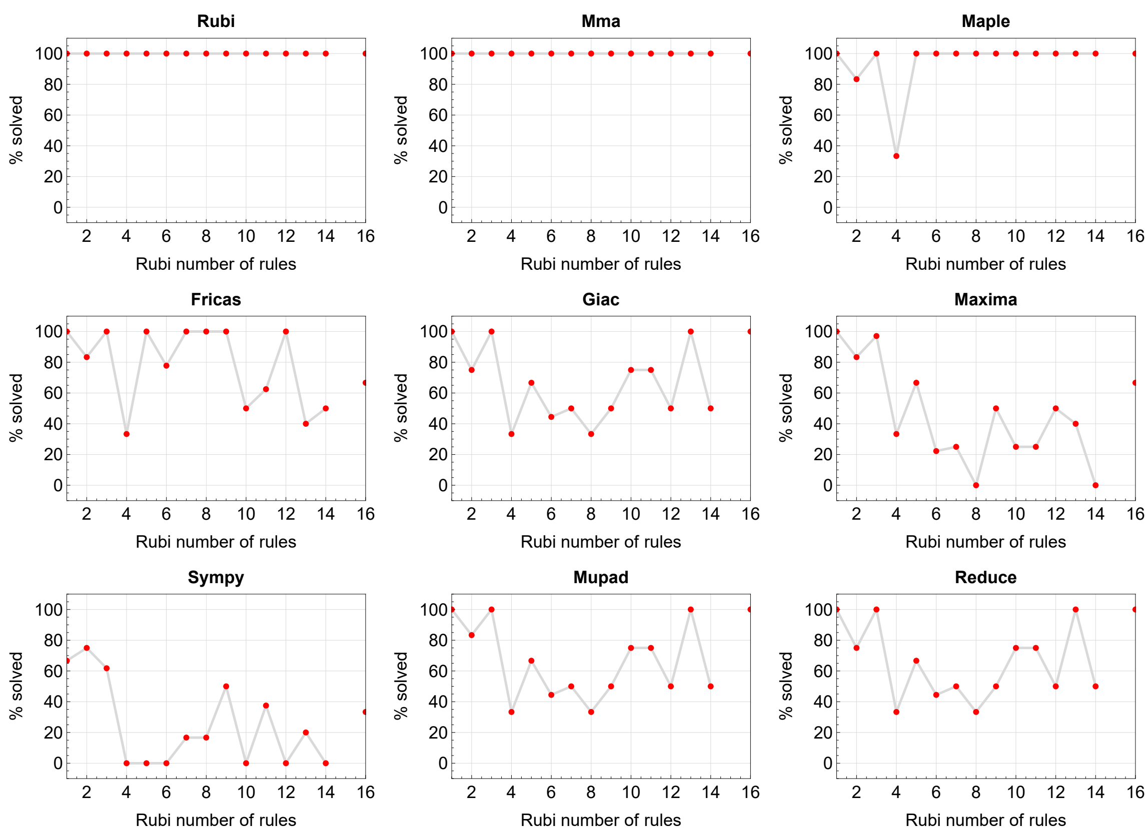


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

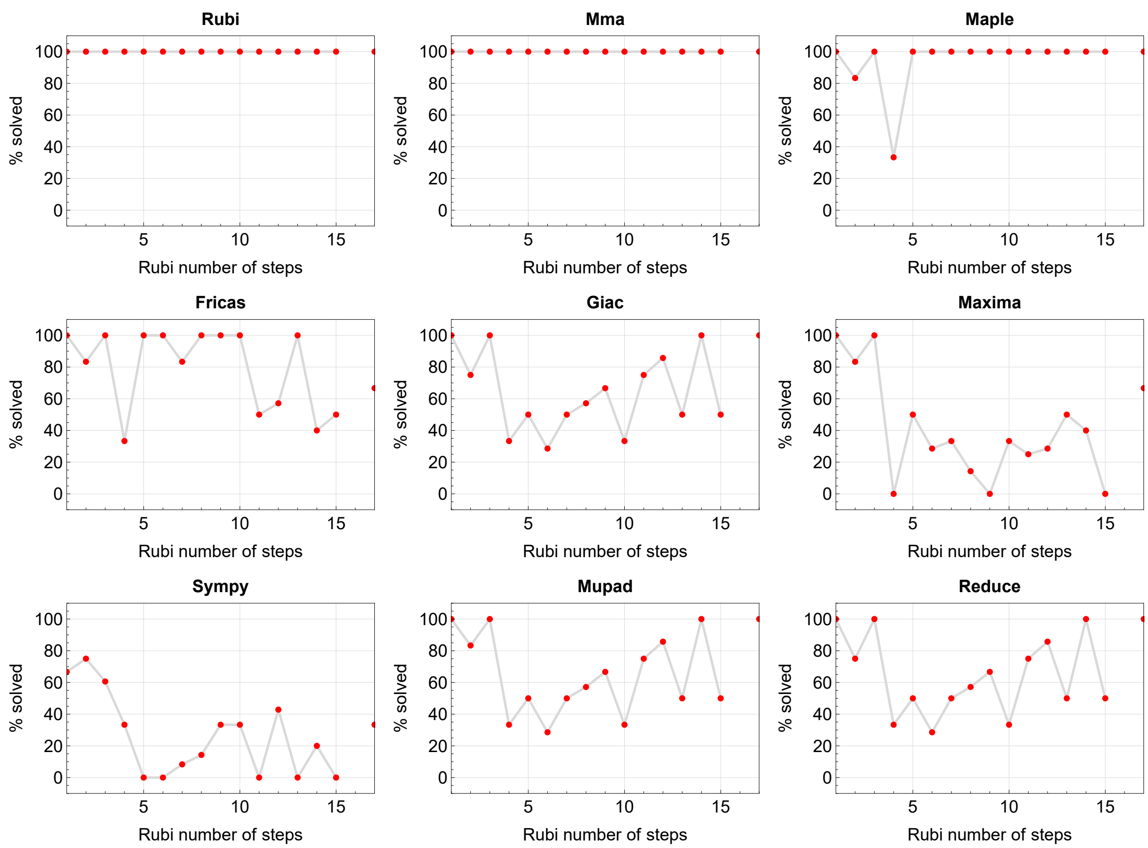


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

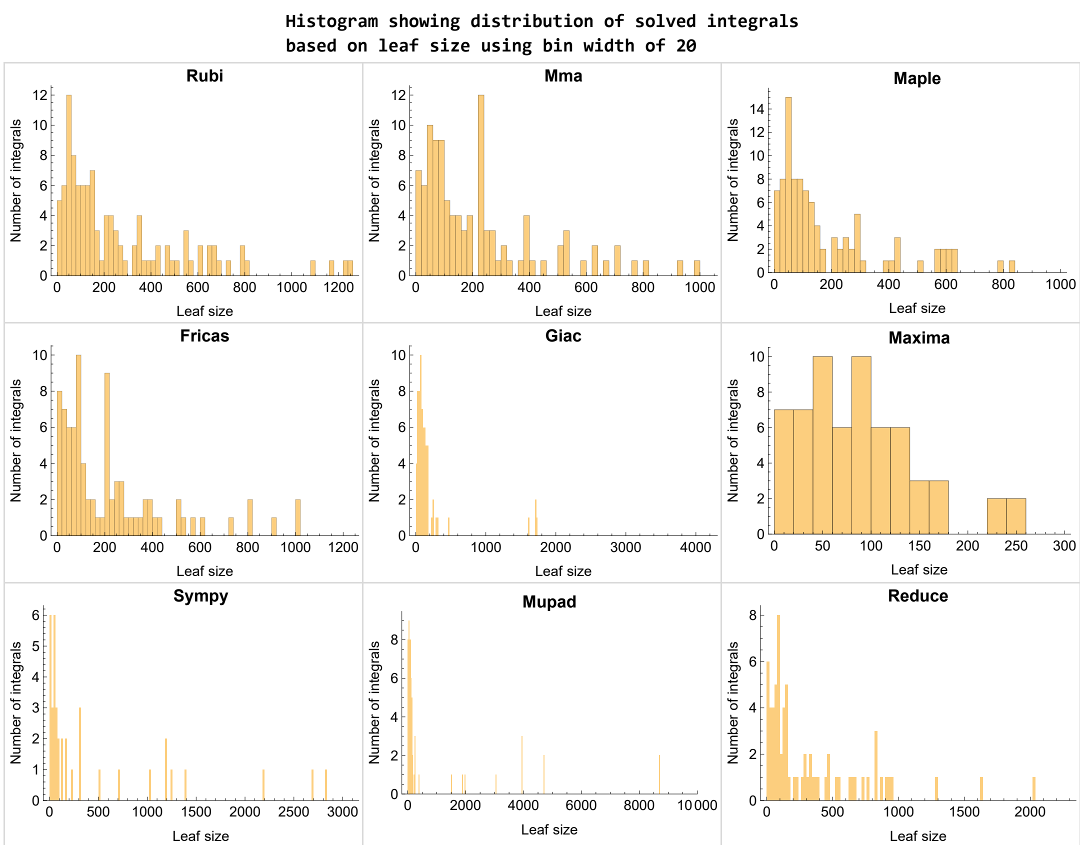


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

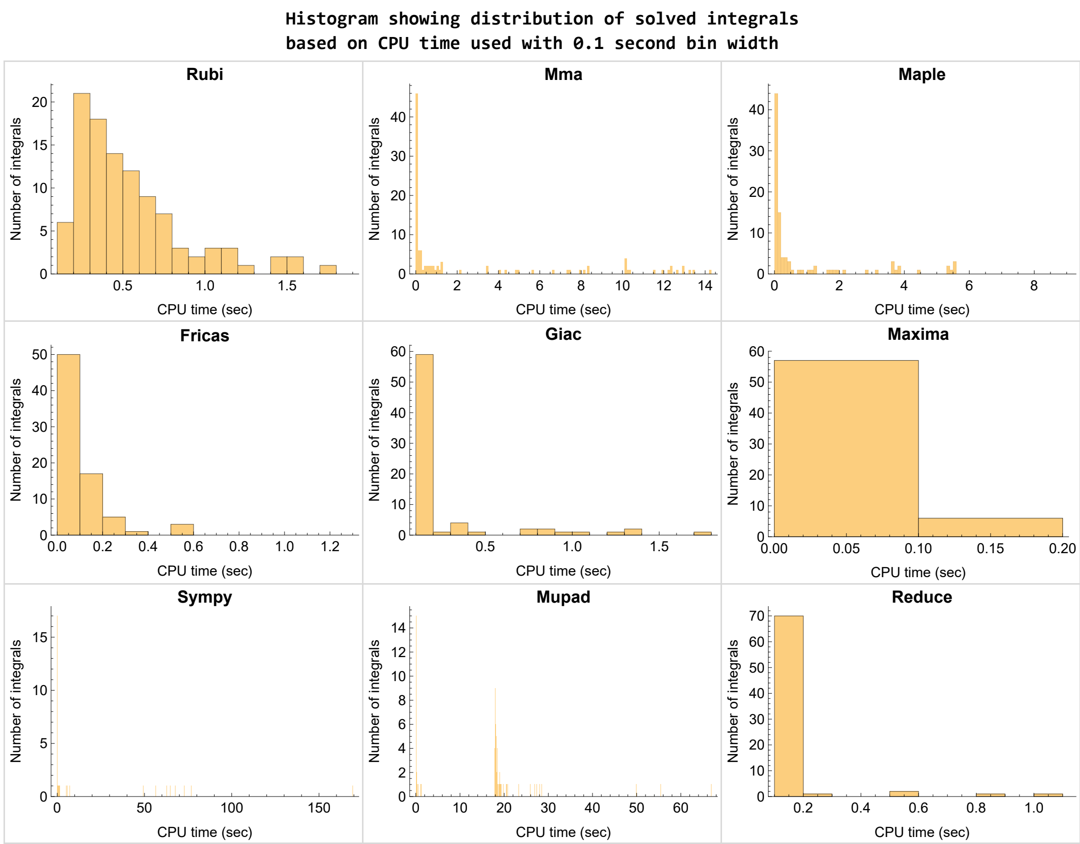


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

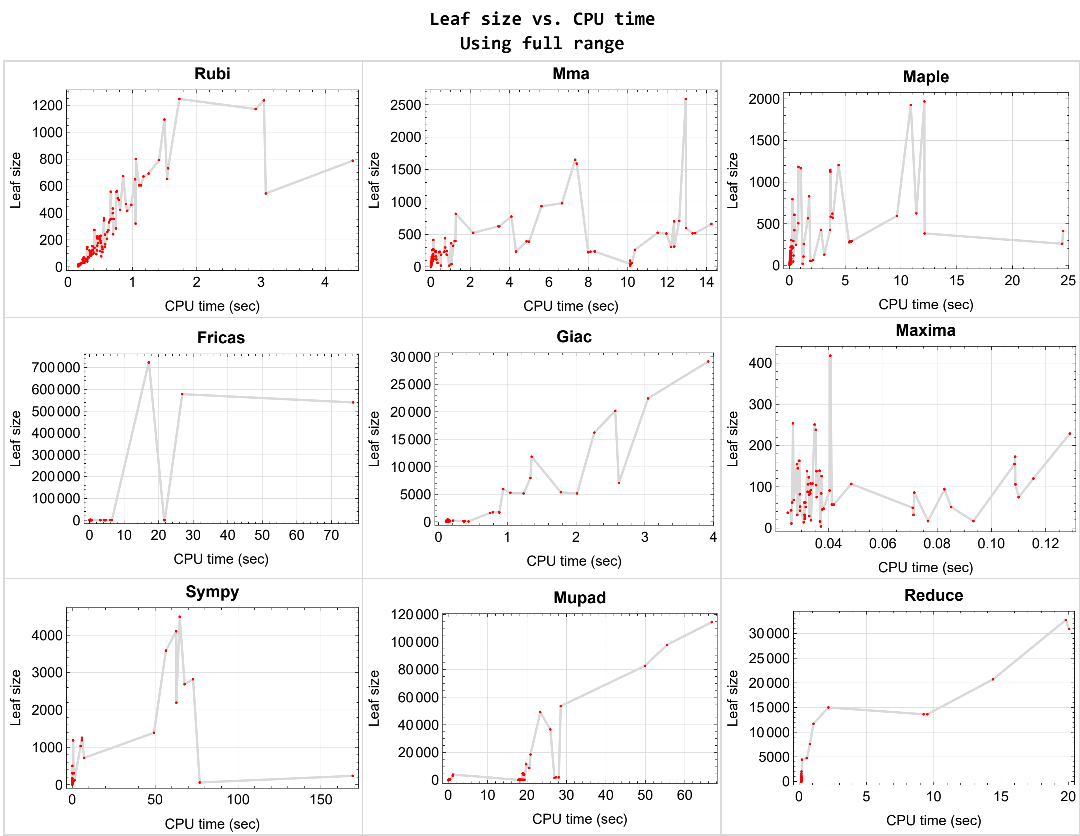


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {22}

Mathematica {19, 20, 21}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

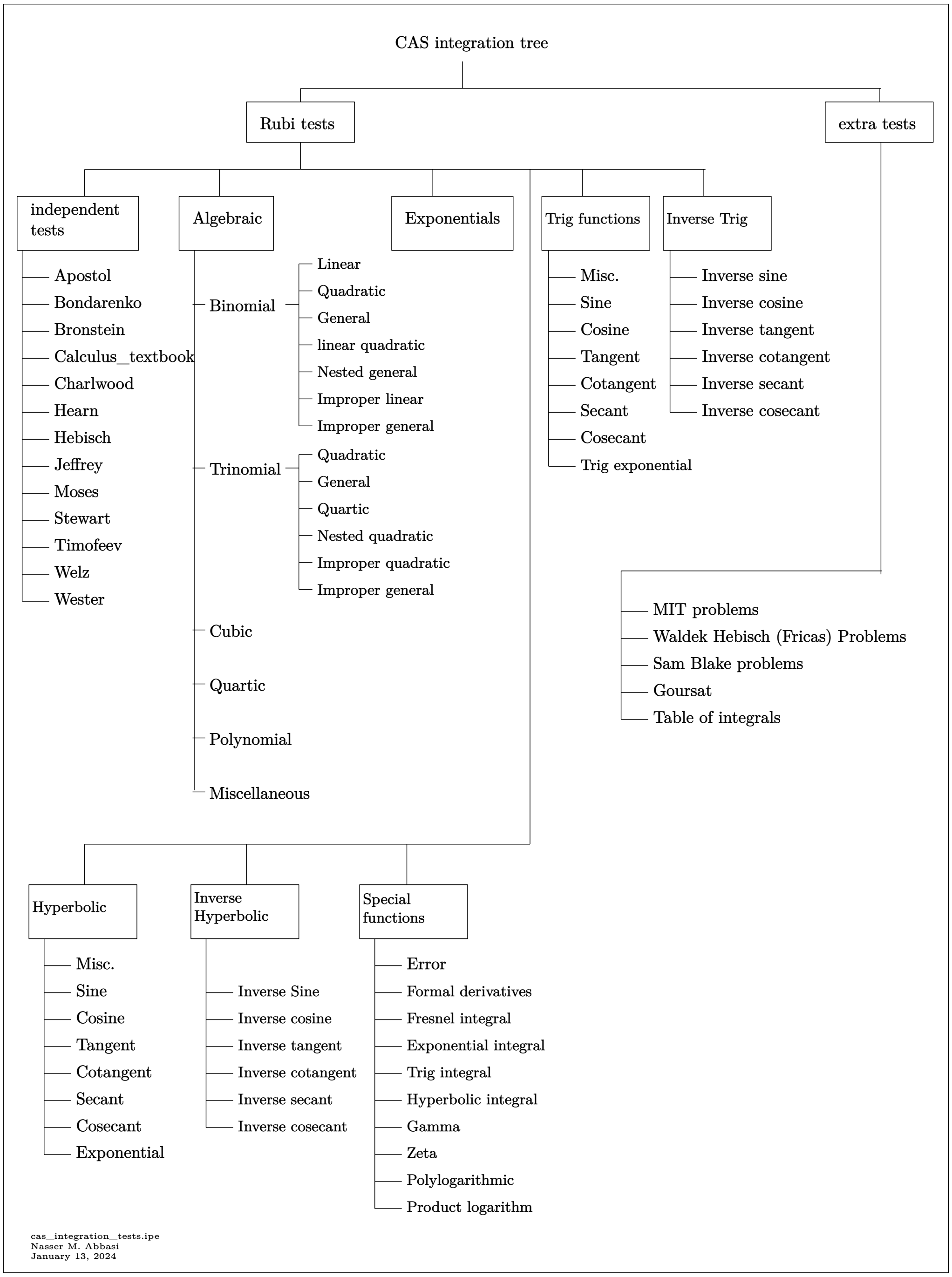
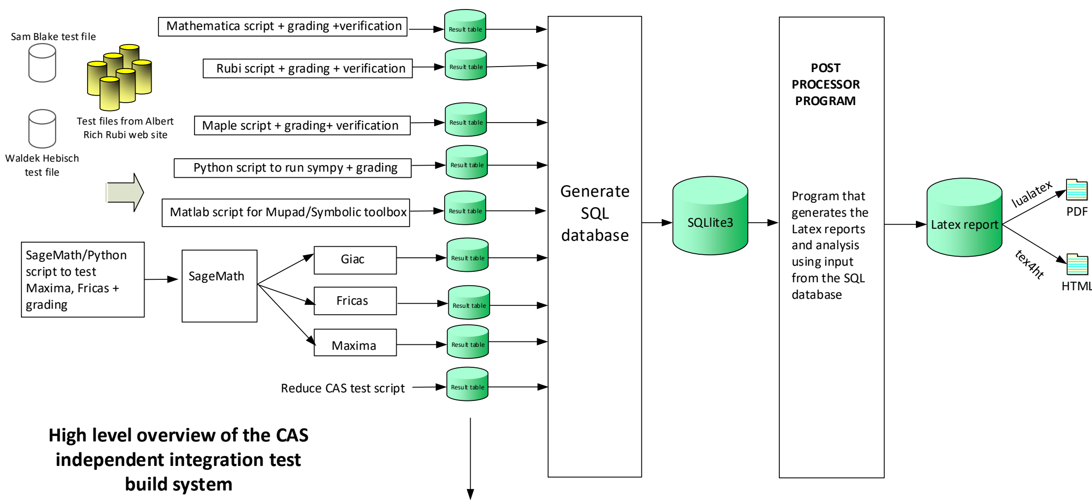


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

B grade { 4, 8, 15, 17 }

C grade { 22 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108 }

B grade { }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 28, 29, 30, 39, 40, 44, 45, 46, 100, 101, 102, 103, 104 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

B grade { 5, 10, 17 }

C grade { 16, 33, 34, 35, 36, 37, 40, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63 }

F normal fail { 18, 19, 20, 21 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 15, 16, 17, 22, 23, 25, 28, 29, 31, 32, 38, 39, 40, 41, 44, 45, 47, 48, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 100, 101, 102, 103, 105, 106, 107, 108 }

B grade { 8, 9, 13, 14, 24, 26, 27, 30, 42, 43, 46, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104 }

C grade { 33, 36, 37, 61 }

F normal fail { 18, 19, 20, 21 }

F(-1) timedout fail { 34, 35, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63 }

F(-2) exception fail { }

Maxima

A grade { 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 41, 42, 43, 44, 45, 46, 47, 48, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108 }

B grade { 24 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 33, 34, 35, 36, 37, 38, 39, 40, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 100, 101, 102, 103, 104 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99 }

B grade { 23, 24, 33, 34, 35, 36, 37, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 105, 106, 107, 108 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 100, 101, 102, 103, 104 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 100, 101, 102, 103, 104 }

F(-2) exception fail { }

Sympy

A grade { 31, 32, 38, 39, 47, 48, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 82, 88, 94 }

B grade { 23, 24, 25, 26, 27, 40, 77, 78, 79, 83, 89, 95 }

C grade { 28, 29, 30 }

F normal fail { 1, 4, 5, 6, 7, 8, 10, 11, 12, 13, 15, 16, 17, 18, 19, 22, 100, 101, 102, 103, 105, 106, 107, 108 }

F(-1) timedout fail { 2, 3, 9, 14, 20, 21, 33, 34, 35, 36, 37, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 80, 81, 84, 85, 86, 87, 90, 91, 92, 93, 96, 97, 98, 99, 104 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 100, 101, 102, 103, 104 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	558	225	278	0	208	0	0	280	0
N.S.	1	1.72	0.69	0.86	0.00	0.64	0.00	0.00	0.86	0.00
time (sec)	N/A	0.751	7.990	5.357	0.000	0.080	0.000	0.000	0.465	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	227	233	287	0	212	0	0	296	0
N.S.	1	0.82	0.84	1.03	0.00	0.76	0.00	0.00	1.06	0.00
time (sec)	N/A	0.499	8.105	5.413	0.000	0.083	0.000	0.000	0.495	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	231	236	287	0	223	0	0	296	0
N.S.	1	0.82	0.84	1.02	0.00	0.79	0.00	0.00	1.05	0.00
time (sec)	N/A	0.510	8.325	5.522	0.000	0.087	0.000	0.000	0.491	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	562	239	291	0	214	0	0	302	0
N.S.	1	2.05	0.87	1.06	0.00	0.78	0.00	0.00	1.10	0.00
time (sec)	N/A	0.761	8.322	5.592	0.000	0.084	0.000	0.000	0.505	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	993	1247	701	1927	0	1009	0	0	0	0
N.S.	1	1.26	0.71	1.94	0.00	1.02	0.00	0.00	0.00	0.00
time (sec)	N/A	1.730	12.358	10.881	0.000	0.097	0.000	0.000	1.775	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	801	388	594	0	517	0	0	903	0
N.S.	1	1.42	0.69	1.06	0.00	0.92	0.00	0.00	1.60	0.00
time (sec)	N/A	1.053	4.986	9.639	0.000	0.085	0.000	0.000	0.797	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	558	225	278	0	208	0	0	280	0
N.S.	1	1.72	0.69	0.86	0.00	0.64	0.00	0.00	0.86	0.00
time (sec)	N/A	0.663	0.407	5.346	0.000	0.082	0.000	0.000	0.454	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	674	307	572	0	807	0	0	1473	0
N.S.	1	2.07	0.94	1.76	0.00	2.48	0.00	0.00	4.53	0.00
time (sec)	N/A	0.856	12.205	3.880	0.000	0.094	0.000	0.000	0.942	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	593	1094	517	1125	0	2688	0	0	0	0
N.S.	1	1.84	0.87	1.90	0.00	4.53	0.00	0.00	0.00	0.00
time (sec)	N/A	1.497	13.298	3.674	0.000	0.301	0.000	0.000	3.534	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	821	793	707	1970	0	1012	0	0	0	0
N.S.	1	0.97	0.86	2.40	0.00	1.23	0.00	0.00	0.00	0.00
time (sec)	N/A	1.415	12.616	12.099	0.000	0.101	0.000	0.000	1.872	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	423	393	624	0	518	0	0	948	0
N.S.	1	0.91	0.85	1.35	0.00	1.12	0.00	0.00	2.05	0.00
time (sec)	N/A	0.808	4.859	11.377	0.000	0.102	0.000	0.000	0.849	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	227	233	287	0	212	0	0	296	0
N.S.	1	0.82	0.84	1.03	0.00	0.76	0.00	0.00	1.06	0.00
time (sec)	N/A	0.456	0.451	5.558	0.000	0.078	0.000	0.000	0.473	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	330	312	584	0	811	0	0	1493	0
N.S.	1	0.87	0.83	1.54	0.00	2.15	0.00	0.00	3.95	0.00
time (sec)	N/A	0.635	12.365	3.700	0.000	0.107	0.000	0.000	1.066	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	713	671	520	1147	0	2641	0	0	0	0
N.S.	1	0.94	0.73	1.61	0.00	3.70	0.00	0.00	0.00	0.00
time (sec)	N/A	1.172	13.428	3.667	0.000	0.260	0.000	0.000	3.984	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	510	262	426	0	283	0	0	170	0
N.S.	1	2.01	1.03	1.68	0.00	1.11	0.00	0.00	0.67	0.00
time (sec)	N/A	0.773	10.370	3.651	0.000	0.081	0.000	0.000	0.477	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	147	96	128	0	90	0	0	277	0
N.S.	1	1.27	0.83	1.10	0.00	0.78	0.00	0.00	2.39	0.00
time (sec)	N/A	0.300	10.117	3.132	0.000	0.080	0.000	0.000	0.194	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	399	236	383	0	242	0	0	50	0
N.S.	1	3.22	1.90	3.09	0.00	1.95	0.00	0.00	0.40	0.00
time (sec)	N/A	0.695	4.325	12.115	0.000	0.087	0.000	0.000	0.608	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	247	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	1.86	0.00
time (sec)	N/A	0.294	0.263	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	232	0	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.409	0.672	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	353	345	298	0	0	0	0	0	0	0
N.S.	1	0.98	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.556	0.725	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	484	466	363	0	0	0	0	0	0	0
N.S.	1	0.96	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.899	1.029	0.000	0.000	0.000	0.000	0.000	16.131	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	350	19	18	17	17	0	0	128	17
N.S.	1	18.42	1.00	0.95	0.89	0.89	0.00	0.00	6.74	0.89
time (sec)	N/A	0.650	0.934	0.149	0.077	0.079	0.000	0.000	0.180	18.034

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	32	32	58	59	29	32
N.S.	1	1.00	1.00	1.05	1.68	1.68	3.05	3.11	1.53	1.68
time (sec)	N/A	0.181	0.508	0.164	0.071	0.076	76.888	0.150	0.154	18.102

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	45	86	90	235	223	98	88
N.S.	1	1.00	1.05	1.12	2.15	2.25	5.88	5.58	2.45	2.20
time (sec)	N/A	0.299	1.047	0.340	0.072	0.085	169.223	0.212	0.179	18.401

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	55	58	62	51	51	2195	55	85	63
N.S.	1	1.08	1.14	1.22	1.00	1.00	43.04	1.08	1.67	1.24
time (sec)	N/A	0.279	0.033	0.060	0.032	0.118	62.792	0.125	0.161	18.158

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	126	112	105	106	217	2689	109	369	107
N.S.	1	1.06	0.94	0.88	0.89	1.82	22.60	0.92	3.10	0.90
time (sec)	N/A	0.385	0.094	0.071	0.032	0.128	67.778	0.132	0.159	17.986

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	193	161	139	155	389	2822	149	655	151
N.S.	1	1.05	0.88	0.76	0.85	2.13	15.42	0.81	3.58	0.83
time (sec)	N/A	0.504	0.134	0.099	0.028	0.138	72.867	0.149	0.165	18.061

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	108	121	82	75	75	3589	75	141	159
N.S.	1	1.46	1.64	1.11	1.01	1.01	48.50	1.01	1.91	2.15
time (sec)	N/A	0.352	0.172	0.169	0.110	0.097	56.500	0.125	0.166	18.272

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	178	186	154	120	212	4106	124	525	201
N.S.	1	1.20	1.26	1.04	0.81	1.43	27.74	0.84	3.55	1.36
time (sec)	N/A	0.464	0.516	0.176	0.115	0.105	62.681	0.112	0.169	0.341

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	240	235	202	173	384	4496	165	905	249
N.S.	1	1.17	1.14	0.98	0.84	1.86	21.83	0.80	4.39	1.21
time (sec)	N/A	0.573	0.692	0.237	0.109	0.128	64.838	0.144	0.155	18.388

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	139	138	138	165	151	152	138
N.S.	1	1.00	1.00	0.90	0.90	0.90	1.07	0.98	0.99	0.90
time (sec)	N/A	0.377	0.058	0.146	0.032	0.063	0.026	0.130	0.157	0.112

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	61	62	59
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.88	0.90	0.86
time (sec)	N/A	0.242	0.011	0.071	0.041	0.060	0.018	0.135	0.169	17.963

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	234	48	0	723401	0	1712	829	3942
N.S.	1	1.00	1.11	0.23	0.00	3428.44	0.00	8.11	3.93	18.68
time (sec)	N/A	0.453	0.243	0.103	0.000	17.186	0.000	0.884	0.173	19.298

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	356	398	232	0	0	0	5159	4754	4707
N.S.	1	0.97	1.08	0.63	0.00	0.00	0.00	14.02	12.92	12.79
time (sec)	N/A	0.675	1.245	0.226	0.000	0.000	0.000	1.237	0.566	18.851

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	605	625	607	0	0	0	5375	13638	8689
N.S.	1	0.97	1.01	0.98	0.00	0.00	0.00	8.66	21.96	13.99
time (sec)	N/A	1.105	3.431	0.455	0.000	0.000	0.000	1.778	9.249	20.571

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	234	48	0	578003	0	1712	829	3942
N.S.	1	1.00	1.12	0.23	0.00	2765.56	0.00	8.19	3.97	18.86
time (sec)	N/A	0.496	0.252	0.122	0.000	26.824	0.000	0.880	0.173	19.010

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	245	54	0	540080	0	1723	863	3046
N.S.	1	1.00	1.09	0.24	0.00	2411.07	0.00	7.69	3.85	13.60
time (sec)	N/A	0.448	0.239	0.135	0.000	76.278	0.000	0.789	0.178	1.135

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	12	8	0	7	7	30	17	7
N.S.	1	1.00	0.52	0.35	0.00	0.30	0.30	1.30	0.74	0.30
time (sec)	N/A	0.184	0.011	0.063	0.000	0.064	0.045	0.138	0.166	17.844

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	46	60	38	0	35	46	37	41	151
N.S.	1	0.98	1.28	0.81	0.00	0.74	0.98	0.79	0.87	3.21
time (sec)	N/A	0.265	0.336	0.069	0.000	0.079	0.091	0.116	0.160	0.109

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	111	114	45	0	68	1185	70	70	88
N.S.	1	1.56	1.61	0.63	0.00	0.96	16.69	0.99	0.99	1.24
time (sec)	N/A	0.358	0.278	0.062	0.000	0.081	0.502	0.134	0.171	0.215

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	78	98	107	88	88	0	92	178	108
N.S.	1	1.03	1.29	1.41	1.16	1.16	0.00	1.21	2.34	1.42
time (sec)	N/A	0.515	0.077	0.087	0.033	4.594	0.000	0.142	0.162	18.419

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	167	185	161	163	346	0	173	721	164
N.S.	1	1.03	1.14	0.99	1.01	2.14	0.00	1.07	4.45	1.01
time (sec)	N/A	0.575	0.092	0.099	0.029	5.903	0.000	0.132	0.162	18.414

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	242	261	217	238	616	0	249	1283	233
N.S.	1	0.96	1.04	0.86	0.94	2.44	0.00	0.99	5.09	0.92
time (sec)	N/A	0.700	0.129	0.108	0.035	6.459	0.000	0.141	0.174	18.529

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	150	187	117	106	106	0	106	290	1509
N.S.	1	1.10	1.38	0.86	0.78	0.78	0.00	0.78	2.13	11.10
time (sec)	N/A	0.526	0.802	0.370	0.109	4.203	0.000	0.121	0.155	26.952

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	208	243	214	155	279	0	165	955	1894
N.S.	1	1.18	1.38	1.22	0.88	1.59	0.00	0.94	5.43	10.76
time (sec)	N/A	0.604	0.812	0.332	0.109	4.269	0.000	0.118	0.163	28.058

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	285	325	292	229	521	0	249	1637	1963
N.S.	1	1.14	1.31	1.17	0.92	2.09	0.00	1.00	6.57	7.88
time (sec)	N/A	0.744	1.130	0.359	0.129	4.780	0.000	0.159	0.171	27.390

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	272	255	254	254	303	304	305	258
N.S.	1	1.00	1.00	0.94	0.93	0.93	1.11	1.12	1.12	0.95
time (sec)	N/A	0.626	0.121	1.289	0.027	0.066	0.037	0.128	0.157	0.188

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	105	104	104	121	124	125	112
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.99	1.02	1.02	0.92
time (sec)	N/A	0.332	0.039	1.263	0.035	0.069	0.020	0.125	0.168	0.088

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	441	99	0	0	0	5944	2036	11383
N.S.	1	1.00	1.37	0.31	0.00	0.00	0.00	18.52	6.34	35.46
time (sec)	N/A	1.050	0.717	0.123	0.000	0.000	0.000	0.940	0.183	19.786

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	460	524	304	0	0	0	7965	7607	18449
N.S.	1	0.98	1.12	0.65	0.00	0.00	0.00	17.02	16.25	39.42
time (sec)	N/A	0.983	2.146	0.170	0.000	0.000	0.000	1.336	0.802	20.884

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	732	732	980	795	0	0	0	7058	20724	36653
N.S.	1	1.00	1.34	1.09	0.00	0.00	0.00	9.64	28.31	50.07
time (sec)	N/A	1.555	6.660	0.271	0.000	0.000	0.000	2.622	14.410	25.900

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	816	246	0	0	0	11830	4445	49150
N.S.	1	1.00	1.50	0.45	0.00	0.00	0.00	21.71	8.16	90.18
time (sec)	N/A	3.074	1.268	0.597	0.000	0.000	0.000	1.354	0.209	23.346

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	770	788	935	506	0	0	0	20158	15020	82785
N.S.	1	1.02	1.21	0.66	0.00	0.00	0.00	26.18	19.51	107.51
time (sec)	N/A	4.427	5.618	0.799	0.000	0.000	0.000	2.570	2.175	49.924

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1150	1172	1590	1167	0	0	0	22427	32760	114377
N.S.	1	1.02	1.38	1.01	0.00	0.00	0.00	19.50	28.49	99.46
time (sec)	N/A	2.916	7.411	1.020	0.000	0.000	0.000	3.046	19.812	66.817

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	641	653	775	422	0	0	0	16214	11710	53538
N.S.	1	1.02	1.21	0.66	0.00	0.00	0.00	25.29	18.27	83.52
time (sec)	N/A	1.540	4.089	0.447	0.000	0.000	0.000	2.265	1.071	28.534

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1177	1236	1649	1182	0	0	0	29140	30922	97905
N.S.	1	1.05	1.40	1.00	0.00	0.00	0.00	24.76	26.27	83.18
time (sec)	N/A	3.045	7.329	0.815	0.000	0.000	0.000	3.921	20.046	55.461

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	416	412	418	418	503	463	467	398
N.S.	1	1.00	1.00	0.99	1.00	1.00	1.21	1.11	1.12	0.96
time (sec)	N/A	0.919	0.124	24.537	0.041	0.106	0.048	0.132	0.181	0.524

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	259	259	251	251	309	285	287	246
N.S.	1	1.00	1.00	1.00	0.97	0.97	1.19	1.10	1.11	0.95
time (sec)	N/A	0.609	0.055	24.451	0.035	0.087	0.034	0.118	0.165	0.205

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	141	138	138	165	151	152	138
N.S.	1	1.00	1.00	0.92	0.90	0.90	1.07	0.98	0.99	0.90
time (sec)	N/A	0.376	0.043	0.134	0.036	0.093	0.025	0.145	0.166	17.983

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	17	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.85	0.80
time (sec)	N/A	0.201	0.003	0.028	0.037	0.202	0.026	0.370	0.178	0.029

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	234	48	0	723401	0	1618	829	3942
N.S.	1	1.00	1.11	0.23	0.00	3428.44	0.00	7.67	3.93	18.68
time (sec)	N/A	0.482	0.226	0.079	0.000	17.128	0.000	0.748	0.184	1.242

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	356	398	232	0	0	0	5158	4754	4707
N.S.	1	0.97	1.08	0.63	0.00	0.00	0.00	14.02	12.92	12.79
time (sec)	N/A	0.709	1.206	0.204	0.000	0.000	0.000	2.014	0.593	19.048

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	605	625	607	0	0	0	5280	13638	8689
N.S.	1	0.97	1.01	0.98	0.00	0.00	0.00	8.50	21.96	13.99
time (sec)	N/A	1.135	3.473	0.412	0.000	0.000	0.000	1.045	9.534	20.453

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	5	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.25	1.00	1.00
time (sec)	N/A	0.154	0.001	0.021	0.037	0.063	0.023	0.120	0.169	0.019

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	15	14	14	12	15	17	14
N.S.	1	1.00	1.14	1.07	1.00	1.00	0.86	1.07	1.21	1.00
time (sec)	N/A	0.186	0.006	0.031	0.031	0.065	0.057	0.134	0.164	0.030

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	28	27	27	26	28	34	27
N.S.	1	1.00	0.97	0.90	0.87	0.87	0.84	0.90	1.10	0.87
time (sec)	N/A	0.226	0.014	0.036	0.031	0.086	0.069	0.131	0.184	17.981

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	51	45	45	43	43	41	47	57	44
N.S.	1	1.09	0.96	0.96	0.91	0.91	0.87	1.00	1.21	0.94
time (sec)	N/A	0.280	0.028	0.039	0.026	0.086	0.086	0.131	0.166	0.040

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	65	62	62	63	72	86	64
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.93	1.06	1.26	0.94
time (sec)	N/A	0.317	0.028	0.042	0.031	0.093	0.097	0.136	0.170	17.986

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	88	84	84	88	103	121	87
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.96	1.12	1.32	0.95
time (sec)	N/A	0.368	0.040	0.044	0.037	0.085	0.114	0.138	0.175	0.040

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	13	11	8
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.18	1.00	0.73
time (sec)	N/A	0.177	0.005	0.029	0.026	0.086	0.045	0.107	0.166	0.082

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	24	22	22	29	24	28	22
N.S.	1	1.00	1.05	1.09	1.00	1.00	1.32	1.09	1.27	1.00
time (sec)	N/A	0.201	0.013	0.038	0.031	0.085	0.151	0.131	0.185	0.067

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	31	29	29	44	31	44	29
N.S.	1	1.00	1.03	1.07	1.00	1.00	1.52	1.07	1.52	1.00
time (sec)	N/A	0.253	0.018	0.045	0.033	0.086	0.281	0.134	0.171	18.003

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	47	45	45	66	47	68	45
N.S.	1	1.00	0.94	1.00	0.96	0.96	1.40	1.00	1.45	0.96
time (sec)	N/A	0.279	0.026	0.049	0.038	0.167	0.473	0.138	0.162	0.072

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	65	62	62	94	67	97	63
N.S.	1	1.00	1.02	0.98	0.94	0.94	1.42	1.02	1.47	0.95
time (sec)	N/A	0.307	0.032	0.053	0.027	0.078	0.820	0.134	0.179	17.967

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	91	88	84	84	122	95	133	86
N.S.	1	1.00	1.01	0.98	0.93	0.93	1.36	1.06	1.48	0.96
time (sec)	N/A	0.342	0.041	0.066	0.033	0.068	1.350	0.112	0.170	17.920

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	19	19	19	22	19	19
N.S.	1	1.00	1.00	0.69	0.66	0.66	0.66	0.76	0.66	0.66
time (sec)	N/A	0.203	0.007	0.032	0.033	0.066	0.063	0.126	0.170	0.063

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	39	38	32	32	304	35	43	38
N.S.	1	1.00	0.93	0.90	0.76	0.76	7.24	0.83	1.02	0.90
time (sec)	N/A	0.250	0.020	0.044	0.028	0.071	1.027	0.119	0.179	0.088

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	47	37	37	716	40	64	47
N.S.	1	1.00	0.94	1.00	0.79	0.79	15.23	0.85	1.36	1.00
time (sec)	N/A	0.273	0.023	0.047	0.025	0.073	7.126	0.131	0.162	0.110

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	59	47	47	1389	50	88	59
N.S.	1	1.00	0.96	1.04	0.82	0.82	24.37	0.88	1.54	1.04
time (sec)	N/A	0.276	0.029	0.059	0.038	0.082	49.289	0.123	0.178	0.101

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	78	62	62	0	65	119	78
N.S.	1	1.00	0.96	1.05	0.84	0.84	0.00	0.88	1.61	1.05
time (sec)	N/A	0.312	0.038	0.064	0.031	0.103	0.000	0.143	0.167	17.844

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	91	99	82	82	0	87	155	99
N.S.	1	1.00	0.95	1.03	0.85	0.85	0.00	0.91	1.61	1.03
time (sec)	N/A	0.355	0.051	0.074	0.029	0.135	0.000	0.128	0.164	17.971

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	33	32	45	34	36	64	32
N.S.	1	1.00	0.91	0.72	0.70	0.98	0.74	0.78	1.39	0.70
time (sec)	N/A	0.236	0.025	0.047	0.031	0.065	0.115	0.111	0.187	0.051

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	64	57	93	1188	61	137	64
N.S.	1	1.00	0.93	0.90	0.80	1.31	16.73	0.86	1.93	0.90
time (sec)	N/A	0.312	0.048	0.062	0.042	0.082	5.785	0.123	0.172	17.847

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	77	79	68	116	0	72	201	79
N.S.	1	1.00	0.94	0.96	0.83	1.41	0.00	0.88	2.45	0.96
time (sec)	N/A	0.350	0.060	0.063	0.027	0.146	0.000	0.133	0.172	17.846

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	90	94	81	141	0	85	265	94
N.S.	1	1.00	0.95	0.99	0.85	1.48	0.00	0.89	2.79	0.99
time (sec)	N/A	0.381	0.061	0.073	0.033	0.555	0.000	0.138	0.174	17.950

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	102	109	92	164	0	96	329	108
N.S.	1	1.00	0.96	1.03	0.87	1.55	0.00	0.91	3.10	1.02
time (sec)	N/A	0.400	0.066	0.084	0.033	3.188	0.000	0.141	0.166	18.437

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	118	127	108	200	0	112	399	127
N.S.	1	1.00	0.97	1.04	0.89	1.64	0.00	0.92	3.27	1.04
time (sec)	N/A	0.451	0.073	0.108	0.033	21.721	0.000	0.131	0.179	18.939

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	40	42	72	46	46	107	42
N.S.	1	1.00	0.87	0.73	0.76	1.31	0.84	0.84	1.95	0.76
time (sec)	N/A	0.225	0.027	0.048	0.029	0.098	0.123	0.117	0.177	0.048

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	94	80	78	75	153	1255	79	231	79
N.S.	1	1.06	0.90	0.88	0.84	1.72	14.10	0.89	2.60	0.89
time (sec)	N/A	0.439	0.057	0.063	0.036	0.199	5.837	0.116	0.170	0.100

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	110	97	96	91	191	0	95	340	97
N.S.	1	1.05	0.92	0.91	0.87	1.82	0.00	0.90	3.24	0.92
time (sec)	N/A	0.533	0.083	0.068	0.040	0.288	0.000	0.130	0.168	18.013

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	117	114	114	107	229	0	111	449	115
N.S.	1	0.97	0.94	0.94	0.88	1.89	0.00	0.92	3.71	0.95
time (sec)	N/A	0.495	0.064	0.094	0.048	0.548	0.000	0.117	0.167	18.069

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	131	136	132	123	267	0	127	558	133
N.S.	1	0.96	0.99	0.96	0.90	1.95	0.00	0.93	4.07	0.97
time (sec)	N/A	0.535	0.073	0.088	0.033	3.090	0.000	0.125	0.174	18.916

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	147	153	150	139	305	0	143	667	151
N.S.	1	0.96	1.00	0.98	0.91	1.99	0.00	0.93	4.36	0.99
time (sec)	N/A	0.569	0.094	0.103	0.037	21.778	0.000	0.120	0.164	19.371

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	47	52	103	53	56	151	52
N.S.	1	1.00	0.88	0.69	0.76	1.51	0.78	0.82	2.22	0.76
time (sec)	N/A	0.250	0.032	0.054	0.029	0.067	0.141	0.123	0.173	0.047

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	92	88	211	1034	92	322	90
N.S.	1	1.00	0.92	0.88	0.84	2.01	9.85	0.88	3.07	0.86
time (sec)	N/A	0.346	0.099	0.070	0.032	0.086	5.036	0.166	0.180	0.099

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	121	113	108	267	0	112	472	113
N.S.	1	1.00	0.99	0.93	0.89	2.19	0.00	0.92	3.87	0.93
time (sec)	N/A	0.375	0.060	0.073	0.034	0.144	0.000	0.114	0.163	18.094

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	144	134	126	321	0	130	625	131
N.S.	1	1.00	1.02	0.95	0.89	2.28	0.00	0.92	4.43	0.93
time (sec)	N/A	0.409	0.079	0.081	0.037	0.544	0.000	0.140	0.171	18.207

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	169	155	145	376	0	149	775	152
N.S.	1	1.00	1.07	0.98	0.92	2.38	0.00	0.94	4.91	0.96
time (sec)	N/A	0.453	0.098	0.090	0.029	3.088	0.000	0.137	0.165	18.783

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	195	176	163	430	0	167	928	170
N.S.	1	1.00	1.10	0.99	0.92	2.43	0.00	0.94	5.24	0.96
time (sec)	N/A	0.507	0.113	0.106	0.029	21.598	0.000	0.144	0.164	19.120

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	724	693	2588	1205	0	911	0	0	32	0
N.S.	1	0.96	3.57	1.66	0.00	1.26	0.00	0.00	0.04	0.00
time (sec)	N/A	1.252	12.950	4.411	0.000	0.260	0.000	0.000	200.030	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	498	661	620	0	574	0	0	1143	0
N.S.	1	0.99	1.31	1.23	0.00	1.14	0.00	0.00	2.26	0.00
time (sec)	N/A	0.788	14.251	3.857	0.000	0.237	0.000	0.000	1.490	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	363	525	426	0	376	0	0	1512	0
N.S.	1	1.01	1.46	1.18	0.00	1.04	0.00	0.00	4.20	0.00
time (sec)	N/A	0.558	11.522	2.832	0.000	0.179	0.000	0.000	0.663	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	433	513	565	0	723	0	0	0	0
N.S.	1	0.97	1.15	1.26	0.00	1.62	0.00	0.00	0.00	0.00
time (sec)	N/A	0.695	11.964	1.664	0.000	0.091	0.000	0.000	0.297	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	650	598	829	0	1948	0	0	0	0
N.S.	1	0.96	0.88	1.22	0.00	2.86	0.00	0.00	0.00	0.00
time (sec)	N/A	1.041	12.965	1.770	0.000	0.129	0.000	0.000	1.020	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	0	60	30	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.00	3.16	1.58	0.89
time (sec)	N/A	0.161	10.122	1.186	0.093	0.075	0.000	0.436	0.180	18.237

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	52	51	82	0	138	87	51
N.S.	1	1.00	0.89	0.91	0.89	1.44	0.00	2.42	1.53	0.89
time (sec)	N/A	0.283	10.178	1.971	0.085	0.081	0.000	0.361	0.184	18.236

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	53	49	80	0	136	88	51
N.S.	1	1.00	0.84	0.93	0.86	1.40	0.00	2.39	1.54	0.89
time (sec)	N/A	0.298	10.113	1.885	0.071	0.074	0.000	0.370	0.199	18.368

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	63	94	92	0	164	98	62
N.S.	1	1.00	0.88	0.91	1.36	1.33	0.00	2.38	1.42	0.90
time (sec)	N/A	0.311	10.220	2.132	0.083	0.082	0.000	0.382	0.179	18.257

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [30] had the largest ratio of [.7619050000000000054]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	7	1.72	32	0.219
2	A	7	7	0.82	33	0.212
3	A	7	7	0.82	33	0.212
4	B	7	7	2.05	34	0.206
5	A	8	8	1.26	38	0.211
6	A	7	7	1.42	38	0.184
7	A	6	6	1.72	38	0.158
8	B	6	6	2.07	38	0.158
9	A	7	7	1.84	38	0.184
10	A	10	10	0.97	40	0.250
11	A	8	8	0.91	40	0.200
12	A	6	6	0.82	40	0.150
13	A	6	6	0.87	40	0.150
14	A	8	8	0.94	40	0.200
15	B	6	6	2.01	52	0.115
16	A	5	5	1.27	27	0.185
17	B	11	11	3.22	48	0.229
18	A	2	2	1.00	14	0.143
19	A	2	2	1.00	22	0.091
20	A	4	4	0.98	27	0.148
21	A	4	4	0.96	32	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	C	2	2	18.42	26	0.077
23	A	1	1	1.00	41	0.024
24	A	1	1	1.00	94	0.011
25	A	8	7	1.08	23	0.304
26	A	10	9	1.06	23	0.391
27	A	12	11	1.05	23	0.478
28	A	12	11	1.46	21	0.524
29	A	14	13	1.20	21	0.619
30	A	17	16	1.17	21	0.762
31	A	2	2	1.00	25	0.080
32	A	2	2	1.00	23	0.087
33	A	8	7	1.00	25	0.280
34	A	11	10	0.97	25	0.400
35	A	14	13	0.97	25	0.520
36	A	8	7	1.00	25	0.280
37	A	8	7	1.00	22	0.318
38	A	4	3	1.00	18	0.167
39	A	9	8	0.98	23	0.348
40	A	12	11	1.56	23	0.478
41	A	7	6	1.03	38	0.158
42	A	11	10	1.03	38	0.263
43	A	13	12	0.96	38	0.316
44	A	7	6	1.10	36	0.167
45	A	14	13	1.18	36	0.361
46	A	17	16	1.14	36	0.444
47	A	2	2	1.00	40	0.050
48	A	2	2	1.00	38	0.053
49	A	7	6	1.00	40	0.150
50	A	11	10	0.98	40	0.250
51	A	14	13	1.00	40	0.325
52	A	7	6	1.00	55	0.109
53	A	12	11	1.02	55	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	14	13	1.02	55	0.236
55	A	12	11	1.02	50	0.220
56	A	17	16	1.05	50	0.320
57	A	2	2	1.00	63	0.032
58	A	2	2	1.00	63	0.032
59	A	2	2	1.00	61	0.033
60	A	2	2	1.00	63	0.032
61	A	9	8	1.00	63	0.127
62	A	12	11	0.97	63	0.175
63	A	15	14	0.97	63	0.222
64	A	2	2	1.00	26	0.077
65	A	3	3	1.00	31	0.097
66	A	3	3	1.00	36	0.083
67	A	3	3	1.09	41	0.073
68	A	3	3	1.00	46	0.065
69	A	3	3	1.00	51	0.059
70	A	3	3	1.00	21	0.143
71	A	3	3	1.00	26	0.115
72	A	3	3	1.00	31	0.097
73	A	3	3	1.00	36	0.083
74	A	3	3	1.00	41	0.073
75	A	3	3	1.00	46	0.065
76	A	3	3	1.00	16	0.188
77	A	3	3	1.00	21	0.143
78	A	3	3	1.00	26	0.115
79	A	3	3	1.00	31	0.097
80	A	3	3	1.00	36	0.083
81	A	3	3	1.00	41	0.073
82	A	3	3	1.00	26	0.115
83	A	3	3	1.00	31	0.097
84	A	3	3	1.00	36	0.083
85	A	3	3	1.00	41	0.073

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	3	3	1.00	46	0.065
87	A	3	3	1.00	51	0.059
88	A	3	3	1.00	21	0.143
89	A	7	7	1.06	26	0.269
90	A	7	7	1.05	31	0.226
91	A	3	3	0.97	36	0.083
92	A	3	3	0.96	41	0.073
93	A	3	3	0.96	46	0.065
94	A	3	3	1.00	16	0.188
95	A	3	3	1.00	21	0.143
96	A	3	3	1.00	26	0.115
97	A	3	3	1.00	31	0.097
98	A	3	3	1.00	36	0.083
99	A	3	3	1.00	41	0.073
100	A	15	14	0.96	32	0.438
101	A	12	11	0.99	32	0.344
102	A	10	9	1.01	32	0.281
103	A	9	8	0.97	32	0.250
104	A	13	12	0.96	32	0.375
105	A	1	1	1.00	28	0.036
106	A	6	5	1.00	31	0.161
107	A	6	5	1.00	33	0.152
108	A	5	4	1.00	36	0.111

CHAPTER 3

Listing of integrals

3.1 $\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 324

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = -\frac{(2bcC-3bBd+2aCd)x(c+dx^2)}{3bd^2\sqrt{ac+(bc+ad)x^2+bdx^4}}+\frac{Cx\sqrt{ac+(bc+ad)x^2+bdx^4}}{3bd} + \frac{c(2bcC-3bBd+2aCd)(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{3/2}d^2\sqrt{ac+(bc+ad)x^2+bdx^4}} - \frac{(acC-3Abd)(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{3/2}d\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output
```
-1/3*(-3*B*b*d+2*C*a*d+2*C*b*c)*x*(d*x^2+c)/b/d^2/(a*c+(a*d+b*c)*x^2+b*d*x
^4)^(1/2)+1/3*C*x*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)/b/d+1/3*c*(-3*B*b*d+2*
C*a*d+2*C*b*c)*(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*EllipticE(b^(1/2)
*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/d^2/(a*c+(
a*d+b*c)*x^2+b*d*x^4)^(1/2)-1/3*(-3*A*b*d+C*a*c)*(b*x^2+a)*(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/
2))/a^(1/2)/b^(3/2)/d/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.99 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \frac{\sqrt{\frac{b}{a}}Cdx(a+bx^2)(c+dx^2)+ic(2bcC-3bBd+2aCd)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-i(}{3b\sqrt{\frac{b}{a}}d^2\sqrt{(a+bx^2)(c+}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
(Sqrt[b/a]*C*d*x*(a + b*x^2)*(c + d*x^2) + I*c*(2*b*c*C - 3*b*B*d + 2*a*C*
d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)] - I*(a*c*C*d + b*(2*c^2*C - 3*B*c*d + 3*A*d^2))*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(3*b*Sqrt[b/a]*d^2*Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.72, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2048, 2207, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{x^2(ad+bc)+ac+bdx^4}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{\int -\frac{(2bcC+2adC-3bBd)x^2+acC-3Abd}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{3bd}+\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}$$

$\Big\downarrow$ 25

$$\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\int \frac{(2bcC+2adC-3bBd)x^2+acC-3Abd}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{3bd}$$

$\Big\downarrow$ 1511

$$\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt{a}\sqrt{c}\left(\frac{\sqrt{b}\sqrt{d}(acC-3Abd)}{\sqrt{a}\sqrt{c}}+2aCd-3bBd+2bcC\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}(2aCd-3bBd+2bcC)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}}{3bd}$$

$\Big\downarrow$ 27

$$\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt{a}\sqrt{c}\left(\frac{\sqrt{b}\sqrt{d}(acC-3Abd)}{\sqrt{a}\sqrt{c}}+2aCd-3bBd+2bcC\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{(2aCd-3bBd+2bcC)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}}{3bd}$$

$\Big\downarrow$ 1416

$$\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{d}(acC-3Abd)}{\sqrt{a}\sqrt{c}}+2aCd-3bBd+2bcC\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right.\right.}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}}{3bd}$$

$\Big\downarrow$ 1509

$$\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-$$

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{d}(acC-3Abd)}{\sqrt{a}\sqrt{c}}+2aCd-3bBd+2bcC\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right.\right.}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
(C*x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(3*b*d) - (-(((2*b*c*C - 3*b*B
*d + 2*a*C*d)*(-((x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c
] + Sqrt[b]*Sqrt[d]*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sq
rt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[
b]*Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4)
)], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/
4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])))/(Sqrt[b]*Sqrt[d])) + (a^(1/4)*
c^(1/4)*(2*b*c*C - 3*b*B*d + 2*a*C*d + (Sqrt[b]*Sqrt[d]*(a*c*C - 3*A*b*d))
/(Sqrt[a]*Sqrt[c]))*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b
*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqr
t[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(3/4)*d^(3/4)*Sqrt[a*c + (b*c + a*
d)*x^2 + b*d*x^4]))/(3*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{Cx\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{3bd}+\frac{\left(A-\frac{Cac}{3bd}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}-\frac{\left(B-\frac{C(2ad+2bc)}{3bd}\right)c\sqrt{1+\frac{b\,x^2}{a}}}{}$
risch	$\frac{Cx\left(b\,x^2+a\right)\left(d\,x^2+c\right)}{3bd\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}}+\frac{-\frac{(3Bbd-2aCd-2Ccb)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}\,d}}{}$
default	$\frac{A\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}-\frac{Bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{EllipticE}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}\,d}$

input
```
int((C*x^4+B*x^2+A)/((b*x^2+a)*(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*C/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(A-1/3*C/b/d*a*c)/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(B-1/3*C/b/d*(2*a*d+
2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$
$$= \frac{(2\,Cbc^3+(2\,Ca-3\,Bb)c^2d)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (2\,Cbc^3+Cacd^2-3\,Abd^3+(2\,Ca-3}{}$$

input
```
integrate((C*x^4+B*x^2+A)/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="fricas
")
```

output
```
1/3*((2*C*b*c^3 + (2*C*a - 3*B*b)*c^2*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e
(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (2*C*b*c^3 + C*a*c*d^2 - 3*A*b*d^3 + (
2*C*a - 3*B*b)*c^2*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/
x), a*d/(b*c)) + (C*b*c*d^2*x^2 - 2*C*b*c^2*d - (2*C*a - 3*B*b)*c*d^2)*sqr
t(b*d*x^4 + (b*c + a*d)*x^2 + a*c))/(b^2*c*d^3*x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/((b*x**2+a)*(d*x**2+c))**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/sqrt((a + b*x**2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{(a + bx^2)(c + dx^2)}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{(bx^2 + a)(dx^2 + c)}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{(a + bx^2)(c + dx^2)}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{(bx^2 + a)(dx^2 + c)}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{(a + bx^2)(c + dx^2)}}\,dx = \int \frac{C\,x^4 + B\,x^2 + A}{\sqrt{(b\,x^2 + a)\,(d\,x^2 + c)}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a + b*x^2)*(c + d*x^2))^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a + b*x^2)*(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx$$
$$= \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,cx-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)acd+3\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)b^2d-2\left(\int \frac{\sqrt{d\,x^2+c}}{bd\,x^4+ad\,x}\right.}{3bd}$$

input
```
int((C*x^4+B*x^2+A)/((b*x^2+a)*(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*c*x - 2*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*c*d + 3*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4
),x)*b**2*d - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x*
*2 + b*c*x**2 + b*d*x**4),x)*b*c**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d - int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*c**2)/(3
*b*d)
```

3.2 $\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 278

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx = -\frac{Cx\sqrt{ac+(bc-ad)x^2-bdx^4}}{3bd} + \frac{a\sqrt{c}(2bcC+3bBd-2aCd)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{ac+(bc-ad)x^2-bdx^4}} + \frac{\sqrt{c}(3Ab^2d+2a^2Cd-ab(cC+3Bd))\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output
```
-1/3*C*x*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)/b/d+1/3*a*c^(1/2)*(3*B*b*d-2*C
*a*d+2*C*b*c)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticE(d^(1/2)*x/c^(1
/2),(-b*c/a/d)^(1/2))/b^2/d^(3/2)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)+1/3*c
^(1/2)*(3*A*b^2*d+2*a^2*C*d-a*b*(3*B*d+C*c))*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)
^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/b^2/d^(3/2)/(a*c+(-a*
d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}Cdx(a+bx^2)(-c+dx^2)-ic(-2bcC-3bBd+2aCd)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right.}{3b\sqrt{\frac{b}{a}}d^2\sqrt{(a+bx^2)}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/Sqrt[(a + b*x^2)*(c - d*x^2)],x]
```

output
```
(Sqrt[b/a]*C*d*x*(a + b*x^2)*(-c + d*x^2) - I*c*(-2*b*c*C - 3*b*B*d + 2*a*
C*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]
*x], -((a*d)/(b*c))] + I*(a*c*C*d - b*(2*c^2*C + 3*B*c*d + 3*A*d^2))*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], -((a*
d)/(b*c))])/(3*b*Sqrt[b/a]*d^2*Sqrt[(a + b*x^2)*(c - d*x^2)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2048, 2207, 25, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx$$
$$\Big\downarrow \text{2048}$$
$$\int \frac{A+Bx^2+Cx^4}{\sqrt{x^2(bc-ad)+ac-bdx^4}}dx$$
$$\Big\downarrow \text{2207}$$

$$-\frac{\int -\frac{(2bcC-2adC+3bBd)x^2+acC+3Abd}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{3bd}-\frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(2bcC-2adC+3bBd)x^2+acC+3Abd}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{3bd}-\frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$$\Big\downarrow \; 1514$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{(2bcC-2adC+3bBd)x^2+acC+3Abd}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{3bd\sqrt{x^2(bc-ad)+ac-bdx^4}}-\frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{(2a^2Cd-ab(3Bd+cC)+3Ab^2d)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{a(-2aCd+3bBd+2bcC)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}\right)}{3bd\sqrt{x^2(bc-ad)+ac-bdx^4}}-\frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(-2aCd+3bBd+2bcC)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{\sqrt{c}(2a^2Cd-ab(3Bd+cC)+3Ab^2d)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}}\right)}{3bd\sqrt{x^2(bc-ad)+ac-bdx^4}}-\frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{\sqrt{c}(2a^2Cd-ab(3Bd+cC)+3Ab^2d)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}}+\frac{a\sqrt{c}(-2aCd+3bBd+2bcC)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\right.}{b\sqrt{d}}\right.}{3bd\sqrt{x^2(bc-ad)+ac-bdx^4}}-\frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$

input
```
Int[(A + B*x^2 + C*x^4)/Sqrt[(a + b*x^2)*(c - d*x^2)],x]
```

output
```
-1/3*(C*x*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4])/(b*d) + (Sqrt[1 + (b*x^2)
/a]*Sqrt[1 - (d*x^2)/c]*((a*Sqrt[c]*(2*b*c*C + 3*b*B*d - 2*a*C*d)*Elliptic
E[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]) + (Sqrt[c]*(3*
A*b^2*d + 2*a^2*C*d - a*b*(c*C + 3*B*d))*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt
[c]], -((b*c)/(a*d))])/(b*Sqrt[d])))/(3*b*d*Sqrt[a*c + (b*c - a*d)*x^2 - b
*d*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 5.41 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.03

method	result
elliptic	$-\frac{Cx\sqrt{-bd\,x^{4}-x^{2}da+bc\,x^{2}+ac}}{3bd}+\frac{\left(A+\frac{Cac}{3bd}\right)\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-x^{2}da+bc\,x^{2}+ac}}-\frac{\left(B+\frac{C(-2ad+2bc)}{3bd}\right)a\sqrt{}}{}$
risch	$-\frac{Cx\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}{3bd\sqrt{-\left(b\,x^{2}+a\right)\left(d\,x^{2}-c\right)}}+\frac{-\frac{(3Bbd-2aCd+2Ccb)a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-a}{a}}\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-x^{2}da+bc\,x^{2}+ac}\,b}}{}$
default	$\frac{A\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-x^{2}da+bc\,x^{2}+ac}}-\frac{Ba\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-\mathrm{EllipticE}\right.}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-x^{2}da+bc\,x^{2}+ac}\,b}$

input
```
int((C*x^4+B*x^2+A)/((b*x^2+a)*(-d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*C/b/d*x*(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)+(A+1/3*C/b/d*a*c)/(d/c)^
(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))-(B+1/3*C/b/d*(-2*a
*d+2*b*c))*a/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d
*x^2+b*c*x^2+a*c)^(1/2)/b*(EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/
2))-EllipticE(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.76

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{(a + bx^2)(c - dx^2)}}\,dx =$$

$$-\frac{(2\,Cbc^3 - (2\,Ca - 3\,Bb)c^2d)\sqrt{-bd}x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc}) - (2\,Cbc^3 + Cacd^2 + 3\,Abd^3 - (2\,Ca}{}$$

input
```
integrate((C*x^4+B*x^2+A)/((b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="frica
s")
```

output
```
-1/3*((2*C*b*c^3 - (2*C*a - 3*B*b)*c^2*d)*sqrt(-b*d)*x*sqrt(c/d)*elliptic_
e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - (2*C*b*c^3 + C*a*c*d^2 + 3*A*b*d^3 -
(2*C*a - 3*B*b)*c^2*d)*sqrt(-b*d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/
x), -a*d/(b*c)) + (C*b*c*d^2*x^2 + 2*C*b*c^2*d - (2*C*a - 3*B*b)*c*d^2)*sq
rt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c))/(b^2*c*d^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{(a + bx^2)(c - dx^2)}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/((b*x**2+a)*(-d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)\,(c-dx^2)}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-(bx^2+a)(dx^2-c)}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/((b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(-(b*x^2 + a)*(d*x^2 - c)), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)\,(c-dx^2)}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-(bx^2+a)(dx^2-c)}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/((b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(-(b*x^2 + a)*(d*x^2 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)\,(c-dx^2)}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{\sqrt{(b\,x^2+a)\,\left(c-d\,x^2\right)}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a + b*x^2)*(c - d*x^2))^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a + b*x^2)*(c - d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(c-dx^2)}}\,dx$$
$$= \frac{-\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,cx-2\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx\right)acd+3\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx\right)b^2d+2\left(\int \frac{\sqrt{}}{-}\right.}{3bd}$$

input
```
int((C*x^4+B*x^2+A)/((b*x^2+a)*(-d*x^2+c))^(1/2),x)
```

output
```
( - sqrt(c - d*x**2)*sqrt(a + b*x**2)*c*x - 2*int((sqrt(c - d*x**2)*sqrt(a
 + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a*c*d + 3*int((
sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x
**4),x)*b**2*d + 2*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d
*x**2 + b*c*x**2 - b*d*x**4),x)*b*c**2 + 3*int((sqrt(c - d*x**2)*sqrt(a +
b*x**2))/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a*b*d + int((sqrt(c - d
*x**2)*sqrt(a + b*x**2))/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a*c**2)
/(3*b*d)
```

3.3 $\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 281

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx = -\frac{Cx\sqrt{ac-(bc-ad)x^2-bdx^4}}{3bd} - \frac{\sqrt{a}c(2bcC-3bBd-2aCd)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{3b^{3/2}d^2\sqrt{ac-(bc-ad)x^2-bdx^4}} - \frac{\sqrt{a}(acCd-b(2c^2C-3Bcd+3Ad^2))\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{3b^{3/2}d^2\sqrt{ac-(bc-ad)x^2-bdx^4}}$$

output
```
-1/3*C*x*(a*c-(-a*d+b*c)*x^2-b*d*x^4)^(1/2)/b/d-1/3*a^(1/2)*c*(-3*B*b*d-2*
C*a*d+2*C*b*c)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(b^(1/2)*x/a^(
1/2),(-a*d/b/c)^(1/2))/b^(3/2)/d^2/(a*c-(-a*d+b*c)*x^2-b*d*x^4)^(1/2)-1/3*
a^(1/2)*(a*c*C*d-b*(3*A*d^2-3*B*c*d+2*C*c^2))*(1-b*x^2/a)^(1/2)*(1+d*x^2/c
)^(1/2)*EllipticF(b^(1/2)*x/a^(1/2),(-a*d/b/c)^(1/2))/b^(3/2)/d^2/(a*c-(-a
*d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.32 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx$$
$$= \frac{-\sqrt{-\frac{b}{a}}Cdx(a-bx^2)(c+dx^2)-ic(-2bcC+3bBd+2aCd)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|\cdots\right.}{3b\sqrt{-\frac{b}{a}}d^2\sqrt{(a-\cdots}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/Sqrt[(a - b*x^2)*(c + d*x^2)],x]
```

output
```
(-(Sqrt[-(b/a)]*C*d*x*(a - b*x^2)*(c + d*x^2)) - I*c*(-2*b*c*C + 3*b*B*d +
 2*a*C*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[-(b/a)]*x], -((a*d)/(b*c))] + I*(a*c*C*d + b*(-2*c^2*C + 3*B*c*d - 3*A*d^
2))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a
)]*x], -((a*d)/(b*c))])/(3*b*Sqrt[-(b/a)]*d^2*Sqrt[(a - b*x^2)*(c + d*x^2)
])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2048, 2207, 25, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx$$
$$\Big\downarrow \ 2048$$
$$\int \frac{A+Bx^2+Cx^4}{\sqrt{x^2(ad-bc)+ac-bdx^4}}dx$$
$$\Big\downarrow \ 2207$$

$$-\frac{\int -\frac{-((2bcC-2adC-3bBd)x^2)+acC+3Abd}{\sqrt{-bdx^4-(bc-ad)x^2+ac}}dx}{3bd}-\frac{Cx\sqrt{-x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$\downarrow$ 25

$$\frac{\int \frac{-((2bcC-2adC-3bBd)x^2)+acC+3Abd}{\sqrt{-bdx^4-(bc-ad)x^2+ac}}dx}{3bd}-\frac{Cx\sqrt{-x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$\downarrow$ 1514

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\int \frac{-((2bcC-2adC-3bBd)x^2)+acC+3Abd}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{3bd\sqrt{-x^2(bc-ad)+ac-bdx^4}}-\frac{Cx\sqrt{-x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$\downarrow$ 399

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(-\frac{(acCd-b(3Ad^2-3Bcd+2c^2C))\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}}dx}{d}-\frac{c(-2aCd-3bBd+2bcC)\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d}\right)}{3bd\sqrt{-x^2(bc-ad)+ac-bdx^4}}-\frac{Cx\sqrt{-x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$\downarrow$ 321

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(-\frac{c(-2aCd-3bBd+2bcC)\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d}-\frac{\sqrt{a}(acCd-b(3Ad^2-3Bcd+2c^2C))\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d}\right)}{3bd\sqrt{-x^2(bc-ad)+ac-bdx^4}}-\frac{Cx\sqrt{-x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$\downarrow$ 327

$$\frac{\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\left(-\frac{\sqrt{a}(acCd-b(3Ad^2-3Bcd+2c^2C))\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d}-\frac{\sqrt{a}c(-2aCd-3bBd+2bcC)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right.}{\sqrt{b}d}\right.}{3bd\sqrt{-x^2(bc-ad)+ac-bdx^4}}-\frac{Cx\sqrt{-x^2(bc-ad)+ac-bdx^4}}{3bd}$$

input
```
Int[(A + B*x^2 + C*x^4)/Sqrt[(a - b*x^2)*(c + d*x^2)],x]
```

output
```
-1/3*(C*x*Sqrt[a*c - (b*c - a*d)*x^2 - b*d*x^4])/(b*d) + (Sqrt[1 - (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*(-((Sqrt[a]*c*(2*b*c*C - 3*b*B*d - 2*a*C*d)*Ellipt
icE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d)) - (Sqrt[a]*
(a*c*C*d - b*(2*c^2*C - 3*B*c*d + 3*A*d^2))*EllipticF[ArcSin[(Sqrt[b]*x)/S
qrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d)))/(3*b*d*Sqrt[a*c - (b*c - a*d)*x^2
- b*d*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.02

method	result
elliptic	$-\frac{Cx\sqrt{-bd\,x^{4}+x^{2}da-bc\,x^{2}+ac}}{3bd}+\frac{\left(A+\frac{Cac}{3bd}\right)\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+x^{2}da-bc\,x^{2}+ac}}-\frac{\left(B+\frac{C(2ad-2bc)}{3bd}\right)c\sqrt{1-\cdots}}{\cdots}$
risch	$-\frac{Cx\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}{3bd\sqrt{-\left(b\,x^{2}-a\right)\left(d\,x^{2}+c\right)}}+\frac{-\frac{(3Bbd+2aCd-2Ccb)c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-b\cdots}{cb}}\cdots\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+x^{2}da-bc\,x^{2}+ac}\,d}}{\cdots}$
default	$\frac{A\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+x^{2}da-bc\,x^{2}+ac}}-\frac{Bc\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-\operatorname{EllipticE}\left(x\cdots\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+x^{2}da-bc\,x^{2}+ac}\,d}$

input
```
int((C*x^4+B*x^2+A)/((-b*x^2+a)*(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*C/b/d*x*(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(1/2)+(A+1/3*C/b/d*a*c)/(b/a)^
(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)^(
1/2)*EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))-(B+1/3*C/b/d*(2*a*d
-2*b*c))*c/(b/a)^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(-b*d*x^4+a*d*x
^2-b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2))
-EllipticE(x*(b/a)^(1/2),(-1-(a*d-b*c)/c/b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)\,(c+dx^2)}}\,dx$$
$$= \frac{(2\,Ca^2bc-(2\,Ca^3+3\,Ba^2b)d)\sqrt{-bd}x\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\,|-\frac{bc}{ad})-((2\,Ca^2b-Cab^2)c-(2\,Ca^3+3\,Ba}{}$$

input
```
integrate((C*x^4+B*x^2+A)/((-b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="frica
s")
```

output
```
1/3*((2*C*a^2*b*c - (2*C*a^3 + 3*B*a^2*b)*d)*sqrt(-b*d)*x*sqrt(a/b)*ellipt
ic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - ((2*C*a^2*b - C*a*b^2)*c - (2*C*a^
3 + 3*B*a^2*b + 3*A*b^3)*d)*sqrt(-b*d)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(
a/b)/x), -b*c/(a*d)) - (C*a*b^2*d*x^2 - 2*C*a*b^2*c + (2*C*a^2*b + 3*B*a*b
^2)*d)*sqrt(-b*d*x^4 - (b*c - a*d)*x^2 + a*c))/(a*b^3*d^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)\,(c+dx^2)}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/((-b*x**2+a)*(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{(a - bx^2)(c + dx^2)}} \, dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{-(bx^2 - a)(dx^2 + c)}} \, dx$$

input
```
integrate((C*x^4+B*x^2+A)/((-b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(-(b*x^2 - a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{(a - bx^2)(c + dx^2)}} \, dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{-(bx^2 - a)(dx^2 + c)}} \, dx$$

input
```
integrate((C*x^4+B*x^2+A)/((-b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(-(b*x^2 - a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{(a - bx^2)(c + dx^2)}} \, dx = \int \frac{C\,x^4 + B\,x^2 + A}{\sqrt{(a - b\,x^2)\,(d\,x^2 + c)}} \, dx$$

input
```
int((A + B*x^2 + C*x^4)/((a - b*x^2)*(c + d*x^2))^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a - b*x^2)*(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c+dx^2)}}\,dx$$
$$= \frac{-\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,cx+2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)acd+3\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^4+ad\,x^2-bc\,x^2+ac}dx\right)b^2d-2\left(\int \cdots\right.}{3bd}$$

input
```
int((C*x^4+B*x^2+A)/((-b*x^2+a)*(d*x^2+c))^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a - b*x**2)*c*x + 2*int((sqrt(c + d*x**2)*sqrt(a
 - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*c*d + 3*int((
sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x**2 - b*c*x**2 - b*d*x
**4),x)*b**2*d - 2*int((sqrt(c + d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c + a*d
*x**2 - b*c*x**2 - b*d*x**4),x)*b*c**2 + 3*int((sqrt(c + d*x**2)*sqrt(a -
b*x**2))/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*b*d + int((sqrt(c + d
*x**2)*sqrt(a - b*x**2))/(a*c + a*d*x**2 - b*c*x**2 - b*d*x**4),x)*a*c**2)
/(3*b*d)
```

3.4 $\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 274

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \frac{Cx\sqrt{ac-(bc+ad)x^2+bdx^4}}{3bd} - \frac{a\sqrt{c}(2bcC+3bBd+2aCd)\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{ac-(bc+ad)x^2+bdx^4}} + \frac{\sqrt{c}(3Ab^2d+2a^2Cd+ab(cC+3Bd))\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{ac-(bc+ad)x^2+bdx^4}}$$

output
```
1/3*C*x*(a*c-(a*d+b*c)*x^2+b*d*x^4)^(1/2)/b/d-1/3*a*c^(1/2)*(3*B*b*d+2*C*a
*d+2*C*b*c)*(1-b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2
),(b*c/a/d)^(1/2))/b^2/d^(3/2)/(a*c-(a*d+b*c)*x^2+b*d*x^4)^(1/2)+1/3*c^(1/
2)*(3*A*b^2*d+2*a^2*C*d+a*b*(3*B*d+C*c))*(1-b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/
2)*EllipticF(d^(1/2)*x/c^(1/2),(b*c/a/d)^(1/2))/b^2/d^(3/2)/(a*c-(a*d+b*c)
*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.32 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \frac{-\sqrt{-\frac{b}{a}}Cdx(a-bx^2)(-c+dx^2)+ic(2bcC+3bBd+2aCd)\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{a}{b}\right.}{3b\sqrt{-\frac{b}{a}}d^2\sqrt{(a-bx}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/Sqrt[(a - b*x^2)*(c - d*x^2)],x]
```

output
```
(-(Sqrt[-(b/a)]*C*d*x*(a - b*x^2)*(-c + d*x^2)) + I*c*(2*b*c*C + 3*b*B*d +
 2*a*C*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[-(b/a)]*x], (a*d)/(b*c)] - I*(a*c*C*d + b*(2*c^2*C + 3*B*c*d + 3*A*d^2))*
Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x
], (a*d)/(b*c)])/(3*b*Sqrt[-(b/a)]*d^2*Sqrt[(a - b*x^2)*(c - d*x^2)])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 562 vs. 2(274) = 548.

Time = 0.76 (sec) , antiderivative size = 562, normalized size of antiderivative = 2.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2048, 2207, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c-dx^2)}}dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{x^2(-ad-bc)+ac+bdx^4}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{\int -\frac{-((2bcC+2adC+3bBd)x^2)+acC-3Abd}{\sqrt{bdx^4-(bc+ad)x^2+ac}}dx}{3bd}+\frac{Cx\sqrt{-x^2(ad+bc)+ac+bdx^4}}{3bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{Cx\sqrt{-x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\int \frac{-((2bcC+2adC+3bBd)x^2)+acC-3Abd}{\sqrt{bdx^4-(bc+ad)x^2+ac}}dx}{3bd}$$

$$\Big\downarrow \text{1511}$$

$$\frac{Cx\sqrt{-x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt{a}\sqrt{c}(2aCd+3bBd+2bcC)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4-(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}\left(-\frac{\sqrt{b}\sqrt{d}(acC-3Abd)}{\sqrt{a}\sqrt{c}}+2aCd+3bBd+2bcC\right)\int \frac{1}{\sqrt{bdx^4-(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{Cx\sqrt{-x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{(2aCd+3bBd+2bcC)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4-(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}\left(-\frac{\sqrt{b}\sqrt{d}(acC-3Abd)}{\sqrt{a}\sqrt{c}}+2aCd+3bBd+2bcC\right)\int \frac{1}{\sqrt{bdx^4-(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}}{3bd}$$

$$\Big\downarrow \text{1416}$$

$$\frac{Cx\sqrt{-x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{(2aCd+3bBd+2bcC)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4-(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{-x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\left(-\frac{\sqrt{b}\sqrt{d}(acC-3Abd)}{\sqrt{a}\sqrt{c}}+2aCd+3bBd+2bc\right.}{2b^{3/4}d^{3/4}\sqrt{-x^2(ad+bc)+ac+bdx}}}{3bd}$$

$$\Big\downarrow \text{1509}$$

$$\frac{Cx\sqrt{-x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{(2aCd+3bBd+2bcC)\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{-x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right)|\frac{1}{4}\left(\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}+2\right)\right)}{\sqrt[4]{b}\sqrt[4]{d}\sqrt{-x^2(ad+bc)+ac+bdx^4}}-\frac{x\sqrt{-x^2(ad+bc)+ac+bdx^4}}{\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2}\right.}{\sqrt{b}\sqrt{d}}}{}$$

input
```
Int[(A + B*x^2 + C*x^4)/Sqrt[(a - b*x^2)*(c - d*x^2)],x]
```

output
```
(C*x*Sqrt[a*c - (b*c + a*d)*x^2 + b*d*x^4])/(3*b*d) - (((2*b*c*C + 3*b*B*d
 + 2*a*C*d)*(-((x*Sqrt[a*c - (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c]
+ Sqrt[b]*Sqrt[d]*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt
[d]*x^2)*Sqrt[(a*c - (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]
*Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))]
, (2 + (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)
*Sqrt[a*c - (b*c + a*d)*x^2 + b*d*x^4])))/(Sqrt[b]*Sqrt[d]) - (a^(1/4)*c^(
1/4)*(2*b*c*C + 3*b*B*d + 2*a*C*d - (Sqrt[b]*Sqrt[d]*(a*c*C - 3*A*b*d))/(S
qrt[a]*Sqrt[c]))*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c - (b*c
+ a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticF
[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 + (b*c + a*d)/(Sqrt[a
]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(3/4)*d^(3/4)*Sqrt[a*c - (b*c + a*d)*
x^2 + b*d*x^4]))/(3*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 5.59 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.06

method	result
elliptic	$\frac{Cx\sqrt{bd\,x^{4}-x^{2}da-bc\,x^{2}+ac}}{3bd}+\frac{\left(A-\frac{Cac}{3bd}\right)\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-x^{2}da-bc\,x^{2}+ac}}+\frac{\left(B-\frac{C(-2ad-2bc)}{3bd}\right)a\sqrt{1-\frac{d}{}}}{}$
risch	$\frac{Cx\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}{3bd\sqrt{\left(b\,x^{2}-a\right)\left(d\,x^{2}-c\right)}}+\frac{\frac{(3Bbd+2aCd+2Ccb)a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-x^{2}da-bc\,x^{2}+ac}\,b}}{}$
default	$\frac{A\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-x^{2}da-bc\,x^{2}+ac}}+\frac{Ba\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-\operatorname{EllipticE}\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-x^{2}da-bc\,x^{2}+ac}\,b}$

input
```
int((C*x^4+B*x^2+A)/((-b*x^2+a)*(-d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*C/b/d*x*(b*d*x^4-a*d*x^2-b*c*x^2+a*c)^(1/2)+(A-1/3*C/b/d*a*c)/(d/c)^(1
/2)*(1-d*x^2/c)^(1/2)*(1-b*x^2/a)^(1/2)/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)^(1/2
)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d-b*c)/a/d)^(1/2))+(B-1/3*C/b/d*(-2*a*d-
2*b*c))*a/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1-b*x^2/a)^(1/2)/(b*d*x^4-a*d*x^2
-b*c*x^2+a*c)^(1/2)/b*(EllipticF(x*(d/c)^(1/2),(-1-(-a*d-b*c)/a/d)^(1/2))-
EllipticE(x*(d/c)^(1/2),(-1-(-a*d-b*c)/a/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \frac{(2\,Ca^2bc+(2\,Ca^3+3\,Ba^2b)d)\sqrt{bd}x\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid\frac{bc}{ad})-((2\,Ca^2b-Cab^2)c+(2\,Ca^3+3\,Ba^2b+$$

input
```
integrate((C*x^4+B*x^2+A)/((-b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/3*((2*C*a^2*b*c + (2*C*a^3 + 3*B*a^2*b)*d)*sqrt(b*d)*x*sqrt(a/b)*ellipti
c_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) - ((2*C*a^2*b - C*a*b^2)*c + (2*C*a^3
+ 3*B*a^2*b + 3*A*b^3)*d)*sqrt(b*d)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b
)/x), b*c/(a*d)) + (C*a*b^2*d*x^2 + 2*C*a*b^2*c + (2*C*a^2*b + 3*B*a*b^2)*
d)*sqrt(b*d*x^4 - (b*c + a*d)*x^2 + a*c))/(a*b^3*d^2*x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \int \frac{A+Bx^2+Cx^4}{\sqrt{(-a+bx^2)(-c+dx^2)}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/((-b*x**2+a)*(-d*x**2+c))**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/sqrt((-a + b*x**2)*(-c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{(bx^2-a)(dx^2-c)}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/((-b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt((b*x^2 - a)*(d*x^2 - c)), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{(bx^2-a)(dx^2-c)}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/((-b*x^2+a)*(-d*x^2+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt((b*x^2 - a)*(d*x^2 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{\sqrt{\left(a-b\,x^2\right)\,\left(c-d\,x^2\right)}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a - b*x^2)*(c - d*x^2))^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a - b*x^2)*(c - d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{(a-bx^2)(c-dx^2)}}\,dx$$
$$= \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}\,cx+2\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{bd\,x^4-ad\,x^2-bc\,x^2+ac}dx\right)acd+3\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}\,x^2}{bd\,x^4-ad\,x^2-bc\,x^2+ac}dx\right)b^2d+2\left(\int \frac{\sqrt{-}}{bc}\right.}{3bd}$$

input
```
int((C*x^4+B*x^2+A)/((-b*x^2+a)*(-d*x^2+c))^(1/2),x)
```

output
```
(sqrt(c - d*x**2)*sqrt(a - b*x**2)*c*x + 2*int((sqrt(c - d*x**2)*sqrt(a -
b*x**2)*x**2)/(a*c - a*d*x**2 - b*c*x**2 + b*d*x**4),x)*a*c*d + 3*int((sqr
t(c - d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c - a*d*x**2 - b*c*x**2 + b*d*x**4
),x)*b**2*d + 2*int((sqrt(c - d*x**2)*sqrt(a - b*x**2)*x**2)/(a*c - a*d*x*
*2 - b*c*x**2 + b*d*x**4),x)*b*c**2 + 3*int((sqrt(c - d*x**2)*sqrt(a - b*x
**2))/(a*c - a*d*x**2 - b*c*x**2 + b*d*x**4),x)*a*b*d - int((sqrt(c - d*x*
*2)*sqrt(a - b*x**2))/(a*c - a*d*x**2 - b*c*x**2 + b*d*x**4),x)*a*c**2)/(3
*b*d)
```

3.5 $\int (A+Bx^2+Cx^4)(ac+(bc+ad)x^2+bdx^4)^{3/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 993

$$\int (A+Bx^2+Cx^4)(ac+(bc+ad)x^2+bdx^4)^{3/2}\,dx =$$

$$-\frac{(48a^5Cd^5-8a^4bd^4(15cC+11Bd)+6a^2b^3cd^2(8c^2C-33Bcd-165Ad^2)+2b^5c^3(24c^2C-44Bcd+99A}{3465b^3d^4\sqrt{ac+(bc+a}}$$

$$+\frac{x(24a^4Cd^4+11a^3bd^3(3cC-4Bd)-3a^2b^2d^2(24c^2C+11Bcd-33Ad^2)+33ab^3cd(c^2C-Bcd+36Ad^2)}{}$$

$$-\frac{x(3(3bd(acC-11Abd)+(bc+ad)(6bcC-11bBd+6aCd))+7bd(6bcC-11bBd+6aCd)x^2)(ac+(b}{693b^2d^2}$$

$$+\frac{Cx(ac+(bc+ad)x^2+bdx^4)^{5/2}}{11bd}$$

$$+\frac{c(48a^5Cd^5-8a^4bd^4(15cC+11Bd)+6a^2b^3cd^2(8c^2C-33Bcd-165Ad^2)+2b^5c^3(24c^2C-44Bcd+99A}{3465\sqrt{a}b}$$

$$-\frac{\sqrt{a}c(24a^4Cd^4-a^3bd^3(57cC+44Bd)+3a^2b^2d^2(6c^2C+44Bcd+33Ad^2)+b^4c^2(24c^2C-44Bcd+99A}{3465b^{7/2}d^3\sqrt{ac+(bc+}}$$

output
```
-1/3465*(48*a^5*C*d^5-8*a^4*b*d^4*(11*B*d+15*C*c)+6*a^2*b^3*c*d^2*(-165*A*
d^2-33*B*c*d+8*C*c^2)+2*b^5*c^3*(99*A*d^2-44*B*c*d+24*C*c^2)-5*a*b^4*c^2*d
*(198*A*d^2-55*B*c*d+24*C*c^2)+a^3*b^2*d^3*(198*A*d^2+275*B*c*d+48*C*c^2))
*x*(d*x^2+c)/b^3/d^4/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)+1/3465*x*(24*a^4*C*
d^4+11*a^3*b*d^3*(-4*B*d+3*C*c)-3*a^2*b^2*d^2*(-33*A*d^2+11*B*c*d+24*C*c^2
)+33*a*b^3*c*d*(36*A*d^2-B*c*d+C*c^2)+b^4*c^2*(99*A*d^2-44*B*c*d+24*C*c^2)
-3*b*d*(9*b*d*(a*d+b*c)*(-11*A*b*d+C*a*c)+14*a*b*c*d*(-11*B*b*d+6*C*a*d+6*
C*b*c)-4*(a*d+b*c)^2*(-11*B*b*d+6*C*a*d+6*C*b*c))*x^2)*(a*c+(a*d+b*c)*x^2+
b*d*x^4)^(1/2)/b^3/d^3-1/693*x*(9*b*d*(-11*A*b*d+C*a*c)+3*(a*d+b*c)*(-11*B
*b*d+6*C*a*d+6*C*b*c)+7*b*d*(-11*B*b*d+6*C*a*d+6*C*b*c)*x^2)*(a*c+(a*d+b*c
)*x^2+b*d*x^4)^(3/2)/b^2/d^2+1/11*C*x*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2)/b/
d+1/3465*c*(48*a^5*C*d^5-8*a^4*b*d^4*(11*B*d+15*C*c)+6*a^2*b^3*c*d^2*(-165
*A*d^2-33*B*c*d+8*C*c^2)+2*b^5*c^3*(99*A*d^2-44*B*c*d+24*C*c^2)-5*a*b^4*c^
2*d*(198*A*d^2-55*B*c*d+24*C*c^2)+a^3*b^2*d^3*(198*A*d^2+275*B*c*d+48*C*c^
2))*(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/
(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(7/2)/d^4/(a*c+(a*d+b*c)*x^
2+b*d*x^4)^(1/2)-1/3465*a^(1/2)*c*(24*a^4*C*d^4-a^3*b*d^3*(44*B*d+57*C*c)+
3*a^2*b^2*d^2*(33*A*d^2+44*B*c*d+6*C*c^2)+b^4*c^2*(99*A*d^2-44*B*c*d+24*C*
c^2)-3*a*b^3*c*d*(594*A*d^2-44*B*c*d+19*C*c^2))*(b*x^2+a)*(a*(d*x^2+c)/c/(
b*x^2+a))^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.36 (sec) , antiderivative size = 701, normalized size of antiderivative = 0.71

$$\int \left(A+Bx^2+Cx^4\right)\left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}\,dx=\frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(24a^4Cd^4-a^3bd^3(57cC+44Bd+18Cdx^2)+3a^2b^2d^2(6c^2C+2\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)*(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(24*a^4*C*d^4 - a^3*b*d^3*(57*c*C +
 44*B*d + 18*C*d*x^2) + 3*a^2*b^2*d^2*(6*c^2*C + 2*c*d*(22*B + 7*C*x^2) +
d^2*(33*A + 11*B*x^2 + 5*C*x^4)) + b^4*(24*c^4*C - 2*c^3*d*(22*B + 9*C*x^2
) + 3*c^2*d^2*(33*A + 11*B*x^2 + 5*C*x^4) + 5*d^4*x^4*(99*A + 77*B*x^2 + 6
3*C*x^4) + 2*c*d^3*x^2*(396*A + 275*B*x^2 + 210*C*x^4)) + a*b^3*d*(-57*c^3
*C + 6*c^2*d*(22*B + 7*C*x^2) + 2*d^3*x^2*(396*A + 275*B*x^2 + 210*C*x^4)
+ c*d^2*(1683*A + 913*B*x^2 + 615*C*x^4))) + I*c*(48*a^5*C*d^5 - 8*a^4*b*d
^4*(15*c*C + 11*B*d) + 6*a^2*b^3*c*d^2*(8*c^2*C - 33*B*c*d - 165*A*d^2) +
2*b^5*c^3*(24*c^2*C - 44*B*c*d + 99*A*d^2) - 5*a*b^4*c^2*d*(24*c^2*C - 55*
B*c*d + 198*A*d^2) + a^3*b^2*d^3*(48*c^2*C + 275*B*c*d + 198*A*d^2))*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)
/(b*c)] - I*c*(-(b*c) + a*d)*(24*a^4*C*d^4 - a^3*b*d^3*(39*c*C + 44*B*d) +
 9*a^2*b^2*d^2*(-(c^2*C) + 11*B*c*d + 11*A*d^2) - 2*b^4*c^2*(24*c^2*C - 44
*B*c*d + 99*A*d^2) + 3*a*b^3*c*d*(32*c^2*C - 77*B*c*d + 297*A*d^2))*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)])/(3465*b^3*Sqrt[b/a]*d^4*Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 1247, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2207, 25, 1490, 1490, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A + Bx^2 + Cx^4) \left(x^2(ad + bc) + ac + bdx^4\right)^{3/2} dx$$

$$\Big\downarrow \; 2207$$

$$\frac{\int -\left(\left((6bcC + 6adC - 11bBd)x^2 + acC - 11Abd\right)\left(bdx^4 + (bc + ad)x^2 + ac\right)^{3/2}\right) dx}{11bd} + \frac{Cx\left(x^2(ad + bc) + ac + bdx^4\right)^{5/2}}{11bd}$$

$$\Big\downarrow \; 25$$

$$\frac{Cx\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}}{11bd}-$$
$$\frac{\int \left((6bcC+6adC-11bBd)x^2+acC-11Abd\right)\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}dx}{11bd}$$

$\Big\downarrow$ 1490

$$\frac{Cx\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}}{11bd}-$$
$$\frac{\int \left(\left(-4(6bcC+6adC-11bBd)(bc+ad)^2+9bd(acC-11Abd)(bc+ad)+14abcd(6bcC+6adC-11bBd)\right)x^2+ac(18bd(acC-11Abd)-(bc+ad)(6bcC+6a\right.}{21bd}$$

$\Big\downarrow$ 1490

$$\frac{Cx\left(x^2(ad+bc)+ac+bdx^4\right)^{5/2}}{11bd}-$$
$$\int \frac{\left(2c^3\left(24Cc^2-44Bdc+99Ad^2\right)b^5-5ac^2d\left(24Cc^2-55Bdc+198Ad^2\right)b^4+6a^2cd^2\left(8Cc^2-33Bdc-165Ad^2\right)b^3+a^3d^3\left(48Cc^2+275Bdc+198Ad^2\right)b^2-8a^4d^4(15cC+1\right.}{\sqrt{bdx^4+(b}}$$
$$1$$

$\Big\downarrow$ 1511

$$\frac{Cx\left(bdx^4+(bc+ad)x^2+ac\right)^{5/2}}{11bd}-$$
$$\frac{x\left(7bd(6bcC+6adC-11bBd)x^2+3(3bd(acC-11Abd)+(bc+ad)(6bcC+6adC-11bBd))\right)\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}{63bd}+\frac{-\frac{\sqrt{a}\sqrt{c}\left(-2c^2\left(24Cc^2-44Bd\right.\right.}{}}{}$$

$\Big\downarrow$ 27

$$\frac{Cx\left(bdx^4+(bc+ad)x^2+ac\right)^{5/2}}{11bd}-$$
$$\frac{x\left(7bd(6bcC+6adC-11bBd)x^2+3(3bd(acC-11Abd)+(bc+ad)(6bcC+6adC-11bBd))\right)\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}{63bd}+\frac{-\frac{\sqrt{a}\sqrt{c}\left(-2c^2\left(24Cc^2-44Bd\right.\right.}{}}{}$$

$\Big\downarrow$ 1416

$$\frac{Cx\left(bdx^4+(bc+ad)x^2+ac\right)^{5/2}}{11bd}-$$
$$\frac{x\left(7bd(6bcC+6adC-11bBd)x^2+3(3bd(acC-11Abd)+(bc+ad)(6bcC+6adC-11bBd))\right)\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}{63bd}+\frac{-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(-2c^2\left(24Cc^2-4\right.\right.}{}}{}$$

$$\Big\downarrow \text{1509}$$

$$\frac{Cx\left(bdx^4+(bc+ad)x^2+ac\right)^{5/2}}{11bd}-$$

$$\frac{x(7bd(6bcC+6adC-11bBd)x^2+3(3bd(acC-11Abd)+(bc+ad)(6bcC+6adC-11bBd)))\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}{63bd}+\frac{-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(-2c^2\left(24Cc^2-4\right.\right.}{}}{}$$

input
```
Int[(A + B*x^2 + C*x^4)*(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(3/2),x]
```

output
```
(C*x*(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(5/2))/(11*b*d) - ((x*(3*(3*b*d*(a*
c*C - 11*A*b*d) + (b*c + a*d)*(6*b*c*C - 11*b*B*d + 6*a*C*d)) + 7*b*d*(6*b
*c*C - 11*b*B*d + 6*a*C*d)*x^2)*(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(3/2))/(
63*b*d) + (-1/15*(x*(24*a^4*C*d^4 + 11*a^3*b*d^3*(3*c*C - 4*B*d) - 3*a^2*b
^2*d^2*(24*c^2*C + 11*B*c*d - 33*A*d^2) + 33*a*b^3*c*d*(c^2*C - B*c*d + 36
*A*d^2) + b^4*c^2*(24*c^2*C - 44*B*c*d + 99*A*d^2) - 3*b*d*(9*b*d*(b*c + a
*d)*(a*c*C - 11*A*b*d) + 14*a*b*c*d*(6*b*c*C - 11*b*B*d + 6*a*C*d) - 4*(b*
c + a*d)^2*(6*b*c*C - 11*b*B*d + 6*a*C*d))*x^2)*Sqrt[a*c + (b*c + a*d)*x^2
 + b*d*x^4])/(b*d) + (-(((48*a^5*C*d^5 - 8*a^4*b*d^4*(15*c*C + 11*B*d) + 6
*a^2*b^3*c*d^2*(8*c^2*C - 33*B*c*d - 165*A*d^2) + 2*b^5*c^3*(24*c^2*C - 44
*B*c*d + 99*A*d^2) - 5*a*b^4*c^2*d*(24*c^2*C - 55*B*c*d + 198*A*d^2) + a^3
*b^2*d^3*(48*c^2*C + 275*B*c*d + 198*A*d^2))*(-((x*Sqrt[a*c + (b*c + a*d)*
x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)) + (a^(1/4)*c^(1/4
)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*
d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(b^(1
/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[
c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])))
/(Sqrt[b]*Sqrt[d])) - (a^(1/4)*c^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqrt[d])
^2*(72*a^(7/2)*Sqrt[b]*Sqrt[c]*C*d^(7/2) - 48*a^4*C*d^4 + 8*a^3*b*d^3*(3*c
*C + 11*B*d) - 3*a^(5/2)*b^(3/2)*Sqrt[c]*d^(5/2)*(21*c*C + 44*B*d) - 9*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1926 vs. $2(962) = 1924$.

Time = 10.88 (sec) , antiderivative size = 1927, normalized size of antiderivative = 1.94

method	result	size
risch	Expression too large to display	1927
elliptic	Expression too large to display	2262
default	Expression too large to display	3080

input
```
int((C*x^4+B*x^2+A)*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
1/3465/b^3/d^3*x*(315*C*b^4*d^4*x^8+385*B*b^4*d^4*x^6+420*C*a*b^3*d^4*x^6+
420*C*b^4*c*d^3*x^6+495*A*b^4*d^4*x^4+550*B*a*b^3*d^4*x^4+550*B*b^4*c*d^3*
x^4+15*C*a^2*b^2*d^4*x^4+615*C*a*b^3*c*d^3*x^4+15*C*b^4*c^2*d^2*x^4+792*A*
a*b^3*d^4*x^2+792*A*b^4*c*d^3*x^2+33*B*a^2*b^2*d^4*x^2+913*B*a*b^3*c*d^3*x
^2+33*B*b^4*c^2*d^2*x^2-18*C*a^3*b*d^4*x^2+42*C*a^2*b^2*c*d^3*x^2+42*C*a*b
^3*c^2*d^2*x^2-18*C*b^4*c^3*d*x^2+99*A*a^2*b^2*d^4+1683*A*a*b^3*c*d^3+99*A
*b^4*c^2*d^2-44*B*a^3*b*d^4+132*B*a^2*b^2*c*d^3+132*B*a*b^3*c^2*d^2-44*B*b
^4*c^3*d+24*C*a^4*d^4-57*C*a^3*b*c*d^3+18*C*a^2*b^2*c^2*d^2-57*C*a*b^3*c^3
*d+24*C*b^4*c^4)*(b*x^2+a)*(d*x^2+c)/((b*x^2+a)*(d*x^2+c))^(1/2)-1/3465/b^
3/d^3*(-(198*A*a^3*b^2*d^5-990*A*a^2*b^3*c*d^4-990*A*a*b^4*c^2*d^3+198*A*b
^5*c^3*d^2-88*B*a^4*b*d^5+275*B*a^3*b^2*c*d^4-198*B*a^2*b^3*c^2*d^3+275*B*
a*b^4*c^3*d^2-88*B*b^5*c^4*d+48*C*a^5*d^5-120*C*a^4*b*c*d^4+48*C*a^3*b^2*c
^2*d^3+48*C*a^2*b^3*c^3*d^2-120*C*a*b^4*c^4*d+48*C*b^5*c^5)*c/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d
*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2)))+24*C*a*b^4*c^5/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+24*C*a^5*c*d^4/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+99*A*a*b^4*c^3*d^2/(-b/a)^(1/2)*(1...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1009, normalized size of antiderivative = 1.02

$$\int \left(A+Bx^2+Cx^4\right)\left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="
fricas")
```

output
```
1/3465*((48*C*b^5*c^6 - 8*(15*C*a*b^4 + 11*B*b^5)*c^5*d + (48*C*a^2*b^3 +
275*B*a*b^4 + 198*A*b^5)*c^4*d^2 + 6*(8*C*a^3*b^2 - 33*B*a^2*b^3 - 165*A*a
*b^4)*c^3*d^3 - 5*(24*C*a^4*b - 55*B*a^3*b^2 + 198*A*a^2*b^3)*c^2*d^4 + 2*
(24*C*a^5 - 44*B*a^4*b + 99*A*a^3*b^2)*c*d^5)*sqrt(b*d)*x*sqrt(-c/d)*ellip
tic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (48*C*b^5*c^6 - 8*(15*C*a*b^4 + 1
1*B*b^5)*c^5*d + (48*C*a^2*b^3 + (275*B + 24*C)*a*b^4 + 198*A*b^5)*c^4*d^2
 + (48*C*a^3*b^2 - 3*(66*B + 19*C)*a^2*b^3 - 22*(45*A + 2*B)*a*b^4)*c^3*d^
3 - (120*C*a^4*b - (275*B + 18*C)*a^3*b^2 + 66*(15*A - 2*B)*a^2*b^3 - 99*A
*a*b^4)*c^2*d^4 + (48*C*a^5 - (88*B + 57*C)*a^4*b + 66*(3*A + 2*B)*a^3*b^2
 - 1782*A*a^2*b^3)*c*d^5 + (24*C*a^5 - 44*B*a^4*b + 99*A*a^3*b^2)*d^6)*sqr
t(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (315*C*b
^5*d^6*x^10 - 48*C*b^5*c^5*d + 35*(12*C*b^5*c*d^5 + (12*C*a*b^4 + 11*B*b^5
)*d^6)*x^8 + 8*(15*C*a*b^4 + 11*B*b^5)*c^4*d^2 - (48*C*a^2*b^3 + 275*B*a*b
^4 + 198*A*b^5)*c^3*d^3 - 6*(8*C*a^3*b^2 - 33*B*a^2*b^3 - 165*A*a*b^4)*c^2
*d^4 + 5*(24*C*a^4*b - 55*B*a^3*b^2 + 198*A*a^2*b^3)*c*d^5 - 2*(24*C*a^5 -
 44*B*a^4*b + 99*A*a^3*b^2)*d^6 + 5*(3*C*b^5*c^2*d^4 + (123*C*a*b^4 + 110*
B*b^5)*c*d^5 + (3*C*a^2*b^3 + 110*B*a*b^4 + 99*A*b^5)*d^6)*x^6 - (18*C*b^5
*c^3*d^3 - 3*(14*C*a*b^4 + 11*B*b^5)*c^2*d^4 - (42*C*a^2*b^3 + 913*B*a*b^4
 + 792*A*b^5)*c*d^5 + 3*(6*C*a^3*b^2 - 11*B*a^2*b^3 - 264*A*a*b^4)*d^6)*x^
4 + (24*C*b^5*c^4*d^2 - (57*C*a*b^4 + 44*B*b^5)*c^3*d^3 + 3*(6*C*a^2*b^...
```

Sympy [F]

$$\int \left(A+Bx^2+Cx^4\right)\left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}\,dx=\int \left(\left(a+bx^2\right)\left(c+dx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2+Cx^4\right)\,dx$$

input
```
integrate((C*x**4+B*x**2+A)*(a*c+(a*d+b*c)*x**2+b*d*x**4)**(3/2),x)
```

output
```
Integral(((a + b*x**2)*(c + d*x**2))**(3/2)*(A + B*x**2 + C*x**4), x)
```

Maxima [F]

$$\int \left(A+Bx^2+Cx^4\right) \left(ac+(bc+ad)x^2+bdx^4\right)^{3/2} \, dx = \int \left(bdx^4+(bc+ad)x^2+ac\right)^{\frac{3}{2}} \left(Cx^4+Bx^2+A\right) \, dx$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="
maxima")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(3/2)*(C*x^4 + B*x^2 + A), x)
```

Giac [F]

$$\int \left(A+Bx^2+Cx^4\right) \left(ac+(bc+ad)x^2+bdx^4\right)^{3/2} \, dx = \int \left(bdx^4+(bc+ad)x^2+ac\right)^{\frac{3}{2}} \left(Cx^4+Bx^2+A\right) \, dx$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="
giac")
```

output
```
integrate((b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(3/2)*(C*x^4 + B*x^2 + A), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx^2+Cx^4\right) \left(ac+(bc+ad)x^2+bdx^4\right)^{3/2} \, dx = \int {\left(b\,d\,x^4+\left(a\,d+b\,c\right)\,x^2+a\,c\right)}^{3/2}\,\left(C\,x^4+B\,x^2+A\right) \,d x$$

input
```
int((a*c + x^2*(a*d + b*c) + b*d*x^4)^(3/2)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a*c + x^2*(a*d + b*c) + b*d*x^4)^(3/2)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int \left(A+Bx^2+Cx^4\right)\left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}\,dx=\text{too large to display}$$

input
```
int((C*x^4+B*x^2+A)*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x)
```

output
```
(24*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**4*c*d**4*x + 55*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a**3*b**2*d**4*x - 57*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a
**3*b*c**2*d**3*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b*c*d**4*x**
3 + 1815*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*c*d**3*x + 825*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*a**2*b**3*d**4*x**3 + 18*sqrt(c + d*x**2)*sqrt
(a + b*x**2)*a**2*b**2*c**3*d**2*x + 42*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
a**2*b**2*c**2*d**3*x**3 + 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b**2*
c*d**4*x**5 + 231*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**4*c**2*d**2*x + 1
705*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**4*c*d**3*x**3 + 1045*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*a*b**4*d**4*x**5 - 57*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*a*b**3*c**4*d*x + 42*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**3*d
**2*x**3 + 615*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c**2*d**3*x**5 + 4
20*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**4*x**7 - 44*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*b**5*c**3*d*x + 33*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**5*c**2*d**2*x**3 + 550*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**5*c*d**3*x*
*5 + 385*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**5*d**4*x**7 + 24*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*b**4*c**5*x - 18*sqrt(c + d*x**2)*sqrt(a + b*x**2)*
b**4*c**4*d*x**3 + 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**3*d**2*x**
5 + 420*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c**2*d**3*x**7 + 315*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*b**4*c*d**4*x**9 - 48*int((sqrt(c + d*x**2)...
```

3.6 $\int (A+Bx^2+Cx^4)\sqrt{ac+(bc+ad)x^2+bdx^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 563

$$\int (A+Bx^2+Cx^4)\sqrt{ac+(bc+ad)x^2+bdx^4}\,dx =$$
$$-\frac{(5bd(bc+ad)(acC-7Abd)+6abcd(4bcC-7bBd+4aCd)-2(bc+ad)^2(4bcC-7bBd+4aCd))\,x(}{105b^2d^3\sqrt{ac+(bc+ad)x^2+bdx^4}}$$
$$-\frac{x(5bd(acC-7Abd)+(bc+ad)(4bcC-7bBd+4aCd)+3bd(4bcC-7bBd+4aCd)x^2)\sqrt{ac+(bc+}}{105b^2d^2}$$
$$+\frac{Cx(ac+(bc+ad)x^2+bdx^4)^{3/2}}{7bd}$$
$$+\frac{c(5bd(bc+ad)(acC-7Abd)+6abcd(4bcC-7bBd+4aCd)-2(bc+ad)^2(4bcC-7bBd+4aCd))\,(}{105\sqrt{a}b^{5/2}d^3\sqrt{ac+(bc+ad)x^2+bdx^4}}$$
$$-\frac{\sqrt{a}c(10bd(acC-7Abd)-(bc+ad)(4bcC-7bBd+4aCd))\,(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{105b^{5/2}d^2\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output

```
-1/105*(5*b*d*(a*d+b*c)*(-7*A*b*d+C*a*c)+6*a*b*c*d*(-7*B*b*d+4*C*a*d+4*C*b
*c)-2*(a*d+b*c)^2*(-7*B*b*d+4*C*a*d+4*C*b*c))*x*(d*x^2+c)/b^2/d^3/(a*c+(a*
d+b*c)*x^2+b*d*x^4)^(1/2)-1/105*x*(5*b*d*(-7*A*b*d+C*a*c)+(a*d+b*c)*(-7*B*
b*d+4*C*a*d+4*C*b*c)+3*b*d*(-7*B*b*d+4*C*a*d+4*C*b*c)*x^2)*(a*c+(a*d+b*c)*
x^2+b*d*x^4)^(1/2)/b^2/d^2+1/7*C*x*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2)/b/d+1
/105*c*(5*b*d*(a*d+b*c)*(-7*A*b*d+C*a*c)+6*a*b*c*d*(-7*B*b*d+4*C*a*d+4*C*b
*c)-2*(a*d+b*c)^2*(-7*B*b*d+4*C*a*d+4*C*b*c))*(b*x^2+a)*(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1
/2))/a^(1/2)/b^(5/2)/d^3/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)-1/105*a^(1/2)*c
*(10*b*d*(-7*A*b*d+C*a*c)-(a*d+b*c)*(-7*B*b*d+4*C*a*d+4*C*b*c))*(b*x^2+a)*
(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),
(1-a*d/b/c)^(1/2))/b^(5/2)/d^2/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.99 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.69

$$\int (A+Bx^2+Cx^4)\sqrt{ac+(bc+ad)x^2+bdx^4}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(4a^2Cd^2-abd(2cC+7Bd+3Cdx^2)+b^2(4c^2C-cd(7B+3Cx^2)-d^2(35A}{}$$

input

```
Integrate[(A + B*x^2 + C*x^4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4],x]
```

output

```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a^2*C*d^2 - a*b*d*(2*c*C + 7*B
*d + 3*C*d*x^2) + b^2*(4*c^2*C - c*d*(7*B + 3*C*x^2) - d^2*(35*A + 21*B*x^
2 + 15*C*x^4)))) - I*c*(8*a^3*C*d^3 - a^2*b*d^2*(5*c*C + 14*B*d) + b^3*c*(
8*c^2*C - 14*B*c*d + 35*A*d^2) + a*b^2*d*(-5*c^2*C + 14*B*c*d + 35*A*d^2))
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] + I*c*(-(b*c) + a*d)*(4*a^2*C*d^2 + a*b*d*(c*C - 7*B*d) + b^
2*(-8*c^2*C + 14*B*c*d - 35*A*d^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*a^2*(b/a)^(5/2)*d^3
*Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 801, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {2207, 25, 1490, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2+Cx^4\right)\sqrt{x^2(ad+bc)+ac+bdx^4}\,dx$$

$$\Big\downarrow \ 2207$$

$$\frac{\int -\left(\left((4bcC+4adC-7bBd)x^2+acC-7Abd\right)\sqrt{bdx^4+(bc+ad)x^2+ac}\right)dx}{7bd}+\frac{Cx\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}{7bd}$$

$$\Big\downarrow \ 25$$

$$\frac{Cx\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}{7bd}-\frac{\int \left((4bcC+4adC-7bBd)x^2+acC-7Abd\right)\sqrt{bdx^4+(bc+ad)x^2+ac}dx}{7bd}$$

$$\Big\downarrow \ 1490$$

$$\frac{Cx\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}{7bd}-\frac{\frac{\int \frac{\left(-2(4bcC+4adC-7bBd)(bc+ad)^2+5bd(acC-7Abd)(bc+ad)+6abcd(4bcC+4adC-7bBd)\right)x^2+ac(10bd(acC-7Abd)-(bc+ad)(4bcC+4adC-7bBd))}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{15bd}+\frac{x\sqrt{\cdots}}{\cdots}}{7bd}$$

$$\Big\downarrow \ 1511$$

$$\frac{Cx\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}{7bd}-\frac{\frac{\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\left(12a^{3/2}\sqrt{b}\sqrt{c}Cd^{3/2}-8a^2Cd^2+3\sqrt{a}b^{3/2}\sqrt{c}\sqrt{d}(4cC-7Bd)-abd(11cC-14Bd)-\left(b^2\left(35Ad^2-14Bcd+8c^2C\right)\right)\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}d\cdots}{\sqrt{b}\sqrt{d}}\cdots}{15bd}\cdots}{\cdots}$$

$$\Big\downarrow \ 27$$

$$\frac{Cx\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}{7bd}-\frac{\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\left(12a^{3/2}\sqrt{b}\sqrt{c}Cd^{3/2}-8a^2Cd^2+3\sqrt{a}b^{3/2}\sqrt{c}\sqrt{d}(4cC-7Bd)-abd(11cC-14Bd)-\left(b^2\left(35Ad^2-14Bcd+8c^2C\right)\right)\right)\int\frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}d}{\sqrt{b}\sqrt{d}}}{15bd}$$

↓ 1416

$$\frac{Cx\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}{7bd}-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\left(12a^{3/2}\sqrt{b}\sqrt{c}Cd^{3/2}-8a^2Cd^2+3\sqrt{a}b^{3/2}\sqrt{c}\sqrt{d}(4cC-7Bd)-abd(11cC-14Bd)-\left(b^2\left(35Ad\right.\right.\right.}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

↓ 1509

$$\frac{Cx\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}{7bd}-$$

$$\frac{x\sqrt{bdx^4+(bc+ad)x^2+ac}\left(3bd(4bcC+4adC-7bBd)x^2+5bd(acC-7Abd)+(bc+ad)(4bcC+4adC-7bBd)\right)}{15bd}+\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)^2\left(-\left(\left(8Cc^2-\right.\right.\right.}{}$$

input
```
Int[(A + B*x^2 + C*x^4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4],x]
```

output
```
(C*x*(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(3/2))/(7*b*d) - ((x*(5*b*d*(a*c*C
- 7*A*b*d) + (b*c + a*d)*(4*b*c*C - 7*b*B*d + 4*a*C*d) + 3*b*d*(4*b*c*C -
7*b*B*d + 4*a*C*d)*x^2)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(15*b*d) +
(-(((5*b*d*(b*c + a*d)*(a*c*C - 7*A*b*d) + 6*a*b*c*d*(4*b*c*C - 7*b*B*d +
4*a*C*d) - 2*(b*c + a*d)^2*(4*b*c*C - 7*b*B*d + 4*a*C*d))*(-((x*Sqrt[a*c +
 (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)) + (a
^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c +
a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticE[2
*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*
Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*x^2
+ b*d*x^4])))/(Sqrt[b]*Sqrt[d])) + (a^(1/4)*c^(1/4)*(Sqrt[b]*Sqrt[c] + Sqr
t[a]*Sqrt[d])^2*(12*a^(3/2)*Sqrt[b]*Sqrt[c]*C*d^(3/2) - 8*a^2*C*d^2 - a*b*
d*(11*c*C - 14*B*d) + 3*Sqrt[a]*b^(3/2)*Sqrt[c]*Sqrt[d]*(4*c*C - 7*B*d) -
b^2*(8*c^2*C - 14*B*c*d + 35*A*d^2))*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^
2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[
d]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 -
 (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(3/4)*d^(3/4)*Sqr
t[a*c + (b*c + a*d)*x^2 + b*d*x^4]))/(15*b*d))/(7*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 9.64 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.06

method	result
elliptic	$\frac{C x^5 \sqrt{b d x^4 + x^2 d a + b c x^2 + a c}}{7} + \frac{\left(B b d + a C d + C c b - \frac{C(6 a d + 6 b c)}{7}\right) x^3 \sqrt{b d x^4 + x^2 d a + b c x^2 + a c}}{5 b d} + \frac{\left(A b d + B a d + B b c + \frac{2 C a c}{7} - \frac{\left(B b d\right.}{}\right.}{}$
risch	$\frac{x(15 C b^2 d^2 x^4 + 21 B b^2 d^2 x^2 + 3 C a b d^2 x^2 + 3 C b^2 c d x^2 + 35 A b^2 d^2 + 7 B b d^2 a + 7 B b^2 c d - 4 a^2 C d^2 + 2 C a b c d - 4 C b^2 c^2)(b x^2 + a)(d x^2 + c}{105 b^2 d^2 \sqrt{(b x^2 + a)(d x^2 + c)}}$
default	Expression too large to display

input
```
int((C*x^4+B*x^2+A)*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/7*C*x^5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(B*b*d+a*C*d+C*c*b-1/7*C
*(6*a*d+6*b*c))/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(A*b*d+B*a
*d+B*b*c+2/7*C*a*c-1/5*(B*b*d+a*C*d+C*c*b-1/7*C*(6*a*d+6*b*c))/b/d*(4*a*d+
4*b*c))/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(A*a*c-1/3*(A*b*d+B*a*d+
B*b*c+2/7*C*a*c-1/5*(B*b*d+a*C*d+C*c*b-1/7*C*(6*a*d+6*b*c))/b/d*(4*a*d+4*b
*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)-(A*a*d+A*b*c+B*a*c-3/5*(B*b*d+a*C*d+C*c*b-1/7*C*(6*a*d+6*b*c))/b/d*a*c-1
/3*(A*b*d+B*a*d+B*b*c+2/7*C*a*c-1/5*(B*b*d+a*C*d+C*c*b-1/7*C*(6*a*d+6*b*c)
)/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 517, normalized size of antiderivative = 0.92

$$\int \left(A+Bx^2+Cx^4\right)\sqrt{ac+(bc+ad)x^2+bdx^4}\,dx =$$

$$-\frac{(8\,Cb^3c^4-(5\,Cab^2+14\,Bb^3)c^3d-(5\,Ca^2b-14\,Bab^2-35\,Ab^3)c^2d^2+(8\,Ca^3-14\,Ba^2b+35\,Aab^2)c}{}$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="
fricas")
```

output
```
-1/105*((8*C*b^3*c^4 - (5*C*a*b^2 + 14*B*b^3)*c^3*d - (5*C*a^2*b - 14*B*a*
b^2 - 35*A*b^3)*c^2*d^2 + (8*C*a^3 - 14*B*a^2*b + 35*A*a*b^2)*c*d^3)*sqrt(
b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (8*C*b^3*c
^4 - (5*C*a*b^2 + 14*B*b^3)*c^3*d - (5*C*a^2*b - 2*(7*B + 2*C)*a*b^2 - 35*
A*b^3)*c^2*d^2 + (8*C*a^3 - 2*(7*B + C)*a^2*b + 7*(5*A - B)*a*b^2)*c*d^3 +
 (4*C*a^3 - 7*B*a^2*b + 70*A*a*b^2)*d^4)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f
(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (15*C*b^3*d^4*x^6 + 8*C*b^3*c^3*d - (5
*C*a*b^2 + 14*B*b^3)*c^2*d^2 - (5*C*a^2*b - 14*B*a*b^2 - 35*A*b^3)*c*d^3 +
 (8*C*a^3 - 14*B*a^2*b + 35*A*a*b^2)*d^4 + 3*(C*b^3*c*d^3 + (C*a*b^2 + 7*B
*b^3)*d^4)*x^4 - (4*C*b^3*c^2*d^2 - (2*C*a*b^2 + 7*B*b^3)*c*d^3 + (4*C*a^2
*b - 7*B*a*b^2 - 35*A*b^3)*d^4)*x^2)*sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c)
)/(b^3*d^4*x)
```

Sympy [F]

$$\int \left(A+Bx^2+Cx^4\right)\sqrt{ac+(bc+ad)x^2+bdx^4}\,dx$$
$$=\int \sqrt{(a+bx^2)\,(c+dx^2)}\left(A+Bx^2+Cx^4\right)\,dx$$

input
```
integrate((C*x**4+B*x**2+A)*(a*c+(a*d+b*c)*x**2+b*d*x**4)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x**2)*(c + d*x**2))*(A + B*x**2 + C*x**4), x)
```

Maxima [F]

$$\int (A+Bx^2+Cx^4)\,\sqrt{ac+(bc+ad)x^2+bdx^4}\,dx$$
$$=\int \sqrt{bdx^4+(bc+ad)x^2+ac}(Cx^4+Bx^2+A)\,dx$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c)*(C*x^4 + B*x^2 + A), x)
```

Giac [F]

$$\int (A+Bx^2+Cx^4)\,\sqrt{ac+(bc+ad)x^2+bdx^4}\,dx$$
$$=\int \sqrt{bdx^4+(bc+ad)x^2+ac}(Cx^4+Bx^2+A)\,dx$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c)*(C*x^4 + B*x^2 + A), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2+Cx^4)\,\sqrt{ac+(bc+ad)x^2+bdx^4}\,dx$$
$$=\int \sqrt{b\,d\,x^4+(a\,d+b\,c)\,x^2+a\,c}\,(C\,x^4+B\,x^2+A)\,dx$$

input
```
int((a*c + x^2*(a*d + b*c) + b*d*x^4)^(1/2)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a*c + x^2*(a*d + b*c) + b*d*x^4)^(1/2)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (A + Bx^2 + Cx^4) \sqrt{ac + (bc + ad)x^2 + bdx^4}\, dx = \text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c*d**2*x + 42*sqrt(c + d*x**2
)*sqrt(a + b*x**2)*a*b**2*d**2*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b
*c**2*d*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d**2*x**3 + 7*sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*b**3*c*d*x + 21*sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*b**3*d**2*x**3 - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**3*x + 3*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c**2*d*x**3 + 15*sqrt(c + d*x**2)*sq
rt(a + b*x**2)*b**2*c*d**2*x**5 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*c*d**3 + 21*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4
),x)*a**2*b**2*d**3 - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
+ a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b*c**2*d**2 + 49*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*
b**3*c*d**2 - 5*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x*
*2 + b*c*x**2 + b*d*x**4),x)*a*b**2*c**3*d - 14*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**4*c**2*d +
 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2
 + b*d*x**4),x)*b**3*c**4 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c
 + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**3*c**2*d**2 + 63*int((sqrt(c + d*
x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a**2*b**
2*c*d**2 - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + ...
```

3.7 $\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 324

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx &= -\frac{(2bcC-3bBd+2aCd)x(c+dx^2)}{3bd^2\sqrt{ac+(bc+ad)x^2+bdx^4}}+\frac{Cx\sqrt{ac+(bc+ad)x^2+bdx^4}}{3bd}\\ &\quad+\frac{c(2bcC-3bBd+2aCd)\,(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{3/2}d^2\sqrt{ac+(bc+ad)x^2+bdx^4}}\\ &\quad-\frac{(acC-3Abd)\,(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{3/2}d\sqrt{ac+(bc+ad)x^2+bdx^4}}\end{aligned}$$

output
```
-1/3*(-3*B*b*d+2*C*a*d+2*C*b*c)*x*(d*x^2+c)/b/d^2/(a*c+(a*d+b*c)*x^2+b*d*x
^4)^(1/2)+1/3*C*x*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)/b/d+1/3*c*(-3*B*b*d+2*
C*a*d+2*C*b*c)*(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*EllipticE(b^(1/2)
*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/d^2/(a*c+(
a*d+b*c)*x^2+b*d*x^4)^(1/2)-1/3*(-3*A*b*d+C*a*c)*(b*x^2+a)*(a*(d*x^2+c)/c/
(b*x^2+a))^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/
2))/a^(1/2)/b^(3/2)/d/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}Cdx(a+bx^2)\,(c+dx^2)+ic(2bcC-3bBd+2aCd)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-i(}{3b\sqrt{\frac{b}{a}}d^2\sqrt{(a+bx^2)\,(c+}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4],x]
```

output
```
(Sqrt[b/a]*C*d*x*(a + b*x^2)*(c + d*x^2) + I*c*(2*b*c*C - 3*b*B*d + 2*a*C*
d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)] - I*(a*c*C*d + b*(2*c^2*C - 3*B*c*d + 3*A*d^2))*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(3*b*Sqrt[b/a]*d^2*Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.72, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2207, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{x^2(ad+bc)+ac+bdx^4}}\,dx$$
$$\Big\downarrow \text{2207}$$
$$\frac{\int -\frac{(2bcC+2adC-3bBd)x^2+acC-3Abd}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{3bd}+\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}$$
$$\Big\downarrow \text{25}$$

$$\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\int \frac{(2bcC+2adC-3bBd)x^2+acC-3Abd}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{3bd}$$

$\downarrow$ 1511

$$\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt{a}\sqrt{c}\left(\frac{\sqrt{b}\sqrt{d}(acC-3Abd)}{\sqrt{a}\sqrt{c}}+2aCd-3bBd+2bcC\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}(2aCd-3bBd+2bcC)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}}{3bd}$$

$\downarrow$ 27

$$\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt{a}\sqrt{c}\left(\frac{\sqrt{b}\sqrt{d}(acC-3Abd)}{\sqrt{a}\sqrt{c}}+2aCd-3bBd+2bcC\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{(2aCd-3bBd+2bcC)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}}{3bd}$$

$\downarrow$ 1416

$$\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{d}(acC-3Abd)}{\sqrt{a}\sqrt{c}}+2aCd-3bBd+2bcC\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}}{3bd}$$

$\downarrow$ 1509

$$\frac{Cx\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{d}(acC-3Abd)}{\sqrt{a}\sqrt{c}}+2aCd-3bBd+2bcC\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}}{}$$

input
```
Int[(A + B*x^2 + C*x^4)/Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4],x]
```

output
```
(C*x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(3*b*d) - (-(((2*b*c*C - 3*b*B
*d + 2*a*C*d)*(-((x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c
] + Sqrt[b]*Sqrt[d]*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sq
rt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[
b]*Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4)
)], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/
4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])))/(Sqrt[b]*Sqrt[d])) + (a^(1/4)*
c^(1/4)*(2*b*c*C - 3*b*B*d + 2*a*C*d + (Sqrt[b]*Sqrt[d]*(a*c*C - 3*A*b*d))
/(Sqrt[a]*Sqrt[c]))*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b
*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqr
t[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(3/4)*d^(3/4)*Sqrt[a*c + (b*c + a*
d)*x^2 + b*d*x^4]))/(3*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{Cx\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}}{3bd}+\frac{\left(A-\frac{Cac}{3bd}\right)\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}}-\frac{\left(B-\frac{C(2ad+2bc)}{3bd}\right)c\sqrt{1+\frac{b\,x^{2}}{a}}}{}$
risch	$\frac{Cx\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}{3bd\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}}+\frac{-\frac{(3Bbd-2aCd-2Ccb)c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}\,d}}{}$
default	$\frac{A\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}}-\frac{Bc\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}\,d}$

input
```
int((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/3*C/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(A-1/3*C/b/d*a*c)/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(B-1/3*C/b/d*(2*a*d+
2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx = \frac{(2\,Cbc^3+(2\,Ca-3\,Bb)c^2d)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (2\,Cbc^3+Cacd^2-3\,Abd^3+(2\,Ca-3$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="
fricas")
```

output
```
1/3*((2*C*b*c^3 + (2*C*a - 3*B*b)*c^2*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e
(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (2*C*b*c^3 + C*a*c*d^2 - 3*A*b*d^3 + (
2*C*a - 3*B*b)*c^2*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/
x), a*d/(b*c)) + (C*b*c*d^2*x^2 - 2*C*b*c^2*d - (2*C*a - 3*B*b)*c*d^2)*sqr
t(b*d*x^4 + (b*c + a*d)*x^2 + a*c))/(b^2*c*d^3*x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(a*c+(a*d+b*c)*x**2+b*d*x**4)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/sqrt((a + b*x**2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{bdx^4+(bc+ad)x^2+ac}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{bdx^4+(bc+ad)x^2+ac}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x, algorithm="
giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{b\,d\,x^4+(a\,d+b\,c)\,x^2+a\,c}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc+ad)x^2+bdx^4}}\,dx$$
$$= \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,cx-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)acd+3\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)b^2d-2\left(\int \frac{\sqrt{d\,x^2+c}}{bd\,x^4+ad\,x}\right.}{3bd}$$

input
```
int((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*c*x - 2*int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*c*d + 3*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4
),x)*b**2*d - 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x*
*2 + b*c*x**2 + b*d*x**4),x)*b*c**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x
**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*d - int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*c**2)/(3
*b*d)
```

3.8 $\int \frac{A+Bx^2+Cx^4}{\left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 325

$$\int \frac{A+Bx^2+Cx^4}{\left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}}\,dx = -\frac{\left(c^2C-Bcd+Ad^2\right)x}{cd(bc-ad)\sqrt{ac+(bc+ad)x^2+bdx^4}} + \frac{\left(Ab^2cd+a^2cCd+ab\left(c^2C-2Bcd+Ad^2\right)\right)\left(a+bx^2\right)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{a^{3/2}\sqrt{b}d(bc-ad)^2\sqrt{ac+(bc+ad)x^2+bdx^4}} + \frac{(bBc-2acC-2Abd+aBd)\left(a+bx^2\right)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}(bc-ad)^2\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output

```
-(A*d^2-B*c*d+C*c^2)*x/c/d/(-a*d+b*c)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)+(A
*b^2*c*d+a^2*c*C*d+a*b*(A*d^2-2*B*c*d+C*c^2))*(b*x^2+a)*(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1
/2))/a^(3/2)/b^(1/2)/d/(-a*d+b*c)^2/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)+(-2*
A*b*d+B*a*d+B*b*c-2*C*a*c)*(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*Inver
seJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/(-
a*d+b*c)^2/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.20 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}dx(A(a^2d^2+abd^2x^2+b^2c(c+dx^2))+ac(bcCx^2-bB(c+\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(3/2),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*d*x*(A*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c + d*x^2)) +
 a*c*(b*c*C*x^2 - b*B*(c + 2*d*x^2) + a*(2*c*C - B*d + C*d*x^2))) + I*c*(A
*b^2*c*d + a^2*c*C*d + a*b*(c^2*C - 2*B*c*d + A*d^2))*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(
-(b*c) + a*d)*(a*c*C + A*b*d - a*B*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c*d*(b*c - a*d)^2*
Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 674 vs. 2(325) = 650.

Time = 0.86 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2206, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(x^2(ad+bc)+ac+bdx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 2206}$$

$$-\frac{\int -\frac{ac(bBc-2aCc-2Abd+aBd)-\left(cCda^2+b\left(Cc^2-2Bdc+Ad^2\right)a+Ab^2cd\right)x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{ac(bc-ad)^2}-\frac{x\left(-\left(x^2\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\right)-A\left(a^2d^2+b^2c^2\right)+ac(aBd-2acC+bBc)\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{ac(bBc-2aCc-2Abd+aBd)-\left(cCda^2+b\left(Cc^2-2Bdc+Ad^2\right)a+Ab^2cd\right)x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{ac(bc-ad)^2}-\frac{x\left(-\left(x^2\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\right)-A\left(a^2d^2+b^2c^2\right)+ac(aBd-2acC+bBc)\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

$$\Big\downarrow \; 1511$$

$$\frac{\frac{\sqrt{a}\sqrt{c}\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\left(-\sqrt{a}\sqrt{b}B\sqrt{c}\sqrt{d}+acC+Abd\right)\int \frac{}{\sqrt{bd}\cdots}}{\sqrt{b}\sqrt{d}}}{ac(bc-ad)^2}$$
$$-\frac{x\left(-\left(x^2\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\right)-A\left(a^2d^2+b^2c^2\right)+ac(aBd-2acC+bBc)\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\left(-\sqrt{a}\sqrt{b}B\sqrt{c}\sqrt{d}+acC+Abd\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x\cdots}}}{\sqrt{b}\sqrt{d}}}{ac(bc-ad)^2}$$
$$-\frac{x\left(-\left(x^2\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\right)-A\left(a^2d^2+b^2c^2\right)+ac(aBd-2acC+bBc)\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

$$\Big\downarrow \; 1416$$

$$\frac{\frac{\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\left(-\sqrt{\cdots}\right.}{2b^{3/4}d^{3/4}\sqrt{\cdots}}}{ac(bc-ad)^2}$$
$$-\frac{x\left(-\left(x^2\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\right)-A\left(a^2d^2+b^2c^2\right)+ac(aBd-2acC+bBc)\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

$$\Big\downarrow \; 1509$$

$$\frac{\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right)|\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{\sqrt[4]{b}\sqrt[4]{d}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{x\sqrt{x^2(a}{\sqrt{a}}\right.}{\sqrt{b}\sqrt{d}}$$

$$\frac{x\left(-\left(x^2\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\right)-A\left(a^2d^2+b^2c^2\right)+ac(aBd-2acC+bBc)\right)}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(3/2),x]
```

output
```
-((x*(a*c*(b*B*c - 2*a*c*C + a*B*d) - A*(b^2*c^2 + a^2*d^2) - (A*b^2*c*d +
 a^2*c*C*d + a*b*(c^2*C - 2*B*c*d + A*d^2))*x^2))/(a*c*(b*c - a*d)^2*Sqrt[
a*c + (b*c + a*d)*x^2 + b*d*x^4])) + (((A*b^2*c*d + a^2*c*C*d + a*b*(c^2*C
 - 2*B*c*d + A*d^2))*(-((x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]
*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqr
t[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c]
+ Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*
c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4
)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])))/(Sqrt[b]*Sqrt[d]) - (a^
(1/4)*c^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqrt[d])^2*(a*c*C - Sqrt[a]*Sqrt[
b]*B*Sqrt[c]*Sqrt[d] + A*b*d)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt
[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c +
 a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(3/4)*d^(3/4)*Sqrt[a*c +
 (b*c + a*d)*x^2 + b*d*x^4]))/(a*c*(b*c - a*d)^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.76

method	result
elliptic	$-\frac{2bd\left(-\frac{\left(Aab\,d^2+A\,b^2cd-2Bacdb+C\,a^2cd+ab\,c^2C\right)x^3}{2bdac\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(A\,a^2d^2+A\,b^2c^2-B\,a^2cd-Bab\,c^2+2C\,a^2c^2\right)x}{2bdac\left(a^2d^2-2abcd+b^2c^2\right)}\right)}{\sqrt{\left(x^4+\frac{(ad+bc)x^2}{bd}+\frac{ac}{bd}\right)bd}}+\frac{\left(\frac{C}{bd}+\frac{Abd-Cac}{bdac}-\frac{A\,a^2d^2+A}{}\right.}{}$
default	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
-2*b*d*(-1/2/b/d*(A*a*b*d^2+A*b^2*c*d-2*B*a*b*c*d+C*a^2*c*d+C*a*b*c^2)/a/c
/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x^3-1/2/b/d*(A*a^2*d^2+A*b^2*c^2-B*a^2*c*d-B*
a*b*c^2+2*C*a^2*c^2)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x)/((x^4+(a*d+b*c)/b/
d*x^2+a*c/b/d)*b*d)^(1/2)+(C/b/d+1/b/d*(A*b*d-C*a*c)/a/c-(A*a^2*d^2+A*b^2*
c^2-B*a^2*c*d-B*a*b*c^2+2*C*a^2*c^2)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2))/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+(A*a*b*d^2+A*b^2
*c*d-2*B*a*b*c*d+C*a^2*c*d+C*a*b*c^2)/a/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(-b/a)
^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. 2(317) = 634.

Time = 0.09 (sec) , antiderivative size = 807, normalized size of antiderivative = 2.48

$$\int \frac{A+Bx^2+Cx^4}{\left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="
fricas")
```

output
```
-((C*a^2*b^2*c^3 + A*a^2*b^2*c*d^2 + (C*a*b^3*c^2*d + A*a*b^3*d^3 + (C*a^2
*b^2 - 2*B*a*b^3 + A*b^4)*c*d^2)*x^4 + (C*a^3*b - 2*B*a^2*b^2 + A*a*b^3)*c
^2*d + (C*a*b^3*c^3 + A*a^2*b^2*d^3 + (2*C*a^2*b^2 - 2*B*a*b^3 + A*b^4)*c^
2*d + (C*a^3*b - 2*B*a^2*b^2 + 2*A*a*b^3)*c*d^2)*x^2)*sqrt(a*c)*sqrt(-b/a)
*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (C*a^2*b^2*c^3 + (C*a*b^3*c
^2*d + (2*C*a^3*b - (B - C)*a^2*b^2 - 2*B*a*b^3 + A*b^4)*c*d^2 - (B*a^3*b
- 2*A*a^2*b^2 - A*a*b^3)*d^3)*x^4 + (2*C*a^4 - (B - C)*a^3*b - 2*B*a^2*b^2
 + A*a*b^3)*c^2*d - (B*a^4 - 2*A*a^3*b - A*a^2*b^2)*c*d^2 + (C*a*b^3*c^3 +
 (2*C*a^3*b - (B - 2*C)*a^2*b^2 - 2*B*a*b^3 + A*b^4)*c^2*d + (2*C*a^4 - (2
*B - C)*a^3*b + 2*(A - B)*a^2*b^2 + 2*A*a*b^3)*c*d^2 - (B*a^4 - 2*A*a^3*b
- A*a^2*b^2)*d^3)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)
), a*d/(b*c)) - sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c)*((C*a^2*b^2*c^2*d +
A*a^2*b^2*d^3 + (C*a^3*b - 2*B*a^2*b^2 + A*a*b^3)*c*d^2)*x^3 - (B*a^3*b*c*
d^2 - A*a^3*b*d^3 - (2*C*a^3*b - B*a^2*b^2 + A*a*b^3)*c^2*d)*x))/(a^3*b^3*
c^4*d - 2*a^4*b^2*c^3*d^2 + a^5*b*c^2*d^3 + (a^2*b^4*c^3*d^2 - 2*a^3*b^3*c
^2*d^3 + a^4*b^2*c*d^4)*x^4 + (a^2*b^4*c^4*d - a^3*b^3*c^3*d^2 - a^4*b^2*c
^2*d^3 + a^5*b*c*d^4)*x^2)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx=\int \frac{A+Bx^2+Cx^4}{\left((a+bx^2)\,(c+dx^2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(a*c+(a*d+b*c)*x**2+b*d*x**4)**(3/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/((a + b*x**2)*(c + d*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bdx^4+(bc+ad)x^2+ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="
maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bdx^4+(bc+ad)x^2+ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x, algorithm="
giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{{\left(b\,d\,x^4+\left(a\,d+b\,c\right)\,x^2+a\,c\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(3/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc+ad)x^2+bdx^4)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(3/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*c*x + int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*x**2)/(a**2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*
x**2 + 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d*x**6
 + b**2*d**2*x**8),x)*a*b**2*c*d + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**2)/(a**2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 + 4*
a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d*x**6 + b**2*d
**2*x**8),x)*a*b**2*d**2*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**
2)/(a**2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 + 4*a*b
*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d*x**6 + b**2*d**2
*x**8),x)*b**3*c*d*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*
*2*c**2 + 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 + 4*a*b*c*d*x
**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d*x**6 + b**2*d**2*x**8)
,x)*b**3*d**2*x**4 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c**2 +
2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 + 4*a*b*c*d*x**4 + 2*a*
b*d**2*x**6 + b**2*c**2*x**4 + 2*b**2*c*d*x**6 + b**2*d**2*x**8),x)*a**2*b
*c*d + int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c**2 + 2*a**2*c*d*x**
2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 + 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 +
b**2*c**2*x**4 + 2*b**2*c*d*x**6 + b**2*d**2*x**8),x)*a**2*b*d**2*x**2 + i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c**2 + 2*a**2*c*d*x**2 + a**2
*d**2*x**4 + 2*a*b*c**2*x**2 + 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*...
```

3.9 $\int \frac{A+Bx^2+Cx^4}{\left(ac+(bc+ad)x^2+bdx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 593

$$\int \frac{A+Bx^2+Cx^4}{\left(ac+(bc+ad)x^2+bdx^4\right)^{5/2}}\,dx =$$

$$-\frac{x\left(ac(bBc-2acC+aBd)-A(b^2c^2+a^2d^2)-\left(Ab^2cd+a^2cCd+ab(c^2C-2Bcd+Ad^2)\right)x^2\right)}{3ac(bc-ad)^2\left(ac+(bc+ad)x^2+bdx^4\right)^{3/2}}$$

$$+\frac{\left(Ab^2c^2d+a^2d(5c^2C-Bcd-2Ad^2)+abc(3c^2C-7Bcd+9Ad^2)\right)x}{3ac^2(bc-ad)^3\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

$$+\frac{\sqrt{b}\left(2A(b^3c^3-5ab^2c^2d-5a^2bcd^2+a^3d^3)+ac(b^2Bc^2-2abc(4cC-7Bd)-a^2d(8cC-Bd))\right)(a+bx^2)}{3a^{5/2}c(bc-ad)^4\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

$$-\frac{\left(Ab^3c^2d-3a^3cCd^2-a^2bd(10c^2C-8Bcd-Ad^2)-ab^2c(3c^2C-8Bcd+18Ad^2)\right)(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\mathrm{E}}{3a^{3/2}\sqrt{b}c(bc-ad)^4\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output
```
-1/3*x*(a*c*(B*a*d+B*b*c-2*C*a*c)-A*(a^2*d^2+b^2*c^2)-(A*b^2*c*d+a^2*c*C*d
+a*b*(A*d^2-2*B*c*d+C*c^2))*x^2)/a/c/(-a*d+b*c)^2/(a*c+(a*d+b*c)*x^2+b*d*x
^4)^(3/2)+1/3*(A*b^2*c^2*d+a^2*d*(-2*A*d^2-B*c*d+5*C*c^2)+a*b*c*(9*A*d^2-7
*B*c*d+3*C*c^2))*x/a/c^2/(-a*d+b*c)^3/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)+1/
3*b^(1/2)*(2*A*(a^3*d^3-5*a^2*b*c*d^2-5*a*b^2*c^2*d+b^3*c^3)+a*c*(b^2*B*c^
2-2*a*b*c*(-7*B*d+4*C*c)-a^2*d*(-B*d+8*C*c)))*(b*x^2+a)*(a*(d*x^2+c)/c/(b*
x^2+a))^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1
/2))/a^(5/2)/c/(-a*d+b*c)^4/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)-1/3*(A*b^3*c
^2*d-3*a^3*c*C*d^2-a^2*b*d*(-A*d^2-8*B*c*d+10*C*c^2)-a*b^2*c*(18*A*d^2-8*B
*c*d+3*C*c^2))*(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*InverseJacobiAM(a
rctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(3/2)/b^(1/2)/c/(-a*d+b*c)^4
/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.30 (sec) , antiderivative size = 517, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx = \frac{i\left(i\sqrt{\frac{b}{a}}x\left(a^2cd(bc-ad)\left(c^2C-Bcd+Ad^2\right)\left(a+bx^2\right)^2+a^2d(-ad(-\right.\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(5/2),x]
```

output
```
((I/3)*(I*Sqrt[b/a]*x*(a^2*c*d*(b*c - a*d)*(c^2*C - B*c*d + A*d^2)*(a + b*
x^2)^2 + a^2*d*(-(a*d*(-4*c^2*C + B*c*d + 2*A*d^2)) + b*c*(4*c^2*C - 7*B*c
*d + 10*A*d^2))*(a + b*x^2)^2*(c + d*x^2) + a*b*c^2*(A*b^2 + a*(-(b*B) + a
*C))*(-(b*c) + a*d)*(c + d*x^2)^2 - b*c^2*(2*A*b^2*(b*c - 5*a*d) + a*(b^2*
B*c - 4*a*b*c*C + 7*a*b*B*d - 4*a^2*C*d))*(a + b*x^2)*(c + d*x^2)^2) - c*(
a + b*x^2)*Sqrt[1 + (b*x^2)/a]*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*(-(b*(2*A*(
b^3*c^3 - 5*a*b^2*c^2*d - 5*a^2*b*c*d^2 + a^3*d^3) + a*c*(b^2*B*c^2 + a^2*
d*(-8*c*C + B*d) + 2*a*b*c*(-4*c*C + 7*B*d)))*EllipticE[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)]) + (b*c - a*d)*(a*c*(b^2*B*c - 5*a*b*c*C + 7*a*b*B*d -
3*a^2*C*d) + A*b*(2*b^2*c^2 - 9*a*b*c*d - a^2*d^2))*EllipticF[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)])))/(a^2*Sqrt[b/a]*c^2*(b*c - a*d)^4*((a + b*x^2)*
(c + d*x^2))^(3/2))
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 1094, normalized size of antiderivative = 1.84, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2206, 25, 1492, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(x^2(ad+bc)+ac+bdx^4)^{5/2}}\,dx$$

$$\Big\downarrow \; 2206$$

$$-\frac{\int -\frac{3(cCda^2+b(Cc^2-2Bdc+Ad^2)a+Ab^2cd)x^2+ac(bBc-2aCc+aBd)+2A(b^2c^2-3abdc+a^2d^2)}{(bdx^4+(bc+ad)x^2+ac)^{3/2}}dx}{3ac(bc-ad)^2}-\frac{x\left(-\left(x^2\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\right)-A\left(a^2d^2+b^2c^2\right)+ac(aBd-2acC+bBc)\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3(cCda^2+b(Cc^2-2Bdc+Ad^2)a+Ab^2cd)x^2+ac(bBc-2aCc+aBd)+2A(b^2c^2-3abdc+a^2d^2)}{(bdx^4+(bc+ad)x^2+ac)^{3/2}}dx}{3ac(bc-ad)^2}-\frac{x\left(-\left(x^2\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\right)-A\left(a^2d^2+b^2c^2\right)+ac(aBd-2acC+bBc)\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}$$

$$\Big\downarrow \; 1492$$

$$\frac{x((a^2d^2+b^2c^2)(2A(a^2d^2-3abcd+b^2c^2)+ac(aBd-2acC+bBc))-3ac(ad+bc)(a^2cCd+ab(Ad^2-2Bcd+c^2C)+Ab^2cd)+bdx^2(ac(a^2(-d)(8cC-B}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

$$\frac{x\left(-\left(x^2\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\right)-A\left(a^2d^2+b^2c^2\right)+ac(aBd-2acC+bBc)\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}$$

$$\Big\downarrow \; 1511$$

$$\frac{x((a^2d^2+b^2c^2)(2A(a^2d^2-3abcd+b^2c^2)+ac(aBd-2acC+bBc))-3ac(ad+bc)(a^2cCd+ab(Ad^2-2Bcd+c^2C)+Ab^2cd)+bdx^2(ac(a^2(-d)(8cC-B}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

$$\frac{x\left(-\left(x^2\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\right)-A\left(a^2d^2+b^2c^2\right)+ac(aBd-2acC+bBc)\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x\left(\left(a^2d^2+b^2c^2\right)\left(2A\left(a^2d^2-3abcd+b^2c^2\right)+ac(aBd-2acC+bBc)\right)-3ac(ad+bc)\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)+bdx^2\left(ac\left(a^2(-d)(8cC-B\right.\right.\right.}{ac(bc-ad)^2\sqrt{x^2(ad+bc)+ac+bdx^4}}}{}$$

$$\frac{x\left(-\left(x^2\left(a^2cCd+ab\left(Ad^2-2Bcd+c^2C\right)+Ab^2cd\right)\right)-A\left(a^2d^2+b^2c^2\right)+ac(aBd-2acC+bBc)\right)}{3ac(bc-ad)^2\left(x^2(ad+bc)+ac+bdx^4\right)^{3/2}}$$

$$\Big\downarrow \; 1416$$

$$\frac{\frac{x\left(bd\left(2A\left(b^3c^3-5ab^2dc^2-5a^2bd^2c+a^3d^3\right)+ac\left(-d(8cC-Bd)a^2-2bc(4cC-7Bd)a+b^2Bc^2\right)\right)x^2+\left(b^2c^2+a^2d^2\right)\left(ac(bBc-2aCc+aBd)+2A\left(b^2c^2-3\right.\right.\right.}{ac(bc-ad)^2\sqrt{bdx^4+(bc+ad)x^2+ac}}}{}$$

$$\frac{x\left(-\left(\left(cCda^2+b\left(Cc^2-2Bdc+Ad^2\right)a+Ab^2cd\right)x^2\right)+ac(bBc-2aCc+aBd)-A\left(b^2c^2+a^2d^2\right)\right)}{3ac(bc-ad)^2\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}$$

$$\Big\downarrow \; 1509$$

$$\frac{\frac{x\left(bd\left(2A\left(b^3c^3-5ab^2dc^2-5a^2bd^2c+a^3d^3\right)+ac\left(-d(8cC-Bd)a^2-2bc(4cC-7Bd)a+b^2Bc^2\right)\right)x^2+\left(b^2c^2+a^2d^2\right)\left(ac(bBc-2aCc+aBd)+2A\left(b^2c^2-3\right.\right.\right.}{ac(bc-ad)^2\sqrt{bdx^4+(bc+ad)x^2+ac}}}{}$$

$$\frac{x\left(-\left(\left(cCda^2+b\left(Cc^2-2Bdc+Ad^2\right)a+Ab^2cd\right)x^2\right)+ac(bBc-2aCc+aBd)-A\left(b^2c^2+a^2d^2\right)\right)}{3ac(bc-ad)^2\left(bdx^4+(bc+ad)x^2+ac\right)^{3/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a*c + (b*c + a*d)*x^2 + b*d*x^4)^(5/2),x]
```

output
```
-1/3*(x*(a*c*(b*B*c - 2*a*c*C + a*B*d) - A*(b^2*c^2 + a^2*d^2) - (A*b^2*c*
d + a^2*c*C*d + a*b*(c^2*C - 2*B*c*d + A*d^2))*x^2))/(a*c*(b*c - a*d)^2*(a
*c + (b*c + a*d)*x^2 + b*d*x^4)^(3/2)) + ((x*((b^2*c^2 + a^2*d^2)*(a*c*(b*
B*c - 2*a*c*C + a*B*d) + 2*A*(b^2*c^2 - 3*a*b*c*d + a^2*d^2)) - 3*a*c*(b*c
 + a*d)*(A*b^2*c*d + a^2*c*C*d + a*b*(c^2*C - 2*B*c*d + A*d^2)) + b*d*(2*A
*(b^3*c^3 - 5*a*b^2*c^2*d - 5*a^2*b*c*d^2 + a^3*d^3) + a*c*(b^2*B*c^2 - 2*
a*b*c*(4*c*C - 7*B*d) - a^2*d*(8*c*C - B*d)))*x^2))/(a*c*(b*c - a*d)^2*Sqr
t[a*c + (b*c + a*d)*x^2 + b*d*x^4]) - (-(Sqrt[b]*Sqrt[d]*(2*A*(b^3*c^3 - 5
*a*b^2*c^2*d - 5*a^2*b*c*d^2 + a^3*d^3) + a*c*(b^2*B*c^2 - 2*a*b*c*(4*c*C
- 7*B*d) - a^2*d*(8*c*C - B*d)))*(-((x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^
4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*S
qrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt
[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*d^(1/4)*
x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))
/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4]))) + (a^(1/4)*
c^(1/4)*(Sqrt[a]*Sqrt[c]*(A*b^3*c^2*d - 3*a^3*c*C*d^2 - a^2*b*d*(10*c^2*C
- 8*B*c*d - A*d^2) - a*b^2*c*(3*c^2*C - 8*B*c*d + 18*A*d^2)) + Sqrt[b]*Sqr
t[d]*(2*A*(b^3*c^3 - 5*a*b^2*c^2*d - 5*a^2*b*c*d^2 + a^3*d^3) + a*c*(b^2*B
*c^2 - 2*a*b*c*(4*c*C - 7*B*d) - a^2*d*(8*c*C - B*d))))*(Sqrt[a]*Sqrt[c] +
 Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*S...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 3.67 (sec) , antiderivative size = 1125, normalized size of antiderivative = 1.90

method	result	size
elliptic	Expression too large to display	1125
default	Expression too large to display	2051

input
```
int((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
(1/3/b^2/d^2*(A*a*b*d^2+A*b^2*c*d-2*B*a*b*c*d+C*a^2*c*d+C*a*b*c^2)/a/c/(a^
2*d^2-2*a*b*c*d+b^2*c^2)*x^3+1/3/b^2/d^2*(A*a^2*d^2+A*b^2*c^2-B*a^2*c*d-B*
a*b*c^2+2*C*a^2*c^2)/a/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)/(x^4+(a*d+b*c)/b/d*x^2+a*c/b/d)^2-2*b*d*(-1/6*(2*A*a^3*d
^3-10*A*a^2*b*c*d^2-10*A*a*b^2*c^2*d+2*A*b^3*c^3+B*a^3*c*d^2+14*B*a^2*b*c^
2*d+B*a*b^2*c^3-8*C*a^3*c^2*d-8*C*a^2*b*c^3)/a^2/c^2/(a^2*d^2-2*a*b*c*d+b^
2*c^2)^2*x^3-1/6*(2*A*a^4*d^4-9*A*a^3*b*c*d^3-2*A*a^2*b^2*c^2*d^2-9*A*a*b^
3*c^3*d+2*A*b^4*c^4+B*a^4*c*d^3+7*B*a^3*b*c^2*d^2+7*B*a^2*b^2*c^3*d+B*a*b^
3*c^4-5*C*a^4*c^2*d^2-6*C*a^3*b*c^3*d-5*C*a^2*b^2*c^4)/a^2/c^2/(a^2*d^2-2*
a*b*c*d+b^2*c^2)^2/b/d*x)/((x^4+(a*d+b*c)/b/d*x^2+a*c/b/d)*b*d)^(1/2)+(1/3
/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(2*A*a^2*d^2-6*A*a*b*c*d+2*A*b^2*c^2+B*a^2*c*
d+B*a*b*c^2-2*C*a^2*c^2)/a^2/c^2-1/3*(2*A*a^4*d^4-9*A*a^3*b*c*d^3-2*A*a^2*
b^2*c^2*d^2-9*A*a*b^3*c^3*d+2*A*b^4*c^4+B*a^4*c*d^3+7*B*a^3*b*c^2*d^2+7*B*
a^2*b^2*c^3*d+B*a*b^3*c^4-5*C*a^4*c^2*d^2-6*C*a^3*b*c^3*d-5*C*a^2*b^2*c^4)
/a^2/c^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))+1/3*b*(2*A*a^3*d^3-10*A*a^2*b*c*d^2-10*A*a*b^2
*c^2*d+2*A*b^3*c^3+B*a^3*c*d^2+14*B*a^2*b*c^2*d+B*a*b^2*c^3-8*C*a^3*c^2*d-
8*C*a^2*b*c^3)/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2/a^2/c/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(Elliptic...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2688 vs. $2(572) = 1144$.

Time = 0.30 (sec) , antiderivative size = 2688, normalized size of antiderivative = 4.53

$$\int \frac{A + Bx^2 + Cx^4}{(ac + (bc + ad)x^2 + bdx^4)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x, algorithm="
fricas")
```

output
```
-1/3*((2*A*a^5*b^2*c^2*d^3 + (2*A*a^3*b^4*d^5 - (8*C*a^2*b^5 - B*a*b^6 - 2
*A*b^7)*c^3*d^2 - 2*(4*C*a^3*b^4 - 7*B*a^2*b^5 + 5*A*a*b^6)*c^2*d^3 + (B*a
^3*b^4 - 10*A*a^2*b^5)*c*d^4)*x^8 + 2*(2*A*a^4*b^3*d^5 - (8*C*a^2*b^5 - B*
a*b^6 - 2*A*b^7)*c^4*d - (16*C*a^3*b^4 - 15*B*a^2*b^5 + 8*A*a*b^6)*c^3*d^2
 - (8*C*a^4*b^3 - 15*B*a^3*b^4 + 20*A*a^2*b^5)*c^2*d^3 + (B*a^4*b^3 - 8*A*
a^3*b^4)*c*d^4)*x^6 - (8*C*a^4*b^3 - B*a^3*b^4 - 2*A*a^2*b^5)*c^5 - 2*(4*C
*a^5*b^2 - 7*B*a^4*b^3 + 5*A*a^3*b^4)*c^4*d + (B*a^5*b^2 - 10*A*a^4*b^3)*c
^3*d^2 + (2*A*a^5*b^2*d^5 - (8*C*a^2*b^5 - B*a*b^6 - 2*A*b^7)*c^5 - 2*(20*
C*a^3*b^4 - 9*B*a^2*b^5 + A*a*b^6)*c^4*d - 2*(20*C*a^4*b^3 - 29*B*a^3*b^4
+ 24*A*a^2*b^5)*c^3*d^2 - 2*(4*C*a^5*b^2 - 9*B*a^4*b^3 + 24*A*a^3*b^4)*c^2
*d^3 + (B*a^5*b^2 - 2*A*a^4*b^3)*c*d^4)*x^4 + 2*(2*A*a^5*b^2*c*d^4 - (8*C*
a^3*b^4 - B*a^2*b^5 - 2*A*a*b^6)*c^5 - (16*C*a^4*b^3 - 15*B*a^3*b^4 + 8*A*
a^2*b^5)*c^4*d - (8*C*a^5*b^2 - 15*B*a^4*b^3 + 20*A*a^3*b^4)*c^3*d^2 + (B*
a^5*b^2 - 8*A*a^4*b^3)*c^2*d^3)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsi
n(x*sqrt(-b/a)), a*d/(b*c)) + (((3*C*a^3*b^4 + 8*C*a^2*b^5 - B*a*b^6 - 2*A
*b^7)*c^3*d^2 + (10*C*a^4*b^3 - 8*(B - C)*a^3*b^4 - (A + 14*B)*a^2*b^5 + 1
0*A*a*b^6)*c^2*d^3 + (3*C*a^5*b^2 - 8*B*a^4*b^3 + (18*A - B)*a^3*b^4 + 10*
A*a^2*b^5)*c*d^4 - (A*a^4*b^3 + 2*A*a^3*b^4)*d^5)*x^8 + 2*((3*C*a^3*b^4 +
8*C*a^2*b^5 - B*a*b^6 - 2*A*b^7)*c^4*d + (13*C*a^4*b^3 - 8*(B - 2*C)*a^3*b
^4 - (A + 15*B)*a^2*b^5 + 8*A*a*b^6)*c^3*d^2 + (13*C*a^5*b^2 - 8*(2*B -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(a*c+(a*d+b*c)*x**2+b*d*x**4)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bdx^4+(bc+ad)x^2+ac)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x, algorithm="
maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(5/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc+ad)x^2+bdx^4)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bdx^4+(bc+ad)x^2+ac)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x, algorithm="
giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*d*x^4 + (b*c + a*d)*x^2 + a*c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\left(ac+(bc+ad)x^2+bdx^4\right)^{5/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\left(b\,d\,x^4+\left(a\,d+b\,c\right)\,x^2+a\,c\right)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(5/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a*c + x^2*(a*d + b*c) + b*d*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\left(ac+(bc+ad)x^2+bdx^4\right)^{5/2}}\,dx=\text{too large to display}$$

input
```
int((C*x^4+B*x^2+A)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(5/2),x)
```

output

```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*x + 2*int((sqrt(c + d*x**2)*sqrt(a
 + b*x**2)*x**4)/(a**4*c**3*d + 3*a**4*c**2*d**2*x**2 + 3*a**4*c*d**3*x**4
 + a**4*d**4*x**6 + a**3*b*c**4 + 6*a**3*b*c**3*d*x**2 + 12*a**3*b*c**2*d*
*2*x**4 + 10*a**3*b*c*d**3*x**6 + 3*a**3*b*d**4*x**8 + 3*a**2*b**2*c**4*x*
*2 + 12*a**2*b**2*c**3*d*x**4 + 18*a**2*b**2*c**2*d**2*x**6 + 12*a**2*b**2
*c*d**3*x**8 + 3*a**2*b**2*d**4*x**10 + 3*a*b**3*c**4*x**4 + 10*a*b**3*c**
3*d*x**6 + 12*a*b**3*c**2*d**2*x**8 + 6*a*b**3*c*d**3*x**10 + a*b**3*d**4*
x**12 + b**4*c**4*x**6 + 3*b**4*c**3*d*x**8 + 3*b**4*c**2*d**2*x**10 + b**
4*c*d**3*x**12),x)*a**4*c**3*d**2 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2)*x**4)/(a**4*c**3*d + 3*a**4*c**2*d**2*x**2 + 3*a**4*c*d**3*x**4 + a**4*
d**4*x**6 + a**3*b*c**4 + 6*a**3*b*c**3*d*x**2 + 12*a**3*b*c**2*d**2*x**4
+ 10*a**3*b*c*d**3*x**6 + 3*a**3*b*d**4*x**8 + 3*a**2*b**2*c**4*x**2 + 12*
a**2*b**2*c**3*d*x**4 + 18*a**2*b**2*c**2*d**2*x**6 + 12*a**2*b**2*c*d**3*
x**8 + 3*a**2*b**2*d**4*x**10 + 3*a*b**3*c**4*x**4 + 10*a*b**3*c**3*d*x**6
 + 12*a*b**3*c**2*d**2*x**8 + 6*a*b**3*c*d**3*x**10 + a*b**3*d**4*x**12 +
b**4*c**4*x**6 + 3*b**4*c**3*d*x**8 + 3*b**4*c**2*d**2*x**10 + b**4*c*d**3
*x**12),x)*a**4*c**2*d**3*x**2 + 2*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**4)/(a**4*c**3*d + 3*a**4*c**2*d**2*x**2 + 3*a**4*c*d**3*x**4 + a**4*d**
4*x**6 + a**3*b*c**4 + 6*a**3*b*c**3*d*x**2 + 12*a**3*b*c**2*d**2*x**4 + 1
0*a**3*b*c*d**3*x**6 + 3*a**3*b*d**4*x**8 + 3*a**2*b**2*c**4*x**2 + 12*...
```

3.10 $\int (A+Bx^2+Cx^4)(ac+(bc-ad)x^2-bdx^4)^{3/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 821

$$\int (A+Bx^2+Cx^4)\left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx =$$

$$-\frac{x(24a^4Cd^4-11a^3bd^3(3cC+4Bd)-3a^2b^2d^2(24c^2C-11Bcd-33Ad^2)-33ab^3cd(c^2C+Bcd+36Ad^2)}{}$$

$$+\frac{x(3(3bd(acC+11Abd)-(bc-ad)(6bcC+11bBd-6aCd))+7bd(6bcC+11bBd-6aCd)x^2)\left(ac+(bc\right.}{693b^2d^2}$$

$$-\frac{Cx(ac+(bc-ad)x^2-bdx^4)^{5/2}}{11bd}$$

$$-\frac{a\sqrt{c}(48a^5Cd^5+8a^4bd^4(15cC-11Bd)-6a^2b^3cd^2(8c^2C+33Bcd-165Ad^2)-2b^5c^3(24c^2C+44Bcd+}{3465b^4}$$

$$+\frac{a\sqrt{c}(bc+ad)\left(48a^4Cd^4+8a^3bd^3(12cC-11Bd)-3ab^3cd(13c^2C+33Bcd-297Ad^2)+3a^2b^2d^2(3c^2C-\right.}{3465b^4d^{7/2}\sqrt{ac+(}}$$

output
```
-1/3465*x*(24*a^4*C*d^4-11*a^3*b*d^3*(4*B*d+3*C*c)-3*a^2*b^2*d^2*(-33*A*d^
2-11*B*c*d+24*C*c^2)-33*a*b^3*c*d*(36*A*d^2+B*c*d+C*c^2)+b^4*c^2*(99*A*d^2
+44*B*c*d+24*C*c^2)-3*b*d*(9*b*d*(-a*d+b*c)*(11*A*b*d+C*a*c)+14*a*b*c*d*(1
1*B*b*d-6*C*a*d+6*C*b*c)+4*(-a*d+b*c)^2*(11*B*b*d-6*C*a*d+6*C*b*c))*x^2)*(
a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)/b^3/d^3+1/693*x*(9*b*d*(11*A*b*d+C*a*c)-
3*(-a*d+b*c)*(11*B*b*d-6*C*a*d+6*C*b*c)+7*b*d*(11*B*b*d-6*C*a*d+6*C*b*c)*x
^2)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2)/b^2/d^2-1/11*C*x*(a*c+(-a*d+b*c)*x^
2-b*d*x^4)^(5/2)/b/d-1/3465*a*c^(1/2)*(48*a^5*C*d^5+8*a^4*b*d^4*(-11*B*d+1
5*C*c)-6*a^2*b^3*c*d^2*(-165*A*d^2+33*B*c*d+8*C*c^2)-2*b^5*c^3*(99*A*d^2+4
4*B*c*d+24*C*c^2)+a^3*b^2*d^3*(198*A*d^2-275*B*c*d+48*C*c^2)-5*a*b^4*c^2*d
*(198*A*d^2+55*B*c*d+24*C*c^2))*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*Ellipt
icE(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/b^4/d^(7/2)/(a*c+(-a*d+b*c)*x^2-b*
d*x^4)^(1/2)+1/3465*a*c^(1/2)*(a*d+b*c)*(48*a^4*C*d^4+8*a^3*b*d^3*(-11*B*d
+12*C*c)-3*a*b^3*c*d*(-297*A*d^2+33*B*c*d+13*C*c^2)+3*a^2*b^2*d^2*(66*A*d^
2-77*B*c*d+3*C*c^2)-b^4*c^2*(99*A*d^2+44*B*c*d+24*C*c^2))*(1+b*x^2/a)^(1/2
)*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/b^4/d^(7
/2)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.62 (sec) , antiderivative size = 707, normalized size of antiderivative = 0.86

$$\int (A+Bx^2+Cx^4)\left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(-c+dx^2\right)\left(24a^4Cd^4+a^3bd^3(57cC-2d(22B+9Cx^2))+3a^2b^2d^2(6c^2C\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)*(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(3/2),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(-c + d*x^2)*(24*a^4*C*d^4 + a^3*b*d^3*(57*c*C
- 2*d*(22*B + 9*C*x^2)) + 3*a^2*b^2*d^2*(6*c^2*C - 2*c*d*(22*B + 7*C*x^2)
+ d^2*(33*A + 11*B*x^2 + 5*C*x^4)) + b^4*(24*c^4*C + 2*c^3*d*(22*B + 9*C*x
^2) + 3*c^2*d^2*(33*A + 11*B*x^2 + 5*C*x^4) + 5*d^4*x^4*(99*A + 77*B*x^2 +
 63*C*x^4) - 2*c*d^3*x^2*(396*A + 275*B*x^2 + 210*C*x^4)) + a*b^3*d*(57*c^
3*C + 6*c^2*d*(22*B + 7*C*x^2) + 2*d^3*x^2*(396*A + 275*B*x^2 + 210*C*x^4)
 - c*d^2*(1683*A + 913*B*x^2 + 615*C*x^4))) - I*c*(48*a^5*C*d^5 + 8*a^4*b*
d^4*(15*c*C - 11*B*d) - 6*a^2*b^3*c*d^2*(8*c^2*C + 33*B*c*d - 165*A*d^2) -
 2*b^5*c^3*(24*c^2*C + 44*B*c*d + 99*A*d^2) + a^3*b^2*d^3*(48*c^2*C - 275*
B*c*d + 198*A*d^2) - 5*a*b^4*c^2*d*(24*c^2*C + 55*B*c*d + 198*A*d^2))*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a
*d)/(b*c))] - I*c*(b*c + a*d)*(-24*a^4*C*d^4 + a^3*b*d^3*(-39*c*C + 44*B*d
) + 9*a^2*b^2*d^2*(c^2*C + 11*B*c*d - 11*A*d^2) + 2*b^4*c^2*(24*c^2*C + 44
*B*c*d + 99*A*d^2) + 3*a*b^3*c*d*(32*c^2*C + 77*B*c*d + 297*A*d^2))*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], -((a*d
)/(b*c))])/(3465*b^3*Sqrt[b/a]*d^4*Sqrt[(a + b*x^2)*(c - d*x^2)])
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 793, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2207, 25, 1490, 25, 1490, 25, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2+Cx^4\right)\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{2207}$$

$$-\frac{\int -\left(\left((6bcC-6adC+11bBd)x^2+acC+11Abd\right)\left(-bdx^4+(bc-ad)x^2+ac\right)^{3/2}\right)dx}{11bd}-\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}{11bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \left((6bcC-6adC+11bBd)x^2+acC+11Abd\right)\left(-bdx^4+(bc-ad)x^2+ac\right)^{3/2}dx}{11bd}-\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}{11bd}$$

$\big\downarrow$ 1490

$$\frac{x\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}\left(3(3bd(acC+11Abd)-(bc-ad)(-6aCd+11bBd+6bcC))+7bdx^2(-6aCd+11bBd+6bcC)\right)}{63bd}-\frac{\int -\Big(((4(6bcC-6adC-}{}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}{11bd}$$

$\big\downarrow$ 25

$$\frac{\int\left(\left(4(6bcC-6adC+11bBd)(bc-ad)^2+9bd(acC+11Abd)(bc-ad)+14abcd(6bcC-6adC+11bBd)\right)x^2+ac(18bd(acC+11Abd)+(bc-ad)(6bcC-6ad\right.}{21bd}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}{11bd}$$

$\big\downarrow$ 1490

$$-\frac{\int -\frac{ac\left(c^2\left(24Cc^2+44Bdc+99Ad^2\right)b^4+3acd\left(19Cc^2+44Bdc+594Ad^2\right)b^3+3a^2d^2\left(6Cc^2-44Bdc+33Ad^2\right)b^2+a^3d^3(57cC-44Bd)b+24a^4Cd^4\right)-\left(-2c^3\left(24Cc^2+\right.\right.}{\sqrt{-bdx}}}{}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}{11bd}$$

$\big\downarrow$ 25

$$\frac{\int \frac{ac\left(c^2\left(24Cc^2+44Bdc+99Ad^2\right)b^4+3acd\left(19Cc^2+44Bdc+594Ad^2\right)b^3+3a^2d^2\left(6Cc^2-44Bdc+33Ad^2\right)b^2+a^3d^3(57cC-44Bd)b+24a^4Cd^4\right)-\left(-2c^3\left(24Cc^2+44B\right.\right.}{\sqrt{-bdx^4+(}}}{}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}{11bd}$$

$\big\downarrow$ 1514

$$\frac{\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{ac\left(c^2\left(24Cc^2+44Bdc+99Ad^2\right)b^4+3acd\left(19Cc^2+44Bdc+594Ad^2\right)b^3+3a^2d^2\left(6Cc^2-44Bdc+33Ad^2\right)b^2+a^3d^3(57cC-44Bd)b+24a^4Cd^4\right)-\cdots}{\cdots}}{\cdots}}{15\cdots}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}{11bd}$$

$\downarrow$ 399

$$\frac{\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(ad+bc)\left(48a^4Cd^4+8a^3bd^3(12cC-11Bd)+3a^2b^2d^2\left(66Ad^2-77Bcd+3c^2C\right)-3ab^3cd\left(-297Ad^2+33Bcd+13c^2C\right)+b^4\left(-c^2\right)\left(99Ad^2+44Bc\cdots\right)\cdots\right)}{b}\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}{11bd}$$

$\downarrow$ 321

$$\frac{\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a\sqrt{c}(ad+bc)\left(48a^4Cd^4+8a^3bd^3(12cC-11Bd)+3a^2b^2d^2\left(66Ad^2-77Bcd+3c^2C\right)-3ab^3cd\left(-297Ad^2+33Bcd+13c^2C\right)+b^4\left(-c^2\right)\left(99Ad^2+44\cdots\right)\cdots\right)}{b\sqrt{d}}\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}{11bd}$$

$\downarrow$ 327

$$\frac{\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a\sqrt{c}(ad+bc)\left(48a^4Cd^4+8a^3bd^3(12cC-11Bd)+3a^2b^2d^2\left(66Ad^2-77Bcd+3c^2C\right)-3ab^3cd\left(-297Ad^2+33Bcd+13c^2C\right)+b^4\left(-c^2\right)\left(99Ad^2+44\cdots\right)\cdots\right)}{b\sqrt{d}}\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{5/2}}{11bd}$$

input
```
Int[(A + B*x^2 + C*x^4)*(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(3/2),x]
```

output
```
-1/11*(C*x*(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(5/2))/(b*d) + ((x*(3*(3*b*d*
(a*c*C + 11*A*b*d) - (b*c - a*d)*(6*b*c*C + 11*b*B*d - 6*a*C*d)) + 7*b*d*(
6*b*c*C + 11*b*B*d - 6*a*C*d)*x^2)*(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(3/2)
)/(63*b*d) + (-1/15*(x*(24*a^4*C*d^4 - 11*a^3*b*d^3*(3*c*C + 4*B*d) - 3*a^
2*b^2*d^2*(24*c^2*C - 11*B*c*d - 33*A*d^2) - 33*a*b^3*c*d*(c^2*C + B*c*d +
 36*A*d^2) + b^4*c^2*(24*c^2*C + 44*B*c*d + 99*A*d^2) - 3*b*d*(9*b*d*(b*c
- a*d)*(a*c*C + 11*A*b*d) + 14*a*b*c*d*(6*b*c*C + 11*b*B*d - 6*a*C*d) + 4*
(b*c - a*d)^2*(6*b*c*C + 11*b*B*d - 6*a*C*d))*x^2)*Sqrt[a*c + (b*c - a*d)*
x^2 - b*d*x^4])/(b*d) + (Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*(-((a*Sqr
t[c]*(48*a^5*C*d^5 + 8*a^4*b*d^4*(15*c*C - 11*B*d) - 6*a^2*b^3*c*d^2*(8*c^
2*C + 33*B*c*d - 165*A*d^2) - 2*b^5*c^3*(24*c^2*C + 44*B*c*d + 99*A*d^2) +
 a^3*b^2*d^3*(48*c^2*C - 275*B*c*d + 198*A*d^2) - 5*a*b^4*c^2*d*(24*c^2*C
+ 55*B*c*d + 198*A*d^2))*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a
*d))])/(b*Sqrt[d])) + (a*Sqrt[c]*(b*c + a*d)*(48*a^4*C*d^4 + 8*a^3*b*d^3*(
12*c*C - 11*B*d) - 3*a*b^3*c*d*(13*c^2*C + 33*B*c*d - 297*A*d^2) + 3*a^2*b
^2*d^2*(3*c^2*C - 77*B*c*d + 66*A*d^2) - b^4*c^2*(24*c^2*C + 44*B*c*d + 99
*A*d^2))*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d
])))/(15*b*d*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4]))/(21*b*d))/(11*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1969 vs. 2(776) = 1552.

Time = 12.10 (sec) , antiderivative size = 1970, normalized size of antiderivative = 2.40

method	result	size
risch	Expression too large to display	1970
elliptic	Expression too large to display	2279
default	Expression too large to display	3125

input
```
int((C*x^4+B*x^2+A)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/3465/b^3/d^3*x*(315*C*b^4*d^4*x^8+385*B*b^4*d^4*x^6+420*C*a*b^3*d^4*x^6
-420*C*b^4*c*d^3*x^6+495*A*b^4*d^4*x^4+550*B*a*b^3*d^4*x^4-550*B*b^4*c*d^3
*x^4+15*C*a^2*b^2*d^4*x^4-615*C*a*b^3*c*d^3*x^4+15*C*b^4*c^2*d^2*x^4+792*A
*a*b^3*d^4*x^2-792*A*b^4*c*d^3*x^2+33*B*a^2*b^2*d^4*x^2-913*B*a*b^3*c*d^3*
x^2+33*B*b^4*c^2*d^2*x^2-18*C*a^3*b*d^4*x^2-42*C*a^2*b^2*c*d^3*x^2+42*C*a*
b^3*c^2*d^2*x^2+18*C*b^4*c^3*d*x^2+99*A*a^2*b^2*d^4-1683*A*a*b^3*c*d^3+99*
A*b^4*c^2*d^2-44*B*a^3*b*d^4-132*B*a^2*b^2*c*d^3+132*B*a*b^3*c^2*d^2+44*B*
b^4*c^3*d+24*C*a^4*d^4+57*C*a^3*b*c*d^3+18*C*a^2*b^2*c^2*d^2+57*C*a*b^3*c^
3*d+24*C*b^4*c^4)*(b*x^2+a)*(-d*x^2+c)/(-(b*x^2+a)*(d*x^2-c))^(1/2)+1/3465
/b^3/d^3*((198*A*a^3*b^2*d^5+990*A*a^2*b^3*c*d^4-990*A*a*b^4*c^2*d^3-198*A
*b^5*c^3*d^2-88*B*a^4*b*d^5-275*B*a^3*b^2*c*d^4-198*B*a^2*b^3*c^2*d^3-275*
B*a*b^4*c^3*d^2-88*B*b^5*c^4*d+48*C*a^5*d^5+120*C*a^4*b*c*d^4+48*C*a^3*b^2
*c^2*d^3-48*C*a^2*b^3*c^3*d^2-120*C*a*b^4*c^4*d-48*C*b^5*c^5)*a/(d/c)^(1/2
)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)
/b*(EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))-EllipticE(x*(d/c)^(
1/2),(-1-(-a*d+b*c)/a/d)^(1/2)))+24*C*a*b^4*c^5/(d/c)^(1/2)*(1-d*x^2/c)^(1
/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(d/
c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))+24*C*a^5*c*d^4/(d/c)^(1/2)*(1-d*x^2/c)
^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))+99*A*a*b^4*c^3*d^2/(d/c)^(1/2)*(...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1012, normalized size of antiderivative = 1.23

$$\int (A+Bx^2+Cx^4)\left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm=
"fricas")
```

output
```
-1/3465*((48*C*b^5*c^6 + 8*(15*C*a*b^4 + 11*B*b^5)*c^5*d + (48*C*a^2*b^3 +
 275*B*a*b^4 + 198*A*b^5)*c^4*d^2 - 6*(8*C*a^3*b^2 - 33*B*a^2*b^3 - 165*A*
a*b^4)*c^3*d^3 - 5*(24*C*a^4*b - 55*B*a^3*b^2 + 198*A*a^2*b^3)*c^2*d^4 - 2
*(24*C*a^5 - 44*B*a^4*b + 99*A*a^3*b^2)*c*d^5)*sqrt(-b*d)*x*sqrt(c/d)*elli
ptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - (48*C*b^5*c^6 + 8*(15*C*a*b^4 +
11*B*b^5)*c^5*d + (48*C*a^2*b^3 + (275*B + 24*C)*a*b^4 + 198*A*b^5)*c^4*d^
2 - (48*C*a^3*b^2 - 3*(66*B + 19*C)*a^2*b^3 - 22*(45*A + 2*B)*a*b^4)*c^3*d
^3 - (120*C*a^4*b - (275*B + 18*C)*a^3*b^2 + 66*(15*A - 2*B)*a^2*b^3 - 99*
A*a*b^4)*c^2*d^4 - (48*C*a^5 - (88*B + 57*C)*a^4*b + 66*(3*A + 2*B)*a^3*b^
2 - 1782*A*a^2*b^3)*c*d^5 + (24*C*a^5 - 44*B*a^4*b + 99*A*a^3*b^2)*d^6)*sq
rt(-b*d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)) + (315*C*
b^5*d^6*x^10 + 48*C*b^5*c^5*d - 35*(12*C*b^5*c*d^5 - (12*C*a*b^4 + 11*B*b^
5)*d^6)*x^8 + 8*(15*C*a*b^4 + 11*B*b^5)*c^4*d^2 + (48*C*a^2*b^3 + 275*B*a*
b^4 + 198*A*b^5)*c^3*d^3 - 6*(8*C*a^3*b^2 - 33*B*a^2*b^3 - 165*A*a*b^4)*c^
2*d^4 - 5*(24*C*a^4*b - 55*B*a^3*b^2 + 198*A*a^2*b^3)*c*d^5 - 2*(24*C*a^5
- 44*B*a^4*b + 99*A*a^3*b^2)*d^6 + 5*(3*C*b^5*c^2*d^4 - (123*C*a*b^4 + 110
*B*b^5)*c*d^5 + (3*C*a^2*b^3 + 110*B*a*b^4 + 99*A*b^5)*d^6)*x^6 + (18*C*b^
5*c^3*d^3 + 3*(14*C*a*b^4 + 11*B*b^5)*c^2*d^4 - (42*C*a^2*b^3 + 913*B*a*b^
4 + 792*A*b^5)*c*d^5 - 3*(6*C*a^3*b^2 - 11*B*a^2*b^3 - 264*A*a*b^4)*d^6)*x
^4 + (24*C*b^5*c^4*d^2 + (57*C*a*b^4 + 44*B*b^5)*c^3*d^3 + 3*(6*C*a^2*b...
```

Sympy [F]

$$\int (A+Bx^2+Cx^4)\left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx=\int\left(-\left(a+bx^2\right)\left(-c+dx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2+Cx^4\right)\,dx$$

input
```
integrate((C*x**4+B*x**2+A)*(a*c+(-a*d+b*c)*x**2-b*d*x**4)**(3/2),x)
```

output
```
Integral((-(a + b*x**2)*(-c + d*x**2))**(3/2)*(A + B*x**2 + C*x**4), x)
```

Maxima [F]

$$\int (A + Bx^2 + Cx^4) (ac + (bc - ad)x^2 - bdx^4)^{3/2} \, dx = \int (-bdx^4 + (bc - ad)x^2 + ac)^{\frac{3}{2}} (Cx^4 + Bx^2 + A) \, dx$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(3/2)*(C*x^4 + B*x^2 + A), x)
```

Giac [F]

$$\int (A + Bx^2 + Cx^4) (ac + (bc - ad)x^2 - bdx^4)^{3/2} \, dx = \int (-bdx^4 + (bc - ad)x^2 + ac)^{\frac{3}{2}} (Cx^4 + Bx^2 + A) \, dx$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(3/2)*(C*x^4 + B*x^2 + A), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2+Cx^4)\left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx=\int \left(-b\,d\,x^4+(b\,c-a\,d)\,x^2+a\,c\right)^{3/2}\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((a*c - x^2*(a*d - b*c) - b*d*x^4)^(3/2)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a*c - x^2*(a*d - b*c) - b*d*x^4)^(3/2)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (A+Bx^2+Cx^4)\left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}\,dx=\text{too large to display}$$

input
```
int((C*x^4+B*x^2+A)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x)
```

output

```
( - 24*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a**4*c*d**4*x - 55*sqrt(c - d*x**
2)*sqrt(a + b*x**2)*a**3*b**2*d**4*x - 57*sqrt(c - d*x**2)*sqrt(a + b*x**2
)*a**3*b*c**2*d**3*x + 18*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a**3*b*c*d**4*
x**3 + 1815*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a**2*b**3*c*d**3*x - 825*sqr
t(c - d*x**2)*sqrt(a + b*x**2)*a**2*b**3*d**4*x**3 - 18*sqrt(c - d*x**2)*s
qrt(a + b*x**2)*a**2*b**2*c**3*d**2*x + 42*sqrt(c - d*x**2)*sqrt(a + b*x**
2)*a**2*b**2*c**2*d**3*x**3 - 15*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a**2*b*
*2*c*d**4*x**5 - 231*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a*b**4*c**2*d**2*x
+ 1705*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a*b**4*c*d**3*x**3 - 1045*sqrt(c
- d*x**2)*sqrt(a + b*x**2)*a*b**4*d**4*x**5 - 57*sqrt(c - d*x**2)*sqrt(a +
 b*x**2)*a*b**3*c**4*d*x - 42*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a*b**3*c**
3*d**2*x**3 + 615*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a*b**3*c**2*d**3*x**5
- 420*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**4*x**7 - 44*sqrt(c - d
*x**2)*sqrt(a + b*x**2)*b**5*c**3*d*x - 33*sqrt(c - d*x**2)*sqrt(a + b*x**
2)*b**5*c**2*d**2*x**3 + 550*sqrt(c - d*x**2)*sqrt(a + b*x**2)*b**5*c*d**3
*x**5 - 385*sqrt(c - d*x**2)*sqrt(a + b*x**2)*b**5*d**4*x**7 - 24*sqrt(c -
 d*x**2)*sqrt(a + b*x**2)*b**4*c**5*x - 18*sqrt(c - d*x**2)*sqrt(a + b*x**
2)*b**4*c**4*d*x**3 - 15*sqrt(c - d*x**2)*sqrt(a + b*x**2)*b**4*c**3*d**2*
x**5 + 420*sqrt(c - d*x**2)*sqrt(a + b*x**2)*b**4*c**2*d**3*x**7 - 315*sqr
t(c - d*x**2)*sqrt(a + b*x**2)*b**4*c*d**4*x**9 - 48*int((sqrt(c - d*x*...
```

3.11 $\int (A + Bx^2 + Cx^4) \sqrt{ac + (bc - ad)x^2 - bdx^4} \, dx$

Optimal result

Integrand size = 40, antiderivative size = 463

$$\int (A + Bx^2 + Cx^4) \sqrt{ac + (bc - ad)x^2 - bdx^4} \, dx$$
$$= \frac{x(5bd(acC + 7Abd) - (bc - ad)(4bcC + 7bBd - 4aCd) + 3bd(4bcC + 7bBd - 4aCd)x^2) \sqrt{ac + (bc - a}{105b^2d^2}$$
$$- \frac{Cx(ac + (bc - ad)x^2 - bdx^4)^{3/2}}{7bd}$$
$$+ \frac{a\sqrt{c}(5bd(bc - ad)(acC + 7Abd) + 6abcd(4bcC + 7bBd - 4aCd) + 2(bc - ad)^2(4bcC + 7bBd - 4aCd}{105b^3d^{5/2}\sqrt{ac + (bc - ad)x^2 - bdx^4}}$$
$$+ \frac{a\sqrt{c}(bc + ad)\left(8a^2Cd^2 + abd(cC - 14Bd) - b^2(4c^2C + 7Bcd - 35Ad^2)\right)\sqrt{1 + \frac{bx^2}{a}}\sqrt{1 - \frac{dx^2}{c}}\,\text{EllipticF}}{105b^3d^{5/2}\sqrt{ac + (bc - ad)x^2 - bdx^4}}$$

output
```
1/105*x*(5*b*d*(7*A*b*d+C*a*c)-(-a*d+b*c)*(7*B*b*d-4*C*a*d+4*C*b*c)+3*b*d*
(7*B*b*d-4*C*a*d+4*C*b*c)*x^2)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)/b^2/d^2-
1/7*C*x*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2)/b/d+1/105*a*c^(1/2)*(5*b*d*(-a*
d+b*c)*(7*A*b*d+C*a*c)+6*a*b*c*d*(7*B*b*d-4*C*a*d+4*C*b*c)+2*(-a*d+b*c)^2*
(7*B*b*d-4*C*a*d+4*C*b*c))*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticE(d
^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/b^3/d^(5/2)/(a*c+(-a*d+b*c)*x^2-b*d*x^4
)^(1/2)+1/105*a*c^(1/2)*(a*d+b*c)*(8*a^2*C*d^2+a*b*d*(-14*B*d+C*c)-b^2*(-3
5*A*d^2+7*B*c*d+4*C*c^2))*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(d^
(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/b^3/d^(5/2)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.86 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.85

$$\int \left(A+Bx^2+Cx^4\right)\sqrt{ac+(bc-ad)x^2-bdx^4}\,dx$$

$$= \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(-c+dx^2\right)\left(4a^2Cd^2+abd(2cC-7Bd-3Cdx^2)+b^2(4c^2C+cd(7B+3Cx^2)-d^2(35A\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(-c + d*x^2)*(4*a^2*C*d^2 + a*b*d*(2*c*C - 7*B*
d - 3*C*d*x^2) + b^2*(4*c^2*C + c*d*(7*B + 3*C*x^2) - d^2*(35*A + 21*B*x^2
 + 15*C*x^4))) - I*c*(8*a^3*C*d^3 + a^2*b*d^2*(5*c*C - 14*B*d) + a*b^2*d*(
-5*c^2*C - 14*B*c*d + 35*A*d^2) - b^3*c*(8*c^2*C + 14*B*c*d + 35*A*d^2))*S
qrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -
((a*d)/(b*c))] + I*c*(b*c + a*d)*(4*a^2*C*d^2 - a*b*d*(c*C + 7*B*d) - b^2*
(8*c^2*C + 14*B*c*d + 35*A*d^2))*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*E
llipticF[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))])/(105*a^2*(b/a)^(5/2)*d^3
*Sqrt[(a + b*x^2)*(c - d*x^2)])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2207, 25, 1490, 25, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2+Cx^4\right) \sqrt{x^2(bc-ad)+ac-bdx^4}\,dx$$

$$\Big\downarrow \ 2207$$

$$-\frac{\int -\left(\left((4bcC-4adC+7bBd)x^2+acC+7Abd\right)\sqrt{-bdx^4+(bc-ad)x^2+ac}\right)dx}{7bd}-\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}{7bd}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \left((4bcC-4adC+7bBd)x^2+acC+7Abd\right)\sqrt{-bdx^4+(bc-ad)x^2+ac}dx}{7bd}-\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}{7bd}$$

$$\Big\downarrow \ 1490$$

$$\frac{\frac{x\sqrt{x^2(bc-ad)+ac-bdx^4}\left(5bd(acC+7Abd)+3bdx^2(-4aCd+7bBd+4bcC)-(bc-ad)(-4aCd+7bBd+4bcC)\right)}{15bd}-\frac{\int -\frac{\left(2(4bcC-4adC+7bBd)(bc-ad)\right.}{}}{}}{7bd}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}{7bd}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\left(2(4bcC-4adC+7bBd)(bc-ad)^2+5bd(acC+7Abd)(bc-ad)+6abcd(4bcC-4adC+7bBd)\right)x^2+ac(10bd(acC+7Abd)+(bc-ad)(4bcC-4adC+7bBd))}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{15bd}+\frac{x\sqrt{x^2}}{}}{7bd}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}{7bd}$$

$$\Big\downarrow \ 1514$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{\left(2(4bcC-4adC+7bBd)(bc-ad)^2+5bd(acC+7Abd)(bc-ad)+6abcd(4bcC-4adC+7bBd)\right)x^2+ac(10bd(acC+7Abd)+(bc-ad)(4bcC-4adC+}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$7bd$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}{7bd}$$

$$\Big\downarrow \text{ 399}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(ad+bc)\left(8a^2Cd^2+abd(cC-14Bd)-\left(b^2\left(-35Ad^2+7Bcd+4c^2C\right)\right)\right)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{a\left(5bd(bc-ad)(acC+7Abd)+2(bc-ad)^2(-4\right.}{}\right.}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}{7bd}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a\left(5bd(bc-ad)(acC+7Abd)+2(bc-ad)^2(-4aCd+7bBd+4bcC)+6abcd(-4aCd+7bBd+4bcC)\right)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{a\sqrt{c}(ad+bc)\left(8a^2Cd^2+abd\right.}{}\right.}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}{7bd}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a\sqrt{c}(ad+bc)\left(8a^2Cd^2+abd(cC-14Bd)-\left(b^2\left(-35Ad^2+7Bcd+4c^2C\right)\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}}+\frac{a\sqrt{c}\left(5bd(bc-ad)(acC+7Abd)+\right.}{}\right.}{15bd\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\frac{Cx\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}{7bd}$$

input
```
Int[(A + B*x^2 + C*x^4)*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4],x]
```

output
```
-1/7*(C*x*(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(3/2))/(b*d) + ((x*(5*b*d*(a*c
*C + 7*A*b*d) - (b*c - a*d)*(4*b*c*C + 7*b*B*d - 4*a*C*d) + 3*b*d*(4*b*c*C
 + 7*b*B*d - 4*a*C*d)*x^2)*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4])/(15*b*d)
 + (Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*((a*Sqrt[c]*(5*b*d*(b*c - a*d)
*(a*c*C + 7*A*b*d) + 6*a*b*c*d*(4*b*c*C + 7*b*B*d - 4*a*C*d) + 2*(b*c - a*
d)^2*(4*b*c*C + 7*b*B*d - 4*a*C*d))*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]],
 -((b*c)/(a*d))])/(b*Sqrt[d]) + (a*Sqrt[c]*(b*c + a*d)*(8*a^2*C*d^2 + a*b*
d*(c*C - 14*B*d) - b^2*(4*c^2*C + 7*B*c*d - 35*A*d^2))*EllipticF[ArcSin[(S
qrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d])))/(15*b*d*Sqrt[a*c + (b*c
 - a*d)*x^2 - b*d*x^4]))/(7*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 11.38 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{C\,x^{5}\sqrt{-bd\,x^{4}-x^{2}da+bc\,x^{2}+ac}}{7}-\frac{\left(-Bbd-aCd+Ccb-\frac{C(-6ad+6bc)}{7}\right)x^{3}\sqrt{-bd\,x^{4}-x^{2}da+bc\,x^{2}+ac}}{5bd}-\frac{\left(-Abd-Bad+Bbc+\frac{2C}{7}\right.}{}$
risch	$\frac{x\left(15C\,b^{2}d^{2}x^{4}+21B\,b^{2}d^{2}x^{2}+3Cab\,d^{2}x^{2}-3C\,b^{2}cd\,x^{2}+35A\,b^{2}d^{2}+7Bb\,d^{2}a-7B\,b^{2}cd-4a^{2}C\,d^{2}-2Cabcd-4C\,b^{2}c^{2}\right)\left(b\,x^{2}+a\right)\left(-d\,x^{2}+\right.}{105b^{2}d^{2}\sqrt{-(b\,x^{2}+a)(d\,x^{2}-c)}}$
default	Expression too large to display

input
```
int((C*x^4+B*x^2+A)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/7*C*x^5*(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)-1/5*(-B*b*d-a*C*d+C*c*b-1/7
*C*(-6*a*d+6*b*c))/b/d*x^3*(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(-A*b*
d-B*a*d+B*b*c+2/7*C*a*c+1/5*(-B*b*d-a*C*d+C*c*b-1/7*C*(-6*a*d+6*b*c))/b/d*
(-4*a*d+4*b*c))/b/d*x*(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)+(A*a*c+1/3*(-A*
b*d-B*a*d+B*b*c+2/7*C*a*c+1/5*(-B*b*d-a*C*d+C*c*b-1/7*C*(-6*a*d+6*b*c))/b/
d*(-4*a*d+4*b*c))/b/d*a*c)/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)
/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*
c)/a/d)^(1/2))-(-A*a*d+A*b*c+B*a*c+3/5*(-B*b*d-a*C*d+C*c*b-1/7*C*(-6*a*d+6
*b*c))/b/d*a*c+1/3*(-A*b*d-B*a*d+B*b*c+2/7*C*a*c+1/5*(-B*b*d-a*C*d+C*c*b-1
/7*C*(-6*a*d+6*b*c))/b/d*(-4*a*d+4*b*c))/b/d*(-2*a*d+2*b*c))*a/(d/c)^(1/2)
*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)/
b*(EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))-EllipticE(x*(d/c)^(1
/2),(-1-(-a*d+b*c)/a/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.12

$$\int (A+Bx^2+Cx^4)\sqrt{ac+(bc-ad)x^2-bdx^4}\,dx =$$

$$-\frac{(8\,Cb^3c^4+(5\,Cab^2+14\,Bb^3)c^3d-(5\,Ca^2b-14\,Bab^2-35\,Ab^3)c^2d^2-(8\,Ca^3-14\,Ba^2b+35\,Aab^2)c}{}$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm=
"fricas")
```

output
```
-1/105*((8*C*b^3*c^4 + (5*C*a*b^2 + 14*B*b^3)*c^3*d - (5*C*a^2*b - 14*B*a*
b^2 - 35*A*b^3)*c^2*d^2 - (8*C*a^3 - 14*B*a^2*b + 35*A*a*b^2)*c*d^3)*sqrt(
-b*d)*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - (8*C*b^3*c
^4 + (5*C*a*b^2 + 14*B*b^3)*c^3*d - (5*C*a^2*b - 2*(7*B + 2*C)*a*b^2 - 35*
A*b^3)*c^2*d^2 - (8*C*a^3 - 2*(7*B + C)*a^2*b + 7*(5*A - B)*a*b^2)*c*d^3 +
 (4*C*a^3 - 7*B*a^2*b + 70*A*a*b^2)*d^4)*sqrt(-b*d)*x*sqrt(c/d)*elliptic_f
(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - (15*C*b^3*d^4*x^6 - 8*C*b^3*c^3*d - (5
*C*a*b^2 + 14*B*b^3)*c^2*d^2 + (5*C*a^2*b - 14*B*a*b^2 - 35*A*b^3)*c*d^3 +
 (8*C*a^3 - 14*B*a^2*b + 35*A*a*b^2)*d^4 - 3*(C*b^3*c*d^3 - (C*a*b^2 + 7*B
*b^3)*d^4)*x^4 - (4*C*b^3*c^2*d^2 + (2*C*a*b^2 + 7*B*b^3)*c*d^3 + (4*C*a^2
*b - 7*B*a*b^2 - 35*A*b^3)*d^4)*x^2)*sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c
))/(b^3*d^4*x)
```

Sympy [F]

$$\int (A+Bx^2+Cx^4)\sqrt{ac+(bc-ad)x^2-bdx^4}\,dx = \int \sqrt{-(a+bx^2)(-c+dx^2)}\left(A+Bx^2+Cx^4\right)\,dx$$

input
```
integrate((C*x**4+B*x**2+A)*(a*c+(-a*d+b*c)*x**2-b*d*x**4)**(1/2),x)
```

output
```
Integral(sqrt(-(a + b*x**2)*(-c + d*x**2))*(A + B*x**2 + C*x**4), x)
```

Maxima [F]

$$\int (A+Bx^2+Cx^4)\sqrt{ac+(bc-ad)x^2-bdx^4}\,dx = \int \sqrt{-bdx^4+(bc-ad)x^2+ac}\left(Cx^4+Bx^2+A\right)dx$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c)*(C*x^4 + B*x^2 + A), x)
```

Giac [F]

$$\int \left(A+Bx^2+Cx^4\right)\sqrt{ac+(bc-ad)x^2-bdx^4}\,dx$$
$$=\int \sqrt{-bdx^4+(bc-ad)x^2+ac}\left(Cx^4+Bx^2+A\right)dx$$

input
```
integrate((C*x^4+B*x^2+A)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c)*(C*x^4 + B*x^2 + A), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx^2+Cx^4\right)\sqrt{ac+(bc-ad)x^2-bdx^4}\,dx$$
$$=\int \sqrt{-b\,d\,x^4+\left(b\,c-a\,d\right)\,x^2+a\,c}\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((a*c - x^2*(a*d - b*c) - b*d*x^4)^(1/2)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a*c - x^2*(a*d - b*c) - b*d*x^4)^(1/2)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int \left(A+Bx^2+Cx^4\right)\sqrt{ac+(bc-ad)x^2-bdx^4}\,dx=\text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x)
```

output
```
( - 4*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a**2*c*d**2*x + 42*sqrt(c - d*x**2
)*sqrt(a + b*x**2)*a*b**2*d**2*x - 2*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a*b
*c**2*d*x + 3*sqrt(c - d*x**2)*sqrt(a + b*x**2)*a*b*c*d**2*x**3 - 7*sqrt(c
 - d*x**2)*sqrt(a + b*x**2)*b**3*c*d*x + 21*sqrt(c - d*x**2)*sqrt(a + b*x*
*2)*b**3*d**2*x**3 - 4*sqrt(c - d*x**2)*sqrt(a + b*x**2)*b**2*c**3*x - 3*s
qrt(c - d*x**2)*sqrt(a + b*x**2)*b**2*c**2*d*x**3 + 15*sqrt(c - d*x**2)*sq
rt(a + b*x**2)*b**2*c*d**2*x**5 - 8*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)
*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a**3*c*d**3 - 21*int((sqr
t(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4
),x)*a**2*b**2*d**3 - 5*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
- a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a**2*b*c**2*d**2 + 49*int((sqrt(c - d
*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a*
b**3*c*d**2 + 5*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x*
*2 + b*c*x**2 - b*d*x**4),x)*a*b**2*c**3*d + 14*int((sqrt(c - d*x**2)*sqrt
(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*b**4*c**2*d +
 8*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2
 - b*d*x**4),x)*b**3*c**4 + 4*int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c
 - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a**3*c**2*d**2 + 63*int((sqrt(c - d*
x**2)*sqrt(a + b*x**2))/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a**2*b**
2*c*d**2 + 2*int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a*c - a*d*x**2 + ...
```

3.12 $\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 278

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx = -\frac{Cx\sqrt{ac+(bc-ad)x^2-bdx^4}}{3bd} + \frac{a\sqrt{c}(2bcC+3bBd-2aCd)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{ac+(bc-ad)x^2-bdx^4}} + \frac{\sqrt{c}(3Ab^2d+2a^2Cd-ab(cC+3Bd))\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output

```
-1/3*C*x*(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)/b/d+1/3*a*c^(1/2)*(3*B*b*d-2*C
*a*d+2*C*b*c)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticE(d^(1/2)*x/c^(1
/2),(-b*c/a/d)^(1/2))/b^2/d^(3/2)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)+1/3*c
^(1/2)*(3*A*b^2*d+2*a^2*C*d-a*b*(3*B*d+C*c))*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)
^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/b^2/d^(3/2)/(a*c+(-a*
d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}Cdx(a+bx^2)\left(-c+dx^2\right)-ic(-2bcC-3bBd+2aCd)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{3b\sqrt{\frac{b}{a}}d^2\sqrt{(a+bx^2)}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4],x]
```

output
```
(Sqrt[b/a]*C*d*x*(a + b*x^2)*(-c + d*x^2) - I*c*(-2*b*c*C - 3*b*B*d + 2*a*
C*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]
*x], -((a*d)/(b*c))] + I*(a*c*C*d - b*(2*c^2*C + 3*B*c*d + 3*A*d^2))*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], -((a*
d)/(b*c))])/(3*b*Sqrt[b/a]*d^2*Sqrt[(a + b*x^2)*(c - d*x^2)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2207, 25, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{x^2(bc-ad)+ac-bdx^4}}\,dx$$
$$\Big\downarrow \text{2207}$$
$$-\frac{\int -\frac{(2bcC-2adC+3bBd)x^2+acC+3Abd}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{3bd}-\frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(2bcC-2adC+3bBd)x^2+acC+3Abd}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{3bd} - \frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$\downarrow$ 1514

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{(2bcC-2adC+3bBd)x^2+acC+3Abd}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{3bd\sqrt{x^2(bc-ad)+ac-bdx^4}} - \frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$\downarrow$ 399

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{(2a^2Cd-ab(3Bd+cC)+3Ab^2d)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{a(-2aCd+3bBd+2bcC)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}\right)}{3bd\sqrt{x^2(bc-ad)+ac-bdx^4}} - \frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$\downarrow$ 321

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a(-2aCd+3bBd+2bcC)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{\sqrt{c}(2a^2Cd-ab(3Bd+cC)+3Ab^2d)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}}\right)}{3bd\sqrt{x^2(bc-ad)+ac-bdx^4}} - \frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$

$\downarrow$ 327

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{\sqrt{c}(2a^2Cd-ab(3Bd+cC)+3Ab^2d)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}}+\frac{a\sqrt{c}(-2aCd+3bBd+2bcC)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\right.}{b\sqrt{d}}\right.}{3bd\sqrt{x^2(bc-ad)+ac-bdx^4}} - \frac{Cx\sqrt{x^2(bc-ad)+ac-bdx^4}}{3bd}$$

input
```
Int[(A + B*x^2 + C*x^4)/Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4],x]
```

output
```
-1/3*(C*x*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4])/(b*d) + (Sqrt[1 + (b*x^2)
/a]*Sqrt[1 - (d*x^2)/c]*((a*Sqrt[c]*(2*b*c*C + 3*b*B*d - 2*a*C*d)*Elliptic
E[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]) + (Sqrt[c]*(3*
A*b^2*d + 2*a^2*C*d - a*b*(c*C + 3*B*d))*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt
[c]], -((b*c)/(a*d))])/(b*Sqrt[d])))/(3*b*d*Sqrt[a*c + (b*c - a*d)*x^2 - b
*d*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.03

method	result
elliptic	$-\frac{Cx\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}}{3bd}+\frac{\left(A+\frac{Cac}{3bd}\right)\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}}-\frac{\left(B+\frac{C(-2ad+2bc)}{3bd}\right)a\sqrt{}}{}$
risch	$-\frac{Cx(b\,x^2+a)(-d\,x^2+c)}{3bd\sqrt{-(b\,x^2+a)(d\,x^2-c)}}+\frac{-\frac{(3Bbd-2aCd+2Ccb)a\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-a}{}}\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}\,b}}{}$
default	$\frac{A\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}}-\frac{Ba\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-\operatorname{EllipticE}\right.}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-x^2da+bc\,x^2+ac}\,b}$

input
```
int((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/3*C/b/d*x*(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)+(A+1/3*C/b/d*a*c)/(d/c)^
(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))-(B+1/3*C/b/d*(-2*a
*d+2*b*c))*a/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d
*x^2+b*c*x^2+a*c)^(1/2)/b*(EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/
2))-EllipticE(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx =$$

$$-\frac{(2\,Cbc^3-(2\,Ca-3\,Bb)c^2d)\sqrt{-bd}x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)|-\frac{ad}{bc})-(2\,Cbc^3+Cacd^2+3\,Abd^3-(2\,Ca\cdots}{\cdots}$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm=
"fricas")
```

output
```
-1/3*((2*C*b*c^3 - (2*C*a - 3*B*b)*c^2*d)*sqrt(-b*d)*x*sqrt(c/d)*elliptic_
e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - (2*C*b*c^3 + C*a*c*d^2 + 3*A*b*d^3 -
(2*C*a - 3*B*b)*c^2*d)*sqrt(-b*d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/
x), -a*d/(b*c)) + (C*b*c*d^2*x^2 + 2*C*b*c^2*d - (2*C*a - 3*B*b)*c*d^2)*sq
rt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c))/(b^2*c*d^3*x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{\sqrt{-\left(a+bx^2\right)\left(-c+dx^2\right)}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(a*c+(-a*d+b*c)*x**2-b*d*x**4)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/sqrt(-(a + b*x**2)*(-c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{-b\,d\,x^4+\left(b\,c-a\,d\right)\,x^2+a\,c}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{ac+(bc-ad)x^2-bdx^4}}\,dx$$
$$= \frac{-\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,cx-2\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx\right)acd+3\left(\int \frac{\sqrt{-d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{-bd\,x^4-ad\,x^2+bc\,x^2+ac}dx\right)b^2d+2\left(\int \frac{\sqrt{}}{-}\right.}{3bd}$$

input
```
int((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2),x)
```

output
```
( - sqrt(c - d*x**2)*sqrt(a + b*x**2)*c*x - 2*int((sqrt(c - d*x**2)*sqrt(a
 + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a*c*d + 3*int((
sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d*x**2 + b*c*x**2 - b*d*x
**4),x)*b**2*d + 2*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c - a*d
*x**2 + b*c*x**2 - b*d*x**4),x)*b*c**2 + 3*int((sqrt(c - d*x**2)*sqrt(a +
b*x**2))/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a*b*d + int((sqrt(c - d
*x**2)*sqrt(a + b*x**2))/(a*c - a*d*x**2 + b*c*x**2 - b*d*x**4),x)*a*c**2)
/(3*b*d)
```

3.13 $\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 378

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx = \frac{x(A(b^2c^2+a^2d^2)-ac(bBc-a(2cC+Bd))-(Ab^2cd+a^2cCd-ab}{ac(bc+ad)^2\sqrt{ac+(bc-ad)x^2-bdx^4}}$$
$$+\frac{(Ab^2cd+a^2cCd-ab(c^2C+2Bcd+Ad^2))\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{c}\sqrt{d}(bc+ad)^2\sqrt{ac+(bc-ad)x^2-bdx^4}}$$
$$+\frac{(bBc-acC+Abd)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{c}\sqrt{d}(bc+ad)\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output
```
x*(A*(a^2*d^2+b^2*c^2)-a*c*(B*b*c-a*(B*d+2*C*c))-(A*b^2*c*d+a^2*c*C*d-a*b*
(A*d^2+2*B*c*d+C*c^2))*x^2)/a/c/(a*d+b*c)^2/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(
1/2)+(A*b^2*c*d+a^2*c*C*d-a*b*(A*d^2+2*B*c*d+C*c^2))*(1+b*x^2/a)^(1/2)*(1-
d*x^2/c)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(1/2))/b/c^(1/2)/d^(
1/2)/(a*d+b*c)^2/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)+(A*b*d+B*b*c-C*a*c)*(1
+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b*c/a/d)^(
1/2))/b/c^(1/2)/d^(1/2)/(a*d+b*c)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.36 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}dx(A(a^2d^2+abd^2x^2+b^2c(c-dx^2))+ac(2acC+ad(B-\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(3/2),x]
```

output
```
(Sqrt[b/a]*(Sqrt[b/a]*d*x*(A*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c - d*x^2)) +
 a*c*(2*a*c*C + a*d*(B - C*x^2) + b*(-(B*c) + c*C*x^2 + 2*B*d*x^2))) + I*c
*(A*b^2*c*d + a^2*c*C*d - a*b*(c^2*C + 2*B*c*d + A*d^2))*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))] +
 I*c*(b*c + a*d)*(a*c*C - A*b*d + a*B*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))]))/(b*c*d*(b*c + a
*d)^2*Sqrt[(a + b*x^2)*(c - d*x^2)])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2206, 25, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(x^2(bc-ad)+ac-bdx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2206$$

$$\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}-\frac{\int -\frac{\left(cCda^2-b\left(Cc^2+2Bdc+Ad^2\right)a+Ab^2cd\right)x^2+ac(bBc-2aCc+2Abd-aBd)}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{ac(ad+bc)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(cCda^2-b(Cc^2+2Bdc+Ad^2)a+Ab^2cd)x^2+ac(bBc-2aCc+2Abd-aBd)}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}dx}{ac(ad+bc)^2}+\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \; 1514$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{(cCda^2-b(Cc^2+2Bdc+Ad^2)a+Ab^2cd)x^2+ac(bBc-2aCc+2Abd-aBd)}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \; 399$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{a(ad+bc)(-acC+Abd+bBc)\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}dx}{b}\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}+\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx}{b}+\frac{a\sqrt{c}(ad+bc)(-acC+Abd+bBc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\right.}{b\sqrt{d}}\right.}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$
$$\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(\frac{a\sqrt{c}(a^2cCd-ab(Ad^2+2Bcd+c^2C)+Ab^2cd)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}}+\frac{a\sqrt{c}(ad+bc)(-acC+Abd+bBc)\operatorname{EllipticF}\left(\operatorname{ar}\right.}{b\sqrt{d}}\right.}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(3/2),x]
```

output
```
(x*(A*(b^2*c^2 + a^2*d^2) - a*c*(b*B*c - a*(2*c*C + B*d)) - (A*b^2*c*d + a
^2*c*C*d - a*b*(c^2*C + 2*B*c*d + A*d^2))*x^2))/(a*c*(b*c + a*d)^2*Sqrt[a*
c + (b*c - a*d)*x^2 - b*d*x^4]) + (Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]
*((a*Sqrt[c]*(A*b^2*c*d + a^2*c*C*d - a*b*(c^2*C + 2*B*c*d + A*d^2))*Ellip
ticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]) + (a*Sqrt[c
]*(b*c + a*d)*(b*B*c - a*c*C + A*b*d)*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]
], -((b*c)/(a*d))])/(b*Sqrt[d])))/(a*c*(b*c + a*d)^2*Sqrt[a*c + (b*c - a*d
)*x^2 - b*d*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{2bd\left(\frac{\left(Aab\,d^2-A\,b^2cd+2Bacdb-C\,a^2cd+ab\,c^2C\right)x^3}{2bdac\left(a^2d^2+2abcd+b^2c^2\right)}+\frac{\left(A\,a^2d^2+A\,b^2c^2+B\,a^2cd-Bab\,c^2+2C\,a^2c^2\right)x}{2bdac\left(a^2d^2+2abcd+b^2c^2\right)}\right)}{\sqrt{-\left(x^4+\frac{(ad-bc)x^2}{bd}-\frac{ac}{bd}\right)bd}}+\frac{\left(-\frac{C}{bd}+\frac{Abd+Cac}{bdac}-\frac{A\,a^2d^2+A\,b^2}{ac(}\right.}{}$
default	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
2*b*d*(1/2/b/d*(A*a*b*d^2-A*b^2*c*d+2*B*a*b*c*d-C*a^2*c*d+C*a*b*c^2)/a/c/(
a^2*d^2+2*a*b*c*d+b^2*c^2)*x^3+1/2/b/d*(A*a^2*d^2+A*b^2*c^2+B*a^2*c*d-B*a*
b*c^2+2*C*a^2*c^2)/a/c/(a^2*d^2+2*a*b*c*d+b^2*c^2)*x)/(-(x^4+(a*d-b*c)/b/d
*x^2-a*c/b/d)*b*d)^(1/2)+(-C/b/d+1/b/d*(A*b*d+C*a*c)/a/c-(A*a^2*d^2+A*b^2*
c^2+B*a^2*c*d-B*a*b*c^2+2*C*a^2*c^2)/a/c/(a^2*d^2+2*a*b*c*d+b^2*c^2))/(d/c
)^(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))+(A*a*b*d^2-A*b^2
*c*d+2*B*a*b*c*d-C*a^2*c*d+C*a*b*c^2)/c/(a^2*d^2+2*a*b*c*d+b^2*c^2)/(d/c)^
(1/2)*(1-d*x^2/c)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(
1/2)/b*(EllipticF(x*(d/c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2))-EllipticE(x*(d/
c)^(1/2),(-1-(-a*d+b*c)/a/d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(352) = 704$.

Time = 0.11 (sec) , antiderivative size = 811, normalized size of antiderivative = 2.15

$$\int \frac{A + Bx^2 + Cx^4}{(ac + (bc - ad)x^2 - bdx^4)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm=
"fricas")
```

output
```
-((C*a^2*b*c^3*d + A*a^2*b*c*d^3 - (C*a^3 - 2*B*a^2*b + A*a*b^2)*c^2*d^2 -
 (C*a*b^2*c^2*d^2 + A*a*b^2*d^4 - (C*a^2*b - 2*B*a*b^2 + A*b^3)*c*d^3)*x^4
 + (C*a*b^2*c^3*d - A*a^2*b*d^4 - (2*C*a^2*b - 2*B*a*b^2 + A*b^3)*c^2*d^2
+ (C*a^3 - 2*B*a^2*b + 2*A*a*b^2)*c*d^3)*x^2)*sqrt(a*c)*sqrt(d/c)*elliptic
_e(arcsin(x*sqrt(d/c)), -b*c/(a*d)) - (A*a^2*b*c*d^3 - (2*C*a^2*b - B*a*b^
2)*c^4 - ((B - C)*a^2*b - 2*A*a*b^2)*c^3*d - (C*a^3 - 2*B*a^2*b + A*a*b^2)
*c^2*d^2 - (A*a*b^2*d^4 - (2*C*a*b^2 - B*b^3)*c^3*d - ((B - C)*a*b^2 - 2*A
*b^3)*c^2*d^2 - (C*a^2*b - 2*B*a*b^2 + A*b^3)*c*d^3)*x^4 - (A*a^2*b*d^4 +
(2*C*a*b^2 - B*b^3)*c^4 - (2*C*a^2*b - (2*B - C)*a*b^2 + 2*A*b^3)*c^3*d -
((B - 2*C)*a^2*b - 2*(A - B)*a*b^2 - A*b^3)*c^2*d^2 - (C*a^3 - 2*B*a^2*b +
 2*A*a*b^2)*c*d^3)*x^2)*sqrt(a*c)*sqrt(d/c)*elliptic_f(arcsin(x*sqrt(d/c))
, -b*c/(a*d)) - sqrt(-b*d*x^4 + (b*c - a*d)*x^2 + a*c)*((C*a*b^2*c^3*d + A
*a*b^2*c*d^3 - (C*a^2*b - 2*B*a*b^2 + A*b^3)*c^2*d^2)*x^3 + (B*a^2*b*c^2*d
^2 + A*a^2*b*c*d^3 + (2*C*a^2*b - B*a*b^2 + A*b^3)*c^3*d)*x))/(a^2*b^3*c^5
*d + 2*a^3*b^2*c^4*d^2 + a^4*b*c^3*d^3 - (a*b^4*c^4*d^2 + 2*a^2*b^3*c^3*d^
3 + a^3*b^2*c^2*d^4)*x^4 + (a*b^4*c^5*d + a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^
3 - a^4*b*c^2*d^4)*x^2)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx=\int \frac{A+Bx^2+Cx^4}{(-(a+bx^2)(-c+dx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(a*c+(-a*d+b*c)*x**2-b*d*x**4)**(3/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(-(a + b*x**2)*(-c + d*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(-bdx^4+(bc-ad)x^2+ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(-bdx^4+(bc-ad)x^2+ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{{\left(-b\,d\,x^4+\left(b\,c-a\,d\right)\,x^2+a\,c\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(3/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(3/2),x)
```

output
```
(sqrt(c - d*x**2)*sqrt(a + b*x**2)*c*x + int((sqrt(c - d*x**2)*sqrt(a + b*
x**2)*x**2)/(a**2*c**2 - 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**
2 - 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 - 2*b**2*c*d*x**6 +
b**2*d**2*x**8),x)*a*b**2*c*d - int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**
2)/(a**2*c**2 - 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 - 4*a*b
*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 - 2*b**2*c*d*x**6 + b**2*d**2
*x**8),x)*a*b**2*d**2*x**2 + int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/
(a**2*c**2 - 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 - 4*a*b*c*
d*x**4 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 - 2*b**2*c*d*x**6 + b**2*d**2*x*
*8),x)*b**3*c*d*x**2 - int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**2)/(a**2*
c**2 - 2*a**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 - 4*a*b*c*d*x**4
 + 2*a*b*d**2*x**6 + b**2*c**2*x**4 - 2*b**2*c*d*x**6 + b**2*d**2*x**8),x)
*b**3*d**2*x**4 + int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a**2*c**2 - 2*a
**2*c*d*x**2 + a**2*d**2*x**4 + 2*a*b*c**2*x**2 - 4*a*b*c*d*x**4 + 2*a*b*d
**2*x**6 + b**2*c**2*x**4 - 2*b**2*c*d*x**6 + b**2*d**2*x**8),x)*a**2*b*c*
d - int((sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a**2*c**2 - 2*a**2*c*d*x**2 +
 a**2*d**2*x**4 + 2*a*b*c**2*x**2 - 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**
2*c**2*x**4 - 2*b**2*c*d*x**6 + b**2*d**2*x**8),x)*a**2*b*d**2*x**2 - int(
(sqrt(c - d*x**2)*sqrt(a + b*x**2))/(a**2*c**2 - 2*a**2*c*d*x**2 + a**2*d*
*2*x**4 + 2*a*b*c**2*x**2 - 4*a*b*c*d*x**4 + 2*a*b*d**2*x**6 + b**2*c**...
```

3.14 $\int \frac{A+Bx^2+Cx^4}{\left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 713

$$\int \frac{A+Bx^2+Cx^4}{\left(ac+(bc-ad)x^2-bdx^4\right)^{5/2}}\,dx = \frac{x(A(b^2c^2+a^2d^2)-ac(bBc-a(2cC+Bd))-(Ab^2cd+a^2cCd-ab}{3ac(bc+ad)^2\left(ac+(bc-ad)x^2-bdx^4\right)^{3/2}}$$
$$+\frac{x(3ac(bc-ad)\left(Ab^2cd+a^2cCd-ab(c^2C+2Bcd+Ad^2)\right)+(b^2c^2+a^2d^2)\left(2A(b^2c^2+3abcd+a^2d^2)+a\right.}{3a^2c^2(bc+}$$
$$+\frac{\sqrt{d}(2A(b^3c^3+5ab^2c^2d-5a^2bcd^2-a^3d^3)+ac(b^2Bc^2+a^2d(8cC+Bd)-2abc(4cC+7Bd)))\sqrt{1+\frac{bx^2}{a}}}{3ac^{3/2}(bc+ad)^4\sqrt{ac+(bc-ad)x^2-bdx^4}}$$
$$-\frac{(Ab^2c^2d+a^2d(5c^2C+Bcd-2Ad^2)-abc(3c^2C+7Bcd+9Ad^2))\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\right.}{3ac^{3/2}\sqrt{d}(bc+ad)^3\sqrt{ac+(bc-ad)x^2-bdx^4}}$$

output
```
1/3*x*(A*(a^2*d^2+b^2*c^2)-a*c*(B*b*c-a*(B*d+2*C*c))-(A*b^2*c*d+a^2*c*C*d-
a*b*(A*d^2+2*B*c*d+C*c^2))*x^2)/a/c/(a*d+b*c)^2/(a*c+(-a*d+b*c)*x^2-b*d*x^
4)^(3/2)+1/3*x*(3*a*c*(-a*d+b*c)*(A*b^2*c*d+a^2*c*C*d-a*b*(A*d^2+2*B*c*d+C
*c^2))+(a^2*d^2+b^2*c^2)*(2*A*(a^2*d^2+3*a*b*c*d+b^2*c^2)+a*c*(B*b*c-a*(B*
d+2*C*c)))-b*d*(2*A*(-a^3*d^3-5*a^2*b*c*d^2+5*a*b^2*c^2*d+b^3*c^3)+a*c*(b^
2*B*c^2+a^2*d*(B*d+8*C*c)-2*a*b*c*(7*B*d+4*C*c)))*x^2)/a^2/c^2/(a*d+b*c)^4
/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)+1/3*d^(1/2)*(2*A*(-a^3*d^3-5*a^2*b*c*d
^2+5*a*b^2*c^2*d+b^3*c^3)+a*c*(b^2*B*c^2+a^2*d*(B*d+8*C*c)-2*a*b*c*(7*B*d+
4*C*c)))*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticE(d^(1/2)*x/c^(1/2),(
-b*c/a/d)^(1/2))/a/c^(3/2)/(a*d+b*c)^4/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(1/2)-
1/3*(A*b^2*c^2*d+a^2*d*(-2*A*d^2+B*c*d+5*C*c^2)-a*b*c*(9*A*d^2+7*B*c*d+3*C
*c^2))*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)*EllipticF(d^(1/2)*x/c^(1/2),(-b
*c/a/d)^(1/2))/a/c^(3/2)/d^(1/2)/(a*d+b*c)^3/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.43 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x\Big(a^2cd(bc+ad)\left(c^2C+Bcd+Ad^2\right)\left(a+bx^2\right)^2+a^2d(ad(-4c^2C$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(5/2),x]
```

output
```
(Sqrt[b/a]*x*(a^2*c*d*(b*c + a*d)*(c^2*C + B*c*d + A*d^2)*(a + b*x^2)^2 +
a^2*d*(a*d*(-4*c^2*C - B*c*d + 2*A*d^2) + b*c*(4*c^2*C + 7*B*c*d + 10*A*d^
2))*(a + b*x^2)^2*(c - d*x^2) + a*b*c^2*(A*b^2 + a*(-(b*B) + a*C))*(b*c +
a*d)*(c - d*x^2)^2 + b*c^2*(2*A*b^2*(b*c + 5*a*d) + a*(b^2*B*c + 4*a^2*C*d
 - a*b*(4*c*C + 7*B*d)))*(a + b*x^2)*(c - d*x^2)^2) + I*c*(a + b*x^2)*Sqrt
[1 + (b*x^2)/a]*(-c + d*x^2)*Sqrt[1 - (d*x^2)/c]*(-(b*(2*A*(b^3*c^3 + 5*a*
b^2*c^2*d - 5*a^2*b*c*d^2 - a^3*d^3) + a*c*(b^2*B*c^2 + a^2*d*(8*c*C + B*d
) - 2*a*b*c*(4*c*C + 7*B*d)))*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b
*c))]) + (b*c + a*d)*(A*b*(2*b^2*c^2 + 9*a*b*c*d - a^2*d^2) + a*c*(b^2*B*c
 + 3*a^2*C*d - a*b*(5*c*C + 7*B*d)))*EllipticF[I*ArcSinh[Sqrt[b/a]*x], -((
a*d)/(b*c))]))/(3*a^2*Sqrt[b/a]*c^2*(b*c + a*d)^4*((a + b*x^2)*(c - d*x^2)
)^(3/2))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 671, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2206, 25, 1492, 25, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(x^2(bc-ad)+ac-bdx^4)^{5/2}}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}-\frac{\int -\frac{-3\left(cCda^2-b\left(Cc^2+2Bdc+Ad^2\right)a+Ab^2cd\right)x^2+ac(bBc-2aCc-aBd)+2A\left(b^2c^2+3abdc+a^2d^2\right)}{\left(-bdx^4+(bc-ad)x^2+ac\right)^{3/2}}dx}{3ac(ad+bc)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-3\left(cCda^2-b\left(Cc^2+2Bdc+Ad^2\right)a+Ab^2cd\right)x^2+2A\left(b^2c^2+3abdc+a^2d^2\right)+ac(bBc-a(2cC+Bd))}{\left(-bdx^4+(bc-ad)x^2+ac\right)^{3/2}}dx}{3ac(ad+bc)^2}+\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{\frac{x\left(3ac(bc-ad)\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)+\left(a^2d^2+b^2c^2\right)\left(2A\left(a^2d^2+3abcd+b^2c^2\right)+ac(bBc-a(Bd+2cC))\right)-bdx^2\left(ac\left(a^2d(Bd+8cC)\right.\right.\right.}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}}{}$$

$$\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{bd\left(2A\left(b^3c^3+5ab^2dc^2-5a^2bd^2c-a^3d^3\right)+ac\left(d(8cC+Bd)a^2-2bc(4cC+7Bd)a+b^2Bc^2\right)\right)x^2+ac\left(3cCd^2a^3-bd\left(10Cc^2+8Bdc-Ad^2\right)a^2+b^2c\left(3Cc^2+8Bdc+18A\right.\right.}{\sqrt{-bdx^4+(bc-ad)x^2+ac}}}{ac(ad+bc)^2}}{}$$

$$\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$$\Big\downarrow \text{1514}$$

$$\frac{\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\int \frac{bd\left(2A\left(b^3c^3+5ab^2dc^2-5a^2bd^2c-a^3d^3\right)+ac\left(d(8cC+Bd)a^2-2bc(4cC+7Bd)a+b^2Bc^2\right)\right)x^2+ac\left(3cCd^2a^3-bd\left(10Cc^2+8Bdc-Ad^2\right)a^2+b^2\right.}{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}}}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}}{}$$

$$\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(ad\left(ac\left(a^2d(Bd+8cC)-2abc(7Bd+4cC)+b^2Bc^2\right)+2A\left(-a^3d^3-5a^2bcd^2+5ab^2c^2d+b^3c^3\right)\right)\int \frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx-a(ad+bc)\left(a^2d(-2\right.\right.}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}}{}$$

$$\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(ad\left(ac\left(a^2d(Bd+8cC)-2abc(7Bd+4cC)+b^2Bc^2\right)+2A\left(-a^3d^3-5a^2bcd^2+5ab^2c^2d+b^3c^3\right)\right)\int\frac{\sqrt{\frac{bx^2}{a}+1}}{\sqrt{1-\frac{dx^2}{c}}}dx-\frac{a\sqrt{c}(ad+bc)\left(a^2d\left(-\right.\right.}{}}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

$$\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}$$

$$\Big\downarrow \; 327$$

$$\frac{x\left(-\left(x^2\left(a^2cCd-ab\left(Ad^2+2Bcd+c^2C\right)+Ab^2cd\right)\right)+A\left(a^2d^2+b^2c^2\right)-ac(bBc-a(Bd+2cC))\right)}{3ac(ad+bc)^2\left(x^2(bc-ad)+ac-bdx^4\right)^{3/2}}+$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\left(a\sqrt{c}\sqrt{d}\left(ac\left(a^2d(Bd+8cC)-2abc(7Bd+4cC)+b^2Bc^2\right)+2A\left(-a^3d^3-5a^2bcd^2+5ab^2c^2d+b^3c^3\right)\right)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)-\frac{a\sqrt{c}}{}\right.}{ac(ad+bc)^2\sqrt{x^2(bc-ad)+ac-bdx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a*c + (b*c - a*d)*x^2 - b*d*x^4)^(5/2),x]
```

output
```
(x*(A*(b^2*c^2 + a^2*d^2) - a*c*(b*B*c - a*(2*c*C + B*d)) - (A*b^2*c*d + a
^2*c*C*d - a*b*(c^2*C + 2*B*c*d + A*d^2))*x^2))/(3*a*c*(b*c + a*d)^2*(a*c
+ (b*c - a*d)*x^2 - b*d*x^4)^(3/2)) + ((x*(3*a*c*(b*c - a*d)*(A*b^2*c*d +
a^2*c*C*d - a*b*(c^2*C + 2*B*c*d + A*d^2)) + (b^2*c^2 + a^2*d^2)*(2*A*(b^2
*c^2 + 3*a*b*c*d + a^2*d^2) + a*c*(b*B*c - a*(2*c*C + B*d))) - b*d*(2*A*(b
^3*c^3 + 5*a*b^2*c^2*d - 5*a^2*b*c*d^2 - a^3*d^3) + a*c*(b^2*B*c^2 + a^2*d
*(8*c*C + B*d) - 2*a*b*c*(4*c*C + 7*B*d)))*x^2))/(a*c*(b*c + a*d)^2*Sqrt[a
*c + (b*c - a*d)*x^2 - b*d*x^4]) + (Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c
]*(a*Sqrt[c]*Sqrt[d]*(2*A*(b^3*c^3 + 5*a*b^2*c^2*d - 5*a^2*b*c*d^2 - a^3*d
^3) + a*c*(b^2*B*c^2 + a^2*d*(8*c*C + B*d) - 2*a*b*c*(4*c*C + 7*B*d)))*Ell
ipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))] - (a*Sqrt[c]*(b*c + a*
d)*(A*b^2*c^2*d + a^2*d*(5*c^2*C + B*c*d - 2*A*d^2) - a*b*c*(3*c^2*C + 7*B
*c*d + 9*A*d^2))*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/S
qrt[d]))/(a*c*(b*c + a*d)^2*Sqrt[a*c + (b*c - a*d)*x^2 - b*d*x^4]))/(3*a*c
*(b*c + a*d)^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 3.67 (sec) , antiderivative size = 1147, normalized size of antiderivative = 1.61

method	result	size
elliptic	Expression too large to display	1147
default	Expression too large to display	2104

input
```
int((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
(1/3/b^2/d^2*(A*a*b*d^2-A*b^2*c*d+2*B*a*b*c*d-C*a^2*c*d+C*a*b*c^2)/a/c/(a^
2*d^2+2*a*b*c*d+b^2*c^2)*x^3+1/3/b^2/d^2*(A*a^2*d^2+A*b^2*c^2+B*a^2*c*d-B*
a*b*c^2+2*C*a^2*c^2)/a/c/(a^2*d^2+2*a*b*c*d+b^2*c^2)*x)*(-b*d*x^4-a*d*x^2+
b*c*x^2+a*c)^(1/2)/(x^4+(a*d-b*c)/b/d*x^2-a*c/b/d)^2+2*b*d*(1/6*(2*A*a^3*d
^3+10*A*a^2*b*c*d^2-10*A*a*b^2*c^2*d-2*A*b^3*c^3-B*a^3*c*d^2+14*B*a^2*b*c^
2*d-B*a*b^2*c^3-8*C*a^3*c^2*d+8*C*a^2*b*c^3)/a^2/c^2/(a^2*d^2+2*a*b*c*d+b^
2*c^2)^2*x^3+1/6*(2*A*a^4*d^4+9*A*a^3*b*c*d^3-2*A*a^2*b^2*c^2*d^2+9*A*a*b^
3*c^3*d+2*A*b^4*c^4-B*a^4*c*d^3+7*B*a^3*b*c^2*d^2-7*B*a^2*b^2*c^3*d+B*a*b^
3*c^4-5*C*a^4*c^2*d^2+6*C*a^3*b*c^3*d-5*C*a^2*b^2*c^4)/a^2/c^2/(a^2*d^2+2*
a*b*c*d+b^2*c^2)^2/b/d*x)/(-(x^4+(a*d-b*c)/b/d*x^2-a*c/b/d)*b*d)^(1/2)+(1/
3/(a^2*d^2+2*a*b*c*d+b^2*c^2)*(2*A*a^2*d^2+6*A*a*b*c*d+2*A*b^2*c^2-B*a^2*c
*d+B*a*b*c^2-2*C*a^2*c^2)/a^2/c^2-1/3*(2*A*a^4*d^4+9*A*a^3*b*c*d^3-2*A*a^2
*b^2*c^2*d^2+9*A*a*b^3*c^3*d+2*A*b^4*c^4-B*a^4*c*d^3+7*B*a^3*b*c^2*d^2-7*B
*a^2*b^2*c^3*d+B*a*b^3*c^4-5*C*a^4*c^2*d^2+6*C*a^3*b*c^3*d-5*C*a^2*b^2*c^4
)/a^2/c^2/(a^2*d^2+2*a*b*c*d+b^2*c^2)^2)/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+
b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(d/c)^(1/2
),(-1-(-a*d+b*c)/a/d)^(1/2))+1/3*d*(2*A*a^3*d^3+10*A*a^2*b*c*d^2-10*A*a*b^
2*c^2*d-2*A*b^3*c^3-B*a^3*c*d^2+14*B*a^2*b*c^2*d-B*a*b^2*c^3-8*C*a^3*c^2*d
+8*C*a^2*b*c^3)/(a^2*d^2+2*a*b*c*d+b^2*c^2)^2/a/c^2/(d/c)^(1/2)*(1-d*x^2/c
)^(1/2)*(1+b*x^2/a)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)^(1/2)*(Ellipti...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2641 vs. $2(674) = 1348$.

Time = 0.26 (sec) , antiderivative size = 2641, normalized size of antiderivative = 3.70

$$\int \frac{A + Bx^2 + Cx^4}{(ac + (bc - ad)x^2 - bdx^4)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x, algorithm=
"fricas")
```

output
```
-1/3*((2*A*a^5*c^2*d^5 + (2*A*a^3*b^2*d^7 + (8*C*a^2*b^3 - B*a*b^4 - 2*A*b
^5)*c^3*d^4 - 2*(4*C*a^3*b^2 - 7*B*a^2*b^3 + 5*A*a*b^4)*c^2*d^5 - (B*a^3*b
^2 - 10*A*a^2*b^3)*c*d^6)*x^8 + (8*C*a^4*b - B*a^3*b^2 - 2*A*a^2*b^3)*c^5*
d^2 - 2*(4*C*a^5 - 7*B*a^4*b + 5*A*a^3*b^2)*c^4*d^3 - (B*a^5 - 10*A*a^4*b)
*c^3*d^4 + 2*(2*A*a^4*b*d^7 - (8*C*a^2*b^3 - B*a*b^4 - 2*A*b^5)*c^4*d^3 +
(16*C*a^3*b^2 - 15*B*a^2*b^3 + 8*A*a*b^4)*c^3*d^4 - (8*C*a^4*b - 15*B*a^3*
b^2 + 20*A*a^2*b^3)*c^2*d^5 - (B*a^4*b - 8*A*a^3*b^2)*c*d^6)*x^6 + (2*A*a^
5*d^7 + (8*C*a^2*b^3 - B*a*b^4 - 2*A*b^5)*c^5*d^2 - 2*(20*C*a^3*b^2 - 9*B*
a^2*b^3 + A*a*b^4)*c^4*d^3 + 2*(20*C*a^4*b - 29*B*a^3*b^2 + 24*A*a^2*b^3)*
c^3*d^4 - 2*(4*C*a^5 - 9*B*a^4*b + 24*A*a^3*b^2)*c^2*d^5 - (B*a^5 - 2*A*a^
4*b)*c*d^6)*x^4 - 2*(2*A*a^5*c*d^6 - (8*C*a^3*b^2 - B*a^2*b^3 - 2*A*a*b^4)
*c^5*d^2 + (16*C*a^4*b - 15*B*a^3*b^2 + 8*A*a^2*b^3)*c^4*d^3 - (8*C*a^5 -
15*B*a^4*b + 20*A*a^3*b^2)*c^3*d^4 - (B*a^5 - 8*A*a^4*b)*c^2*d^5)*x^2)*sqr
t(a*c)*sqrt(d/c)*elliptic_e(arcsin(x*sqrt(d/c)), -b*c/(a*d)) - (3*C*a^3*b^
2*c^7 + 2*A*a^5*c^2*d^5 + (3*C*a*b^4*c^5*d^2 + 2*A*a^3*b^2*d^7 - (10*C*a^2
*b^3 - 8*B*a*b^4 - A*b^5)*c^4*d^3 + (3*C*a^3*b^2 - 8*(B - C)*a^2*b^3 + (18
*A - B)*a*b^4 - 2*A*b^5)*c^3*d^4 - (8*C*a^3*b^2 - (A + 14*B)*a^2*b^3 + 10*
A*a*b^4)*c^2*d^5 - (B*a^3*b^2 - 10*A*a^2*b^3)*c*d^6)*x^8 - (10*C*a^4*b - 8
*B*a^3*b^2 - A*a^2*b^3)*c^6*d + (3*C*a^5 - 8*(B - C)*a^4*b + (18*A - B)*a^
3*b^2 - 2*A*a^2*b^3)*c^5*d^2 - (8*C*a^5 - (A + 14*B)*a^4*b + 10*A*a^3*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{(ac + (bc - ad)x^2 - bdx^4)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(a*c+(-a*d+b*c)*x**2-b*d*x**4)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{5/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(-bdx^4+(bc-ad)x^2+ac)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(5/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{5/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(-bdx^4+(bc-ad)x^2+ac)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(-b*d*x^4 + (b*c - a*d)*x^2 + a*c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{5/2}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{{\left(-b\,d\,x^4+\left(b\,c-a\,d\right)\,x^2+a\,c\right)}^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(5/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a*c - x^2*(a*d - b*c) - b*d*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(ac+(bc-ad)x^2-bdx^4)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((C*x^4+B*x^2+A)/(a*c+(-a*d+b*c)*x^2-b*d*x^4)^(5/2),x)
```

output
```
(sqrt(c - d*x**2)*sqrt(a + b*x**2)*b*x + 2*int((sqrt(c - d*x**2)*sqrt(a +
b*x**2)*x**4)/(a**4*c**3*d - 3*a**4*c**2*d**2*x**2 + 3*a**4*c*d**3*x**4 -
a**4*d**4*x**6 - a**3*b*c**4 + 6*a**3*b*c**3*d*x**2 - 12*a**3*b*c**2*d**2*
x**4 + 10*a**3*b*c*d**3*x**6 - 3*a**3*b*d**4*x**8 - 3*a**2*b**2*c**4*x**2
+ 12*a**2*b**2*c**3*d*x**4 - 18*a**2*b**2*c**2*d**2*x**6 + 12*a**2*b**2*c*
d**3*x**8 - 3*a**2*b**2*d**4*x**10 - 3*a*b**3*c**4*x**4 + 10*a*b**3*c**3*d
*x**6 - 12*a*b**3*c**2*d**2*x**8 + 6*a*b**3*c*d**3*x**10 - a*b**3*d**4*x**
12 - b**4*c**4*x**6 + 3*b**4*c**3*d*x**8 - 3*b**4*c**2*d**2*x**10 + b**4*c
*d**3*x**12),x)*a**4*c**3*d**2 - 4*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*
x**4)/(a**4*c**3*d - 3*a**4*c**2*d**2*x**2 + 3*a**4*c*d**3*x**4 - a**4*d**
4*x**6 - a**3*b*c**4 + 6*a**3*b*c**3*d*x**2 - 12*a**3*b*c**2*d**2*x**4 + 1
0*a**3*b*c*d**3*x**6 - 3*a**3*b*d**4*x**8 - 3*a**2*b**2*c**4*x**2 + 12*a**
2*b**2*c**3*d*x**4 - 18*a**2*b**2*c**2*d**2*x**6 + 12*a**2*b**2*c*d**3*x**
8 - 3*a**2*b**2*d**4*x**10 - 3*a*b**3*c**4*x**4 + 10*a*b**3*c**3*d*x**6 -
12*a*b**3*c**2*d**2*x**8 + 6*a*b**3*c*d**3*x**10 - a*b**3*d**4*x**12 - b**
4*c**4*x**6 + 3*b**4*c**3*d*x**8 - 3*b**4*c**2*d**2*x**10 + b**4*c*d**3*x*
*12),x)*a**4*c**2*d**3*x**2 + 2*int((sqrt(c - d*x**2)*sqrt(a + b*x**2)*x**
4)/(a**4*c**3*d - 3*a**4*c**2*d**2*x**2 + 3*a**4*c*d**3*x**4 - a**4*d**4*x
**6 - a**3*b*c**4 + 6*a**3*b*c**3*d*x**2 - 12*a**3*b*c**2*d**2*x**4 + 10*a
**3*b*c*d**3*x**6 - 3*a**3*b*d**4*x**8 - 3*a**2*b**2*c**4*x**2 + 12*a**...
```

3.15 $\int \frac{-a^2e^2+c^2d^2x^4}{\left(a+\frac{\left(cd^2+ae^2\right)x^2}{de}+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 254

$$\int \frac{-a^2e^2+c^2d^2x^4}{\left(a+\frac{(cd^2+ae^2)x^2}{de}+cx^4\right)^{3/2}}\,dx = \frac{ae^{3/2}(cd^2+ae^2)\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{cd^2}{ae^2}\right)}{\sqrt{d}\,(cd^2-ae^2)\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}} - \frac{2acd^{3/2}e^{3/2}\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),1-\frac{cd^2}{ae^2}\right)}{(cd^2-ae^2)\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}$$

output

```
a*e^(3/2)*(a*e^2+c*d^2)*(d*(c*d*x^2+a*e)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*El
lipticE(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),(1-c*d^2/a/e^2)^(1/2))/d^(1/2)
/(-a*e^2+c*d^2)/(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)-2*a*c*d^(3/2)*e^(3/2)*(d
*(c*d*x^2+a*e)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*InverseJacobiAM(arctan(e^(1/
2)*x/d^(1/2)),(1-c*d^2/a/e^2)^(1/2))/(-a*e^2+c*d^2)/(a+(c*d/e+a*e/d)*x^2+c
*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.37 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.03

$$\int \frac{-a^2e^2+c^2d^2x^4}{\left(a+\frac{(cd^2+ae^2)x^2}{de}+cx^4\right)^{3/2}}\,dx = \frac{-\sqrt{\frac{cd}{ae}}e(cd^2+ae^2)\,x(ae+cdx^2)-icd^2(cd^2+ae^2)\sqrt{1+\frac{cdx^2}{ae}}\sqrt{1+\frac{ex^2}{d}}\ldots}{\sqrt{\ldots}}$$

input
```
Integrate[(-(a^2*e^2) + c^2*d^2*x^4)/(a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*
x^4)^(3/2),x]
```

output
```
(-(Sqrt[(c*d)/(a*e)]*e*(c*d^2 + a*e^2)*x*(a*e + c*d*x^2)) - I*c*d^2*(c*d^2
 + a*e^2)*Sqrt[1 + (c*d*x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*ArcSin
h[Sqrt[(c*d)/(a*e)]*x], (a*e^2)/(c*d^2)] - I*c*d^2*(-(c*d^2) + a*e^2)*Sqrt
[1 + (c*d*x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[(c*d)/(
a*e)]*x], (a*e^2)/(c*d^2)])/(Sqrt[(c*d)/(a*e)]*(-(c*d^2) + a*e^2)*Sqrt[((a
*e + c*d*x^2)*(d + e*x^2))/(d*e)])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 510 vs. 2(254) = 508.

Time = 0.77 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2206, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c^2d^2x^4-a^2e^2}{\left(\frac{x^2(ae^2+cd^2)}{de}+a+cx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{ex\left(cdx^2\left(c^2d^4-a^2e^4\right)+ae\left(c^2d^4-a^2e^4\right)\right)}{\left(cd^2-ae^2\right)^2\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{d^2e^2\int\frac{ac\left(\left(c^2d^4-a^2e^4\right)x^2+2ade\left(cd^2-ae^2\right)\right)}{de\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{a\left(cd^2-ae^2\right)^2}$$

$\downarrow$ 27

$$\frac{ex\left(cdx^2\left(c^2d^4-a^2e^4\right)+ae\left(c^2d^4-a^2e^4\right)\right)}{\left(cd^2-ae^2\right)^2\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{cde\int\frac{\left(c^2d^4-a^2e^4\right)x^2+2ade\left(cd^2-ae^2\right)}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\left(cd^2-ae^2\right)^2}$$

$\downarrow$ 1511

$$\frac{ex\left(cdx^2\left(c^2d^4-a^2e^4\right)+ae\left(c^2d^4-a^2e^4\right)\right)}{\left(cd^2-ae^2\right)^2\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{cde\left(\frac{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\sqrt{a}e+\sqrt{c}d\right)^3\int\frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(c^2d^4-a^2e^4\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}\right)}{\left(cd^2-ae^2\right)^2}$$

$\downarrow$ 27

$$\frac{ex\left(cdx^2\left(c^2d^4-a^2e^4\right)+ae\left(c^2d^4-a^2e^4\right)\right)}{\left(cd^2-ae^2\right)^2\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{cde\left(\frac{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(\sqrt{a}e+\sqrt{c}d\right)^3\int\frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\left(c^2d^4-a^2e^4\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}\right)}{\left(cd^2-ae^2\right)^2}$$

$\downarrow$ 1416

$$\frac{ex\left(cdx^2\left(c^2d^4-a^2e^4\right)+ae\left(c^2d^4-a^2e^4\right)\right)}{\left(cd^2-ae^2\right)^2\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{cde\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{c}d-\sqrt{a}e\right)\left(\sqrt{a}e+\sqrt{c}d\right)^3\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{\left(c^2d^4-a^2e^4\right)\int\frac{\ldots}{\sqrt{cx^4\ldots}}}{\sqrt{\ldots}}\right)}{\left(cd^2-ae^2\right)^2}$$

$\downarrow$ 1509

$$\frac{ex\left(cdx^2\left(c^2d^4-a^2e^4\right)+ae\left(c^2d^4-a^2e^4\right)\right)}{\left(cd^2-ae^2\right)^2\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-$$

$$\frac{cde\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{c}d-\sqrt{a}e\right)\left(\sqrt{a}e+\sqrt{c}d\right)^3\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{\left(c^2d^4-a^2e^4\right)\left(\sqrt[4]{a}\ldots\right.}{\ldots}\right.}{\left(cd^2-ae^2\right)^2}$$

input
```
Int[(-(a^2*e^2) + c^2*d^2*x^4)/(a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4)^(
3/2),x]
```

output
```
(e*x*(a*e*(c^2*d^4 - a^2*e^4) + c*d*(c^2*d^4 - a^2*e^4)*x^2))/((c*d^2 - a*
e^2)^2*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4]) - (c*d*e*(-(((c^2*d^4 -
a^2*e^4)*(-((x*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(Sqrt[a] + Sqrt[
c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e + (a*e)/d)*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], (2 - ((c*d)/e + (a*e)/d)/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + (
(c*d)/e + (a*e)/d)*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(Sqrt[c]*d - Sqrt[a
]*e)*(Sqrt[c]*d + Sqrt[a]*e)^3*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e
+ (a*e)/d)*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], (2 - ((c*d)/e + (a*e)/d)/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])))/(c*d^2 - a*e^2)^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.68

method	result
elliptic	$-\frac{(cd\,x^2+ae)ex(a\,e^2+c\,d^2)}{(a\,e^2-c\,d^2)\sqrt{\frac{\left(x^2+\frac{d}{e}\right)\left(cd\,x^2+ae\right)}{d}}}+\frac{\left(-a\,e^2+\frac{a\,e^2\left(a\,e^2+c\,d^2\right)}{a\,e^2-c\,d^2}\right)\sqrt{1+\frac{x^2dc}{ae}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \text{EllipticF}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\frac{ae}{d}\right)e}{dc}}\right)}{\sqrt{-\frac{cd}{ae}}\,\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+c\,x^4}}$ -
default	Expression too large to display

input
```
int((c^2*d^2*x^4-a^2*e^2)/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(3/2),x,method=_
RETURNVERBOSE)
```

output
```
-(c*d*x^2+a*e)*e/(a*e^2-c*d^2)*x*(a*e^2+c*d^2)/((x^2+d/e)*(c*d*x^2+a*e)/d)
^(1/2)+(-a*e^2+a*e^2/(a*e^2-c*d^2)*(a*e^2+c*d^2))/(-c*d/a/e)^(1/2)*(1+1/a*
x^2*d/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)*Ell
ipticF(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2))-2*(a*e^2+c*d^2
)*e*c*d/(a*e^2-c*d^2)*a/(-c*d/a/e)^(1/2)*(1+1/a*x^2*d/e*c)^(1/2)*(1+e*x^2/
d)^(1/2)/(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)/(c*d/e+1/d*a*e+(a*e^2-c*d^2)/
d/e)*(EllipticF(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2))-Ellip
ticE(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.11

$$\int \frac{-a^2e^2+c^2d^2x^4}{\left(a+\frac{(cd^2+ae^2)x^2}{de}+cx^4\right)^{3/2}}\,dx =$$

$$-\frac{(c^2d^5+acd^3e^2+(c^2d^4e+acd^2e^3)x^2)\sqrt{a}\sqrt{-\frac{cd}{ae}}E(\arcsin\left(x\sqrt{-\frac{cd}{ae}}\right) \mid \frac{ae^2}{cd^2}) - (c^2d^5+acd^3e^2+2\,a^2d^2e^3+}{acd^3-a^2de^2+}$$

input
```
integrate((c^2*d^2*x^4-a^2*e^2)/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(3/2),x, a
lgorithm="fricas")
```

output
```
-((c^2*d^5 + a*c*d^3*e^2 + (c^2*d^4*e + a*c*d^2*e^3)*x^2)*sqrt(a)*sqrt(-c*
d/(a*e))*elliptic_e(arcsin(x*sqrt(-c*d/(a*e))), a*e^2/(c*d^2)) - (c^2*d^5
+ a*c*d^3*e^2 + 2*a^2*d^2*e^3 + (c^2*d^4*e + a*c*d^2*e^3 + 2*a^2*d*e^4)*x^
2)*sqrt(a)*sqrt(-c*d/(a*e))*elliptic_f(arcsin(x*sqrt(-c*d/(a*e))), a*e^2/(
c*d^2)) - (a*c*d^3*e^2 + a^2*d*e^4)*x*sqrt((c*d*e*x^4 + a*d*e + (c*d^2 + a
*e^2)*x^2)/(d*e)))/(a*c*d^3 - a^2*d*e^2 + (a*c*d^2*e - a^2*e^3)*x^2)
```

Sympy [F]

$$\int \frac{-a^2e^2 + c^2d^2x^4}{\left(a + \frac{(cd^2+ae^2)x^2}{de} + cx^4\right)^{3/2}}\,dx = \int \frac{(-ae + cdx^2)\,(ae + cdx^2)}{\left(a + \frac{aex^2}{d} + \frac{cdx^2}{e} + cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c**2*d**2*x**4-a**2*e**2)/(a+(a*e**2+c*d**2)*x**2/d/e+c*x**4)**
(3/2),x)
```

output
```
Integral((-a*e + c*d*x**2)*(a*e + c*d*x**2)/(a + a*e*x**2/d + c*d*x**2/e +
 c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{-a^2e^2 + c^2d^2x^4}{\left(a + \frac{(cd^2+ae^2)x^2}{de} + cx^4\right)^{3/2}}\,dx = \int \frac{c^2d^2x^4 - a^2e^2}{\left(cx^4 + a + \frac{(cd^2+ae^2)x^2}{de}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d^2*x^4-a^2*e^2)/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(3/2),x, a
lgorithm="maxima")
```

output
```
integrate((c^2*d^2*x^4 - a^2*e^2)/(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e))^
(3/2), x)
```

Giac [F]

$$\int \frac{-a^2e^2 + c^2d^2x^4}{\left(a + \frac{(cd^2+ae^2)x^2}{de} + cx^4\right)^{3/2}}\,dx = \int \frac{c^2d^2x^4 - a^2e^2}{\left(cx^4 + a + \frac{(cd^2+ae^2)x^2}{de}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c^2*d^2*x^4-a^2*e^2)/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(3/2),x, a
lgorithm="giac")
```

output
```
integrate((c^2*d^2*x^4 - a^2*e^2)/(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e))^
(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-a^2e^2+c^2d^2x^4}{\left(a+\frac{(cd^2+ae^2)x^2}{de}+cx^4\right)^{3/2}}\,dx=\int -\frac{a^2\,e^2-c^2\,d^2\,x^4}{\left(a+c\,x^4+\frac{x^2\,(c\,d^2+a\,e^2)}{d\,e}\right)^{3/2}}\,dx$$

input
```
int(-(a^2*e^2 - c^2*d^2*x^4)/(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(3/
2),x)
```

output
```
int(-(a^2*e^2 - c^2*d^2*x^4)/(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(3/
2), x)
```

Reduce [F]

$$\int \frac{-a^2e^2+c^2d^2x^4}{\left(a+\frac{(cd^2+ae^2)x^2}{de}+cx^4\right)^{3/2}}\,dx=\sqrt{e}\,\sqrt{d}\,de\left(\left(\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}\,x^2}{cd\,e^2x^6+a\,e^3x^4+2c\,d^2e\,x^4+2ad\,e^2x^2+c\,d^3x^2+a\,d^2e}\right.\right.\right.$$
$$\left.-\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}}{cd\,e^2x^6+a\,e^3x^4+2c\,d^2e\,x^4+2ad\,e^2x^2+c\,d^3x^2+a\,d^2e}dx\right)ae\right)$$

input
```
int((c^2*d^2*x^4-a^2*e^2)/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(3/2),x)
```

output
```
sqrt(e)*sqrt(d)*d*e*(int((sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2)*x**2)/(a*d
**2*e + 2*a*d*e**2*x**2 + a*e**3*x**4 + c*d**3*x**2 + 2*c*d**2*e*x**4 + c*
d*e**2*x**6),x)*c*d - int((sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2))/(a*d**2*
e + 2*a*d*e**2*x**2 + a*e**3*x**4 + c*d**3*x**2 + 2*c*d**2*e*x**4 + c*d*e*
*2*x**6),x)*a*e)
```

3.16 $\int \frac{-34-65x^2-25x^4}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 116

$$\int \frac{-34-65x^2-25x^4}{\left(2+3x^2+x^4\right)^{3/2}}\,dx = \frac{2x}{\sqrt{2+3x^2+x^4}} + \frac{4\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2+3x^2+x^4}} - \frac{27(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

output
```
2*x/(x^4+3*x^2+2)^(1/2)+4*2^(1/2)*(x^2+1)*((x^2+2)/(x^2+1))^(1/2)*Elliptic
E(x/(x^2+1)^(1/2),1/2*2^(1/2))/(x^4+3*x^2+2)^(1/2)-27/2*(x^2+1)*((x^2+2)/(
x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*2^(1/2))*2^(1/2)/(x^4+3*x^2+2)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{-34-65x^2-25x^4}{\left(2+3x^2+x^4\right)^{3/2}}\,dx = \frac{10x+4x^3+4i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\Big|2\right)+23i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{E}}{\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(-34 - 65*x^2 - 25*x^4)/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(10*x + 4*x^3 + (4*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sq
rt[2]], 2] + (23*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt
[2]], 2])/Sqrt[2 + 3*x^2 + x^4]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2206, 27, 1503, 1412, 1455}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-25x^4-65x^2-34}{\left(x^4+3x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2206}$$

$$\frac{2x\left(2x^2+5\right)}{\sqrt{x^4+3x^2+2}}-\frac{1}{2}\int \frac{2\left(4x^2+27\right)}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2x\left(2x^2+5\right)}{\sqrt{x^4+3x^2+2}}-\int \frac{4x^2+27}{\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 1503}$$

$$-27\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-4\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx+\frac{2x\left(2x^2+5\right)}{\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{ 1412}$$

$$-4\int \frac{x^2}{\sqrt{x^4+3x^2+2}}dx-\frac{27\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{2x\left(2x^2+5\right)}{\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{ 1455}$$

$$-\frac{27(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-4\left(\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}\right)+\frac{2x(2x^2+5)}{\sqrt{x^4+3x^2+2}}$$

input
```
Int[(-34 - 65*x^2 - 25*x^4)/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(2*x*(5 + 2*x^2))/Sqrt[2 + 3*x^2 + x^4] - 4*((x*(2 + x^2))/Sqrt[2 + 3*x^2
+ x^4] - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
 1/2])/Sqrt[2 + 3*x^2 + x^4]) - (27*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*El
lipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1455
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

method	result
risch	$\frac{2x(2x^2+5)}{\sqrt{x^4+3x^2+2}}+\frac{27i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{ix\sqrt{2}}{2},\right.\right.}{\sqrt{x^4+3x^2+2}}$
elliptic	$-\frac{2(-2x^3-5x)}{\sqrt{x^4+3x^2+2}}+\frac{27i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-\text{EllipticE}\left(\right.\right.}{\sqrt{x^4+3x^2+2}}$
default	$\frac{-51x^3-85x}{\sqrt{x^4+3x^2+2}}+\frac{27i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}-\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{ix\sqrt{2}}{2},\right.\right.}{\sqrt{x^4+3x^2+2}}$

input
```
int((-25*x^4-65*x^2-34)/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*x*(2*x^2+5)/(x^4+3*x^2+2)^(1/2)+27/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-2*I*2^(1/2)*(2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2)
,2^(1/2))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{-34-65x^2-25x^4}{(2+3x^2+x^4)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{2}\sqrt{-\frac{1}{2}}(x^4+3x^2+2)E(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)|2)-29\sqrt{2}\sqrt{-\frac{1}{2}}(x^4+3x^2+2)F(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)|2)-}{x^4+3x^2+2}$$

input
```
integrate((-25*x^4-65*x^2-34)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(2)*sqrt(-1/2)*(x^4 + 3*x^2 + 2)*elliptic_e(arcsin(sqrt(-1/2)*x),
2) - 29*sqrt(2)*sqrt(-1/2)*(x^4 + 3*x^2 + 2)*elliptic_f(arcsin(sqrt(-1/2)*
x), 2) - 2*sqrt(x^4 + 3*x^2 + 2)*(2*x^3 + 5*x))/(x^4 + 3*x^2 + 2)
```

Sympy [F]

$$\int \frac{-34-65x^2-25x^4}{(2+3x^2+x^4)^{3/2}}\,dx =$$
$$-\int \frac{65x^2}{x^4\sqrt{x^4+3x^2+2}+3x^2\sqrt{x^4+3x^2+2}+2\sqrt{x^4+3x^2+2}}\,dx$$
$$-\int \frac{25x^4}{x^4\sqrt{x^4+3x^2+2}+3x^2\sqrt{x^4+3x^2+2}+2\sqrt{x^4+3x^2+2}}\,dx$$
$$-\int \frac{34}{x^4\sqrt{x^4+3x^2+2}+3x^2\sqrt{x^4+3x^2+2}+2\sqrt{x^4+3x^2+2}}\,dx$$

input
```
integrate((-25*x**4-65*x**2-34)/(x**4+3*x**2+2)**(3/2),x)
```

output
```
-Integral(65*x**2/(x**4*sqrt(x**4 + 3*x**2 + 2) + 3*x**2*sqrt(x**4 + 3*x**
2 + 2) + 2*sqrt(x**4 + 3*x**2 + 2)), x) - Integral(25*x**4/(x**4*sqrt(x**4
 + 3*x**2 + 2) + 3*x**2*sqrt(x**4 + 3*x**2 + 2) + 2*sqrt(x**4 + 3*x**2 + 2
)), x) - Integral(34/(x**4*sqrt(x**4 + 3*x**2 + 2) + 3*x**2*sqrt(x**4 + 3*
x**2 + 2) + 2*sqrt(x**4 + 3*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{-34-65x^2-25x^4}{(2+3x^2+x^4)^{3/2}}\,dx = \int -\frac{25\,x^4+65\,x^2+34}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-25*x^4-65*x^2-34)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((25*x^4 + 65*x^2 + 34)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{-34-65x^2-25x^4}{(2+3x^2+x^4)^{3/2}}\,dx = \int -\frac{25\,x^4+65\,x^2+34}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-25*x^4-65*x^2-34)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(25*x^4 + 65*x^2 + 34)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-34-65x^2-25x^4}{(2+3x^2+x^4)^{3/2}}\,dx = \int -\frac{25\,x^4+65\,x^2+34}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int(-(65*x^2 + 25*x^4 + 34)/(3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int(-(65*x^2 + 25*x^4 + 34)/(3*x^2 + x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{-34-65x^2-25x^4}{\left(2+3x^2+x^4\right)^{3/2}}\,dx = \frac{25\sqrt{x^4+3x^2+2}\,x-84\left(\int \frac{\sqrt{x^4+3x^2+2}}{x^8+6x^6+13x^4+12x^2+4}dx\right)x^4-252\left(\int \frac{\sqrt{x^4+3x^2+2}}{x^8+6x^6+13x^4+12x^2}\right.}{}$$

input
```
int((-25*x^4-65*x^2-34)/(x^4+3*x^2+2)^(3/2),x)
```

output
```
(25*sqrt(x**4 + 3*x**2 + 2)*x - 84*int(sqrt(x**4 + 3*x**2 + 2)/(x**8 + 6*x
**6 + 13*x**4 + 12*x**2 + 4),x)*x**4 - 252*int(sqrt(x**4 + 3*x**2 + 2)/(x*
*8 + 6*x**6 + 13*x**4 + 12*x**2 + 4),x)*x**2 - 168*int(sqrt(x**4 + 3*x**2
+ 2)/(x**8 + 6*x**6 + 13*x**4 + 12*x**2 + 4),x) - 65*int((sqrt(x**4 + 3*x*
*2 + 2)*x**2)/(x**8 + 6*x**6 + 13*x**4 + 12*x**2 + 4),x)*x**4 - 195*int((s
qrt(x**4 + 3*x**2 + 2)*x**2)/(x**8 + 6*x**6 + 13*x**4 + 12*x**2 + 4),x)*x*
*2 - 130*int((sqrt(x**4 + 3*x**2 + 2)*x**2)/(x**8 + 6*x**6 + 13*x**4 + 12*
x**2 + 4),x))/(x**4 + 3*x**2 + 2)
```

3.17 $\int \frac{(cd-be-cex^2)^2}{\left(\frac{-cd^2+bde}{e^2}+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 124

$$\int \frac{(cd-be-cex^2)^2}{\left(\frac{-cd^2+bde}{e^2}+bx^2+cx^4\right)^{3/2}}\,dx = -\frac{e^{3/2}(cd-be)\,(d+ex^2)\sqrt{\frac{d\left(1-\frac{cex^2}{cd-be}\right)}{d+ex^2}}\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|\frac{2cd-be}{cd-be}\right)}{d^{3/2}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

output

```
-e^(3/2)*(-b*e+c*d)*(e*x^2+d)*(d*(1-c*e*x^2/(-b*e+c*d))/(e*x^2+d))^(1/2)*E
llipticE(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),((-b*e+2*c*d)/(-b*e+c*d))^(1/
2))/d^(3/2)/(-d*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.90

$$\int \frac{(cd-be-cex^2)^2}{\left(\frac{-cd^2+bde}{e^2}+bx^2+cx^4\right)^{3/2}}\,dx = \frac{e\sqrt{\frac{e}{d}}\left(\sqrt{\frac{e}{d}}x(-cd+be+cex^2)-i(cd-be)\sqrt{\frac{-cd+be+cex^2}{-cd+be}}\sqrt{1+\frac{ex^2}{d}}E\big(i\text{ar}$$

input
```
Integrate[(c*d - b*e - c*e*x^2)^2/((-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4)
^(3/2),x]
```

output
```
(e*Sqrt[e/d]*(Sqrt[e/d]*x*(-(c*d) + b*e + c*e*x^2) - I*(c*d - b*e)*Sqrt[(-
(c*d) + b*e + c*e*x^2)/(-(c*d) + b*e)]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*Arc
Sinh[Sqrt[e/d]*x], (c*d)/(-(c*d) + b*e)] + I*(c*d - b*e)*Sqrt[(-(c*d) + b*
e + c*e*x^2)/(-(c*d) + b*e)]*Sqrt[1 + (e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[
e/d]*x], (c*d)/(-(c*d) + b*e)]))/Sqrt[((d + e*x^2)*(-(c*d) + b*e + c*e*x^2
))/e^2]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 399 vs. 2(124) = 248.

Time = 0.69 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.22, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1395, 314, 25, 27, 389, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-be+cd-cex^2)^2}{\left(\frac{bde-cd^2}{e^2}+bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1395}$$

$$\frac{\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}\int \frac{\sqrt{-cex^2+cd-be}}{\left(-\frac{x^2}{e}-\frac{d}{e^2}\right)^{3/2}}dx}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\Big\downarrow \ 314$$

$$\frac{\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}\left(\frac{e^2\int -\frac{cex^2}{\sqrt{-\frac{x^2}{e}-\frac{d}{e^2}}\sqrt{-cex^2+cd-be}}dx}{d}-\frac{e^2x\sqrt{-be+cd-cex^2}}{d\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}}\right)}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}\left(-\frac{e^2\int \frac{cex^2}{\sqrt{-\frac{x^2}{e}-\frac{d}{e^2}}\sqrt{-cex^2+cd-be}}dx}{d}-\frac{e^2x\sqrt{-be+cd-cex^2}}{d\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}}\right)}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}\left(-\frac{ce^3\int \frac{x^2}{\sqrt{-\frac{x^2}{e}-\frac{d}{e^2}}\sqrt{-cex^2+cd-be}}dx}{d}-\frac{e^2x\sqrt{-be+cd-cex^2}}{d\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}}\right)}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\Big\downarrow \ 389$$

$$\frac{\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}\left(-\frac{ce^3\left(-\frac{d\int \frac{1}{\sqrt{-\frac{x^2}{e}-\frac{d}{e^2}}\sqrt{-cex^2+cd-be}}dx}{e}-e\int \frac{\sqrt{-\frac{x^2}{e}-\frac{d}{e^2}}}{\sqrt{-cex^2+cd-be}}dx\right)}{d}-\frac{e^2x\sqrt{-be+cd-cex^2}}{d\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}}\right)}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\Big\downarrow \ 323$$

$$\frac{\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}\left(-\frac{ce^3\left(-e\int \frac{\sqrt{-\frac{x^2}{e}-\frac{d}{e^2}}}{\sqrt{-cex^2+cd-be}}dx-\frac{d\sqrt{1-\frac{cex^2}{cd-be}}\int \frac{1}{\sqrt{-\frac{x^2}{e}-\frac{d}{e^2}}\sqrt{1-\frac{cex^2}{cd-be}}}dx}{e\sqrt{-be+cd-cex^2}}\right)}{d}-\frac{e^2x\sqrt{-be+cd-cex^2}}{d\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}}\right)}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$\big\downarrow$ 323

$$\frac{\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}\left(-\frac{ce^3\left(-e\int \frac{\sqrt{-\frac{x^2}{e}-\frac{d}{e^2}}}{\sqrt{-cex^2+cd-be}}dx-\frac{d\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\int \frac{1}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx}{e\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}}\right)}{d}-\frac{e^2x\sqrt{-be+cd-cex^2}}{d\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}}\right)}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$\big\downarrow$ 321

$$\frac{\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}\left(-\frac{ce^3\left(-e\int \frac{\sqrt{-\frac{x^2}{e}-\frac{d}{e^2}}}{\sqrt{-cex^2+cd-be}}dx-\frac{d\sqrt{\frac{ex^2}{d}+1}\sqrt{cd-be}\sqrt{1-\frac{cex^2}{cd-be}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}e^{3/2}\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}}\right)}{d}-e^2\cdots\right)}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$\big\downarrow$ 331

$$\frac{\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}\left(-\frac{ce^3\left(-\frac{e\sqrt{1-\frac{cex^2}{cd-be}}\int \frac{\sqrt{-\frac{x^2}{e}-\frac{d}{e^2}}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx}{\sqrt{-be+cd-cex^2}}-\frac{d\sqrt{\frac{ex^2}{d}+1}\sqrt{cd-be}\sqrt{1-\frac{cex^2}{cd-be}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}e^{3/2}\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}}\right)}{d}\cdots\right)}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$\Big\downarrow$ 330

$$\frac{\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}\left(-\frac{ce^3\left(-\frac{e\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{1-\frac{cex^2}{cd-be}}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx}{\sqrt{\frac{ex^2}{d}+1}\sqrt{-be+cd-cex^2}}-\frac{d\sqrt{\frac{ex^2}{d}+1}\sqrt{cd-be}\sqrt{1-\frac{cex^2}{cd-be}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)\right.}{\sqrt{c}e^{3/2}\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}}\right.}{d}\right.}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{-be+cd-cex^2}\left(-\frac{ce^3\left(-\frac{\sqrt{e}\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}\sqrt{cd-be}\sqrt{1-\frac{cex^2}{cd-be}}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)|\frac{be}{cd}-1\right)}{\sqrt{c}\sqrt{\frac{ex^2}{d}+1}\sqrt{-be+cd-cex^2}}-\frac{d\sqrt{\frac{ex^2}{d}+1}\sqrt{cd-be}\sqrt{1-\frac{cex^2}{cd-be}}\,\text{Ellipt}}{\sqrt{c}e^{3/2}\sqrt{-\frac{d}{e^2}-\frac{x^2}{e}}}\right.}{d}\right.}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

input

```
Int[(c*d - b*e - c*e*x^2)^2/((-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4)^(3/2)
,x]
```

output

```
(Sqrt[-(d/e^2) - x^2/e]*Sqrt[c*d - b*e - c*e*x^2]*(-((e^2*x*Sqrt[c*d - b*e
 - c*e*x^2])/(d*Sqrt[-(d/e^2) - x^2/e])) - (c*e^3*(-((Sqrt[e]*Sqrt[c*d - b
*e]*Sqrt[-(d/e^2) - x^2/e]*Sqrt[1 - (c*e*x^2)/(c*d - b*e)]*EllipticE[ArcSi
n[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]], -1 + (b*e)/(c*d)])/(Sqrt[c]*Sqrt[c
*d - b*e - c*e*x^2]*Sqrt[1 + (e*x^2)/d])) - (d*Sqrt[c*d - b*e]*Sqrt[1 + (e
*x^2)/d]*Sqrt[1 - (c*e*x^2)/(c*d - b*e)]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[e]
*x)/Sqrt[c*d - b*e]], -1 + (b*e)/(c*d)])/(Sqrt[c]*e^(3/2)*Sqrt[-(d/e^2) -
x^2/e]*Sqrt[c*d - b*e - c*e*x^2])))/d))/Sqrt[-((d*(c*d - b*e))/e^2) + b*x^
2 + c*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] && !SimplerSqrtQ[-b/a, -d/c]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(125) = 250$.

Time = 12.12 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.09

method	result
elliptic	$\frac{(ce\,x^2+be-cd)e^2x}{d\sqrt{\frac{\left(x^2+\frac{d}{e}\right)\left(ce\,x^2+be-cd\right)}{e}}}+\frac{\left(c\,e^2+\frac{(be-2cd)e^2}{d}-\frac{(be-cd)e^2}{d}\right)\sqrt{1+\frac{ce\,x^2}{be-cd}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \text{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)}{\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^4+b\,x^2+\frac{bd}{e}-\frac{c\,d^2}{e^2}}}+\frac{2e^3c(}{}$
default	Expression too large to display

input
```
int((-c*e*x^2-b*e+c*d)^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
(c*e*x^2+b*e-c*d)*e^2/d*x/((x^2+d/e)*(c*e*x^2+b*e-c*d)/e)^(1/2)+(c*e^2+(b*
e-2*c*d)*e^2/d-(b*e-c*d)*e^2/d)/(-c*e/(b*e-c*d))^(1/2)*(1+c*e/(b*e-c*d)*x^
2)^(1/2)*(1+e*x^2/d)^(1/2)/(c*x^4+b*x^2+b*d/e-c*d^2/e^2)^(1/2)*EllipticF(x
*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c/d)^(1/2))+2*e^3*c/d*(b*d/e-c*d^2/e^2)/(-
c*e/(b*e-c*d))^(1/2)*(1+c*e/(b*e-c*d)*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(c*x^4+
b*x^2+b*d/e-c*d^2/e^2)^(1/2)/(b+(b*e-2*c*d)/e)*(EllipticF(x*(-c*e/(b*e-c*d
))^(1/2),(-1+b*e/c/d)^(1/2))-EllipticE(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c/
d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.95

$$\int \frac{(cd-be-cex^2)^2}{\left(\frac{-cd^2+bde}{e^2}+bx^2+cx^4\right)^{3/2}}\,dx = \frac{(ce^4x^2+cde^3)\sqrt{-cd^2+bde}\sqrt{\frac{ce}{cd-be}}E\left(\arcsin\left(\sqrt{\frac{ce}{cd-be}}x\right)\mid-\frac{cd-be}{cd}\right)-(ce}{c}$$

input
```
integrate((-c*e*x^2-b*e+c*d)^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(3/2),x, al
gorithm="fricas")
```

output
```
((c*e^4*x^2 + c*d*e^3)*sqrt(-c*d^2 + b*d*e)*sqrt(c*e/(c*d - b*e))*elliptic
_e(arcsin(sqrt(c*e/(c*d - b*e))*x), -(c*d - b*e)/(c*d)) - (c*e^4*x^2 + c*d
*e^3)*sqrt(-c*d^2 + b*d*e)*sqrt(c*e/(c*d - b*e))*elliptic_f(arcsin(sqrt(c*
e/(c*d - b*e))*x), -(c*d - b*e)/(c*d)) + (c*d*e^4 - b*e^5)*x*sqrt((c*e^2*x
^4 + b*e^2*x^2 - c*d^2 + b*d*e)/e^2))/(c*d^3 - b*d^2*e + (c*d^2*e - b*d*e^
2)*x^2)
```

Sympy [F]

$$\int \frac{(cd-be-cex^2)^2}{\left(\frac{-cd^2+bde}{e^2}+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{(be-cd+cex^2)^2}{\left(\left(\frac{d}{e}+x^2\right)\left(b-\frac{cd}{e}+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e*x**2-b*e+c*d)**2/((b*d*e-c*d**2)/e**2+b*x**2+c*x**4)**(3/2
),x)
```

output
```
Integral((b*e - c*d + c*e*x**2)**2/((d/e + x**2)*(b - c*d/e + c*x**2))**(3
/2), x)
```

Maxima [F]

$$\int \frac{(cd-be-cex^2)^2}{\left(\frac{-cd^2+bde}{e^2}+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{(cex^2-cd+be)^2}{\left(cx^4+bx^2-\frac{cd^2-bde}{e^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e*x^2-b*e+c*d)^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((c*e*x^2 - c*d + b*e)^2/(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2)^(3
/2), x)
```

Giac [F]

$$\int \frac{(cd-be-cex^2)^2}{\left(\frac{-cd^2+bde}{e^2}+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{(cex^2-cd+be)^2}{\left(cx^4+bx^2-\frac{cd^2-bde}{e^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e*x^2-b*e+c*d)^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(3/2),x, al
gorithm="giac")
```

output
```
integrate((c*e*x^2 - c*d + b*e)^2/(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2)^(3
/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd-be-cex^2)^2}{\left(\frac{-cd^2+bde}{e^2}+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{\left(c\,e\,x^2+b\,e-c\,d\right)^2}{\left(b\,x^2-\frac{c\,d^2-b\,d\,e}{e^2}+c\,x^4\right)^{3/2}}\,dx$$

input
```
int((b*e - c*d + c*e*x^2)^2/(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(3/2),x)
```

output
```
int((b*e - c*d + c*e*x^2)^2/(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(3/2), x
)
```

Reduce [F]

$$\int \frac{(cd-be-cex^2)^2}{\left(\frac{-cd^2+bde}{e^2}+bx^2+cx^4\right)^{3/2}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{ce\,x^2+be-cd}}{e^2x^4+2de\,x^2+d^2}dx\right)e^3$$

input
```
int((-c*e*x^2-b*e+c*d)^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(3/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(b*e - c*d + c*e*x**2))/(d**2 + 2*d*e*x**2 + e**
2*x**4),x)*e**3
```

3.18 $\int (a+cx^2+bx^4)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 133

$$\int (a+cx^2+bx^4)^p \, dx = x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)$$

output
```
x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2))
,-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/(
(1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$\int (a+cx^2+bx^4)^p\,dx$$
$$=x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)$$

input
```
Integrate[(a + c*x^2 + b*x^4)^p,x]
```

output
```
(x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-
4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/(((c - Sqrt[-4*a*b +
c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b
*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1418, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4+cx^2)^p\,dx$$
$$\Big\downarrow \text{1418}$$
$$\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}(a+bx^4+cx^2)^p\int\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^p\left(\frac{2bx^2}{c+\sqrt{c^2-4ab}}+\right.$$
$$\Big\downarrow \text{333}$$

$$x\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}(a+bx^4+cx^2)^p\,\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{c^2-4ab}},\cdot\right.$$

input
```
Int[(a + c*x^2 + b*x^4)^p,x]
```

output
```
(x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-
4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c -
Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1418
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p]))   Int[(1
 + 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b,
c, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \left(b\,x^{4}+c\,x^{2}+a\right)^{p}dx$$

input
```
int((b*x^4+c*x^2+a)^p,x)
```

output
```
int((b*x^4+c*x^2+a)^p,x)
```

Fricas [F]

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(bx^4+cx^2+a\right)^p\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F]

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(a+bx^4+cx^2\right)^p\,dx$$

input
```
integrate((b*x**4+c*x**2+a)**p,x)
```

output
```
Integral((a + b*x**4 + c*x**2)**p, x)
```

Maxima [F]

$$\int \left(a+cx^2+bx^4\right)^p\,dx=\int \left(bx^4+cx^2+a\right)^p\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + c*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+cx^2+bx^4\right)^p \,dx=\int \left(bx^4+cx^2+a\right)^p \,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + c*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+cx^2+bx^4\right)^p \,dx=\int \left(b\,x^4+c\,x^2+a\right)^p \,dx$$

input
```
int((a + b*x^4 + c*x^2)^p,x)
```

output
```
int((a + b*x^4 + c*x^2)^p, x)
```

Reduce [F]

$$\int \left(a+cx^2+bx^4\right)^p \,dx$$
$$=\frac{\left(b\,x^4+c\,x^2+a\right)^p x+16\left(\int \frac{\left(b\,x^4+c\,x^2+a\right)^p}{4bp\,x^4+b\,x^4+4cp\,x^2+c\,x^2+4ap+a}dx\right) a\,p^2+4\left(\int \frac{\left(b\,x^4+c\,x^2+a\right)^p}{4bp\,x^4+b\,x^4+4cp\,x^2+c\,x^2+4ap+a}dx\right) ap+}{4p+1}$$

input
```
int((b*x^4+c*x^2+a)^p,x)
```

output
```
((a + b*x**4 + c*x**2)**p*x + 16*int((a + b*x**4 + c*x**2)**p/(4*a*p + a +
 4*b*p*x**4 + b*x**4 + 4*c*p*x**2 + c*x**2),x)*a*p**2 + 4*int((a + b*x**4
+ c*x**2)**p/(4*a*p + a + 4*b*p*x**4 + b*x**4 + 4*c*p*x**2 + c*x**2),x)*a*
p + 8*int(((a + b*x**4 + c*x**2)**p*x**2)/(4*a*p + a + 4*b*p*x**4 + b*x**4
 + 4*c*p*x**2 + c*x**2),x)*c*p**2 + 2*int(((a + b*x**4 + c*x**2)**p*x**2)/
(4*a*p + a + 4*b*p*x**4 + b*x**4 + 4*c*p*x**2 + c*x**2),x)*c*p)/(4*p + 1)
```

3.19 $\int (A+Bx^2)(a+cx^2+bx^4)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int (A+Bx^2)(a+cx^2+bx^4)^p\,dx = Ax\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)+\frac{1}{3}Bx^3\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c-\sqrt{-4ab+c^2}},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)$$

output
```
A*x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2
)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)
/((1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)+1/3*B*x^3*(b*x^4+c*x^2+a)^p*Appell
F1(3/2,-p,-p,5/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^
(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c^2)
^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.85

$$\int (A+Bx^2)\left(a+cx^2+bx^4\right)^p\,dx$$
$$=\frac{1}{3}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\left(3A\operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+Bx^2\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right)$$

input
```
Integrate[(A + B*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

output
```
(x*(a + c*x^2 + b*x^4)^p*(3*A*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + S
qrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + B*x^2*AppellF1[
3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqr
t[-4*a*b + c^2])]))/(3*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*
b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))
^p)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2)\left(a+bx^4+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{1515}$$

$$\int \left(A\left(a+bx^4+cx^2\right)^p+Bx^2\left(a+bx^4+cx^2\right)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$Ax\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{c^2-4ab}}\right.$$
$$\frac{1}{3}Bx^3\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c-\sqrt{c^2-4}}\right.$$

input
```
Int[(A + B*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

output
```
(A*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt
[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c
- Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (B*
x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[
-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c
 - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Defintions of rubi rules used

rule 1515
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right)\left(b\,x^{4}+c\,x^{2}+a\right)^{p}dx$$

input
```
int((B*x^2+A)*(b*x^4+c*x^2+a)^p,x)
```

output
```
int((B*x^2+A)*(b*x^4+c*x^2+a)^p,x)
```

Fricas [F]

$$\int (A+Bx^2)\left(a+cx^2+bx^4\right)^p\,dx=\int (Bx^2+A)\left(bx^4+cx^2+a\right)^p\,dx$$

input
```
integrate((B*x^2+A)*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F]

$$\int (A+Bx^2)\left(a+cx^2+bx^4\right)^p\,dx=\int (A+Bx^2)\left(a+bx^4+cx^2\right)^p\,dx$$

input
```
integrate((B*x**2+A)*(b*x**4+c*x**2+a)**p,x)
```

output
```
Integral((A + B*x**2)*(a + b*x**4 + c*x**2)**p, x)
```

Maxima [F]

$$\int (A+Bx^2)\left(a+cx^2+bx^4\right)^p\,dx=\int (Bx^2+A)(bx^4+cx^2+a)^p\,dx$$

input
```
integrate((B*x^2+A)*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^4 + c*x^2 + a)^p, x)
```

Giac [F]

$$\int (A+Bx^2)\left(a+cx^2+bx^4\right)^p\,dx=\int (Bx^2+A)(bx^4+cx^2+a)^p\,dx$$

input
```
integrate((B*x^2+A)*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^4 + c*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2)\left(a+cx^2+bx^4\right)^p\,dx=\int \left(B\,x^2+A\right)\left(b\,x^4+c\,x^2+a\right)^p\,dx$$

input
```
int((A + B*x^2)*(a + b*x^4 + c*x^2)^p,x)
```

output
```
int((A + B*x^2)*(a + b*x^4 + c*x^2)^p, x)
```

Reduce [F]

$$\int (A+Bx^2)\left(a+cx^2+bx^4\right)^p\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(b*x^4+c*x^2+a)^p,x)
```

output
```
(4*(a + b*x**4 + c*x**2)**p*a*p*x + 3*(a + b*x**4 + c*x**2)**p*a*x + 4*(a
+ b*x**4 + c*x**2)**p*b*p*x**3 + (a + b*x**4 + c*x**2)**p*b*x**3 + 2*(a +
b*x**4 + c*x**2)**p*c*p*x + 256*int((a + b*x**4 + c*x**2)**p/(16*a*p**2 +
16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4 + 16*c*p**2*x**2 +
16*c*p*x**2 + 3*c*x**2),x)*a**2*p**4 + 448*int((a + b*x**4 + c*x**2)**p/(1
6*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4 + 16*c*p
**2*x**2 + 16*c*p*x**2 + 3*c*x**2),x)*a**2*p**3 + 240*int((a + b*x**4 + c*
x**2)**p/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x*
*4 + 16*c*p**2*x**2 + 16*c*p*x**2 + 3*c*x**2),x)*a**2*p**2 + 36*int((a + b
*x**4 + c*x**2)**p/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**
4 + 3*b*x**4 + 16*c*p**2*x**2 + 16*c*p*x**2 + 3*c*x**2),x)*a**2*p - 32*int
((a + b*x**4 + c*x**2)**p/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*
b*p*x**4 + 3*b*x**4 + 16*c*p**2*x**2 + 16*c*p*x**2 + 3*c*x**2),x)*a*c*p**3
 - 32*int((a + b*x**4 + c*x**2)**p/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x
**4 + 16*b*p*x**4 + 3*b*x**4 + 16*c*p**2*x**2 + 16*c*p*x**2 + 3*c*x**2),x)
*a*c*p**2 - 6*int((a + b*x**4 + c*x**2)**p/(16*a*p**2 + 16*a*p + 3*a + 16*
b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4 + 16*c*p**2*x**2 + 16*c*p*x**2 + 3*c*
x**2),x)*a*c*p + 256*int(((a + b*x**4 + c*x**2)**p*x**2)/(16*a*p**2 + 16*a
*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4 + 16*c*p**2*x**2 + 16*c
*p*x**2 + 3*c*x**2),x)*a*b*p**4 + 320*int(((a + b*x**4 + c*x**2)**p*x**...
```

3.20 $\int (a+cx^2+bx^4)^p (A+Bx^2+Cx^4)\, dx$

Optimal result

Integrand size = 27, antiderivative size = 353

$$\int (a+cx^2+bx^4)^p (A+Bx^2+Cx^4)\, dx = \frac{Cx(a+cx^2+bx^4)^{1+p}}{b(5+4p)} - \frac{(aC-Ab(5+4p))x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2}\right.}{b(5+4p)} - \frac{(cC(3+2p)-bB(5+4p))x^3\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p \operatorname{AppellF1}\left(\frac{3}{2},\right.}{3b(5+4p)}$$

output
```
C*x*(b*x^4+c*x^2+a)^(p+1)/b/(5+4*p)-(a*C-A*b*(5+4*p))*x*(b*x^4+c*x^2+a)^p*
AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b
+c^2)^(1/2)))/b/(5+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2
/(c+(-4*a*b+c^2)^(1/2)))^p)-1/3*(c*C*(3+2*p)-b*B*(5+4*p))*x^3*(b*x^4+c*x^2
+a)^p*AppellF1(3/2,-p,-p,5/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(
-4*a*b+c^2)^(1/2)))/b/(5+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2
*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.84

$$\int (a+cx^2+bx^4)^p (A+Bx^2+Cx^4)\,dx$$
$$=\frac{1}{15}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\left(15A\,\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+5Bx^2\,\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+3Cx^4\,\text{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right)$$

input
```
Integrate[(a + c*x^2 + b*x^4)^p*(A + B*x^2 + C*x^4),x]
```

output
```
(x*(a + c*x^2 + b*x^4)^p*(15*A*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c +
Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + 5*B*x^2*Appell
F1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c +
Sqrt[-4*a*b + c^2])] + 3*C*x^4*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^2)/(c +
Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])]))/(15*((c - Sqrt
[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b +
c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2207, 25, 1515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2+Cx^4)(a+bx^4+cx^2)^p\,dx$$

$$\Big\downarrow \; 2207$$

$$\frac{\int -\left(\left(\left(cC(2p+3)-bB(4p+5)\right)x^2+aC-Ab(4p+5)\right)\left(bx^4+cx^2+a\right)^p\right)dx}{b(4p+5)}+\frac{Cx\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+5)}$$

$$\Big\downarrow \; 25$$

$$\frac{Cx\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+5)}-\frac{\int \left(\left(cC(2p+3)-bB(4p+5)\right)x^2+aC-Ab(4p+5)\right)\left(bx^4+cx^2+a\right)^p dx}{b(4p+5)}$$

$$\Big\downarrow \; 1515$$

$$\frac{Cx\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+5)}-\frac{\int \left(\left(cC(2p+3)-bB(4p+5)\right)x^2\left(bx^4+cx^2+a\right)^p+aC\left(1-\frac{Ab(4p+5)}{aC}\right)\left(bx^4+cx^2+a\right)^p\right)dx}{b(4p+5)}$$

$$\Big\downarrow \; 2009$$

$$\frac{Cx\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+5)}-\frac{x(aC-Ab(4p+5))\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{c^2-4a}}\right.}{}$$

input
```
Int[(a + c*x^2 + b*x^4)^p*(A + B*x^2 + C*x^4),x]
```

output
```
(C*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) - (((a*C - A*b*(5 + 4*p))*
x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4
*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c - S
qrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + ((c*C*
(3 + 2*p) - b*B*(5 + 4*p))*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p,
 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c
^2])])/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + S
qrt[-4*a*b + c^2]))^p))/(b*(5 + 4*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1515
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [F]

$$\int \left(b\,x^{4}+c\,x^{2}+a\right)^{p} \left(C\,x^{4}+B\,x^{2}+A\right)dx$$

input
```
int((b*x^4+c*x^2+a)^p*(C*x^4+B*x^2+A),x)
```

output
```
int((b*x^4+c*x^2+a)^p*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+cx^2+a\right)^p dx$$

input
```
integrate((b*x^4+c*x^2+a)^p*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\text{Timed out}$$

input
```
integrate((b*x**4+c*x**2+a)**p*(C*x**4+B*x**2+A),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+cx^2+a\right)^p dx$$

input
```
integrate((b*x^4+c*x^2+a)^p*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + c*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+cx^2+a\right)^p\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + c*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\int \left(C\,x^4+B\,x^2+A\right)\,\left(b\,x^4+c\,x^2+a\right)^p\,dx$$

input
```
int((A + B*x^2 + C*x^4)*(a + b*x^4 + c*x^2)^p,x)
```

output
```
int((A + B*x^2 + C*x^4)*(a + b*x^4 + c*x^2)^p, x)
```

Reduce [F]

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\text{too large to display}$$

input
```
int((b*x^4+c*x^2+a)^p*(C*x^4+B*x^2+A),x)
```

output
```
(16*(a + b*x**4 + c*x**2)**p*a*b**2*p**2*x + 32*(a + b*x**4 + c*x**2)**p*a
*b**2*p*x + 15*(a + b*x**4 + c*x**2)**p*a*b**2*x + 16*(a + b*x**4 + c*x**2
)**p*a*b*c*p**2*x + 12*(a + b*x**4 + c*x**2)**p*a*b*c*p*x + 16*(a + b*x**4
 + c*x**2)**p*b**3*p**2*x**3 + 24*(a + b*x**4 + c*x**2)**p*b**3*p*x**3 + 5
*(a + b*x**4 + c*x**2)**p*b**3*x**3 + 16*(a + b*x**4 + c*x**2)**p*b**2*c*p
**2*x**5 + 8*(a + b*x**4 + c*x**2)**p*b**2*c*p**2*x + 16*(a + b*x**4 + c*x
**2)**p*b**2*c*p*x**5 + 10*(a + b*x**4 + c*x**2)**p*b**2*c*p*x + 3*(a + b*
x**4 + c*x**2)**p*b**2*c*x**5 + 8*(a + b*x**4 + c*x**2)**p*b*c**2*p**2*x**
3 + 2*(a + b*x**4 + c*x**2)**p*b*c**2*p*x**3 - 4*(a + b*x**4 + c*x**2)**p*
c**3*p**2*x - 6*(a + b*x**4 + c*x**2)**p*c**3*p*x + 4096*int((a + b*x**4 +
 c*x**2)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**4 + 144
*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4 + 64*c*p**3*x**2 + 144*c*p**2*x**2
+ 92*c*p*x**2 + 15*c*x**2),x)*a**2*b**2*p**6 + 17408*int((a + b*x**4 + c*x
**2)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**4 + 144*b*p
**2*x**4 + 92*b*p*x**4 + 15*b*x**4 + 64*c*p**3*x**2 + 144*c*p**2*x**2 + 92
*c*p*x**2 + 15*c*x**2),x)*a**2*b**2*p**5 + 28160*int((a + b*x**4 + c*x**2)
**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**4 + 144*b*p**2*
x**4 + 92*b*p*x**4 + 15*b*x**4 + 64*c*p**3*x**2 + 144*c*p**2*x**2 + 92*c*p
*x**2 + 15*c*x**2),x)*a**2*b**2*p**4 + 21376*int((a + b*x**4 + c*x**2)**p/
(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**4 + 144*b*p**2*x...
```

3.21 $\int (a+cx^2+bx^4)^p (A+Bx^2+Cx^4+Dx^6)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 484

$$\int (a+cx^2+bx^4)^p (A+Bx^2+Cx^4+Dx^6)\,dx$$
$$= -\frac{(cD(5+2p)-bC(7+4p))x(a+cx^2+bx^4)^{1+p}}{b^2(5+4p)(7+4p)} + \frac{Dx^3(a+cx^2+bx^4)^{1+p}}{b(7+4p)}$$
$$+ \frac{(Ab^2(35+48p+16p^2)+a(cD(5+2p)-bC(7+4p)))\,x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+ \cdots}{b^2(5+4p)(7+4p)}$$
$$+ \frac{(c^2D(15+16p+4p^2)+b^2B(35+48p+16p^2)-b(3aD(5+4p)+cC(21+26p+8p^2)))\,x^3\left(1+\frac{2 \cdots}{c-\sqrt{- \cdots}}\right.}{3b^2(5+4p \cdots}$$

output

```
-(c*D*(5+2*p)-b*C*(7+4*p))*x*(b*x^4+c*x^2+a)^(p+1)/b^2/(5+4*p)/(7+4*p)+D*x
^3*(b*x^4+c*x^2+a)^(p+1)/b/(7+4*p)+(A*b^2*(16*p^2+48*p+35)+a*(c*D*(5+2*p)-
b*C*(7+4*p)))*x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a
*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/b^2/(5+4*p)/(7+4*p)/((1+2*
b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)+1/
3*(c^2*D*(4*p^2+16*p+15)+b^2*B*(16*p^2+48*p+35)-b*(3*a*D*(5+4*p)+c*C*(8*p^
2+26*p+21)))*x^3*(b*x^4+c*x^2+a)^p*AppellF1(3/2,-p,-p,5/2,-2*b*x^2/(c-(-4*
a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/b^2/(5+4*p)/(7+4*p)/((1+2
*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.75

$$\int (a+cx^2+bx^4)^p (A+Bx^2+Cx^4+Dx^6)\,dx$$
$$= \frac{1}{105}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\left(105A\,\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+35Bx^2\,\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+21Cx^4\,\text{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+15Dx^6\,\text{AppellF1}\left(\frac{7}{2},-p,-p,\frac{9}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right)$$

input
```
Integrate[(a + c*x^2 + b*x^4)^p*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(x*(a + c*x^2 + b*x^4)^p*(105*A*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c +
 Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + 35*B*x^2*Appe
llF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c
+ Sqrt[-4*a*b + c^2])] + 21*C*x^4*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^2)/(c
 + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + 15*D*x^6*Ap
pellF1[7/2, -p, -p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-
c + Sqrt[-4*a*b + c^2])]))/(105*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c - S
qrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b
 + c^2]))^p)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2207, 2207, 1515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4+cx^2\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{\int \left(bx^4+cx^2+a\right)^p \left(-\left((cD(2p+5)-bC(4p+7))x^4\right)-(3aD-bB(4p+7))x^2+Ab(4p+7)\right) dx}{b(4p+7)}+\frac{Dx^3\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+7)}$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{\frac{\int \left(A\left(16p^2+48p+35\right)b^2-aC(4p+7)b+\left(B\left(16p^2+48p+35\right)b^2-\left(3aD(4p+5)+cC\left(8p^2+26p+21\right)\right)b+c^2D\left(4p^2+16p+15\right)\right)x^2+acD(2p+5)\right)\left(bx^4+cx\right.}{b(4p+5)}}{b(4p+7)}$$

$$\frac{Dx^3\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+7)}$$

$$\Big\downarrow \text{ 1515}$$

$$\frac{\frac{\int \left(\left(B\left(16p^2+48p+35\right)b^2-\left(3aD(4p+5)+cC\left(8p^2+26p+21\right)\right)b+c^2D\left(4p^2+16p+15\right)\right)x^2\left(bx^4+cx^2+a\right)^p+acD(2p+5)\left(\frac{b(4p+7)(Ab(4p+5)-aC)}{acD(2p+5)}+1\right)\left(\right.}{b(4p+5)}}{b(4p+7)}$$

$$\frac{Dx^3\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+7)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(a+bx^4+cx^2\right)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)\left(-abC(4p+7)+acD(2p+5)+Ab\right.}{}$$

$$\frac{Dx^3\left(a+bx^4+cx^2\right)^{p+1}}{b(4p+7)}$$

input
```
Int[(a + c*x^2 + b*x^4)^p*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(D*x^3*(a + c*x^2 + b*x^4)^(1 + p))/(b*(7 + 4*p)) + (-(((c*D*(5 + 2*p) - b
*C*(7 + 4*p))*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p))) + (((a*c*D*(5
+ 2*p) - a*b*C*(7 + 4*p) + A*b^2*(35 + 48*p + 16*p^2))*x*(a + c*x^2 + b*x^
4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b
*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))
^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + ((c^2*D*(15 + 16*p + 4*p^
2) + b^2*B*(35 + 48*p + 16*p^2) - b*(3*a*D*(5 + 4*p) + c*C*(21 + 26*p + 8*
p^2)))*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c
- Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*(1 + (2*b*
x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^
p))/(b*(5 + 4*p)))/(b*(7 + 4*p))
```

Defintions of rubi rules used

rule 1515
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [F]

$$\int \left(b\,x^{4}+c\,x^{2}+a\right)^{p} \left(D x^{6}+C\,x^{4}+B\,x^{2}+A\right)dx$$

input
```
int((b*x^4+c*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
int((b*x^4+c*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \int \left(Dx^{6}+Cx^{4}+Bx^{2}+A\right)\left(bx^{4}+cx^{2}+a\right)^{p}\,dx$$

input
```
integrate((b*x^4+c*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+c*x**2+a)**p*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx = \int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^4+cx^2+a\right)^p dx$$

input
```
integrate((b*x^4+c*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^4 + c*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx = \int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^4+cx^2+a\right)^p dx$$

input
```
integrate((b*x^4+c*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^4 + c*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx = \int \left(b\,x^4+c\,x^2+a\right)^p \left(A+B\,x^2+C\,x^4+x^6\,D\right) dx$$

input
```
int((a + b*x^4 + c*x^2)^p*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((a + b*x^4 + c*x^2)^p*(A + B*x^2 + C*x^4 + x^6*D), x)
```

Reduce [F]

$$\int \left(a+cx^2+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx = \text{too large to display}$$

input
```
int((b*x^4+c*x^2+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(64*(a + b*x**4 + c*x**2)**p*a*b**3*p**3*x + 240*(a + b*x**4 + c*x**2)**p*
a*b**3*p**2*x + 284*(a + b*x**4 + c*x**2)**p*a*b**3*p*x + 105*(a + b*x**4
+ c*x**2)**p*a*b**3*x + 64*(a + b*x**4 + c*x**2)**p*a*b**2*c*p**3*x + 160*
(a + b*x**4 + c*x**2)**p*a*b**2*c*p**2*x + 84*(a + b*x**4 + c*x**2)**p*a*b
**2*c*p*x + 64*(a + b*x**4 + c*x**2)**p*a*b**2*d*p**3*x**3 + 96*(a + b*x**
4 + c*x**2)**p*a*b**2*d*p**2*x**3 + 20*(a + b*x**4 + c*x**2)**p*a*b**2*d*p
*x**3 - 32*(a + b*x**4 + c*x**2)**p*a*b*c*d*p**3*x - 128*(a + b*x**4 + c*x
**2)**p*a*b*c*d*p**2*x - 90*(a + b*x**4 + c*x**2)**p*a*b*c*d*p*x + 64*(a +
 b*x**4 + c*x**2)**p*b**4*p**3*x**3 + 208*(a + b*x**4 + c*x**2)**p*b**4*p*
*2*x**3 + 188*(a + b*x**4 + c*x**2)**p*b**4*p*x**3 + 35*(a + b*x**4 + c*x*
*2)**p*b**4*x**3 + 64*(a + b*x**4 + c*x**2)**p*b**3*c*p**3*x**5 + 32*(a +
b*x**4 + c*x**2)**p*b**3*c*p**3*x + 176*(a + b*x**4 + c*x**2)**p*b**3*c*p*
*2*x**5 + 96*(a + b*x**4 + c*x**2)**p*b**3*c*p**2*x + 124*(a + b*x**4 + c*
x**2)**p*b**3*c*p*x**5 + 70*(a + b*x**4 + c*x**2)**p*b**3*c*p*x + 21*(a +
b*x**4 + c*x**2)**p*b**3*c*x**5 + 64*(a + b*x**4 + c*x**2)**p*b**3*d*p**3*
x**7 + 144*(a + b*x**4 + c*x**2)**p*b**3*d*p**2*x**7 + 92*(a + b*x**4 + c*
x**2)**p*b**3*d*p*x**7 + 15*(a + b*x**4 + c*x**2)**p*b**3*d*x**7 + 32*(a +
 b*x**4 + c*x**2)**p*b**2*c**2*p**3*x**3 + 64*(a + b*x**4 + c*x**2)**p*b**
2*c**2*p**2*x**3 + 14*(a + b*x**4 + c*x**2)**p*b**2*c**2*p*x**3 + 32*(a +
b*x**4 + c*x**2)**p*b**2*c*d*p**3*x**5 + 32*(a + b*x**4 + c*x**2)**p*b*...
```

3.22 $\int \frac{2a+bx^2}{(a+bx^2+cx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 19

$$\int \frac{2a+bx^2}{(a+bx^2+cx^4)^{5/4}}\,dx = \frac{2x}{\sqrt[4]{a+bx^2+cx^4}}$$

output
```
2*x/(c*x^4+b*x^2+a)^(1/4)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{2a+bx^2}{(a+bx^2+cx^4)^{5/4}}\,dx = \frac{2x}{\sqrt[4]{a+bx^2+cx^4}}$$

input
```
Integrate[(2*a + b*x^2)/(a + b*x^2 + c*x^4)^(5/4),x]
```

output
```
(2*x)/(a + b*x^2 + c*x^4)^(1/4)
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 2 in optimal.

Time = 0.65 (sec) , antiderivative size = 350, normalized size of antiderivative = 18.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2a+bx^2}{(a+bx^2+cx^4)^{5/4}}\,dx$$

$$\Big\downarrow \text{1515}$$

$$\int \left(\frac{bx^2}{(a+bx^2+cx^4)^{5/4}}+\frac{2a}{(a+bx^2+cx^4)^{5/4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2bx\sqrt[4]{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{\sqrt{b^2-4ac}\,x^2}{a\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)}\right)}{\sqrt{b^2-4ac}\sqrt[4]{a+bx^2+cx^4}}+\frac{bx^3\sqrt[4]{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{3}{2},\frac{5}{2},-\frac{2\left(\frac{cx^2}{b-\sqrt{b^2-4ac}}-\frac{cx^2}{b+\sqrt{b^2-4ac}}\right)}{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}\right)}{3a\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{5/4}\sqrt[4]{a+bx^2+cx^4}}+\frac{2x\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)}{\sqrt[4]{a+bx^2+cx^4}}$$

input
```
Int[(2*a + b*x^2)/(a + b*x^2 + c*x^4)^(5/4),x]
```

output
```
(2*(1 + b/Sqrt[b^2 - 4*a*c])*x)/(a + b*x^2 + c*x^4)^(1/4) - (2*b*x*((1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])))
^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((Sqrt[b^2 - 4*a*c]*x^2)/(a*(1 +
(2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))))])/(Sqrt[b^2 - 4*a*c]*(a + b*x^2 + c*x
^4)^(1/4)) + (b*x^3*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^(1/4)*Hypergeo
metric2F1[5/4, 3/2, 5/2, (-2*((c*x^2)/(b - Sqrt[b^2 - 4*a*c]) - (c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))])/(3*a*(1
+ (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^(5/4)*(a + b*x^2 + c*x^4)^(1/4))
```

Defintions of rubi rules used

rule 1515
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{2x}{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{1}{4}}}$	18
trager	$\frac{2x}{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{1}{4}}}$	18
pseudoelliptic	$\frac{2x}{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{1}{4}}}$	18
orering	$\frac{2x}{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{1}{4}}}$	18

input
```
int((b*x^2+2*a)/(c*x^4+b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
2*x/(c*x^4+b*x^2+a)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{2a+bx^2}{(a+bx^2+cx^4)^{5/4}}\,dx = \frac{2\,x}{(cx^4+bx^2+a)^{\frac{1}{4}}}$$

input
```
integrate((b*x^2+2*a)/(c*x^4+b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
2*x/(c*x^4 + b*x^2 + a)^(1/4)
```

Sympy [F]

$$\int \frac{2a+bx^2}{(a+bx^2+cx^4)^{5/4}}\,dx = \int \frac{2a+bx^2}{(a+bx^2+cx^4)^{\frac{5}{4}}}\,dx$$

input
```
integrate((b*x**2+2*a)/(c*x**4+b*x**2+a)**(5/4),x)
```

output
```
Integral((2*a + b*x**2)/(a + b*x**2 + c*x**4)**(5/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{2a+bx^2}{(a+bx^2+cx^4)^{5/4}}\,dx = \frac{2\,x}{(cx^4+bx^2+a)^{\frac{1}{4}}}$$

input
```
integrate((b*x^2+2*a)/(c*x^4+b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
2*x/(c*x^4 + b*x^2 + a)^(1/4)
```

Giac [F]

$$\int \frac{2a+bx^2}{(a+bx^2+cx^4)^{5/4}}\,dx=\int \frac{bx^2+2\,a}{(cx^4+bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((b*x^2+2*a)/(c*x^4+b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 2*a)/(c*x^4 + b*x^2 + a)^(5/4), x)
```

Mupad [B] (verification not implemented)

Time = 18.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{2a+bx^2}{(a+bx^2+cx^4)^{5/4}}\,dx=\frac{2\,x}{(c\,x^4+b\,x^2+a)^{1/4}}$$

input
```
int((2*a + b*x^2)/(a + b*x^2 + c*x^4)^(5/4),x)
```

output
```
(2*x)/(a + b*x^2 + c*x^4)^(1/4)
```

Reduce [F]

$$\int \frac{2a+bx^2}{(a+bx^2+cx^4)^{5/4}}\,dx=\left(\int \frac{x^2}{(c\,x^4+b\,x^2+a)^{\frac{1}{4}}\,a+(c\,x^4+b\,x^2+a)^{\frac{1}{4}}\,b\,x^2+(c\,x^4+b\,x^2+a)^{\frac{1}{4}}\,c\,x^4}dx\right)b$$
$$+2\left(\int \frac{1}{(c\,x^4+b\,x^2+a)^{\frac{1}{4}}\,a+(c\,x^4+b\,x^2+a)^{\frac{1}{4}}\,b\,x^2+(c\,x^4+b\,x^2+a)^{\frac{1}{4}}\,c\,x^4}dx\right)a$$

input
```
int((b*x^2+2*a)/(c*x^4+b*x^2+a)^(5/4),x)
```

output
```
int(x**2/((a + b*x**2 + c*x**4)**(1/4)*a + (a + b*x**2 + c*x**4)**(1/4)*b*
x**2 + (a + b*x**2 + c*x**4)**(1/4)*c*x**4),x)*b + 2*int(1/((a + b*x**2 +
c*x**4)**(1/4)*a + (a + b*x**2 + c*x**4)**(1/4)*b*x**2 + (a + b*x**2 + c*x
**4)**(1/4)*c*x**4),x)*a
```

3.23 $\int (a+bx^2+cx^4)^p (ad+bd(3+2p)x^2+cd(5+4p)x$

Optimal result

Integrand size = 41, antiderivative size = 19

$$\int (a+bx^2+cx^4)^p (ad+bd(3+2p)x^2+cd(5+4p)x^4)\, dx = dx(a+bx^2+cx^4)^{1+p}$$

output
```
d*x*(c*x^4+b*x^2+a)^(p+1)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (a+bx^2+cx^4)^p (ad+bd(3+2p)x^2+cd(5+4p)x^4)\, dx = dx(a+bx^2+cx^4)^{1+p}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^p*(a*d + b*d*(3 + 2*p)*x^2 + c*d*(5 + 4*p)*x
^4),x]
```

output
```
d*x*(a + b*x^2 + c*x^4)^(1 + p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2+cx^4)^p \left(ad+bd(2p+3)x^2+cd(4p+5)x^4\right) \, dx$$

$$\Big\downarrow \ 2021$$

$$dx\left(a+bx^2+cx^4\right)^{p+1}$$

input
```
Int[(a + b*x^2 + c*x^4)^p*(a*d + b*d*(3 + 2*p)*x^2 + c*d*(5 + 4*p)*x^4),x]
```

output
```
d*x*(a + b*x^2 + c*x^4)^(1 + p)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
gosper	$dx(c\,x^4+b\,x^2+a)^{p+1}$	20
risch	$d(c\,x^4+b\,x^2+a)^p\,x(c\,x^4+b\,x^2+a)$	30
norman	$adx\,e^{p\ln(c\,x^4+b\,x^2+a)}+bd\,x^3e^{p\ln(c\,x^4+b\,x^2+a)}+cd\,x^5e^{p\ln(c\,x^4+b\,x^2+a)}$	66
parallelrisch	$\frac{x^5(c\,x^4+b\,x^2+a)^p acd+x^3(c\,x^4+b\,x^2+a)^p abd+a^2d(c\,x^4+b\,x^2+a)^p x}{a}$	68
orering	$\frac{(c\,x^4+b\,x^2+a)x(c\,x^4+b\,x^2+a)^p(ad+bd(3+2p)x^2+cd(5+4p)x^4)}{4c\,x^4p+5c\,x^4+2b\,x^2p+3b\,x^2+a}$	85

input
```
int((c*x^4+b*x^2+a)^p*(a*d+b*d*(3+2*p)*x^2+c*d*(5+4*p)*x^4),x,method=_RETU
RNVERBOSE)
```

output
```
d*x*(c*x^4+b*x^2+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int (a+bx^2+cx^4)^p\,(ad+bd(3+2p)x^2+cd(5+4p)x^4)\;dx$$
$$=(cdx^5+bdx^3+adx)\,(cx^4+bx^2+a)^p$$

input
```
integrate((c*x^4+b*x^2+a)^p*(a*d+b*d*(3+2*p)*x^2+c*d*(5+4*p)*x^4),x, algor
ithm="fricas")
```

output
```
(c*d*x^5 + b*d*x^3 + a*d*x)*(c*x^4 + b*x^2 + a)^p
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(17) = 34$.

Time = 76.89 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

$$\int (a+bx^2+cx^4)^p (ad+bd(3+2p)x^2+cd(5+4p)x^4)\,dx = adx(a+bx^2+cx^4)^p + bdx^3(a+bx^2+cx^4)^p + cdx^5(a+bx^2+cx^4)^p$$

input
```
integrate((c*x**4+b*x**2+a)**p*(a*d+b*d*(3+2*p)*x**2+c*d*(5+4*p)*x**4),x)
```

output
```
a*d*x*(a + b*x**2 + c*x**4)**p + b*d*x**3*(a + b*x**2 + c*x**4)**p + c*d*x
**5*(a + b*x**2 + c*x**4)**p
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int (a+bx^2+cx^4)^p (ad+bd(3+2p)x^2+cd(5+4p)x^4)\,dx = (cdx^5+bdx^3+adx)(cx^4+bx^2+a)^p$$

input
```
integrate((c*x^4+b*x^2+a)^p*(a*d+b*d*(3+2*p)*x^2+c*d*(5+4*p)*x^4),x, algor
ithm="maxima")
```

output
```
(c*d*x^5 + b*d*x^3 + a*d*x)*(c*x^4 + b*x^2 + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(19) = 38$.

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.11

$$\int (a+bx^2+cx^4)^p \left(ad+bd(3+2p)x^2+cd(5+4p)x^4\right) dx = \left(cx^4+bx^2+a\right)^p cdx^5 + \left(cx^4+bx^2+a\right)^p bdx^3 + \left(cx^4+bx^2+a\right)^p adx$$

input
```
integrate((c*x^4+b*x^2+a)^p*(a*d+b*d*(3+2*p)*x^2+c*d*(5+4*p)*x^4),x, algor
ithm="giac")
```

output
```
(c*x^4 + b*x^2 + a)^p*c*d*x^5 + (c*x^4 + b*x^2 + a)^p*b*d*x^3 + (c*x^4 + b
*x^2 + a)^p*a*d*x
```

Mupad [B] (verification not implemented)

Time = 18.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int (a+bx^2+cx^4)^p \left(ad+bd(3+2p)x^2+cd(5+4p)x^4\right) dx = \left(c\,x^4+b\,x^2+a\right)^p \left(c\,d\,x^5+b\,d\,x^3+a\,d\,x\right)$$

input
```
int((a*d + b*d*x^2*(2*p + 3) + c*d*x^4*(4*p + 5))*(a + b*x^2 + c*x^4)^p,x)
```

output
```
(a + b*x^2 + c*x^4)^p*(a*d*x + b*d*x^3 + c*d*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int (a+bx^2+cx^4)^p (ad+bd(3+2p)x^2+cd(5+4p)x^4)\ dx$$
$$=(c\,x^4+b\,x^2+a)^p\,dx(c\,x^4+b\,x^2+a)$$

input
```
int((c*x^4+b*x^2+a)^p*(a*d+b*d*(3+2*p)*x^2+c*d*(5+4*p)*x^4),x)
```

output
```
(a + b*x**2 + c*x**4)**p*d*x*(a + b*x**2 + c*x**4)
```

3.24 $\int (a+bx^2+cx^4)^p (3a^2d+3a^2ex^2+(abe(5+2p)+3$

Optimal result

Integrand size = 94, antiderivative size = 40

$$\int (a+bx^2+cx^4)^p \left(3a^2d+3a^2ex^2+\left(abe(5+2p)+3acd(5+4p)-b^2d\left(15+16p+4p^2\right)\right)x^4 + c(7+4p)(ae-bd(3+2p))x^6\right)\,dx = x\left(3ad+(ae-bd(3+2p))x^2\right)\left(a+bx^2+cx^4\right)^{1+p}$$

output
```
x*(3*a*d+(a*e-b*d*(3+2*p))*x^2)*(c*x^4+b*x^2+a)^(p+1)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int (a+bx^2+cx^4)^p \left(3a^2d+3a^2ex^2+\left(abe(5+2p)+3acd(5+4p)-b^2d\left(15+16p+4p^2\right)\right)x^4 + c(7+4p)(ae-bd(3+2p))x^6\right)\,dx = x\left(a+bx^2+cx^4\right)^{1+p}\left(-bd(3+2p)x^2+a\left(3d+ex^2\right)\right)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^p*(3*a^2*d + 3*a^2*e*x^2 + (a*b*e*(5 + 2*p)
+ 3*a*c*d*(5 + 4*p) - b^2*d*(15 + 16*p + 4*p^2))*x^4 + c*(7 + 4*p)*(a*e -
b*d*(3 + 2*p))*x^6),x]
```

output
```
x*(a + b*x^2 + c*x^4)^(1 + p)*(-(b*d*(3 + 2*p)*x^2) + a*(3*d + e*x^2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.011$, Rules used = {2204}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2+cx^4)^p \left(3a^2d+3a^2ex^2+x^4\left(abe(2p+5)+3acd(4p+5)+b^2(-d)\left(4p^2+16p+15\right)\right)+c(4p+7)x^6\right.$$

$$\Big\downarrow \text{ 2204}$$

$$x\left(a+bx^2+cx^4\right)^{p+1}\left(x^2(ae-bd(2p+3))+3ad\right)$$

input
```
Int[(a + b*x^2 + c*x^4)^p*(3*a^2*d + 3*a^2*e*x^2 + (a*b*e*(5 + 2*p) + 3*a*
c*d*(5 + 4*p) - b^2*d*(15 + 16*p + 4*p^2))*x^4 + c*(7 + 4*p)*(a*e - b*d*(3
 + 2*p))*x^6),x]
```

output
```
x*(3*a*d + (a*e - b*d*(3 + 2*p))*x^2)*(a + b*x^2 + c*x^4)^(1 + p)
```

Defintions of rubi rules used

rule 2204
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{d
= Coeff[Px, x, 0], e = Coeff[Px, x, 2], f = Coeff[Px, x, 4], g = Coeff[Px,
x, 6]}, Simp[x*(3*a*d + (a*e - b*d*(2*p + 3))*x^2)*((a + b*x^2 + c*x^4)^(p
+ 1)/(3*a^2)), x] /; EqQ[3*a^2*g - c*(4*p + 7)*(a*e - b*d*(2*p + 3)), 0] &&
 EqQ[3*a^2*f - 3*a*c*d*(4*p + 5) - b*(2*p + 5)*(a*e - b*d*(2*p + 3)), 0]] /
; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && EqQ[Expon[Px, x], 6]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

method	result
gosper	$x(c\,x^4+b\,x^2+a)^{p+1}\,(-2bdp\,x^2+ae\,x^2-3bd\,x^2+3ad)$
risch	$x(-2bcdp\,x^6+ace\,x^6-3bcd\,x^6-2b^2dp\,x^4+abe\,x^4+3acd\,x^4-3b^2d\,x^4-2abdp\,x^2+a^2e$
norman	$(-2b^2dp+abe+3dac-3b^2d)\,x^5\mathrm{e}^{p\ln(c\,x^4+b\,x^2+a)}+a(-2bdp+ae)\,x^3\mathrm{e}^{p\ln(c\,x^4+b\,x^2+a)}+c(-2$
parallelrisch	$\frac{-2x^7(c\,x^4+b\,x^2+a)^p b\,c^2dp+x^7(c\,x^4+b\,x^2+a)^p a\,c^2e-3x^7(c\,x^4+b\,x^2+a)^p b\,c^2d-2x^5(c\,x^4+b\,x^2+a)^p b^2cdp+x^5(c\,x^4+b\,x^2+a}{}$
orering	$\frac{(-2bdp\,x^2+ae\,x^2-3bd\,x^2+3ad)x(c\,x^4+b\,x^2+a)(c\,x^4+b\,x^2+a)^p(3a^2d+3a^2e\,x^2+(abe(5+2p)+3acd(5+4p)-b^2d(4p^2+16p+}{-8bcd\,p^2x^6+4acep\,x^6-26bcdp\,x^6+7ace\,x^6-4b^2d\,p^2x^4-21bcd\,x^6+2abep\,x^4+12acdp\,x^4-16b^2dp\,x^4+5abe\,x^4+15acd}$

input
```
int((c*x^4+b*x^2+a)^p*(3*a^2*d+3*a^2*e*x^2+(a*b*e*(5+2*p)+3*a*c*d*(5+4*p)-
b^2*d*(4*p^2+16*p+15))*x^4+c*(7+4*p)*(a*e-b*d*(3+2*p))*x^6),x,method=_RETU
RNVERBOSE)
```

output
```
x*(c*x^4+b*x^2+a)^(p+1)*(-2*b*d*p*x^2+a*e*x^2-3*b*d*x^2+3*a*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(41) = 82.

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \left(a+bx^2+cx^4\right)^p \left(3a^2d+3a^2ex^2+\left(abe(5+2p)+3acd(5+4p)-b^2d\left(15+16p+4p^2\right)\right)x^4 + c(7+4p)(ae-bd(3+2p))x^6\right)\,dx = -\left((2\,bcdp+3\,bcd-ace)x^7+\left(2\,b^2dp-abe+3\,\left(b^2-ac\right)d\right)x^5-3\,a^2dx+\left(2\,abdp-a^2e\right)x^3\right)\left(cx^4+bx^2\right.$$

input
```
integrate((c*x^4+b*x^2+a)^p*(3*a^2*d+3*a^2*e*x^2+(a*b*e*(5+2*p)+3*a*c*d*(5
+4*p)-b^2*d*(4*p^2+16*p+15))*x^4+c*(7+4*p)*(a*e-b*d*(3+2*p))*x^6),x, algor
ithm="fricas")
```

output
```
-((2*b*c*d*p + 3*b*c*d - a*c*e)*x^7 + (2*b^2*d*p - a*b*e + 3*(b^2 - a*c)*d
)*x^5 - 3*a^2*d*x + (2*a*b*d*p - a^2*e)*x^3)*(c*x^4 + b*x^2 + a)^p
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(36) = 72$.

Time = 169.22 (sec) , antiderivative size = 235, normalized size of antiderivative = 5.88

$$\int (a+bx^2+cx^4)^p \left(3a^2d+3a^2ex^2+\left(abe(5+2p)+3acd(5+4p)-b^2d\left(15+16p+4p^2\right)\right)x^4 + c(7+4p)(ae-bd(3+2p))x^6\right) dx = 3a^2dx\left(a+bx^2+cx^4\right)^p + a^2ex^3\left(a+bx^2+cx^4\right)^p - 2abdpx^3\left(a+bx^2+cx^4\right)^p + abex^5\left(a+bx^2+cx^4\right)^p + 3acdx^5\left(a+bx^2+cx^4\right)^p + acex^7\left(a+bx^2+cx^4\right)^p - 2b^2dpx^5\left(a+bx^2+cx^4\right)^p - 3b^2dx^5\left(a+bx^2+cx^4\right)^p - 2bcdpx^7\left(a+bx^2+cx^4\right)^p - 3bcdx^7\left(a+bx^2+cx^4\right)^p$$

input
```
integrate((c*x**4+b*x**2+a)**p*(3*a**2*d+3*a**2*e*x**2+(a*b*e*(5+2*p)+3*a*
c*d*(5+4*p)-b**2*d*(4*p**2+16*p+15))*x**4+c*(7+4*p)*(a*e-b*d*(3+2*p))*x**6
),x)
```

output
```
3*a**2*d*x*(a + b*x**2 + c*x**4)**p + a**2*e*x**3*(a + b*x**2 + c*x**4)**p
 - 2*a*b*d*p*x**3*(a + b*x**2 + c*x**4)**p + a*b*e*x**5*(a + b*x**2 + c*x*
*4)**p + 3*a*c*d*x**5*(a + b*x**2 + c*x**4)**p + a*c*e*x**7*(a + b*x**2 +
c*x**4)**p - 2*b**2*d*p*x**5*(a + b*x**2 + c*x**4)**p - 3*b**2*d*x**5*(a +
 b*x**2 + c*x**4)**p - 2*b*c*d*p*x**7*(a + b*x**2 + c*x**4)**p - 3*b*c*d*x
**7*(a + b*x**2 + c*x**4)**p
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(41) = 82$.

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.15

$$\int (a+bx^2+cx^4)^p \left(3a^2d+3a^2ex^2+\left(abe(5+2p)+3acd(5+4p)-b^2d\left(15+16p+4p^2\right)\right)x^4 + c(7+4p)(ae-bd(3+2p))x^6\right) dx = -\left((bcd(2\,p+3)-ace)x^7+\left(b^2d(2\,p+3)-(3\,cd+be)a\right)x^5-3\,a^2dx+\left(2\,abdp-a^2e\right)x^3\right)\left(cx^4+bx^2+\right.$$

input
```
integrate((c*x^4+b*x^2+a)^p*(3*a^2*d+3*a^2*e*x^2+(a*b*e*(5+2*p)+3*a*c*d*(5
+4*p)-b^2*d*(4*p^2+16*p+15))*x^4+c*(7+4*p)*(a*e-b*d*(3+2*p))*x^6),x, algor
ithm="maxima")
```

output
```
-((b*c*d*(2*p + 3) - a*c*e)*x^7 + (b^2*d*(2*p + 3) - (3*c*d + b*e)*a)*x^5
- 3*a^2*d*x + (2*a*b*d*p - a^2*e)*x^3)*(c*x^4 + b*x^2 + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(41) = 82$.

Time = 0.21 (sec) , antiderivative size = 223, normalized size of antiderivative = 5.58

$$\int (a+bx^2+cx^4)^p \left(3a^2d+3a^2ex^2+\left(abe(5+2p)+3acd(5+4p)-b^2d\left(15+16p+4p^2\right)\right)x^4 + c(7+4p)(ae-bd(3+2p))x^6\right) dx = -2\left(cx^4+bx^2+a\right)^p bcdpx^7 - 3\left(cx^4+bx^2+a\right)^p bcdx^7 + \left(cx^4+bx^2+a\right)^p acex^7 - 2\left(cx^4+bx^2+a\right)^p b^2dpx^5 - 3\left(cx^4+bx^2+a\right)^p b^2dx^5 + 3\left(cx^4+bx^2+a\right)^p acdx^5 + \left(cx^4+bx^2+a\right)^p abex^5 - 2\left(cx^4+bx^2+a\right)^p abdpx^3 + \left(cx^4+bx^2+a\right)^p a^2ex^3 + 3\left(cx^4+bx^2+a\right)^p a^2dx$$

input
```
integrate((c*x^4+b*x^2+a)^p*(3*a^2*d+3*a^2*e*x^2+(a*b*e*(5+2*p)+3*a*c*d*(5
+4*p)-b^2*d*(4*p^2+16*p+15))*x^4+c*(7+4*p)*(a*e-b*d*(3+2*p))*x^6),x, algor
ithm="giac")
```

output
```
-2*(c*x^4 + b*x^2 + a)^p*b*c*d*p*x^7 - 3*(c*x^4 + b*x^2 + a)^p*b*c*d*x^7 +
 (c*x^4 + b*x^2 + a)^p*a*c*e*x^7 - 2*(c*x^4 + b*x^2 + a)^p*b^2*d*p*x^5 - 3
*(c*x^4 + b*x^2 + a)^p*b^2*d*x^5 + 3*(c*x^4 + b*x^2 + a)^p*a*c*d*x^5 + (c*
x^4 + b*x^2 + a)^p*a*b*e*x^5 - 2*(c*x^4 + b*x^2 + a)^p*a*b*d*p*x^3 + (c*x^
4 + b*x^2 + a)^p*a^2*e*x^3 + 3*(c*x^4 + b*x^2 + a)^p*a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 18.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.20

$$\int (a+bx^2+cx^4)^p \left(3a^2d+3a^2ex^2+\left(abe(5+2p)+3acd(5+4p)-b^2d\left(15+16p+4p^2\right)\right)x^4 + c(7+4p)(ae-bd(3+2p))x^6\right) dx = \left(c\,x^4+b\,x^2+a\right)^p \left(x^3\left(a^2\,e-2\,a\,b\,d\,p\right) - x^5\left(3\,b^2\,d-a\,b\,e-3\,a\,c\,d+2\,b^2\,d\,p\right) - c\,x^7\left(3\,b\,d-a\,e+2\,b\,d\,p\right) + 3\,a^2\,d\,x\right)$$

input
```
int((a + b*x^2 + c*x^4)^p*(3*a^2*d + x^4*(a*b*e*(2*p + 5) - b^2*d*(16*p +
4*p^2 + 15) + 3*a*c*d*(4*p + 5)) + 3*a^2*e*x^2 + c*x^6*(4*p + 7)*(a*e - b*
d*(2*p + 3))),x)
```

output
```
(a + b*x^2 + c*x^4)^p*(x^3*(a^2*e - 2*a*b*d*p) - x^5*(3*b^2*d - a*b*e - 3*
a*c*d + 2*b^2*d*p) - c*x^7*(3*b*d - a*e + 2*b*d*p) + 3*a^2*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.45

$$\int (a+bx^2+cx^4)^p \left(3a^2d+3a^2ex^2+\left(abe(5+2p)+3acd(5+4p)-b^2d\left(15+16p+4p^2\right)\right)x^4 + c(7+4p)(ae-bd(3+2p))x^6\right)\,dx = \left(c\,x^4+b\,x^2+a\right)^p x\left(-2bcdp\,x^6+ace\,x^6-3bcd\,x^6 - 2b^2dp\,x^4+abe\,x^4+3acd\,x^4-3b^2d\,x^4-2abdp\,x^2+a^2e\,x^2+3a^2d\right)$$

input
```
int((c*x^4+b*x^2+a)^p*(3*a^2*d+3*a^2*e*x^2+(a*b*e*(5+2*p)+3*a*c*d*(5+4*p)-
b^2*d*(4*p^2+16*p+15))*x^4+c*(7+4*p)*(a*e-b*d*(3+2*p))*x^6),x)
```

output
```
(a + b*x**2 + c*x**4)**p*x*(3*a**2*d + a**2*e*x**2 - 2*a*b*d*p*x**2 + a*b*
e*x**4 + 3*a*c*d*x**4 + a*c*e*x**6 - 2*b**2*d*p*x**4 - 3*b**2*d*x**4 - 2*b
*c*d*p*x**6 - 3*b*c*d*x**6)
```

3.25 $\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx = -\frac{1}{6}(d+4f)\text{arctanh}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\text{arctanh}(x) -\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

output
```
-1/6*(d+4*f)*arctanh(1/2*x)+1/3*(d+f)*arctanh(x)-1/6*e*ln(-x^2+1)+1/6*e*ln
(-x^2+4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx = \frac{1}{12}(-2(d+e+f)\log(1-x)+(d+2e+4f)\log(2-x) +2(d-e+f)\log(1+x)-(d-2e+4f)\log(2+x))$$

input
```
Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4),x]
```

output
```
(-2*(d + e + f)*Log[1 - x] + (d + 2*e + 4*f)*Log[2 - x] + 2*(d - e + f)*Lo
g[1 + x] - (d - 2*e + 4*f)*Log[2 + x])/12
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2202, 27, 1432, 1081, 1480, 220, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{x^4-5x^2+4} \, dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{fx^2+d}{x^4-5x^2+4}dx+\int \frac{ex}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{fx^2+d}{x^4-5x^2+4}dx+e\int \frac{x}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{fx^2+d}{x^4-5x^2+4}dx+\frac{1}{2}e\int \frac{1}{x^4-5x^2+4}dx^2$$

$$\Big\downarrow \text{1081}$$

$$\int \frac{fx^2+d}{x^4-5x^2+4}dx+\frac{1}{2}e\int \left(\frac{1}{3(1-x^2)}-\frac{1}{3(4-x^2)}\right)dx^2$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{3}(d+4f)\int \frac{1}{x^2-4}dx-\frac{1}{3}(d+f)\int \frac{1}{x^2-1}dx+\frac{1}{2}e\int \left(\frac{1}{3(1-x^2)}-\frac{1}{3(4-x^2)}\right)dx^2$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}e\int \left(\frac{1}{3(1-x^2)}-\frac{1}{3(4-x^2)}\right)dx^2-\frac{1}{6}\text{arctanh}\left(\frac{x}{2}\right)(d+4f)+\frac{1}{3}\text{arctanh}(x)(d+f)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}\text{arctanh}\left(\frac{x}{2}\right)(d+4f)+\frac{1}{3}\text{arctanh}(x)(d+f)+\frac{1}{2}e\left(\frac{1}{3}\log\left(4-x^2\right)-\frac{1}{3}\log\left(1-x^2\right)\right)$$

input
```
Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4),x]
```

output
```
-1/6*((d + 4*f)*ArcTanh[x/2]) + ((d + f)*ArcTanh[x])/3 + (e*(-1/3*Log[1 -
x^2] + Log[4 - x^2]/3))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

method	result
default	$\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{6}-\frac{e}{6}-\frac{f}{6}\right)\ln\left(x-1\right)+\left(-\frac{d}{12}+\frac{e}{6}-\right.$
norman	$\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{6}-\frac{e}{6}-\frac{f}{6}\right)\ln\left(x-1\right)+\left(-\frac{d}{12}+\frac{e}{6}-\right.$
parallelrisch	$\frac{\ln(x-2)d}{12}+\frac{\ln(x-2)e}{6}+\frac{\ln(x-2)f}{3}-\frac{\ln(x-1)d}{6}-\frac{\ln(x-1)e}{6}-\frac{\ln(x-1)f}{6}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}$
risch	$\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}+\frac{\ln(2-x)d}{12}+\frac{\ln(2-x)e}{6}+\frac{\ln(2-x)f}{3}-\frac{\ln(1-x)d}{6}-\frac{\ln(1-x)e}{6}-\frac{\ln(1-x)f}{6}$

input
```
int((f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
(1/12*d+1/6*e+1/3*f)*ln(x-2)+(1/6*d-1/6*e+1/6*f)*ln(1+x)+(-1/6*d-1/6*e-1/6
*f)*ln(x-1)+(-1/12*d+1/6*e-1/3*f)*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx = -\frac{1}{12}\left(d-2\,e+4\,f\right)\log\left(x+2\right)+\frac{1}{6}\left(d-e+f\right)\log\left(x+1\right) -\frac{1}{6}\left(d+e+f\right)\log\left(x-1\right)+\frac{1}{12}\left(d+2\,e+4\,f\right)\log\left(x-2\right)$$

input
```
integrate((f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
-1/12*(d - 2*e + 4*f)*log(x + 2) + 1/6*(d - e + f)*log(x + 1) - 1/6*(d + e
 + f)*log(x - 1) + 1/12*(d + 2*e + 4*f)*log(x - 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2195 vs. 2(44) = 88.

Time = 62.79 (sec) , antiderivative size = 2195, normalized size of antiderivative = 43.04

$$\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

output
```
-(d - 2*e + 4*f)*log(x + (-35*d**5*e + 51*d**5*(d - 2*e + 4*f)/2 - 820*d**
4*e*f + 90*d**4*f*(d - 2*e + 4*f) - 180*d**3*e**3 - 90*d**3*e**2*(d - 2*e
+ 4*f) - 4100*d**3*e*f**2 + 41*d**3*e*(d - 2*e + 4*f)**2 + 42*d**3*f**2*(d
 - 2*e + 4*f) - 15*d**3*(d - 2*e + 4*f)**3/2 - 432*d**2*e**2*f*(d - 2*e +
4*f) - 8000*d**2*e*f**3 + 240*d**2*e*f*(d - 2*e + 4*f)**2 - 240*d**2*f**3*
(d - 2*e + 4*f) - 12*d**2*f*(d - 2*e + 4*f)**3 + 320*d*e**5 - 96*d*e**4*(d
 - 2*e + 4*f) + 720*d*e**3*f**2 - 80*d*e**3*(d - 2*e + 4*f)**2 - 1080*d*e*
*2*f**2*(d - 2*e + 4*f) + 24*d*e**2*(d - 2*e + 4*f)**3 - 6400*d*e*f**4 + 4
92*d*e*f**2*(d - 2*e + 4*f)**2 - 576*d*f**4*(d - 2*e + 4*f) + 30*d*f**2*(d
 - 2*e + 4*f)**3 + 512*e**5*f - 128*e**3*f*(d - 2*e + 4*f)**2 - 576*e**2*f
**3*(d - 2*e + 4*f) - 1472*e*f**5 + 320*e*f**3*(d - 2*e + 4*f)**2 - 480*f*
*5*(d - 2*e + 4*f) + 48*f**3*(d - 2*e + 4*f)**3)/(9*d**6 + 45*d**5*f - 160
*d**4*e**2 - 36*d**4*f**2 - 1312*d**3*e**2*f - 360*d**3*f**3 + 256*d**2*e*
*4 - 3840*d**2*e**2*f**2 - 144*d**2*f**4 + 1280*d*e**4*f - 5248*d*e**2*f**
3 + 720*d*f**5 + 1024*e**4*f**2 - 2560*e**2*f**4 + 576*f**6))/12 + (d - e
+ f)*log(x + (-35*d**5*e - 51*d**5*(d - e + f) - 820*d**4*e*f - 180*d**4*f
*(d - e + f) - 180*d**3*e**3 + 180*d**3*e**2*(d - e + f) - 4100*d**3*e*f**
2 + 164*d**3*e*(d - e + f)**2 - 84*d**3*f**2*(d - e + f) + 60*d**3*(d - e
+ f)**3 + 864*d**2*e**2*f*(d - e + f) - 8000*d**2*e*f**3 + 960*d**2*e*f*(d
 - e + f)**2 + 480*d**2*f**3*(d - e + f) + 96*d**2*f*(d - e + f)**3 + 3...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx = -\frac{1}{12}\,(d-2\,e+4\,f)\log\left(x+2\right)+\frac{1}{6}\,(d-e+f)\log\left(x+1\right) -\frac{1}{6}\,(d+e+f)\log\left(x-1\right)+\frac{1}{12}\,(d+2\,e+4\,f)\log\left(x-2\right)$$

input
```
integrate((f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
-1/12*(d - 2*e + 4*f)*log(x + 2) + 1/6*(d - e + f)*log(x + 1) - 1/6*(d + e
 + f)*log(x - 1) + 1/12*(d + 2*e + 4*f)*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx = -\frac{1}{12}\,(d-2\,e+4\,f)\log\left(|x+2|\right)+\frac{1}{6}\,(d-e+f)\log\left(|x+1|\right) -\frac{1}{6}\,(d+e+f)\log\left(|x-1|\right)+\frac{1}{12}\,(d+2\,e+4\,f)\log\left(|x-2|\right)$$

input
```
integrate((f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
-1/12*(d - 2*e + 4*f)*log(abs(x + 2)) + 1/6*(d - e + f)*log(abs(x + 1)) -
1/6*(d + e + f)*log(abs(x - 1)) + 1/12*(d + 2*e + 4*f)*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx = \ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)-\ln(x-1)\left(\frac{d}{6}+\frac{e}{6}+\frac{f}{6}\right) + \ln(x-2)\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}\right)-\ln(x+2)\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}\right)$$

input
```
int((d + e*x + f*x^2)/(x^4 - 5*x^2 + 4),x)
```

output
```
log(x + 1)*(d/6 - e/6 + f/6) - log(x - 1)*(d/6 + e/6 + f/6) + log(x - 2)*(
d/12 + e/6 + f/3) - log(x + 2)*(d/12 - e/6 + f/3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

$$\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx = \frac{\log(x-2)\,d}{12}+\frac{\log(x-2)\,e}{6}+\frac{\log(x-2)\,f}{3}-\frac{\log(x-1)\,d}{6} - \frac{\log(x-1)\,e}{6}-\frac{\log(x-1)\,f}{6}-\frac{\log(x+2)\,d}{12}+\frac{\log(x+2)\,e}{6} - \frac{\log(x+2)\,f}{3}+\frac{\log(x+1)\,d}{6}-\frac{\log(x+1)\,e}{6}+\frac{\log(x+1)\,f}{6}$$

input
```
int((f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
(log(x - 2)*d + 2*log(x - 2)*e + 4*log(x - 2)*f - 2*log(x - 1)*d - 2*log(x
 - 1)*e - 2*log(x - 1)*f - log(x + 2)*d + 2*log(x + 2)*e - 4*log(x + 2)*f
+ 2*log(x + 1)*d - 2*log(x + 1)*e + 2*log(x + 1)*f)/12
```

3.26 $\int \frac{d+ex+fx^2}{\left(4-5x^2+x^4\right)^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\begin{aligned}\int \frac{d+ex+fx^2}{\left(4-5x^2+x^4\right)^2} \, dx = {} & \frac{e}{18\left(1-x^2\right)}+\frac{e}{18\left(4-x^2\right)}+\frac{x(17d+20f-(5d+8f)x^2)}{72\left(4-5x^2+x^4\right)} \\ & +\frac{1}{432}(19d+52f)\text{arctanh}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\text{arctanh}(x) \\ & +\frac{1}{27}e\log\left(1-x^2\right)-\frac{1}{27}e\log\left(4-x^2\right)\end{aligned}$$

output
```
e/(-18*x^2+18)+e/(-18*x^2+72)+x*(17*d+20*f-(5*d+8*f)*x^2)/(72*x^4-360*x^2+
288)+1/432*(19*d+52*f)*arctanh(1/2*x)-1/54*(d+7*f)*arctanh(x)+1/27*e*ln(-x
^2+1)-1/27*e*ln(-x^2+4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{d+ex+fx^2}{\left(4-5x^2+x^4\right)^2} \, dx = {} & \frac{1}{864}\left(\frac{12(17dx+20fx-5dx^3-8fx^3+e(20-8x^2))}{4-5x^2+x^4}\right. \\ & +8(d+4e+7f)\log(1-x)-(19d+32e+52f)\log(2-x) \\ & \left.-8(d-4e+7f)\log(1+x)+(19d-32e+52f)\log(2+x)\right)\end{aligned}$$

input
```
Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^2,x]
```

output
```
((12*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2)))/(4 - 5*x^2 +
x^4) + 8*(d + 4*e + 7*f)*Log[1 - x] - (19*d + 32*e + 52*f)*Log[2 - x] - 8*
(d - 4*e + 7*f)*Log[1 + x] + (19*d - 32*e + 52*f)*Log[2 + x])/864
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2202, 27, 1432, 1084, 1492, 25, 1480, 220, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{\left(x^4-5x^2+4\right)^2}dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{fx^2+d}{\left(x^4-5x^2+4\right)^2}dx+\int \frac{ex}{\left(x^4-5x^2+4\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{fx^2+d}{\left(x^4-5x^2+4\right)^2}dx+e\int \frac{x}{\left(x^4-5x^2+4\right)^2}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{fx^2+d}{\left(x^4-5x^2+4\right)^2}dx+\frac{1}{2}e\int \frac{1}{\left(x^4-5x^2+4\right)^2}dx^2$$

$$\Big\downarrow \text{1084}$$

$$\int \frac{fx^2+d}{\left(x^4-5x^2+4\right)^2}dx+\frac{1}{2}e\int \left(\frac{2}{27\left(4-x^2\right)}+\frac{1}{9\left(4-x^2\right)^2}-\frac{2}{27\left(1-x^2\right)}+\frac{1}{9\left(1-x^2\right)^2}\right)dx^2$$

$$\Big\downarrow \text{1492}$$

$$-\frac{1}{72}\int -\frac{-\left((5d+8f)x^2\right)+d-20f}{x^4-5x^2+4}dx+$$
$$\frac{1}{2}e\int \left(\frac{2}{27\left(4-x^2\right)}+\frac{1}{9\left(4-x^2\right)^2}-\frac{2}{27\left(1-x^2\right)}+\frac{1}{9\left(1-x^2\right)^2}\right)dx^2+$$
$$\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{72\left(x^4-5x^2+4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{72}\int \frac{-\left((5d+8f)x^2\right)+d-20f}{x^4-5x^2+4}dx+$$
$$\frac{1}{2}e\int \left(\frac{2}{27\left(4-x^2\right)}+\frac{1}{9\left(4-x^2\right)^2}-\frac{2}{27\left(1-x^2\right)}+\frac{1}{9\left(1-x^2\right)^2}\right)dx^2+$$
$$\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{72\left(x^4-5x^2+4\right)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{72}\left(\frac{4}{3}(d+7f)\int \frac{1}{x^2-1}dx-\frac{1}{3}(19d+52f)\int \frac{1}{x^2-4}dx\right)+$$
$$\frac{1}{2}e\int \left(\frac{2}{27\left(4-x^2\right)}+\frac{1}{9\left(4-x^2\right)^2}-\frac{2}{27\left(1-x^2\right)}+\frac{1}{9\left(1-x^2\right)^2}\right)dx^2+$$
$$\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{72\left(x^4-5x^2+4\right)}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}e\int \left(\frac{2}{27\left(4-x^2\right)}+\frac{1}{9\left(4-x^2\right)^2}-\frac{2}{27\left(1-x^2\right)}+\frac{1}{9\left(1-x^2\right)^2}\right)dx^2+$$
$$\frac{1}{72}\left(\frac{1}{6}\text{arctanh}\left(\frac{x}{2}\right)(19d+52f)-\frac{4}{3}\text{arctanh}(x)(d+7f)\right)+\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{72\left(x^4-5x^2+4\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{72}\left(\frac{1}{6}\text{arctanh}\left(\frac{x}{2}\right)(19d+52f)-\frac{4}{3}\text{arctanh}(x)(d+7f)\right)+\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{72\left(x^4-5x^2+4\right)}+$$
$$\frac{1}{2}e\left(\frac{1}{9\left(1-x^2\right)}+\frac{1}{9\left(4-x^2\right)}+\frac{2}{27}\log\left(1-x^2\right)-\frac{2}{27}\log\left(4-x^2\right)\right)$$

input
```
Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^2,x]
```

output
```
(x*(17*d + 20*f - (5*d + 8*f)*x^2))/(72*(4 - 5*x^2 + x^4)) + (((19*d + 52*
f)*ArcTanh[x/2])/6 - (4*(d + 7*f)*ArcTanh[x])/3)/72 + (e*(1/(9*(1 - x^2))
+ 1/(9*(4 - x^2)) + (2*Log[1 - x^2])/27 - (2*Log[4 - x^2])/27))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

method	result
norman	$\frac{\left(-\frac{5d}{72}-\frac{f}{9}\right)x^3+\left(\frac{17d}{72}+\frac{5f}{18}\right)x-\frac{e\,x^2}{9}+\frac{5e}{18}}{x^4-5x^2+4}+\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}\right)\ln\left(x-2\right)+\left(-\frac{d}{108}+\frac{e}{27}-\frac{7f}{108}\right)\ln\left(1+x\right.$
default	$\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}\right)\ln\left(x-2\right)-\frac{\frac{d}{144}+\frac{e}{72}+\frac{f}{36}}{x-2}+\left(-\frac{d}{108}+\frac{e}{27}-\frac{7f}{108}\right)\ln\left(1+x\right)-\frac{\frac{d}{36}-\frac{e}{36}+\frac{f}{36}}{1+x}-\frac{d}{36}$
risch	$\frac{\left(-\frac{5d}{72}-\frac{f}{9}\right)x^3+\left(\frac{17d}{72}+\frac{5f}{18}\right)x-\frac{e\,x^2}{9}+\frac{5e}{18}}{x^4-5x^2+4}+\frac{19\ln(x+2)d}{864}-\frac{\ln(x+2)e}{27}+\frac{13\ln(x+2)f}{216}+\frac{\ln(1-x)d}{108}+\frac{\ln(1-x)e}{27}+\frac{7\ln(}{1}$
parallelrisch	$-\frac{-204dx-240fx-240e+128\ln(x-2)e+128\ln(x+2)e+40\ln(x-1)x^2d+160\ln(x-1)x^2e+60d\,x^3+96e\,x^2+208\ln(x-2)f+5}{}$

input
```
int((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
((-5/72*d-1/9*f)*x^3+(17/72*d+5/18*f)*x-1/9*e*x^2+5/18*e)/(x^4-5*x^2+4)+(-
19/864*d-1/27*e-13/216*f)*ln(x-2)+(-1/108*d+1/27*e-7/108*f)*ln(1+x)+(1/108
*d+1/27*e+7/108*f)*ln(x-1)+(19/864*d-1/27*e+13/216*f)*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(98) = 196.

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.82

$$\int \frac{d+ex+fx^2}{\left(4-5x^2+x^4\right)^2}\,dx =$$
$$-\frac{12\,(5\,d+8\,f)x^3+96\,ex^2-12\,(17\,d+20\,f)x-((19\,d-32\,e+52\,f)x^4-5\,(19\,d-32\,e+52\,f)x^2+7}{}$$

input
```
integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

output
```
-1/864*(12*(5*d + 8*f)*x^3 + 96*e*x^2 - 12*(17*d + 20*f)*x - ((19*d - 32*e
 + 52*f)*x^4 - 5*(19*d - 32*e + 52*f)*x^2 + 76*d - 128*e + 208*f)*log(x +
2) + 8*((d - 4*e + 7*f)*x^4 - 5*(d - 4*e + 7*f)*x^2 + 4*d - 16*e + 28*f)*l
og(x + 1) - 8*((d + 4*e + 7*f)*x^4 - 5*(d + 4*e + 7*f)*x^2 + 4*d + 16*e +
28*f)*log(x - 1) + ((19*d + 32*e + 52*f)*x^4 - 5*(19*d + 32*e + 52*f)*x^2
+ 76*d + 128*e + 208*f)*log(x - 2) - 240*e)/(x^4 - 5*x^2 + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2689 vs. 2(94) = 188.

Time = 67.78 (sec) , antiderivative size = 2689, normalized size of antiderivative = 22.60

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

output
```
-(d - 4*e + 7*f)*log(x + (-6006260*d**5*e + 2341251*d**5*(d - 4*e + 7*f) -
 246016240*d**4*e*f + 31626180*d**4*f*(d - 4*e + 7*f) - 18247680*d**3*e**3
 + 24099840*d**3*e**2*(d - 4*e + 7*f) - 2758371200*d**3*e*f**2 + 7387904*d
**3*e*(d - 4*e + 7*f)**2 + 171122976*d**3*f**2*(d - 4*e + 7*f) - 665280*d*
*3*(d - 4*e + 7*f)**3 + 298598400*d**2*e**3*f + 369487872*d**2*e**2*f*(d -
 4*e + 7*f) - 13192256000*d**2*e*f**3 + 90885120*d**2*e*f*(d - 4*e + 7*f)*
*2 + 441486720*d**2*f**3*(d - 4*e + 7*f) - 5536512*d**2*f*(d - 4*e + 7*f)*
*3 + 587202560*d*e**5 - 12582912*d*e**4*(d - 4*e + 7*f) + 1353646080*d*e**
3*f**2 - 36700160*d*e**3*(d - 4*e + 7*f)**2 + 1448755200*d*e**2*f**2*(d -
4*e + 7*f) + 786432*d*e**2*(d - 4*e + 7*f)**3 - 28282393600*d*e*f**4 + 362
729472*d*e*f**2*(d - 4*e + 7*f)**2 + 399575808*d*f**4*(d - 4*e + 7*f) - 10
368000*d*f**2*(d - 4*e + 7*f)**3 + 2751463424*e**5*f + 251658240*e**4*f*(d
 - 4*e + 7*f) - 530841600*e**3*f**3 - 171966464*e**3*f*(d - 4*e + 7*f)**2
+ 1935212544*e**2*f**3*(d - 4*e + 7*f) - 15728640*e**2*f*(d - 4*e + 7*f)**
3 - 21886889984*e*f**5 + 483737600*e*f**3*(d - 4*e + 7*f)**2 - 212474880*f
**5*(d - 4*e + 7*f) + 4534272*f**3*(d - 4*e + 7*f)**3)/(1675971*d**6 + 285
07545*d**5*f - 66150400*d**4*e**2 + 168075324*d**4*f**2 - 1091117056*d**3*
e**2*f + 384095520*d**3*f**3 + 318767104*d**2*e**4 - 6528860160*d**2*e**2*
f**2 + 162082944*d**2*f**4 + 3103784960*d*e**4*f - 17414619136*d*e**2*f**3
 - 305130240*d*f**5 + 6106906624*e**4*f**2 - 17414225920*e**2*f**4 + 67...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^2}\,dx = \frac{1}{864}\,(19\,d-32\,e+52\,f)\log\left(x+2\right) - \frac{1}{108}\,(d-4\,e+7\,f)\log\left(x+1\right)+\frac{1}{108}\,(d+4\,e+7\,f)\log\left(x-1\right) - \frac{1}{864}\,(19\,d+32\,e+52\,f)\log\left(x-2\right) - \frac{(5\,d+8\,f)x^3+8\,ex^2-(17\,d+20\,f)x-20\,e}{72\,(x^4-5\,x^2+4)}$$

input
```
integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

output
```
1/864*(19*d - 32*e + 52*f)*log(x + 2) - 1/108*(d - 4*e + 7*f)*log(x + 1) +
 1/108*(d + 4*e + 7*f)*log(x - 1) - 1/864*(19*d + 32*e + 52*f)*log(x - 2)
- 1/72*((5*d + 8*f)*x^3 + 8*e*x^2 - (17*d + 20*f)*x - 20*e)/(x^4 - 5*x^2 +
 4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^2}\,dx = \frac{1}{864}(19\,d-32\,e+52\,f)\log(|x+2|) - \frac{1}{108}(d-4\,e+7\,f)\log(|x+1|) + \frac{1}{108}(d+4\,e+7\,f)\log(|x-1|) - \frac{1}{864}(19\,d+32\,e+52\,f)\log(|x-2|) - \frac{5\,dx^3+8\,fx^3+8\,ex^2-17\,dx-20\,fx-20\,e}{72\,(x^4-5\,x^2+4)}$$

input
```
integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

output
```
1/864*(19*d - 32*e + 52*f)*log(abs(x + 2)) - 1/108*(d - 4*e + 7*f)*log(abs
(x + 1)) + 1/108*(d + 4*e + 7*f)*log(abs(x - 1)) - 1/864*(19*d + 32*e + 52
*f)*log(abs(x - 2)) - 1/72*(5*d*x^3 + 8*f*x^3 + 8*e*x^2 - 17*d*x - 20*f*x
- 20*e)/(x^4 - 5*x^2 + 4)
```

Mupad [B] (verification not implemented)

Time = 17.99 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^2}\,dx = \ln(x-1)\left(\frac{d}{108}+\frac{e}{27}+\frac{7\,f}{108}\right) - \ln(x+1)\left(\frac{d}{108}-\frac{e}{27}+\frac{7\,f}{108}\right) - \ln(x-2)\left(\frac{19\,d}{864}+\frac{e}{27}+\frac{13\,f}{216}\right) + \ln(x+2)\left(\frac{19\,d}{864}-\frac{e}{27}+\frac{13\,f}{216}\right) + \frac{\left(-\frac{5\,d}{72}-\frac{f}{9}\right)x^3-\frac{e\,x^2}{9}+\left(\frac{17\,d}{72}+\frac{5\,f}{18}\right)x+\frac{5\,e}{18}}{x^4-5\,x^2+4}$$

input
```
int((d + e*x + f*x^2)/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
log(x - 1)*(d/108 + e/27 + (7*f)/108) - log(x + 1)*(d/108 - e/27 + (7*f)/1
08) - log(x - 2)*((19*d)/864 + e/27 + (13*f)/216) + log(x + 2)*((19*d)/864
 - e/27 + (13*f)/216) + ((5*e)/18 - x^3*((5*d)/72 + f/9) - (e*x^2)/9 + x*(
(17*d)/72 + (5*f)/18))/(x^4 - 5*x^2 + 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.10

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{816e - 380\log(x-2)\,d + 380\log(x+2)\,d - 640\log(x-2)\,e - 1040\log(x-2)\,f + 160\log(x-1)\,d + 6}{}$$

input
```
int((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
( - 95*log(x - 2)*d*x**4 + 475*log(x - 2)*d*x**2 - 380*log(x - 2)*d - 160*
log(x - 2)*e*x**4 + 800*log(x - 2)*e*x**2 - 640*log(x - 2)*e - 260*log(x -
 2)*f*x**4 + 1300*log(x - 2)*f*x**2 - 1040*log(x - 2)*f + 40*log(x - 1)*d*
x**4 - 200*log(x - 1)*d*x**2 + 160*log(x - 1)*d + 160*log(x - 1)*e*x**4 -
800*log(x - 1)*e*x**2 + 640*log(x - 1)*e + 280*log(x - 1)*f*x**4 - 1400*lo
g(x - 1)*f*x**2 + 1120*log(x - 1)*f + 95*log(x + 2)*d*x**4 - 475*log(x + 2
)*d*x**2 + 380*log(x + 2)*d - 160*log(x + 2)*e*x**4 + 800*log(x + 2)*e*x**
2 - 640*log(x + 2)*e + 260*log(x + 2)*f*x**4 - 1300*log(x + 2)*f*x**2 + 10
40*log(x + 2)*f - 40*log(x + 1)*d*x**4 + 200*log(x + 1)*d*x**2 - 160*log(x
 + 1)*d + 160*log(x + 1)*e*x**4 - 800*log(x + 1)*e*x**2 + 640*log(x + 1)*e
 - 280*log(x + 1)*f*x**4 + 1400*log(x + 1)*f*x**2 - 1120*log(x + 1)*f - 30
0*d*x**3 + 1020*d*x - 96*e*x**4 + 816*e - 480*f*x**3 + 1200*f*x)/(4320*(x*
*4 - 5*x**2 + 4))
```

3.27 $\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\begin{aligned}\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^3} dx &= \frac{e}{108(1-x^2)^2} - \frac{e}{54(1-x^2)} - \frac{e}{108(4-x^2)^2} \\ &- \frac{e}{54(4-x^2)} + \frac{x(17d+20f-(5d+8f)x^2)}{144(4-5x^2+x^4)^2} \\ &- \frac{x(59d+380f-35(d+4f)x^2)}{3456(4-5x^2+x^4)} \\ &- \frac{(313d+820f)\operatorname{arctanh}\left(\frac{x}{2}\right)}{20736} + \frac{1}{648}(13d+25f)\operatorname{arctanh}(x) \\ &- \frac{1}{81}e\log\left(1-x^2\right) + \frac{1}{81}e\log\left(4-x^2\right)\end{aligned}$$

output
```
1/108*e/(-x^2+1)^2-e/(-54*x^2+54)-1/108*e/(-x^2+4)^2-e/(-54*x^2+216)+1/144
*x*(17*d+20*f-(5*d+8*f)*x^2)/(x^4-5*x^2+4)^2-x*(59*d+380*f-35*(d+4*f)*x^2)
/(3456*x^4-17280*x^2+13824)-1/20736*(313*d+820*f)*arctanh(1/2*x)+1/648*(13
*d+25*f)*arctanh(x)-1/81*e*ln(-x^2+1)+1/81*e*ln(-x^2+4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^3}\,dx = \frac{\frac{288(17dx+20fx-5dx^3-8fx^3+e(20-8x^2))}{(4-5x^2+x^4)^2}+\frac{12(64e(-5+2x^2)+20fx(-19+7x^2)+dx(-59+35x^2))}{4-5x^2+x^4}-32(13d+16e+25f)\log(\cdots}{\cdots}$$

input
```
Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^3,x]
```

output
```
((288*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2)))/(4 - 5*x^2 +
 x^4)^2 + (12*(64*e*(-5 + 2*x^2) + 20*f*x*(-19 + 7*x^2) + d*x*(-59 + 35*x^
2)))/(4 - 5*x^2 + x^4) - 32*(13*d + 16*e + 25*f)*Log[1 - x] + (313*d + 512
*e + 820*f)*Log[2 - x] + 32*(13*d - 16*e + 25*f)*Log[1 + x] + (-313*d + 51
2*e - 820*f)*Log[2 + x])/41472
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {2202, 27, 1432, 1084, 1492, 25, 1492, 27, 1480, 220, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(x^4-5x^2+4)^3}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{fx^2+d}{(x^4-5x^2+4)^3}dx+\int \frac{ex}{(x^4-5x^2+4)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{fx^2+d}{(x^4-5x^2+4)^3}dx+e\int \frac{x}{(x^4-5x^2+4)^3}dx$$

$$\Big\downarrow \; 1432$$

$$\int \frac{fx^2+d}{(x^4-5x^2+4)^3}dx+\frac{1}{2}e\int \frac{1}{(x^4-5x^2+4)^3}dx^2$$

$$\Big\downarrow \; 1084$$

$$\int \frac{fx^2+d}{(x^4-5x^2+4)^3}dx+\frac{1}{2}e\int \left(-\frac{2}{81(4-x^2)}-\frac{1}{27(4-x^2)^2}-\frac{1}{27(4-x^2)^3}+\frac{2}{81(1-x^2)}-\frac{1}{27(1-x^2)^2}+\frac{1}{27(1-x^2)^3}\right)dx^2$$

$$\Big\downarrow \; 1492$$

$$-\frac{1}{144}\int -\frac{-5(5d+8f)x^2+19d-20f}{(x^4-5x^2+4)^2}dx+\frac{1}{2}e\int \left(-\frac{2}{81(4-x^2)}-\frac{1}{27(4-x^2)^2}-\frac{1}{27(4-x^2)^3}+\frac{2}{81(1-x^2)}-\frac{1}{27(1-x^2)^2}+\frac{1}{27(1-x^2)^3}\right)dx^2+\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{144(x^4-5x^2+4)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{144}\int \frac{-5(5d+8f)x^2+19d-20f}{(x^4-5x^2+4)^2}dx+\frac{1}{2}e\int \left(-\frac{2}{81(4-x^2)}-\frac{1}{27(4-x^2)^2}-\frac{1}{27(4-x^2)^3}+\frac{2}{81(1-x^2)}-\frac{1}{27(1-x^2)^2}+\frac{1}{27(1-x^2)^3}\right)dx^2+\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{144(x^4-5x^2+4)^2}$$

$$\Big\downarrow \; 1492$$

$$\frac{1}{144}\left(-\frac{1}{72}\int -\frac{3\left(35(d+4f)x^2+173d+260f\right)}{x^4-5x^2+4}dx-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{24(x^4-5x^2+4)}\right)+\frac{1}{2}e\int \left(-\frac{2}{81(4-x^2)}-\frac{1}{27(4-x^2)^2}-\frac{1}{27(4-x^2)^3}+\frac{2}{81(1-x^2)}-\frac{1}{27(1-x^2)^2}+\frac{1}{27(1-x^2)^3}\right)dx^2+\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{144(x^4-5x^2+4)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{144}\left(\frac{1}{24}\int \frac{35(d+4f)x^2+173d+260f}{x^4-5x^2+4}dx-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{24\left(x^4-5x^2+4\right)}\right)+$$
$$\frac{1}{2}e\int \left(-\frac{2}{81\left(4-x^2\right)}-\frac{1}{27\left(4-x^2\right)^2}-\frac{1}{27\left(4-x^2\right)^3}+\frac{2}{81\left(1-x^2\right)}-\frac{1}{27\left(1-x^2\right)^2}+\frac{1}{27\left(1-x^2\right)^3}\right)dx^2+$$
$$\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{144}\left(\frac{1}{24}\left(\frac{1}{3}(313d+820f)\int \frac{1}{x^2-4}dx-\frac{16}{3}(13d+25f)\int \frac{1}{x^2-1}dx\right)-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{24\left(x^4-5x^2+4\right)}\right)+$$
$$\frac{1}{2}e\int \left(-\frac{2}{81\left(4-x^2\right)}-\frac{1}{27\left(4-x^2\right)^2}-\frac{1}{27\left(4-x^2\right)^3}+\frac{2}{81\left(1-x^2\right)}-\frac{1}{27\left(1-x^2\right)^2}+\frac{1}{27\left(1-x^2\right)^3}\right)dx^2+$$
$$\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{1}{2}e\int \left(-\frac{2}{81\left(4-x^2\right)}-\frac{1}{27\left(4-x^2\right)^2}-\frac{1}{27\left(4-x^2\right)^3}+\frac{2}{81\left(1-x^2\right)}-\frac{1}{27\left(1-x^2\right)^2}+\frac{1}{27\left(1-x^2\right)^3}\right)dx^2+$$
$$\frac{1}{144}\left(\frac{1}{24}\left(\frac{16}{3}\text{arctanh}(x)(13d+25f)-\frac{1}{6}\text{arctanh}\left(\frac{x}{2}\right)(313d+820f)\right)-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{24\left(x^4-5x^2+4\right)}\right)+$$
$$\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{144}\left(\frac{1}{24}\left(\frac{16}{3}\text{arctanh}(x)(13d+25f)-\frac{1}{6}\text{arctanh}\left(\frac{x}{2}\right)(313d+820f)\right)-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{24\left(x^4-5x^2+4\right)}\right)+$$
$$\frac{x\left(-\left(x^2(5d+8f)\right)+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}+$$
$$\frac{1}{2}e\left(-\frac{1}{27\left(1-x^2\right)}-\frac{1}{27\left(4-x^2\right)}+\frac{1}{54\left(1-x^2\right)^2}-\frac{1}{54\left(4-x^2\right)^2}-\frac{2}{81}\log\left(1-x^2\right)+\frac{2}{81}\log\left(4-x^2\right)\right)$$

input
```
Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^3,x]
```

output
```
(x*(17*d + 20*f - (5*d + 8*f)*x^2))/(144*(4 - 5*x^2 + x^4)^2) + (-1/24*(x*
(59*d + 380*f - 35*(d + 4*f)*x^2))/(4 - 5*x^2 + x^4) + (-1/6*((313*d + 820
*f)*ArcTanh[x/2]) + (16*(13*d + 25*f)*ArcTanh[x])/3)/24)/144 + (e*(1/(54*(
1 - x^2)^2) - 1/(27*(1 - x^2)) - 1/(54*(4 - x^2)^2) - 1/(27*(4 - x^2)) - (
2*Log[1 - x^2])/81 + (2*Log[4 - x^2])/81))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

method	result
norman	$\frac{\left(-\frac{13d}{192}-\frac{5f}{16}\right)x^5+\left(\frac{35d}{384}+\frac{21f}{32}\right)x^3+\left(\frac{35d}{3456}+\frac{35f}{864}\right)x^7+\left(\frac{43d}{864}-\frac{65f}{216}\right)x-\frac{5e\,x^4}{18}+\frac{e\,x^6}{27}+\frac{5e\,x^2}{9}-\frac{25e}{108}}{\left(x^4-5x^2+4\right)^2}+\left(-\frac{313d}{41472}+\frac{e}{81}-\frac{205f}{10368}\right.$
risch	$\frac{\left(-\frac{13d}{192}-\frac{5f}{16}\right)x^5+\left(\frac{35d}{384}+\frac{21f}{32}\right)x^3+\left(\frac{35d}{3456}+\frac{35f}{864}\right)x^7+\left(\frac{43d}{864}-\frac{65f}{216}\right)x-\frac{5e\,x^4}{18}+\frac{e\,x^6}{27}+\frac{5e\,x^2}{9}-\frac{25e}{108}}{\left(x^4-5x^2+4\right)^2}+\frac{13\ln(1+x)d}{1296}-\frac{\ln(1+x)e}{81}$
default	$-\frac{-\frac{19d}{6912}-\frac{17e}{3456}-\frac{5f}{576}}{x-2}-\frac{\frac{d}{1728}+\frac{e}{864}+\frac{f}{432}}{2(x-2)^2}+\left(\frac{313d}{41472}+\frac{e}{81}+\frac{205f}{10368}\right)\ln(x-2)-\frac{-\frac{d}{432}+\frac{e}{144}-\frac{5f}{432}}{1+x}-\frac{\frac{d}{216}-\frac{e}{216}+}{2(1+x)}$
parallelrisch	$2064dx-12480fx-9600e-12960f\,x^5+1680f\,x^7+1536e\,x^6+8192\ln(x-2)e+420d\,x^7+8192\ln(x+2)e+16640\ln(x-1)x^2d-4$

input
```
int((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x,method=_RETURNVERBOSE)
```

output
```
((-13/192*d-5/16*f)*x^5+(35/384*d+21/32*f)*x^3+(35/3456*d+35/864*f)*x^7+(4
3/864*d-65/216*f)*x-5/18*e*x^4+1/27*e*x^6+5/9*e*x^2-25/108*e)/(x^4-5*x^2+4
)^2+(-313/41472*d+1/81*e-205/10368*f)*ln(x+2)+(-13/1296*d-1/81*e-25/1296*f
)*ln(x-1)+(13/1296*d-1/81*e+25/1296*f)*ln(1+x)+(313/41472*d+1/81*e+205/103
68*f)*ln(x-2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(152) = 304$.

Time = 0.14 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.13

$$\int \frac{d + ex + fx^2}{(4 - 5x^2 + x^4)^3} \, dx$$
$$= \frac{420\,(d + 4\,f)x^7 + 1536\,ex^6 - 216\,(13\,d + 60\,f)x^5 - 11520\,ex^4 + 756\,(5\,d + 36\,f)x^3 + 23040\,ex^2 + 48\,(43}{}$$

input
```
integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fricas")
```

output
```
1/41472*(420*(d + 4*f)*x^7 + 1536*e*x^6 - 216*(13*d + 60*f)*x^5 - 11520*e*
x^4 + 756*(5*d + 36*f)*x^3 + 23040*e*x^2 + 48*(43*d - 260*f)*x - ((313*d -
 512*e + 820*f)*x^8 - 10*(313*d - 512*e + 820*f)*x^6 + 33*(313*d - 512*e +
 820*f)*x^4 - 40*(313*d - 512*e + 820*f)*x^2 + 5008*d - 8192*e + 13120*f)*
log(x + 2) + 32*((13*d - 16*e + 25*f)*x^8 - 10*(13*d - 16*e + 25*f)*x^6 +
33*(13*d - 16*e + 25*f)*x^4 - 40*(13*d - 16*e + 25*f)*x^2 + 208*d - 256*e
+ 400*f)*log(x + 1) - 32*((13*d + 16*e + 25*f)*x^8 - 10*(13*d + 16*e + 25*
f)*x^6 + 33*(13*d + 16*e + 25*f)*x^4 - 40*(13*d + 16*e + 25*f)*x^2 + 208*d
 + 256*e + 400*f)*log(x - 1) + ((313*d + 512*e + 820*f)*x^8 - 10*(313*d +
512*e + 820*f)*x^6 + 33*(313*d + 512*e + 820*f)*x^4 - 40*(313*d + 512*e +
820*f)*x^2 + 5008*d + 8192*e + 13120*f)*log(x - 2) - 9600*e)/(x^8 - 10*x^6
 + 33*x^4 - 40*x^2 + 16)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2822 vs. 2(148) = 296.

Time = 72.87 (sec) , antiderivative size = 2822, normalized size of antiderivative = 15.42

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

output
```
(13*d - 16*e + 25*f)*log(x + (-1106258459719280*d**5*e - 13113710954343*d*
*5*(13*d - 16*e + 25*f) - 12929482401572800*d**4*e*f - 107063904267900*d**
4*f*(13*d - 16*e + 25*f) - 817263343042560*d**3*e**3 + 153628968222720*d**
3*e**2*(13*d - 16*e + 25*f) - 59478343838144000*d**3*e*f**2 + 953019755724
8*d**3*e*(13*d - 16*e + 25*f)**2 - 324891412840800*d**3*f**2*(13*d - 16*e
+ 25*f) + 88038005760*d**3*(13*d - 16*e + 25*f)**3 - 2885705898393600*d**2
*e**3*f + 1014848673546240*d**2*e**2*f*(13*d - 16*e + 25*f) - 134905286808
320000*d**2*e*f**3 + 63469758382080*d**2*e*f*(13*d - 16*e + 25*f)**2 - 422
972724528000*d**2*f**3*(13*d - 16*e + 25*f) + 364616847360*d**2*f*(13*d -
16*e + 25*f)**3 + 5035763255214080*d*e**5 + 142661633703936*d*e**4*(13*d -
 16*e + 25*f) - 2138314899456000*d*e**3*f**2 - 19670950215680*d*e**3*(13*d
 - 16*e + 25*f)**2 + 2257033730457600*d*e**2*f**2*(13*d - 16*e + 25*f) - 5
57272006656*d*e**2*(13*d - 16*e + 25*f)**3 - 151082645593600000*d*e*f**4 +
 141056507904000*d*e*f**2*(13*d - 16*e + 25*f)**2 - 167683154400000*d*f**4
*(13*d - 16*e + 25*f) + 339373670400*d*f**2*(13*d - 16*e + 25*f)**3 + 1064
3272556871680*e**5*f + 214404767416320*e**4*f*(13*d - 16*e + 25*f) + 52999
2253440000*e**3*f**3 - 41575283425280*e**3*f*(13*d - 16*e + 25*f)**2 + 167
1759396864000*e**2*f**3*(13*d - 16*e + 25*f) - 837518622720*e**2*f*(13*d -
 16*e + 25*f)**3 - 66895452108800000*e*f**5 + 104485486592000*e*f**3*(13*d
 - 16*e + 25*f)**2 + 51041923200000*f**5*(13*d - 16*e + 25*f) - 8028979...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^3}\,dx$$
$$= -\frac{1}{41472}\,(313\,d-512\,e+820\,f)\log\left(x+2\right)+\frac{1}{1296}\,(13\,d-16\,e+25\,f)\log\left(x+1\right)$$
$$-\frac{1}{1296}\,(13\,d+16\,e+25\,f)\log\left(x-1\right)+\frac{1}{41472}\,(313\,d+512\,e+820\,f)\log\left(x-2\right)$$
$$+\frac{35\,(d+4\,f)x^7+128\,ex^6-18\,(13\,d+60\,f)x^5-960\,ex^4+63\,(5\,d+36\,f)x^3+1920\,ex^2+4\,(43\,d-26}{3456\,(x^8-10\,x^6+33\,x^4-40\,x^2+16)}$$

input
```
integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="maxima")
```

output
```
-1/41472*(313*d - 512*e + 820*f)*log(x + 2) + 1/1296*(13*d - 16*e + 25*f)*
log(x + 1) - 1/1296*(13*d + 16*e + 25*f)*log(x - 1) + 1/41472*(313*d + 512
*e + 820*f)*log(x - 2) + 1/3456*(35*(d + 4*f)*x^7 + 128*e*x^6 - 18*(13*d +
 60*f)*x^5 - 960*e*x^4 + 63*(5*d + 36*f)*x^3 + 1920*e*x^2 + 4*(43*d - 260*
f)*x - 800*e)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^3}\,dx$$
$$= -\frac{1}{41472}\,(313\,d-512\,e+820\,f)\log\left(|x+2|\right)+\frac{1}{1296}\,(13\,d-16\,e+25\,f)\log\left(|x+1|\right)$$
$$-\frac{1}{1296}\,(13\,d+16\,e+25\,f)\log\left(|x-1|\right)+\frac{1}{41472}\,(313\,d+512\,e+820\,f)\log\left(|x-2|\right)$$
$$+\frac{35\,dx^7+140\,fx^7+128\,ex^6-234\,dx^5-1080\,fx^5-960\,ex^4+315\,dx^3+2268\,fx^3+1920\,ex^2+172\,d}{3456\,(x^4-5\,x^2+4)^2}$$

input
```
integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="giac")
```

output
```
-1/41472*(313*d - 512*e + 820*f)*log(abs(x + 2)) + 1/1296*(13*d - 16*e + 2
5*f)*log(abs(x + 1)) - 1/1296*(13*d + 16*e + 25*f)*log(abs(x - 1)) + 1/414
72*(313*d + 512*e + 820*f)*log(abs(x - 2)) + 1/3456*(35*d*x^7 + 140*f*x^7
+ 128*e*x^6 - 234*d*x^5 - 1080*f*x^5 - 960*e*x^4 + 315*d*x^3 + 2268*f*x^3
+ 1920*e*x^2 + 172*d*x - 1040*f*x - 800*e)/(x^4 - 5*x^2 + 4)^2
```

Mupad [B] (verification not implemented)

Time = 18.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^3}\,dx = \ln(x+1)\left(\frac{13\,d}{1296}-\frac{e}{81}+\frac{25\,f}{1296}\right)-\ln(x-1)\left(\frac{13\,d}{1296}+\frac{e}{81}+\frac{25\,f}{1296}\right) + \ln(x-2)\left(\frac{313\,d}{41472}+\frac{e}{81}+\frac{205\,f}{10368}\right)-\ln(x+2)\left(\frac{313\,d}{41472}-\frac{e}{81}+\frac{205\,f}{10368}\right) + \frac{\left(\frac{35\,d}{3456}+\frac{35\,f}{864}\right)x^7+\frac{e\,x^6}{27}+\left(-\frac{13\,d}{192}-\frac{5\,f}{16}\right)x^5-\frac{5\,e\,x^4}{18}+\left(\frac{35\,d}{384}+\frac{21\,f}{32}\right)x^3+\frac{5\,e\,x^2}{9}+\left(\frac{43\,d}{864}-\frac{65\,f}{216}\right)x-\frac{25\,e}{108}}{x^8-10\,x^6+33\,x^4-40\,x^2+16}$$

input
```
int((d + e*x + f*x^2)/(x^4 - 5*x^2 + 4)^3,x)
```

output
```
log(x + 1)*((13*d)/1296 - e/81 + (25*f)/1296) - log(x - 1)*((13*d)/1296 +
e/81 + (25*f)/1296) + log(x - 2)*((313*d)/41472 + e/81 + (205*f)/10368) -
log(x + 2)*((313*d)/41472 - e/81 + (205*f)/10368) + (x^3*((35*d)/384 + (21
*f)/32) - x^5*((13*d)/192 + (5*f)/16) - (25*e)/108 + x^7*((35*d)/3456 + (3
5*f)/864) + (5*e*x^2)/9 - (5*e*x^4)/18 + (e*x^6)/27 + x*((43*d)/864 - (65*
f)/216))/(33*x^4 - 40*x^2 - 10*x^6 + x^8 + 16)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 655, normalized size of antiderivative = 3.58

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x)
```

output
```
(1565*log(x - 2)*d*x**8 - 15650*log(x - 2)*d*x**6 + 51645*log(x - 2)*d*x**
4 - 62600*log(x - 2)*d*x**2 + 25040*log(x - 2)*d + 2560*log(x - 2)*e*x**8
- 25600*log(x - 2)*e*x**6 + 84480*log(x - 2)*e*x**4 - 102400*log(x - 2)*e*
x**2 + 40960*log(x - 2)*e + 4100*log(x - 2)*f*x**8 - 41000*log(x - 2)*f*x*
*6 + 135300*log(x - 2)*f*x**4 - 164000*log(x - 2)*f*x**2 + 65600*log(x - 2
)*f - 2080*log(x - 1)*d*x**8 + 20800*log(x - 1)*d*x**6 - 68640*log(x - 1)*
d*x**4 + 83200*log(x - 1)*d*x**2 - 33280*log(x - 1)*d - 2560*log(x - 1)*e*
x**8 + 25600*log(x - 1)*e*x**6 - 84480*log(x - 1)*e*x**4 + 102400*log(x -
1)*e*x**2 - 40960*log(x - 1)*e - 4000*log(x - 1)*f*x**8 + 40000*log(x - 1)
*f*x**6 - 132000*log(x - 1)*f*x**4 + 160000*log(x - 1)*f*x**2 - 64000*log(
x - 1)*f - 1565*log(x + 2)*d*x**8 + 15650*log(x + 2)*d*x**6 - 51645*log(x
+ 2)*d*x**4 + 62600*log(x + 2)*d*x**2 - 25040*log(x + 2)*d + 2560*log(x +
2)*e*x**8 - 25600*log(x + 2)*e*x**6 + 84480*log(x + 2)*e*x**4 - 102400*log
(x + 2)*e*x**2 + 40960*log(x + 2)*e - 4100*log(x + 2)*f*x**8 + 41000*log(x
 + 2)*f*x**6 - 135300*log(x + 2)*f*x**4 + 164000*log(x + 2)*f*x**2 - 65600
*log(x + 2)*f + 2080*log(x + 1)*d*x**8 - 20800*log(x + 1)*d*x**6 + 68640*l
og(x + 1)*d*x**4 - 83200*log(x + 1)*d*x**2 + 33280*log(x + 1)*d - 2560*log
(x + 1)*e*x**8 + 25600*log(x + 1)*e*x**6 - 84480*log(x + 1)*e*x**4 + 10240
0*log(x + 1)*e*x**2 - 40960*log(x + 1)*e + 4000*log(x + 1)*f*x**8 - 40000*
log(x + 1)*f*x**6 + 132000*log(x + 1)*f*x**4 - 160000*log(x + 1)*f*x**2...
```

3.28 $\int \frac{d+ex+fx^2}{1+x^2+x^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{d+ex+fx^2}{1+x^2+x^4} dx = -\frac{(d+2e+f)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(d-2e+f)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{2}(d-f)\text{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
-1/6*(d+2*e+f)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*(d-2*e+f)*arctan(1/
3*(1+2*x)*3^(1/2))*3^(1/2)+1/2*(d-f)*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.64

$$\int \frac{d+ex+fx^2}{1+x^2+x^4} dx = \frac{\left(2id+\left(-i+\sqrt{3}\right)f\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}} + \frac{\left(-2id+\left(i+\sqrt{3}\right)f\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}} - \frac{e\arctan\left(\frac{\sqrt{3}}{1+2x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4),x]
```

output
```
(((2*I)*d + (-I + Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)
*Sqrt[3]] + (((-2*I)*d + (I + Sqrt[3])*f)*ArcTan[((I + Sqrt[3])*x)/2])/Sqr
t[6 - (6*I)*Sqrt[3]] - (e*ArcTan[Sqrt[3]/(1 + 2*x^2)])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2202, 27, 1432, 1083, 217, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{fx^2+d}{x^4+x^2+1}dx+\int \frac{ex}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{fx^2+d}{x^4+x^2+1}dx+e\int \frac{x}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{ 1432}$$

$$\int \frac{fx^2+d}{x^4+x^2+1}dx+\frac{1}{2}e\int \frac{1}{x^4+x^2+1}dx^2$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{fx^2+d}{x^4+x^2+1}dx-e\int \frac{1}{-x^4-3}d\left(2x^2+1\right)$$

$$\Big\downarrow \text{ 217}$$

$$\int \frac{fx^2+d}{x^4+x^2+1}dx+\frac{e\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{1}{2}\int \frac{d-(d-f)x}{x^2-x+1}dx+\frac{1}{2}\int \frac{d+(d-f)x}{x^2+x+1}dx+\frac{e\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{1}{2}(d+f)\int \frac{1}{x^2-x+1}dx-\frac{1}{2}(d-f)\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(d+f)\int \frac{1}{x^2+x+1}dx+\frac{1}{2}(d-f)\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{e\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{1}{2}(d+f)\int \frac{1}{x^2-x+1}dx+\frac{1}{2}(d-f)\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(d+f)\int \frac{1}{x^2+x+1}dx+\frac{1}{2}(d-f)\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{e\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{1}{2}(d-f)\int \frac{1-2x}{x^2-x+1}dx-(d+f)\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{1}{2}(d-f)\int \frac{2x+1}{x^2+x+1}dx-(d+f)\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)+\frac{e\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{1}{2}(d-f)\int \frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)(d+f)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}(d-f)\int \frac{2x+1}{x^2+x+1}dx+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)(d+f)}{\sqrt{3}}\right)+\frac{e\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)(d+f)}{\sqrt{3}}-\frac{1}{2}(d-f)\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)(d+f)}{\sqrt{3}}+\frac{1}{2}(d-f)\log\left(x^2+x+1\right)\right)+\frac{e\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(d + e*x + f*x^2)/(1 + x^2 + x^4),x]
```

output
```
(e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/Sqrt[3] + (((d + f)*ArcTan[(-1 + 2*x)/Sqrt
[3]])/Sqrt[3] - ((d - f)*Log[1 - x + x^2])/2)/2 + (((d + f)*ArcTan[(1 + 2*
x)/Sqrt[3]])/Sqrt[3] + ((d - f)*Log[1 + x + x^2])/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{(d-f)\ln(x^2+x+1)}{4}+\frac{\left(\frac{d}{2}-e+\frac{f}{2}\right)\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{(f-d)\ln(x^2-x+1)}{4}+\frac{\left(\frac{d}{2}+e+\frac{f}{2}\right)\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	82
risch	Expression too large to display	7878

input
```
int((f*x^2+e*x+d)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*(d-f)*ln(x^2+x+1)+1/3*(1/2*d-e+1/2*f)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1
/2)+1/4*(f-d)*ln(x^2-x+1)+1/3*(1/2*d+e+1/2*f)*3^(1/2)*arctan(1/3*(2*x-1)*3
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{d+ex+fx^2}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}(d-2e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}(d+2e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}(d-f)\log\left(x^2+x+1\right) - \frac{1}{4}(d-f)\log\left(x^2-x+1\right)$$

input
```
integrate((f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*(d - 2*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d +
 2*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f)*log(x^2 + x + 1) - 1
/4*(d - f)*log(x^2 - x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 56.50 (sec) , antiderivative size = 3589, normalized size of antiderivative = 48.50

$$\int \frac{d+ex+fx^2}{1+x^2+x^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x**2+e*x+d)/(x**4+x**2+1),x)
```

output
```
(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)*log(x + (-7*d**5*e + 6*d**5*(-d/
4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 25*d**4*e*f + 18*d**4*f*(-d/4 + f/
4 - sqrt(3)*I*(d + 2*e + f)/12) - 15*d**3*e**3 - 18*d**3*e**2*(-d/4 + f/4
- sqrt(3)*I*(d + 2*e + f)/12) - 25*d**3*e*f**2 + 60*d**3*e*(-d/4 + f/4 - s
qrt(3)*I*(d + 2*e + f)/12)**2 - 42*d**3*f**2*(-d/4 + f/4 - sqrt(3)*I*(d +
2*e + f)/12) + 72*d**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**3 + 108*
d**2*e**2*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 20*d**2*e*f**3 - 1
44*d**2*e*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 - 12*d**2*f**3*(-
d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) - 144*d**2*f*(-d/4 + f/4 - sqrt(3)
*I*(d + 2*e + f)/12)**3 + 4*d*e**5 + 24*d*e**4*(-d/4 + f/4 - sqrt(3)*I*(d
+ 2*e + f)/12) + 15*d*e**3*f**2 + 48*d*e**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2
*e + f)/12)**2 - 54*d*e**2*f**2*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)
+ 288*d*e**2*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**3 - 20*d*e*f**4 +
180*d*e*f**2*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 + 36*d*f**4*(-d/
4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) - 72*d*f**2*(-d/4 + f/4 - sqrt(3)*I*
(d + 2*e + f)/12)**3 - 8*e**5*f - 96*e**3*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2
*e + f)/12)**2 + 36*e**2*f**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) +
11*e*f**5 - 48*e*f**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 - 6*f**
5*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 144*f**3*(-d/4 + f/4 - sqrt(
3)*I*(d + 2*e + f)/12)**3)/(3*d**6 - 3*d**5*f - 8*d**4*e**2 - 3*d**4*f*...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{d+ex+fx^2}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}(d-2e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}(d+2e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}(d-f)\log\left(x^2+x+1\right) - \frac{1}{4}(d-f)\log\left(x^2-x+1\right)$$

input
```
integrate((f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*(d - 2*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d +
 2*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f)*log(x^2 + x + 1) - 1
/4*(d - f)*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{d+ex+fx^2}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}(d-2e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}(d+2e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}(d-f)\log\left(x^2+x+1\right) - \frac{1}{4}(d-f)\log\left(x^2-x+1\right)$$

input
```
integrate((f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*(d - 2*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d +
 2*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f)*log(x^2 + x + 1) - 1
/4*(d - f)*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 18.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.15

$$\int \frac{d+ex+fx^2}{1+x^2+x^4}\,dx = -\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{d}{4}-\frac{f}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{12}\right) - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{4}-\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{12}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{4}-\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{12}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{d}{4}-\frac{f}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{12}\right)$$

input

```
int((d + e*x + f*x^2)/(x^2 + x^4 + 1),x)
```

output

```
log(x + (3^(1/2)*1i)/2 - 1/2)*(f/4 - d/4 + (3^(1/2)*d*1i)/12 + (3^(1/2)*e*
1i)/6 + (3^(1/2)*f*1i)/12) - log(x - (3^(1/2)*1i)/2 + 1/2)*(f/4 - d/4 + (3
^(1/2)*d*1i)/12 - (3^(1/2)*e*1i)/6 + (3^(1/2)*f*1i)/12) - log(x - (3^(1/2)
*1i)/2 - 1/2)*(d/4 - f/4 + (3^(1/2)*d*1i)/12 + (3^(1/2)*e*1i)/6 + (3^(1/2)
*f*1i)/12) + log(x + (3^(1/2)*1i)/2 + 1/2)*(d/4 - f/4 + (3^(1/2)*d*1i)/12
- (3^(1/2)*e*1i)/6 + (3^(1/2)*f*1i)/12)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.91

$$\int \frac{d+ex+fx^2}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)d}{6}+\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)e}{3}+\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)f}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)d}{6}-\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)e}{3}+\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)f}{6} - \frac{\log(x^2-x+1)\,d}{4}+\frac{\log(x^2-x+1)\,f}{4} + \frac{\log(x^2+x+1)\,d}{4}-\frac{\log(x^2+x+1)\,f}{4}$$

input
```
int((f*x^2+e*x+d)/(x^4+x^2+1),x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3))*d + 4*sqrt(3)*atan((2*x - 1)/sqrt(3))*e
 + 2*sqrt(3)*atan((2*x - 1)/sqrt(3))*f + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))
*d - 4*sqrt(3)*atan((2*x + 1)/sqrt(3))*e + 2*sqrt(3)*atan((2*x + 1)/sqrt(3
))*f - 3*log(x**2 - x + 1)*d + 3*log(x**2 - x + 1)*f + 3*log(x**2 + x + 1)
*d - 3*log(x**2 + x + 1)*f)/12
```

3.29 $\int \frac{d+ex+fx^2}{(1+x^2+x^4)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\begin{aligned}\int \frac{d+ex+fx^2}{(1+x^2+x^4)^2} dx &= \frac{e(1+2x^2)}{6(1+x^2+x^4)} + \frac{x(d+f-(d-2f)x^2)}{6(1+x^2+x^4)} \\ &\quad - \frac{(4d+f)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{(4d+f)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}} \\ &\quad + \frac{2e\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{4}(2d-f)\operatorname{arctanh}\left(\frac{x}{1+x^2}\right)\end{aligned}$$

output
```
e*(2*x^2+1)/(6*x^4+6*x^2+6)+x*(d+f-(d-2*f)*x^2)/(6*x^4+6*x^2+6)-1/36*(4*d+
f)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/36*(4*d+f)*arctan(1/3*(1+2*x)*3^(
1/2))*3^(1/2)+2/9*e*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)+1/4*(2*d-f)*arct
anh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.26

$$\int \frac{d+ex+fx^2}{(1+x^2+x^4)^2}\,dx = \frac{1}{36}\left(\frac{6(e+2ex^2+x(d+f-dx^2+2fx^2))}{1+x^2+x^4} - \frac{\left(\left(-11i+\sqrt{3}\right)d-2\left(-2i+\sqrt{3}\right)f\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}} - \frac{\left(\left(11i+\sqrt{3}\right)d-2\left(2i+\sqrt{3}\right)f\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}} - 8\sqrt{3}e\arctan\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

input
```
Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4)^2,x]
```

output
```
((6*(e + 2*e*x^2 + x*(d + f - d*x^2 + 2*f*x^2)))/(1 + x^2 + x^4) - (((-11*
I + Sqrt[3])*d - 2*(-2*I + Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[
(1 + I*Sqrt[3])/6] - (((11*I + Sqrt[3])*d - 2*(2*I + Sqrt[3])*f)*ArcTan[((
I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 8*Sqrt[3]*e*ArcTan[Sqrt[3]/(
1 + 2*x^2)])/36
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {2202, 27, 1432, 1086, 1083, 217, 1492, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{\left(x^4+x^2+1\right)^2}dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^2}dx+\int \frac{ex}{(x^4+x^2+1)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^2}dx+e\int \frac{x}{(x^4+x^2+1)^2}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^2}dx+\frac{1}{2}e\int \frac{1}{(x^4+x^2+1)^2}dx^2$$

$$\Big\downarrow \text{1086}$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^2}dx+\frac{1}{2}e\left(\frac{2}{3}\int \frac{1}{x^4+x^2+1}dx^2+\frac{2x^2+1}{3(x^4+x^2+1)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^2}dx+\frac{1}{2}e\left(\frac{2x^2+1}{3(x^4+x^2+1)}-\frac{4}{3}\int \frac{1}{-x^4-3}d(2x^2+1)\right)$$

$$\Big\downarrow \text{217}$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^2}dx+\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3(x^4+x^2+1)}\right)$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{6}\int \frac{-((d-2f)x^2)+5d-f}{x^4+x^2+1}dx+\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3(x^4+x^2+1)}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{6(x^4+x^2+1)}$$

$$\Big\downarrow \text{1483}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{5d-f-3(2d-f)x}{x^2-x+1}dx+\frac{1}{2}\int \frac{5d-f+3(2d-f)x}{x^2+x+1}dx\right)+\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3(x^4+x^2+1)}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{6(x^4+x^2+1)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{2}(4d+f)\int\frac{1}{x^2-x+1}dx-\frac{3}{2}(2d-f)\int-\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(4d+f)\int\frac{1}{x^2+x+1}dx+\frac{3}{2}(2d-\right.\right.$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3(x^4+x^2+1)}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{6(x^4+x^2+1)}$$

$\downarrow$ 25

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{2}(4d+f)\int\frac{1}{x^2-x+1}dx+\frac{3}{2}(2d-f)\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(4d+f)\int\frac{1}{x^2+x+1}dx+\frac{3}{2}(2d-f)\right.\right.$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3(x^4+x^2+1)}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{6(x^4+x^2+1)}$$

$\downarrow$ 1083

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{3}{2}(2d-f)\int\frac{1-2x}{x^2-x+1}dx-(4d+f)\int\frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{3}{2}(2d-f)\int\frac{2x+1}{x^2+x+1}dx-\right.\right.$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3(x^4+x^2+1)}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{6(x^4+x^2+1)}$$

$\downarrow$ 217

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{3}{2}(2d-f)\int\frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)(4d+f)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{3}{2}(2d-f)\int\frac{2x+1}{x^2+x+1}dx+\frac{\arctan\left(\frac{2x+}{\sqrt{3}}\right.}{\sqrt{}}\right.\right.$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3(x^4+x^2+1)}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{6(x^4+x^2+1)}$$

$\downarrow$ 1103

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)(4d+f)}{\sqrt{3}}-\frac{3}{2}(2d-f)\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)(4d+f)}{\sqrt{3}}+\frac{3}{2}(2d-f)\log\left(\right.\right.\right.$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3(x^4+x^2+1)}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{6(x^4+x^2+1)}$$

input
```
Int[(d + e*x + f*x^2)/(1 + x^2 + x^4)^2,x]
```

output
```
(x*(d + f - (d - 2*f)*x^2))/(6*(1 + x^2 + x^4)) + (e*((1 + 2*x^2)/(3*(1 +
x^2 + x^4)) + (4*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3])))/2 + ((((4*d +
f)*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - (3*(2*d - f)*Log[1 - x + x^2])/2)
/2 + (((4*d + f)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*(2*d - f)*Log[1 +
 x + x^2])/2)/2)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(-\frac{d}{3}-\frac{e}{3}+\frac{2f}{3}\right)x-\frac{2d}{3}+\frac{e}{3}+\frac{f}{3}}{4x^2+4x+4}+\frac{(6d-3f)\ln(x^2+x+1)}{24}+\frac{\left(2d-4e+\frac{f}{2}\right)\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{18}-\frac{\left(\frac{d}{3}-\frac{e}{3}-\frac{2f}{3}\right)x-\frac{2d}{3}-\frac{e}{3}+\frac{f}{3}}{4(x^2-x+1)}-$
risch	Expression too large to display

input
```
int((f*x^2+e*x+d)/(x^4+x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*((-1/3*d-1/3*e+2/3*f)*x-2/3*d+1/3*e+1/3*f)/(x^2+x+1)+1/24*(6*d-3*f)*ln
(x^2+x+1)+1/18*(2*d-4*e+1/2*f)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*((1
/3*d-1/3*e-2/3*f)*x-2/3*d-1/3*e+1/3*f)/(x^2-x+1)-1/24*(6*d-3*f)*ln(x^2-x+1
)-1/18*(-2*d-4*e-1/2*f)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.43

$$\int \frac{d+ex+fx^2}{(1+x^2+x^4)^2}\,dx = -\frac{12\,(d-2\,f)x^3-24\,ex^2-2\,\sqrt{3}((4\,d-8\,e+f)x^4+(4\,d-8\,e+f)x^2+4\,d-8\,e+f)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\right.}{}$$

input
```
integrate((f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/72*(12*(d - 2*f)*x^3 - 24*e*x^2 - 2*sqrt(3)*((4*d - 8*e + f)*x^4 + (4*d
 - 8*e + f)*x^2 + 4*d - 8*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)
*((4*d + 8*e + f)*x^4 + (4*d + 8*e + f)*x^2 + 4*d + 8*e + f)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 12*(d + f)*x - 9*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*d -
 f)*log(x^2 + x + 1) + 9*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*d - f)*log(x^2
 - x + 1) - 12*e)/(x^4 + x^2 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 62.68 (sec) , antiderivative size = 4106, normalized size of antiderivative = 27.74

$$\int \frac{d+ex+fx^2}{(1+x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

output
```
(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)*log(x + (-164944*d**5*e + 1641
6*d**5*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 336520*d**4*e*f + 200
664*d**4*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) - 115200*d**3*e**3
- 504576*d**3*e**2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) - 272380*d*
*3*e*f**2 + 1734912*d**3*e*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2
- 229500*d**3*f**2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 2612736*d
**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 51840*d**2*e**3*f + 8
81280*d**2*e**2*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 119420*d**
2*e*f**3 - 2477952*d**2*e*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2
 + 50436*d**2*f**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) - 2519424*d
**2*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 28672*d*e**5 + 1843
20*d*e**4*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 8640*d*e**3*f**2 +
 774144*d*e**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 409536*d*e
**2*f**2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 4976640*d*e**2*(-d/
4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 - 31040*d*e*f**4 + 1270080*d*e*
f**2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 + 14040*d*f**4*(-d/4 +
 f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 139968*d*f**2*(-d/4 + f/8 - sqrt(3)
*I*(4*d + 8*e + f)/72)**3 - 20480*e**5*f - 36864*e**4*f*(-d/4 + f/8 - sqrt
(3)*I*(4*d + 8*e + f)/72) - 2880*e**3*f**3 - 552960*e**3*f*(-d/4 + f/8 - s
qrt(3)*I*(4*d + 8*e + f)/72)**2 + 70848*e**2*f**3*(-d/4 + f/8 - sqrt(3)...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \frac{d+ex+fx^2}{(1+x^2+x^4)^2}\,dx = \frac{1}{36}\sqrt{3}(4\,d-8\,e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) + \frac{1}{36}\sqrt{3}(4\,d+8\,e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{8}\,(2\,d-f)\log\left(x^2+x+1\right) - \frac{1}{8}\,(2\,d-f)\log\left(x^2-x+1\right) - \frac{(d-2\,f)x^3-2\,ex^2-(d+f)x-e}{6\,(x^4+x^2+1)}$$

input
```
integrate((f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

output
```
1/36*sqrt(3)*(4*d - 8*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*
(4*d + 8*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f)*log(x^2 + x
+ 1) - 1/8*(2*d - f)*log(x^2 - x + 1) - 1/6*((d - 2*f)*x^3 - 2*e*x^2 - (d
+ f)*x - e)/(x^4 + x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{d+ex+fx^2}{(1+x^2+x^4)^2}\,dx = \frac{1}{36}\sqrt{3}(4\,d-8\,e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) + \frac{1}{36}\sqrt{3}(4\,d+8\,e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{8}(2\,d-f)\log\left(x^2+x+1\right) - \frac{1}{8}(2\,d-f)\log\left(x^2-x+1\right) - \frac{dx^3-2\,fx^3-2\,ex^2-dx-fx-e}{6\,(x^4+x^2+1)}$$

input
```
integrate((f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac")
```

output
```
1/36*sqrt(3)*(4*d - 8*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*
(4*d + 8*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f)*log(x^2 + x
+ 1) - 1/8*(2*d - f)*log(x^2 - x + 1) - 1/6*(d*x^3 - 2*f*x^3 - 2*e*x^2 - d
*x - f*x - e)/(x^4 + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.36

$$\int \frac{d+ex+fx^2}{\left(1+x^2+x^4\right)^2}\,dx = \frac{\left(\frac{f}{3}-\frac{d}{6}\right)x^3+\frac{e\,x^2}{3}+\left(\frac{d}{6}+\frac{f}{6}\right)x+\frac{e}{6}}{x^4+x^2+1} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{d}{4}-\frac{f}{8}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{72}\right) - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{8}-\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{72}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{8}-\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{72}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{d}{4}-\frac{f}{8}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{72}\right)$$

input
```
int((d + e*x + f*x^2)/(x^2 + x^4 + 1)^2,x)
```

output
```
(e/6 - x^3*(d/6 - f/3) + (e*x^2)/3 + x*(d/6 + f/6))/(x^2 + x^4 + 1) - log(
x - (3^(1/2)*1i)/2 - 1/2)*(d/4 - f/8 + (3^(1/2)*d*1i)/18 + (3^(1/2)*e*1i)/
9 + (3^(1/2)*f*1i)/72) - log(x - (3^(1/2)*1i)/2 + 1/2)*(f/8 - d/4 + (3^(1/
2)*d*1i)/18 - (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/72) + log(x + (3^(1/2)*1i)
/2 - 1/2)*(f/8 - d/4 + (3^(1/2)*d*1i)/18 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1
i)/72) + log(x + (3^(1/2)*1i)/2 + 1/2)*(d/4 - f/8 + (3^(1/2)*d*1i)/18 - (3
^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/72)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 525, normalized size of antiderivative = 3.55

$$\int \frac{d+ex+fx^2}{\left(1+x^2+x^4\right)^2}\,dx = \frac{-18\log(x^2-x+1)\,d\,x^4-18\log(x^2-x+1)\,d\,x^2+9\log(x^2-x+1)\,f\,x^4+9\log(x^2-x+1)\,f\,x^2+18}{}$$

input
```
int((f*x^2+e*x+d)/(x^4+x^2+1)^2,x)
```

output
```
(8*sqrt(3)*atan((2*x - 1)/sqrt(3))*d*x**4 + 8*sqrt(3)*atan((2*x - 1)/sqrt(
3))*d*x**2 + 8*sqrt(3)*atan((2*x - 1)/sqrt(3))*d + 16*sqrt(3)*atan((2*x -
1)/sqrt(3))*e*x**4 + 16*sqrt(3)*atan((2*x - 1)/sqrt(3))*e*x**2 + 16*sqrt(3
)*atan((2*x - 1)/sqrt(3))*e + 2*sqrt(3)*atan((2*x - 1)/sqrt(3))*f*x**4 + 2
*sqrt(3)*atan((2*x - 1)/sqrt(3))*f*x**2 + 2*sqrt(3)*atan((2*x - 1)/sqrt(3)
)*f + 8*sqrt(3)*atan((2*x + 1)/sqrt(3))*d*x**4 + 8*sqrt(3)*atan((2*x + 1)/
sqrt(3))*d*x**2 + 8*sqrt(3)*atan((2*x + 1)/sqrt(3))*d - 16*sqrt(3)*atan((2
*x + 1)/sqrt(3))*e*x**4 - 16*sqrt(3)*atan((2*x + 1)/sqrt(3))*e*x**2 - 16*s
qrt(3)*atan((2*x + 1)/sqrt(3))*e + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*f*x**
4 + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*f*x**2 + 2*sqrt(3)*atan((2*x + 1)/sq
rt(3))*f - 18*log(x**2 - x + 1)*d*x**4 - 18*log(x**2 - x + 1)*d*x**2 - 18*
log(x**2 - x + 1)*d + 9*log(x**2 - x + 1)*f*x**4 + 9*log(x**2 - x + 1)*f*x
**2 + 9*log(x**2 - x + 1)*f + 18*log(x**2 + x + 1)*d*x**4 + 18*log(x**2 +
x + 1)*d*x**2 + 18*log(x**2 + x + 1)*d - 9*log(x**2 + x + 1)*f*x**4 - 9*lo
g(x**2 + x + 1)*f*x**2 - 9*log(x**2 + x + 1)*f - 12*d*x**3 + 12*d*x - 24*e
*x**4 - 12*e + 24*f*x**3 + 12*f*x)/(72*(x**4 + x**2 + 1))
```

3.30 $\int \frac{d+ex+fx^2}{(1+x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 206

$$\begin{aligned}\int \frac{d+ex+fx^2}{(1+x^2+x^4)^3}\,dx = {} & \frac{e(1+2x^2)}{12(1+x^2+x^4)^2}+\frac{x(d+f-(d-2f)x^2)}{12(1+x^2+x^4)^2} \\ & +\frac{e(1+2x^2)}{6(1+x^2+x^4)}+\frac{x(2d+3f-7(d-f)x^2)}{24(1+x^2+x^4)} \\ & -\frac{(13d+2f)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{(13d+2f)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{48\sqrt{3}} \\ & +\frac{2e\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{16}(9d-4f)\operatorname{arctanh}\left(\frac{x}{1+x^2}\right)\end{aligned}$$

output
```
1/12*e*(2*x^2+1)/(x^4+x^2+1)^2+1/12*x*(d+f-(d-2*f)*x^2)/(x^4+x^2+1)^2+e*(2
*x^2+1)/(6*x^4+6*x^2+6)+x*(2*d+3*f-7*(d-f)*x^2)/(24*x^4+24*x^2+24)-1/144*(
13*d+2*f)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/144*(13*d+2*f)*arctan(1/3*
(1+2*x)*3^(1/2))*3^(1/2)+2/9*e*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)+1/16*
(9*d-4*f)*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.14

$$\int \frac{d+ex+fx^2}{(1+x^2+x^4)^3}\,dx = \frac{1}{144}\left(\frac{6(2dx+3fx-7dx^3+7fx^3+e(4+8x^2))}{1+x^2+x^4} + \frac{12(e+2ex^2+x(d+f-dx^2+2fx^2))}{(1+x^2+x^4)^2} - \frac{\left(\left(-47i+7\sqrt{3}\right)d+\left(17i-7\sqrt{3}\right)f\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}} - \frac{\left(\left(47i+7\sqrt{3}\right)d-\left(17i+7\sqrt{3}\right)f\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}} - 32\sqrt{3}e\arctan\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

input
```
Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4)^3,x]
```

output
```
((6*(2*d*x + 3*f*x - 7*d*x^3 + 7*f*x^3 + e*(4 + 8*x^2)))/(1 + x^2 + x^4) +
 (12*(e + 2*e*x^2 + x*(d + f - d*x^2 + 2*f*x^2)))/(1 + x^2 + x^4)^2 - (((-
47*I + 7*Sqrt[3])*d + (17*I - 7*Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*x)/2])/
Sqrt[(1 + I*Sqrt[3])/6] - (((47*I + 7*Sqrt[3])*d - (17*I + 7*Sqrt[3])*f)*A
rcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 32*Sqrt[3]*e*ArcTan[
Sqrt[3]/(1 + 2*x^2)])/144
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.17, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {2202, 27, 1432, 1086, 1086, 1083, 217, 1492, 1492, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(x^4+x^2+1)^3}\,dx$$

$$\Big\downarrow \; 2202$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^3}dx+\int \frac{ex}{(x^4+x^2+1)^3}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^3}dx+e\int \frac{x}{(x^4+x^2+1)^3}dx$$

$$\Big\downarrow \; 1432$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^3}dx+\frac{1}{2}e\int \frac{1}{(x^4+x^2+1)^3}dx^2$$

$$\Big\downarrow \; 1086$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^3}dx+\frac{1}{2}e\left(\int \frac{1}{(x^4+x^2+1)^2}dx^2+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)$$

$$\Big\downarrow \; 1086$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^3}dx+\frac{1}{2}e\left(\frac{2}{3}\int \frac{1}{x^4+x^2+1}dx^2+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)$$

$$\Big\downarrow \; 1083$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^3}dx+\frac{1}{2}e\left(-\frac{4}{3}\int \frac{1}{-x^4-3}d(2x^2+1)+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)$$

$$\Big\downarrow \; 217$$

$$\int \frac{fx^2+d}{(x^4+x^2+1)^3}dx+\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)$$

$$\Big\downarrow \; 1492$$

$$\frac{1}{12}\int \frac{-5(d-2f)x^2+11d-f}{(x^4+x^2+1)^2}dx+$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{12\left(x^4+x^2+1\right)^2}$$

$$\Big\downarrow \; 1492$$

$$\frac{1}{12}\left(\frac{1}{6}\int \frac{3\left(5(4d-f)-7(d-f)x^2\right)}{x^4+x^2+1}dx+\frac{x\left(-7x^2(d-f)+2d+3f\right)}{2\left(x^4+x^2+1\right)}\right)+$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{12\left(x^4+x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\left(\frac{1}{2}\int \frac{5(4d-f)-7(d-f)x^2}{x^4+x^2+1}dx+\frac{x\left(-7x^2(d-f)+2d+3f\right)}{2\left(x^4+x^2+1\right)}\right)+$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{12\left(x^4+x^2+1\right)^2}$$

$$\Big\downarrow \text{1483}$$

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{5(4d-f)-3(9d-4f)x}{x^2-x+1}dx+\frac{1}{2}\int \frac{5(4d-f)+3(9d-4f)x}{x^2+x+1}dx\right)+\frac{x\left(-7x^2(d-f)+2d+3f\right)}{2\left(x^4+x^2+1\right)}\right)+$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{12\left(x^4+x^2+1\right)^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(13d+2f)\int \frac{1}{x^2-x+1}dx-\frac{3}{2}(9d-4f)\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(13d+2f)\int \frac{1}{x^2+x+1}dx\right.\right.\right.$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{12\left(x^4+x^2+1\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(13d+2f)\int \frac{1}{x^2-x+1}dx+\frac{3}{2}(9d-4f)\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(13d+2f)\int \frac{1}{x^2+x+1}dx+\right.\right.\right.$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{12\left(x^4+x^2+1\right)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{3}{2}(9d-4f)\int\frac{1-2x}{x^2-x+1}dx-(13d+2f)\int\frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{3}{2}(9d-4f)\int\frac{2x+}{x^2+x}\right.\right.\right.$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{12\left(x^4+x^2+1\right)^2}$$

$\downarrow$ 217

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{3}{2}(9d-4f)\int\frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)(13d+2f)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{3}{2}(9d-4f)\int\frac{2x+1}{x^2+x+1}dx+\frac{\text{ar}}{}\right.\right.\right.$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{12\left(x^4+x^2+1\right)^2}$$

$\downarrow$ 1103

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)(13d+2f)}{\sqrt{3}}-\frac{3}{2}(9d-4f)\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)(13d+2f)}{\sqrt{3}}+\frac{3}{2}(9c\right.\right.\right.$$
$$\frac{1}{2}e\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}+\frac{2x^2+1}{6\left(x^4+x^2+1\right)^2}\right)+\frac{x\left(-\left(x^2(d-2f)\right)+d+f\right)}{12\left(x^4+x^2+1\right)^2}$$

input
```
Int[(d + e*x + f*x^2)/(1 + x^2 + x^4)^3,x]
```

output
```
(x*(d + f - (d - 2*f)*x^2))/(12*(1 + x^2 + x^4)^2) + (e*((1 + 2*x^2)/(6*(1
 + x^2 + x^4)^2) + (1 + 2*x^2)/(3*(1 + x^2 + x^4)) + (4*ArcTan[(1 + 2*x^2)
/Sqrt[3]])/(3*Sqrt[3])))/2 + ((x*(2*d + 3*f - 7*(d - f)*x^2))/(2*(1 + x^2
+ x^4)) + ((((13*d + 2*f)*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - (3*(9*d -
4*f)*Log[1 - x + x^2])/2)/2 + (((13*d + 2*f)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sq
rt[3] + (3*(9*d - 4*f)*Log[1 + x + x^2])/2)/2)/2)/12
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98

method	result
default	$\frac{\left(-\frac{7d}{3}+\frac{7f}{3}-\frac{4e}{3}\right)x^3+(-6d+4f)x^2+\left(-\frac{20d}{3}+\frac{13f}{3}+\frac{e}{3}\right)x-4d+\frac{4f}{3}+2e}{16(x^2+x+1)^2}+\frac{(27d-12f)\ln(x^2+x+1)}{96}+\frac{\left(\frac{13d}{2}-16e+f\right)\arctan\left(\frac{(1+2x}{3}\right.}{72}$
risch	Expression too large to display

input
```
int((f*x^2+e*x+d)/(x^4+x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*((-7/3*d+7/3*f-4/3*e)*x^3+(-6*d+4*f)*x^2+(-20/3*d+13/3*f+1/3*e)*x-4*d
+4/3*f+2*e)/(x^2+x+1)^2+1/96*(27*d-12*f)*ln(x^2+x+1)+1/72*(13/2*d-16*e+f)*
arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/16*((7/3*d-7/3*f-4/3*e)*x^3+(-6*d+4*
f)*x^2+(20/3*d-13/3*f+1/3*e)*x-4*d+4/3*f-2*e)/(x^2-x+1)^2-1/96*(27*d-12*f)
*ln(x^2-x+1)-1/72*(-13/2*d-16*e-f)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(184) = 368$.

Time = 0.13 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.86

$$\int \frac{d+ex+fx^2}{(1+x^2+x^4)^3}\,dx = -\frac{84\,(d-f)x^7-96\,ex^6+60\,(d-2\,f)x^5-144\,ex^4+84\,(d-2\,f)x^3-192\,ex^2-2\,\sqrt{3}((13\,d-32\,e+2\,\ldots}{}$$

input
```
integrate((f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/288*(84*(d - f)*x^7 - 96*e*x^6 + 60*(d - 2*f)*x^5 - 144*e*x^4 + 84*(d -
 2*f)*x^3 - 192*e*x^2 - 2*sqrt(3)*((13*d - 32*e + 2*f)*x^8 + 2*(13*d - 32*
e + 2*f)*x^6 + 3*(13*d - 32*e + 2*f)*x^4 + 2*(13*d - 32*e + 2*f)*x^2 + 13*
d - 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((13*d + 32*e +
2*f)*x^8 + 2*(13*d + 32*e + 2*f)*x^6 + 3*(13*d + 32*e + 2*f)*x^4 + 2*(13*d
 + 32*e + 2*f)*x^2 + 13*d + 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12
*(4*d + 5*f)*x - 9*((9*d - 4*f)*x^8 + 2*(9*d - 4*f)*x^6 + 3*(9*d - 4*f)*x^
4 + 2*(9*d - 4*f)*x^2 + 9*d - 4*f)*log(x^2 + x + 1) + 9*((9*d - 4*f)*x^8 +
 2*(9*d - 4*f)*x^6 + 3*(9*d - 4*f)*x^4 + 2*(9*d - 4*f)*x^2 + 9*d - 4*f)*lo
g(x^2 - x + 1) - 72*e)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 64.84 (sec) , antiderivative size = 4496, normalized size of antiderivative = 21.83

$$\int \frac{d+ex+fx^2}{(1+x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

output
```
(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)*log(x + (-1025428432*d
**5*e - 334752912*d**5*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)
 + 2008961360*d**4*e*f + 1151575920*d**4*f*(-9*d/32 + f/8 - sqrt(3)*I*(13*
d + 32*e + 2*f)/288) - 431308800*d**3*e**3 - 3143688192*d**3*e**2*(-9*d/32
 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 1598857120*d**3*e*f**2 + 991
7005824*d**3*e*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 - 94
4300160*d**3*f**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 11
878244352*d**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 23
3164800*d**2*e**3*f + 4409634816*d**2*e**2*f*(-9*d/32 + f/8 - sqrt(3)*I*(1
3*d + 32*e + 2*f)/288) + 662937520*d**2*e*f**3 - 13004623872*d**2*e*f*(-9*
d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 + 231796080*d**2*f**3*(
-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 10089639936*d**2*f*(-
9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 142606336*d*e**5 +
754974720*d*e**4*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 184
3200*d*e**3*f**2 + 3850371072*d*e**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32
*e + 2*f)/288)**2 - 1926291456*d*e**2*f**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*
d + 32*e + 2*f)/288) + 20384317440*d*e**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*d
 + 32*e + 2*f)/288)**3 - 146756960*d*e*f**4 + 5813379072*d*e*f**2*(-9*d/32
 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 + 12679200*d*f**4*(-9*d/32
+ f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 1116758016*d*f**2*(-9*d/32...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

$$\int \frac{d+ex+fx^2}{\left(1+x^2+x^4\right)^3}\,dx = \frac{1}{144}\,\sqrt{3}(13\,d-32\,e+2\,f)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{144}\,\sqrt{3}(13\,d+32\,e+2\,f)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{32}\,(9\,d-4\,f)\log\left(x^2+x+1\right) - \frac{1}{32}\,(9\,d-4\,f)\log\left(x^2-x+1\right) - \frac{7\,(d-f)x^7-8\,ex^6+5\,(d-2\,f)x^5-12\,ex^4+7\,(d-2\,f)x^3-16\,ex^2-(4\,d+5\,f)x-6\,e}{24\,(x^8+2\,x^6+3\,x^4+2\,x^2+1)}$$

input
```
integrate((f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxima")
```

output
```
1/144*sqrt(3)*(13*d - 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144*sq
rt(3)*(13*d + 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f)
*log(x^2 + x + 1) - 1/32*(9*d - 4*f)*log(x^2 - x + 1) - 1/24*(7*(d - f)*x^
7 - 8*e*x^6 + 5*(d - 2*f)*x^5 - 12*e*x^4 + 7*(d - 2*f)*x^3 - 16*e*x^2 - (4
*d + 5*f)*x - 6*e)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int \frac{d+ex+fx^2}{\left(1+x^2+x^4\right)^3}\,dx = \frac{1}{144}\,\sqrt{3}(13\,d-32\,e+2\,f)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{144}\,\sqrt{3}(13\,d+32\,e+2\,f)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{32}\,(9\,d-4\,f)\log\left(x^2+x+1\right) - \frac{1}{32}\,(9\,d-4\,f)\log\left(x^2-x+1\right) - \frac{7\,dx^7-7\,fx^7-8\,ex^6+5\,dx^5-10\,fx^5-12\,ex^4+7\,dx^3-14\,fx^3-16\,ex^2-4\,dx-5\,fx-6\,e}{24\,\left(x^4+x^2+1\right)^2}$$

input
```
integrate((f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac")
```

output
```
1/144*sqrt(3)*(13*d - 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144*sq
rt(3)*(13*d + 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f)
*log(x^2 + x + 1) - 1/32*(9*d - 4*f)*log(x^2 - x + 1) - 1/24*(7*d*x^7 - 7*
f*x^7 - 8*e*x^6 + 5*d*x^5 - 10*f*x^5 - 12*e*x^4 + 7*d*x^3 - 14*f*x^3 - 16*
e*x^2 - 4*d*x - 5*f*x - 6*e)/(x^4 + x^2 + 1)^2
```

Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.21

$$\int \frac{d+ex+fx^2}{(1+x^2+x^4)^3}\,dx$$
$$= \frac{\left(\frac{7f}{24}-\frac{7d}{24}\right)x^7+\frac{e\,x^6}{3}+\left(\frac{5f}{12}-\frac{5d}{24}\right)x^5+\frac{e\,x^4}{2}+\left(\frac{7f}{12}-\frac{7d}{24}\right)x^3+\frac{2\,e\,x^2}{3}+\left(\frac{d}{6}+\frac{5f}{24}\right)x+\frac{e}{4}}{x^8+2\,x^6+3\,x^4+2\,x^2+1}$$
$$-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{9\,d}{32}-\frac{f}{8}+\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{144}\right)$$
$$-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{8}-\frac{9\,d}{32}+\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{144}\right)$$
$$+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{8}-\frac{9\,d}{32}+\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{144}\right)$$
$$+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{9\,d}{32}-\frac{f}{8}+\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{144}\right)$$

input
```
int((d + e*x + f*x^2)/(x^2 + x^4 + 1)^3,x)
```

output
```
(e/4 - x^5*((5*d)/24 - (5*f)/12) - x^3*((7*d)/24 - (7*f)/12) - x^7*((7*d)/
24 - (7*f)/24) + (2*e*x^2)/3 + (e*x^4)/2 + (e*x^6)/3 + x*(d/6 + (5*f)/24))
/(2*x^2 + 3*x^4 + 2*x^6 + x^8 + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((9*d)/
32 - f/8 + (3^(1/2)*d*13i)/288 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144) -
log(x - (3^(1/2)*1i)/2 + 1/2)*(f/8 - (9*d)/32 + (3^(1/2)*d*13i)/288 - (3^(
1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144) + log(x + (3^(1/2)*1i)/2 - 1/2)*(f/8 -
(9*d)/32 + (3^(1/2)*d*13i)/288 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144) +
log(x + (3^(1/2)*1i)/2 + 1/2)*((9*d)/32 - f/8 + (3^(1/2)*d*13i)/288 - (3^(
1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 905, normalized size of antiderivative = 4.39

$$\int \frac{d+ex+fx^2}{(1+x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(x^4+x^2+1)^3,x)
```

output
```
(26*sqrt(3)*atan((2*x - 1)/sqrt(3))*d*x**8 + 52*sqrt(3)*atan((2*x - 1)/sqr
t(3))*d*x**6 + 78*sqrt(3)*atan((2*x - 1)/sqrt(3))*d*x**4 + 52*sqrt(3)*atan
((2*x - 1)/sqrt(3))*d*x**2 + 26*sqrt(3)*atan((2*x - 1)/sqrt(3))*d + 64*sqr
t(3)*atan((2*x - 1)/sqrt(3))*e*x**8 + 128*sqrt(3)*atan((2*x - 1)/sqrt(3))*
e*x**6 + 192*sqrt(3)*atan((2*x - 1)/sqrt(3))*e*x**4 + 128*sqrt(3)*atan((2*
x - 1)/sqrt(3))*e*x**2 + 64*sqrt(3)*atan((2*x - 1)/sqrt(3))*e + 4*sqrt(3)*
atan((2*x - 1)/sqrt(3))*f*x**8 + 8*sqrt(3)*atan((2*x - 1)/sqrt(3))*f*x**6
+ 12*sqrt(3)*atan((2*x - 1)/sqrt(3))*f*x**4 + 8*sqrt(3)*atan((2*x - 1)/sqr
t(3))*f*x**2 + 4*sqrt(3)*atan((2*x - 1)/sqrt(3))*f + 26*sqrt(3)*atan((2*x
+ 1)/sqrt(3))*d*x**8 + 52*sqrt(3)*atan((2*x + 1)/sqrt(3))*d*x**6 + 78*sqrt
(3)*atan((2*x + 1)/sqrt(3))*d*x**4 + 52*sqrt(3)*atan((2*x + 1)/sqrt(3))*d*
x**2 + 26*sqrt(3)*atan((2*x + 1)/sqrt(3))*d - 64*sqrt(3)*atan((2*x + 1)/sq
rt(3))*e*x**8 - 128*sqrt(3)*atan((2*x + 1)/sqrt(3))*e*x**6 - 192*sqrt(3)*a
tan((2*x + 1)/sqrt(3))*e*x**4 - 128*sqrt(3)*atan((2*x + 1)/sqrt(3))*e*x**2
 - 64*sqrt(3)*atan((2*x + 1)/sqrt(3))*e + 4*sqrt(3)*atan((2*x + 1)/sqrt(3)
)*f*x**8 + 8*sqrt(3)*atan((2*x + 1)/sqrt(3))*f*x**6 + 12*sqrt(3)*atan((2*x
 + 1)/sqrt(3))*f*x**4 + 8*sqrt(3)*atan((2*x + 1)/sqrt(3))*f*x**2 + 4*sqrt(
3)*atan((2*x + 1)/sqrt(3))*f - 81*log(x**2 - x + 1)*d*x**8 - 162*log(x**2
- x + 1)*d*x**6 - 243*log(x**2 - x + 1)*d*x**4 - 162*log(x**2 - x + 1)*d*x
**2 - 81*log(x**2 - x + 1)*d + 36*log(x**2 - x + 1)*f*x**8 + 72*log(x**...
```

3.31 $\int (d+ex+fx^2)(a+bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 154

$$\int (d+ex+fx^2)(a+bx^2+cx^4)^2\,dx = a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{2}abex^4 +\frac{1}{5}(b^2d+2acd+2abf)x^5+\frac{1}{6}(b^2+2ac)ex^6 +\frac{1}{7}(2bcd+b^2f+2acf)x^7+\frac{1}{4}bcex^8 +\frac{1}{9}c(cd+2bf)x^9+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

output
```
a^2*d*x+1/2*a^2*e*x^2+1/3*a*(a*f+2*b*d)*x^3+1/2*a*b*e*x^4+1/5*(2*a*b*f+2*a
*c*d+b^2*d)*x^5+1/6*(2*a*c+b^2)*e*x^6+1/7*(2*a*c*f+b^2*f+2*b*c*d)*x^7+1/4*
b*c*e*x^8+1/9*c*(2*b*f+c*d)*x^9+1/10*c^2*e*x^10+1/11*c^2*f*x^11
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\int (d+ex+fx^2)(a+bx^2+cx^4)^2\,dx = a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{2}abex^4+\frac{1}{5}(b^2d+2acd+2abf)x^5+\frac{1}{6}(b^2+2ac)ex^6+\frac{1}{7}(2bcd+b^2f+2acf)x^7+\frac{1}{4}bcex^8+\frac{1}{9}c(cd+2bf)x^9+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

input

```
Integrate[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output

```
a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*
d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*
f + 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^1
0)/10 + (c^2*f*x^11)/11
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2+cx^4)^2(d+ex+fx^2)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int (a^2d+a^2ex+x^6(2acf+b^2f+2bcd)+x^4(2abf+2acd+b^2d)+ex^5(2ac+b^2)+ax^2(af+2bd)+2abex^3+$$

$$\Big\downarrow \text{2009}$$

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{4}bcex^8+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

input

```
Int[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output

```
a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*
d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*
f + 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^1
0)/10 + (c^2*f*x^11)/11
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^2 f\,x^{11}}{11}+\frac{c^2 e\,x^{10}}{10}+\frac{\left(2bcf+d\,c^2\right)x^9}{9}+\frac{bce\,x^8}{4}+\frac{\left(2bcd+f\left(2ac+b^2\right)\right)x^7}{7}+\frac{\left(2ac+b^2\right)e\,x^6}{6}+\frac{\left(d\left(2ac+b^2\right)+2abf\right)x^5}{5}+$
norman	$\frac{c^2 f\,x^{11}}{11}+\frac{c^2 e\,x^{10}}{10}+\left(\frac{2}{9}bcf+\frac{1}{9}d\,c^2\right)x^9+\frac{bce\,x^8}{4}+\left(\frac{2}{7}acf+\frac{1}{7}b^2f+\frac{2}{7}bcd\right)x^7+\left(\frac{1}{3}ace+\frac{1}{6}b^2e\right)x^6$
gosper	$\frac{1}{11}c^2f\,x^{11}+\frac{1}{10}c^2e\,x^{10}+\frac{2}{9}x^9bcf+\frac{1}{9}c^2d\,x^9+\frac{1}{4}bce\,x^8+\frac{2}{7}x^7acf+\frac{1}{7}x^7b^2f+\frac{2}{7}x^7bcd+\frac{1}{3}ace\,x$
risch	$\frac{1}{11}c^2f\,x^{11}+\frac{1}{10}c^2e\,x^{10}+\frac{2}{9}x^9bcf+\frac{1}{9}c^2d\,x^9+\frac{1}{4}bce\,x^8+\frac{2}{7}x^7acf+\frac{1}{7}x^7b^2f+\frac{2}{7}x^7bcd+\frac{1}{3}ace\,x$
parallelrisch	$\frac{1}{11}c^2f\,x^{11}+\frac{1}{10}c^2e\,x^{10}+\frac{2}{9}x^9bcf+\frac{1}{9}c^2d\,x^9+\frac{1}{4}bce\,x^8+\frac{2}{7}x^7acf+\frac{1}{7}x^7b^2f+\frac{2}{7}x^7bcd+\frac{1}{3}ace\,x$
orering	$\frac{x\left(1260f\,c^2x^{10}+1386e\,c^2x^9+3080bcf\,x^8+1540c^2d\,x^8+3465ceb\,x^7+3960acf\,x^6+1980b^2f\,x^6+3960bcd\,x^6+4620ace\,x^5+2310\right.}{13860}$

input

```
int((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*c^2*f*x^11+1/10*c^2*e*x^10+1/9*(2*b*c*f+c^2*d)*x^9+1/4*b*c*e*x^8+1/7*
(2*b*c*d+f*(2*a*c+b^2))*x^7+1/6*(2*a*c+b^2)*e*x^6+1/5*(d*(2*a*c+b^2)+2*a*b
*f)*x^5+1/2*a*b*e*x^4+1/3*(a^2*f+2*a*b*d)*x^3+1/2*a^2*e*x^2+a^2*d*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int (d+ex+fx^2) (a+bx^2+cx^4)^2 dx = \frac{1}{11} c^2 f x^{11} + \frac{1}{10} c^2 e x^{10} + \frac{1}{4} bcex^8 + \frac{1}{9} (c^2 d + 2 bcf) x^9 + \frac{1}{6} (b^2 + 2 ac) e x^6 + \frac{1}{7} (2 bcd + (b^2 + 2 ac) f) x^7 + \frac{1}{2} abex^4 + \frac{1}{5} (2 abf + (b^2 + 2 ac) d) x^5 + \frac{1}{2} a^2 e x^2 + a^2 dx + \frac{1}{3} (2 abd + a^2 f) x^3$$

input
```
integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/4*b*c*e*x^8 + 1/9*(c^2*d + 2*b*c*f)*
x^9 + 1/6*(b^2 + 2*a*c)*e*x^6 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*f)*x^7 + 1/2*
a*b*e*x^4 + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2 + a^2*d*x
+ 1/3*(2*a*b*d + a^2*f)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.07

$$\int (d+ex+fx^2) (a+bx^2+cx^4)^2 dx = a^2 dx + \frac{a^2 e x^2}{2} + \frac{abex^4}{2} + \frac{bcex^8}{4} + \frac{c^2 e x^{10}}{10} + \frac{c^2 f x^{11}}{11} + x^9 \cdot \left(\frac{2bcf}{9} + \frac{c^2 d}{9}\right) + x^7 \cdot \left(\frac{2acf}{7} + \frac{b^2 f}{7} + \frac{2bcd}{7}\right) + x^6 \left(\frac{ace}{3} + \frac{b^2 e}{6}\right) + x^5 \cdot \left(\frac{2abf}{5} + \frac{2acd}{5} + \frac{b^2 d}{5}\right) + x^3 \left(\frac{a^2 f}{3} + \frac{2abd}{3}\right)$$

input
```
integrate((f*x**2+e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*d*x + a**2*e*x**2/2 + a*b*e*x**4/2 + b*c*e*x**8/4 + c**2*e*x**10/10 +
 c**2*f*x**11/11 + x**9*(2*b*c*f/9 + c**2*d/9) + x**7*(2*a*c*f/7 + b**2*f/
7 + 2*b*c*d/7) + x**6*(a*c*e/3 + b**2*e/6) + x**5*(2*a*b*f/5 + 2*a*c*d/5 +
 b**2*d/5) + x**3*(a**2*f/3 + 2*a*b*d/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int (d+ex+fx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{11}\,c^2fx^{11} + \frac{1}{10}\,c^2ex^{10} + \frac{1}{4}\,bcex^8 + \frac{1}{9}\left(c^2d+2\,bcf\right)x^9 + \frac{1}{6}\left(b^2+2\,ac\right)ex^6 + \frac{1}{7}\left(2\,bcd+\left(b^2+2\,ac\right)f\right)x^7 + \frac{1}{2}\,abex^4 + \frac{1}{5}\left(2\,abf+\left(b^2+2\,ac\right)d\right)x^5 + \frac{1}{2}\,a^2ex^2 + a^2dx + \frac{1}{3}\left(2\,abd+a^2f\right)x^3$$

input
```
integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/4*b*c*e*x^8 + 1/9*(c^2*d + 2*b*c*f)*
x^9 + 1/6*(b^2 + 2*a*c)*e*x^6 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*f)*x^7 + 1/2*
a*b*e*x^4 + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2 + a^2*d*x
+ 1/3*(2*a*b*d + a^2*f)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.98

$$\int (d+ex+fx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{11}c^2fx^{11}+\frac{1}{10}c^2ex^{10}+\frac{1}{9}c^2dx^9+\frac{2}{9}bcfx^9+\frac{1}{4}bcex^8+\frac{2}{7}bcdx^7+\frac{1}{7}b^2fx^7+\frac{2}{7}acfx^7+\frac{1}{6}b^2ex^6+\frac{1}{3}acex^6+\frac{1}{5}b^2dx^5+\frac{2}{5}acdx^5+\frac{2}{5}abfx^5+\frac{1}{2}abex^4+\frac{2}{3}abdx^3+\frac{1}{3}a^2fx^3+\frac{1}{2}a^2ex^2+a^2dx$$

input
```
integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/4*b*
c*e*x^8 + 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7 + 1/6*b^2*e*x^6 +
1/3*a*c*e*x^6 + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^5 + 1/2*a*b*e*
x^4 + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*e*x^2 + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int (d+ex+fx^2)\left(a+bx^2+cx^4\right)^2\,dx = x^5\left(\frac{d\,b^2}{5}+\frac{2\,a\,f\,b}{5}+\frac{2\,a\,c\,d}{5}\right)+x^7\left(\frac{f\,b^2}{7}+\frac{2\,c\,d\,b}{7}+\frac{2\,a\,c\,f}{7}\right)+x^3\left(\frac{f\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)+x^9\left(\frac{d\,c^2}{9}+\frac{2\,b\,f\,c}{9}\right)+\frac{a^2\,e\,x^2}{2}+\frac{c^2\,e\,x^{10}}{10}+\frac{c^2\,f\,x^{11}}{11}+\frac{e\,x^6\,\left(b^2+2\,a\,c\right)}{6}+a^2\,d\,x+\frac{a\,b\,e\,x^4}{2}+\frac{b\,c\,e\,x^8}{4}$$

input
```
int((d + e*x + f*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^5*((b^2*d)/5 + (2*a*c*d)/5 + (2*a*b*f)/5) + x^7*((b^2*f)/7 + (2*b*c*d)/7
 + (2*a*c*f)/7) + x^3*((a^2*f)/3 + (2*a*b*d)/3) + x^9*((c^2*d)/9 + (2*b*c*
f)/9) + (a^2*e*x^2)/2 + (c^2*e*x^10)/10 + (c^2*f*x^11)/11 + (e*x^6*(2*a*c
+ b^2))/6 + a^2*d*x + (a*b*e*x^4)/2 + (b*c*e*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int (d + ex + fx^2) (a + bx^2 + cx^4)^2 \, dx$$
$$= \frac{x(1260c^2f\,x^{10} + 1386c^2e\,x^9 + 3080bcf\,x^8 + 1540c^2d\,x^8 + 3465bce\,x^7 + 3960acf\,x^6 + 1980b^2f\,x^6 + 3960\ldots}{}$$

input
```
int((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x)
```

output
```
(x*(13860*a**2*d + 6930*a**2*e*x + 4620*a**2*f*x**2 + 9240*a*b*d*x**2 + 69
30*a*b*e*x**3 + 5544*a*b*f*x**4 + 5544*a*c*d*x**4 + 4620*a*c*e*x**5 + 3960
*a*c*f*x**6 + 2772*b**2*d*x**4 + 2310*b**2*e*x**5 + 1980*b**2*f*x**6 + 396
0*b*c*d*x**6 + 3465*b*c*e*x**7 + 3080*b*c*f*x**8 + 1540*c**2*d*x**8 + 1386
*c**2*e*x**9 + 1260*c**2*f*x**10))/13860
```

3.32 $\int (d+ex+fx^2) (a+bx^2+cx^4) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int (d+ex+fx^2) (a+bx^2+cx^4) \, dx = adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}bex^4 +\frac{1}{5}(cd+bf)x^5+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

output
```
a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*b*e*x^4+1/5*(b*f+c*d)*x^5+1/6*c*e*
x^6+1/7*c*f*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int (d+ex+fx^2) (a+bx^2+cx^4) \, dx = adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}bex^4 +\frac{1}{5}(cd+bf)x^5+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

input
```
Integrate[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + (b*e*x^4)/4 + ((c*d + b*f)*x^5
)/5 + (c*e*x^6)/6 + (c*f*x^7)/7
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(x^2(af+bd)+ad+aex+x^4(bf+cd)+bex^3+cex^5+cfx^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3(af+bd)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{4}bex^4+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

input
```
Int[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + (b*e*x^4)/4 + ((c*d + b*f)*x^5
)/5 + (c*e*x^6)/6 + (c*f*x^7)/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
default	$adx+\frac{ae\,x^2}{2}+\frac{(af+bd)x^3}{3}+\frac{be\,x^4}{4}+\frac{(bf+cd)x^5}{5}+\frac{ce\,x^6}{6}+\frac{cf\,x^7}{7}$	58
norman	$\frac{cf\,x^7}{7}+\frac{ce\,x^6}{6}+\left(\frac{bf}{5}+\frac{cd}{5}\right)x^5+\frac{be\,x^4}{4}+\left(\frac{af}{3}+\frac{bd}{3}\right)x^3+\frac{ae\,x^2}{2}+adx$	60
gosper	$\frac{1}{7}cf\,x^7+\frac{1}{6}ce\,x^6+\frac{1}{5}x^5bf+\frac{1}{5}cd\,x^5+\frac{1}{4}be\,x^4+\frac{1}{3}x^3af+\frac{1}{3}bd\,x^3+\frac{1}{2}ae\,x^2+adx$	62
risch	$\frac{1}{7}cf\,x^7+\frac{1}{6}ce\,x^6+\frac{1}{5}x^5bf+\frac{1}{5}cd\,x^5+\frac{1}{4}be\,x^4+\frac{1}{3}x^3af+\frac{1}{3}bd\,x^3+\frac{1}{2}ae\,x^2+adx$	62
parallelrisch	$\frac{1}{7}cf\,x^7+\frac{1}{6}ce\,x^6+\frac{1}{5}x^5bf+\frac{1}{5}cd\,x^5+\frac{1}{4}be\,x^4+\frac{1}{3}x^3af+\frac{1}{3}bd\,x^3+\frac{1}{2}ae\,x^2+adx$	62
orering	$\frac{x(60cf\,x^6+70ce\,x^5+84bf\,x^4+84cd\,x^4+105be\,x^3+140af\,x^2+140bd\,x^2+210aex+420ad)}{420}$	63

input
```
int((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*b*e*x^4+1/5*(b*f+c*d)*x^5+1/6*c*e*
x^6+1/7*c*f*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \left(d+ex+fx^2\right)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{7}\,cfx^7+\frac{1}{6}\,cex^6+\frac{1}{4}\,bex^4+\frac{1}{5}\,(cd+bf)x^5+\frac{1}{2}\,aex^2+\frac{1}{3}\,(bd+af)x^3+adx$$

input
```
integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/7*c*f*x^7 + 1/6*c*e*x^6 + 1/4*b*e*x^4 + 1/5*(c*d + b*f)*x^5 + 1/2*a*e*x^
2 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int (d+ex+fx^2)(a+bx^2+cx^4)\,dx = adx+\frac{aex^2}{2}+\frac{bex^4}{4}+\frac{cex^6}{6}+\frac{cfx^7}{7}+x^5\left(\frac{bf}{5}+\frac{cd}{5}\right)+x^3\left(\frac{af}{3}+\frac{bd}{3}\right)$$

input
```
integrate((f*x**2+e*x+d)*(c*x**4+b*x**2+a),x)
```

output
```
a*d*x + a*e*x**2/2 + b*e*x**4/4 + c*e*x**6/6 + c*f*x**7/7 + x**5*(b*f/5 +
c*d/5) + x**3*(a*f/3 + b*d/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int (d+ex+fx^2)(a+bx^2+cx^4)\,dx = \frac{1}{7}cfx^7+\frac{1}{6}cex^6+\frac{1}{4}bex^4+\frac{1}{5}(cd+bf)x^5+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+adx$$

input
```
integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/7*c*f*x^7 + 1/6*c*e*x^6 + 1/4*b*e*x^4 + 1/5*(c*d + b*f)*x^5 + 1/2*a*e*x^
2 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int (d+ex+fx^2)(a+bx^2+cx^4)\,dx = \frac{1}{7}cfx^7+\frac{1}{6}cex^6+\frac{1}{5}cdx^5+\frac{1}{5}bfx^5 + \frac{1}{4}bex^4+\frac{1}{3}bdx^3+\frac{1}{3}afx^3+\frac{1}{2}aex^2+adx$$

input
```
integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/7*c*f*x^7 + 1/6*c*e*x^6 + 1/5*c*d*x^5 + 1/5*b*f*x^5 + 1/4*b*e*x^4 + 1/3*
b*d*x^3 + 1/3*a*f*x^3 + 1/2*a*e*x^2 + a*d*x
```

Mupad [B] (verification not implemented)

Time = 17.96 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int (d+ex+fx^2)(a+bx^2+cx^4)\,dx = \frac{c\,f\,x^7}{7}+\frac{c\,e\,x^6}{6}+\left(\frac{c\,d}{5}+\frac{b\,f}{5}\right)x^5+\frac{b\,e\,x^4}{4} + \left(\frac{b\,d}{3}+\frac{a\,f}{3}\right)x^3+\frac{a\,e\,x^2}{2}+a\,d\,x$$

input
```
int((d + e*x + f*x^2)*(a + b*x^2 + c*x^4),x)
```

output
```
x^3*((b*d)/3 + (a*f)/3) + x^5*((c*d)/5 + (b*f)/5) + a*d*x + (a*e*x^2)/2 +
(b*e*x^4)/4 + (c*e*x^6)/6 + (c*f*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int (d + ex + fx^2) (a + bx^2 + cx^4) \, dx$$
$$= \frac{x(60cf\,x^6 + 70ce\,x^5 + 84bf\,x^4 + 84cd\,x^4 + 105be\,x^3 + 140af\,x^2 + 140bd\,x^2 + 210aex + 420ad)}{420}$$

input
```
int((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x)
```

output
```
(x*(420*a*d + 210*a*e*x + 140*a*f*x**2 + 140*b*d*x**2 + 105*b*e*x**3 + 84*
b*f*x**4 + 84*c*d*x**4 + 70*c*e*x**5 + 60*c*f*x**6))/420
```

3.33 $\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx = \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{e\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
1/2*(f+(-b*f+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*(f-(
-b*f+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(
1/2))^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-e*arctanh((2*c*x
^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.11

$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx = \frac{\frac{\sqrt{2}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)f\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)f\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}+e\log\left(-b+\sqrt{b^2-\right.}{2\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4),x]
```

output
```
((Sqrt[2]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + e*Log[-b
+ Sqrt[b^2 - 4*a*c] - 2*c*x^2] - e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
2*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2202, 27, 1432, 1083, 219, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{fx^2+d}{cx^4+bx^2+a}dx+\int \frac{ex}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{fx^2+d}{cx^4+bx^2+a}dx+e\int \frac{x}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{fx^2+d}{cx^4+bx^2+a}dx+\frac{1}{2}e\int \frac{1}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{fx^2+d}{cx^4+bx^2+a}dx-e\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{fx^2+d}{cx^4+bx^2+a}dx-\frac{e\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx-\frac{e\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4),x]
```

output
```
((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((f
- (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^2f+\_Re+d\right)\ln\left(x-\_R\right)}{2\_R^3c+\_Rb}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(-\frac{e\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{2}+\frac{\left(\sqrt{-4ac+b^2}\,f+bf-2cd\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4c(4ac-b^2)}-\frac{\sqrt{-4ac+b^2}\left(\frac{e\ln(\cdots}{}\right.}{}\right.$

input
```
int((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2*f+_R*e+d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.19 (sec) , antiderivative size = 723401, normalized size of antiderivative = 3428.44

$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx = \int \frac{fx^2+ex+d}{cx^4+bx^2+a}\,dx$$

input
```
integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1712 vs. $2(173) = 346$.

Time $= 0.88$ (sec) , antiderivative size $= 1712$, normalized size of antiderivative $= 8.11$

$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*e*log(x^2 + 1/2*
(b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*
b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*
b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8
*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2
*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*d - 2*(2*a*b^2*c^2 - 8
*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a
*c^2)*f)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8
*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)
*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2...
```

Mupad [B] (verification not implemented)

Time = 19.30 (sec) , antiderivative size = 3942, normalized size of antiderivative = 18.68

$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2)/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log(c^2*d*e^2 - c^2*d^2*f + c^2*e^3*x - a*c*f^3 - 8*root(16*a*b^4*c
*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2 + 64*a^2
*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2*d^2*z^2
 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*c*e*f^2
*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*e^2*f
+ 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f^2 +
a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^3*b^3*c^2*x + b*c*d*f^2 -
 16*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2
*c*d*f*z^2 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 -
 16*a*b*c^2*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z
^2 + 16*a^2*c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e
*z - 4*a*c*d*e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e
^2 + a*b*e^2*f^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^2*a*c^
3*d - 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a
*b^2*c*d*f*z^2 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z
^2 - 16*a*b*c^2*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f
^2*z^2 + 16*a^2*c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d
^2*e*z - 4*a*c*d*e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d
^2*e^2 + a*b*e^2*f^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)*c^
3*d^2*x + 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 829, normalized size of antiderivative = 3.93

$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

output

```
( - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*e -
4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*f + 2*sqrt(a)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*b*c*d + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*f
- 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d - 4*sqrt(2*sqrt(c)*sqrt(
a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*e + 4*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a*c*f - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c*d -
2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +
2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*f + 4*sqrt(c)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*a*c*d + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqr
t(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c*f - sqrt(a)*sqrt(
2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) +...
```

3.34 $\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 368

$$\begin{aligned}\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}\,dx = &-\frac{e(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\ &+\frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} \\ &+\frac{\sqrt{c}\left(bd-2af+\frac{b^2d-12acd+4abf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\ &+\frac{\sqrt{c}\left(bd-2af-\frac{b^2d-12acd+4abf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} \\ &+\frac{2ce\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

output
```
-1/2*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*(b^2*d-2*a*c*d-a*b*f
+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/2)*(b*d-2*a*f
+(4*a*b*f-12*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-
(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2)+1/4*c^(1/2)*(b*d-2*a*f-(4*a*b*f-12*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*a
rctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+2*c*e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.08

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{2ab(e+fx)-2bdx(b+cx^2)+4acx(d+x(e+fx))}{a(-b^2+4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}d+4af\right)-2a\left(6cd+\sqrt{b^2-4ac}f\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2d+12acd+b\sqrt{b^2-4ac}d-4abf-2a\sqrt{b^2-4ac}f\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{4ce\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{4ce\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*a*b*(e + f*x) - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x)))/(a*(-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2
- 4*a*c]*d + 4*a*f) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4
*a*c]*d - 4*a*b*f - 2*a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]) - (4*c*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2)
+ (4*c*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2202, 27, 1432, 1086, 1083, 219, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx+\int \frac{ex}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx+e\int \frac{x}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}e\int \frac{1}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}e\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)\,(a+bx^2+cx^4)}\right)+\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}e\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)\,(a+bx^2+cx^4)}\right)+\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{219}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}e\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)\,(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{1492}$$

$$-\frac{\int -\frac{db^2+afb+c(bd-2af)x^2-6acd}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{1}{2}e\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{db^2+afb+c(bd-2af)x^2-6acd}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{1}{2}e\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{1}{2}c\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a(b^2-4ac)}$$

$$\frac{1}{2}e\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a(b^2-4ac)}+$$

$$\frac{1}{2}e\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

input
```
Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + ((Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d + 4*a*b*f)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d -
 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 -
4*a*c)) + (e*(-((b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (4*c*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.63

method	result
risch	$\frac{\frac{c(2af-bd)x^3}{2(4ac-b^2)a}+\frac{c\,x^2e}{4ac-b^2}+\frac{(abf+2dac-b^2d)x}{2a(4ac-b^2)}+\frac{be}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{c(2af-bd)\_R^2}{(4ac-b^2)a}+\frac{4\_Rce}{4ac-b^2}-\frac{abf-6dac+b^2}{(4ac-b^2)a}\right.}{2\_R^3c+\_Rb}\right.}{4}$
default	$16c^2\left(\frac{\frac{\frac{\left(4\sqrt{-4ac+b^2}\,acd-\sqrt{-4ac+b^2}\,b^2d+8a^2cf-2a\,b^2f-4abcd+b^3d\right)x}{16ac}+\frac{e(4ac-b^2)}{8c}}{x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}}+\frac{2\sqrt{-4ac+b^2}\,ae\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4\sqrt{-\right.}{}}{}}{4c(4ac-b^2)^2}\right.$

input
```
int((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*c*(2*a*f-b*d)/(4*a*c-b^2)/a*x^3+c/(4*a*c-b^2)*x^2*e+1/2*(a*b*f+2*a*c*
d-b^2*d)/a/(4*a*c-b^2)*x+1/2/(4*a*c-b^2)*b*e)/(c*x^4+b*x^2+a)+1/4*sum((c*(
2*a*f-b*d)/(4*a*c-b^2)/a*_R^2+4/(4*a*c-b^2)*_R*c*e-(a*b*f-6*a*c*d+b^2*d)/(
4*a*c-b^2)/a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}\,dx = \int \frac{fx^2+ex+d}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*a*c*e*x^2 - (b*c*d - 2*a*c*f)*x^3 + a*b*e + (a*b*f - (b^2 - 2*a*c)
*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c
)*x^2) - 1/2*integrate((4*a*c*e*x - a*b*f - (b*c*d - 2*a*c*f)*x^2 - (b^2 -
 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5159 vs. $2(320) = 640$.

Time = 1.24 (sec) , antiderivative size = 5159, normalized size of antiderivative = 14.02

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*d*x^3 - 2*a*c*f*x^3 - 2*a*c*e*x^2 + b^2*d*x - 2*a*c*d*x - a*b*f*x
 - a*b*e)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a
*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*f +
2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c
^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 -
128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192
*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*...
```

Mupad [B] (verification not implemented)

Time = 18.85 (sec) , antiderivative size = 4707, normalized size of antiderivative = 12.79

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((5*b^3*c^4*d^3 + 8*a^3*c^4*f^3 - 96*a^2*c^5*d*e^2 + 72*a^2*c^5*
d^2*f - 3*b^4*c^3*d^2*f + 6*a^2*b^2*c^3*f^3 - 36*a*b*c^5*d^3 + 16*a*b^2*c^
4*d*e^2 + 18*a*b^2*c^4*d^2*f + 3*a*b^3*c^3*d*f^2 - 60*a^2*b*c^4*d*f^2 + 16
*a^2*b*c^4*e^2*f)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
)) - root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^
6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*
z^4 - 256*a^3*b^12*z^4 + 576*a^2*b^8*c*d*f*z^2 + 24576*a^5*b^2*c^4*d*f*z^2
 - 3072*a^3*b^6*c^2*d*f*z^2 + 2048*a^4*b^4*c^3*d*f*z^2 + 12288*a^6*b*c^4*f
^2*z^2 + 61440*a^5*b*c^5*d^2*z^2 - 49152*a^6*c^5*d*f*z^2 + 432*a*b^9*c*d^2
*z^2 - 8192*a^5*b^3*c^3*f^2*z^2 + 1536*a^4*b^5*c^2*f^2*z^2 + 24576*a^5*b^2
*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 512*a^3*b^6*c^2*e^2*z^2 - 61440*
a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*z^2 - 4608*a^2*b^7*c^2*d^2*z^2
 - 32*a*b^10*d*f*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*a^2*b^9*f^2*z^2 - 16*b^1
1*d^2*z^2 - 4096*a^4*b*c^4*d*e*f*z + 64*a*b^7*c*d*e*f*z + 3072*a^3*b^3*c^3
*d*e*f*z - 768*a^2*b^5*c^2*d*e*f*z + 32*a^2*b^6*c*e*f^2*z - 672*a*b^6*c^2*
d^2*e*z + 1536*a^4*b^2*c^3*e*f^2*z - 384*a^3*b^4*c^2*e*f^2*z - 15872*a^3*b
^2*c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*e*z - 2048*a^5*c^4*e*f^2*z + 18432*a
^4*c^5*d^2*e*z + 32*b^8*c*d^2*e*z - 32*a*b^4*c^2*d*e^2*f + 192*a^2*b^2*c^3
*d*e^2*f - 192*a^3*b*c^3*e^2*f^2 + 198*a*b^4*c^2*d^2*f^2 + 144*a^2*b^3*c^2
*d*f^3 - 960*a^2*b*c^4*d^2*e^2 + 240*a*b^3*c^3*d^2*e^2 + 768*a^3*c^4*d*...
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 4754, normalized size of antiderivative = 12.92

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
( - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*
c*e - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*
b**2*c*e*x**2 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*a**2*b*c**2*e*x**4 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b
*c*f - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*f + 16*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c*d - 8*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a**2*b**2*c*f*x**2 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*a**2*b*c**2*f*x**4 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4
*d - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*f*x**2 + 16*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(...
```

3.35 $\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 621

$$\begin{aligned}
\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}\,dx &= -\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} \\
&+ \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3ce(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)} \\
&+ \frac{x(3b^4d-25ab^2cd+28a^2c^2d+ab^3f+8a^2bcf+c(3b^3d-24abcd+ab^2f+20a^2cf)x^2)}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)} \\
&+ \frac{\sqrt{c}\left(3b^4d+b^3\left(3\sqrt{b^2-4ac}\,d+af\right)-4abc\left(6\sqrt{b^2-4ac}\,d+13af\right)-ab^2\left(30cd-\sqrt{b^2-4ac}\,f\right)+4a^2c(4}{8\sqrt{2}a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&+ \frac{\sqrt{c}\left(3b^3d-24abcd+ab^2f+20a^2cf-\frac{3b^4d-30ab^2cd+168a^2c^2d+ab^3f-52a^2bcf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}} \\
&- \frac{6c^2e\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}
\end{aligned}$$

output
```
-1/4*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*x*(b^2*d-2*a*c*d-a*b
*f+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/2*c*e*(2*c*x^2+b
)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/8*x*(3*b^4*d-25*a*b^2*c*d+28*a^2*c^2*d+
a*b^3*f+8*a^2*b*c*f+c*(20*a^2*c*f+a*b^2*f-24*a*b*c*d+3*b^3*d)*x^2)/a^2/(-4
*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*c^(1/2)*(3*b^4*d+b^3*(3*(-4*a*c+b^2)^(1/2
)*d+a*f)-4*a*b*c*(6*(-4*a*c+b^2)^(1/2)*d+13*a*f)-a*b^2*(30*c*d-f*(-4*a*c+b
^2)^(1/2))+4*a^2*c*(42*c*d+5*f*(-4*a*c+b^2)^(1/2)))*arctan(2^(1/2)*c^(1/2)
*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(5/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)+1/16*c^(1/2)*(3*b^3*d-24*a*b*c*d+a*b^2*f+20*a^2*c*f-(-5
2*a^2*b*c*f+168*a^2*c^2*d+a*b^3*f-30*a*b^2*c*d+3*b^4*d)/(-4*a*c+b^2)^(1/2)
)*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a
*c+b^2)^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)-6*c^2*e*arctanh((2*c*x^2+b)/(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 3.43 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.01

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{16}\left(\frac{4ab(e+fx)-4bdx(b+cx^2)+8acx(d+x(e+fx))}{a(-b^2+4ac)(a+bx^2+cx^4)^2}\right.$$
$$+\frac{6b^3dx(b+cx^2)+2abx(-25bcd+b^2f-24c^2dx^2+bcfx^2)+8a^2c(b(3e+2fx)+cx(7d+6ex+5fx^2)}{a^2(b^2-4ac)^2(a+bx^2+cx^4)}$$
$$+\frac{\sqrt{2}\sqrt{c}\left(3b^4d+b^3\left(3\sqrt{b^2-4ac}\,d+af\right)-4abc\left(6\sqrt{b^2-4ac}\,d+13af\right)+ab^2\left(-30cd+\sqrt{b^2-4ac}\,f\right)+4}{a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\sqrt{c}\left(-3b^4d+b^3\left(3\sqrt{b^2-4ac}\,d-af\right)+4abc\left(-6\sqrt{b^2-4ac}\,d+13af\right)+ab^2\left(30cd+\sqrt{b^2-4ac}\,f\right)+}{a^2(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.+\frac{48c^2e\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{5/2}}-\frac{48c^2e\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}}\right)$$

input
```
Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((4*a*b*(e + f*x) - 4*b*d*x*(b + c*x^2) + 8*a*c*x*(d + x*(e + f*x)))/(a*(-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (6*b^3*d*x*(b + c*x^2) + 2*a*b*x*(-2
5*b*c*d + b^2*f - 24*c^2*d*x^2 + b*c*f*x^2) + 8*a^2*c*(b*(3*e + 2*f*x) + c
*x*(7*d + 6*e*x + 5*f*x^2)))/(a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
Sqrt[2]*Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6*
Sqrt[b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(-30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*
a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) + (Sqrt[2]*Sqrt[c]*(-3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d - a*f) + 4*a*
b*c*(-6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(30*c*d + Sqrt[b^2 - 4*a*c]*
f) + 4*a^2*c*(-42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + (48*c^2*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c
)^(5/2) - (48*c^2*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5
/2))/16
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 605, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2202, 27, 1432, 1086, 1086, 1083, 219, 1492, 25, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^3}dx+\int \frac{ex}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^3}dx+e\int \frac{x}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{ 1432}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^3}dx+\frac{1}{2}e\int \frac{1}{(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}e\left(-\frac{3c\int\frac{1}{(cx^4+bx^2+a)^2}dx^2}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)+\int\frac{fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}e\left(-\frac{3c\left(-\frac{2c\int\frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)+$$
$$\int\frac{fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\int\frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)+$$
$$\int\frac{fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{219}$$

$$\int\frac{fx^2+d}{(cx^4+bx^2+a)^3}dx+$$
$$\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{1492}$$

$$-\frac{\int-\frac{3db^2+afb+5c(bd-2af)x^2-14acd}{(cx^4+bx^2+a)^2}dx}{4a(b^2-4ac)}+$$
$$\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)+$$
$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{3db^2+afb+5c(bd-2af)x^2-14acd}{(cx^4+bx^2+a)^2}dx}{4a\left(b^2-4ac\right)}+\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \ 1492$$

$$\frac{\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d+ab^3f-25ab^2cd+3b^4d\right)}{2a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{3db^4+afb^3-27acdb^2-16a^2cfb+c\left(3db^3+afb^2-24acdb+20a^2c\right.}{cx^4+bx^2+a}}{2a(b^2-4ac)}}{4a\left(b^2-4ac\right)}+\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{3db^4+afb^3-27acdb^2-16a^2cfb+c\left(3db^3+afb^2-24acdb+20a^2cf\right)x^2+84a^2c^2d}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d+ab^3f-2\right.}{2a(b^2-4ac)(a+bx^2+cx^4)}}{4a\left(b^2-4ac\right)}+\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \ 1480$$

$$\frac{\frac{1}{2}c\left(\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd}{\sqrt{b^2-4ac}}\right.}{2a(b^2-4ac)}$$

$$4a\,(b$$

$$\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2\,(b^2-4ac)\,(a+bx^2+cx^4)^2}\right)+$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-52a^2bcf+168a^2c}{\sqrt{}}\right.}{\sqrt{2}\sqrt{}}}{2a(b^2-4ac)}$$

$$4a\,(b^2-4$$

$$\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2\,(b^2-4ac)\,(a+bx^2+cx^4)^2}\right)+$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

input
```
Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)^2) + ((x*(3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f
 + 8*a^2*b*c*f + c*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(2*
a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((Sqrt[c]*(3*b^3*d - 24*a*b*c*d + a
*b^2*f + 20*a^2*c*f + (3*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f -
52*a^2*b*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d
- 24*a*b*c*d + a*b^2*f + 20*a^2*c*f - (3*b^4*d - 30*a*b^2*c*d + 168*a^2*c^
2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*
a*(b^2 - 4*a*c)))/(4*a*(b^2 - 4*a*c)) + (e*(-1/2*(b + 2*c*x^2)/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (3*c*(-((b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x
^2 + c*x^4))) + (4*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*
c)^(3/2)))/(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 607, normalized size of antiderivative = 0.98

method	result
risch	$\frac{\frac{c^2\left(20a^2cf+a\,b^2f-24abcd+3b^3d\right)x^7}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{3c^3e\,x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{c\left(28a^2bcf+28a^2c^2d+2a\,b^3f-49a\,b^2cd+6b^4d\right)x^5}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{9b\,c^2e\,x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(36a^3\right.}{}}{(c\,x^4+b}$
default	Expression too large to display

input
```
int((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*c^2*(20*a^2*c*f+a*b^2*f-24*a*b*c*d+3*b^3*d)/a^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^7+3*c^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/8/a^2*c*(28*a^2*b*c*f+2
8*a^2*c^2*d+2*a*b^3*f-49*a*b^2*c*d+6*b^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5
+9/2*b*c^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+1/8*(36*a^3*c^2*f+5*a^2*b^2*c*
f-4*a^2*b*c^2*d+a*b^4*f-20*a*b^3*c*d+3*b^5*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3+(5*a*c+b^2)*c*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/8*(16*a^2*b*c*f+44
*a^2*c^2*d-a*b^3*f-37*a*b^2*c*d+5*b^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x+1/
4*b*(10*a*c-b^2)*e/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/16*sum(
(c*(20*a^2*c*f+a*b^2*f-24*a*b*c*d+3*b^3*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*
_R^2+48*c^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*_R-(16*a^2*b*c*f-84*a^2*c^2*d-a*b
^3*f+27*a*b^2*c*d-3*b^4*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)
*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{\left(a+bx^2+cx^4\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}\,dx = \int \frac{fx^2+ex+d}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(24*a^2*c^3*e*x^6 + 36*a^2*b*c^2*e*x^4 + (3*(b^3*c^2 - 8*a*b*c^3)*d +
(a*b^2*c^2 + 20*a^2*c^3)*f)*x^7 + ((6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d
 + 2*(a*b^3*c + 14*a^2*b*c^2)*f)*x^5 + 8*(a^2*b^2*c + 5*a^3*c^2)*e*x^2 + (
(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d + (a*b^4 + 5*a^2*b^2*c + 36*a^3*c^2)*
f)*x^3 - 2*(a^2*b^3 - 10*a^3*b*c)*e + ((5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^
2)*d - (a^2*b^3 - 16*a^3*b*c)*f)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4
*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*
c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^
3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + 1/8*integrate((48*a^2*c^2*e*x +
 (3*(b^3*c - 8*a*b*c^2)*d + (a*b^2*c + 20*a^2*c^2)*f)*x^2 + 3*(b^4 - 9*a*b
^2*c + 28*a^2*c^2)*d + (a*b^3 - 16*a^2*b*c)*f)/(c*x^4 + b*x^2 + a), x)/(a^
2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5375 vs. $2(561) = 1122$.

Time = 1.78 (sec) , antiderivative size = 5375, normalized size of antiderivative = 8.66

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3*(b^2*c^4 - 4*a*c^5 - 2*b*c^5 + c^6)*sqrt(b^2 - 4*a*c)*e*log(x^2 + 1/2*(
a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2
*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))/((b^8 - 16*
a*b^6*c - 2*b^7*c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2*
c^3 - 96*a^2*b^3*c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^2
*b^2*c^4 - 64*a^3*c^5)*c^2) + 1/32*(3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^8 - 17*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*
c^2 - 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3
- 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 232*a^2*b^4*c^3 + 30*a*b^5*c^3 +
448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 - 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^4*c^5 + 352*a^3*b*c^5 + sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 15*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b^2 ...
```

Mupad [B] (verification not implemented)

Time = 20.57 (sec) , antiderivative size = 8689, normalized size of antiderivative = 13.99

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((x^2*(5*a*c^2*e + b^2*c*e))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3*e - 10*
a*b*c*e)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(28*a^2*c^3*d + 6*b^4*c
*d + 2*a*b^3*c*f - 49*a*b^2*c^2*d + 28*a^2*b*c^2*f))/(8*a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) + (x*(5*b^4*d + 44*a^2*c^2*d - a*b^3*f - 37*a*b^2*c*d +
16*a^2*b*c*f))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^3*e*x^6)/(b^4 +
 16*a^2*c^2 - 8*a*b^2*c) + (x^3*(3*b^5*d + 36*a^3*c^2*f + a*b^4*f - 20*a*b
^3*c*d - 4*a^2*b*c^2*d + 5*a^2*b^2*c*f))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)) + (9*b*c^2*e*x^4)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*a^
2*c^2*f + 3*b^3*c*d - 24*a*b*c^2*d + a*b^2*c*f))/(8*a^2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
 + symsum(log(root(56371445760*a^11*b^8*c^6*z^4 - 503316480*a^8*b^14*c^3*z
^4 + 47185920*a^7*b^16*c^2*z^4 - 171798691840*a^14*b^2*c^9*z^4 + 193273528
320*a^13*b^4*c^8*z^4 - 128849018880*a^12*b^6*c^7*z^4 - 16911433728*a^10*b^
10*c^5*z^4 + 3523215360*a^9*b^12*c^4*z^4 - 2621440*a^6*b^18*c*z^4 + 687194
76736*a^15*c^10*z^4 + 65536*a^5*b^20*z^4 - 73728*a^2*b^16*c*d*f*z^2 - 1321
205760*a^9*b^2*c^8*d*f*z^2 + 732168192*a^7*b^6*c^6*d*f*z^2 - 366280704*a^6
*b^8*c^5*d*f*z^2 - 330301440*a^8*b^4*c^7*d*f*z^2 + 96583680*a^5*b^10*c^4*d
*f*z^2 - 15175680*a^4*b^12*c^3*d*f*z^2 + 1428480*a^3*b^14*c^2*d*f*z^2 - 44
0401920*a^10*b*c^8*f^2*z^2 + 1761607680*a^10*c^9*d*f*z^2 - 14080*a^3*b^15*
c*f^2*z^2 + 6936330240*a^8*b^3*c^8*d^2*z^2 + 2464874496*a^6*b^7*c^6*d^2...
```

Reduce [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 13638, normalized size of antiderivative = 21.96

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

output
```
( - 192*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b
*c**2*e - 384*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**4*b**2*c**2*e*x**2 - 384*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqr
t(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a**4*b*c**3*e*x**4 - 192*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*
sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt
(2*sqrt(c)*sqrt(a) + b))*a**3*b**3*c**2*e*x**4 - 384*sqrt(2*sqrt(c)*sqrt(a
) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**3*e*x**6 - 192*sqrt(2*
sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**4*e*x**8 -
80*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b*c**2*f - 36*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**3*c*f + 264*sqrt(a)*sqrt(2*sqrt(c)*sqr
t(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a**4*b**2*c**2*d - 160*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) ...
```

3.36 $\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx=\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}}-\frac{b\,\text{arctanh}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

output
```
1/2*(c-(-2*a*f+c*e)/(-4*d*f+e^2)^(1/2))*arctan(2^(1/2)*f^(1/2)*x/(e-(-4*d*
f+e^2)^(1/2))^(1/2))*2^(1/2)/f^(1/2)/(e-(-4*d*f+e^2)^(1/2))^(1/2)+1/2*(c+(
-2*a*f+c*e)/(-4*d*f+e^2)^(1/2))*arctan(2^(1/2)*f^(1/2)*x/(e+(-4*d*f+e^2)^(
1/2))^(1/2))*2^(1/2)/f^(1/2)/(e+(-4*d*f+e^2)^(1/2))^(1/2)-b*arctanh((2*f*x
^2+e)/(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.12

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \frac{\frac{\sqrt{2}\left(2af+c\left(-e+\sqrt{e^2-4df}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\sqrt{2}\left(-2af+c\left(e+\sqrt{e^2-4df}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}}+b\log\left(-e+\sqrt{e^2-\cdots\right.}{2\sqrt{e^2-4df}}$$

input
```
Integrate[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

output
```
((Sqrt[2]*(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/
Sqrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + (Sqr
t[2]*(-2*a*f + c*(e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[
e + Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) + b*Log[-e
+ Sqrt[e^2 - 4*d*f] - 2*f*x^2] - b*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x^2])/(
2*Sqrt[e^2 - 4*d*f])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2202, 27, 1432, 1083, 219, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{cx^2+a}{fx^4+ex^2+d}dx+\int \frac{bx}{fx^4+ex^2+d}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{cx^2+a}{fx^4+ex^2+d}dx+b\int \frac{x}{fx^4+ex^2+d}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{cx^2+a}{fx^4+ex^2+d}dx+\frac{1}{2}b\int \frac{1}{fx^4+ex^2+d}dx^2$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{cx^2+a}{fx^4+ex^2+d}dx-b\int \frac{1}{-x^4+e^2-4df}d\left(2fx^2+e\right)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{cx^2+a}{fx^4+ex^2+d}dx-\frac{b\text{arctanh}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\int \frac{1}{fx^2+\frac{1}{2}\left(e-\sqrt{e^2-4df}\right)}dx+\frac{1}{2}\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\int \frac{1}{fx^2+\frac{1}{2}\left(e+\sqrt{e^2-4df}\right)}dx-\frac{b\text{arctanh}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}-\frac{b\text{arctanh}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

input
```
Int[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

output
```
((c - (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e -
 Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + ((c
+ (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqr
t[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) - (b*ArcTa
nh[(e + 2*f*x^2)/Sqrt[e^2 - 4*d*f]])/Sqrt[e^2 - 4*d*f]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(f\_Z^4+\_Z^2e+d\right)}\frac{\left(c\_R^2+\_Rb+a\right)\ln\left(x-\_R\right)}{2\_R^3f+\_Re}\right)}{2}$
default	$4f\left(-\frac{\sqrt{-4df+e^2}\left(-\frac{b\ln\left(2fx^2+\sqrt{-4df+e^2}+e\right)}{2}+\frac{\left(\sqrt{-4df+e^2}\,c-2af+ce\right)\sqrt{2}\arctan\left(\frac{fx\sqrt{2}}{\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)}{2\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)}{4f(4df-e^2)}-\frac{\sqrt{-4df+e^2}\left(b\right.}{}\right.$

input
```
int((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2*c+_R*b+a)/(2*_R^3*f+_R*e)*ln(x-_R),_R=RootOf(_Z^4*f+_Z^2*e+d
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.82 (sec) , antiderivative size = 578003, normalized size of antiderivative = 2765.56

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(f*x**4+e*x**2+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \int \frac{cx^2+bx+a}{fx^4+ex^2+d}\,dx$$

input
```
integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)/(f*x^4 + e*x^2 + d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1712 vs. $2(171) = 342$.

Time $= 0.88$ (sec) , antiderivative size $= 1712$, normalized size of antiderivative $= 8.19$

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="giac")
```

output
```
1/2*(e^2*f^2 - 4*d*f^3 - 2*e*f^3 + f^4)*sqrt(e^2 - 4*d*f)*b*log(x^2 + 1/2*
(e - sqrt(e^2 - 4*d*f))/f)/((e^4 - 8*d*e^2*f - 2*e^3*f + 16*d^2*f^2 + 8*d*
e*f^2 + e^2*f^2 - 4*d*f^3)*f^2) + 1/4*((sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*
f)*f)*e^4 - 8*sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e^2*f - 2*sqrt(2)*
sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e^3*f - 2*e^4*f + 16*sqrt(2)*sqrt(e*f + sq
rt(e^2 - 4*d*f)*f)*d^2*f^2 + 8*sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e
*f^2 + sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e^2*f^2 + 16*d*e^2*f^2 + 2*
e^3*f^2 - 4*sqrt(2)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*f^3 - 32*d^2*f^3 - 8
*d*e*f^3 - sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e^3 +
 4*sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e*f + 2*sqr
t(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e^2*f - sqrt(2)*sqr
t(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*e*f^2 + 2*(e^2 - 4*d*f)*e^2
*f - 8*(e^2 - 4*d*f)*d*f^2 - 2*(e^2 - 4*d*f)*e*f^2)*a - 2*(2*d*e^2*f^2 - 8
*d^2*f^3 - sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e^2
 + 4*sqrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d^2*f + 2*s
qrt(2)*sqrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*e*f - sqrt(2)*s
qrt(e^2 - 4*d*f)*sqrt(e*f + sqrt(e^2 - 4*d*f)*f)*d*f^2 - 2*(e^2 - 4*d*f)*d
*f^2)*c)*arctan(2*sqrt(1/2)*x/sqrt((e + sqrt(e^2 - 4*d*f))/f))/((d*e^4 - 8
*d^2*e^2*f - 2*d*e^3*f + 16*d^3*f^2 + 8*d^2*e*f^2 + d*e^2*f^2 - 4*d^2*f^3)
*abs(f)) + 1/4*((sqrt(2)*sqrt(e*f - sqrt(e^2 - 4*d*f)*f)*e^4 - 8*sqrt(2...
```

Mupad [B] (verification not implemented)

Time = 19.01 (sec) , antiderivative size = 3942, normalized size of antiderivative = 18.86

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x)
```

output
```
symsum(log(a*b^2*f^2 - a^2*c*f^2 + b^3*f^2*x - c^3*d*f - 8*root(16*d*e^4*f
*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2
*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2
 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f
*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f
+ 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f +
b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*e^3*f^2*x + a*c^2*e*f -
 16*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d
*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 +
 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z
^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2
*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d
*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*d*
f^3 - 4*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a
*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z
^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3
*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d
*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c
^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a^
2*f^3*x + 4*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 829, normalized size of antiderivative = 3.97

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x)
```

output
```
( - 4*sqrt(2*sqrt(f)*sqrt(d) + e)*sqrt(2*sqrt(f)*sqrt(d) - e)*atan((sqrt(2
*sqrt(f)*sqrt(d) - e) - 2*sqrt(f)*x)/sqrt(2*sqrt(f)*sqrt(d) + e))*b*d*f +
2*sqrt(d)*sqrt(2*sqrt(f)*sqrt(d) + e)*atan((sqrt(2*sqrt(f)*sqrt(d) - e) -
2*sqrt(f)*x)/sqrt(2*sqrt(f)*sqrt(d) + e))*a*e*f - 4*sqrt(d)*sqrt(2*sqrt(f)
*sqrt(d) + e)*atan((sqrt(2*sqrt(f)*sqrt(d) - e) - 2*sqrt(f)*x)/sqrt(2*sqrt
(f)*sqrt(d) + e))*c*d*f - 4*sqrt(f)*sqrt(2*sqrt(f)*sqrt(d) + e)*atan((sqrt
(2*sqrt(f)*sqrt(d) - e) - 2*sqrt(f)*x)/sqrt(2*sqrt(f)*sqrt(d) + e))*a*d*f
+ 2*sqrt(f)*sqrt(2*sqrt(f)*sqrt(d) + e)*atan((sqrt(2*sqrt(f)*sqrt(d) - e)
- 2*sqrt(f)*x)/sqrt(2*sqrt(f)*sqrt(d) + e))*c*d*e - 4*sqrt(2*sqrt(f)*sqrt(
d) + e)*sqrt(2*sqrt(f)*sqrt(d) - e)*atan((sqrt(2*sqrt(f)*sqrt(d) - e) + 2*
sqrt(f)*x)/sqrt(2*sqrt(f)*sqrt(d) + e))*b*d*f - 2*sqrt(d)*sqrt(2*sqrt(f)*s
qrt(d) + e)*atan((sqrt(2*sqrt(f)*sqrt(d) - e) + 2*sqrt(f)*x)/sqrt(2*sqrt(f
)*sqrt(d) + e))*a*e*f + 4*sqrt(d)*sqrt(2*sqrt(f)*sqrt(d) + e)*atan((sqrt(2
*sqrt(f)*sqrt(d) - e) + 2*sqrt(f)*x)/sqrt(2*sqrt(f)*sqrt(d) + e))*c*d*f +
4*sqrt(f)*sqrt(2*sqrt(f)*sqrt(d) + e)*atan((sqrt(2*sqrt(f)*sqrt(d) - e) +
2*sqrt(f)*x)/sqrt(2*sqrt(f)*sqrt(d) + e))*a*d*f - 2*sqrt(f)*sqrt(2*sqrt(f)
*sqrt(d) + e)*atan((sqrt(2*sqrt(f)*sqrt(d) - e) + 2*sqrt(f)*x)/sqrt(2*sqrt
(f)*sqrt(d) + e))*c*d*e - sqrt(d)*sqrt(2*sqrt(f)*sqrt(d) - e)*log( - sqrt(
2*sqrt(f)*sqrt(d) - e)*x + sqrt(d) + sqrt(f)*x**2)*a*e*f + 2*sqrt(d)*sqrt(
2*sqrt(f)*sqrt(d) - e)*log( - sqrt(2*sqrt(f)*sqrt(d) - e)*x + sqrt(d) +...
```

3.37 $\int \frac{(d+ex)^2}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 224

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{2de\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
1/2*(e^2+(-b*e^2+2*c*d^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-
(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/
2*(e^2-(-b*e^2+2*c*d^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-
4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-2*d*
e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx$$
$$= \frac{\frac{\sqrt{2}\left(2cd^2+\left(-b+\sqrt{b^2-4ac}\right)e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}+2de\log\left(-b+\sqrt{\right.}}{2\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

output
```
((Sqrt[2]*(2*c*d^2 + (-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[2]*(-2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 2
*d*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - 2*d*e*Log[b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2202, 27, 1432, 1083, 219, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 2202$$
$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx+\int \frac{2dex}{cx^4+bx^2+a}dx$$
$$\Big\downarrow \; 27$$
$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx+2de\int \frac{x}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx+de\int \frac{1}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx-2de\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx-\frac{2de\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx-\frac{2de\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2de\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

output
```
((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) - (2*d*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2\_R^3c+\_Rb}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(-ed\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(\sqrt{-4ac+b^2}\,e^2+b\,e^2-2c\,d^2\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4c(4ac-b^2)}-\frac{\sqrt{-4ac+b^2}}{}\right.$

input
```
int((e*x+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+
_Z^2*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 76.28 (sec) , antiderivative size = 540080, normalized size of antiderivative = 2411.07

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \int \frac{(ex+d)^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1723 vs. $2(186) = 372$.

Time = 0.79 (sec) , antiderivative size = 1723, normalized size of antiderivative = 7.69

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*d*e*log(x^2 + 1/2*(b
 - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*
c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/2*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*
a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 + (b^4*c - 6*a*b^2*c^2 - 2*b
^3*c^2 + 8*a^2*c^3 + 4*a*b*c^3 + b^2*c^3 - 2*a*c^4)*sqrt(b^2 - 4*a*c))*d*e
*log(x^2 + 1/2*(b + sqrt(b^2 - 4*a*c))/c)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2) + 1/4*((sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c
 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*d^2 - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b...
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 3046, normalized size of antiderivative = 13.60

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^2/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log(3*c^2*d^4*e^2 - a*c*e^6 - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c
^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 1
6*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^
4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*
e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*
d^8 + a^2*e^8, z, k)^3*b^3*c^2*x + 4*c^2*d^3*e^3*x + 4*root(16*a*b^4*c*z^4
 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2
*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*
z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*
e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^
2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*b^2*c^2*d^2 + b*c*d^2*e^4 - 4*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^
2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2
+ 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z -
 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*
b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(16*
a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z
^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4
*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32
*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 863, normalized size of antiderivative = 3.85

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4} \, dx = \text{Too large to display}$$

input
```
int((e*x+d)^2/(c*x^4+b*x^2+a),x)
```

output
```
( - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d*e
- 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*e**2 + 2*sqrt(a)*sqrt(2*sq
rt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2
*sqrt(c)*sqrt(a) + b))*b*c*d**2 + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b)
)*a*b*e**2 - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d**2 - 8*sqrt(2
*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqr
t(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d*e + 4*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*e**2 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a
) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*b*c*d**2 - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*e**2 +
 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d**2 + 2*sqrt(a)*sqrt(2*sqr
t(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)
*x**2)*a*c*e**2 - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqr...
```

3.38 $\int \frac{1+x^2}{1-x^2+x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 23

$$\int \frac{1+x^2}{1-x^2+x^4} \, dx = -\arctan\left(\sqrt{3}-2x\right)+\arctan\left(\sqrt{3}+2x\right)$$

output
```
arctan(-3^(1/2)+2*x)+arctan(3^(1/2)+2*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{1+x^2}{1-x^2+x^4} \, dx = -\arctan\left(\frac{x}{-1+x^2}\right)$$

input
```
Integrate[(1 + x^2)/(1 - x^2 + x^4),x]
```

output
```
-ArcTan[x/(-1 + x^2)]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^4-x^2+1} \, dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)-\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)$$

$$\Big\downarrow \text{217}$$

$$\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)$$

input
```
Int[(1 + x^2)/(1 - x^2 + x^4),x]
```

output
```
-ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.35

method	result	size
risch	$\arctan\left(x^{3}\right)+\arctan\left(x\right)$	8
default	$\arctan\left(2x-\sqrt{3}\right)+\arctan\left(2x+\sqrt{3}\right)$	20
parallelrisch	$\frac{i\ln\left(x^{2}+ix-1\right)}{2}-\frac{i\ln\left(x^{2}-ix-1\right)}{2}$	28

input
```
int((x^2+1)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
arctan(x^3)+arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx=\arctan\left(x^3\right)+\arctan\left(x\right)$$

input
```
integrate((x^2+1)/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
arctan(x^3) + arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \operatorname{atan}(x) + \operatorname{atan}(x^3)$$

input
```
integrate((x**2+1)/(x**4-x**2+1),x)
```

output
```
atan(x) + atan(x**3)
```

Maxima [F]

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \int \frac{x^2+1}{x^4-x^2+1}\,dx$$

input
```
integrate((x^2+1)/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \frac{1}{4}\,\pi\operatorname{sgn}(x) + \frac{1}{2}\,\arctan\left(\frac{x^4-3\,x^2+1}{2\,(x^3-x)}\right)$$

input
```
integrate((x^2+1)/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/4*pi*sgn(x) + 1/2*arctan(1/2*(x^4 - 3*x^2 + 1)/(x^3 - x))
```

Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.30

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \operatorname{atan}(x^3) + \operatorname{atan}(x)$$

input
```
int((x^2 + 1)/(x^4 - x^2 + 1),x)
```

output
```
atan(x^3) + atan(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = -atan\left(\sqrt{3}-2x\right) + atan\left(\sqrt{3}+2x\right)$$

input
```
int((x^2+1)/(x^4-x^2+1),x)
```

output
```
- atan(sqrt(3) - 2*x) + atan(sqrt(3) + 2*x)
```

3.39 $\int \frac{5+8x+5x^2}{1-x^2+x^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{5+8x+5x^2}{1-x^2+x^4} dx = -\frac{1}{3}\left(15+8\sqrt{3}\right)\arctan\left(\sqrt{3}-2x\right)+\frac{1}{3}\left(15-8\sqrt{3}\right)\arctan\left(\sqrt{3}+2x\right)$$

output
```
1/3*(15+8*3^(1/2))*arctan(-3^(1/2)+2*x)+1/3*(15-8*3^(1/2))*arctan(3^(1/2)+
2*x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \frac{5+8x+5x^2}{1-x^2+x^4} dx = 5\arctan\left(\frac{1}{2}\left(x-i\sqrt{3}x\right)\right) + 5\arctan\left(\frac{1}{2}\left(x+i\sqrt{3}x\right)\right) - \frac{8\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(5 + 8*x + 5*x^2)/(1 - x^2 + x^4),x]
```

output
```
5*ArcTan[(x - I*Sqrt[3]*x)/2] + 5*ArcTan[(x + I*Sqrt[3]*x)/2] - (8*ArcTan[
(1 - 2*x^2)/Sqrt[3]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2202, 27, 1432, 1083, 217, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+8x+5}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{8x}{x^4-x^2+1}dx+\int \frac{5x^2+5}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 27}$$

$$8\int \frac{x}{x^4-x^2+1}dx+\int \frac{5x^2+5}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 1432}$$

$$4\int \frac{1}{x^4-x^2+1}dx^2+\int \frac{5x^2+5}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{5x^2+5}{x^4-x^2+1}dx-8\int \frac{1}{-x^4-3}d(2x^2-1)$$

$$\Big\downarrow \text{ 217}$$

$$\int \frac{5x^2+5}{x^4-x^2+1}dx+\frac{8\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{5}{2}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{5}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{8\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{1083}$$

$$-5\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)-5\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)+\frac{8\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{217}$$

$$\frac{8\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}-5\arctan\left(\sqrt{3}-2x\right)+5\arctan\left(2x+\sqrt{3}\right)$$

input
```
Int[(5 + 8*x + 5*x^2)/(1 - x^2 + x^4),x]
```

output
```
-5*ArcTan[Sqrt[3] - 2*x] + 5*ArcTan[Sqrt[3] + 2*x] + (8*ArcTan[(-1 + 2*x^2
)/Sqrt[3]])/Sqrt[3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
default	$-2\left(-\frac{5}{2}-\frac{4\sqrt{3}}{3}\right)\arctan\left(2x-\sqrt{3}\right)+2\left(\frac{5}{2}-\frac{4\sqrt{3}}{3}\right)\arctan\left(2x+\sqrt{3}\right)$	38
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-\_Z^2+1\right)}\frac{\left(5\_R^2+8\_R+5\right)\ln\left(x-\_R\right)}{2\_R^3-\_R}\right)}{2}$	45

input
```
int((5*x^2+8*x+5)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-2*(-5/2-4/3*3^(1/2))*arctan(2*x-3^(1/2))+2*(5/2-4/3*3^(1/2))*arctan(2*x+3
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{5+8x+5x^2}{1-x^2+x^4} \, dx = -\frac{1}{3} \left(8 \sqrt{3}-15\right) \arctan \left(2 x+\sqrt{3}\right) - \frac{1}{3} \left(8 \sqrt{3}+15\right) \arctan \left(-2 x+\sqrt{3}\right)$$

input
```
integrate((5*x^2+8*x+5)/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
-1/3*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) - 1/3*(8*sqrt(3) + 15)*arctan(
-2*x + sqrt(3))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{5+8x+5x^2}{1-x^2+x^4} \, dx = 2 \cdot \left(\frac{4\sqrt{3}}{3}+\frac{5}{2}\right) \operatorname{atan}\left(2x-\sqrt{3}\right) + 2 \cdot \left(\frac{5}{2}-\frac{4\sqrt{3}}{3}\right) \operatorname{atan}\left(2x+\sqrt{3}\right)$$

input
```
integrate((5*x**2+8*x+5)/(x**4-x**2+1),x)
```

output
```
2*(4*sqrt(3)/3 + 5/2)*atan(2*x - sqrt(3)) + 2*(5/2 - 4*sqrt(3)/3)*atan(2*x
 + sqrt(3))
```

Maxima [F]

$$\int \frac{5+8x+5x^2}{1-x^2+x^4} \, dx = \int \frac{5 \, x^{2} + 8 \, x + 5}{x^{4} - x^{2} + 1} \, dx$$

input
```
integrate((5*x^2+8*x+5)/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
integrate((5*x^2 + 8*x + 5)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{5+8x+5x^2}{1-x^2+x^4}\,dx = -\frac{1}{3}\left(8\sqrt{3}-15\right)\arctan\left(2\,x+\sqrt{3}\right) + \frac{1}{3}\left(8\sqrt{3}+15\right)\arctan\left(2\,x-\sqrt{3}\right)$$

input
```
integrate((5*x^2+8*x+5)/(x^4-x^2+1),x, algorithm="giac")
```

output
```
-1/3*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) + 1/3*(8*sqrt(3) + 15)*arctan(
2*x - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.21

$$\int \frac{5+8x+5x^2}{1-x^2+x^4}\,dx = 2\,\mathrm{atanh}\left(\frac{960\sqrt{-\frac{20\sqrt{3}}{3}-\frac{139}{12}}}{2400\,x+1280\sqrt{3}\,x-800\sqrt{3}-1280} - \frac{320\sqrt{3}\,x\sqrt{-\frac{20\sqrt{3}}{3}-\frac{139}{12}}}{2400\,x+1280\sqrt{3}\,x-800\sqrt{3}-1280}\right)\sqrt{-\frac{20\sqrt{3}}{3}-\frac{139}{12}} + 2\,\mathrm{atanh}\left(\frac{960\sqrt{\frac{20\sqrt{3}}{3}-\frac{139}{12}}}{2400\,x-1280\sqrt{3}\,x+800\sqrt{3}-1280} + \frac{320\sqrt{3}\,x\sqrt{\frac{20\sqrt{3}}{3}-\frac{139}{12}}}{2400\,x-1280\sqrt{3}\,x+800\sqrt{3}-1280}\right)\sqrt{\frac{20\sqrt{3}}{3}-\frac{139}{12}}$$

input
```
int((8*x + 5*x^2 + 5)/(x^4 - x^2 + 1),x)
```

output
```
2*atanh((960*(- (20*3^(1/2))/3 - 139/12)^(1/2))/(2400*x + 1280*3^(1/2)*x -
 800*3^(1/2) - 1280) - (320*3^(1/2)*x*(- (20*3^(1/2))/3 - 139/12)^(1/2))/(
2400*x + 1280*3^(1/2)*x - 800*3^(1/2) - 1280))*(- (20*3^(1/2))/3 - 139/12)
^(1/2) + 2*atanh((960*((20*3^(1/2))/3 - 139/12)^(1/2))/(2400*x - 1280*3^(1
/2)*x + 800*3^(1/2) - 1280) + (320*3^(1/2)*x*((20*3^(1/2))/3 - 139/12)^(1/
2))/(2400*x - 1280*3^(1/2)*x + 800*3^(1/2) - 1280))*((20*3^(1/2))/3 - 139/
12)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{5+8x+5x^2}{1-x^2+x^4}\,dx = -\frac{8\sqrt{3}\,atan\left(\sqrt{3}-2x\right)}{3} - 5atan\left(\sqrt{3}-2x\right) - \frac{8\sqrt{3}\,atan\left(\sqrt{3}+2x\right)}{3} + 5atan\left(\sqrt{3}+2x\right)$$

input
```
int((5*x^2+8*x+5)/(x^4-x^2+1),x)
```

output
```
( - 8*sqrt(3)*atan(sqrt(3) - 2*x) - 15*atan(sqrt(3) - 2*x) - 8*sqrt(3)*ata
n(sqrt(3) + 2*x) + 15*atan(sqrt(3) + 2*x))/3
```

3.40 $\int \frac{3+4x+2x^2}{1-x^2+x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{3+4x+2x^2}{1-x^2+x^4}\,dx = -\frac{1}{6}\left(15+8\sqrt{3}\right)\arctan\left(\sqrt{3}-2x\right) + \frac{1}{6}\left(15-8\sqrt{3}\right)\arctan\left(\sqrt{3}+2x\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
1/6*(15+8*3^(1/2))*arctan(-3^(1/2)+2*x)+1/6*(15-8*3^(1/2))*arctan(3^(1/2)+
2*x)+1/6*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int \frac{3+4x+2x^2}{1-x^2+x^4}\,dx = \frac{1}{6}\left(\sqrt{-2-2i\sqrt{3}}\left(3+4i\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(x-i\sqrt{3}x\right)\right) + \sqrt{-2+2i\sqrt{3}}\left(3-4i\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(x+i\sqrt{3}x\right)\right) - 8\sqrt{3}\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)\right)$$

input
```
Integrate[(3 + 4*x + 2*x^2)/(1 - x^2 + x^4),x]
```

output
```
(Sqrt[-2 - (2*I)*Sqrt[3]]*(3 + (4*I)*Sqrt[3])*ArcTan[(x - I*Sqrt[3]*x)/2]
+ Sqrt[-2 + (2*I)*Sqrt[3]]*(3 - (4*I)*Sqrt[3])*ArcTan[(x + I*Sqrt[3]*x)/2]
 - 8*Sqrt[3]*ArcTan[(1 - 2*x^2)/Sqrt[3]])/6
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.56, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2202, 27, 1432, 1083, 217, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+4x+3}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{4x}{x^4-x^2+1}dx+\int \frac{2x^2+3}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 27}$$

$$4\int \frac{x}{x^4-x^2+1}dx+\int \frac{2x^2+3}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 1432}$$

$$2\int \frac{1}{x^4-x^2+1}dx^2+\int \frac{2x^2+3}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{2x^2+3}{x^4-x^2+1}dx-4\int \frac{1}{-x^4-3}d\left(2x^2-1\right)$$

$$\Big\downarrow \text{ 217}$$

$$\int \frac{2x^2+3}{x^4-x^2+1}dx+\frac{4\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{\int \frac{3\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{x+3\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{4\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{5}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{5}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{4\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{5}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{5}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{4\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \ 1083$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-5\sqrt{3}\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-5\sqrt{3}\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)}{2\sqrt{3}}+\frac{4\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-5\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+5\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}+\frac{4\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \ 1103$$

$$\frac{4\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{-5\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{5\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}$$

input
```
Int[(3 + 4*x + 2*x^2)/(1 - x^2 + x^4),x]
```

output
```
(4*ArcTan[(-1 + 2*x^2)/Sqrt[3]])/Sqrt[3] + (-5*Sqrt[3]*ArcTan[Sqrt[3] - 2*
x] - Log[1 - Sqrt[3]*x + x^2]/2)/(2*Sqrt[3]) + (5*Sqrt[3]*ArcTan[Sqrt[3] +
 2*x] + Log[1 + Sqrt[3]*x + x^2]/2)/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-\_Z^{2}+1\right)}\frac{\left(2\_R^{2}+4\_R+3\right)\ln\left(x-\_R\right)}{2\_R^{3}-\_R}\right)}{2}$	45
default	$-\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{12}-\frac{\left(-\frac{15}{2}-4\sqrt{3}\right)\arctan\left(2x-\sqrt{3}\right)}{3}+\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{12}+\frac{\left(\frac{15}{2}-4\sqrt{3}\right)\arctan\left(2x+\sqrt{3}\right)}{3}$	71

input
```
int((2*x^2+4*x+3)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((2*_R^2+4*_R+3)/(2*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{3+4x+2x^2}{1-x^2+x^4}\,dx = -\frac{1}{6}\left(8\sqrt{3}-15\right)\arctan\left(2x+\sqrt{3}\right) - \frac{1}{6}\left(8\sqrt{3}+15\right)\arctan\left(-2x+\sqrt{3}\right) + \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)$$

input

```
integrate((2*x^2+4*x+3)/(x^4-x^2+1),x, algorithm="fricas")
```

output

```
-1/6*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) - 1/6*(8*sqrt(3) + 15)*arctan(
-2*x + sqrt(3)) + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log
(x^2 - sqrt(3)*x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. 2(63) = 126.

Time = 0.50 (sec) , antiderivative size = 1185, normalized size of antiderivative = 16.69

$$\int \frac{3+4x+2x^2}{1-x^2+x^4}\,dx = \text{Too large to display}$$

input

```
integrate((2*x**2+4*x+3)/(x**4-x**2+1),x)
```

output
```
sqrt(3)*log(x**2 + x*(-5744/2717 - 777*sqrt(3)/2090 + 471*sqrt(3)*sqrt(80*
sqrt(3) + 4801)/27170 + 92*sqrt(80*sqrt(3) + 4801)/2717) - 221660*sqrt(3)*
sqrt(80*sqrt(3) + 4801)/7382089 - 1139267*sqrt(80*sqrt(3) + 4801)/28392650
 + 60350184*sqrt(3)/36910445 + 155806811/33554950)/12 - sqrt(3)*log(x**2 +
 x*(-92*sqrt(4801 - 80*sqrt(3))/2717 - 5744/2717 + 777*sqrt(3)/2090 + 471*
sqrt(3)*sqrt(4801 - 80*sqrt(3))/27170) - 221660*sqrt(3)*sqrt(4801 - 80*sqr
t(3))/7382089 - 60350184*sqrt(3)/36910445 + 1139267*sqrt(4801 - 80*sqrt(3)
)/28392650 + 155806811/33554950)/12 - 2*sqrt(sqrt(4801 - 80*sqrt(3))/24 +
47/16)*atan(54340*x/(-12193*sqrt(3)*sqrt(2*sqrt(4801 - 80*sqrt(3)) + 141)
+ 920*sqrt(2*sqrt(4801 - 80*sqrt(3)) + 141) + 157*sqrt(3)*sqrt(4801 - 80*s
qrt(3))*sqrt(2*sqrt(4801 - 80*sqrt(3)) + 141)) + 471*sqrt(3)*sqrt(4801 - 8
0*sqrt(3))/(-12193*sqrt(3)*sqrt(2*sqrt(4801 - 80*sqrt(3)) + 141) + 920*sqr
t(2*sqrt(4801 - 80*sqrt(3)) + 141) + 157*sqrt(3)*sqrt(4801 - 80*sqrt(3))*s
qrt(2*sqrt(4801 - 80*sqrt(3)) + 141)) + 10101*sqrt(3)/(-12193*sqrt(3)*sqrt
(2*sqrt(4801 - 80*sqrt(3)) + 141) + 920*sqrt(2*sqrt(4801 - 80*sqrt(3)) + 1
41) + 157*sqrt(3)*sqrt(4801 - 80*sqrt(3))*sqrt(2*sqrt(4801 - 80*sqrt(3)) +
 141)) - 57440/(-12193*sqrt(3)*sqrt(2*sqrt(4801 - 80*sqrt(3)) + 141) + 920
*sqrt(2*sqrt(4801 - 80*sqrt(3)) + 141) + 157*sqrt(3)*sqrt(4801 - 80*sqrt(3
))*sqrt(2*sqrt(4801 - 80*sqrt(3)) + 141)) - 920*sqrt(4801 - 80*sqrt(3))/(-
12193*sqrt(3)*sqrt(2*sqrt(4801 - 80*sqrt(3)) + 141) + 920*sqrt(2*sqrt(4...
```

Maxima [F]

$$\int \frac{3+4x+2x^2}{1-x^2+x^4}\,dx=\int \frac{2\,x^2+4\,x+3}{x^4-x^2+1}\,dx$$

input
```
integrate((2*x^2+4*x+3)/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 4*x + 3)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{3+4x+2x^2}{1-x^2+x^4}\,dx = -\frac{1}{6}\left(8\sqrt{3}-15\right)\arctan\left(2x+\sqrt{3}\right) + \frac{1}{6}\left(8\sqrt{3}+15\right)\arctan\left(2x-\sqrt{3}\right) + \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)$$

input
```
integrate((2*x^2+4*x+3)/(x^4-x^2+1),x, algorithm="giac")
```

output
```
-1/6*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) + 1/6*(8*sqrt(3) + 15)*arctan(
2*x - sqrt(3)) + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(
x^2 - sqrt(3)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

$$\int \frac{3+4x+2x^2}{1-x^2+x^4}\,dx = \sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(z^4+\frac{23\,z^2}{4}-\frac{5\,z}{3}+\frac{25}{144},z,k\right)\left(84\,x+\mathrm{root}\left(z^4+\frac{23\,z^2}{4}-\frac{5\,z}{3}+\frac{25}{144},z,k\right)\left(-48\,x+\mathrm{root}\left(z^4+\frac{23\,z^2}{4}-\frac{5\,z}{3}+\frac{25}{144},z,k\right)x\,24+36\right)+112\right)+10\right)\mathrm{root}\left(z^4+\frac{23\,z^2}{4}-\frac{5\,z}{3}+\frac{25}{144},z,k\right)$$

input
```
int((4*x + 2*x^2 + 3)/(x^4 - x^2 + 1),x)
```

output
```
symsum(log(10 - root(z^4 + (23*z^2)/4 - (5*z)/3 + 25/144, z, k)*(84*x + ro
ot(z^4 + (23*z^2)/4 - (5*z)/3 + 25/144, z, k)*(24*root(z^4 + (23*z^2)/4 -
(5*z)/3 + 25/144, z, k)*x - 48*x + 36) + 112))*root(z^4 + (23*z^2)/4 - (5*
z)/3 + 25/144, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{3+4x+2x^2}{1-x^2+x^4}\,dx = -\frac{4\sqrt{3}\,atan\left(\sqrt{3}-2x\right)}{3}-\frac{5atan\left(\sqrt{3}-2x\right)}{2} - \frac{4\sqrt{3}\,atan\left(\sqrt{3}+2x\right)}{3}+\frac{5atan\left(\sqrt{3}+2x\right)}{2} - \frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{12}$$

input
```
int((2*x^2+4*x+3)/(x^4-x^2+1),x)
```

output
```
( - 16*sqrt(3)*atan(sqrt(3) - 2*x) - 30*atan(sqrt(3) - 2*x) - 16*sqrt(3)*a
tan(sqrt(3) + 2*x) + 30*atan(sqrt(3) + 2*x) - sqrt(3)*log( - sqrt(3)*x + x
**2 + 1) + sqrt(3)*log(sqrt(3)*x + x**2 + 1))/12
```

3.41 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4} dx$

Optimal result

Integrand size = 38, antiderivative size = 76

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4} dx = hx+\frac{ix^2}{2}-\frac{1}{6}(d+4f+16h)\text{arctanh}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f+h)\text{arctanh}(x)-\frac{1}{6}(e+g+i)\log\left(1-x^2\right)+\frac{1}{6}(e+4g+16i)\log\left(4-x^2\right)$$

output
```
h*x+1/2*i*x^2-1/6*(d+4*f+16*h)*arctanh(1/2*x)+1/3*(d+f+h)*arctanh(x)-1/6*(
e+g+i)*ln(-x^2+1)+1/6*(e+4*g+16*i)*ln(-x^2+4)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.29

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4} dx = \frac{1}{12}\left(12hx+6ix^2-2(d+e+f+g+h+i)\log(1-x)+(d+2e+4(f+2g+4h+8i))\log(2-x)+2(d-e+f-g+h-i)\log(1+x)-(d-2(e-2f+4g-8h+16i))\log(2+x)\right)$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4),x]
```

output
```
(12*h*x + 6*i*x^2 - 2*(d + e + f + g + h + i)*Log[1 - x] + (d + 2*e + 4*(f
 + 2*g + 4*h + 8*i))*Log[2 - x] + 2*(d - e + f - g + h - i)*Log[1 + x] - (
d - 2*(e - 2*f + 4*g - 8*h + 16*i))*Log[2 + x])/12
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2202, 2194, 2188, 2009, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{x^4-5x^2+4} \, dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{hx^4+fx^2+d}{x^4-5x^2+4}dx+\int \frac{x\left(ix^4+gx^2+e\right)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\int \frac{hx^4+fx^2+d}{x^4-5x^2+4}dx+\frac{1}{2}\int \frac{ix^4+gx^2+e}{x^4-5x^2+4}dx^2$$

$$\Big\downarrow \text{ 2188}$$

$$\int \frac{hx^4+fx^2+d}{x^4-5x^2+4}dx+\frac{1}{2}\int \left(i+\frac{(g+5i)x^2+e-4i}{x^4-5x^2+4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\int \frac{hx^4+fx^2+d}{x^4-5x^2+4}dx+\frac{1}{2}\left(-\frac{1}{3}\log\left(1-x^2\right)(e+g+i)+\frac{1}{3}\log\left(4-x^2\right)(e+4g+16i)+ix^2\right)$$

$$\Big\downarrow \text{ 2205}$$

$$\int \left(h+\frac{(f+5h)x^2+d-4h}{x^4-5x^2+4}\right)dx+$$
$$\frac{1}{2}\left(-\frac{1}{3}\log\left(1-x^2\right)(e+g+i)+\frac{1}{3}\log\left(4-x^2\right)(e+4g+16i)+ix^2\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}\text{arctanh}\left(\frac{x}{2}\right)(d+4f+16h)+\frac{1}{3}\text{arctanh}(x)(d+f+h)+\frac{1}{2}\left(-\frac{1}{3}\log\left(1-x^2\right)(e+g+i)+\frac{1}{3}\log\left(4-x^2\right)(e+4g+16i)+ix^2\right)+hx$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4),x]
```

output
```
h*x - ((d + 4*f + 16*h)*ArcTanh[x/2])/6 + ((d + f + h)*ArcTanh[x])/3 + (i*
x^2 - ((e + g + i)*Log[1 - x^2])/3 + ((e + 4*g + 16*i)*Log[4 - x^2])/3)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.41

method	result
default	$\frac{i\,x^{2}}{2}+hx+\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}+\frac{2g}{3}+\frac{4h}{3}+\frac{8i}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}-\frac{i}{6}\right)\ln\left(1+x\right)+$
norman	$\frac{i\,x^{2}}{2}+hx+\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}+\frac{2g}{3}+\frac{4h}{3}+\frac{8i}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}-\frac{i}{6}\right)\ln\left(1+x\right)+$
parallelrisch	$\frac{\ln(x-2)e}{6}+\frac{i\,x^{2}}{2}+\frac{\ln(x+2)e}{6}+\frac{\ln(x-2)f}{3}+\frac{\ln(1+x)d}{6}-\frac{\ln(x+2)f}{3}-\frac{\ln(x-1)d}{6}-\frac{\ln(x+2)d}{12}+\frac{\ln(x-2)d}{12}-\ln($
risch	$\frac{\ln(2-x)e}{6}+\frac{i\,x^{2}}{2}+\frac{\ln(x+2)e}{6}-\frac{\ln(1-x)d}{6}+\frac{\ln(2-x)d}{12}+\frac{\ln(2-x)f}{3}+\frac{\ln(1+x)d}{6}-\frac{\ln(x+2)f}{3}-\frac{\ln(x+2)d}{12}-\ln($

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/2*i*x^2+h*x+(1/12*d+1/6*e+1/3*f+2/3*g+4/3*h+8/3*i)*ln(x-2)+(1/6*d-1/6*e+
1/6*f-1/6*g+1/6*h-1/6*i)*ln(1+x)+(-1/6*d-1/6*e-1/6*f-1/6*g-1/6*h-1/6*i)*ln
(x-1)+(-1/12*d+1/6*e-1/3*f+2/3*g-4/3*h+8/3*i)*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\begin{aligned}
&\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4}\,dx \\
&= \frac{1}{2}\,ix^2+hx-\frac{1}{12}\,(d-2\,e+4\,f-8\,g+16\,h-32\,i)\log\left(x+2\right) \\
&\quad+\frac{1}{6}\,(d-e+f-g+h-i)\log\left(x+1\right)-\frac{1}{6}\,(d+e+f+g+h+i)\log\left(x-1\right) \\
&\quad+\frac{1}{12}\,(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log\left(x-2\right)
\end{aligned}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fric
as")
```

output
```
1/2*i*x^2 + h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + 1/
6*(d - e + f - g + h - i)*log(x + 1) - 1/6*(d + e + f + g + h + i)*log(x -
 1) + 1/12*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4}\,dx = \text{Timed out}$$

input
```
integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\begin{aligned}
&\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4}\,dx \\
&= \frac{1}{2}\,ix^2+hx-\frac{1}{12}\,(d-2\,e+4\,f-8\,g+16\,h-32\,i)\log(x+2) \\
&\quad+\frac{1}{6}\,(d-e+f-g+h-i)\log(x+1)-\frac{1}{6}\,(d+e+f+g+h+i)\log(x-1) \\
&\quad+\frac{1}{12}\,(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log(x-2)
\end{aligned}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxi
ma")
```

output
```
1/2*i*x^2 + h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + 1/
6*(d - e + f - g + h - i)*log(x + 1) - 1/6*(d + e + f + g + h + i)*log(x -
 1) + 1/12*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{2}ix^2+hx-\frac{1}{12}(d-2e+4f-8g+16h-32i)\log(|x+2|)$$
$$+\frac{1}{6}(d-e+f-g+h-i)\log(|x+1|)-\frac{1}{6}(d+e+f+g+h+i)\log(|x-1|)$$
$$+\frac{1}{12}(d+2e+4f+8g+16h+32i)\log(|x-2|)$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac
")
```

output
```
1/2*i*x^2 + h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(abs(x + 2))
 + 1/6*(d - e + f - g + h - i)*log(abs(x + 1)) - 1/6*(d + e + f + g + h +
i)*log(abs(x - 1)) + 1/12*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(abs(x -
2))
```

Mupad [B] (verification not implemented)

Time = 18.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.42

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4}\,dx=h\,x+\frac{i\,x^2}{2}-\ln(x-1)\left(\frac{d}{6}+\frac{e}{6}+\frac{f}{6}+\frac{g}{6}+\frac{h}{6}+\frac{i}{6}\right)$$
$$+\ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}-\frac{i}{6}\right)$$
$$+\ln(x-2)\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}+\frac{2\,g}{3}+\frac{4\,h}{3}+\frac{8\,i}{3}\right)$$
$$-\ln(x+2)\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2\,g}{3}+\frac{4\,h}{3}-\frac{8\,i}{3}\right)$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^4 - 5*x^2 + 4),x)
```

output
```
h*x + (i*x^2)/2 - log(x - 1)*(d/6 + e/6 + f/6 + g/6 + h/6 + i/6) + log(x +
 1)*(d/6 - e/6 + f/6 - g/6 + h/6 - i/6) + log(x - 2)*(d/12 + e/6 + f/3 + (
2*g)/3 + (4*h)/3 + (8*i)/3) - log(x + 2)*(d/12 - e/6 + f/3 - (2*g)/3 + (4*
h)/3 - (8*i)/3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.34

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4}\,dx = \frac{2\log(x-2)\,g}{3}+\frac{4\log(x-2)\,h}{3}+\frac{8\log(x-2)\,i}{3}-\frac{\log(x-1)\,g}{6}-\frac{\log(x-1)\,h}{6}-\frac{\log(x-1)\,i}{6}+\frac{2\log(x+2)\,g}{3}-\frac{4\log(x+2)\,h}{3}+\frac{8\log(x+2)\,i}{3}-\frac{\log(x+1)\,g}{6}+\frac{\log(x+1)\,h}{6}-\frac{\log(x+1)\,i}{6}+hx+\frac{i\,x^2}{2}+\frac{\log(x-2)\,d}{12}-\frac{\log(x+2)\,d}{12}+\frac{\log(x-2)\,e}{6}+\frac{\log(x-2)\,f}{3}-\frac{\log(x-1)\,d}{6}-\frac{\log(x-1)\,e}{6}-\frac{\log(x-1)\,f}{6}+\frac{\log(x+2)\,e}{6}-\frac{\log(x+2)\,f}{3}+\frac{\log(x+1)\,d}{6}-\frac{\log(x+1)\,e}{6}+\frac{\log(x+1)\,f}{6}$$

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
(log(x - 2)*d + 2*log(x - 2)*e + 4*log(x - 2)*f + 8*log(x - 2)*g + 16*log(
x - 2)*h + 32*log(x - 2)*i - 2*log(x - 1)*d - 2*log(x - 1)*e - 2*log(x - 1
)*f - 2*log(x - 1)*g - 2*log(x - 1)*h - 2*log(x - 1)*i - log(x + 2)*d + 2*
log(x + 2)*e - 4*log(x + 2)*f + 8*log(x + 2)*g - 16*log(x + 2)*h + 32*log(
x + 2)*i + 2*log(x + 1)*d - 2*log(x + 1)*e + 2*log(x + 1)*f - 2*log(x + 1)
*g + 2*log(x + 1)*h - 2*log(x + 1)*i + 12*h*x + 6*i*x**2)/12
```

3.42 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^2} dx$

Optimal result

Integrand size = 38, antiderivative size = 162

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^2}\,dx &= \frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{72\,(4-5x^2+x^4)} \\ &+ \frac{5e+8g+20i-(2e+5g+17i)x^2}{18\,(4-5x^2+x^4)} \\ &+ \frac{1}{432}(19d+52f+112h)\text{arctanh}\left(\frac{x}{2}\right) \\ &- \frac{1}{54}(d+7f+13h)\text{arctanh}(x) \\ &+ \frac{1}{54}(2e+5g+8i)\log\left(1-x^2\right) \\ &- \frac{1}{54}(2e+5g+8i)\log\left(4-x^2\right)\end{aligned}$$

output
```
x*(17*d+20*f+32*h-(5*d+8*f+20*h)*x^2)/(72*x^4-360*x^2+288)+(5*e+8*g+20*i-(
2*e+5*g+17*i)*x^2)/(18*x^4-90*x^2+72)+1/432*(19*d+52*f+112*h)*arctanh(1/2*
x)-1/54*(d+7*f+13*h)*arctanh(x)+1/54*(2*e+5*g+8*i)*ln(-x^2+1)-1/54*(2*e+5*
g+8*i)*ln(-x^2+4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.14

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(4 - 5x^2 + x^4)^2} dx$$
$$= \frac{20e + 32g + 80i + 17dx + 20fx + 32hx - 8ex^2 - 20gx^2 - 68ix^2 - 5dx^3 - 8fx^3 - 20hx^3}{72(4 - 5x^2 + x^4)}$$
$$+ \frac{1}{108}(d + 4e + 7f + 10g + 13h + 16i)\log(1 - x)$$
$$+ \frac{1}{864}(-19d - 32e - 52f - 80g - 112h - 128i)\log(2 - x)$$
$$+ \frac{1}{108}(-d + 4e - 7f + 10g - 13h + 16i)\log(1 + x)$$
$$+ \frac{1}{864}(19d - 32e + 52f - 80g + 112h - 128i)\log(2 + x)$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^2,x]
```

output
```
(20*e + 32*g + 80*i + 17*d*x + 20*f*x + 32*h*x - 8*e*x^2 - 20*g*x^2 - 68*i
*x^2 - 5*d*x^3 - 8*f*x^3 - 20*h*x^3)/(72*(4 - 5*x^2 + x^4)) + ((d + 4*e +
7*f + 10*g + 13*h + 16*i)*Log[1 - x])/108 + ((-19*d - 32*e - 52*f - 80*g -
 112*h - 128*i)*Log[2 - x])/864 + ((-d + 4*e - 7*f + 10*g - 13*h + 16*i)*L
og[1 + x])/108 + ((19*d - 32*e + 52*f - 80*g + 112*h - 128*i)*Log[2 + x])/
864
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2202, 2194, 2191, 27, 1081, 2009, 2206, 25, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(x^4 - 5x^2 + 4)^2} dx$$
$$\Big\downarrow \; 2202$$

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^2}dx+\int \frac{x\left(ix^4+gx^2+e\right)}{\left(x^4-5x^2+4\right)^2}dx$$

$\big\downarrow$ 2194

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^2}dx+\frac{1}{2}\int \frac{ix^4+gx^2+e}{\left(x^4-5x^2+4\right)^2}dx^2$$

$\big\downarrow$ 2191

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^2}dx+\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{9\left(x^4-5x^2+4\right)}-\frac{1}{9}\int \frac{2e+5g+8i}{x^4-5x^2+4}dx^2\right)$$

$\big\downarrow$ 27

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^2}dx+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{9\left(x^4-5x^2+4\right)}-\frac{1}{9}(2e+5g+8i)\int \frac{1}{x^4-5x^2+4}dx^2\right)$$

$\big\downarrow$ 1081

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^2}dx+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{9\left(x^4-5x^2+4\right)}-\frac{1}{9}(2e+5g+8i)\int \left(\frac{1}{3\left(1-x^2\right)}-\frac{1}{3\left(4-x^2\right)}\right)dx^2\right)$$

$\big\downarrow$ 2009

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^2}dx+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{9\left(x^4-5x^2+4\right)}-\frac{1}{9}\left(\frac{1}{3}\log\left(4-x^2\right)-\frac{1}{3}\log\left(1-x^2\right)\right)(2e+5g+8i)\right)$$

$\big\downarrow$ 2206

$$-\frac{1}{72}\int -\frac{-\left((5d+8f+20h)x^2\right)+d-20f-32h}{x^4-5x^2+4}dx+$$
$$\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{9\left(x^4-5x^2+4\right)}-\frac{1}{9}\left(\frac{1}{3}\log\left(4-x^2\right)-\frac{1}{3}\log\left(1-x^2\right)\right)(2e+5g+8i)\right)$$

$\big\downarrow$ 25

$$\frac{1}{72}\int \frac{-\left((5d+8f+20h)x^2\right)+d-20f-32h}{x^4-5x^2+4}dx+\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{9\left(x^4-5x^2+4\right)}-\frac{1}{9}\left(\frac{1}{3}\log\left(4-x^2\right)-\frac{1}{3}\log\left(1-x^2\right)\right)(2e+5g+8i)\right)$$

$$\Big\downarrow \; 1480$$

$$\frac{1}{72}\left(\frac{4}{3}(d+7f+13h)\int \frac{1}{x^2-1}dx-\frac{1}{3}(19d+52f+112h)\int \frac{1}{x^2-4}dx\right)+\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{9\left(x^4-5x^2+4\right)}-\frac{1}{9}\left(\frac{1}{3}\log\left(4-x^2\right)-\frac{1}{3}\log\left(1-x^2\right)\right)(2e+5g+8i)\right)$$

$$\Big\downarrow \; 220$$

$$\frac{1}{72}\left(\frac{1}{6}\text{arctanh}\left(\frac{x}{2}\right)(19d+52f+112h)-\frac{4}{3}\text{arctanh}(x)(d+7f+13h)\right)+\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{9\left(x^4-5x^2+4\right)}-\frac{1}{9}\left(\frac{1}{3}\log\left(4-x^2\right)-\frac{1}{3}\log\left(1-x^2\right)\right)(2e+5g+8i)\right)$$

input

```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^2,x]
```

output

```
(x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(72*(4 - 5*x^2 + x^4)) +
 (((19*d + 52*f + 112*h)*ArcTanh[x/2])/6 - (4*(d + 7*f + 13*h)*ArcTanh[x])
/3)/72 + ((5*e + 8*g + 20*i - (2*e + 5*g + 17*i)*x^2)/(9*(4 - 5*x^2 + x^4)
) - ((2*e + 5*g + 8*i)*(-1/3*Log[1 - x^2] + Log[4 - x^2]/3))/9)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.99

method	result
norman	$\frac{\left(-\frac{5d}{72}-\frac{f}{9}-\frac{5h}{18}\right)x^3+\left(\frac{17d}{72}+\frac{5f}{18}+\frac{4h}{9}\right)x+\left(-\frac{5g}{18}-\frac{e}{9}-\frac{17i}{18}\right)x^2+\frac{4g}{9}+\frac{5e}{18}+\frac{10i}{9}}{x^4-5x^2+4}+\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}-\frac{5g}{54}-\frac{7h}{54}-\frac{4i}{27}\right)$
default	$\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}-\frac{5g}{54}-\frac{7h}{54}-\frac{4i}{27}\right)\ln\left(x-2\right)-\frac{\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}+\frac{h}{9}+\frac{2i}{9}}{x-2}+\left(-\frac{d}{108}+\frac{e}{27}-\frac{7f}{108}+\frac{5}{5}\right.$
risch	$-\frac{\ln(2-x)e}{27}-\frac{\ln(x+2)e}{27}+\frac{\ln(1-x)d}{108}-\frac{19\ln(2-x)d}{864}-\frac{13\ln(2-x)f}{216}-\frac{\ln(1+x)d}{108}+\frac{13\ln(x+2)f}{216}+\frac{19\ln(x+2)d}{864}-$
parallelrisch	$-\frac{-960i-384g-204dx-240fx-240e+128\ln(x-2)e+816i\,x^2-128\ln(x-1)x^4i-640\ln(x+2)x^2i+128\ln(x+2)e+40\ln(x-1)}{}$

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOS
E)
```

output
```
((-5/72*d-1/9*f-5/18*h)*x^3+(17/72*d+5/18*f+4/9*h)*x+(-5/18*g-1/9*e-17/18*
i)*x^2+4/9*g+5/18*e+10/9*i)/(x^4-5*x^2+4)+(-19/864*d-1/27*e-13/216*f-5/54*
g-7/54*h-4/27*i)*ln(x-2)+(-1/108*d+1/27*e-7/108*f+5/54*g-13/108*h+4/27*i)*
ln(1+x)+(1/108*d+1/27*e+7/108*f+5/54*g+13/108*h+4/27*i)*ln(x-1)+(19/864*d-
1/27*e+13/216*f-5/54*g+7/54*h-4/27*i)*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(146) = 292$.

Time = 5.90 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.14

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^2}\,dx = -\frac{12\,(5\,d+8\,f+20\,h)x^3+48\,(2\,e+5\,g+17\,i)x^2-12\,(17\,d+20\,f+32\,h)x-((19\,d-32\,e+52\,f-8}{}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fr
icas")
```

output
```
-1/864*(12*(5*d + 8*f + 20*h)*x^3 + 48*(2*e + 5*g + 17*i)*x^2 - 12*(17*d +
 20*f + 32*h)*x - ((19*d - 32*e + 52*f - 80*g + 112*h - 128*i)*x^4 - 5*(19
*d - 32*e + 52*f - 80*g + 112*h - 128*i)*x^2 + 76*d - 128*e + 208*f - 320*
g + 448*h - 512*i)*log(x + 2) + 8*((d - 4*e + 7*f - 10*g + 13*h - 16*i)*x^
4 - 5*(d - 4*e + 7*f - 10*g + 13*h - 16*i)*x^2 + 4*d - 16*e + 28*f - 40*g
+ 52*h - 64*i)*log(x + 1) - 8*((d + 4*e + 7*f + 10*g + 13*h + 16*i)*x^4 -
5*(d + 4*e + 7*f + 10*g + 13*h + 16*i)*x^2 + 4*d + 16*e + 28*f + 40*g + 52
*h + 64*i)*log(x - 1) + ((19*d + 32*e + 52*f + 80*g + 112*h + 128*i)*x^4 -
 5*(19*d + 32*e + 52*f + 80*g + 112*h + 128*i)*x^2 + 76*d + 128*e + 208*f
+ 320*g + 448*h + 512*i)*log(x - 2) - 240*e - 384*g - 960*i)/(x^4 - 5*x^2
+ 4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$\begin{aligned}&\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^2}\,dx\\&=\frac{1}{864}\,(19\,d-32\,e+52\,f-80\,g+112\,h-128\,i)\log{(x+2)}\\&\quad-\frac{1}{108}\,(d-4\,e+7\,f-10\,g+13\,h-16\,i)\log{(x+1)}\\&\quad+\frac{1}{108}\,(d+4\,e+7\,f+10\,g+13\,h+16\,i)\log{(x-1)}\\&\quad-\frac{1}{864}\,(19\,d+32\,e+52\,f+80\,g+112\,h+128\,i)\log{(x-2)}\\&\quad-\frac{(5\,d+8\,f+20\,h)x^3+4\,(2\,e+5\,g+17\,i)x^2-(17\,d+20\,f+32\,h)x-20\,e-32\,g-80\,i}{72\,(x^4-5\,x^2+4)}\end{aligned}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="ma
xima")
```

output
```
1/864*(19*d - 32*e + 52*f - 80*g + 112*h - 128*i)*log(x + 2) - 1/108*(d -
4*e + 7*f - 10*g + 13*h - 16*i)*log(x + 1) + 1/108*(d + 4*e + 7*f + 10*g +
 13*h + 16*i)*log(x - 1) - 1/864*(19*d + 32*e + 52*f + 80*g + 112*h + 128*
i)*log(x - 2) - 1/72*((5*d + 8*f + 20*h)*x^3 + 4*(2*e + 5*g + 17*i)*x^2 -
(17*d + 20*f + 32*h)*x - 20*e - 32*g - 80*i)/(x^4 - 5*x^2 + 4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.07

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{1}{864}\,(19\,d-32\,e+52\,f-80\,g+112\,h-128\,i)\log\left(|x+2|\right)$$
$$-\frac{1}{108}\,(d-4\,e+7\,f-10\,g+13\,h-16\,i)\log\left(|x+1|\right)$$
$$+\frac{1}{108}\,(d+4\,e+7\,f+10\,g+13\,h+16\,i)\log\left(|x-1|\right)$$
$$-\frac{1}{864}\,(19\,d+32\,e+52\,f+80\,g+112\,h+128\,i)\log\left(|x-2|\right)$$
$$-\frac{5\,dx^3+8\,fx^3+20\,hx^3+8\,ex^2+20\,gx^2+68\,ix^2-17\,dx-20\,fx-32\,hx-20\,e-32\,g-80\,i}{72\,(x^4-5\,x^2+4)}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="gi
ac")
```

output
```
1/864*(19*d - 32*e + 52*f - 80*g + 112*h - 128*i)*log(abs(x + 2)) - 1/108*
(d - 4*e + 7*f - 10*g + 13*h - 16*i)*log(abs(x + 1)) + 1/108*(d + 4*e + 7*
f + 10*g + 13*h + 16*i)*log(abs(x - 1)) - 1/864*(19*d + 32*e + 52*f + 80*g
 + 112*h + 128*i)*log(abs(x - 2)) - 1/72*(5*d*x^3 + 8*f*x^3 + 20*h*x^3 + 8
*e*x^2 + 20*g*x^2 + 68*i*x^2 - 17*d*x - 20*f*x - 32*h*x - 20*e - 32*g - 80
*i)/(x^4 - 5*x^2 + 4)
```

Mupad [B] (verification not implemented)

Time = 18.41 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{\left(-\frac{5\,d}{72}-\frac{f}{9}-\frac{5\,h}{18}\right)\,x^3+\left(-\frac{e}{9}-\frac{5\,g}{18}-\frac{17\,i}{18}\right)\,x^2+\left(\frac{17\,d}{72}+\frac{5\,f}{18}+\frac{4\,h}{9}\right)\,x+\frac{5\,e}{18}+\frac{4\,g}{9}+\frac{10\,i}{9}}{x^4-5\,x^2+4}$$
$$+\ln\left(x-1\right)\left(\frac{d}{108}+\frac{e}{27}+\frac{7\,f}{108}+\frac{5\,g}{54}+\frac{13\,h}{108}+\frac{4\,i}{27}\right)$$
$$-\ln\left(x+1\right)\left(\frac{d}{108}-\frac{e}{27}+\frac{7\,f}{108}-\frac{5\,g}{54}+\frac{13\,h}{108}-\frac{4\,i}{27}\right)$$
$$-\ln\left(x-2\right)\left(\frac{19\,d}{864}+\frac{e}{27}+\frac{13\,f}{216}+\frac{5\,g}{54}+\frac{7\,h}{54}+\frac{4\,i}{27}\right)$$
$$+\ln\left(x+2\right)\left(\frac{19\,d}{864}-\frac{e}{27}+\frac{13\,f}{216}-\frac{5\,g}{54}+\frac{7\,h}{54}-\frac{4\,i}{27}\right)$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
((5*e)/18 + (4*g)/9 + (10*i)/9 + x*((17*d)/72 + (5*f)/18 + (4*h)/9) - x^3*
((5*d)/72 + f/9 + (5*h)/18) - x^2*(e/9 + (5*g)/18 + (17*i)/18))/(x^4 - 5*x
^2 + 4) + log(x - 1)*(d/108 + e/27 + (7*f)/108 + (5*g)/54 + (13*h)/108 + (
4*i)/27) - log(x + 1)*(d/108 - e/27 + (7*f)/108 - (5*g)/54 + (13*h)/108 -
(4*i)/27) - log(x - 2)*((19*d)/864 + e/27 + (13*f)/216 + (5*g)/54 + (7*h)/
54 + (4*i)/27) + log(x + 2)*((19*d)/864 - e/27 + (13*f)/216 - (5*g)/54 + (
7*h)/54 - (4*i)/27)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 721, normalized size of antiderivative = 4.45

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output

```
( - 95*log(x - 2)*d*x**4 + 475*log(x - 2)*d*x**2 - 380*log(x - 2)*d - 160*
log(x - 2)*e*x**4 + 800*log(x - 2)*e*x**2 - 640*log(x - 2)*e - 260*log(x -
 2)*f*x**4 + 1300*log(x - 2)*f*x**2 - 1040*log(x - 2)*f - 400*log(x - 2)*g
*x**4 + 2000*log(x - 2)*g*x**2 - 1600*log(x - 2)*g - 560*log(x - 2)*h*x**4
 + 2800*log(x - 2)*h*x**2 - 2240*log(x - 2)*h - 640*log(x - 2)*i*x**4 + 32
00*log(x - 2)*i*x**2 - 2560*log(x - 2)*i + 40*log(x - 1)*d*x**4 - 200*log(
x - 1)*d*x**2 + 160*log(x - 1)*d + 160*log(x - 1)*e*x**4 - 800*log(x - 1)*
e*x**2 + 640*log(x - 1)*e + 280*log(x - 1)*f*x**4 - 1400*log(x - 1)*f*x**2
 + 1120*log(x - 1)*f + 400*log(x - 1)*g*x**4 - 2000*log(x - 1)*g*x**2 + 16
00*log(x - 1)*g + 520*log(x - 1)*h*x**4 - 2600*log(x - 1)*h*x**2 + 2080*lo
g(x - 1)*h + 640*log(x - 1)*i*x**4 - 3200*log(x - 1)*i*x**2 + 2560*log(x -
 1)*i + 95*log(x + 2)*d*x**4 - 475*log(x + 2)*d*x**2 + 380*log(x + 2)*d -
160*log(x + 2)*e*x**4 + 800*log(x + 2)*e*x**2 - 640*log(x + 2)*e + 260*log
(x + 2)*f*x**4 - 1300*log(x + 2)*f*x**2 + 1040*log(x + 2)*f - 400*log(x +
2)*g*x**4 + 2000*log(x + 2)*g*x**2 - 1600*log(x + 2)*g + 560*log(x + 2)*h*
x**4 - 2800*log(x + 2)*h*x**2 + 2240*log(x + 2)*h - 640*log(x + 2)*i*x**4
+ 3200*log(x + 2)*i*x**2 - 2560*log(x + 2)*i - 40*log(x + 1)*d*x**4 + 200*
log(x + 1)*d*x**2 - 160*log(x + 1)*d + 160*log(x + 1)*e*x**4 - 800*log(x +
 1)*e*x**2 + 640*log(x + 1)*e - 280*log(x + 1)*f*x**4 + 1400*log(x + 1)*f*
x**2 - 1120*log(x + 1)*f + 400*log(x + 1)*g*x**4 - 2000*log(x + 1)*g*x*...
```

3.43 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^3} dx$

Optimal result

Integrand size = 38, antiderivative size = 252

$$\begin{aligned}
&\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^3} dx \\
&= -\frac{2e+5g+11i}{108(1-x^2)} - \frac{2e+5g+11i}{108(4-x^2)} + \frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{144(4-5x^2+x^4)^2} \\
&\quad + \frac{5e+8g+20i-(2e+5g+17i)x^2}{36(4-5x^2+x^4)^2} \\
&\quad - \frac{x(59d+380f+848h-5(7d+28f+64h)x^2)}{3456(4-5x^2+x^4)} \\
&\quad - \frac{(313d+820f+1936h)\operatorname{arctanh}\left(\frac{x}{2}\right)}{20736} + \frac{1}{648}(13d+25f+61h)\operatorname{arctanh}(x) \\
&\quad - \frac{1}{162}(2e+5g+11i)\log\left(1-x^2\right) + \frac{1}{162}(2e+5g+11i)\log\left(4-x^2\right)
\end{aligned}$$

output
```
-1/108*(2*e+5*g+11*i)/(-x^2+1)-(2*e+5*g+11*i)/(-108*x^2+432)+1/144*x*(17*d
+20*f+32*h-(5*d+8*f+20*h)*x^2)/(x^4-5*x^2+4)^2+1/36*(5*e+8*g+20*i-(2*e+5*g
+17*i)*x^2)/(x^4-5*x^2+4)^2-x*(59*d+380*f+848*h-5*(7*d+28*f+64*h)*x^2)/(34
56*x^4-17280*x^2+13824)-1/20736*(313*d+820*f+1936*h)*arctanh(1/2*x)+1/648*
(13*d+25*f+61*h)*arctanh(x)-1/162*(2*e+5*g+11*i)*ln(-x^2+1)+1/162*(2*e+5*g
+11*i)*ln(-x^2+4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.04

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^3}\,dx$$
$$= \frac{20e+32g+80i+17dx+20fx+32hx-8ex^2-20gx^2-68ix^2-5dx^3-8fx^3-20hx^3}{144\left(4-5x^2+x^4\right)^2}$$
$$+\frac{-320e-800g-1760i-59dx-380fx-848hx+128ex^2+320gx^2+704ix^2+35dx^3+140fx^3+3}{3456\left(4-5x^2+x^4\right)}$$
$$+\frac{(-13d-16e-25f-40g-61h-88i)\log(1-x)}{1296}$$
$$+\frac{(313d+512e+820f+1280g+1936h+2816i)\log(2-x)}{41472}$$
$$+\frac{(13d-16e+25f-40g+61h-88i)\log(1+x)}{1296}$$
$$+\frac{(-313d+512e-820f+1280g-1936h+2816i)\log(2+x)}{41472}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^3,x]
```

output
```
(20*e + 32*g + 80*i + 17*d*x + 20*f*x + 32*h*x - 8*e*x^2 - 20*g*x^2 - 68*i
*x^2 - 5*d*x^3 - 8*f*x^3 - 20*h*x^3)/(144*(4 - 5*x^2 + x^4)^2) + (-320*e -
 800*g - 1760*i - 59*d*x - 380*f*x - 848*h*x + 128*e*x^2 + 320*g*x^2 + 704
*i*x^2 + 35*d*x^3 + 140*f*x^3 + 320*h*x^3)/(3456*(4 - 5*x^2 + x^4)) + ((-1
3*d - 16*e - 25*f - 40*g - 61*h - 88*i)*Log[1 - x])/1296 + ((313*d + 512*e
 + 820*f + 1280*g + 1936*h + 2816*i)*Log[2 - x])/41472 + ((13*d - 16*e + 2
5*f - 40*g + 61*h - 88*i)*Log[1 + x])/1296 + ((-313*d + 512*e - 820*f + 12
80*g - 1936*h + 2816*i)*Log[2 + x])/41472
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2202, 2194, 2191, 27, 1084, 2009, 2206, 25, 1492, 27, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(x^4-5x^2+4\right)^3}dx$$

$\Big\downarrow$ 2202

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^3}dx+\int \frac{x\left(ix^4+gx^2+e\right)}{\left(x^4-5x^2+4\right)^3}dx$$

$\Big\downarrow$ 2194

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^3}dx+\frac{1}{2}\int \frac{ix^4+gx^2+e}{\left(x^4-5x^2+4\right)^3}dx^2$$

$\Big\downarrow$ 2191

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^3}dx+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{18\left(x^4-5x^2+4\right)^2}-\frac{1}{18}\int \frac{3(2e+5g+11i)}{\left(x^4-5x^2+4\right)^2}dx^2\right)$$

$\Big\downarrow$ 27

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^3}dx+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{18\left(x^4-5x^2+4\right)^2}-\frac{1}{6}(2e+5g+11i)\int \frac{1}{\left(x^4-5x^2+4\right)^2}dx^2\right)$$

$\Big\downarrow$ 1084

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^3}dx+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{18\left(x^4-5x^2+4\right)^2}-\frac{1}{6}(2e+5g+11i)\int \left(\frac{2}{27\left(4-x^2\right)}+\frac{1}{9\left(4-x^2\right)^2}-\frac{2}{27\left(1-x^2\right)}+\frac{}{9}\right.\right.$$

$\Big\downarrow$ 2009

$$\int \frac{hx^4+fx^2+d}{\left(x^4-5x^2+4\right)^3}dx+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{18\left(x^4-5x^2+4\right)^2}-\frac{1}{6}\left(\frac{1}{9\left(1-x^2\right)}+\frac{1}{9\left(4-x^2\right)}+\frac{2}{27}\log\left(1-x^2\right)-\frac{2}{27}\log\left(4-x^2\right)\right)(2\right.$$

$\Big\downarrow$ 2206

$$-\frac{1}{144}\int -\frac{-5(5d+8f+20h)x^2+19d-20f-32h}{(x^4-5x^2+4)^2}dx+$$
$$\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{18\left(x^4-5x^2+4\right)^2}-\frac{1}{6}\left(\frac{1}{9\left(1-x^2\right)}+\frac{1}{9\left(4-x^2\right)}+\frac{2}{27}\log\left(1-x^2\right)-\frac{2}{27}\log\left(4-x^2\right)\right)(2$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{144}\int \frac{-5(5d+8f+20h)x^2+19d-20f-32h}{(x^4-5x^2+4)^2}dx+$$
$$\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{18\left(x^4-5x^2+4\right)^2}-\frac{1}{6}\left(\frac{1}{9\left(1-x^2\right)}+\frac{1}{9\left(4-x^2\right)}+\frac{2}{27}\log\left(1-x^2\right)-\frac{2}{27}\log\left(4-x^2\right)\right)(2$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{144}\left(-\frac{1}{72}\int -\frac{3\left(5(7d+28f+64h)x^2+173d+260f+656h\right)}{x^4-5x^2+4}dx-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+8\right.}{24\left(x^4-5x^2+4\right)}\right.$$
$$\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{18\left(x^4-5x^2+4\right)^2}-\frac{1}{6}\left(\frac{1}{9\left(1-x^2\right)}+\frac{1}{9\left(4-x^2\right)}+\frac{2}{27}\log\left(1-x^2\right)-\frac{2}{27}\log\left(4-x^2\right)\right)(2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{144}\left(\frac{1}{24}\int \frac{5(7d+28f+64h)x^2+173d+260f+656h}{x^4-5x^2+4}dx-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+848h\right)}{24\left(x^4-5x^2+4\right)}\right)$$
$$\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{18\left(x^4-5x^2+4\right)^2}-\frac{1}{6}\left(\frac{1}{9\left(1-x^2\right)}+\frac{1}{9\left(4-x^2\right)}+\frac{2}{27}\log\left(1-x^2\right)-\frac{2}{27}\log\left(4-x^2\right)\right)(2$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{144}\left(\frac{1}{24}\left(\frac{1}{3}(313d+820f+1936h)\int\frac{1}{x^2-4}dx-\frac{16}{3}(13d+25f+61h)\int\frac{1}{x^2-1}dx\right)-\frac{x\left(-5x^2(7d+28f+6\right.}{24\left(x^4\right.}\right.$$
$$\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{18\left(x^4-5x^2+4\right)^2}-\frac{1}{6}\left(\frac{1}{9(1-x^2)}+\frac{1}{9(4-x^2)}+\frac{2}{27}\log\left(1-x^2\right)-\frac{2}{27}\log\left(4-x^2\right)\right)(2\right.$$

$$\Big\downarrow 220$$

$$\frac{1}{144}\left(\frac{1}{24}\left(\frac{16}{3}\text{arctanh}(x)(13d+25f+61h)-\frac{1}{6}\text{arctanh}\left(\frac{x}{2}\right)(313d+820f+1936h)\right)-\frac{x\left(-5x^2(7d+28f+64\right.}{24\left(x^4-\right.}\right.$$
$$\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}+$$
$$\frac{1}{2}\left(\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{18\left(x^4-5x^2+4\right)^2}-\frac{1}{6}\left(\frac{1}{9(1-x^2)}+\frac{1}{9(4-x^2)}+\frac{2}{27}\log\left(1-x^2\right)-\frac{2}{27}\log\left(4-x^2\right)\right)(2\right.$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^3,x]
```

output
```
(x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(144*(4 - 5*x^2 + x^4)^2
) + (-1/24*(x*(59*d + 380*f + 848*h - 5*(7*d + 28*f + 64*h)*x^2))/(4 - 5*x
^2 + x^4) + (-1/6*((313*d + 820*f + 1936*h)*ArcTanh[x/2]) + (16*(13*d + 25
*f + 61*h)*ArcTanh[x])/3)/24)/144 + ((5*e + 8*g + 20*i - (2*e + 5*g + 17*i
)*x^2)/(18*(4 - 5*x^2 + x^4)^2) - ((2*e + 5*g + 11*i)*(1/(9*(1 - x^2)) + 1
/(9*(4 - x^2)) + (2*Log[1 - x^2])/27 - (2*Log[4 - x^2])/27))/6)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1084
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(b/2 - q/2 + c*x)^p*(b/2 + q
/2 + c*x)^p, x], x], x] /;  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c},
 x] && IntegerQ[p] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.86

method	result
norman	$\frac{\left(-\frac{13d}{192}-\frac{5f}{16}-\frac{17h}{24}\right)x^5+\left(\frac{35d}{384}+\frac{21f}{32}+\frac{35h}{24}\right)x^3+\left(\frac{35d}{3456}+\frac{35f}{864}+\frac{5h}{54}\right)x^7+\left(\frac{43d}{864}-\frac{65f}{216}-\frac{41h}{54}\right)x+\left(-\frac{5e}{18}-\frac{25g}{36}-\frac{55i}{36}\right)x^4+\left(\frac{e}{27}+\frac{5g}{54}+\frac{1}{5}\right.}{(x^4-5x^2+4)^2}$
default	$-\frac{-\frac{19d}{6912}-\frac{17e}{3456}-\frac{5f}{576}-\frac{13g}{864}-\frac{11h}{432}-\frac{i}{24}}{x-2}-\frac{\frac{d}{1728}+\frac{e}{864}+\frac{f}{432}+\frac{g}{216}+\frac{h}{108}+\frac{i}{54}}{2(x-2)^2}+\left(\frac{313d}{41472}+\frac{e}{81}+\frac{205f}{10368}+\frac{5g}{162}+\frac{121h}{2592}+\right.$
risch	$\frac{\ln(2-x)e}{81}+\frac{\ln(x+2)e}{81}-\frac{13\ln(1-x)d}{1296}+\frac{313\ln(2-x)d}{41472}+\frac{205\ln(2-x)f}{10368}+\frac{13\ln(1+x)d}{1296}+\frac{\left(-\frac{13d}{192}-\frac{5f}{16}-\frac{17h}{24}\right)x^5+\left(\frac{3}{3}\right.}{}$
parallelrisch	Expression too large to display

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x,method=_RETURNVERBOS
E)
```

output
```
((-13/192*d-5/16*f-17/24*h)*x^5+(35/384*d+21/32*f+35/24*h)*x^3+(35/3456*d+
35/864*f+5/54*h)*x^7+(43/864*d-65/216*f-41/54*h)*x+(-5/18*e-25/36*g-55/36*
i)*x^4+(1/27*e+5/54*g+11/54*i)*x^6+(5/9*e+25/18*g+26/9*i)*x^2-25/108*e-19/
27*g-40/27*i)/(x^4-5*x^2+4)^2+(-313/41472*d+1/81*e-205/10368*f+5/162*g-121
/2592*h+11/162*i)*ln(x+2)+(-13/1296*d-1/81*e-25/1296*f-5/162*g-61/1296*h-1
1/162*i)*ln(x-1)+(13/1296*d-1/81*e+25/1296*f-5/162*g+61/1296*h-11/162*i)*l
n(1+x)+(313/41472*d+1/81*e+205/10368*f+5/162*g+121/2592*h+11/162*i)*ln(x-2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(226) = 452$.

Time = 6.46 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.44

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(4 - 5x^2 + x^4)^3}\, dx = \text{Too large to display}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fr
icas")
```

output
```
1/41472*(60*(7*d + 28*f + 64*h)*x^7 + 768*(2*e + 5*g + 11*i)*x^6 - 216*(13
*d + 60*f + 136*h)*x^5 - 5760*(2*e + 5*g + 11*i)*x^4 + 756*(5*d + 36*f + 8
0*h)*x^3 + 2304*(10*e + 25*g + 52*i)*x^2 + 48*(43*d - 260*f - 656*h)*x - (
(313*d - 512*e + 820*f - 1280*g + 1936*h - 2816*i)*x^8 - 10*(313*d - 512*e
 + 820*f - 1280*g + 1936*h - 2816*i)*x^6 + 33*(313*d - 512*e + 820*f - 128
0*g + 1936*h - 2816*i)*x^4 - 40*(313*d - 512*e + 820*f - 1280*g + 1936*h -
 2816*i)*x^2 + 5008*d - 8192*e + 13120*f - 20480*g + 30976*h - 45056*i)*lo
g(x + 2) + 32*((13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*x^8 - 10*(13*d -
16*e + 25*f - 40*g + 61*h - 88*i)*x^6 + 33*(13*d - 16*e + 25*f - 40*g + 61
*h - 88*i)*x^4 - 40*(13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*x^2 + 208*d
- 256*e + 400*f - 640*g + 976*h - 1408*i)*log(x + 1) - 32*((13*d + 16*e +
25*f + 40*g + 61*h + 88*i)*x^8 - 10*(13*d + 16*e + 25*f + 40*g + 61*h + 88
*i)*x^6 + 33*(13*d + 16*e + 25*f + 40*g + 61*h + 88*i)*x^4 - 40*(13*d + 16
*e + 25*f + 40*g + 61*h + 88*i)*x^2 + 208*d + 256*e + 400*f + 640*g + 976*
h + 1408*i)*log(x - 1) + ((313*d + 512*e + 820*f + 1280*g + 1936*h + 2816*
i)*x^8 - 10*(313*d + 512*e + 820*f + 1280*g + 1936*h + 2816*i)*x^6 + 33*(3
13*d + 512*e + 820*f + 1280*g + 1936*h + 2816*i)*x^4 - 40*(313*d + 512*e +
 820*f + 1280*g + 1936*h + 2816*i)*x^2 + 5008*d + 8192*e + 13120*f + 20480
*g + 30976*h + 45056*i)*log(x - 2) - 9600*e - 29184*g - 61440*i)/(x^8 - 10
*x^6 + 33*x^4 - 40*x^2 + 16)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.94

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^3}\,dx$$
$$= -\frac{1}{41472}\left(313\,d-512\,e+820\,f-1280\,g+1936\,h-2816\,i\right)\log\left(x+2\right)$$
$$+\frac{1}{1296}\left(13\,d-16\,e+25\,f-40\,g+61\,h-88\,i\right)\log\left(x+1\right)$$
$$-\frac{1}{1296}\left(13\,d+16\,e+25\,f+40\,g+61\,h+88\,i\right)\log\left(x-1\right)$$
$$+\frac{1}{41472}\left(313\,d+512\,e+820\,f+1280\,g+1936\,h+2816\,i\right)\log\left(x-2\right)$$
$$+\frac{5\,(7\,d+28\,f+64\,h)x^7+64\,(2\,e+5\,g+11\,i)x^6-18\,(13\,d+60\,f+136\,h)x^5-480\,(2\,e+5\,g+11\,i)x}{3456\,(x^8-10\,x}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="ma
xima")
```

output
```
-1/41472*(313*d - 512*e + 820*f - 1280*g + 1936*h - 2816*i)*log(x + 2) + 1
/1296*(13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*log(x + 1) - 1/1296*(13*d
+ 16*e + 25*f + 40*g + 61*h + 88*i)*log(x - 1) + 1/41472*(313*d + 512*e +
820*f + 1280*g + 1936*h + 2816*i)*log(x - 2) + 1/3456*(5*(7*d + 28*f + 64*
h)*x^7 + 64*(2*e + 5*g + 11*i)*x^6 - 18*(13*d + 60*f + 136*h)*x^5 - 480*(2
*e + 5*g + 11*i)*x^4 + 63*(5*d + 36*f + 80*h)*x^3 + 192*(10*e + 25*g + 52*
i)*x^2 + 4*(43*d - 260*f - 656*h)*x - 800*e - 2432*g - 5120*i)/(x^8 - 10*x
^6 + 33*x^4 - 40*x^2 + 16)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.99

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^3}\,dx$$
$$= -\frac{1}{41472}\,(313\,d-512\,e+820\,f-1280\,g+1936\,h-2816\,i)\log(|x+2|) + \frac{1}{1296}\,(13\,d-16\,e+25\,f-40\,g+61\,h-88\,i)\log(|x+1|) - \frac{1}{1296}\,(13\,d+16\,e+25\,f+40\,g+61\,h+88\,i)\log(|x-1|) + \frac{1}{41472}\,(313\,d+512\,e+820\,f+1280\,g+1936\,h+2816\,i)\log(|x-2|) + \frac{35\,dx^7+140\,fx^7+320\,hx^7+128\,ex^6+320\,gx^6+704\,ix^6-234\,dx^5-1080\,fx^5-2448\,hx^5-960\,ex}{}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="gi
ac")
```

output
```
-1/41472*(313*d - 512*e + 820*f - 1280*g + 1936*h - 2816*i)*log(abs(x + 2)
) + 1/1296*(13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*log(abs(x + 1)) - 1/1
296*(13*d + 16*e + 25*f + 40*g + 61*h + 88*i)*log(abs(x - 1)) + 1/41472*(3
13*d + 512*e + 820*f + 1280*g + 1936*h + 2816*i)*log(abs(x - 2)) + 1/3456*
(35*d*x^7 + 140*f*x^7 + 320*h*x^7 + 128*e*x^6 + 320*g*x^6 + 704*i*x^6 - 23
4*d*x^5 - 1080*f*x^5 - 2448*h*x^5 - 960*e*x^4 - 2400*g*x^4 - 5280*i*x^4 +
315*d*x^3 + 2268*f*x^3 + 5040*h*x^3 + 1920*e*x^2 + 4800*g*x^2 + 9984*i*x^2
 + 172*d*x - 1040*f*x - 2624*h*x - 800*e - 2432*g - 5120*i)/(x^4 - 5*x^2 +
 4)^2
```

Mupad [B] (verification not implemented)

Time = 18.53 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.92

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^3}\,dx$$
$$= \ln(x+1)\left(\frac{13\,d}{1296}-\frac{e}{81}+\frac{25\,f}{1296}-\frac{5\,g}{162}+\frac{61\,h}{1296}-\frac{11\,i}{162}\right)$$
$$-\ln(x-1)\left(\frac{13\,d}{1296}+\frac{e}{81}+\frac{25\,f}{1296}+\frac{5\,g}{162}+\frac{61\,h}{1296}+\frac{11\,i}{162}\right)$$
$$-\frac{\left(-\frac{35\,d}{3456}-\frac{35\,f}{864}-\frac{5\,h}{54}\right)x^7+\left(-\frac{e}{27}-\frac{5\,g}{54}-\frac{11\,i}{54}\right)x^6+\left(\frac{13\,d}{192}+\frac{5\,f}{16}+\frac{17\,h}{24}\right)x^5+\left(\frac{5\,e}{18}+\frac{25\,g}{36}+\frac{55\,i}{36}\right)x^4+\left(-\frac{35}{38\cdots}\right.}{x^8-10\,x^6+33\,x^4-40\cdots}$$
$$+\ln(x-2)\left(\frac{313\,d}{41472}+\frac{e}{81}+\frac{205\,f}{10368}+\frac{5\,g}{162}+\frac{121\,h}{2592}+\frac{11\,i}{162}\right)$$
$$-\ln(x+2)\left(\frac{313\,d}{41472}-\frac{e}{81}+\frac{205\,f}{10368}-\frac{5\,g}{162}+\frac{121\,h}{2592}-\frac{11\,i}{162}\right)$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^4 - 5*x^2 + 4)^3,x)
```

output
```
log(x + 1)*((13*d)/1296 - e/81 + (25*f)/1296 - (5*g)/162 + (61*h)/1296 - (
11*i)/162) - log(x - 1)*((13*d)/1296 + e/81 + (25*f)/1296 + (5*g)/162 + (6
1*h)/1296 + (11*i)/162) - ((25*e)/108 + (19*g)/27 + (40*i)/27 + x*((65*f)/
216 - (43*d)/864 + (41*h)/54) + x^5*((13*d)/192 + (5*f)/16 + (17*h)/24) -
x^3*((35*d)/384 + (21*f)/32 + (35*h)/24) - x^7*((35*d)/3456 + (35*f)/864 +
 (5*h)/54) - x^2*((5*e)/9 + (25*g)/18 + (26*i)/9) - x^6*(e/27 + (5*g)/54 +
 (11*i)/54) + x^4*((5*e)/18 + (25*g)/36 + (55*i)/36))/(33*x^4 - 40*x^2 - 1
0*x^6 + x^8 + 16) + log(x - 2)*((313*d)/41472 + e/81 + (205*f)/10368 + (5*
g)/162 + (121*h)/2592 + (11*i)/162) - log(x + 2)*((313*d)/41472 - e/81 + (
205*f)/10368 - (5*g)/162 + (121*h)/2592 - (11*i)/162)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1283, normalized size of antiderivative = 5.09

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(4 - 5x^2 + x^4)^3} dx = \text{Too large to display}$$

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x)
```

output
```
(1565*log(x - 2)*d*x**8 - 15650*log(x - 2)*d*x**6 + 51645*log(x - 2)*d*x**
4 - 62600*log(x - 2)*d*x**2 + 25040*log(x - 2)*d + 2560*log(x - 2)*e*x**8
- 25600*log(x - 2)*e*x**6 + 84480*log(x - 2)*e*x**4 - 102400*log(x - 2)*e*
x**2 + 40960*log(x - 2)*e + 4100*log(x - 2)*f*x**8 - 41000*log(x - 2)*f*x*
*6 + 135300*log(x - 2)*f*x**4 - 164000*log(x - 2)*f*x**2 + 65600*log(x - 2
)*f + 6400*log(x - 2)*g*x**8 - 64000*log(x - 2)*g*x**6 + 211200*log(x - 2)
*g*x**4 - 256000*log(x - 2)*g*x**2 + 102400*log(x - 2)*g + 9680*log(x - 2)
*h*x**8 - 96800*log(x - 2)*h*x**6 + 319440*log(x - 2)*h*x**4 - 387200*log(
x - 2)*h*x**2 + 154880*log(x - 2)*h + 14080*log(x - 2)*i*x**8 - 140800*log
(x - 2)*i*x**6 + 464640*log(x - 2)*i*x**4 - 563200*log(x - 2)*i*x**2 + 225
280*log(x - 2)*i - 2080*log(x - 1)*d*x**8 + 20800*log(x - 1)*d*x**6 - 6864
0*log(x - 1)*d*x**4 + 83200*log(x - 1)*d*x**2 - 33280*log(x - 1)*d - 2560*
log(x - 1)*e*x**8 + 25600*log(x - 1)*e*x**6 - 84480*log(x - 1)*e*x**4 + 10
2400*log(x - 1)*e*x**2 - 40960*log(x - 1)*e - 4000*log(x - 1)*f*x**8 + 400
00*log(x - 1)*f*x**6 - 132000*log(x - 1)*f*x**4 + 160000*log(x - 1)*f*x**2
 - 64000*log(x - 1)*f - 6400*log(x - 1)*g*x**8 + 64000*log(x - 1)*g*x**6 -
 211200*log(x - 1)*g*x**4 + 256000*log(x - 1)*g*x**2 - 102400*log(x - 1)*g
 - 9760*log(x - 1)*h*x**8 + 97600*log(x - 1)*h*x**6 - 322080*log(x - 1)*h*
x**4 + 390400*log(x - 1)*h*x**2 - 156160*log(x - 1)*h - 14080*log(x - 1)*i
*x**8 + 140800*log(x - 1)*i*x**6 - 464640*log(x - 1)*i*x**4 + 563200*lo...
```

3.44 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4} dx$

Optimal result

Integrand size = 36, antiderivative size = 136

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4}\,dx = hx+\frac{ix^2}{2}-\frac{(d+f-2h)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f-2h)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(2e-g-i)\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}(d-f)\text{arctanh}\left(\frac{x}{1+x^2}\right)+\frac{1}{4}(g-i)\log\left(1+x^2+x^4\right)$$

output
```
h*x+1/2*i*x^2-1/6*(d+f-2*h)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*(d+f-2
*h)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/6*(2*e-g-i)*arctan(1/3*(2*x^2+1)
*3^(1/2))*3^(1/2)+1/2*(d-f)*arctanh(x/(x^2+1))+1/4*(g-i)*ln(x^4+x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.38

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4}\,dx = \frac{1}{12}\Bigg(6x(2h+ix) + \left(1+i\sqrt{3}\right)\left(2\sqrt{3}d-\left(3i+\sqrt{3}\right)f - \left(-3i+\sqrt{3}\right)h\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right) + \left(i+\sqrt{3}\right)\left(-2i\sqrt{3}d+\left(3+i\sqrt{3}\right)f + i\left(3i+\sqrt{3}\right)h\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right) - 2\sqrt{3}(2e-g-i)\arctan\left(\frac{\sqrt{3}}{1+2x^2}\right) + 3(g-i)\log\left(1+x^2+x^4\right)\Bigg)$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4),x]
```

output
```
(6*x*(2*h + i*x) + (1 + I*Sqrt[3])*(2*Sqrt[3]*d - (3*I + Sqrt[3])*f - (-3*
I + Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2] + (I + Sqrt[3])*((-2*I)*Sqrt[
3]*d + (3 + I*Sqrt[3])*f + I*(3*I + Sqrt[3])*h)*ArcTan[((I + Sqrt[3])*x)/2
] - 2*Sqrt[3]*(2*e - g - i)*ArcTan[Sqrt[3]/(1 + 2*x^2)] + 3*(g - i)*Log[1
+ x^2 + x^4])/12
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2202, 2194, 2188, 2009, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{hx^4+fx^2+d}{x^4+x^2+1}dx+\int \frac{x\left(ix^4+gx^2+e\right)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{2194}$$

$$\int \frac{hx^4+fx^2+d}{x^4+x^2+1}dx+\frac{1}{2}\int \frac{ix^4+gx^2+e}{x^4+x^2+1}dx^2$$

$$\Big\downarrow \text{2188}$$

$$\int \frac{hx^4+fx^2+d}{x^4+x^2+1}dx+\frac{1}{2}\int \left(i+\frac{(g-i)x^2+e-i}{x^4+x^2+1}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{hx^4+fx^2+d}{x^4+x^2+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g-i)}{\sqrt{3}}+\frac{1}{2}(g-i)\log\left(x^4+x^2+1\right)+ix^2\right)$$

$$\Big\downarrow \text{2205}$$

$$\int \left(h+\frac{(f-h)x^2+d-h}{x^4+x^2+1}\right)dx+$$
$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g-i)}{\sqrt{3}}+\frac{1}{2}(g-i)\log\left(x^4+x^2+1\right)+ix^2\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+$$
$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g-i)}{\sqrt{3}}+\frac{1}{2}(g-i)\log\left(x^4+x^2+1\right)+ix^2\right)-\frac{1}{4}(d-$$
$$f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+hx$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4),x]
```

output
```
h*x - ((d + f - 2*h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f - 2*
h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) - ((d - f)*Log[1 - x + x^2])/4 +
 ((d - f)*Log[1 + x + x^2])/4 + (i*x^2 + ((2*e - g - i)*ArcTan[(1 + 2*x^2)
/Sqrt[3]])/Sqrt[3] + ((g - i)*Log[1 + x^2 + x^4])/2)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

method	result
default	$\frac{i x^2}{2}+hx+\frac{(d-f+g-i)\ln(x^2+x+1)}{4}+\frac{\left(\frac{d}{2}-e+\frac{f}{2}+\frac{g}{2}-h+\frac{i}{2}\right)\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{(g-i+f-d)\ln(x^2-x+1)}{4}+\frac{\left(\frac{d}{2}+e\right.}{}$
risch	Expression too large to display

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*i*x^2+h*x+1/4*(d-f+g-i)*ln(x^2+x+1)+1/3*(1/2*d-e+1/2*f+1/2*g-h+1/2*i)*
arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*(g-i+f-d)*ln(x^2-x+1)+1/3*(1/2*d+e
+1/2*f-1/2*g-h-1/2*i)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4}\,dx &= \frac{1}{2}\,ix^2+\frac{1}{6}\,\sqrt{3}(d-2\,e+f+g-2\,h+i)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)\\ &\quad+\frac{1}{6}\,\sqrt{3}(d+2\,e+f-g-2\,h-i)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+hx\\ &\quad+\frac{1}{4}\,(d-f+g-i)\log\left(x^2+x+1\right)-\frac{1}{4}\,(d-f-g+i)\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="fricas
")
```

output
```
1/2*i*x^2 + 1/6*sqrt(3)*(d - 2*e + f + g - 2*h + i)*arctan(1/3*sqrt(3)*(2*
x + 1)) + 1/6*sqrt(3)*(d + 2*e + f - g - 2*h - i)*arctan(1/3*sqrt(3)*(2*x
- 1)) + h*x + 1/4*(d - f + g - i)*log(x^2 + x + 1) - 1/4*(d - f - g + i)*l
og(x^2 - x + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4}\,dx = \text{Timed out}$$

input
```
integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4}\,dx &= \frac{1}{2}\,ix^2+\frac{1}{6}\,\sqrt{3}(d-2\,e+f+g-2\,h+i)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) \\ &\quad+\frac{1}{6}\,\sqrt{3}(d+2\,e+f-g-2\,h-i)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+hx \\ &\quad+\frac{1}{4}\,(d-f+g-i)\log\left(x^2+x+1\right)-\frac{1}{4}\,(d-f-g+i)\log\left(x^2-x+1\right)\end{aligned}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="maxima
")
```

output
```
1/2*i*x^2 + 1/6*sqrt(3)*(d - 2*e + f + g - 2*h + i)*arctan(1/3*sqrt(3)*(2*
x + 1)) + 1/6*sqrt(3)*(d + 2*e + f - g - 2*h - i)*arctan(1/3*sqrt(3)*(2*x
- 1)) + h*x + 1/4*(d - f + g - i)*log(x^2 + x + 1) - 1/4*(d - f - g + i)*l
og(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4}\,dx = \frac{1}{2}\,ix^2+\frac{1}{6}\,\sqrt{3}(d-2\,e+f+g-2\,h+i)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}(d+2\,e+f-g-2\,h-i)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+hx + \frac{1}{4}\,(d-f+g-i)\log\left(x^2+x+1\right)-\frac{1}{4}\,(d-f-g+i)\log\left(x^2-x+1\right)$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/2*i*x^2 + 1/6*sqrt(3)*(d - 2*e + f + g - 2*h + i)*arctan(1/3*sqrt(3)*(2*
x + 1)) + 1/6*sqrt(3)*(d + 2*e + f - g - 2*h - i)*arctan(1/3*sqrt(3)*(2*x
- 1)) + h*x + 1/4*(d - f + g - i)*log(x^2 + x + 1) - 1/4*(d - f - g + i)*l
og(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 26.95 (sec) , antiderivative size = 1509, normalized size of antiderivative = 11.10

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^2 + x^4 + 1),x)
```

output
```
h*x - log(d*g*3i - d*f*9i - d*e*6i + d*h*3i + d*i*3i + e*h*6i - f*h*3i - g
*h*3i - h*i*3i - 3*3^(1/2)*d^2 - d^2*x*6i - f^2*x*3i + d^2*3i + f^2*6i - 2
*3^(1/2)*d*e + 3*3^(1/2)*d*f + 3^(1/2)*d*g + 4*3^(1/2)*e*f + 3*3^(1/2)*d*h
 + 3^(1/2)*d*i - 2*3^(1/2)*e*h - 2*3^(1/2)*f*g - 3*3^(1/2)*f*h - 2*3^(1/2)
*f*i + 3^(1/2)*g*h + 3^(1/2)*h*i + d*f*x*9i + e*f*x*6i + d*h*x*3i - e*h*x*
6i - f*g*x*3i - f*h*x*3i - f*i*x*3i + g*h*x*3i + h*i*x*3i - 3*3^(1/2)*f^2*
x + 3*3^(1/2)*d*f*x - 2*3^(1/2)*d*g*x - 2*3^(1/2)*e*f*x - 3*3^(1/2)*d*h*x
- 2*3^(1/2)*d*i*x - 2*3^(1/2)*e*h*x + 3^(1/2)*f*g*x + 3*3^(1/2)*f*h*x + 3^
(1/2)*f*i*x + 3^(1/2)*g*h*x + 3^(1/2)*h*i*x + 4*3^(1/2)*d*e*x)*(d/4 - f/4
- g/4 + i/4 + (3^(1/2)*d*1i)/12 + (3^(1/2)*e*1i)/6 + (3^(1/2)*f*1i)/12 - (
3^(1/2)*g*1i)/12 - (3^(1/2)*h*1i)/6 - (3^(1/2)*i*1i)/12) - log(d*e*6i + d*
f*9i - d*g*3i - d*h*3i - d*i*3i - e*h*6i + f*h*3i + g*h*3i + h*i*3i - 3*3^
(1/2)*d^2 + d^2*x*6i + f^2*x*3i - d^2*3i - f^2*6i - 2*3^(1/2)*d*e + 3*3^(1
/2)*d*f + 3^(1/2)*d*g + 4*3^(1/2)*e*f + 3*3^(1/2)*d*h + 3^(1/2)*d*i - 2*3^
(1/2)*e*h - 2*3^(1/2)*f*g - 3*3^(1/2)*f*h - 2*3^(1/2)*f*i + 3^(1/2)*g*h +
3^(1/2)*h*i - d*f*x*9i - e*f*x*6i - d*h*x*3i + e*h*x*6i + f*g*x*3i + f*h*x
*3i + f*i*x*3i - g*h*x*3i - h*i*x*3i - 3*3^(1/2)*f^2*x + 3*3^(1/2)*d*f*x -
 2*3^(1/2)*d*g*x - 2*3^(1/2)*e*f*x - 3*3^(1/2)*d*h*x - 2*3^(1/2)*d*i*x - 2
*3^(1/2)*e*h*x + 3^(1/2)*f*g*x + 3*3^(1/2)*f*h*x + 3^(1/2)*f*i*x + 3^(1/2)
*g*h*x + 3^(1/2)*h*i*x + 4*3^(1/2)*d*e*x)*(d/4 - f/4 - g/4 + i/4 - (3^(...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.13

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)d}{6}+\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)e}{3} + \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)f}{6} - \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)g}{6} - \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)h}{3} - \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)i}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)d}{6} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)e}{3} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)f}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)g}{6} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)h}{3} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)i}{6} - \frac{\log(x^2-x+1)\,d}{4} + \frac{\log(x^2-x+1)\,f}{4} + \frac{\log(x^2-x+1)\,g}{4} - \frac{\log(x^2-x+1)\,i}{4} + \frac{\log(x^2+x+1)\,d}{4} - \frac{\log(x^2+x+1)\,f}{4} + \frac{\log(x^2+x+1)\,g}{4} - \frac{\log(x^2+x+1)\,i}{4} + hx + \frac{i\,x^2}{2}$$

input

```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x)
```

output

```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3))*d + 4*sqrt(3)*atan((2*x - 1)/sqrt(3))*e
 + 2*sqrt(3)*atan((2*x - 1)/sqrt(3))*f - 2*sqrt(3)*atan((2*x - 1)/sqrt(3))
*g - 4*sqrt(3)*atan((2*x - 1)/sqrt(3))*h - 2*sqrt(3)*atan((2*x - 1)/sqrt(3
))*i + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*d - 4*sqrt(3)*atan((2*x + 1)/sqrt
(3))*e + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*f + 2*sqrt(3)*atan((2*x + 1)/sq
rt(3))*g - 4*sqrt(3)*atan((2*x + 1)/sqrt(3))*h + 2*sqrt(3)*atan((2*x + 1)/
sqrt(3))*i - 3*log(x**2 - x + 1)*d + 3*log(x**2 - x + 1)*f + 3*log(x**2 -
x + 1)*g - 3*log(x**2 - x + 1)*i + 3*log(x**2 + x + 1)*d - 3*log(x**2 + x
+ 1)*f + 3*log(x**2 + x + 1)*g - 3*log(x**2 + x + 1)*i + 12*h*x + 6*i*x**2
)/12
```

3.45 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 176

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^2}\,dx &= \frac{x(d+f-2h-(d-2f+h)x^2)}{6\,(1+x^2+x^4)} \\ &\quad + \frac{e-2g+i+(2e-g-i)x^2}{6\,(1+x^2+x^4)} \\ &\quad - \frac{(4d+f+h)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} \\ &\quad + \frac{(4d+f+h)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}} \\ &\quad + \frac{(2e-g+2i)\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}} \\ &\quad + \frac{1}{4}(2d-f+h)\operatorname{arctanh}\left(\frac{x}{1+x^2}\right)\end{aligned}$$

output
```
x*(d+f-2*h-(d-2*f+h)*x^2)/(6*x^4+6*x^2+6)+(e-2*g+i+(2*e-g-i)*x^2)/(6*x^4+6
*x^2+6)-1/36*(4*d+f+h)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/36*(4*d+f+h)*
arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/9*(2*e-g+2*i)*arctan(1/3*(2*x^2+1)*3
^(1/2))*3^(1/2)+1/4*(2*d-f+h)*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.38

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^2}\,dx = \frac{1}{36}\left(\frac{6(e+i+dx+fx-2hx+2ex^2-ix^2-dx^3+2fx^3-hx^3-g(2+x^2))}{1+x^2+x^4} - \frac{\left(\left(-11i+\sqrt{3}\right)d-2\left(-2i+\sqrt{3}\right)f+\left(-5i+\sqrt{3}\right)h\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}} - \frac{\left(\left(11i+\sqrt{3}\right)d-2\left(2i+\sqrt{3}\right)f+\left(5i+\sqrt{3}\right)h\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}} - 4\sqrt{3}(2e-g+2i)\arctan\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^2,x]
```

output
```
((6*(e + i + d*x + f*x - 2*h*x + 2*e*x^2 - i*x^2 - d*x^3 + 2*f*x^3 - h*x^3
 - g*(2 + x^2)))/(1 + x^2 + x^4) - (((-11*I + Sqrt[3])*d - 2*(-2*I + Sqrt[
3])*f + (-5*I + Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt
[3])/6] - (((11*I + Sqrt[3])*d - 2*(2*I + Sqrt[3])*f + (5*I + Sqrt[3])*h)*
ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 4*Sqrt[3]*(2*e - g
+ 2*i)*ArcTan[Sqrt[3]/(1 + 2*x^2)])/36
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {2202, 2194, 2191, 27, 1083, 217, 2206, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(x^4+x^2+1)^2}\,dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{hx^4+fx^2+d}{(x^4+x^2+1)^2}dx+\int \frac{x(ix^4+gx^2+e)}{(x^4+x^2+1)^2}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\int \frac{hx^4+fx^2+d}{(x^4+x^2+1)^2}dx+\frac{1}{2}\int \frac{ix^4+gx^2+e}{(x^4+x^2+1)^2}dx^2$$

$$\Big\downarrow \text{ 2191}$$

$$\int \frac{hx^4+fx^2+d}{(x^4+x^2+1)^2}dx+\frac{1}{2}\left(\frac{1}{3}\int \frac{2e-g+2i}{x^4+x^2+1}dx^2+\frac{x^2(2e-g-i)+e-2g+i}{3(x^4+x^2+1)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{hx^4+fx^2+d}{(x^4+x^2+1)^2}dx+\frac{1}{2}\left(\frac{1}{3}(2e-g+2i)\int \frac{1}{x^4+x^2+1}dx^2+\frac{x^2(2e-g-i)+e-2g+i}{3(x^4+x^2+1)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{hx^4+fx^2+d}{(x^4+x^2+1)^2}dx+ \frac{1}{2}\left(\frac{x^2(2e-g-i)+e-2g+i}{3(x^4+x^2+1)}-\frac{2}{3}(2e-g+2i)\int \frac{1}{-x^4-3}d(2x^2+1)\right)$$

$$\Big\downarrow \text{ 217}$$

$$\int \frac{hx^4+fx^2+d}{(x^4+x^2+1)^2}dx+\frac{1}{2}\left(\frac{2\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+2i)}{3\sqrt{3}}+\frac{x^2(2e-g-i)+e-2g+i}{3(x^4+x^2+1)}\right)$$

$$\Big\downarrow \text{ 2206}$$

$$\frac{1}{6}\int \frac{-\left((d-2f+h)x^2\right)+5d-f+2h}{x^4+x^2+1}dx+ \frac{1}{2}\left(\frac{2\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+2i)}{3\sqrt{3}}+\frac{x^2(2e-g-i)+e-2g+i}{3(x^4+x^2+1)}\right)+ \frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{6(x^4+x^2+1)}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{1}{6}\left(\frac{1}{2}\int\frac{5d-f+2h-3(2d-f+h)x}{x^2-x+1}dx+\frac{1}{2}\int\frac{5d-f+2h+3(2d-f+h)x}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{2\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+2i)}{3\sqrt{3}}+\frac{x^2(2e-g-i)+e-2g+i}{3\left(x^4+x^2+1\right)}\right)+\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{6\left(x^4+x^2+1\right)}$$

$\downarrow$ 1142

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{2}(4d+f+h)\int\frac{1}{x^2-x+1}dx-\frac{3}{2}(2d-f+h)\int-\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(4d+f+h)\int\frac{1}{x^2+x+1}d\right.\right.$$
$$\frac{1}{2}\left(\frac{2\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+2i)}{3\sqrt{3}}+\frac{x^2(2e-g-i)+e-2g+i}{3\left(x^4+x^2+1\right)}\right)+\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{6\left(x^4+x^2+1\right)}$$

$\downarrow$ 25

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{2}(4d+f+h)\int\frac{1}{x^2-x+1}dx+\frac{3}{2}(2d-f+h)\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(4d+f+h)\int\frac{1}{x^2+x+1}dx\right.\right.$$
$$\frac{1}{2}\left(\frac{2\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+2i)}{3\sqrt{3}}+\frac{x^2(2e-g-i)+e-2g+i}{3\left(x^4+x^2+1\right)}\right)+\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{6\left(x^4+x^2+1\right)}$$

$\downarrow$ 1083

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{3}{2}(2d-f+h)\int\frac{1-2x}{x^2-x+1}dx-(4d+f+h)\int\frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{3}{2}(2d-f+h)\int\frac{2x}{x^2+}\right.\right.$$
$$\frac{1}{2}\left(\frac{2\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+2i)}{3\sqrt{3}}+\frac{x^2(2e-g-i)+e-2g+i}{3\left(x^4+x^2+1\right)}\right)+\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{6\left(x^4+x^2+1\right)}$$

$\downarrow$ 217

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{3}{2}(2d-f+h)\int\frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)(4d+f+h)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{3}{2}(2d-f+h)\int\frac{2x+1}{x^2+x+1}dx+\right.\right.$$
$$\frac{1}{2}\left(\frac{2\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+2i)}{3\sqrt{3}}+\frac{x^2(2e-g-i)+e-2g+i}{3(x^4+x^2+1)}\right)+$$
$$\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{6(x^4+x^2+1)}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)(4d+f+h)}{\sqrt{3}}-\frac{3}{2}\log\left(x^2-x+1\right)(2d-f+h)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)(4d+f+h)}{\sqrt{3}}+\frac{3}{2}\right.\right.$$
$$\frac{1}{2}\left(\frac{2\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+2i)}{3\sqrt{3}}+\frac{x^2(2e-g-i)+e-2g+i}{3(x^4+x^2+1)}\right)+$$
$$\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{6(x^4+x^2+1)}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^2,x]
```

output
```
(x*(d + f - 2*h - (d - 2*f + h)*x^2))/(6*(1 + x^2 + x^4)) + ((e - 2*g + i
+ (2*e - g - i)*x^2)/(3*(1 + x^2 + x^4)) + (2*(2*e - g + 2*i)*ArcTan[(1 +
2*x^2)/Sqrt[3]])/(3*Sqrt[3]))/2 + ((((4*d + f + h)*ArcTan[(-1 + 2*x)/Sqrt[
3]])/Sqrt[3] - (3*(2*d - f + h)*Log[1 - x + x^2])/2)/2 + (((4*d + f + h)*A
rcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*(2*d - f + h)*Log[1 + x + x^2])/2)/
2)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.22

method	result
default	$\frac{\left(-\frac{d}{3}-\frac{e}{3}-\frac{g}{3}-\frac{h}{3}+\frac{2f}{3}+\frac{2i}{3}\right)x-\frac{2d}{3}+\frac{e}{3}-\frac{2g}{3}+\frac{h}{3}+\frac{f}{3}+\frac{i}{3}}{4x^2+4x+4}+\frac{(6d-3f+3h)\ln\left(x^2+x+1\right)}{24}+\frac{\left(2d-4e+\frac{f}{2}+2g+\frac{h}{2}-4i\right)\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)}{18}$
risch	Expression too large to display

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*((-1/3*d-1/3*e-1/3*g-1/3*h+2/3*f+2/3*i)*x-2/3*d+1/3*e-2/3*g+1/3*h+1/3*
f+1/3*i)/(x^2+x+1)+1/24*(6*d-3*f+3*h)*ln(x^2+x+1)+1/18*(2*d-4*e+1/2*f+2*g+
1/2*h-4*i)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*((1/3*d-1/3*e-1/3*g+1/3
*h-2/3*f+2/3*i)*x-2/3*d-1/3*e+2/3*g+1/3*h+1/3*f-1/3*i)/(x^2-x+1)-1/24*(6*d
-3*f+3*h)*ln(x^2-x+1)-1/18*(-2*d-4*e-1/2*f+2*g-1/2*h-4*i)*3^(1/2)*arctan(1
/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.59

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^2}\,dx =$$
$$-\frac{12\,(d-2\,f+h)x^3-12\,(2\,e-g-i)x^2-2\,\sqrt{3}((4\,d-8\,e+f+4\,g+h-8\,i)x^4+(4\,d-8\,e+f+4\,g$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="fric
as")
```

output
```
-1/72*(12*(d - 2*f + h)*x^3 - 12*(2*e - g - i)*x^2 - 2*sqrt(3)*((4*d - 8*e
 + f + 4*g + h - 8*i)*x^4 + (4*d - 8*e + f + 4*g + h - 8*i)*x^2 + 4*d - 8*
e + f + 4*g + h - 8*i)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((4*d + 8
*e + f - 4*g + h + 8*i)*x^4 + (4*d + 8*e + f - 4*g + h + 8*i)*x^2 + 4*d +
8*e + f - 4*g + h + 8*i)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(d + f - 2*h)*
x - 9*((2*d - f + h)*x^4 + (2*d - f + h)*x^2 + 2*d - f + h)*log(x^2 + x +
1) + 9*((2*d - f + h)*x^4 + (2*d - f + h)*x^2 + 2*d - f + h)*log(x^2 - x +
 1) - 12*e + 24*g - 12*i)/(x^4 + x^2 + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^2}\,dx$$
$$= \frac{1}{36}\sqrt{3}(4d-8e+f+4g+h-8i)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{36}\sqrt{3}(4d+8e+f-4g+h+8i)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{8}(2d-f+h)\log\left(x^2+x+1\right) - \frac{1}{8}(2d-f+h)\log\left(x^2-x+1\right) - \frac{(d-2f+h)x^3-(2e-g-i)x^2-(d+f-2h)x-e+2g-i}{6(x^4+x^2+1)}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxi
ma")
```

output
```
1/36*sqrt(3)*(4*d - 8*e + f + 4*g + h - 8*i)*arctan(1/3*sqrt(3)*(2*x + 1))
 + 1/36*sqrt(3)*(4*d + 8*e + f - 4*g + h + 8*i)*arctan(1/3*sqrt(3)*(2*x -
1)) + 1/8*(2*d - f + h)*log(x^2 + x + 1) - 1/8*(2*d - f + h)*log(x^2 - x +
 1) - 1/6*((d - 2*f + h)*x^3 - (2*e - g - i)*x^2 - (d + f - 2*h)*x - e + 2
*g - i)/(x^4 + x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^2}\,dx$$
$$= \frac{1}{36}\sqrt{3}(4\,d-8\,e+f+4\,g+h-8\,i)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)$$
$$+ \frac{1}{36}\sqrt{3}(4\,d+8\,e+f-4\,g+h+8\,i)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)$$
$$+ \frac{1}{8}(2\,d-f+h)\log\left(x^2+x+1\right)-\frac{1}{8}(2\,d-f+h)\log\left(x^2-x+1\right)$$
$$- \frac{dx^3-2\,fx^3+hx^3-2\,ex^2+gx^2+ix^2-dx-fx+2\,hx-e+2\,g-i}{6\,(x^4+x^2+1)}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac
")
```

output
```
1/36*sqrt(3)*(4*d - 8*e + f + 4*g + h - 8*i)*arctan(1/3*sqrt(3)*(2*x + 1))
 + 1/36*sqrt(3)*(4*d + 8*e + f - 4*g + h + 8*i)*arctan(1/3*sqrt(3)*(2*x -
1)) + 1/8*(2*d - f + h)*log(x^2 + x + 1) - 1/8*(2*d - f + h)*log(x^2 - x +
 1) - 1/6*(d*x^3 - 2*f*x^3 + h*x^3 - 2*e*x^2 + g*x^2 + i*x^2 - d*x - f*x +
 2*h*x - e + 2*g - i)/(x^4 + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 28.06 (sec) , antiderivative size = 1894, normalized size of antiderivative = 10.76

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^2 + x^4 + 1)^2,x)
```

output
```
(e/6 - g/3 + i/6 + x*(d/6 + f/6 - h/3) - x^3*(d/6 - f/3 + h/6) - x^2*(g/6
- e/3 + i/6))/(x^2 + x^4 + 1) - log(60*d*g - 153*d*f - 120*d*e + 24*e*f +
135*d*h - 120*d*i - 48*e*h - 12*f*g - 81*f*h + 24*f*i + 24*g*h - 48*h*i +
3^(1/2)*d^2*90i + 3^(1/2)*f^2*9i + 3^(1/2)*h^2*18i - 198*d^2*x - 36*f^2*x
- 45*h^2*x + 126*d^2 + 45*f^2 + 36*h^2 + 3^(1/2)*d*e*56i - 3^(1/2)*d*f*63i
 - 3^(1/2)*d*g*28i - 3^(1/2)*e*f*40i + 3^(1/2)*d*h*81i + 3^(1/2)*d*i*56i +
 3^(1/2)*e*h*32i + 3^(1/2)*f*g*20i - 3^(1/2)*f*h*27i - 3^(1/2)*f*i*40i - 3
^(1/2)*g*h*16i + 3^(1/2)*h*i*32i - 24*d*e*x + 171*d*f*x + 12*d*g*x + 48*e*
f*x - 189*d*h*x - 24*d*i*x - 24*e*h*x - 24*f*g*x + 81*f*h*x + 48*f*i*x + 1
2*g*h*x - 24*h*i*x + 3^(1/2)*d^2*x*18i + 3^(1/2)*f^2*x*18i + 3^(1/2)*h^2*x
*9i - 3^(1/2)*d*f*x*45i + 3^(1/2)*d*g*x*44i + 3^(1/2)*e*f*x*32i + 3^(1/2)*
d*h*x*27i - 3^(1/2)*d*i*x*88i - 3^(1/2)*e*h*x*40i - 3^(1/2)*f*g*x*16i - 3^
(1/2)*f*h*x*27i + 3^(1/2)*f*i*x*32i + 3^(1/2)*g*h*x*20i - 3^(1/2)*h*i*x*40
i - 3^(1/2)*d*e*x*88i)*(d/4 - f/8 + h/8 + (3^(1/2)*d*1i)/18 + (3^(1/2)*e*1
i)/9 + (3^(1/2)*f*1i)/72 - (3^(1/2)*g*1i)/18 + (3^(1/2)*h*1i)/72 + (3^(1/2
)*i*1i)/9) - log(120*d*e - 153*d*f - 60*d*g - 24*e*f + 135*d*h + 120*d*i +
 48*e*h + 12*f*g - 81*f*h - 24*f*i - 24*g*h + 48*h*i - 3^(1/2)*d^2*90i - 3
^(1/2)*f^2*9i - 3^(1/2)*h^2*18i + 198*d^2*x + 36*f^2*x + 45*h^2*x + 126*d^
2 + 45*f^2 + 36*h^2 + 3^(1/2)*d*e*56i + 3^(1/2)*d*f*63i - 3^(1/2)*d*g*28i
- 3^(1/2)*e*f*40i - 3^(1/2)*d*h*81i + 3^(1/2)*d*i*56i + 3^(1/2)*e*h*32i...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 955, normalized size of antiderivative = 5.43

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(1 + x^2 + x^4)^2} \, dx = \text{Too large to display}$$

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x)
```

output
```
(8*sqrt(3)*atan((2*x - 1)/sqrt(3))*d*x**4 + 8*sqrt(3)*atan((2*x - 1)/sqrt(
3))*d*x**2 + 8*sqrt(3)*atan((2*x - 1)/sqrt(3))*d + 16*sqrt(3)*atan((2*x -
1)/sqrt(3))*e*x**4 + 16*sqrt(3)*atan((2*x - 1)/sqrt(3))*e*x**2 + 16*sqrt(3
)*atan((2*x - 1)/sqrt(3))*e + 2*sqrt(3)*atan((2*x - 1)/sqrt(3))*f*x**4 + 2
*sqrt(3)*atan((2*x - 1)/sqrt(3))*f*x**2 + 2*sqrt(3)*atan((2*x - 1)/sqrt(3)
)*f - 8*sqrt(3)*atan((2*x - 1)/sqrt(3))*g*x**4 - 8*sqrt(3)*atan((2*x - 1)/
sqrt(3))*g*x**2 - 8*sqrt(3)*atan((2*x - 1)/sqrt(3))*g + 2*sqrt(3)*atan((2*
x - 1)/sqrt(3))*h*x**4 + 2*sqrt(3)*atan((2*x - 1)/sqrt(3))*h*x**2 + 2*sqrt
(3)*atan((2*x - 1)/sqrt(3))*h + 16*sqrt(3)*atan((2*x - 1)/sqrt(3))*i*x**4
+ 16*sqrt(3)*atan((2*x - 1)/sqrt(3))*i*x**2 + 16*sqrt(3)*atan((2*x - 1)/sq
rt(3))*i + 8*sqrt(3)*atan((2*x + 1)/sqrt(3))*d*x**4 + 8*sqrt(3)*atan((2*x
+ 1)/sqrt(3))*d*x**2 + 8*sqrt(3)*atan((2*x + 1)/sqrt(3))*d - 16*sqrt(3)*at
an((2*x + 1)/sqrt(3))*e*x**4 - 16*sqrt(3)*atan((2*x + 1)/sqrt(3))*e*x**2 -
 16*sqrt(3)*atan((2*x + 1)/sqrt(3))*e + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*
f*x**4 + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*f*x**2 + 2*sqrt(3)*atan((2*x +
1)/sqrt(3))*f + 8*sqrt(3)*atan((2*x + 1)/sqrt(3))*g*x**4 + 8*sqrt(3)*atan(
(2*x + 1)/sqrt(3))*g*x**2 + 8*sqrt(3)*atan((2*x + 1)/sqrt(3))*g + 2*sqrt(3
)*atan((2*x + 1)/sqrt(3))*h*x**4 + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*h*x**
2 + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*h - 16*sqrt(3)*atan((2*x + 1)/sqrt(3
))*i*x**4 - 16*sqrt(3)*atan((2*x + 1)/sqrt(3))*i*x**2 - 16*sqrt(3)*atan...
```

3.46 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 249

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^3}\,dx &= \frac{x(d+f-2h-(d-2f+h)x^2)}{12\,(1+x^2+x^4)^2} \\ &+ \frac{e-2g+i+(2e-g-i)x^2}{12\,(1+x^2+x^4)^2} \\ &+ \frac{(2e-g+i)\,(1+2x^2)}{12\,(1+x^2+x^4)} \\ &+ \frac{x(2d+3f-h-(7d-7f+4h)x^2)}{24\,(1+x^2+x^4)} \\ &- \frac{(13d+2f+h)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}} \\ &+ \frac{(13d+2f+h)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{48\sqrt{3}} \\ &+ \frac{(2e-g+i)\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}} \\ &+ \frac{1}{16}(9d-4f+3h)\operatorname{arctanh}\left(\frac{x}{1+x^2}\right)\end{aligned}$$

output
```
1/12*x*(d+f-2*h-(d-2*f+h)*x^2)/(x^4+x^2+1)^2+1/12*(e-2*g+i+(2*e-g-i)*x^2)/
(x^4+x^2+1)^2+(2*e-g+i)*(2*x^2+1)/(12*x^4+12*x^2+12)+x*(2*d+3*f-h-(7*d-7*f
+4*h)*x^2)/(24*x^4+24*x^2+24)-1/144*(13*d+2*f+h)*arctan(1/3*(1-2*x)*3^(1/2
))*3^(1/2)+1/144*(13*d+2*f+h)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/9*(2*e
-g+i)*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)+1/16*(9*d-4*f+3*h)*arctanh(x/(
x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.31

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^3}\,dx$$

$$= \frac{1}{144}\left(\frac{12(e+i+dx+fx-2hx+2ex^2-ix^2-dx^3+2fx^3-hx^3-g(2+x^2))}{(1+x^2+x^4)^2}\right.$$
$$+\frac{6(2i+2dx+3fx-hx+4ix^2-7dx^3+7fx^3-4hx^3-2g(1+2x^2)+e(4+8x^2))}{1+x^2+x^4}$$
$$-\frac{\left(\left(-47i+7\sqrt{3}\right)d+\left(17i-7\sqrt{3}\right)f+2\left(-7i+2\sqrt{3}\right)h\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}$$
$$-\frac{\left(\left(47i+7\sqrt{3}\right)d-\left(17i+7\sqrt{3}\right)f+2\left(7i+2\sqrt{3}\right)h\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}$$
$$\left.-16\sqrt{3}(2e-g+i)\arctan\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^3,x]
```

output
```
((12*(e + i + d*x + f*x - 2*h*x + 2*e*x^2 - i*x^2 - d*x^3 + 2*f*x^3 - h*x^
3 - g*(2 + x^2)))/(1 + x^2 + x^4)^2 + (6*(2*i + 2*d*x + 3*f*x - h*x + 4*i*
x^2 - 7*d*x^3 + 7*f*x^3 - 4*h*x^3 - 2*g*(1 + 2*x^2) + e*(4 + 8*x^2)))/(1 +
 x^2 + x^4) - (((-47*I + 7*Sqrt[3])*d + (17*I - 7*Sqrt[3])*f + 2*(-7*I + 2
*Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((47
*I + 7*Sqrt[3])*d - (17*I + 7*Sqrt[3])*f + 2*(7*I + 2*Sqrt[3])*h)*ArcTan[(
(I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 16*Sqrt[3]*(2*e - g + i)*Ar
cTan[Sqrt[3]/(1 + 2*x^2)])/144
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2202, 2194, 2191, 27, 1086, 1083, 217, 2206, 1492, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(x^4+x^2+1)^3}dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{hx^4+fx^2+d}{(x^4+x^2+1)^3}dx+\int \frac{x(ix^4+gx^2+e)}{(x^4+x^2+1)^3}dx$$

$$\Big\downarrow \text{2194}$$

$$\int \frac{hx^4+fx^2+d}{(x^4+x^2+1)^3}dx+\frac{1}{2}\int \frac{ix^4+gx^2+e}{(x^4+x^2+1)^3}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\int \frac{hx^4+fx^2+d}{(x^4+x^2+1)^3}dx+\frac{1}{2}\left(\frac{1}{6}\int \frac{3(2e-g+i)}{(x^4+x^2+1)^2}dx^2+\frac{x^2(2e-g-i)+e-2g+i}{6(x^4+x^2+1)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{hx^4+fx^2+d}{(x^4+x^2+1)^3}dx+\frac{1}{2}\left(\frac{1}{2}(2e-g+i)\int \frac{1}{(x^4+x^2+1)^2}dx^2+\frac{x^2(2e-g-i)+e-2g+i}{6(x^4+x^2+1)^2}\right)$$

$$\Big\downarrow \text{1086}$$

$$\int \frac{hx^4+fx^2+d}{\left(x^4+x^2+1\right)^3}dx+ \frac{1}{2}\left(\frac{1}{2}(2e-g+i)\left(\frac{2}{3}\int \frac{1}{x^4+x^2+1}dx^2+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}\right)+\frac{x^2(2e-g-i)+e-2g+i}{6\left(x^4+x^2+1\right)^2}\right)$$

$\Big\downarrow$ 1083

$$\int \frac{hx^4+fx^2+d}{\left(x^4+x^2+1\right)^3}dx+ \frac{1}{2}\left(\frac{1}{2}(2e-g+i)\left(\frac{2x^2+1}{3\left(x^4+x^2+1\right)}-\frac{4}{3}\int \frac{1}{-x^4-3}d\left(2x^2+1\right)\right)+\frac{x^2(2e-g-i)+e-2g+i}{6\left(x^4+x^2+1\right)^2}\right)$$

$\Big\downarrow$ 217

$$\int \frac{hx^4+fx^2+d}{\left(x^4+x^2+1\right)^3}dx+ \frac{1}{2}\left(\frac{1}{2}\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}\right)(2e-g+i)+\frac{x^2(2e-g-i)+e-2g+i}{6\left(x^4+x^2+1\right)^2}\right)$$

$\Big\downarrow$ 2206

$$\frac{1}{12}\int \frac{-5(d-2f+h)x^2+11d-f+2h}{\left(x^4+x^2+1\right)^2}dx+ \frac{1}{2}\left(\frac{1}{2}\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}\right)(2e-g+i)+\frac{x^2(2e-g-i)+e-2g+i}{6\left(x^4+x^2+1\right)^2}\right)+ \frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}$$

$\Big\downarrow$ 1492

$$\frac{1}{12}\left(\frac{1}{6}\int \frac{3\left(5(4d-f+h)-(7d-7f+4h)x^2\right)}{x^4+x^2+1}dx+\frac{x\left(-\left(x^2(7d-7f+4h)\right)+2d+3f-h\right)}{2\left(x^4+x^2+1\right)}\right)+ \frac{1}{2}\left(\frac{1}{2}\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}\right)(2e-g+i)+\frac{x^2(2e-g-i)+e-2g+i}{6\left(x^4+x^2+1\right)^2}\right)+ \frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}$$

$\Big\downarrow$ 27

$$\frac{1}{12}\left(\frac{1}{2}\int \frac{5(4d-f+h)-(7d-7f+4h)x^2}{x^4+x^2+1}dx+\frac{x\left(-\left(x^2(7d-7f+4h)\right)+2d+3f-h\right)}{2\left(x^4+x^2+1\right)}\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}\right)(2e-g+i)+\frac{x^2(2e-g-i)+e-2g+i}{6\left(x^4+x^2+1\right)^2}\right)+$$
$$\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}$$

$\downarrow$ 1483

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{5(4d-f+h)-3(9d-4f+3h)x}{x^2-x+1}dx+\frac{1}{2}\int \frac{5(4d-f+h)+3(9d-4f+3h)x}{x^2+x+1}dx\right)+\frac{x\left(-\left(x^2(7d\right.\right.}{}\right.$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}\right)(2e-g+i)+\frac{x^2(2e-g-i)+e-2g+i}{6\left(x^4+x^2+1\right)^2}\right)+$$
$$\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}$$

$\downarrow$ 1142

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(13d+2f+h)\int \frac{1}{x^2-x+1}dx-\frac{3}{2}(9d-4f+3h)\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(13d+2f+h)\int\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}\right)(2e-g+i)+\frac{x^2(2e-g-i)+e-2g+i}{6\left(x^4+x^2+1\right)^2}\right)+$$
$$\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}$$

$\downarrow$ 25

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}(13d+2f+h)\int \frac{1}{x^2-x+1}dx+\frac{3}{2}(9d-4f+3h)\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}(13d+2f+h)\int \frac{}{x^2}\right.\right.\right.$$
$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3\left(x^4+x^2+1\right)}\right)(2e-g+i)+\frac{x^2(2e-g-i)+e-2g+i}{6\left(x^4+x^2+1\right)^2}\right)+$$
$$\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}$$

$\downarrow$ 1083

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{3}{2}(9d-4f+3h)\int\frac{1-2x}{x^2-x+1}dx-(13d+2f+h)\int\frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{3}{2}(9d-4f+\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3(x^4+x^2+1)}\right)(2e-g+i)+\frac{x^2(2e-g-i)+e-2g+i}{6(x^4+x^2+1)^2}\right)+$$

$$\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{12(x^4+x^2+1)^2}$$

$$\Big\downarrow \ 217$$

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{3}{2}(9d-4f+3h)\int\frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)(13d+2f+h)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{3}{2}(9d-4f+3h)\int\frac{2x}{x^2-}\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3(x^4+x^2+1)}\right)(2e-g+i)+\frac{x^2(2e-g-i)+e-2g+i}{6(x^4+x^2+1)^2}\right)+$$

$$\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{12(x^4+x^2+1)^2}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{12}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)(13d+2f+h)}{\sqrt{3}}-\frac{3}{2}\log\left(x^2-x+1\right)(9d-4f+3h)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)(13d+2}{\sqrt{3}}\right.\right.\right.$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{4\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2x^2+1}{3(x^4+x^2+1)}\right)(2e-g+i)+\frac{x^2(2e-g-i)+e-2g+i}{6(x^4+x^2+1)^2}\right)+$$

$$\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{12(x^4+x^2+1)^2}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^3,x]
```

output
```
(x*(d + f - 2*h - (d - 2*f + h)*x^2))/(12*(1 + x^2 + x^4)^2) + ((e - 2*g +
 i + (2*e - g - i)*x^2)/(6*(1 + x^2 + x^4)^2) + ((2*e - g + i)*((1 + 2*x^2
)/(3*(1 + x^2 + x^4)) + (4*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3])))/2)/2
 + ((x*(2*d + 3*f - h - (7*d - 7*f + 4*h)*x^2))/(2*(1 + x^2 + x^4)) + ((((
13*d + 2*f + h)*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - (3*(9*d - 4*f + 3*h)
*Log[1 - x + x^2])/2)/2 + (((13*d + 2*f + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/Sq
rt[3] + (3*(9*d - 4*f + 3*h)*Log[1 + x + x^2])/2)/2)/2)/12
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.17

method	result
default	$\frac{\left(-\frac{7d}{3}+\frac{7f}{3}-\frac{4h}{3}-\frac{4e}{3}-\frac{g}{3}+\frac{i}{3}\right)x^3+(-6d+4f-2h-2g+2i)x^2+\left(-\frac{20d}{3}+\frac{13f}{3}-\frac{5h}{3}+\frac{e}{3}-\frac{8g}{3}+\frac{7i}{3}\right)x-4d+\frac{4f}{3}+2e-2g+\frac{4i}{3}}{16(x^2+x+1)^2}+\frac{(27d-12f+$
risch	Expression too large to display

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*((-7/3*d+7/3*f-4/3*h-4/3*e-1/3*g+1/3*i)*x^3+(-6*d+4*f-2*h-2*g+2*i)*x^
2+(-20/3*d+13/3*f-5/3*h+1/3*e-8/3*g+7/3*i)*x-4*d+4/3*f+2*e-2*g+4/3*i)/(x^2
+x+1)^2+1/96*(27*d-12*f+9*h)*ln(x^2+x+1)+1/72*(13/2*d-16*e+f+8*g+1/2*h-8*i
)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/16*((7/3*d-7/3*f+4/3*h-4/3*e-1/3*g
+1/3*i)*x^3+(-6*d+4*f-2*h+2*g-2*i)*x^2+(20/3*d-13/3*f+5/3*h+1/3*e-8/3*g+7/
3*i)*x-4*d+4/3*f-2*e+2*g-4/3*i)/(x^2-x+1)^2-1/96*(27*d-12*f+9*h)*ln(x^2-x+
1)-1/72*(-13/2*d-16*e-f+8*g-1/2*h-8*i)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(224) = 448$.

Time = 4.78 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.09

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(1 + x^2 + x^4)^3} dx =$$
$$-\frac{12\,(7\,d - 7\,f + 4\,h)x^7 - 48\,(2\,e - g + i)x^6 + 60\,(d - 2\,f + h)x^5 - 72\,(2\,e - g + i)x^4 + 84\,(d - 2\,f + h}{}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="fric
as")
```

output
```
-1/288*(12*(7*d - 7*f + 4*h)*x^7 - 48*(2*e - g + i)*x^6 + 60*(d - 2*f + h)
*x^5 - 72*(2*e - g + i)*x^4 + 84*(d - 2*f + h)*x^3 - 48*(4*e - 2*g + i)*x^
2 - 2*sqrt(3)*((13*d - 32*e + 2*f + 16*g + h - 16*i)*x^8 + 2*(13*d - 32*e
+ 2*f + 16*g + h - 16*i)*x^6 + 3*(13*d - 32*e + 2*f + 16*g + h - 16*i)*x^4
 + 2*(13*d - 32*e + 2*f + 16*g + h - 16*i)*x^2 + 13*d - 32*e + 2*f + 16*g
+ h - 16*i)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((13*d + 32*e + 2*f
- 16*g + h + 16*i)*x^8 + 2*(13*d + 32*e + 2*f - 16*g + h + 16*i)*x^6 + 3*(
13*d + 32*e + 2*f - 16*g + h + 16*i)*x^4 + 2*(13*d + 32*e + 2*f - 16*g + h
 + 16*i)*x^2 + 13*d + 32*e + 2*f - 16*g + h + 16*i)*arctan(1/3*sqrt(3)*(2*
x - 1)) - 12*(4*d + 5*f - 5*h)*x - 9*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f
 + 3*h)*x^6 + 3*(9*d - 4*f + 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*d - 4*
f + 3*h)*log(x^2 + x + 1) + 9*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f + 3*h)
*x^6 + 3*(9*d - 4*f + 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*d - 4*f + 3*h
)*log(x^2 - x + 1) - 72*e + 72*g - 48*i)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(1 + x^2 + x^4)^3} dx = \text{Timed out}$$

input
```
integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.92

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^3}\,dx$$
$$= \frac{1}{144}\sqrt{3}(13\,d-32\,e+2\,f+16\,g+h-16\,i)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)$$
$$+\frac{1}{144}\sqrt{3}(13\,d+32\,e+2\,f-16\,g+h+16\,i)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)$$
$$+\frac{1}{32}\,(9\,d-4\,f+3\,h)\log\left(x^2+x+1\right)-\frac{1}{32}\,(9\,d-4\,f+3\,h)\log\left(x^2-x+1\right)$$
$$-\frac{(7\,d-7\,f+4\,h)x^7-4\,(2\,e-g+i)x^6+5\,(d-2\,f+h)x^5-6\,(2\,e-g+i)x^4+7\,(d-2\,f+h)x^3-4}{24\,(x^8+2\,x^6+3\,x^4+2\,x^2+1)}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxi
ma")
```

output
```
1/144*sqrt(3)*(13*d - 32*e + 2*f + 16*g + h - 16*i)*arctan(1/3*sqrt(3)*(2*
x + 1)) + 1/144*sqrt(3)*(13*d + 32*e + 2*f - 16*g + h + 16*i)*arctan(1/3*s
qrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f + 3*h)*log(x^2 + x + 1) - 1/32*(9*d -
4*f + 3*h)*log(x^2 - x + 1) - 1/24*((7*d - 7*f + 4*h)*x^7 - 4*(2*e - g + i
)*x^6 + 5*(d - 2*f + h)*x^5 - 6*(2*e - g + i)*x^4 + 7*(d - 2*f + h)*x^3 -
4*(4*e - 2*g + i)*x^2 - (4*d + 5*f - 5*h)*x - 6*e + 6*g - 4*i)/(x^8 + 2*x^
6 + 3*x^4 + 2*x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^3}\,dx$$
$$= \frac{1}{144}\sqrt{3}(13\,d-32\,e+2\,f+16\,g+h-16\,i)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)$$
$$+\frac{1}{144}\sqrt{3}(13\,d+32\,e+2\,f-16\,g+h+16\,i)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)$$
$$+\frac{1}{32}(9\,d-4\,f+3\,h)\log\left(x^2+x+1\right)-\frac{1}{32}(9\,d-4\,f+3\,h)\log\left(x^2-x+1\right)$$
$$-\frac{7\,dx^7-7\,fx^7+4\,hx^7-8\,ex^6+4\,gx^6-4\,ix^6+5\,dx^5-10\,fx^5+5\,hx^5-12\,ex^4+6\,gx^4-6\,ix^4+7\,d}{24\,(x^4+x^2+1)^2}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac
")
```

output
```
1/144*sqrt(3)*(13*d - 32*e + 2*f + 16*g + h - 16*i)*arctan(1/3*sqrt(3)*(2*
x + 1)) + 1/144*sqrt(3)*(13*d + 32*e + 2*f - 16*g + h + 16*i)*arctan(1/3*s
qrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f + 3*h)*log(x^2 + x + 1) - 1/32*(9*d -
4*f + 3*h)*log(x^2 - x + 1) - 1/24*(7*d*x^7 - 7*f*x^7 + 4*h*x^7 - 8*e*x^6
+ 4*g*x^6 - 4*i*x^6 + 5*d*x^5 - 10*f*x^5 + 5*h*x^5 - 12*e*x^4 + 6*g*x^4 -
6*i*x^4 + 7*d*x^3 - 14*f*x^3 + 7*h*x^3 - 16*e*x^2 + 8*g*x^2 - 4*i*x^2 - 4*
d*x - 5*f*x + 5*h*x - 6*e + 6*g - 4*i)/(x^4 + x^2 + 1)^2
```

Mupad [B] (verification not implemented)

Time = 27.39 (sec) , antiderivative size = 1963, normalized size of antiderivative = 7.88

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^2 + x^4 + 1)^3,x)
```

output
```
(e/4 - g/4 + i/6 + x*(d/6 + (5*f)/24 - (5*h)/24) - x^7*((7*d)/24 - (7*f)/2
4 + h/6) - x^5*((5*d)/24 - (5*f)/12 + (5*h)/24) - x^3*((7*d)/24 - (7*f)/12
 + (7*h)/24) + x^4*(e/2 - g/4 + i/4) + x^2*((2*e)/3 - g/3 + i/6) + x^6*(e/
3 - g/6 + i/6))/(2*x^2 + 3*x^4 + 2*x^6 + x^8 + 1) - log(960*d*g - 2763*d*f
 - 1920*d*e + 480*e*f + 1971*d*h - 960*d*i - 480*e*h - 240*f*g - 981*f*h +
 240*f*i + 240*g*h - 240*h*i + 3^(1/2)*d^2*1620i + 3^(1/2)*f^2*180i + 3^(1
/2)*h^2*135i - 3807*d^2*x - 612*f^2*x - 378*h^2*x + 2754*d^2 + 684*f^2 + 3
51*h^2 + 3^(1/2)*d*e*1088i - 3^(1/2)*d*f*1125i - 3^(1/2)*d*g*544i - 3^(1/2
)*e*f*608i + 3^(1/2)*d*h*945i + 3^(1/2)*d*i*544i + 3^(1/2)*e*h*416i + 3^(1
/2)*f*g*304i - 3^(1/2)*f*h*315i - 3^(1/2)*f*i*304i - 3^(1/2)*g*h*208i + 3^
(1/2)*h*i*208i - 672*d*e*x + 3069*d*f*x + 336*d*g*x + 672*e*f*x - 2403*d*h
*x - 336*d*i*x - 384*e*h*x - 336*f*g*x + 963*f*h*x + 336*f*i*x + 192*g*h*x
 - 192*h*i*x + 3^(1/2)*d^2*x*567i + 3^(1/2)*f^2*x*252i + 3^(1/2)*h^2*x*108
i - 3^(1/2)*d*f*x*819i + 3^(1/2)*d*g*x*752i + 3^(1/2)*e*f*x*544i + 3^(1/2)
*d*h*x*513i - 3^(1/2)*d*i*x*752i - 3^(1/2)*e*h*x*448i - 3^(1/2)*f*g*x*272i
 - 3^(1/2)*f*h*x*333i + 3^(1/2)*f*i*x*272i + 3^(1/2)*g*h*x*224i - 3^(1/2)*
h*i*x*224i - 3^(1/2)*d*e*x*1504i)*((9*d)/32 - f/8 + (3*h)/32 + (3^(1/2)*d*
13i)/288 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144 - (3^(1/2)*g*1i)/18 + (3^
(1/2)*h*1i)/288 + (3^(1/2)*i*1i)/18) - log(1920*d*e - 2763*d*f - 960*d*g -
 480*e*f + 1971*d*h + 960*d*i + 480*e*h + 240*f*g - 981*f*h - 240*f*i -...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1637, normalized size of antiderivative = 6.57

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x)
```

output

```
(26*sqrt(3)*atan((2*x - 1)/sqrt(3))*d*x**8 + 52*sqrt(3)*atan((2*x - 1)/sqr
t(3))*d*x**6 + 78*sqrt(3)*atan((2*x - 1)/sqrt(3))*d*x**4 + 52*sqrt(3)*atan
((2*x - 1)/sqrt(3))*d*x**2 + 26*sqrt(3)*atan((2*x - 1)/sqrt(3))*d + 64*sqr
t(3)*atan((2*x - 1)/sqrt(3))*e*x**8 + 128*sqrt(3)*atan((2*x - 1)/sqrt(3))*
e*x**6 + 192*sqrt(3)*atan((2*x - 1)/sqrt(3))*e*x**4 + 128*sqrt(3)*atan((2*
x - 1)/sqrt(3))*e*x**2 + 64*sqrt(3)*atan((2*x - 1)/sqrt(3))*e + 4*sqrt(3)*
atan((2*x - 1)/sqrt(3))*f*x**8 + 8*sqrt(3)*atan((2*x - 1)/sqrt(3))*f*x**6
+ 12*sqrt(3)*atan((2*x - 1)/sqrt(3))*f*x**4 + 8*sqrt(3)*atan((2*x - 1)/sqr
t(3))*f*x**2 + 4*sqrt(3)*atan((2*x - 1)/sqrt(3))*f - 32*sqrt(3)*atan((2*x
- 1)/sqrt(3))*g*x**8 - 64*sqrt(3)*atan((2*x - 1)/sqrt(3))*g*x**6 - 96*sqrt
(3)*atan((2*x - 1)/sqrt(3))*g*x**4 - 64*sqrt(3)*atan((2*x - 1)/sqrt(3))*g*
x**2 - 32*sqrt(3)*atan((2*x - 1)/sqrt(3))*g + 2*sqrt(3)*atan((2*x - 1)/sqr
t(3))*h*x**8 + 4*sqrt(3)*atan((2*x - 1)/sqrt(3))*h*x**6 + 6*sqrt(3)*atan((
2*x - 1)/sqrt(3))*h*x**4 + 4*sqrt(3)*atan((2*x - 1)/sqrt(3))*h*x**2 + 2*sq
rt(3)*atan((2*x - 1)/sqrt(3))*h + 32*sqrt(3)*atan((2*x - 1)/sqrt(3))*i*x**
8 + 64*sqrt(3)*atan((2*x - 1)/sqrt(3))*i*x**6 + 96*sqrt(3)*atan((2*x - 1)/
sqrt(3))*i*x**4 + 64*sqrt(3)*atan((2*x - 1)/sqrt(3))*i*x**2 + 32*sqrt(3)*a
tan((2*x - 1)/sqrt(3))*i + 26*sqrt(3)*atan((2*x + 1)/sqrt(3))*d*x**8 + 52*
sqrt(3)*atan((2*x + 1)/sqrt(3))*d*x**6 + 78*sqrt(3)*atan((2*x + 1)/sqrt(3)
)*d*x**4 + 52*sqrt(3)*atan((2*x + 1)/sqrt(3))*d*x**2 + 26*sqrt(3)*atan(...
```

3.47 $\int (a+bx^2+cx^4)^2 (d+ex+fx^2+gx^3+hx^4+ix^5)$

Optimal result

Integrand size = 40, antiderivative size = 272

$$\begin{aligned}\int (a+bx^2+cx^4)^2 (d+ex+fx^2+gx^3+hx^4+ix^5)\,dx &= a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{4}a(2be+ag)x^4\\ &\quad+\frac{1}{5}\left(b^2d+2abf+a(2cd+ah)\right)x^5+\frac{1}{6}\left(b^2e+2abg+a(2ce+ai)\right)x^6\\ &\quad+\frac{1}{7}\left(b^2f+2acf+2b(cd+ah)\right)x^7+\frac{1}{8}\left(b^2g+2acg+2b(ce+ai)\right)x^8\\ &\quad+\frac{1}{9}\left(c^2d+b^2h+2c(bf+ah)\right)x^9+\frac{1}{10}\left(c^2e+b^2i+2c(bg+ai)\right)x^{10}\\ &\quad+\frac{1}{11}c(cf+2bh)x^{11}+\frac{1}{12}c(cg+2bi)x^{12}+\frac{1}{13}c^2hx^{13}+\frac{1}{14}c^2ix^{14}\end{aligned}$$

output
```
a^2*d*x+1/2*a^2*e*x^2+1/3*a*(a*f+2*b*d)*x^3+1/4*a*(a*g+2*b*e)*x^4+1/5*(b^2
*d+2*a*b*f+a*(a*h+2*c*d))*x^5+1/6*(b^2*e+2*a*b*g+a*(a*i+2*c*e))*x^6+1/7*(b
^2*f+2*a*c*f+2*b*(a*h+c*d))*x^7+1/8*(b^2*g+2*a*c*g+2*b*(a*i+c*e))*x^8+1/9*
(c^2*d+b^2*h+2*c*(a*h+b*f))*x^9+1/10*(c^2*e+b^2*i+2*c*(a*i+b*g))*x^10+1/11
*c*(2*b*h+c*f)*x^11+1/12*c*(2*b*i+c*g)*x^12+1/13*c^2*h*x^13+1/14*c^2*i*x^1
4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int \left(a+bx^2+cx^4\right)^2 \left(d+ex+fx^2+gx^3+hx^4+ix^5\right)\,dx \\
&= a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{4}a(2be+ag)x^4 \\
&\quad+\frac{1}{5}\left(b^2d+2acd+2abf+a^2h\right)x^5+\frac{1}{6}\left(b^2e+2ace+2abg+a^2i\right)x^6 \\
&\quad+\frac{1}{7}\left(2bcd+b^2f+2acf+2abh\right)x^7+\frac{1}{8}\left(2bce+b^2g+2acg+2abi\right)x^8 \\
&\quad+\frac{1}{9}\left(c^2d+2bcf+b^2h+2ach\right)x^9+\frac{1}{10}\left(c^2e+2bcg+b^2i+2aci\right)x^{10} \\
&\quad+\frac{1}{11}c(cf+2bh)x^{11}+\frac{1}{12}c(cg+2bi)x^{12}+\frac{1}{13}c^2hx^{13}+\frac{1}{14}c^2ix^{14}
\end{aligned}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),
x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/
4 + ((b^2*d + 2*a*c*d + 2*a*b*f + a^2*h)*x^5)/5 + ((b^2*e + 2*a*c*e + 2*a*
b*g + a^2*i)*x^6)/6 + ((2*b*c*d + b^2*f + 2*a*c*f + 2*a*b*h)*x^7)/7 + ((2*
b*c*e + b^2*g + 2*a*c*g + 2*a*b*i)*x^8)/8 + ((c^2*d + 2*b*c*f + b^2*h + 2*
a*c*h)*x^9)/9 + ((c^2*e + 2*b*c*g + b^2*i + 2*a*c*i)*x^10)/10 + (c*(c*f +
2*b*h)*x^11)/11 + (c*(c*g + 2*b*i)*x^12)/12 + (c^2*h*x^13)/13 + (c^2*i*x^1
4)/14
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^2 \left(d+ex+fx^2+gx^3+hx^4+ix^5\right)\,dx$$

$$\Big\downarrow \text{2200}$$

$$\int \left(a^2d + a^2ex + x^8\left(2c(ah + bf) + b^2h + c^2d\right) + x^9\left(2c(ai + bg) + b^2i + c^2e\right) + x^6\left(2b(ah + cd) + 2acf + b^2f\right) +\right.$$

$$\Big\downarrow \text{2009}$$

$$a^2dx + \frac{1}{2}a^2ex^2 + \frac{1}{9}x^9\left(2c(ah + bf) + b^2h + c^2d\right) + \frac{1}{10}x^{10}\left(2c(ai + bg) + b^2i + c^2e\right) + \frac{1}{7}x^7\left(2b(ah + cd) + 2acf + b^2f\right) + \frac{1}{5}x^5\left(2abf + a(ah + 2cd) + b^2d\right) + \frac{1}{8}x^8\left(2b(ai + ce) + 2acg + b^2g\right) + \frac{1}{6}x^6\left(2abg + a(ai + 2ce) + b^2e\right) + \frac{1}{3}ax^3(af + 2bd) + \frac{1}{4}ax^4(ag + 2be) + \frac{1}{11}cx^{11}(2bh + cf) + \frac{1}{12}cx^{12}(2bi + cg) + \frac{1}{13}c^2hx^{13} + \frac{1}{14}c^2ix^{14}$$

input
```
Int[(a + b*x^2 + c*x^4)^2*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/
4 + ((b^2*d + 2*a*b*f + a*(2*c*d + a*h))*x^5)/5 + ((b^2*e + 2*a*b*g + a*(2
*c*e + a*i))*x^6)/6 + ((b^2*f + 2*a*c*f + 2*b*(c*d + a*h))*x^7)/7 + ((b^2*
g + 2*a*c*g + 2*b*(c*e + a*i))*x^8)/8 + ((c^2*d + b^2*h + 2*c*(b*f + a*h))
*x^9)/9 + ((c^2*e + b^2*i + 2*c*(b*g + a*i))*x^10)/10 + (c*(c*f + 2*b*h)*x
^11)/11 + (c*(c*g + 2*b*i)*x^12)/12 + (c^2*h*x^13)/13 + (c^2*i*x^14)/14
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^2 i\,x^{14}}{14}+\frac{c^2 h\,x^{13}}{13}+\frac{(2bci+c^2 g)x^{12}}{12}+\frac{(2bch+f\,c^2)x^{11}}{11}+\frac{((2ac+b^2)i+2bcg+e\,c^2)x^{10}}{10}+\frac{((2ac+b^2)h+2bcf+d\,c^2)x^9}{9}$
norman	$a^2dx+\frac{a^2e\,x^2}{2}+\left(\frac{1}{3}f\,a^2+\frac{2}{3}abd\right)x^3+\left(\frac{1}{4}a^2g+\frac{1}{2}abe\right)x^4+\left(\frac{1}{5}a^2h+\frac{2}{5}abf+\frac{2}{5}dac+\frac{1}{5}b^2d\right)x^5$
gosper	$a^2dx+\frac{1}{3}ace\,x^6+\frac{1}{2}a^2e\,x^2+\frac{2}{9}x^9ach+\frac{1}{5}x^{10}aci+\frac{1}{5}x^{10}bcg+\frac{2}{11}x^{11}bch+\frac{1}{6}x^{12}bci+\frac{2}{7}x^7abh$
risch	$a^2dx+\frac{1}{3}ace\,x^6+\frac{1}{2}a^2e\,x^2+\frac{2}{9}x^9ach+\frac{1}{5}x^{10}aci+\frac{1}{5}x^{10}bcg+\frac{2}{11}x^{11}bch+\frac{1}{6}x^{12}bci+\frac{2}{7}x^7abh$
parallelrisch	$a^2dx+\frac{1}{3}ace\,x^6+\frac{1}{2}a^2e\,x^2+\frac{2}{9}x^9ach+\frac{1}{5}x^{10}aci+\frac{1}{5}x^{10}bcg+\frac{2}{11}x^{11}bch+\frac{1}{6}x^{12}bci+\frac{2}{7}x^7abh$
orering	$x(25740c^2i\,x^{13}+27720c^2h\,x^{12}+60060bci\,x^{11}+30030c^2g\,x^{11}+65520bch\,x^{10}+32760f\,c^2x^{10}+72072aci\,x^9+36036b^2i\,x^9+7207$

input
```
int((c*x^4+b*x^2+a)^2*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x,method=_RETURNVERB
OSE)
```

output
```
1/14*c^2*i*x^14+1/13*c^2*h*x^13+1/12*(2*b*c*i+c^2*g)*x^12+1/11*(2*b*c*h+c^
2*f)*x^11+1/10*((2*a*c+b^2)*i+2*b*c*g+e*c^2)*x^10+1/9*((2*a*c+b^2)*h+2*b*c
*f+d*c^2)*x^9+1/8*(2*a*b*i+(2*a*c+b^2)*g+2*c*e*b)*x^8+1/7*(2*a*b*h+f*(2*a*
c+b^2)+2*b*c*d)*x^7+1/6*(a^2*i+2*a*b*g+(2*a*c+b^2)*e)*x^6+1/5*(a^2*h+2*a*b
*f+d*(2*a*c+b^2))*x^5+1/4*(a^2*g+2*a*b*e)*x^4+1/3*(a^2*f+2*a*b*d)*x^3+1/2*
a^2*e*x^2+a^2*d*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \left(a+bx^2+cx^4\right)^2\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)\,dx\\
&=\frac{1}{14}c^2ix^{14}+\frac{1}{13}c^2hx^{13}+\frac{1}{12}\left(c^2g+2\,bci\right)x^{12}+\frac{1}{11}\left(c^2f+2\,bch\right)x^{11}\\
&+\frac{1}{10}\left(c^2e+2\,bcg+\left(b^2+2\,ac\right)i\right)x^{10}+\frac{1}{9}\left(c^2d+2\,bcf+\left(b^2+2\,ac\right)h\right)x^9\\
&+\frac{1}{8}\left(2\,bce+2\,abi+\left(b^2+2\,ac\right)g\right)x^8+\frac{1}{7}\left(2\,bcd+2\,abh+\left(b^2+2\,ac\right)f\right)x^7\\
&+\frac{1}{6}\left(2\,abg+a^2i+\left(b^2+2\,ac\right)e\right)x^6+\frac{1}{5}\left(2\,abf+a^2h+\left(b^2+2\,ac\right)d\right)x^5\\
&+\frac{1}{2}a^2ex^2+\frac{1}{4}\left(2\,abe+a^2g\right)x^4+a^2dx+\frac{1}{3}\left(2\,abd+a^2f\right)x^3\end{aligned}$$

input
```
integrate((c*x^4+b*x^2+a)^2*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="
fricas")
```

output
```
1/14*c^2*i*x^14 + 1/13*c^2*h*x^13 + 1/12*(c^2*g + 2*b*c*i)*x^12 + 1/11*(c^
2*f + 2*b*c*h)*x^11 + 1/10*(c^2*e + 2*b*c*g + (b^2 + 2*a*c)*i)*x^10 + 1/9*
(c^2*d + 2*b*c*f + (b^2 + 2*a*c)*h)*x^9 + 1/8*(2*b*c*e + 2*a*b*i + (b^2 +
2*a*c)*g)*x^8 + 1/7*(2*b*c*d + 2*a*b*h + (b^2 + 2*a*c)*f)*x^7 + 1/6*(2*a*b
*g + a^2*i + (b^2 + 2*a*c)*e)*x^6 + 1/5*(2*a*b*f + a^2*h + (b^2 + 2*a*c)*d
)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*e + a^2*g)*x^4 + a^2*d*x + 1/3*(2*a*b*d
 + a^2*f)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.11

$$\int (a+bx^2+cx^4)^2 (d+ex+fx^2+gx^3+hx^4+ix^5)\,dx = a^2dx + \frac{a^2ex^2}{2} + \frac{c^2hx^{13}}{13} + \frac{c^2ix^{14}}{14} + x^{12}\left(\frac{bci}{6}+\frac{c^2g}{12}\right) + x^{11} \cdot \left(\frac{2bch}{11}+\frac{c^2f}{11}\right) + x^{10}\left(\frac{aci}{5}+\frac{b^2i}{10}+\frac{bcg}{5}+\frac{c^2e}{10}\right) + x^9 \cdot \left(\frac{2ach}{9}+\frac{b^2h}{9}+\frac{2bcf}{9}+\frac{c^2d}{9}\right) + x^8\left(\frac{abi}{4}+\frac{acg}{4}+\frac{b^2g}{8}+\frac{bce}{4}\right) + x^7 \cdot \left(\frac{2abh}{7}+\frac{2acf}{7}+\frac{b^2f}{7}+\frac{2bcd}{7}\right) + x^6\left(\frac{a^2i}{6}+\frac{abg}{3}+\frac{ace}{3}+\frac{b^2e}{6}\right) + x^5\left(\frac{a^2h}{5}+\frac{2abf}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right) + x^4\left(\frac{a^2g}{4}+\frac{abe}{2}\right) + x^3\left(\frac{a^2f}{3}+\frac{2abd}{3}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**2*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d),x)
```

output
```
a**2*d*x + a**2*e*x**2/2 + c**2*h*x**13/13 + c**2*i*x**14/14 + x**12*(b*c*
i/6 + c**2*g/12) + x**11*(2*b*c*h/11 + c**2*f/11) + x**10*(a*c*i/5 + b**2*
i/10 + b*c*g/5 + c**2*e/10) + x**9*(2*a*c*h/9 + b**2*h/9 + 2*b*c*f/9 + c**
2*d/9) + x**8*(a*b*i/4 + a*c*g/4 + b**2*g/8 + b*c*e/4) + x**7*(2*a*b*h/7 +
 2*a*c*f/7 + b**2*f/7 + 2*b*c*d/7) + x**6*(a**2*i/6 + a*b*g/3 + a*c*e/3 +
b**2*e/6) + x**5*(a**2*h/5 + 2*a*b*f/5 + 2*a*c*d/5 + b**2*d/5) + x**4*(a**
2*g/4 + a*b*e/2) + x**3*(a**2*f/3 + 2*a*b*d/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.93

$$\begin{aligned}
&\int \left(a+bx^2+cx^4\right)^2\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)\,dx \\
&= \frac{1}{14}\,c^2ix^{14}+\frac{1}{13}\,c^2hx^{13}+\frac{1}{12}\left(c^2g+2\,bci\right)x^{12}+\frac{1}{11}\left(c^2f+2\,bch\right)x^{11} \\
&\quad+\frac{1}{10}\left(c^2e+2\,bcg+\left(b^2+2\,ac\right)i\right)x^{10}+\frac{1}{9}\left(c^2d+2\,bcf+\left(b^2+2\,ac\right)h\right)x^9 \\
&\quad+\frac{1}{8}\left(2\,bce+2\,abi+\left(b^2+2\,ac\right)g\right)x^8+\frac{1}{7}\left(2\,bcd+2\,abh+\left(b^2+2\,ac\right)f\right)x^7 \\
&\quad+\frac{1}{6}\left(2\,abg+a^2i+\left(b^2+2\,ac\right)e\right)x^6+\frac{1}{5}\left(2\,abf+a^2h+\left(b^2+2\,ac\right)d\right)x^5 \\
&\quad+\frac{1}{2}\,a^2ex^2+\frac{1}{4}\left(2\,abe+a^2g\right)x^4+a^2dx+\frac{1}{3}\left(2\,abd+a^2f\right)x^3
\end{aligned}$$

input
```
integrate((c*x^4+b*x^2+a)^2*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="
maxima")
```

output
```
1/14*c^2*i*x^14 + 1/13*c^2*h*x^13 + 1/12*(c^2*g + 2*b*c*i)*x^12 + 1/11*(c^
2*f + 2*b*c*h)*x^11 + 1/10*(c^2*e + 2*b*c*g + (b^2 + 2*a*c)*i)*x^10 + 1/9*
(c^2*d + 2*b*c*f + (b^2 + 2*a*c)*h)*x^9 + 1/8*(2*b*c*e + 2*a*b*i + (b^2 +
2*a*c)*g)*x^8 + 1/7*(2*b*c*d + 2*a*b*h + (b^2 + 2*a*c)*f)*x^7 + 1/6*(2*a*b
*g + a^2*i + (b^2 + 2*a*c)*e)*x^6 + 1/5*(2*a*b*f + a^2*h + (b^2 + 2*a*c)*d
)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*e + a^2*g)*x^4 + a^2*d*x + 1/3*(2*a*b*d
 + a^2*f)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2+cx^4\right)^2 \left(d+ex+fx^2+gx^3+hx^4+ix^5\right)\,dx$$
$$=\frac{1}{14}c^2ix^{14}+\frac{1}{13}c^2hx^{13}+\frac{1}{12}c^2gx^{12}+\frac{1}{6}bcix^{12}+\frac{1}{11}c^2fx^{11}+\frac{2}{11}bchx^{11}+\frac{1}{10}c^2ex^{10}$$
$$+\frac{1}{5}bcgx^{10}+\frac{1}{10}b^2ix^{10}+\frac{1}{5}acix^{10}+\frac{1}{9}c^2dx^9+\frac{2}{9}bcfx^9+\frac{1}{9}b^2hx^9+\frac{2}{9}achx^9$$
$$+\frac{1}{4}bcex^8+\frac{1}{8}b^2gx^8+\frac{1}{4}acgx^8+\frac{1}{4}abix^8+\frac{2}{7}bcdx^7+\frac{1}{7}b^2fx^7+\frac{2}{7}acfx^7$$
$$+\frac{2}{7}abhx^7+\frac{1}{6}b^2ex^6+\frac{1}{3}acex^6+\frac{1}{3}abgx^6+\frac{1}{6}a^2ix^6+\frac{1}{5}b^2dx^5+\frac{2}{5}acdx^5$$
$$+\frac{2}{5}abfx^5+\frac{1}{5}a^2hx^5+\frac{1}{2}abex^4+\frac{1}{4}a^2gx^4+\frac{2}{3}abdx^3+\frac{1}{3}a^2fx^3+\frac{1}{2}a^2ex^2+a^2dx$$

input
```
integrate((c*x^4+b*x^2+a)^2*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="
giac")
```

output
```
1/14*c^2*i*x^14 + 1/13*c^2*h*x^13 + 1/12*c^2*g*x^12 + 1/6*b*c*i*x^12 + 1/1
1*c^2*f*x^11 + 2/11*b*c*h*x^11 + 1/10*c^2*e*x^10 + 1/5*b*c*g*x^10 + 1/10*b
^2*i*x^10 + 1/5*a*c*i*x^10 + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/9*b^2*h*x^9
 + 2/9*a*c*h*x^9 + 1/4*b*c*e*x^8 + 1/8*b^2*g*x^8 + 1/4*a*c*g*x^8 + 1/4*a*b
*i*x^8 + 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7 + 2/7*a*b*h*x^7 + 1
/6*b^2*e*x^6 + 1/3*a*c*e*x^6 + 1/3*a*b*g*x^6 + 1/6*a^2*i*x^6 + 1/5*b^2*d*x
^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^5 + 1/5*a^2*h*x^5 + 1/2*a*b*e*x^4 + 1/4*a
^2*g*x^4 + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*e*x^2 + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2+cx^4\right)^2\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)\,dx$$
$$= x^3\left(\frac{f\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)+x^4\left(\frac{g\,a^2}{4}+\frac{b\,e\,a}{2}\right)+x^{11}\left(\frac{f\,c^2}{11}+\frac{2\,b\,h\,c}{11}\right)$$
$$+x^{12}\left(\frac{g\,c^2}{12}+\frac{b\,i\,c}{6}\right)+x^5\left(\frac{h\,a^2}{5}+\frac{2\,f\,a\,b}{5}+\frac{2\,c\,d\,a}{5}+\frac{d\,b^2}{5}\right)$$
$$+x^6\left(\frac{i\,a^2}{6}+\frac{g\,a\,b}{3}+\frac{c\,e\,a}{3}+\frac{e\,b^2}{6}\right)+x^7\left(\frac{b^2\,f}{7}+\frac{2\,b\,c\,d}{7}+\frac{2\,a\,c\,f}{7}+\frac{2\,a\,b\,h}{7}\right)$$
$$+x^9\left(\frac{h\,b^2}{9}+\frac{2\,f\,b\,c}{9}+\frac{d\,c^2}{9}+\frac{2\,a\,h\,c}{9}\right)+x^8\left(\frac{b^2\,g}{8}+\frac{b\,c\,e}{4}+\frac{a\,c\,g}{4}+\frac{a\,b\,i}{4}\right)$$
$$+x^{10}\left(\frac{i\,b^2}{10}+\frac{g\,b\,c}{5}+\frac{e\,c^2}{10}+\frac{a\,i\,c}{5}\right)+\frac{a^2\,e\,x^2}{2}+\frac{c^2\,h\,x^{13}}{13}+\frac{c^2\,i\,x^{14}}{14}+a^2\,d\,x$$

input
```
int((a + b*x^2 + c*x^4)^2*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x)
```

output
```
x^3*((a^2*f)/3 + (2*a*b*d)/3) + x^4*((a^2*g)/4 + (a*b*e)/2) + x^11*((c^2*f
)/11 + (2*b*c*h)/11) + x^12*((c^2*g)/12 + (b*c*i)/6) + x^5*((b^2*d)/5 + (a
^2*h)/5 + (2*a*c*d)/5 + (2*a*b*f)/5) + x^6*((b^2*e)/6 + (a^2*i)/6 + (a*c*e
)/3 + (a*b*g)/3) + x^7*((b^2*f)/7 + (2*b*c*d)/7 + (2*a*c*f)/7 + (2*a*b*h)/
7) + x^9*((c^2*d)/9 + (b^2*h)/9 + (2*b*c*f)/9 + (2*a*c*h)/9) + x^8*((b^2*g
)/8 + (b*c*e)/4 + (a*c*g)/4 + (a*b*i)/4) + x^10*((c^2*e)/10 + (b^2*i)/10 +
 (b*c*g)/5 + (a*c*i)/5) + (a^2*e*x^2)/2 + (c^2*h*x^13)/13 + (c^2*i*x^14)/1
4 + a^2*d*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2+cx^4\right)^2\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)\,dx$$
$$= \frac{x(25740c^2i\,x^{13}+27720c^2h\,x^{12}+60060bci\,x^{11}+30030c^2g\,x^{11}+65520bch\,x^{10}+32760c^2f\,x^{10}+72072ac}{}$$

input
```
int((c*x^4+b*x^2+a)^2*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x)
```

output

```
(x*(360360*a**2*d + 180180*a**2*e*x + 120120*a**2*f*x**2 + 90090*a**2*g*x*
*3 + 72072*a**2*h*x**4 + 60060*a**2*i*x**5 + 240240*a*b*d*x**2 + 180180*a*
b*e*x**3 + 144144*a*b*f*x**4 + 120120*a*b*g*x**5 + 102960*a*b*h*x**6 + 900
90*a*b*i*x**7 + 144144*a*c*d*x**4 + 120120*a*c*e*x**5 + 102960*a*c*f*x**6
+ 90090*a*c*g*x**7 + 80080*a*c*h*x**8 + 72072*a*c*i*x**9 + 72072*b**2*d*x*
*4 + 60060*b**2*e*x**5 + 51480*b**2*f*x**6 + 45045*b**2*g*x**7 + 40040*b**
2*h*x**8 + 36036*b**2*i*x**9 + 102960*b*c*d*x**6 + 90090*b*c*e*x**7 + 8008
0*b*c*f*x**8 + 72072*b*c*g*x**9 + 65520*b*c*h*x**10 + 60060*b*c*i*x**11 +
40040*c**2*d*x**8 + 36036*c**2*e*x**9 + 32760*c**2*f*x**10 + 30030*c**2*g*
x**11 + 27720*c**2*h*x**12 + 25740*c**2*i*x**13))/360360
```

3.48 $\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4+ix^5)$

Optimal result

Integrand size = 38, antiderivative size = 122

$$\begin{aligned}\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4+ix^5)\,dx \\ = adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf+ah)x^5 \\ +\frac{1}{6}(ce+bg+ai)x^6+\frac{1}{7}(cf+bh)x^7+\frac{1}{8}(cg+bi)x^8+\frac{1}{9}chx^9+\frac{1}{10}cix^{10}\end{aligned}$$

output
```
a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(a*h+b*f+c*d)*x^
5+1/6*(a*i+b*g+c*e)*x^6+1/7*(b*h+c*f)*x^7+1/8*(b*i+c*g)*x^8+1/9*c*h*x^9+1/
10*c*i*x^10
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4+ix^5)\,dx \\ = adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf+ah)x^5 \\ +\frac{1}{6}(ce+bg+ai)x^6+\frac{1}{7}(cf+bh)x^7+\frac{1}{8}(cg+bi)x^8+\frac{1}{9}chx^9+\frac{1}{10}cix^{10}\end{aligned}$$

input
```
Integrate[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x]
```

output
```
a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d +
b*f + a*h)*x^5)/5 + ((c*e + b*g + a*i)*x^6)/6 + ((c*f + b*h)*x^7)/7 + ((c*
g + b*i)*x^8)/8 + (c*h*x^9)/9 + (c*i*x^10)/10
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)dx$$

$$\Big\downarrow \text{ 2200}$$

$$\int \left(x^4(ah+bf+cd)+x^5(ai+bg+ce)+x^2(af+bd)+x^3(ag+be)+ad+aex+x^6(bh+cf)+x^7(bi+cg)+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{6}x^6(ai+bg+ce)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}x^8(bi+cg)+\frac{1}{9}chx^9+\frac{1}{10}cix^{10}$$

input
```
Int[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x]
```

output
```
a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d +
b*f + a*h)*x^5)/5 + ((c*e + b*g + a*i)*x^6)/6 + ((c*f + b*h)*x^7)/7 + ((c*
g + b*i)*x^8)/8 + (c*h*x^9)/9 + (c*i*x^10)/10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.86

method	result
default	$adx+\frac{ae\,x^{2}}{2}+\frac{(af+bd)x^{3}}{3}+\frac{(ag+be)x^{4}}{4}+\frac{(ah+bf+cd)x^{5}}{5}+\frac{(ai+bg+ce)x^{6}}{6}+\frac{(bh+cf)x^{7}}{7}+\frac{(bi+cg)x^{8}}{8}+\frac{ch\,x}{9}$
norman	$\frac{ci\,x^{10}}{10}+\frac{ch\,x^{9}}{9}+\left(\frac{bi}{8}+\frac{cg}{8}\right)x^{8}+\left(\frac{bh}{7}+\frac{cf}{7}\right)x^{7}+\left(\frac{ai}{6}+\frac{bg}{6}+\frac{ce}{6}\right)x^{6}+\left(\frac{ah}{5}+\frac{bf}{5}+\frac{cd}{5}\right)x^{5}+\left(\frac{ag}{4}+\right.$
gosper	$\frac{1}{10}ci\,x^{10}+\frac{1}{9}ch\,x^{9}+\frac{1}{8}x^{8}bi+\frac{1}{8}x^{8}cg+\frac{1}{7}x^{7}bh+\frac{1}{7}cf\,x^{7}+\frac{1}{6}x^{6}ai+\frac{1}{6}x^{6}bg+\frac{1}{6}ce\,x^{6}+\frac{1}{5}x^{5}ah+$
risch	$\frac{1}{10}ci\,x^{10}+\frac{1}{9}ch\,x^{9}+\frac{1}{8}x^{8}bi+\frac{1}{8}x^{8}cg+\frac{1}{7}x^{7}bh+\frac{1}{7}cf\,x^{7}+\frac{1}{6}x^{6}ai+\frac{1}{6}x^{6}bg+\frac{1}{6}ce\,x^{6}+\frac{1}{5}x^{5}ah+$
parallelrisch	$\frac{1}{10}ci\,x^{10}+\frac{1}{9}ch\,x^{9}+\frac{1}{8}x^{8}bi+\frac{1}{8}x^{8}cg+\frac{1}{7}x^{7}bh+\frac{1}{7}cf\,x^{7}+\frac{1}{6}x^{6}ai+\frac{1}{6}x^{6}bg+\frac{1}{6}ce\,x^{6}+\frac{1}{5}x^{5}ah+$
orering	$\frac{x(252ci\,x^{9}+280ch\,x^{8}+315bi\,x^{7}+315cg\,x^{7}+360bh\,x^{6}+360cf\,x^{6}+420ai\,x^{5}+420bg\,x^{5}+420ce\,x^{5}+504ah\,x^{4}+504bf\,x^{4}+504cd}{2520}$

input
```
int((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(a*h+b*f+c*d)*x^
5+1/6*(a*i+b*g+c*e)*x^6+1/7*(b*h+c*f)*x^7+1/8*(b*i+c*g)*x^8+1/9*c*h*x^9+1/
10*c*i*x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4+ix^5)\,dx = \frac{1}{10}cix^{10}+\frac{1}{9}chx^9+\frac{1}{8}(cg+bi)x^8+\frac{1}{7}(cf+bh)x^7+\frac{1}{6}(ce+bg+ai)x^6 + \frac{1}{5}(cd+bf+ah)x^5+\frac{1}{4}(be+ag)x^4+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+adx$$

input
```
integrate((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="fr
icas")
```

output
```
1/10*c*i*x^10 + 1/9*c*h*x^9 + 1/8*(c*g + b*i)*x^8 + 1/7*(c*f + b*h)*x^7 +
1/6*(c*e + b*g + a*i)*x^6 + 1/5*(c*d + b*f + a*h)*x^5 + 1/4*(b*e + a*g)*x^
4 + 1/2*a*e*x^2 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4+ix^5)\,dx = adx+\frac{aex^2}{2}+\frac{chx^9}{9}+\frac{cix^{10}}{10}+x^8\left(\frac{bi}{8}+\frac{cg}{8}\right)+x^7\left(\frac{bh}{7}+\frac{cf}{7}\right) + x^6\left(\frac{ai}{6}+\frac{bg}{6}+\frac{ce}{6}\right)+x^5\left(\frac{ah}{5}+\frac{bf}{5}+\frac{cd}{5}\right)+x^4\left(\frac{ag}{4}+\frac{be}{4}\right)+x^3\left(\frac{af}{3}+\frac{bd}{3}\right)$$

input
```
integrate((c*x**4+b*x**2+a)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d),x)
```

output
```
a*d*x + a*e*x**2/2 + c*h*x**9/9 + c*i*x**10/10 + x**8*(b*i/8 + c*g/8) + x*
*7*(b*h/7 + c*f/7) + x**6*(a*i/6 + b*g/6 + c*e/6) + x**5*(a*h/5 + b*f/5 +
c*d/5) + x**4*(a*g/4 + b*e/4) + x**3*(a*f/3 + b*d/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4+ix^5)\,dx = \frac{1}{10}cix^{10}+\frac{1}{9}chx^9+\frac{1}{8}(cg+bi)x^8+\frac{1}{7}(cf+bh)x^7+\frac{1}{6}(ce+bg+ai)x^6 + \frac{1}{5}(cd+bf+ah)x^5+\frac{1}{4}(be+ag)x^4+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+adx$$

input
```
integrate((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="ma
xima")
```

output
```
1/10*c*i*x^10 + 1/9*c*h*x^9 + 1/8*(c*g + b*i)*x^8 + 1/7*(c*f + b*h)*x^7 +
1/6*(c*e + b*g + a*i)*x^6 + 1/5*(c*d + b*f + a*h)*x^5 + 1/4*(b*e + a*g)*x^
4 + 1/2*a*e*x^2 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4+ix^5)\,dx = \frac{1}{10}cix^{10}+\frac{1}{9}chx^9+\frac{1}{8}cgx^8+\frac{1}{8}bix^8+\frac{1}{7}cfx^7+\frac{1}{7}bhx^7+\frac{1}{6}cex^6+\frac{1}{6}bgx^6+\frac{1}{6}aix^6 + \frac{1}{5}cdx^5+\frac{1}{5}bfx^5+\frac{1}{5}ahx^5+\frac{1}{4}bex^4+\frac{1}{4}agx^4+\frac{1}{3}bdx^3+\frac{1}{3}afx^3+\frac{1}{2}aex^2+adx$$

input
```
integrate((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="gi
ac")
```

output
```
1/10*c*i*x^10 + 1/9*c*h*x^9 + 1/8*c*g*x^8 + 1/8*b*i*x^8 + 1/7*c*f*x^7 + 1/
7*b*h*x^7 + 1/6*c*e*x^6 + 1/6*b*g*x^6 + 1/6*a*i*x^6 + 1/5*c*d*x^5 + 1/5*b*
f*x^5 + 1/5*a*h*x^5 + 1/4*b*e*x^4 + 1/4*a*g*x^4 + 1/3*b*d*x^3 + 1/3*a*f*x^
3 + 1/2*a*e*x^2 + a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4+ix^5)\,dx$$
$$=\frac{c\,i\,x^{10}}{10}+\frac{c\,h\,x^9}{9}+\left(\frac{c\,g}{8}+\frac{b\,i}{8}\right)x^8+\left(\frac{c\,f}{7}+\frac{b\,h}{7}\right)x^7+\left(\frac{c\,e}{6}+\frac{b\,g}{6}+\frac{a\,i}{6}\right)x^6+\left(\frac{c\,d}{5}+\frac{b\,f}{5}+\frac{a\,h}{5}\right)x^5+\left(\frac{b\,e}{4}+\frac{a\,g}{4}\right)x^4+\left(\frac{b\,d}{3}+\frac{a\,f}{3}\right)x^3+\frac{a\,e\,x^2}{2}+a\,d\,x$$

input
```
int((a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x)
```

output
```
x^5*((c*d)/5 + (b*f)/5 + (a*h)/5) + x^6*((c*e)/6 + (b*g)/6 + (a*i)/6) + x^
3*((b*d)/3 + (a*f)/3) + x^4*((b*e)/4 + (a*g)/4) + x^7*((c*f)/7 + (b*h)/7)
+ x^8*((c*g)/8 + (b*i)/8) + (c*h*x^9)/9 + (c*i*x^10)/10 + a*d*x + (a*e*x^2
)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4+ix^5)\,dx$$
$$=\frac{x(252ci\,x^9+280ch\,x^8+315bi\,x^7+315cg\,x^7+360bh\,x^6+360cf\,x^6+420ai\,x^5+420bg\,x^5+420ce\,x^5+$$
$$2520$$

input
```
int((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x)
```

output
```
(x*(2520*a*d + 1260*a*e*x + 840*a*f*x**2 + 630*a*g*x**3 + 504*a*h*x**4 + 4
20*a*i*x**5 + 840*b*d*x**2 + 630*b*e*x**3 + 504*b*f*x**4 + 420*b*g*x**5 +
360*b*h*x**6 + 315*b*i*x**7 + 504*c*d*x**4 + 420*c*e*x**5 + 360*c*f*x**6 +
 315*c*g*x**7 + 280*c*h*x**8 + 252*c*i*x**9))/2520
```

3.49 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 321

$$
\begin{aligned}
&\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{a+bx^2+cx^4}\,dx \\
&= \frac{hx}{c}+\frac{ix^2}{2c}+\frac{\left(cf-bh+\frac{2c^2d+b^2h-c(bf+2ah)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad+\frac{\left(cf-bh-\frac{2c^2d-bcf+b^2h-2ach}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad-\frac{(2c^2e-bcg+b^2i-2aci)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(cg-bi)\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}
$$

output

```
h*x/c+1/2*i*x^2/c+1/2*(c*f-b*h+(2*c^2*d+b^2*h-c*(2*a*h+b*f))/(-4*a*c+b^2)^
(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*(c*f-b*h-(-2*a*c*h+b^2*h-b*c*f+2*c^2*
d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/
2))*2^(1/2)/c^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-1/2*(-2*a*c*i+b^2*i-b*c*g
+2*c^2*e)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)+1
/4*(-b*i+c*g)*ln(c*x^4+b*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.37

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{a+bx^2+cx^4}\,dx$$

$$= \frac{4chx+2cix^2+\frac{2\sqrt{2}\sqrt{c}\left(2c^2d+b\left(b-\sqrt{b^2-4ac}\right)h+c\left(-bf+\sqrt{b^2-4ac}f-2ah\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\left(2c^2d+b\left(b+\sqrt{b^2-4ac}\right.\right.}{}}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4),x]
```

output
```
(4*c*h*x + 2*c*i*x^2 + (2*Sqrt[2]*Sqrt[c]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a
*c])*h + c*(-(b*f) + Sqrt[b^2 - 4*a*c]*f - 2*a*h))*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) - (2*Sqrt[2]*Sqrt[c]*(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*h - c*(b
*f + Sqrt[b^2 - 4*a*c]*f + 2*a*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c^
2*e + b*(b - Sqrt[b^2 - 4*a*c])*i + c*(-(b*g) + Sqrt[b^2 - 4*a*c]*g - 2*a*
i))*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((2*c^2*e +
 b*(b + Sqrt[b^2 - 4*a*c])*i - c*(b*g + Sqrt[b^2 - 4*a*c]*g + 2*a*i))*Log[
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2202, 2194, 2188, 2009, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 2202$$

$$\int \frac{hx^4+fx^2+d}{cx^4+bx^2+a}dx+\int \frac{x\left(ix^4+gx^2+e\right)}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{2194}$$

$$\int \frac{hx^4+fx^2+d}{cx^4+bx^2+a}dx+\frac{1}{2}\int \frac{ix^4+gx^2+e}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{2188}$$

$$\int \frac{hx^4+fx^2+d}{cx^4+bx^2+a}dx+\frac{1}{2}\int \left(\frac{i}{c}+\frac{(cg-bi)x^2+ce-ai}{c\left(cx^4+bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{hx^4+fx^2+d}{cx^4+bx^2+a}dx+\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2aci+b^2i-bcg+2c^2e\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bi)\log\left(a+bx^2+cx^4\right)}{2c^2}+\frac{ix^2}{c}\right)$$

$$\Big\downarrow \text{2205}$$

$$\int \left(\frac{h}{c}+\frac{(cf-bh)x^2+cd-ah}{c\left(cx^4+bx^2+a\right)}\right)dx+\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2aci+b^2i-bcg+2c^2e\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bi)\log\left(a+bx^2+cx^4\right)}{2c^2}+\frac{ix^2}{c}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2ah+bf)+b^2h+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2ach+b^2h-bcf+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2aci+b^2i-bcg+2c^2e\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bi)\log\left(a+bx^2+cx^4\right)}{2c^2}+\frac{ix^2}{c}\right)+\frac{hx}{c}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4),x]
```

output
```
(h*x)/c + ((c*f - b*h + (2*c^2*d + b^2*h - c*(b*f + 2*a*h))/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(
3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*f - b*h - (2*c^2*d - b*c*f + b^2*h
 - 2*a*c*h)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((i*x^2)/c -
 ((2*c^2*e - b*c*g + b^2*i - 2*a*c*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a
*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*i)*Log[a + b*x^2 + c*x^4])/(2*c^
2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.31

method	result
risch	$\frac{hx}{c}+\frac{i\,x^{2}}{2c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(\left(-bi+cg\right)\_R^{3}+\left(-bh+cf\right)\_R^{2}+\left(-ai+ce\right)\_R-ah+cd\right)\ln\left(x-\_R\right)}{2\_R^{3}c+\_Rb}}{2c}$
default	$\frac{hx+\frac{1}{2}i\,x^{2}}{c}+\frac{\sqrt{-4ac+b^{2}}\left(\frac{\left(\sqrt{-4ac+b^{2}}\,bi-\sqrt{-4ac+b^{2}}\,gc-2aci+b^{2}i-bcg+2e\,c^{2}\right)\ln\left(2c\,x^{2}+\sqrt{-4ac+b^{2}}+b\right)}{4c}+\frac{\left(\sqrt{-4ac+b^{2}}\,bh-\sqrt{-4ac+b^{2}}\,f\right.}{}\right.}{c\left(4ac-b^{2}\right)}$

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOS
E)
```

output
```
h*x/c+1/2*i*x^2/c+1/2/c*sum(((-b*i+c*g)*_R^3+(-b*h+c*f)*_R^2+(-a*i+c*e)*_R
-a*h+c*d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{a+bx^2+cx^4}\,dx=\text{Timed out}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{a + bx^2 + cx^4} \, dx = \text{Timed out}$$

input
```
integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{a + bx^2 + cx^4} \, dx = \int \frac{ix^5 + hx^4 + gx^3 + fx^2 + ex + d}{cx^4 + bx^2 + a} \, dx$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="ma
xima")
```

output
```
1/2*(i*x^2 + 2*h*x)/c - integrate(-((c*g - b*i)*x^3 + (c*f - b*h)*x^2 + c*
d - a*h + (c*e - a*i)*x)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5944 vs. 2(277) = 554.

Time = 0.94 (sec) , antiderivative size = 5944, normalized size of antiderivative = 18.52

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="gi
ac")
```

output
```
1/4*(c*g - b*i)*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/2*(c*i*x^2 + 2*c*h*x)/
c^2 + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*f
- (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*h + 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
3*c^4 - 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 ...
```

Mupad [B] (verification not implemented)

Time = 19.79 (sec) , antiderivative size = 11383, normalized size of antiderivative = 35.46

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{a + bx^2 + cx^4} dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log((x*(c^4*e^3 - a^3*c*i^3 + c^4*d^2*g + b^4*e*i^2 + a^2*b^2*i^3 +
 b^2*c^2*e*g^2 + 3*a^2*c^2*e*i^2 + a^2*c^2*g*h^2 + 2*b^2*c^2*e^2*i - a^2*c
^2*g^2*i - 2*c^4*d*e*f - a*b*c^2*g^3 + a*c^3*e*g^2 + b*c^3*e*f^2 - a*c^3*f
^2*g - 2*b*c^3*e^2*g - 3*a*c^3*e^2*i - b*c^3*d^2*i + b^3*c*e*h^2 - a*b^3*g
*i^2 - 2*a*b*c^2*e*h^2 - 3*a*b^2*c*e*i^2 - a*b^2*c*g*h^2 + 2*a*b^2*c*g^2*i
 + a^2*b*c*h^2*i - 2*b^2*c^2*e*f*h - 2*a^2*c^2*f*h*i + 2*b*c^3*d*e*h + 2*a
*c^3*d*f*i - 2*a*c^3*d*g*h + 2*a*c^3*e*f*h - 2*b^3*c*e*g*i + 2*a*b*c^2*e*g
*i + 2*a*b*c^2*f*g*h))/c^2 - (a*c^3*f^3 - c^4*d*e^2 + c^4*d^2*f - b^4*d*i^
2 - b^2*c^2*d*g^2 - a^2*c^2*d*i^2 + a^2*c^2*f*h^2 - a^2*c^2*g^2*h - a^2*b^
2*h*i^2 - a^2*b*c*h^3 + a*c^3*d*g^2 - b*c^3*d*f^2 + a*c^3*e^2*h - b*c^3*d^
2*h - b^3*c*d*h^2 + a*b^3*f*i^2 + a^3*c*h*i^2 + 2*a*b*c^2*d*h^2 + a*b*c^2*
f*g^2 + 3*a*b^2*c*d*i^2 - 2*a*b*c^2*f^2*h + a*b^2*c*f*h^2 - 2*a^2*b*c*f*i^
2 - 2*b^2*c^2*d*e*i + 2*b^2*c^2*d*f*h - 2*a^2*c^2*e*h*i + 2*a^2*c^2*f*g*i
+ 2*b*c^3*d*e*g + 2*a*c^3*d*e*i - 2*a*c^3*d*f*h - 2*a*c^3*e*f*g + 2*b^3*c*
d*g*i - 4*a*b*c^2*d*g*i + 2*a*b*c^2*e*f*i - 2*a*b^2*c*f*g*i + 2*a^2*b*c*g*
h*i)/c^2 - root(128*a^2*b^2*c^5*z^4 - 16*a*b^4*c^4*z^4 - 256*a^3*c^6*z^4 +
 128*a^2*b^3*c^3*i*z^3 - 128*a^2*b^2*c^4*g*z^3 - 256*a^3*b*c^4*i*z^3 - 16*
a*b^5*c^2*i*z^3 + 16*a*b^4*c^3*g*z^3 + 256*a^3*c^5*g*z^3 + 160*a^3*b*c^3*g
*i*z^2 + 8*a*b^4*c^2*f*h*z^2 + 8*a*b^4*c^2*e*i*z^2 + 32*a^2*b*c^4*e*g*z^2
+ 32*a^2*b*c^4*d*h*z^2 - 8*a*b^3*c^3*e*g*z^2 - 8*a*b^3*c^3*d*h*z^2 + 16...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2036, normalized size of antiderivative = 6.34

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

output

```
(4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c*i -
2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*i + 2
*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*g - 4*s
qrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*e + 2*sq
rt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*h - 4*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a*c**2*f + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c**
2*d + 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c*h - 2*sqrt(c)*sqrt(
2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*a*b**2*h + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a*b*c*f - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*d + 4*...
```

3.50 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 468

$$\begin{aligned}
&\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&\quad+\frac{2acg-b(ce+ai)-(2c^2e-bcg+b^2i-2aci)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\
&\quad+\frac{\left(bcd-2acf+abh+\frac{4abcf+b^2(cd-ah)-4ac(3cd+ah)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad+\frac{\left(bcd-2acf+abh-\frac{4abcf+b^2(cd-ah)-4ac(3cd+ah)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad+\frac{(2ce-bg+2ai)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}$$

output
```
1/2*x*(b^2*d-a*b*f-2*a*(-a*h+c*d)+(a*b*h-2*a*c*f+b*c*d)*x^2)/a/(-4*a*c+b^2
)/(c*x^4+b*x^2+a)+1/2*(2*a*c*g-b*(a*i+c*e)-(-2*a*c*i+b^2*i-b*c*g+2*c^2*e)*
x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*(b*c*d-2*a*c*f+a*b*h+(4*a*b*c*f+b^
2*(-a*h+c*d)-4*a*c*(a*h+3*c*d))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)
*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)+1/4*(b*c*d-2*a*c*f+a*b*h-(4*a*b*c*f+b^2*(-a*h+c*d)-4*a*
c*(a*h+3*c*d))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2
)^(1/2))^(1/2))*2^(1/2)/a/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2
)+(2*a*i-b*g+2*c*e)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
3/2)
```

Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.12

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^2}\,dx$$

$$=\frac{1}{4}\left(\frac{2(-bcdx(b+cx^2)+a^2(bi-2c(g+x(h+ix)))+a(b^2ix^2+2c^2x(d+x(e+fx))+bc(e+x(f-x(g\right.}{ac\,(-b^2+4ac)\,(a+bx^2+cx^4)}$$

$$+\frac{\sqrt{2}\left(b^2(cd-ah)-2ac\left(6cd+\sqrt{b^2-4ac}f+2ah\right)+b\left(c\sqrt{b^2-4ac}d+4acf+a\sqrt{b^2-4ac}h\right)\right)\arctan\Big(}{a\sqrt{c}\,(b^2-4ac)^{3/2}\,\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\sqrt{2}\left(b^2(-cd+ah)+2ac\left(6cd-\sqrt{b^2-4ac}f+2ah\right)+b\left(c\sqrt{b^2-4ac}d-4acf+a\sqrt{b^2-4ac}h\right)\right)\text{arctan}}{a\sqrt{c}\,(b^2-4ac)^{3/2}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

$$+\frac{2(-2ce+bg-2ai)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}$$

$$\left.+\frac{2(2ce-bg+2ai)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^2,
x]
```

output
```
((2*(-(b*c*d*x*(b + c*x^2)) + a^2*(b*i - 2*c*(g + x*(h + i*x))) + a*(b^2*i
*x^2 + 2*c^2*x*(d + x*(e + f*x)) + b*c*(e + x*(f - x*(g + h*x))))))/(a*c*(
-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d - a*h) - 2*a*c*(6*
c*d + Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*f +
a*Sqrt[b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(a*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt
[2]*(b^2*(-(c*d) + a*h) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b*
(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*f + a*Sqrt[b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*S
qrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(-2*c*e + b*g - 2*a*i)*Log[-b + Sqrt[b^2
- 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (2*(2*c*e - b*g + 2*a*i)*Log[b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2202, 2194, 2191, 27, 1083, 219, 2206, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{hx^4+fx^2+d}{(cx^4+bx^2+a)^2}dx+\int \frac{x(ix^4+gx^2+e)}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\int \frac{hx^4+fx^2+d}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}\int \frac{ix^4+gx^2+e}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{c\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{\int \frac{2ce-bg+2ai}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)+\int \frac{hx^4+fx^2+d}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(2ai-bg+2ce)\int\frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)+\int\frac{hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{2(2ai-bg+2ce)\int\frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}+\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\int\frac{hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{219}$$

$$\int\frac{hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2ai-bg+2ce)}{\left(b^2-4ac\right)^{3/2}}+\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{2206}$$

$$-\frac{\int-\frac{db^2+afb+(bcd-2acf+abh)x^2-2a(3cd+ah)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2ai-bg+2ce)}{\left(b^2-4ac\right)^{3/2}}+\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int\frac{db^2+afb+(bcd-2acf+abh)x^2-2a(3cd+ah)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2ai-bg+2ce)}{\left(b^2-4ac\right)^{3/2}}+\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{1}{2}\left(\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(-\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+ab\right.}{2a\left(b^2-4ac\right)}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2ai-bg+2ce)}{\left(b^2-4ac\right)^{3/2}}+\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+$$
$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2ai-bg+2ce)}{\left(b^2-4ac\right)^{3/2}}+\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+$$
$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(2*a
*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (((b*c*d - 2*a*c*f + a*b*h + (4*a*b*
c*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*f + a*b*h - (4*a*b*c*f + b^2*(c*d - a
*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
)/(2*a*(b^2 - 4*a*c)) + ((c*(2*a*g - b*(e + (a*i)/c)) - (2*c^2*e - b*c*g +
 b^2*i - 2*a*c*i)*x^2)/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(2*c*e -
 b*g + 2*a*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2
))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.65

method	result
risch	$\frac{-\frac{(abh-2acf+bcd)x^3}{2a(4ac-b^2)}-\frac{(2aci-b^2i+bcg-2e\,c^2)x^2}{2c(4ac-b^2)}-\frac{(2a^2h-abf-2dac+b^2d)x}{2a(4ac-b^2)}+\frac{abi-2acg+ceb}{2c(4ac-b^2)}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{(ab}{}\right.}{}\right.}{}$
default	$\frac{-\frac{(abh-2acf+bcd)x^3}{2a(4ac-b^2)}-\frac{(2aci-b^2i+bcg-2e\,c^2)x^2}{2c(4ac-b^2)}-\frac{(2a^2h-abf-2dac+b^2d)x}{2a(4ac-b^2)}+\frac{abi-2acg+ceb}{2c(4ac-b^2)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(8\sqrt{-4ac+b^2}\,a^2ci-4\sqrt{-4ac+b^2}\,abcg\right.}{}\right.}{}$

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERB
OSE)
```

output
```
(-1/2/a*(a*b*h-2*a*c*f+b*c*d)/(4*a*c-b^2)*x^3-1/2*(2*a*c*i-b^2*i+b*c*g-2*c
^2*e)/c/(4*a*c-b^2)*x^2-1/2*(2*a^2*h-a*b*f-2*a*c*d+b^2*d)/a/(4*a*c-b^2)*x+
1/2/c*(a*b*i-2*a*c*g+b*c*e)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum((-1/a*(a*
b*h-2*a*c*f+b*c*d)/(4*a*c-b^2)*_R^2+2*(2*a*i-b*g+2*c*e)/(4*a*c-b^2)*_R+(2*
a^2*h-a*b*f+6*a*c*d-b^2*d)/a/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=Root
Of(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx^2 + cx^4)^2} \, dx = \int \frac{ix^5 + hx^4 + gx^3 + fx^2 + ex + d}{(cx^4 + bx^2 + a)^2} \, dx$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="
maxima")
```

output
```
-1/2*(a*b*c*e - 2*a^2*c*g + a^2*b*i - (b*c^2*d - 2*a*c^2*f + a*b*c*h)*x^3
+ (2*a*c^2*e - a*b*c*g + (a*b^2 - 2*a^2*c)*i)*x^2 + (a*b*c*f - 2*a^2*c*h -
 (b^2*c - 2*a*c^2)*d)*x)/(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*
x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2) + 1/2*integrate((a*b*f - 2*a^2*h + (b*c
*d - 2*a*c*f + a*b*h)*x^2 + (b^2 - 6*a*c)*d - 2*(2*a*c*e - a*b*g + 2*a^2*i
)*x)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7965 vs. $2(420) = 840$.

Time = 1.34 (sec) , antiderivative size = 7965, normalized size of antiderivative = 17.02

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="
giac")
```

output
```
1/2*(b*c^2*d*x^3 - 2*a*c^2*f*x^3 + a*b*c*h*x^3 - 2*a*c^2*e*x^2 + a*b*c*g*x
^2 - a*b^2*i*x^2 + 2*a^2*c*i*x^2 + b^2*c*d*x - 2*a*c^2*d*x - a*b*c*f*x + 2
*a^2*c*h*x - a*b*c*e + 2*a^2*c*g - a^2*b*i)/((c*x^4 + b*x^2 + a)*(a*b^2*c
- 4*a^2*c^2)) + 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d - 2*(2
*a*b^2*c^3 - 8*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3
 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4*a^2*c)^2*f + (2*a*b^3*c^2 - 8*a^2*b*c
^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a
*b*c^2)*(a*b^2 - 4*a^2*c)^2*h + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^6*c - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 2*a*b^6*c^2 + 64*sqrt(2...
```

Mupad [B] (verification not implemented)

Time = 20.88 (sec) , antiderivative size = 18449, normalized size of antiderivative = 39.42

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx^2 + cx^4)^2}\, dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((b*c*e - 2*a*c*g + a*b*i)/(2*c*(4*a*c - b^2)) - (x*(b^2*d + 2*a^2*h - 2*a
*c*d - a*b*f))/(2*a*(4*a*c - b^2)) + (x^2*(2*c^2*e + b^2*i - b*c*g - 2*a*c
*i))/(2*c*(4*a*c - b^2)) - (x^3*(b*c*d - 2*a*c*f + a*b*h))/(2*a*(4*a*c - b
^2)))/(a + b*x^2 + c*x^4) + symsum(log((5*b^3*c^4*d^3 + 8*a^3*c^4*f^3 - 96
*a^2*c^5*d*e^2 + 72*a^2*c^5*d^2*f - 3*a^3*b^3*c*h^3 - 4*a^4*b*c^2*h^3 - 3*
b^4*c^3*d^2*f - 32*a^3*c^4*e^2*h - 96*a^4*c^3*d*i^2 + b^5*c^2*d^2*h + 8*a^
4*c^3*f*h^2 - 32*a^5*c^2*h*i^2 + 6*a^2*b^2*c^3*f^3 - 36*a*b*c^5*d^3 + a*b^
5*c*d*h^2 - 192*a^3*c^4*d*e*i + 48*a^3*c^4*d*f*h - 64*a^4*c^3*e*h*i + 16*a
*b^2*c^4*d*e^2 + 18*a*b^2*c^4*d^2*f + 3*a*b^3*c^3*d*f^2 - 60*a^2*b*c^4*d*f
^2 + 4*a*b^4*c^2*d*g^2 + 16*a^2*b*c^4*e^2*f - a*b^3*c^3*d^2*h - 60*a^2*b*c
^4*d^2*h - 28*a^3*b*c^3*d*h^2 + a^2*b^4*c*f*h^2 - 28*a^3*b*c^3*f^2*h + 16*
a^4*b*c^2*f*i^2 - 24*a^2*b^2*c^3*d*g^2 - 9*a^2*b^3*c^2*d*h^2 + 4*a^2*b^3*c
^2*f*g^2 + 16*a^3*b^2*c^2*d*i^2 - 5*a^2*b^3*c^2*f^2*h + 18*a^3*b^2*c^2*f*h
^2 - 8*a^3*b^2*c^2*g^2*h - 16*a*b^3*c^3*d*e*g + 96*a^2*b*c^4*d*e*g - 4*a*b
^4*c^2*d*f*h + 96*a^3*b*c^3*d*g*i + 32*a^3*b*c^3*e*f*i + 32*a^3*b*c^3*e*g*
h + 32*a^4*b*c^2*g*h*i + 32*a^2*b^2*c^3*d*e*i + 52*a^2*b^2*c^3*d*f*h - 16*
a^2*b^2*c^3*e*f*g - 16*a^2*b^3*c^2*d*g*i - 16*a^3*b^2*c^2*f*g*i)/(8*(a^2*b
^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - root(1572864*a^8*b^2*c
^6*z^4 - 983040*a^7*b^4*c^5*z^4 + 327680*a^6*b^6*c^4*z^4 - 61440*a^5*b^8*c
^3*z^4 + 6144*a^4*b^10*c^2*z^4 - 256*a^3*b^12*c*z^4 - 1048576*a^9*c^7*z...
```

Reduce [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 7607, normalized size of antiderivative = 16.25

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

output

```
( - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*
c*i + 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b
**2*c*g - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a
**3*b**2*c*i*x**2 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
- b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a**3*b*c**2*e - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqr
t(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a**3*b*c**2*i*x**4 + 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sq
rt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2
*sqrt(c)*sqrt(a) + b))*a**2*b**3*c*g*x**2 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)
*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**2*e*x**2 + 8*sqrt(2*sqrt(c)*
sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**2*g*x**4 - 16*sq
rt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**3*e*x*
*4 + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)...
```

3.51 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 732

$$
\begin{aligned}
&\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^3}\,dx \\
&= \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} \\
&\quad+\frac{2acg-b(ce+ai)-\left(2c^2e-bcg+b^2i-2aci\right)x^2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} \\
&\quad+\frac{\left(6c^2e-3bcg+b^2i+2aci\right)\left(b+2cx^2\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} \\
&\quad+\frac{x(3b^4d+ab^3f+8a^2bcf+4a^2c(7cd+ah)-ab^2(25cd+7ah)+c(3b^3d+ab^2f+20a^2cf-12ab(2cd+}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} \\
&\quad+\frac{\sqrt{c}\left(3b^3d+ab^2f+20a^2cf-12ab(2cd+ah)+\frac{3b^4d+ab^3f-52a^2bcf-6ab^2(5cd-3ah)+24a^2c(7cd+ah)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{}{\sqrt{b}}\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad+\frac{\sqrt{c}\left(3b^3d+ab^2f+20a^2cf-12ab(2cd+ah)-\frac{3b^4d+ab^3f-52a^2bcf-6ab^2(5cd-3ah)+24a^2c(7cd+ah)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{}{\sqrt{b}}\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad-\frac{\left(6c^2e-3bcg+b^2i+2aci\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}
\end{aligned}
$$

output
```
1/4*x*(b^2*d-a*b*f-2*a*(-a*h+c*d)+(a*b*h-2*a*c*f+b*c*d)*x^2)/a/(-4*a*c+b^2
)/(c*x^4+b*x^2+a)^2+1/4*(2*a*c*g-b*(a*i+c*e)-(-2*a*c*i+b^2*i-b*c*g+2*c^2*e
)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*(2*a*c*i+b^2*i-3*b*c*g+6*c^2*e
)*(2*c*x^2+b)/c/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/8*x*(3*b^4*d+a*b^3*f+8*a^
2*b*c*f+4*a^2*c*(a*h+7*c*d)-a*b^2*(7*a*h+25*c*d)+c*(3*b^3*d+a*b^2*f+20*a^2
*c*f-12*a*b*(a*h+2*c*d))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*c^(1
/2)*(3*b^3*d+a*b^2*f+20*a^2*c*f-12*a*b*(a*h+2*c*d)+(3*b^4*d+a*b^3*f-52*a^2
*b*c*f-6*a*b^2*(-3*a*h+5*c*d)+24*a^2*c*(a*h+7*c*d))/(-4*a*c+b^2)^(1/2))*ar
ctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b
^2)^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*c^(1/2)*(3*b^3*d+a*b^2*f+20*a^2*c*
f-12*a*b*(a*h+2*c*d)-(3*b^4*d+a*b^3*f-52*a^2*b*c*f-6*a*b^2*(-3*a*h+5*c*d)+
24*a^2*c*(a*h+7*c*d))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*
a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^2/(b+(-4*a*c+b^2)^(1/2))^(
1/2)-(2*a*c*i+b^2*i-3*b*c*g+6*c^2*e)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 6.66 (sec) , antiderivative size = 980, normalized size of antiderivative = 1.34

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{abce-2a^2cg+a^2bi-b^2cdx+2ac^2dx+abcfx-2a^2chx+2ac^2ex^2-abcgx^2+ab^2ix^2-2a^2cix^2-bc^2\cdots}{4ac\left(-b^2+4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$+\frac{12a^2bc^2e-6a^2b^2cg+2a^2b^3i+4a^3bci+3b^4cdx-25ab^2c^2dx+28a^2c^3dx+ab^3cfx+8a^2bc^2fx-7a^2b\cdots}{8a\cdots}$$

$$+\frac{\sqrt{c}\left(3b^4d-30ab^2cd+168a^2c^2d+3b^3\sqrt{b^2-4ac}d-24abc\sqrt{b^2-4ac}d+ab^3f-52a^2bcf+ab^2\sqrt{b^2-4\cdots}\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\cdots}}$$

$$+\frac{\sqrt{c}\left(-3b^4d+30ab^2cd-168a^2c^2d+3b^3\sqrt{b^2-4ac}d-24abc\sqrt{b^2-4ac}d-ab^3f+52a^2bcf+ab^2\sqrt{b^2-\cdots}\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b+\cdots}}$$

$$+\frac{\left(6c^2e-3bcg+b^2i+2aci\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{2\left(b^2-4ac\right)^{5/2}}$$

$$+\frac{\left(-6c^2e+3bcg-b^2i-2aci\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{2\left(b^2-4ac\right)^{5/2}}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^3,
x]
```

output
```
(a*b*c*e - 2*a^2*c*g + a^2*b*i - b^2*c*d*x + 2*a*c^2*d*x + a*b*c*f*x - 2*a
^2*c*h*x + 2*a*c^2*e*x^2 - a*b*c*g*x^2 + a*b^2*i*x^2 - 2*a^2*c*i*x^2 - b*c
^2*d*x^3 + 2*a*c^2*f*x^3 - a*b*c*h*x^3)/(4*a*c*(-b^2 + 4*a*c)*(a + b*x^2 +
 c*x^4)^2) + (12*a^2*b*c^2*e - 6*a^2*b^2*c*g + 2*a^2*b^3*i + 4*a^3*b*c*i +
 3*b^4*c*d*x - 25*a*b^2*c^2*d*x + 28*a^2*c^3*d*x + a*b^3*c*f*x + 8*a^2*b*c
^2*f*x - 7*a^2*b^2*c*h*x + 4*a^3*c^2*h*x + 24*a^2*c^3*e*x^2 - 12*a^2*b*c^2
*g*x^2 + 4*a^2*b^2*c*i*x^2 + 8*a^3*c^2*i*x^2 + 3*b^3*c^2*d*x^3 - 24*a*b*c^
3*d*x^3 + a*b^2*c^2*f*x^3 + 20*a^2*c^3*f*x^3 - 12*a^2*b*c^2*h*x^3)/(8*a^2*
c*(-b^2 + 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d - 30*a*b^2*c*d
 + 168*a^2*c^2*d + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]*
d + a*b^3*f - 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c*Sqrt[b^2
 - 4*a*c]*f + 18*a^2*b^2*h + 24*a^3*c*h - 12*a^2*b*Sqrt[b^2 - 4*a*c]*h)*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2
 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(-3*b^4*d + 30*a*b
^2*c*d - 168*a^2*c^2*d + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4
*a*c]*d - a*b^3*f + 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c*Sq
rt[b^2 - 4*a*c]*f - 18*a^2*b^2*h - 24*a^3*c*h - 12*a^2*b*Sqrt[b^2 - 4*a*c]
*h)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^
2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((6*c^2*e - 3*b*c*g +
 b^2*i + 2*a*c*i)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(2*(b^2 - 4*a*...
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2202, 2194, 2191, 27, 1086, 1083, 219, 2206, 25, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^3}dx+\int \frac{x\left(ix^4+gx^2+e\right)}{\left(cx^4+bx^2+a\right)^3}dx$$

$$\downarrow \text{2194}$$

$$\int \frac{hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^3}dx+\frac{1}{2}\int \frac{ix^4+gx^2+e}{\left(cx^4+bx^2+a\right)^3}dx^2$$

$$\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\int \frac{\frac{ib^2}{c}-3gb+6ce+2ai}{\left(cx^4+bx^2+a\right)^2}dx^2}{2\left(b^2-4ac\right)}\right)+\int \frac{hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^3}dx$$

$$\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\left(2ai+\frac{b^2i}{c}-3bg+6ce\right)\int \frac{1}{\left(cx^4+bx^2+a\right)^2}dx^2}{2\left(b^2-4ac\right)}\right)+\int \frac{hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^3}dx$$

$$\downarrow \text{1086}$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\left(2ai+\frac{b^2i}{c}-3bg+6ce\right)\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{\cdots}{(b^2-4ac\cdots}\right.}{2\left(b^2-4ac\right)}\right.$$
$$\int \frac{hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^3}dx$$

$$\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\left(2ai+\frac{b^2i}{c}-3bg+6ce\right)\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}-\frac{\cdots}{(b\cdots}\right.}{2\left(b^2-4ac\right)}\right.$$
$$\int \frac{hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^3}dx$$

$$\downarrow \text{219}$$

$$\int \frac{hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx+$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)\left(2ai+\right.}{2\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \; 2206$$

$$-\frac{\int -\frac{3db^2+afb+5(bcd-2acf+abh)x^2-2a(7cd+ah)}{(cx^4+bx^2+a)^2}dx}{4a\left(b^2-4ac\right)}+$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)\left(2ai+\right.}{2\left(b^2-4ac\right)}\right.$$

$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3db^2+afb+5(bcd-2acf+abh)x^2-2a(7cd+ah)}{(cx^4+bx^2+a)^2}dx}{4a\left(b^2-4ac\right)}+$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)\left(2ai+\right.}{2\left(b^2-4ac\right)}\right.$$

$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \; 1492$$

$$\frac{\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+8a^2bcf+4a^2c(ah+7cd)+ab^3f-ab^2(7ah+25cd)+3b^4d\right)}{2a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{3db^4+afb^3-3a(9cd-ah)b^2-16a^2cfb+\cdots}{\cdots}}{\cdots}}{4a\left(b^2-4ac\right)}$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\left(\frac{4c\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)\left(2ai+\cdots\right.}{2\left(b^2-4ac\right)}\cdots\right.$$

$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3db^4+afb^3-3a(9cd-ah)b^2-16a^2cfb+c\left(3db^3+afb^2-12a(2cd+ah)b+20a^2cf\right)x^2+12a^2c(7cd+ah)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+\cdots\right.}{2a(b^2-4\cdots}}{4a\left(b^2-4ac\right)}$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\left(\frac{4c\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)\left(2ai+\cdots\right.}{2\left(b^2-4ac\right)}\cdots\right.$$

$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{\frac{1}{2}c\left(\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{-52a^2bcf+24a\cdots}{\cdots}\right.}{2a(b^2-4ac)}}{\cdots}$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\left(\frac{4c\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)\left(2ai+\cdots\right.}{2\left(b^2-4ac\right)}\cdots\right.$$

$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\right.}{}}{2a(b^2-4ac)}$$

$$\frac{1}{2}\left(\frac{c\left(2ag-b\left(\frac{ai}{c}+e\right)\right)-x^2\left(-2aci+b^2i-bcg+2c^2e\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\left(\frac{4c\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)\left(2ai+\right.}{2\left(b^2-4ac\right)}\right.$$

$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(4*a
*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((x*(3*b^4*d + a*b^3*f + 8*a^2*b*c
*f + 4*a^2*c*(7*c*d + a*h) - a*b^2*(25*c*d + 7*a*h) + c*(3*b^3*d + a*b^2*f
 + 20*a^2*c*f - 12*a*b*(2*c*d + a*h))*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2
+ c*x^4)) + ((Sqrt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*
h) + (3*b^4*d + a*b^3*f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*
c*(7*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3
*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) - (3*b^4*d + a*b^3*f - 52
*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c)))/(4*a*(b^2 - 4*a*c)) + (
(c*(2*a*g - b*(e + (a*i)/c)) - (2*c^2*e - b*c*g + b^2*i - 2*a*c*i)*x^2)/(2
*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - ((6*c*e - 3*b*g + 2*a*i + (b^2*i
)/c)*(-((b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (4*c*ArcTanh[
(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c)))
/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.09

method	result
risch	$\frac{-\frac{c^{2}\left(12a^{2}bh-20a^{2}cf-a\,b^{2}f+24abcd-3b^{3}d\right)x^{7}}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c\left(2aci+b^{2}i-3bcg+6e\,c^{2}\right)x^{6}}{32a^{2}c^{2}-16a\,b^{2}c+2b^{4}}+\frac{c\left(4a^{3}ch-19a^{2}b^{2}h+28a^{2}bcf+28a^{2}c^{2}d+2a\,b^{3}f-49a\,b^{2}cd+6b^{4}d\right.}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{}$
default	Expression too large to display

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERB
OSE)
```

output
```
(-1/8*c^2*(12*a^2*b*h-20*a^2*c*f-a*b^2*f+24*a*b*c*d-3*b^3*d)/a^2/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^7+1/2*c*(2*a*c*i+b^2*i-3*b*c*g+6*c^2*e)/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^6+1/8/a^2*c*(4*a^3*c*h-19*a^2*b^2*h+28*a^2*b*c*f+28*a^2*c^2
*d+2*a*b^3*f-49*a*b^2*c*d+6*b^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+3/4*b*(2
*a*c*i+b^2*i-3*b*c*g+6*c^2*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-1/8*(16*a^3*b
*c*h-36*a^3*c^2*f+5*a^2*b^3*h-5*a^2*b^2*c*f+4*a^2*b*c^2*d-a*b^4*f+20*a*b^3
*c*d-3*b^5*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(2*a^2*c*i-5*a*b^2*i+
5*a*b*c*g-10*a*c^2*e+b^3*g-2*b^2*c*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(
12*a^3*c*h+3*a^2*b^2*h-16*a^2*b*c*f-44*a^2*c^2*d+a*b^3*f+37*a*b^2*c*d-5*b^
4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x+1/4*(6*a^2*b*i-8*a^2*c*g-a*b^2*g+10*a*
b*c*e-b^3*e)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/16*sum((-c*(1
2*a^2*b*h-20*a^2*c*f-a*b^2*f+24*a*b*c*d-3*b^3*d)/a^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*_R^2+8*(2*a*c*i+b^2*i-3*b*c*g+6*c^2*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R
+(12*a^3*c*h+3*a^2*b^2*h-16*a^2*b*c*f+84*a^2*c^2*d+a*b^3*f-27*a*b^2*c*d+3*
b^4*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(
_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx^2 + cx^4)^3} \, dx = \int \frac{ix^5 + hx^4 + gx^3 + fx^2 + ex + d}{(cx^4 + bx^2 + a)^3} \, dx$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="
maxima")
```

output
```
-1/8*((12*a^2*b*c^2*h - 3*(b^3*c^2 - 8*a*b*c^3)*d - (a*b^2*c^2 + 20*a^2*c^
3)*f)*x^7 - 4*(6*a^2*c^3*e - 3*a^2*b*c^2*g + (a^2*b^2*c + 2*a^3*c^2)*i)*x^
6 - 12*a^4*b*i - ((6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d + 2*(a*b^3*c + 1
4*a^2*b*c^2)*f - (19*a^2*b^2*c - 4*a^3*c^2)*h)*x^5 - 6*(6*a^2*b*c^2*e - 3*
a^2*b^2*c*g + (a^2*b^3 + 2*a^3*b*c)*i)*x^4 - ((3*b^5 - 20*a*b^3*c - 4*a^2*
b*c^2)*d + (a*b^4 + 5*a^2*b^2*c + 36*a^3*c^2)*f - (5*a^2*b^3 + 16*a^3*b*c)
*h)*x^3 - 4*(2*(a^2*b^2*c + 5*a^3*c^2)*e - (a^2*b^3 + 5*a^3*b*c)*g + (5*a^
3*b^2 - 2*a^4*c)*i)*x^2 + 2*(a^2*b^3 - 10*a^3*b*c)*e + 2*(a^3*b^2 + 8*a^4*
c)*g - ((5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d - (a^2*b^3 - 16*a^3*b*c)*f
 - 3*(a^3*b^2 + 4*a^4*c)*h)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)
*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 +
 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5
 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) - 1/8*integrate((((12*a^2*b*c*h - 3*(b^
3*c - 8*a*b*c^2)*d - (a*b^2*c + 20*a^2*c^2)*f)*x^2 - 3*(b^4 - 9*a*b^2*c +
28*a^2*c^2)*d - (a*b^3 - 16*a^2*b*c)*f - 3*(a^2*b^2 + 4*a^3*c)*h - 8*(6*a^
2*c^2*e - 3*a^2*b*c*g + (a^2*b^2 + 2*a^3*c)*i)*x)/(c*x^4 + b*x^2 + a), x)/
(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7058 vs. $2(680) = 1360$.

Time = 2.62 (sec) , antiderivative size = 7058, normalized size of antiderivative = 9.64

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx^2 + cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="
giac")
```

output
```
1/32*(3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^
2 + 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 - 2*b^7*c^2 - 368*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^3 - 232*a^2*b^4*c^3 + 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2
*c^4 - 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c
^5 - 896*a^4*c^5 + 352*a^3*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^7 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^6*c + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c^2 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^5*c^2 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b*c^3 - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^3 - 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)...
```

Mupad [B] (verification not implemented)

Time = 25.90 (sec) , antiderivative size = 36653, normalized size of antiderivative = 50.07

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((x^5*(28*a^2*c^3*d + 4*a^3*c^2*h + 6*b^4*c*d + 2*a*b^3*c*f - 49*a*b^2*c^2
*d + 28*a^2*b*c^2*f - 19*a^2*b^2*c*h))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)) - (x^2*(b^3*g - 10*a*c^2*e - 2*b^2*c*e - 5*a*b^2*i + 2*a^2*c*i + 5*a*b
*c*g))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b^3*e + a*b^2*g + 8*a^2*c*g -
 6*a^2*b*i - 10*a*b*c*e)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*x^4*(6*
c^2*e + b^2*i - 3*b*c*g + 2*a*c*i))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (
c*x^6*(6*c^2*e + b^2*i - 3*b*c*g + 2*a*c*i))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)) + (x^3*(3*b^5*d + 36*a^3*c^2*f - 5*a^2*b^3*h + a*b^4*f - 20*a*b^3*c*
d - 16*a^3*b*c*h - 4*a^2*b*c^2*d + 5*a^2*b^2*c*f))/(8*a^2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) - (x*(3*a^2*b^2*h - 44*a^2*c^2*d - 5*b^4*d + a*b^3*f + 12*
a^3*c*h + 37*a*b^2*c*d - 16*a^2*b*c*f))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (c*x^7*(20*a^2*c^2*f + 3*b^3*c*d - 24*a*b*c^2*d + a*b^2*c*f - 12*a^2*
b*c*h))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 +
 c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + symsum(log((10368*a*b^5*c^6*d^3 - 8000
*a^5*c^7*f^3 - 567*b^7*c^5*d^3 + 169344*a^3*b*c^8*d^3 + 193536*a^4*c^8*d*e
^2 - 141120*a^4*c^8*d^2*f + 1728*a^6*b*c^5*h^3 + 315*b^8*c^4*d^2*f + 27648
*a^5*c^7*e^2*h + 21504*a^6*c^6*d*i^2 - 135*b^9*c^3*d^2*h - 2880*a^6*c^6*f*
h^2 + 3072*a^7*c^5*h*i^2 - 67824*a^2*b^3*c^7*d^3 + 35*a^2*b^6*c^4*f^3 + 84
*a^3*b^4*c^5*f^3 - 12720*a^4*b^2*c^6*f^3 + 540*a^4*b^5*c^3*h^3 + 4320*a^5*
b^3*c^4*h^3 + 129024*a^5*c^7*d*e*i - 40320*a^5*c^7*d*f*h + 18432*a^6*c^...
```

Reduce [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 20724, normalized size of antiderivative = 28.31

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

output
```
( - 64*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**6*b*
c*i - 32*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*
b**3*i + 96*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*5*b**2*c*g - 128*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*
atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
b))*a**5*b**2*c*i*x**2 - 192*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sq
rt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)
*sqrt(a) + b))*a**5*b*c**2*e - 128*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt
(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*s
qrt(c)*sqrt(a) + b))*a**5*b*c**2*i*x**4 - 64*sqrt(2*sqrt(c)*sqrt(a) + b)*s
qrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x
)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**4*i*x**2 + 192*sqrt(2*sqrt(c)*sqrt(
a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**3*c*g*x**2 - 128*sqrt(2*sq
rt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**3*c*i*x**4 - 38
4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*...
```

3.52 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 55, antiderivative size = 545

$$\begin{aligned}
&\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4} dx \\
&= \frac{(c^2h+b^2m-c(bk+am))\,x}{c^3}+\frac{(cj-bl)x^2}{2c^2}+\frac{(ck-bm)x^3}{3c^2}+\frac{lx^4}{4c}+\frac{mx^5}{5c} \\
&+\frac{\left(c^3f-c^2(bh+ak)-b^3m+bc(bk+2am)+\frac{2c^4d-c^3(bf+2ah)+b^4m-b^2c(bk+4am)+c^2(b^2h+3abk+2a^2m)}{\sqrt{b^2-4ac}}\right)\arctan}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&+\frac{\left(c^3f-c^2(bh+ak)-b^3m+bc(bk+2am)-\frac{2c^4d-c^3(bf+2ah)+b^4m-b^2c(bk+4am)+c^2(b^2h+3abk+2a^2m)}{\sqrt{b^2-4ac}}\right)\arctan}{\sqrt{2}c^{7/2}\sqrt{b+\sqrt{b^2-4ac}}} \\
&-\frac{(2c^3e-c^2(bg+2aj)-b^3l+bc(bj+3al))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} \\
&+\frac{(c^2g+b^2l-c(bj+al))\log\left(a+bx^2+cx^4\right)}{4c^3}
\end{aligned}$$

output
```
(c^2*h+b^2*m-c*(a*m+b*k))*x/c^3+1/2*(-b*l+c*j)*x^2/c^2+1/3*(-b*m+c*k)*x^3/
c^2+1/4*l*x^4/c+1/5*m*x^5/c+1/2*(c^3*f-c^2*(a*k+b*h)-b^3*m+b*c*(2*a*m+b*k)
+(2*c^4*d-c^3*(2*a*h+b*f)+b^4*m-b^2*c*(4*a*m+b*k)+c^2*(2*a^2*m+3*a*b*k+b^2
*h))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(
1/2))*2^(1/2)/c^(7/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*(c^3*f-c^2*(a*k+b*h
)-b^3*m+b*c*(2*a*m+b*k)-(2*c^4*d-c^3*(2*a*h+b*f)+b^4*m-b^2*c*(4*a*m+b*k)+c
^2*(2*a^2*m+3*a*b*k+b^2*h))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(
b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(7/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-
1/2*(2*c^3*e-c^2*(2*a*j+b*g)-b^3*l+b*c*(3*a*l+b*j))*arctanh((2*c*x^2+b)/(-
4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)+1/4*(c^2*g+b^2*l-c*(a*l+b*j))*ln(
c*x^4+b*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.50

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx$$
$$= \frac{(c^2h+b^2m-c(bk+am))\,x}{c^3}+\frac{(cj-bl)x^2}{2c^2}+\frac{(ck-bm)x^3}{3c^2}+\frac{lx^4}{4c}+\frac{mx^5}{5c}$$
$$+\frac{\left(2c^4d+c^3\left(-bf+\sqrt{b^2-4ac}f-2ah\right)+b^3\left(b-\sqrt{b^2-4ac}\right)m+c^2\left(b^2h-b\sqrt{b^2-4ac}h+3abk-a\sqrt{b\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b}}$$
$$-\frac{\left(2c^4d-c^3\left(bf+\sqrt{b^2-4ac}f+2ah\right)+b^3\left(b+\sqrt{b^2-4ac}\right)m+c^2\left(b^2h+b\sqrt{b^2-4ac}h+3abk+a\sqrt{b^2\right.\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b-}}$$
$$+\frac{\left(2c^3e+c^2\left(-bg+\sqrt{b^2-4ac}g-2aj\right)+b^2\left(-b+\sqrt{b^2-4ac}\right)l+c\left(b^2j-b\sqrt{b^2-4ac}j+3abl-a\sqrt{b^2\right.\right.}{4c^3\sqrt{b^2-4ac}}$$
$$+\frac{\left(-2c^3e+c^2\left(bg+\sqrt{b^2-4ac}g+2aj\right)+b^2\left(b+\sqrt{b^2-4ac}\right)l-c\left(b^2j+b\sqrt{b^2-4ac}j+3abl+a\sqrt{b^2-\right.\right.}{4c^3\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8
)/(a + b*x^2 + c*x^4),x]
```

output
```
((c^2*h + b^2*m - c*(b*k + a*m))*x)/c^3 + ((c*j - b*l)*x^2)/(2*c^2) + ((c*
k - b*m)*x^3)/(3*c^2) + (l*x^4)/(4*c) + (m*x^5)/(5*c) + ((2*c^4*d + c^3*(-
(b*f) + Sqrt[b^2 - 4*a*c]*f - 2*a*h) + b^3*(b - Sqrt[b^2 - 4*a*c])*m + c^2
*(b^2*h - b*Sqrt[b^2 - 4*a*c]*h + 3*a*b*k - a*Sqrt[b^2 - 4*a*c]*k + 2*a^2*
m) + b*c*(-(b^2*k) + b*Sqrt[b^2 - 4*a*c]*k - 4*a*b*m + 2*a*Sqrt[b^2 - 4*a*
c]*m))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c^4*d - c^3*(b
*f + Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b^3*(b + Sqrt[b^2 - 4*a*c])*m + c^2*(b
^2*h + b*Sqrt[b^2 - 4*a*c]*h + 3*a*b*k + a*Sqrt[b^2 - 4*a*c]*k + 2*a^2*m)
- b*c*(b^2*k + b*Sqrt[b^2 - 4*a*c]*k + 4*a*b*m + 2*a*Sqrt[b^2 - 4*a*c]*m))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)
*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c^3*e + c^2*(-(b*g)
+ Sqrt[b^2 - 4*a*c]*g - 2*a*j) + b^2*(-b + Sqrt[b^2 - 4*a*c])*l + c*(b^2*j
 - b*Sqrt[b^2 - 4*a*c]*j + 3*a*b*l - a*Sqrt[b^2 - 4*a*c]*l))*Log[-b + Sqrt
[b^2 - 4*a*c] - 2*c*x^2])/(4*c^3*Sqrt[b^2 - 4*a*c]) + ((-2*c^3*e + c^2*(b*
g + Sqrt[b^2 - 4*a*c]*g + 2*a*j) + b^2*(b + Sqrt[b^2 - 4*a*c])*l - c*(b^2*
j + b*Sqrt[b^2 - 4*a*c]*j + 3*a*b*l + a*Sqrt[b^2 - 4*a*c]*l))*Log[b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2])/(4*c^3*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 3.07 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2202, 2194, 2188, 2009, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{cx^4+bx^2+a}dx+\int \frac{x\left(lx^6+jx^4+gx^2+e\right)}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{2194}$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{cx^4+bx^2+a}dx+\frac{1}{2}\int \frac{lx^6+jx^4+gx^2+e}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\int \left(\frac{lx^2}{c}+\frac{cj-bl}{c^2}+\frac{ec^2-ajc+\left(lb^2+c^2g-c(bj+al)\right)x^2+abl}{c^2\left(cx^4+bx^2+a\right)}\right)dx^2+$$
$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{cx^4+bx^2+a}dx+$$
$$\frac{1}{2}\left(-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2aj+bg)+bc(3al+bj)+b^3(-l)+2c^3e\right)}{c^3\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(al+bj)+b\right.}{2c^3}\right.$$

$$\Big\downarrow \text{2205}$$

$$\int \left(\frac{mx^4}{c}+\frac{(ck-bm)x^2}{c^2}+\frac{mb^2+c^2h-c(bk+am)}{c^3}+\frac{dc^3-ahc^2+a(bk+am)c+\left(-mb^3+c(bk+2am)b+\right.}{c^3\left(cx^4+bx^2+a\right)}\right.$$
$$\frac{1}{2}\left(-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2aj+bg)+bc(3al+bj)+b^3(-l)+2c^3e\right)}{c^3\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(al+bj)+b\right.}{2c^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{c^2\left(2a^2m+3abk+b^2h\right)-b^2c(4am+bk)-c^3(2ah+bf)+b^4m+2c^4d}{\sqrt{b^2-4ac}}-c^2(ak+bh)+bc(2am+bk)+b^3(-\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{c^2\left(2a^2m+3abk+b^2h\right)-b^2c(4am+bk)-c^3(2ah+bf)+b^4m+2c^4d}{\sqrt{b^2-4ac}}-c^2(ak+bh)+bc(2am+bk)+b^3(\right.}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}$$
$$\frac{1}{2}\left(-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2aj+bg)+bc(3al+bj)+b^3(-l)+2c^3e\right)}{c^3\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(al+bj)+b\right.}{2c^3}\right.$$
$$\frac{x\left(-c(am+bk)+b^2m+c^2h\right)}{c^3}+\frac{x^3(ck-bm)}{3c^2}+\frac{mx^5}{5c}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
 b*x^2 + c*x^4),x]
```

output
```
((c^2*h + b^2*m - c*(b*k + a*m))*x)/c^3 + ((c*k - b*m)*x^3)/(3*c^2) + (m*x
^5)/(5*c) + ((c^3*f - c^2*(b*h + a*k) - b^3*m + b*c*(b*k + 2*a*m) + (2*c^4
*d - c^3*(b*f + 2*a*h) + b^4*m - b^2*c*(b*k + 4*a*m) + c^2*(b^2*h + 3*a*b*
k + 2*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c^3*f -
c^2*(b*h + a*k) - b^3*m + b*c*(b*k + 2*a*m) - (2*c^4*d - c^3*(b*f + 2*a*h)
 + b^4*m - b^2*c*(b*k + 4*a*m) + c^2*(b^2*h + 3*a*b*k + 2*a^2*m))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (((c*j - b*l)*x^2)/c^2 + (l*x^4)
/(2*c) - ((2*c^3*e - c^2*(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*g + b^
2*l - c*(b*j + a*l))*Log[a + b*x^2 + c*x^4])/(2*c^3))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.45

method	result
risch	$\frac{m x^5}{5c} + \frac{l x^4}{4c} - \frac{bm x^3}{3c^2} + \frac{k x^3}{3c} - \frac{bl x^2}{2c^2} + \frac{j x^2}{2c} - \frac{xam}{c^2} + \frac{x b^2 m}{c^3} - \frac{xbk}{c^2} + \frac{hx}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(c\left(-acl+\cdots\right.\right.}{\cdots}}{\cdots}$
default	$-\frac{-\frac{1}{5} m x^5 c^2 - \frac{1}{4} l x^4 c^2 + \frac{1}{3} bcm x^3 - \frac{1}{3} c^2 k x^3 + \frac{1}{2} bcl x^2 - \frac{1}{2} c^2 j x^2 + xacm - x b^2 m + xbck - x c^2 h}{c^3} + \frac{-\frac{\sqrt{-4ac+b^2}\left(\frac{\left(-\sqrt{-4ac+b^2}\, a c^2 l + \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$

input
```
int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x,me
thod=_RETURNVERBOSE)
```

output
```
1/5*m*x^5/c+1/4*l*x^4/c-1/3/c^2*b*m*x^3+1/3/c*k*x^3-1/2/c^2*b*l*x^2+1/2/c*
j*x^2-1/c^2*x*a*m+1/c^3*x*b^2*m-1/c^2*x*b*k+h*x/c+1/2/c^3*sum((c*(-a*c*l+b
^2*l-b*c*j+c^2*g)*_R^3+(2*a*b*c*m-a*c^2*k-b^3*m+b^2*c*k-b*c^2*h+c^3*f)*_R^
2+c*(a*b*l-a*c*j+c^2*e)*_R+a^2*c*m-a*b^2*m+a*b*c*k-a*c^2*h+c^3*d)/(2*_R^3*
c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a
),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4
+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx = \int \frac{mx^8+lx^7+kx^6+jx^5+hx^4+gx^3+fx^2+ex+d}{cx^4+bx^2+a}\,dx$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a
),x, algorithm="maxima")
```

output
```
1/60*(12*c^2*m*x^5 + 15*c^2*l*x^4 + 20*(c^2*k - b*c*m)*x^3 + 30*(c^2*j - b
*c*l)*x^2 + 60*(c^2*h - b*c*k + (b^2 - a*c)*m)*x)/c^3 - integrate(-(c^3*d
- a*c^2*h + a*b*c*k + (c^3*g - b*c^2*j + (b^2*c - a*c^2)*l)*x^3 + (c^3*f -
 b*c^2*h + (b^2*c - a*c^2)*k - (b^3 - 2*a*b*c)*m)*x^2 - (a*b^2 - a^2*c)*m
+ (c^3*e - a*c^2*j + a*b*c*l)*x)/(c*x^4 + b*x^2 + a), x)/c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11830 vs. $2(495) = 990$.

Time = 1.35 (sec) , antiderivative size = 11830, normalized size of antiderivative = 21.71

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{a + bx^2 + cx^4}\, dx = \text{Too large to display}$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a
),x, algorithm="giac")
```

output
```
-1/8*((2*b^4*c^5 - 16*a*b^2*c^6 + 32*a^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*c^6 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6)*c^2*f - (
2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*h
+ (2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 9*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a...
```

Mupad [B] (verification not implemented)

Time = 23.35 (sec) , antiderivative size = 49150, normalized size of antiderivative = 90.18

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
 b*x^2 + c*x^4),x)
```

output
```
x^2*(j/(2*c) - (b*l)/(2*c^2)) - x*((b*(k/c - (b*m)/c^2))/c - h/c + (a*m)/c
^2) + x^3*(k/(3*c) - (b*m)/(3*c^2)) + symsum(log((c^7*d*e^2 - a*c^6*f^3 -
c^7*d^2*f + b^7*d*m^2 + a^4*c^3*k^3 + a^4*b^3*m^3 + a^2*b*c^4*h^3 + b^2*c^
5*d*g^2 + b^3*c^4*d*h^2 + a^2*c^5*d*j^2 - a^2*c^5*f*h^2 + a^2*c^5*g^2*h +
b^4*c^3*d*j^2 - a^3*c^4*d*l^2 - b^2*c^5*d^2*k + b^5*c^2*d*k^2 + 3*a^2*c^5*
f^2*k - 3*a^3*c^4*f*k^2 + a^2*c^5*e^2*m - a^3*c^4*h*j^2 + b^3*c^4*d^2*m +
a^3*c^4*h^2*k - a^4*c^3*f*m^2 + a^2*b^5*h*m^2 - a^3*c^4*g^2*m + a^4*c^3*h*
l^2 - a^3*b^4*k*m^2 + a^4*c^3*j^2*m + a^5*c^2*k*m^2 - a^5*c^2*l^2*m - a^3*
b^2*c^2*k^3 - a*c^6*d*g^2 + b*c^6*d*f^2 - a*c^6*e^2*h + b*c^6*d^2*h + a*c^
6*d^2*k - 2*a^5*b*c*m^3 + b^6*c*d*l^2 - a*b^6*f*m^2 - 2*a*b*c^5*d*h^2 - a*
b*c^5*f*g^2 + 2*a*b*c^5*f^2*h + a*b*c^5*e^2*k - 2*a*b*c^5*d^2*m - 6*a*b^5*
c*d*m^2 - 2*b^2*c^5*d*f*h - a*b^5*c*f*l^2 + 2*b^2*c^5*d*e*j - 2*b^3*c^4*d*
e*l + 2*b^3*c^4*d*f*k - 2*b^3*c^4*d*g*j - 2*a^2*c^5*d*f*m + 2*a^2*c^5*d*g*
l - 2*a^2*c^5*d*h*k - 2*a^2*c^5*e*f*l - 2*a^2*c^5*e*g*k + 2*a^2*c^5*e*h*j
- 2*a^2*c^5*f*g*j - 2*b^4*c^3*d*f*m + 2*b^4*c^3*d*g*l - 2*b^4*c^3*d*h*k +
2*b^5*c^2*d*h*m + 2*a^3*c^4*f*h*m - 2*a^3*c^4*g*h*l - 2*b^5*c^2*d*j*l + 2*
a^3*c^4*d*k*m - 2*a^3*c^4*e*j*m + 2*a^3*c^4*e*k*l + 2*a^3*c^4*f*j*l + 2*a^
3*c^4*g*j*k + 2*a^4*c^3*g*l*m - 2*a^4*c^3*h*k*m - 2*a^4*c^3*j*k*l - 3*a*b^
2*c^4*d*j^2 - a*b^2*c^4*f*h^2 - 4*a*b^3*c^3*d*k^2 + 3*a^2*b*c^4*d*k^2 - a*
b^3*c^3*f*j^2 - 5*a*b^4*c^2*d*l^2 + 2*a^2*b*c^4*f*j^2 - 2*a*b^2*c^4*f^2...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 4445, normalized size of antiderivative = 8.16

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

output
```
( - 90*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*
c**2*l + 60*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*2*c**3*j + 30*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a*b**3*c*l - 30*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b
))*a*b**2*c**2*j + 30*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) -
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*a*b*c**3*g - 60*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
 - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*a*c**4*e - 90*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**
2*m + 60*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**3*k + 30*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c*m - 30*sqrt(a)*sqrt(2*sqrt(c)*sqr
t(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a*b**2*c**2*k + 30*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*at...
```

3.53 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 770

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^2}\,dx = \frac{mx}{c^2}$$
$$- \frac{bc(ce+aj)-ab^2l-2ac(cg-al)+(2c^3e-c^2(bg+2aj)-b^3l+bc(bj+3al))x^2}{2c^2(b^2-4ac)(a+bx^2+cx^4)}$$
$$- \frac{x(abc(cf+ak)-b^2(c^2d+a^2m)+2ac(c^2d-ach+a^2m)+(ab^2ck+2ac^2(cf-ak)-ab^3m-bc(c^2d}{2ac^2(b^2-4ac)(a+bx^2+cx^4)}$$
$$+ \frac{\left(ab^2ck-2ac^2(cf+3ak)-3ab^3m+bc(c^2d+ach+13a^2m)-\frac{ab^3ck-4abc^2(cf+2ak)-3ab^4m-b^2c(c^2d-ach-19a}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}ac^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\left(ab^2ck-2ac^2(cf+3ak)-3ab^3m+bc(c^2d+ach+13a^2m)+\frac{ab^3ck-4abc^2(cf+2ak)-3ab^4m-b^2c(c^2d-ach-19a}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}ac^{5/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{(4c^3e-c^2(2bg-4aj)+b^3l-6abcl)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}} + \frac{l\log(a+bx^2+cx^4)}{4c^2}$$

output
```
m*x/c^2-1/2*(b*c*(a*j+c*e)-a*b^2*l-2*a*c*(-a*l+c*g)+(2*c^3*e-c^2*(2*a*j+b*
g)-b^3*l+b*c*(3*a*l+b*j))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(a*b
*c*(a*k+c*f)-b^2*(a^2*m+c^2*d)+2*a*c*(a^2*m-a*c*h+c^2*d)+(a*b^2*c*k+2*a*c^
2*(-a*k+c*f)-a*b^3*m-b*c*(-3*a^2*m+a*c*h+c^2*d))*x^2)/a/c^2/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)+1/4*(a*b^2*c*k-2*a*c^2*(3*a*k+c*f)-3*a*b^3*m+b*c*(13*a^2*m+
a*c*h+c^2*d)-(a*b^3*c*k-4*a*b*c^2*(2*a*k+c*f)-3*a*b^4*m-b^2*c*(-19*a^2*m-a
*c*h+c^2*d)+4*a*c^2*(-5*a^2*m+a*c*h+3*c^2*d))/(-4*a*c+b^2)^(1/2))*arctan(2
^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/c^(5/2)/(-4*a*c+b
^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*(a*b^2*c*k-2*a*c^2*(3*a*k+c*f)-3*a*b^
3*m+b*c*(13*a^2*m+a*c*h+c^2*d)+(a*b^3*c*k-4*a*b*c^2*(2*a*k+c*f)-3*a*b^4*m-
b^2*c*(-19*a^2*m-a*c*h+c^2*d)+4*a*c^2*(-5*a^2*m+a*c*h+3*c^2*d))/(-4*a*c+b^
2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a
/c^(5/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+1/2*(4*c^3*e-c^2*(-4*a*
j+2*b*g)+b^3*l-6*a*b*c*l)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*
a*c+b^2)^(3/2)+1/4*l*ln(c*x^4+b*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 5.62 (sec) , antiderivative size = 935, normalized size of antiderivative = 1.21

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{4\sqrt{c}mx+\frac{2\sqrt{c}(2a^3c(l+mx)-bc^2dx(b+cx^2)+a(b^2cx^2(j+kx)-b^3x^2(l+mx)+2c^3x(d+x(e+fx))+bc^2(e+x(f-x(g+hx))))-a^2(b^2(l+mx)+}{a(-b^2+4ac)(a+bx^2+cx^4)}}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8
)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(4*Sqrt[c]*m*x + (2*Sqrt[c]*(2*a^3*c*(l + m*x) - b*c^2*d*x*(b + c*x^2) + a
*(b^2*c*x^2*(j + k*x) - b^3*x^2*(l + m*x) + 2*c^3*x*(d + x*(e + f*x)) + b*
c^2*(e + x*(f - x*(g + h*x)))) - a^2*(b^2*(l + m*x) + 2*c^2*(g + x*(h + x*
(j + k*x))) - b*c*(j + x*(k + 3*x*(l + m*x))))))/(a*(-b^2 + 4*a*c)*(a + b*
x^2 + c*x^4)) - (Sqrt[2]*(-3*a*b^4*m + 2*a*c^2*(6*c^2*d + c*Sqrt[b^2 - 4*a
*c]*f + 2*a*c*h + 3*a*Sqrt[b^2 - 4*a*c]*k - 10*a^2*m) + a*b^3*(c*k + 3*Sqr
t[b^2 - 4*a*c]*m) - b*c*(c^2*(Sqrt[b^2 - 4*a*c]*d + 4*a*f) + a*c*(Sqrt[b^2
 - 4*a*c]*h + 8*a*k) + 13*a^2*Sqrt[b^2 - 4*a*c]*m) + b^2*c*(-(c^2*d) + a*c
*h + a*(-(Sqrt[b^2 - 4*a*c]*k) + 19*a*m)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) - (Sqrt[2]*(3*a*b^4*m + 2*a*c^2*(-6*c^2*d + c*Sqrt[b^2 - 4*a*c]*f - 2*
a*c*h + 3*a*Sqrt[b^2 - 4*a*c]*k + 10*a^2*m) + a*b^3*(-(c*k) + 3*Sqrt[b^2 -
 4*a*c]*m) - b*c*(c^2*(Sqrt[b^2 - 4*a*c]*d - 4*a*f) + a*c*(Sqrt[b^2 - 4*a*
c]*h - 8*a*k) + 13*a^2*Sqrt[b^2 - 4*a*c]*m) + b^2*c*(c^2*d - a*c*h - a*(Sq
rt[b^2 - 4*a*c]*k + 19*a*m)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (Sqrt[c
]*(-4*c^3*e + 2*c^2*(b*g - 2*a*j) + b^2*(-b + Sqrt[b^2 - 4*a*c])*l + a*c*(
6*b*l - 4*Sqrt[b^2 - 4*a*c]*l))*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^
2 - 4*a*c)^(3/2) + (Sqrt[c]*(4*c^3*e + c^2*(-2*b*g + 4*a*j) + b^2*(b + Sqr
t[b^2 - 4*a*c])*l - 2*a*c*(3*b + 2*Sqrt[b^2 - 4*a*c])*l)*Log[b + Sqrt[b...
```

Rubi [A] (verified)

Time = 4.43 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2202, 2194, 2191, 1142, 1083, 219, 1103, 2206, 25, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \ 2202$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^2}dx+\int \frac{x(lx^6+jx^4+gx^2+e)}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \ 2194$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}\int \frac{lx^6+jx^4+gx^2+e}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \ 2191$$

$$\frac{1}{2}\left(-\frac{\int \frac{\left(4a-\frac{b^2}{c}\right)lx^2+2ce-bg+2aj-\frac{abl}{c}}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{x^2\left(-c^2(2aj+bg)+bc(3al+bj)+b^3(-l)+2c^3e\right)-ab^2l+bc(aj+ce)}{c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right.$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{2}\left(-\frac{\frac{\left(-c^2(2bg-4aj)-6abcl+b^3l+4c^3e\right)\int \frac{1}{cx^4+bx^2+a}dx^2}{2c^2}-\frac{l\left(b^2-4ac\right)\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2aj+bg)+bc(3al+bj)+b^3\right.}{c^2\left(b^2-\right.}\right.$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{2}\left(-\frac{-\frac{l\left(b^2-4ac\right)\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c^2}-\frac{\left(-c^2(2bg-4aj)-6abcl+b^3l+4c^3e\right)\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2aj+bg)+bc(3al+\right.}{}\right.$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left(-\frac{-\frac{l\left(b^2-4ac\right)\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c^2}-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4aj)-6abcl+b^3l+4c^3e\right)}{c^2\sqrt{b^2-4ac}}}{b^2-4ac}-\frac{x^2\left(-c^2(2aj+bg)+bc(3al+b\right.}{c^2}\right.$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \ 1103$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^2}dx+$$

$$\frac{1}{2}\left(-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4aj)-6abcl+b^3l+4c^3e\right)}{c^2\sqrt{b^2-4ac}}-\frac{l\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2aj+bg)+bc(3al+b\ldots\right.}{c^2\ldots}\right.$$

$$\Big\downarrow \ 2206$$

$$-\frac{\int -\frac{-2a\left(4a-\frac{b^2}{c}\right)mx^4+\frac{\left(-amb^3+ackb^2+c\left(5ma^2+cha+c^2d\right)b-2ac^2(cf+3ak)\right)x^2}{c^2}+\frac{\left(c^2d-a^2m\right)b^2+ac(cf+ak)b-2ac\left(-ma^2+cha+3c^2d\right)}{c^2}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}-$$

$$\frac{x\left(-\left(b^2\left(a^2m+c^2d\right)\right)+x^2\left(-bc\left(-3a^2m+ach+c^2d\right)-ab^3m+ab^2ck+2ac^2(cf-ak)\right)+2ac\left(a^2m-ach+c^2\ldots\right.\right.}{2ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{1}{2}\left(-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4aj)-6abcl+b^3l+4c^3e\right)}{c^2\sqrt{b^2-4ac}}-\frac{l\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2aj+bg)+bc(3al+b\ldots\right.}{c^2\ldots}\right.$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-2a\left(4a-\frac{b^2}{c}\right)mx^4+\frac{\left(-amb^3+ackb^2+c\left(5ma^2+cha+c^2d\right)b-2ac^2(cf+3ak)\right)x^2}{c^2}+\frac{\left(c^2d-a^2m\right)b^2+ac(cf+ak)b-2ac\left(-ma^2+cha+3c^2d\right)}{c^2}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}-$$

$$\frac{x\left(-\left(b^2\left(a^2m+c^2d\right)\right)+x^2\left(-bc\left(-3a^2m+ach+c^2d\right)-ab^3m+ab^2ck+2ac^2(cf-ak)\right)+2ac\left(a^2m-ach+c^2\ldots\right.\right.}{2ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{1}{2}\left(-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4aj)-6abcl+b^3l+4c^3e\right)}{c^2\sqrt{b^2-4ac}}-\frac{l\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2aj+bg)+bc(3al+b\ldots\right.}{c^2\ldots}\right.$$

$$\Big\downarrow \ 2205$$

$$\frac{\int \left(\frac{2a\left(b^2-4ac\right)m}{c^2}+\frac{\left(c^2d-3a^2m\right)b^2+ac(cf+ak)b+\left(-3amb^3+ackb^2+c\left(13ma^2+cha+c^2d\right)b-2ac^2(cf+3ak)\right)x^2-2ac\left(-5ma^2+cha+3c^2d\right)}{c^2\left(cx^4+bx^2+a\right)}\right)\ldots}{2a\left(b^2-4ac\right)}$$

$$\frac{x\left(-\left(b^2\left(a^2m+c^2d\right)\right)+x^2\left(-bc\left(-3a^2m+ach+c^2d\right)-ab^3m+ab^2ck+2ac^2(cf-ak)\right)+2ac\left(a^2m-ach+c^2\ldots\right.\right.}{2ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{1}{2}\left(-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4aj)-6abcl+b^3l+4c^3e\right)}{c^2\sqrt{b^2-4ac}}-\frac{l\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2aj+bg)+bc(3al+b\ldots\right.}{c^2\ldots}\right.$$

$\downarrow$ 2009

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c\left(-19a^2m-ach+c^2d\right)+4ac^2\left(-5a^2m+ach+3c^2d\right)-3ab^4m+ab^3ck-4abc^2(2ak+cf)}{\sqrt{b^2-4ac}}+bc(13a^2m+ach+c^2d)-3ab^3m+ab\right.}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$\frac{x(-(b^2(a^2m+c^2d))+x^2(-bc(-3a^2m+ach+c^2d)-ab^3m+ab^2ck+2ac^2(cf-ak))+2ac(a^2m-ach+c^2d}{2ac^2(b^2-4ac)(a+bx^2+cx^4)}$$

$$\frac{1}{2}\left(-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-c^2(2bg-4aj)-6abcl+b^3l+4c^3e)}{c^2\sqrt{b^2-4ac}}-\frac{l(b^2-4ac)\log(a+bx^2+cx^4)}{2c^2}}{b^2-4ac}-\frac{x^2(-c^2(2aj+bg)+bc(3al+b}{c^2}\right.$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
 b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(a*b*c*(c*f + a*k) - b^2*(c^2*d + a^2*m) + 2*a*c*(c^2*d - a*c*h +
a^2*m) + (a*b^2*c*k + 2*a*c^2*(c*f - a*k) - a*b^3*m - b*c*(c^2*d + a*c*h -
 3*a^2*m))*x^2))/(a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((2*a*(b^2 -
4*a*c)*m*x)/c^2 + ((a*b^2*c*k - 2*a*c^2*(c*f + 3*a*k) - 3*a*b^3*m + b*c*(c
^2*d + a*c*h + 13*a^2*m) - (a*b^3*c*k - 4*a*b*c^2*(c*f + 2*a*k) - 3*a*b^4*
m - b^2*c*(c^2*d - a*c*h - 19*a^2*m) + 4*a*c^2*(3*c^2*d + a*c*h - 5*a^2*m)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((a*b^2*c*k - 2*a*c^2*
(c*f + 3*a*k) - 3*a*b^3*m + b*c*(c^2*d + a*c*h + 13*a^2*m) + (a*b^3*c*k -
4*a*b*c^2*(c*f + 2*a*k) - 3*a*b^4*m - b^2*c*(c^2*d - a*c*h - 19*a^2*m) + 4
*a*c^2*(3*c^2*d + a*c*h - 5*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 -
 4*a*c]]))/(2*a*(b^2 - 4*a*c)) + (-((b*c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*
g - a*l) + (2*c^3*e - c^2*(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*x^2)/
(c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) - (-(((4*c^3*e - c^2*(2*b*g - 4*a
*j) + b^3*l - 6*a*b*c*l)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sq
rt[b^2 - 4*a*c])) - ((b^2 - 4*a*c)*l*Log[a + b*x^2 + c*x^4])/(2*c^2))/(b^2
 - 4*a*c))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.66

method	result
risch	$\frac{mx}{c^2}+\frac{\frac{\left(3a^2bcm-2a^2c^2k-a\,b^3m+a\,b^2ck-ab\,c^2h+2a\,c^3f-b\,c^3d\right)x^3}{2a\left(4ac-b^2\right)}+\frac{\left(3abcl-2a\,c^2j-b^3l+b^2cj-b\,c^2g+2c^3e\right)x^2}{8ac-2b^2}+\frac{\left(2a^3cm-a^2b^2m+a^2bck-2\right.}{2a\left(\right.}}{c^2(c\,x^4+b\,x^2+a)}$
default	Expression too large to display

input
```
int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,
method=_RETURNVERBOSE)
```

output
```
m*x/c^2+(1/2/a*(3*a^2*b*c*m-2*a^2*c^2*k-a*b^3*m+a*b^2*c*k-a*b*c^2*h+2*a*c^
3*f-b*c^3*d)/(4*a*c-b^2)*x^3+1/2*(3*a*b*c*l-2*a*c^2*j-b^3*l+b^2*c*j-b*c^2*
g+2*c^3*e)/(4*a*c-b^2)*x^2+1/2*(2*a^3*c*m-a^2*b^2*m+a^2*b*c*k-2*a^2*c^2*h+
a*b*c^2*f+2*a*c^3*d-b^2*c^2*d)/a/(4*a*c-b^2)*x+1/2*(2*a^2*c*l-a*b^2*l+a*b*
c*j-2*a*c^2*g+b*c^2*e)/(4*a*c-b^2))/c^2/(c*x^4+b*x^2+a)+1/4/c^2*sum((2*l*c
*_R^3-(13*a^2*b*c*m-6*a^2*c^2*k-3*a*b^3*m+a*b^2*c*k+a*b*c^2*h-2*a*c^3*f+b*
c^3*d)/a/(4*a*c-b^2)*_R^2-2*c*(a*b*l-2*a*c*j+b*c*g-2*c^2*e)/(4*a*c-b^2)*_R
-(10*a^3*c*m-3*a^2*b^2*m+a^2*b*c*k-2*a^2*c^2*h+a*b*c^2*f-6*a*c^3*d+b^2*c^2
*d)/a/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a
)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4
+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^2}\,dx$$
$$=\int \frac{mx^8+lx^7+kx^6+jx^5+hx^4+gx^3+fx^2+ex+d}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a
)^2,x, algorithm="maxima")
```

output
```
-1/2*(a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*j - (b*c^3*d - 2*a*c^3*f + a*b*c^2
*h - (a*b^2*c - 2*a^2*c^2)*k + (a*b^3 - 3*a^2*b*c)*m)*x^3 + (2*a*c^3*e - a
*b*c^2*g + (a*b^2*c - 2*a^2*c^2)*j - (a*b^3 - 3*a^2*b*c)*l)*x^2 - (a^2*b^2
 - 2*a^3*c)*l + (a*b*c^2*f - 2*a^2*c^2*h + a^2*b*c*k - (b^2*c^2 - 2*a*c^3)
*d - (a^2*b^2 - 2*a^3*c)*m)*x)/(a^2*b^2*c^2 - 4*a^3*c^3 + (a*b^2*c^3 - 4*a
^2*c^4)*x^4 + (a*b^3*c^2 - 4*a^2*b*c^3)*x^2) + m*x/c^2 - 1/2*integrate(-(a
*b*c^2*f - 2*a^2*c^2*h + a^2*b*c*k + 2*(a*b^2*c - 4*a^2*c^2)*l*x^3 + (b*c^
3*d - 2*a*c^3*f + a*b*c^2*h + (a*b^2*c - 6*a^2*c^2)*k - (3*a*b^3 - 13*a^2*
b*c)*m)*x^2 + (b^2*c^2 - 6*a*c^3)*d - (3*a^2*b^2 - 10*a^3*c)*m - 2*(2*a*c^
3*e - a*b*c^2*g + 2*a^2*c^2*j - a^2*b*c*l)*x)/(c*x^4 + b*x^2 + a), x)/(a*b
^2*c^2 - 4*a^2*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20158 vs. 2(718) = 1436.

Time = 2.57 (sec) , antiderivative size = 20158, normalized size of antiderivative = 26.18

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a
)^2,x, algorithm="giac")
```

output
```
m*x/c^2 + 1/4*l*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/16*((a^2*b^4*c^5 - 8*a
^3*b^2*c^6 + 16*a^4*c^7)^2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d - 2*(a^2*b^4*c^5 -
8*a^3*b^2*c^6 + 16*a^4*c^7)^2*(2*a*b^2*c^5 - 8*a^2*c^6 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*a*c^5)*f + (a^2*b^4*c
^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7)^2*(2*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*a*b*c^4)
*h + (a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7)^2*(2*a*b^4*c^3 - 20*a^2*b^
2*c^4 + 48*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c...
```

Mupad [B] (verification not implemented)

Time = 49.92 (sec) , antiderivative size = 82785, normalized size of antiderivative = 107.51

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
 b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log(root(1572864*a^8*b^2*c^10*z^4 - 983040*a^7*b^4*c^9*z^4 + 327680
*a^6*b^6*c^8*z^4 - 61440*a^5*b^8*c^7*z^4 + 6144*a^4*b^10*c^6*z^4 - 256*a^3
*b^12*c^5*z^4 - 1048576*a^9*c^11*z^4 - 1572864*a^8*b^2*c^8*l*z^3 + 983040*
a^7*b^4*c^7*l*z^3 - 327680*a^6*b^6*c^6*l*z^3 + 61440*a^5*b^8*c^5*l*z^3 - 6
144*a^4*b^10*c^4*l*z^3 + 256*a^3*b^12*c^3*l*z^3 + 1048576*a^9*c^9*l*z^3 +
96*a^3*b^12*c*k*m*z^2 + 98304*a^8*b*c^7*j*l*z^2 + 24576*a^8*b*c^7*h*m*z^2
+ 155648*a^7*b*c^8*d*m*z^2 + 98304*a^7*b*c^8*e*l*z^2 + 57344*a^7*b*c^8*f*k
*z^2 + 32768*a^7*b*c^8*g*j*z^2 + 57344*a^6*b*c^9*d*h*z^2 + 32768*a^6*b*c^9
*e*g*z^2 - 32*a*b^10*c^5*d*f*z^2 - 491520*a^8*b^2*c^6*k*m*z^2 + 358400*a^7
*b^4*c^5*k*m*z^2 - 129024*a^6*b^6*c^4*k*m*z^2 + 24768*a^5*b^8*c^3*k*m*z^2
- 2432*a^4*b^10*c^2*k*m*z^2 - 90112*a^7*b^3*c^6*j*l*z^2 + 30720*a^6*b^5*c^
5*j*l*z^2 - 4608*a^5*b^7*c^4*j*l*z^2 + 256*a^4*b^9*c^3*j*l*z^2 - 21504*a^6
*b^5*c^5*h*m*z^2 + 9216*a^5*b^7*c^4*h*m*z^2 + 8192*a^7*b^3*c^6*h*m*z^2 - 1
568*a^4*b^9*c^3*h*m*z^2 + 96*a^3*b^11*c^2*h*m*z^2 - 172032*a^7*b^2*c^7*f*m
*z^2 + 116736*a^6*b^4*c^6*f*m*z^2 - 49152*a^7*b^2*c^7*g*l*z^2 + 45056*a^6*
b^4*c^6*g*l*z^2 - 35840*a^5*b^6*c^5*f*m*z^2 + 24576*a^7*b^2*c^7*h*k*z^2 -
15360*a^5*b^6*c^5*g*l*z^2 + 5184*a^4*b^8*c^4*f*m*z^2 - 3072*a^5*b^6*c^5*h*
k*z^2 + 2304*a^4*b^8*c^4*g*l*z^2 + 2048*a^6*b^4*c^6*h*k*z^2 + 576*a^4*b^8*
c^4*h*k*z^2 - 288*a^3*b^10*c^3*f*m*z^2 - 128*a^3*b^10*c^3*g*l*z^2 - 32*a^3
*b^10*c^3*h*k*z^2 - 147456*a^6*b^3*c^7*d*m*z^2 - 90112*a^6*b^3*c^7*e*l*...
```

Reduce [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 15020, normalized size of antiderivative = 19.51

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
(24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**2*
c**2*l - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*4*b*c**3*j - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b)
)*a**3*b**4*c*l + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) -
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a**3*b**3*c**2*l*x**2 + 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c
)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqr
t(c)*sqrt(a) + b))*a**3*b**2*c**3*g - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(
2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**3*j*x**2 + 24*sqrt(2*sqrt(c)*sqrt(
a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**3*l*x**4 - 16*sqrt(2*
sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**4*e - 16*sq
rt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**4*j*x*
*4 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq...
```

3.54 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 1150

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

output
```
-1/4*(b*c*(a*j+c*e)-a*b^2*l-2*a*c*(-a*l+c*g)+(2*c^3*e-c^2*(2*a*j+b*g)-b^3*
l+b*c*(3*a*l+b*j))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/4*x*(a*b*c*(a
*k+c*f)-b^2*(a^2*m+c^2*d)+2*a*c*(a^2*m-a*c*h+c^2*d)+(a*b^2*c*k+2*a*c^2*(-a
*k+c*f)-a*b^3*m-b*c*(-3*a^2*m+a*c*h+c^2*d))*x^2)/a/c^2/(-4*a*c+b^2)/(c*x^4
+b*x^2+a)^2+1/4*(b^3*j/c+2*b*(a*j+3*c*e)-16*a^2*l-b^4*l/c^2-b^2*(3*g-5*a*l
/c)+2*(-3*a*b*l+2*a*c*j+b^2*j-3*b*c*g+6*c^2*e)*x^2)/(-4*a*c+b^2)^2/(c*x^4+
b*x^2+a)+1/8*x*(4*a^2*b*c^2*(a*k+2*c*f)+a*b^3*c*(2*a*k+c*f)-a*b^2*c*(-11*a
^2*m+7*a*c*h+25*c^2*d)+4*a^2*c^2*(-9*a^2*m+a*c*h+7*c^2*d)+b^4*(-2*a^2*m+3*
c^2*d)+c*(a*b^2*c*(3*a*k+c*f)+4*a^2*c^2*(3*a*k+5*c*f)+b^3*(a^2*m+3*c^2*d)-
4*a*b*c*(4*a^2*m+3*a*c*h+6*c^2*d))*x^2)/a^2/c^2/(-4*a*c+b^2)^2/(c*x^4+b*x^
2+a)+1/16*(a*b^2*c*(3*a*k+c*f)+4*a^2*c^2*(3*a*k+5*c*f)+b^3*(a^2*m+3*c^2*d)
-4*a*b*c*(4*a^2*m+3*a*c*h+6*c^2*d)+(a*b^3*c*(-3*a*k+c*f)-4*a^2*b*c^2*(9*a*
k+13*c*f)-6*a*b^2*c*(-3*a^2*m-3*a*c*h+5*c^2*d)+b^4*(-a^2*m+3*c^2*d)+8*a^2*
c^2*(5*a^2*m+3*a*c*h+21*c^2*d))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)
*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/c^(3/2)/(-4*a*c+b^2)^2/(b-(-4
*a*c+b^2)^(1/2))^(1/2)+1/16*(a*b^2*c*(3*a*k+c*f)+4*a^2*c^2*(3*a*k+5*c*f)+b
^3*(a^2*m+3*c^2*d)-4*a*b*c*(4*a^2*m+3*a*c*h+6*c^2*d)-(a*b^3*c*(-3*a*k+c*f)
-4*a^2*b*c^2*(9*a*k+13*c*f)-6*a*b^2*c*(-3*a^2*m-3*a*c*h+5*c^2*d)+b^4*(-a^2
*m+3*c^2*d)+8*a^2*c^2*(5*a^2*m+3*a*c*h+21*c^2*d))/(-4*a*c+b^2)^(1/2))*arct
an(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/c^(3/2)/...
```

Mathematica [A] (verified)

Time = 7.41 (sec) , antiderivative size = 1590, normalized size of antiderivative = 1.38

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8
)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*j - a^2*b^2*l + 2*a^3*c*l - b^2*c^2*d*x
 + 2*a*c^3*d*x + a*b*c^2*f*x - 2*a^2*c^2*h*x + a^2*b*c*k*x - a^2*b^2*m*x +
 2*a^3*c*m*x + 2*a*c^3*e*x^2 - a*b*c^2*g*x^2 + a*b^2*c*j*x^2 - 2*a^2*c^2*j
*x^2 - a*b^3*l*x^2 + 3*a^2*b*c*l*x^2 - b*c^3*d*x^3 + 2*a*c^3*f*x^3 - a*b*c
^2*h*x^3 + a*b^2*c*k*x^3 - 2*a^2*c^2*k*x^3 - a*b^3*m*x^3 + 3*a^2*b*c*m*x^3
)/(4*a*c^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a^2*b*c^3*e - 6*a^2
*b^2*c^2*g + 2*a^2*b^3*c*j + 4*a^3*b*c^2*j - 2*a^2*b^4*l + 10*a^3*b^2*c*l
- 32*a^4*c^2*l + 3*b^4*c^2*d*x - 25*a*b^2*c^3*d*x + 28*a^2*c^4*d*x + a*b^3
*c^2*f*x + 8*a^2*b*c^3*f*x - 7*a^2*b^2*c^2*h*x + 4*a^3*c^3*h*x + 2*a^2*b^3
*c*k*x + 4*a^3*b*c^2*k*x - 2*a^2*b^4*m*x + 11*a^3*b^2*c*m*x - 36*a^4*c^2*m
*x + 24*a^2*c^4*e*x^2 - 12*a^2*b*c^3*g*x^2 + 4*a^2*b^2*c^2*j*x^2 + 8*a^3*c
^3*j*x^2 - 12*a^3*b*c^2*l*x^2 + 3*b^3*c^3*d*x^3 - 24*a*b*c^4*d*x^3 + a*b^2
*c^3*f*x^3 + 20*a^2*c^4*f*x^3 - 12*a^2*b*c^3*h*x^3 + 3*a^2*b^2*c^2*k*x^3 +
 12*a^3*c^3*k*x^3 + a^2*b^3*c*m*x^3 - 16*a^3*b*c^2*m*x^3)/(8*a^2*c^2*(-b^2
 + 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*b^4*c^2*d - 30*a*b^2*c^3*d + 168*a^
2*c^4*d + 3*b^3*c^2*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^2 - 4*a*c]*d +
 a*b^3*c^2*f - 52*a^2*b*c^3*f + a*b^2*c^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c^3
*Sqrt[b^2 - 4*a*c]*f + 18*a^2*b^2*c^2*h + 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt
[b^2 - 4*a*c]*h - 3*a^2*b^3*c*k - 36*a^3*b*c^2*k + 3*a^2*b^2*c*Sqrt[b^2 -
4*a*c]*k + 12*a^3*c^2*Sqrt[b^2 - 4*a*c]*k - a^2*b^4*m + 18*a^3*b^2*c*m ...
```

Rubi [A] (verified)

Time = 2.92 (sec) , antiderivative size = 1172, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.236$, Rules used = {2202, 2194, 2191, 1159, 1083, 219, 2206, 25, 2206, 25, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx+\int \frac{x(lx^6+jx^4+gx^2+e)}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{2194}$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx+\frac{1}{2}\int \frac{lx^6+jx^4+gx^2+e}{(cx^4+bx^2+a)^3}dx^2$$

$\big\downarrow$ 2191

$$\frac{1}{2}\left(-\frac{\int \frac{-\frac{lb^3}{c^2}-3gb+\frac{(bj+al)b}{c}+2\left(4a-\frac{b^2}{c}\right)lx^2+6ce+2aj}{(cx^4+bx^2+a)^2}dx^2}{2(b^2-4ac)}-\frac{x^2(-c^2(2aj+bg)+bc(3al+bj)+b^3(-l)+2c^3e)-ab^2l+}{2c^2(b^2-4ac)(a+bx^2+cx^4)^2}\right.$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$\big\downarrow$ 1159

$$\frac{1}{2}\left(-\frac{-\frac{2(-3abl+2acj+b^2j-3bcg+6c^2e)\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{-16a^2l+2x^2(-3abl+2acj+b^2j-3bcg+6c^2e)-b^2\left(3g-\frac{5al}{c}\right)+2b(aj+3ce)-\frac{b^4l}{c^2}+}{(b^2-4ac)(a+bx^2+cx^4)}}{2(b^2-4ac)}\right.$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$\big\downarrow$ 1083

$$\frac{1}{2}\left(-\frac{\frac{4(-3abl+2acj+b^2j-3bcg+6c^2e)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{-16a^2l+2x^2(-3abl+2acj+b^2j-3bcg+6c^2e)-b^2\left(3g-\frac{5al}{c}\right)+2b(aj+3ce)}{(b^2-4ac)(a+bx^2+cx^4)}}{2(b^2-4ac)}\right.$$

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$\big\downarrow$ 219

$$\int \frac{mx^8+kx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx+$$

$$\frac{1}{2}\left(-\frac{\frac{4\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-3abl+2acj+b^2j-3bcg+6c^2e)}{(b^2-4ac)^{3/2}}-\frac{-16a^2l+2x^2(-3abl+2acj+b^2j-3bcg+6c^2e)-b^2\left(3g-\frac{5al}{c}\right)+2b(aj+3ce)-\frac{b^4l}{c^2}}{(b^2-4ac)(a+bx^2+cx^4)}}{2(b^2-4ac)}\right.$$

$\big\downarrow$ 2206

$$-\frac{\int -\frac{-4a\left(4a-\frac{b^2}{c}\right)mx^4-\frac{\left(-amb^3+ackb^2-c\left(ma^2+5cha+5c^2d\right)b+2ac^2(5cf+3ak)\right)x^2}{c^2}+\frac{\left(3c^2d-a^2m\right)b^2+ac(cf+ak)b-2ac\left(-ma^2+cha+7c^2d\right)}{c^2}}{(cx^4+bx^2+a)^2}dx}{4a\,(b^2-4ac)}+$$

$$\frac{1}{2}\left(-\frac{\frac{4\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-3abl+2acj+b^2j-3bcg+6c^2e)}{(b^2-4ac)^{3/2}}-\frac{-16a^2l+2x^2(-3abl+2acj+b^2j-3bcg+6c^2e)-b^2\left(3g-\frac{5al}{c}\right)+2b(aj+3ce)-\frac{b^{\cdots}}{c}}{(b^2-4ac)(a+bx^2+cx^4)}}{2\,(b^2-4ac)}\right.$$

$$\frac{x\left(-\left(b^2\left(a^2m+c^2d\right)\right)+x^2\left(-bc\left(-3a^2m+ach+c^2d\right)-ab^3m+ab^2ck+2ac^2(cf-ak)\right)+2ac\left(a^2m-ach+c^2\cdots\right)\right.}{4ac^2\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-4a\left(4a-\frac{b^2}{c}\right)mx^4-\frac{\left(-amb^3+ackb^2-c\left(ma^2+5cha+5c^2d\right)b+2ac^2(5cf+3ak)\right)x^2}{c^2}+\frac{\left(3c^2d-a^2m\right)b^2+ac(cf+ak)b-2ac\left(-ma^2+cha+7c^2d\right)}{c^2}}{(cx^4+bx^2+a)^2}dx}{4a\,(b^2-4ac)}+$$

$$\frac{1}{2}\left(-\frac{\frac{4\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-3abl+2acj+b^2j-3bcg+6c^2e)}{(b^2-4ac)^{3/2}}-\frac{-16a^2l+2x^2(-3abl+2acj+b^2j-3bcg+6c^2e)-b^2\left(3g-\frac{5al}{c}\right)+2b(aj+3ce)-\frac{b^{\cdots}}{c}}{(b^2-4ac)(a+bx^2+cx^4)}}{2\,(b^2-4ac)}\right.$$

$$\frac{x\left(-\left(b^2\left(a^2m+c^2d\right)\right)+x^2\left(-bc\left(-3a^2m+ach+c^2d\right)-ab^3m+ab^2ck+2ac^2(cf-ak)\right)+2ac\left(a^2m-ach+c^2\cdots\right)\right.}{4ac^2\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \; 2206$$

$$-\frac{x\left(-\left(\left(ma^2+c^2d\right)b^2\right)+ac(cf+ak)b+\left(-amb^3+ackb^2-c\left(-3ma^2+cha+c^2d\right)b+2ac^2(cf-ak)\right)x^2+2a\cdots\right.}{4ac^2\,(b^2-4ac)\,(cx^4+bx^2+a)^2}$$

$$\frac{1}{2}\left(-\frac{-alb^2+c(ce+aj)b+\left(-lb^3+c(bj+3al)b+2c^3e-c^2(bg+2aj)\right)x^2-2ac(cg-al)}{2c^2\,(b^2-4ac)\,(cx^4+bx^2+a)^2}-\frac{\frac{4(jb^2-3cgb-3alb+6c^2\cdots}{\cdots}}{(b\cdots}\right.$$

$$\frac{x\left(\left((ma^2+3c^2d)b^3+ac(cf+3ak)b^2-4ac(4ma^2+3cha+6c^2d)b+4a^2c^2(5cf+3ak)\right)x^2+c\left(\left(3d-\frac{2a^2m}{c^2}\right)b^4+\frac{a(cf+2ak)b^3}{c}-a\left(-\frac{11ma^2}{c}+7ha+25cd\cdots\right)\right.\right.}{2ac(b^2-4ac)(cx^4+bx^2+a)}$$

$$\Big\downarrow \; 25$$

$$-\frac{x\left(-\left(\left(ma^2+c^2d\right)b^2\right)+ac(cf+ak)b+\left(-amb^3+ackb^2-c\left(-3ma^2+cha+c^2d\right)b+2ac^2(cf-ak)\right)x^2+2a\cdots\right.}{4ac^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}$$

$$\frac{1}{2}\left(-\frac{-alb^2+c(ce+aj)b+\left(-lb^3+c(bj+3al)b+2c^3e-c^2(bg+2aj)\right)x^2-2ac(cg-al)}{2c^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}-\frac{\frac{4\left(jb^2-3cgb-3alb+6c^2\cdots\right.}{(b\cdots}}{\cdots}\right.$$

$$\frac{\frac{x\left(\left(\left(ma^2+3c^2d\right)b^3+ac(cf+3ak)b^2-4ac\left(4ma^2+3cha+6c^2d\right)b+4a^2c^2(5cf+3ak)\right)x^2+c\left(\left(3d-\frac{2a^2m}{c^2}\right)b^4+\frac{a(cf+2ak)b^3}{c}-a\left(-\frac{11ma^2}{c}+7ha+25cd\cdots\right.\right.\right.}{2ac\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)}}{\cdots}$$

$\downarrow$ 27

$$-\frac{x\left(-\left(\left(ma^2+c^2d\right)b^2\right)+ac(cf+ak)b+\left(-amb^3+ackb^2-c\left(-3ma^2+cha+c^2d\right)b+2ac^2(cf-ak)\right)x^2+2a\cdots\right.}{4ac^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}$$

$$\frac{1}{2}\left(-\frac{-alb^2+c(ce+aj)b+\left(-lb^3+c(bj+3al)b+2c^3e-c^2(bg+2aj)\right)x^2-2ac(cg-al)}{2c^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}-\frac{\frac{4\left(jb^2-3cgb-3alb+6c^2\cdots\right.}{(b\cdots}}{\cdots}\right.$$

$$\frac{\frac{x\left(\left(\left(ma^2+3c^2d\right)b^3+ac(cf+3ak)b^2-4ac\left(4ma^2+3cha+6c^2d\right)b+4a^2c^2(5cf+3ak)\right)x^2+c\left(\left(3d-\frac{2a^2m}{c^2}\right)b^4+\frac{a(cf+2ak)b^3}{c}-a\left(-\frac{11ma^2}{c}+7ha+25cd\cdots\right.\right.\right.}{2ac\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)}}{\cdots}$$

$\downarrow$ 1480

$$-\frac{x\left(-\left(\left(ma^2+c^2d\right)b^2\right)+ac(cf+ak)b+\left(-amb^3+ackb^2-c\left(-3ma^2+cha+c^2d\right)b+2ac^2(cf-ak)\right)x^2+2a\cdots\right.}{4ac^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}$$

$$\frac{1}{2}\left(-\frac{-alb^2+c(ce+aj)b+\left(-lb^3+c(bj+3al)b+2c^3e-c^2(bg+2aj)\right)x^2-2ac(cg-al)}{2c^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}-\frac{\frac{4\left(jb^2-3cgb-3alb+6c^2\cdots\right.}{(b\cdots}}{\cdots}\right.$$

$$\frac{\frac{x\left(\left(\left(ma^2+3c^2d\right)b^3+ac(cf+3ak)b^2-4ac\left(4ma^2+3cha+6c^2d\right)b+4a^2c^2(5cf+3ak)\right)x^2+c\left(\left(3d-\frac{2a^2m}{c^2}\right)b^4+\frac{a(cf+2ak)b^3}{c}-a\left(-\frac{11ma^2}{c}+7ha+25cd\cdots\right.\right.\right.}{2ac\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)}}{\cdots}$$

$\downarrow$ 218

$$-\frac{x(-((ma^2+c^2d)\,b^2)+ac(cf+ak)b+(-amb^3+ackb^2-c(-3ma^2+cha+c^2d)\,b+2ac^2(cf-ak))\,x^2+2a}{4ac^2\,(b^2-4ac)\,(cx^4+bx^2+a)^2}$$

$$\frac{\frac{x(((ma^2+3c^2d)b^3+ac(cf+3ak)b^2-4ac(4ma^2+3cha+6c^2d)b+4a^2c^2(5cf+3ak))x^2+c((3d-\frac{2a^2m}{c^2})b^4+\frac{a(cf+2ak)b^3}{c}-a(-\frac{11ma^2}{c}+7ha+25cd}{2ac(b^2-4ac)(cx^4+bx^2+a)}}{}$$

$$\frac{1}{2}\Bigg(-\frac{-alb^2+c(ce+aj)b+(-lb^3+c(bj+3al)b+2c^3e-c^2(bg+2aj))\,x^2-2ac(cg-al)}{2c^2\,(b^2-4ac)\,(cx^4+bx^2+a)^2}-\frac{4(jb^2-3cgb-3alb+6c^2}{(b}$$

input

```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
 b*x^2 + c*x^4)^3,x]
```

output

```
-1/4*(x*(a*b*c*(c*f + a*k) - b^2*(c^2*d + a^2*m) + 2*a*c*(c^2*d - a*c*h +
a^2*m) + (a*b^2*c*k + 2*a*c^2*(c*f - a*k) - a*b^3*m - b*c*(c^2*d + a*c*h -
 3*a^2*m))*x^2))/(a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((x*(c*(4*a
^2*b*(2*c*f + a*k) + (a*b^3*(c*f + 2*a*k))/c + 4*a^2*(7*c^2*d + a*c*h - 9*
a^2*m) + b^4*(3*d - (2*a^2*m)/c^2) - a*b^2*(25*c*d + 7*a*h - (11*a^2*m)/c)
) + (a*b^2*c*(c*f + 3*a*k) + 4*a^2*c^2*(5*c*f + 3*a*k) + b^3*(3*c^2*d + a^
2*m) - 4*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*m))*x^2))/(2*a*c*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) + (((a*b^2*c*(c*f + 3*a*k) + 4*a^2*c^2*(5*c*f + 3*a*k)
 + b^3*(3*c^2*d + a^2*m) - 4*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*m) + (a*b^3*
c*(c*f - 3*a*k) - 4*a^2*b*c^2*(13*c*f + 9*a*k) - 6*a*b^2*c*(5*c^2*d - 3*a*
c*h - 3*a^2*m) + b^4*(3*c^2*d - a^2*m) + 8*a^2*c^2*(21*c^2*d + 3*a*c*h + 5
*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((a*b^2*c*(c*f
+ 3*a*k) + 4*a^2*c^2*(5*c*f + 3*a*k) + b^3*(3*c^2*d + a^2*m) - 4*a*b*c*(6*
c^2*d + 3*a*c*h + 4*a^2*m) - (a*b^3*c*(c*f - 3*a*k) - 4*a^2*b*c^2*(13*c*f
+ 9*a*k) - 6*a*b^2*c*(5*c^2*d - 3*a*c*h - 3*a^2*m) + b^4*(3*c^2*d - a^2*m)
 + 8*a^2*c^2*(21*c^2*d + 3*a*c*h + 5*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(2*a*c*(b^2 - 4*a*c)))/(4*a*(b^2 - 4*a*c)) + (-1/2*(b*
c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*g - a*l) + (2*c^3*e - c^2*(b*g + 2*a...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.02 (sec) , antiderivative size = 1167, normalized size of antiderivative = 1.01

method	result	size
risch	Expression too large to display	1167
default	Expression too large to display	1987

input
```
int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,
method=_RETURNVERBOSE)
```

output
```
(-1/8*(16*a^3*b*c*m-12*a^3*c^2*k-a^2*b^3*m-3*a^2*b^2*c*k+12*a^2*b*c^2*h-20
*a^2*c^3*f-a*b^2*c^2*f+24*a*b*c^3*d-3*b^3*c^2*d)/a^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^7-1/2*c*(3*a*b*l-2*a*c*j-b^2*j+3*b*c*g-6*c^2*e)/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^6-1/8/a^2*(36*a^4*c^2*m+5*a^3*b^2*c*m-16*a^3*b*c^2*k-4*a^3*c^3*
h+a^2*b^4*m-5*a^2*b^3*c*k+19*a^2*b^2*c^2*h-28*a^2*b*c^3*f-28*a^2*c^4*d-2*a
*b^3*c^2*f+49*a*b^2*c^3*d-6*b^4*c^2*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^5-1/
4*(16*a^2*c^2*l+a*b^2*c*l-6*a*b*c^2*j+b^4*l-3*b^3*c*j+9*b^2*c^2*g-18*b*c^3
*e)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^4-1/8/c*(28*a^4*b*c*m+4*a^4*c^2*k+2*a^3
*b^3*m-19*a^3*b^2*c*k+16*a^3*b*c^2*h-36*a^3*c^3*f+5*a^2*b^3*c*h-5*a^2*b^2*
c^2*f+4*a^2*b*c^3*d-a*b^4*c*f+20*a*b^3*c^2*d-3*b^5*c*d)/a^2/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^3-1/2/c*(5*a^2*b*c*l+2*a^2*c^2*j+a*b^3*l-5*a*b^2*c*j+5*a*b*
c^2*g-10*a*c^3*e+b^3*c*g-2*b^2*c^2*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(
20*a^4*c*m+a^3*b^2*m-12*a^3*b*c*k+12*a^3*c^2*h+3*a^2*b^2*c*h-16*a^2*b*c^2*
f-44*a^2*c^3*d+a*b^3*c*f+37*a*b^2*c^2*d-5*b^4*c*d)/(16*a^2*c^2-8*a*b^2*c+b
^4)/c/a*x-1/4/c*(8*a^3*c*l+a^2*b^2*l-6*a^2*b*c*j+8*a^2*c^2*g+a*b^2*c*g-10*
a*b*c^2*e+b^3*c*e)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/16*sum(
(-(16*a^3*b*c*m-12*a^3*c^2*k-a^2*b^3*m-3*a^2*b^2*c*k+12*a^2*b*c^2*h-20*a^2
*c^3*f-a*b^2*c^2*f+24*a*b*c^3*d-3*b^3*c^2*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4
)/c*_R^2-8*(3*a*b*l-2*a*c*j-b^2*j+3*b*c*g-6*c^2*e)/(16*a^2*c^2-8*a*b^2*c+b
^4)*_R+1/c*(20*a^4*c*m+a^3*b^2*m-12*a^3*b*c*k+12*a^3*c^2*h+3*a^2*b^2*c*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{(a + bx^2 + cx^4)^3} dx = \text{Timed out}$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a
)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{(a + bx^2 + cx^4)^3} dx = \text{Timed out}$$

input
```
integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4
+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{(a + bx^2 + cx^4)^3} dx$$
$$= \int \frac{mx^8 + lx^7 + kx^6 + jx^5 + hx^4 + gx^3 + fx^2 + ex + d}{(cx^4 + bx^2 + a)^3} dx$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a
)^3,x, algorithm="maxima")
```

output
```
-1/8*((12*a^2*b*c^3*h - 3*(b^3*c^3 - 8*a*b*c^4)*d - (a*b^2*c^3 + 20*a^2*c^
4)*f - 3*(a^2*b^2*c^2 + 4*a^3*c^3)*k - (a^2*b^3*c - 16*a^3*b*c^2)*m)*x^7 -
 12*a^4*b*c*j - 4*(6*a^2*c^4*e - 3*a^2*b*c^3*g - 3*a^3*b*c^2*l + (a^2*b^2*
c^2 + 2*a^3*c^3)*j)*x^6 - ((6*b^4*c^2 - 49*a*b^2*c^3 + 28*a^2*c^4)*d + 2*(
a*b^3*c^2 + 14*a^2*b*c^3)*f - (19*a^2*b^2*c^2 - 4*a^3*c^3)*h + (5*a^2*b^3*
c + 16*a^3*b*c^2)*k - (a^2*b^4 + 5*a^3*b^2*c + 36*a^4*c^2)*m)*x^5 - 2*(18*
a^2*b*c^3*e - 9*a^2*b^2*c^2*g + 3*(a^2*b^3*c + 2*a^3*b*c^2)*j - (a^2*b^4 +
 a^3*b^2*c + 16*a^4*c^2)*l)*x^4 - ((3*b^5*c - 20*a*b^3*c^2 - 4*a^2*b*c^3)*
d + (a*b^4*c + 5*a^2*b^2*c^2 + 36*a^3*c^3)*f - (5*a^2*b^3*c + 16*a^3*b*c^2
)*h + (19*a^3*b^2*c - 4*a^4*c^2)*k - 2*(a^3*b^3 + 14*a^4*b*c)*m)*x^3 - 4*(
2*(a^2*b^2*c^2 + 5*a^3*c^3)*e - (a^2*b^3*c + 5*a^3*b*c^2)*g + (5*a^3*b^2*c
 - 2*a^4*c^2)*j - (a^3*b^3 + 5*a^4*b*c)*l)*x^2 + 2*(a^2*b^3*c - 10*a^3*b*c
^2)*e + 2*(a^3*b^2*c + 8*a^4*c^2)*g + 2*(a^4*b^2 + 8*a^5*c)*l - (12*a^4*b*
c*k + (5*a*b^4*c - 37*a^2*b^2*c^2 + 44*a^3*c^3)*d - (a^2*b^3*c - 16*a^3*b*
c^2)*f - 3*(a^3*b^2*c + 4*a^4*c^2)*h - (a^4*b^2 + 20*a^5*c)*m)*x)/(a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3 + (a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^
5)*x^8 + 2*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*x^6 + (a^2*b^6*c -
 6*a^3*b^4*c^2 + 32*a^5*c^4)*x^4 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b
*c^3)*x^2) - 1/8*integrate((12*a^3*b*c*k + (12*a^2*b*c^2*h - 3*(b^3*c^2 -
8*a*b*c^3)*d - (a*b^2*c^2 + 20*a^2*c^3)*f - 3*(a^2*b^2*c + 4*a^3*c^2)*k...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22427 vs. 2(1098) = 2196.

Time = 3.05 (sec) , antiderivative size = 22427, normalized size of antiderivative = 19.50

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{(a + bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a
)^3,x, algorithm="giac")
```

output
```
1/64*(3*(2*b^5*c^4 - 24*a*b^3*c^5 + 64*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 16*(b^2 - 4*a*c)*a*b
*c^5)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)^2*d + (2*a*b^4*c^4 + 32*a^2
*b^2*c^5 - 160*a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^3*c^3 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*c^4 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^4 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^4 - 40*(b^2 - 4*a*c)*a^2*c^5)*(a^2*b^4*c
- 8*a^3*b^2*c^2 + 16*a^4*c^3)^2*f - 12*(2*a^2*b^3*c^4 - 8*a^3*b*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 2*sqr...
```

Mupad [B] (verification not implemented)

Time = 66.82 (sec) , antiderivative size = 114377, normalized size of antiderivative = 99.46

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
 b*x^2 + c*x^4)^3,x)
```

output
```
symsum(log(root(56371445760*a^11*b^8*c^9*z^4 - 503316480*a^8*b^14*c^6*z^4
+ 47185920*a^7*b^16*c^5*z^4 - 2621440*a^6*b^18*c^4*z^4 + 65536*a^5*b^20*c^
3*z^4 - 171798691840*a^14*b^2*c^12*z^4 + 193273528320*a^13*b^4*c^11*z^4 -
128849018880*a^12*b^6*c^10*z^4 - 16911433728*a^10*b^10*c^8*z^4 + 352321536
0*a^9*b^12*c^7*z^4 + 68719476736*a^15*c^13*z^4 + 1536*a^5*b^16*c*k*m*z^2 +
 1536*a*b^18*c^3*d*f*z^2 - 2571632640*a^9*b^5*c^8*d*m*z^2 + 2548039680*a^9
*b^3*c^10*d*h*z^2 + 1509949440*a^10*b^3*c^9*e*l*z^2 + 1509949440*a^9*b^3*c
^10*e*g*z^2 - 1401421824*a^8*b^5*c^9*d*h*z^2 - 1321205760*a^9*b^2*c^11*d*f
*z^2 - 2793406464*a^11*b*c^10*d*m*z^2 + 890634240*a^8*b^7*c^7*d*m*z^2 - 75
4974720*a^10*b^4*c^8*g*l*z^2 - 754974720*a^9*b^5*c^8*e*l*z^2 + 719585280*a
^8*b^6*c^8*d*k*z^2 - 707788800*a^9*b^4*c^9*d*k*z^2 - 754974720*a^8*b^5*c^9
*e*g*z^2 + 603979776*a^11*b^2*c^9*g*l*z^2 - 581959680*a^10*b^4*c^8*f*m*z^2
 + 732168192*a^7*b^6*c^9*d*f*z^2 + 534773760*a^11*b^3*c^8*h*m*z^2 - 456130
560*a^11*b^4*c^7*k*m*z^2 - 603979776*a^10*b^2*c^10*e*j*z^2 + 534773760*a^1
0*b^3*c^9*f*k*z^2 + 384040960*a^9*b^6*c^7*f*m*z^2 + 377487360*a^9*b^6*c^7*
g*l*z^2 - 456130560*a^9*b^4*c^9*f*h*z^2 + 301989888*a^11*b^3*c^8*j*l*z^2 -
 415236096*a^10*b^2*c^10*d*k*z^2 + 254017536*a^10*b^6*c^6*k*m*z^2 - 330301
440*a^10*b^4*c^8*h*k*z^2 + 390463488*a^7*b^7*c^8*d*h*z^2 + 188743680*a^12*
b^2*c^8*k*m*z^2 + 301989888*a^10*b^3*c^9*g*j*z^2 - 297861120*a^7*b^8*c^7*d
*k*z^2 - 366280704*a^6*b^8*c^8*d*f*z^2 + 188743680*a^11*b^2*c^9*h*k*z^2...
```

Reduce [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 32760, normalized size of antiderivative = 28.49

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

output
```
(96*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**6*b**2*
c**2*l - 64*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*6*b*c**3*j - 32*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b
))*a**5*b**3*c**2*j + 192*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(
a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*a**5*b**3*c**2*l*x**2 + 96*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*
sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt
(2*sqrt(c)*sqrt(a) + b))*a**5*b**2*c**3*g - 128*sqrt(2*sqrt(c)*sqrt(a) + b
)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b**2*c**3*j*x**2 + 192*sqrt(2*sqrt(
c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b**2*c**3*l*x**4 - 19
2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b*c**4*
e - 128*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b
*c**4*j*x**4 - 64*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - ...
```

3.55 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^6+kx^7}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 641

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^6+kx^7}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{x(b^2cd-2ac(cd-ah)-ab(cf+aj)+(bc(cd+ah)-ab^2j-2ac(cf-aj))x^2)}{2ac(b^2-4ac)(a+bx^2+cx^4)}$$
$$-\frac{bc(ce+ai)-ab^2k-2ac(cg-ak)+(2c^3e-c^2(bg+2ai)-b^3k+bc(bi+3ak))x^2}{2c^2(b^2-4ac)(a+bx^2+cx^4)}$$
$$+\frac{\left(b(cd+ah)+\frac{ab^2j}{c}-2a(cf+3aj)+\frac{b^2c(cd-ah)-4ac^2(3cd+ah)-ab^3j+4abc(cf+2aj)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(b(cd+ah)+\frac{ab^2j}{c}-2a(cf+3aj)-\frac{b^2c(cd-ah)-4ac^2(3cd+ah)-ab^3j+4abc(cf+2aj)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+\frac{(4c^3e-c^2(2bg-4ai)+b^3k-6abck)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}}+\frac{k\log(a+bx^2+cx^4)}{4c^2}$$

output
```
1/2*x*(b^2*c*d-2*a*c*(-a*h+c*d)-a*b*(a*j+c*f)+(b*c*(a*h+c*d)-a*b^2*j-2*a*c
*(-a*j+c*f))*x^2)/a/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(b*c*(a*i+c*e)-a*b^
2*k-2*a*c*(-a*k+c*g)+(2*c^3*e-c^2*(2*a*i+b*g)-b^3*k+b*c*(3*a*k+b*i))*x^2)/
c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*(b*(a*h+c*d)+a*b^2*j/c-2*a*(3*a*j+c*f
)+(b^2*c*(-a*h+c*d)-4*a*c^2*(a*h+3*c*d)-a*b^3*j+4*a*b*c*(2*a*j+c*f))/c/(-4
*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^
(1/2)/a/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*(b*(a*h+c*d)
+a*b^2*j/c-2*a*(3*a*j+c*f)-(b^2*c*(-a*h+c*d)-4*a*c^2*(a*h+3*c*d)-a*b^3*j+4
*a*b*c*(2*a*j+c*f))/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*
a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2)+1/2*(4*c^3*e-c^2*(-4*a*i+2*b*g)+b^3*k-6*a*b*c*k)*arctanh((2*c*x^2
+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*k*ln(c*x^4+b*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 4.09 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.21

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^6+kx^7}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2(2a^3ck-bc^2dx(b+cx^2)+a(-b^3kx^2+b^2cx^2(i+jx)+2c^3x(d+x(e+fx))+bc^2(e+x(f-x(g+hx))))+a^2(-b^2k+bc(i+x(j+3kx))-2c^2(g+x(h+x(\cdots}{a(-b^2+4ac)(a+bx^2+cx^4)}}{\cdots}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^6 + k*x^7)/(a + b
*x^2 + c*x^4)^2,x]
```

output

```
((2*(2*a^3*c*k - b*c^2*d*x*(b + c*x^2) + a*(-(b^3*k*x^2) + b^2*c*x^2*(i +
j*x) + 2*c^3*x*(d + x*(e + f*x)) + b*c^2*(e + x*(f - x*(g + h*x)))) + a^2*
(-(b^2*k) + b*c*(i + x*(j + 3*k*x)) - 2*c^2*(g + x*(h + x*(i + j*x))))))/(
a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*Sqrt[c]*(a*b^3*j - b*c*(c
*Sqrt[b^2 - 4*a*c]*d + 4*a*c*f + a*Sqrt[b^2 - 4*a*c]*h + 8*a^2*j) - b^2*(c
^2*d - a*c*h + a*Sqrt[b^2 - 4*a*c]*j) + 2*a*c*(6*c^2*d + c*Sqrt[b^2 - 4*a*
c]*f + 2*a*c*h + 3*a*Sqrt[b^2 - 4*a*c]*j))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[2]*Sqrt[c]*(a*b^3*j + b*c*(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*f + a
*Sqrt[b^2 - 4*a*c]*h - 8*a^2*j) + 2*a*c*(6*c^2*d - c*Sqrt[b^2 - 4*a*c]*f +
 2*a*c*h - 3*a*Sqrt[b^2 - 4*a*c]*j) + b^2*(-(c^2*d) + a*c*h + a*Sqrt[b^2 -
 4*a*c]*j))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((-4*c^3*e + 2*c^2*(b*g -
 2*a*i) + b^2*(-b + Sqrt[b^2 - 4*a*c])*k + a*c*(6*b*k - 4*Sqrt[b^2 - 4*a*c
]*k))*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((4*c^3
*e + c^2*(-2*b*g + 4*a*i) + b^2*(b + Sqrt[b^2 - 4*a*c])*k - 2*a*c*(3*b + 2
*Sqrt[b^2 - 4*a*c])*k)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)
^(3/2))/(4*c^2)
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.220$, Rules used = {2202, 2194, 2191, 1142, 1083, 219, 1103, 2206, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^6+kx^7}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{jx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^2}dx+\int \frac{x(kx^6+ix^4+gx^2+e)}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\int \frac{jx^6+hx^4+fx^2+d}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}\int \frac{kx^6+ix^4+gx^2+e}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow 2191$$

$$\frac{1}{2}\left(-\frac{\int \frac{\left(4a-\frac{b^2}{c}\right)kx^2+2ce-bg+2ai-\frac{abk}{c}}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak+bi)+b^3(-k)+2c^3e\right)-ab^2k+bc(ai+ce}{c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right.$$

$$\int \frac{jx^6+hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow 1142$$

$$\frac{1}{2}\left(-\frac{\frac{\left(-c^2(2bg-4ai)-6abck+b^3k+4c^3e\right)\int \frac{1}{cx^4+bx^2+a}dx^2}{2c^2}-\frac{k\left(b^2-4ac\right)\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak+bi)+b\right.}{c^2\left(b^2\right.}\right.$$

$$\int \frac{jx^6+hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow 1083$$

$$\frac{1}{2}\left(-\frac{-\frac{k\left(b^2-4ac\right)\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c^2}-\frac{\left(-c^2(2bg-4ai)-6abck+b^3k+4c^3e\right)\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak\right.}{}\right.$$

$$\int \frac{jx^6+hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow 219$$

$$\frac{1}{2}\left(-\frac{-\frac{k\left(b^2-4ac\right)\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c^2}-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4ai)-6abck+b^3k+4c^3e\right)}{c^2\sqrt{b^2-4ac}}}{b^2-4ac}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak+\right.}{}\right.$$

$$\int \frac{jx^6+hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow 1103$$

$$\int \frac{jx^6+hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^2}dx+$$

$$\frac{1}{2}\left(-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4ai)-6abck+b^3k+4c^3e\right)}{c^2\sqrt{b^2-4ac}}-\frac{k\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak+\right.}{}\right.$$

$$\Big\downarrow \ 2206$$

$$-\frac{\int -\frac{db^2+\frac{a(cf+aj)b}{c}+\left(\frac{ajb^2}{c}+(cd+ah)b-2a(cf+3aj)\right)x^2-2a(3cd+ah)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+$$

$$\frac{1}{2}\left(-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4ai)-6abck+b^3k+4c^3e\right)}{c^2\sqrt{b^2-4ac}}-\frac{k\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak+\right.}{c}\right.$$

$$\frac{x\left(x^2\left(-ab^2j+bc(ah+cd)-2ac(cf-aj)\right)+c\left(-\frac{ab(aj+cf)}{c}-2a(cd-ah)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{db^2+\frac{a(cf+aj)b}{c}+\left(\frac{ajb^2}{c}+(cd+ah)b-2a(cf+3aj)\right)x^2-2a(3cd+ah)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+$$

$$\frac{1}{2}\left(-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4ai)-6abck+b^3k+4c^3e\right)}{c^2\sqrt{b^2-4ac}}-\frac{k\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak+\right.}{c}\right.$$

$$\frac{x\left(x^2\left(-ab^2j+bc(ah+cd)-2ac(cf-aj)\right)+c\left(-\frac{ab(aj+cf)}{c}-2a(cd-ah)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 1480$$

$$\frac{\frac{1}{2}\left(\frac{ab^2j}{c}+\frac{-ab^3j+b^2c(cd-ah)+4abc(2aj+cf)-4ac^2(ah+3cd)}{c\sqrt{b^2-4ac}}+b(ah+cd)-2a(3aj+cf)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{ab^2}{c}\right.}{2a\left(b^2-4ac\right)}$$

$$\frac{1}{2}\left(-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4ai)-6abck+b^3k+4c^3e\right)}{c^2\sqrt{b^2-4ac}}-\frac{k\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak+\right.}{c}\right.$$

$$\frac{x\left(x^2\left(-ab^2j+bc(ah+cd)-2ac(cf-aj)\right)+c\left(-\frac{ab(aj+cf)}{c}-2a(cd-ah)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2j}{c}+\frac{-ab^3j+b^2c(cd-ah)+4abc(2aj+cf)-4ac^2(ah+3cd)}{c\sqrt{b^2-4ac}}+b(ah+cd)-2a(3aj+cf)\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{ab^2j}{c}-\cdots\right.}{\cdots}}{2a\left(b^2-4ac\right)}$$

$$\frac{1}{2}\left(-\frac{-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4ai)-6abck+b^3k+4c^3e\right)}{c^2\sqrt{b^2-4ac}}-\frac{k\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2c^2}}{b^2-4ac}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak+\cdots\right.}{\cdots}\right.$$

$$\frac{x\left(x^2\left(-ab^2j+bc(ah+cd)-2ac(cf-aj)\right)+c\left(-\frac{ab(aj+cf)}{c}-2a(cd-ah)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input

```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^6 + k*x^7)/(a + b*x^2 +
 c*x^4)^2,x]
```

output

```
(x*(c*(b^2*d - 2*a*(c*d - a*h) - (a*b*(c*f + a*j))/c) + (b*c*(c*d + a*h) -
 a*b^2*j - 2*a*c*(c*f - a*j))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (((b*(c*d + a*h) + (a*b^2*j)/c - 2*a*(c*f + 3*a*j) + (b^2*c*(c*d - a
*h) - 4*a*c^2*(3*c*d + a*h) - a*b^3*j + 4*a*b*c*(c*f + 2*a*j))/(c*Sqrt[b^2
 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*h) + (a*b^2*j)/c -
 2*a*(c*f + 3*a*j) - (b^2*c*(c*d - a*h) - 4*a*c^2*(3*c*d + a*h) - a*b^3*j
+ 4*a*b*c*(c*f + 2*a*j))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
]))/(2*a*(b^2 - 4*a*c)) + (-((b*c*(c*e + a*i) - a*b^2*k - 2*a*c*(c*g - a*k
) + (2*c^3*e - c^2*(b*g + 2*a*i) - b^3*k + b*c*(b*i + 3*a*k))*x^2)/(c^2*(b
^2 - 4*a*c)*(a + b*x^2 + c*x^4))) - (-(((4*c^3*e - c^2*(2*b*g - 4*a*i) + b
^3*k - 6*a*b*c*k)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
- 4*a*c])) - ((b^2 - 4*a*c)*k*Log[a + b*x^2 + c*x^4])/(2*c^2))/(b^2 - 4*a*
c))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.66

method	result
risch	$\frac{-\frac{\left(2a^2cj-a\,b^2j+abch-2a\,c^2f+b\,c^2d\right)x^3}{2a\left(4ac-b^2\right)c}+\frac{\left(3abck-2a\,c^2i-b^3k+b^2ci-b\,c^2g+2c^3e\right)x^2}{2\left(4ac-b^2\right)c^2}+\frac{\left(a^2bj-2a^2ch+abcf+2a\,c^2d-b^2cd\right)x}{2ac\left(4ac-b^2\right)}+\frac{2a^2ck-a\,b^2k+c}{2\left(4ac\right.}}{c\,x^4+b\,x^2+a}$
default	Expression too large to display

input
```
int((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,method
=_RETURNVERBOSE)
```

output
```
(-1/2/a*(2*a^2*c*j-a*b^2*j+a*b*c*h-2*a*c^2*f+b*c^2*d)/(4*a*c-b^2)/c*x^3+1/
2*(3*a*b*c*k-2*a*c^2*i-b^3*k+b^2*c*i-b*c^2*g+2*c^3*e)/(4*a*c-b^2)/c^2*x^2+
1/2*(a^2*b*j-2*a^2*c*h+a*b*c*f+2*a*c^2*d-b^2*c*d)/a/c/(4*a*c-b^2)*x+1/2*(2
*a^2*c*k-a*b^2*k+a*b*c*i-2*a*c^2*g+b*c^2*e)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+
a)+1/4/c*sum((2*k*_R^3+1/a*(6*a^2*c*j-a*b^2*j-a*b*c*h+2*a*c^2*f-b*c^2*d)/(
4*a*c-b^2)*_R^2-2*(a*b*k-2*a*c*i+b*c*g-2*c^2*e)/(4*a*c-b^2)*_R-1/a*(a^2*b*
j-2*a^2*c*h+a*b*c*f-6*a*c^2*d+b^2*c*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_
R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^6+kx^7}{(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,
 algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^6+kx^7}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((k*x**7+j*x**6+i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2
+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^6+kx^7}{(a+bx^2+cx^4)^2}\,dx = \int \frac{kx^7+jx^6+ix^5+hx^4+gx^3+fx^2+ex+d}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,
 algorithm="maxima")
```

output
```
-1/2*(a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*i - (b*c^3*d - 2*a*c^3*f + a*b*c^2
*h - (a*b^2*c - 2*a^2*c^2)*j)*x^3 + (2*a*c^3*e - a*b*c^2*g + (a*b^2*c - 2*
a^2*c^2)*i - (a*b^3 - 3*a^2*b*c)*k)*x^2 - (a^2*b^2 - 2*a^3*c)*k + (a*b*c^2
*f - 2*a^2*c^2*h + a^2*b*c*j - (b^2*c^2 - 2*a*c^3)*d)*x)/(a^2*b^2*c^2 - 4*
a^3*c^3 + (a*b^2*c^3 - 4*a^2*c^4)*x^4 + (a*b^3*c^2 - 4*a^2*b*c^3)*x^2) - 1
/2*integrate(-(2*(a*b^2 - 4*a^2*c)*k*x^3 + a*b*c*f - 2*a^2*c*h + a^2*b*j +
 (b*c^2*d - 2*a*c^2*f + a*b*c*h + (a*b^2 - 6*a^2*c)*j)*x^2 + (b^2*c - 6*a*
c^2)*d - 2*(2*a*c^2*e - a*b*c*g + 2*a^2*c*i - a^2*b*k)*x)/(c*x^4 + b*x^2 +
 a), x)/(a*b^2*c - 4*a^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16214 vs. $2(590) = 1180$.

Time = 2.27 (sec) , antiderivative size = 16214, normalized size of antiderivative = 25.29

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5 + jx^6 + kx^7}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,
 algorithm="giac")
```

output
```
1/4*k*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/16*((a^2*b^4*c^3 - 8*a^3*b^2*c^4
 + 16*a^4*c^5)^2*(2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*d - 2*(a^2*b^4*c^3 - 8*a^3*b^2*
c^4 + 16*a^4*c^5)^2*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*f + (a^2*b^4*c^3 - 8*a^3
*b^2*c^4 + 16*a^4*c^5)^2*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*h + (a^2*b^
4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)^2*(2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a
^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 +
 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2...
```

Mupad [B] (verification not implemented)

Time = 28.53 (sec) , antiderivative size = 53538, normalized size of antiderivative = 83.52

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^6+kx^7}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input

```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^6 + k*x^7)/(a + b*x^2 +
 c*x^4)^2,x)
```

output

```
((b*c^2*e - 2*a*c^2*g - a*b^2*k + 2*a^2*c*k + a*b*c*i)/(2*c^2*(4*a*c - b^2
)) + (x^2*(2*c^3*e - b^3*k - b*c^2*g - 2*a*c^2*i + b^2*c*i + 3*a*b*c*k))/(
2*c^2*(4*a*c - b^2)) + (x*(2*a*c^2*d - b^2*c*d - 2*a^2*c*h + a^2*b*j + a*b
*c*f))/(2*a*c*(4*a*c - b^2)) - (x^3*(b*c^2*d - 2*a*c^2*f - a*b^2*j + 2*a^2
*c*j + a*b*c*h))/(2*a*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + symsum(log(r
oot(1572864*a^8*b^2*c^9*z^4 - 983040*a^7*b^4*c^8*z^4 + 327680*a^6*b^6*c^7*
z^4 - 61440*a^5*b^8*c^6*z^4 + 6144*a^4*b^10*c^5*z^4 - 256*a^3*b^12*c^4*z^4
 - 1048576*a^9*c^10*z^4 - 1572864*a^8*b^2*c^7*k*z^3 + 983040*a^7*b^4*c^6*k
*z^3 - 327680*a^6*b^6*c^5*k*z^3 + 61440*a^5*b^8*c^4*k*z^3 - 6144*a^4*b^10*
c^3*k*z^3 + 256*a^3*b^12*c^2*k*z^3 + 1048576*a^9*c^8*k*z^3 + 98304*a^8*b*c
^6*i*k*z^2 + 98304*a^7*b*c^7*e*k*z^2 + 57344*a^7*b*c^7*f*j*z^2 + 32768*a^7
*b*c^7*g*i*z^2 + 57344*a^6*b*c^8*d*h*z^2 + 32768*a^6*b*c^8*e*g*z^2 - 32*a*
b^10*c^4*d*f*z^2 - 90112*a^7*b^3*c^5*i*k*z^2 + 30720*a^6*b^5*c^4*i*k*z^2 -
 4608*a^5*b^7*c^3*i*k*z^2 + 256*a^4*b^9*c^2*i*k*z^2 - 49152*a^7*b^2*c^6*g*
k*z^2 + 45056*a^6*b^4*c^5*g*k*z^2 + 24576*a^7*b^2*c^6*h*j*z^2 - 15360*a^5*
b^6*c^4*g*k*z^2 - 3072*a^5*b^6*c^4*h*j*z^2 + 2304*a^4*b^8*c^3*g*k*z^2 + 20
48*a^6*b^4*c^5*h*j*z^2 + 576*a^4*b^8*c^3*h*j*z^2 - 128*a^3*b^10*c^2*g*k*z^
2 - 32*a^3*b^10*c^2*h*j*z^2 - 90112*a^6*b^3*c^6*e*k*z^2 - 49152*a^6*b^3*c^
6*f*j*z^2 + 30720*a^5*b^5*c^5*e*k*z^2 - 24576*a^6*b^3*c^6*g*i*z^2 + 15360*
a^5*b^5*c^5*f*j*z^2 + 6144*a^5*b^5*c^5*g*i*z^2 - 4608*a^4*b^7*c^4*e*k*z...
```

Reduce [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 11710, normalized size of antiderivative = 18.27

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5 + jx^6 + kx^7}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input

```
int((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

output

```
(24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**2*
c*k - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*
b*c**2*i - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a
**3*b**4*k + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b)
)*a**3*b**3*c*k*x**2 + 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a
) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*a**3*b**2*c**2*g - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c
)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqr
t(c)*sqrt(a) + b))*a**3*b**2*c**2*i*x**2 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*
sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**2*k*x**4 - 16*sqrt(2*sqrt(c)*
sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**3*e - 16*sqrt(2*sqr
t(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a)
 - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**3*i*x**4 - 4*s
qrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqr...
```

3.56 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^8+kx^{11}}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 1177

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^8+kx^{11}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

output
```
-1/4*x*(c^2*(a*b*f-b^2*(d+a^2*j/c^2)+2*a*(c*d-a*h+a^2*j/c))+(2*a*c^3*f-a*b
^3*j-b*c*(-3*a^2*j+a*c*h+c^2*d))*x^2)/a/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2
-1/4*(b*c^3*(a*i+c*e)-a*b^4*k+4*a^2*b^2*c*k-2*a^3*c^2*k-2*a*c^4*g+(2*c^5*e
+b^2*c^3*i-c^4*(2*a*i+b*g)-b^5*k+5*a*b^3*c*k-5*a^2*b*c^2*k)*x^2)/c^4/(-4*a
*c+b^2)/(c*x^4+b*x^2+a)^2+1/8*x*(c*(a*b^3*f+8*a^2*b*c*f+4*a^2*(-9*a^2*j+a*
c*h+7*c^2*d)+b^4*(3*d-2*a^2*j/c^2)-a*b^2*(25*c*d+7*a*h-11*a^2*j/c))+(a*b^2
*c^2*f+20*a^2*c^3*f+b^3*(a^2*j+3*c^2*d)-4*a*b*c*(4*a^2*j+3*a*c*h+6*c^2*d))
*x^2)/a^2/c/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/4*(b^3*c^2*i+2*b*c^3*(a*i+3*c
*e)+11*a*b^4*k-b^6*k/c+32*a^3*c^2*k-3*b^2*(13*a^2*c*k+c^3*g)+2*(6*c^5*e+b^
2*c^3*i-c^4*(-2*a*i+3*b*g)+2*b^5*k-15*a*b^3*c*k+25*a^2*b*c^2*k)*x^2)/c^3/(
-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*(a*b^2*c^2*f+20*a^2*c^3*f+b^3*(a^2*j+3*
c^2*d)-4*a*b*c*(4*a^2*j+3*a*c*h+6*c^2*d)+(a*b^3*c^2*f-52*a^2*b*c^3*f-6*a*b
^2*c*(-3*a^2*j-3*a*c*h+5*c^2*d)+b^4*(-a^2*j+3*c^2*d)+8*a^2*c^2*(5*a^2*j+3*
a*c*h+21*c^2*d))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b
^2)^(1/2))^(1/2))*2^(1/2)/a^2/c^(3/2)/(-4*a*c+b^2)^2/(b-(-4*a*c+b^2)^(1/2)
)^(1/2)+1/16*(a*b^2*c^2*f+20*a^2*c^3*f+b^3*(a^2*j+3*c^2*d)-4*a*b*c*(4*a^2*
j+3*a*c*h+6*c^2*d)-(a*b^3*c^2*f-52*a^2*b*c^3*f-6*a*b^2*c*(-3*a^2*j-3*a*c*h
+5*c^2*d)+b^4*(-a^2*j+3*c^2*d)+8*a^2*c^2*(5*a^2*j+3*a*c*h+21*c^2*d))/(-4*a
*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1
/2)/a^2/c^(3/2)/(-4*a*c+b^2)^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)-1/2*(12*c^5...
```

Mathematica [A] (verified)

Time = 7.33 (sec) , antiderivative size = 1649, normalized size of antiderivative = 1.40

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^8+kx^{11}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^8 + k*x^11)/(a +
b*x^2 + c*x^4)^3,x]
```

output
```
(a*b*c^4*e - 2*a^2*c^4*g + a^2*b*c^3*i - a^2*b^4*k + 4*a^3*b^2*c*k - 2*a^4
*c^2*k - b^2*c^4*d*x + 2*a*c^5*d*x + a*b*c^4*f*x - 2*a^2*c^4*h*x - a^2*b^2
*c^2*j*x + 2*a^3*c^3*j*x + 2*a*c^5*e*x^2 - a*b*c^4*g*x^2 + a*b^2*c^3*i*x^2
 - 2*a^2*c^4*i*x^2 - a*b^5*k*x^2 + 5*a^2*b^3*c*k*x^2 - 5*a^3*b*c^2*k*x^2 -
 b*c^5*d*x^3 + 2*a*c^5*f*x^3 - a*b*c^4*h*x^3 - a*b^3*c^2*j*x^3 + 3*a^2*b*c
^3*j*x^3)/(4*a*c^4*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a^2*b*c^5*e
 - 6*a^2*b^2*c^4*g + 2*a^2*b^3*c^3*i + 4*a^3*b*c^4*i - 2*a^2*b^6*k + 22*a^
3*b^4*c*k - 78*a^4*b^2*c^2*k + 64*a^5*c^3*k + 3*b^4*c^4*d*x - 25*a*b^2*c^5
*d*x + 28*a^2*c^6*d*x + a*b^3*c^4*f*x + 8*a^2*b*c^5*f*x - 7*a^2*b^2*c^4*h*
x + 4*a^3*c^5*h*x - 2*a^2*b^4*c^2*j*x + 11*a^3*b^2*c^3*j*x - 36*a^4*c^4*j*
x + 24*a^2*c^6*e*x^2 - 12*a^2*b*c^5*g*x^2 + 4*a^2*b^2*c^4*i*x^2 + 8*a^3*c^
5*i*x^2 + 8*a^2*b^5*c*k*x^2 - 60*a^3*b^3*c^2*k*x^2 + 100*a^4*b*c^3*k*x^2 +
 3*b^3*c^5*d*x^3 - 24*a*b*c^6*d*x^3 + a*b^2*c^5*f*x^3 + 20*a^2*c^6*f*x^3 -
 12*a^2*b*c^5*h*x^3 + a^2*b^3*c^3*j*x^3 - 16*a^3*b*c^4*j*x^3)/(8*a^2*c^4*(
-b^2 + 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*b^4*c^2*d - 30*a*b^2*c^3*d + 16
8*a^2*c^4*d + 3*b^3*c^2*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^2 - 4*a*c]
*d + a*b^3*c^2*f - 52*a^2*b*c^3*f + a*b^2*c^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2
*c^3*Sqrt[b^2 - 4*a*c]*f + 18*a^2*b^2*c^2*h + 24*a^3*c^3*h - 12*a^2*b*c^2*
Sqrt[b^2 - 4*a*c]*h - a^2*b^4*j + 18*a^3*b^2*c*j + 40*a^4*c^2*j + a^2*b^3*
Sqrt[b^2 - 4*a*c]*j - 16*a^3*b*c*Sqrt[b^2 - 4*a*c]*j)*ArcTan[(Sqrt[2]*S...
```

Rubi [A] (verified)

Time = 3.04 (sec) , antiderivative size = 1236, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2202, 2194, 2191, 2191, 27, 1142, 1083, 219, 1103, 2206, 25, 2206, 25, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^8+kx^{11}}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{jx^8+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx+\int \frac{x\left(kx^{10}+ix^4+gx^2+e\right)}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{2194}$$

$$\int \frac{jx^8+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx+\frac{1}{2}\int \frac{kx^{10}+ix^4+gx^2+e}{(cx^4+bx^2+a)^3}dx^2$$

$\big\downarrow$ 2191

$$\frac{1}{2}\left(\frac{c^4\left(\frac{2a^3k}{c^2}-\frac{4a^2b^2k}{c^3}+\frac{ab^4k}{c^4}-b\left(\frac{ai}{c}+e\right)+2ag\right)-x^2\left(-5a^2bc^2k+5ab^3ck-c^4(2ai+bg)+b^5(-k)+b^2c^3i+2c^5\right.}{2c^4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right.$$

$$\int \frac{jx^8+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$\big\downarrow$ 2191

$$\frac{1}{2}\left(\frac{c^4\left(\frac{2a^3k}{c^2}-\frac{4a^2b^2k}{c^3}+\frac{ab^4k}{c^4}-b\left(\frac{ai}{c}+e\right)+2ag\right)-x^2\left(-5a^2bc^2k+5ab^3ck-c^4(2ai+bg)+b^5(-k)+b^2c^3i+2c^5\right.}{2c^4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right.$$

$$\int \frac{jx^8+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(\frac{c^4\left(\frac{2a^3k}{c^2}-\frac{4a^2b^2k}{c^3}+\frac{ab^4k}{c^4}-b\left(\frac{ai}{c}+e\right)+2ag\right)-x^2\left(-5a^2bc^2k+5ab^3ck-c^4(2ai+bg)+b^5(-k)+b^2c^3i+2c^5\right.}{2c^4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right.$$

$$\int \frac{jx^8+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$\big\downarrow$ 1142

$$\frac{1}{2}\left(\frac{c^4\left(\frac{2a^3k}{c^2}-\frac{4a^2b^2k}{c^3}+\frac{ab^4k}{c^4}-b\left(\frac{ai}{c}+e\right)+2ag\right)-x^2\left(-5a^2bc^2k+5ab^3ck-c^4(2ai+bg)+b^5(-k)+b^2c^3i+2c^5\right.}{2c^4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right.$$

$$\int \frac{jx^8+hx^4+fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$\big\downarrow$ 1083

$$\frac{1}{2}\left(\frac{c^4\left(\frac{2a^3k}{c^2}-\frac{4a^2b^2k}{c^3}+\frac{ab^4k}{c^4}-b\left(\frac{ai}{c}+e\right)+2ag\right)-x^2\left(-5a^2bc^2k+5ab^3ck-c^4(2ai+bg)+b^5(-k)+b^2c^3i+2c^5\right.}{2c^4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right.$$

$$\int \frac{jx^8+hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^3}dx$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{c^4\left(\frac{2a^3k}{c^2}-\frac{4a^2b^2k}{c^3}+\frac{ab^4k}{c^4}-b\left(\frac{ai}{c}+e\right)+2ag\right)-x^2\left(-5a^2bc^2k+5ab^3ck-c^4(2ai+bg)+b^5(-k)+b^2c^3i+2c^5\right.}{2c^4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right.$$

$$\int \frac{jx^8+hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^3}dx$$

$$\Big\downarrow \text{ 1103}$$

$$\int \frac{jx^8+hx^4+fx^2+d}{\left(cx^4+bx^2+a\right)^3}dx+$$

$$\frac{1}{2}\left(\frac{c^4\left(\frac{2a^3k}{c^2}-\frac{4a^2b^2k}{c^3}+\frac{ab^4k}{c^4}-b\left(\frac{ai}{c}+e\right)+2ag\right)-x^2\left(-5a^2bc^2k+5ab^3ck-c^4(2ai+bg)+b^5(-k)+b^2c^3i+2c^5\right.}{2c^4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right.$$

$$\Big\downarrow \text{ 2206}$$

$$-\frac{\int -\frac{-4a\left(4a-\frac{b^2}{c}\right)jx^4-\frac{\left(-ajb^3-c\left(ja^2+5cha+5c^2d\right)b+10ac^3f\right)x^2}{c^2}+abf+b^2\left(3d-\frac{a^2j}{c^2}\right)-2a\left(-\frac{ja^2}{c}+ha+7cd\right)}{(cx^4+bx^2+a)^2}dx}{4a\left(b^2-4ac\right)}-$$

$$\frac{x\left(x^2\left(-bc(-3a^2j+ach+c^2d)-ab^3j+2ac^3f\right)+c^2\left(-\left(b^2\left(\frac{a^2j}{c^2}+d\right)\right)+2a\left(\frac{a^2j}{c}-ah+cd\right)+abf\right)\right)}{4ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+$$

$$\frac{1}{2}\left(\frac{c^4\left(\frac{2a^3k}{c^2}-\frac{4a^2b^2k}{c^3}+\frac{ab^4k}{c^4}-b\left(\frac{ai}{c}+e\right)+2ag\right)-x^2\left(-5a^2bc^2k+5ab^3ck-c^4(2ai+bg)+b^5(-k)+b^2c^3i+2c^5\right.}{2c^4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right.$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\frac{\int \frac{-4a\left(4a-\frac{b^2}{c}\right)jx^4-\frac{\left(-ajb^3-c\left(ja^2+5cha+5c^2d\right)b+10ac^3f\right)x^2}{c^2}+abf+b^2\left(3d-\frac{a^2j}{c^2}\right)-2a\left(-\frac{ja^2}{c}+ha+7cd\right)}{(cx^4+bx^2+a)^2}dx}{4a\left(b^2-4ac\right)}-$$

$$\frac{x\left(x^2\left(-bc(-3a^2j+ach+c^2d)-ab^3j+2ac^3f\right)+c^2\left(-\left(b^2\left(\frac{a^2j}{c^2}+d\right)\right)+2a\left(\frac{a^2j}{c}-ah+cd\right)+abf\right)\right)}{4ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+$$

$$\frac{1}{2}\left(\frac{c^4\left(\frac{2a^3k}{c^2}-\frac{4a^2b^2k}{c^3}+\frac{ab^4k}{c^4}-b\left(\frac{ai}{c}+e\right)+2ag\right)-x^2\left(-5a^2bc^2k+5ab^3ck-c^4(2ai+bg)+b^5(-k)+b^2c^3i+2c^5\right.}{2c^4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right.$$

$$\Big\downarrow \textcolor{blue}{2206}$$

$$-\frac{x\left(\left(-\left(\left(\frac{ja^2}{c^2}+d\right)b^2\right)+afb+2a\left(\frac{ja^2}{c}-ha+cd\right)\right)c^2+\left(-ajb^3-c\left(-3ja^2+cha+c^2d\right)b+2ac^3f\right)x^2\right)}{4ac^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}+$$

$$\frac{1}{2}\left(\frac{c^4\left(\frac{akb^4}{c^4}-\frac{4a^2kb^2}{c^3}-\left(e+\frac{ai}{c}\right)b+2ag+\frac{2a^3k}{c^2}\right)-\left(-kb^5+5ackb^3+c^3ib^2-5a^2c^2kb+2c^5e-c^4(bg+2ai)\right)x^2}{2c^4\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}\right.$$

$$\frac{\frac{x\left(\left(\left(ja^2+3c^2d\right)b^3+ac^2fb^2-4ac\left(4ja^2+3cha+6c^2d\right)b+20a^2c^3f\right)x^2+c\left(\left(3d-\frac{2a^2j}{c^2}\right)b^4+afb^3-a\left(-\frac{11ja^2}{c}+7ha+25cd\right)b^2+8a^2cfb+4a^2\left(-9ja^2\right.\right.\right.}{2ac\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)}}{4}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$-\frac{x\left(\left(-\left(\left(\frac{ja^2}{c^2}+d\right)b^2\right)+afb+2a\left(\frac{ja^2}{c}-ha+cd\right)\right)c^2+\left(-ajb^3-c\left(-3ja^2+cha+c^2d\right)b+2ac^3f\right)x^2\right)}{4ac^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}+$$

$$\frac{1}{2}\left(\frac{c^4\left(\frac{akb^4}{c^4}-\frac{4a^2kb^2}{c^3}-\left(e+\frac{ai}{c}\right)b+2ag+\frac{2a^3k}{c^2}\right)-\left(-kb^5+5ackb^3+c^3ib^2-5a^2c^2kb+2c^5e-c^4(bg+2ai)\right)x^2}{2c^4\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}\cdots\right.$$

$$\frac{\frac{x\left(\left((ja^2+3c^2d)b^3+ac^2fb^2-4ac(4ja^2+3cha+6c^2d)b+20a^2c^3f\right)x^2+c\left(\left(3d-\frac{2a^2j}{c^2}\right)b^4+afb^3-a\left(-\frac{11ja^2}{c}+7ha+25cd\right)b^2+8a^2cfb+4a^2(-9ja^2-\cdots\right.\right.}{2ac(b^2-4ac)(cx^4+bx^2+a)}}{4a\cdots}$$

$\downarrow$ 27

$$-\frac{x\left(\left(-\left(\left(\frac{ja^2}{c^2}+d\right)b^2\right)+afb+2a\left(\frac{ja^2}{c}-ha+cd\right)\right)c^2+\left(-ajb^3-c\left(-3ja^2+cha+c^2d\right)b+2ac^3f\right)x^2\right)}{4ac^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}+$$

$$\frac{1}{2}\left(\frac{c^4\left(\frac{akb^4}{c^4}-\frac{4a^2kb^2}{c^3}-\left(e+\frac{ai}{c}\right)b+2ag+\frac{2a^3k}{c^2}\right)-\left(-kb^5+5ackb^3+c^3ib^2-5a^2c^2kb+2c^5e-c^4(bg+2ai)\right)x^2}{2c^4\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}\cdots\right.$$

$$\frac{\frac{x\left(\left((ja^2+3c^2d)b^3+ac^2fb^2-4ac(4ja^2+3cha+6c^2d)b+20a^2c^3f\right)x^2+c\left(\left(3d-\frac{2a^2j}{c^2}\right)b^4+afb^3-a\left(-\frac{11ja^2}{c}+7ha+25cd\right)b^2+8a^2cfb+4a^2(-9ja^2-\cdots\right.\right.}{2ac(b^2-4ac)(cx^4+bx^2+a)}}{4a\cdots}$$

$\downarrow$ 1480

$$-\frac{x\left(\left(-\left(\left(\frac{ja^2}{c^2}+d\right)b^2\right)+afb+2a\left(\frac{ja^2}{c}-ha+cd\right)\right)c^2+\left(-ajb^3-c\left(-3ja^2+cha+c^2d\right)b+2ac^3f\right)x^2\right)}{4ac^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}+$$

$$\frac{1}{2}\left(\frac{c^4\left(\frac{akb^4}{c^4}-\frac{4a^2kb^2}{c^3}-\left(e+\frac{ai}{c}\right)b+2ag+\frac{2a^3k}{c^2}\right)-\left(-kb^5+5ackb^3+c^3ib^2-5a^2c^2kb+2c^5e-c^4(bg+2ai)\right)x^2}{2c^4\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}\cdots\right.$$

$$\frac{x\left(\left((ja^2+3c^2d)b^3+ac^2fb^2-4ac(4ja^2+3cha+6c^2d)b+20a^2c^3f\right)x^2+c\left(\left(3d-\frac{2a^2j}{c^2}\right)b^4+afb^3-a\left(-\frac{11ja^2}{c}+7ha+25cd\right)b^2+8a^2cfb+4a^2(-9ja^2-\cdots\right.\right.}{2ac(b^2-4ac)(cx^4+bx^2+a)}\cdots$$

$$\Big\downarrow \text{218}$$

$$-\frac{x\left(\left(-\left(\left(\frac{ja^2}{c^2}+d\right)b^2\right)+afb+2a\left(\frac{ja^2}{c}-ha+cd\right)\right)c^2+\left(-ajb^3-c\left(-3ja^2+cha+c^2d\right)b+2ac^3f\right)x^2\right)}{4ac^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}+$$

$$\frac{\frac{x\Big(((ja^2+3c^2d)b^3+ac^2fb^2-4ac(4ja^2+3cha+6c^2d)b+20a^2c^3f)x^2+c\Big(\Big(3d-\frac{2a^2j}{c^2}\Big)b^4+afb^3-a\Big(-\frac{11ja^2}{c}+7ha+25cd\Big)b^2+8a^2cfb+4a^2(-9ja^2-}{2ac(b^2-4ac)(cx^4+bx^2+a)}}{}$$

$$\frac{1}{2}\Bigg(\frac{c^4\left(\frac{akb^4}{c^4}-\frac{4a^2kb^2}{c^3}-\left(e+\frac{ai}{c}\right)b+2ag+\frac{2a^3k}{c^2}\right)-\left(-kb^5+5ackb^3+c^3ib^2-5a^2c^2kb+2c^5e-c^4(bg+2ai)\right)x^2}{2c^4\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}$$

input

```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^8 + k*x^11)/(a + b*x^2
+ c*x^4)^3,x]
```

output

```
-1/4*(x*(c^2*(a*b*f - b^2*(d + (a^2*j)/c^2) + 2*a*(c*d - a*h + (a^2*j)/c))
 + (2*a*c^3*f - a*b^3*j - b*c*(c^2*d + a*c*h - 3*a^2*j))*x^2))/(a*c^2*(b^2
 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((x*(c*(a*b^3*f + 8*a^2*b*c*f + 4*a^2*(
7*c^2*d + a*c*h - 9*a^2*j) + b^4*(3*d - (2*a^2*j)/c^2) - a*b^2*(25*c*d + 7
*a*h - (11*a^2*j)/c)) + (a*b^2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j
) - 4*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*j))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (((a*b^2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j) -
 4*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*j) + (a*b^3*c^2*f - 52*a^2*b*c^3*f - 6
*a*b^2*c*(5*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) + 8*a^2*c^2
*(21*c^2*d + 3*a*c*h + 5*a^2*j))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) + ((a*b^2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j) - 4*a*b*c*(6
*c^2*d + 3*a*c*h + 4*a^2*j) - (a*b^3*c^2*f - 52*a^2*b*c^3*f - 6*a*b^2*c*(5
*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) + 8*a^2*c^2*(21*c^2*d
+ 3*a*c*h + 5*a^2*j))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*
a*c*(b^2 - 4*a*c)))/(4*a*(b^2 - 4*a*c)) + ((c^4*(2*a*g - b*(e + (a*i)/c) +
 (a*b^4*k)/c^4 - (4*a^2*b^2*k)/c^3 + (2*a^3*k)/c^2) - (2*c^5*e + b^2*c^3*i
 - c^4*(b*g + 2*a*i) - b^5*k + 5*a*b^3*c*k - 5*a^2*b*c^2*k)*x^2)/(2*c^4*(b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (-((c^3*((b^3*i)/c + 2*b*(3*c*e + ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 1182, normalized size of antiderivative = 1.00

method	result	size
risch	Expression too large to display	1182
default	Expression too large to display	2058

input
```
int((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,metho
d=_RETURNVERBOSE)
```

output
```
(-1/8*(16*a^3*b*c*j-a^2*b^3*j+12*a^2*b*c^2*h-20*a^2*c^3*f-a*b^2*c^2*f+24*a
*b*c^3*d-3*b^3*c^2*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/2*(25*a^2*b*c^2
*k-15*a*b^3*c*k+2*a*c^4*i+2*b^5*k+b^2*c^3*i-3*b*c^4*g+6*c^5*e)/c^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^6-1/8/a^2*(36*a^4*c^2*j+5*a^3*b^2*c*j-4*a^3*c^3*h+a^
2*b^4*j+19*a^2*b^2*c^2*h-28*a^2*b*c^3*f-28*a^2*c^4*d-2*a*b^3*c^2*f+49*a*b^
2*c^3*d-6*b^4*c^2*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^5+1/4*(32*a^3*c^3*k+11
*a^2*b^2*c^2*k-19*a*b^4*c*k+6*a*b*c^4*i+3*b^6*k+3*b^3*c^3*i-9*b^2*c^4*g+18
*b*c^5*e)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^4-1/8/c*(28*a^4*b*c*j+2*a^3*b^3
*j+16*a^3*b*c^2*h-36*a^3*c^3*f+5*a^2*b^3*c*h-5*a^2*b^2*c^2*f+4*a^2*b*c^3*d
-a*b^4*c*f+20*a*b^3*c^2*d-3*b^5*c*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/
2/c^3*(31*a^3*b*c^2*k-22*a^2*b^3*c*k-2*a^2*c^4*i+3*a*b^5*k+5*a*b^2*c^3*i-5
*a*b*c^4*g+10*a*c^5*e-b^3*c^3*g+2*b^2*c^4*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
2-1/8*(20*a^4*c*j+a^3*b^2*j+12*a^3*c^2*h+3*a^2*b^2*c*h-16*a^2*b*c^2*f-44*a
^2*c^3*d+a*b^3*c*f+37*a*b^2*c^2*d-5*b^4*c*d)/c/(16*a^2*c^2-8*a*b^2*c+b^4)/
a*x+1/4*(24*a^4*c^2*k-21*a^3*b^2*c*k+3*a^2*b^4*k+6*a^2*b*c^3*i-8*a^2*c^4*g
-a*b^2*c^3*g+10*a*b*c^4*e-b^3*c^3*e)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3)/(c*x^
4+b*x^2+a)^2+1/16/c*sum((8/c*k*_R^3-1/a^2*(16*a^3*b*c*j-a^2*b^3*j+12*a^2*b
*c^2*h-20*a^2*c^3*f-a*b^2*c^2*f+24*a*b*c^3*d-3*b^3*c^2*d)/(16*a^2*c^2-8*a*
b^2*c+b^4)*_R^2-8/c*(7*a^2*b*c*k-a*b^3*k-2*a*c^3*i-b^2*c^2*i+3*b*c^3*g-6*c
^4*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R+1/a^2*(20*a^4*c*j+a^3*b^2*j+12*a^3*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^8+kx^{11}}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x
, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^8+kx^{11}}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((k*x**11+j*x**8+i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**
2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^8+kx^{11}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{kx^{11}+jx^8+ix^5+hx^4+gx^3+fx^2+ex+d}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x
, algorithm="maxima")
```

output
```
1/8*(12*a^4*b*c^3*i - (12*a^2*b*c^5*h - 3*(b^3*c^5 - 8*a*b*c^6)*d - (a*b^2
*c^5 + 20*a^2*c^6)*f - (a^2*b^3*c^3 - 16*a^3*b*c^4)*j)*x^7 + 4*(6*a^2*c^6*
e - 3*a^2*b*c^5*g + (a^2*b^2*c^4 + 2*a^3*c^5)*i + (2*a^2*b^5*c - 15*a^3*b^
3*c^2 + 25*a^4*b*c^3)*k)*x^6 + ((6*b^4*c^4 - 49*a*b^2*c^5 + 28*a^2*c^6)*d
+ 2*(a*b^3*c^4 + 14*a^2*b*c^5)*f - (19*a^2*b^2*c^4 - 4*a^3*c^5)*h - (a^2*b
^4*c^2 + 5*a^3*b^2*c^3 + 36*a^4*c^4)*j)*x^5 + 2*(18*a^2*b*c^5*e - 9*a^2*b^
2*c^4*g + 3*(a^2*b^3*c^3 + 2*a^3*b*c^4)*i + (3*a^2*b^6 - 19*a^3*b^4*c + 11
*a^4*b^2*c^2 + 32*a^5*c^3)*k)*x^4 + ((3*b^5*c^3 - 20*a*b^3*c^4 - 4*a^2*b*c
^5)*d + (a*b^4*c^3 + 5*a^2*b^2*c^4 + 36*a^3*c^5)*f - (5*a^2*b^3*c^3 + 16*a
^3*b*c^4)*h - 2*(a^3*b^3*c^2 + 14*a^4*b*c^3)*j)*x^3 + 4*(2*(a^2*b^2*c^4 +
5*a^3*c^5)*e - (a^2*b^3*c^3 + 5*a^3*b*c^4)*g + (5*a^3*b^2*c^3 - 2*a^4*c^4)
*i + (3*a^3*b^5 - 22*a^4*b^3*c + 31*a^5*b*c^2)*k)*x^2 - 2*(a^2*b^3*c^3 - 1
0*a^3*b*c^4)*e - 2*(a^3*b^2*c^3 + 8*a^4*c^4)*g + 6*(a^4*b^4 - 7*a^5*b^2*c
+ 8*a^6*c^2)*k + ((5*a*b^4*c^3 - 37*a^2*b^2*c^4 + 44*a^3*c^5)*d - (a^2*b^3
*c^3 - 16*a^3*b*c^4)*f - 3*(a^3*b^2*c^3 + 4*a^4*c^4)*h - (a^4*b^2*c^2 + 20
*a^5*c^3)*j)*x)/(a^4*b^4*c^3 - 8*a^5*b^2*c^4 + 16*a^6*c^5 + (a^2*b^4*c^5 -
 8*a^3*b^2*c^6 + 16*a^4*c^7)*x^8 + 2*(a^2*b^5*c^4 - 8*a^3*b^3*c^5 + 16*a^4
*b*c^6)*x^6 + (a^2*b^6*c^3 - 6*a^3*b^4*c^4 + 32*a^5*c^6)*x^4 + 2*(a^3*b^5*
c^3 - 8*a^4*b^3*c^4 + 16*a^5*b*c^5)*x^2) + 1/8*integrate((8*(a^2*b^4 - 8*a
^3*b^2*c + 16*a^4*c^2)*k*x^3 - (12*a^2*b*c^3*h - 3*(b^3*c^3 - 8*a*b*c^4...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29140 vs. 2(1122) = 2244.

Time = 3.92 (sec) , antiderivative size = 29140, normalized size of antiderivative = 24.76

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5 + jx^8 + kx^{11}}{(a + bx^2 + cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x
, algorithm="giac")
```

output
```
-1/64*(3*(a^4*b^8*c^5 - 16*a^5*b^6*c^6 + 96*a^6*b^4*c^7 - 256*a^7*b^2*c^8
+ 256*a^8*c^9)^2*(2*b^5*c^4 - 24*a*b^3*c^5 + 64*a^2*b*c^6 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 12*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 16*(b^2 - 4
*a*c)*a*b*c^5)*d + (a^4*b^8*c^5 - 16*a^5*b^6*c^6 + 96*a^6*b^4*c^7 - 256*a^
7*b^2*c^8 + 256*a^8*c^9)^2*(2*a*b^4*c^4 + 32*a^2*b^2*c^5 - 160*a^3*c^6 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 80*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 40*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 20*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*a
*b^2*c^4 - 40*(b^2 - 4*a*c)*a^2*c^5)*f - 12*(a^4*b^8*c^5 - 16*a^5*b^6*c^6
+ 96*a^6*b^4*c^7 - 256*a^7*b^2*c^8 + 256*a^8*c^9)^2*(2*a^2*b^3*c^4 - 8*...
```

Mupad [B] (verification not implemented)

Time = 55.46 (sec) , antiderivative size = 97905, normalized size of antiderivative = 83.18

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5 + jx^8 + kx^{11}}{(a + bx^2 + cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^8 + k*x^11)/(a + b*x^2
+ c*x^4)^3,x)
```

output
```
((x^7*(3*b^3*c^2*d + 20*a^2*c^3*f + a^2*b^3*j - 24*a*b*c^3*d - 16*a^3*b*c*
j + a*b^2*c^2*f - 12*a^2*b*c^2*h))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
- (b^3*c^3*e + 8*a^2*c^4*g - 3*a^2*b^4*k - 24*a^4*c^2*k - 10*a*b*c^4*e + a
*b^2*c^3*g - 6*a^2*b*c^3*i + 21*a^3*b^2*c*k)/(4*c^3*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (x^4*(3*b^6*k - 9*b^2*c^4*g + 3*b^3*c^3*i + 32*a^3*c^3*k + 18*
b*c^5*e + 11*a^2*b^2*c^2*k + 6*a*b*c^4*i - 19*a*b^4*c*k))/(4*c^3*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (x^2*(2*b^2*c^4*e - b^3*c^3*g - 2*a^2*c^4*i + 10*
a*c^5*e + 3*a*b^5*k - 5*a*b*c^4*g + 5*a*b^2*c^3*i - 22*a^2*b^3*c*k + 31*a^
3*b*c^2*k))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^6*(6*c^5*e + 2*b^5
*k + b^2*c^3*i - 3*b*c^4*g + 2*a*c^4*i - 15*a*b^3*c*k + 25*a^2*b*c^2*k))/(
2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^3*(2*a^3*b^3*j - 36*a^3*c^3*f -
 3*b^5*c*d - 5*a^2*b^2*c^2*f - a*b^4*c*f + 28*a^4*b*c*j + 20*a*b^3*c^2*d +
 4*a^2*b*c^3*d + 5*a^2*b^3*c*h + 16*a^3*b*c^2*h))/(8*a^2*c*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)) + (x^5*(28*a^2*c^4*d + 6*b^4*c^2*d + 4*a^3*c^3*h - a^2*b^
4*j - 36*a^4*c^2*j - 19*a^2*b^2*c^2*h - 49*a*b^2*c^3*d + 2*a*b^3*c^2*f + 2
8*a^2*b*c^3*f - 5*a^3*b^2*c*j))/(8*a^2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) -
 (x*(12*a^3*c^2*h - 44*a^2*c^3*d + a^3*b^2*j - 5*b^4*c*d + 20*a^4*c*j + a*
b^3*c*f + 37*a*b^2*c^2*d - 16*a^2*b*c^2*f + 3*a^2*b^2*c*h))/(8*a*c*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 +
 2*b*c*x^6) + symsum(log((10368*a*b^5*c^10*d^3 - 8000*a^5*c^11*f^3 - 56...
```

Reduce [B] (verification not implemented)

Time = 20.05 (sec) , antiderivative size = 30922, normalized size of antiderivative = 26.27

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^8+kx^{11}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

output
```
(480*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**7*b**2
*c**2*k - 160*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**6*b**4*c*k + 960*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a**6*b**3*c**2*k*x**2 + 960*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(
c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sq
rt(c)*sqrt(a) + b))*a**6*b**2*c**3*k*x**4 - 64*sqrt(2*sqrt(c)*sqrt(a) + b)
*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**6*b*c**4*i + 16*sqrt(2*sqrt(c)*sqrt(a)
 + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b**6*k - 320*sqrt(2*sqrt(c)*sqr
t(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b**5*c*k*x**2 + 160*sqrt(2*
sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b**4*c**2*k*x**4
 - 32*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b**
3*c**3*i + 960*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*...
```

3.57 $\int (a+bx^2+cx^4)^3 (ad+aex+(bd+af)x^2+bex^3+$

Optimal result

Integrand size = 63, antiderivative size = 416

$$\begin{aligned}
&\int (a+bx^2+cx^4)^3 (ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6)\,dx \\
&= a^4dx+\frac{1}{2}a^4ex^2+\frac{1}{3}a^3(4bd+af)x^3+a^3bex^4+\frac{2}{5}a^2(3b^2d+2acd+2abf)\,x^5 \\
&\quad+\frac{1}{3}a^2(3b^2+2ac)\,ex^6+\frac{2}{7}a(2b^3d+6abcd+3ab^2f+2a^2cf)\,x^7+\frac{1}{2}ab(b^2+3ac)\,ex^8 \\
&\quad+\frac{1}{9}(b^4d+12ab^2cd+6a^2c^2d+4ab^3f+12a^2bcf)\,x^9+\frac{1}{10}(b^4+12ab^2c+6a^2c^2)\,ex^{10} \\
&\quad+\frac{1}{11}(4b^3cd+12abc^2d+b^4f+12ab^2cf+6a^2c^2f)\,x^{11}+\frac{1}{3}bc(b^2+3ac)\,ex^{12} \\
&\quad+\frac{2}{13}c(3b^2cd+2ac^2d+2b^3f+6abcf)\,x^{13}+\frac{1}{7}c^2(3b^2+2ac)\,ex^{14} \\
&\quad+\frac{2}{15}c^2(2bcd+3b^2f+2acf)\,x^{15}+\frac{1}{4}bc^3ex^{16}+\frac{1}{17}c^3(cd+4bf)x^{17}+\frac{1}{18}c^4ex^{18}+\frac{1}{19}c^4fx^{19}
\end{aligned}$$

output
```
a^4*d*x+1/2*a^4*e*x^2+1/3*a^3*(a*f+4*b*d)*x^3+a^3*b*e*x^4+2/5*a^2*(2*a*b*f
+2*a*c*d+3*b^2*d)*x^5+1/3*a^2*(2*a*c+3*b^2)*e*x^6+2/7*a*(2*a^2*c*f+3*a*b^2
*f+6*a*b*c*d+2*b^3*d)*x^7+1/2*a*b*(3*a*c+b^2)*e*x^8+1/9*(12*a^2*b*c*f+6*a^
2*c^2*d+4*a*b^3*f+12*a*b^2*c*d+b^4*d)*x^9+1/10*(6*a^2*c^2+12*a*b^2*c+b^4)*
e*x^10+1/11*(6*a^2*c^2*f+12*a*b^2*c*f+12*a*b*c^2*d+b^4*f+4*b^3*c*d)*x^11+1
/3*b*c*(3*a*c+b^2)*e*x^12+2/13*c*(6*a*b*c*f+2*a*c^2*d+2*b^3*f+3*b^2*c*d)*x
^13+1/7*c^2*(2*a*c+3*b^2)*e*x^14+2/15*c^2*(2*a*c*f+3*b^2*f+2*b*c*d)*x^15+1
/4*b*c^3*e*x^16+1/17*c^3*(4*b*f+c*d)*x^17+1/18*c^4*e*x^18+1/19*c^4*f*x^19
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00

$$\int (a+bx^2+cx^4)^3 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right) dx$$
$$= a^4dx+\frac{1}{2}a^4ex^2+\frac{1}{3}a^3(4bd+af)x^3+a^3bex^4+\frac{2}{5}a^2\left(3b^2d+2acd+2abf\right)x^5$$
$$+\frac{1}{3}a^2\left(3b^2+2ac\right)ex^6+\frac{2}{7}a\left(2b^3d+6abcd+3ab^2f+2a^2cf\right)x^7+\frac{1}{2}ab\left(b^2+3ac\right)ex^8$$
$$+\frac{1}{9}\left(b^4d+12ab^2cd+6a^2c^2d+4ab^3f+12a^2bcf\right)x^9+\frac{1}{10}\left(b^4+12ab^2c+6a^2c^2\right)ex^{10}$$
$$+\frac{1}{11}\left(4b^3cd+12abc^2d+b^4f+12ab^2cf+6a^2c^2f\right)x^{11}+\frac{1}{3}bc\left(b^2+3ac\right)ex^{12}$$
$$+\frac{2}{13}c\left(3b^2cd+2ac^2d+2b^3f+6abcf\right)x^{13}+\frac{1}{7}c^2\left(3b^2+2ac\right)ex^{14}$$
$$+\frac{2}{15}c^2\left(2bcd+3b^2f+2acf\right)x^{15}+\frac{1}{4}bc^3ex^{16}+\frac{1}{17}c^3(cd+4bf)x^{17}+\frac{1}{18}c^4ex^{18}+\frac{1}{19}c^4fx^{19}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^3*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 +
 (c*d + b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

output
```
a^4*d*x + (a^4*e*x^2)/2 + (a^3*(4*b*d + a*f)*x^3)/3 + a^3*b*e*x^4 + (2*a^2
*(3*b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + (a^2*(3*b^2 + 2*a*c)*e*x^6)/3 + (2
*a*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*f + 2*a^2*c*f)*x^7)/7 + (a*b*(b^2 + 3*a*
c)*e*x^8)/2 + ((b^4*d + 12*a*b^2*c*d + 6*a^2*c^2*d + 4*a*b^3*f + 12*a^2*b*
c*f)*x^9)/9 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*e*x^10)/10 + ((4*b^3*c*d + 1
2*a*b*c^2*d + b^4*f + 12*a*b^2*c*f + 6*a^2*c^2*f)*x^11)/11 + (b*c*(b^2 + 3
*a*c)*e*x^12)/3 + (2*c*(3*b^2*c*d + 2*a*c^2*d + 2*b^3*f + 6*a*b*c*f)*x^13)
/13 + (c^2*(3*b^2 + 2*a*c)*e*x^14)/7 + (2*c^2*(2*b*c*d + 3*b^2*f + 2*a*c*f
)*x^15)/15 + (b*c^3*e*x^16)/4 + (c^3*(c*d + 4*b*f)*x^17)/17 + (c^4*e*x^18)
/18 + (c^4*f*x^19)/19
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^3 \left(x^2(af+bd)+ad+aex+x^4(bf+cd)+bex^3+cex^5+cfx^6\right) \, dx$$

$$\Big\downarrow \text{2200}$$

$$\int \left(a^4d+a^4ex+a^3x^2(af+4bd)+4a^3bex^3+2a^2x^4\left(2abf+2acd+3b^2d\right)+2a^2ex^5\left(2ac+3b^2\right)+ex^9\left(6a^2c^2+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$a^4dx+\frac{1}{2}a^4ex^2+\frac{1}{3}a^3x^3(af+4bd)+a^3bex^4+\frac{2}{5}a^2x^5\left(2abf+2acd+3b^2d\right)+\frac{1}{3}a^2ex^6\left(2ac+3b^2\right)+\frac{1}{10}ex^{10}\left(6a^2c^2+12ab^2c+b^4\right)+\frac{2}{7}ax^7\left(2a^2cf+3ab^2f+6abcd+2b^3d\right)+\frac{1}{11}x^{11}\left(6a^2c^2f+12ab^2cf+12abc^2d+b^4f+4b^3cd\right)+\frac{1}{9}x^9\left(12a^2bcf+6a^2c^2d+4ab^3f+12ab^2cd+b^4d\right)+\frac{2}{15}c^2x^{15}\left(2acf+3b^2f+2bcd\right)+\frac{1}{7}c^2ex^{14}\left(2ac+3b^2\right)+\frac{1}{3}bcex^{12}\left(3ac+b^2\right)+\frac{1}{2}abex^8\left(3ac+b^2\right)+\frac{2}{13}cx^{13}\left(6abcf+2ac^2d+2b^3f+3b^2cd\right)+\frac{1}{17}c^3x^{17}(4bf+cd)+\frac{1}{4}bc^3ex^{16}+\frac{1}{18}c^4ex^{18}+\frac{1}{19}c^4fx^{19}$$

input
```
Int[(a + b*x^2 + c*x^4)^3*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d
 + b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

output
```
a^4*d*x + (a^4*e*x^2)/2 + (a^3*(4*b*d + a*f)*x^3)/3 + a^3*b*e*x^4 + (2*a^2
*(3*b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + (a^2*(3*b^2 + 2*a*c)*e*x^6)/3 + (2
*a*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*f + 2*a^2*c*f)*x^7)/7 + (a*b*(b^2 + 3*a*
c)*e*x^8)/2 + ((b^4*d + 12*a*b^2*c*d + 6*a^2*c^2*d + 4*a*b^3*f + 12*a^2*b*
c*f)*x^9)/9 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*e*x^10)/10 + ((4*b^3*c*d + 1
2*a*b*c^2*d + b^4*f + 12*a*b^2*c*f + 6*a^2*c^2*f)*x^11)/11 + (b*c*(b^2 + 3
*a*c)*e*x^12)/3 + (2*c*(3*b^2*c*d + 2*a*c^2*d + 2*b^3*f + 6*a*b*c*f)*x^13)
/13 + (c^2*(3*b^2 + 2*a*c)*e*x^14)/7 + (2*c^2*(2*b*c*d + 3*b^2*f + 2*a*c*f
)*x^15)/15 + (b*c^3*e*x^16)/4 + (c^3*(c*d + 4*b*f)*x^17)/17 + (c^4*e*x^18)
/18 + (c^4*f*x^19)/19
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 24.54 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.99

method	result
norman	$\left(\frac{1}{3} f\, a^{4}+\frac{4}{3} d\, a^{3} b\right) x^{3}+\left(\frac{2}{7} a\, c^{3} e+\frac{3}{7} b^{2} c^{2} e\right) x^{14}+\left(\frac{2}{3} a^{3} c e+a^{2} b^{2} e\right) x^{6}+\left(\frac{4}{17} c^{3} b f+\frac{1}{17} c^{4} d\right) x^{17}$
risch	$\frac{4}{13} a\, c^{3} d\, x^{13}+\frac{2}{3} a^{2} c^{2} d\, x^{9}+\frac{4}{5} a^{3} c d\, x^{5}+\frac{1}{4} b\, c^{3} e\, x^{16}+\frac{1}{18} c^{4} e\, x^{18}+\frac{1}{19} c^{4} f\, x^{19}+a^{3} b e\, x^{4}+\frac{6}{5} x^{5} a^{2} b$
parallelrisch	$\frac{4}{13} a\, c^{3} d\, x^{13}+\frac{2}{3} a^{2} c^{2} d\, x^{9}+\frac{4}{5} a^{3} c d\, x^{5}+\frac{1}{4} b\, c^{3} e\, x^{16}+\frac{1}{18} c^{4} e\, x^{18}+\frac{1}{19} c^{4} f\, x^{19}+a^{3} b e\, x^{4}+\frac{6}{5} x^{5} a^{2} b$
gosper	$x \left(3063060 f\, c^{4} x^{18}+3233230 c^{4} e\, x^{17}+13693680 b\, c^{3} f\, x^{16}+3423420 c^{4} d\, x^{16}+14549535 e b\, c^{3} x^{15}+15519504 a\, c^{3} f\, x^{14}+23279256$
orering	$x \left(3063060 f\, c^{4} x^{18}+3233230 c^{4} e\, x^{17}+13693680 b\, c^{3} f\, x^{16}+3423420 c^{4} d\, x^{16}+14549535 e b\, c^{3} x^{15}+15519504 a\, c^{3} f\, x^{14}+23279256$
default	$\frac{c^{4} f\, x^{19}}{19}+\frac{c^{4} e\, x^{18}}{18}+\frac{\left(3 c^{3} b f+c^{3} \left(b f+c d\right)\right) x^{17}}{17}+\frac{b\, c^{3} e\, x^{16}}{4}+\frac{\left(\left(c^{2} a+2 b^{2} c+c \left(2 a c+b^{2}\right)\right) c f+3 b\, c^{2} \left(b f+c d\right)+c^{3} \left(a f+b d\right)\right) x^{1}}{15}$

input
```
int((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x
^5+c*f*x^6),x,method=_RETURNVERBOSE)
```

output
```
(1/3*f*a^4+4/3*d*a^3*b)*x^3+(2/7*a*c^3*e+3/7*b^2*c^2*e)*x^14+(2/3*a^3*c*e+
a^2*b^2*e)*x^6+(4/17*c^3*b*f+1/17*c^4*d)*x^17+(a*b*c^2*e+1/3*b^3*c*e)*x^12
+(3/2*a^2*b*c*e+1/2*a*b^3*e)*x^8+(4/15*a*c^3*f+2/5*b^2*c^2*f+4/15*b*c^3*d)
*x^15+(3/5*a^2*c^2*e+6/5*a*b^2*c*e+1/10*b^4*e)*x^10+(4/5*f*a^3*b+4/5*a^3*c
*d+6/5*a^2*b^2*d)*x^5+(4/7*a^3*c*f+6/7*a^2*b^2*f+12/7*a^2*b*c*d+4/7*a*b^3*
d)*x^7+(12/13*a*b*c^2*f+4/13*a*c^3*d+4/13*b^3*c*f+6/13*b^2*c^2*d)*x^13+(6/
11*a^2*c^2*f+12/11*a*b^2*c*f+12/11*a*b*c^2*d+1/11*b^4*f+4/11*b^3*c*d)*x^11
+(4/3*a^2*b*c*f+2/3*a^2*c^2*d+4/9*a*b^3*f+4/3*a*b^2*c*d+1/9*b^4*d)*x^9+x*a
^4*d+a^3*b*e*x^4+1/2*a^4*e*x^2+1/18*c^4*e*x^18+1/19*c^4*f*x^19+1/4*b*c^3*e
*x^16
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int \left(a+bx^2+cx^4\right)^3 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx \\
&= \frac{1}{19}\,c^4fx^{19}+\frac{1}{18}\,c^4ex^{18}+\frac{1}{4}\,bc^3ex^{16}+\frac{1}{17}\left(c^4d+4\,bc^3f\right)x^{17}+\frac{1}{7}\left(3\,b^2c^2+2\,ac^3\right)ex^{14} \\
&\quad+\frac{2}{15}\left(2\,bc^3d+\left(3\,b^2c^2+2\,ac^3\right)f\right)x^{15}+\frac{1}{3}\left(b^3c+3\,abc^2\right)ex^{12} \\
&\quad+\frac{2}{13}\left(\left(3\,b^2c^2+2\,ac^3\right)d+2\left(b^3c+3\,abc^2\right)f\right)x^{13}+\frac{1}{10}\left(b^4+12\,ab^2c+6\,a^2c^2\right)ex^{10} \\
&\quad+\frac{1}{11}\left(4\left(b^3c+3\,abc^2\right)d+\left(b^4+12\,ab^2c+6\,a^2c^2\right)f\right)x^{11}+\frac{1}{2}\left(ab^3+3\,a^2bc\right)ex^8 \\
&\quad+\frac{1}{9}\left(\left(b^4+12\,ab^2c+6\,a^2c^2\right)d+4\left(ab^3+3\,a^2bc\right)f\right)x^9+a^3bex^4 \\
&\quad+\frac{1}{3}\left(3\,a^2b^2+2\,a^3c\right)ex^6+\frac{2}{7}\left(2\left(ab^3+3\,a^2bc\right)d+\left(3\,a^2b^2+2\,a^3c\right)f\right)x^7 \\
&\quad+\frac{1}{2}\,a^4ex^2+a^4dx+\frac{2}{5}\left(2\,a^3bf+\left(3\,a^2b^2+2\,a^3c\right)d\right)x^5+\frac{1}{3}\left(4\,a^3bd+a^4f\right)x^3
\end{aligned}$$

input
```
integrate((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4
+c*e*x^5+c*f*x^6),x, algorithm="fricas")
```

output
```
1/19*c^4*f*x^19 + 1/18*c^4*e*x^18 + 1/4*b*c^3*e*x^16 + 1/17*(c^4*d + 4*b*c
^3*f)*x^17 + 1/7*(3*b^2*c^2 + 2*a*c^3)*e*x^14 + 2/15*(2*b*c^3*d + (3*b^2*c
^2 + 2*a*c^3)*f)*x^15 + 1/3*(b^3*c + 3*a*b*c^2)*e*x^12 + 2/13*((3*b^2*c^2
+ 2*a*c^3)*d + 2*(b^3*c + 3*a*b*c^2)*f)*x^13 + 1/10*(b^4 + 12*a*b^2*c + 6*
a^2*c^2)*e*x^10 + 1/11*(4*(b^3*c + 3*a*b*c^2)*d + (b^4 + 12*a*b^2*c + 6*a^
2*c^2)*f)*x^11 + 1/2*(a*b^3 + 3*a^2*b*c)*e*x^8 + 1/9*((b^4 + 12*a*b^2*c +
6*a^2*c^2)*d + 4*(a*b^3 + 3*a^2*b*c)*f)*x^9 + a^3*b*e*x^4 + 1/3*(3*a^2*b^2
 + 2*a^3*c)*e*x^6 + 2/7*(2*(a*b^3 + 3*a^2*b*c)*d + (3*a^2*b^2 + 2*a^3*c)*f
)*x^7 + 1/2*a^4*e*x^2 + a^4*d*x + 2/5*(2*a^3*b*f + (3*a^2*b^2 + 2*a^3*c)*d
)*x^5 + 1/3*(4*a^3*b*d + a^4*f)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.21

$$\int (a+bx^2+cx^4)^3\,(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6)\,dx = a^4dx+\frac{a^4ex^2}{2}+a^3bex^4+\frac{bc^3ex^{16}}{4}+\frac{c^4ex^{18}}{18}+\frac{c^4fx^{19}}{19}+x^{17}\cdot\left(\frac{4bc^3f}{17}+\frac{c^4d}{17}\right)+x^{15}\cdot\left(\frac{4ac^3f}{15}+\frac{2b^2c^2f}{5}+\frac{4bc^3d}{15}\right)+x^{14}\cdot\left(\frac{2ac^3e}{7}+\frac{3b^2c^2e}{7}\right)+x^{13}\cdot\left(\frac{12abc^2f}{13}+\frac{4ac^3d}{13}+\frac{4b^3cf}{13}+\frac{6b^2c^2d}{13}\right)+x^{12}\left(abc^2e+\frac{b^3ce}{3}\right)+x^{11}\cdot\left(\frac{6a^2c^2f}{11}+\frac{12ab^2cf}{11}+\frac{12abc^2d}{11}+\frac{b^4f}{11}+\frac{4b^3cd}{11}\right)+x^{10}\cdot\left(\frac{3a^2c^2e}{5}+\frac{6ab^2ce}{5}+\frac{b^4e}{10}\right)+x^9\cdot\left(\frac{4a^2bcf}{3}+\frac{2a^2c^2d}{3}+\frac{4ab^3f}{9}+\frac{4ab^2cd}{3}+\frac{b^4d}{9}\right)+x^8\cdot\left(\frac{3a^2bce}{2}+\frac{ab^3e}{2}\right)+x^7\cdot\left(\frac{4a^3cf}{7}+\frac{6a^2b^2f}{7}+\frac{12a^2bcd}{7}+\frac{4ab^3d}{7}\right)+x^6\cdot\left(\frac{2a^3ce}{3}+a^2b^2e\right)+x^5\cdot\left(\frac{4a^3bf}{5}+\frac{4a^3cd}{5}+\frac{6a^2b^2d}{5}\right)+x^3\left(\frac{a^4f}{3}+\frac{4a^3bd}{3}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**3*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d
)*x**4+c*e*x**5+c*f*x**6),x)
```

output
```
a**4*d*x + a**4*e*x**2/2 + a**3*b*e*x**4 + b*c**3*e*x**16/4 + c**4*e*x**18
/18 + c**4*f*x**19/19 + x**17*(4*b*c**3*f/17 + c**4*d/17) + x**15*(4*a*c**
3*f/15 + 2*b**2*c**2*f/5 + 4*b*c**3*d/15) + x**14*(2*a*c**3*e/7 + 3*b**2*c
**2*e/7) + x**13*(12*a*b*c**2*f/13 + 4*a*c**3*d/13 + 4*b**3*c*f/13 + 6*b**
2*c**2*d/13) + x**12*(a*b*c**2*e + b**3*c*e/3) + x**11*(6*a**2*c**2*f/11 +
 12*a*b**2*c*f/11 + 12*a*b*c**2*d/11 + b**4*f/11 + 4*b**3*c*d/11) + x**10*
(3*a**2*c**2*e/5 + 6*a*b**2*c*e/5 + b**4*e/10) + x**9*(4*a**2*b*c*f/3 + 2*
a**2*c**2*d/3 + 4*a*b**3*f/9 + 4*a*b**2*c*d/3 + b**4*d/9) + x**8*(3*a**2*b
*c*e/2 + a*b**3*e/2) + x**7*(4*a**3*c*f/7 + 6*a**2*b**2*f/7 + 12*a**2*b*c*
d/7 + 4*a*b**3*d/7) + x**6*(2*a**3*c*e/3 + a**2*b**2*e) + x**5*(4*a**3*b*f
/5 + 4*a**3*c*d/5 + 6*a**2*b**2*d/5) + x**3*(a**4*f/3 + 4*a**3*b*d/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int \left(a+bx^2+cx^4\right)^3 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right) dx \\
&= \frac{1}{19}c^4fx^{19}+\frac{1}{18}c^4ex^{18}+\frac{1}{4}bc^3ex^{16}+\frac{1}{17}\left(c^4d+4\,bc^3f\right)x^{17}+\frac{1}{7}\left(3\,b^2c^2+2\,ac^3\right)ex^{14} \\
&+\frac{2}{15}\left(2\,bc^3d+\left(3\,b^2c^2+2\,ac^3\right)f\right)x^{15}+\frac{1}{3}\left(b^3c+3\,abc^2\right)ex^{12} \\
&+\frac{2}{13}\left(\left(3\,b^2c^2+2\,ac^3\right)d+2\left(b^3c+3\,abc^2\right)f\right)x^{13}+\frac{1}{10}\left(b^4+12\,ab^2c+6\,a^2c^2\right)ex^{10} \\
&+\frac{1}{11}\left(4\left(b^3c+3\,abc^2\right)d+\left(b^4+12\,ab^2c+6\,a^2c^2\right)f\right)x^{11}+\frac{1}{2}\left(ab^3+3\,a^2bc\right)ex^8 \\
&+\frac{1}{9}\left(\left(b^4+12\,ab^2c+6\,a^2c^2\right)d+4\left(ab^3+3\,a^2bc\right)f\right)x^9+a^3bex^4 \\
&+\frac{1}{3}\left(3\,a^2b^2+2\,a^3c\right)ex^6+\frac{2}{7}\left(2\left(ab^3+3\,a^2bc\right)d+\left(3\,a^2b^2+2\,a^3c\right)f\right)x^7 \\
&+\frac{1}{2}a^4ex^2+a^4dx+\frac{2}{5}\left(2\,a^3bf+\left(3\,a^2b^2+2\,a^3c\right)d\right)x^5+\frac{1}{3}\left(4\,a^3bd+a^4f\right)x^3
\end{aligned}$$

input
```
integrate((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4
+c*e*x^5+c*f*x^6),x, algorithm="maxima")
```

output
```
1/19*c^4*f*x^19 + 1/18*c^4*e*x^18 + 1/4*b*c^3*e*x^16 + 1/17*(c^4*d + 4*b*c
^3*f)*x^17 + 1/7*(3*b^2*c^2 + 2*a*c^3)*e*x^14 + 2/15*(2*b*c^3*d + (3*b^2*c
^2 + 2*a*c^3)*f)*x^15 + 1/3*(b^3*c + 3*a*b*c^2)*e*x^12 + 2/13*((3*b^2*c^2
+ 2*a*c^3)*d + 2*(b^3*c + 3*a*b*c^2)*f)*x^13 + 1/10*(b^4 + 12*a*b^2*c + 6*
a^2*c^2)*e*x^10 + 1/11*(4*(b^3*c + 3*a*b*c^2)*d + (b^4 + 12*a*b^2*c + 6*a^
2*c^2)*f)*x^11 + 1/2*(a*b^3 + 3*a^2*b*c)*e*x^8 + 1/9*((b^4 + 12*a*b^2*c +
6*a^2*c^2)*d + 4*(a*b^3 + 3*a^2*b*c)*f)*x^9 + a^3*b*e*x^4 + 1/3*(3*a^2*b^2
 + 2*a^3*c)*e*x^6 + 2/7*(2*(a*b^3 + 3*a^2*b*c)*d + (3*a^2*b^2 + 2*a^3*c)*f
)*x^7 + 1/2*a^4*e*x^2 + a^4*d*x + 2/5*(2*a^3*b*f + (3*a^2*b^2 + 2*a^3*c)*d
)*x^5 + 1/3*(4*a^3*b*d + a^4*f)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.11

$$\int \left(a+bx^2+cx^4\right)^3 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx$$
$$\begin{aligned}
&= \frac{1}{19}c^4fx^{19}+\frac{1}{18}c^4ex^{18}+\frac{1}{17}c^4dx^{17}+\frac{4}{17}bc^3fx^{17}+\frac{1}{4}bc^3ex^{16}+\frac{4}{15}bc^3dx^{15}\\
&+\frac{2}{5}b^2c^2fx^{15}+\frac{4}{15}ac^3fx^{15}+\frac{3}{7}b^2c^2ex^{14}+\frac{2}{7}ac^3ex^{14}+\frac{6}{13}b^2c^2dx^{13}+\frac{4}{13}ac^3dx^{13}\\
&+\frac{4}{13}b^3cfx^{13}+\frac{12}{13}abc^2fx^{13}+\frac{1}{3}b^3cex^{12}+abc^2ex^{12}+\frac{4}{11}b^3cdx^{11}+\frac{12}{11}abc^2dx^{11}\\
&+\frac{1}{11}b^4fx^{11}+\frac{12}{11}ab^2cfx^{11}+\frac{6}{11}a^2c^2fx^{11}+\frac{1}{10}b^4ex^{10}+\frac{6}{5}ab^2cex^{10}+\frac{3}{5}a^2c^2ex^{10}\\
&+\frac{1}{9}b^4dx^9+\frac{4}{3}ab^2cdx^9+\frac{2}{3}a^2c^2dx^9+\frac{4}{9}ab^3fx^9+\frac{4}{3}a^2bcfx^9+\frac{1}{2}ab^3ex^8+\frac{3}{2}a^2bcex^8\\
&+\frac{4}{7}ab^3dx^7+\frac{12}{7}a^2bcdx^7+\frac{6}{7}a^2b^2fx^7+\frac{4}{7}a^3cfx^7+a^2b^2ex^6+\frac{2}{3}a^3cex^6+\frac{6}{5}a^2b^2dx^5\\
&+\frac{4}{5}a^3cdx^5+\frac{4}{5}a^3bfx^5+a^3bex^4+\frac{4}{3}a^3bdx^3+\frac{1}{3}a^4fx^3+\frac{1}{2}a^4ex^2+a^4dx
\end{aligned}$$

input
```
integrate((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4
+c*e*x^5+c*f*x^6),x, algorithm="giac")
```

output
```
1/19*c^4*f*x^19 + 1/18*c^4*e*x^18 + 1/17*c^4*d*x^17 + 4/17*b*c^3*f*x^17 +
1/4*b*c^3*e*x^16 + 4/15*b*c^3*d*x^15 + 2/5*b^2*c^2*f*x^15 + 4/15*a*c^3*f*x
^15 + 3/7*b^2*c^2*e*x^14 + 2/7*a*c^3*e*x^14 + 6/13*b^2*c^2*d*x^13 + 4/13*a
*c^3*d*x^13 + 4/13*b^3*c*f*x^13 + 12/13*a*b*c^2*f*x^13 + 1/3*b^3*c*e*x^12
+ a*b*c^2*e*x^12 + 4/11*b^3*c*d*x^11 + 12/11*a*b*c^2*d*x^11 + 1/11*b^4*f*x
^11 + 12/11*a*b^2*c*f*x^11 + 6/11*a^2*c^2*f*x^11 + 1/10*b^4*e*x^10 + 6/5*a
*b^2*c*e*x^10 + 3/5*a^2*c^2*e*x^10 + 1/9*b^4*d*x^9 + 4/3*a*b^2*c*d*x^9 + 2
/3*a^2*c^2*d*x^9 + 4/9*a*b^3*f*x^9 + 4/3*a^2*b*c*f*x^9 + 1/2*a*b^3*e*x^8 +
 3/2*a^2*b*c*e*x^8 + 4/7*a*b^3*d*x^7 + 12/7*a^2*b*c*d*x^7 + 6/7*a^2*b^2*f*
x^7 + 4/7*a^3*c*f*x^7 + a^2*b^2*e*x^6 + 2/3*a^3*c*e*x^6 + 6/5*a^2*b^2*d*x^
5 + 4/5*a^3*c*d*x^5 + 4/5*a^3*b*f*x^5 + a^3*b*e*x^4 + 4/3*a^3*b*d*x^3 + 1/
3*a^4*f*x^3 + 1/2*a^4*e*x^2 + a^4*d*x
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2+cx^4\right)^3 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx$$
$$=x^3\left(\frac{f\,a^4}{3}+\frac{4\,b\,d\,a^3}{3}\right)+x^{17}\left(\frac{d\,c^4}{17}+\frac{4\,b\,f\,c^3}{17}\right)+x^5\left(\frac{4\,f\,a^3\,b}{5}+\frac{4\,c\,d\,a^3}{5}+\frac{6\,d\,a^2\,b^2}{5}\right)+x^{15}\left(\frac{2\,f\,b^2\,c^2}{5}+\frac{4\,d\,b\,c^3}{15}+\frac{4\,a\,f\,c^3}{15}\right)+x^9\left(\frac{4\,f\,a^2\,b\,c}{3}+\frac{2\,d\,a^2\,c^2}{3}+\frac{4\,f\,a\,b^3}{9}+\frac{4\,d\,a\,b^2\,c}{3}+\frac{d\,b^4}{9}\right)+x^{11}\left(\frac{6\,f\,a^2\,c^2}{11}+\frac{12\,f\,a\,b^2\,c}{11}+\frac{12\,d\,a\,b\,c^2}{11}+\frac{f\,b^4}{11}+\frac{4\,d\,b^3\,c}{11}\right)+x^7\left(\frac{4\,c\,f\,a^3}{7}+\frac{6\,f\,a^2\,b^2}{7}+\frac{12\,c\,d\,a^2\,b}{7}+\frac{4\,d\,a\,b^3}{7}\right)+x^{13}\left(\frac{4\,f\,b^3\,c}{13}+\frac{6\,d\,b^2\,c^2}{13}+\frac{12\,a\,f\,b\,c^2}{13}+\frac{4\,a\,d\,c^3}{13}\right)+\frac{a^4\,e\,x^2}{2}+\frac{c^4\,e\,x^{18}}{18}+\frac{c^4\,f\,x^{19}}{19}+\frac{e\,x^{10}\,\left(6\,a^2\,c^2+12\,a\,b^2\,c+b^4\right)}{10}+a^4\,d\,x+\frac{a^2\,e\,x^6\,\left(3\,b^2+2\,a\,c\right)}{3}+\frac{c^2\,e\,x^{14}\,\left(3\,b^2+2\,a\,c\right)}{7}+a^3\,b\,e\,x^4+\frac{b\,c^3\,e\,x^{16}}{4}+\frac{a\,b\,e\,x^8\,\left(b^2+3\,a\,c\right)}{2}+\frac{b\,c\,e\,x^{12}\,\left(b^2+3\,a\,c\right)}{3}$$

input
```
int((a + b*x^2 + c*x^4)^3*(a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x
 + b*e*x^3 + c*e*x^5 + c*f*x^6),x)
```

output
```
x^3*((a^4*f)/3 + (4*a^3*b*d)/3) + x^17*((c^4*d)/17 + (4*b*c^3*f)/17) + x^5
*((6*a^2*b^2*d)/5 + (4*a^3*c*d)/5 + (4*a^3*b*f)/5) + x^15*((2*b^2*c^2*f)/5
 + (4*b*c^3*d)/15 + (4*a*c^3*f)/15) + x^9*((b^4*d)/9 + (2*a^2*c^2*d)/3 + (
4*a*b^3*f)/9 + (4*a*b^2*c*d)/3 + (4*a^2*b*c*f)/3) + x^11*((b^4*f)/11 + (6*
a^2*c^2*f)/11 + (4*b^3*c*d)/11 + (12*a*b*c^2*d)/11 + (12*a*b^2*c*f)/11) +
x^7*((6*a^2*b^2*f)/7 + (4*a*b^3*d)/7 + (4*a^3*c*f)/7 + (12*a^2*b*c*d)/7) +
 x^13*((6*b^2*c^2*d)/13 + (4*a*c^3*d)/13 + (4*b^3*c*f)/13 + (12*a*b*c^2*f)
/13) + (a^4*e*x^2)/2 + (c^4*e*x^18)/18 + (c^4*f*x^19)/19 + (e*x^10*(b^4 +
6*a^2*c^2 + 12*a*b^2*c))/10 + a^4*d*x + (a^2*e*x^6*(2*a*c + 3*b^2))/3 + (c
^2*e*x^14*(2*a*c + 3*b^2))/7 + a^3*b*e*x^4 + (b*c^3*e*x^16)/4 + (a*b*e*x^8
*(3*a*c + b^2))/2 + (b*c*e*x^12*(3*a*c + b^2))/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2+cx^4\right)^3 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right) dx$$

$$= \frac{x(3063060c^4f\,x^{18}+3233230c^4e\,x^{17}+13693680b\,c^3f\,x^{16}+3423420c^4d\,x^{16}+145495335b\,c^3e\,x^{15}+155195}{}$$

input
```
int((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x
^5+c*f*x^6),x)
```

output

```
(x*(58198140*a**4*d + 29099070*a**4*e*x + 19399380*a**4*f*x**2 + 77597520*
a**3*b*d*x**2 + 58198140*a**3*b*e*x**3 + 46558512*a**3*b*f*x**4 + 46558512
*a**3*c*d*x**4 + 38798760*a**3*c*e*x**5 + 33256080*a**3*c*f*x**6 + 6983776
8*a**2*b**2*d*x**4 + 58198140*a**2*b**2*e*x**5 + 49884120*a**2*b**2*f*x**6
 + 99768240*a**2*b*c*d*x**6 + 87297210*a**2*b*c*e*x**7 + 77597520*a**2*b*c
*f*x**8 + 38798760*a**2*c**2*d*x**8 + 34918884*a**2*c**2*e*x**9 + 31744440
*a**2*c**2*f*x**10 + 33256080*a*b**3*d*x**6 + 29099070*a*b**3*e*x**7 + 258
65840*a*b**3*f*x**8 + 77597520*a*b**2*c*d*x**8 + 69837768*a*b**2*c*e*x**9
+ 63488880*a*b**2*c*f*x**10 + 63488880*a*b*c**2*d*x**10 + 58198140*a*b*c**
2*e*x**11 + 53721360*a*b*c**2*f*x**12 + 17907120*a*c**3*d*x**12 + 16628040
*a*c**3*e*x**13 + 15519504*a*c**3*f*x**14 + 6466460*b**4*d*x**8 + 5819814*
b**4*e*x**9 + 5290740*b**4*f*x**10 + 21162960*b**3*c*d*x**10 + 19399380*b*
*3*c*e*x**11 + 17907120*b**3*c*f*x**12 + 26860680*b**2*c**2*d*x**12 + 2494
2060*b**2*c**2*e*x**13 + 23279256*b**2*c**2*f*x**14 + 15519504*b*c**3*d*x*
*14 + 14549535*b*c**3*e*x**15 + 13693680*b*c**3*f*x**16 + 3423420*c**4*d*x
**16 + 3233230*c**4*e*x**17 + 3063060*c**4*f*x**18))/58198140
```

3.58 $\int (a+bx^2+cx^4)^2 (ad+aex+(bd+af)x^2+bex^3+$

Optimal result

Integrand size = 63, antiderivative size = 259

$$\begin{aligned}\int (a+bx^2+cx^4)^2 (ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6)\,dx \\ = a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{3}a^2(3bd+af)x^3+\frac{3}{4}a^2bex^4+\frac{3}{5}a\left(b^2d+acd+abf\right)x^5 \\ +\frac{1}{2}a\left(b^2+ac\right)ex^6+\frac{1}{7}\left(b^3d+6abcd+3ab^2f+3a^2cf\right)x^7+\frac{1}{8}b\left(b^2+6ac\right)ex^8 \\ +\frac{1}{9}\left(3b^2cd+3ac^2d+b^3f+6abcf\right)x^9+\frac{3}{10}c\left(b^2+ac\right)ex^{10} \\ +\frac{3}{11}c\left(bcd+b^2f+acf\right)x^{11}+\frac{1}{4}bc^2ex^{12}+\frac{1}{13}c^2(cd+3bf)x^{13}+\frac{1}{14}c^3ex^{14}+\frac{1}{15}c^3fx^{15}\end{aligned}$$

output
```
a^3*d*x+1/2*a^3*e*x^2+1/3*a^2*(a*f+3*b*d)*x^3+3/4*a^2*b*e*x^4+3/5*a*(a*b*f
+a*c*d+b^2*d)*x^5+1/2*a*(a*c+b^2)*e*x^6+1/7*(3*a^2*c*f+3*a*b^2*f+6*a*b*c*d
+b^3*d)*x^7+1/8*b*(6*a*c+b^2)*e*x^8+1/9*(6*a*b*c*f+3*a*c^2*d+b^3*f+3*b^2*c
*d)*x^9+3/10*c*(a*c+b^2)*e*x^10+3/11*c*(a*c*f+b^2*f+b*c*d)*x^11+1/4*b*c^2*
e*x^12+1/13*c^2*(3*b*f+c*d)*x^13+1/14*c^3*e*x^14+1/15*c^3*f*x^15
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2+cx^4\right)^2 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right) dx$$
$$= a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{3}a^2(3bd+af)x^3+\frac{3}{4}a^2bex^4+\frac{3}{5}a\left(b^2d+acd+abf\right)x^5 + \frac{1}{2}a\left(b^2+ac\right)ex^6+\frac{1}{7}\left(b^3d+6abcd+3ab^2f+3a^2cf\right)x^7+\frac{1}{8}b\left(b^2+6ac\right)ex^8 + \frac{1}{9}\left(3b^2cd+3ac^2d+b^3f+6abcf\right)x^9+\frac{3}{10}c\left(b^2+ac\right)ex^{10} + \frac{3}{11}c\left(bcd+b^2f+acf\right)x^{11}+\frac{1}{4}bc^2ex^{12}+\frac{1}{13}c^2(cd+3bf)x^{13}+\frac{1}{14}c^3ex^{14}+\frac{1}{15}c^3fx^{15}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 +
 (c*d + b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

output
```
a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*d + a*f)*x^3)/3 + (3*a^2*b*e*x^4)/4 +
(3*a*(b^2*d + a*c*d + a*b*f)*x^5)/5 + (a*(b^2 + a*c)*e*x^6)/2 + ((b^3*d +
6*a*b*c*d + 3*a*b^2*f + 3*a^2*c*f)*x^7)/7 + (b*(b^2 + 6*a*c)*e*x^8)/8 + ((
3*b^2*c*d + 3*a*c^2*d + b^3*f + 6*a*b*c*f)*x^9)/9 + (3*c*(b^2 + a*c)*e*x^1
0)/10 + (3*c*(b*c*d + b^2*f + a*c*f)*x^11)/11 + (b*c^2*e*x^12)/4 + (c^2*(c
*d + 3*b*f)*x^13)/13 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^2 \left(x^2(af+bd)+ad+aex+x^4(bf+cd)+bex^3+cex^5+cfx^6\right) dx$$

$$\Big\downarrow \text{ 2200}$$

$$\int \left(a^3d + a^3ex + x^6\left(3a^2cf + 3ab^2f + 6abcd + b^3d\right) + a^2x^2(af + 3bd) + 3a^2bex^3 + 3cx^{10}\left(acf + b^2f + bcd\right) + 3a$$

$$\Big\downarrow \text{2009}$$

$$a^3dx + \frac{1}{2}a^3ex^2 + \frac{1}{7}x^7\left(3a^2cf + 3ab^2f + 6abcd + b^3d\right) + \frac{1}{3}a^2x^3(af + 3bd) + \frac{3}{4}a^2bex^4 + \frac{3}{11}cx^{11}\left(acf + b^2f + bcd\right) + \frac{3}{5}ax^5\left(abf + acd + b^2d\right) + \frac{3}{10}cex^{10}\left(ac + b^2\right) + \frac{1}{8}bex^8\left(6ac + b^2\right) + \frac{1}{2}aex^6\left(ac + b^2\right) + \frac{1}{9}x^9\left(6abcf + 3ac^2d + b^3f + 3b^2cd\right) + \frac{1}{13}c^2x^{13}(3bf + cd) + \frac{1}{4}bc^2ex^{12} + \frac{1}{14}c^3ex^{14} + \frac{1}{15}c^3fx^{15}$$

input
```
Int[(a + b*x^2 + c*x^4)^2*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d
+ b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

output
```
a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*d + a*f)*x^3)/3 + (3*a^2*b*e*x^4)/4 +
(3*a*(b^2*d + a*c*d + a*b*f)*x^5)/5 + (a*(b^2 + a*c)*e*x^6)/2 + ((b^3*d +
6*a*b*c*d + 3*a*b^2*f + 3*a^2*c*f)*x^7)/7 + (b*(b^2 + 6*a*c)*e*x^8)/8 + ((
3*b^2*c*d + 3*a*c^2*d + b^3*f + 6*a*b*c*f)*x^9)/9 + (3*c*(b^2 + a*c)*e*x^1
0)/10 + (3*c*(b*c*d + b^2*f + a*c*f)*x^11)/11 + (b*c^2*e*x^12)/4 + (c^2*(c
*d + 3*b*f)*x^13)/13 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 24.45 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00

method	result
norman	$\left(\frac{1}{3}a^{3} f+a^{2} b d\right) x^{3}+\left(\frac{3}{10}a \,c^{2} e+\frac{3}{10}b^{2} c e\right) x^{10}+\left(\frac{1}{2}a^{2} c e+\frac{1}{2}a \,b^{2} e\right) x^{6}+\left(\frac{3}{13}b \,c^{2} f+\frac{1}{13}c^{3} d\right) x^{13}+$
risch	$\frac{3}{4}a^{2} b e \,x^{4}+\frac{1}{2}a \,b^{2} e \,x^{6}+\frac{1}{8}b^{3} e \,x^{8}+\frac{1}{2}a^{2} c e \,x^{6}+\frac{1}{4}b \,c^{2} e \,x^{12}+a^{3} d x +\frac{3}{13}x^{13} b \,c^{2} f +\frac{3}{11}x^{11} a \,c^{2} f +$
parallelrisch	$\frac{3}{4}a^{2} b e \,x^{4}+\frac{1}{2}a \,b^{2} e \,x^{6}+\frac{1}{8}b^{3} e \,x^{8}+\frac{1}{2}a^{2} c e \,x^{6}+\frac{1}{4}b \,c^{2} e \,x^{12}+a^{3} d x +\frac{3}{13}x^{13} b \,c^{2} f +\frac{3}{11}x^{11} a \,c^{2} f +$
gosper	$x \left(24024 c^{3} f \,x^{14}+25740 c^{3} e \,x^{13}+83160 b \,c^{2} f \,x^{12}+27720 c^{3} d \,x^{12}+90090 b \,c^{2} e \,x^{11}+98280 a \,c^{2} f \,x^{10}+98280 b^{2} c f \,x^{10}+98280 b \,c^{2} d$
default	$\frac{c^{3} f \,x^{15}}{15}+\frac{c^{3} e \,x^{14}}{14}+\frac{\left(2 b \,c^{2} f +c^{2} \left(b f +c d \right)\right) x^{13}}{13}+\frac{b \,c^{2} e \,x^{12}}{4}+\frac{\left(\left(2 a c +b^{2}\right) c f +2 b c \left(b f +c d \right)+c^{2} \left(a f +b d \right)\right) x^{11}}{11}+\frac{\left(\left(2 a c +b^{2}\right.\right.}{}$
orering	$x \left(24024 c^{3} f \,x^{14}+25740 c^{3} e \,x^{13}+83160 b \,c^{2} f \,x^{12}+27720 c^{3} d \,x^{12}+90090 b \,c^{2} e \,x^{11}+98280 a \,c^{2} f \,x^{10}+98280 b^{2} c f \,x^{10}+98280 b \,c^{2} d$

input
```
int((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x
^5+c*f*x^6),x,method=_RETURNVERBOSE)
```

output
```
(1/3*a^3*f+a^2*b*d)*x^3+(3/10*a*c^2*e+3/10*b^2*c*e)*x^10+(1/2*a^2*c*e+1/2*
a*b^2*e)*x^6+(3/13*b*c^2*f+1/13*c^3*d)*x^13+(3/4*a*b*c*e+1/8*b^3*e)*x^8+(3
/11*a*c^2*f+3/11*b^2*c*f+3/11*b*c^2*d)*x^11+(3/5*a^2*b*f+3/5*a^2*c*d+3/5*a
*b^2*d)*x^5+(3/7*a^2*c*f+3/7*a*b^2*f+6/7*a*b*c*d+1/7*b^3*d)*x^7+(2/3*a*b*c
*f+1/3*a*c^2*d+1/9*b^3*f+1/3*b^2*c*d)*x^9+a^3*d*x+1/2*a^3*e*x^2+1/14*c^3*e
*x^14+1/15*c^3*f*x^15+3/4*a^2*b*e*x^4+1/4*b*c^2*e*x^12
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.97

$$\begin{aligned}
&\int \left(a+bx^2+cx^4\right)^2 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right) dx \\
&= \frac{1}{15}c^3fx^{15}+\frac{1}{14}c^3ex^{14}+\frac{1}{4}bc^2ex^{12}+\frac{1}{13}\left(c^3d+3\,bc^2f\right)x^{13} \\
&\quad+\frac{3}{10}\left(b^2c+ac^2\right)ex^{10}+\frac{3}{11}\left(bc^2d+\left(b^2c+ac^2\right)f\right)x^{11} \\
&\quad+\frac{1}{8}\left(b^3+6\,abc\right)ex^8+\frac{1}{9}\left(3\left(b^2c+ac^2\right)d+\left(b^3+6\,abc\right)f\right)x^9 \\
&\quad+\frac{3}{4}a^2bex^4+\frac{1}{2}\left(ab^2+a^2c\right)ex^6+\frac{1}{7}\left(\left(b^3+6\,abc\right)d+3\left(ab^2+a^2c\right)f\right)x^7 \\
&\quad+\frac{1}{2}a^3ex^2+\frac{3}{5}\left(a^2bf+\left(ab^2+a^2c\right)d\right)x^5+a^3dx+\frac{1}{3}\left(3\,a^2bd+a^3f\right)x^3
\end{aligned}$$

input
```
integrate((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4
+c*e*x^5+c*f*x^6),x, algorithm="fricas")
```

output
```
1/15*c^3*f*x^15 + 1/14*c^3*e*x^14 + 1/4*b*c^2*e*x^12 + 1/13*(c^3*d + 3*b*c
^2*f)*x^13 + 3/10*(b^2*c + a*c^2)*e*x^10 + 3/11*(b*c^2*d + (b^2*c + a*c^2)
*f)*x^11 + 1/8*(b^3 + 6*a*b*c)*e*x^8 + 1/9*(3*(b^2*c + a*c^2)*d + (b^3 + 6
*a*b*c)*f)*x^9 + 3/4*a^2*b*e*x^4 + 1/2*(a*b^2 + a^2*c)*e*x^6 + 1/7*((b^3 +
 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*f)*x^7 + 1/2*a^3*e*x^2 + 3/5*(a^2*b*f + (a
*b^2 + a^2*c)*d)*x^5 + a^3*d*x + 1/3*(3*a^2*b*d + a^3*f)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.19

$$\int (a + bx^2 + cx^4)^2 (ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6) \, dx$$
$$= a^3dx + \frac{a^3ex^2}{2} + \frac{3a^2bex^4}{4} + \frac{bc^2ex^{12}}{4} + \frac{c^3ex^{14}}{14} + \frac{c^3fx^{15}}{15} + x^{13} \cdot \left(\frac{3bc^2f}{13} + \frac{c^3d}{13}\right) + x^{11} \cdot \left(\frac{3ac^2f}{11} + \frac{3b^2cf}{11} + \frac{3bc^2d}{11}\right) + x^{10} \cdot \left(\frac{3ac^2e}{10} + \frac{3b^2ce}{10}\right) + x^9 \cdot \left(\frac{2abcf}{3} + \frac{ac^2d}{3} + \frac{b^3f}{9} + \frac{b^2cd}{3}\right) + x^8 \cdot \left(\frac{3abce}{4} + \frac{b^3e}{8}\right) + x^7 \cdot \left(\frac{3a^2cf}{7} + \frac{3ab^2f}{7} + \frac{6abcd}{7} + \frac{b^3d}{7}\right) + x^6\left(\frac{a^2ce}{2} + \frac{ab^2e}{2}\right) + x^5 \cdot \left(\frac{3a^2bf}{5} + \frac{3a^2cd}{5} + \frac{3ab^2d}{5}\right) + x^3\left(\frac{a^3f}{3} + a^2bd\right)$$

input
```
integrate((c*x**4+b*x**2+a)**2*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d
)*x**4+c*e*x**5+c*f*x**6),x)
```

output
```
a**3*d*x + a**3*e*x**2/2 + 3*a**2*b*e*x**4/4 + b*c**2*e*x**12/4 + c**3*e*x
**14/14 + c**3*f*x**15/15 + x**13*(3*b*c**2*f/13 + c**3*d/13) + x**11*(3*a
*c**2*f/11 + 3*b**2*c*f/11 + 3*b*c**2*d/11) + x**10*(3*a*c**2*e/10 + 3*b**
2*c*e/10) + x**9*(2*a*b*c*f/3 + a*c**2*d/3 + b**3*f/9 + b**2*c*d/3) + x**8
*(3*a*b*c*e/4 + b**3*e/8) + x**7*(3*a**2*c*f/7 + 3*a*b**2*f/7 + 6*a*b*c*d/
7 + b**3*d/7) + x**6*(a**2*c*e/2 + a*b**2*e/2) + x**5*(3*a**2*b*f/5 + 3*a*
*2*c*d/5 + 3*a*b**2*d/5) + x**3*(a**3*f/3 + a**2*b*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.97

$$\int (a+bx^2+cx^4)^2 (ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6)\,dx$$
$$= \frac{1}{15}c^3fx^{15}+\frac{1}{14}c^3ex^{14}+\frac{1}{4}bc^2ex^{12}+\frac{1}{13}(c^3d+3\,bc^2f)x^{13} + \frac{3}{10}(b^2c+ac^2)ex^{10}+\frac{3}{11}(bc^2d+(b^2c+ac^2)f)x^{11} + \frac{1}{8}(b^3+6\,abc)ex^8+\frac{1}{9}(3(b^2c+ac^2)d+(b^3+6\,abc)f)x^9 + \frac{3}{4}a^2bex^4+\frac{1}{2}(ab^2+a^2c)ex^6+\frac{1}{7}((b^3+6\,abc)d+3(ab^2+a^2c)f)x^7 + \frac{1}{2}a^3ex^2+\frac{3}{5}(a^2bf+(ab^2+a^2c)d)x^5+a^3dx+\frac{1}{3}(3\,a^2bd+a^3f)x^3$$

input
```
integrate((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4
+c*e*x^5+c*f*x^6),x, algorithm="maxima")
```

output
```
1/15*c^3*f*x^15 + 1/14*c^3*e*x^14 + 1/4*b*c^2*e*x^12 + 1/13*(c^3*d + 3*b*c
^2*f)*x^13 + 3/10*(b^2*c + a*c^2)*e*x^10 + 3/11*(b*c^2*d + (b^2*c + a*c^2)
*f)*x^11 + 1/8*(b^3 + 6*a*b*c)*e*x^8 + 1/9*(3*(b^2*c + a*c^2)*d + (b^3 + 6
*a*b*c)*f)*x^9 + 3/4*a^2*b*e*x^4 + 1/2*(a*b^2 + a^2*c)*e*x^6 + 1/7*((b^3 +
 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*f)*x^7 + 1/2*a^3*e*x^2 + 3/5*(a^2*b*f + (a
*b^2 + a^2*c)*d)*x^5 + a^3*d*x + 1/3*(3*a^2*b*d + a^3*f)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.10

$$\int (a+bx^2+cx^4)^2 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx$$
$$= \frac{1}{15}c^3fx^{15}+\frac{1}{14}c^3ex^{14}+\frac{1}{13}c^3dx^{13}+\frac{3}{13}bc^2fx^{13}+\frac{1}{4}bc^2ex^{12}+\frac{3}{11}bc^2dx^{11}+\frac{3}{11}b^2cfx^{11}+\frac{3}{11}ac^2fx^{11}+\frac{3}{10}b^2cex^{10}+\frac{3}{10}ac^2ex^{10}+\frac{1}{3}b^2cdx^9+\frac{1}{3}ac^2dx^9+\frac{1}{9}b^3fx^9+\frac{2}{3}abcfx^9+\frac{1}{8}b^3ex^8+\frac{3}{4}abcex^8+\frac{1}{7}b^3dx^7+\frac{6}{7}abcdx^7+\frac{3}{7}ab^2fx^7+\frac{3}{7}a^2cfx^7+\frac{1}{2}ab^2ex^6+\frac{1}{2}a^2cex^6+\frac{3}{5}ab^2dx^5+\frac{3}{5}a^2cdx^5+\frac{3}{5}a^2bfx^5+\frac{3}{4}a^2bex^4+a^2bdx^3+\frac{1}{3}a^3fx^3+\frac{1}{2}a^3ex^2+a^3dx$$

input
```
integrate((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4
+c*e*x^5+c*f*x^6),x, algorithm="giac")
```

output
```
1/15*c^3*f*x^15 + 1/14*c^3*e*x^14 + 1/13*c^3*d*x^13 + 3/13*b*c^2*f*x^13 +
1/4*b*c^2*e*x^12 + 3/11*b*c^2*d*x^11 + 3/11*b^2*c*f*x^11 + 3/11*a*c^2*f*x^
11 + 3/10*b^2*c*e*x^10 + 3/10*a*c^2*e*x^10 + 1/3*b^2*c*d*x^9 + 1/3*a*c^2*d
*x^9 + 1/9*b^3*f*x^9 + 2/3*a*b*c*f*x^9 + 1/8*b^3*e*x^8 + 3/4*a*b*c*e*x^8 +
 1/7*b^3*d*x^7 + 6/7*a*b*c*d*x^7 + 3/7*a*b^2*f*x^7 + 3/7*a^2*c*f*x^7 + 1/2
*a*b^2*e*x^6 + 1/2*a^2*c*e*x^6 + 3/5*a*b^2*d*x^5 + 3/5*a^2*c*d*x^5 + 3/5*a
^2*b*f*x^5 + 3/4*a^2*b*e*x^4 + a^2*b*d*x^3 + 1/3*a^3*f*x^3 + 1/2*a^3*e*x^2
 + a^3*d*x
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2+cx^4\right)^2 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right) dx = x^3\left(\frac{f\,a^3}{3}+b\,d\,a^2\right)+x^{13}\left(\frac{d\,c^3}{13}+\frac{3\,b\,f\,c^2}{13}\right)+x^5\left(\frac{3\,f\,a^2\,b}{5}+\frac{3\,c\,d\,a^2}{5}+\frac{3\,d\,a\,b^2}{5}\right)+x^{11}\left(\frac{3\,f\,b^2\,c}{11}+\frac{3\,d\,b\,c^2}{11}+\frac{3\,a\,f\,c^2}{11}\right)+x^7\left(\frac{3\,c\,f\,a^2}{7}+\frac{3\,f\,a\,b^2}{7}+\frac{6\,c\,d\,a\,b}{7}+\frac{d\,b^3}{7}\right)+x^9\left(\frac{f\,b^3}{9}+\frac{d\,b^2\,c}{3}+\frac{2\,a\,f\,b\,c}{3}+\frac{a\,d\,c^2}{3}\right)+\frac{a^3\,e\,x^2}{2}+\frac{c^3\,e\,x^{14}}{14}+\frac{c^3\,f\,x^{15}}{15}+a^3\,d\,x+\frac{a\,e\,x^6\,(b^2+a\,c)}{2}+\frac{b\,e\,x^8\,(b^2+6\,a\,c)}{8}+\frac{3\,c\,e\,x^{10}\,(b^2+a\,c)}{10}+\frac{3\,a^2\,b\,e\,x^4}{4}+\frac{b\,c^2\,e\,x^{12}}{4}$$

input
```
int((a + b*x^2 + c*x^4)^2*(a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x
 + b*e*x^3 + c*e*x^5 + c*f*x^6),x)
```

output
```
x^3*((a^3*f)/3 + a^2*b*d) + x^13*((c^3*d)/13 + (3*b*c^2*f)/13) + x^5*((3*a
*b^2*d)/5 + (3*a^2*c*d)/5 + (3*a^2*b*f)/5) + x^11*((3*b*c^2*d)/11 + (3*a*c
^2*f)/11 + (3*b^2*c*f)/11) + x^7*((b^3*d)/7 + (3*a*b^2*f)/7 + (3*a^2*c*f)/
7 + (6*a*b*c*d)/7) + x^9*((b^3*f)/9 + (a*c^2*d)/3 + (b^2*c*d)/3 + (2*a*b*c
*f)/3) + (a^3*e*x^2)/2 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15 + a^3*d*x + (a*
e*x^6*(a*c + b^2))/2 + (b*e*x^8*(6*a*c + b^2))/8 + (3*c*e*x^10*(a*c + b^2)
)/10 + (3*a^2*b*e*x^4)/4 + (b*c^2*e*x^12)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.11

$$\int \left(a+bx^2+cx^4\right)^2 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right) dx = \frac{x(24024c^3f\,x^{14}+25740c^3e\,x^{13}+83160b\,c^2f\,x^{12}+27720c^3d\,x^{12}+90090b\,c^2e\,x^{11}+98280a\,c^2f\,x^{10}+982}{}$$

input
```
int((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x
^5+c*f*x^6),x)
```

output

```
(x*(360360*a**3*d + 180180*a**3*e*x + 120120*a**3*f*x**2 + 360360*a**2*b*d
*x**2 + 270270*a**2*b*e*x**3 + 216216*a**2*b*f*x**4 + 216216*a**2*c*d*x**4
 + 180180*a**2*c*e*x**5 + 154440*a**2*c*f*x**6 + 216216*a*b**2*d*x**4 + 18
0180*a*b**2*e*x**5 + 154440*a*b**2*f*x**6 + 308880*a*b*c*d*x**6 + 270270*a
*b*c*e*x**7 + 240240*a*b*c*f*x**8 + 120120*a*c**2*d*x**8 + 108108*a*c**2*e
*x**9 + 98280*a*c**2*f*x**10 + 51480*b**3*d*x**6 + 45045*b**3*e*x**7 + 400
40*b**3*f*x**8 + 120120*b**2*c*d*x**8 + 108108*b**2*c*e*x**9 + 98280*b**2*
c*f*x**10 + 98280*b*c**2*d*x**10 + 90090*b*c**2*e*x**11 + 83160*b*c**2*f*x
**12 + 27720*c**3*d*x**12 + 25740*c**3*e*x**13 + 24024*c**3*f*x**14))/3603
60
```

3.59 $\int (a+bx^2+cx^4)(ad+aex+(bd+af)x^2+bex^3+$

Optimal result

Integrand size = 61, antiderivative size = 154

$$\int (a+bx^2+cx^4)\left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx$$
$$=a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{2}abex^4+\frac{1}{5}\left(b^2d+2acd+2abf\right)x^5+\frac{1}{6}\left(b^2+2ac\right)ex^6+\frac{1}{7}\left(2bcd+b^2f+2acf\right)x^7+\frac{1}{4}bcex^8+\frac{1}{9}c(cd+2bf)x^9+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

output
```
a^2*d*x+1/2*a^2*e*x^2+1/3*a*(a*f+2*b*d)*x^3+1/2*a*b*e*x^4+1/5*(2*a*b*f+2*a
*c*d+b^2*d)*x^5+1/6*(2*a*c+b^2)*e*x^6+1/7*(2*a*c*f+b^2*f+2*b*c*d)*x^7+1/4*
b*c*e*x^8+1/9*c*(2*b*f+c*d)*x^9+1/10*c^2*e*x^10+1/11*c^2*f*x^11
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\int (a+bx^2+cx^4)\left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx$$
$$=a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{2}abex^4+\frac{1}{5}\left(b^2d+2acd+2abf\right)x^5+\frac{1}{6}\left(b^2+2ac\right)ex^6+\frac{1}{7}\left(2bcd+b^2f+2acf\right)x^7+\frac{1}{4}bcex^8+\frac{1}{9}c(cd+2bf)x^9+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

input
```
Integrate[(a + b*x^2 + c*x^4)*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (
c*d + b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*
d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*
f + 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^1
0)/10 + (c^2*f*x^11)/11
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right) \left(x^2(af+bd)+ad+aex+x^4(bf+cd)+bex^3+cex^5+cfx^6\right) \, dx$$

$$\Big\downarrow \text{2200}$$

$$\int \left(a^2d+a^2ex+x^6\left(2acf+b^2f+2bcd\right)+x^4\left(2abf+2acd+b^2d\right)+ex^5\left(2ac+b^2\right)+ax^2(af+2bd)+2abex^3+$$

$$\Big\downarrow \text{2009}$$

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{4}bcex^8+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

input
```
Int[(a + b*x^2 + c*x^4)*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d +
b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*
d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*
f + 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^1
0)/10 + (c^2*f*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

method	result
norman	$\frac{c^2 f x^{11}}{11}+\frac{c^2 e x^{10}}{10}+\left(\frac{2}{9} b c f+\frac{1}{9} d c^2\right) x^9+\frac{b c e x^8}{4}+\left(\frac{2}{7} a c f+\frac{1}{7} b^2 f+\frac{2}{7} b c d\right) x^7+\left(\frac{1}{3} a c e+\frac{1}{6} b^2 e\right) x^6$
risch	$\frac{1}{11} c^2 f x^{11}+\frac{1}{10} c^2 e x^{10}+\frac{2}{9} x^9 b c f+\frac{1}{9} c^2 d x^9+\frac{1}{4} b c e x^8+\frac{2}{7} x^7 a c f+\frac{1}{7} x^7 b^2 f+\frac{2}{7} x^7 b c d+\frac{1}{3} a c e x$
parallelrisch	$\frac{1}{11} c^2 f x^{11}+\frac{1}{10} c^2 e x^{10}+\frac{2}{9} x^9 b c f+\frac{1}{9} c^2 d x^9+\frac{1}{4} b c e x^8+\frac{2}{7} x^7 a c f+\frac{1}{7} x^7 b^2 f+\frac{2}{7} x^7 b c d+\frac{1}{3} a c e x$
gosper	$\frac{x\left(1260 f c^2 x^{10}+1386 c^2 e x^9+3080 x^8 b c f+1540 x^8 d c^2+3465 b c e x^7+3960 a c f x^6+1980 x^6 b^2 f+3960 b c d x^6+4620 a c e x^5+2310 x\right.}{13860}$
default	$\frac{c^2 f x^{11}}{11}+\frac{c^2 e x^{10}}{10}+\frac{(b c f+c(b f+c d)) x^9}{9}+\frac{b c e x^8}{4}+\frac{(a c f+b(b f+c d)+c(a f+b d)) x^7}{7}+\frac{(2 a c e+b^2 e) x^6}{6}+\frac{(a(b f+c d)+}{}$
orering	$\frac{x\left(1260 f c^2 x^{10}+1386 c^2 e x^9+3080 x^8 b c f+1540 x^8 d c^2+3465 b c e x^7+3960 a c f x^6+1980 x^6 b^2 f+3960 b c d x^6+4620 a c e x^5+2310 x\right.}{}$

input
```
int((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5
+c*f*x^6),x,method=_RETURNVERBOSE)
```

output
```
1/11*c^2*f*x^11+1/10*c^2*e*x^10+(2/9*b*c*f+1/9*d*c^2)*x^9+1/4*b*c*e*x^8+(2
/7*a*c*f+1/7*b^2*f+2/7*b*c*d)*x^7+(1/3*a*c*e+1/6*b^2*e)*x^6+(2/5*a*b*f+2/5
*d*a*c+1/5*b^2*d)*x^5+1/2*a*b*e*x^4+(1/3*f*a^2+2/3*a*b*d)*x^3+1/2*a^2*e*x^
2+a^2*d*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int (a+bx^2+cx^4)\left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx$$
$$=\frac{1}{11}c^2fx^{11}+\frac{1}{10}c^2ex^{10}+\frac{1}{4}bcex^8+\frac{1}{9}\left(c^2d+2\,bcf\right)x^9+\frac{1}{6}\left(b^2+2\,ac\right)ex^6+\frac{1}{7}\left(2\,bcd+\left(b^2+2\,ac\right)f\right)x^7+\frac{1}{2}abex^4+\frac{1}{5}\left(2\,abf+\left(b^2+2\,ac\right)d\right)x^5+\frac{1}{2}a^2ex^2+a^2dx+\frac{1}{3}\left(2\,abd+a^2f\right)x^3$$

input
```
integrate((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c
*e*x^5+c*f*x^6),x, algorithm="fricas")
```

output
```
1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/4*b*c*e*x^8 + 1/9*(c^2*d + 2*b*c*f)*
x^9 + 1/6*(b^2 + 2*a*c)*e*x^6 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*f)*x^7 + 1/2*
a*b*e*x^4 + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2 + a^2*d*x
+ 1/3*(2*a*b*d + a^2*f)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.07

$$\int (a+bx^2+cx^4)\left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx$$
$$=a^2dx+\frac{a^2ex^2}{2}+\frac{abex^4}{2}+\frac{bcex^8}{4}+\frac{c^2ex^{10}}{10}+\frac{c^2fx^{11}}{11}+x^9\cdot\left(\frac{2bcf}{9}+\frac{c^2d}{9}\right)+x^7\cdot\left(\frac{2acf}{7}+\frac{b^2f}{7}+\frac{2bcd}{7}\right)+x^6\left(\frac{ace}{3}+\frac{b^2e}{6}\right)+x^5\cdot\left(\frac{2abf}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right)+x^3\left(\frac{a^2f}{3}+\frac{2abd}{3}\right)$$

input
```
integrate((c*x**4+b*x**2+a)*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x
**4+c*e*x**5+c*f*x**6),x)
```

output
```
a**2*d*x + a**2*e*x**2/2 + a*b*e*x**4/2 + b*c*e*x**8/4 + c**2*e*x**10/10 +
 c**2*f*x**11/11 + x**9*(2*b*c*f/9 + c**2*d/9) + x**7*(2*a*c*f/7 + b**2*f/
7 + 2*b*c*d/7) + x**6*(a*c*e/3 + b**2*e/6) + x**5*(2*a*b*f/5 + 2*a*c*d/5 +
 b**2*d/5) + x**3*(a**2*f/3 + 2*a*b*d/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int (a+bx^2+cx^4)\,(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6)\;dx$$
$$=\frac{1}{11}\,c^2fx^{11}+\frac{1}{10}\,c^2ex^{10}+\frac{1}{4}\,bcex^8+\frac{1}{9}\,(c^2d+2\,bcf)x^9+\frac{1}{6}\,(b^2+2\,ac)ex^6+\frac{1}{7}\,(2\,bcd+(b^2+2\,ac)f)x^7+\frac{1}{2}\,abex^4+\frac{1}{5}\,(2\,abf+(b^2+2\,ac)d)x^5+\frac{1}{2}\,a^2ex^2+a^2dx+\frac{1}{3}\,(2\,abd+a^2f)x^3$$

input
```
integrate((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c
*e*x^5+c*f*x^6),x, algorithm="maxima")
```

output
```
1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/4*b*c*e*x^8 + 1/9*(c^2*d + 2*b*c*f)*
x^9 + 1/6*(b^2 + 2*a*c)*e*x^6 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*f)*x^7 + 1/2*
a*b*e*x^4 + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2 + a^2*d*x
+ 1/3*(2*a*b*d + a^2*f)*x^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.98

$$\int (a+bx^2+cx^4)\,(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6)\;dx$$
$$=\frac{1}{11}\,c^2fx^{11}+\frac{1}{10}\,c^2ex^{10}+\frac{1}{9}\,c^2dx^9+\frac{2}{9}\,bcfx^9+\frac{1}{4}\,bcex^8+\frac{2}{7}\,bcdx^7+\frac{1}{7}\,b^2fx^7+\frac{2}{7}\,acfx^7+\frac{1}{6}\,b^2ex^6+\frac{1}{3}\,acex^6+\frac{1}{5}\,b^2dx^5+\frac{2}{5}\,acdx^5+\frac{2}{5}\,abfx^5+\frac{1}{2}\,abex^4+\frac{2}{3}\,abdx^3+\frac{1}{3}\,a^2fx^3+\frac{1}{2}\,a^2ex^2+a^2dx$$

input
```
integrate((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c
*e*x^5+c*f*x^6),x, algorithm="giac")
```

output
```
1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/4*b*
c*e*x^8 + 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7 + 1/6*b^2*e*x^6 +
1/3*a*c*e*x^6 + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^5 + 1/2*a*b*e*
x^4 + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*e*x^2 + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 17.98 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int (a+bx^2+cx^4)\left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx$$
$$=x^5\left(\frac{d\,b^2}{5}+\frac{2\,a\,f\,b}{5}+\frac{2\,a\,c\,d}{5}\right)+x^7\left(\frac{f\,b^2}{7}+\frac{2\,c\,d\,b}{7}+\frac{2\,a\,c\,f}{7}\right)+x^3\left(\frac{f\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)+x^9\left(\frac{d\,c^2}{9}+\frac{2\,b\,f\,c}{9}\right)+\frac{a^2\,e\,x^2}{2}+\frac{c^2\,e\,x^{10}}{10}+\frac{c^2\,f\,x^{11}}{11}+\frac{e\,x^6\,(b^2+2\,a\,c)}{6}+a^2\,d\,x+\frac{a\,b\,e\,x^4}{2}+\frac{b\,c\,e\,x^8}{4}$$

input
```
int((a + b*x^2 + c*x^4)*(a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x +
 b*e*x^3 + c*e*x^5 + c*f*x^6),x)
```

output
```
x^5*((b^2*d)/5 + (2*a*c*d)/5 + (2*a*b*f)/5) + x^7*((b^2*f)/7 + (2*b*c*d)/7
 + (2*a*c*f)/7) + x^3*((a^2*f)/3 + (2*a*b*d)/3) + x^9*((c^2*d)/9 + (2*b*c*
f)/9) + (a^2*e*x^2)/2 + (c^2*e*x^10)/10 + (c^2*f*x^11)/11 + (e*x^6*(2*a*c
+ b^2))/6 + a^2*d*x + (a*b*e*x^4)/2 + (b*c*e*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int (a+bx^2+cx^4)\left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)dx$$
$$= \frac{x(1260c^2f\,x^{10}+1386c^2e\,x^9+3080bcf\,x^8+1540c^2d\,x^8+3465bce\,x^7+3960acf\,x^6+1980b^2f\,x^6+3960}{}$$

input
```
int((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5
+c*f*x^6),x)
```

output
```
(x*(13860*a**2*d + 6930*a**2*e*x + 4620*a**2*f*x**2 + 9240*a*b*d*x**2 + 69
30*a*b*e*x**3 + 5544*a*b*f*x**4 + 5544*a*c*d*x**4 + 4620*a*c*e*x**5 + 3960
*a*c*f*x**6 + 2772*b**2*d*x**4 + 2310*b**2*e*x**5 + 1980*b**2*f*x**6 + 396
0*b*c*d*x**6 + 3465*b*c*e*x**7 + 3080*b*c*f*x**8 + 1540*c**2*d*x**8 + 1386
*c**2*e*x**9 + 1260*c**2*f*x**10))/13860
```

3.60 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 63, antiderivative size = 20

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{a+bx^2+cx^4} dx = dx+\frac{ex^2}{2}+\frac{fx^3}{3}$$

output
```
d*x+1/2*e*x^2+1/3*f*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{a+bx^2+cx^4} dx = dx+\frac{ex^2}{2}+\frac{fx^3}{3}$$

input
```
Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e
*x^5 + c*f*x^6)/(a + b*x^2 + c*x^4),x]
```

output
```
d*x + (e*x^2)/2 + (f*x^3)/3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(af+bd)+ad+aex+x^4(bf+cd)+bex^3+cex^5+cfx^6}{a+bx^2+cx^4}dx$$
$$\Big\downarrow \text{2019}$$
$$\int \left(d+ex+fx^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$dx+\frac{ex^2}{2}+\frac{fx^3}{3}$$

input
```
Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
 c*f*x^6)/(a + b*x^2 + c*x^4),x]
```

output
```
d*x + (e*x^2)/2 + (f*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$dx+\frac{1}{2}e\,x^{2}+\frac{1}{3}f\,x^{3}$	17
norman	$dx+\frac{1}{2}e\,x^{2}+\frac{1}{3}f\,x^{3}$	17
risch	$dx+\frac{1}{2}e\,x^{2}+\frac{1}{3}f\,x^{3}$	17
parallelrisch	$dx+\frac{1}{2}e\,x^{2}+\frac{1}{3}f\,x^{3}$	17
parts	$dx+\frac{1}{2}e\,x^{2}+\frac{1}{3}f\,x^{3}$	17
gosper	$\frac{x(2f\,x^{2}+3ex+6d)}{6}$	18
orering	$\frac{x(2f\,x^{2}+3ex+6d)\left(ad+aex+(af+bd)x^{2}+be\,x^{3}+(bf+cd)x^{4}+ce\,x^{5}+cf\,x^{6}\right)}{6(f\,x^{2}+ex+d)(c\,x^{4}+b\,x^{2}+a)}$	92

input
```
int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4
+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d*x+1/2*e*x^2+1/3*f*x^3
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{a+bx^2+cx^4}\,dx=\frac{1}{3}\,fx^3+\frac{1}{2}\,ex^2+dx$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/3*f*x^3 + 1/2*e*x^2 + d*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{a + bx^2 + cx^4} dx = dx + \frac{ex^2}{2} + \frac{fx^3}{3}$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x
**6)/(c*x**4+b*x**2+a),x)
```

output
```
d*x + e*x**2/2 + f*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{a + bx^2 + cx^4} dx = \frac{1}{3} fx^3 + \frac{1}{2} ex^2 + dx$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/3*f*x^3 + 1/2*e*x^2 + d*x
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{a + bx^2 + cx^4} dx = \frac{1}{3} fx^3 + \frac{1}{2} ex^2 + dx$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/3*f*x^3 + 1/2*e*x^2 + d*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{a+bx^2+cx^4}\,dx=\frac{f\,x^3}{3}+\frac{e\,x^2}{2}+d\,x$$

input
```
int((a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x + b*e*x^3 + c*e*x^5 +
 c*f*x^6)/(a + b*x^2 + c*x^4),x)
```

output
```
d*x + (e*x^2)/2 + (f*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{a+bx^2+cx^4}\,dx$$
$$=\frac{x(2f\,x^2+3ex+6d)}{6}$$

input
```
int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4
+b*x^2+a),x)
```

output
```
(x*(6*d + 3*e*x + 2*f*x**2))/6
```

3.61 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 63, antiderivative size = 211

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^2}\,dx = \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{e\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
1/2*(f+(-b*f+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*(f-(
-b*f+2*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(
1/2))^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-e*arctanh((2*c*x
^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.11

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{\frac{\sqrt{2}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)f\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)f\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}+e\log\left(-b+\sqrt{b^2-\right.}{2\sqrt{b^2-4ac}}$$

input
```
Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e
*x^5 + c*f*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((Sqrt[2]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + e*Log[-b
+ Sqrt[b^2 - 4*a*c] - 2*c*x^2] - e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
2*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.127$, Rules used = {2019, 2202, 27, 1432, 1083, 219, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(af+bd)+ad+aex+x^4(bf+cd)+bex^3+cex^5+cfx^6}{(a+bx^2+cx^4)^2}\,dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}dx$$
$$\Big\downarrow \text{2202}$$

$$\int \frac{fx^2+d}{cx^4+bx^2+a}dx+\int \frac{ex}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{fx^2+d}{cx^4+bx^2+a}dx+e\int \frac{x}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{ 1432}$$

$$\int \frac{fx^2+d}{cx^4+bx^2+a}dx+\frac{1}{2}e\int \frac{1}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{fx^2+d}{cx^4+bx^2+a}dx-e\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{fx^2+d}{cx^4+bx^2+a}dx-\frac{e\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{2}\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx-\frac{e\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
 c*f*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((f
- (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(\_R^2 f+\_R e+d\right)\ln\left(x-\_R\right)}{2\_R^3 c+\_R b}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(-\frac{e\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{2}+\frac{\left(\sqrt{-4ac+b^2}\,f+bf-2cd\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4c(4ac-b^2)}-\frac{\sqrt{-4ac+b^2}\left(\frac{e\ln(\ldots}{\ldots}\right.}{\ldots}\right.$

input
```
int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4
+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2*f+_R*e+d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.13 (sec) , antiderivative size = 723401, normalized size of antiderivative = 3428.44

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^2}\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x
**6)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^2}\,dx$$
$$= \int \frac{cfx^6+cex^5+bex^3+(cd+bf)x^4+aex+(bd+af)x^2+ad}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d +
a*f)*x^2 + a*d)/(c*x^4 + b*x^2 + a)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(173) = 346$.

Time = 0.75 (sec) , antiderivative size = 1618, normalized size of antiderivative = 7.67

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*e*log(x^2 + 1/2
*(b + sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a
*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)
*sqrt(b^2 - 4*a*c)*e*log(x^2 + 1/2*(b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*
b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*(
(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c -
 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4
*a*c)*b*c^2)*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*f)*arctan(2*sqrt(1/2)*x/sqrt((...
```

Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 3942, normalized size of antiderivative = 18.68

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x + b*e*x^3 + c*e*x^5 +
 c*f*x^6)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log(c^2*d*e^2 - c^2*d^2*f + c^2*e^3*x - a*c*f^3 - 8*root(16*a*b^4*c
*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2 + 64*a^2
*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2*d^2*z^2
 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*c*e*f^2
*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*e^2*f
+ 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f^2 +
a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^3*b^3*c^2*x + b*c*d*f^2 -
 16*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2
*c*d*f*z^2 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 -
 16*a*b*c^2*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z
^2 + 16*a^2*c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e
*z - 4*a*c*d*e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e
^2 + a*b*e^2*f^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^2*a*c^
3*d - 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a
*b^2*c*d*f*z^2 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z
^2 - 16*a*b*c^2*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f
^2*z^2 + 16*a^2*c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d
^2*e*z - 4*a*c*d*e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d
^2*e^2 + a*b*e^2*f^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)*c^
3*d^2*x + 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 829, normalized size of antiderivative = 3.93

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^2} dx$$
$$= \text{Too large to display}$$

input
```
int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4
+b*x^2+a)^2,x)
```

output
```
( - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*e -
4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*f + 2*sqrt(a)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*b*c*d + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*f
- 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d - 4*sqrt(2*sqrt(c)*sqrt(
a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*e + 4*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a*c*f - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c*d -
2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +
2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*f + 4*sqrt(c)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*a*c*d + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqr
t(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c*f - sqrt(a)*sqrt(
2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) +...
```

3.62 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 63, antiderivative size = 368

$$\begin{aligned}
&\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^3}\,dx \\
&= -\frac{e(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad+\frac{\sqrt{c}\left(bd-2af+\frac{b^2d-12acd+4abf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad+\frac{\sqrt{c}\left(bd-2af-\frac{b^2d-12acd+4abf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}+\frac{2ce\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}$$

output
```
-1/2*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*(b^2*d-2*a*c*d-a*b*f
+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/2)*(b*d-2*a*f
+(4*a*b*f-12*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-
(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2)+1/4*c^(1/2)*(b*d-2*a*f-(4*a*b*f-12*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*a
rctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+2*c*e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.08

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2ab(e+fx)-2bdx(b+cx^2)+4acx(d+x(e+fx))}{a(-b^2+4ac)(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}d+4af\right)-2a\left(6cd+\sqrt{b^2-4ac}f\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\sqrt{c}\left(-b^2d+12acd+b\sqrt{b^2-4ac}d-4abf-2a\sqrt{b^2-4ac}f\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.-\frac{4ce\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{4ce\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e
*x^5 + c*f*x^6)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((2*a*b*(e + f*x) - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x)))/(a*(-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2
- 4*a*c]*d + 4*a*f) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4
*a*c]*d - 4*a*b*f - 2*a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]) - (4*c*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2)
+ (4*c*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2019, 2202, 27, 1432, 1086, 1083, 219, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(af+bd)+ad+aex+x^4(bf+cd)+bex^3+cex^5+cfx^6}{(a+bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx+\int \frac{ex}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx+e\int \frac{x}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{ 1432}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}e\int \frac{1}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{1}{2}e\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)+\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}e\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)+\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}e\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \; 1492$$

$$-\frac{\int -\frac{db^2+afb+c(bd-2af)x^2-6acd}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{1}{2}e\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{db^2+afb+c(bd-2af)x^2-6acd}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{1}{2}e\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{\frac{1}{2}c\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a\left(b^2-4ac\right)}$$
$$\frac{1}{2}e\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{1}{2}e\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
 c*f*x^6)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + ((Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d + 4*a*b*f)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d -
 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 -
4*a*c)) + (e*(-((b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (4*c*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.63

method	result
risch	$\frac{\frac{c(2af-bd)x^3}{2(4ac-b^2)a}+\frac{c x^2 e}{4ac-b^2}+\frac{(abf+2dac-b^2d)x}{2a(4ac-b^2)}+\frac{be}{8ac-2b^2}}{c x^4+b x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{c(2af-bd)\_R^2}{(4ac-b^2)a}+\frac{4\_Rce}{4ac-b^2}-\frac{abf-6dac+b^2}{(4ac-b^2)a}\right.}{2\_R^3c+\_Rb}\right.}{4}$
default	$16c^2\left(\frac{\frac{\frac{(4\sqrt{-4ac+b^2}\,acd-\sqrt{-4ac+b^2}\,b^2d+8a^2cf-2ab^2f-4abcd+b^3d)x}{16ac}+\frac{e(4ac-b^2)}{8c}}{x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}}+\frac{2\sqrt{-4ac+b^2}\,ae\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)+\frac{(-4\sqrt{-}}{}}{}}{4c(4ac-b^2)^2}\right.$

input
```
int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4
+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*c*(2*a*f-b*d)/(4*a*c-b^2)/a*x^3+c/(4*a*c-b^2)*x^2*e+1/2*(a*b*f+2*a*c*
d-b^2*d)/a/(4*a*c-b^2)*x+1/2/(4*a*c-b^2)*b*e)/(c*x^4+b*x^2+a)+1/4*sum((c*(
2*a*f-b*d)/(4*a*c-b^2)/a*_R^2+4/(4*a*c-b^2)*_R*c*e-(a*b*f-6*a*c*d+b^2*d)/(
4*a*c-b^2)/a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^3}\,dx=\text{Timed out}$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x
**6)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^3}\,dx$$
$$= \int \frac{cfx^6 + cex^5 + bex^3 + (cd + bf)x^4 + aex + (bd + af)x^2 + ad}{(cx^4 + bx^2 + a)^3}\,dx$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*a*c*e*x^2 - (b*c*d - 2*a*c*f)*x^3 + a*b*e + (a*b*f - (b^2 - 2*a*c)
*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c
)*x^2) - 1/2*integrate((4*a*c*e*x - a*b*f - (b*c*d - 2*a*c*f)*x^2 - (b^2 -
 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5158 vs. $2(320) = 640$.

Time = 2.01 (sec) , antiderivative size = 5158, normalized size of antiderivative = 14.02

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^3}\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*(b*c*d*x^3 - 2*a*c*f*x^3 - 2*a*c*e*x^2 + b^2*d*x - 2*a*c*d*x - a*b*f*x
 - a*b*e)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a
*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*f +
2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c
^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 -
128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192
*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*...
```

Mupad [B] (verification not implemented)

Time = 19.05 (sec) , antiderivative size = 4707, normalized size of antiderivative = 12.79

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^3}\,dx$$
$$= \text{Too large to display}$$

input
```
int((a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x + b*e*x^3 + c*e*x^5 +
 c*f*x^6)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
symsum(log((5*b^3*c^4*d^3 + 8*a^3*c^4*f^3 - 96*a^2*c^5*d*e^2 + 72*a^2*c^5*
d^2*f - 3*b^4*c^3*d^2*f + 6*a^2*b^2*c^3*f^3 - 36*a*b*c^5*d^3 + 16*a*b^2*c^
4*d*e^2 + 18*a*b^2*c^4*d^2*f + 3*a*b^3*c^3*d*f^2 - 60*a^2*b*c^4*d*f^2 + 16
*a^2*b*c^4*e^2*f)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
)) - root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^
6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*
z^4 - 256*a^3*b^12*z^4 + 576*a^2*b^8*c*d*f*z^2 + 24576*a^5*b^2*c^4*d*f*z^2
 - 3072*a^3*b^6*c^2*d*f*z^2 + 2048*a^4*b^4*c^3*d*f*z^2 + 12288*a^6*b*c^4*f
^2*z^2 + 61440*a^5*b*c^5*d^2*z^2 - 49152*a^6*c^5*d*f*z^2 + 432*a*b^9*c*d^2
*z^2 - 8192*a^5*b^3*c^3*f^2*z^2 + 1536*a^4*b^5*c^2*f^2*z^2 + 24576*a^5*b^2
*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 512*a^3*b^6*c^2*e^2*z^2 - 61440*
a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*z^2 - 4608*a^2*b^7*c^2*d^2*z^2
 - 32*a*b^10*d*f*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*a^2*b^9*f^2*z^2 - 16*b^1
1*d^2*z^2 - 4096*a^4*b*c^4*d*e*f*z + 64*a*b^7*c*d*e*f*z + 3072*a^3*b^3*c^3
*d*e*f*z - 768*a^2*b^5*c^2*d*e*f*z + 32*a^2*b^6*c*e*f^2*z - 672*a*b^6*c^2*
d^2*e*z + 1536*a^4*b^2*c^3*e*f^2*z - 384*a^3*b^4*c^2*e*f^2*z - 15872*a^3*b
^2*c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*e*z - 2048*a^5*c^4*e*f^2*z + 18432*a
^4*c^5*d^2*e*z + 32*b^8*c*d^2*e*z - 32*a*b^4*c^2*d*e^2*f + 192*a^2*b^2*c^3
*d*e^2*f - 192*a^3*b*c^3*e^2*f^2 + 198*a*b^4*c^2*d^2*f^2 + 144*a^2*b^3*c^2
*d*f^3 - 960*a^2*b*c^4*d^2*e^2 + 240*a*b^3*c^3*d^2*e^2 + 768*a^3*c^4*d*...
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 4754, normalized size of antiderivative = 12.92

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^3} dx$$
$$= \text{Too large to display}$$

input
```
int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4
+b*x^2+a)^3,x)
```

output
```
( - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*
c*e - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*
b**2*c*e*x**2 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*a**2*b*c**2*e*x**4 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b
*c*f - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*f + 16*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c*d - 8*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a**2*b**2*c*f*x**2 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*a**2*b*c**2*f*x**4 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4
*d - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*f*x**2 + 16*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(...
```

3.63 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^4}\,dx$

Optimal result

Integrand size = 63, antiderivative size = 621

$$
\begin{aligned}
&\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^4}\,dx \\
&= -\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} \\
&\quad + \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3ce(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)} \\
&\quad + \frac{x(3b^4d-25ab^2cd+28a^2c^2d+ab^3f+8a^2bcf+c(3b^3d-24abcd+ab^2f+20a^2cf)x^2)}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)} \\
&\quad + \frac{\sqrt{c}(3b^4d+b^3(3\sqrt{b^2-4ac}d+af)-4abc(6\sqrt{b^2-4ac}d+13af)-ab^2(30cd-\sqrt{b^2-4ac}f)+4a^2c(4}{8\sqrt{2}a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad + \frac{\sqrt{c}\left(3b^3d-24abcd+ab^2f+20a^2cf-\frac{3b^4d-30ab^2cd+168a^2c^2d+ab^3f-52a^2bcf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad - \frac{6c^2e\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}
\end{aligned}
$$

output
```
-1/4*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*x*(b^2*d-2*a*c*d-a*b
*f+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/2*c*e*(2*c*x^2+b
)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/8*x*(3*b^4*d-25*a*b^2*c*d+28*a^2*c^2*d+
a*b^3*f+8*a^2*b*c*f+c*(20*a^2*c*f+a*b^2*f-24*a*b*c*d+3*b^3*d)*x^2)/a^2/(-4
*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*c^(1/2)*(3*b^4*d+b^3*(3*(-4*a*c+b^2)^(1/2
)*d+a*f)-4*a*b*c*(6*(-4*a*c+b^2)^(1/2)*d+13*a*f)-a*b^2*(30*c*d-f*(-4*a*c+b
^2)^(1/2))+4*a^2*c*(42*c*d+5*f*(-4*a*c+b^2)^(1/2)))*arctan(2^(1/2)*c^(1/2)
*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(5/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)+1/16*c^(1/2)*(3*b^3*d-24*a*b*c*d+a*b^2*f+20*a^2*c*f-(-5
2*a^2*b*c*f+168*a^2*c^2*d+a*b^3*f-30*a*b^2*c*d+3*b^4*d)/(-4*a*c+b^2)^(1/2)
)*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a
*c+b^2)^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)-6*c^2*e*arctanh((2*c*x^2+b)/(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 3.47 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.01

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^4}\,dx$$

$$= \frac{1}{16}\left(\frac{4ab(e+fx)-4bdx(b+cx^2)+8acx(d+x(e+fx))}{a(-b^2+4ac)(a+bx^2+cx^4)^2}\right.$$

$$+\frac{6b^3dx(b+cx^2)+2abx(-25bcd+b^2f-24c^2dx^2+bcfx^2)+8a^2c(b(3e+2fx)+cx(7d+6ex+5fx^2)}{a^2(b^2-4ac)^2(a+bx^2+cx^4)}$$

$$+\frac{\sqrt{2}\sqrt{c}\left(3b^4d+b^3\left(3\sqrt{b^2-4ac}d+af\right)-4abc\left(6\sqrt{b^2-4ac}d+13af\right)+ab^2\left(-30cd+\sqrt{b^2-4ac}f\right)+4}{a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\sqrt{2}\sqrt{c}\left(-3b^4d+b^3\left(3\sqrt{b^2-4ac}d-af\right)+4abc\left(-6\sqrt{b^2-4ac}d+13af\right)+ab^2\left(30cd+\sqrt{b^2-4ac}f\right)+}{a^2(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

$$\left.+\frac{48c^2e\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{5/2}}-\frac{48c^2e\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}}\right)$$

input
```
Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e
*x^5 + c*f*x^6)/(a + b*x^2 + c*x^4)^4,x]
```

output
```
((4*a*b*(e + f*x) - 4*b*d*x*(b + c*x^2) + 8*a*c*x*(d + x*(e + f*x)))/(a*(-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (6*b^3*d*x*(b + c*x^2) + 2*a*b*x*(-2
5*b*c*d + b^2*f - 24*c^2*d*x^2 + b*c*f*x^2) + 8*a^2*c*(b*(3*e + 2*f*x) + c
*x*(7*d + 6*e*x + 5*f*x^2)))/(a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
Sqrt[2]*Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6*
Sqrt[b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(-30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*
a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) + (Sqrt[2]*Sqrt[c]*(-3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d - a*f) + 4*a*
b*c*(-6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(30*c*d + Sqrt[b^2 - 4*a*c]*
f) + 4*a^2*c*(-42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + (48*c^2*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c
)^(5/2) - (48*c^2*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5
/2))/16
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 605, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2019, 2202, 27, 1432, 1086, 1086, 1083, 219, 1492, 25, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(af+bd)+ad+aex+x^4(bf+cd)+bex^3+cex^5+cfx^6}{(a+bx^2+cx^4)^4}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^3}dx+\int \frac{ex}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^3}dx+e\int \frac{x}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^3}dx+\frac{1}{2}e\int \frac{1}{(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}e\left(-\frac{3c\int \frac{1}{(cx^4+bx^2+a)^2}dx^2}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)+\int \frac{fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}e\left(-\frac{3c\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)+$$
$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)+$$
$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^3}dx$$

$$\Big\downarrow \text{219}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^3}dx+$$
$$\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{1492}$$

$$-\frac{\int -\frac{3db^2+afb+5c(bd-2af)x^2-14acd}{(cx^4+bx^2+a)^2}dx}{4a(b^2-4ac)}+\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3db^2+afb+5c(bd-2af)x^2-14acd}{(cx^4+bx^2+a)^2}dx}{4a(b^2-4ac)}+\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \; 1492$$

$$\frac{\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d+ab^3f-25ab^2cd+3b^4d\right)}{2a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{3db^4+afb^3-27acdb^2-16a^2cfb+c\left(3db^3+afb^2-24acdb+20a^2c\right.}{cx^4+bx^2+a}}{2a(b^2-4ac)}}{4a(b^2-4ac)}+\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{3db^4+afb^3-27acdb^2-16a^2cfb+c\left(3db^3+afb^2-24acdb+20a^2cf\right)x^2+84a^2c^2d}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x(cx^2(20a^2cf+ab^2f-24abcd+3b^3d)+8a^2bcf+28a^2c^2d+ab^3f-2}{2a(b^2-4ac)(a+bx^2+cx^4)}}{4a\left(b^2-4ac\right)}$$

$$\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)+$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \ 1480$$

$$\frac{\frac{\frac{1}{2}c\left(\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2c}{\sqrt{b^2-4ac}}\right.}{2a(b^2-4ac)}}{4a\left(b\right.}$$

$$\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)+$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-52a^2bcf+168a^2c}{\sqrt{}}\right.}{\sqrt{2}\sqrt{}}}{2a(b^2-4ac)}}{4a\left(b^2-4\right.}$$

$$\frac{1}{2}e\left(-\frac{3c\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)+$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

input
```
Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
 c*f*x^6)/(a + b*x^2 + c*x^4)^4,x]
```

output
```
(x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)^2) + ((x*(3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f
 + 8*a^2*b*c*f + c*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(2*
a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((Sqrt[c]*(3*b^3*d - 24*a*b*c*d + a
*b^2*f + 20*a^2*c*f + (3*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f -
52*a^2*b*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d
- 24*a*b*c*d + a*b^2*f + 20*a^2*c*f - (3*b^4*d - 30*a*b^2*c*d + 168*a^2*c^
2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*
a*(b^2 - 4*a*c)))/(4*a*(b^2 - 4*a*c)) + (e*(-1/2*(b + 2*c*x^2)/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (3*c*(-((b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x
^2 + c*x^4))) + (4*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*
c)^(3/2)))/(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.41 (sec) , antiderivative size = 607, normalized size of antiderivative = 0.98

method	result
risch	$\frac{\frac{c^2\left(20a^2cf+a\,b^2f-24abcd+3b^3d\right)x^7}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{3c^3e\,x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{c\left(28a^2bcf+28a^2c^2d+2a\,b^3f-49a\,b^2cd+6b^4d\right)x^5}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{9b\,c^2e\,x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{(36a^3}{}}{(c\,x^4+b}$
default	Expression too large to display

input
```
int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4
+b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/8*c^2*(20*a^2*c*f+a*b^2*f-24*a*b*c*d+3*b^3*d)/a^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^7+3*c^3*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/8/a^2*c*(28*a^2*b*c*f+2
8*a^2*c^2*d+2*a*b^3*f-49*a*b^2*c*d+6*b^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5
+9/2*b*c^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+1/8*(36*a^3*c^2*f+5*a^2*b^2*c*
f-4*a^2*b*c^2*d+a*b^4*f-20*a*b^3*c*d+3*b^5*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3+(5*a*c+b^2)*c*e/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/8*(16*a^2*b*c*f+44
*a^2*c^2*d-a*b^3*f-37*a*b^2*c*d+5*b^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x+1/
4*b*(10*a*c-b^2)*e/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/16*sum(
(c*(20*a^2*c*f+a*b^2*f-24*a*b*c*d+3*b^3*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*
_R^2+48*c^2*e/(16*a^2*c^2-8*a*b^2*c+b^4)*_R-(16*a^2*b*c*f-84*a^2*c^2*d-a*b
^3*f+27*a*b^2*c*d-3*b^4*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)
*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^4} dx = \text{Timed out}$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a)^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^4} dx = \text{Timed out}$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x
**6)/(c*x**4+b*x**2+a)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^4} dx$$
$$= \int \frac{cfx^6 + cex^5 + bex^3 + (cd + bf)x^4 + aex + (bd + af)x^2 + ad}{(cx^4 + bx^2 + a)^4} dx$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/8*(24*a^2*c^3*e*x^6 + 36*a^2*b*c^2*e*x^4 + (3*(b^3*c^2 - 8*a*b*c^3)*d +
(a*b^2*c^2 + 20*a^2*c^3)*f)*x^7 + ((6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d
 + 2*(a*b^3*c + 14*a^2*b*c^2)*f)*x^5 + 8*(a^2*b^2*c + 5*a^3*c^2)*e*x^2 + (
(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d + (a*b^4 + 5*a^2*b^2*c + 36*a^3*c^2)*
f)*x^3 - 2*(a^2*b^3 - 10*a^3*b*c)*e + ((5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^
2)*d - (a^2*b^3 - 16*a^3*b*c)*f)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4
*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*
c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^
3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + 1/8*integrate((48*a^2*c^2*e*x +
 (3*(b^3*c - 8*a*b*c^2)*d + (a*b^2*c + 20*a^2*c^2)*f)*x^2 + 3*(b^4 - 9*a*b
^2*c + 28*a^2*c^2)*d + (a*b^3 - 16*a^2*b*c)*f)/(c*x^4 + b*x^2 + a), x)/(a^
2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5280 vs. $2(561) = 1122$.

Time = 1.04 (sec) , antiderivative size = 5280, normalized size of antiderivative = 8.50

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^4} \, dx$$
$$= \text{Too large to display}$$

input
```
integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/
(c*x^4+b*x^2+a)^4,x, algorithm="giac")
```

output
```
-3*(b^2*c^4 - 4*a*c^5 - 2*b*c^5 + c^6)*sqrt(b^2 - 4*a*c)*e*log(x^2 + 1/2*(
a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2
*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))/((b^8 - 16*
a*b^6*c - 2*b^7*c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2*
c^3 - 96*a^2*b^3*c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^2
*b^2*c^4 - 64*a^3*c^5)*c^2) + 3*(b^2*c^4 - 4*a*c^5 - 2*b*c^5 + c^6)*sqrt(b
^2 - 4*a*c)*e*log(x^2 + 1/2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt((
a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c
^2 + 16*a^4*c^3))/((b^8 - 16*a*b^6*c - 2*b^7*c + 96*a^2*b^4*c^2 + 24*a*b^5
*c^2 + b^6*c^2 - 256*a^3*b^2*c^3 - 96*a^2*b^3*c^3 - 12*a*b^4*c^3 + 256*a^4
*c^4 + 128*a^3*b*c^4 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*c^2) + 1/32*(3*(sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c - 2*b^8
*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2...
```

Mupad [B] (verification not implemented)

Time = 20.45 (sec) , antiderivative size = 8689, normalized size of antiderivative = 13.99

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^4} dx$$
$$= \text{Too large to display}$$

input
```
int((a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x + b*e*x^3 + c*e*x^5 +
 c*f*x^6)/(a + b*x^2 + c*x^4)^4,x)
```

output
```
((x^2*(5*a*c^2*e + b^2*c*e))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3*e - 10*
a*b*c*e)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(28*a^2*c^3*d + 6*b^4*c
*d + 2*a*b^3*c*f - 49*a*b^2*c^2*d + 28*a^2*b*c^2*f))/(8*a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) + (x*(5*b^4*d + 44*a^2*c^2*d - a*b^3*f - 37*a*b^2*c*d +
16*a^2*b*c*f))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^3*e*x^6)/(b^4 +
 16*a^2*c^2 - 8*a*b^2*c) + (x^3*(3*b^5*d + 36*a^3*c^2*f + a*b^4*f - 20*a*b
^3*c*d - 4*a^2*b*c^2*d + 5*a^2*b^2*c*f))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)) + (9*b*c^2*e*x^4)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*a^
2*c^2*f + 3*b^3*c*d - 24*a*b*c^2*d + a*b^2*c*f))/(8*a^2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
 + symsum(log(root(56371445760*a^11*b^8*c^6*z^4 - 503316480*a^8*b^14*c^3*z
^4 + 47185920*a^7*b^16*c^2*z^4 - 171798691840*a^14*b^2*c^9*z^4 + 193273528
320*a^13*b^4*c^8*z^4 - 128849018880*a^12*b^6*c^7*z^4 - 16911433728*a^10*b^
10*c^5*z^4 + 3523215360*a^9*b^12*c^4*z^4 - 2621440*a^6*b^18*c*z^4 + 687194
76736*a^15*c^10*z^4 + 65536*a^5*b^20*z^4 - 73728*a^2*b^16*c*d*f*z^2 - 1321
205760*a^9*b^2*c^8*d*f*z^2 + 732168192*a^7*b^6*c^6*d*f*z^2 - 366280704*a^6
*b^8*c^5*d*f*z^2 - 330301440*a^8*b^4*c^7*d*f*z^2 + 96583680*a^5*b^10*c^4*d
*f*z^2 - 15175680*a^4*b^12*c^3*d*f*z^2 + 1428480*a^3*b^14*c^2*d*f*z^2 - 44
0401920*a^10*b*c^8*f^2*z^2 + 1761607680*a^10*c^9*d*f*z^2 - 14080*a^3*b^15*
c*f^2*z^2 + 6936330240*a^8*b^3*c^8*d^2*z^2 + 2464874496*a^6*b^7*c^6*d^2...
```

Reduce [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 13638, normalized size of antiderivative = 21.96

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{(a + bx^2 + cx^4)^4} dx$$
$$= \text{Too large to display}$$

input
```
int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4
+b*x^2+a)^4,x)
```

output

```
( - 192*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b
*c**2*e - 384*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**4*b**2*c**2*e*x**2 - 384*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqr
t(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a**4*b*c**3*e*x**4 - 192*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*
sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt
(2*sqrt(c)*sqrt(a) + b))*a**3*b**3*c**2*e*x**4 - 384*sqrt(2*sqrt(c)*sqrt(a
) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**3*e*x**6 - 192*sqrt(2*
sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**4*e*x**8 -
80*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b*c**2*f - 36*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**3*c*f + 264*sqrt(a)*sqrt(2*sqrt(c)*sqr
t(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a**4*b**2*c**2*d - 160*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) ...
```

3.64 $\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 4

$$\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4} dx = \log(2+x)$$

output
```
ln(2+x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4} dx = \log(2+x)$$

input
```
Integrate[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4),x]
```

output
```
Log[2 + x]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-2x^2-x+2}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{x+2}dx$$

$$\Big\downarrow \text{16}$$

$$\log(x+2)$$

input
```
Int[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4),x]
```

output
```
Log[2 + x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$\ln(x+2)$	5
norman	$\ln(x+2)$	5
risch	$\ln(x+2)$	5
parallelrisch	$\ln(x+2)$	5

input
```
int((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4}\,dx = \log(x+2)$$

input
```
integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4}\,dx = \log(x+2)$$

input
```
integrate((x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

output
```
log(x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4}\,dx=\log{(x+2)}$$

input
```
integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4}\,dx=\log{(|x+2|)}$$

input
```
integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4}\,dx=\ln{(x+2)}$$

input
```
int(-(x + 2*x^2 - x^3 - 2)/(x^4 - 5*x^2 + 4),x)
```

output
```
log(x + 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4}\,dx=\log(x+2)$$

input
```
int((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x)
```

output
```
log(x + 2)
```

3.65 $\int \frac{(d+ex)(2-x-2x^2+x^3)}{4-5x^2+x^4} dx$

Optimal result

Integrand size = 31, antiderivative size = 14

$$\int \frac{(d+ex)\,(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx = ex+(d-2e)\log(2+x)$$

output
```
e*x+(d-2*e)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)\,(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx = e(2+x)+(d-2e)\log(2+x)$$

input
```
Integrate[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

output
```
e*(2 + x) + (d - 2*e)*Log[2 + x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2019, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3-2x^2-x+2)(d+ex)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{d+ex}{x+2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{d-2e}{x+2}+e\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$(d-2e)\log(x+2)+ex$$

input
```
Int[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

output
```
e*x + (d - 2*e)*Log[2 + x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$ex + (d - 2e) \ln(x + 2)$	15
norman	$ex + (d - 2e) \ln(x + 2)$	15
risch	$ex + \ln(x + 2) d - 2 \ln(x + 2) e$	18
parallelrisch	$ex + \ln(x + 2) d - 2 \ln(x + 2) e$	18

input
```
int((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
e*x+(d-2*e)*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(d + ex) (2 - x - 2x^2 + x^3)}{4 - 5x^2 + x^4} dx = ex + (d - 2e) \log(x + 2)$$

input
```
integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
e*x + (d - 2*e)*log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)\left(2-x-2x^2+x^3\right)}{4-5x^2+x^4}\,dx = ex+(d-2e)\log{(x+2)}$$

input
```
integrate((e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

output
```
e*x + (d - 2*e)*log(x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(2-x-2x^2+x^3\right)}{4-5x^2+x^4}\,dx = ex+(d-2\,e)\log\left(x+2\right)$$

input
```
integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
e*x + (d - 2*e)*log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)\left(2-x-2x^2+x^3\right)}{4-5x^2+x^4}\,dx = ex+(d-2\,e)\log\left(|x+2|\right)$$

input
```
integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
e*x + (d - 2*e)*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(2-x-2x^2+x^3\right)}{4-5x^2+x^4}\,dx=\ln\left(x+2\right)\,\left(d-2\,e\right)+e\,x$$

input
```
int(-((d + e*x)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4),x)
```

output
```
log(x + 2)*(d - 2*e) + e*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)\left(2-x-2x^2+x^3\right)}{4-5x^2+x^4}\,dx=\log(x+2)\,d-2\log(x+2)\,e+ex$$

input
```
int((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x)
```

output
```
log(x + 2)*d - 2*log(x + 2)*e + e*x
```

3.66 $\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4} dx$

Optimal result

Integrand size = 36, antiderivative size = 31

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4} dx = (e-4f)x+\frac{1}{2}f(2+x)^2+(d-2e+4f)\log(2+x)$$

output
```
(e-4*f)*x+1/2*f*(2+x)^2+(d-2*e+4*f)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4} dx = \frac{1}{2}(2e+f(-6+x))(2+x)+(d-2e+4f)\log(2+x)$$

input
```
Integrate[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

output
```
((2*e + f*(-6 + x))*(2 + x))/2 + (d - 2*e + 4*f)*Log[2 + x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2019, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3-2x^2-x+2)(d+ex+fx^2)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2}{x+2}dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{d-2e+4f}{x+2}+e+f(x+2)-4f\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x+2)(d-2e+4f)+x(e-4f)+\frac{1}{2}f(x+2)^2$$

input
```
Int[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

output
```
(e - 4*f)*x + (f*(2 + x)^2)/2 + (d - 2*e + 4*f)*Log[2 + x]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{f\,x^{2}}{2}+ex-2fx+\left(d-2e+4f\right)\ln\left(x+2\right)$	28
norman	$\left(e-2f\right)x+\frac{f\,x^{2}}{2}+\left(d-2e+4f\right)\ln\left(x+2\right)$	28
risch	$\frac{f\,x^{2}}{2}+ex-2fx+\ln\left(x+2\right)d-2\ln\left(x+2\right)e+4\ln\left(x+2\right)f$	35
parallelrisch	$\frac{f\,x^{2}}{2}+ex-2fx+\ln\left(x+2\right)d-2\ln\left(x+2\right)e+4\ln\left(x+2\right)f$	35

input
```
int((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/2*f*x^2+e*x-2*f*x+(d-2*e+4*f)*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int\frac{(d+ex+fx^2)\,(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx=\frac{1}{2}\,fx^2+(e-2\,f)x+(d-2\,e+4\,f)\log\left(x+2\right)$$

input
```
integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas
")
```

output
```
1/2*f*x^2 + (e - 2*f)*x + (d - 2*e + 4*f)*log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx = \frac{fx^2}{2} + x(e-2f) + (d-2e+4f)\log(x+2)$$

input
```
integrate((f*x**2+e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

output
```
f*x**2/2 + x*(e - 2*f) + (d - 2*e + 4*f)*log(x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx = \frac{1}{2}fx^2 + (e-2f)x + (d-2e+4f)\log(x+2)$$

input
```
integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima
")
```

output
```
1/2*f*x^2 + (e - 2*f)*x + (d - 2*e + 4*f)*log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx = \frac{1}{2}fx^2 + ex - 2fx + (d-2e+4f)\log(|x+2|)$$

input
```
integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
1/2*f*x^2 + e*x - 2*f*x + (d - 2*e + 4*f)*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 17.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx = x\,(e-2\,f)+\frac{f\,x^2}{2}+\ln\left(x+2\right)\,\left(d-2\,e+4\,f\right)$$

input
```
int(-((d + e*x + f*x^2)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4),x)
```

output
```
x*(e - 2*f) + (f*x^2)/2 + log(x + 2)*(d - 2*e + 4*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx = \log(x+2)\,d-2\log(x+2)\,e + 4\log(x+2)\,f+ex+\frac{f\,x^2}{2}-2fx$$

input
```
int((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x)
```

output
```
(2*log(x + 2)*d - 4*log(x + 2)*e + 8*log(x + 2)*f + 2*e*x + f*x**2 - 4*f*x
)/2
```

3.67 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4} \, dx$

Optimal result

Integrand size = 41, antiderivative size = 47

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4} \, dx = (e-2f+4g)x+\frac{1}{2}(f-2g)x^2+\frac{gx^3}{3}+(d-2e+4f-8g)\log(2+x)$$

output
```
(e-2*f+4*g)*x+1/2*(f-2*g)*x^2+1/3*g*x^3+(d-2*e+4*f-8*g)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4} \, dx = \frac{1}{6}(2+x)\left(6e+3f(-6+x)+2g\left(22-5x+x^2\right)\right)+(d-2e+4f-8g)\log(2+x)$$

input
```
Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x
^4),x]
```

output
```
((2 + x)*(6*e + 3*f*(-6 + x) + 2*g*(22 - 5*x + x^2)))/6 + (d - 2*e + 4*f -
8*g)*Log[2 + x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {2019, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3-2x^2-x+2)(d+ex+fx^2+gx^3)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2+gx^3}{x+2}dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{d-2e+4f-8g}{x+2}+e+(x+2)(f-6g)-4f+g(x+2)^2+12g\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x+2)(d-2e+4f-8g)+x(e-4f+12g)+\frac{1}{2}(x+2)^2(f-6g)+\frac{1}{3}g(x+2)^3$$

input
```
Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

output
```
(e - 4*f + 12*g)*x + ((f - 6*g)*(2 + x)^2)/2 + (g*(2 + x)^3)/3 + (d - 2*e
+ 4*f - 8*g)*Log[2 + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result
norman	$\left(\frac{f}{2}-g\right)x^{2}+\left(e-2f+4g\right)x+\frac{g\,x^{3}}{3}+\left(d-2e+4f-8g\right)\ln\left(x+2\right)$
default	$\frac{g\,x^{3}}{3}+\frac{f\,x^{2}}{2}-g\,x^{2}+ex-2fx+4gx+\left(d-2e+4f-8g\right)\ln\left(x+2\right)$
risch	$\frac{g\,x^{3}}{3}+\frac{f\,x^{2}}{2}-g\,x^{2}+ex-2fx+4gx+\ln\left(x+2\right)d-2\ln\left(x+2\right)e+4\ln\left(x+2\right)f-8\ln($
parallelrisch	$\frac{g\,x^{3}}{3}+\frac{f\,x^{2}}{2}-g\,x^{2}+ex-2fx+4gx+\ln\left(x+2\right)d-2\ln\left(x+2\right)e+4\ln\left(x+2\right)f-8\ln($

input
```
int((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERB
OSE)
```

output
```
(1/2*f-g)*x^2+(e-2*f+4*g)*x+1/3*g*x^3+(d-2*e+4*f-8*g)*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx = \frac{1}{3}\,gx^3+\frac{1}{2}\,(f-2\,g)x^2+(e-2\,f+4\,g)x+(d-2\,e+4\,f-8\,g)\log{(x+2)}$$

input
```
integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="
fricas")
```

output
```
1/3*g*x^3 + 1/2*(f - 2*g)*x^2 + (e - 2*f + 4*g)*x + (d - 2*e + 4*f - 8*g)*
log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx = \frac{gx^3}{3}+x^2\left(\frac{f}{2}-g\right)+x(e-2f+4g)+(d-2e+4f-8g)\log{(x+2)}$$

input
```
integrate((x**3-2*x**2-x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

output
```
g*x**3/3 + x**2*(f/2 - g) + x*(e - 2*f + 4*g) + (d - 2*e + 4*f - 8*g)*log(
x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{3}\,gx^3+\frac{1}{2}\,(f-2\,g)x^2+(e-2\,f+4\,g)x+(d-2\,e+4\,f-8\,g)\log(x+2)$$

input
```
integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="
maxima")
```

output
```
1/3*g*x^3 + 1/2*(f - 2*g)*x^2 + (e - 2*f + 4*g)*x + (d - 2*e + 4*f - 8*g)*
log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{3}\,gx^3+\frac{1}{2}\,fx^2-gx^2+ex-2\,fx+4\,gx+(d-2\,e+4\,f-8\,g)\log(|x+2|)$$

input
```
integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="
giac")
```

output
```
1/3*g*x^3 + 1/2*f*x^2 - g*x^2 + e*x - 2*f*x + 4*g*x + (d - 2*e + 4*f - 8*g
)*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx$$
$$= x^2\left(\frac{f}{2}-g\right)+x\,(e-2\,f+4\,g)+\frac{g\,x^3}{3}+\ln\left(x+2\right)\,\left(d-2\,e+4\,f-8\,g\right)$$

input
```
int(-((d + e*x + f*x^2 + g*x^3)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4),x
)
```

output
```
x^2*(f/2 - g) + x*(e - 2*f + 4*g) + (g*x^3)/3 + log(x + 2)*(d - 2*e + 4*f
- 8*g)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx$$
$$= \log(x+2)\,d-2\log(x+2)\,e+4\log(x+2)\,f$$
$$-8\log(x+2)\,g+ex+\frac{f\,x^2}{2}-2fx+\frac{g\,x^3}{3}-g\,x^2+4gx$$

input
```
int((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
(6*log(x + 2)*d - 12*log(x + 2)*e + 24*log(x + 2)*f - 48*log(x + 2)*g + 6*
e*x + 3*f*x**2 - 12*f*x + 2*g*x**3 - 6*g*x**2 + 24*g*x)/6
```

3.68 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4} dx$

Optimal result

Integrand size = 46, antiderivative size = 68

$$\begin{aligned}\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4} dx\\ &= (e-2f+4g-8h)x+\frac{1}{2}(f-2g+4h)x^2+\frac{1}{3}(g-2h)x^3\\ &\quad+\frac{hx^4}{4}+(d-2e+4f-8g+16h)\log(2+x)\end{aligned}$$

output
```
(e-2*f+4*g-8*h)*x+1/2*(f-2*g+4*h)*x^2+1/3*(g-2*h)*x^3+1/4*h*x^4+(d-2*e+4*f
-8*g+16*h)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4} dx\\ &= (e-2f+4g-8h)x+\frac{1}{2}(f-2g+4h)x^2+\frac{1}{3}(g-2h)x^3\\ &\quad+\frac{hx^4}{4}+(d-2e+4f-8g+16h)\log(2+x)\end{aligned}$$

input
```
Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5
*x^2 + x^4),x]
```

output
```
(e - 2*f + 4*g - 8*h)*x + ((f - 2*g + 4*h)*x^2)/2 + ((g - 2*h)*x^3)/3 + (h
*x^4)/4 + (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2019, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3-2x^2-x+2)(d+ex+fx^2+gx^3+hx^4)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{x+2}dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{d-2e+4f-8g+16h}{x+2}+e\left(1-\frac{2(f-2g+4h)}{e}\right)+x(f-2g+4h)+x^2(g-2h)+hx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x+2)(d-2e+4f-8g+16h)+x(e-2f+4g-8h)+\frac{1}{2}x^2(f-2g+4h)+\frac{1}{3}x^3(g-2h)+\frac{hx^4}{4}$$

input
```
Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
 x^4),x]
```

output
```
(e - 2*f + 4*g - 8*h)*x + ((f - 2*g + 4*h)*x^2)/2 + ((g - 2*h)*x^3)/3 + (h
*x^4)/4 + (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

method	result
norman	$\left(\frac{g}{3}-\frac{2h}{3}\right)x^3+\left(\frac{f}{2}-g+2h\right)x^2+\left(e-2f+4g-8h\right)x+\frac{h\,x^4}{4}+\left(d-2e+4f-8g+16h\right)$
default	$\frac{h\,x^4}{4}+\frac{g\,x^3}{3}-\frac{2h\,x^3}{3}+\frac{f\,x^2}{2}-g\,x^2+2h\,x^2+ex-2fx+4gx-8hx+\left(d-2e+4f-8g+1\right.$
risch	$\frac{h\,x^4}{4}+\frac{g\,x^3}{3}-\frac{2h\,x^3}{3}+\frac{f\,x^2}{2}-g\,x^2+2h\,x^2+ex-2fx+4gx-8hx+\ln\left(x+2\right)d-2\ln\left(x+\right.$
parallelrisch	$\frac{h\,x^4}{4}+\frac{g\,x^3}{3}-\frac{2h\,x^3}{3}+\frac{f\,x^2}{2}-g\,x^2+2h\,x^2+ex-2fx+4gx-8hx+\ln\left(x+2\right)d-2\ln\left(x+\right.$

input
```
int((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETU
RNVERBOSE)
```

output
```
(1/3*g-2/3*h)*x^3+(1/2*f-g+2*h)*x^2+(e-2*f+4*g-8*h)*x+1/4*h*x^4+(d-2*e+4*f
-8*g+16*h)*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = \frac{1}{4}hx^4+\frac{1}{3}(g-2h)x^3+\frac{1}{2}(f-2g+4h)x^2 + (e-2f+4g-8h)x+(d-2e+4f-8g+16h)\log(x+2)$$

input
```
integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algor
ithm="fricas")
```

output
```
1/4*h*x^4 + 1/3*(g - 2*h)*x^3 + 1/2*(f - 2*g + 4*h)*x^2 + (e - 2*f + 4*g -
 8*h)*x + (d - 2*e + 4*f - 8*g + 16*h)*log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = \frac{hx^4}{4}+x^3\left(\frac{g}{3}-\frac{2h}{3}\right)+x^2\left(\frac{f}{2}-g+2h\right) + x(e-2f+4g-8h)+(d-2e+4f-8g+16h)\log(x+2)$$

input
```
integrate((x**3-2*x**2-x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x
)
```

output
```
h*x**4/4 + x**3*(g/3 - 2*h/3) + x**2*(f/2 - g + 2*h) + x*(e - 2*f + 4*g -
8*h) + (d - 2*e + 4*f - 8*g + 16*h)*log(x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = \frac{1}{4}hx^4+\frac{1}{3}(g-2h)x^3+\frac{1}{2}(f-2g+4h)x^2 + (e-2f+4g-8h)x+(d-2e+4f-8g+16h)\log(x+2)$$

input
```
integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algor
ithm="maxima")
```

output
```
1/4*h*x^4 + 1/3*(g - 2*h)*x^3 + 1/2*(f - 2*g + 4*h)*x^2 + (e - 2*f + 4*g -
 8*h)*x + (d - 2*e + 4*f - 8*g + 16*h)*log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = \frac{1}{4}hx^4+\frac{1}{3}gx^3-\frac{2}{3}hx^3+\frac{1}{2}fx^2-gx^2+2hx^2+ex-2fx + 4gx-8hx+(d-2e+4f-8g+16h)\log(|x+2|)$$

input
```
integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algor
ithm="giac")
```

output
```
1/4*h*x^4 + 1/3*g*x^3 - 2/3*h*x^3 + 1/2*f*x^2 - g*x^2 + 2*h*x^2 + e*x - 2*
f*x + 4*g*x - 8*h*x + (d - 2*e + 4*f - 8*g + 16*h)*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 17.99 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$$
$$= x^3\left(\frac{g}{3}-\frac{2h}{3}\right)+\ln(x+2)\,(d-2e+4f-8g+16h) + \frac{hx^4}{4}+x^2\left(\frac{f}{2}-g+2h\right)+x\,(e-2f+4g-8h)$$

input
```
int(-((x + 2*x^2 - x^3 - 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^
2 + 4),x)
```

output
```
x^3*(g/3 - (2*h)/3) + log(x + 2)*(d - 2*e + 4*f - 8*g + 16*h) + (h*x^4)/4
+ x^2*(f/2 - g + 2*h) + x*(e - 2*f + 4*g - 8*h)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$$
$$= \log(x+2)\,d-2\log(x+2)\,e+4\log(x+2)\,f-8\log(x+2)\,g+16\log(x+2)\,h + ex+\frac{fx^2}{2}-2fx+\frac{gx^3}{3}-gx^2+4gx+\frac{hx^4}{4}-\frac{2hx^3}{3}+2hx^2-8hx$$

input
```
int((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
(12*log(x + 2)*d - 24*log(x + 2)*e + 48*log(x + 2)*f - 96*log(x + 2)*g + 1
92*log(x + 2)*h + 12*e*x + 6*f*x**2 - 24*f*x + 4*g*x**3 - 12*g*x**2 + 48*g
*x + 3*h*x**4 - 8*h*x**3 + 24*h*x**2 - 96*h*x)/12
```

3.69 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 92

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx = (e-2f+4g-8h+16i)x+\frac{1}{2}(f-2g+4h-8i)x^2+\frac{1}{3}(g-2h+4i)x^3 + \frac{1}{4}(h-2i)x^4+\frac{ix^5}{5}+(d-2e+4f-8g+16h-32i)\log(2+x)$$

output
```
(e-2*f+4*g-8*h+16*i)*x+1/2*(f-2*g+4*h-8*i)*x^2+1/3*(g-2*h+4*i)*x^3+1/4*(h-
2*i)*x^4+1/5*i*x^5+(d-2*e+4*f-8*g+16*h-32*i)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx = (e-2f+4g-8h+16i)x+\frac{1}{2}(f-2g+4h-8i)x^2+\frac{1}{3}(g-2h+4i)x^3 + \frac{1}{4}(h-2i)x^4+\frac{ix^5}{5}+(d-2e+4f-8g+16h-32i)\log(2+x)$$

input
```
Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)
)/(4 - 5*x^2 + x^4),x]
```

output
```
(e - 2*f + 4*g - 8*h + 16*i)*x + ((f - 2*g + 4*h - 8*i)*x^2)/2 + ((g - 2*h
 + 4*i)*x^3)/3 + ((h - 2*i)*x^4)/4 + (i*x^5)/5 + (d - 2*e + 4*f - 8*g + 16
*h - 32*i)*Log[2 + x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2019, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^3-2x^2-x+2\right)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{x+2}dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{d-2e+4f-8g+16h-32i}{x+2}+e\left(1-\frac{2(f-2g+4h-8i)}{e}\right)+x(f-2g+4h-8i)+x^2(g-2h+4i)+x$$

$$\Big\downarrow \text{2009}$$

$$\log(x+2)(d-2e+4f-8g+16h-32i)+x(e-2f+4g-8h+16i)+\frac{1}{2}x^2(f-2g+4h-8i)+\frac{1}{3}x^3(g-2h+4i)+\frac{1}{4}x^4(h-2i)+\frac{ix^5}{5}$$

input
```
Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
 5*x^2 + x^4),x]
```

output
```
(e - 2*f + 4*g - 8*h + 16*i)*x + ((f - 2*g + 4*h - 8*i)*x^2)/2 + ((g - 2*h
 + 4*i)*x^3)/3 + ((h - 2*i)*x^4)/4 + (i*x^5)/5 + (d - 2*e + 4*f - 8*g + 16
*h - 32*i)*Log[2 + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

method	result
norman	$\left(\frac{h}{4}-\frac{i}{2}\right) x^{4}+\left(\frac{g}{3}-\frac{2h}{3}+\frac{4i}{3}\right) x^{3}+\left(\frac{f}{2}-g+2h-4i\right) x^{2}+\left(e-2f+4g-8h+16i\right) x+\frac{i\,x^{5}}{5}$
default	$\frac{i\,x^{5}}{5}+\frac{h\,x^{4}}{4}-\frac{i\,x^{4}}{2}+\frac{g\,x^{3}}{3}-\frac{2h\,x^{3}}{3}+\frac{4i\,x^{3}}{3}+\frac{f\,x^{2}}{2}-g\,x^{2}+2h\,x^{2}-4i\,x^{2}+ex-2fx+4gx-8h$
risch	$\frac{i\,x^{5}}{5}+\frac{h\,x^{4}}{4}-\frac{i\,x^{4}}{2}+\frac{g\,x^{3}}{3}-\frac{2h\,x^{3}}{3}+\frac{4i\,x^{3}}{3}+\frac{f\,x^{2}}{2}-g\,x^{2}+2h\,x^{2}-4i\,x^{2}+ex-2fx+4gx-8h$
parallelrisch	$\frac{i\,x^{5}}{5}+\frac{h\,x^{4}}{4}-\frac{i\,x^{4}}{2}+\frac{g\,x^{3}}{3}-\frac{2h\,x^{3}}{3}+\frac{4i\,x^{3}}{3}+\frac{f\,x^{2}}{2}-g\,x^{2}+2h\,x^{2}-4i\,x^{2}+ex-2fx+4gx-8h$

input
```
int((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method
=_RETURNVERBOSE)
```

output
```
(1/4*h-1/2*i)*x^4+(1/3*g-2/3*h+4/3*i)*x^3+(1/2*f-g+2*h-4*i)*x^2+(e-2*f+4*g
-8*h+16*i)*x+1/5*i*x^5+(d-2*e+4*f-8*g+16*h-32*i)*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{5}\,ix^5+\frac{1}{4}\,(h-2\,i)x^4+\frac{1}{3}\,(g-2\,h+4\,i)x^3+\frac{1}{2}\,(f-2\,g+4\,h-8\,i)x^2$$
$$+(e-2\,f+4\,g-8\,h+16\,i)x+(d-2\,e+4\,f-8\,g+16\,h-32\,i)\log(x+2)$$

input
```
integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,
 algorithm="fricas")
```

output
```
1/5*i*x^5 + 1/4*(h - 2*i)*x^4 + 1/3*(g - 2*h + 4*i)*x^3 + 1/2*(f - 2*g + 4
*h - 8*i)*x^2 + (e - 2*f + 4*g - 8*h + 16*i)*x + (d - 2*e + 4*f - 8*g + 16
*h - 32*i)*log(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$=\frac{ix^5}{5}+x^4\left(\frac{h}{4}-\frac{i}{2}\right)+x^3\left(\frac{g}{3}-\frac{2h}{3}+\frac{4i}{3}\right)+x^2\left(\frac{f}{2}-g+2h-4i\right)$$
$$+x(e-2f+4g-8h+16i)+(d-2e+4f-8g+16h-32i)\log(x+2)$$

input
```
integrate((x**3-2*x**2-x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x*
*2+4),x)
```

output
```
i*x**5/5 + x**4*(h/4 - i/2) + x**3*(g/3 - 2*h/3 + 4*i/3) + x**2*(f/2 - g +
 2*h - 4*i) + x*(e - 2*f + 4*g - 8*h + 16*i) + (d - 2*e + 4*f - 8*g + 16*h
 - 32*i)*log(x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{5}\,ix^5+\frac{1}{4}\,(h-2\,i)x^4+\frac{1}{3}\,(g-2\,h+4\,i)x^3+\frac{1}{2}\,(f-2\,g+4\,h-8\,i)x^2$$
$$+(e-2\,f+4\,g-8\,h+16\,i)x+(d-2\,e+4\,f-8\,g+16\,h-32\,i)\log(x+2)$$

input
```
integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,
 algorithm="maxima")
```

output
```
1/5*i*x^5 + 1/4*(h - 2*i)*x^4 + 1/3*(g - 2*h + 4*i)*x^3 + 1/2*(f - 2*g + 4
*h - 8*i)*x^2 + (e - 2*f + 4*g - 8*h + 16*i)*x + (d - 2*e + 4*f - 8*g + 16
*h - 32*i)*log(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{5}\,ix^5+\frac{1}{4}\,hx^4-\frac{1}{2}\,ix^4+\frac{1}{3}\,gx^3-\frac{2}{3}\,hx^3+\frac{4}{3}\,ix^3+\frac{1}{2}\,fx^2-gx^2+2\,hx^2-4\,ix^2+ex$$
$$-2\,fx+4\,gx-8\,hx+16\,ix+(d-2\,e+4\,f-8\,g+16\,h-32\,i)\log(|x+2|)$$

input
```
integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,
 algorithm="giac")
```

output
```
1/5*i*x^5 + 1/4*h*x^4 - 1/2*i*x^4 + 1/3*g*x^3 - 2/3*h*x^3 + 4/3*i*x^3 + 1/
2*f*x^2 - g*x^2 + 2*h*x^2 - 4*i*x^2 + e*x - 2*f*x + 4*g*x - 8*h*x + 16*i*x
 + (d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(abs(x + 2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$= x^4\left(\frac{h}{4}-\frac{i}{2}\right)+\ln(x+2)\,(d-2\,e+4\,f-8\,g+16\,h-32\,i)+\frac{i\,x^5}{5}$$
$$+x^2\left(\frac{f}{2}-g+2\,h-4\,i\right)+x\,(e-2\,f+4\,g-8\,h+16\,i)+x^3\left(\frac{g}{3}-\frac{2\,h}{3}+\frac{4\,i}{3}\right)$$

input
```
int(-((x + 2*x^2 - x^3 - 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^
4 - 5*x^2 + 4),x)
```

output
```
x^4*(h/4 - i/2) + log(x + 2)*(d - 2*e + 4*f - 8*g + 16*h - 32*i) + (i*x^5)
/5 + x^2*(f/2 - g + 2*h - 4*i) + x*(e - 2*f + 4*g - 8*h + 16*i) + x^3*(g/3
 - (2*h)/3 + (4*i)/3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.32

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$= \log(x+2)\,d-2\log(x+2)\,e+4\log(x+2)\,f-8\log(x+2)\,g$$
$$+16\log(x+2)\,h-32\log(x+2)\,i+ex+\frac{f\,x^2}{2}-2fx+\frac{g\,x^3}{3}-g\,x^2$$
$$+4gx+\frac{h\,x^4}{4}-\frac{2h\,x^3}{3}+2h\,x^2-8hx+\frac{i\,x^5}{5}-\frac{i\,x^4}{2}+\frac{4i\,x^3}{3}-4i\,x^2+16ix$$

input
```
int((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
(60*log(x + 2)*d - 120*log(x + 2)*e + 240*log(x + 2)*f - 480*log(x + 2)*g
+ 960*log(x + 2)*h - 1920*log(x + 2)*i + 60*e*x + 30*f*x**2 - 120*f*x + 20
*g*x**3 - 60*g*x**2 + 240*g*x + 15*h*x**4 - 40*h*x**3 + 120*h*x**2 - 480*h
*x + 12*i*x**5 - 30*i*x**4 + 80*i*x**3 - 240*i*x**2 + 960*i*x)/60
```

3.70 $\int \frac{2-3x+x^2}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{2-3x+x^2}{4-5x^2+x^4}\,dx = \log(1+x)-\log(2+x)$$

output
```
ln(1+x)-ln(2+x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2-3x+x^2}{4-5x^2+x^4}\,dx = \log(1+x)-\log(2+x)$$

input
```
Integrate[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4),x]
```

output
```
Log[1 + x] - Log[2 + x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2019, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-3x+2}{x^4-5x^2+4} \, dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{x^2+3x+2}dx$$

$$\Big\downarrow \text{ 1081}$$

$$\int \left(\frac{1}{x+1}+\frac{1}{-x-2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\log(x+1)-\log(x+2)$$

input
```
Int[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4),x]
```

output
```
Log[1 + x] - Log[2 + x]
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\ln (1+x)-\ln (x+2)$	12
norman	$\ln (1+x)-\ln (x+2)$	12
risch	$\ln (1+x)-\ln (x+2)$	12
parallelrisch	$\ln (1+x)-\ln (x+2)$	12

input
```
int((x^2-3*x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
ln(1+x)-ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2-3x+x^2}{4-5x^2+x^4}\,dx = -\log (x+2)+\log (x+1)$$

input
```
integrate((x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
-log(x + 2) + log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{2-3x+x^2}{4-5x^2+x^4}\,dx = \log(x+1) - \log(x+2)$$

input
```
integrate((x**2-3*x+2)/(x**4-5*x**2+4),x)
```

output
```
log(x + 1) - log(x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2-3x+x^2}{4-5x^2+x^4}\,dx = -\log(x+2) + \log(x+1)$$

input
```
integrate((x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
-log(x + 2) + log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{2-3x+x^2}{4-5x^2+x^4}\,dx = -\log(|x+2|) + \log(|x+1|)$$

input
```
integrate((x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
-log(abs(x + 2)) + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{2-3x+x^2}{4-5x^2+x^4}\,dx = -2\,\mathrm{atanh}(2\,x+3)$$

input
```
int((x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x)
```

output
```
-2*atanh(2*x + 3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2-3x+x^2}{4-5x^2+x^4}\,dx = -\log(x+2)+\log(x+1)$$

input
```
int((x^2-3*x+2)/(x^4-5*x^2+4),x)
```

output
```
 - log(x + 2) + log(x + 1)
```

3.71 $\int \frac{(d+ex)(2-3x+x^2)}{4-5x^2+x^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 22

$$\int \frac{(d+ex)\left(2-3x+x^2\right)}{4-5x^2+x^4}\,dx=(d-e)\log(1+x)-(d-2e)\log(2+x)$$

output
```
(d-e)*ln(1+x)-(d-2*e)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)\left(2-3x+x^2\right)}{4-5x^2+x^4}\,dx=(d-e)\log(1+x)+(-d+2e)\log(2+x)$$

input
```
Integrate[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4),x]
```

output
```
(d - e)*Log[1 + x] + (-d + 2*e)*Log[2 + x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2019, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-3x+2)(d+ex)}{x^4-5x^2+4}\,dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{d+ex}{x^2+3x+2}dx$$
$$\Big\downarrow \text{1141}$$
$$\int \left(\frac{d-e}{x+1}-\frac{d-2e}{x+2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$(d-e)\log(x+1)-(d-2e)\log(x+2)$$

input
```
Int[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4),x]
```

output
```
(d - e)*Log[1 + x] - (d - 2*e)*Log[2 + x]
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

method	result	size
default	$(d-e)\ln(1+x)+(-d+2e)\ln(x+2)$	24
norman	$(d-e)\ln(1+x)+(-d+2e)\ln(x+2)$	24
parallelrisch	$\ln(1+x)\,d-\ln(1+x)\,e-\ln(x+2)\,d+2\ln(x+2)\,e$	29
risch	$\ln(-x-1)\,d-\ln(-x-1)\,e-\ln(x+2)\,d+2\ln(x+2)\,e$	33

input
```
int((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
(d-e)*ln(1+x)+(-d+2*e)*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\,(2-3x+x^2)}{4-5x^2+x^4}\,dx=-(d-2\,e)\log(x+2)+(d-e)\log(x+1)$$

input
```
integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
-(d - 2*e)*log(x + 2) + (d - e)*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)\,(2-3x+x^2)}{4-5x^2+x^4}\,dx = (-d+2e)\log\left(x+\frac{4d-6e}{2d-3e}\right)+(d-e)\log(x+1)$$

input
```
integrate((e*x+d)*(x**2-3*x+2)/(x**4-5*x**2+4),x)
```

output
```
(-d + 2*e)*log(x + (4*d - 6*e)/(2*d - 3*e)) + (d - e)*log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\,(2-3x+x^2)}{4-5x^2+x^4}\,dx = -(d-2\,e)\log(x+2)+(d-e)\log(x+1)$$

input
```
integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
-(d - 2*e)*log(x + 2) + (d - e)*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)\,(2-3x+x^2)}{4-5x^2+x^4}\,dx = -(d-2\,e)\log(|x+2|)+(d-e)\log(|x+1|)$$

input
```
integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
-(d - 2*e)*log(abs(x + 2)) + (d - e)*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)(2-3x+x^2)}{4-5x^2+x^4}\,dx = \ln(x+1)\ (d-e) - \ln(x+2)\ (d-2\,e)$$

input
```
int(((d + e*x)*(x^2 - 3*x + 2))/(x^4 - 5*x^2 + 4),x)
```

output
```
log(x + 1)*(d - e) - log(x + 2)*(d - 2*e)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)(2-3x+x^2)}{4-5x^2+x^4}\,dx = -\log(x+2)\,d+2\log(x+2)\,e+\log(x+1)\,d-\log(x+1)\,e$$

input
```
int((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x)
```

output
```
- log(x + 2)*d + 2*log(x + 2)*e + log(x + 1)*d - log(x + 1)*e
```

3.72 $\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 29

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx = fx+(d-e+f)\log(1+x)-(d-2e+4f)\log(2+x)$$

output
```
f*x+(d-e+f)*ln(1+x)-(d-2*e+4*f)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx = fx+(d-e+f)\log(1+x)+(-d+2e-4f)\log(2+x)$$

input
```
Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

output
```
f*x + (d - e + f)*Log[1 + x] + (-d + 2*e - 4*f)*Log[2 + x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2019, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-3x+2)(d+ex+fx^2)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{d+ex+fx^2}{x^2+3x+2}dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(\frac{d+x(e-3f)-2f}{x^2+3x+2}+f\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\log(x+1)(d-e+f)-\log(x+2)(d-2e+4f)+fx$$

input
```
Int[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

output
```
f*x + (d - e + f)*Log[1 + x] - (d - 2*e + 4*f)*Log[2 + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

method	result
default	$fx+(d-e+f)\ln(1+x)+(-d+2e-4f)\ln(x+2)$
norman	$fx+(d-e+f)\ln(1+x)+(-d+2e-4f)\ln(x+2)$
parallelrisch	$\ln(1+x)d-\ln(1+x)e+\ln(1+x)f-\ln(x+2)d+2\ln(x+2)e-4\ln(x+2)f+f$
risch	$fx-\ln(x+2)d+2\ln(x+2)e-4\ln(x+2)f+\ln(-x-1)d-\ln(-x-1)e+\ln(-x$

input
```
int((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
f*x+(d-e+f)*ln(1+x)+(-d+2*e-4*f)*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx = fx-(d-2e+4f)\log(x+2) + (d-e+f)\log(x+1)$$

input
```
integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
f*x - (d - 2*e + 4*f)*log(x + 2) + (d - e + f)*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx = fx + (-d+2e-4f)\log\left(x+\frac{4d-6e+10f}{2d-3e+5f}\right) + (d-e+f)\log(x+1)$$

input
```
integrate((x**2-3*x+2)*(f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

output
```
f*x + (-d + 2*e - 4*f)*log(x + (4*d - 6*e + 10*f)/(2*d - 3*e + 5*f)) + (d
- e + f)*log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx = fx - (d-2e+4f)\log(x+2) + (d-e+f)\log(x+1)$$

input
```
integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
f*x - (d - 2*e + 4*f)*log(x + 2) + (d - e + f)*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx = fx - (d-2e+4f)\log(|x+2|) + (d-e+f)\log(|x+1|)$$

input
```
integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
f*x - (d - 2*e + 4*f)*log(abs(x + 2)) + (d - e + f)*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 18.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx = f\,x+\ln(x+1)\,(d-e+f)-\ln(x+2)\,(d-2\,e+4\,f)$$

input
```
int(((x^2 - 3*x + 2)*(d + e*x + f*x^2))/(x^4 - 5*x^2 + 4),x)
```

output
```
f*x + log(x + 1)*(d - e + f) - log(x + 2)*(d - 2*e + 4*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx = -\log(x+2)\,d+2\log(x+2)\,e-4\log(x+2)\,f + \log(x+1)\,d-\log(x+1)\,e+\log(x+1)\,f+fx$$

input
```
int((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
- log(x + 2)*d + 2*log(x + 2)*e - 4*log(x + 2)*f + log(x + 1)*d - log(x +
1)*e + log(x + 1)*f + f*x
```

3.73 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 47

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx = (f-3g)x+\frac{gx^2}{2}+(d-e+f-g)\log(1+x) - (d-2e+4f-8g)\log(2+x)$$

output
```
(f-3*g)*x+1/2*g*x^2+(d-e+f-g)*ln(1+x)-(d-2*e+4*f-8*g)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx = fx+\frac{1}{2}g(-6+x)x+(d-e+f-g)\log(1+x) - (d-2e+4f-8g)\log(2+x)$$

input
```
Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

output
```
f*x + (g*(-6 + x)*x)/2 + (d - e + f - g)*Log[1 + x] - (d - 2*e + 4*f - 8*g
)*Log[2 + x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2019, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-3x+2)(d+ex+fx^2+gx^3)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2+gx^3}{x^2+3x+2}dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{d+x(e-3f+7g)-2f+6g}{x^2+3x+2}+f+gx-3g\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x+1)(d-e+f-g)-\log(x+2)(d-2e+4f-8g)+x(f-3g)+\frac{gx^2}{2}$$

input
```
Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

output
```
(f - 3*g)*x + (g*x^2)/2 + (d - e + f - g)*Log[1 + x] - (d - 2*e + 4*f - 8*
g)*Log[2 + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result
default	$\frac{g\,x^2}{2}+fx-3gx+(d-e+f-g)\ln(1+x)+(-d+2e-4f+8g)\ln(x+2)$
norman	$(f-3g)\,x+\frac{g\,x^2}{2}+(-d+2e-4f+8g)\ln(x+2)+(d-e+f-g)\ln(1+x)$
parallelrisch	$\frac{g\,x^2}{2}+fx-3gx+\ln(1+x)\,d-\ln(1+x)\,e+\ln(1+x)\,f-\ln(1+x)\,g-\ln(x+2)\,d+$
risch	$\frac{g\,x^2}{2}+fx-3gx+\ln(-x-1)\,d-\ln(-x-1)\,e+\ln(-x-1)\,f-\ln(-x-1)\,g-\ln(x+$

input
```
int((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/2*g*x^2+f*x-3*g*x+(d-e+f-g)*ln(1+x)+(-d+2*e-4*f+8*g)*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{(2-3x+x^2)\,(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx = \frac{1}{2}\,gx^2+(f-3\,g)x - (d-2\,e+4\,f-8\,g)\log(x+2) + (d-e+f-g)\log(x+1)$$

input
```
integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fric
as")
```

output
```
1/2*g*x^2 + (f - 3*g)*x - (d - 2*e + 4*f - 8*g)*log(x + 2) + (d - e + f -
g)*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx = \frac{gx^2}{2} + x(f-3g) + (-d+2e-4f+8g)\log\left(x+\frac{4d-6e+10f-18g}{2d-3e+5f-9g}\right) + (d-e+f-g)\log(x+1)$$

input
```
integrate((x**2-3*x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

output
```
g*x**2/2 + x*(f - 3*g) + (-d + 2*e - 4*f + 8*g)*log(x + (4*d - 6*e + 10*f
- 18*g)/(2*d - 3*e + 5*f - 9*g)) + (d - e + f - g)*log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx = \frac{1}{2}\,gx^2 + (f-3\,g)x - (d-2\,e+4\,f-8\,g)\log(x+2) + (d-e+f-g)\log(x+1)$$

input
```
integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxi
ma")
```

output
```
1/2*g*x^2 + (f - 3*g)*x - (d - 2*e + 4*f - 8*g)*log(x + 2) + (d - e + f -
g)*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx = \frac{1}{2}\,gx^2 + fx - 3\,gx - (d-2\,e+4\,f-8\,g)\log\left(|x+2|\right) + (d-e+f-g)\log\left(|x+1|\right)$$

input
```
integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac
")
```

output
```
1/2*g*x^2 + f*x - 3*g*x - (d - 2*e + 4*f - 8*g)*log(abs(x + 2)) + (d - e +
 f - g)*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx = \ln(x+1)\,(d-e+f-g) + x\,(f-3\,g) + \frac{g\,x^2}{2} - \ln(x+2)\,(d-2\,e+4\,f-8\,g)$$

input
```
int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3))/(x^4 - 5*x^2 + 4),x)
```

output
```
log(x + 1)*(d - e + f - g) + x*(f - 3*g) + (g*x^2)/2 - log(x + 2)*(d - 2*e
 + 4*f - 8*g)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx = -\log(x+2)\,d+2\log(x+2)\,e-4\log(x+2)\,f+8\log(x+2)\,g+\log(x+1)\,d-\log(x+1)\,e+\log(x+1)\,f-\log(x+1)\,g+fx+\frac{g\,x^2}{2}-3gx$$

input
```
int((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
( - 2*log(x + 2)*d + 4*log(x + 2)*e - 8*log(x + 2)*f + 16*log(x + 2)*g + 2
*log(x + 1)*d - 2*log(x + 1)*e + 2*log(x + 1)*f - 2*log(x + 1)*g + 2*f*x +
 g*x**2 - 6*g*x)/2
```

3.74 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 66

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = (f-3g+7h)x+\frac{1}{2}(g-3h)x^2+\frac{hx^3}{3}+(d-e+f-g+h)\log(1+x) - (d-2e+4f-8g+16h)\log(2+x)$$

output
```
(f-3*g+7*h)*x+1/2*(g-3*h)*x^2+1/3*h*x^3+(d-e+f-g+h)*ln(1+x)-(d-2*e+4*f-8*g
+16*h)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = (f-3g+7h)x+\frac{1}{2}(g-3h)x^2+\frac{hx^3}{3}+(d-e+f-g+h)\log(1+x) + (-d+2e-4f+8g-16h)\log(2+x)$$

input
```
Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
 x^4),x]
```

output
```
(f - 3*g + 7*h)*x + ((g - 3*h)*x^2)/2 + (h*x^3)/3 + (d - e + f - g + h)*Lo
g[1 + x] + (-d + 2*e - 4*f + 8*g - 16*h)*Log[2 + x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {2019, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-3x+2)(d+ex+fx^2+gx^3+hx^4)}{x^4-5x^2+4}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{x^2+3x+2}dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{d+x(e-3f+7g-15h)-2f+6g-14h}{x^2+3x+2}+f+x(g-3h)-3g+hx^2+7h\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x+1)(d-e+f-g+h)-\log(x+2)(d-2e+4f-8g+16h)+x(f-3g+7h)+\frac{1}{2}x^2(g-3h)+\frac{hx^3}{3}$$

input
```
Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),
x]
```

output
```
(f - 3*g + 7*h)*x + ((g - 3*h)*x^2)/2 + (h*x^3)/3 + (d - e + f - g + h)*Lo
g[1 + x] - (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

method	result
norman	$\left(\frac{g}{2}-\frac{3h}{2}\right)x^{2}+(f-3g+7h)\,x+\frac{h\,x^{3}}{3}+(-d+2e-4f+8g-16h)\ln(x+2)+(d-e+$
default	$\frac{h\,x^{3}}{3}+\frac{g\,x^{2}}{2}-\frac{3h\,x^{2}}{2}+fx-3gx+7hx+(d-e+f-g+h)\ln(1+x)+(-d+2e-4f+$
parallelrisch	$\frac{h\,x^{3}}{3}+\frac{g\,x^{2}}{2}-\frac{3h\,x^{2}}{2}+fx-3gx+7hx+\ln(1+x)\,d-\ln(1+x)\,e+\ln(1+x)\,f-\ln(1+$
risch	$\frac{h\,x^{3}}{3}+\frac{g\,x^{2}}{2}-\frac{3h\,x^{2}}{2}+fx-3gx+7hx-\ln(x+2)\,d+2\ln(x+2)\,e-4\ln(x+2)\,f+8\ln($

input
```
int((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVE
RBOSE)
```

output
```
(1/2*g-3/2*h)*x^2+(f-3*g+7*h)*x+1/3*h*x^3+(-d+2*e-4*f+8*g-16*h)*ln(x+2)+(d
-e+f-g+h)*ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{(2-3x+x^2)\,(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{3}\,hx^3+\frac{1}{2}\,(g-3\,h)x^2+(f-3\,g+7\,h)x$$
$$-(d-2\,e+4\,f-8\,g+16\,h)\log{(x+2)}+(d-e+f-g+h)\log{(x+1)}$$

input
```
integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm
="fricas")
```

output
```
1/3*h*x^3 + 1/2*(g - 3*h)*x^2 + (f - 3*g + 7*h)*x - (d - 2*e + 4*f - 8*g +
 16*h)*log(x + 2) + (d - e + f - g + h)*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \frac{(2-3x+x^2)\,(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$$
$$=\frac{hx^3}{3}+x^2\left(\frac{g}{2}-\frac{3h}{2}\right)+x(f-3g+7h)$$
$$+(-d+2e-4f+8g-16h)\log{\left(x+\frac{4d-6e+10f-18g+34h}{2d-3e+5f-9g+17h}\right)}$$
$$+(d-e+f-g+h)\log{(x+1)}$$

input
```
integrate((x**2-3*x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

output
```
h*x**3/3 + x**2*(g/2 - 3*h/2) + x*(f - 3*g + 7*h) + (-d + 2*e - 4*f + 8*g
- 16*h)*log(x + (4*d - 6*e + 10*f - 18*g + 34*h)/(2*d - 3*e + 5*f - 9*g +
17*h)) + (d - e + f - g + h)*log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{3}hx^3+\frac{1}{2}(g-3h)x^2+(f-3g+7h)x$$
$$-(d-2e+4f-8g+16h)\log(x+2)+(d-e+f-g+h)\log(x+1)$$

input
```
integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm
="maxima")
```

output
```
1/3*h*x^3 + 1/2*(g - 3*h)*x^2 + (f - 3*g + 7*h)*x - (d - 2*e + 4*f - 8*g +
 16*h)*log(x + 2) + (d - e + f - g + h)*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{3}hx^3+\frac{1}{2}gx^2-\frac{3}{2}hx^2+fx-3gx+7hx$$
$$-(d-2e+4f-8g+16h)\log(|x+2|)+(d-e+f-g+h)\log(|x+1|)$$

input
```
integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm
="giac")
```

output
```
1/3*h*x^3 + 1/2*g*x^2 - 3/2*h*x^2 + f*x - 3*g*x + 7*h*x - (d - 2*e + 4*f -
 8*g + 16*h)*log(abs(x + 2)) + (d - e + f - g + h)*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = x^2\left(\frac{g}{2}-\frac{3\,h}{2}\right)+x\,(f-3\,g+7\,h)-\ln(x+2)\,(d-2\,e+4\,f-8\,g+16\,h) + \frac{h\,x^3}{3}+\ln(x+1)\,(d-e+f-g+h)$$

input
```
int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^2 + 4),
x)
```

output
```
x^2*(g/2 - (3*h)/2) + x*(f - 3*g + 7*h) - log(x + 2)*(d - 2*e + 4*f - 8*g
+ 16*h) + (h*x^3)/3 + log(x + 1)*(d - e + f - g + h)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.47

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = -\log(x+2)\,d+2\log(x+2)\,e-4\log(x+2)\,f+8\log(x+2)\,g - 16\log(x+2)\,h+\log(x+1)\,d-\log(x+1)\,e+\log(x+1)\,f - \log(x+1)\,g+\log(x+1)\,h+fx+\frac{g\,x^2}{2}-3gx+\frac{h\,x^3}{3}-\frac{3h\,x^2}{2}+7hx$$

input
```
int((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
( - 6*log(x + 2)*d + 12*log(x + 2)*e - 24*log(x + 2)*f + 48*log(x + 2)*g -
 96*log(x + 2)*h + 6*log(x + 1)*d - 6*log(x + 1)*e + 6*log(x + 1)*f - 6*lo
g(x + 1)*g + 6*log(x + 1)*h + 6*f*x + 3*g*x**2 - 18*g*x + 2*h*x**3 - 9*h*x
**2 + 42*h*x)/6
```

3.75 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4} \, dx$

Optimal result

Integrand size = 46, antiderivative size = 90

$$\begin{aligned} &\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4} \, dx \\ &= (f-3g+7h-15i)x+\frac{1}{2}(g-3h+7i)x^2+\frac{1}{3}(h-3i)x^3+\frac{ix^4}{4} \\ &\quad +(d-e+f-g+h-i)\log(1+x)-(d-2e+4f-8g+16h-32i)\log(2+x) \end{aligned}$$

output
```
(f-3*g+7*h-15*i)*x+1/2*(g-3*h+7*i)*x^2+1/3*(h-3*i)*x^3+1/4*i*x^4+(d-e+f-g+
h-i)*ln(1+x)-(d-2*e+4*f-8*g+16*h-32*i)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\begin{aligned} &\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4} \, dx \\ &= (f-3g+7h-15i)x+\frac{1}{2}(g-3h+7i)x^2+\frac{1}{3}(h-3i)x^3+\frac{ix^4}{4} \\ &\quad +(d-e+f-g+h-i)\log(1+x)+(-d+2e-4f+8g-16h+32i)\log(2+x) \end{aligned}$$

input
```
Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
 5*x^2 + x^4),x]
```

output
```
(f - 3*g + 7*h - 15*i)*x + ((g - 3*h + 7*i)*x^2)/2 + ((h - 3*i)*x^3)/3 + (
i*x^4)/4 + (d - e + f - g + h - i)*Log[1 + x] + (-d + 2*e - 4*f + 8*g - 16
*h + 32*i)*Log[2 + x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2019, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-3x+2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{x^2+3x+2}dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{d+x(e-3f+7g-15h+31i)-2f+6g-14h+30i}{x^2+3x+2}+f+x(g-3h+7i)-3g+x^2(h-3i)+7h+ix^3\right.$$

$$\Big\downarrow \text{2009}$$

$$\log(x+1)(d-e+f-g+h-i)-\log(x+2)(d-2e+4f-8g+16h-32i)+x(f-3g+7h-15i)+\frac{1}{2}x^2(g-3h+7i)+\frac{1}{3}x^3(h-3i)+\frac{ix^4}{4}$$

input
```
Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2
 + x^4),x]
```

output
```
(f - 3*g + 7*h - 15*i)*x + ((g - 3*h + 7*i)*x^2)/2 + ((h - 3*i)*x^3)/3 + (
i*x^4)/4 + (d - e + f - g + h - i)*Log[1 + x] - (d - 2*e + 4*f - 8*g + 16*
h - 32*i)*Log[2 + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

method	result
norman	$\left(\frac{h}{3}-i\right)x^3+\left(\frac{g}{2}-\frac{3h}{2}+\frac{7i}{2}\right)x^2+\left(f-3g+7h-15i\right)x+\frac{i\,x^4}{4}+\left(-d+2e-4f+8g-16h\right.$
default	$\frac{i\,x^4}{4}+\frac{h\,x^3}{3}-i\,x^3+\frac{g\,x^2}{2}-\frac{3h\,x^2}{2}+\frac{7i\,x^2}{2}+fx-3gx+7hx-15ix+\left(d-e+f-g+h-i\right)$
parallelrisch	$\frac{i\,x^4}{4}+\frac{h\,x^3}{3}-i\,x^3+\frac{g\,x^2}{2}-\frac{3h\,x^2}{2}+\frac{7i\,x^2}{2}+fx-3gx+7hx-15ix+\ln\left(1+x\right)d-\ln\left(1+x\right)$
risch	$\frac{i\,x^4}{4}+\frac{h\,x^3}{3}-i\,x^3+\frac{g\,x^2}{2}-\frac{3h\,x^2}{2}+\frac{7i\,x^2}{2}+fx-3gx+7hx-15ix+\ln\left(-x-1\right)d-\ln\left(-x\right.$

input
```
int((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RE
TURNVERBOSE)
```

output
```
(1/3*h-i)*x^3+(1/2*g-3/2*h+7/2*i)*x^2+(f-3*g+7*h-15*i)*x+1/4*i*x^4+(-d+2*e
-4*f+8*g-16*h+32*i)*ln(x+2)+(d-e+f-g+h-i)*ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{4}\,ix^4+\frac{1}{3}\,(h-3\,i)x^3+\frac{1}{2}\,(g-3\,h+7\,i)x^2+(f-3\,g+7\,h-15\,i)x$$
$$-(d-2\,e+4\,f-8\,g+16\,h-32\,i)\log(x+2)+(d-e+f-g+h-i)\log(x+1)$$

input
```
integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, alg
orithm="fricas")
```

output
```
1/4*i*x^4 + 1/3*(h - 3*i)*x^3 + 1/2*(g - 3*h + 7*i)*x^2 + (f - 3*g + 7*h -
 15*i)*x - (d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + (d - e + f - g
 + h - i)*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.36

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$=\frac{ix^4}{4}+x^3\left(\frac{h}{3}-i\right)+x^2\left(\frac{g}{2}-\frac{3h}{2}+\frac{7i}{2}\right)+x(f-3g+7h-15i)$$
$$+(-d+2e-4f+8g-16h+32i)\log\left(x+\frac{4d-6e+10f-18g+34h-66i}{2d-3e+5f-9g+17h-33i}\right)$$
$$+(d-e+f-g+h-i)\log(x+1)$$

input
```
integrate((x**2-3*x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)
,x)
```

output
```
i*x**4/4 + x**3*(h/3 - i) + x**2*(g/2 - 3*h/2 + 7*i/2) + x*(f - 3*g + 7*h
- 15*i) + (-d + 2*e - 4*f + 8*g - 16*h + 32*i)*log(x + (4*d - 6*e + 10*f -
 18*g + 34*h - 66*i)/(2*d - 3*e + 5*f - 9*g + 17*h - 33*i)) + (d - e + f -
 g + h - i)*log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$= \frac{1}{4}\,ix^4 + \frac{1}{3}\,(h-3\,i)x^3 + \frac{1}{2}\,(g-3\,h+7\,i)x^2 + (f-3\,g+7\,h-15\,i)x - (d-2\,e+4\,f-8\,g+16\,h-32\,i)\log{(x+2)} + (d-e+f-g+h-i)\log{(x+1)}$$

input
```
integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, alg
orithm="maxima")
```

output
```
1/4*i*x^4 + 1/3*(h - 3*i)*x^3 + 1/2*(g - 3*h + 7*i)*x^2 + (f - 3*g + 7*h -
 15*i)*x - (d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + (d - e + f - g
 + h - i)*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$= \frac{1}{4}\,ix^4 + \frac{1}{3}\,hx^3 - ix^3 + \frac{1}{2}\,gx^2 - \frac{3}{2}\,hx^2 + \frac{7}{2}\,ix^2 + fx - 3\,gx + 7\,hx - 15\,ix - (d-2\,e+4\,f-8\,g+16\,h-32\,i)\log{(|x+2|)} + (d-e+f-g+h-i)\log{(|x+1|)}$$

input
```
integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, alg
orithm="giac")
```

output
```
1/4*i*x^4 + 1/3*h*x^3 - i*x^3 + 1/2*g*x^2 - 3/2*h*x^2 + 7/2*i*x^2 + f*x -
3*g*x + 7*h*x - 15*i*x - (d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(abs(x + 2
)) + (d - e + f - g + h - i)*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 17.92 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$= x^3\left(\frac{h}{3}-i\right) - \ln(x+2)\,(d-2\,e+4\,f-8\,g+16\,h-32\,i) + \ln(x+1)\,(d-e+f-g+h-i) + \frac{i\,x^4}{4} + x^2\left(\frac{g}{2}-\frac{3\,h}{2}+\frac{7\,i}{2}\right) + x\,(f-3\,g+7\,h-15\,i)$$

input
```
int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^4 - 5*x
^2 + 4),x)
```

output
```
x^3*(h/3 - i) - log(x + 2)*(d - 2*e + 4*f - 8*g + 16*h - 32*i) + log(x + 1
)*(d - e + f - g + h - i) + (i*x^4)/4 + x^2*(g/2 - (3*h)/2 + (7*i)/2) + x*
(f - 3*g + 7*h - 15*i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.48

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$$
$$= -\log(x+2)\,d + 2\log(x+2)\,e - 4\log(x+2)\,f + 8\log(x+2)\,g - 16\log(x+2)\,h + 32\log(x+2)\,i + \log(x+1)\,d - \log(x+1)\,e + \log(x+1)\,f - \log(x+1)\,g + \log(x+1)\,h - \log(x+1)\,i + fx + \frac{g\,x^2}{2} - 3gx + \frac{h\,x^3}{3} - \frac{3h\,x^2}{2} + 7hx + \frac{i\,x^4}{4} - i\,x^3 + \frac{7i\,x^2}{2} - 15ix$$

input
```
int((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
( - 12*log(x + 2)*d + 24*log(x + 2)*e - 48*log(x + 2)*f + 96*log(x + 2)*g
- 192*log(x + 2)*h + 384*log(x + 2)*i + 12*log(x + 1)*d - 12*log(x + 1)*e
+ 12*log(x + 1)*f - 12*log(x + 1)*g + 12*log(x + 1)*h - 12*log(x + 1)*i +
12*f*x + 6*g*x**2 - 36*g*x + 4*h*x**3 - 18*h*x**2 + 84*h*x + 3*i*x**4 - 12
*i*x**3 + 42*i*x**2 - 180*i*x)/12
```

3.76 $\int \frac{2+x}{4-5x^2+x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{2+x}{4-5x^2+x^4} dx = -\frac{1}{2}\log(1-x) + \frac{1}{3}\log(2-x) + \frac{1}{6}\log(1+x)$$

output
```
-1/2*ln(1-x)+1/3*ln(2-x)+1/6*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{4-5x^2+x^4} dx = -\frac{1}{2}\log(1-x) + \frac{1}{3}\log(2-x) + \frac{1}{6}\log(1+x)$$

input
```
Integrate[(2 + x)/(4 - 5*x^2 + x^4),x]
```

output
```
-1/2*Log[1 - x] + Log[2 - x]/3 + Log[1 + x]/6
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{x^3-2x^2-x+2}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(-\frac{1}{2(x-1)}+\frac{1}{6(x+1)}+\frac{1}{3(x-2)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(x+1)$$

input
```
Int[(2 + x)/(4 - 5*x^2 + x^4),x]
```

output
```
-1/2*Log[1 - x] + Log[2 - x]/3 + Log[1 + x]/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\ln(x-2)}{3}+\frac{\ln(1+x)}{6}-\frac{\ln(x-1)}{2}$	20
norman	$\frac{\ln(x-2)}{3}+\frac{\ln(1+x)}{6}-\frac{\ln(x-1)}{2}$	20
risch	$\frac{\ln(x-2)}{3}+\frac{\ln(1+x)}{6}-\frac{\ln(x-1)}{2}$	20
parallelrisch	$\frac{\ln(x-2)}{3}+\frac{\ln(1+x)}{6}-\frac{\ln(x-1)}{2}$	20

input
```
int((x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x-2)+1/6*ln(1+x)-1/2*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x}{4-5x^2+x^4}\,dx=\frac{1}{6}\log(x+1)-\frac{1}{2}\log(x-1)+\frac{1}{3}\log(x-2)$$

input
```
integrate((2+x)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
1/6*log(x + 1) - 1/2*log(x - 1) + 1/3*log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x}{4-5x^2+x^4}\,dx = \frac{\log(x-2)}{3} - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{6}$$

input
```
integrate((2+x)/(x**4-5*x**2+4),x)
```

output
```
log(x - 2)/3 - log(x - 1)/2 + log(x + 1)/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x}{4-5x^2+x^4}\,dx = \frac{1}{6}\log(x+1) - \frac{1}{2}\log(x-1) + \frac{1}{3}\log(x-2)$$

input
```
integrate((2+x)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
1/6*log(x + 1) - 1/2*log(x - 1) + 1/3*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{2+x}{4-5x^2+x^4}\,dx = \frac{1}{6}\log(|x+1|) - \frac{1}{2}\log(|x-1|) + \frac{1}{3}\log(|x-2|)$$

input
```
integrate((2+x)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
1/6*log(abs(x + 1)) - 1/2*log(abs(x - 1)) + 1/3*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x}{4-5x^2+x^4}\,dx = \frac{\ln(x+1)}{6} - \frac{\ln(x-1)}{2} + \frac{\ln(x-2)}{3}$$

input
```
int((x + 2)/(x^4 - 5*x^2 + 4),x)
```

output
```
log(x + 1)/6 - log(x - 1)/2 + log(x - 2)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{2+x}{4-5x^2+x^4}\,dx = \frac{\log(x-2)}{3} - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{6}$$

input
```
int((2+x)/(x^4-5*x^2+4),x)
```

output
```
(2*log(x - 2) - 3*log(x - 1) + log(x + 1))/6
```

3.77 $\int \frac{(2+x)(d+ex)}{4-5x^2+x^4} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{(2+x)(d+ex)}{4-5x^2+x^4} \, dx = -\frac{1}{2}(d+e)\log(1-x)+\frac{1}{3}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(1+x)$$

output
```
-1/2*(d+e)*ln(1-x)+1/3*(d+2*e)*ln(2-x)+1/6*(d-e)*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{(2+x)(d+ex)}{4-5x^2+x^4} \, dx = \frac{1}{6}(-3(d+e)\log(1-x)+2(d+2e)\log(2-x)+(d-e)\log(1+x))$$

input
```
Integrate[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4),x]
```

output
```
(-3*(d + e)*Log[1 - x] + 2*(d + 2*e)*Log[2 - x] + (d - e)*Log[1 + x])/6
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+2)(d+ex)}{x^4-5x^2+4} \, dx$$

$$\Big\downarrow \; 2019$$

$$\int \frac{d+ex}{x^3-2x^2-x+2} dx$$

$$\Big\downarrow \; 2462$$

$$\int \left(\frac{-d-e}{2(x-1)}+\frac{d+2e}{3(x-2)}+\frac{d-e}{6(x+1)}\right) dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{2}(d+e)\log(1-x)+\frac{1}{3}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(x+1)$$

input
```
Int[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4),x]
```

output
```
-1/2*((d + e)*Log[1 - x]) + ((d + 2*e)*Log[2 - x])/3 + ((d - e)*Log[1 + x]
)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$\left(\frac{d}{3}+\frac{2e}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{2}-\frac{e}{2}\right)\ln\left(x-1\right)$	38
norman	$\left(\frac{d}{3}+\frac{2e}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{2}-\frac{e}{2}\right)\ln\left(x-1\right)$	38
parallelrisch	$\frac{\ln(x-2)d}{3}+\frac{2\ln(x-2)e}{3}-\frac{\ln(x-1)d}{2}-\frac{\ln(x-1)e}{2}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}$	44
risch	$\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}-\frac{\ln(1-x)d}{2}-\frac{\ln(1-x)e}{2}+\frac{\ln(2-x)d}{3}+\frac{2\ln(2-x)e}{3}$	52

input
```
int((x+2)*(e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
(1/3*d+2/3*e)*ln(x-2)+(1/6*d-1/6*e)*ln(1+x)+(-1/2*d-1/2*e)*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{(2+x)(d+ex)}{4-5x^2+x^4}\,dx=\frac{1}{6}\,(d-e)\log\left(x+1\right)-\frac{1}{2}\,(d+e)\log\left(x-1\right)+\frac{1}{3}\,(d+2\,e)\log\left(x-2\right)$$

input
```
integrate((2+x)*(e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
1/6*(d - e)*log(x + 1) - 1/2*(d + e)*log(x - 1) + 1/3*(d + 2*e)*log(x - 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(37) = 74$.

Time = 1.03 (sec) , antiderivative size = 304, normalized size of antiderivative = 7.24

$$\int \frac{(2+x)(d+ex)}{4-5x^2+x^4}\,dx$$
$$= \frac{(d-e)\log\left(x+\frac{26d^3+66d^2e-9d^2(d-e)+78de^2-12de(d-e)-7d(d-e)^2+46e^3+3e^2(d-e)-8e(d-e)^2}{10d^3+69d^2e+102de^2+35e^3}\right)}{6}$$
$$- \frac{(d+e)\log\left(x+\frac{26d^3+66d^2e+27d^2(d+e)+78de^2+36de(d+e)-63d(d+e)^2+46e^3-9e^2(d+e)-72e(d+e)^2}{10d^3+69d^2e+102de^2+35e^3}\right)}{2}$$
$$+ \frac{(d+2e)\log\left(x+\frac{26d^3+66d^2e-18d^2(d+2e)+78de^2-24de(d+2e)-28d(d+2e)^2+46e^3+6e^2(d+2e)-32e(d+2e)^2}{10d^3+69d^2e+102de^2+35e^3}\right)}{3}$$

input
```
integrate((2+x)*(e*x+d)/(x**4-5*x**2+4),x)
```

output
```
(d - e)*log(x + (26*d**3 + 66*d**2*e - 9*d**2*(d - e) + 78*d*e**2 - 12*d*e
*(d - e) - 7*d*(d - e)**2 + 46*e**3 + 3*e**2*(d - e) - 8*e*(d - e)**2)/(10
*d**3 + 69*d**2*e + 102*d*e**2 + 35*e**3))/6 - (d + e)*log(x + (26*d**3 +
66*d**2*e + 27*d**2*(d + e) + 78*d*e**2 + 36*d*e*(d + e) - 63*d*(d + e)**2
 + 46*e**3 - 9*e**2*(d + e) - 72*e*(d + e)**2)/(10*d**3 + 69*d**2*e + 102*
d*e**2 + 35*e**3))/2 + (d + 2*e)*log(x + (26*d**3 + 66*d**2*e - 18*d**2*(d
 + 2*e) + 78*d*e**2 - 24*d*e*(d + 2*e) - 28*d*(d + 2*e)**2 + 46*e**3 + 6*e
**2*(d + 2*e) - 32*e*(d + 2*e)**2)/(10*d**3 + 69*d**2*e + 102*d*e**2 + 35*
e**3))/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{(2+x)(d+ex)}{4-5x^2+x^4}\,dx = \frac{1}{6}(d-e)\log(x+1) - \frac{1}{2}(d+e)\log(x-1) + \frac{1}{3}(d+2e)\log(x-2)$$

input
```
integrate((2+x)*(e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
1/6*(d - e)*log(x + 1) - 1/2*(d + e)*log(x - 1) + 1/3*(d + 2*e)*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{(2+x)(d+ex)}{4-5x^2+x^4}\,dx = \frac{1}{6}(d-e)\log(|x+1|) - \frac{1}{2}(d+e)\log(|x-1|) + \frac{1}{3}(d+2e)\log(|x-2|)$$

input
```
integrate((2+x)*(e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
1/6*(d - e)*log(abs(x + 1)) - 1/2*(d + e)*log(abs(x - 1)) + 1/3*(d + 2*e)*
log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{(2+x)(d+ex)}{4-5x^2+x^4}\,dx = \ln(x-2)\left(\frac{d}{3}+\frac{2e}{3}\right) - \ln(x-1)\left(\frac{d}{2}+\frac{e}{2}\right) + \ln(x+1)\left(\frac{d}{6}-\frac{e}{6}\right)$$

input
```
int(((x + 2)*(d + e*x))/(x^4 - 5*x^2 + 4),x)
```

output
```
log(x - 2)*(d/3 + (2*e)/3) - log(x - 1)*(d/2 + e/2) + log(x + 1)*(d/6 - e/
6)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{(2+x)(d+ex)}{4-5x^2+x^4}\,dx = \frac{\log(x-2)\,d}{3} + \frac{2\log(x-2)\,e}{3} - \frac{\log(x-1)\,d}{2} - \frac{\log(x-1)\,e}{2} + \frac{\log(x+1)\,d}{6} - \frac{\log(x+1)\,e}{6}$$

input
```
int((2+x)*(e*x+d)/(x^4-5*x^2+4),x)
```

output
```
(2*log(x - 2)*d + 4*log(x - 2)*e - 3*log(x - 1)*d - 3*log(x - 1)*e + log(x
 + 1)*d - log(x + 1)*e)/6
```

3.78 $\int \frac{(2+x)\left(d+ex+fx^2\right)}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 47

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{4-5x^2+x^4}\,dx = -\frac{1}{2}(d+e+f)\log(1-x)+\frac{1}{3}(d+2e+4f)\log(2-x)+\frac{1}{6}(d-e+f)\log(1+x)$$

output
```
-1/2*(d+e+f)*ln(1-x)+1/3*(d+2*e+4*f)*ln(2-x)+1/6*(d-e+f)*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{4-5x^2+x^4}\,dx = \frac{1}{6}(-3(d+e+f)\log(1-x)+2(d+2e+4f)\log(2-x)+(d-e+f)\log(1+x))$$

input
```
Integrate[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

output
```
(-3*(d + e + f)*Log[1 - x] + 2*(d + 2*e + 4*f)*Log[2 - x] + (d - e + f)*Lo
g[1 + x])/6
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+2)\left(d+ex+fx^2\right)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2}{x^3-2x^2-x+2}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-d-e-f}{2(x-1)}+\frac{d+2e+4f}{3(x-2)}+\frac{d-e+f}{6(x+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\log(1-x)(d+e+f)+\frac{1}{3}\log(2-x)(d+2e+4f)+\frac{1}{6}\log(x+1)(d-e+f)$$

input
```
Int[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

output
```
-1/2*((d + e + f)*Log[1 - x]) + ((d + 2*e + 4*f)*Log[2 - x])/3 + ((d - e +
 f)*Log[1 + x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result
default	$\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}\right)\ln\left(x-1\right)$
norman	$\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}\right)\ln\left(x-1\right)$
parallelrisch	$\frac{\ln(x-2)d}{3}+\frac{2\ln(x-2)e}{3}+\frac{4\ln(x-2)f}{3}-\frac{\ln(x-1)d}{2}-\frac{\ln(x-1)e}{2}-\frac{\ln(x-1)f}{2}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)}{6}$
risch	$-\frac{\ln(1-x)d}{2}-\frac{\ln(1-x)e}{2}-\frac{\ln(1-x)f}{2}+\frac{\ln(2-x)d}{3}+\frac{2\ln(2-x)e}{3}+\frac{4\ln(2-x)f}{3}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+}{6}$

input
```
int((x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
(1/3*d+2/3*e+4/3*f)*ln(x-2)+(1/6*d-1/6*e+1/6*f)*ln(1+x)+(-1/2*d-1/2*e-1/2*
f)*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{4-5x^2+x^4}\,dx=\frac{1}{6}\left(d-e+f\right)\log\left(x+1\right)-\frac{1}{2}\left(d+e+f\right)\log\left(x-1\right)+\frac{1}{3}\left(d+2\,e+4\,f\right)\log\left(x-2\right)$$

input
```
integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
1/6*(d - e + f)*log(x + 1) - 1/2*(d + e + f)*log(x - 1) + 1/3*(d + 2*e + 4
*f)*log(x - 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(49) = 98$.

Time = 7.13 (sec) , antiderivative size = 716, normalized size of antiderivative = 15.23

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{4-5x^2+x^4}\,dx$$
$$= \frac{(d-e+f)\log\left(x+\frac{26d^3+66d^2e+132d^2f-9d^2(d-e+f)+78de^2+276def-12de(d-e+f)+222df^2+6df(d-e+f)-7d(d-e+f)^2+46e^3+}{10d^3+69d^2e+102d^2f+102de^2+318def+246d}\right.}{6}$$
$$-\frac{(d+e+f)\log\left(x+\frac{26d^3+66d^2e+132d^2f+27d^2(d+e+f)+78de^2+276def+36de(d+e+f)+222df^2-18df(d+e+f)-63d(d+e+f)^2-}{10d^3+69d^2e+102d^2f+102de^2+318def}\right.}{2}$$
$$+\frac{(d+2e+4f)\log\left(x+\frac{26d^3+66d^2e+132d^2f-18d^2(d+2e+4f)+78de^2+276def-24de(d+2e+4f)+222df^2+12df(d+2e+4f)-28d(}{10d^3+69d^2e+102d^2f+102}\right.}{3}$$

input
```
integrate((2+x)*(f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

output
```
(d - e + f)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f - 9*d**2*(d - e + f)
 + 78*d*e**2 + 276*d*e*f - 12*d*e*(d - e + f) + 222*d*f**2 + 6*d*f*(d - e
+ f) - 7*d*(d - e + f)**2 + 46*e**3 + 204*e**2*f + 3*e**2*(d - e + f) + 28
2*e*f**2 + 36*e*f*(d - e + f) - 8*e*(d - e + f)**2 + 116*f**3 + 51*f**2*(d
 - e + f) - 13*f*(d - e + f)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 102*d
*e**2 + 318*d*e*f + 246*d*f**2 + 35*e**3 + 174*e**2*f + 285*e*f**2 + 154*f
**3))/6 - (d + e + f)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f + 27*d**2*
(d + e + f) + 78*d*e**2 + 276*d*e*f + 36*d*e*(d + e + f) + 222*d*f**2 - 18
*d*f*(d + e + f) - 63*d*(d + e + f)**2 + 46*e**3 + 204*e**2*f - 9*e**2*(d
+ e + f) + 282*e*f**2 - 108*e*f*(d + e + f) - 72*e*(d + e + f)**2 + 116*f*
*3 - 153*f**2*(d + e + f) - 117*f*(d + e + f)**2)/(10*d**3 + 69*d**2*e + 1
02*d**2*f + 102*d*e**2 + 318*d*e*f + 246*d*f**2 + 35*e**3 + 174*e**2*f + 2
85*e*f**2 + 154*f**3))/2 + (d + 2*e + 4*f)*log(x + (26*d**3 + 66*d**2*e +
132*d**2*f - 18*d**2*(d + 2*e + 4*f) + 78*d*e**2 + 276*d*e*f - 24*d*e*(d +
 2*e + 4*f) + 222*d*f**2 + 12*d*f*(d + 2*e + 4*f) - 28*d*(d + 2*e + 4*f)**
2 + 46*e**3 + 204*e**2*f + 6*e**2*(d + 2*e + 4*f) + 282*e*f**2 + 72*e*f*(d
 + 2*e + 4*f) - 32*e*(d + 2*e + 4*f)**2 + 116*f**3 + 102*f**2*(d + 2*e + 4
*f) - 52*f*(d + 2*e + 4*f)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 102*d*e
**2 + 318*d*e*f + 246*d*f**2 + 35*e**3 + 174*e**2*f + 285*e*f**2 + 154*f**
3))/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{(2+x)\,(d+ex+fx^2)}{4-5x^2+x^4}\,dx = \frac{1}{6}\,(d-e+f)\log{(x+1)} - \frac{1}{2}\,(d+e+f)\log{(x-1)} + \frac{1}{3}\,(d+2\,e+4\,f)\log{(x-2)}$$

input
```
integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
1/6*(d - e + f)*log(x + 1) - 1/2*(d + e + f)*log(x - 1) + 1/3*(d + 2*e + 4
*f)*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{(2+x)\,(d+ex+fx^2)}{4-5x^2+x^4}\,dx = \frac{1}{6}\,(d-e+f)\log{(|x+1|)} - \frac{1}{2}\,(d+e+f)\log{(|x-1|)} + \frac{1}{3}\,(d+2\,e+4\,f)\log{(|x-2|)}$$

input
```
integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
1/6*(d - e + f)*log(abs(x + 1)) - 1/2*(d + e + f)*log(abs(x - 1)) + 1/3*(d
 + 2*e + 4*f)*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{4-5x^2+x^4}\,dx = \ln\left(x-2\right)\left(\frac{d}{3}+\frac{2\,e}{3}+\frac{4\,f}{3}\right) - \ln\left(x-1\right)\left(\frac{d}{2}+\frac{e}{2}+\frac{f}{2}\right) + \ln\left(x+1\right)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)$$

input
```
int(((x + 2)*(d + e*x + f*x^2))/(x^4 - 5*x^2 + 4),x)
```

output
```
log(x - 2)*(d/3 + (2*e)/3 + (4*f)/3) - log(x - 1)*(d/2 + e/2 + f/2) + log(
x + 1)*(d/6 - e/6 + f/6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{4-5x^2+x^4}\,dx = \frac{\log(x-2)\,d}{3}+\frac{2\log(x-2)\,e}{3}+\frac{4\log(x-2)\,f}{3} - \frac{\log(x-1)\,d}{2}-\frac{\log(x-1)\,e}{2}-\frac{\log(x-1)\,f}{2} + \frac{\log(x+1)\,d}{6}-\frac{\log(x+1)\,e}{6}+\frac{\log(x+1)\,f}{6}$$

input
```
int((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
(2*log(x - 2)*d + 4*log(x - 2)*e + 8*log(x - 2)*f - 3*log(x - 1)*d - 3*log
(x - 1)*e - 3*log(x - 1)*f + log(x + 1)*d - log(x + 1)*e + log(x + 1)*f)/6
```

3.79 $\int \frac{(2+x)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 57

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx = gx-\frac{1}{2}(d+e+f+g)\log(1-x)+\frac{1}{3}(d+2e+4f+8g)\log(2-x)+\frac{1}{6}(d-e+f-g)\log(1+x)$$

output
```
g*x-1/2*(d+e+f+g)*ln(1-x)+1/3*(d+2*e+4*f+8*g)*ln(2-x)+1/6*(d-e+f-g)*ln(1+x
)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx = \frac{1}{6}(6gx-3(d+e+f+g)\log(1-x)+2(d+2e+4f+8g)\log(2-x)+(d-e+f-g)\log(1+x))$$

input
```
Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

output
```
(6*g*x - 3*(d + e + f + g)*Log[1 - x] + 2*(d + 2*e + 4*f + 8*g)*Log[2 - x]
 + (d - e + f - g)*Log[1 + x])/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+2)\left(d+ex+fx^2+gx^3\right)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2+gx^3}{x^3-2x^2-x+2}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-d-e-f-g}{2(x-1)}+\frac{d+2e+4f+8g}{3(x-2)}+\frac{d-e+f-g}{6(x+1)}+g\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\log(1-x)(d+e+f+g)+\frac{1}{3}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)+gx$$

input
```
Int[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

output
```
g*x - ((d + e + f + g)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g)*Log[2 - x])/
3 + ((d - e + f - g)*Log[1 + x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

method	result
default	$gx+\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8g}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}-\frac{g}{2}\right)\ln(x$
norman	$gx+\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8g}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}-\frac{g}{2}\right)\ln(x$
parallelrisch	$gx+\frac{\ln(x-2)d}{3}+\frac{2\ln(x-2)e}{3}+\frac{4\ln(x-2)f}{3}+\frac{8\ln(x-2)g}{3}-\frac{\ln(x-1)d}{2}-\frac{\ln(x-1)e}{2}-\frac{\ln(x-1)f}{2}-\frac{\ln(x-1)g}{2}+$
risch	$gx-\frac{\ln(1-x)d}{2}-\frac{\ln(1-x)e}{2}-\frac{\ln(1-x)f}{2}-\frac{\ln(1-x)g}{2}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(1+x)g}{6}+\ln(2$

input
```
int((x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
g*x+(1/3*d+2/3*e+4/3*f+8/3*g)*ln(x-2)+(1/6*d-1/6*e+1/6*f-1/6*g)*ln(1+x)+(-
1/2*d-1/2*e-1/2*f-1/2*g)*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx = gx+\frac{1}{6}\,(d-e+f-g)\log\left(x+1\right) - \frac{1}{2}\,(d+e+f+g)\log\left(x-1\right) + \frac{1}{3}\,(d+2\,e+4\,f+8\,g)\log\left(x-2\right)$$

input
```
integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

output
```
g*x + 1/6*(d - e + f - g)*log(x + 1) - 1/2*(d + e + f + g)*log(x - 1) + 1/
3*(d + 2*e + 4*f + 8*g)*log(x - 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1389 vs. 2(63) = 126.

Time = 49.29 (sec) , antiderivative size = 1389, normalized size of antiderivative = 24.37

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx = \text{Too large to display}$$

input
```
integrate((2+x)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

output
```
g*x + (d - e + f - g)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f + 174*d**2
*g - 9*d**2*(d - e + f - g) + 78*d*e**2 + 276*d*e*f + 444*d*e*g - 12*d*e*(
d - e + f - g) + 222*d*f**2 + 636*d*f*g + 6*d*f*(d - e + f - g) + 510*d*g*
*2 + 36*d*g*(d - e + f - g) - 7*d*(d - e + f - g)**2 + 46*e**3 + 204*e**2*
f + 390*e**2*g + 3*e**2*(d - e + f - g) + 282*e*f**2 + 984*e*f*g + 36*e*f*
(d - e + f - g) + 930*e*g**2 + 102*e*g*(d - e + f - g) - 8*e*(d - e + f -
g)**2 + 116*f**3 + 534*f**2*g + 51*f**2*(d - e + f - g) + 924*f*g**2 + 228
*f*g*(d - e + f - g) - 13*f*(d - e + f - g)**2 + 586*g**3 + 243*g**2*(d -
e + f - g) - 20*g*(d - e + f - g)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f +
213*d**2*g + 102*d*e**2 + 318*d*e*f + 564*d*e*g + 246*d*f**2 + 894*d*f*g +
 750*d*g**2 + 35*e**3 + 174*e**2*f + 249*e**2*g + 285*e*f**2 + 852*e*f*g +
 537*e*g**2 + 154*f**3 + 717*f**2*g + 966*f*g**2 + 323*g**3))/6 - (d + e +
 f + g)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f + 174*d**2*g + 27*d**2*(
d + e + f + g) + 78*d*e**2 + 276*d*e*f + 444*d*e*g + 36*d*e*(d + e + f + g
) + 222*d*f**2 + 636*d*f*g - 18*d*f*(d + e + f + g) + 510*d*g**2 - 108*d*g
*(d + e + f + g) - 63*d*(d + e + f + g)**2 + 46*e**3 + 204*e**2*f + 390*e*
*2*g - 9*e**2*(d + e + f + g) + 282*e*f**2 + 984*e*f*g - 108*e*f*(d + e +
f + g) + 930*e*g**2 - 306*e*g*(d + e + f + g) - 72*e*(d + e + f + g)**2 +
116*f**3 + 534*f**2*g - 153*f**2*(d + e + f + g) + 924*f*g**2 - 684*f*g*(d
 + e + f + g) - 117*f*(d + e + f + g)**2 + 586*g**3 - 729*g**2*(d + e +...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx = gx + \frac{1}{6}\,(d-e+f-g)\log\left(x+1\right) - \frac{1}{2}\,(d+e+f+g)\log\left(x-1\right) + \frac{1}{3}\,(d+2\,e+4\,f+8\,g)\log\left(x-2\right)$$

input
```
integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

output
```
g*x + 1/6*(d - e + f - g)*log(x + 1) - 1/2*(d + e + f + g)*log(x - 1) + 1/
3*(d + 2*e + 4*f + 8*g)*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx = gx+\frac{1}{6}(d-e+f-g)\log(|x+1|) - \frac{1}{2}(d+e+f+g)\log(|x-1|) + \frac{1}{3}(d+2\,e+4\,f+8\,g)\log(|x-2|)$$

input
```
integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

output
```
g*x + 1/6*(d - e + f - g)*log(abs(x + 1)) - 1/2*(d + e + f + g)*log(abs(x
- 1)) + 1/3*(d + 2*e + 4*f + 8*g)*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx = \ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}\right) - \ln(x-1)\left(\frac{d}{2}+\frac{e}{2}+\frac{f}{2}+\frac{g}{2}\right) + \ln(x-2)\left(\frac{d}{3}+\frac{2\,e}{3}+\frac{4\,f}{3}+\frac{8\,g}{3}\right)+g\,x$$

input
```
int(((x + 2)*(d + e*x + f*x^2 + g*x^3))/(x^4 - 5*x^2 + 4),x)
```

output
```
log(x + 1)*(d/6 - e/6 + f/6 - g/6) - log(x - 1)*(d/2 + e/2 + f/2 + g/2) +
log(x - 2)*(d/3 + (2*e)/3 + (4*f)/3 + (8*g)/3) + g*x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.54

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx = \frac{\log(x-2)\,d}{3}+\frac{2\log(x-2)\,e}{3}+\frac{4\log(x-2)\,f}{3}+\frac{8\log(x-2)\,g}{3}-\frac{\log(x-1)\,d}{2}-\frac{\log(x-1)\,e}{2}-\frac{\log(x-1)\,f}{2}-\frac{\log(x-1)\,g}{2}+\frac{\log(x+1)\,d}{6}-\frac{\log(x+1)\,e}{6}+\frac{\log(x+1)\,f}{6}-\frac{\log(x+1)\,g}{6}+gx$$

input
```
int((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
(2*log(x - 2)*d + 4*log(x - 2)*e + 8*log(x - 2)*f + 16*log(x - 2)*g - 3*lo
g(x - 1)*d - 3*log(x - 1)*e - 3*log(x - 1)*f - 3*log(x - 1)*g + log(x + 1)
*d - log(x + 1)*e + log(x + 1)*f - log(x + 1)*g + 6*g*x)/6
```

3.80 $\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 74

$$\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = (g+2h)x+\frac{hx^2}{2} - \frac{1}{2}(d+e+f+g+h)\log(1-x) + \frac{1}{3}(d+2e+4f+8g+16h)\log(2-x) + \frac{1}{6}(d-e+f-g+h)\log(1+x)$$

output
```
(g+2*h)*x+1/2*h*x^2-1/2*(d+e+f+g+h)*ln(1-x)+1/3*(d+2*e+4*f+8*g+16*h)*ln(2-
x)+1/6*(d-e+f-g+h)*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = \frac{1}{6}\left(6(g+2h)x+3hx^2 - 3(d+e+f+g+h)\log(1-x) + 2(d+2(e+2f+4g+8h))\log(2-x) + (d-e+f-g+h)\log(1+x)\right)$$

input
```
Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x]
```

output
```
(6*(g + 2*h)*x + 3*h*x^2 - 3*(d + e + f + g + h)*Log[1 - x] + 2*(d + 2*(e
+ 2*f + 4*g + 8*h))*Log[2 - x] + (d - e + f - g + h)*Log[1 + x])/6
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+2)\left(d+ex+fx^2+gx^3+hx^4\right)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{x^3-2x^2-x+2}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{-d-e-f-g-h}{2(x-1)}+\frac{d+2e+4f+8g+16h}{3(x-2)}+\frac{d-e+f-g+h}{6(x+1)}+g\left(\frac{2h}{g}+1\right)+hx\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h)+\frac{1}{6}\log(x+1)(d-e+f-g+h)+x(g+2h)+\frac{hx^2}{2}$$

input
```
Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x]
```

output
```
(g + 2*h)*x + (h*x^2)/2 - ((d + e + f + g + h)*Log[1 - x])/2 + ((d + 2*e +
 4*f + 8*g + 16*h)*Log[2 - x])/3 + ((d - e + f - g + h)*Log[1 + x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

method	result
default	$\frac{h\,x^2}{2}+gx+2hx+\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8g}{3}+\frac{16h}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}\right)\ln\left(1+x\right)+$
norman	$\left(g+2h\right)x+\frac{h\,x^2}{2}+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}-\frac{g}{2}-\frac{h}{2}\right)\ln\left(x-1\right)+\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8g}{3}+\frac{16h}{3}\right)\ln\left(x-2\right.$
parallelrisch	$\frac{h\,x^2}{2}+gx+2hx+\frac{\ln(x-2)d}{3}+\frac{2\ln(x-2)e}{3}+\frac{4\ln(x-2)f}{3}+\frac{8\ln(x-2)g}{3}+\frac{16\ln(x-2)h}{3}-\frac{\ln(x-1)d}{2}-\frac{\ln(x-}{2}$
risch	$\frac{h\,x^2}{2}+gx+2hx+\frac{\ln(2-x)d}{3}+\frac{2\ln(2-x)e}{3}+\frac{4\ln(2-x)f}{3}+\frac{8\ln(2-x)g}{3}+\frac{16\ln(2-x)h}{3}-\frac{\ln(1-x)d}{2}-\frac{\ln(1-}{2}$

input
```
int((x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/2*h*x^2+g*x+2*h*x+(1/3*d+2/3*e+4/3*f+8/3*g+16/3*h)*ln(x-2)+(1/6*d-1/6*e+
1/6*f-1/6*g+1/6*h)*ln(1+x)+(-1/2*d-1/2*e-1/2*f-1/2*g-1/2*h)*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{4-5x^2+x^4}\,dx = \frac{1}{2}hx^2+(g+2h)x+\frac{1}{6}(d-e+f-g+h)\log(x+1)-\frac{1}{2}(d+e+f+g+h)\log(x-1)+\frac{1}{3}(d+2e+4f+8g+16h)\log(x-2)$$

input
```
integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fric
as")
```

output
```
1/2*h*x^2 + (g + 2*h)*x + 1/6*(d - e + f - g + h)*log(x + 1) - 1/2*(d + e
+ f + g + h)*log(x - 1) + 1/3*(d + 2*e + 4*f + 8*g + 16*h)*log(x - 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{4-5x^2+x^4}\,dx = \text{Timed out}$$

input
```
integrate((2+x)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{4-5x^2+x^4}\,dx = \frac{1}{2}\,hx^2 + (g+2\,h)x + \frac{1}{6}\,(d-e+f-g+h)\log(x+1) - \frac{1}{2}\,(d+e+f+g+h)\log(x-1) + \frac{1}{3}\,(d+2\,e+4\,f+8\,g+16\,h)\log(x-2)$$

input
```
integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxi
ma")
```

output
```
1/2*h*x^2 + (g + 2*h)*x + 1/6*(d - e + f - g + h)*log(x + 1) - 1/2*(d + e
+ f + g + h)*log(x - 1) + 1/3*(d + 2*e + 4*f + 8*g + 16*h)*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{4-5x^2+x^4}\,dx = \frac{1}{2}\,hx^2 + gx + 2\,hx + \frac{1}{6}\,(d-e+f-g+h)\log(|x+1|) - \frac{1}{2}\,(d+e+f+g+h)\log(|x-1|) + \frac{1}{3}\,(d+2\,e+4\,f+8\,g+16\,h)\log(|x-2|)$$

input
```
integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac
")
```

output
```
1/2*h*x^2 + g*x + 2*h*x + 1/6*(d - e + f - g + h)*log(abs(x + 1)) - 1/2*(d
 + e + f + g + h)*log(abs(x - 1)) + 1/3*(d + 2*e + 4*f + 8*g + 16*h)*log(a
bs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{4-5x^2+x^4}\,dx = x\,(g+2\,h)+\frac{h\,x^2}{2} - \ln(x-1)\left(\frac{d}{2}+\frac{e}{2}+\frac{f}{2}+\frac{g}{2}+\frac{h}{2}\right) + \ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}\right) + \ln(x-2)\left(\frac{d}{3}+\frac{2\,e}{3}+\frac{4\,f}{3}+\frac{8\,g}{3}+\frac{16\,h}{3}\right)$$

input
```
int(((x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^2 + 4),x)
```

output
```
x*(g + 2*h) + (h*x^2)/2 - log(x - 1)*(d/2 + e/2 + f/2 + g/2 + h/2) + log(x
 + 1)*(d/6 - e/6 + f/6 - g/6 + h/6) + log(x - 2)*(d/3 + (2*e)/3 + (4*f)/3
+ (8*g)/3 + (16*h)/3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{4-5x^2+x^4}\,dx = \frac{\log(x-2)\,d}{3}+\frac{2\log(x-2)\,e}{3}+\frac{4\log(x-2)\,f}{3}+\frac{8\log(x-2)\,g}{3}+\frac{16\log(x-2)\,h}{3}-\frac{\log(x-1)\,d}{2}-\frac{\log(x-1)\,e}{2}-\frac{\log(x-1)\,f}{2}-\frac{\log(x-1)\,g}{2}-\frac{\log(x-1)\,h}{2}+\frac{\log(x+1)\,d}{6}-\frac{\log(x+1)\,e}{6}+\frac{\log(x+1)\,f}{6}-\frac{\log(x+1)\,g}{6}+\frac{\log(x+1)\,h}{6}+gx+\frac{h\,x^2}{2}+2hx$$

input
```
int((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
(2*log(x - 2)*d + 4*log(x - 2)*e + 8*log(x - 2)*f + 16*log(x - 2)*g + 32*l
og(x - 2)*h - 3*log(x - 1)*d - 3*log(x - 1)*e - 3*log(x - 1)*f - 3*log(x -
 1)*g - 3*log(x - 1)*h + log(x + 1)*d - log(x + 1)*e + log(x + 1)*f - log(
x + 1)*g + log(x + 1)*h + 6*g*x + 3*h*x**2 + 12*h*x)/6
```

3.81 $\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4} \, dx$

Optimal result

Integrand size = 41, antiderivative size = 96

$$\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4} \, dx = (g+2h+5i)x+\frac{1}{2}(h+2i)x^2+\frac{ix^3}{3}-\frac{1}{2}(d+e+f+g+h+i)\log(1-x) + \frac{1}{3}(d+2e+4f+8g+16h+32i)\log(2-x)+\frac{1}{6}(d-e+f-g+h-i)\log(1+x)$$

output
```
(g+2*h+5*i)*x+1/2*(h+2*i)*x^2+1/3*i*x^3-1/2*(d+e+f+g+h+i)*ln(1-x)+1/3*(d+2
*e+4*f+8*g+16*h+32*i)*ln(2-x)+1/6*(d-e+f-g+h-i)*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4} \, dx = \frac{1}{6}\left(6(g+2h+5i)x+3(h+2i)x^2+2ix^3-3(d+e+f+g+h+i)\log(1-x) + 2(d+2e+4(f+2g+4h+8i))\log(2-x)+(d-e+f-g+h-i)\log(1+x)\right)$$

input
```
Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 +
 x^4),x]
```

output
```
(6*(g + 2*h + 5*i)*x + 3*(h + 2*i)*x^2 + 2*i*x^3 - 3*(d + e + f + g + h +
i)*Log[1 - x] + 2*(d + 2*e + 4*(f + 2*g + 4*h + 8*i))*Log[2 - x] + (d - e
+ f - g + h - i)*Log[1 + x])/6
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+2)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{x^4-5x^2+4}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{x^3-2x^2-x+2}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{d+2e+4f+8g+16h+32i}{3(x-2)}+\frac{-d-e-f-g-h-i}{2(x-1)}+\frac{d-e+f-g+h-i}{6(x+1)}+g\left(\frac{2h+5i}{g}+1\right)+x(h$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h+i)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h+32i)+\frac{1}{6}\log(x+1)(d-e+f-g+h-i)+x(g+2h+5i)+\frac{1}{2}x^2(h+2i)+\frac{ix^3}{3}$$

input
```
Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4),
x]
```

output
```
(g + 2*h + 5*i)*x + ((h + 2*i)*x^2)/2 + (i*x^3)/3 - ((d + e + f + g + h +
i)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*Log[2 - x])/3 + ((
d - e + f - g + h - i)*Log[1 + x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

method	result
norman	$\left(\frac{h}{2}+i\right)x^2+(g+2h+5i)x+\frac{i\,x^3}{3}+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}-\frac{g}{2}-\frac{h}{2}-\frac{i}{2}\right)\ln(x-1)+\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}\right.$
default	$\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8g}{3}+\frac{16h}{3}+\frac{32i}{3}\right)\ln(x-2)+\frac{i\,x^3}{3}+\frac{h\,x^2}{2}+i\,x^2+gx+2hx+5ix+\left(\frac{d}{6}-\frac{e}{6}+\right.$
parallelrisch	$gx+\frac{2\ln(x-2)e}{3}+i\,x^2+\frac{4\ln(x-2)f}{3}+\frac{\ln(1+x)d}{6}-\frac{\ln(x-1)d}{2}+\frac{\ln(x-2)d}{3}+\frac{h\,x^2}{2}-\frac{\ln(x-1)e}{2}+\frac{\ln(1+x)f}{6}$
risch	$\frac{i\,x^3}{3}+\frac{h\,x^2}{2}+i\,x^2+gx+2hx+5ix+\frac{\ln(2-x)d}{3}+\frac{2\ln(2-x)e}{3}+\frac{4\ln(2-x)f}{3}+\frac{8\ln(2-x)g}{3}+\frac{16\ln(2-x)}{3}$

input
```
int((x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVE
RBOSE)
```

output
```
(1/2*h+i)*x^2+(g+2*h+5*i)*x+1/3*i*x^3+(-1/2*d-1/2*e-1/2*f-1/2*g-1/2*h-1/2*
i)*ln(x-1)+(1/3*d+2/3*e+4/3*f+8/3*g+16/3*h+32/3*i)*ln(x-2)+(1/6*d-1/6*e+1/
6*f-1/6*g+1/6*h-1/6*i)*ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{3}\,ix^3+\frac{1}{2}\,(h+2\,i)x^2+(g+2\,h+5\,i)x$$
$$+\frac{1}{6}\,(d-e+f-g+h-i)\log\left(x+1\right)-\frac{1}{2}\,(d+e+f+g+h+i)\log\left(x-1\right)$$
$$+\frac{1}{3}\,(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log\left(x-2\right)$$

input
```
integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm
="fricas")
```

output
```
1/3*i*x^3 + 1/2*(h + 2*i)*x^2 + (g + 2*h + 5*i)*x + 1/6*(d - e + f - g + h
 - i)*log(x + 1) - 1/2*(d + e + f + g + h + i)*log(x - 1) + 1/3*(d + 2*e +
 4*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{4-5x^2+x^4}\,dx=\text{Timed out}$$

input
```
integrate((2+x)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{3}\,ix^3+\frac{1}{2}\,(h+2\,i)x^2+(g+2\,h+5\,i)x$$
$$+\frac{1}{6}\,(d-e+f-g+h-i)\log(x+1)-\frac{1}{2}\,(d+e+f+g+h+i)\log(x-1)$$
$$+\frac{1}{3}\,(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log(x-2)$$

input
```
integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm
="maxima")
```

output
```
1/3*i*x^3 + 1/2*(h + 2*i)*x^2 + (g + 2*h + 5*i)*x + 1/6*(d - e + f - g + h
 - i)*log(x + 1) - 1/2*(d + e + f + g + h + i)*log(x - 1) + 1/3*(d + 2*e +
 4*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{4-5x^2+x^4}\,dx$$
$$=\frac{1}{3}\,ix^3+\frac{1}{2}\,hx^2+ix^2+gx+2\,hx+5\,ix+\frac{1}{6}\,(d-e+f-g+h-i)\log(|x+1|)$$
$$-\frac{1}{2}\,(d+e+f+g+h+i)\log(|x-1|)$$
$$+\frac{1}{3}\,(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log(|x-2|)$$

input
```
integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm
="giac")
```

output
```
1/3*i*x^3 + 1/2*h*x^2 + i*x^2 + g*x + 2*h*x + 5*i*x + 1/6*(d - e + f - g +
 h - i)*log(abs(x + 1)) - 1/2*(d + e + f + g + h + i)*log(abs(x - 1)) + 1/
3*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{4-5x^2+x^4}\,dx = x\,(g+2\,h+5\,i)+\frac{i\,x^3}{3}-\ln(x-1)\,\left(\frac{d}{2}+\frac{e}{2}+\frac{f}{2}+\frac{g}{2}+\frac{h}{2}+\frac{i}{2}\right) + \ln(x+1)\,\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}-\frac{i}{6}\right) + \ln(x-2)\,\left(\frac{d}{3}+\frac{2\,e}{3}+\frac{4\,f}{3}+\frac{8\,g}{3}+\frac{16\,h}{3}+\frac{32\,i}{3}\right)+x^2\,\left(\frac{h}{2}+i\right)$$

input
```
int(((x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^4 - 5*x^2 + 4),
x)
```

output
```
x*(g + 2*h + 5*i) + (i*x^3)/3 - log(x - 1)*(d/2 + e/2 + f/2 + g/2 + h/2 +
i/2) + log(x + 1)*(d/6 - e/6 + f/6 - g/6 + h/6 - i/6) + log(x - 2)*(d/3 +
(2*e)/3 + (4*f)/3 + (8*g)/3 + (16*h)/3 + (32*i)/3) + x^2*(h/2 + i)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.61

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{4-5x^2+x^4}\,dx = \frac{\log(x-2)\,d}{3}+\frac{2\log(x-2)\,e}{3}+\frac{4\log(x-2)\,f}{3}+\frac{8\log(x-2)\,g}{3}+\frac{16\log(x-2)\,h}{3} + \frac{32\log(x-2)\,i}{3}-\frac{\log(x-1)\,d}{2}-\frac{\log(x-1)\,e}{2}-\frac{\log(x-1)\,f}{2}-\frac{\log(x-1)\,g}{2} - \frac{\log(x-1)\,h}{2}-\frac{\log(x-1)\,i}{2}+\frac{\log(x+1)\,d}{6}-\frac{\log(x+1)\,e}{6}+\frac{\log(x+1)\,f}{6} - \frac{\log(x+1)\,g}{6}+\frac{\log(x+1)\,h}{6}-\frac{\log(x+1)\,i}{6}+gx+\frac{h\,x^2}{2}+2hx+\frac{i\,x^3}{3}+i\,x^2+5ix$$

input
```
int((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

output
```
(2*log(x - 2)*d + 4*log(x - 2)*e + 8*log(x - 2)*f + 16*log(x - 2)*g + 32*l
og(x - 2)*h + 64*log(x - 2)*i - 3*log(x - 1)*d - 3*log(x - 1)*e - 3*log(x
- 1)*f - 3*log(x - 1)*g - 3*log(x - 1)*h - 3*log(x - 1)*i + log(x + 1)*d -
 log(x + 1)*e + log(x + 1)*f - log(x + 1)*g + log(x + 1)*h - log(x + 1)*i
+ 6*g*x + 3*h*x**2 + 12*h*x + 2*i*x**3 + 6*i*x**2 + 30*i*x)/6
```

3.82 $\int \frac{2-x-2x^2+x^3}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 46

$$\int \frac{2-x-2x^2+x^3}{(4-5x^2+x^4)^2}\,dx = \frac{1}{12(2+x)}-\frac{1}{18}\log(1-x)+\frac{1}{48}\log(2-x)+\frac{1}{6}\log(1+x)-\frac{19}{144}\log(2+x)$$

output
```
1/(24+12*x)-1/18*ln(1-x)+1/48*ln(2-x)+1/6*ln(1+x)-19/144*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{2-x-2x^2+x^3}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\left(\frac{12}{2+x}+24\log(-1-x)-8\log(1-x)+3\log(2-x)-19\log(2+x)\right)$$

input
```
Integrate[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4)^2,x]
```

output
```
(12/(2 + x) + 24*Log[-1 - x] - 8*Log[1 - x] + 3*Log[2 - x] - 19*Log[2 + x]
)/144
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-2x^2-x+2}{\left(x^4-5x^2+4\right)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(x+2)^2\left(x^3-2x^2-x+2\right)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(-\frac{1}{18(x-1)}+\frac{1}{6(x+1)}-\frac{19}{144(x+2)}-\frac{1}{12(x+2)^2}+\frac{1}{48(x-2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12(x+2)}-\frac{1}{18}\log(1-x)+\frac{1}{48}\log(2-x)+\frac{1}{6}\log(x+1)-\frac{19}{144}\log(x+2)$$

input
```
Int[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4)^2,x]
```

output
```
1/(12*(2 + x)) - Log[1 - x]/18 + Log[2 - x]/48 + Log[1 + x]/6 - (19*Log[2
+ x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\ln(x-2)}{48}+\frac{\ln(1+x)}{6}-\frac{\ln(x-1)}{18}+\frac{1}{24+12x}-\frac{19\ln(x+2)}{144}$	33
risch	$\frac{\ln(x-2)}{48}+\frac{\ln(1+x)}{6}-\frac{\ln(x-1)}{18}+\frac{1}{24+12x}-\frac{19\ln(x+2)}{144}$	33
norman	$\frac{-\frac{1}{6}x^2-\frac{1}{12}x+\frac{1}{12}x^3+\frac{1}{6}}{x^4-5x^2+4}+\frac{\ln(x-2)}{48}-\frac{\ln(x-1)}{18}+\frac{\ln(1+x)}{6}-\frac{19\ln(x+2)}{144}$	54
parallelrisch	$\frac{3\ln(x-2)x-8\ln(x-1)x+24\ln(1+x)x-19\ln(x+2)x+12+6\ln(x-2)-16\ln(x-1)+48\ln(1+x)-38\ln(x+2)}{144x+288}$	62

input
```
int((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/48*ln(x-2)+1/6*ln(1+x)-1/18*ln(x-1)+1/12/(x+2)-19/144*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{2-x-2x^2+x^3}{\left(4-5x^2+x^4\right)^2}\,dx = -\frac{19\,(x+2)\log\left(x+2\right)-24\,(x+2)\log\left(x+1\right)+8\,(x+2)\log\left(x-1\right)-3\,(x+2)\log\left(x-2\right)-12}{144\,(x+2)}$$

input
```
integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

output
```
-1/144*(19*(x + 2)*log(x + 2) - 24*(x + 2)*log(x + 1) + 8*(x + 2)*log(x -
1) - 3*(x + 2)*log(x - 2) - 12)/(x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{2-x-2x^2+x^3}{\left(4-5x^2+x^4\right)^2}\,dx = \frac{\log\left(x-2\right)}{48}-\frac{\log\left(x-1\right)}{18}+\frac{\log\left(x+1\right)}{6} - \frac{19\log\left(x+2\right)}{144}+\frac{1}{12x+24}$$

input
```
integrate((x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

output
```
log(x - 2)/48 - log(x - 1)/18 + log(x + 1)/6 - 19*log(x + 2)/144 + 1/(12*x
 + 24)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{2-x-2x^2+x^3}{(4-5x^2+x^4)^2}\,dx = \frac{1}{12\,(x+2)} - \frac{19}{144}\,\log{(x+2)} + \frac{1}{6}\,\log{(x+1)} - \frac{1}{18}\,\log{(x-1)} + \frac{1}{48}\,\log{(x-2)}$$

input
```
integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

output
```
1/12/(x + 2) - 19/144*log(x + 2) + 1/6*log(x + 1) - 1/18*log(x - 1) + 1/48
*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{2-x-2x^2+x^3}{(4-5x^2+x^4)^2}\,dx = \frac{1}{12\,(x+2)} - \frac{19}{144}\,\log{(|x+2|)} + \frac{1}{6}\,\log{(|x+1|)} - \frac{1}{18}\,\log{(|x-1|)} + \frac{1}{48}\,\log{(|x-2|)}$$

input
```
integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

output
```
1/12/(x + 2) - 19/144*log(abs(x + 2)) + 1/6*log(abs(x + 1)) - 1/18*log(abs
(x - 1)) + 1/48*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{2-x-2x^2+x^3}{(4-5x^2+x^4)^2}\,dx = \frac{\ln(x+1)}{6} - \frac{\ln(x-1)}{18} + \frac{\ln(x-2)}{48} - \frac{19\,\ln(x+2)}{144} + \frac{1}{12\,(x+2)}$$

input
```
int(-(x + 2*x^2 - x^3 - 2)/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
log(x + 1)/6 - log(x - 1)/18 + log(x - 2)/48 - (19*log(x + 2))/144 + 1/(12
*(x + 2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int \frac{2-x-2x^2+x^3}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{3\log(x-2)\,x+6\log(x-2)-8\log(x-1)\,x-16\log(x-1)-19\log(x+2)\,x-38\log(x+2)+24\log}{144x+288}$$

input
```
int((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x)
```

output
```
(3*log(x - 2)*x + 6*log(x - 2) - 8*log(x - 1)*x - 16*log(x - 1) - 19*log(x
 + 2)*x - 38*log(x + 2) + 24*log(x + 1)*x + 48*log(x + 1) - 6*x)/(144*(x +
 2))
```

3.83 $\int \frac{(d+ex)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 71

$$\int \frac{(d+ex)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx = \frac{d-2e}{12(2+x)}-\frac{1}{18}(d+e)\log(1-x) + \frac{1}{48}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(1+x) - \frac{1}{144}(19d-26e)\log(2+x)$$

output
```
(d-2*e)/(24+12*x)-1/18*(d+e)*ln(1-x)+1/48*(d+2*e)*ln(2-x)+1/6*(d-e)*ln(1+x
)-1/144*(19*d-26*e)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\left(\frac{12(d-2e)}{2+x}+24(d-e)\log(-1-x) - 8(d+e)\log(1-x)+3(d+2e)\log(2-x) + (-19d+26e)\log(2+x)\right)$$

input
```
Integrate[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
((12*(d - 2*e))/(2 + x) + 24*(d - e)*Log[-1 - x] - 8*(d + e)*Log[1 - x] +
3*(d + 2*e)*Log[2 - x] + (-19*d + 26*e)*Log[2 + x])/144
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3-2x^2-x+2)(d+ex)}{(x^4-5x^2+4)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex}{(x+2)^2(x^3-2x^2-x+2)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-d-e}{18(x-1)}+\frac{d+2e}{48(x-2)}+\frac{d-e}{6(x+1)}+\frac{26e-19d}{144(x+2)}+\frac{2e-d}{12(x+2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d-2e}{12(x+2)}-\frac{1}{18}(d+e)\log(1-x)+\frac{1}{48}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(x+1)-\frac{1}{144}(19d-26e)\log(x+2)$$

input
```
Int[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
(d - 2*e)/(12*(2 + x)) - ((d + e)*Log[1 - x])/18 + ((d + 2*e)*Log[2 - x])/
48 + ((d - e)*Log[1 + x])/6 - ((19*d - 26*e)*Log[2 + x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

method	result
default	$\left(\frac{d}{48}+\frac{e}{24}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{18}-\frac{e}{18}\right)\ln\left(x-1\right)+\left(-\frac{19d}{144}+\frac{13e}{72}\right)\ln\left(x+2\right.$
risch	$\frac{d}{24+12x}-\frac{e}{6(x+2)}-\frac{\ln(x-1)d}{18}-\frac{\ln(x-1)e}{18}+\frac{\ln(2-x)d}{48}+\frac{\ln(2-x)e}{24}-\frac{19\ln(-x-2)d}{144}+\frac{13\ln(-x-2)e}{72}+\frac{\ln(1+}{6}$
norman	$\frac{\left(-\frac{d}{12}+\frac{e}{6}\right)x+\left(\frac{d}{12}-\frac{e}{6}\right)x^3+\left(-\frac{d}{6}+\frac{e}{3}\right)x^2+\frac{d}{6}-\frac{e}{3}}{x^4-5x^2+4}+\left(-\frac{19d}{144}+\frac{13e}{72}\right)\ln\left(x+2\right)+\left(-\frac{d}{18}-\frac{e}{18}\right)\ln\left(x-1\right)+\left(\frac{d}{6}\right.$
parallelrisch	$\frac{3\ln(x-2)xd+6\ln(x-2)xe-8\ln(x-1)xd-8\ln(x-1)xe+24\ln(1+x)xd-24\ln(1+x)xe-19\ln(x+2)xd+26\ln(x+2)xe+6\ln(x-}{144x+288}$

input
```
int((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/48*d+1/24*e)*ln(x-2)+(1/6*d-1/6*e)*ln(1+x)+(-1/18*d-1/18*e)*ln(x-1)+(-1
9/144*d+13/72*e)*ln(x+2)-(-1/12*d+1/6*e)/(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)\left(2-x-2x^2+x^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx = -\frac{((19\,d-26\,e)x+38\,d-52\,e)\log\left(x+2\right)-24\,((d-e)x+2\,d-2\,e)\log\left(x+1\right)+8\,((d+e)x+2\,d+}{144\,(x+2)}$$

input
```
integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

output
```
-1/144*(((19*d - 26*e)*x + 38*d - 52*e)*log(x + 2) - 24*((d - e)*x + 2*d -
 2*e)*log(x + 1) + 8*((d + e)*x + 2*d + 2*e)*log(x - 1) - 3*((d + 2*e)*x +
 2*d + 4*e)*log(x - 2) - 12*d + 24*e)/(x + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. 2(60) = 120.

Time = 5.79 (sec) , antiderivative size = 1188, normalized size of antiderivative = 16.73

$$\int \frac{(d+ex)\left(2-x-2x^2+x^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

output
```
(d - 2*e)/(12*x + 24) + (d - e)*log(x + (-1534775*d**6 + 8032360*d**5*e -
984027*d**5*(d - e) - 12991180*d**4*e**2 + 11797266*d**4*e*(d - e) + 35671
68*d**4*(d - e)**2 + 1075200*d**3*e**3 - 32721528*d**3*e**2*(d - e) - 8725
248*d**3*e*(d - e)**2 - 247104*d**3*(d - e)**3 + 16959280*d**2*e**4 + 3897
7296*d**2*e**3*(d - e) - 2820096*d**2*e**2*(d - e)**2 - 10357632*d**2*e*(d
 - e)**3 - 15836800*d*e**5 - 21294960*d*e**4*(d - e) + 15436800*d*e**3*(d
- e)**2 + 16277760*d*e**2*(d - e)**3 + 4283840*e**6 + 3876000*e**5*(d - e)
 - 6865920*e**4*(d - e)**2 - 4078080*e**3*(d - e)**3)/(801262*d**6 - 46622
51*d**5*e + 7296938*d**4*e**2 + 1388616*d**3*e**3 - 12447440*d**2*e**4 + 9
990800*d*e**5 - 2380000*e**6))/6 - (d + e)*log(x + (-1534775*d**6 + 803236
0*d**5*e + 328009*d**5*(d + e) - 12991180*d**4*e**2 - 3932422*d**4*e*(d +
e) + 396352*d**4*(d + e)**2 + 1075200*d**3*e**3 + 10907176*d**3*e**2*(d +
e) - 969472*d**3*e*(d + e)**2 + 9152*d**3*(d + e)**3 + 16959280*d**2*e**4
- 12992432*d**2*e**3*(d + e) - 313344*d**2*e**2*(d + e)**2 + 383616*d**2*e
*(d + e)**3 - 15836800*d*e**5 + 7098320*d*e**4*(d + e) + 1715200*d*e**3*(d
 + e)**2 - 602880*d*e**2*(d + e)**3 + 4283840*e**6 - 1292000*e**5*(d + e)
- 762880*e**4*(d + e)**2 + 151040*e**3*(d + e)**3)/(801262*d**6 - 4662251*
d**5*e + 7296938*d**4*e**2 + 1388616*d**3*e**3 - 12447440*d**2*e**4 + 9990
800*d*e**5 - 2380000*e**6))/18 + (d + 2*e)*log(x + (-1534775*d**6 + 803236
0*d**5*e - 984027*d**5*(d + 2*e)/8 - 12991180*d**4*e**2 + 5898633*d**4*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)\left(2-x-2x^2+x^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx = -\frac{1}{144}\,(19\,d-26\,e)\log\left(x+2\right) + \frac{1}{6}\,(d-e)\log\left(x+1\right) - \frac{1}{18}\,(d+e)\log\left(x-1\right) + \frac{1}{48}\,(d+2\,e)\log\left(x-2\right) + \frac{d-2\,e}{12\,(x+2)}$$

input
```
integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

output
```
-1/144*(19*d - 26*e)*log(x + 2) + 1/6*(d - e)*log(x + 1) - 1/18*(d + e)*lo
g(x - 1) + 1/48*(d + 2*e)*log(x - 2) + 1/12*(d - 2*e)/(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx = -\frac{1}{144}(19d-26e)\log(|x+2|) + \frac{1}{6}(d-e)\log(|x+1|) - \frac{1}{18}(d+e)\log(|x-1|) + \frac{1}{48}(d+2e)\log(|x-2|) + \frac{d-2e}{12(x+2)}$$

input
```
integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

output
```
-1/144*(19*d - 26*e)*log(abs(x + 2)) + 1/6*(d - e)*log(abs(x + 1)) - 1/18*
(d + e)*log(abs(x - 1)) + 1/48*(d + 2*e)*log(abs(x - 2)) + 1/12*(d - 2*e)/
(x + 2)
```

Mupad [B] (verification not implemented)

Time = 17.85 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx = \frac{\frac{d}{12}-\frac{e}{6}}{x+2} + \ln(x+1)\left(\frac{d}{6}-\frac{e}{6}\right) - \ln(x-1)\left(\frac{d}{18}+\frac{e}{18}\right) + \ln(x-2)\left(\frac{d}{48}+\frac{e}{24}\right) - \ln(x+2)\left(\frac{19d}{144}-\frac{13e}{72}\right)$$

input
```
int(-((d + e*x)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
(d/12 - e/6)/(x + 2) + log(x + 1)*(d/6 - e/6) - log(x - 1)*(d/18 + e/18) +
 log(x - 2)*(d/48 + e/24) - log(x + 2)*((19*d)/144 - (13*e)/72)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.93

$$\int \frac{(d+ex)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{3\log(x-2)\,dx + 6\log(x-2)\,d + 6\log(x-2)\,ex + 12\log(x-2)\,e - 8\log(x-1)\,dx - 16\log(x-1)\,d -}{}$$

input
```
int((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x)
```

output
```
(3*log(x - 2)*d*x + 6*log(x - 2)*d + 6*log(x - 2)*e*x + 12*log(x - 2)*e -
8*log(x - 1)*d*x - 16*log(x - 1)*d - 8*log(x - 1)*e*x - 16*log(x - 1)*e -
19*log(x + 2)*d*x - 38*log(x + 2)*d + 26*log(x + 2)*e*x + 52*log(x + 2)*e
+ 24*log(x + 1)*d*x + 48*log(x + 1)*d - 24*log(x + 1)*e*x - 48*log(x + 1)*
e - 6*d*x + 12*e*x)/(144*(x + 2))
```

3.84 $\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 82

$$
\begin{aligned}
\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx = {} & \frac{d-2e+4f}{12(2+x)} - \frac{1}{18}(d+e+f)\log(1-x) \\
& + \frac{1}{48}(d+2e+4f)\log(2-x) \\
& + \frac{1}{6}(d-e+f)\log(1+x) \\
& - \frac{1}{144}(19d-26e+28f)\log(2+x)
\end{aligned}
$$

output
```
(d-2*e+4*f)/(24+12*x)-1/18*(d+e+f)*ln(1-x)+1/48*(d+2*e+4*f)*ln(2-x)+1/6*(d
-e+f)*ln(1+x)-1/144*(19*d-26*e+28*f)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\left(\frac{12(d-2e+4f)}{2+x} + 24(d-e+f)\log(-1-x) - 8(d+e+f)\log(1-x) + 3(d+2e+4f)\log(2-x) + (-19d+26e-28f)\log(2+x)\right)$$

input
```
Integrate[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
((12*(d - 2*e + 4*f))/(2 + x) + 24*(d - e + f)*Log[-1 - x] - 8*(d + e + f)
*Log[1 - x] + 3*(d + 2*e + 4*f)*Log[2 - x] + (-19*d + 26*e - 28*f)*Log[2 +
 x])/144
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3-2x^2-x+2)(d+ex+fx^2)}{(x^4-5x^2+4)^2}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2}{(x+2)^2(x^3-2x^2-x+2)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-19d+26e-28f}{144(x+2)} + \frac{d+2e+4f}{48(x-2)} + \frac{-d-e-f}{18(x-1)} + \frac{d-e+f}{6(x+1)} + \frac{-d+2e-4f}{12(x+2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d-2e+4f}{12(x+2)} - \frac{1}{18}\log(1-x)(d+e+f) + \frac{1}{48}\log(2-x)(d+2e+4f) + \frac{1}{6}\log(x+1)(d-e+f) - \frac{1}{144}\log(x+2)(19d-26e+28f)$$

input
```
Int[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
(d - 2*e + 4*f)/(12*(2 + x)) - ((d + e + f)*Log[1 - x])/18 + ((d + 2*e + 4
*f)*Log[2 - x])/48 + ((d - e + f)*Log[1 + x])/6 - ((19*d - 26*e + 28*f)*Lo
g[2 + x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result
default	$\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{18}-\frac{e}{18}-\frac{f}{18}\right)\ln\left(x-1\right)+\left(\frac{13e}{72}-\right.$
risch	$\frac{d}{24+12x}-\frac{e}{6(x+2)}+\frac{f}{3x+6}+\frac{13\ln(-x-2)e}{72}-\frac{7\ln(-x-2)f}{36}-\frac{19\ln(-x-2)d}{144}+\frac{\ln(2-x)d}{48}+\frac{\ln(2-x)e}{24}+\frac{\ln(2-}{12}$
norman	$\frac{\left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}\right)x+\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}\right)x^3+\left(\frac{e}{3}-\frac{2f}{3}-\frac{d}{6}\right)x^2-\frac{e}{3}+\frac{2f}{3}+\frac{d}{6}}{x^4-5x^2+4}+\left(-\frac{d}{18}-\frac{e}{18}-\frac{f}{18}\right)\ln\left(x-1\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)$
parallelrisch	$\frac{48f+12d-24e+12\ln(x-2)e+52\ln(x+2)e+24\ln(1+x)xd-24\ln(1+x)xe-19\ln(x+2)xd+26\ln(x+2)xe-28\ln(x+2)xf+3\ln}{}$

input
```
int((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/48*d+1/24*e+1/12*f)*ln(x-2)+(1/6*d-1/6*e+1/6*f)*ln(1+x)+(-1/18*d-1/18*e
-1/18*f)*ln(x-1)+(13/72*e-7/36*f-19/144*d)*ln(x+2)-(-1/12*d+1/6*e-1/3*f)/(
x+2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.41

$$\int\frac{(d+ex+fx^2)\left(2-x-2x^2+x^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx=$$
$$-\frac{((19\,d-26\,e+28\,f)x+38\,d-52\,e+56\,f)\log\left(x+2\right)-24\left((d-e+f)x+2\,d-2\,e+2\,f\right)\log\left(x+\right.}{}$$

input
```
integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="fric
as")
```

output
```
-1/144*((((19*d - 26*e + 28*f)*x + 38*d - 52*e + 56*f)*log(x + 2) - 24*((d
- e + f)*x + 2*d - 2*e + 2*f)*log(x + 1) + 8*((d + e + f)*x + 2*d + 2*e +
2*f)*log(x - 1) - 3*((d + 2*e + 4*f)*x + 2*d + 4*e + 8*f)*log(x - 2) - 12*
d + 24*e - 48*f)/(x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)\,(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{(d+ex+fx^2)\,(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx = & -\frac{1}{144}\,(19\,d-26\,e+28\,f)\log\left(x+2\right) \\ & +\frac{1}{6}\,(d-e+f)\log\left(x+1\right) \\ & -\frac{1}{18}\,(d+e+f)\log\left(x-1\right) \\ & +\frac{1}{48}\,(d+2\,e+4\,f)\log\left(x-2\right) \\ & +\frac{d-2\,e+4\,f}{12\,(x+2)}\end{aligned}$$

input
```
integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxi
ma")
```

output
```
-1/144*(19*d - 26*e + 28*f)*log(x + 2) + 1/6*(d - e + f)*log(x + 1) - 1/18
*(d + e + f)*log(x - 1) + 1/48*(d + 2*e + 4*f)*log(x - 2) + 1/12*(d - 2*e
+ 4*f)/(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx = -\frac{1}{144}(19\,d-26\,e+28\,f)\log(|x+2|) + \frac{1}{6}(d-e+f)\log(|x+1|) - \frac{1}{18}(d+e+f)\log(|x-1|) + \frac{1}{48}(d+2\,e+4\,f)\log(|x-2|) + \frac{d-2\,e+4\,f}{12\,(x+2)}$$

input
```
integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac
")
```

output
```
-1/144*(19*d - 26*e + 28*f)*log(abs(x + 2)) + 1/6*(d - e + f)*log(abs(x +
1)) - 1/18*(d + e + f)*log(abs(x - 1)) + 1/48*(d + 2*e + 4*f)*log(abs(x -
2)) + 1/12*(d - 2*e + 4*f)/(x + 2)
```

Mupad [B] (verification not implemented)

Time = 17.85 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx = \frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}}{x+2} + \ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right) - \ln(x-1)\left(\frac{d}{18}+\frac{e}{18}+\frac{f}{18}\right) + \ln(x-2)\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}\right) - \ln(x+2)\left(\frac{19\,d}{144}-\frac{13\,e}{72}+\frac{7\,f}{36}\right)$$

input
```
int(-((d + e*x + f*x^2)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
(d/12 - e/6 + f/3)/(x + 2) + log(x + 1)*(d/6 - e/6 + f/6) - log(x - 1)*(d/
18 + e/18 + f/18) + log(x - 2)*(d/48 + e/24 + f/12) - log(x + 2)*((19*d)/1
44 - (13*e)/72 + (7*f)/36)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.45

$$\int \frac{(d+ex+fx^2)\,(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{12ex+24\log(x+1)\,fx-28\log(x+2)\,fx+12\log(x-2)\,fx-8\log(x-1)\,fx+3\log(x-2)\,dx+6\,\mathrm{l}}{}$$

input
```
int((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x)
```

output
```
(3*log(x - 2)*d*x + 6*log(x - 2)*d + 6*log(x - 2)*e*x + 12*log(x - 2)*e +
12*log(x - 2)*f*x + 24*log(x - 2)*f - 8*log(x - 1)*d*x - 16*log(x - 1)*d -
 8*log(x - 1)*e*x - 16*log(x - 1)*e - 8*log(x - 1)*f*x - 16*log(x - 1)*f -
 19*log(x + 2)*d*x - 38*log(x + 2)*d + 26*log(x + 2)*e*x + 52*log(x + 2)*e
 - 28*log(x + 2)*f*x - 56*log(x + 2)*f + 24*log(x + 1)*d*x + 48*log(x + 1)
*d - 24*log(x + 1)*e*x - 48*log(x + 1)*e + 24*log(x + 1)*f*x + 48*log(x +
1)*f - 6*d*x + 12*e*x - 24*f*x)/(144*(x + 2))
```

3.85 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 95

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = \frac{d-2e+4f-8g}{12(2+x)} - \frac{1}{18}(d+e+f+g)\log(1-x) + \frac{1}{48}(d+2e+4f+8g)\log(2-x) + \frac{1}{6}(d-e+f-g)\log(1+x) - \frac{1}{144}(19d-26e+28f-8g)\log(2+x)$$

output
```
(d-2*e+4*f-8*g)/(24+12*x)-1/18*(d+e+f+g)*ln(1-x)+1/48*(d+2*e+4*f+8*g)*ln(2
-x)+1/6*(d-e+f-g)*ln(1+x)-1/144*(19*d-26*e+28*f-8*g)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\left(\frac{12(d-2e+4f-8g)}{2+x} + 24(d-e+f-g)\log(-1-x) - 8(d+e+f+g)\log(1-x) + 3(d+2e+4f+8g)\log(2-x) + (-19d+26e-28f+8g)\log(2+x)\right)$$

input
```
Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x
^4)^2,x]
```

output
```
((12*(d - 2*e + 4*f - 8*g))/(2 + x) + 24*(d - e + f - g)*Log[-1 - x] - 8*(
d + e + f + g)*Log[1 - x] + 3*(d + 2*e + 4*f + 8*g)*Log[2 - x] + (-19*d +
26*e - 28*f + 8*g)*Log[2 + x])/144
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3-2x^2-x+2)(d+ex+fx^2+gx^3)}{(x^4-5x^2+4)^2}dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{d+ex+fx^2+gx^3}{(x+2)^2(x^3-2x^2-x+2)}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{-d-e-f-g}{18(x-1)}+\frac{d+2e+4f+8g}{48(x-2)}+\frac{d-e+f-g}{6(x+1)}+\frac{-19d+26e-28f+8g}{144(x+2)}+\frac{-d+2e-4f+8g}{12(x+2)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{d-2e+4f-8g}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g)+\frac{1}{48}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)-\frac{1}{144}\log(x+2)(19d-26e+28f-8g)$$

input
```
Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,
x]
```

output
```
(d - 2*e + 4*f - 8*g)/(12*(2 + x)) - ((d + e + f + g)*Log[1 - x])/18 + ((d
 + 2*e + 4*f + 8*g)*Log[2 - x])/48 + ((d - e + f - g)*Log[1 + x])/6 - ((19
*d - 26*e + 28*f - 8*g)*Log[2 + x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

method	result
default	$\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}+\frac{g}{6}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{18}-\frac{e}{18}-\frac{f}{18}-\frac{g}{18}\right)\ln\left(x-\right.$
norman	$\frac{\left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}+\frac{2g}{3}\right)x+\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}\right)x^3+\left(\frac{4g}{3}-\frac{2f}{3}+\frac{e}{3}-\frac{d}{6}\right)x^2-\frac{4g}{3}+\frac{2f}{3}-\frac{e}{3}+\frac{d}{6}}{x^4-5x^2+4}+\left(-\frac{d}{18}-\frac{e}{18}-\frac{f}{18}-\frac{g}{18}\right)\ln\left(x-\right.$
risch	$\frac{d}{24+12x}-\frac{e}{6(x+2)}+\frac{f}{3x+6}-\frac{2g}{3(x+2)}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(1+x)g}{6}+\frac{\ln(2-x)d}{48}+\frac{\ln(2-}{24}$
parallelrisch	$48f-96g+12d-24e+12\ln(x-2)e+52\ln(x+2)e+24\ln(1+x)xd-24\ln(1+x)xe-19\ln(x+2)xd+26\ln(x+2)xe-28\ln(x+2)xf$

input
```
int((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVE
RBOSE)
```

output
```
(1/48*d+1/24*e+1/12*f+1/6*g)*ln(x-2)+(1/6*d-1/6*e+1/6*f-1/6*g)*ln(1+x)+(-1
/18*d-1/18*e-1/18*f-1/18*g)*ln(x-1)+(1/18*g-7/36*f+13/72*e-19/144*d)*ln(x+
2)-(-1/12*d+1/6*e-1/3*f+2/3*g)/(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.48

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = -\frac{((19\,d-26\,e+28\,f-8\,g)x+38\,d-52\,e+56\,f-16\,g)\log(x+2)-24\,((d-e+f-g)x+2\,d-2$$

input
```
integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm
="fricas")
```

output
```
-1/144*(((19*d - 26*e + 28*f - 8*g)*x + 38*d - 52*e + 56*f - 16*g)*log(x +
 2) - 24*((d - e + f - g)*x + 2*d - 2*e + 2*f - 2*g)*log(x + 1) + 8*((d +
e + f + g)*x + 2*d + 2*e + 2*f + 2*g)*log(x - 1) - 3*((d + 2*e + 4*f + 8*g
)*x + 2*d + 4*e + 8*f + 16*g)*log(x - 2) - 12*d + 24*e - 48*f + 96*g)/(x +
 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((x**3-2*x**2-x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$$
$$= -\frac{1}{144}(19\,d-26\,e+28\,f-8\,g)\log(x+2) + \frac{1}{6}(d-e+f-g)\log(x+1) - \frac{1}{18}(d+e+f+g)\log(x-1) + \frac{1}{48}(d+2\,e+4\,f+8\,g)\log(x-2) + \frac{d-2\,e+4\,f-8\,g}{12\,(x+2)}$$

input
```
integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm
="maxima")
```

output
```
-1/144*(19*d - 26*e + 28*f - 8*g)*log(x + 2) + 1/6*(d - e + f - g)*log(x +
 1) - 1/18*(d + e + f + g)*log(x - 1) + 1/48*(d + 2*e + 4*f + 8*g)*log(x -
 2) + 1/12*(d - 2*e + 4*f - 8*g)/(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$$
$$= -\frac{1}{144}(19\,d-26\,e+28\,f-8\,g)\log(|x+2|) + \frac{1}{6}(d-e+f-g)\log(|x+1|) - \frac{1}{18}(d+e+f+g)\log(|x-1|) + \frac{1}{48}(d+2\,e+4\,f+8\,g)\log(|x-2|) + \frac{d-2\,e+4\,f-8\,g}{12\,(x+2)}$$

input
```
integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm
="giac")
```

output
```
-1/144*(19*d - 26*e + 28*f - 8*g)*log(abs(x + 2)) + 1/6*(d - e + f - g)*lo
g(abs(x + 1)) - 1/18*(d + e + f + g)*log(abs(x - 1)) + 1/48*(d + 2*e + 4*f
 + 8*g)*log(abs(x - 2)) + 1/12*(d - 2*e + 4*f - 8*g)/(x + 2)
```

Mupad [B] (verification not implemented)

Time = 17.95 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$$
$$=\frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}}{x+2}+\ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}\right)-\ln(x-1)\left(\frac{d}{18}+\frac{e}{18}+\frac{f}{18}+\frac{g}{18}\right)$$
$$+\ln(x-2)\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}+\frac{g}{6}\right)-\ln(x+2)\left(\frac{19\,d}{144}-\frac{13\,e}{72}+\frac{7\,f}{36}-\frac{g}{18}\right)$$

input
```
int(-((d + e*x + f*x^2 + g*x^3)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4)^2
,x)
```

output
```
(d/12 - e/6 + f/3 - (2*g)/3)/(x + 2) + log(x + 1)*(d/6 - e/6 + f/6 - g/6)
- log(x - 1)*(d/18 + e/18 + f/18 + g/18) + log(x - 2)*(d/48 + e/24 + f/12
+ g/6) - log(x + 2)*((19*d)/144 - (13*e)/72 + (7*f)/36 - g/18)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.79

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$$
$$=\frac{12ex+24\log(x+1)\,fx-28\log(x+2)\,fx+12\log(x-2)\,fx-8\log(x-1)\,fx+48\log(x-2)\,g-16}{}$$

input
```
int((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
(3*log(x - 2)*d*x + 6*log(x - 2)*d + 6*log(x - 2)*e*x + 12*log(x - 2)*e +
12*log(x - 2)*f*x + 24*log(x - 2)*f + 24*log(x - 2)*g*x + 48*log(x - 2)*g
- 8*log(x - 1)*d*x - 16*log(x - 1)*d - 8*log(x - 1)*e*x - 16*log(x - 1)*e
- 8*log(x - 1)*f*x - 16*log(x - 1)*f - 8*log(x - 1)*g*x - 16*log(x - 1)*g
- 19*log(x + 2)*d*x - 38*log(x + 2)*d + 26*log(x + 2)*e*x + 52*log(x + 2)*
e - 28*log(x + 2)*f*x - 56*log(x + 2)*f + 8*log(x + 2)*g*x + 16*log(x + 2)
*g + 24*log(x + 1)*d*x + 48*log(x + 1)*d - 24*log(x + 1)*e*x - 48*log(x +
1)*e + 24*log(x + 1)*f*x + 48*log(x + 1)*f - 24*log(x + 1)*g*x - 48*log(x
+ 1)*g - 6*d*x + 12*e*x - 24*f*x + 48*g*x)/(144*(x + 2))
```

3.86 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 106

$$\begin{aligned}\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx &= \frac{d-2e+4f-8g+16h}{12(2+x)}-\frac{1}{18}(d+e+f+g+h)\log(1-x)\\ &\quad+\frac{1}{48}(d+2e+4f+8g+16h)\log(2-x)+\frac{1}{6}(d-e+f-g+h)\log(1+x)\\ &\quad-\frac{1}{144}(19d-26e+28f-8g-80h)\log(2+x)\end{aligned}$$

output

```
(d-2*e+4*f-8*g+16*h)/(24+12*x)-1/18*(d+e+f+g+h)*ln(1-x)+1/48*(d+2*e+4*f+8*
g+16*h)*ln(2-x)+1/6*(d-e+f-g+h)*ln(1+x)-1/144*(19*d-26*e+28*f-8*g-80*h)*ln
(2+x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$$
$$=\frac{1}{144}\left(\frac{12(d-2e+4f-8g+16h)}{2+x}+24(d-e+f-g+h)\log(-1-x)\right.$$
$$-8(d+e+f+g+h)\log(1-x)+3(d+2(e+2f+4g+8h))\log(2-x)$$
$$\left.+(-19d+26e-28f+8g+80h)\log(2+x)\right)$$

input
```
Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5
*x^2 + x^4)^2,x]
```

output
```
((12*(d - 2*e + 4*f - 8*g + 16*h))/(2 + x) + 24*(d - e + f - g + h)*Log[-1
 - x] - 8*(d + e + f + g + h)*Log[1 - x] + 3*(d + 2*(e + 2*f + 4*g + 8*h))
*Log[2 - x] + (-19*d + 26*e - 28*f + 8*g + 80*h)*Log[2 + x])/144
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3-2x^2-x+2)(d+ex+fx^2+gx^3+hx^4)}{(x^4-5x^2+4)^2}\,dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{(x+2)^2(x^3-2x^2-x+2)}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{-d+2e-4f+8g-16h}{12(x+2)^2} + \frac{d+2e+4f+8g+16h}{48(x-2)} + \frac{-d-e-f-g-h}{18(x-1)} + \frac{d-e+f-g+h}{6(x+1)} + \frac{-19d+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{d-2e+4f-8g+16h}{12(x+2)} - \frac{1}{18}\log(1-x)(d+e+f+g+h) + \frac{1}{48}\log(2-x)(d+2e+4f+8g+16h) + \frac{1}{6}\log(x+1)(d-e+f-g+h) - \frac{1}{144}\log(x+2)(19d-26e+28f-8g-80h)$$

input
```
Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
 x^4)^2,x]
```

output
```
(d - 2*e + 4*f - 8*g + 16*h)/(12*(2 + x)) - ((d + e + f + g + h)*Log[1 - x
])/18 + ((d + 2*e + 4*f + 8*g + 16*h)*Log[2 - x])/48 + ((d - e + f - g + h
)*Log[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g - 80*h)*Log[2 + x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03

method	result
default	$\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}+\frac{g}{6}+\frac{h}{3}\right)\ln\left(x-2\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{18}-\frac{e}{18}-\frac{f}{18}-\frac{g}{18}\right.$
norman	$\frac{\left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}+\frac{2g}{3}-\frac{4h}{3}\right)x+\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}+\frac{4h}{3}\right)x^3+\left(-\frac{8h}{3}+\frac{4g}{3}-\frac{2f}{3}+\frac{e}{3}-\frac{d}{6}\right)x^2+\frac{8h}{3}-\frac{4g}{3}+\frac{2f}{3}-\frac{e}{3}+\frac{d}{6}}{x^4-5x^2+4}+\left(-\frac{d}{18}-\frac{e}{18}-\right.$
risch	$\frac{\ln(2-x)e}{24}+\frac{\ln(2-x)g}{6}-\frac{7\ln(-x-2)f}{36}-\frac{19\ln(-x-2)d}{144}+\frac{13\ln(-x-2)e}{72}+\frac{\ln(2-x)d}{48}+\frac{\ln(2-x)f}{12}+\frac{4h}{3(x+2)}+$
parallelrisch	$48f-96g+192h+12d-24e+12\ln(x-2)e+52\ln(x+2)e+24\ln(1+x)xd-24\ln(1+x)xe-19\ln(x+2)xd+26\ln(x+2)xe-28\ln(x$

input
```
int((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RE
TURNVERBOSE)
```

output
```
(1/48*d+1/24*e+1/12*f+1/6*g+1/3*h)*ln(x-2)+(1/6*d-1/6*e+1/6*f-1/6*g+1/6*h)
*ln(1+x)+(-1/18*d-1/18*e-1/18*f-1/18*g-1/18*h)*ln(x-1)+(5/9*h+1/18*g-7/36*
f+13/72*e-19/144*d)*ln(x+2)-(-1/12*d+1/6*e-1/3*f+2/3*g-4/3*h)/(x+2)
```

Fricas [A] (verification not implemented)

Time = 3.19 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.55

$$\int\frac{(2-x-2x^2+x^3)\left(d+ex+fx^2+gx^3+hx^4\right)}{(4-5x^2+x^4)^2}\,dx = -\frac{((19\,d-26\,e+28\,f-8\,g-80\,h)x+38\,d-52\,e+56\,f-16\,g-160\,h)\log\left(x+2\right)-24\left((d-e+f\right.}{}$$

input
```
integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, alg
orithm="fricas")
```

output
```
-1/144*(((19*d - 26*e + 28*f - 8*g - 80*h)*x + 38*d - 52*e + 56*f - 16*g -
 160*h)*log(x + 2) - 24*((d - e + f - g + h)*x + 2*d - 2*e + 2*f - 2*g + 2
*h)*log(x + 1) + 8*((d + e + f + g + h)*x + 2*d + 2*e + 2*f + 2*g + 2*h)*l
og(x - 1) - 3*((d + 2*e + 4*f + 8*g + 16*h)*x + 2*d + 4*e + 8*f + 16*g + 3
2*h)*log(x - 2) - 12*d + 24*e - 48*f + 96*g - 192*h)/(x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-x-2x^2+x^3)\left(d+ex+fx^2+gx^3+hx^4\right)}{(4-5x^2+x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((x**3-2*x**2-x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**
2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\begin{aligned}
&\int \frac{(2-x-2x^2+x^3)\left(d+ex+fx^2+gx^3+hx^4\right)}{(4-5x^2+x^4)^2}\,dx \\
&= -\frac{1}{144}\,(19\,d-26\,e+28\,f-8\,g-80\,h)\log\left(x+2\right) \\
&\quad+\frac{1}{6}\,(d-e+f-g+h)\log\left(x+1\right)-\frac{1}{18}\,(d+e+f+g+h)\log\left(x-1\right) \\
&\quad+\frac{1}{48}\,(d+2\,e+4\,f+8\,g+16\,h)\log\left(x-2\right)+\frac{d-2\,e+4\,f-8\,g+16\,h}{12\,(x+2)}
\end{aligned}$$

input
```
integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, alg
orithm="maxima")
```

output
```
-1/144*(19*d - 26*e + 28*f - 8*g - 80*h)*log(x + 2) + 1/6*(d - e + f - g +
 h)*log(x + 1) - 1/18*(d + e + f + g + h)*log(x - 1) + 1/48*(d + 2*e + 4*f
 + 8*g + 16*h)*log(x - 2) + 1/12*(d - 2*e + 4*f - 8*g + 16*h)/(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$$
$$= -\frac{1}{144}(19\,d-26\,e+28\,f-8\,g-80\,h)\log(|x+2|) + \frac{1}{6}(d-e+f-g+h)\log(|x+1|) - \frac{1}{18}(d+e+f+g+h)\log(|x-1|) + \frac{1}{48}(d+2\,e+4\,f+8\,g+16\,h)\log(|x-2|) + \frac{d-2\,e+4\,f-8\,g+16\,h}{12\,(x+2)}$$

input
```
integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, alg
orithm="giac")
```

output
```
-1/144*(19*d - 26*e + 28*f - 8*g - 80*h)*log(abs(x + 2)) + 1/6*(d - e + f
- g + h)*log(abs(x + 1)) - 1/18*(d + e + f + g + h)*log(abs(x - 1)) + 1/48
*(d + 2*e + 4*f + 8*g + 16*h)*log(abs(x - 2)) + 1/12*(d - 2*e + 4*f - 8*g
+ 16*h)/(x + 2)
```

Mupad [B] (verification not implemented)

Time = 18.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2\,g}{3}+\frac{4\,h}{3}}{x+2} + \ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}\right) - \ln(x-1)\left(\frac{d}{18}+\frac{e}{18}+\frac{f}{18}+\frac{g}{18}+\frac{h}{18}\right) + \ln(x-2)\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}+\frac{g}{6}+\frac{h}{3}\right) + \ln(x+2)\left(\frac{13\,e}{72}-\frac{19\,d}{144}-\frac{7\,f}{36}+\frac{g}{18}+\frac{5\,h}{9}\right)$$

input
```
int(-((x + 2*x^2 - x^3 - 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^
2 + 4)^2,x)
```

output
```
(d/12 - e/6 + f/3 - (2*g)/3 + (4*h)/3)/(x + 2) + log(x + 1)*(d/6 - e/6 + f
/6 - g/6 + h/6) - log(x - 1)*(d/18 + e/18 + f/18 + g/18 + h/18) + log(x -
2)*(d/48 + e/24 + f/12 + g/6 + h/3) + log(x + 2)*((13*e)/72 - (19*d)/144 -
 (7*f)/36 + g/18 + (5*h)/9)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.10

$$\int \frac{(2-x-2x^2+x^3)\,(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{12ex+24\log(x+1)\,fx-28\log(x+2)\,fx+12\log(x-2)\,fx-8\log(x-1)\,fx+48\log(x-2)\,g+96}{}$$

input
```
int((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
(3*log(x - 2)*d*x + 6*log(x - 2)*d + 6*log(x - 2)*e*x + 12*log(x - 2)*e +
12*log(x - 2)*f*x + 24*log(x - 2)*f + 24*log(x - 2)*g*x + 48*log(x - 2)*g
+ 48*log(x - 2)*h*x + 96*log(x - 2)*h - 8*log(x - 1)*d*x - 16*log(x - 1)*d
 - 8*log(x - 1)*e*x - 16*log(x - 1)*e - 8*log(x - 1)*f*x - 16*log(x - 1)*f
 - 8*log(x - 1)*g*x - 16*log(x - 1)*g - 8*log(x - 1)*h*x - 16*log(x - 1)*h
 - 19*log(x + 2)*d*x - 38*log(x + 2)*d + 26*log(x + 2)*e*x + 52*log(x + 2)
*e - 28*log(x + 2)*f*x - 56*log(x + 2)*f + 8*log(x + 2)*g*x + 16*log(x + 2
)*g + 80*log(x + 2)*h*x + 160*log(x + 2)*h + 24*log(x + 1)*d*x + 48*log(x
+ 1)*d - 24*log(x + 1)*e*x - 48*log(x + 1)*e + 24*log(x + 1)*f*x + 48*log(
x + 1)*f - 24*log(x + 1)*g*x - 48*log(x + 1)*g + 24*log(x + 1)*h*x + 48*lo
g(x + 1)*h - 6*d*x + 12*e*x - 24*f*x + 48*g*x - 96*h*x)/(144*(x + 2))
```

3.87 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 122

$$\begin{aligned}
&\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx\\
&= ix+\frac{d-2e+4f-8g+16h-32i}{12(2+x)}-\frac{1}{18}(d+e+f+g+h+i)\log(1-x)\\
&\quad+\frac{1}{48}(d+2e+4f+8g+16h+32i)\log(2-x)+\frac{1}{6}(d-e+f-g+h-i)\log(1+x)\\
&\quad-\frac{1}{144}(19d-26e+28f-8g-80h+352i)\log(2+x)
\end{aligned}$$

output
```
i*x+(d-2*e+4*f-8*g+16*h-32*i)/(24+12*x)-1/18*(d+e+f+g+h+i)*ln(1-x)+1/48*(d
+2*e+4*f+8*g+16*h+32*i)*ln(2-x)+1/6*(d-e+f-g+h-i)*ln(1+x)-1/144*(19*d-26*e
+28*f-8*g-80*h+352*i)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.97

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$$
$$=\frac{1}{144}\left(144ix+\frac{12(d-2(e-2f+4g-8h+16i))}{2+x}-8(d+e+f+g+h+i)\log(1-x)\right.$$
$$+3(d+2e+4(f+2g+4h+8i))\log(2-x)+24(d-e+f-g+h-i)\log(1+x)$$
$$\left.+(-19d+26e-28f+8g+80h-352i)\log(2+x)\right)$$

input
```
Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)
)/(4 - 5*x^2 + x^4)^2,x]
```

output
```
(144*i*x + (12*(d - 2*(e - 2*f + 4*g - 8*h + 16*i)))/(2 + x) - 8*(d + e +
f + g + h + i)*Log[1 - x] + 3*(d + 2*e + 4*(f + 2*g + 4*h + 8*i))*Log[2 -
x] + 24*(d - e + f - g + h - i)*Log[1 + x] + (-19*d + 26*e - 28*f + 8*g +
80*h - 352*i)*Log[2 + x])/144
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3-2x^2-x+2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(x^4-5x^2+4)^2}dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(x+2)^2(x^3-2x^2-x+2)}dx$$
$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-19d+26e-28f+8g+80h-352i}{144(x+2)} + \frac{d+2e+4f+8g+16h+32i}{48(x-2)} + \frac{-d-e-f-g-h-i}{18(x-1)} + \frac{d-e+\ldots}{\ldots} \right.$$

$\downarrow$ 2009

$$\frac{d-2e+4f-8g+16h-32i}{12(x+2)} - \frac{1}{18}\log(1-x)(d+e+f+g+h+i) + \frac{1}{48}\log(2-x)(d+2e+4f+8g+16h+32i) + \frac{1}{6}\log(x+1)(d-e+f-g+h-i) - \frac{1}{144}\log(x+2)(19d-26e+28f-8g-80h+352i) + ix$$

input
```
Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
 5*x^2 + x^4)^2,x]
```

output
```
i*x + (d - 2*e + 4*f - 8*g + 16*h - 32*i)/(12*(2 + x)) - ((d + e + f + g +
 h + i)*Log[1 - x])/18 + ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*Log[2 - x])/
48 + ((d - e + f - g + h - i)*Log[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g -
 80*h + 352*i)*Log[2 + x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04

method	result
default	$\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}+\frac{g}{6}+\frac{h}{3}+\frac{2i}{3}\right)\ln\left(x-2\right)+ix+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}-\frac{i}{6}\right)\ln\left(1+x\right)+\left(-\frac{d}{18}\right.$
norman	$\frac{i\,x^5+\left(-\frac{4h}{3}+\frac{20i}{3}+\frac{2g}{3}-\frac{f}{3}+\frac{e}{6}-\frac{d}{12}\right)x+\left(\frac{4h}{3}-\frac{23i}{3}-\frac{2g}{3}+\frac{f}{3}-\frac{e}{6}+\frac{d}{12}\right)x^3+\left(-\frac{d}{6}+\frac{e}{3}-\frac{2f}{3}+\frac{4g}{3}-\frac{8h}{3}+\frac{16i}{3}\right)x^2+\frac{8h}{3}-\frac{16i}{3}+\frac{d}{6}-\frac{e}{3}+\frac{2f}{3}}{x^4-5x^2+4}$
risch	$\frac{\ln(2-x)e}{24}+\frac{\ln(2-x)g}{6}-\frac{7\ln(-x-2)f}{36}-\frac{22\ln(-x-2)i}{9}-\frac{19\ln(-x-2)d}{144}+\frac{13\ln(-x-2)e}{72}+\frac{\ln(2-x)d}{48}+\frac{\ln(2-x)}{12}$
parallelrisch	$\frac{-960i+48f-96g+192h+12d-24e+12\ln(x-2)e+144i\,x^2+52\ln(x+2)e+24\ln(1+x)xd-24\ln(1+x)xe-19\ln(x+2)xd+26\ln}{}$

input
```
int((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,meth
od=_RETURNVERBOSE)
```

output
```
(1/48*d+1/24*e+1/12*f+1/6*g+1/3*h+2/3*i)*ln(x-2)+i*x+(1/6*d-1/6*e+1/6*f-1/
6*g+1/6*h-1/6*i)*ln(1+x)+(-1/18*d-1/18*e-1/18*f-1/18*g-1/18*h-1/18*i)*ln(x
-1)+(5/9*h-22/9*i+1/18*g-7/36*f+13/72*e-19/144*d)*ln(x+2)-(-1/12*d+1/6*e-1
/3*f+2/3*g-4/3*h+8/3*i)/(x+2)
```

Fricas [A] (verification not implemented)

Time = 21.72 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.64

$$\int \frac{(2-x-2x^2+x^3)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx$$
$$=\frac{144\,ix^2+288\,ix-((19\,d-26\,e+28\,f-8\,g-80\,h+352\,i)x+38\,d-52\,e+56\,f-16\,g-160\,h+704}{}$$

input
```
integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,
x, algorithm="fricas")
```

output
```
1/144*(144*i*x^2 + 288*i*x - ((19*d - 26*e + 28*f - 8*g - 80*h + 352*i)*x
+ 38*d - 52*e + 56*f - 16*g - 160*h + 704*i)*log(x + 2) + 24*((d - e + f -
 g + h - i)*x + 2*d - 2*e + 2*f - 2*g + 2*h - 2*i)*log(x + 1) - 8*((d + e
+ f + g + h + i)*x + 2*d + 2*e + 2*f + 2*g + 2*h + 2*i)*log(x - 1) + 3*((d
 + 2*e + 4*f + 8*g + 16*h + 32*i)*x + 2*d + 4*e + 8*f + 16*g + 32*h + 64*i
)*log(x - 2) + 12*d - 24*e + 48*f - 96*g + 192*h - 384*i)/(x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-x-2x^2+x^3)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((x**3-2*x**2-x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x*
*2+4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\begin{aligned}&\int \frac{(2-x-2x^2+x^3)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx\\&=ix-\frac{1}{144}\,(19\,d-26\,e+28\,f-8\,g-80\,h+352\,i)\log\left(x+2\right)\\&\quad+\frac{1}{6}\,(d-e+f-g+h-i)\log\left(x+1\right)-\frac{1}{18}\,(d+e+f+g+h+i)\log\left(x-1\right)\\&\quad+\frac{1}{48}\,(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log\left(x-2\right)+\frac{d-2\,e+4\,f-8\,g+16\,h-32\,i}{12\,(x+2)}\end{aligned}$$

input
```
integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,
x, algorithm="maxima")
```

output
```
i*x - 1/144*(19*d - 26*e + 28*f - 8*g - 80*h + 352*i)*log(x + 2) + 1/6*(d
- e + f - g + h - i)*log(x + 1) - 1/18*(d + e + f + g + h + i)*log(x - 1)
+ 1/48*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(x - 2) + 1/12*(d - 2*e + 4*
f - 8*g + 16*h - 32*i)/(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$$
$$= ix - \frac{1}{144}(19\,d-26\,e+28\,f-8\,g-80\,h+352\,i)\log(|x+2|)$$
$$+ \frac{1}{6}(d-e+f-g+h-i)\log(|x+1|) - \frac{1}{18}(d+e+f+g+h+i)\log(|x-1|)$$
$$+ \frac{1}{48}(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log(|x-2|)$$
$$+ \frac{d-2\,e+4\,f-8\,g+16\,h-32\,i}{12\,(x+2)}$$

input
```
integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,
x, algorithm="giac")
```

output
```
i*x - 1/144*(19*d - 26*e + 28*f - 8*g - 80*h + 352*i)*log(abs(x + 2)) + 1/
6*(d - e + f - g + h - i)*log(abs(x + 1)) - 1/18*(d + e + f + g + h + i)*l
og(abs(x - 1)) + 1/48*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(abs(x - 2))
+ 1/12*(d - 2*e + 4*f - 8*g + 16*h - 32*i)/(x + 2)
```

Mupad [B] (verification not implemented)

Time = 18.94 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$$
$$= i\,x + \frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2\,g}{3}+\frac{4\,h}{3}-\frac{8\,i}{3}}{x+2} + \ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}-\frac{i}{6}\right) + \ln(x-2)\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}+\frac{g}{6}+\frac{h}{3}+\frac{2\,i}{3}\right) - \ln(x-1)\left(\frac{d}{18}+\frac{e}{18}+\frac{f}{18}+\frac{g}{18}+\frac{h}{18}+\frac{i}{18}\right) - \ln(x+2)\left(\frac{19\,d}{144}-\frac{13\,e}{72}+\frac{7\,f}{36}-\frac{g}{18}-\frac{5\,h}{9}+\frac{22\,i}{9}\right)$$

input
```
int(-((x + 2*x^2 - x^3 - 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^
4 - 5*x^2 + 4)^2,x)
```

output
```
i*x + (d/12 - e/6 + f/3 - (2*g)/3 + (4*h)/3 - (8*i)/3)/(x + 2) + log(x + 1
)*(d/6 - e/6 + f/6 - g/6 + h/6 - i/6) + log(x - 2)*(d/48 + e/24 + f/12 + g
/6 + h/3 + (2*i)/3) - log(x - 1)*(d/18 + e/18 + f/18 + g/18 + h/18 + i/18)
 - log(x + 2)*((19*d)/144 - (13*e)/72 + (7*f)/36 - g/18 - (5*h)/9 + (22*i)
/9)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.27

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{12ex + 24\log(x+1)\,fx - 28\log(x+2)\,fx + 12\log(x-2)\,fx - 8\log(x-1)\,fx + 48\log(x-2)\,g + 96}{}$$

input
```
int((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
(3*log(x - 2)*d*x + 6*log(x - 2)*d + 6*log(x - 2)*e*x + 12*log(x - 2)*e +
12*log(x - 2)*f*x + 24*log(x - 2)*f + 24*log(x - 2)*g*x + 48*log(x - 2)*g
+ 48*log(x - 2)*h*x + 96*log(x - 2)*h + 96*log(x - 2)*i*x + 192*log(x - 2)
*i - 8*log(x - 1)*d*x - 16*log(x - 1)*d - 8*log(x - 1)*e*x - 16*log(x - 1)
*e - 8*log(x - 1)*f*x - 16*log(x - 1)*f - 8*log(x - 1)*g*x - 16*log(x - 1)
*g - 8*log(x - 1)*h*x - 16*log(x - 1)*h - 8*log(x - 1)*i*x - 16*log(x - 1)
*i - 19*log(x + 2)*d*x - 38*log(x + 2)*d + 26*log(x + 2)*e*x + 52*log(x +
2)*e - 28*log(x + 2)*f*x - 56*log(x + 2)*f + 8*log(x + 2)*g*x + 16*log(x +
 2)*g + 80*log(x + 2)*h*x + 160*log(x + 2)*h - 352*log(x + 2)*i*x - 704*lo
g(x + 2)*i + 24*log(x + 1)*d*x + 48*log(x + 1)*d - 24*log(x + 1)*e*x - 48*
log(x + 1)*e + 24*log(x + 1)*f*x + 48*log(x + 1)*f - 24*log(x + 1)*g*x - 4
8*log(x + 1)*g + 24*log(x + 1)*h*x + 48*log(x + 1)*h - 24*log(x + 1)*i*x -
 48*log(x + 1)*i - 6*d*x + 12*e*x - 24*f*x + 48*g*x - 96*h*x + 144*i*x**2
+ 480*i*x)/(144*(x + 2))
```

3.88 $\int \frac{2-3x+x^2}{\left(4-5x^2+x^4\right)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{2-3x+x^2}{\left(4-5x^2+x^4\right)^2} \, dx = -\frac{1}{6(1+x)}-\frac{1}{12(2+x)}-\frac{1}{36}\log(1-x) +\frac{1}{144}\log(2-x)-\frac{7}{36}\log(1+x)+\frac{31}{144}\log(2+x)$$

output
```
-1/6/(1+x)-1/(24+12*x)-1/36*ln(1-x)+1/144*ln(2-x)-7/36*ln(1+x)+31/144*ln(2
+x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{2-3x+x^2}{\left(4-5x^2+x^4\right)^2} \, dx = \frac{1}{144}\left(-\frac{12(5+3x)}{2+3x+x^2}-4\log(1-x)+\log(2-x)-28\log(1+x) +31\log(2+x)\right)$$

input
```
Integrate[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4)^2,x]
```

output
```
((-12*(5 + 3*x))/(2 + 3*x + x^2) - 4*Log[1 - x] + Log[2 - x] - 28*Log[1 +
x] + 31*Log[2 + x])/144
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2019, 1299, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-3x+2}{(x^4-5x^2+4)^2}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(x^2-3x+2)(x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{1299}$$

$$\int \left(-\frac{1}{144(2-x)}-\frac{7}{36(x+1)}+\frac{31}{144(x+2)}+\frac{1}{6(x+1)^2}+\frac{1}{12(x+2)^2}+\frac{1}{36(1-x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6(x+1)}-\frac{1}{12(x+2)}-\frac{1}{36}\log(1-x)+\frac{1}{144}\log(2-x)-\frac{7}{36}\log(x+1)+\frac{31}{144}\log(x+2)$$

input
```
Int[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4)^2,x]
```

output
```
-1/6*1/(1 + x) - 1/(12*(2 + x)) - Log[1 - x]/36 + Log[2 - x]/144 - (7*Log[
1 + x])/36 + (31*Log[2 + x])/144
```

Defintions of rubi rules used

rule 1299
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[Ex
pandIntegrand[(b/2 - r/2 + c*x)^p*(b/2 + r/2 + c*x)^p*(d + e*x + f*x^2)^q,
x], x], x] /; EqQ[p, -1] ||  !FractionalPowerFactorQ[r]] /; FreeQ[{a, b, c,
 d, e, f}, x] && ILtQ[p, 0] && IntegerQ[q] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result
default	$\frac{\ln(x-2)}{144}-\frac{1}{6(1+x)}-\frac{7\ln(1+x)}{36}-\frac{\ln(x-1)}{36}-\frac{1}{12(x+2)}+\frac{31\ln(x+2)}{144}$
risch	$\frac{-\frac{x}{4}-\frac{5}{12}}{x^2+3x+2}+\frac{\ln(x-2)}{144}-\frac{\ln(x-1)}{36}-\frac{7\ln(1+x)}{36}+\frac{31\ln(x+2)}{144}$
norman	$\frac{\frac{1}{3}x^2+\frac{3}{4}x-\frac{1}{4}x^3-\frac{5}{6}}{x^4-5x^2+4}+\frac{\ln(x-2)}{144}-\frac{\ln(x-1)}{36}-\frac{7\ln(1+x)}{36}+\frac{31\ln(x+2)}{144}$
parallelrisch	$\frac{31\ln(x+2)x^2+\ln(x-2)x^2-4\ln(x-1)x^2-28\ln(1+x)x^2-60+93\ln(x+2)x+3\ln(x-2)x-12\ln(x-1)x-84\ln(1+x)x+62\ln(x-}{144x^2+432x+288}$

input
```
int((x^2-3*x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/144*ln(x-2)-1/6/(1+x)-7/36*ln(1+x)-1/36*ln(x-1)-1/12/(x+2)+31/144*ln(x+2
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int \frac{2-3x+x^2}{\left(4-5x^2+x^4\right)^2}\,dx$$
$$= \frac{31\,(x^2+3\,x+2)\log{(x+2)}-28\,(x^2+3\,x+2)\log{(x+1)}-4\,(x^2+3\,x+2)\log{(x-1)}+(x^2+3\,x+}{144\,(x^2+3\,x+2)}$$

input
```
integrate((x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

output
```
1/144*(31*(x^2 + 3*x + 2)*log(x + 2) - 28*(x^2 + 3*x + 2)*log(x + 1) - 4*(
x^2 + 3*x + 2)*log(x - 1) + (x^2 + 3*x + 2)*log(x - 2) - 36*x - 60)/(x^2 +
 3*x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{2-3x+x^2}{\left(4-5x^2+x^4\right)^2}\,dx = \frac{-3x-5}{12x^2+36x+24}+\frac{\log{(x-2)}}{144}-\frac{\log{(x-1)}}{36}-\frac{7\log{(x+1)}}{36}+\frac{31\log{(x+2)}}{144}$$

input
```
integrate((x**2-3*x+2)/(x**4-5*x**2+4)**2,x)
```

output
```
(-3*x - 5)/(12*x**2 + 36*x + 24) + log(x - 2)/144 - log(x - 1)/36 - 7*log(
x + 1)/36 + 31*log(x + 2)/144
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{2-3x+x^2}{(4-5x^2+x^4)^2}\,dx = -\frac{3\,x+5}{12\,(x^2+3\,x+2)} + \frac{31}{144}\,\log{(x+2)} - \frac{7}{36}\,\log{(x+1)} - \frac{1}{36}\,\log{(x-1)} + \frac{1}{144}\,\log{(x-2)}$$

input
```
integrate((x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

output
```
-1/12*(3*x + 5)/(x^2 + 3*x + 2) + 31/144*log(x + 2) - 7/36*log(x + 1) - 1/
36*log(x - 1) + 1/144*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{2-3x+x^2}{(4-5x^2+x^4)^2}\,dx = -\frac{3\,x+5}{12\,(x+2)(x+1)} + \frac{31}{144}\,\log{(|x+2|)} - \frac{7}{36}\,\log{(|x+1|)} - \frac{1}{36}\,\log{(|x-1|)} + \frac{1}{144}\,\log{(|x-2|)}$$

input
```
integrate((x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

output
```
-1/12*(3*x + 5)/((x + 2)*(x + 1)) + 31/144*log(abs(x + 2)) - 7/36*log(abs(
x + 1)) - 1/36*log(abs(x - 1)) + 1/144*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{2-3x+x^2}{(4-5x^2+x^4)^2}\,dx = \frac{\ln(x-2)}{144} - \frac{7\,\ln(x+1)}{36} - \frac{\ln(x-1)}{36} + \frac{31\,\ln(x+2)}{144} - \frac{\frac{x}{4}+\frac{5}{12}}{x^2+3\,x+2}$$

input
```
int((x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
log(x - 2)/144 - (7*log(x + 1))/36 - log(x - 1)/36 + (31*log(x + 2))/144 -
 (x/4 + 5/12)/(3*x + x^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.95

$$\int \frac{2-3x+x^2}{(4-5x^2+x^4)^2}\,dx = \frac{\log(x-2)\,x^2+3\log(x-2)\,x+2\log(x-2)-4\log(x-1)\,x^2-12\log(x-1)\,x-8\log(x-1)+31\log}{144x^2+}$$

input
```
int((x^2-3*x+2)/(x^4-5*x^2+4)^2,x)
```

output
```
(log(x - 2)*x**2 + 3*log(x - 2)*x + 2*log(x - 2) - 4*log(x - 1)*x**2 - 12*
log(x - 1)*x - 8*log(x - 1) + 31*log(x + 2)*x**2 + 93*log(x + 2)*x + 62*lo
g(x + 2) - 28*log(x + 1)*x**2 - 84*log(x + 1)*x - 56*log(x + 1) + 12*x**2
- 36)/(144*(x**2 + 3*x + 2))
```

3.89 $\int \frac{(d+ex)(2-3x+x^2)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{(d+ex)(2-3x+x^2)}{(4-5x^2+x^4)^2}\,dx = -\frac{5d-6e+(3d-4e)x}{12(2+3x+x^2)} - \frac{1}{36}(d+e)\log(1-x) + \frac{1}{144}(d+2e)\log(2-x) - \frac{1}{36}(7d-13e)\log(1+x) + \frac{1}{144}(31d-50e)\log(2+x)$$

output
```
-1/12*(5*d-6*e+(3*d-4*e)*x)/(x^2+3*x+2)-1/36*(d+e)*ln(1-x)+1/144*(d+2*e)*l
n(2-x)-1/36*(7*d-13*e)*ln(1+x)+1/144*(31*d-50*e)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)(2-3x+x^2)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\left(\frac{12(-5d+6e-3dx+4ex)}{2+3x+x^2} - 4(d+e)\log(1-x) + (d+2e)\log(2-x) + 4(-7d+13e)\log(1+x) + (31d-50e)\log(2+x)\right)$$

input
```
Integrate[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
((12*(-5*d + 6*e - 3*d*x + 4*e*x))/(2 + 3*x + x^2) - 4*(d + e)*Log[1 - x]
+ (d + 2*e)*Log[2 - x] + 4*(-7*d + 13*e)*Log[1 + x] + (31*d - 50*e)*Log[2
+ x])/144
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2019, 1349, 27, 2141, 27, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-3x+2)(d+ex)}{(x^4-5x^2+4)^2}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{d+ex}{(x^2-3x+2)(x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{ 1349}$$

$$-\frac{1}{72}\int \frac{6((3d-4e)x^2-4(2d-3e)x+3d-10e)}{(x^2-3x+2)(x^2+3x+2)}dx-\frac{x(3d-4e)+5d-6e}{12(x^2+3x+2)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{12}\int \frac{(3d-4e)x^2-4(2d-3e)x+3d-10e}{(x^2-3x+2)(x^2+3x+2)}dx-\frac{x(3d-4e)+5d-6e}{12(x^2+3x+2)}$$

$$\Big\downarrow \text{ 2141}$$

$$\frac{1}{12}\left(-\frac{1}{72}\int -\frac{6(7d+6e-(3d+2e)x)}{x^2-3x+2}dx-\frac{1}{72}\int \frac{6(25d-54e-(3d+2e)x)}{x^2+3x+2}dx\right)-\frac{x(3d-4e)+5d-6e}{12(x^2+3x+2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{12}\left(\frac{1}{12}\int \frac{7d+6e-(3d+2e)x}{x^2-3x+2}dx-\frac{1}{12}\int \frac{25d-54e-(3d+2e)x}{x^2+3x+2}dx\right)-\frac{x(3d-4e)+5d-6e}{12\left(x^2+3x+2\right)}$$

$$\Big\downarrow \; 1141$$

$$\frac{1}{12}\left(\frac{1}{12}\int \left(\frac{4(d+e)}{1-x}-\frac{d+2e}{2-x}\right)dx-\frac{1}{12}\int \left(\frac{4(7d-13e)}{x+1}-\frac{31d-50e}{x+2}\right)dx\right)-\frac{x(3d-4e)+5d-6e}{12\left(x^2+3x+2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{12}\left(\frac{1}{12}((d+2e)\log(2-x)-4(d+e)\log(1-x))+\frac{1}{12}((31d-50e)\log(x+2)-4(7d-13e)\log(x+1))\right)-\frac{x(3d-4e)+5d-6e}{12\left(x^2+3x+2\right)}$$

input
```
Int[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
-1/12*(5*d - 6*e + (3*d - 4*e)*x)/(2 + 3*x + x^2) + ((-4*(d + e)*Log[1 - x
] + (d + 2*e)*Log[2 - x])/12 + (-4*(7*d - 13*e)*Log[1 + x] + (31*d - 50*e)
*Log[2 + x])/12)/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1349
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f
*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1
) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c
*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g
*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2
*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a
*((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*
c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*
e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

method	result
default	$\left(\frac{d}{144}+\frac{e}{72}\right)\ln\left(x-2\right)+\left(-\frac{7d}{36}+\frac{13e}{36}\right)\ln\left(1+x\right)-\frac{\frac{d}{6}-\frac{e}{6}}{1+x}+\left(-\frac{d}{36}-\frac{e}{36}\right)\ln\left(x-1\right)-\frac{\frac{d}{12}-\frac{e}{6}}{x+2}+($
risch	$\frac{\left(-\frac{d}{4}+\frac{e}{3}\right)x-\frac{5d}{12}+\frac{e}{2}}{x^2+3x+2}+\frac{\ln(2-x)d}{144}+\frac{\ln(2-x)e}{72}-\frac{7\ln(-x-1)d}{36}+\frac{13\ln(-x-1)e}{36}-\frac{\ln(x-1)d}{36}-\frac{\ln(x-1)e}{36}+\frac{31\ln(x-}{144}$
norman	$\frac{\left(-\frac{d}{4}+\frac{e}{3}\right)x^3+\left(\frac{3d}{4}-\frac{5e}{6}\right)x+\left(\frac{d}{3}-\frac{e}{2}\right)x^2-\frac{5d}{6}+e}{x^4-5x^2+4}+\left(-\frac{7d}{36}+\frac{13e}{36}\right)\ln\left(1+x\right)+\left(-\frac{d}{36}-\frac{e}{36}\right)\ln\left(x-1\right)+\left(\frac{d}{144}\right.$
parallelrisch	$-36dx-60d+72e+48ex+4\ln(x-2)e-100\ln(x+2)e-4\ln(x-1)x^2d-84\ln(1+x)xd+156\ln(1+x)xe+93\ln(x+2)xd-150\ln(x$

input
```
int((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/144*d+1/72*e)*ln(x-2)+(-7/36*d+13/36*e)*ln(1+x)-(1/6*d-1/6*e)/(1+x)+(-1
/36*d-1/36*e)*ln(x-1)-(1/12*d-1/6*e)/(x+2)+(31/144*d-25/72*e)*ln(x+2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)\left(2-3x+x^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx =$$
$$-\frac{12\,(3\,d-4\,e)x-((31\,d-50\,e)x^2+3\,(31\,d-50\,e)x+62\,d-100\,e)\log\left(x+2\right)+4\left((7\,d-13\,e)x^2+3\right.}{}$$

input
```
integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

output
```
-1/144*(12*(3*d - 4*e)*x - ((31*d - 50*e)*x^2 + 3*(31*d - 50*e)*x + 62*d -
 100*e)*log(x + 2) + 4*((7*d - 13*e)*x^2 + 3*(7*d - 13*e)*x + 14*d - 26*e)
*log(x + 1) + 4*((d + e)*x^2 + 3*(d + e)*x + 2*d + 2*e)*log(x - 1) - ((d +
 2*e)*x^2 + 3*(d + 2*e)*x + 2*d + 4*e)*log(x - 2) + 60*d - 72*e)/(x^2 + 3*
x + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1255 vs. $2(80) = 160$.

Time = 5.84 (sec) , antiderivative size = 1255, normalized size of antiderivative = 14.10

$$\int \frac{(d+ex)\,(2-3x+x^2)}{(4-5x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(x**2-3*x+2)/(x**4-5*x**2+4)**2,x)
```

output
```
-(d + e)*log(x + (-24383100*d**6 + 187408066*d**5*e + 10439775*d**5*(d + e
) - 511591980*d**4*e**2 - 94132290*d**4*e*(d + e) + 667200*d**4*(d + e)**2
 + 469491120*d**3*e**3 + 333672552*d**3*e**2*(d + e) - 2703328*d**3*e*(d +
 e)**2 - 198000*d**3*(d + e)**3 + 322778400*d**2*e**4 - 582497712*d**2*e**
3*(d + e) + 1752768*d**2*e**2*(d + e)**2 + 1107552*d**2*e*(d + e)**3 - 863
493856*d*e**5 + 500776560*d*e**4*(d + e) + 4226944*d*e**3*(d + e)**2 - 188
0640*d*e**2*(d + e)**3 + 429000000*e**6 - 169242912*e**5*(d + e) - 4538112
*e**4*(d + e)**2 + 964224*e**3*(d + e)**3)/(13474125*d**6 - 102860175*d**5
*e + 274190390*d**4*e**2 - 224142072*d**3*e**3 - 245084096*d**2*e**4 + 535
797456*d*e**5 - 256183200*e**6))/36 + (d + 2*e)*log(x + (-24383100*d**6 +
187408066*d**5*e - 10439775*d**5*(d + 2*e)/4 - 511591980*d**4*e**2 + 47066
145*d**4*e*(d + 2*e)/2 + 41700*d**4*(d + 2*e)**2 + 469491120*d**3*e**3 - 8
3418138*d**3*e**2*(d + 2*e) - 168958*d**3*e*(d + 2*e)**2 + 12375*d**3*(d +
 2*e)**3/4 + 322778400*d**2*e**4 + 145624428*d**2*e**3*(d + 2*e) + 109548*
d**2*e**2*(d + 2*e)**2 - 34611*d**2*e*(d + 2*e)**3/2 - 863493856*d*e**5 -
125194140*d*e**4*(d + 2*e) + 264184*d*e**3*(d + 2*e)**2 + 29385*d*e**2*(d
+ 2*e)**3 + 429000000*e**6 + 42310728*e**5*(d + 2*e) - 283632*e**4*(d + 2*
e)**2 - 15066*e**3*(d + 2*e)**3)/(13474125*d**6 - 102860175*d**5*e + 27419
0390*d**4*e**2 - 224142072*d**3*e**3 - 245084096*d**2*e**4 + 535797456*d*e
**5 - 256183200*e**6))/144 - (7*d - 13*e)*log(x + (-24383100*d**6 + 187...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)\,(2-3x+x^2)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\,(31\,d-50\,e)\log\left(x+2\right) - \frac{1}{36}\,(7\,d-13\,e)\log\left(x+1\right) - \frac{1}{36}\,(d+e)\log\left(x-1\right) + \frac{1}{144}\,(d+2\,e)\log\left(x-2\right) - \frac{(3\,d-4\,e)x+5\,d-6\,e}{12\,(x^2+3\,x+2)}$$

input
```
integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

output
```
1/144*(31*d - 50*e)*log(x + 2) - 1/36*(7*d - 13*e)*log(x + 1) - 1/36*(d +
e)*log(x - 1) + 1/144*(d + 2*e)*log(x - 2) - 1/12*((3*d - 4*e)*x + 5*d - 6
*e)/(x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)\,(2-3x+x^2)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\,(31\,d-50\,e)\log\left(|x+2|\right) - \frac{1}{36}\,(7\,d-13\,e)\log\left(|x+1|\right) - \frac{1}{36}\,(d+e)\log\left(|x-1|\right) + \frac{1}{144}\,(d+2\,e)\log\left(|x-2|\right) - \frac{(3\,d-4\,e)x+5\,d-6\,e}{12\,(x+2)(x+1)}$$

input
```
integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

output
```
1/144*(31*d - 50*e)*log(abs(x + 2)) - 1/36*(7*d - 13*e)*log(abs(x + 1)) -
1/36*(d + e)*log(abs(x - 1)) + 1/144*(d + 2*e)*log(abs(x - 2)) - 1/12*((3*
d - 4*e)*x + 5*d - 6*e)/((x + 2)*(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)(2-3x+x^2)}{(4-5x^2+x^4)^2}\,dx = \ln(x-2)\left(\frac{d}{144}+\frac{e}{72}\right)-\ln(x-1)\left(\frac{d}{36}+\frac{e}{36}\right) - \ln(x+1)\left(\frac{7\,d}{36}-\frac{13\,e}{36}\right)-\frac{\frac{5\,d}{12}-\frac{e}{2}+x\left(\frac{d}{4}-\frac{e}{3}\right)}{x^2+3\,x+2} + \ln(x+2)\left(\frac{31\,d}{144}-\frac{25\,e}{72}\right)$$

input
```
int(((d + e*x)*(x^2 - 3*x + 2))/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
log(x - 2)*(d/144 + e/72) - log(x - 1)*(d/36 + e/36) - log(x + 1)*((7*d)/3
6 - (13*e)/36) - ((5*d)/12 - e/2 + x*(d/4 - e/3))/(3*x + x^2 + 2) + log(x
+ 2)*((31*d)/144 - (25*e)/72)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.60

$$\int \frac{(d+ex)(2-3x+x^2)}{(4-5x^2+x^4)^2}\,dx = \frac{-36d+40e+3\log(x-2)\,dx+6\log(x-2)\,ex-12\log(x-1)\,dx-12\log(x-1)\,ex+93\log(x+2)\,dx}{}$$

input
```
int((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x)
```

output
```
(log(x - 2)*d*x**2 + 3*log(x - 2)*d*x + 2*log(x - 2)*d + 2*log(x - 2)*e*x*
*2 + 6*log(x - 2)*e*x + 4*log(x - 2)*e - 4*log(x - 1)*d*x**2 - 12*log(x -
1)*d*x - 8*log(x - 1)*d - 4*log(x - 1)*e*x**2 - 12*log(x - 1)*e*x - 8*log(
x - 1)*e + 31*log(x + 2)*d*x**2 + 93*log(x + 2)*d*x + 62*log(x + 2)*d - 50
*log(x + 2)*e*x**2 - 150*log(x + 2)*e*x - 100*log(x + 2)*e - 28*log(x + 1)
*d*x**2 - 84*log(x + 1)*d*x - 56*log(x + 1)*d + 52*log(x + 1)*e*x**2 + 156
*log(x + 1)*e*x + 104*log(x + 1)*e + 12*d*x**2 - 36*d - 16*e*x**2 + 40*e)/
(144*(x**2 + 3*x + 2))
```

3.90 $\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 105

$$\begin{aligned}\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2} dx = & -\frac{5d-6e+8f+(3d-4e+6f)x}{12(2+3x+x^2)} \\ & -\frac{1}{36}(d+e+f)\log(1-x) \\ & +\frac{1}{144}(d+2e+4f)\log(2-x) \\ & -\frac{1}{36}(7d-13e+19f)\log(1+x) \\ & +\frac{1}{144}(31d-50e+76f)\log(2+x)\end{aligned}$$

output
```
-1/12*(5*d-6*e+8*f+(3*d-4*e+6*f)*x)/(x^2+3*x+2)-1/36*(d+e+f)*ln(1-x)+1/144
*(d+2*e+4*f)*ln(2-x)-1/36*(7*d-13*e+19*f)*ln(1+x)+1/144*(31*d-50*e+76*f)*l
n(2+x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\left(-\frac{12(-6e+8f-4ex+6fx+d(5+3x))}{2+3x+x^2} - 4(d+e+f)\log(1-x) + (d+2e+4f)\log(2-x) - 4(7d-13e+19f)\log(1+x) + (31d-50e+76f)\log(2+x)\right)$$

input
```
Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
((-12*(-6*e + 8*f - 4*e*x + 6*f*x + d*(5 + 3*x)))/(2 + 3*x + x^2) - 4*(d +
 e + f)*Log[1 - x] + (d + 2*e + 4*f)*Log[2 - x] - 4*(7*d - 13*e + 19*f)*Lo
g[1 + x] + (31*d - 50*e + 76*f)*Log[2 + x])/144
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2019, 2135, 27, 2141, 27, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-3x+2)(d+ex+fx^2)}{(x^4-5x^2+4)^2}dx$$

$$\Big\downarrow \ 2019$$

$$\int \frac{d+ex+fx^2}{(x^2-3x+2)(x^2+3x+2)^2}dx$$

$$\Big\downarrow \ 2135$$

$$-\frac{1}{72}\int \frac{6\left((3d-4e+6f)x^2-4(2d-3e+5f)x+3d-10e+12f\right)}{(x^2-3x+2)(x^2+3x+2)}dx - \frac{x(3d-4e+6f)+5d-6e+8f}{12(x^2+3x+2)}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{12}\int \frac{(3d-4e+6f)x^2-4(2d-3e+5f)x+3d-10e+12f}{(x^2-3x+2)(x^2+3x+2)}dx-\frac{x(3d-4e+6f)+5d-6e+8f}{12(x^2+3x+2)}$$

$$\Big\downarrow \; 2141$$

$$\frac{1}{12}\left(-\frac{1}{72}\int -\frac{6(7d+6e+4f-(3d+2e)x)}{x^2-3x+2}dx-\frac{1}{72}\int \frac{6(25d-54e+76f-(3d+2e)x)}{x^2+3x+2}dx\right)-\frac{x(3d-4e+6f)+5d-6e+8f}{12(x^2+3x+2)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{12}\left(\frac{1}{12}\int \frac{7d+6e+4f-(3d+2e)x}{x^2-3x+2}dx-\frac{1}{12}\int \frac{25d-54e+76f-(3d+2e)x}{x^2+3x+2}dx\right)-\frac{x(3d-4e+6f)+5d-6e+8f}{12(x^2+3x+2)}$$

$$\Big\downarrow \; 1141$$

$$\frac{1}{12}\left(\frac{1}{12}\int \left(\frac{4(d+e+f)}{1-x}-\frac{d+2e+4f}{2-x}\right)dx-\frac{1}{12}\int \left(\frac{4(7d-13e+19f)}{x+1}-\frac{31d-50e+76f}{x+2}\right)dx\right)-\frac{x(3d-4e+6f)+5d-6e+8f}{12(x^2+3x+2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{12}\left(\frac{1}{12}(\log(2-x)(d+2e+4f)-4\log(1-x)(d+e+f))+\frac{1}{12}(\log(x+2)(31d-50e+76f)-4\log(x+1)(7d\right.$$
$$-\frac{x(3d-4e+6f)+5d-6e+8f}{12(x^2+3x+2)}$$

input
```
Int[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
-1/12*(5*d - 6*e + 8*f + (3*d - 4*e + 6*f)*x)/(2 + 3*x + x^2) + ((-4*(d +
e + f)*Log[1 - x] + (d + 2*e + 4*f)*Log[2 - x])/12 + (-4*(7*d - 13*e + 19*
f)*Log[1 + x] + (31*d - 50*e + 76*f)*Log[2 + x])/12)/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2135

```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

rule 2141

```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

method	result
default	$\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}\right)\ln\left(x-2\right)+\left(-\frac{7d}{36}+\frac{13e}{36}-\frac{19f}{36}\right)\ln\left(1+x\right)-\frac{\frac{d}{6}-\frac{e}{6}+\frac{f}{6}}{1+x}+\left(-\frac{d}{36}-\frac{e}{36}-\frac{f}{36}\right)\ln($
norman	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}\right)x^3+\left(\frac{3d}{4}-\frac{5e}{6}+f\right)x+\left(\frac{d}{3}-\frac{e}{2}+\frac{5f}{6}\right)x^2-\frac{5d}{6}+e-\frac{4f}{3}}{x^4-5x^2+4}+\left(-\frac{7d}{36}+\frac{13e}{36}-\frac{19f}{36}\right)\ln\left(1+x\right)+\left(-\frac{d}{36}-\frac{e}{36}\right.$
risch	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}\right)x-\frac{5d}{12}+\frac{e}{2}-\frac{2f}{3}}{x^2+3x+2}+\frac{31\ln(x+2)d}{144}-\frac{25\ln(x+2)e}{72}+\frac{19\ln(x+2)f}{36}-\frac{7\ln(-x-1)d}{36}+\frac{13\ln(-x-1)e}{36}-\frac{19\ln}{}$
parallelrisch	$\frac{-96f-36dx-72fx-60d+72e+48ex+4\ln(x-2)e-100\ln(x+2)e-4\ln(x-1)x^2d-84\ln(1+x)xd+156\ln(1+x)xe+93\ln(x+2)}{}$

input
```
int((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/144*d+1/72*e+1/36*f)*ln(x-2)+(-7/36*d+13/36*e-19/36*f)*ln(1+x)-(1/6*d-1
/6*e+1/6*f)/(1+x)+(-1/36*d-1/36*e-1/36*f)*ln(x-1)-(1/12*d-1/6*e+1/3*f)/(x+
2)+(31/144*d-25/72*e+19/36*f)*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(95) = 190$.

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.82

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx =$$
$$-\frac{12\,(3\,d-4\,e+6\,f)x-((31\,d-50\,e+76\,f)x^2+3\,(31\,d-50\,e+76\,f)x+62\,d-100\,e+152\,f)\log\left(x\right.}{}$$

input
```
integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

output
```
-1/144*(12*(3*d - 4*e + 6*f)*x - ((31*d - 50*e + 76*f)*x^2 + 3*(31*d - 50*
e + 76*f)*x + 62*d - 100*e + 152*f)*log(x + 2) + 4*((7*d - 13*e + 19*f)*x^
2 + 3*(7*d - 13*e + 19*f)*x + 14*d - 26*e + 38*f)*log(x + 1) + 4*((d + e +
 f)*x^2 + 3*(d + e + f)*x + 2*d + 2*e + 2*f)*log(x - 1) - ((d + 2*e + 4*f)
*x^2 + 3*(d + 2*e + 4*f)*x + 2*d + 4*e + 8*f)*log(x - 2) + 60*d - 72*e + 9
6*f)/(x^2 + 3*x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((x**2-3*x+2)*(f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\,(31\,d-50\,e+76\,f)\log(x+2) - \frac{1}{36}\,(7\,d-13\,e+19\,f)\log(x+1) - \frac{1}{36}\,(d+e+f)\log(x-1) + \frac{1}{144}\,(d+2\,e+4\,f)\log(x-2) - \frac{(3\,d-4\,e+6\,f)x+5\,d-6\,e+8\,f}{12\,(x^2+3\,x+2)}$$

input
```
integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

output
```
1/144*(31*d - 50*e + 76*f)*log(x + 2) - 1/36*(7*d - 13*e + 19*f)*log(x + 1
) - 1/36*(d + e + f)*log(x - 1) + 1/144*(d + 2*e + 4*f)*log(x - 2) - 1/12*
((3*d - 4*e + 6*f)*x + 5*d - 6*e + 8*f)/(x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}(31\,d-50\,e+76\,f)\log(|x+2|) - \frac{1}{36}(7\,d-13\,e+19\,f)\log(|x+1|) - \frac{1}{36}(d+e+f)\log(|x-1|) + \frac{1}{144}(d+2\,e+4\,f)\log(|x-2|) - \frac{(3\,d-4\,e+6\,f)x+5\,d-6\,e+8\,f}{12\,(x+2)(x+1)}$$

input
```
integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

output
```
1/144*(31*d - 50*e + 76*f)*log(abs(x + 2)) - 1/36*(7*d - 13*e + 19*f)*log(
abs(x + 1)) - 1/36*(d + e + f)*log(abs(x - 1)) + 1/144*(d + 2*e + 4*f)*log
(abs(x - 2)) - 1/12*((3*d - 4*e + 6*f)*x + 5*d - 6*e + 8*f)/((x + 2)*(x +
1))
```

Mupad [B] (verification not implemented)

Time = 18.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx = \ln(x-2)\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}\right) - \ln(x+1)\left(\frac{7\,d}{36}-\frac{13\,e}{36}+\frac{19\,f}{36}\right) - \ln(x-1)\left(\frac{d}{36}+\frac{e}{36}+\frac{f}{36}\right) + \ln(x+2)\left(\frac{31\,d}{144}-\frac{25\,e}{72}+\frac{19\,f}{36}\right) - \frac{\frac{5\,d}{12}-\frac{e}{2}+\frac{2\,f}{3}+x\left(\frac{d}{4}-\frac{e}{3}+\frac{f}{2}\right)}{x^2+3\,x+2}$$

input
```
int(((x^2 - 3*x + 2)*(d + e*x + f*x^2))/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
log(x - 2)*(d/144 + e/72 + f/36) - log(x + 1)*((7*d)/36 - (13*e)/36 + (19*
f)/36) - log(x - 1)*(d/36 + e/36 + f/36) + log(x + 2)*((31*d)/144 - (25*e)
/72 + (19*f)/36) - ((5*d)/12 - e/2 + (2*f)/3 + x*(d/4 - e/3 + f/2))/(3*x +
 x^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.24

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx = \frac{-48f-36d+40e-228\log(x+1)\,fx+228\log(x+2)\,fx+12\log(x-2)\,fx-12\log(x-1)\,fx+3\,\mathrm{lo}}{}$$

input
```
int((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
(log(x - 2)*d*x**2 + 3*log(x - 2)*d*x + 2*log(x - 2)*d + 2*log(x - 2)*e*x*
*2 + 6*log(x - 2)*e*x + 4*log(x - 2)*e + 4*log(x - 2)*f*x**2 + 12*log(x -
2)*f*x + 8*log(x - 2)*f - 4*log(x - 1)*d*x**2 - 12*log(x - 1)*d*x - 8*log(
x - 1)*d - 4*log(x - 1)*e*x**2 - 12*log(x - 1)*e*x - 8*log(x - 1)*e - 4*lo
g(x - 1)*f*x**2 - 12*log(x - 1)*f*x - 8*log(x - 1)*f + 31*log(x + 2)*d*x**
2 + 93*log(x + 2)*d*x + 62*log(x + 2)*d - 50*log(x + 2)*e*x**2 - 150*log(x
 + 2)*e*x - 100*log(x + 2)*e + 76*log(x + 2)*f*x**2 + 228*log(x + 2)*f*x +
 152*log(x + 2)*f - 28*log(x + 1)*d*x**2 - 84*log(x + 1)*d*x - 56*log(x +
1)*d + 52*log(x + 1)*e*x**2 + 156*log(x + 1)*e*x + 104*log(x + 1)*e - 76*l
og(x + 1)*f*x**2 - 228*log(x + 1)*f*x - 152*log(x + 1)*f + 12*d*x**2 - 36*
d - 16*e*x**2 + 40*e + 24*f*x**2 - 48*f)/(144*(x**2 + 3*x + 2))
```

3.91 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 121

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = -\frac{5d-6e+8f-12g+(3d-4e+6f-10g)x}{12(2+3x+x^2)} - \frac{1}{36}(d+e+f+g)\log(1-x) + \frac{1}{144}(d+2e+4f+8g)\log(2-x) - \frac{1}{36}(7d-13e+19f-25g)\log(1+x) + \frac{1}{144}(31d-50e+76f-104g)\log(2+x)$$

output

```
-1/12*(5*d-6*e+8*f-12*g+(3*d-4*e+6*f-10*g)*x)/(x^2+3*x+2)-1/36*(d+e+f+g)*l
n(1-x)+1/144*(d+2*e+4*f+8*g)*ln(2-x)-1/36*(7*d-13*e+19*f-25*g)*ln(1+x)+1/1
44*(31*d-50*e+76*f-104*g)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\left(\frac{12(-5d+6e-8f+12g-3dx+4ex-6fx+10gx)}{2+3x+x^2} - 4(d+e+f+g)\log(1-x)+(d+2e+4f+8g)\log(2-x) + 4(-7d+13e-19f+25g)\log(1+x)+(31d-50e+76f-104g)\log(2+x)\right)$$

input
```
Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,
x]
```

output
```
((12*(-5*d + 6*e - 8*f + 12*g - 3*d*x + 4*e*x - 6*f*x + 10*g*x))/(2 + 3*x
+ x^2) - 4*(d + e + f + g)*Log[1 - x] + (d + 2*e + 4*f + 8*g)*Log[2 - x] +
 4*(-7*d + 13*e - 19*f + 25*g)*Log[1 + x] + (31*d - 50*e + 76*f - 104*g)*L
og[2 + x])/144
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2019, 7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-3x+2)(d+ex+fx^2+gx^3)}{(x^4-5x^2+4)^2}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{d+ex+fx^2+gx^3}{(x^2-3x+2)(x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{ 7279}$$

$$\int \left(\frac{31d-50e+76f-104g}{144(x+2)}+\frac{d+2e+4f+8g}{144(x-2)}+\frac{-d-e-f-g}{36(x-1)}+\frac{-7d+13e-19f+25g}{36(x+1)}+\frac{d-e+f-g}{6(x+1)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d-2e+4f-8g}{12(x+2)}-\frac{d-e+f-g}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g)+\frac{1}{144}\log(2-x)(d+2e+4f+8g)-\frac{1}{36}\log(x+1)(7d-13e+19f-25g)+\frac{1}{144}\log(x+2)(31d-50e+76f-104g)$$

input
```
Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
-1/6*(d - e + f - g)/(1 + x) - (d - 2*e + 4*f - 8*g)/(12*(2 + x)) - ((d +
e + f + g)*Log[1 - x])/36 + ((d + 2*e + 4*f + 8*g)*Log[2 - x])/144 - ((7*d
 - 13*e + 19*f - 25*g)*Log[1 + x])/36 + ((31*d - 50*e + 76*f - 104*g)*Log[
2 + x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

method	result
default	$\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}\right)\ln\left(x-2\right)+\left(-\frac{7d}{36}+\frac{13e}{36}-\frac{19f}{36}+\frac{25g}{36}\right)\ln\left(1+x\right)-\frac{\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}}{1+x}+\left(-\frac{d}{36}\right.$
norman	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}+\frac{5g}{6}\right)x^{3}+\left(\frac{3d}{4}-\frac{5e}{6}+f-\frac{4g}{3}\right)x+\left(\frac{d}{3}-\frac{e}{2}+\frac{5f}{6}-\frac{3g}{2}\right)x^{2}-\frac{5d}{6}+e+2g-\frac{4f}{3}}{x^{4}-5x^{2}+4}+\left(-\frac{7d}{36}+\frac{13e}{36}-\frac{19f}{36}+\frac{25g}{36}\right)\ln\left(1\right.$
risch	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}+\frac{5g}{6}\right)x-\frac{5d}{12}+\frac{e}{2}-\frac{2f}{3}+g}{x^{2}+3x+2}+\frac{31\ln\left(x+2\right)d}{144}-\frac{25\ln\left(x+2\right)e}{72}+\frac{19\ln\left(x+2\right)f}{36}-\frac{13\ln\left(x+2\right)g}{18}+\frac{\ln\left(2-x\right)d}{144}+\ln($
parallelrisch	$\frac{-96f+144g-36dx-72fx-60d+72e+120gx+48ex+4\ln\left(x-2\right)e-100\ln\left(x+2\right)e-4\ln\left(x-1\right)x^{2}d-84\ln\left(1+x\right)xd+156\ln\left(1+x\right)x}{}$

input
```
int((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOS
E)
```

output
```
(1/144*d+1/72*e+1/36*f+1/18*g)*ln(x-2)+(-7/36*d+13/36*e-19/36*f+25/36*g)*l
n(1+x)-(1/6*d-1/6*e+1/6*f-1/6*g)/(1+x)+(-1/36*d-1/36*e-1/36*f-1/36*g)*ln(x
-1)-(1/12*d-1/6*e+1/3*f-2/3*g)/(x+2)+(31/144*d-25/72*e+19/36*f-13/18*g)*ln
(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(111) = 222$.

Time = 0.55 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.89

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx =$$
$$-\frac{12\,(3\,d-4\,e+6\,f-10\,g)x-((31\,d-50\,e+76\,f-104\,g)x^2+3\,(31\,d-50\,e+76\,f-104\,g)x+62\,d}{}$$

input
```
integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fr
icas")
```

output
```
-1/144*(12*(3*d - 4*e + 6*f - 10*g)*x - ((31*d - 50*e + 76*f - 104*g)*x^2
+ 3*(31*d - 50*e + 76*f - 104*g)*x + 62*d - 100*e + 152*f - 208*g)*log(x +
 2) + 4*((7*d - 13*e + 19*f - 25*g)*x^2 + 3*(7*d - 13*e + 19*f - 25*g)*x +
 14*d - 26*e + 38*f - 50*g)*log(x + 1) + 4*((d + e + f + g)*x^2 + 3*(d + e
 + f + g)*x + 2*d + 2*e + 2*f + 2*g)*log(x - 1) - ((d + 2*e + 4*f + 8*g)*x
^2 + 3*(d + 2*e + 4*f + 8*g)*x + 2*d + 4*e + 8*f + 16*g)*log(x - 2) + 60*d
 - 72*e + 96*f - 144*g)/(x^2 + 3*x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-3x+x^2)\left(d+ex+fx^2+gx^3\right)}{(4-5x^2+x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((x**2-3*x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\begin{aligned}
&\int \frac{(2-3x+x^2)\left(d+ex+fx^2+gx^3\right)}{(4-5x^2+x^4)^2}\,dx \\
&= \frac{1}{144}\,(31\,d-50\,e+76\,f-104\,g)\log(x+2)-\frac{1}{36}\,(7\,d-13\,e+19\,f-25\,g)\log(x+1) \\
&\quad -\frac{1}{36}\,(d+e+f+g)\log(x-1)+\frac{1}{144}\,(d+2\,e+4\,f+8\,g)\log(x-2) \\
&\quad -\frac{(3\,d-4\,e+6\,f-10\,g)x+5\,d-6\,e+8\,f-12\,g}{12\,(x^2+3\,x+2)}
\end{aligned}$$

input
```
integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="ma
xima")
```

output
```
1/144*(31*d - 50*e + 76*f - 104*g)*log(x + 2) - 1/36*(7*d - 13*e + 19*f -
25*g)*log(x + 1) - 1/36*(d + e + f + g)*log(x - 1) + 1/144*(d + 2*e + 4*f
+ 8*g)*log(x - 2) - 1/12*((3*d - 4*e + 6*f - 10*g)*x + 5*d - 6*e + 8*f - 1
2*g)/(x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}(31\,d-50\,e+76\,f-104\,g)\log(|x+2|) - \frac{1}{36}(7\,d-13\,e+19\,f-25\,g)\log(|x+1|) - \frac{1}{36}(d+e+f+g)\log(|x-1|) + \frac{1}{144}(d+2\,e+4\,f+8\,g)\log(|x-2|) - \frac{(3\,d-4\,e+6\,f-10\,g)x+5\,d-6\,e+8\,f-12\,g}{12\,(x+2)(x+1)}$$

input
```
integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="gi
ac")
```

output
```
1/144*(31*d - 50*e + 76*f - 104*g)*log(abs(x + 2)) - 1/36*(7*d - 13*e + 19
*f - 25*g)*log(abs(x + 1)) - 1/36*(d + e + f + g)*log(abs(x - 1)) + 1/144*
(d + 2*e + 4*f + 8*g)*log(abs(x - 2)) - 1/12*((3*d - 4*e + 6*f - 10*g)*x +
 5*d - 6*e + 8*f - 12*g)/((x + 2)*(x + 1))
```

Mupad [B] (verification not implemented)

Time = 18.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = \ln(x-2)\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}\right) - \ln(x+1)\left(\frac{7\,d}{36}-\frac{13\,e}{36}+\frac{19\,f}{36}-\frac{25\,g}{36}\right) - \ln(x-1)\left(\frac{d}{36}+\frac{e}{36}+\frac{f}{36}+\frac{g}{36}\right) + \ln(x+2)\left(\frac{31\,d}{144}-\frac{25\,e}{72}+\frac{19\,f}{36}-\frac{13\,g}{18}\right) - \frac{\frac{5\,d}{12}-\frac{e}{2}+\frac{2\,f}{3}-g+x\left(\frac{d}{4}-\frac{e}{3}+\frac{f}{2}-\frac{5\,g}{6}\right)}{x^2+3\,x+2}$$

input
```
int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3))/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
log(x - 2)*(d/144 + e/72 + f/36 + g/18) - log(x + 1)*((7*d)/36 - (13*e)/36
 + (19*f)/36 - (25*g)/36) - log(x - 1)*(d/36 + e/36 + f/36 + g/36) + log(x
 + 2)*((31*d)/144 - (25*e)/72 + (19*f)/36 - (13*g)/18) - ((5*d)/12 - e/2 +
 (2*f)/3 - g + x*(d/4 - e/3 + f/2 - (5*g)/6))/(3*x + x^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.71

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
(log(x - 2)*d*x**2 + 3*log(x - 2)*d*x + 2*log(x - 2)*d + 2*log(x - 2)*e*x*
*2 + 6*log(x - 2)*e*x + 4*log(x - 2)*e + 4*log(x - 2)*f*x**2 + 12*log(x -
2)*f*x + 8*log(x - 2)*f + 8*log(x - 2)*g*x**2 + 24*log(x - 2)*g*x + 16*log
(x - 2)*g - 4*log(x - 1)*d*x**2 - 12*log(x - 1)*d*x - 8*log(x - 1)*d - 4*l
og(x - 1)*e*x**2 - 12*log(x - 1)*e*x - 8*log(x - 1)*e - 4*log(x - 1)*f*x**
2 - 12*log(x - 1)*f*x - 8*log(x - 1)*f - 4*log(x - 1)*g*x**2 - 12*log(x -
1)*g*x - 8*log(x - 1)*g + 31*log(x + 2)*d*x**2 + 93*log(x + 2)*d*x + 62*lo
g(x + 2)*d - 50*log(x + 2)*e*x**2 - 150*log(x + 2)*e*x - 100*log(x + 2)*e
+ 76*log(x + 2)*f*x**2 + 228*log(x + 2)*f*x + 152*log(x + 2)*f - 104*log(x
 + 2)*g*x**2 - 312*log(x + 2)*g*x - 208*log(x + 2)*g - 28*log(x + 1)*d*x**
2 - 84*log(x + 1)*d*x - 56*log(x + 1)*d + 52*log(x + 1)*e*x**2 + 156*log(x
 + 1)*e*x + 104*log(x + 1)*e - 76*log(x + 1)*f*x**2 - 228*log(x + 1)*f*x -
 152*log(x + 1)*f + 100*log(x + 1)*g*x**2 + 300*log(x + 1)*g*x + 200*log(x
 + 1)*g + 12*d*x**2 - 36*d - 16*e*x**2 + 40*e + 24*f*x**2 - 48*f - 40*g*x*
*2 + 64*g)/(144*(x**2 + 3*x + 2))
```

3.92 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 137

$$\begin{aligned}
\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx
&= -\frac{5d-6e+8f-12g+20h+(3d-4e+6f-10g+18h)x}{12(2+3x+x^2)} \\
&\quad -\frac{1}{36}(d+e+f+g+h)\log(1-x)+\frac{1}{144}(d+2e+4f+8g+16h)\log(2-x) \\
&\quad -\frac{1}{36}(7d-13e+19f-25g+31h)\log(1+x) \\
&\quad +\frac{1}{144}(31d-50e+76f-104g+112h)\log(2+x)
\end{aligned}$$

output
```
-1/12*(5*d-6*e+8*f-12*g+20*h+(3*d-4*e+6*f-10*g+18*h)*x)/(x^2+3*x+2)-1/36*(
d+e+f+g+h)*ln(1-x)+1/144*(d+2*e+4*f+8*g+16*h)*ln(2-x)-1/36*(7*d-13*e+19*f-
25*g+31*h)*ln(1+x)+1/144*(31*d-50*e+76*f-104*g+112*h)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$$
$$=\frac{1}{144}\left(-\frac{12(d(5+3x)+2(4f-6g+10h+3fx-5gx+9hx-e(3+2x)))}{2+3x+x^2}-4(d+e+f+g+h)\log(1-x)+(d+2(e+2f+4g+8h))\log(2-x)-4(7d-13e+19f-25g+31h)\log(1+x)+(31d-50e+76f-104g+112h)\log(2+x)\right)$$

input
```
Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
 x^4)^2,x]
```

output
```
((-12*(d*(5 + 3*x) + 2*(4*f - 6*g + 10*h + 3*f*x - 5*g*x + 9*h*x - e*(3 +
2*x))))/(2 + 3*x + x^2) - 4*(d + e + f + g + h)*Log[1 - x] + (d + 2*(e + 2
*f + 4*g + 8*h))*Log[2 - x] - 4*(7*d - 13*e + 19*f - 25*g + 31*h)*Log[1 +
x] + (31*d - 50*e + 76*f - 104*g + 112*h)*Log[2 + x])/144
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {2019, 7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-3x+2)\left(d+ex+fx^2+gx^3+hx^4\right)}{(x^4-5x^2+4)^2}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{(x^2-3x+2)(x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{ 7279}$$

$$\int \left(\frac{-7d+13e-19f+25g-31h}{36(x+1)} + \frac{d+2e+4f+8g+16h}{144(x-2)} + \frac{-d-e-f-g-h}{36(x-1)} + \frac{31d-50e+76f-104g}{144(x+2)} \right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d-e+f-g+h}{6(x+1)} - \frac{d-2e+4f-8g+16h}{12(x+2)} - \frac{1}{36}\log(1-x)(d+e+f+g+h) + \frac{1}{144}\log(2-x)(d+2e+4f+8g+16h) - \frac{1}{36}\log(x+1)(7d-13e+19f-25g+31h) + \frac{1}{144}\log(x+2)(31d-50e+76f-104g+112h)$$

input

```
Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^
2,x]
```

output

```
-1/6*(d - e + f - g + h)/(1 + x) - (d - 2*e + 4*f - 8*g + 16*h)/(12*(2 + x
)) - ((d + e + f + g + h)*Log[1 - x])/36 + ((d + 2*e + 4*f + 8*g + 16*h)*L
og[2 - x])/144 - ((7*d - 13*e + 19*f - 25*g + 31*h)*Log[1 + x])/36 + ((31*
d - 50*e + 76*f - 104*g + 112*h)*Log[2 + x])/144
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 7279

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

method	result
default	$\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}+\frac{h}{9}\right)\ln\left(x-2\right)+\left(-\frac{7d}{36}+\frac{13e}{36}-\frac{19f}{36}+\frac{25g}{36}-\frac{31h}{36}\right)\ln\left(1+x\right)-\frac{\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-}{1+x}$
norman	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}+\frac{5g}{6}-\frac{3h}{2}\right)x^3+\left(\frac{3d}{4}-\frac{5e}{6}+f-\frac{4g}{3}+2h\right)x+\left(\frac{d}{3}-\frac{e}{2}+\frac{5f}{6}-\frac{3g}{2}+\frac{17h}{6}\right)x^2-\frac{5d}{6}+e+2g-\frac{10h}{3}-\frac{4f}{3}}{x^4-5x^2+4}+\left(-\frac{7d}{36}+\frac{13e}{36}-\right.$
risch	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}+\frac{5g}{6}-\frac{3h}{2}\right)x-\frac{5d}{12}+\frac{e}{2}-\frac{2f}{3}+g-\frac{5h}{3}}{x^2+3x+2}-\frac{7\ln(-x-1)d}{36}+\frac{13\ln(-x-1)e}{36}-\frac{19\ln(-x-1)f}{36}+\frac{25\ln(-x-1)g}{36}-$
parallelrisch	$\frac{-96f+144g-36dx-72fx-240h-60d+72e+120gx+48ex+4\ln(x-2)e-100\ln(x+2)e-4\ln(x-1)x^2d-84\ln(1+x)xd+156\ln}{}$

input
```
int((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURN
VERBOSE)
```

output
```
(1/144*d+1/72*e+1/36*f+1/18*g+1/9*h)*ln(x-2)+(-7/36*d+13/36*e-19/36*f+25/3
6*g-31/36*h)*ln(1+x)-(1/6*d-1/6*e+1/6*f-1/6*g+1/6*h)/(1+x)+(-1/36*d-1/36*e
-1/36*f-1/36*g-1/36*h)*ln(x-1)-(1/12*d-1/6*e+1/3*f-2/3*g+4/3*h)/(x+2)+(31/
144*d-25/72*e+19/36*f-13/18*g+7/9*h)*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(127) = 254$.

Time = 3.09 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.95

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx =$$
$$-\frac{12\,(3\,d-4\,e+6\,f-10\,g+18\,h)x-((31\,d-50\,e+76\,f-104\,g+112\,h)x^2+3\,(31\,d-50\,e+76\,f-}{}$$

input
```
integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorit
hm="fricas")
```

output
```
-1/144*(12*(3*d - 4*e + 6*f - 10*g + 18*h)*x - ((31*d - 50*e + 76*f - 104*
g + 112*h)*x^2 + 3*(31*d - 50*e + 76*f - 104*g + 112*h)*x + 62*d - 100*e +
 152*f - 208*g + 224*h)*log(x + 2) + 4*((7*d - 13*e + 19*f - 25*g + 31*h)*
x^2 + 3*(7*d - 13*e + 19*f - 25*g + 31*h)*x + 14*d - 26*e + 38*f - 50*g +
62*h)*log(x + 1) + 4*((d + e + f + g + h)*x^2 + 3*(d + e + f + g + h)*x +
2*d + 2*e + 2*f + 2*g + 2*h)*log(x - 1) - ((d + 2*e + 4*f + 8*g + 16*h)*x^
2 + 3*(d + 2*e + 4*f + 8*g + 16*h)*x + 2*d + 4*e + 8*f + 16*g + 32*h)*log(
x - 2) + 60*d - 72*e + 96*f - 144*g + 240*h)/(x^2 + 3*x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((x**2-3*x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\begin{aligned}
&\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx \\
&= \frac{1}{144}(31\,d-50\,e+76\,f-104\,g+112\,h)\log(x+2) \\
&\quad - \frac{1}{36}(7\,d-13\,e+19\,f-25\,g+31\,h)\log(x+1) \\
&\quad - \frac{1}{36}(d+e+f+g+h)\log(x-1) + \frac{1}{144}(d+2\,e+4\,f+8\,g+16\,h)\log(x-2) \\
&\quad - \frac{(3\,d-4\,e+6\,f-10\,g+18\,h)x+5\,d-6\,e+8\,f-12\,g+20\,h}{12\,(x^2+3\,x+2)}
\end{aligned}$$

input
```
integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorit
hm="maxima")
```

output
```
1/144*(31*d - 50*e + 76*f - 104*g + 112*h)*log(x + 2) - 1/36*(7*d - 13*e +
 19*f - 25*g + 31*h)*log(x + 1) - 1/36*(d + e + f + g + h)*log(x - 1) + 1/
144*(d + 2*e + 4*f + 8*g + 16*h)*log(x - 2) - 1/12*((3*d - 4*e + 6*f - 10*
g + 18*h)*x + 5*d - 6*e + 8*f - 12*g + 20*h)/(x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}(31\,d-50\,e+76\,f-104\,g+112\,h)\log(|x+2|) - \frac{1}{36}(7\,d-13\,e+19\,f-25\,g+31\,h)\log(|x+1|) - \frac{1}{36}(d+e+f+g+h)\log(|x-1|) + \frac{1}{144}(d+2\,e+4\,f+8\,g+16\,h)\log(|x-2|) - \frac{(3\,d-4\,e+6\,f-10\,g+18\,h)x+5\,d-6\,e+8\,f-12\,g+20\,h}{12\,(x+2)(x+1)}$$

input
```
integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorit
hm="giac")
```

output
```
1/144*(31*d - 50*e + 76*f - 104*g + 112*h)*log(abs(x + 2)) - 1/36*(7*d - 1
3*e + 19*f - 25*g + 31*h)*log(abs(x + 1)) - 1/36*(d + e + f + g + h)*log(a
bs(x - 1)) + 1/144*(d + 2*e + 4*f + 8*g + 16*h)*log(abs(x - 2)) - 1/12*((3
*d - 4*e + 6*f - 10*g + 18*h)*x + 5*d - 6*e + 8*f - 12*g + 20*h)/((x + 2)*
(x + 1))
```

Mupad [B] (verification not implemented)

Time = 18.92 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.97

$$\int \frac{(2-3x+x^2)\,(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$$
$$= \ln(x-2)\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}+\frac{h}{9}\right)-\ln(x-1)\left(\frac{d}{36}+\frac{e}{36}+\frac{f}{36}+\frac{g}{36}+\frac{h}{36}\right)$$
$$-\ln(x+1)\left(\frac{7\,d}{36}-\frac{13\,e}{36}+\frac{19\,f}{36}-\frac{25\,g}{36}+\frac{31\,h}{36}\right)$$
$$-\frac{\frac{5\,d}{12}-\frac{e}{2}+\frac{2\,f}{3}-g+\frac{5\,h}{3}+x\left(\frac{d}{4}-\frac{e}{3}+\frac{f}{2}-\frac{5\,g}{6}+\frac{3\,h}{2}\right)}{x^2+3\,x+2}$$
$$+\ln(x+2)\left(\frac{31\,d}{144}-\frac{25\,e}{72}+\frac{19\,f}{36}-\frac{13\,g}{18}+\frac{7\,h}{9}\right)$$

input
```
int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^2 + 4)^
2,x)
```

output
```
log(x - 2)*(d/144 + e/72 + f/36 + g/18 + h/9) - log(x - 1)*(d/36 + e/36 +
f/36 + g/36 + h/36) - log(x + 1)*((7*d)/36 - (13*e)/36 + (19*f)/36 - (25*g
)/36 + (31*h)/36) - ((5*d)/12 - e/2 + (2*f)/3 - g + (5*h)/3 + x*(d/4 - e/3
 + f/2 - (5*g)/6 + (3*h)/2))/(3*x + x^2 + 2) + log(x + 2)*((31*d)/144 - (2
5*e)/72 + (19*f)/36 - (13*g)/18 + (7*h)/9)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 558, normalized size of antiderivative = 4.07

$$\int \frac{(2-3x+x^2)\,(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
(log(x - 2)*d*x**2 + 3*log(x - 2)*d*x + 2*log(x - 2)*d + 2*log(x - 2)*e*x*
*2 + 6*log(x - 2)*e*x + 4*log(x - 2)*e + 4*log(x - 2)*f*x**2 + 12*log(x -
2)*f*x + 8*log(x - 2)*f + 8*log(x - 2)*g*x**2 + 24*log(x - 2)*g*x + 16*log
(x - 2)*g + 16*log(x - 2)*h*x**2 + 48*log(x - 2)*h*x + 32*log(x - 2)*h - 4
*log(x - 1)*d*x**2 - 12*log(x - 1)*d*x - 8*log(x - 1)*d - 4*log(x - 1)*e*x
**2 - 12*log(x - 1)*e*x - 8*log(x - 1)*e - 4*log(x - 1)*f*x**2 - 12*log(x
- 1)*f*x - 8*log(x - 1)*f - 4*log(x - 1)*g*x**2 - 12*log(x - 1)*g*x - 8*lo
g(x - 1)*g - 4*log(x - 1)*h*x**2 - 12*log(x - 1)*h*x - 8*log(x - 1)*h + 31
*log(x + 2)*d*x**2 + 93*log(x + 2)*d*x + 62*log(x + 2)*d - 50*log(x + 2)*e
*x**2 - 150*log(x + 2)*e*x - 100*log(x + 2)*e + 76*log(x + 2)*f*x**2 + 228
*log(x + 2)*f*x + 152*log(x + 2)*f - 104*log(x + 2)*g*x**2 - 312*log(x + 2
)*g*x - 208*log(x + 2)*g + 112*log(x + 2)*h*x**2 + 336*log(x + 2)*h*x + 22
4*log(x + 2)*h - 28*log(x + 1)*d*x**2 - 84*log(x + 1)*d*x - 56*log(x + 1)*
d + 52*log(x + 1)*e*x**2 + 156*log(x + 1)*e*x + 104*log(x + 1)*e - 76*log(
x + 1)*f*x**2 - 228*log(x + 1)*f*x - 152*log(x + 1)*f + 100*log(x + 1)*g*x
**2 + 300*log(x + 1)*g*x + 200*log(x + 1)*g - 124*log(x + 1)*h*x**2 - 372*
log(x + 1)*h*x - 248*log(x + 1)*h + 12*d*x**2 - 36*d - 16*e*x**2 + 40*e +
24*f*x**2 - 48*f - 40*g*x**2 + 64*g + 72*h*x**2 - 96*h)/(144*(x**2 + 3*x +
 2))
```

3.93 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 153

$$\begin{aligned}
&\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx \\
&= -\frac{5d-6e+8f-12g+20h-36i+(3d-4e+6f-10g+18h-34i)x}{12(2+3x+x^2)} \\
&\quad -\frac{1}{36}(d+e+f+g+h+i)\log(1-x)+\frac{1}{144}(d+2e+4f+8g+16h+32i)\log(2-x) \\
&\quad -\frac{1}{36}(7d-13e+19f-25g+31h-37i)\log(1+x) \\
&\quad +\frac{1}{144}(31d-50e+76f-104g+112h-32i)\log(2+x)
\end{aligned}$$

output
```
-1/12*(5*d-6*e+8*f-12*g+20*h-36*i+(3*d-4*e+6*f-10*g+18*h-34*i)*x)/(x^2+3*x
+2)-1/36*(d+e+f+g+h+i)*ln(1-x)+1/144*(d+2*e+4*f+8*g+16*h+32*i)*ln(2-x)-1/3
6*(7*d-13*e+19*f-25*g+31*h-37*i)*ln(1+x)+1/144*(31*d-50*e+76*f-104*g+112*h
-32*i)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{1}{144}\left(\frac{12(-d(5+3x)+2(-4f+6g-10h+18i-3fx+5gx-9hx+17ix+e(3+2x)))}{2+3x+x^2}\right.$$
$$-4(d+e+f+g+h+i)\log(1-x)+(d+2e+4(f+2g+4h+8i))\log(2-x)$$
$$+4(-7d+13e-19f+25g-31h+37i)\log(1+x)$$
$$\left.+(31d-50e+76f-104g+112h-32i)\log(2+x)\right)$$

input
```
Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
 5*x^2 + x^4)^2,x]
```

output
```
((12*(-(d*(5 + 3*x)) + 2*(-4*f + 6*g - 10*h + 18*i - 3*f*x + 5*g*x - 9*h*x
 + 17*i*x + e*(3 + 2*x))))/(2 + 3*x + x^2) - 4*(d + e + f + g + h + i)*Log
[1 - x] + (d + 2*e + 4*(f + 2*g + 4*h + 8*i))*Log[2 - x] + 4*(-7*d + 13*e
- 19*f + 25*g - 31*h + 37*i)*Log[1 + x] + (31*d - 50*e + 76*f - 104*g + 11
2*h - 32*i)*Log[2 + x])/144
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2019, 7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2-3x+2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(x^4-5x^2+4)^2}\,dx$$
$$\Big\downarrow \ 2019$$
$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(x^2-3x+2)(x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{d-2e+4f-8g+16h-32i}{12(x+2)^2}+\frac{d+2e+4f+8g+16h+32i}{144(x-2)}+\frac{-d-e-f-g-h-i}{36(x-1)}+\frac{-7d+13e-1}{3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d-2e+4f-8g+16h-32i}{12(x+2)}-\frac{d-e+f-g+h-i}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g+h+i)+\frac{1}{144}\log(2-x)(d+2e+4f+8g+16h+32i)-\frac{1}{36}\log(x+1)(7d-13e+19f-25g+31h-37i)+\frac{1}{144}\log(x+2)(31d-50e+76f-104g+112h-32i)$$

input
```
Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2
 + x^4)^2,x]
```

output
```
-1/6*(d - e + f - g + h - i)/(1 + x) - (d - 2*e + 4*f - 8*g + 16*h - 32*i)
/(12*(2 + x)) - ((d + e + f + g + h + i)*Log[1 - x])/36 + ((d + 2*e + 4*f
+ 8*g + 16*h + 32*i)*Log[2 - x])/144 - ((7*d - 13*e + 19*f - 25*g + 31*h -
 37*i)*Log[1 + x])/36 + ((31*d - 50*e + 76*f - 104*g + 112*h - 32*i)*Log[2
 + x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98

method	result
default	$\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}+\frac{h}{9}+\frac{2i}{9}\right)\ln\left(x-2\right)+\left(-\frac{7d}{36}+\frac{13e}{36}-\frac{19f}{36}+\frac{25g}{36}-\frac{31h}{36}+\frac{37i}{36}\right)\ln\left(1+x\right)$
norman	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}+\frac{5g}{6}+\frac{17i}{6}-\frac{3h}{2}\right)x^3+\left(\frac{3d}{4}-\frac{5e}{6}+f-\frac{4g}{3}+2h-\frac{10i}{3}\right)x+\left(\frac{d}{3}-\frac{e}{2}+\frac{5f}{6}-\frac{3g}{2}+\frac{17h}{6}-\frac{11i}{2}\right)x^2+6i-\frac{5d}{6}+e+2g-\frac{10h}{3}-\frac{4f}{3}}{x^4-5x^2+4}+$
risch	$\frac{\ln(2-x)e}{72}+\frac{\ln(2-x)g}{18}-\frac{25\ln(x+2)e}{72}-\frac{31\ln(-x-1)h}{36}+\frac{25\ln(-x-1)g}{36}+\frac{\ln(2-x)d}{144}+\frac{\ln(2-x)f}{36}-\frac{19\ln(-x-1)}{36}$
parallelrisch	$432i-96f+144g-36dx-72fx-240h-60d+72e+120gx+48ex+4\ln(x-2)e-32\ln(x+2)x^2i-100\ln(x+2)e-4\ln(x-1)x^2d-8$

input
```
int((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_
RETURNVERBOSE)
```

output
```
(1/144*d+1/72*e+1/36*f+1/18*g+1/9*h+2/9*i)*ln(x-2)+(-7/36*d+13/36*e-19/36*
f+25/36*g-31/36*h+37/36*i)*ln(1+x)-(1/6*d-1/6*e+1/6*f-1/6*g+1/6*h-1/6*i)/(
1+x)+(-1/36*d-1/36*e-1/36*f-1/36*g-1/36*h-1/36*i)*ln(x-1)-(1/12*d-1/6*e+1/
3*f-2/3*g+4/3*h-8/3*i)/(x+2)+(31/144*d-25/72*e+19/36*f-13/18*g-2/9*i+7/9*h
)*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(143) = 286.

Time = 21.78 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.99

$$\int \frac{(2-3x+x^2)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx =$$
$$-\frac{12\,(3\,d-4\,e+6\,f-10\,g+18\,h-34\,i)x-((31\,d-50\,e+76\,f-104\,g+112\,h-32\,i)x^2+3\,(31\,d-}{}$$

input
```
integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, a
lgorithm="fricas")
```

output
```
-1/144*(12*(3*d - 4*e + 6*f - 10*g + 18*h - 34*i)*x - ((31*d - 50*e + 76*f
 - 104*g + 112*h - 32*i)*x^2 + 3*(31*d - 50*e + 76*f - 104*g + 112*h - 32*
i)*x + 62*d - 100*e + 152*f - 208*g + 224*h - 64*i)*log(x + 2) + 4*((7*d -
 13*e + 19*f - 25*g + 31*h - 37*i)*x^2 + 3*(7*d - 13*e + 19*f - 25*g + 31*
h - 37*i)*x + 14*d - 26*e + 38*f - 50*g + 62*h - 74*i)*log(x + 1) + 4*((d
+ e + f + g + h + i)*x^2 + 3*(d + e + f + g + h + i)*x + 2*d + 2*e + 2*f +
 2*g + 2*h + 2*i)*log(x - 1) - ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*x^2 +
3*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*x + 2*d + 4*e + 8*f + 16*g + 32*h +
64*i)*log(x - 2) + 60*d - 72*e + 96*f - 144*g + 240*h - 432*i)/(x^2 + 3*x
+ 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((x**2-3*x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)
**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

$$\begin{aligned}
&\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx \\
&= \frac{1}{144}(31\,d-50\,e+76\,f-104\,g+112\,h-32\,i)\log(x+2) \\
&\quad -\frac{1}{36}(7\,d-13\,e+19\,f-25\,g+31\,h-37\,i)\log(x+1) \\
&\quad -\frac{1}{36}(d+e+f+g+h+i)\log(x-1) \\
&\quad +\frac{1}{144}(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log(x-2) \\
&\quad -\frac{(3\,d-4\,e+6\,f-10\,g+18\,h-34\,i)x+5\,d-6\,e+8\,f-12\,g+20\,h-36\,i}{12\,(x^2+3\,x+2)}
\end{aligned}$$

input
```
integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, a
lgorithm="maxima")
```

output
```
1/144*(31*d - 50*e + 76*f - 104*g + 112*h - 32*i)*log(x + 2) - 1/36*(7*d -
 13*e + 19*f - 25*g + 31*h - 37*i)*log(x + 1) - 1/36*(d + e + f + g + h +
i)*log(x - 1) + 1/144*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(x - 2) - 1/1
2*((3*d - 4*e + 6*f - 10*g + 18*h - 34*i)*x + 5*d - 6*e + 8*f - 12*g + 20*
h - 36*i)/(x^2 + 3*x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{(2-3x+x^2)\,(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx &= \frac{1}{144}\,(31\,d-50\,e+76\,f-104\,g+112\,h-32\,i)\log\left(|x+2|\right)\\ &\quad-\frac{1}{36}\,(7\,d-13\,e+19\,f-25\,g+31\,h-37\,i)\log\left(|x+1|\right)\\ &\quad-\frac{1}{36}\,(d+e+f+g+h+i)\log\left(|x-1|\right)\\ &\quad+\frac{1}{144}\,(d+2\,e+4\,f+8\,g+16\,h+32\,i)\log\left(|x-2|\right)\\ &\quad-\frac{(3\,d-4\,e+6\,f-10\,g+18\,h-34\,i)x+5\,d-6\,e+8\,f-12\,g+20\,h-36\,i}{12\,(x+2)(x+1)}\end{aligned}$$

input
```
integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, a
lgorithm="giac")
```

output
```
1/144*(31*d - 50*e + 76*f - 104*g + 112*h - 32*i)*log(abs(x + 2)) - 1/36*(
7*d - 13*e + 19*f - 25*g + 31*h - 37*i)*log(abs(x + 1)) - 1/36*(d + e + f
+ g + h + i)*log(abs(x - 1)) + 1/144*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*l
og(abs(x - 2)) - 1/12*((3*d - 4*e + 6*f - 10*g + 18*h - 34*i)*x + 5*d - 6*
e + 8*f - 12*g + 20*h - 36*i)/((x + 2)*(x + 1))
```

Mupad [B] (verification not implemented)

Time = 19.37 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$$
$$=\ln(x-2)\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}+\frac{h}{9}+\frac{2i}{9}\right)-\ln(x-1)\left(\frac{d}{36}+\frac{e}{36}+\frac{f}{36}+\frac{g}{36}+\frac{h}{36}+\frac{i}{36}\right)$$
$$-\ln(x+1)\left(\frac{7d}{36}-\frac{13e}{36}+\frac{19f}{36}-\frac{25g}{36}+\frac{31h}{36}-\frac{37i}{36}\right)$$
$$+\ln(x+2)\left(\frac{31d}{144}-\frac{25e}{72}+\frac{19f}{36}-\frac{13g}{18}+\frac{7h}{9}-\frac{2i}{9}\right)$$
$$-\frac{\frac{5d}{12}-\frac{e}{2}+\frac{2f}{3}-g+\frac{5h}{3}-3i+x\left(\frac{d}{4}-\frac{e}{3}+\frac{f}{2}-\frac{5g}{6}+\frac{3h}{2}-\frac{17i}{6}\right)}{x^2+3x+2}$$

input
```
int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^4 - 5*x
^2 + 4)^2,x)
```

output
```
log(x - 2)*(d/144 + e/72 + f/36 + g/18 + h/9 + (2*i)/9) - log(x - 1)*(d/36
 + e/36 + f/36 + g/36 + h/36 + i/36) - log(x + 1)*((7*d)/36 - (13*e)/36 +
(19*f)/36 - (25*g)/36 + (31*h)/36 - (37*i)/36) + log(x + 2)*((31*d)/144 -
(25*e)/72 + (19*f)/36 - (13*g)/18 + (7*h)/9 - (2*i)/9) - ((5*d)/12 - e/2 +
 (2*f)/3 - g + (5*h)/3 - 3*i + x*(d/4 - e/3 + f/2 - (5*g)/6 + (3*h)/2 - (1
7*i)/6))/(3*x + x^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 667, normalized size of antiderivative = 4.36

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
(log(x - 2)*d*x**2 + 3*log(x - 2)*d*x + 2*log(x - 2)*d + 2*log(x - 2)*e*x*
*2 + 6*log(x - 2)*e*x + 4*log(x - 2)*e + 4*log(x - 2)*f*x**2 + 12*log(x -
2)*f*x + 8*log(x - 2)*f + 8*log(x - 2)*g*x**2 + 24*log(x - 2)*g*x + 16*log
(x - 2)*g + 16*log(x - 2)*h*x**2 + 48*log(x - 2)*h*x + 32*log(x - 2)*h + 3
2*log(x - 2)*i*x**2 + 96*log(x - 2)*i*x + 64*log(x - 2)*i - 4*log(x - 1)*d
*x**2 - 12*log(x - 1)*d*x - 8*log(x - 1)*d - 4*log(x - 1)*e*x**2 - 12*log(
x - 1)*e*x - 8*log(x - 1)*e - 4*log(x - 1)*f*x**2 - 12*log(x - 1)*f*x - 8*
log(x - 1)*f - 4*log(x - 1)*g*x**2 - 12*log(x - 1)*g*x - 8*log(x - 1)*g -
4*log(x - 1)*h*x**2 - 12*log(x - 1)*h*x - 8*log(x - 1)*h - 4*log(x - 1)*i*
x**2 - 12*log(x - 1)*i*x - 8*log(x - 1)*i + 31*log(x + 2)*d*x**2 + 93*log(
x + 2)*d*x + 62*log(x + 2)*d - 50*log(x + 2)*e*x**2 - 150*log(x + 2)*e*x -
 100*log(x + 2)*e + 76*log(x + 2)*f*x**2 + 228*log(x + 2)*f*x + 152*log(x
+ 2)*f - 104*log(x + 2)*g*x**2 - 312*log(x + 2)*g*x - 208*log(x + 2)*g + 1
12*log(x + 2)*h*x**2 + 336*log(x + 2)*h*x + 224*log(x + 2)*h - 32*log(x +
2)*i*x**2 - 96*log(x + 2)*i*x - 64*log(x + 2)*i - 28*log(x + 1)*d*x**2 - 8
4*log(x + 1)*d*x - 56*log(x + 1)*d + 52*log(x + 1)*e*x**2 + 156*log(x + 1)
*e*x + 104*log(x + 1)*e - 76*log(x + 1)*f*x**2 - 228*log(x + 1)*f*x - 152*
log(x + 1)*f + 100*log(x + 1)*g*x**2 + 300*log(x + 1)*g*x + 200*log(x + 1)
*g - 124*log(x + 1)*h*x**2 - 372*log(x + 1)*h*x - 248*log(x + 1)*h + 148*l
og(x + 1)*i*x**2 + 444*log(x + 1)*i*x + 296*log(x + 1)*i + 12*d*x**2 - ...
```

3.94 $\int \frac{2+x}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 68

$$\int \frac{2+x}{(4-5x^2+x^4)^2}\,dx=\frac{1}{12(1-x)}+\frac{1}{36(2-x)}-\frac{1}{36(1+x)}+\frac{1}{18}\log(1-x)-\frac{35}{432}\log(2-x)+\frac{1}{54}\log(1+x)+\frac{1}{144}\log(2+x)$$

output
```
1/(12-12*x)+1/(72-36*x)-1/(36+36*x)+1/18*ln(1-x)-35/432*ln(2-x)+1/54*ln(1+
x)+1/144*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{2+x}{(4-5x^2+x^4)^2}\,dx=\frac{1}{432}\left(\frac{12(5+6x-5x^2)}{2-x-2x^2+x^3}+24\log(1-x)-35\log(2-x)+8\log(1+x)+3\log(2+x)\right)$$

input
```
Integrate[(2 + x)/(4 - 5*x^2 + x^4)^2,x]
```

output
```
((12*(5 + 6*x - 5*x^2))/(2 - x - 2*x^2 + x^3) + 24*Log[1 - x] - 35*Log[2 -
 x] + 8*Log[1 + x] + 3*Log[2 + x])/432
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{(x^4-5x^2+4)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(x+2)(x^3-2x^2-x+2)^2}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{1}{18(x-1)}+\frac{1}{54(x+1)}+\frac{1}{144(x+2)}+\frac{1}{12(x-1)^2}+\frac{1}{36(x+1)^2}-\frac{35}{432(x-2)}+\frac{1}{36(x-2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12(1-x)}+\frac{1}{36(2-x)}-\frac{1}{36(x+1)}+\frac{1}{18}\log(1-x)-\frac{35}{432}\log(2-x)+\frac{1}{54}\log(x+1)+\frac{1}{144}\log(x+2)$$

input
```
Int[(2 + x)/(4 - 5*x^2 + x^4)^2,x]
```

output
```
1/(12*(1 - x)) + 1/(36*(2 - x)) - 1/(36*(1 + x)) + Log[1 - x]/18 - (35*Log
[2 - x])/432 + Log[1 + x]/54 + Log[2 + x]/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69

method	result
default	$-\frac{1}{36(x-2)}-\frac{35\ln(x-2)}{432}-\frac{1}{36(1+x)}+\frac{\ln(1+x)}{54}-\frac{1}{12(x-1)}+\frac{\ln(x-1)}{18}+\frac{\ln(x+2)}{144}$
risch	$\frac{-\frac{5}{36}x^2+\frac{1}{6}x+\frac{5}{36}}{x^3-2x^2-x+2}-\frac{35\ln(x-2)}{432}+\frac{\ln(x-1)}{18}+\frac{\ln(1+x)}{54}+\frac{\ln(x+2)}{144}$
norman	$\frac{-\frac{1}{9}x^2+\frac{17}{36}x-\frac{5}{36}x^3+\frac{5}{18}}{x^4-5x^2+4}-\frac{35\ln(x-2)}{432}+\frac{\ln(x-1)}{18}+\frac{\ln(1+x)}{54}+\frac{\ln(x+2)}{144}$
parallelrisch	$-\frac{35\ln(x-2)x^3-24\ln(x-1)x^3-8\ln(1+x)x^3-3\ln(x+2)x^3-60-70\ln(x-2)x^2+48\ln(x-1)x^2+16\ln(1+x)x^2+6\ln(x+2)x^2}{432(x^3-2x^2}$

input
```
int((x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/36/(x-2)-35/432*ln(x-2)-1/36/(1+x)+1/54*ln(1+x)-1/12/(x-1)+1/18*ln(x-1)
+1/144*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(50) = 100$.

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.51

$$\int \frac{2+x}{(4-5x^2+x^4)^2}\,dx = -\frac{60\,x^2-3\,(x^3-2\,x^2-x+2)\log(x+2)-8\,(x^3-2\,x^2-x+2)\log(x+1)-24\,(x^3-2\,x^2-x+2)\,\mathrm{l}}{432\,(x^3-2\,x^2-x+2)}$$

input
```
integrate((2+x)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

output
```
-1/432*(60*x^2 - 3*(x^3 - 2*x^2 - x + 2)*log(x + 2) - 8*(x^3 - 2*x^2 - x +
 2)*log(x + 1) - 24*(x^3 - 2*x^2 - x + 2)*log(x - 1) + 35*(x^3 - 2*x^2 - x
 + 2)*log(x - 2) - 72*x - 60)/(x^3 - 2*x^2 - x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{2+x}{(4-5x^2+x^4)^2}\,dx = \frac{-5x^2+6x+5}{36x^3-72x^2-36x+72} - \frac{35\log(x-2)}{432} + \frac{\log(x-1)}{18} + \frac{\log(x+1)}{54} + \frac{\log(x+2)}{144}$$

input
```
integrate((2+x)/(x**4-5*x**2+4)**2,x)
```

output
```
(-5*x**2 + 6*x + 5)/(36*x**3 - 72*x**2 - 36*x + 72) - 35*log(x - 2)/432 +
log(x - 1)/18 + log(x + 1)/54 + log(x + 2)/144
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{2+x}{(4-5x^2+x^4)^2}\,dx = -\frac{5\,x^2-6\,x-5}{36\,(x^3-2\,x^2-x+2)}+\frac{1}{144}\,\log\left(x+2\right)+\frac{1}{54}\,\log\left(x+1\right)+\frac{1}{18}\,\log\left(x-1\right)-\frac{35}{432}\,\log\left(x-2\right)$$

input
```
integrate((2+x)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

output
```
-1/36*(5*x^2 - 6*x - 5)/(x^3 - 2*x^2 - x + 2) + 1/144*log(x + 2) + 1/54*lo
g(x + 1) + 1/18*log(x - 1) - 35/432*log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{2+x}{(4-5x^2+x^4)^2}\,dx = -\frac{5\,x^2-6\,x-5}{36\,(x+1)(x-1)(x-2)}+\frac{1}{144}\,\log\left(|x+2|\right)+\frac{1}{54}\,\log\left(|x+1|\right)+\frac{1}{18}\,\log\left(|x-1|\right)-\frac{35}{432}\,\log\left(|x-2|\right)$$

input
```
integrate((2+x)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

output
```
-1/36*(5*x^2 - 6*x - 5)/((x + 1)*(x - 1)*(x - 2)) + 1/144*log(abs(x + 2))
+ 1/54*log(abs(x + 1)) + 1/18*log(abs(x - 1)) - 35/432*log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{2+x}{(4-5x^2+x^4)^2}\,dx = \frac{\ln(x-1)}{18}+\frac{\ln(x+1)}{54}-\frac{35\ln(x-2)}{432}+\frac{\ln(x+2)}{144}-\frac{-\frac{5x^2}{36}+\frac{x}{6}+\frac{5}{36}}{-x^3+2x^2+x-2}$$

input
```
int((x + 2)/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
log(x - 1)/18 + log(x + 1)/54 - (35*log(x - 2))/432 + log(x + 2)/144 - (x/
6 - (5*x^2)/36 + 5/36)/(x + 2*x^2 - x^3 - 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.22

$$\int \frac{2+x}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{-35\log(x-2)\,x^3+70\log(x-2)\,x^2+35\log(x-2)\,x-70\log(x-2)+24\log(x-1)\,x^3-48\log(x-1}{}$$

input
```
int((2+x)/(x^4-5*x^2+4)^2,x)
```

output
```
( - 35*log(x - 2)*x**3 + 70*log(x - 2)*x**2 + 35*log(x - 2)*x - 70*log(x -
 2) + 24*log(x - 1)*x**3 - 48*log(x - 1)*x**2 - 24*log(x - 1)*x + 48*log(x
 - 1) + 3*log(x + 2)*x**3 - 6*log(x + 2)*x**2 - 3*log(x + 2)*x + 6*log(x +
 2) + 8*log(x + 1)*x**3 - 16*log(x + 1)*x**2 - 8*log(x + 1)*x + 16*log(x +
 1) - 30*x**3 + 102*x)/(432*(x**3 - 2*x**2 - x + 2))
```

3.95 $\int \frac{(2+x)(d+ex)}{(4-5x^2+x^4)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{(2+x)(d+ex)}{(4-5x^2+x^4)^2} dx = \frac{d+e}{12(1-x)} + \frac{d+2e}{36(2-x)} - \frac{d-e}{36(1+x)} + \frac{1}{36}(2d+5e)\log(1-x) - \frac{1}{432}(35d+58e)\log(2-x) + \frac{1}{108}(2d+e)\log(1+x) + \frac{1}{144}(d-2e)\log(2+x)$$

output
```
(d+e)/(12-12*x)+(d+2*e)/(72-36*x)-(d-e)/(36+36*x)+1/36*(2*d+5*e)*ln(1-x)-1
/432*(35*d+58*e)*ln(2-x)+1/108*(2*d+e)*ln(1+x)+1/144*(d-2*e)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{(2+x)(d+ex)}{(4-5x^2+x^4)^2} dx = \frac{1}{432}\left(\frac{12(d(5+6x-5x^2)+2e(5-2x^2))}{2-x-2x^2+x^3} + 12(2d+5e)\log(1-x) - (35d+58e)\log(2-x) + 4(2d+e)\log(1+x) + 3(d-2e)\log(2+x)\right)$$

input
```
Integrate[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
((12*(d*(5 + 6*x - 5*x^2) + 2*e*(5 - 2*x^2)))/(2 - x - 2*x^2 + x^3) + 12*(
2*d + 5*e)*Log[1 - x] - (35*d + 58*e)*Log[2 - x] + 4*(2*d + e)*Log[1 + x]
+ 3*(d - 2*e)*Log[2 + x])/432
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+2)(d+ex)}{(x^4-5x^2+4)^2}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{d+ex}{(x+2)\left(x^3-2x^2-x+2\right)^2}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{-35d-58e}{432(x-2)}+\frac{2d+5e}{36(x-1)}+\frac{2d+e}{108(x+1)}+\frac{d-2e}{144(x+2)}+\frac{d+2e}{36(x-2)^2}+\frac{d+e}{12(x-1)^2}+\frac{d-e}{36(x+1)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d-e}{36(x+1)}+\frac{d+e}{12(1-x)}+\frac{d+2e}{36(2-x)}+\frac{1}{36}(2d+5e)\log(1-x)-\frac{1}{432}(35d+58e)\log(2-x)+\frac{1}{108}(2d+e)\log(x+1)+\frac{1}{144}(d-2e)\log(x+2)$$

input
```
Int[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
(d + e)/(12*(1 - x)) + (d + 2*e)/(36*(2 - x)) - (d - e)/(36*(1 + x)) + ((2
*d + 5*e)*Log[1 - x])/36 - ((35*d + 58*e)*Log[2 - x])/432 + ((2*d + e)*Log
[1 + x])/108 + ((d - 2*e)*Log[2 + x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

method	result
default	$\left(-\frac{35d}{432}-\frac{29e}{216}\right)\ln\left(x-2\right)-\frac{\frac{d}{36}+\frac{e}{18}}{x-2}-\frac{\frac{d}{36}-\frac{e}{36}}{1+x}+\left(\frac{d}{54}+\frac{e}{108}\right)\ln\left(1+x\right)-\frac{\frac{d}{12}+\frac{e}{12}}{x-1}+\left(\frac{d}{18}+\frac{5e}{36}\right)\ln\left(x\right.$
norman	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}\right)x^3+\left(\frac{17d}{36}+\frac{5e}{18}\right)x+\left(-\frac{d}{9}-\frac{2e}{9}\right)x^2+\frac{5d}{18}+\frac{5e}{9}}{x^4-5x^2+4}+\left(-\frac{35d}{432}-\frac{29e}{216}\right)\ln\left(x-2\right)+\left(\frac{d}{18}+\frac{5e}{36}\right)\ln\left(x-1\right)+$
risch	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}\right)x^2+\frac{dx}{6}+\frac{5d}{36}+\frac{5e}{18}}{x^3-2x^2-x+2}+\frac{\ln(x+2)d}{144}-\frac{\ln(x+2)e}{72}+\frac{\ln(-x-1)d}{54}+\frac{\ln(-x-1)e}{108}+\frac{\ln(x-1)d}{18}+\frac{5\ln(x-1)e}{36}-\frac{35}{}$
parallelrisch	$-\frac{-72dx-60d-120e+116\ln(x-2)e+12\ln(x+2)e+48\ln(x-1)x^2d+8\ln(1+x)xd+4\ln(1+x)xe+3\ln(x+2)xd-6\ln(x+2)xe+}{}$

input
```
int((x+2)*(e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
(-35/432*d-29/216*e)*ln(x-2)-(1/36*d+1/18*e)/(x-2)-(1/36*d-1/36*e)/(1+x)+(
1/54*d+1/108*e)*ln(1+x)-(1/12*d+1/12*e)/(x-1)+(1/18*d+5/36*e)*ln(x-1)+(1/1
44*d-1/72*e)*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(87) = 174$.

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.01

$$\int \frac{(2+x)(d+ex)}{(4-5x^2+x^4)^2}\,dx = -\frac{12\,(5\,d+4\,e)x^2 - 72\,dx - 3\,((d-2\,e)x^3 - 2\,(d-2\,e)x^2 - (d-2\,e)x + 2\,d - 4\,e)\log(x+2) - 4\,((2\,d}{}$$

input
```
integrate((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

output
```
-1/432*(12*(5*d + 4*e)*x^2 - 72*d*x - 3*((d - 2*e)*x^3 - 2*(d - 2*e)*x^2 -
 (d - 2*e)*x + 2*d - 4*e)*log(x + 2) - 4*((2*d + e)*x^3 - 2*(2*d + e)*x^2
- (2*d + e)*x + 4*d + 2*e)*log(x + 1) - 12*((2*d + 5*e)*x^3 - 2*(2*d + 5*e
)*x^2 - (2*d + 5*e)*x + 4*d + 10*e)*log(x - 1) + ((35*d + 58*e)*x^3 - 2*(3
5*d + 58*e)*x^2 - (35*d + 58*e)*x + 70*d + 116*e)*log(x - 2) - 60*d - 120*
e)/(x^3 - 2*x^2 - x + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. $2(78) = 156$.

Time = 5.04 (sec) , antiderivative size = 1034, normalized size of antiderivative = 9.85

$$\int \frac{(2+x)(d+ex)}{(4-5x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((2+x)*(e*x+d)/(x**4-5*x**2+4)**2,x)
```

output
```
(d - 2*e)*log(x + (8710660*d**5 + 91884504*d**4*e - 7579779*d**4*(d - 2*e)
/4 + 364910432*d**3*e**2 - 18128055*d**3*e*(d - 2*e) - 83772*d**3*(d - 2*e
)**2 + 686697536*d**2*e**3 - 60296868*d**2*e**2*(d - 2*e) - 597816*d**2*e*
(d - 2*e)**2 + 65907*d**2*(d - 2*e)**3/4 + 614357568*d*e**4 - 85949220*d*e
**3*(d - 2*e) - 1500048*d*e**2*(d - 2*e)**2 + 105840*d*e*(d - 2*e)**3 + 20
8470400*e**5 - 45136356*e**4*(d - 2*e) - 1196064*e**3*(d - 2*e)**2 + 12827
7*e**2*(d - 2*e)**3)/(3374210*d**5 + 38645295*d**4*e + 170558380*d**3*e**2
 + 362061760*d**2*e**3 + 370298160*d*e**4 + 146466320*e**5))/144 + (2*d +
e)*log(x + (8710660*d**5 + 91884504*d**4*e - 2526593*d**4*(2*d + e) + 3649
10432*d**3*e**2 - 24170740*d**3*e*(2*d + e) - 148928*d**3*(2*d + e)**2 + 6
86697536*d**2*e**3 - 80395824*d**2*e**2*(2*d + e) - 1062784*d**2*e*(2*d +
e)**2 + 39056*d**2*(2*d + e)**3 + 614357568*d*e**4 - 114598960*d*e**3*(2*d
 + e) - 2666752*d*e**2*(2*d + e)**2 + 250880*d*e*(2*d + e)**3 + 208470400*
e**5 - 60181808*e**4*(2*d + e) - 2126336*e**3*(2*d + e)**2 + 304064*e**2*(
2*d + e)**3)/(3374210*d**5 + 38645295*d**4*e + 170558380*d**3*e**2 + 36206
1760*d**2*e**3 + 370298160*d*e**4 + 146466320*e**5))/108 + (2*d + 5*e)*log
(x + (8710660*d**5 + 91884504*d**4*e - 7579779*d**4*(2*d + 5*e) + 36491043
2*d**3*e**2 - 72512220*d**3*e*(2*d + 5*e) - 1340352*d**3*(2*d + 5*e)**2 +
686697536*d**2*e**3 - 241187472*d**2*e**2*(2*d + 5*e) - 9565056*d**2*e*(2*
d + 5*e)**2 + 1054512*d**2*(2*d + 5*e)**3 + 614357568*d*e**4 - 34379688...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int \frac{(2+x)(d+ex)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\,(d-2\,e)\log\left(x+2\right)+\frac{1}{108}\,(2\,d+e)\log\left(x+1\right) + \frac{1}{36}\,(2\,d+5\,e)\log\left(x-1\right)-\frac{1}{432}\,(35\,d+58\,e)\log\left(x-2\right) - \frac{(5\,d+4\,e)x^2-6\,dx-5\,d-10\,e}{36\,(x^3-2\,x^2-x+2)}$$

input
```
integrate((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

output
```
1/144*(d - 2*e)*log(x + 2) + 1/108*(2*d + e)*log(x + 1) + 1/36*(2*d + 5*e)
*log(x - 1) - 1/432*(35*d + 58*e)*log(x - 2) - 1/36*((5*d + 4*e)*x^2 - 6*d
*x - 5*d - 10*e)/(x^3 - 2*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(2+x)(d+ex)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{144}\,(d-2\,e)\log\left(|x+2|\right)+\frac{1}{108}\,(2\,d+e)\log\left(|x+1|\right) + \frac{1}{36}\,(2\,d+5\,e)\log\left(|x-1|\right)-\frac{1}{432}\,(35\,d+58\,e)\log\left(|x-2|\right) - \frac{(5\,d+4\,e)x^2-6\,dx-5\,d-10\,e}{36\,(x+1)(x-1)(x-2)}$$

input
```
integrate((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

output
```
1/144*(d - 2*e)*log(abs(x + 2)) + 1/108*(2*d + e)*log(abs(x + 1)) + 1/36*(
2*d + 5*e)*log(abs(x - 1)) - 1/432*(35*d + 58*e)*log(abs(x - 2)) - 1/36*((
5*d + 4*e)*x^2 - 6*d*x - 5*d - 10*e)/((x + 1)*(x - 1)*(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int \frac{(2+x)(d+ex)}{(4-5x^2+x^4)^2}\,dx = \ln(x-1)\left(\frac{d}{18}+\frac{5\,e}{36}\right)-\frac{\left(-\frac{5\,d}{36}-\frac{e}{9}\right)\,x^2+\frac{d\,x}{6}+\frac{5\,d}{36}+\frac{5\,e}{18}}{-x^3+2\,x^2+x-2} + \ln(x+1)\left(\frac{d}{54}+\frac{e}{108}\right)+\ln(x+2)\left(\frac{d}{144}-\frac{e}{72}\right) - \ln(x-2)\left(\frac{35\,d}{432}+\frac{29\,e}{216}\right)$$

input
```
int(((x + 2)*(d + e*x))/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
log(x - 1)*(d/18 + (5*e)/36) - ((5*d)/36 + (5*e)/18 - x^2*((5*d)/36 + e/9)
 + (d*x)/6)/(x + 2*x^2 - x^3 - 2) + log(x + 1)*(d/54 + e/108) + log(x + 2)
*(d/144 - e/72) - log(x - 2)*((35*d)/432 + (29*e)/216)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.07

$$\int \frac{(2+x)(d+ex)}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{24ex + 72e + 35\log(x-2)\,dx + 58\log(x-2)\,ex - 24\log(x-1)\,dx - 60\log(x-1)\,ex - 3\log(x+2)\,d\ldots}{}$$

input
```
int((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
( - 35*log(x - 2)*d*x**3 + 70*log(x - 2)*d*x**2 + 35*log(x - 2)*d*x - 70*l
og(x - 2)*d - 58*log(x - 2)*e*x**3 + 116*log(x - 2)*e*x**2 + 58*log(x - 2)
*e*x - 116*log(x - 2)*e + 24*log(x - 1)*d*x**3 - 48*log(x - 1)*d*x**2 - 24
*log(x - 1)*d*x + 48*log(x - 1)*d + 60*log(x - 1)*e*x**3 - 120*log(x - 1)*
e*x**2 - 60*log(x - 1)*e*x + 120*log(x - 1)*e + 3*log(x + 2)*d*x**3 - 6*lo
g(x + 2)*d*x**2 - 3*log(x + 2)*d*x + 6*log(x + 2)*d - 6*log(x + 2)*e*x**3
+ 12*log(x + 2)*e*x**2 + 6*log(x + 2)*e*x - 12*log(x + 2)*e + 8*log(x + 1)
*d*x**3 - 16*log(x + 1)*d*x**2 - 8*log(x + 1)*d*x + 16*log(x + 1)*d + 4*lo
g(x + 1)*e*x**3 - 8*log(x + 1)*e*x**2 - 4*log(x + 1)*e*x + 8*log(x + 1)*e
- 30*d*x**3 + 102*d*x - 24*e*x**3 + 24*e*x + 72*e)/(432*(x**3 - 2*x**2 - x
 + 2))
```

3.96 $\int \frac{(2+x)(d+ex+fx^2)}{(4-5x^2+x^4)^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 122

$$\begin{aligned}\int \frac{(2+x)(d+ex+fx^2)}{(4-5x^2+x^4)^2} dx &= \frac{d+e+f}{12(1-x)} + \frac{d+2e+4f}{36(2-x)} - \frac{d-e+f}{36(1+x)} \\ &\quad + \frac{1}{36}(2d+5e+8f)\log(1-x) \\ &\quad - \frac{1}{432}(35d+58e+92f)\log(2-x) \\ &\quad + \frac{1}{108}(2d+e-4f)\log(1+x) + \frac{1}{144}(d-2e+4f)\log(2+x)\end{aligned}$$

output
```
(d+e+f)/(12-12*x)+(d+2*e+4*f)/(72-36*x)-(d-e+f)/(36+36*x)+1/36*(2*d+5*e+8*
f)*ln(1-x)-1/432*(35*d+58*e+92*f)*ln(2-x)+1/108*(2*d+e-4*f)*ln(1+x)+1/144*
(d-2*e+4*f)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{(2+x)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx = \frac{1}{432}\left(\frac{12(d(5+6x-5x^2)+e(10-4x^2)+2f(4+3x-4x^2))}{2-x-2x^2+x^3} + 12(2d+5e+8f)\log(1-x)-(35d+58e+92f)\log(2-x) + 4(2d+e-4f)\log(1+x)+3(d-2e+4f)\log(2+x)\right)$$

input
```
Integrate[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
((12*(d*(5 + 6*x - 5*x^2) + e*(10 - 4*x^2) + 2*f*(4 + 3*x - 4*x^2)))/(2 -
x - 2*x^2 + x^3) + 12*(2*d + 5*e + 8*f)*Log[1 - x] - (35*d + 58*e + 92*f)*
Log[2 - x] + 4*(2*d + e - 4*f)*Log[1 + x] + 3*(d - 2*e + 4*f)*Log[2 + x])/
432
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+2)(d+ex+fx^2)}{(x^4-5x^2+4)^2}\,dx$$

$$\Big\downarrow \; 2019$$

$$\int \frac{d+ex+fx^2}{(x+2)(x^3-2x^2-x+2)^2}dx$$

$$\Big\downarrow \; 2462$$

$$\int \left(\frac{-35d-58e-92f}{432(x-2)} + \frac{2d+5e+8f}{36(x-1)} + \frac{2d+e-4f}{108(x+1)} + \frac{d-2e+4f}{144(x+2)} + \frac{d+2e+4f}{36(x-2)^2} + \frac{d+e+f}{12(x-1)^2} + \frac{d-e+f}{36(x+1)}\right.$$

$\big\downarrow$ 2009

$$-\frac{d-e+f}{36(x+1)} + \frac{d+e+f}{12(1-x)} + \frac{d+2e+4f}{36(2-x)} + \frac{1}{36}\log(1-x)(2d+5e+8f) - \frac{1}{432}\log(2-x)(35d+58e+92f) + \frac{1}{108}\log(x+1)(2d+e-4f) + \frac{1}{144}\log(x+2)(d-2e+4f)$$

input
```
Int[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
(d + e + f)/(12*(1 - x)) + (d + 2*e + 4*f)/(36*(2 - x)) - (d - e + f)/(36*
(1 + x)) + ((2*d + 5*e + 8*f)*Log[1 - x])/36 - ((35*d + 58*e + 92*f)*Log[2
 - x])/432 + ((2*d + e - 4*f)*Log[1 + x])/108 + ((d - 2*e + 4*f)*Log[2 + x
])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

method	result
default	$\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}\right)\ln\left(x-2\right)-\frac{\frac{d}{36}+\frac{e}{18}+\frac{f}{9}}{x-2}-\frac{\frac{d}{36}-\frac{e}{36}+\frac{f}{36}}{1+x}+\left(\frac{d}{54}+\frac{e}{108}-\frac{f}{27}\right)\ln\left(1+x\right)-\frac{\frac{d}{12}+\frac{e}{12}}{x-1}$
norman	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}\right)x^3+\left(\frac{17d}{36}+\frac{5e}{18}+\frac{5f}{9}\right)x+\left(-\frac{d}{9}-\frac{2e}{9}-\frac{5f}{18}\right)x^2+\frac{5d}{18}+\frac{5e}{9}+\frac{4f}{9}}{x^4-5x^2+4}+\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}\right)\ln\left(x-2\right)+\left(\frac{d}{18}\right.$
risch	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}\right)x^2+\left(\frac{d}{6}+\frac{f}{6}\right)x+\frac{5d}{36}+\frac{5e}{18}+\frac{2f}{9}}{x^3-2x^2-x+2}-\frac{35\ln(2-x)d}{432}-\frac{29\ln(2-x)e}{216}-\frac{23\ln(2-x)f}{108}+\frac{\ln(x+2)d}{144}-\frac{\ln(x+2)e}{72}$
parallelrisch	$-\frac{-96f-72dx-72fx-60d-120e+96f\,x^2+116\ln(x-2)e+12\ln(x+2)e+48\ln(x-1)x^2d+8\ln(1+x)xd+4\ln(1+x)xe+3\ln(x-}{}$

input
```
int((x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
(-35/432*d-29/216*e-23/108*f)*ln(x-2)-(1/36*d+1/18*e+1/9*f)/(x-2)-(1/36*d-
1/36*e+1/36*f)/(1+x)+(1/54*d+1/108*e-1/27*f)*ln(1+x)-(1/12*d+1/12*e+1/12*f
)/(x-1)+(1/18*d+5/36*e+2/9*f)*ln(x-1)+(1/144*d-1/72*e+1/36*f)*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(104) = 208.

Time = 0.14 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.19

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx = -\frac{12\,(5\,d+4\,e+8\,f)x^2-72\,(d+f)x-3\,((d-2\,e+4\,f)x^3-2\,(d-2\,e+4\,f)x^2-(d-2\,e+4\,f)x+}{}$$

input
```
integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

output
```
-1/432*(12*(5*d + 4*e + 8*f)*x^2 - 72*(d + f)*x - 3*((d - 2*e + 4*f)*x^3 -
 2*(d - 2*e + 4*f)*x^2 - (d - 2*e + 4*f)*x + 2*d - 4*e + 8*f)*log(x + 2) -
 4*((2*d + e - 4*f)*x^3 - 2*(2*d + e - 4*f)*x^2 - (2*d + e - 4*f)*x + 4*d
+ 2*e - 8*f)*log(x + 1) - 12*((2*d + 5*e + 8*f)*x^3 - 2*(2*d + 5*e + 8*f)*
x^2 - (2*d + 5*e + 8*f)*x + 4*d + 10*e + 16*f)*log(x - 1) + ((35*d + 58*e
+ 92*f)*x^3 - 2*(35*d + 58*e + 92*f)*x^2 - (35*d + 58*e + 92*f)*x + 70*d +
 116*e + 184*f)*log(x - 2) - 60*d - 120*e - 96*f)/(x^3 - 2*x^2 - x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2+x)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((2+x)*(f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{(2+x)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx &= \frac{1}{144}\,(d-2\,e+4\,f)\log(x+2) \\ &\quad+\frac{1}{108}\,(2\,d+e-4\,f)\log(x+1) \\ &\quad+\frac{1}{36}\,(2\,d+5\,e+8\,f)\log(x-1) \\ &\quad-\frac{1}{432}\,(35\,d+58\,e+92\,f)\log(x-2) \\ &\quad-\frac{(5\,d+4\,e+8\,f)x^2-6\,(d+f)x-5\,d-10\,e-8\,f}{36\,(x^3-2\,x^2-x+2)}\end{aligned}$$

input
```
integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

output
```
1/144*(d - 2*e + 4*f)*log(x + 2) + 1/108*(2*d + e - 4*f)*log(x + 1) + 1/36
*(2*d + 5*e + 8*f)*log(x - 1) - 1/432*(35*d + 58*e + 92*f)*log(x - 2) - 1/
36*((5*d + 4*e + 8*f)*x^2 - 6*(d + f)*x - 5*d - 10*e - 8*f)/(x^3 - 2*x^2 -
 x + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \frac{1}{144}\left(d-2\,e+4\,f\right)\log\left(|x+2|\right) + \frac{1}{108}\left(2\,d+e-4\,f\right)\log\left(|x+1|\right) + \frac{1}{36}\left(2\,d+5\,e+8\,f\right)\log\left(|x-1|\right) - \frac{1}{432}\left(35\,d+58\,e+92\,f\right)\log\left(|x-2|\right) - \frac{(5\,d+4\,e+8\,f)x^2-6\,(d+f)x-5\,d-10\,e-8\,f}{36\,(x+1)(x-1)(x-2)}$$

input
```
integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

output
```
1/144*(d - 2*e + 4*f)*log(abs(x + 2)) + 1/108*(2*d + e - 4*f)*log(abs(x +
1)) + 1/36*(2*d + 5*e + 8*f)*log(abs(x - 1)) - 1/432*(35*d + 58*e + 92*f)*
log(abs(x - 2)) - 1/36*((5*d + 4*e + 8*f)*x^2 - 6*(d + f)*x - 5*d - 10*e -
 8*f)/((x + 1)*(x - 1)*(x - 2))
```

Mupad [B] (verification not implemented)

Time = 18.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \ln(x-1)\left(\frac{d}{18}+\frac{5\,e}{36}+\frac{2\,f}{9}\right)+\ln(x+1)\left(\frac{d}{54}+\frac{e}{108}-\frac{f}{27}\right) + \ln(x+2)\left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}\right) - \ln(x-2)\left(\frac{35\,d}{432}+\frac{29\,e}{216}+\frac{23\,f}{108}\right) - \frac{\left(-\frac{5\,d}{36}-\frac{e}{9}-\frac{2\,f}{9}\right)x^2+\left(\frac{d}{6}+\frac{f}{6}\right)x+\frac{5\,d}{36}+\frac{5\,e}{18}+\frac{2\,f}{9}}{-x^3+2\,x^2+x-2}$$

input
```
int(((x + 2)*(d + e*x + f*x^2))/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
log(x - 1)*(d/18 + (5*e)/36 + (2*f)/9) + log(x + 1)*(d/54 + e/108 - f/27)
+ log(x + 2)*(d/144 - e/72 + f/36) - log(x - 2)*((35*d)/432 + (29*e)/216 +
 (23*f)/108) - ((5*d)/36 + (5*e)/18 + (2*f)/9 + x*(d/6 + f/6) - x^2*((5*d)
/36 + e/9 + (2*f)/9))/(x + 2*x^2 - x^3 - 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.87

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
( - 35*log(x - 2)*d*x**3 + 70*log(x - 2)*d*x**2 + 35*log(x - 2)*d*x - 70*l
og(x - 2)*d - 58*log(x - 2)*e*x**3 + 116*log(x - 2)*e*x**2 + 58*log(x - 2)
*e*x - 116*log(x - 2)*e - 92*log(x - 2)*f*x**3 + 184*log(x - 2)*f*x**2 + 9
2*log(x - 2)*f*x - 184*log(x - 2)*f + 24*log(x - 1)*d*x**3 - 48*log(x - 1)
*d*x**2 - 24*log(x - 1)*d*x + 48*log(x - 1)*d + 60*log(x - 1)*e*x**3 - 120
*log(x - 1)*e*x**2 - 60*log(x - 1)*e*x + 120*log(x - 1)*e + 96*log(x - 1)*
f*x**3 - 192*log(x - 1)*f*x**2 - 96*log(x - 1)*f*x + 192*log(x - 1)*f + 3*
log(x + 2)*d*x**3 - 6*log(x + 2)*d*x**2 - 3*log(x + 2)*d*x + 6*log(x + 2)*
d - 6*log(x + 2)*e*x**3 + 12*log(x + 2)*e*x**2 + 6*log(x + 2)*e*x - 12*log
(x + 2)*e + 12*log(x + 2)*f*x**3 - 24*log(x + 2)*f*x**2 - 12*log(x + 2)*f*
x + 24*log(x + 2)*f + 8*log(x + 1)*d*x**3 - 16*log(x + 1)*d*x**2 - 8*log(x
 + 1)*d*x + 16*log(x + 1)*d + 4*log(x + 1)*e*x**3 - 8*log(x + 1)*e*x**2 -
4*log(x + 1)*e*x + 8*log(x + 1)*e - 16*log(x + 1)*f*x**3 + 32*log(x + 1)*f
*x**2 + 16*log(x + 1)*f*x - 32*log(x + 1)*f - 30*d*x**3 + 102*d*x - 24*e*x
**3 + 24*e*x + 72*e - 48*f*x**3 + 120*f*x)/(432*(x**3 - 2*x**2 - x + 2))
```

3.97 $\int \frac{(2+x)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 141

$$\int \frac{(2+x)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = \frac{d+e+f+g}{12(1-x)}+\frac{d+2e+4f+8g}{36(2-x)}-\frac{d-e+f-g}{36(1+x)} + \frac{1}{36}(2d+5e+8f+11g)\log(1-x) - \frac{1}{432}(35d+58e+92f+136g)\log(2-x) + \frac{1}{108}(2d+e-4f+7g)\log(1+x) + \frac{1}{144}(d-2e+4f-8g)\log(2+x)$$

output

```
(d+e+f+g)/(12-12*x)+(d+2*e+4*f+8*g)/(72-36*x)-(d-e+f-g)/(36+36*x)+1/36*(2*
d+5*e+8*f+11*g)*ln(1-x)-1/432*(35*d+58*e+92*f+136*g)*ln(2-x)+1/108*(2*d+e-
4*f+7*g)*ln(1+x)+1/144*(d-2*e+4*f-8*g)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.02

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{1}{432}\left(\frac{12(d(5+6x-5x^2)+2(g(8-5x^2)+f(4+3x-4x^2)+e(5-2x^2)))}{2-x-2x^2+x^3} + 12(2d+5e+8f+11g)\log(1-x)-(35d+58e+92f+136g)\log(2-x) + 4(2d+e-4f+7g)\log(1+x)+3(d-2e+4f-8g)\log(2+x)\right)$$

input
```
Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
((12*(d*(5 + 6*x - 5*x^2) + 2*(g*(8 - 5*x^2) + f*(4 + 3*x - 4*x^2) + e*(5
- 2*x^2))))/(2 - x - 2*x^2 + x^3) + 12*(2*d + 5*e + 8*f + 11*g)*Log[1 - x]
 - (35*d + 58*e + 92*f + 136*g)*Log[2 - x] + 4*(2*d + e - 4*f + 7*g)*Log[1
 + x] + 3*(d - 2*e + 4*f - 8*g)*Log[2 + x])/432
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+2)\left(d+ex+fx^2+gx^3\right)}{(x^4-5x^2+4)^2}\,dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{d+ex+fx^2+gx^3}{(x+2)\left(x^3-2x^2-x+2\right)^2}dx$$
$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-35d-58e-92f-136g}{432(x-2)} + \frac{2d+5e+8f+11g}{36(x-1)} + \frac{2d+e-4f+7g}{108(x+1)} + \frac{d-2e+4f-8g}{144(x+2)} + \frac{d+2e+4f+}{36(x-2)^2} \right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d-e+f-g}{36(x+1)} + \frac{d+e+f+g}{12(1-x)} + \frac{d+2e+4f+8g}{36(2-x)} + \frac{1}{36}\log(1-x)(2d+5e+8f+11g) - \frac{1}{432}\log(2-x)(35d+58e+92f+136g) + \frac{1}{108}\log(x+1)(2d+e-4f+7g) + \frac{1}{144}\log(x+2)(d-2e+4f-8g)$$

input
```
Int[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
(d + e + f + g)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g)/(36*(2 - x)) - (d - e
 + f - g)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 11*g)*Log[1 - x])/36 - ((35*d
 + 58*e + 92*f + 136*g)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g)*Log[1 + x
])/108 + ((d - 2*e + 4*f - 8*g)*Log[2 + x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.95

method	result
default	$\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}-\frac{17g}{54}\right)\ln\left(x-2\right)-\frac{\frac{d}{36}+\frac{e}{18}+\frac{f}{9}+\frac{2g}{9}}{x-2}-\frac{\frac{d}{36}-\frac{e}{36}+\frac{f}{36}-\frac{g}{36}}{1+x}+\left(\frac{d}{54}+\frac{e}{108}-\frac{f}{27}+\frac{7g}{108}\right)$
norman	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}\right)x^3+\left(\frac{17d}{36}+\frac{5e}{18}+\frac{5f}{9}+\frac{4g}{9}\right)x+\left(-\frac{d}{9}-\frac{2e}{9}-\frac{5f}{18}-\frac{5g}{9}\right)x^2+\frac{5d}{18}+\frac{5e}{9}+\frac{4f}{9}+\frac{8g}{9}}{x^4-5x^2+4}+\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}-\right.$
risch	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}\right)x^2+\left(\frac{d}{6}+\frac{f}{6}\right)x+\frac{5d}{36}+\frac{5e}{18}+\frac{2f}{9}+\frac{4g}{9}}{x^3-2x^2-x+2}-\frac{35\ln(2-x)d}{432}-\frac{29\ln(2-x)e}{216}-\frac{23\ln(2-x)f}{108}-\frac{17\ln(2-x)g}{54}+$
parallelrisch	$-\frac{-96f-192g-72dx-72fx-60d-120e+96f\,x^2+116\ln(x-2)e+12\ln(x+2)e+48\ln(x-1)x^2d+8\ln(1+x)xd+4\ln(1+x)xe+}{}$

input
```
int((x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
(-35/432*d-29/216*e-23/108*f-17/54*g)*ln(x-2)-(1/36*d+1/18*e+1/9*f+2/9*g)/
(x-2)-(1/36*d-1/36*e+1/36*f-1/36*g)/(1+x)+(1/54*d+1/108*e-1/27*f+7/108*g)*
ln(1+x)-(1/12*d+1/12*e+1/12*f+1/12*g)/(x-1)+(1/18*d+5/36*e+2/9*f+11/36*g)*
ln(x-1)+(1/144*d-1/72*e+1/36*f-1/18*g)*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(123) = 246$.

Time = 0.54 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.28

$$\int\frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx = -\frac{12\,(5\,d+4\,e+8\,f+10\,g)x^2-72\,(d+f)x-3\,((d-2\,e+4\,f-8\,g)x^3-2\,(d-2\,e+4\,f-8\,g)x^2-(}{}$$

input
```
integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

output
```
-1/432*(12*(5*d + 4*e + 8*f + 10*g)*x^2 - 72*(d + f)*x - 3*((d - 2*e + 4*f
 - 8*g)*x^3 - 2*(d - 2*e + 4*f - 8*g)*x^2 - (d - 2*e + 4*f - 8*g)*x + 2*d
- 4*e + 8*f - 16*g)*log(x + 2) - 4*((2*d + e - 4*f + 7*g)*x^3 - 2*(2*d + e
 - 4*f + 7*g)*x^2 - (2*d + e - 4*f + 7*g)*x + 4*d + 2*e - 8*f + 14*g)*log(
x + 1) - 12*((2*d + 5*e + 8*f + 11*g)*x^3 - 2*(2*d + 5*e + 8*f + 11*g)*x^2
 - (2*d + 5*e + 8*f + 11*g)*x + 4*d + 10*e + 16*f + 22*g)*log(x - 1) + ((3
5*d + 58*e + 92*f + 136*g)*x^3 - 2*(35*d + 58*e + 92*f + 136*g)*x^2 - (35*
d + 58*e + 92*f + 136*g)*x + 70*d + 116*e + 184*f + 272*g)*log(x - 2) - 60
*d - 120*e - 96*f - 192*g)/(x^3 - 2*x^2 - x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((2+x)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.89

$$\begin{aligned}
&\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx \\
&= \frac{1}{144}\,(d-2\,e+4\,f-8\,g)\log\left(x+2\right)+\frac{1}{108}\,(2\,d+e-4\,f+7\,g)\log\left(x+1\right) \\
&\quad+\frac{1}{36}\,(2\,d+5\,e+8\,f+11\,g)\log\left(x-1\right)-\frac{1}{432}\,(35\,d+58\,e+92\,f+136\,g)\log\left(x-2\right) \\
&\quad-\frac{(5\,d+4\,e+8\,f+10\,g)x^2-6\,(d+f)x-5\,d-10\,e-8\,f-16\,g}{36\,(x^3-2\,x^2-x+2)}
\end{aligned}$$

input
```
integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

output
```
1/144*(d - 2*e + 4*f - 8*g)*log(x + 2) + 1/108*(2*d + e - 4*f + 7*g)*log(x
 + 1) + 1/36*(2*d + 5*e + 8*f + 11*g)*log(x - 1) - 1/432*(35*d + 58*e + 92
*f + 136*g)*log(x - 2) - 1/36*((5*d + 4*e + 8*f + 10*g)*x^2 - 6*(d + f)*x
- 5*d - 10*e - 8*f - 16*g)/(x^3 - 2*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{(2+x)\,(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx &= \frac{1}{144}\,(d-2\,e+4\,f-8\,g)\log\left(|x+2|\right)+\frac{1}{108}\,(2\,d+e-4\,f+7\,g)\log\left(|x+1|\right)\\ &\quad+\frac{1}{36}\,(2\,d+5\,e+8\,f+11\,g)\log\left(|x-1|\right)\\ &\quad-\frac{1}{432}\,(35\,d+58\,e+92\,f+136\,g)\log\left(|x-2|\right)\\ &\quad-\frac{(5\,d+4\,e+8\,f+10\,g)x^2-6\,(d+f)x-5\,d-10\,e-8\,f-16\,g}{36\,(x+1)(x-1)(x-2)}\end{aligned}$$

input
```
integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

output
```
1/144*(d - 2*e + 4*f - 8*g)*log(abs(x + 2)) + 1/108*(2*d + e - 4*f + 7*g)*
log(abs(x + 1)) + 1/36*(2*d + 5*e + 8*f + 11*g)*log(abs(x - 1)) - 1/432*(3
5*d + 58*e + 92*f + 136*g)*log(abs(x - 2)) - 1/36*((5*d + 4*e + 8*f + 10*g
)*x^2 - 6*(d + f)*x - 5*d - 10*e - 8*f - 16*g)/((x + 1)*(x - 1)*(x - 2))
```

Mupad [B] (verification not implemented)

Time = 18.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx$$
$$=\ln(x-1)\left(\frac{d}{18}+\frac{5\,e}{36}+\frac{2\,f}{9}+\frac{11\,g}{36}\right)+\ln(x+2)\left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}\right)$$
$$+\ln(x+1)\left(\frac{d}{54}+\frac{e}{108}-\frac{f}{27}+\frac{7\,g}{108}\right)-\ln(x-2)\left(\frac{35\,d}{432}+\frac{29\,e}{216}+\frac{23\,f}{108}+\frac{17\,g}{54}\right)$$
$$-\frac{\left(-\frac{5\,d}{36}-\frac{e}{9}-\frac{2\,f}{9}-\frac{5\,g}{18}\right)\,x^2+\left(\frac{d}{6}+\frac{f}{6}\right)\,x+\frac{5\,d}{36}+\frac{5\,e}{18}+\frac{2\,f}{9}+\frac{4\,g}{9}}{-x^3+2\,x^2+x-2}$$

input
```
int(((x + 2)*(d + e*x + f*x^2 + g*x^3))/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
log(x - 1)*(d/18 + (5*e)/36 + (2*f)/9 + (11*g)/36) + log(x + 2)*(d/144 - e
/72 + f/36 - g/18) + log(x + 1)*(d/54 + e/108 - f/27 + (7*g)/108) - log(x
- 2)*((35*d)/432 + (29*e)/216 + (23*f)/108 + (17*g)/54) - ((5*d)/36 + (5*e
)/18 + (2*f)/9 + (4*g)/9 - x^2*((5*d)/36 + e/9 + (2*f)/9 + (5*g)/18) + x*(
d/6 + f/6))/(x + 2*x^2 - x^3 - 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 625, normalized size of antiderivative = 4.43

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
( - 35*log(x - 2)*d*x**3 + 70*log(x - 2)*d*x**2 + 35*log(x - 2)*d*x - 70*l
og(x - 2)*d - 58*log(x - 2)*e*x**3 + 116*log(x - 2)*e*x**2 + 58*log(x - 2)
*e*x - 116*log(x - 2)*e - 92*log(x - 2)*f*x**3 + 184*log(x - 2)*f*x**2 + 9
2*log(x - 2)*f*x - 184*log(x - 2)*f - 136*log(x - 2)*g*x**3 + 272*log(x -
2)*g*x**2 + 136*log(x - 2)*g*x - 272*log(x - 2)*g + 24*log(x - 1)*d*x**3 -
 48*log(x - 1)*d*x**2 - 24*log(x - 1)*d*x + 48*log(x - 1)*d + 60*log(x - 1
)*e*x**3 - 120*log(x - 1)*e*x**2 - 60*log(x - 1)*e*x + 120*log(x - 1)*e +
96*log(x - 1)*f*x**3 - 192*log(x - 1)*f*x**2 - 96*log(x - 1)*f*x + 192*log
(x - 1)*f + 132*log(x - 1)*g*x**3 - 264*log(x - 1)*g*x**2 - 132*log(x - 1)
*g*x + 264*log(x - 1)*g + 3*log(x + 2)*d*x**3 - 6*log(x + 2)*d*x**2 - 3*lo
g(x + 2)*d*x + 6*log(x + 2)*d - 6*log(x + 2)*e*x**3 + 12*log(x + 2)*e*x**2
 + 6*log(x + 2)*e*x - 12*log(x + 2)*e + 12*log(x + 2)*f*x**3 - 24*log(x +
2)*f*x**2 - 12*log(x + 2)*f*x + 24*log(x + 2)*f - 24*log(x + 2)*g*x**3 + 4
8*log(x + 2)*g*x**2 + 24*log(x + 2)*g*x - 48*log(x + 2)*g + 8*log(x + 1)*d
*x**3 - 16*log(x + 1)*d*x**2 - 8*log(x + 1)*d*x + 16*log(x + 1)*d + 4*log(
x + 1)*e*x**3 - 8*log(x + 1)*e*x**2 - 4*log(x + 1)*e*x + 8*log(x + 1)*e -
16*log(x + 1)*f*x**3 + 32*log(x + 1)*f*x**2 + 16*log(x + 1)*f*x - 32*log(x
 + 1)*f + 28*log(x + 1)*g*x**3 - 56*log(x + 1)*g*x**2 - 28*log(x + 1)*g*x
+ 56*log(x + 1)*g - 30*d*x**3 + 102*d*x - 24*e*x**3 + 24*e*x + 72*e - 48*f
*x**3 + 120*f*x - 60*g*x**3 + 60*g*x + 72*g)/(432*(x**3 - 2*x**2 - x + ...
```

3.98 $\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 158

$$\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx = \frac{d+e+f+g+h}{12(1-x)} + \frac{d+2e+4f+8g+16h}{36(2-x)} - \frac{d-e+f-g+h}{36(1+x)} + \frac{1}{36}(2d+5e+8f+11g+14h)\log(1-x) - \frac{1}{432}(35d+58e+92f+136g+176h)\log(2-x) + \frac{1}{108}(2d+e-4f+7g-10h)\log(1+x) + \frac{1}{144}(d-2e+4f-8g+16h)\log(2+x)$$

output

```
(d+e+f+g+h)/(12-12*x)+(d+2*e+4*f+8*g+16*h)/(72-36*x)-(d-e+f-g+h)/(36+36*x)
+1/36*(2*d+5*e+8*f+11*g+14*h)*ln(1-x)-1/432*(35*d+58*e+92*f+136*g+176*h)*l
n(2-x)+1/108*(2*d+e-4*f+7*g-10*h)*ln(1+x)+1/144*(d-2*e+4*f-8*g+16*h)*ln(2+
x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.07

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{\left(4-5x^2+x^4\right)^2}\,dx$$
$$= \frac{1}{432}\left(\frac{12(d(5+6x-5x^2)+2(8g+10h+3hx-5gx^2-10hx^2+f(4+3x-4x^2)+e(5-2x^2)))}{2-x-2x^2+x^3} + 12(2d+5e+8f+11g+14h)\log(1-x)-(35d+58e+92f+136g+176h)\log(2-x) + 4(2d+e-4f+7g-10h)\log(1+x)+3(d-2e+4f-8g+16h)\log(2+x)\right)$$

input
```
Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2,
x]
```

output
```
((12*(d*(5 + 6*x - 5*x^2) + 2*(8*g + 10*h + 3*h*x - 5*g*x^2 - 10*h*x^2 + f
*(4 + 3*x - 4*x^2) + e*(5 - 2*x^2))))/(2 - x - 2*x^2 + x^3) + 12*(2*d + 5*
e + 8*f + 11*g + 14*h)*Log[1 - x] - (35*d + 58*e + 92*f + 136*g + 176*h)*L
og[2 - x] + 4*(2*d + e - 4*f + 7*g - 10*h)*Log[1 + x] + 3*(d - 2*e + 4*f -
 8*g + 16*h)*Log[2 + x])/432
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+2)\left(d+ex+fx^2+gx^3+hx^4\right)}{\left(x^4-5x^2+4\right)^2}dx$$
$$\Big\downarrow \; 2019$$
$$\int \frac{d+ex+fx^2+gx^3+hx^4}{(x+2)\left(x^3-2x^2-x+2\right)^2}dx$$
$$\Big\downarrow \; 2462$$

$$\int \left(\frac{-35d - 58e - 92f - 136g - 176h}{432(x-2)} + \frac{2d + 5e + 8f + 11g + 14h}{36(x-1)} + \frac{2d + e - 4f + 7g - 10h}{108(x+1)} + \frac{d - 2e + 4f -}{144(x+}\right.$$

$\downarrow$ 2009

$$-\frac{d-e+f-g+h}{36(x+1)} + \frac{d+e+f+g+h}{12(1-x)} + \frac{d+2e+4f+8g+16h}{36(2-x)} + \frac{1}{36}\log(1-x)(2d+5e+ 8f+11g+14h) - \frac{1}{432}\log(2-x)(35d+58e+92f+136g+176h) + \frac{1}{108}\log(x+1)(2d+e- 4f+7g-10h) + \frac{1}{144}\log(x+2)(d-2e+4f-8g+16h)$$

input
```
Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2,x]
```

output
```
(d + e + f + g + h)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*h)/(36*(2 - x
)) - (d - e + f - g + h)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 11*g + 14*h)*L
og[1 - x])/36 - ((35*d + 58*e + 92*f + 136*g + 176*h)*Log[2 - x])/432 + ((
2*d + e - 4*f + 7*g - 10*h)*Log[1 + x])/108 + ((d - 2*e + 4*f - 8*g + 16*h
)*Log[2 + x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

method	result
default	$\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}-\frac{17g}{54}-\frac{11h}{27}\right)\ln(x-2)-\frac{\frac{d}{36}+\frac{e}{18}+\frac{f}{9}+\frac{2g}{9}+\frac{4h}{9}}{x-2}-\frac{\frac{d}{36}-\frac{e}{36}+\frac{f}{36}-\frac{g}{36}+\frac{h}{36}}{1+x}+\left(\frac{d}{54}+\frac{e}{108}\right.$
norman	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}-\frac{5h}{9}\right)x^3+\left(\frac{17d}{36}+\frac{5e}{18}+\frac{5f}{9}+\frac{4g}{9}+\frac{8h}{9}\right)x+\left(-\frac{d}{9}-\frac{2e}{9}-\frac{5f}{18}-\frac{5g}{9}-\frac{17h}{18}\right)x^2+\frac{5d}{18}+\frac{5e}{9}+\frac{10h}{9}+\frac{4f}{9}+\frac{8g}{9}}{x^4-5x^2+4}+\left(-\frac{35d}{432}\right.$
risch	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}-\frac{5h}{9}\right)x^2+\left(\frac{h}{6}+\frac{f}{6}+\frac{d}{6}\right)x+\frac{5d}{36}+\frac{5e}{18}+\frac{2f}{9}+\frac{4g}{9}+\frac{5h}{9}}{x^3-2x^2-x+2}+\frac{\ln(x-1)d}{18}+\frac{5\ln(x-1)e}{36}+\frac{2\ln(x-1)f}{9}+\frac{11\ln(x-}{36}$
parallelrisch	$\frac{96f+192g+72dx+72fx+240h+60d+120e-96fx^2-116\ln(x-2)e-12\ln(x+2)e-48\ln(x-1)x^2d-8\ln(1+x)xd-4\ln(1+x)xe}{}$

input
```
int((x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOS
E)
```

output
```
(-35/432*d-29/216*e-23/108*f-17/54*g-11/27*h)*ln(x-2)-(1/36*d+1/18*e+1/9*f
+2/9*g+4/9*h)/(x-2)-(1/36*d-1/36*e+1/36*f-1/36*g+1/36*h)/(1+x)+(1/54*d+1/1
08*e-1/27*f+7/108*g-5/54*h)*ln(1+x)-(1/12*d+1/12*e+1/12*f+1/12*g+1/12*h)/(
x-1)+(1/18*d+5/36*e+2/9*f+11/36*g+7/18*h)*ln(x-1)+(1/144*d-1/72*e+1/36*f-1
/18*g+1/9*h)*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(140) = 280.

Time = 3.09 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.38

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{\left(4-5x^2+x^4\right)^2}\,dx = -\frac{12\,(5\,d+4\,e+8\,f+10\,g+20\,h)x^2-72\,(d+f+h)x-3\,((d-2\,e+4\,f-8\,g+16\,h)x^3-2\,(d-2\,e}{}$$

input
```
integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fr
icas")
```

output
```
-1/432*(12*(5*d + 4*e + 8*f + 10*g + 20*h)*x^2 - 72*(d + f + h)*x - 3*((d
- 2*e + 4*f - 8*g + 16*h)*x^3 - 2*(d - 2*e + 4*f - 8*g + 16*h)*x^2 - (d -
2*e + 4*f - 8*g + 16*h)*x + 2*d - 4*e + 8*f - 16*g + 32*h)*log(x + 2) - 4*
((2*d + e - 4*f + 7*g - 10*h)*x^3 - 2*(2*d + e - 4*f + 7*g - 10*h)*x^2 - (
2*d + e - 4*f + 7*g - 10*h)*x + 4*d + 2*e - 8*f + 14*g - 20*h)*log(x + 1)
- 12*((2*d + 5*e + 8*f + 11*g + 14*h)*x^3 - 2*(2*d + 5*e + 8*f + 11*g + 14
*h)*x^2 - (2*d + 5*e + 8*f + 11*g + 14*h)*x + 4*d + 10*e + 16*f + 22*g + 2
8*h)*log(x - 1) + ((35*d + 58*e + 92*f + 136*g + 176*h)*x^3 - 2*(35*d + 58
*e + 92*f + 136*g + 176*h)*x^2 - (35*d + 58*e + 92*f + 136*g + 176*h)*x +
70*d + 116*e + 184*f + 272*g + 352*h)*log(x - 2) - 60*d - 120*e - 96*f - 1
92*g - 240*h)/(x^3 - 2*x^2 - x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((2+x)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{\left(4-5x^2+x^4\right)^2}\,dx \\
&= \frac{1}{144}\,(d-2\,e+4\,f-8\,g+16\,h)\log(x+2) \\
&\quad + \frac{1}{108}\,(2\,d+e-4\,f+7\,g-10\,h)\log(x+1) \\
&\quad + \frac{1}{36}\,(2\,d+5\,e+8\,f+11\,g+14\,h)\log(x-1) \\
&\quad - \frac{1}{432}\,(35\,d+58\,e+92\,f+136\,g+176\,h)\log(x-2) \\
&\quad - \frac{(5\,d+4\,e+8\,f+10\,g+20\,h)x^2-6\,(d+f+h)x-5\,d-10\,e-8\,f-16\,g-20\,h}{36\,(x^3-2\,x^2-x+2)}
\end{aligned}$$

input
```
integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="ma
xima")
```

output
```
1/144*(d - 2*e + 4*f - 8*g + 16*h)*log(x + 2) + 1/108*(2*d + e - 4*f + 7*g
 - 10*h)*log(x + 1) + 1/36*(2*d + 5*e + 8*f + 11*g + 14*h)*log(x - 1) - 1/
432*(35*d + 58*e + 92*f + 136*g + 176*h)*log(x - 2) - 1/36*((5*d + 4*e + 8
*f + 10*g + 20*h)*x^2 - 6*(d + f + h)*x - 5*d - 10*e - 8*f - 16*g - 20*h)/
(x^3 - 2*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int \frac{(2+x)\,(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx \\
&= \frac{1}{144}\,(d-2\,e+4\,f-8\,g+16\,h)\log\left(|x+2|\right) \\
&\quad+\frac{1}{108}\,(2\,d+e-4\,f+7\,g-10\,h)\log\left(|x+1|\right) \\
&\quad+\frac{1}{36}\,(2\,d+5\,e+8\,f+11\,g+14\,h)\log\left(|x-1|\right) \\
&\quad-\frac{1}{432}\,(35\,d+58\,e+92\,f+136\,g+176\,h)\log\left(|x-2|\right) \\
&\quad-\frac{(5\,d+4\,e+8\,f+10\,g+20\,h)x^2-6\,(d+f+h)x-5\,d-10\,e-8\,f-16\,g-20\,h}{36\,(x+1)(x-1)(x-2)}
\end{aligned}$$

input
```
integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="gi
ac")
```

output
```
1/144*(d - 2*e + 4*f - 8*g + 16*h)*log(abs(x + 2)) + 1/108*(2*d + e - 4*f
+ 7*g - 10*h)*log(abs(x + 1)) + 1/36*(2*d + 5*e + 8*f + 11*g + 14*h)*log(a
bs(x - 1)) - 1/432*(35*d + 58*e + 92*f + 136*g + 176*h)*log(abs(x - 2)) -
1/36*((5*d + 4*e + 8*f + 10*g + 20*h)*x^2 - 6*(d + f + h)*x - 5*d - 10*e -
 8*f - 16*g - 20*h)/((x + 1)*(x - 1)*(x - 2))
```

Mupad [B] (verification not implemented)

Time = 18.78 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$$
$$= \ln(x-1)\left(\frac{d}{18}+\frac{5\,e}{36}+\frac{2\,f}{9}+\frac{11\,g}{36}+\frac{7\,h}{18}\right)$$
$$-\frac{\left(-\frac{5\,d}{36}-\frac{e}{9}-\frac{2\,f}{9}-\frac{5\,g}{18}-\frac{5\,h}{9}\right)x^2+\left(\frac{d}{6}+\frac{f}{6}+\frac{h}{6}\right)x+\frac{5\,d}{36}+\frac{5\,e}{18}+\frac{2\,f}{9}+\frac{4\,g}{9}+\frac{5\,h}{9}}{-x^3+2\,x^2+x-2}$$
$$+\ln(x+2)\left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}+\frac{h}{9}\right)+\ln(x+1)\left(\frac{d}{54}+\frac{e}{108}-\frac{f}{27}+\frac{7\,g}{108}-\frac{5\,h}{54}\right)$$
$$-\ln(x-2)\left(\frac{35\,d}{432}+\frac{29\,e}{216}+\frac{23\,f}{108}+\frac{17\,g}{54}+\frac{11\,h}{27}\right)$$

input
```
int(((x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^2 + 4)^2,x)
```

output
```
log(x - 1)*(d/18 + (5*e)/36 + (2*f)/9 + (11*g)/36 + (7*h)/18) - ((5*d)/36
+ (5*e)/18 + (2*f)/9 + (4*g)/9 + (5*h)/9 - x^2*((5*d)/36 + e/9 + (2*f)/9 +
 (5*g)/18 + (5*h)/9) + x*(d/6 + f/6 + h/6))/(x + 2*x^2 - x^3 - 2) + log(x
+ 2)*(d/144 - e/72 + f/36 - g/18 + h/9) + log(x + 1)*(d/54 + e/108 - f/27
+ (7*g)/108 - (5*h)/54) - log(x - 2)*((35*d)/432 + (29*e)/216 + (23*f)/108
 + (17*g)/54 + (11*h)/27)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 775, normalized size of antiderivative = 4.91

$$\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output
```
( - 35*log(x - 2)*d*x**3 + 70*log(x - 2)*d*x**2 + 35*log(x - 2)*d*x - 70*l
og(x - 2)*d - 58*log(x - 2)*e*x**3 + 116*log(x - 2)*e*x**2 + 58*log(x - 2)
*e*x - 116*log(x - 2)*e - 92*log(x - 2)*f*x**3 + 184*log(x - 2)*f*x**2 + 9
2*log(x - 2)*f*x - 184*log(x - 2)*f - 136*log(x - 2)*g*x**3 + 272*log(x -
2)*g*x**2 + 136*log(x - 2)*g*x - 272*log(x - 2)*g - 176*log(x - 2)*h*x**3
+ 352*log(x - 2)*h*x**2 + 176*log(x - 2)*h*x - 352*log(x - 2)*h + 24*log(x
 - 1)*d*x**3 - 48*log(x - 1)*d*x**2 - 24*log(x - 1)*d*x + 48*log(x - 1)*d
+ 60*log(x - 1)*e*x**3 - 120*log(x - 1)*e*x**2 - 60*log(x - 1)*e*x + 120*l
og(x - 1)*e + 96*log(x - 1)*f*x**3 - 192*log(x - 1)*f*x**2 - 96*log(x - 1)
*f*x + 192*log(x - 1)*f + 132*log(x - 1)*g*x**3 - 264*log(x - 1)*g*x**2 -
132*log(x - 1)*g*x + 264*log(x - 1)*g + 168*log(x - 1)*h*x**3 - 336*log(x
- 1)*h*x**2 - 168*log(x - 1)*h*x + 336*log(x - 1)*h + 3*log(x + 2)*d*x**3
- 6*log(x + 2)*d*x**2 - 3*log(x + 2)*d*x + 6*log(x + 2)*d - 6*log(x + 2)*e
*x**3 + 12*log(x + 2)*e*x**2 + 6*log(x + 2)*e*x - 12*log(x + 2)*e + 12*log
(x + 2)*f*x**3 - 24*log(x + 2)*f*x**2 - 12*log(x + 2)*f*x + 24*log(x + 2)*
f - 24*log(x + 2)*g*x**3 + 48*log(x + 2)*g*x**2 + 24*log(x + 2)*g*x - 48*l
og(x + 2)*g + 48*log(x + 2)*h*x**3 - 96*log(x + 2)*h*x**2 - 48*log(x + 2)*
h*x + 96*log(x + 2)*h + 8*log(x + 1)*d*x**3 - 16*log(x + 1)*d*x**2 - 8*log
(x + 1)*d*x + 16*log(x + 1)*d + 4*log(x + 1)*e*x**3 - 8*log(x + 1)*e*x**2
- 4*log(x + 1)*e*x + 8*log(x + 1)*e - 16*log(x + 1)*f*x**3 + 32*log(x +...
```

3.99 $\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 177

$$\begin{aligned}
&\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx \\
&= \frac{d+e+f+g+h+i}{12(1-x)}+\frac{d+2e+4f+8g+16h+32i}{36(2-x)} \\
&\quad -\frac{d-e+f-g+h-i}{36(1+x)}+\frac{1}{36}(2d+5e+8f+11g+14h+17i)\log(1-x) \\
&\quad -\frac{1}{432}(35d+58e+92f+136g+176h+160i)\log(2-x) \\
&\quad +\frac{1}{108}(2d+e-4f+7g-10h+13i)\log(1+x) \\
&\quad +\frac{1}{144}(d-2e+4f-8g+16h-32i)\log(2+x)
\end{aligned}$$

output

```
(d+e+f+g+h+i)/(12-12*x)+(d+2*e+4*f+8*g+16*h+32*i)/(72-36*x)-(d-e+f-g+h-i)/
(36+36*x)+1/36*(2*d+5*e+8*f+11*g+14*h+17*i)*ln(1-x)-1/432*(35*d+58*e+92*f+
136*g+176*h+160*i)*ln(2-x)+1/108*(2*d+e-4*f+7*g-10*h+13*i)*ln(1+x)+1/144*(
d-2*e+4*f-8*g+16*h-32*i)*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx$$
$$= \frac{5d+10e+8f+16g+20h+40i+6dx+6fx+6hx-5dx^2-4ex^2-8fx^2-10gx^2-20hx^2-34ix^2}{36\left(2-x-2x^2+x^3\right)}$$
$$+\frac{1}{36}(2d+5e+8f+11g+14h+17i)\log(1-x)$$
$$+\frac{1}{432}(-35d-58e-92f-136g-176h-160i)\log(2-x)$$
$$+\frac{1}{108}(2d+e-4f+7g-10h+13i)\log(1+x)$$
$$+\frac{1}{144}(d-2e+4f-8g+16h-32i)\log(2+x)$$

input
```
Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 +
 x^4)^2,x]
```

output
```
(5*d + 10*e + 8*f + 16*g + 20*h + 40*i + 6*d*x + 6*f*x + 6*h*x - 5*d*x^2 -
 4*e*x^2 - 8*f*x^2 - 10*g*x^2 - 20*h*x^2 - 34*i*x^2)/(36*(2 - x - 2*x^2 +
x^3)) + ((2*d + 5*e + 8*f + 11*g + 14*h + 17*i)*Log[1 - x])/36 + ((-35*d -
 58*e - 92*f - 136*g - 176*h - 160*i)*Log[2 - x])/432 + ((2*d + e - 4*f +
7*g - 10*h + 13*i)*Log[1 + x])/108 + ((d - 2*e + 4*f - 8*g + 16*h - 32*i)*
Log[2 + x])/144
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+2)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(x^4-5x^2+4\right)^2}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(x+2)\left(x^3-2x^2-x+2\right)^2}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-35d-58e-92f-136g-176h-160i}{432(x-2)}+\frac{2d+5e+8f+11g+14h+17i}{36(x-1)}+\frac{2d+e-4f+7g-10h+1}{108(x+1)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d-e+f-g+h-i}{36(x+1)}+\frac{d+e+f+g+h+i}{12(1-x)}+\frac{d+2e+4f+8g+16h+32i}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g+14h+17i)-\frac{1}{432}\log(2-x)(35d+58e+92f+136g+176h+160i)+\frac{1}{108}\log(x+1)(2d+e-4f+7g-10h+13i)+\frac{1}{144}\log(x+2)(d-2e+4f-8g+16h-32i)$$

input
```
Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4)^
2,x]
```

output
```
(d + e + f + g + h + i)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*h + 32*i)
/(36*(2 - x)) - (d - e + f - g + h - i)/(36*(1 + x)) + ((2*d + 5*e + 8*f +
 11*g + 14*h + 17*i)*Log[1 - x])/36 - ((35*d + 58*e + 92*f + 136*g + 176*h
 + 160*i)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g - 10*h + 13*i)*Log[1 + x
])/108 + ((d - 2*e + 4*f - 8*g + 16*h - 32*i)*Log[2 + x])/144
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

method	result
default	$\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}-\frac{17g}{54}-\frac{11h}{27}-\frac{10i}{27}\right)\ln\left(x-2\right)-\frac{\frac{d}{36}+\frac{e}{18}+\frac{f}{9}+\frac{2g}{9}+\frac{4h}{9}+\frac{8i}{9}}{x-2}-\frac{\frac{d}{36}-\frac{e}{36}+\frac{f}{36}-\frac{g}{36}+\frac{h}{36}-\frac{i}{36}}{1+x}$
norman	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}-\frac{5h}{9}-\frac{17i}{18}\right)x^3+\left(\frac{17d}{36}+\frac{5e}{18}+\frac{5f}{9}+\frac{4g}{9}+\frac{8h}{9}+\frac{10i}{9}\right)x+\left(-\frac{d}{9}-\frac{2e}{9}-\frac{5f}{18}-\frac{5g}{9}-\frac{17h}{18}-\frac{17i}{9}\right)x^2+\frac{5d}{18}+\frac{5e}{9}+\frac{10h}{9}+\frac{20i}{9}+}{x^4-5x^2+4}$
risch	$-\frac{29\ln(2-x)e}{216}-\frac{17\ln(2-x)g}{54}-\frac{\ln(x+2)e}{72}-\frac{5\ln(-x-1)h}{54}+\frac{7\ln(-x-1)g}{108}-\frac{35\ln(2-x)d}{432}-\frac{23\ln(2-x)f}{108}-\ln(-$
parallelrisch	Expression too large to display

input
```
int((x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURN
VERBOSE)
```

output
```
(-35/432*d-29/216*e-23/108*f-17/54*g-11/27*h-10/27*i)*ln(x-2)-(1/36*d+1/18
*e+1/9*f+2/9*g+4/9*h+8/9*i)/(x-2)-(1/36*d-1/36*e+1/36*f-1/36*g+1/36*h-1/36
*i)/(1+x)+(1/54*d+1/108*e-1/27*f+7/108*g-5/54*h+13/108*i)*ln(1+x)-(1/12*d+
1/12*e+1/12*f+1/12*g+1/12*h+1/12*i)/(x-1)+(1/18*d+5/36*e+2/9*f+11/36*g+7/1
8*h+17/36*i)*ln(x-1)+(1/144*d-1/72*e+1/36*f-1/18*g+1/9*h-2/9*i)*ln(x+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(159) = 318.

Time = 21.60 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.43

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx =$$
$$-\frac{12\,(5\,d+4\,e+8\,f+10\,g+20\,h+34\,i)x^2-72\,(d+f+h)x-3\,((d-2\,e+4\,f-8\,g+16\,h-32\,i)x}{}$$

input
```
integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorit
hm="fricas")
```

output
```
-1/432*(12*(5*d + 4*e + 8*f + 10*g + 20*h + 34*i)*x^2 - 72*(d + f + h)*x -
 3*((d - 2*e + 4*f - 8*g + 16*h - 32*i)*x^3 - 2*(d - 2*e + 4*f - 8*g + 16*
h - 32*i)*x^2 - (d - 2*e + 4*f - 8*g + 16*h - 32*i)*x + 2*d - 4*e + 8*f -
16*g + 32*h - 64*i)*log(x + 2) - 4*((2*d + e - 4*f + 7*g - 10*h + 13*i)*x^
3 - 2*(2*d + e - 4*f + 7*g - 10*h + 13*i)*x^2 - (2*d + e - 4*f + 7*g - 10*
h + 13*i)*x + 4*d + 2*e - 8*f + 14*g - 20*h + 26*i)*log(x + 1) - 12*((2*d
+ 5*e + 8*f + 11*g + 14*h + 17*i)*x^3 - 2*(2*d + 5*e + 8*f + 11*g + 14*h +
 17*i)*x^2 - (2*d + 5*e + 8*f + 11*g + 14*h + 17*i)*x + 4*d + 10*e + 16*f
+ 22*g + 28*h + 34*i)*log(x - 1) + ((35*d + 58*e + 92*f + 136*g + 176*h +
160*i)*x^3 - 2*(35*d + 58*e + 92*f + 136*g + 176*h + 160*i)*x^2 - (35*d +
58*e + 92*f + 136*g + 176*h + 160*i)*x + 70*d + 116*e + 184*f + 272*g + 35
2*h + 320*i)*log(x - 2) - 60*d - 120*e - 96*f - 192*g - 240*h - 480*i)/(x^
3 - 2*x^2 - x + 2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((2+x)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.92

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx$$
$$= \frac{1}{144}\left(d-2\,e+4\,f-8\,g+16\,h-32\,i\right)\log\left(x+2\right)$$
$$+ \frac{1}{108}\left(2\,d+e-4\,f+7\,g-10\,h+13\,i\right)\log\left(x+1\right)$$
$$+ \frac{1}{36}\left(2\,d+5\,e+8\,f+11\,g+14\,h+17\,i\right)\log\left(x-1\right)$$
$$- \frac{1}{432}\left(35\,d+58\,e+92\,f+136\,g+176\,h+160\,i\right)\log\left(x-2\right)$$
$$- \frac{(5\,d+4\,e+8\,f+10\,g+20\,h+34\,i)x^2-6\,(d+f+h)x-5\,d-10\,e-8\,f-16\,g-20\,h-40\,i}{36\,(x^3-2\,x^2-x+2)}$$

input
```
integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorit
hm="maxima")
```

output
```
1/144*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + 1/108*(2*d + e - 4*
f + 7*g - 10*h + 13*i)*log(x + 1) + 1/36*(2*d + 5*e + 8*f + 11*g + 14*h +
17*i)*log(x - 1) - 1/432*(35*d + 58*e + 92*f + 136*g + 176*h + 160*i)*log(
x - 2) - 1/36*((5*d + 4*e + 8*f + 10*g + 20*h + 34*i)*x^2 - 6*(d + f + h)*
x - 5*d - 10*e - 8*f - 16*g - 20*h - 40*i)/(x^3 - 2*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx$$
$$= \frac{1}{144}\left(d-2\,e+4\,f-8\,g+16\,h-32\,i\right)\log\left(|x+2|\right)$$
$$+ \frac{1}{108}\left(2\,d+e-4\,f+7\,g-10\,h+13\,i\right)\log\left(|x+1|\right)$$
$$+ \frac{1}{36}\left(2\,d+5\,e+8\,f+11\,g+14\,h+17\,i\right)\log\left(|x-1|\right)$$
$$- \frac{1}{432}\left(35\,d+58\,e+92\,f+136\,g+176\,h+160\,i\right)\log\left(|x-2|\right)$$
$$- \frac{(5\,d+4\,e+8\,f+10\,g+20\,h+34\,i)x^2-6\,(d+f+h)x-5\,d-10\,e-8\,f-16\,g-20\,h-40\,i}{36\,(x+1)(x-1)(x-2)}$$

input
```
integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorit
hm="giac")
```

output
```
1/144*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(abs(x + 2)) + 1/108*(2*d + e
 - 4*f + 7*g - 10*h + 13*i)*log(abs(x + 1)) + 1/36*(2*d + 5*e + 8*f + 11*g
 + 14*h + 17*i)*log(abs(x - 1)) - 1/432*(35*d + 58*e + 92*f + 136*g + 176*
h + 160*i)*log(abs(x - 2)) - 1/36*((5*d + 4*e + 8*f + 10*g + 20*h + 34*i)*
x^2 - 6*(d + f + h)*x - 5*d - 10*e - 8*f - 16*g - 20*h - 40*i)/((x + 1)*(x
 - 1)*(x - 2))
```

Mupad [B] (verification not implemented)

Time = 19.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.96

$$\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$$
$$= \ln(x-1)\left(\frac{d}{18}+\frac{5e}{36}+\frac{2f}{9}+\frac{11g}{36}+\frac{7h}{18}+\frac{17i}{36}\right)$$
$$+\ln(x+2)\left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}+\frac{h}{9}-\frac{2i}{9}\right)$$
$$+\ln(x+1)\left(\frac{d}{54}+\frac{e}{108}-\frac{f}{27}+\frac{7g}{108}-\frac{5h}{54}+\frac{13i}{108}\right)$$
$$-\ln(x-2)\left(\frac{35d}{432}+\frac{29e}{216}+\frac{23f}{108}+\frac{17g}{54}+\frac{11h}{27}+\frac{10i}{27}\right)$$
$$-\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}-\frac{5h}{9}-\frac{17i}{18}\right)x^2+\left(\frac{d}{6}+\frac{f}{6}+\frac{h}{6}\right)x+\frac{5d}{36}+\frac{5e}{18}+\frac{2f}{9}+\frac{4g}{9}+\frac{5h}{9}+\frac{10i}{9}}{-x^3+2x^2+x-2}$$

input
```
int(((x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^4 - 5*x^2 + 4)^
2,x)
```

output
```
log(x - 1)*(d/18 + (5*e)/36 + (2*f)/9 + (11*g)/36 + (7*h)/18 + (17*i)/36)
+ log(x + 2)*(d/144 - e/72 + f/36 - g/18 + h/9 - (2*i)/9) + log(x + 1)*(d/
54 + e/108 - f/27 + (7*g)/108 - (5*h)/54 + (13*i)/108) - log(x - 2)*((35*d
)/432 + (29*e)/216 + (23*f)/108 + (17*g)/54 + (11*h)/27 + (10*i)/27) - ((5
*d)/36 + (5*e)/18 + (2*f)/9 + (4*g)/9 + (5*h)/9 + (10*i)/9 - x^2*((5*d)/36
 + e/9 + (2*f)/9 + (5*g)/18 + (5*h)/9 + (17*i)/18) + x*(d/6 + f/6 + h/6))/
(x + 2*x^2 - x^3 - 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 928, normalized size of antiderivative = 5.24

$$\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

output

```
( - 35*log(x - 2)*d*x**3 + 70*log(x - 2)*d*x**2 + 35*log(x - 2)*d*x - 70*l
og(x - 2)*d - 58*log(x - 2)*e*x**3 + 116*log(x - 2)*e*x**2 + 58*log(x - 2)
*e*x - 116*log(x - 2)*e - 92*log(x - 2)*f*x**3 + 184*log(x - 2)*f*x**2 + 9
2*log(x - 2)*f*x - 184*log(x - 2)*f - 136*log(x - 2)*g*x**3 + 272*log(x -
2)*g*x**2 + 136*log(x - 2)*g*x - 272*log(x - 2)*g - 176*log(x - 2)*h*x**3
+ 352*log(x - 2)*h*x**2 + 176*log(x - 2)*h*x - 352*log(x - 2)*h - 160*log(
x - 2)*i*x**3 + 320*log(x - 2)*i*x**2 + 160*log(x - 2)*i*x - 320*log(x - 2
)*i + 24*log(x - 1)*d*x**3 - 48*log(x - 1)*d*x**2 - 24*log(x - 1)*d*x + 48
*log(x - 1)*d + 60*log(x - 1)*e*x**3 - 120*log(x - 1)*e*x**2 - 60*log(x -
1)*e*x + 120*log(x - 1)*e + 96*log(x - 1)*f*x**3 - 192*log(x - 1)*f*x**2 -
 96*log(x - 1)*f*x + 192*log(x - 1)*f + 132*log(x - 1)*g*x**3 - 264*log(x
- 1)*g*x**2 - 132*log(x - 1)*g*x + 264*log(x - 1)*g + 168*log(x - 1)*h*x**
3 - 336*log(x - 1)*h*x**2 - 168*log(x - 1)*h*x + 336*log(x - 1)*h + 204*lo
g(x - 1)*i*x**3 - 408*log(x - 1)*i*x**2 - 204*log(x - 1)*i*x + 408*log(x -
 1)*i + 3*log(x + 2)*d*x**3 - 6*log(x + 2)*d*x**2 - 3*log(x + 2)*d*x + 6*l
og(x + 2)*d - 6*log(x + 2)*e*x**3 + 12*log(x + 2)*e*x**2 + 6*log(x + 2)*e*
x - 12*log(x + 2)*e + 12*log(x + 2)*f*x**3 - 24*log(x + 2)*f*x**2 - 12*log
(x + 2)*f*x + 24*log(x + 2)*f - 24*log(x + 2)*g*x**3 + 48*log(x + 2)*g*x**
2 + 24*log(x + 2)*g*x - 48*log(x + 2)*g + 48*log(x + 2)*h*x**3 - 96*log(x
+ 2)*h*x**2 - 48*log(x + 2)*h*x + 96*log(x + 2)*h - 96*log(x + 2)*i*x**...
```

3.100 $\int (d+ex+fx^2+gx^3)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 724

$$\int (d+ex+fx^2+gx^3)\left(a+bx^2+cx^4\right)^{3/2}\,dx =$$

$$-\frac{(18b^3cd-144abc^2d-8b^4f+57ab^2cf-84a^2c^2f)\,x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{3(b^2-4ac)(2ce-bg)(b+2cx^2)\sqrt{a+bx^2+cx^4}}{256c^3}$$

$$+\frac{x(9b^2cd+90ac^2d-4b^3f+9abcf+3c(9bcd-4b^2f+14acf)x^2)\sqrt{a+bx^2+cx^4}}{315c^2}$$

$$+\frac{(2ce-bg)(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{x(3(3cd+bf)+7cfx^2)\left(a+bx^2+cx^4\right)^{3/2}}{63c}$$

$$+\frac{g\left(a+bx^2+cx^4\right)^{5/2}}{10c}+\frac{3\left(b^2-4ac\right)^2(2ce-bg)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}$$

$$+\frac{\sqrt[4]{a}(18b^3cd-144abc^2d-8b^4f+57ab^2cf-84a^2c^2f)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\right.\right.}{315c^{11/4}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(18b^2cd-27\sqrt{a}bc^{3/2}d-90ac^2d-8b^3f+12\sqrt{a}b^2\sqrt{c}f+33abcf-42a^{3/2}c^{3/2}f\right)\left(\sqrt{a}+\right.}{630c^{11/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/315*(-84*a^2*c^2*f+57*a*b^2*c*f-144*a*b*c^2*d-8*b^4*f+18*b^3*c*d)*x*(c*
x^4+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+c^(1/2)*x^2)-3/256*(-4*a*c+b^2)*(-b*g+
2*c*e)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^3+1/315*x*(9*b^2*c*d+90*a*c^2*d
-4*b^3*f+9*a*b*c*f+3*c*(14*a*c*f-4*b^2*f+9*b*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/
2)/c^2+1/32*(-b*g+2*c*e)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c^2+1/63*x*(7*c
*f*x^2+3*b*f+9*c*d)*(c*x^4+b*x^2+a)^(3/2)/c+1/10*g*(c*x^4+b*x^2+a)^(5/2)/c
+3/512*(-4*a*c+b^2)^2*(-b*g+2*c*e)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+
b*x^2+a)^(1/2))/c^(7/2)+1/315*a^(1/4)*(-84*a^2*c^2*f+57*a*b^2*c*f-144*a*b*
c^2*d-8*b^4*f+18*b^3*c*d)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+
c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)-1/630*a^(1/4)*(b+2*
a^(1/2)*c^(1/2))*(18*b^2*c*d-27*a^(1/2)*b*c^(3/2)*d-90*a*c^2*d-8*b^3*f+12*
a^(1/2)*b^2*c^(1/2)*f+33*a*b*c*f-42*a^(3/2)*c^(3/2)*f)*(a^(1/2)+c^(1/2)*x^
2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arcta
n(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(11/4)/(c*x^4+b*x^
2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.95 (sec) , antiderivative size = 2588, normalized size of antiderivative = 3.57

$$\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\text{Result too large to show}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*Sqrt[c]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(-945*b^4*
g + 2*b^3*c*(945*e + x*(512*f + 315*g*x)) - 12*b^2*c*(-525*a*g + c*x*(192*
d + 105*e*x + 64*f*x^2 + 42*g*x^3)) - 8*b*c^2*(3*a*(525*e + 256*f*x + 147*
g*x^2) + 2*c*x^3*(1152*d + 945*e*x + 800*f*x^2 + 693*g*x^3)) - 16*c^2*(504
*a^2*g + 2*c^2*x^5*(360*d + 7*x*(45*e + 40*f*x + 36*g*x^2)) + a*c*x*(2160*
d + 7*x*(225*e + 16*x*(11*f + 9*g*x))))) + (2304*I)*Sqrt[2]*b^3*c^(3/2)*(b
 - Sqrt[b^2 - 4*a*c])*d*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*Arc
Sinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (18432
*I)*Sqrt[2]*a*b*c^(5/2)*(-b + Sqrt[b^2 - 4*a*c])*d*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2
- 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x]
, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])]) + (7296*I)*Sqrt[2]*a*b^2*c^(3/2)*(b - Sqrt[b^2 - 4*a*c]
)*f*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*...
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 693, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2202, 1490, 1490, 25, 1511, 27, 1416, 1509, 1576, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^{3/2} \left(d+ex+fx^2+gx^3\right) dx$$

$$\Big\downarrow \; 2202$$

$$\int \left(fx^2+d\right) \left(cx^4+bx^2+a\right)^{3/2} dx+\int x\left(gx^2+e\right) \left(cx^4+bx^2+a\right)^{3/2} dx$$

$$\Big\downarrow \; 1490$$

$$\frac{\int \left(\left(-4fb^2+9cdb+14acf\right)x^2+a(18cd-bf)\right)\sqrt{cx^4+bx^2+a}dx}{21c}+$$

$$\int x\left(gx^2+e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3(bf+3cd)+7cfx^2\right)}{63c}$$

$\downarrow$ 1490

$$\frac{\frac{\int -\frac{\left(-8fb^4+18cdb^3+57acfb^2-144ac^2db-84a^2c^2f\right)x^2+a\left(-4fb^3+9cdb^2+24acfb-180ac^2d\right)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14acf-4b^2f+9bcd\right)+9abcf+9\cdots\right)}{15c}}{21c}$$

$$\int x\left(gx^2+e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3(bf+3cd)+7cfx^2\right)}{63c}$$

$\downarrow$ 25

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14acf-4b^2f+9bcd\right)+9abcf+90ac^2d-4b^3f+9b^2cd\right)}{15c}-\frac{\int \frac{\left(-8fb^4+18cdb^3+57acfb^2-144ac^2db-84a^2c^2f\right)x^2+a\left(-4fb^3+9cdb^2+\cdots\right)}{\sqrt{cx^4+bx^2+a}}}{15c}}{21c}$$

$$\int x\left(gx^2+e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3(bf+3cd)+7cfx^2\right)}{63c}$$

$\downarrow$ 1511

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14acf-4b^2f+9bcd\right)+9abcf+90ac^2d-4b^3f+9b^2cd\right)}{15c}-\frac{\frac{\sqrt{a}\left(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}\left(24abcf-180ac^2d-4b^3f+9b^2cd\right)-144ab\cdots\right)}{\sqrt{c}}}{\cdots}}{21c}$$

$$\int x\left(gx^2+e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3(bf+3cd)+7cfx^2\right)}{63c}$$

$\downarrow$ 27

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14acf-4b^2f+9bcd\right)+9abcf+90ac^2d-4b^3f+9b^2cd\right)}{15c}-\frac{\frac{\sqrt{a}\left(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}\left(24abcf-180ac^2d-4b^3f+9b^2cd\right)-144ab\cdots\right)}{\sqrt{c}}}{\cdots}}{21c}$$

$$\int x\left(gx^2+e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3(bf+3cd)+7cfx^2\right)}{63c}$$

$\downarrow$ 1416

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14acf-4b^2f+9bcd\right)+9abcf+90ac^2d-4b^3f+9b^2cd\right)}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}\left(24abcf-\right.\right.}{}}{}$$

$$\int x\left(gx^2+e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3(bf+3cd)+7cfx^2\right)}{63c}$$

$\big\downarrow$ 1509

$$\int x\left(gx^2+e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+$$

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14acf-4b^2f+9bcd\right)+9abcf+90ac^2d-4b^3f+9b^2cd\right)}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}\left(24abcf-\right.\right.}{}}{}$$

$$\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3(bf+3cd)+7cfx^2\right)}{63c}$$

$\big\downarrow$ 1576

$$\frac{1}{2}\int\left(gx^2+e\right)\left(cx^4+bx^2+a\right)^{3/2}dx^2+$$

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14acf-4b^2f+9bcd\right)+9abcf+90ac^2d-4b^3f+9b^2cd\right)}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}\left(24abcf-\right.\right.}{}}{}$$

$$\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3(bf+3cd)+7cfx^2\right)}{63c}$$

$\big\downarrow$ 1160

$$\frac{1}{2}\left(\frac{(2ce-bg)\int\left(cx^4+bx^2+a\right)^{3/2}dx^2}{2c}+\frac{g\left(a+bx^2+cx^4\right)^{5/2}}{5c}\right)+$$

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14acf-4b^2f+9bcd\right)+9abcf+90ac^2d-4b^3f+9b^2cd\right)}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}\left(24abcf-\right.\right.}{}}{}$$

$$\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3(bf+3cd)+7cfx^2\right)}{63c}$$

$\big\downarrow$ 1087

$$\frac{1}{2}\left(\frac{(2ce-bg)\left(\frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{8c}-\frac{3(b^2-4ac)\int\sqrt{cx^4+bx^2+a}\,dx^2}{16c}\right)}{2c}+\frac{g(a+bx^2+cx^4)^{5/2}}{5c}\right)+$$

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}(3cx^2(14acf-4b^2f+9bcd)+9abcf+90ac^2d-4b^3f+9b^2cd)}{15c}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}\left(24abcf-\right.\right.}{}}{}$$

$$\frac{x(a+bx^2+cx^4)^{3/2}(3(bf+3cd)+7cfx^2)}{63c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{(2ce-bg)\left(\frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)}{16c}\right)}{2c}+\frac{g(a+bx^2+\right.}{5c}$$

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}(3cx^2(14acf-4b^2f+9bcd)+9abcf+90ac^2d-4b^3f+9b^2cd)}{15c}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}\left(24abcf-\right.\right.}{}}{}$$

$$\frac{x(a+bx^2+cx^4)^{3/2}(3(bf+3cd)+7cfx^2)}{63c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{(2ce-bg)\left(\frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)}{16c}\right)}{2c}+\frac{g(a+bx^2}{5}\right.$$

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}(3cx^2(14acf-4b^2f+9bcd)+9abcf+90ac^2d-4b^3f+9b^2cd)}{15c}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}(24abcf-}{}}{}$$

$$\frac{x(a+bx^2+cx^4)^{3/2}(3(bf+3cd)+7cfx^2)}{63c}$$

$\downarrow$ 219

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}(3cx^2(14acf-4b^2f+9bcd)+9abcf+90ac^2d-4b^3f+9b^2cd)}{15c}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(-84a^2c^2f+57ab^2cf+\sqrt{a}\sqrt{c}(24abcf-}{}}{}$$

$$\frac{1}{2}\left(\frac{(2ce-bg)\left(\frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)}{2c}+\frac{g(a}{}\right.$$

$$\frac{x(a+bx^2+cx^4)^{3/2}(3(bf+3cd)+7cfx^2)}{63c}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(3*(3*c*d + b*f) + 7*c*f*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(63*c) + ((g*(
a + b*x^2 + c*x^4)^(5/2))/(5*c) + ((2*c*e - b*g)*(((b + 2*c*x^2)*(a + b*x^
2 + c*x^4)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x^2)*Sqrt[a + b*x^2
+ c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4])])/(8*c^(3/2))))/(16*c)))/(2*c))/2 + ((x*(9*b^2*c*d + 90*a
*c^2*d - 4*b^3*f + 9*a*b*c*f + 3*c*(9*b*c*d - 4*b^2*f + 14*a*c*f)*x^2)*Sqr
t[a + b*x^2 + c*x^4])/(15*c) - (-(((18*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f +
 57*a*b^2*c*f - 84*a^2*c^2*f)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sq
rt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1
/4)*(18*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*f +
Sqrt[a]*Sqrt[c]*(9*b^2*c*d - 180*a*c^2*d - 4*b^3*f + 24*a*b*c*f))*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4
)*Sqrt[a + b*x^2 + c*x^4]))/(15*c))/(21*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 1205, normalized size of antiderivative = 1.66

method	result	size
risch	Expression too large to display	1205
elliptic	Expression too large to display	1376
default	Expression too large to display	1580

input
```
int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/80640/c^3*(8064*c^4*g*x^8+8960*c^4*f*x^7+11088*b*c^3*g*x^6+10080*c^4*e*x
^6+12800*b*c^3*f*x^5+11520*c^4*d*x^5+16128*a*c^3*g*x^4+504*b^2*c^2*g*x^4+1
5120*b*c^3*e*x^4+19712*a*c^3*f*x^3+768*b^2*c^2*f*x^3+18432*b*c^3*d*x^3+352
8*a*b*c^2*g*x^2+25200*a*c^3*e*x^2-630*b^3*c*g*x^2+1260*b^2*c^2*e*x^2+6144*
a*b*c^2*f*x+34560*a*c^3*d*x-1024*b^3*c*f*x+2304*b^2*c^2*d*x+8064*a^2*c^2*g
-6300*a*b^2*c*g+12600*a*b*c^2*e+945*b^4*g-1890*b^3*c*e)*(c*x^4+b*x^2+a)^(1
/2)-1/80640/c^3*(128*c*(84*a^2*c^2*f-57*a*b^2*c*f+144*a*b*c^2*d+8*b^4*f-18
*b^3*c*d)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^
2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elliptic
E(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2)))-1/2*(-15120*a^2*b*c^2*g+30240*a^2*c^3*e+7560*a*b^3
*c*g-15120*a*b^2*c^2*e-945*b^5*g+1890*b^4*c*e)*ln((1/2*b+c*x^2)/c^(1/2)+(c
*x^4+b*x^2+a)^(1/2))/c^(1/2)-11520*d*a^2*c^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
+576*a*b^2*c^2*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*...
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 911, normalized size of antiderivative = 1.26

$$\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/322560*(512*sqrt(1/2)*((18*(b^3*c^2 - 8*a*b*c^3)*d - (8*b^4*c - 57*a*b^
2*c^2 + 84*a^2*c^3)*f)*x*sqrt((b^2 - 4*a*c)/c^2) - (18*(b^4*c - 8*a*b^2*c^
2)*d - (8*b^5 - 57*a*b^3*c + 84*a^2*b*c^2)*f)*x)*sqrt(c)*sqrt((c*sqrt((b^2
 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a
*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)
) - 512*sqrt(1/2)*((9*(2*b^3*c^2 + 20*a*c^4 - (16*a*b + b^2)*c^3)*d - (8*b
^4*c + 12*(7*a^2 + 2*a*b)*c^3 - (57*a*b^2 + 4*b^3)*c^2)*f)*x*sqrt((b^2 - 4
*a*c)/c^2) - (9*(2*b^4*c - 20*a*b*c^3 - (16*a*b^2 - b^3)*c^2)*d - (8*b^5 +
 12*(7*a^2*b - 2*a*b^2)*c^2 - (57*a*b^3 - 4*b^4)*c)*f)*x)*sqrt(c)*sqrt((c*
sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((
b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a
*c)/(a*c)) + 945*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e - (b^5 - 8*a*b^3*
c + 16*a^2*b*c^2)*g)*sqrt(c)*x*log(8*c^2*x^4 + 8*b*c*x^2 + b^2 - 4*sqrt(c*
x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) + 4*a*c) - 4*(8064*c^5*g*x^9 + 8960
*c^5*f*x^8 + 1008*(10*c^5*e + 11*b*c^4*g)*x^7 + 1280*(9*c^5*d + 10*b*c^4*f
)*x^6 + 504*(30*b*c^4*e + (b^2*c^3 + 32*a*c^4)*g)*x^5 + 256*(72*b*c^4*d +
(3*b^2*c^3 + 77*a*c^4)*f)*x^4 + 126*(10*(b^2*c^3 + 20*a*c^4)*e - (5*b^3*c^
2 - 28*a*b*c^3)*g)*x^3 + 256*(9*(b^2*c^3 + 15*a*c^4)*d - 4*(b^3*c^2 - 6*a*
b*c^3)*f)*x^2 - 4608*(b^3*c^2 - 8*a*b*c^3)*d + 256*(8*b^4*c - 57*a*b^2*c^2
 + 84*a^2*c^3)*f - 63*(10*(3*b^3*c^2 - 20*a*b*c^3)*e - (15*b^4*c - 100*...
```

Sympy [F]

$$\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(a+bx^2+cx^4\right)^{\frac{3}{2}}\left(d+ex+fx^2+gx^3\right)\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)*(d + e*x + f*x**2 + g*x**3), x)
```

Maxima [F]

$$\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(gx^3+fx^2+ex+d\right)dx$$

input
```
integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(g*x^3 + f*x^2 + e*x + d), x)
```

Giac [F]

$$\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(gx^3+fx^2+ex+d\right)dx$$

input
```
integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(g*x^3 + f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int {\left(c\,x^4+b\,x^2+a\right)}^{3/2}\,\left(g\,x^3+f\,x^2+e\,x+d\right)\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)*(d + e*x + f*x^2 + g*x^3),x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)*(d + e*x + f*x^2 + g*x^3), x)
```

Reduce [F]

$$\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(g\,x^3+f\,x^2+ex+d\right)\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}dx$$

input
```
int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

3.101 $\int \left(d+ex+fx^2+gx^3\right)\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 505

$$\int \left(d+ex+fx^2+gx^3\right)\sqrt{a+bx^2+cx^4}\,dx = \frac{\left(5bcd-2b^2f+6acf\right)x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{(2ce-bg)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{x\left(5cd+bf+3cfx^2\right)\sqrt{a+bx^2+cx^4}}{15c}$$
$$+\frac{g\left(a+bx^2+cx^4\right)^{3/2}}{6c}-\frac{\left(b^2-4ac\right)(2ce-bg)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}$$
$$-\frac{\sqrt[4]{a}\left(5bcd-2b^2f+6acf\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(5cd-2bf+3\sqrt{a}\sqrt{c}f\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\Big(\right.\right.}{30c^{7/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/15*(6*a*c*f-2*b^2*f+5*b*c*d)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(1/2)+c^
(1/2)*x^2)+1/16*(-b*g+2*c*e)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/15*x*
(3*c*f*x^2+b*f+5*c*d)*(c*x^4+b*x^2+a)^(1/2)/c+1/6*g*(c*x^4+b*x^2+a)^(3/2)/
c-1/32*(-4*a*c+b^2)*(-b*g+2*c*e)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*
x^2+a)^(1/2))/c^(5/2)-1/15*a^(1/4)*(6*a*c*f-2*b^2*f+5*b*c*d)*(a^(1/2)+c^(1
/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(c*x^4+
b*x^2+a)^(1/2)+1/30*a^(1/4)*(b+2*a^(1/2)*c^(1/2))*(5*c*d-2*b*f+3*a^(1/2)*c
^(1/2)*f)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^
(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2)
)^(1/2))/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.25 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.31

$$\int \left(d+ex+fx^2+gx^3\right)\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{2\sqrt{c}(a+bx^2+cx^4)\left(-15b^2g+2bc(15e+x(8f+5gx))+4c(10ag+cx(20d+x(15e+2x(6f+5gx))))\right.}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[c]*(a + b*x^2 + c*x^4)*(-15*b^2*g + 2*b*c*(15*e + x*(8*f + 5*g*x))
 + 4*c*(10*a*g + c*x*(20*d + x*(15*e + 2*x*(6*f + 5*g*x))))) + ((-8*I)*Sqr
t[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*(-5*b*c*d + 2*b^2*f - 6*a*c*f)*Sqrt[
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b
^2 - 4*a*c])] + (8*I)*Sqrt[2]*Sqrt[c]*(-2*b^3*f + b*c*(-5*Sqrt[b^2 - 4*a*c
]*d + 8*a*f) + b^2*(5*c*d + 2*Sqrt[b^2 - 4*a*c]*f) - 2*a*c*(10*c*d + 3*Sqr
t[b^2 - 4*a*c]*f))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*E
llipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - 15*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*(-2*c*e + b*g)*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(480*c^
(5/2)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2202, 1490, 1511, 27, 1416, 1509, 1576, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2+cx^4}\left(d+ex+fx^2+gx^3\right)\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \left(fx^2+d\right)\sqrt{cx^4+bx^2+a}dx+\int x\left(gx^2+e\right)\sqrt{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1490}$$

$$\frac{\int \frac{\left(-2fb^2+5cdb+6acf\right)x^2+a(10cd-bf)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\int x\left(gx^2+e\right)\sqrt{cx^4+bx^2+a}dx+\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}$$

$$\Big\downarrow \text{1511}$$

$$\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(3\sqrt{a}\sqrt{c}f-2bf+5cd\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(6acf-2b^2f+5bcd\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{15c}+$$

$$\int x\left(gx^2+e\right)\sqrt{cx^4+bx^2+a}dx+\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(3\sqrt{a}\sqrt{c}f-2bf+5cd\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(6acf-2b^2f+5bcd\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{15c}+$$

$$\int x\left(gx^2+e\right)\sqrt{cx^4+bx^2+a}dx+\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}$$

$$\Big\downarrow \; 1416$$

$$\frac{\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}\sqrt{c}f-2bf+5cd\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(6acf-2b^2f+5bcd\right)\int \frac{\sqrt{a}-}{\sqrt{cx^4}}}{\sqrt{c}}}{15c}$$

$$\int x\left(gx^2+e\right)\sqrt{cx^4+bx^2+a}dx+\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}$$

$$\Big\downarrow \; 1509$$

$$\int x\left(gx^2+e\right)\sqrt{cx^4+bx^2+a}dx+$$

$$\frac{\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}\sqrt{c}f-2bf+5cd\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(6acf-2b^2f+5bcd\right)\left(\frac{\sqrt[4]{a}(}{}\right.}{}}{15c}$$

$$\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}$$

$$\Big\downarrow \; 1576$$

$$\frac{1}{2}\int \left(gx^2+e\right)\sqrt{cx^4+bx^2+a}dx^2+$$

$$\frac{\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}\sqrt{c}f-2bf+5cd\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(6acf-2b^2f+5bcd\right)\left(\frac{\sqrt[4]{a}(}{}\right.}{}}{15c}$$

$$\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{2}\left(\frac{(2ce-bg)\int \sqrt{cx^4+bx^2+a}dx^2}{2c}+\frac{g\left(a+bx^2+cx^4\right)^{3/2}}{3c}\right)+$$

$$\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(3\sqrt{a}\sqrt{c}f-2bf+5cd)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(6acf-2b^2f+5bcd)\left(\sqrt[4]{a}(\right.}{}}{15c}$$

$$\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}$$

$\downarrow$ 1087

$$\frac{1}{2}\left(\frac{(2ce-bg)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)}{2c}+\frac{g\left(a+bx^2+cx^4\right)^{3/2}}{3c}\right)+$$

$$\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(3\sqrt{a}\sqrt{c}f-2bf+5cd)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(6acf-2b^2f+5bcd)\left(\sqrt[4]{a}(\right.}{}}{15c}$$

$$\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}$$

$\downarrow$ 1092

$$\frac{1}{2}\left(\frac{(2ce-bg)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)}{2c}+\frac{g\left(a+bx^2+cx^4\right)^{3/2}}{3c}\right)+$$

$$\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(3\sqrt{a}\sqrt{c}f-2bf+5cd)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(6acf-2b^2f+5bcd)\left(\sqrt[4]{a}(\right.}{}}{15c}$$

$$\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}$$

$\downarrow$ 219

$$\frac{\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}\sqrt{c}f-2bf+5cd\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(6acf-2b^2f+5bcd)\left(\frac{\sqrt[4]{a}(\cdots}{\cdots}\right.}{\cdots}}{15c}$$

$$\frac{1}{2}\left(\frac{(2ce-bg)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{2c}+\frac{g\left(a+bx^2+cx^4\right)^{3/2}}{3c}\right)+$$

$$\frac{x\sqrt{a+bx^2+cx^4}\left(bf+5cd+3cfx^2\right)}{15c}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(x*(5*c*d + b*f + 3*c*f*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) + ((g*(a + b*
x^2 + c*x^4)^(3/2))/(3*c) + ((2*c*e - b*g)*(((b + 2*c*x^2)*Sqrt[a + b*x^2
+ c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4])])/(8*c^(3/2))))/(2*c))/2 + (-(((5*b*c*d - 2*b^2*f + 6*a*c
*f)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(
1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c]
)*(5*c*d - 2*b*f + 3*Sqrt[a]*Sqrt[c]*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
)/(15*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 3.86 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.23

method	result
elliptic	$\frac{g x^4 \sqrt{c x^4+b x^2+a}}{6}+\frac{f x^3 \sqrt{c x^4+b x^2+a}}{5}+\frac{\left(\frac{b g}{6}+c e\right) x^2 \sqrt{c x^4+b x^2+a}}{4 c}+\frac{\left(\frac{b f}{5}+c d\right) x \sqrt{c x^4+b x^2+a}}{3 c}+\frac{\left(\frac{a g}{3}+b e-\frac{3 b\left(\frac{b g}{6}+c e\right)}{4 c}\right.}{2 c}$
risch	$\frac{\left(40 g x^4 c^2+48 f x^3 c^2+10 b c g x^2+60 c^2 e x^2+16 b f x c+80 c^2 x d+40 a c g-15 b^2 g+30 c e b\right) \sqrt{c x^4+b x^2+a}}{240 c^2}-\frac{8 c\left(6 a c f-2 b^2 f+5 b c d\right) a \sqrt{2} \sqrt{4-}}{}$
default	$d\left(\frac{x \sqrt{c x^4+b x^2+a}}{3}+\frac{a \sqrt{2} \sqrt{4-\frac{2\left(-b+\sqrt{-4 a c+b^2}\right) x^2}{a}} \sqrt{4+\frac{2\left(b+\sqrt{-4 a c+b^2}\right) x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x \sqrt{2} \sqrt{\frac{-b+\sqrt{-4 a c+b^2}}{a}}}{2}, \frac{\sqrt{-4+\frac{2 b\left(b+\sqrt{-}\right.}{a}}}{2}\right.}{6 \sqrt{\frac{-b+\sqrt{-4 a c+b^2}}{a}} \sqrt{c x^4+b x^2+a}}\right.$

input
```
int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*g*x^4*(c*x^4+b*x^2+a)^(1/2)+1/5*f*x^3*(c*x^4+b*x^2+a)^(1/2)+1/4*(1/6*b
*g+c*e)/c*x^2*(c*x^4+b*x^2+a)^(1/2)+1/3*(1/5*b*f+c*d)/c*x*(c*x^4+b*x^2+a)^
(1/2)+1/2*(1/3*a*g+b*e-3/4*b/c*(1/6*b*g+c*e))/c*(c*x^4+b*x^2+a)^(1/2)+1/4*
(a*d-1/3*a/c*(1/5*b*f+c*d))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2
*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*(a*e-1/2*a/c
*(1/6*b*g+c*e)-1/2*b/c*(1/3*a*g+b*e-3/4*b/c*(1/6*b*g+c*e)))*ln((2*c*x^2+b)
/c^(1/2)+2*(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*(2/5*a*f+b*d-2/3*b/c*(1/5*b*
f+c*d))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.14

$$\int \left(d+ex+fx^2+gx^3\right)\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{32\sqrt{\frac{1}{2}}\left(\left(5\,bc^2d-2\,(b^2c-3\,ac^2)f\right)x\sqrt{\frac{b^2-4\,ac}{c^2}}-\left(5\,b^2cd-2\,(b^3-3\,abc)f\right)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\right.}{}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/960*(32*sqrt(1/2)*((5*b*c^2*d - 2*(b^2*c - 3*a*c^2)*f)*x*sqrt((b^2 - 4*a
*c)/c^2) - (5*b^2*c*d - 2*(b^3 - 3*a*b*c)*f)*x)*sqrt(c)*sqrt((c*sqrt((b^2
- 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*
c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c))
 - 32*sqrt(1/2)*((5*(b*c^2 - 2*c^3)*d - (2*b^2*c - (6*a + b)*c^2)*f)*x*sqr
t((b^2 - 4*a*c)/c^2) - (5*(b^2*c + 2*b*c^2)*d - (2*b^3 - (6*a*b - b^2)*c)*
f)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sq
rt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4
*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 15*(2*(b^2*c - 4*a*c^2)*e - (b^3 - 4*a*
b*c)*g)*sqrt(c)*x*log(8*c^2*x^4 + 8*b*c*x^2 + b^2 - 4*sqrt(c*x^4 + b*x^2 +
 a)*(2*c*x^2 + b)*sqrt(c) + 4*a*c) + 4*(40*c^3*g*x^5 + 48*c^3*f*x^4 + 80*b
*c^2*d + 10*(6*c^3*e + b*c^2*g)*x^3 + 16*(5*c^3*d + b*c^2*f)*x^2 - 32*(b^2
*c - 3*a*c^2)*f + 5*(6*b*c^2*e - (3*b^2*c - 8*a*c^2)*g)*x)*sqrt(c*x^4 + b*
x^2 + a))/(c^3*x)
```

Sympy [F]

$$\int \left(d+ex+fx^2+gx^3\right)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{a+bx^2+cx^4}\left(d+ex+fx^2+gx^3\right)\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)*(d + e*x + f*x**2 + g*x**3), x)
```

Maxima [F]

$$\int \left(d+ex+fx^2+gx^3\right) \sqrt{a+bx^2+cx^4}\, dx = \int \sqrt{cx^4+bx^2+a}\left(gx^3+fx^2+ex+d\right) dx$$

input
```
integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d), x)
```

Giac [F]

$$\int \left(d+ex+fx^2+gx^3\right) \sqrt{a+bx^2+cx^4}\, dx = \int \sqrt{cx^4+bx^2+a}\left(gx^3+fx^2+ex+d\right) dx$$

input
```
integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+fx^2+gx^3\right) \sqrt{a+bx^2+cx^4}\, dx = \int \sqrt{c\,x^4+b\,x^2+a}\,\left(g\,x^3+f\,x^2+e\,x+d\right)\, dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)*(d + e*x + f*x^2 + g*x^3),x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)*(d + e*x + f*x^2 + g*x^3), x)
```

Reduce [F]

$$\int \left(d+ex+fx^2+gx^3\right)\sqrt{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(80*sqrt(a + b*x**2 + c*x**4)*a*c**2*g - 30*sqrt(a + b*x**2 + c*x**4)*b**2
*c*g + 60*sqrt(a + b*x**2 + c*x**4)*b*c**2*e + 32*sqrt(a + b*x**2 + c*x**4
)*b*c**2*f*x + 20*sqrt(a + b*x**2 + c*x**4)*b*c**2*g*x**2 + 160*sqrt(a + b
*x**2 + c*x**4)*c**3*d*x + 120*sqrt(a + b*x**2 + c*x**4)*c**3*e*x**2 + 96*
sqrt(a + b*x**2 + c*x**4)*c**3*f*x**3 + 80*sqrt(a + b*x**2 + c*x**4)*c**3*
g*x**4 + 60*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) - sqrt(c)*x**2)*a*b*c*g
- 120*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) - sqrt(c)*x**2)*a*c**2*e - 15*
sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) - sqrt(c)*x**2)*b**3*g + 30*sqrt(c)*
log(sqrt(a + b*x**2 + c*x**4) - sqrt(c)*x**2)*b**2*c*e - 60*sqrt(c)*log(sq
rt(a + b*x**2 + c*x**4) + sqrt(c)*x**2)*a*b*c*g + 120*sqrt(c)*log(sqrt(a +
 b*x**2 + c*x**4) + sqrt(c)*x**2)*a*c**2*e + 15*sqrt(c)*log(sqrt(a + b*x**
2 + c*x**4) + sqrt(c)*x**2)*b**3*g - 30*sqrt(c)*log(sqrt(a + b*x**2 + c*x*
*4) + sqrt(c)*x**2)*b**2*c*e - 32*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*
a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a**2*b*c**2*f + 320*int(sqr
t(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**
6),x)*a**2*c**3*d + 192*int((sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2 + 2*a*b
*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*b*c**3*f - 64*int((sqrt(a +
b*x**2 + c*x**4)*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**
6),x)*b**3*c**2*f + 160*int((sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2 + 2*a*b
*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*b**2*c**3*d + 192*int((sqrt...
```

3.102 $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 360

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{g\sqrt{a+bx^2+cx^4}}{2c}+\frac{fx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{(2ce-bg)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$
$$-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(\sqrt{c}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/2*g*(c*x^4+b*x^2+a)^(1/2)/c+f*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+c
^(1/2)*x^2)+1/4*(-b*g+2*c*e)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+
a)^(1/2))/c^(3/2)-a^(1/4)*f*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2
)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-
b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/2*(c^(1/2)*d+a^(
1/2)*f)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1
/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^
(1/2))/a^(1/4)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.52 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.46

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{i\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right) f\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(I*Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*f*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[
2]*Sqrt[c]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*(2*Sqrt[c]*g*(a + b*x^2 + c*x^4) + (-2*c*e + b*
g)*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*
x^4]]))/(4*c^(3/2)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4]
)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2202, 1511, 27, 1416, 1509, 1576, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}dx$$

$\Big\downarrow$ 2202

$$\int \frac{fx^2+d}{\sqrt{cx^4+bx^2+a}}dx+\int \frac{x(gx^2+e)}{\sqrt{cx^4+bx^2+a}}dx$$

$\Big\downarrow$ 1511

$$\left(\frac{\sqrt{a}f}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx+\int \frac{x(gx^2+e)}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}f\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$\Big\downarrow$ 27

$$\left(\frac{\sqrt{a}f}{\sqrt{c}}+d\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx+\int \frac{x(gx^2+e)}{\sqrt{cx^4+bx^2+a}}dx-\frac{f\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$\Big\downarrow$ 1416

$$\int \frac{x(gx^2+e)}{\sqrt{cx^4+bx^2+a}}dx-\frac{f\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}f}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1509

$$\int \frac{x(gx^2+e)}{\sqrt{cx^4+bx^2+a}}dx+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}f}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{f\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

$\Big\downarrow$ 1576

$$\frac{1}{2}\int \frac{gx^2+e}{\sqrt{cx^4+bx^2+a}}dx^2+$$
$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}f}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-$$
$$\frac{f\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

$\downarrow$ 1160

$$\frac{1}{2}\left(\frac{(2ce-bg)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{2c}+\frac{g\sqrt{a+bx^2+cx^4}}{c}\right)+$$
$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}f}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-$$
$$\frac{f\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

$\downarrow$ 1092

$$\frac{1}{2}\left(\frac{(2ce-bg)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{c}+\frac{g\sqrt{a+bx^2+cx^4}}{c}\right)+$$
$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}f}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-$$
$$\frac{f\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

$\downarrow$ 219

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}f}{\sqrt{c}}+d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{f\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}+\frac{1}{2}\left(\frac{(2ce-bg)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}+\frac{g\sqrt{a+bx^2+cx^4}}{c}\right)$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((g*Sqrt[a + b*x^2 + c*x^4])/c + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/(2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*c^(3/2)))/2 - (f*(-((x*Sqrt[a + b*x^2
 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/
4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*
x^4])))/Sqrt[c] + ((d + (Sqrt[a]*f)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*
x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.18

method	result
elliptic	$\frac{g\sqrt{c\,x^4+b\,x^2+a}}{2c}+\frac{d\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+\cdots}\right.}{ac}}}{2}\cdots\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\cdots$
risch	$\frac{g\sqrt{c\,x^4+b\,x^2+a}}{2c}-\frac{-\frac{(-bg+2ce)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}-\frac{cd\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\cdots}{\cdots}\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x\cdots}}\cdots}{\cdots}$
default	$\frac{d\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+\frac{e\ln\left(\frac{\frac{b}{2}+c\cdots}{\sqrt{\cdots}}\right.}{\cdots}\cdots$

input
```
int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*g*(c*x^4+b*x^2+a)^(1/2)/c+1/4*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*(e-1
/2*b/c*g)*ln((2*c*x^2+b)/c^(1/2)+2*(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*f*a*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.04

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{4\sqrt{\frac{1}{2}}\left(acfx\sqrt{\frac{b^2-4\,ac}{c^2}}-abfx\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right) \mid \frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac}) + 4\sqrt{\frac{1}{2}}\Big(\cdots}{\cdots}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(4*sqrt(1/2)*(a*c*f*x*sqrt((b^2 - 4*a*c)/c^2) - a*b*f*x)*sqrt(c)*sqrt(
(c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqr
t((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 -
2*a*c)/(a*c)) + 4*sqrt(1/2)*((c^2*d - a*c*f)*x*sqrt((b^2 - 4*a*c)/c^2) + (
b*c*d + a*b*f)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic
_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*s
qrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - (2*a*c*e - a*b*g)*sqrt(c)*x
*log(8*c^2*x^4 + 8*b*c*x^2 + b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)
*sqrt(c) + 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*(a*c*g*x + 2*a*c*f))/(a*c^2*
x)
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{gx^3+fx^2+ex+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x^3 + f*x^2 + e*x + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{gx^3+fx^2+ex+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x^3 + f*x^2 + e*x + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d + ex + fx^2 + gx^3}{\sqrt{a + bx^2 + cx^4}} \, dx = \text{Too large to display}$$

input
```
int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(4*sqrt(4*a*c - b**2)*sqrt( - 4*a*c + b**2)*atan((4*sqrt(c)*sqrt(a + b*x**
2 + c*x**4)*a*b*c - sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b**3 + 4*a**2*c**2 -
 a*b**2*c + 4*a*b*c**2*x**2 - b**3*c*x**2)/(sqrt(4*a*c - b**2)*sqrt( - 4*a
*c + b**2)*a*c))*a*c*g + 2*sqrt(4*a*c - b**2)*sqrt( - 4*a*c + b**2)*atan((
4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*b*c - sqrt(c)*sqrt(a + b*x**2 + c*x*
*4)*b**3 + 4*a**2*c**2 - a*b**2*c + 4*a*b*c**2*x**2 - b**3*c*x**2)/(sqrt(4
*a*c - b**2)*sqrt( - 4*a*c + b**2)*a*c))*b**2*g + 16*sqrt(c)*int(sqrt(a +
b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*
a**2*b*c**2*d - 4*sqrt(c)*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2
 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*b**3*c*d + 16*sqrt(c)*int((sqrt(a
 + b*x**2 + c*x**4)*x**5)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*
x**6),x)*a*b**2*c**2*g - 4*sqrt(c)*int((sqrt(a + b*x**2 + c*x**4)*x**5)/(a
**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*b**4*c*g + 16*sqrt(
c)*int((sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b*
*2*x**4 + b*c*x**6),x)*a*b**2*c**2*f - 4*sqrt(c)*int((sqrt(a + b*x**2 + c*
x**4)*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*b**4*
c*f + 16*sqrt(c)*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 +
 a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a**2*b*c**2*f - 4*sqrt(c)*int((sqrt(a
 + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*
x**6),x)*a*b**3*c*f + 16*sqrt(c)*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(...
```

3.103 $\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 447

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{be-2ag+(2ce-bg)x^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{c}(bd-2af)x\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\sqrt[4]{c}(bd-2af)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{\left(\sqrt{c}d-\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
x*(b^2*d-2*a*c*d-a*b*f+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^
(1/2)-(b*e-2*a*g+(-b*g+2*c*e)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-c^(1
/2)*(-2*a*f+b*d)*x*(c*x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x
^2)+c^(1/4)*(-2*a*f+b*d)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c
^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a
^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*(c^(
1/2)*d-a^(1/2)*f)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*
x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)
/c^(1/2))^(1/2))/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/c^(1/4)/(c*x^4+b*x^2+a)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.96 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.15

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(-2a^2g-bdx(b+cx^2)+2acx(d+x(e+fx))+ab(e+x(f-gx)))+i\left(-b+\sqrt{b^2-4ac}\right)}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-1/4*(4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-2*a^2*g - b*d*x*(b + c*x^2) + 2*
a*c*x*(d + x*(e + f*x)) + a*b*(e + x*(f - g*x))) + I*(-b + Sqrt[b^2 - 4*a*
c])*(b*d - 2*a*f)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqr
t[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-(b^2*d) + 4*a*c*d + b*Sqrt[
b^2 - 4*a*c]*d - 2*a*Sqrt[b^2 - 4*a*c]*f)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2
- 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2202, 1492, 1511, 27, 1416, 1509, 1576, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^{3/2}}dx+\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 1492}$$

$$-\frac{\int \frac{c(bd-2af)x^2+a(2cd-bf)}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}+\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{3/2}}dx+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1511}$$

$$-\frac{\sqrt{a}\left(\sqrt{c}(bd-2af)+\sqrt{a}(2cd-bf)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{a}\sqrt{c}(bd-2af)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}+$$
$$\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{3/2}}dx+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt{a}\left(\sqrt{c}(bd-2af)+\sqrt{a}(2cd-bf)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}(bd-2af)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}+$$
$$\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{3/2}}dx+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1416}$$

$$-\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{c}(bd-2af)+\sqrt{a}(2cd-bf))\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}(bd-2af)\int \frac{\sqrt{a}-\sqrt{\cdots}}{\sqrt{cx^4+b\cdots}}}{a(b^2-4ac)}$$
$$\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{3/2}}dx+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1509

$$\int \frac{x\left(gx^2+e\right)}{\left(cx^4+bx^2+a\right)^{3/2}}dx - \frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(\sqrt{c}(bd-2af)+\sqrt{a}(2cd-bf))\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}} - \sqrt{c}(bd-2af)\Bigg(\frac{\sqrt[4]{a}(\sqrt{a}+}{}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1576

$$\frac{1}{2}\int \frac{gx^2+e}{\left(cx^4+bx^2+a\right)^{3/2}}dx^2 - \frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(\sqrt{c}(bd-2af)+\sqrt{a}(2cd-bf))\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}} - \sqrt{c}(bd-2af)\Bigg(\frac{\sqrt[4]{a}(\sqrt{a}+}{}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1158

$$-\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(\sqrt{c}(bd-2af)+\sqrt{a}(2cd-bf))\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}} - \sqrt{c}(bd-2af)\Bigg(\frac{\sqrt[4]{a}(\sqrt{a}}{}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{-2ag+x^2(2ce-bg)+be}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(a*(b^2 - 4*a*c)*Sqrt[
a + b*x^2 + c*x^4]) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/((b^2 - 4*a*c)*Sqr
t[a + b*x^2 + c*x^4]) - (-(Sqrt[c]*(b*d - 2*a*f)*(-((x*Sqrt[a + b*x^2 + c*
x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
)) + (a^(1/4)*(Sqrt[c]*(b*d - 2*a*f) + Sqrt[a]*(2*c*d - b*f))*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sq
rt[a + b*x^2 + c*x^4]))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.26

method	result
elliptic	$-\frac{2c\left(-\frac{(2af-bd)x^3}{2a\left(4ac-b^2\right)}+\frac{(bg-2ce)x^2}{2c\left(4ac-b^2\right)}-\frac{\left(abf+2dac-b^2d\right)x}{2ac\left(4ac-b^2\right)}+\frac{2ag-be}{2\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{d}{a}-\frac{abf+2dac-b^2d}{a\left(4ac-b^2\right)}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\cdots}}{4\sqrt{\cdots}}$
default	Expression too large to display

input
```
int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/2*(2*a*f-b*d)/a/(4*a*c-b^2)*x^3+1/2*(b*g-2*c*e)/c/(4*a*c-b^2)*x^2
-1/2*(a*b*f+2*a*c*d-b^2*d)/a/c/(4*a*c-b^2)*x+1/2*(2*a*g-b*e)/(4*a*c-b^2)/c
)/((x^4+1/c*b*x^2+1/c*a)*c)^(1/2)+1/4*(d/a-(a*b*f+2*a*c*d-b^2*d)/a/(4*a*c-
b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*c*(2*a*f-b*d)/(4*a*c-b^2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*
c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2
)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.62

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(a*b^2*c*d - 2*a^2*b*c*f + (b^2*c^2*d - 2*a*b*c^2*f)*x^4 +
 (b^3*c*d - 2*a*b^2*c*f)*x^2 - (a^2*b*c*d - 2*a^3*c*f + (a*b*c^2*d - 2*a^2
*c^2*f)*x^4 + (a*b^2*c*d - 2*a^2*b*c*f)*x^2)*sqrt((b^2 - 4*a*c)/a^2))*sqrt
(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*
sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2)
 + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*(((2*a*b + b^2)*c^2*d - (a*b^2*c + 2*a*
b*c^2)*f)*x^4 + (2*a^2*b + a*b^2)*c*d + ((2*a*b^2 + b^3)*c*d - (a*b^3 + 2*
a*b^2*c)*f)*x^2 - (a^2*b^2 + 2*a^2*b*c)*f + (((2*a^2 - a*b)*c^2*d - (a^2*b
*c - 2*a^2*c^2)*f)*x^4 + (2*a^3 - a^2*b)*c*d + ((2*a^2*b - a*b^2)*c*d - (a
^2*b^2 - 2*a^2*b*c)*f)*x^2 - (a^3*b - 2*a^3*c)*f)*sqrt((b^2 - 4*a*c)/a^2))
*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/
2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)
/a^2) + b^2 - 2*a*c)/(a*c)) + 2*(a^2*b*c*e - 2*a^3*c*g - (a*b*c^2*d - 2*a^
2*c^2*f)*x^3 + (2*a^2*c^2*e - a^2*b*c*g)*x^2 + (a^2*b*c*f - (a*b^2*c - 2*a
^2*c^2)*d)*x)*sqrt(c*x^4 + b*x^2 + a))/(a^3*b^2*c - 4*a^4*c^2 + (a^2*b^2*c
^2 - 4*a^3*c^3)*x^4 + (a^2*b^3*c - 4*a^3*b*c^2)*x^2)
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{gx^3+fx^2+ex+d}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{gx^3+fx^2+ex+d}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{3/2}}\,dx=\text{too large to display}$$

input
```
int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(48*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a**2
+ 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**3*b*
c**2*d + 96*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4
)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)
*a**3*c**3*d*x**2 - 8*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**
2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**
2*x**8),x)*a**2*b**3*c*d + 96*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a
 + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x*
*6 + c**2*x**8),x)*a**2*b**2*c**2*d*x**2 + 336*sqrt(c)*sqrt(a + b*x**2 + c
*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**
2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*b*c**3*d*x**4 + 224*sqrt(c)*sqrt(
a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*
a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*c**4*d*x**6 - sqrt(
c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x
**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b**5*d - 26*sq
rt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*
b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b**4*c*d*x*
*2 - 8*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a*
*2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b*
*3*c**2*d*x**4 + 208*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x...
```

3.104 $\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 680

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{3a\,(b^2-4ac)\,(a+bx^2+cx^4)^{3/2}}$$
$$-\frac{be-2ag+(2ce-bg)x^2}{3\,(b^2-4ac)\,(a+bx^2+cx^4)^{3/2}}+\frac{4(2ce-bg)\,(b+2cx^2)}{3\,(b^2-4ac)^2\,\sqrt{a+bx^2+cx^4}}$$
$$+\frac{x(2b^4d-17ab^2cd+20a^2c^2d+ab^3f+4a^2bcf+c(2b^3d-16abcd+ab^2f+12a^2cf)\,x^2)}{3a^2\,(b^2-4ac)^2\,\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt{c}(2b^3d-16abcd+ab^2f+12a^2cf)\,x\sqrt{a+bx^2+cx^4}}{3a^2\,(b^2-4ac)^2\,(\sqrt{a}+\sqrt{c}x^2)}$$
$$+\frac{\sqrt[4]{c}(2b^3d-16abcd+ab^2f+12a^2cf)\,(\sqrt{a}+\sqrt{c}x^2)\,\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\,(b^2-4ac)^2\,\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{c}\left(2b^2d-3\sqrt{a}b\sqrt{c}d-10acd+abf+6a^{3/2}\sqrt{c}f\right)\,(\sqrt{a}+\sqrt{c}x^2)\,\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{6a^{7/4}\,(b-2\sqrt{a}\sqrt{c})\,(b^2-4ac)\,\sqrt{a+bx^2+cx^4}}$$

output
```
1/3*x*(b^2*d-2*a*c*d-a*b*f+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2
+a)^(3/2)-1/3*(b*e-2*a*g+(-b*g+2*c*e)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(3
/2)+4/3*(-b*g+2*c*e)*(2*c*x^2+b)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)+1/3*
x*(2*b^4*d-17*a*b^2*c*d+20*a^2*c^2*d+a*b^3*f+4*a^2*b*c*f+c*(12*a^2*c*f+a*b
^2*f-16*a*b*c*d+2*b^3*d)*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/3
*c^(1/2)*(12*a^2*c*f+a*b^2*f-16*a*b*c*d+2*b^3*d)*x*(c*x^4+b*x^2+a)^(1/2)/a
^2/(-4*a*c+b^2)^2/(a^(1/2)+c^(1/2)*x^2)+1/3*c^(1/4)*(12*a^2*c*f+a*b^2*f-16
*a*b*c*d+2*b^3*d)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*
x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/
c^(1/2))^(1/2))/a^(7/4)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/6*c^(1/4)*(
2*b^2*d-3*a^(1/2)*b*c^(1/2)*d-10*a*c*d+a*b*f+6*a^(3/2)*c^(1/2)*f)*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacob
iAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(7/4)/(
b-2*a^(1/2)*c^(1/2))/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.97 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.88

$$\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^{5/2}}\,dx = \frac{-4a(b^2-4ac)\left(-2a^2g-bdx(b+cx^2)+2acx(d+x(e+fx))+ab(e+x(f-\right.}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
(-4*a*(b^2 - 4*a*c)*(-2*a^2*g - b*d*x*(b + c*x^2) + 2*a*c*x*(d + x*(e + f*
x)) + a*b*(e + x*(f - g*x))) + 4*(a + b*x^2 + c*x^4)*(2*b^3*d*x*(b + c*x^2
) + a*b*x*(-17*b*c*d + b^2*f - 16*c^2*d*x^2 + b*c*f*x^2) + 4*a^2*(-(b^2*g)
 + c^2*x*(5*d + x*(4*e + 3*f*x)) + b*c*(2*e + x*(f - 2*g*x)))) + (I*Sqrt[2
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(-((-b + Sqrt[b^2
- 4*a*c])*(2*b^3*d - 16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*EllipticE[I*ArcSin
h[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])]) + (-2*b^4*d + b^3*(2*Sqrt[b^2 - 4*a*c]*d - a*f) + 4*
a*b*c*(-4*Sqrt[b^2 - 4*a*c]*d + a*f) + a*b^2*(18*c*d + Sqrt[b^2 - 4*a*c]*f
) + 4*a^2*c*(-10*c*d + 3*Sqrt[b^2 - 4*a*c]*f))*EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(12*a^2*(b^2 - 4*a*c)^2*(a
+ b*x^2 + c*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 650, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2202, 1492, 25, 1492, 27, 1511, 27, 1416, 1509, 1576, 1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{fx^2+d}{(cx^4+bx^2+a)^{5/2}}dx+\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{5/2}}dx$$

$$\Big\downarrow \text{1492}$$

$$-\frac{\int -\frac{2db^2+afb+3c(bd-2af)x^2-10acd}{(cx^4+bx^2+a)^{3/2}}dx}{3a(b^2-4ac)}+\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{5/2}}dx+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{3a(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2db^2+afb+3c(bd-2af)x^2-10acd}{(cx^4+bx^2+a)^{3/2}}dx}{3a(b^2-4ac)}+\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{5/2}}dx+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{3a(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$\Big\downarrow$ 1492

$$\frac{\frac{x(cx^2(12a^2cf+ab^2f-16abcd+2b^3d)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\int \frac{c\left(\left(2db^3+afb^2-16acdb+12a^2cf\right)x^2+a\left(db^2+8afb-20acd\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}}{3a(b^2-4ac)}$$
$$\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{5/2}}dx+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{3a(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{x(cx^2(12a^2cf+ab^2f-16abcd+2b^3d)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{c\int \frac{\left(2db^3+afb^2-16acdb+12a^2cf\right)x^2+a\left(db^2+8afb-20acd\right)}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}}{3a(b^2-4ac)}+$$
$$\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{5/2}}dx+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{3a(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$\Big\downarrow$ 1511

$$\frac{\frac{x(cx^2(12a^2cf+ab^2f-16abcd+2b^3d)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\frac{\sqrt{a}\left(12a^2cf+\sqrt{a}\sqrt{c}\left(8abf-20acd+b^2d\right)+ab^2f-16abcd+2b^3d\right)}{\sqrt{c}}\right.}{}}{3a(b^2-4ac)}$$
$$\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{5/2}}dx+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{3a(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{x(cx^2(12a^2cf+ab^2f-16abcd+2b^3d)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\frac{\sqrt{a}\left(12a^2cf+\sqrt{a}\sqrt{c}\left(8abf-20acd+b^2d\right)+ab^2f-16abcd+2b^3d\right)}{\sqrt{c}}\right.}{a(}}{3a(b^2-4ac)}$$
$$\int \frac{x(gx^2+e)}{(cx^4+bx^2+a)^{5/2}}dx+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{3a(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$\Big\downarrow$ 1416

$$\frac{x\left(cx^2\left(12a^2cf+ab^2f-16abcd+2b^3d\right)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d\right)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(12a^2cf+\sqrt{a}\sqrt{c}\left(8abf-20a\ldots\right.\right.}{2c\ldots}\right.}{\ldots}$$

$$3a\left(b^2-4ac\right)$$

$$\int \frac{x\left(gx^2+e\right)}{\left(cx^4+bx^2+a\right)^{5/2}}dx+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 1509

$$\int \frac{x\left(gx^2+e\right)}{\left(cx^4+bx^2+a\right)^{5/2}}dx+$$

$$\frac{x\left(cx^2\left(12a^2cf+ab^2f-16abcd+2b^3d\right)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d\right)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(12a^2cf+\sqrt{a}\sqrt{c}\left(8abf-20a\ldots\right.\right.}{2c\ldots}\right.}{\ldots}$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 1576

$$\frac{1}{2}\int \frac{gx^2+e}{\left(cx^4+bx^2+a\right)^{5/2}}dx^2+$$

$$\frac{x\left(cx^2\left(12a^2cf+ab^2f-16abcd+2b^3d\right)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d\right)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(12a^2cf+\sqrt{a}\sqrt{c}\left(8abf-20a\ldots\right.\right.}{2c\ldots}\right.}{\ldots}$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 1159

$$\frac{1}{2}\left(-\frac{4(2ce-bg)\int \frac{1}{(cx^4+bx^2+a)^{3/2}}dx^2}{3\left(b^2-4ac\right)}-\frac{2\left(-2ag+x^2(2ce-bg)+be\right)}{3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}\right)+$$

$$\frac{\frac{x\left(cx^2\left(12a^2cf+ab^2f-16abcd+2b^3d\right)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d\right)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(12a^2cf+\sqrt{a}\sqrt{c}\left(8abf-20a\ldots\right.\right.}{2c\ldots}\right.}{\ldots}$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 1088$$

$$\frac{\frac{x\left(cx^2\left(12a^2cf+ab^2f-16abcd+2b^3d\right)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d\right)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(12a^2cf+\sqrt{a}\sqrt{c}\left(8abf-20a\ldots\right.\right.}{2c\ldots}\right.}{\ldots}$$

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}+$$

$$\frac{1}{2}\left(\frac{8\left(b+2cx^2\right)(2ce-bg)}{3\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}}-\frac{2\left(-2ag+x^2(2ce-bg)+be\right)}{3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}\right)$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
(x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(3*a*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)^(3/2)) + ((-2*(b*e - 2*a*g + (2*c*e - b*g)*x^2))/(3*(b^2
- 4*a*c)*(a + b*x^2 + c*x^4)^(3/2)) + (8*(2*c*e - b*g)*(b + 2*c*x^2))/(3*(
b^2 - 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]))/2 + ((x*(2*b^4*d - 17*a*b^2*c*d +
 20*a^2*c^2*d + a*b^3*f + 4*a^2*b*c*f + c*(2*b^3*d - 16*a*b*c*d + a*b^2*f
+ 12*a^2*c*f)*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (c*(-(((2*
b^3*d - 16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*(-((x*Sqrt[a + b*x^2 + c*x^4])/
(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqr
t[c]) + (a^(1/4)*(2*b^3*d - 16*a*b*c*d + a*b^2*f + 12*a^2*c*f + Sqrt[a]*Sq
rt[c]*(b^2*d - 20*a*c*d + 8*a*b*f))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])))/(
a*(b^2 - 4*a*c)))/(3*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\left(\frac{(2af-bd)x^3}{3ca\left(4ac-b^2\right)}-\frac{(bg-2ce)x^2}{3\left(4ac-b^2\right)c^2}+\frac{\left(abf+2dac-b^2d\right)x}{3a\left(4ac-b^2\right)c^2}-\frac{2ag-be}{3\left(4ac-b^2\right)c^2}\right)\sqrt{c x^4+b x^2+a}}{\left(x^4+\frac{b x^2}{c}+\frac{a}{c}\right)^2}-\frac{2c\left(-\frac{\left(12a^2cf+a b^2f-16abcd+2b^3d\right)x^3}{6a^2\left(4ac-b^2\right)^2}+\frac{4(bg-}{3(4ac-}\right.}{}$
default	Expression too large to display

input
```
int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3/c*(2*a*f-b*d)/a/(4*a*c-b^2)*x^3-1/3*(b*g-2*c*e)/(4*a*c-b^2)/c^2*x^2+1
/3*(a*b*f+2*a*c*d-b^2*d)/a/(4*a*c-b^2)/c^2*x-1/3*(2*a*g-b*e)/(4*a*c-b^2)/c
^2)*(c*x^4+b*x^2+a)^(1/2)/(x^4+1/c*b*x^2+1/c*a)^2-2*c*(-1/6*(12*a^2*c*f+a*
b^2*f-16*a*b*c*d+2*b^3*d)/a^2/(4*a*c-b^2)^2*x^3+4/3*(b*g-2*c*e)/(4*a*c-b^2
)^2*x^2-1/6*(4*a^2*b*c*f+20*a^2*c^2*d+a*b^3*f-17*a*b^2*c*d+2*b^4*d)/a^2/(4
*a*c-b^2)^2/c*x+2/3*b*(b*g-2*c*e)/(4*a*c-b^2)^2/c)/((x^4+1/c*b*x^2+1/c*a)*
c)^(1/2)+1/4*(-1/3/(4*a*c-b^2)*(a*b*f-10*a*c*d+2*b^2*d)/a^2-1/3*(4*a^2*b*c
*f+20*a^2*c^2*d+a*b^3*f-17*a*b^2*c*d+2*b^4*d)/a^2/(4*a*c-b^2)^2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2))+1/6*c*(12*a^2*c*f+a*b^2*f-16*a*b*c*d+2*b^3*d)/(4*a*c-
b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(591) = 1182$.

Time = 0.13 (sec) , antiderivative size = 1948, normalized size of antiderivative = 2.86

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fricas")
```

output

```
-1/6*(sqrt(1/2)*((2*(b^4*c^2 - 8*a*b^2*c^3)*d + (a*b^3*c^2 + 12*a^2*b*c^3)
*f)*x^8 + 2*(2*(b^5*c - 8*a*b^3*c^2)*d + (a*b^4*c + 12*a^2*b^2*c^2)*f)*x^6
 + (2*(b^6 - 6*a*b^4*c - 16*a^2*b^2*c^2)*d + (a*b^5 + 14*a^2*b^3*c + 24*a^
3*b*c^2)*f)*x^4 + 2*(2*(a*b^5 - 8*a^2*b^3*c)*d + (a^2*b^4 + 12*a^3*b^2*c)*
f)*x^2 + 2*(a^2*b^4 - 8*a^3*b^2*c)*d + (a^3*b^3 + 12*a^4*b*c)*f - ((2*(a*b
^3*c^2 - 8*a^2*b*c^3)*d + (a^2*b^2*c^2 + 12*a^3*c^3)*f)*x^8 + 2*(2*(a*b^4*
c - 8*a^2*b^2*c^2)*d + (a^2*b^3*c + 12*a^3*b*c^2)*f)*x^6 + (2*(a*b^5 - 6*a
^2*b^3*c - 16*a^3*b*c^2)*d + (a^2*b^4 + 14*a^3*b^2*c + 24*a^4*c^2)*f)*x^4
+ 2*(2*(a^2*b^4 - 8*a^3*b^2*c)*d + (a^3*b^3 + 12*a^4*b*c)*f)*x^2 + 2*(a^3*
b^3 - 8*a^4*b*c)*d + (a^4*b^2 + 12*a^5*c)*f)*sqrt((b^2 - 4*a*c)/a^2))*sqrt
(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*
sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2)
 + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*(((4*(5*a^2*b + 4*a*b^2)*c^3 - (a*b^3 +
 2*b^4)*c^2)*d - (12*a^2*b*c^3 + (8*a^2*b^2 + a*b^3)*c^2)*f)*x^8 + 2*((4*(
5*a^2*b^2 + 4*a*b^3)*c^2 - (a*b^4 + 2*b^5)*c)*d - (12*a^2*b^2*c^2 + (8*a^2
*b^3 + a*b^4)*c)*f)*x^6 - ((a*b^5 + 2*b^6 - 8*(5*a^3*b + 4*a^2*b^2)*c^2 -
6*(3*a^2*b^3 + 2*a*b^4)*c)*d + (8*a^2*b^4 + a*b^5 + 24*a^3*b*c^2 + 2*(8*a^
3*b^2 + 7*a^2*b^3)*c)*f)*x^4 - 2*((a^2*b^4 + 2*a*b^5 - 4*(5*a^3*b^2 + 4*a^
2*b^3)*c)*d + (8*a^3*b^3 + a^2*b^4 + 12*a^3*b^2*c)*f)*x^2 - (a^3*b^3 + 2*a
^2*b^4 - 4*(5*a^4*b + 4*a^3*b^2)*c)*d - (8*a^4*b^2 + a^3*b^3 + 12*a^4*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{gx^3+fx^2+ex+d}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{gx^3+fx^2+ex+d}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{\left(c\,x^4+b\,x^2+a\right)^{5/2}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\text{too large to display}$$

input
```
int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x)
```

output

```
( - 8*sqrt(a + b*x**2 + c*x**4)*a**3*c*g - 2*sqrt(a + b*x**2 + c*x**4)*a**
2*b**2*g + 12*sqrt(a + b*x**2 + c*x**4)*a**2*b*c*e - 12*sqrt(a + b*x**2 +
c*x**4)*a**2*b*c*g*x**2 + 24*sqrt(a + b*x**2 + c*x**4)*a**2*c**2*e*x**2 +
16*sqrt(a + b*x**2 + c*x**4)*a**2*c**2*f*x**3 - sqrt(a + b*x**2 + c*x**4)*
a*b**3*e - 3*sqrt(a + b*x**2 + c*x**4)*a*b**3*g*x**2 + 6*sqrt(a + b*x**2 +
 c*x**4)*a*b**2*c*e*x**2 - 8*sqrt(a + b*x**2 + c*x**4)*a*b**2*c*f*x**3 - 1
2*sqrt(a + b*x**2 + c*x**4)*a*b**2*c*g*x**4 + 24*sqrt(a + b*x**2 + c*x**4)
*a*b*c**2*e*x**4 - 8*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*g*x**6 + 16*sqrt(a
 + b*x**2 + c*x**4)*a*c**3*e*x**6 + sqrt(a + b*x**2 + c*x**4)*b**4*f*x**3
+ 48*int(sqrt(a + b*x**2 + c*x**4)/(a**3 + 3*a**2*b*x**2 + 3*a**2*c*x**4 +
 3*a*b**2*x**4 + 6*a*b*c*x**6 + 3*a*c**2*x**8 + b**3*x**6 + 3*b**2*c*x**8
+ 3*b*c**2*x**10 + c**3*x**12),x)*a**5*c**2*d - 24*int(sqrt(a + b*x**2 + c
*x**4)/(a**3 + 3*a**2*b*x**2 + 3*a**2*c*x**4 + 3*a*b**2*x**4 + 6*a*b*c*x**
6 + 3*a*c**2*x**8 + b**3*x**6 + 3*b**2*c*x**8 + 3*b*c**2*x**10 + c**3*x**1
2),x)*a**4*b**2*c*d + 96*int(sqrt(a + b*x**2 + c*x**4)/(a**3 + 3*a**2*b*x*
*2 + 3*a**2*c*x**4 + 3*a*b**2*x**4 + 6*a*b*c*x**6 + 3*a*c**2*x**8 + b**3*x
**6 + 3*b**2*c*x**8 + 3*b*c**2*x**10 + c**3*x**12),x)*a**4*b*c**2*d*x**2 +
 96*int(sqrt(a + b*x**2 + c*x**4)/(a**3 + 3*a**2*b*x**2 + 3*a**2*c*x**4 +
3*a*b**2*x**4 + 6*a*b*c*x**6 + 3*a*c**2*x**8 + b**3*x**6 + 3*b**2*c*x**8 +
 3*b*c**2*x**10 + c**3*x**12),x)*a**4*c**3*d*x**4 + 3*int(sqrt(a + b*x*...
```

3.105 $\int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 19

$$\int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx=\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

output
```
g*x/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx=\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[(a*g - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(g*x)/Sqrt[a + b*x^2 + c*x^4]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$$
$$\Big\downarrow \text{2021}$$
$$\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(a*g - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(g*x)/Sqrt[a + b*x^2 + c*x^4]
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{gx}{\sqrt{c\,x^4+b\,x^2+a}}$	18
default	$\frac{gx}{\sqrt{c\,x^4+b\,x^2+a}}$	18
trager	$\frac{gx}{\sqrt{c\,x^4+b\,x^2+a}}$	18
elliptic	$\frac{gx}{\sqrt{c\,x^4+b\,x^2+a}}$	18
pseudoelliptic	$\frac{gx}{\sqrt{c\,x^4+b\,x^2+a}}$	18
orering	$\frac{x(-cg\,x^4+ag)}{\sqrt{c\,x^4+b\,x^2+a}\,(-c\,x^4+a)}$	38

input
```
int((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
g*x/(c*x^4+b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{cx^4+bx^2+a}}$$

input
```
integrate((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
g*x/sqrt(c*x^4 + b*x^2 + a)
```

Sympy [F]

$$\int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = -g\left(\int\left(-\frac{a}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx + \int\frac{cx^4}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\,dx\right)$$

input
```
integrate((-c*g*x**4+a*g)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
-g*(Integral(-a/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*
x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) + Integral(c*x**4/(a*sqrt(a
+ b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*
x**2 + c*x**4)), x))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{ag - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\, dx = \frac{gx}{\sqrt{cx^4 + bx^2 + a}}$$

input
```
integrate((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
g*x/sqrt(c*x^4 + b*x^2 + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(17) = 34.

Time = 0.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.16

$$\int \frac{ag - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\, dx = \frac{(b^4g - 8\,ab^2cg + 16\,a^2c^2g)x}{\sqrt{cx^4 + bx^2 + a}(b^4 - 8\,ab^2c + 16\,a^2c^2)}$$

input
```
integrate((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(b^4*g - 8*a*b^2*c*g + 16*a^2*c^2*g)*x/(sqrt(c*x^4 + b*x^2 + a)*(b^4 - 8*a
*b^2*c + 16*a^2*c^2))
```

Mupad [B] (verification not implemented)

Time = 18.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{ag - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\,dx = \frac{g\,x}{\sqrt{c\,x^4 + b\,x^2 + a}}$$

input
```
int((a*g - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
(g*x)/(a + b*x^2 + c*x^4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{ag - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4 + b\,x^2 + a}\,gx}{c\,x^4 + b\,x^2 + a}$$

input
```
int((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4)*g*x)/(a + b*x**2 + c*x**4)
```

3.106 $\int \frac{ag+ex-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 57

$$\int \frac{ag+ex-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{e(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

output
```
g*x/(c*x^4+b*x^2+a)^(1/2)-e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{ag+ex-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{-be+b^2gx-4acgx-2cex^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[(a*g + e*x - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-(b*e) + b^2*g*x - 4*a*c*g*x - 2*c*e*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 +
 c*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2202, 27, 1432, 1088, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag-cgx^4+ex}{(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{ex}{(cx^4+bx^2+a)^{3/2}}dx+\int \frac{ag-cgx^4}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$e\int \frac{x}{(cx^4+bx^2+a)^{3/2}}dx+\int \frac{ag-cgx^4}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{1}{2}e\int \frac{1}{(cx^4+bx^2+a)^{3/2}}dx^2+\int \frac{ag-cgx^4}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 1088}$$

$$\int \frac{ag-cgx^4}{(cx^4+bx^2+a)^{3/2}}dx-\frac{e(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{e(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(a*g + e*x - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(g*x)/Sqrt[a + b*x^2 + c*x^4] - (e*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + 
b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

method	result
gosper	$\frac{4acgx-b^2gx+2ce\,x^2+be}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$
trager	$\frac{4acgx-b^2gx+2ce\,x^2+be}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$
orering	$\frac{4acgx-b^2gx+2ce\,x^2+be}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$
elliptic	$\frac{e(2c\,x^2+b)}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}+\frac{gx}{\sqrt{c\,x^4+b\,x^2+a}}$
default	$ag\left(-\frac{2c\left(\frac{b\,x^3}{2a(4ac-b^2)}-\frac{(2ac-b^2)x}{2a(4ac-b^2)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^2}{a(4ac-b^2)}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\cdots\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}\cdots}\right.$

input
```
int((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(4*a*c*g*x-b^2*g*x+2*c*e*x^2+b*e)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44

$$\int \frac{ag+ex-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{\sqrt{cx^4+bx^2+a}(2\,cex^2-(b^2-4\,ac)gx+be)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

input
```
integrate((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(c*x^4 + b*x^2 + a)*(2*c*e*x^2 - (b^2 - 4*a*c)*g*x + b*e)/((b^2*c - 4
*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F]

$$\int \frac{ag + ex - cgx^4}{(a + bx^2 + cx^4)^{3/2}} \, dx = -\int \left(-\frac{ag}{a\sqrt{a + bx^2 + cx^4} + bx^2\sqrt{a + bx^2 + cx^4} + cx^4\sqrt{a + bx^2 + cx^4}} \right) dx - \int \left(-\frac{ex}{a\sqrt{a + bx^2 + cx^4} + bx^2\sqrt{a + bx^2 + cx^4} + cx^4\sqrt{a + bx^2 + cx^4}} \right) dx - \int \frac{cgx^4}{a\sqrt{a + bx^2 + cx^4} + bx^2\sqrt{a + bx^2 + cx^4} + cx^4\sqrt{a + bx^2 + cx^4}} \, dx$$

input
```
integrate((-c*g*x**4+a*g+e*x)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
-Integral(-a*g/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x
**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(-e*x/(a*sqrt(a + b
*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**
2 + c*x**4)), x) - Integral(c*g*x**4/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2
*sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{ag + ex - cgx^4}{(a + bx^2 + cx^4)^{3/2}} \, dx = -\frac{2\,cex^2 + be - (b^2g - 4\,acg)x}{\sqrt{cx^4 + bx^2 + a}(b^2 - 4\,ac)}$$

input
```
integrate((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-(2*c*e*x^2 + b*e - (b^2*g - 4*a*c*g)*x)/(sqrt(c*x^4 + b*x^2 + a)*(b^2 - 4
*a*c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(53) = 106$.

Time = 0.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.42

$$\int \frac{ag+ex-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{\left(\frac{2\,(b^2ce-4\,ac^2e)x}{b^4-8\,ab^2c+16\,a^2c^2}-\frac{b^4g-8\,ab^2cg+16\,a^2c^2g}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{b^3e-4\,abce}{b^4-8\,ab^2c+16\,a^2c^2}}{\sqrt{cx^4+bx^2+a}}$$

input
```
integrate((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-((2*(b^2*c*e - 4*a*c^2*e)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2) - (b^4*g - 8*a
*b^2*c*g + 16*a^2*c^2*g)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + (b^3*e - 4*a*
b*c*e)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 18.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{ag+ex-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-g\,b^2\,x+e\,b+2\,c\,e\,x^2+4\,a\,c\,g\,x}{(4\,a\,c-b^2)\,\sqrt{c\,x^4+b\,x^2+a}}$$

input
```
int((a*g + e*x - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
(b*e + 2*c*e*x^2 - b^2*g*x + 4*a*c*g*x)/((4*a*c - b^2)*(a + b*x^2 + c*x^4)
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.53

$$\int \frac{ag+ex-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2+a}\,(4acgx-b^2gx+2ce\,x^2+be)}{4a\,c^2x^4-b^2c\,x^4+4abc\,x^2-b^3x^2+4a^2c-a\,b^2}$$

input
```
int((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4)*(4*a*c*g*x - b**2*g*x + b*e + 2*c*e*x**2))/(4*a
**2*c - a*b**2 + 4*a*b*c*x**2 + 4*a*c**2*x**4 - b**3*x**2 - b**2*c*x**4)
```

3.107 $\int \frac{ag+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 57

$$\int \frac{ag+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{a+bx^2+cx^4}}+\frac{f(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

output
```
g*x/(c*x^4+b*x^2+a)^(1/2)+f*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{ag+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{bx(bg+fx)+2a(f-2cgx)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[(a*g + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(b*x*(b*g + f*x) + 2*a*(f - 2*c*g*x))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^
4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2202, 27, 1434, 1158, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag-cgx^4+fx^3}{(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{fx^3}{(cx^4+bx^2+a)^{3/2}}dx+\int \frac{ag-cgx^4}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$f\int \frac{x^3}{(cx^4+bx^2+a)^{3/2}}dx+\int \frac{ag-cgx^4}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}f\int \frac{x^2}{(cx^4+bx^2+a)^{3/2}}dx^2+\int \frac{ag-cgx^4}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{1158}$$

$$\int \frac{ag-cgx^4}{(cx^4+bx^2+a)^{3/2}}dx+\frac{f(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{2021}$$

$$\frac{f(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(a*g + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(g*x)/Sqrt[a + b*x^2 + c*x^4] + (f*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a +
b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{4acgx-b^{2}gx-bf\,x^{2}-2af}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$
trager	$\frac{4acgx-b^{2}gx-bf\,x^{2}-2af}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$
orering	$\frac{4acgx-b^{2}gx-bf\,x^{2}-2af}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$
elliptic	$-\frac{f\left(b\,x^{2}+2a\right)}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}+\frac{gx}{\sqrt{c\,x^{4}+b\,x^{2}+a}}$
default	$ag\left(-\frac{2c\left(\frac{b\,x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)x}{2a\left(4ac-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^{2}}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\cdots\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\cdots}\right.$

input
```
int((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(4*a*c*g*x-b^2*g*x-b*f*x^2-2*a*f)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{ag+fx^{3}-cgx^{4}}{\left(a+bx^{2}+cx^{4}\right)^{3/2}}\,dx=\frac{\sqrt{cx^{4}+bx^{2}+a}\left(bfx^{2}+\left(b^{2}-4\,ac\right)gx+2\,af\right)}{\left(b^{2}c-4\,ac^{2}\right)x^{4}+ab^{2}-4\,a^{2}c+\left(b^{3}-4\,abc\right)x^{2}}$$

input
```
integrate((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

output
```
sqrt(c*x^4 + b*x^2 + a)*(b*f*x^2 + (b^2 - 4*a*c)*g*x + 2*a*f)/((b^2*c - 4*
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F]

$$\int \frac{ag + fx^3 - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\, dx = -\int \left(-\frac{ag}{a\sqrt{a + bx^2 + cx^4} + bx^2\sqrt{a + bx^2 + cx^4} + cx^4\sqrt{a + bx^2 + cx^4}}\right) dx - \int \left(-\frac{fx^3}{a\sqrt{a + bx^2 + cx^4} + bx^2\sqrt{a + bx^2 + cx^4} + cx^4\sqrt{a + bx^2 + cx^4}}\right) dx - \int \frac{cgx^4}{a\sqrt{a + bx^2 + cx^4} + bx^2\sqrt{a + bx^2 + cx^4} + cx^4\sqrt{a + bx^2 + cx^4}}\, dx$$

input
```
integrate((-c*g*x**4+f*x**3+a*g)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
-Integral(-a*g/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x
**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(-f*x**3/(a*sqrt(a
+ b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*
x**2 + c*x**4)), x) - Integral(c*g*x**4/(a*sqrt(a + b*x**2 + c*x**4) + b*x
**2*sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{ag + fx^3 - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\, dx = \frac{bfx^2 + 2\,af + (b^2g - 4\,acg)x}{\sqrt{cx^4 + bx^2 + a}(b^2 - 4\,ac)}$$

input
```
integrate((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

output
```
(b*f*x^2 + 2*a*f + (b^2*g - 4*a*c*g)*x)/(sqrt(c*x^4 + b*x^2 + a)*(b^2 - 4*
a*c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(53) = 106$.

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

$$\int \frac{ag + fx^3 - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\,dx = \frac{\left(\frac{(b^3f - 4\,abcf)x}{b^4 - 8\,ab^2c + 16\,a^2c^2} + \frac{b^4g - 8\,ab^2cg + 16\,a^2c^2g}{b^4 - 8\,ab^2c + 16\,a^2c^2}\right)x + \frac{2\,(ab^2f - 4\,a^2cf)}{b^4 - 8\,ab^2c + 16\,a^2c^2}}{\sqrt{cx^4 + bx^2 + a}}$$

input
```
integrate((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(((b^3*f - 4*a*b*c*f)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2) + (b^4*g - 8*a*b^2*
c*g + 16*a^2*c^2*g)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + 2*(a*b^2*f - 4*a^2
*c*f)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 18.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{ag + fx^3 - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\,dx = -\frac{g\,b^2\,x + f\,b\,x^2 - 4\,a\,c\,g\,x + 2\,a\,f}{(4\,a\,c - b^2)\,\sqrt{c\,x^4 + b\,x^2 + a}}$$

input
```
int((a*g + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
-(2*a*f + b*f*x^2 + b^2*g*x - 4*a*c*g*x)/((4*a*c - b^2)*(a + b*x^2 + c*x^4
)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.54

$$\int \frac{ag + fx^3 - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4 + b\,x^2 + a}\,(4acgx - b^2gx - bf\,x^2 - 2af)}{4a\,c^2x^4 - b^2c\,x^4 + 4abc\,x^2 - b^3x^2 + 4a^2c - a\,b^2}$$

input
```
int((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x)
```

output

```
(sqrt(a + b*x**2 + c*x**4)*(4*a*c*g*x - 2*a*f - b**2*g*x - b*f*x**2))/(4*a
**2*c - a*b**2 + 4*a*b*c*x**2 + 4*a*c**2*x**4 - b**3*x**2 - b**2*c*x**4)
```

3.108 $\int \frac{ag+ex+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 69

$$\int \frac{ag+ex+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{a+bx^2+cx^4}} - \frac{be-2af+(2ce-bf)x^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

output
```
g*x/(c*x^4+b*x^2+a)^(1/2)-(b*e-2*a*f+(-b*f+2*c*e)*x^2)/(-4*a*c+b^2)/(c*x^4
+b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{ag+ex+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-be+2af+b^2gx-4acgx-2cex^2+bfx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[(a*g + e*x + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-(b*e) + 2*a*f + b^2*g*x - 4*a*c*g*x - 2*c*e*x^2 + b*f*x^2)/((b^2 - 4*a*c
)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2202, 1576, 1158, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag-cgx^4+ex+fx^3}{(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{x(fx^2+e)}{(cx^4+bx^2+a)^{3/2}}dx+\int \frac{ag-cgx^4}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int \frac{fx^2+e}{(cx^4+bx^2+a)^{3/2}}dx^2+\int \frac{ag-cgx^4}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 1158}$$

$$\int \frac{ag-cgx^4}{(cx^4+bx^2+a)^{3/2}}dx-\frac{-2af+x^2(2ce-bf)+be}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{-2af+x^2(2ce-bf)+be}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(a*g + e*x + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(g*x)/Sqrt[a + b*x^2 + c*x^4] - (b*e - 2*a*f + (2*c*e - b*f)*x^2)/((b^2 -
4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result	size
gosper	$\frac{4acgx-b^{2}gx-bf\,x^{2}+2ce\,x^{2}-2af+be}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$	63
trager	$\frac{4acgx-b^{2}gx-bf\,x^{2}+2ce\,x^{2}-2af+be}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$	63
orering	$\frac{4acgx-b^{2}gx-bf\,x^{2}+2ce\,x^{2}-2af+be}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$	63
elliptic	$-\frac{bf\,x^{2}-2ce\,x^{2}+2af-be}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}+\frac{gx}{\sqrt{c\,x^{4}+b\,x^{2}+a}}$	69
default	Expression too large to display	1012

input
```
int((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
(4*a*c*g*x-b^2*g*x-b*f*x^2+2*c*e*x^2-2*a*f+b*e)/(c*x^4+b*x^2+a)^(1/2)/(4*a
*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \frac{ag + ex + fx^3 - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\,dx = \frac{\sqrt{cx^4 + bx^2 + a}((b^2 - 4\,ac)gx - (2\,ce - bf)x^2 - be + 2\,af)}{(b^2c - 4\,ac^2)x^4 + ab^2 - 4\,a^2c + (b^3 - 4\,abc)x^2}$$

input
```
integrate((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fri
cas")
```

output
```
sqrt(c*x^4 + b*x^2 + a)*((b^2 - 4*a*c)*g*x - (2*c*e - b*f)*x^2 - b*e + 2*a
*f)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F]

$$\int \frac{ag + ex + fx^3 - cgx^4}{(a + bx^2 + cx^4)^{3/2}}\,dx = -\int \left(-\frac{ag}{a\sqrt{a + bx^2 + cx^4} + bx^2\sqrt{a + bx^2 + cx^4} + cx^4\sqrt{a + bx^2 + cx^4}}\right)dx - \int \left(-\frac{ex}{a\sqrt{a + bx^2 + cx^4} + bx^2\sqrt{a + bx^2 + cx^4} + cx^4\sqrt{a + bx^2 + cx^4}}\right)dx - \int \left(-\frac{fx^3}{a\sqrt{a + bx^2 + cx^4} + bx^2\sqrt{a + bx^2 + cx^4} + cx^4\sqrt{a + bx^2 + cx^4}}\right)dx - \int \frac{cgx^4}{a\sqrt{a + bx^2 + cx^4} + bx^2\sqrt{a + bx^2 + cx^4} + cx^4\sqrt{a + bx^2 + cx^4}}\,dx$$

input
```
integrate((-c*g*x**4+f*x**3+a*g+e*x)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
-Integral(-a*g/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x
**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(-e*x/(a*sqrt(a + b
*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**
2 + c*x**4)), x) - Integral(-f*x**3/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*
sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integr
al(c*g*x**4/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4
) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.36

$$\int \frac{ag+ex+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{\sqrt{cx^4+bx^2+a}((2\,ce-bf)x^2+be-2\,af-(b^2g-4\,acg)x)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

input
```
integrate((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="max
ima")
```

output
```
-sqrt(c*x^4 + b*x^2 + a)*((2*c*e - b*f)*x^2 + b*e - 2*a*f - (b^2*g - 4*a*c
*g)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(65) = 130$.

Time = 0.38 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.38

$$\int \frac{ag+ex+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{\left(\frac{(2\,b^2ce-8\,ac^2e-b^3f+4\,abcf)x}{b^4-8\,ab^2c+16\,a^2c^2}-\frac{b^4g-8\,ab^2cg+16\,a^2c^2g}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{b^3e-4\,abce-2\,ab^2f+8\,a^2cf}{b^4-8\,ab^2c+16\,a^2c^2}}{\sqrt{cx^4+bx^2+a}}$$

input
```
integrate((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="gia
c")
```

output
```
-(((2*b^2*c*e - 8*a*c^2*e - b^3*f + 4*a*b*c*f)*x/(b^4 - 8*a*b^2*c + 16*a^2
*c^2) - (b^4*g - 8*a*b^2*c*g + 16*a^2*c^2*g)/(b^4 - 8*a*b^2*c + 16*a^2*c^2
))*x + (b^3*e - 4*a*b*c*e - 2*a*b^2*f + 8*a^2*c*f)/(b^4 - 8*a*b^2*c + 16*a
^2*c^2))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 18.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{ag+ex+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{g\,b^2\,x+f\,b\,x^2-e\,b-2\,c\,e\,x^2-4\,a\,c\,g\,x+2\,a\,f}{(4\,a\,c-b^2)\,\sqrt{c\,x^4+b\,x^2+a}}$$

input
```
int((a*g + e*x + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
-(2*a*f - b*e + b*f*x^2 - 2*c*e*x^2 + b^2*g*x - 4*a*c*g*x)/((4*a*c - b^2)*
(a + b*x^2 + c*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \frac{ag+ex+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2+a}\,(4acgx-b^2gx-bf\,x^2+2ce\,x^2-2af+be)}{4a\,c^2x^4-b^2c\,x^4+4abc\,x^2-b^3x^2+4a^2c-a\,b^2}$$

input
```
int((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4)*(4*a*c*g*x - 2*a*f - b**2*g*x + b*e - b*f*x**2
+ 2*c*e*x**2))/(4*a**2*c - a*b**2 + 4*a*b*c*x**2 + 4*a*c**2*x**4 - b**3*x*
*2 - b**2*c*x**4)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*   "F" if the result fails to integrate an expression that*)
(*       is integrable*)
(*   "C" if result involves higher level functions than necessary*)
(*   "B" if result is more than twice the size of the optimal*)
(*       antiderivative*)
(*   "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
         return True
      else:
         return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file