Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.2-Quartic-trinomial/120-1.2.2.6

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**190**]. This is test number [120].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (190)	0.00 (0)
Mathematica	100.00 (190)	0.00 (0)
Maple	98.42 (187)	1.58 (3)
Fricas	83.16 (158)	16.84 (32)
Mupad	68.95 (131)	31.05 (59)
Giac	68.95 (131)	31.05 (59)
Reduce	59.47 (113)	40.53 (77)
Sympy	40.00 (76)	60.00 (114)
Maxima	31.58 (60)	68.42 (130)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

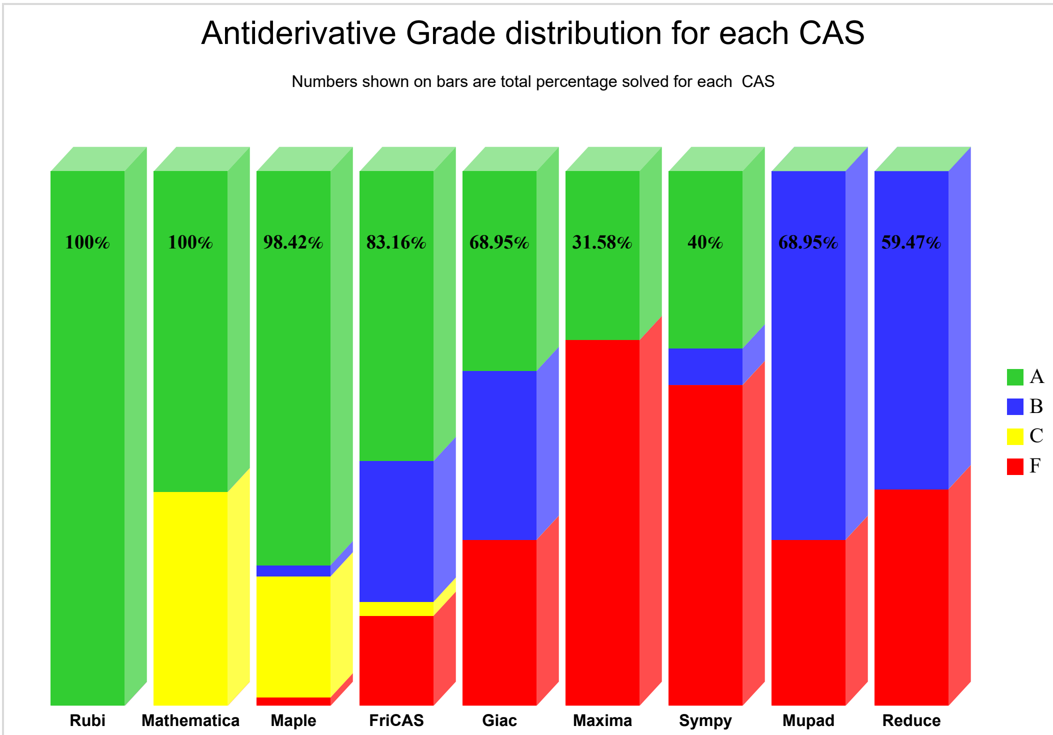
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

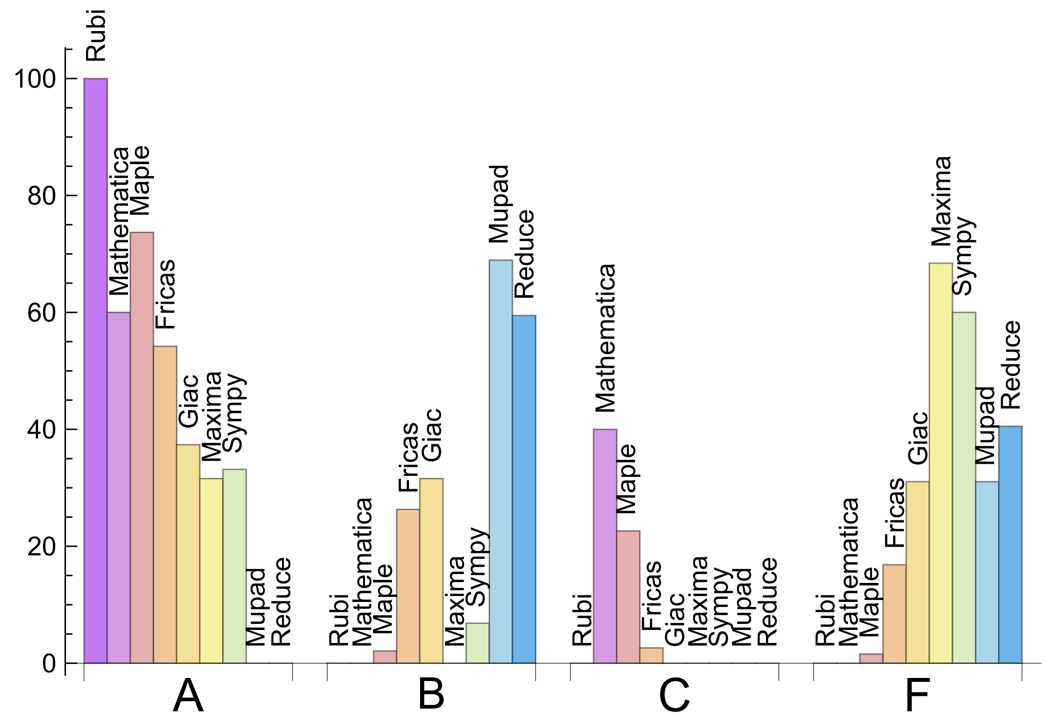
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	73.684	2.105	22.632	1.579
Mathematica	60.000	0.000	40.000	0.000
Fricas	54.211	26.316	2.632	16.842
Giac	37.368	31.579	0.000	31.053
Sympy	33.158	6.842	0.000	60.000
Maxima	31.579	0.000	0.000	68.421
Mupad	0.000	68.947	0.000	31.053
Reduce	0.000	59.474	0.000	40.526

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	3	100.00	0.00	0.00
Fricas	32	37.50	62.50	0.00
Mupad	59	0.00	100.00	0.00
Giac	59	100.00	0.00	0.00
Reduce	77	100.00	0.00	0.00
Sympy	114	43.86	56.14	0.00
Maxima	130	87.69	0.00	12.31

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Reduce	0.33
Giac	0.54
Rubi	0.83
Maple	2.51
Sympy	3.27
Mathematica	4.00
Fricas	6.03
Mupad	13.34

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	86.07	0.92	59.50	0.88
Rubi	287.07	1.07	254.50	1.01
Maple	300.24	0.98	162.00	0.88
Mathematica	335.08	1.07	194.00	1.01
Sympy	1088.36	4.22	70.50	0.97
Reduce	1404.36	5.47	429.00	4.25
Giac	1911.90	5.68	208.00	1.18
Mupad	5009.55	14.27	173.00	0.93
Fricas	24628.25	96.79	272.00	1.38

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

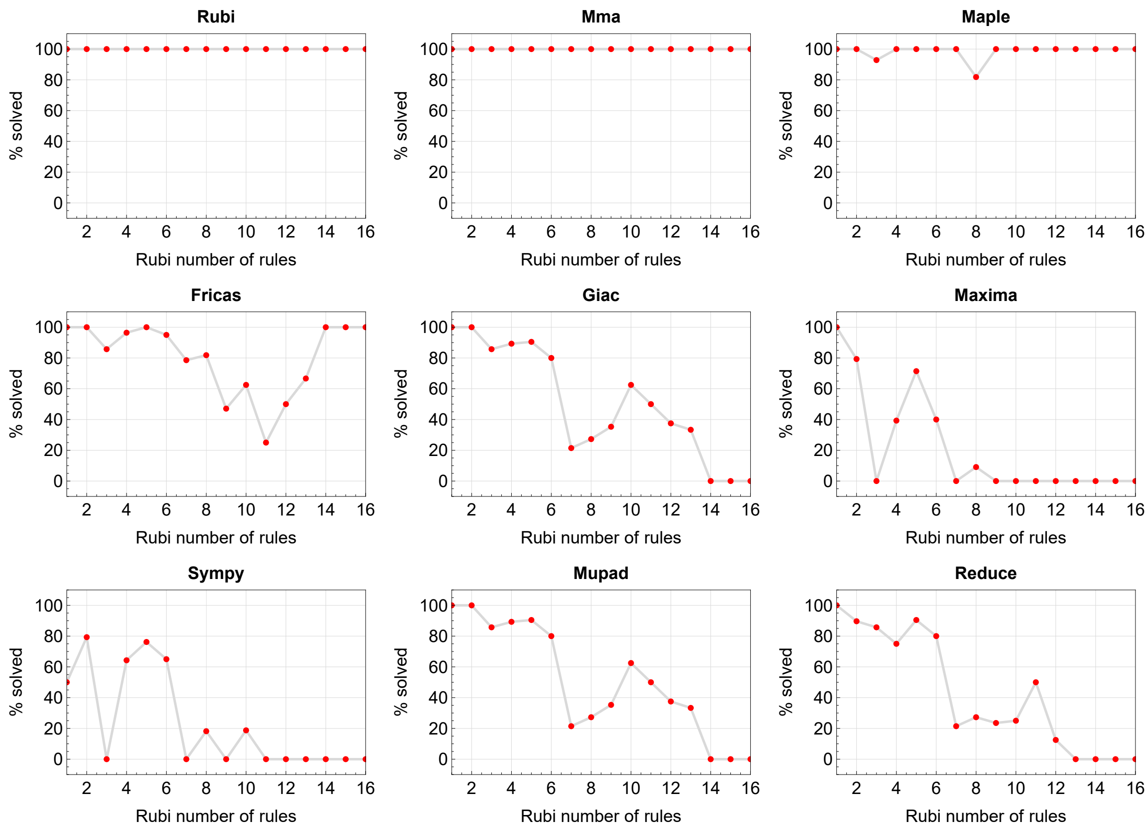


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

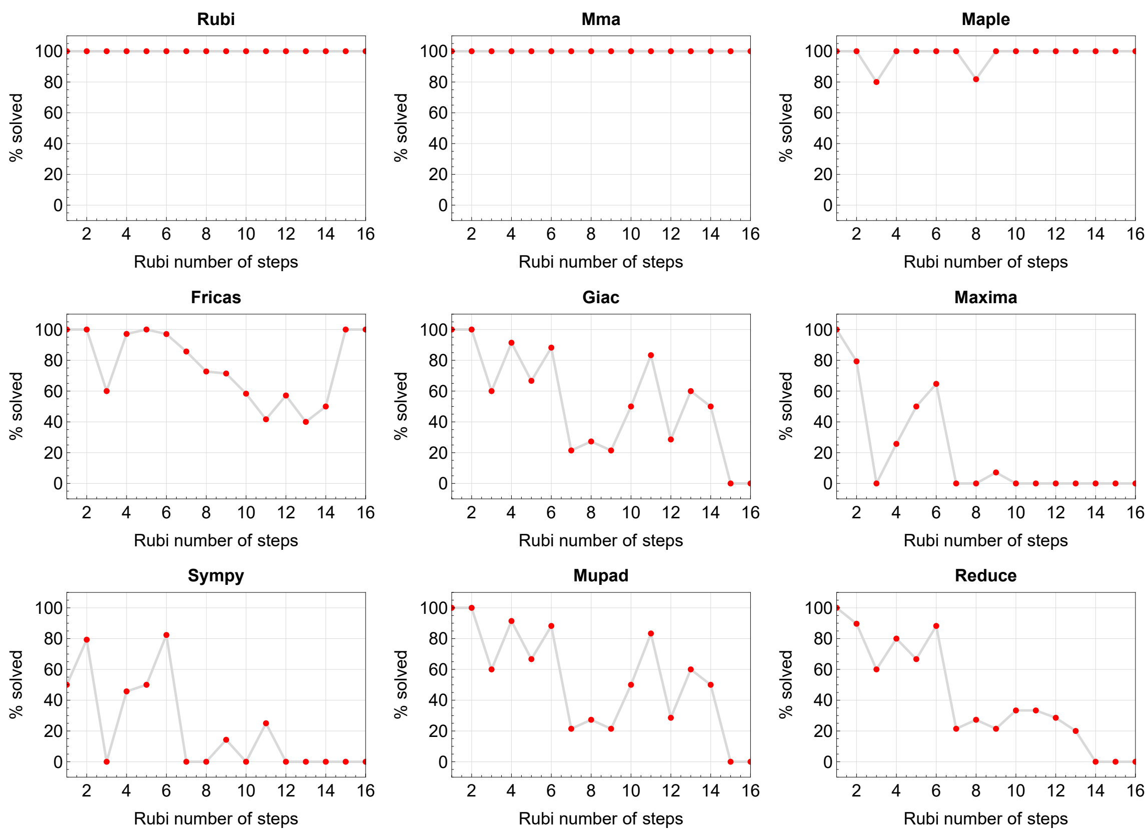


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

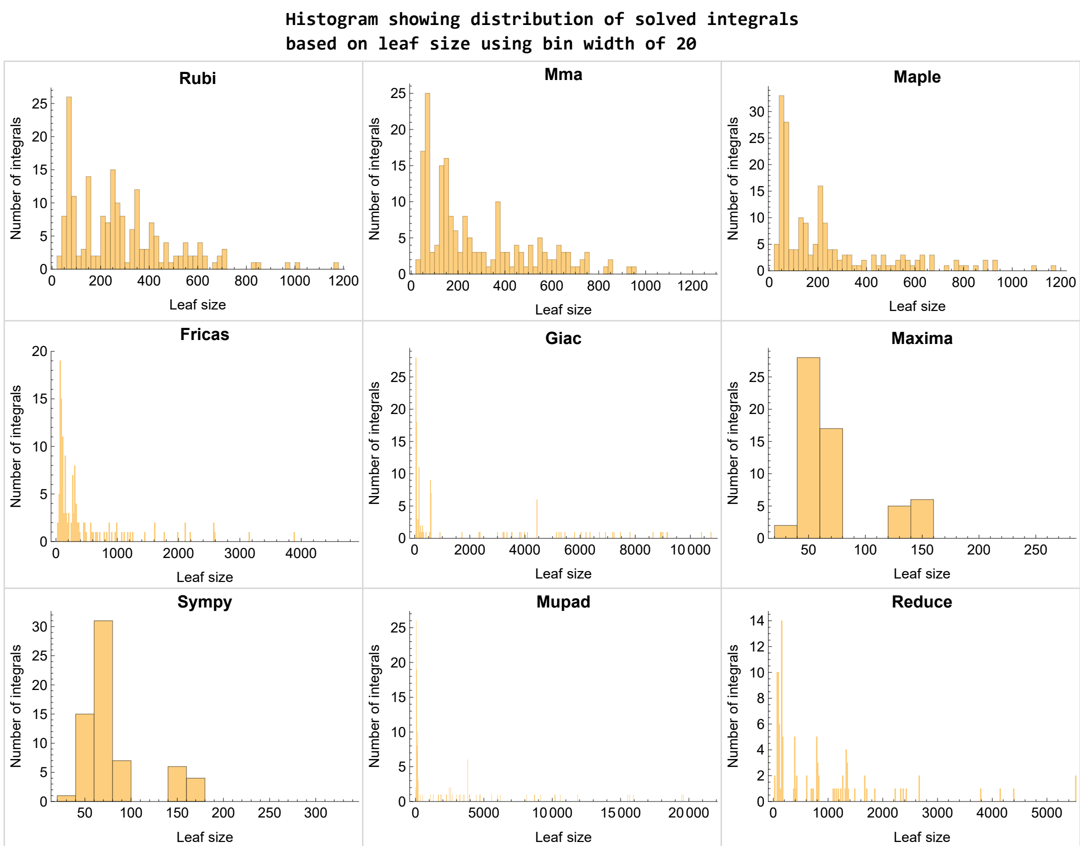


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

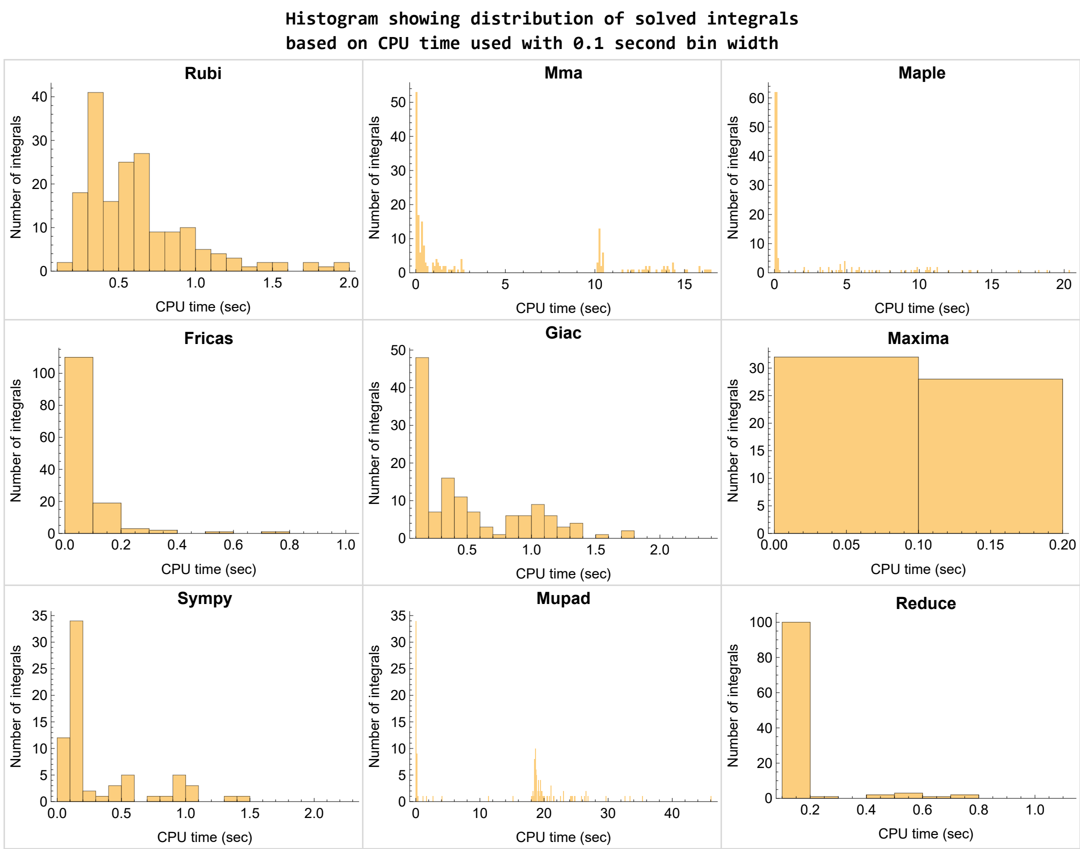


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

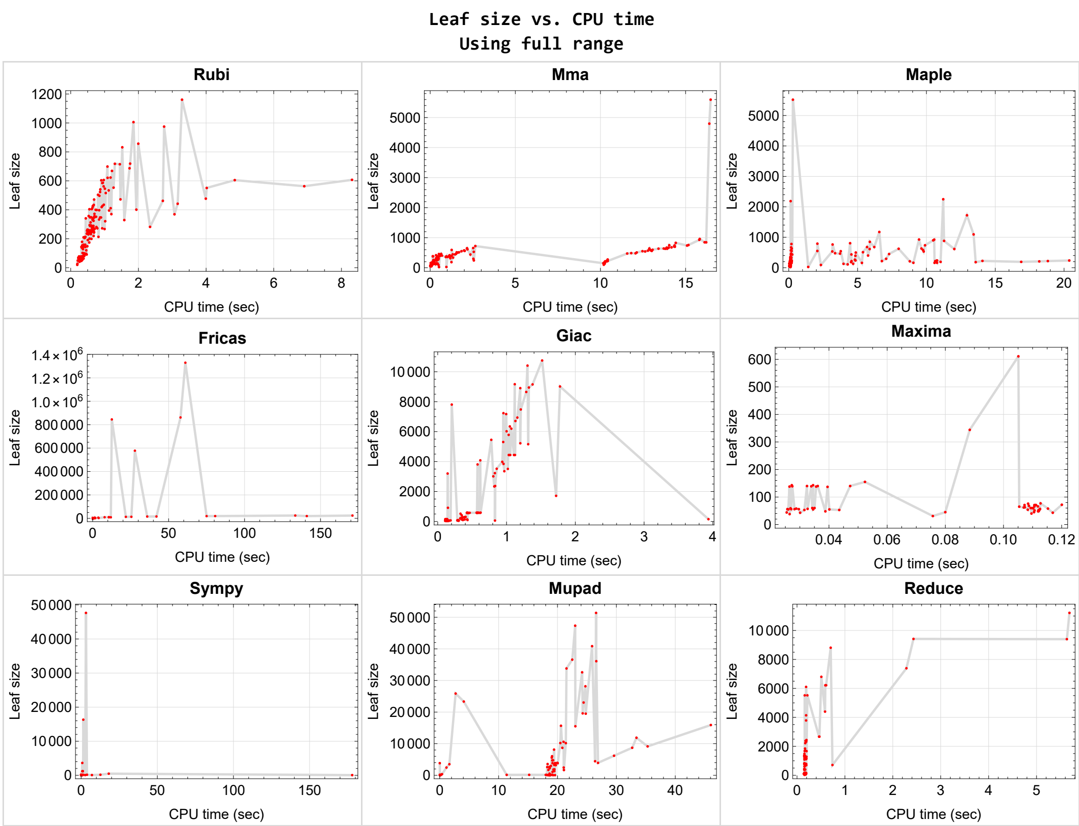


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {41, 42, 188}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

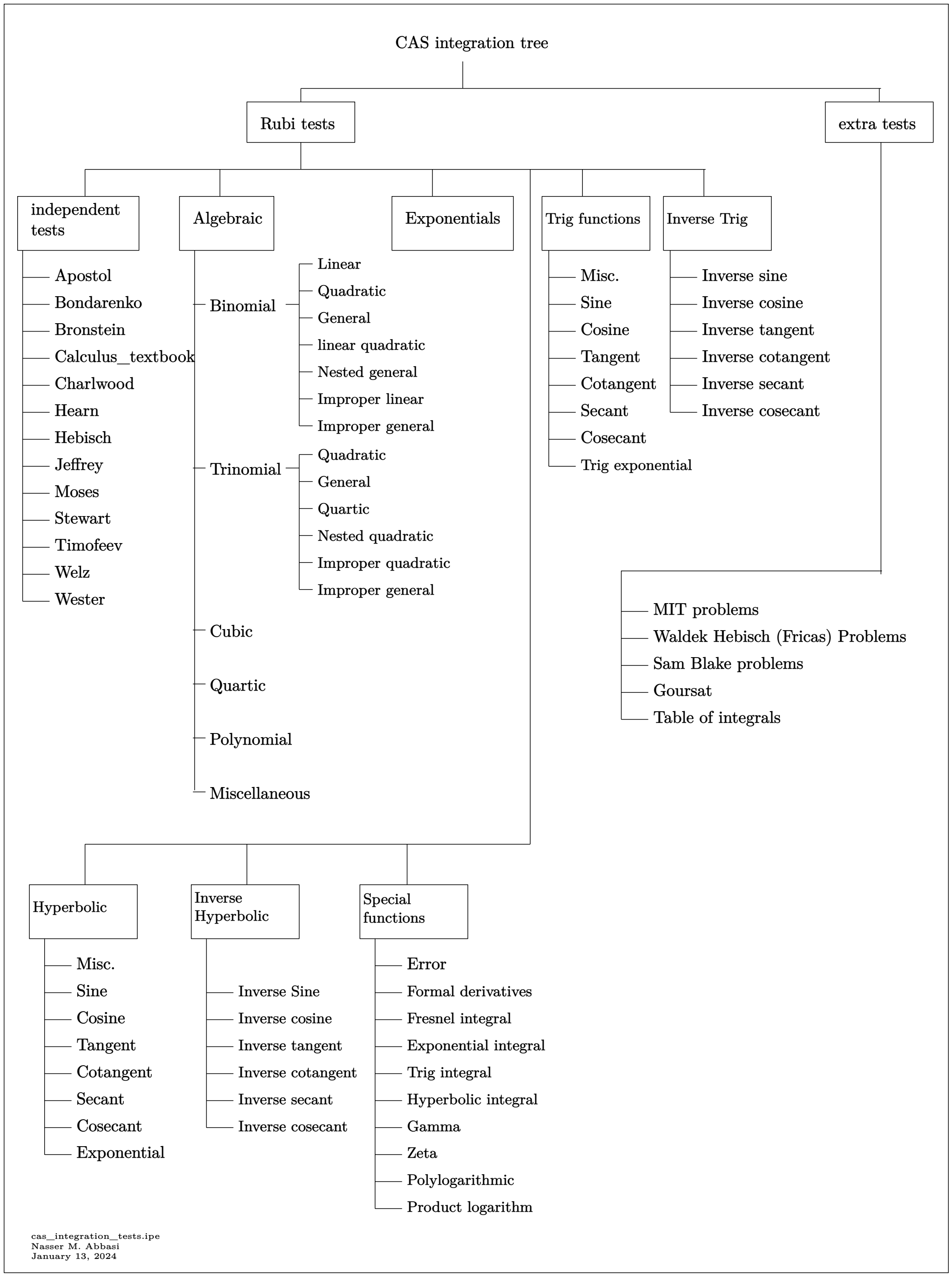
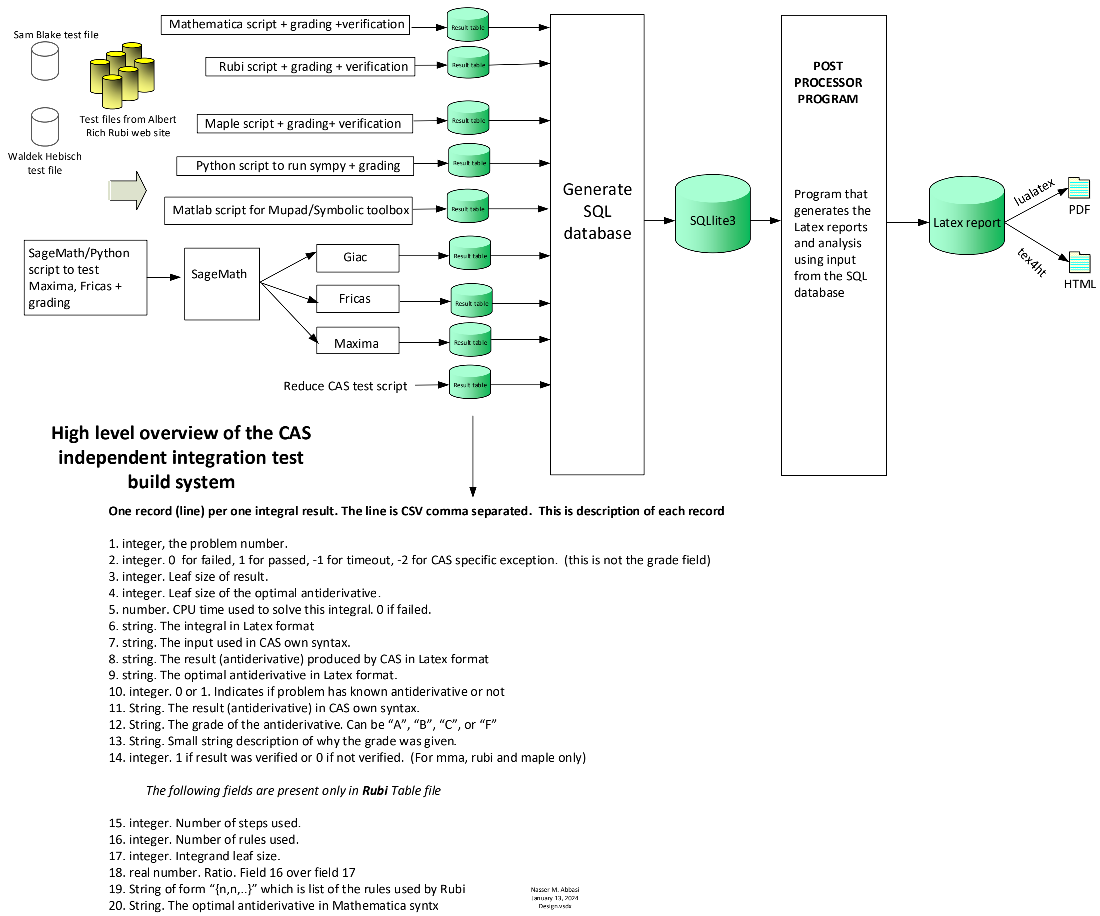


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 126, 127, 128, 129, 130, 131, 188, 189, 190 }

B grade { }

C grade { 41, 42, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 26, 27, 28, 35, 36, 37, 40, 48, 49, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190 }

B grade { 38, 39, 139, 140 }

C grade { 21, 22, 23, 24, 25, 29, 30, 31, 32, 33, 34, 43, 44, 45, 46, 47, 56, 57, 58, 69, 70, 71, 72, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 164 }

F normal fail { 41, 42, 188 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 48, 49, 50, 51, 52, 53, 54, 55, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 116, 126, 132, 133, 134, 138, 139, 140, 141, 146, 147, 148, 150, 151, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190 }

B grade { 38, 39, 40, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 81, 111, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 130, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163 }

C grade { 22, 23, 24, 25, 164 }

F normal fail { 41, 42, 135, 136, 137, 142, 143, 144, 145, 149, 168, 188 }

F(-1) timedout fail { 21, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 127, 131 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 38, 39, 40, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 189, 190 }

B grade { }

C grade { }

F normal fail { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 41, 42, 43, 44, 45, 46, 47, 56, 57, 58, 59, 60, 61, 69, 70, 71, 72, 73, 74, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188 }

F(-1) timedout fail { }

F(-2) exception fail { 48, 49, 50, 51, 52, 53, 54, 55, 62, 63, 64, 65, 66, 67, 68, 126 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 48, 49, 50, 51, 52, 53, 54, 55, 62, 63, 64, 65, 66, 67, 68, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 126 }

B grade { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 56, 57, 58, 59, 60, 61, 69, 70, 71, 72, 73, 74, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 189, 190 }

C grade { }

F normal fail { 41, 42, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 189, 190 }

C grade { }

F normal fail { }

F(-1) timedout fail { 41, 42, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 116, 119, 120, 124, 125 }

B grade { 38, 39, 40, 65, 111, 114, 115, 117, 118, 121, 122, 123, 190 }

C grade { }

F normal fail { 41, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187 }

F(-1) timedout fail { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 126, 127, 128, 129, 130, 131, 157, 158, 159, 160, 161, 162, 163, 188, 189 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 31, 33, 35, 37, 38, 39, 40, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 189, 190 }

C grade { }

F normal fail { 27, 29, 30, 32, 34, 36, 41, 42, 43, 44, 45, 46, 47, 59, 60, 61, 73, 74, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	60	60	68	64	67	62
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.92	0.86	0.91	0.84
time (sec)	N/A	0.254	0.018	0.115	0.034	0.068	0.018	0.132	0.174	0.036

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	60	60	68	64	67	62
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.92	0.86	0.91	0.84
time (sec)	N/A	0.239	0.014	0.108	0.035	0.089	0.020	0.140	0.166	0.030

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	61	65	59
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.88	0.94	0.86
time (sec)	N/A	0.222	0.016	0.070	0.033	0.085	0.023	0.130	0.170	0.030

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	55	55	63	60	62	57
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.97	0.92	0.95	0.88
time (sec)	N/A	0.228	0.019	0.038	0.026	0.098	0.064	0.130	0.181	0.036

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	55	62	58	57	69	56
N.S.	1	1.00	1.00	0.90	0.87	0.98	0.92	0.90	1.10	0.89
time (sec)	N/A	0.243	0.025	0.040	0.040	0.102	0.076	0.126	0.165	0.036

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	58	55	62	61	58	71	56
N.S.	1	1.00	0.92	0.92	0.87	0.98	0.97	0.92	1.13	0.89
time (sec)	N/A	0.242	0.043	0.041	0.035	0.086	0.152	0.134	0.174	0.035

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	55	56	62	63	56	69	55
N.S.	1	1.00	0.95	0.87	0.89	0.98	1.00	0.89	1.10	0.87
time (sec)	N/A	0.247	0.049	0.045	0.026	0.091	0.285	0.116	0.176	0.033

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	56	56	62	63	57	71	56
N.S.	1	1.00	0.98	0.89	0.89	0.98	1.00	0.90	1.13	0.89
time (sec)	N/A	0.242	0.030	0.041	0.030	0.089	0.941	0.127	0.170	0.046

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	56	62	66	57	69	56
N.S.	1	1.00	1.00	0.89	0.89	0.98	1.05	0.90	1.10	0.89
time (sec)	N/A	0.243	0.062	0.044	0.028	0.097	2.656	0.138	0.187	18.260

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	59	59	62	70	60	69	60
N.S.	1	1.00	1.00	0.87	0.87	0.91	1.03	0.88	1.01	0.88
time (sec)	N/A	0.240	0.056	0.038	0.027	0.088	7.092	0.832	0.171	0.068

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	143	143	168	154	157	141
N.S.	1	1.00	1.00	0.89	0.90	0.90	1.06	0.97	0.99	0.89
time (sec)	N/A	0.427	0.054	0.128	0.027	0.080	0.028	0.134	0.172	15.193

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	143	143	163	154	157	141
N.S.	1	1.00	1.00	0.89	0.90	0.90	1.03	0.97	0.99	0.89
time (sec)	N/A	0.376	0.045	0.125	0.035	0.146	0.035	3.936	0.186	0.055

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	139	140	140	165	151	155	138
N.S.	1	1.00	1.00	0.90	0.91	0.91	1.07	0.98	1.01	0.90
time (sec)	N/A	0.342	0.035	0.142	0.032	0.063	0.034	0.121	0.172	0.056

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	150	138	138	138	156	149	151	135
N.S.	1	1.00	1.00	0.92	0.92	0.92	1.04	0.99	1.01	0.90
time (sec)	N/A	0.338	0.043	0.047	0.026	0.068	0.155	0.109	0.177	11.372

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	145	142	137	145	156	147	159	135
N.S.	1	1.00	1.00	0.98	0.94	1.00	1.08	1.01	1.10	0.93
time (sec)	N/A	0.356	0.109	0.049	0.040	0.082	0.149	0.114	0.181	0.101

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	139	140	139	145	153	148	161	135
N.S.	1	1.00	0.93	0.94	0.93	0.97	1.03	0.99	1.08	0.91
time (sec)	N/A	0.361	0.108	0.048	0.027	0.068	0.236	0.138	0.173	18.175

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	151	141	140	145	160	146	159	137
N.S.	1	1.00	1.01	0.95	0.94	0.97	1.07	0.98	1.07	0.92
time (sec)	N/A	0.371	0.090	0.049	0.036	0.072	0.382	0.113	0.182	0.060

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	130	139	139	145	153	142	163	134
N.S.	1	1.00	0.88	0.94	0.94	0.98	1.03	0.96	1.10	0.91
time (sec)	N/A	0.363	0.095	0.051	0.034	0.061	1.315	0.128	0.170	18.003

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	142	135	138	145	155	140	161	136
N.S.	1	1.00	0.99	0.94	0.97	1.01	1.08	0.98	1.13	0.95
time (sec)	N/A	0.368	0.093	0.053	0.036	0.063	3.829	0.111	0.174	0.054

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	144	136	140	145	158	141	163	136
N.S.	1	1.00	0.97	0.91	0.94	0.97	1.06	0.95	1.09	0.91
time (sec)	N/A	0.368	0.107	0.050	0.047	0.065	12.664	0.134	0.186	0.055

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	347	460	118	0	0	0	5307	1668	2588
N.S.	1	1.02	1.36	0.35	0.00	0.00	0.00	15.65	4.92	7.63
time (sec)	N/A	0.866	0.646	0.118	0.000	0.000	0.000	0.952	0.199	18.257

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	281	377	86	0	1329593	0	3521	1156	2696
N.S.	1	1.01	1.36	0.31	0.00	4782.71	0.00	12.67	4.16	9.70
time (sec)	N/A	0.631	0.449	0.100	0.000	61.037	0.000	0.857	0.197	18.706

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	279	360	71	0	861800	0	3845	1112	1890
N.S.	1	1.03	1.33	0.26	0.00	3191.85	0.00	14.24	4.12	7.00
time (sec)	N/A	0.667	0.411	0.104	0.000	57.842	0.000	0.956	0.189	19.483

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	235	240	52	0	845032	0	2370	697	5594
N.S.	1	1.05	1.08	0.23	0.00	3789.38	0.00	10.63	3.13	25.09
time (sec)	N/A	0.474	0.388	0.095	0.000	12.630	0.000	0.835	0.195	19.027

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	234	48	0	578003	0	1712	786	3942
N.S.	1	1.00	1.11	0.23	0.00	2739.35	0.00	8.11	3.73	18.68
time (sec)	N/A	0.411	0.237	0.079	0.000	27.819	0.000	1.721	0.192	19.967

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	234	285	251	0	0	0	2338	711	2258
N.S.	1	1.02	1.24	1.10	0.00	0.00	0.00	10.21	3.10	9.86
time (sec)	N/A	0.514	0.489	0.101	0.000	0.000	0.000	0.822	0.740	18.676

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	261	315	325	0	0	0	3510	30	2588
N.S.	1	1.00	1.21	1.25	0.00	0.00	0.00	13.50	0.12	9.95
time (sec)	N/A	0.644	1.184	0.108	0.000	0.000	0.000	1.019	200.030	18.602

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	289	377	300	0	0	0	3355	1279	3563
N.S.	1	1.00	1.31	1.04	0.00	0.00	0.00	11.65	4.44	12.37
time (sec)	N/A	0.670	0.948	0.125	0.000	0.000	0.000	0.968	0.198	18.320

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	427	502	266	0	0	0	5784	30	6033
N.S.	1	1.01	1.18	0.63	0.00	0.00	0.00	13.64	0.07	14.23
time (sec)	N/A	0.938	1.616	0.138	0.000	0.000	0.000	1.031	200.027	19.076

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	397	444	249	0	0	0	5221	30	4754
N.S.	1	0.98	1.10	0.61	0.00	0.00	0.00	12.89	0.07	11.74
time (sec)	N/A	0.840	1.461	0.129	0.000	0.000	0.000	1.200	200.027	19.261

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	339	358	213	0	0	0	3228	2670	3278
N.S.	1	0.98	1.03	0.61	0.00	0.00	0.00	9.30	7.69	9.45
time (sec)	N/A	0.663	1.026	0.109	0.000	0.000	0.000	0.830	0.468	19.452

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4439	30	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	0.08	10.77
time (sec)	N/A	0.684	1.138	0.112	0.000	0.000	0.000	1.036	200.030	20.037

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	313	335	198	0	0	0	3014	2667	3198
N.S.	1	0.99	1.06	0.62	0.00	0.00	0.00	9.51	8.41	10.09
time (sec)	N/A	0.587	1.343	0.215	0.000	0.000	0.000	0.808	0.460	19.715

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	356	393	232	0	0	0	5159	27	4707
N.S.	1	0.97	1.07	0.63	0.00	0.00	0.00	14.02	0.07	12.79
time (sec)	N/A	0.700	1.253	0.219	0.000	0.000	0.000	1.315	200.034	19.108

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	419	458	566	0	0	0	6023	4396	8129
N.S.	1	1.04	1.14	1.40	0.00	0.00	0.00	14.95	10.91	20.17
time (sec)	N/A	0.913	1.517	0.150	0.000	0.000	0.000	1.001	0.585	19.417

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	534	559	667	0	0	0	9015	30	8684
N.S.	1	1.03	1.08	1.29	0.00	0.00	0.00	17.37	0.06	16.73
time (sec)	N/A	1.121	2.026	0.171	0.000	0.000	0.000	1.777	200.026	20.824

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	553	655	778	0	0	0	6939	6801	10595
N.S.	1	1.09	1.29	1.53	0.00	0.00	0.00	13.69	13.41	20.90
time (sec)	N/A	1.265	2.526	0.194	0.000	0.000	0.000	1.158	0.508	21.030

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	296	5520	611	3898	47658	7808	5520	2443
N.S.	1	1.00	0.74	13.83	1.53	9.77	119.44	19.57	13.83	6.12
time (sec)	N/A	0.766	2.538	0.300	0.105	0.184	3.175	0.204	0.161	21.058

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	185	2187	344	1603	16323	3203	2220	1314
N.S.	1	1.00	0.71	8.41	1.32	6.17	62.78	12.32	8.54	5.05
time (sec)	N/A	0.525	1.251	0.133	0.088	0.118	1.424	0.143	0.172	19.463

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	90	136	155	444	3628	914	606	527
N.S.	1	1.00	0.66	0.99	1.13	3.24	26.48	6.67	4.42	3.85
time (sec)	N/A	0.331	0.490	0.079	0.052	0.098	0.770	0.148	0.158	18.646

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	368	372	438	0	0	0	0	0	75	0
N.S.	1	1.01	1.19	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.701	2.362	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	685	687	242	0	0	0	0	0	156	0
N.S.	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.737	2.565	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4439	30	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	0.08	10.77
time (sec)	N/A	0.700	1.120	0.072	0.000	0.000	0.000	1.040	200.023	0.004

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4439	32	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	0.09	10.77
time (sec)	N/A	0.586	0.163	0.098	0.000	0.000	0.000	1.123	200.027	19.627

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4439	33	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	0.09	10.77
time (sec)	N/A	0.607	0.159	0.093	0.000	0.000	0.000	1.054	200.028	19.553

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4439	36	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	0.10	10.77
time (sec)	N/A	0.580	0.160	0.092	0.000	0.000	0.000	1.034	200.031	19.657

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	348	378	204	0	0	0	4439	36	3835
N.S.	1	0.98	1.06	0.57	0.00	0.00	0.00	12.47	0.10	10.77
time (sec)	N/A	0.567	0.160	0.095	0.000	0.000	0.000	1.108	200.029	19.150

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	272	260	330	0	900	0	297	1847	2972
N.S.	1	1.00	0.95	1.21	0.00	3.30	0.00	1.09	6.77	10.89
time (sec)	N/A	0.785	0.193	0.188	0.000	0.317	0.000	0.389	0.167	18.885

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	201	193	224	0	677	0	208	1483	2295
N.S.	1	0.99	0.95	1.10	0.00	3.33	0.00	1.02	7.31	11.31
time (sec)	N/A	0.556	0.136	0.165	0.000	0.229	0.000	0.327	0.155	18.688

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	143	136	146	0	473	0	137	1084	1689
N.S.	1	0.99	0.94	1.01	0.00	3.28	0.00	0.95	7.53	11.73
time (sec)	N/A	0.429	0.094	0.182	0.000	0.122	0.000	0.428	0.155	18.448

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	102	100	101	0	318	0	97	720	1081
N.S.	1	0.99	0.97	0.98	0.00	3.09	0.00	0.94	6.99	10.50
time (sec)	N/A	0.348	0.066	0.123	0.000	0.099	0.000	0.433	0.166	19.151

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	100	178	99	0	309	0	96	611	3927
N.S.	1	1.03	1.84	1.02	0.00	3.19	0.00	0.99	6.30	40.48
time (sec)	N/A	0.392	0.139	0.080	0.000	0.241	0.000	0.365	0.157	26.884

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	120	203	132	0	399	0	131	811	4437
N.S.	1	1.02	1.72	1.12	0.00	3.38	0.00	1.11	6.87	37.60
time (sec)	N/A	0.457	0.147	0.089	0.000	0.282	0.000	0.392	0.155	26.386

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	314	203	0	609	0	206	1239	6187
N.S.	1	1.00	1.80	1.17	0.00	3.50	0.00	1.18	7.12	35.56
time (sec)	N/A	0.547	0.346	0.106	0.000	0.589	0.000	0.346	0.171	29.606

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	246	416	294	0	834	0	303	1677	9141
N.S.	1	1.01	1.70	1.20	0.00	3.42	0.00	1.24	6.87	37.46
time (sec)	N/A	0.687	0.347	0.126	0.000	1.339	0.000	0.405	0.160	35.310

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	456	164	0	15467	0	7235	2379	23332
N.S.	1	1.00	1.24	0.44	0.00	41.92	0.00	19.61	6.45	63.23
time (sec)	N/A	3.065	0.514	0.108	0.000	35.919	0.000	0.955	0.177	4.084

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	365	100	0	9364	0	5454	1710	15674
N.S.	1	1.00	1.29	0.35	0.00	33.21	0.00	19.34	6.06	55.58
time (sec)	N/A	2.343	0.498	0.099	0.000	7.888	0.000	0.778	0.172	20.591

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	258	68	0	5788	0	4082	1309	10209
N.S.	1	1.00	1.18	0.31	0.00	26.43	0.00	18.64	5.98	46.62
time (sec)	N/A	0.675	0.334	0.082	0.000	3.738	0.000	0.618	0.170	20.485

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	253	220	0	5930	0	3984	32	10170
N.S.	1	1.00	1.19	1.03	0.00	27.84	0.00	18.70	0.15	47.75
time (sec)	N/A	0.819	0.305	0.102	0.000	1.455	0.000	0.938	200.025	21.419

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	284	244	0	9850	0	3810	32	15505
N.S.	1	1.00	1.06	0.91	0.00	36.89	0.00	14.27	0.12	58.07
time (sec)	N/A	0.999	0.352	0.111	0.000	10.681	0.000	0.579	200.039	23.039

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	394	360	0	15830	0	6710	32	23019
N.S.	1	1.00	1.20	1.09	0.00	48.12	0.00	20.40	0.10	69.97
time (sec)	N/A	1.582	0.578	0.123	0.000	42.010	0.000	1.129	200.029	24.435

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	321	309	432	0	2111	0	415	5521	3499
N.S.	1	0.99	0.96	1.34	0.00	6.54	0.00	1.28	17.09	10.83
time (sec)	N/A	1.018	0.495	0.253	0.000	0.337	0.000	0.349	0.216	1.637

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	243	236	309	0	1455	0	271	3787	2450
N.S.	1	1.00	0.97	1.27	0.00	5.96	0.00	1.11	15.52	10.04
time (sec)	N/A	0.686	0.375	0.169	0.000	0.181	0.000	0.370	0.189	1.140

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	191	175	228	0	970	0	191	2320	1651
N.S.	1	1.05	0.96	1.25	0.00	5.33	0.00	1.05	12.75	9.07
time (sec)	N/A	0.504	0.245	0.142	0.000	0.120	0.000	0.347	0.192	21.094

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	128	130	139	0	650	474	137	1274	342
N.S.	1	1.04	1.06	1.13	0.00	5.28	3.85	1.11	10.36	2.78
time (sec)	N/A	0.361	0.103	0.085	0.000	0.083	18.150	0.349	0.180	0.385

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	199	268	228	0	1103	0	224	2436	8706
N.S.	1	1.20	1.61	1.37	0.00	6.64	0.00	1.35	14.67	52.45
time (sec)	N/A	0.611	0.469	0.113	0.000	0.780	0.000	0.340	0.197	32.690

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	272	403	316	0	1764	0	280	4142	11879
N.S.	1	1.16	1.72	1.35	0.00	7.54	0.00	1.20	17.70	50.76
time (sec)	N/A	0.930	0.691	0.134	0.000	1.764	0.000	0.358	0.190	33.416

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	370	592	466	0	2567	0	521	6104	15905
N.S.	1	1.12	1.80	1.42	0.00	7.80	0.00	1.58	18.55	48.34
time (sec)	N/A	1.201	1.262	0.167	0.000	3.975	0.000	0.331	0.190	46.048

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	563	648	323	0	18909	0	8946	9400	33799
N.S.	1	1.02	1.18	0.59	0.00	34.38	0.00	16.27	17.09	61.45
time (sec)	N/A	6.901	2.165	0.141	0.000	75.017	0.000	1.325	5.633	21.530

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	442	511	242	0	12597	0	7479	7390	25862
N.S.	1	1.01	1.17	0.55	0.00	28.83	0.00	17.11	16.91	59.18
time (sec)	N/A	3.160	1.556	0.125	0.000	21.862	0.000	1.207	2.284	2.698

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	352	414	200	0	8951	0	6200	6216	19494
N.S.	1	0.97	1.14	0.55	0.00	24.73	0.00	17.13	17.17	53.85
time (sec)	N/A	0.748	1.136	0.111	0.000	11.975	0.000	1.070	0.608	24.804

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	332	382	200	0	8991	0	6348	6214	19589
N.S.	1	0.96	1.10	0.58	0.00	25.99	0.00	18.35	17.96	56.62
time (sec)	N/A	0.641	1.094	0.120	0.000	10.665	0.000	1.048	0.591	24.282

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	401	444	438	0	13111	0	7173	32	28164
N.S.	1	0.93	1.03	1.02	0.00	30.49	0.00	16.68	0.07	65.50
time (sec)	N/A	1.939	1.327	0.151	0.000	25.491	0.000	0.995	200.020	24.742

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	605	548	570	0	19333	0	8649	32	36097
N.S.	1	1.06	0.96	1.00	0.00	33.92	0.00	15.17	0.06	63.33
time (sec)	N/A	4.847	1.839	0.168	0.000	80.655	0.000	1.287	200.023	26.593

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	63	62	56	58	82	61	63	102	57
N.S.	1	0.97	0.95	0.86	0.89	1.26	0.94	0.97	1.57	0.88
time (sec)	N/A	0.378	0.031	0.065	0.029	0.066	0.089	0.116	0.160	0.055

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	61	61	51	53	77	56	58	97	53
N.S.	1	1.05	1.05	0.88	0.91	1.33	0.97	1.00	1.67	0.91
time (sec)	N/A	0.351	0.032	0.058	0.032	0.071	0.100	0.118	0.153	0.040

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	53	54	46	48	72	48	53	92	47
N.S.	1	1.04	1.06	0.90	0.94	1.41	0.94	1.04	1.80	0.92
time (sec)	N/A	0.339	0.026	0.066	0.039	0.065	0.084	0.134	0.156	18.533

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	47	49	41	43	67	44	45	87	43
N.S.	1	1.02	1.07	0.89	0.93	1.46	0.96	0.98	1.89	0.93
time (sec)	N/A	0.314	0.027	0.056	0.030	0.066	0.094	0.133	0.152	18.561

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	41	42	36	38	57	36	40	82	37
N.S.	1	1.05	1.08	0.92	0.97	1.46	0.92	1.03	2.10	0.95
time (sec)	N/A	0.260	0.020	0.052	0.026	0.066	0.100	0.109	0.152	0.051

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	46	44	38	44	71	41	47	100	40
N.S.	1	1.07	1.02	0.88	1.02	1.65	0.95	1.09	2.33	0.93
time (sec)	N/A	0.291	0.023	0.055	0.025	0.067	0.127	0.133	0.145	0.042

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	64	50	45	53	92	51	53	116	50
N.S.	1	1.19	0.93	0.83	0.98	1.70	0.94	0.98	2.15	0.93
time (sec)	N/A	0.352	0.027	0.061	0.044	0.066	0.125	0.152	0.149	18.627

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	56	50	56	97	56	66	121	55
N.S.	1	1.03	0.89	0.79	0.89	1.54	0.89	1.05	1.92	0.87
time (sec)	N/A	0.350	0.034	0.063	0.027	0.064	0.122	0.131	0.154	18.609

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	73	71	56	58	79	68	58	101	58
N.S.	1	1.04	1.01	0.80	0.83	1.13	0.97	0.83	1.44	0.83
time (sec)	N/A	0.331	0.056	0.121	0.115	0.073	0.114	0.129	0.146	0.058

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	62	58	49	51	74	54	51	96	50
N.S.	1	1.09	1.02	0.86	0.89	1.30	0.95	0.89	1.68	0.88
time (sec)	N/A	0.335	0.055	0.115	0.111	0.074	0.118	0.108	0.169	0.055

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	59	57	46	48	69	54	48	91	48
N.S.	1	1.05	1.02	0.82	0.86	1.23	0.96	0.86	1.62	0.86
time (sec)	N/A	0.291	0.054	0.102	0.112	0.080	0.128	0.155	0.174	0.052

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	52	50	41	43	64	48	43	86	42
N.S.	1	1.06	1.02	0.84	0.88	1.31	0.98	0.88	1.76	0.86
time (sec)	N/A	0.289	0.045	0.094	0.117	0.074	0.114	0.147	0.149	18.576

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	49	46	38	40	59	46	40	81	40
N.S.	1	1.02	0.96	0.79	0.83	1.23	0.96	0.83	1.69	0.83
time (sec)	N/A	0.206	0.046	0.092	0.110	0.072	0.118	0.127	0.157	0.074

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	54	51	43	45	68	49	45	88	45
N.S.	1	1.02	0.96	0.81	0.85	1.28	0.92	0.85	1.66	0.85
time (sec)	N/A	0.292	0.058	0.114	0.110	0.079	0.171	0.129	0.150	18.535

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	61	56	48	52	79	56	52	97	51
N.S.	1	0.98	0.90	0.77	0.84	1.27	0.90	0.84	1.56	0.82
time (sec)	N/A	0.317	0.068	0.126	0.112	0.077	0.165	0.141	0.144	18.503

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	68	61	53	57	84	61	57	102	57
N.S.	1	0.99	0.88	0.77	0.83	1.22	0.88	0.83	1.48	0.83
time (sec)	N/A	0.331	0.072	0.130	0.113	0.071	0.157	0.122	0.170	0.079

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	75	77	58	62	89	66	62	107	61
N.S.	1	0.99	1.01	0.76	0.82	1.17	0.87	0.82	1.41	0.80
time (sec)	N/A	0.344	0.070	0.140	0.107	0.075	0.151	0.131	0.159	18.543

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	91	71	61	71	114	75	61	158	70
N.S.	1	1.12	0.88	0.75	0.88	1.41	0.93	0.75	1.95	0.86
time (sec)	N/A	0.470	0.073	0.130	0.111	0.081	0.141	0.166	0.156	18.601

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	88	66	58	68	109	76	58	153	68
N.S.	1	1.10	0.82	0.72	0.85	1.36	0.95	0.72	1.91	0.85
time (sec)	N/A	0.397	0.074	0.122	0.109	0.071	0.198	0.138	0.169	0.055

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	81	60	53	63	104	70	53	148	63
N.S.	1	1.08	0.80	0.71	0.84	1.39	0.93	0.71	1.97	0.84
time (sec)	N/A	0.399	0.069	0.113	0.107	0.073	0.146	0.108	0.150	0.071

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	78	55	50	60	99	65	50	143	59
N.S.	1	1.08	0.76	0.69	0.83	1.38	0.90	0.69	1.99	0.82
time (sec)	N/A	0.316	0.073	0.107	0.110	0.076	0.157	0.147	0.193	18.601

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	78	56	50	60	99	66	50	143	60
N.S.	1	1.08	0.78	0.69	0.83	1.38	0.92	0.69	1.99	0.83
time (sec)	N/A	0.303	0.071	0.110	0.109	0.075	0.123	0.150	0.177	0.080

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	78	56	50	60	99	65	50	143	59
N.S.	1	1.08	0.78	0.69	0.83	1.38	0.90	0.69	1.99	0.82
time (sec)	N/A	0.246	0.069	0.109	0.110	0.074	0.145	0.164	0.170	0.080

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	83	63	56	65	108	71	55	150	65
N.S.	1	1.05	0.80	0.71	0.82	1.37	0.90	0.70	1.90	0.82
time (sec)	N/A	0.388	0.082	0.129	0.105	0.072	0.138	0.160	0.161	18.770

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	90	78	61	72	119	76	62	159	71
N.S.	1	1.05	0.91	0.71	0.84	1.38	0.88	0.72	1.85	0.83
time (sec)	N/A	0.418	0.065	0.141	0.120	0.071	0.180	0.133	0.154	0.083

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	97	73	66	77	124	82	67	164	77
N.S.	1	1.04	0.78	0.71	0.83	1.33	0.88	0.72	1.76	0.83
time (sec)	N/A	0.447	0.087	0.147	0.113	0.074	0.181	0.134	0.170	0.089

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	87	78	71	71	95	87	76	399	75
N.S.	1	1.01	0.91	0.83	0.83	1.10	1.01	0.88	4.64	0.87
time (sec)	N/A	0.383	0.054	0.058	0.107	0.069	0.142	0.284	0.160	18.880

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	82	73	66	66	90	80	71	394	69
N.S.	1	1.01	0.90	0.81	0.81	1.11	0.99	0.88	4.86	0.85
time (sec)	N/A	0.366	0.037	0.053	0.111	0.069	0.096	0.301	0.172	0.054

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	75	66	62	59	85	73	66	389	65
N.S.	1	1.01	0.89	0.84	0.80	1.15	0.99	0.89	5.26	0.88
time (sec)	N/A	0.344	0.035	0.068	0.110	0.069	0.106	0.291	0.154	0.047

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	68	61	56	54	80	68	54	384	60
N.S.	1	1.05	0.94	0.86	0.83	1.23	1.05	0.83	5.91	0.92
time (sec)	N/A	0.326	0.032	0.052	0.108	0.068	0.109	0.302	0.157	18.709

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	68	58	51	49	70	60	49	379	69
N.S.	1	1.17	1.00	0.88	0.84	1.21	1.03	0.84	6.53	1.19
time (sec)	N/A	0.284	0.028	0.045	0.112	0.069	0.140	0.304	0.154	0.049

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	77	72	58	55	84	65	62	397	59
N.S.	1	1.17	1.09	0.88	0.83	1.27	0.98	0.94	6.02	0.89
time (sec)	N/A	0.319	0.093	0.059	0.112	0.067	0.145	0.290	0.157	18.693

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	80	101	63	66	105	76	66	419	68
N.S.	1	1.13	1.42	0.89	0.93	1.48	1.07	0.93	5.90	0.96
time (sec)	N/A	0.362	0.066	0.057	0.112	0.070	0.137	0.298	0.152	0.064

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	81	82	68	71	110	80	79	424	72
N.S.	1	1.01	1.02	0.85	0.89	1.38	1.00	0.99	5.30	0.90
time (sec)	N/A	0.372	0.106	0.061	0.111	0.070	0.178	0.297	0.150	18.693

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	92	114	73	76	115	85	84	429	78
N.S.	1	1.06	1.31	0.84	0.87	1.32	0.98	0.97	4.93	0.90
time (sec)	N/A	0.399	0.085	0.063	0.108	0.071	0.152	0.288	0.156	18.594

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	253	145	79	0	268	71	585	806	171
N.S.	1	1.25	0.71	0.39	0.00	1.32	0.35	2.88	3.97	0.84
time (sec)	N/A	0.604	0.216	0.142	0.000	0.076	0.467	0.438	0.152	0.135

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	244	132	72	0	308	1205	576	801	164
N.S.	1	1.27	0.69	0.38	0.00	1.60	6.28	3.00	4.17	0.85
time (sec)	N/A	0.546	0.187	0.080	0.000	0.081	1.021	0.425	0.160	18.705

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	237	129	69	0	258	60	573	796	162
N.S.	1	1.27	0.69	0.37	0.00	1.38	0.32	3.06	4.26	0.87
time (sec)	N/A	0.516	0.199	0.066	0.000	0.080	0.491	0.428	0.158	19.141

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	230	121	64	0	253	51	566	791	156
N.S.	1	1.28	0.67	0.36	0.00	1.41	0.28	3.14	4.39	0.87
time (sec)	N/A	0.519	0.188	0.068	0.000	0.076	0.523	0.445	0.155	0.159

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	263	115	61	0	268	1185	565	786	153
N.S.	1	1.47	0.64	0.34	0.00	1.50	6.62	3.16	4.39	0.85
time (sec)	N/A	0.493	0.321	0.069	0.000	0.077	0.879	0.448	0.151	0.177

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	232	126	63	0	281	1192	572	799	159
N.S.	1	1.26	0.68	0.34	0.00	1.53	6.48	3.11	4.34	0.86
time (sec)	N/A	0.594	0.216	0.075	0.000	0.080	1.078	0.435	0.162	0.173

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	239	131	68	0	276	60	579	820	165
N.S.	1	1.24	0.68	0.35	0.00	1.43	0.31	3.00	4.25	0.85
time (sec)	N/A	0.549	0.351	0.081	0.000	0.082	0.528	0.464	0.152	0.177

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	246	140	73	0	302	1202	584	825	171
N.S.	1	1.23	0.70	0.36	0.00	1.51	6.01	2.92	4.12	0.86
time (sec)	N/A	0.590	0.355	0.077	0.000	0.082	1.067	0.424	0.161	18.762

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	255	156	82	0	344	1204	588	1345	184
N.S.	1	1.26	0.77	0.41	0.00	1.70	5.96	2.91	6.66	0.91
time (sec)	N/A	0.657	0.258	0.076	0.000	0.082	0.954	0.623	0.167	0.146

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	252	155	79	0	318	82	585	1340	182
N.S.	1	1.25	0.77	0.39	0.00	1.58	0.41	2.91	6.67	0.91
time (sec)	N/A	0.622	0.237	0.072	0.000	0.076	0.510	0.565	0.153	18.768

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	245	138	74	0	311	71	580	1335	176
N.S.	1	1.26	0.71	0.38	0.00	1.60	0.37	2.99	6.88	0.91
time (sec)	N/A	0.609	0.392	0.069	0.000	0.083	0.503	0.594	0.151	18.827

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	292	129	71	0	327	1198	577	1330	173
N.S.	1	1.50	0.66	0.36	0.00	1.68	6.14	2.96	6.82	0.89
time (sec)	N/A	0.598	0.362	0.082	0.000	0.081	0.953	0.614	0.161	0.179

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	290	133	71	0	353	1200	577	1330	174
N.S.	1	1.44	0.66	0.35	0.00	1.76	5.97	2.87	6.62	0.87
time (sec)	N/A	0.612	0.352	0.074	0.000	0.084	0.935	0.568	0.149	0.178

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	294	129	71	0	328	1195	577	1330	173
N.S.	1	1.45	0.64	0.35	0.00	1.62	5.89	2.84	6.55	0.85
time (sec)	N/A	0.559	0.349	0.069	0.000	0.083	0.935	0.570	0.158	18.857

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	253	140	73	0	320	75	582	1343	179
N.S.	1	1.22	0.67	0.35	0.00	1.54	0.36	2.80	6.46	0.86
time (sec)	N/A	0.675	0.438	0.098	0.000	0.079	0.509	0.581	0.162	18.857

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	268	139	78	0	335	80	589	1364	185
N.S.	1	1.24	0.64	0.36	0.00	1.54	0.37	2.71	6.29	0.85
time (sec)	N/A	0.653	0.388	0.089	0.000	0.078	0.491	0.585	0.153	18.593

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	148	142	151	0	486	0	144	1196	1834
N.S.	1	0.99	0.95	1.01	0.00	3.26	0.00	0.97	8.03	12.31
time (sec)	N/A	0.490	0.125	0.199	0.000	0.151	0.000	0.345	0.155	19.307

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	607	721	348	0	0	0	10752	11215	47339
N.S.	1	1.02	1.21	0.59	0.00	0.00	0.00	18.10	18.88	79.70
time (sec)	N/A	8.312	2.653	0.164	0.000	0.000	0.000	1.518	5.687	23.006

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	477	575	274	0	23774	0	9152	9418	36589
N.S.	1	1.01	1.22	0.58	0.00	50.48	0.00	19.43	20.00	77.68
time (sec)	N/A	3.992	1.992	0.148	0.000	171.019	0.000	1.379	2.431	22.511

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	435	512	269	0	19375	0	8905	8810	32587
N.S.	1	0.97	1.14	0.60	0.00	43.15	0.00	19.83	19.62	72.58
time (sec)	N/A	0.977	1.690	0.130	0.000	140.974	0.000	1.200	0.703	24.186

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	462	529	550	0	23991	0	9167	37	40860
N.S.	1	1.00	1.15	1.20	0.00	52.15	0.00	19.93	0.08	88.83
time (sec)	N/A	2.721	2.560	0.205	0.000	133.377	0.000	1.120	200.016	25.877

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	537	551	612	629	0	0	0	10411	37	51386
N.S.	1	1.03	1.14	1.17	0.00	0.00	0.00	19.39	0.07	95.69
time (sec)	N/A	4.017	2.191	0.213	0.000	0.000	0.000	1.305	200.022	26.554

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	761	718	845	899	0	876	0	0	435	0
N.S.	1	0.94	1.11	1.18	0.00	1.15	0.00	0.00	0.57	0.00
time (sec)	N/A	1.304	16.208	10.494	0.000	0.099	0.000	0.000	0.233	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	623	587	737	678	0	717	0	0	323	0
N.S.	1	0.94	1.18	1.09	0.00	1.15	0.00	0.00	0.52	0.00
time (sec)	N/A	0.948	15.085	5.687	0.000	0.093	0.000	0.000	0.207	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	471	630	542	0	574	0	0	231	0
N.S.	1	0.96	1.28	1.10	0.00	1.17	0.00	0.00	0.47	0.00
time (sec)	N/A	0.690	13.806	3.777	0.000	0.094	0.000	0.000	0.216	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	427	567	475	0	0	0	0	200	0
N.S.	1	0.96	1.28	1.07	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.657	12.787	3.364	0.000	0.000	0.000	0.000	0.206	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	434	540	462	0	0	0	0	139	0
N.S.	1	1.08	1.35	1.15	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.680	12.624	3.724	0.000	0.000	0.000	0.000	0.230	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	523	597	509	0	0	0	0	215	0
N.S.	1	1.08	1.23	1.05	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.866	13.061	5.389	0.000	0.000	0.000	0.000	0.317	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	574	621	697	581	0	580	0	0	165	0
N.S.	1	1.08	1.21	1.01	0.00	1.01	0.00	0.00	0.29	0.00
time (sec)	N/A	1.090	14.370	9.753	0.000	0.092	0.000	0.000	0.460	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1073	1006	5595	2249	0	1253	0	0	692	0
N.S.	1	0.94	5.21	2.10	0.00	1.17	0.00	0.00	0.64	0.00
time (sec)	N/A	1.854	16.470	11.222	0.000	0.105	0.000	0.000	0.281	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	903	832	4797	1722	0	1063	0	0	553	0
N.S.	1	0.92	5.31	1.91	0.00	1.18	0.00	0.00	0.61	0.00
time (sec)	N/A	1.523	16.394	12.947	0.000	0.110	0.000	0.000	0.256	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	741	699	846	1174	0	874	0	0	435	0
N.S.	1	0.94	1.14	1.58	0.00	1.18	0.00	0.00	0.59	0.00
time (sec)	N/A	1.085	16.116	6.578	0.000	0.100	0.000	0.000	0.249	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	641	606	755	849	0	0	0	0	407	0
N.S.	1	0.95	1.18	1.32	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.976	15.167	5.888	0.000	0.000	0.000	0.000	0.257	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	566	541	676	679	0	0	0	0	345	0
N.S.	1	0.96	1.19	1.20	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.874	14.171	6.211	0.000	0.000	0.000	0.000	0.265	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	562	668	622	0	0	0	0	409	0
N.S.	1	1.07	1.27	1.18	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.971	13.891	7.983	0.000	0.000	0.000	0.000	0.408	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	623	669	707	629	0	0	0	0	375	0
N.S.	1	1.07	1.13	1.01	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	1.218	14.326	9.640	0.000	0.000	0.000	0.000	0.617	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	497	632	532	0	572	0	0	232	0
N.S.	1	0.96	1.22	1.03	0.00	1.11	0.00	0.00	0.45	0.00
time (sec)	N/A	0.846	13.918	9.802	0.000	0.093	0.000	0.000	0.197	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	405	559	449	0	460	0	0	150	0
N.S.	1	0.94	1.30	1.05	0.00	1.07	0.00	0.00	0.35	0.00
time (sec)	N/A	0.622	13.012	7.288	0.000	0.090	0.000	0.000	0.177	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	341	485	399	0	373	0	0	86	0
N.S.	1	0.99	1.41	1.16	0.00	1.08	0.00	0.00	0.25	0.00
time (sec)	N/A	0.445	11.807	5.761	0.000	0.088	0.000	0.000	0.182	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	357	479	392	0	0	0	0	89	0
N.S.	1	1.09	1.46	1.20	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.530	12.061	4.841	0.000	0.000	0.000	0.000	0.185	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	415	507	435	0	374	0	0	96	0
N.S.	1	1.20	1.46	1.25	0.00	1.08	0.00	0.00	0.28	0.00
time (sec)	N/A	0.642	12.130	4.551	0.000	0.089	0.000	0.000	0.189	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	504	472	498	0	458	0	0	125	0
N.S.	1	1.17	1.10	1.16	0.00	1.07	0.00	0.00	0.29	0.00
time (sec)	N/A	0.806	11.570	4.869	0.000	0.087	0.000	0.000	0.246	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	538	516	666	614	0	1175	0	0	90	0
N.S.	1	0.96	1.24	1.14	0.00	2.18	0.00	0.00	0.17	0.00
time (sec)	N/A	1.034	14.007	12.028	0.000	0.098	0.000	0.000	0.174	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	425	579	533	0	992	0	0	33	0
N.S.	1	0.94	1.28	1.18	0.00	2.19	0.00	0.00	0.07	0.00
time (sec)	N/A	0.652	12.947	3.162	0.000	0.096	0.000	0.000	0.158	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	404	550	550	0	794	0	0	30	0
N.S.	1	0.95	1.29	1.29	0.00	1.86	0.00	0.00	0.07	0.00
time (sec)	N/A	0.551	12.466	2.075	0.000	0.095	0.000	0.000	0.155	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	546	619	613	0	989	0	0	34	0
N.S.	1	1.08	1.23	1.22	0.00	1.96	0.00	0.00	0.07	0.00
time (sec)	N/A	1.008	12.883	5.875	0.000	0.098	0.000	0.000	0.155	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	719	748	736	0	1203	0	0	34	0
N.S.	1	1.18	1.23	1.21	0.00	1.97	0.00	0.00	0.06	0.00
time (sec)	N/A	1.755	14.039	9.899	0.000	0.102	0.000	0.000	0.167	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	773	975	927	881	0	1606	0	0	126	0
N.S.	1	1.26	1.20	1.14	0.00	2.08	0.00	0.00	0.16	0.00
time (sec)	N/A	2.759	15.806	11.282	0.000	0.105	0.000	0.000	0.202	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	776	715	740	926	0	2560	0	0	451	0
N.S.	1	0.92	0.95	1.19	0.00	3.30	0.00	0.00	0.58	0.00
time (sec)	N/A	1.451	14.349	9.464	0.000	0.130	0.000	0.000	0.255	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	623	638	805	0	2116	0	0	222	0
N.S.	1	0.95	0.97	1.22	0.00	3.22	0.00	0.00	0.34	0.00
time (sec)	N/A	1.176	13.716	4.455	0.000	0.119	0.000	0.000	0.243	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	627	585	640	762	0	1985	0	0	60	0
N.S.	1	0.93	1.02	1.22	0.00	3.17	0.00	0.00	0.10	0.00
time (sec)	N/A	0.932	12.872	3.182	0.000	0.119	0.000	0.000	0.161	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	604	634	790	0	2198	0	0	57	0
N.S.	1	0.93	0.98	1.22	0.00	3.39	0.00	0.00	0.09	0.00
time (sec)	N/A	0.902	13.409	2.078	0.000	0.124	0.000	0.000	0.225	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	787	857	827	922	0	2597	0	0	61	0
N.S.	1	1.09	1.05	1.17	0.00	3.30	0.00	0.00	0.08	0.00
time (sec)	N/A	1.996	14.423	10.566	0.000	0.135	0.000	0.000	0.167	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	956	1161	957	1094	0	3140	0	0	713	0
N.S.	1	1.21	1.00	1.14	0.00	3.28	0.00	0.00	0.75	0.00
time (sec)	N/A	3.288	15.830	13.429	0.000	0.150	0.000	0.000	0.284	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	154	99	90	0	47	0	0	98	0
N.S.	1	1.31	0.84	0.76	0.00	0.40	0.00	0.00	0.83	0.00
time (sec)	N/A	0.362	10.196	2.334	0.000	0.086	0.000	0.000	0.194	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	179	192	0	123	0	0	316	0
N.S.	1	1.00	0.70	0.76	0.00	0.48	0.00	0.00	1.24	0.00
time (sec)	N/A	0.517	10.262	16.888	0.000	0.079	0.000	0.000	0.186	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	213	163	159	0	108	0	0	227	0
N.S.	1	0.98	0.75	0.73	0.00	0.50	0.00	0.00	1.04	0.00
time (sec)	N/A	0.434	10.217	10.592	0.000	0.077	0.000	0.000	0.170	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	179	148	129	0	90	0	0	154	0
N.S.	1	0.98	0.81	0.71	0.00	0.49	0.00	0.00	0.85	0.00
time (sec)	N/A	0.334	10.177	4.676	0.000	0.080	0.000	0.000	0.171	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	205	152	127	0	0	0	0	131	0
N.S.	1	1.17	0.87	0.73	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.408	10.177	3.977	0.000	0.000	0.000	0.000	0.161	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	237	169	159	0	82	0	0	174	0
N.S.	1	1.29	0.92	0.86	0.00	0.45	0.00	0.00	0.95	0.00
time (sec)	N/A	0.479	10.227	5.313	0.000	0.087	0.000	0.000	0.164	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	288	192	191	0	100	0	0	218	0
N.S.	1	1.30	0.86	0.86	0.00	0.45	0.00	0.00	0.98	0.00
time (sec)	N/A	0.573	10.253	11.030	0.000	0.081	0.000	0.000	0.187	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	324	209	223	0	115	0	0	182	0
N.S.	1	1.26	0.81	0.86	0.00	0.45	0.00	0.00	0.71	0.00
time (sec)	N/A	0.673	10.286	14.072	0.000	0.082	0.000	0.000	0.190	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	268	183	207	0	215	0	0	1018	0
N.S.	1	1.00	0.69	0.78	0.00	0.81	0.00	0.00	3.81	0.00
time (sec)	N/A	0.708	10.283	18.201	0.000	0.086	0.000	0.000	0.194	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	240	173	182	0	202	0	0	817	0
N.S.	1	1.00	0.72	0.76	0.00	0.85	0.00	0.00	3.42	0.00
time (sec)	N/A	0.537	10.256	13.580	0.000	0.082	0.000	0.000	0.192	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	208	166	162	0	189	0	0	627	0
N.S.	1	0.97	0.78	0.76	0.00	0.88	0.00	0.00	2.93	0.00
time (sec)	N/A	0.421	10.224	10.770	0.000	0.081	0.000	0.000	0.183	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	210	168	162	0	160	0	0	447	0
N.S.	1	1.31	1.05	1.01	0.00	1.00	0.00	0.00	2.79	0.00
time (sec)	N/A	0.379	10.233	9.053	0.000	0.087	0.000	0.000	0.195	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	274	182	182	0	176	0	0	622	0
N.S.	1	1.44	0.96	0.96	0.00	0.93	0.00	0.00	3.27	0.00
time (sec)	N/A	0.627	10.258	4.526	0.000	0.091	0.000	0.000	0.181	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	347	195	210	0	193	0	0	1294	0
N.S.	1	1.52	0.86	0.92	0.00	0.85	0.00	0.00	5.68	0.00
time (sec)	N/A	0.894	10.262	8.793	0.000	0.085	0.000	0.000	0.194	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	411	213	239	0	211	0	0	636	0
N.S.	1	1.36	0.71	0.79	0.00	0.70	0.00	0.00	2.11	0.00
time (sec)	N/A	1.179	10.279	10.770	0.000	0.089	0.000	0.000	0.192	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	281	227	236	0	302	0	0	0	0
N.S.	1	0.99	0.80	0.83	0.00	1.06	0.00	0.00	0.00	0.00
time (sec)	N/A	0.758	10.424	20.379	0.000	0.083	0.000	0.000	0.208	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	258	222	218	0	290	0	0	0	0
N.S.	1	0.98	0.84	0.83	0.00	1.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.599	10.404	18.824	0.000	0.080	0.000	0.000	0.202	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	258	222	218	0	263	0	0	1255	0
N.S.	1	1.23	1.06	1.04	0.00	1.25	0.00	0.00	5.98	0.00
time (sec)	N/A	0.583	10.425	10.644	0.000	0.084	0.000	0.000	0.200	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	259	222	218	0	261	0	0	0	0
N.S.	1	1.25	1.07	1.05	0.00	1.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.510	10.399	6.765	0.000	0.084	0.000	0.000	0.212	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	255	221	218	0	264	0	0	1181	0
N.S.	1	1.23	1.07	1.05	0.00	1.28	0.00	0.00	5.71	0.00
time (sec)	N/A	0.479	10.397	4.562	0.000	0.089	0.000	0.000	0.199	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	333	239	238	0	276	0	0	1226	0
N.S.	1	1.16	0.83	0.83	0.00	0.96	0.00	0.00	4.26	0.00
time (sec)	N/A	0.838	10.401	4.850	0.000	0.086	0.000	0.000	0.210	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	395	255	264	0	297	0	0	1206	0
N.S.	1	1.25	0.81	0.84	0.00	0.94	0.00	0.00	3.83	0.00
time (sec)	N/A	1.132	10.409	4.801	0.000	0.086	0.000	0.000	0.207	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	472	273	293	0	309	0	0	0	0
N.S.	1	1.36	0.78	0.84	0.00	0.89	0.00	0.00	0.00	0.00
time (sec)	N/A	1.469	10.424	7.075	0.000	0.087	0.000	0.000	0.229	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	144	132	109	0	160	0	0	904	0
N.S.	1	1.48	1.36	1.12	0.00	1.65	0.00	0.00	9.32	0.00
time (sec)	N/A	0.314	10.248	4.278	0.000	0.081	0.000	0.000	0.204	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	411	400	355	0	0	0	0	0	0	0
N.S.	1	0.97	0.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.735	0.921	0.000	0.000	0.000	0.000	0.000	0.345	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	24	45	34	0	71	34	44
N.S.	1	1.00	0.85	0.89	1.67	1.26	0.00	2.63	1.26	1.63
time (sec)	N/A	0.197	0.954	1.413	0.080	0.098	0.000	0.187	0.154	19.580

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	31	31	54	58	30	31
N.S.	1	1.00	1.00	1.05	1.55	1.55	2.70	2.90	1.50	1.55
time (sec)	N/A	0.192	0.519	0.131	0.076	0.075	177.982	0.164	0.161	19.248

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [186] had the largest ratio of [.5000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	26	0.077
2	A	2	2	1.00	24	0.083
3	A	2	2	1.00	23	0.087
4	A	2	2	1.00	26	0.077
5	A	2	2	1.00	26	0.077
6	A	2	2	1.00	26	0.077
7	A	2	2	1.00	26	0.077
8	A	2	2	1.00	26	0.077
9	A	2	2	1.00	26	0.077
10	A	2	2	1.00	26	0.077
11	A	2	2	1.00	28	0.071
12	A	2	2	1.00	26	0.077
13	A	2	2	1.00	25	0.080
14	A	2	2	1.00	28	0.071
15	A	2	2	1.00	28	0.071
16	A	2	2	1.00	28	0.071
17	A	2	2	1.00	28	0.071
18	A	2	2	1.00	28	0.071
19	A	2	2	1.00	28	0.071
20	A	2	2	1.00	28	0.071
21	A	12	11	1.02	28	0.393

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	9	8	1.01	28	0.286
23	A	11	10	1.03	28	0.357
24	A	10	9	1.05	26	0.346
25	A	8	7	1.00	25	0.280
26	A	8	7	1.02	28	0.250
27	A	13	12	1.00	28	0.429
28	A	10	9	1.00	28	0.321
29	A	14	13	1.01	28	0.464
30	A	11	10	0.98	28	0.357
31	A	10	9	0.98	28	0.321
32	A	10	9	0.98	28	0.321
33	A	10	9	0.99	26	0.346
34	A	11	10	0.97	25	0.400
35	A	12	11	1.04	28	0.393
36	A	13	12	1.03	28	0.429
37	A	13	12	1.09	28	0.429
38	A	2	2	1.00	30	0.067
39	A	2	2	1.00	30	0.067
40	A	2	2	1.00	28	0.071
41	A	8	8	1.01	30	0.267
42	A	8	8	1.00	30	0.267
43	A	10	9	0.98	28	0.321
44	A	11	10	0.98	30	0.333
45	A	11	10	0.98	31	0.323
46	A	11	10	0.98	34	0.294
47	A	11	10	0.98	34	0.294
48	A	4	3	1.00	30	0.100
49	A	4	3	0.99	30	0.100
50	A	4	3	0.99	30	0.100
51	A	4	3	0.99	28	0.107
52	A	4	3	1.03	30	0.100
53	A	4	3	1.02	30	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	3	1.00	30	0.100
55	A	4	3	1.01	30	0.100
56	A	2	2	1.00	30	0.067
57	A	2	2	1.00	30	0.067
58	A	2	2	1.00	27	0.074
59	A	2	2	1.00	30	0.067
60	A	2	2	1.00	30	0.067
61	A	2	2	1.00	30	0.067
62	A	7	6	0.99	30	0.200
63	A	7	6	1.00	30	0.200
64	A	8	7	1.05	30	0.233
65	A	6	5	1.04	28	0.179
66	A	7	6	1.20	30	0.200
67	A	6	5	1.16	30	0.167
68	A	6	5	1.12	30	0.167
69	A	3	3	1.02	30	0.100
70	A	3	3	1.01	30	0.100
71	A	5	5	0.97	30	0.167
72	A	4	4	0.96	27	0.148
73	A	4	4	0.93	30	0.133
74	A	4	4	1.06	30	0.133
75	A	6	5	0.97	31	0.161
76	A	5	4	1.05	31	0.129
77	A	6	5	1.04	31	0.161
78	A	5	4	1.02	31	0.129
79	A	6	5	1.05	29	0.172
80	A	6	5	1.07	31	0.161
81	A	6	5	1.19	31	0.161
82	A	6	5	1.03	31	0.161
83	A	4	4	1.04	31	0.129
84	A	4	4	1.09	31	0.129
85	A	4	4	1.05	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	4	1.06	31	0.129
87	A	4	4	1.02	28	0.143
88	A	4	4	1.02	31	0.129
89	A	4	4	0.98	31	0.129
90	A	4	4	0.99	31	0.129
91	A	4	4	0.99	31	0.129
92	A	6	6	1.12	31	0.194
93	A	6	6	1.10	31	0.194
94	A	6	6	1.08	31	0.194
95	A	6	6	1.08	31	0.194
96	A	6	6	1.08	31	0.194
97	A	5	5	1.08	28	0.179
98	A	6	6	1.05	31	0.194
99	A	6	6	1.05	31	0.194
100	A	6	6	1.04	31	0.194
101	A	6	5	1.01	31	0.161
102	A	6	5	1.01	31	0.161
103	A	6	5	1.01	31	0.161
104	A	6	5	1.05	31	0.161
105	A	9	8	1.17	29	0.276
106	A	6	5	1.17	31	0.161
107	A	6	5	1.13	31	0.161
108	A	6	5	1.01	31	0.161
109	A	6	5	1.06	31	0.161
110	A	4	4	1.25	31	0.129
111	A	4	4	1.27	31	0.129
112	A	4	4	1.27	31	0.129
113	A	4	4	1.28	31	0.129
114	A	9	8	1.47	28	0.286
115	A	4	4	1.26	31	0.129
116	A	4	4	1.24	31	0.129
117	A	4	4	1.23	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	6	1.26	31	0.194
119	A	6	6	1.25	31	0.194
120	A	6	6	1.26	31	0.194
121	A	11	10	1.50	31	0.323
122	A	11	10	1.44	31	0.323
123	A	11	10	1.45	28	0.357
124	A	6	6	1.22	31	0.194
125	A	6	6	1.24	31	0.194
126	A	4	3	0.99	33	0.091
127	A	3	3	1.02	35	0.086
128	A	4	4	1.01	35	0.114
129	A	4	4	0.97	32	0.125
130	A	4	4	1.00	35	0.114
131	A	4	4	1.03	35	0.114
132	A	10	10	0.94	32	0.312
133	A	8	8	0.94	32	0.250
134	A	7	7	0.96	29	0.241
135	A	7	7	0.96	32	0.219
136	A	9	9	1.08	32	0.281
137	A	9	9	1.08	32	0.281
138	A	12	12	1.08	32	0.375
139	A	12	12	0.94	32	0.375
140	A	10	10	0.92	32	0.312
141	A	9	9	0.94	29	0.310
142	A	9	9	0.95	32	0.281
143	A	9	9	0.96	32	0.281
144	A	11	11	1.07	32	0.344
145	A	12	12	1.07	32	0.375
146	A	9	9	0.96	32	0.281
147	A	7	7	0.94	32	0.219
148	A	5	5	0.99	29	0.172
149	A	7	7	1.09	32	0.219

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	8	8	1.20	32	0.250
151	A	10	10	1.17	32	0.312
152	A	8	8	0.96	32	0.250
153	A	7	7	0.94	32	0.219
154	A	6	6	0.95	29	0.207
155	A	9	9	1.08	32	0.281
156	A	12	12	1.18	32	0.375
157	A	14	14	1.26	32	0.438
158	A	8	8	0.92	32	0.250
159	A	9	9	0.95	32	0.281
160	A	10	10	0.93	32	0.312
161	A	7	7	0.93	29	0.241
162	A	12	12	1.09	32	0.375
163	A	15	15	1.21	32	0.469
164	A	4	4	1.31	39	0.103
165	A	8	8	1.00	32	0.250
166	A	6	6	0.98	32	0.188
167	A	4	4	0.98	29	0.138
168	A	6	6	1.17	32	0.188
169	A	7	7	1.29	32	0.219
170	A	9	9	1.30	32	0.281
171	A	10	10	1.26	32	0.312
172	A	9	9	1.00	32	0.281
173	A	7	7	1.00	32	0.219
174	A	5	5	0.97	32	0.156
175	A	4	4	1.31	29	0.138
176	A	8	8	1.44	32	0.250
177	A	11	11	1.52	32	0.344
178	A	13	13	1.36	32	0.406
179	A	9	9	0.99	32	0.281
180	A	7	7	0.98	32	0.219
181	A	7	7	1.23	32	0.219

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	7	7	1.25	32	0.219
183	A	6	6	1.23	29	0.207
184	A	10	10	1.16	32	0.312
185	A	13	13	1.25	32	0.406
186	A	16	16	1.36	32	0.500
187	A	3	3	1.48	56	0.054
188	A	3	3	0.97	32	0.094
189	A	1	1	1.00	48	0.021
190	A	1	1	1.00	42	0.024

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^2(A+Bx+Cx^2)(a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 74

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx = \frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}bBx^6+\frac{1}{7}(Ac+bC)x^7+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

output
```
1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/
8*B*c*x^8+1/9*c*C*x^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx = \frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}bBx^6+\frac{1}{7}(Ac+bC)x^7+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

input
```
Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
(a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*
C)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2+cx^4\right)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(x^4(aC+Ab)+aAx^2+aBx^3+x^6(Ac+bC)+bBx^5+Bcx^7+cCx^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

input
```
Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
(a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*
C)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{aA\,x^{3}}{3}+\frac{aB\,x^{4}}{4}+\frac{(Ab+aC)x^{5}}{5}+\frac{bB\,x^{6}}{6}+\frac{(Ac+Cb)x^{7}}{7}+\frac{Bc\,x^{8}}{8}+\frac{cC\,x^{9}}{9}$	61
norman	$\frac{cC\,x^{9}}{9}+\frac{Bc\,x^{8}}{8}+\left(\frac{Ac}{7}+\frac{Cb}{7}\right)x^{7}+\frac{bB\,x^{6}}{6}+\left(\frac{Ab}{5}+\frac{aC}{5}\right)x^{5}+\frac{aB\,x^{4}}{4}+\frac{aA\,x^{3}}{3}$	63
gosper	$\frac{1}{9}cC\,x^{9}+\frac{1}{8}Bc\,x^{8}+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Cb+\frac{1}{6}bB\,x^{6}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}aC+\frac{1}{4}aB\,x^{4}+\frac{1}{3}aA\,x^{3}$	65
risch	$\frac{1}{9}cC\,x^{9}+\frac{1}{8}Bc\,x^{8}+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Cb+\frac{1}{6}bB\,x^{6}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}aC+\frac{1}{4}aB\,x^{4}+\frac{1}{3}aA\,x^{3}$	65
parallelrisch	$\frac{1}{9}cC\,x^{9}+\frac{1}{8}Bc\,x^{8}+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Cb+\frac{1}{6}bB\,x^{6}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}aC+\frac{1}{4}aB\,x^{4}+\frac{1}{3}aA\,x^{3}$	65
orering	$\frac{x^{3}\left(280cC\,x^{6}+315Bc\,x^{5}+360Ac\,x^{4}+360Cb\,x^{4}+420Bb\,x^{3}+504Ab\,x^{2}+504Ca\,x^{2}+630Bax+840Aa\right)}{2520}$	65

input
```
int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/
8*B*c*x^8+1/9*c*C*x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{9}\,Ccx^9+\frac{1}{8}\,Bcx^8+\frac{1}{6}\,Bbx^6+\frac{1}{7}\,(Cb+Ac)x^7+\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1/4*B*a*x^
4 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{Aax^3}{3}+\frac{Bax^4}{4}+\frac{Bbx^6}{6}+\frac{Bcx^8}{8}+\frac{Ccx^9}{9}+x^7\left(\frac{Ac}{7}+\frac{Cb}{7}\right)+x^5\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

input
```
integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

output
```
A*a*x**3/3 + B*a*x**4/4 + B*b*x**6/6 + B*c*x**8/8 + C*c*x**9/9 + x**7*(A*c
/7 + C*b/7) + x**5*(A*b/5 + C*a/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{9}\,Ccx^9+\frac{1}{8}\,Bcx^8+\frac{1}{6}\,Bbx^6+\frac{1}{7}\,(Cb+Ac)x^7+\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1/4*B*a*x^
4 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{9}\,Ccx^9+\frac{1}{8}\,Bcx^8+\frac{1}{7}\,Cbx^7+\frac{1}{7}\,Acx^7+\frac{1}{6}\,Bbx^6+\frac{1}{5}\,Cax^5+\frac{1}{5}\,Abx^5+\frac{1}{4}\,Bax^4+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/7*C*b*x^7 + 1/7*A*c*x^7 + 1/6*B*b*x^6 + 1/5*
C*a*x^5 + 1/5*A*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{C\,c\,x^9}{9}+\frac{B\,c\,x^8}{8}+\left(\frac{A\,c}{7}+\frac{C\,b}{7}\right)\,x^7+\frac{B\,b\,x^6}{6}+\left(\frac{A\,b}{5}+\frac{C\,a}{5}\right)\,x^5+\frac{B\,a\,x^4}{4}+\frac{A\,a\,x^3}{3}$$

input
```
int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

output
```
x^5*((A*b)/5 + (C*a)/5) + x^7*((A*c)/7 + (C*b)/7) + (A*a*x^3)/3 + (B*a*x^4
)/4 + (B*b*x^6)/6 + (B*c*x^8)/8 + (C*c*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx$$
$$=\frac{x^3(280c^2x^6+315bc\,x^5+360ac\,x^4+360bc\,x^4+420b^2x^3+504ab\,x^2+504ac\,x^2+630abx+840a^2)}{2520}$$

input
```
int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

output
```
(x**3*(840*a**2 + 504*a*b*x**2 + 630*a*b*x + 360*a*c*x**4 + 504*a*c*x**2 +
 420*b**2*x**3 + 315*b*c*x**5 + 360*b*c*x**4 + 280*c**2*x**6))/2520
```

3.2 $\int x(A+Bx+Cx^2)(a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int x(A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = \frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}bBx^5+\frac{1}{6}(Ac+bC)x^6+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

output
```
1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*b)*x^6+1/
7*B*c*x^7+1/8*c*C*x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int x(A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = \frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}bBx^5+\frac{1}{6}(Ac+bC)x^6+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

input
```
Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
(a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*
C)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2+cx^4\right)\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(x^3(aC+Ab)+aAx+aBx^2+x^5(Ac+bC)+bBx^4+Bcx^6+cCx^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

input
```
Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
(a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*
C)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	siz
default	$\frac{aA\,x^{2}}{2}+\frac{aB\,x^{3}}{3}+\frac{(Ab+aC)x^{4}}{4}+\frac{bB\,x^{5}}{5}+\frac{(Ac+Cb)x^{6}}{6}+\frac{Bc\,x^{7}}{7}+\frac{cC\,x^{8}}{8}$	61
norman	$\frac{cC\,x^{8}}{8}+\frac{Bc\,x^{7}}{7}+\left(\frac{Ac}{6}+\frac{Cb}{6}\right)x^{6}+\frac{bB\,x^{5}}{5}+\left(\frac{Ab}{4}+\frac{aC}{4}\right)x^{4}+\frac{aB\,x^{3}}{3}+\frac{aA\,x^{2}}{2}$	63
gosper	$\frac{1}{8}cC\,x^{8}+\frac{1}{7}Bc\,x^{7}+\frac{1}{6}x^{6}Ac+\frac{1}{6}Cb\,x^{6}+\frac{1}{5}bB\,x^{5}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}aC+\frac{1}{3}aB\,x^{3}+\frac{1}{2}aA\,x^{2}$	65
risch	$\frac{1}{8}cC\,x^{8}+\frac{1}{7}Bc\,x^{7}+\frac{1}{6}x^{6}Ac+\frac{1}{6}Cb\,x^{6}+\frac{1}{5}bB\,x^{5}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}aC+\frac{1}{3}aB\,x^{3}+\frac{1}{2}aA\,x^{2}$	65
parallelrisch	$\frac{1}{8}cC\,x^{8}+\frac{1}{7}Bc\,x^{7}+\frac{1}{6}x^{6}Ac+\frac{1}{6}Cb\,x^{6}+\frac{1}{5}bB\,x^{5}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}aC+\frac{1}{3}aB\,x^{3}+\frac{1}{2}aA\,x^{2}$	65
orering	$\frac{x^{2}\left(105cC\,x^{6}+120Bc\,x^{5}+140Ac\,x^{4}+140Cb\,x^{4}+168Bb\,x^{3}+210Ab\,x^{2}+210Ca\,x^{2}+280Bax+420Aa\right)}{840}$	65

input
```
int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*b)*x^6+1/
7*B*c*x^7+1/8*c*C*x^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x\left(A+Bx+Cx^{2}\right)\left(a+bx^{2}+cx^{4}\right)\,dx=\frac{1}{8}\,Ccx^{8}+\frac{1}{7}\,Bcx^{7}+\frac{1}{5}\,Bbx^{5}+\frac{1}{6}\,(Cb+Ac)x^{6}+\frac{1}{3}\,Bax^{3}+\frac{1}{4}\,(Ca+Ab)x^{4}+\frac{1}{2}\,Aax^{2}$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/6*(C*b + A*c)*x^6 + 1/3*B*a*x^
3 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{Aax^2}{2}+\frac{Bax^3}{3}+\frac{Bbx^5}{5}+\frac{Bcx^7}{7}+\frac{Ccx^8}{8}+x^6\left(\frac{Ac}{6}+\frac{Cb}{6}\right)+x^4\left(\frac{Ab}{4}+\frac{Ca}{4}\right)$$

input
```
integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

output
```
A*a*x**2/2 + B*a*x**3/3 + B*b*x**5/5 + B*c*x**7/7 + C*c*x**8/8 + x**6*(A*c
/6 + C*b/6) + x**4*(A*b/4 + C*a/4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{8}\,Ccx^8+\frac{1}{7}\,Bcx^7+\frac{1}{5}\,Bbx^5+\frac{1}{6}\,(Cb+Ac)x^6+\frac{1}{3}\,Bax^3+\frac{1}{4}\,(Ca+Ab)x^4+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/6*(C*b + A*c)*x^6 + 1/3*B*a*x^
3 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{8}\,Ccx^8+\frac{1}{7}\,Bcx^7+\frac{1}{6}\,Cbx^6+\frac{1}{6}\,Acx^6+\frac{1}{5}\,Bbx^5+\frac{1}{4}\,Cax^4+\frac{1}{4}\,Abx^4+\frac{1}{3}\,Bax^3+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/6*C*b*x^6 + 1/6*A*c*x^6 + 1/5*B*b*x^5 + 1/4*
C*a*x^4 + 1/4*A*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx=\frac{C\,c\,x^8}{8}+\frac{B\,c\,x^7}{7}+\left(\frac{A\,c}{6}+\frac{C\,b}{6}\right)\,x^6+\frac{B\,b\,x^5}{5}+\left(\frac{A\,b}{4}+\frac{C\,a}{4}\right)\,x^4+\frac{B\,a\,x^3}{3}+\frac{A\,a\,x^2}{2}$$

input
```
int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

output
```
x^4*((A*b)/4 + (C*a)/4) + x^6*((A*c)/6 + (C*b)/6) + (A*a*x^2)/2 + (B*a*x^3
)/3 + (B*b*x^5)/5 + (B*c*x^7)/7 + (C*c*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)\,dx$$
$$=\frac{x^2(105c^2x^6+120bc\,x^5+140ac\,x^4+140bc\,x^4+168b^2x^3+210ab\,x^2+210ac\,x^2+280abx+420a^2)}{840}$$

input
```
int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

output
```
(x**2*(420*a**2 + 210*a*b*x**2 + 280*a*b*x + 140*a*c*x**4 + 210*a*c*x**2 +
 168*b**2*x**3 + 120*b*c*x**5 + 140*b*c*x**4 + 105*c**2*x**6))/840
```

3.3 $\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4+\frac{1}{5}(Ac+bC)x^5+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

output
```
a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*(A*c+C*b)*x^5+1/6*B*c*
x^6+1/7*c*C*x^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)(a+bx^2+cx^4)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4+\frac{1}{5}(Ac+bC)x^5+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

input
```
Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5
)/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2+cx^4)(A+Bx+Cx^2)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(x^2(aC+Ab)+aA+aBx+x^4(Ac+bC)+bBx^3+Bcx^5+cCx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

input
```
Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5
)/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
default	$aAx+\frac{Ba\,x^{2}}{2}+\frac{(Ab+aC)x^{3}}{3}+\frac{Bb\,x^{4}}{4}+\frac{(Ac+Cb)x^{5}}{5}+\frac{Bc\,x^{6}}{6}+\frac{cC\,x^{7}}{7}$	58
norman	$\frac{cC\,x^{7}}{7}+\frac{Bc\,x^{6}}{6}+\left(\frac{Ac}{5}+\frac{Cb}{5}\right)x^{5}+\frac{Bb\,x^{4}}{4}+\left(\frac{Ab}{3}+\frac{aC}{3}\right)x^{3}+\frac{Ba\,x^{2}}{2}+aAx$	60
gosper	$\frac{1}{7}cC\,x^{7}+\frac{1}{6}Bc\,x^{6}+\frac{1}{5}Ac\,x^{5}+\frac{1}{5}Cb\,x^{5}+\frac{1}{4}Bb\,x^{4}+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}Ca\,x^{3}+\frac{1}{2}Ba\,x^{2}+aAx$	62
risch	$\frac{1}{7}cC\,x^{7}+\frac{1}{6}Bc\,x^{6}+\frac{1}{5}Ac\,x^{5}+\frac{1}{5}Cb\,x^{5}+\frac{1}{4}Bb\,x^{4}+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}Ca\,x^{3}+\frac{1}{2}Ba\,x^{2}+aAx$	62
parallelrisch	$\frac{1}{7}cC\,x^{7}+\frac{1}{6}Bc\,x^{6}+\frac{1}{5}Ac\,x^{5}+\frac{1}{5}Cb\,x^{5}+\frac{1}{4}Bb\,x^{4}+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}Ca\,x^{3}+\frac{1}{2}Ba\,x^{2}+aAx$	62
orering	$\frac{x(60cC\,x^{6}+70Bc\,x^{5}+84Ac\,x^{4}+84Cb\,x^{4}+105Bb\,x^{3}+140Ab\,x^{2}+140Ca\,x^{2}+210Bax+420Aa)}{420}$	63

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*B*b*x^4+1/5*(A*c+C*b)*x^5+1/6*B*c*
x^6+1/7*c*C*x^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{7}\,Ccx^7+\frac{1}{6}\,Bcx^6+\frac{1}{4}\,Bbx^4+\frac{1}{5}\,(Cb+Ac)x^5+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/5*(C*b + A*c)*x^5 + 1/2*B*a*x^
2 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int (A+Bx+Cx^2)\,(a+bx^2+cx^4)\;dx = Aax+\frac{Bax^2}{2}+\frac{Bbx^4}{4}+\frac{Bcx^6}{6}+\frac{Ccx^7}{7}+x^5\left(\frac{Ac}{5}+\frac{Cb}{5}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

output
```
A*a*x + B*a*x**2/2 + B*b*x**4/4 + B*c*x**6/6 + C*c*x**7/7 + x**5*(A*c/5 +
C*b/5) + x**3*(A*b/3 + C*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int (A+Bx+Cx^2)\,(a+bx^2+cx^4)\;dx = \frac{1}{7}\,Ccx^7+\frac{1}{6}\,Bcx^6+\frac{1}{4}\,Bbx^4+\frac{1}{5}\,(Cb+Ac)x^5+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/5*(C*b + A*c)*x^5 + 1/2*B*a*x^
2 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx = \frac{1}{7}Ccx^7+\frac{1}{6}Bcx^6+\frac{1}{5}Cbx^5+\frac{1}{5}Acx^5+\frac{1}{4}Bbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Abx^3+\frac{1}{2}Bax^2+Aax$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/5*C*b*x^5 + 1/5*A*c*x^5 + 1/4*B*b*x^4 + 1/3*
C*a*x^3 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx = \frac{C\,c\,x^7}{7}+\frac{B\,c\,x^6}{6}+\left(\frac{A\,c}{5}+\frac{C\,b}{5}\right)x^5+\frac{B\,b\,x^4}{4}+\left(\frac{A\,b}{3}+\frac{C\,a}{3}\right)x^3+\frac{B\,a\,x^2}{2}+A\,a\,x$$

input
```
int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

output
```
x^3*((A*b)/3 + (C*a)/3) + x^5*((A*c)/5 + (C*b)/5) + A*a*x + (B*a*x^2)/2 +
(B*b*x^4)/4 + (B*c*x^6)/6 + (C*c*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int (A + Bx + Cx^2)(a + bx^2 + cx^4) \, dx$$
$$= \frac{x(60c^2x^6 + 70bc\,x^5 + 84ac\,x^4 + 84bc\,x^4 + 105b^2x^3 + 140ab\,x^2 + 140ac\,x^2 + 210abx + 420a^2)}{420}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

output
```
(x*(420*a**2 + 140*a*b*x**2 + 210*a*b*x + 84*a*c*x**4 + 140*a*c*x**2 + 105
*b**2*x**3 + 70*b*c*x**5 + 84*b*c*x**4 + 60*c**2*x**6))/420
```

3.4 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} dx$

Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} dx = aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}bBx^3+\frac{1}{4}(Ac+bC)x^4 + \frac{1}{5}Bcx^5+\frac{1}{6}cCx^6+aA\log(x)$$

output
```
a*B*x+1/2*(A*b+C*a)*x^2+1/3*b*B*x^3+1/4*(A*c+C*b)*x^4+1/5*B*c*x^5+1/6*c*C*
x^6+a*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x} dx = aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}bBx^3+\frac{1}{4}(Ac+bC)x^4 + \frac{1}{5}Bcx^5+\frac{1}{6}cCx^6+aA\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

output
```
a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5
)/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(x(aC+Ab)+\frac{aA}{x}+aB+x^3(Ac+bC)+bBx^2+Bcx^4+cCx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

output

```
a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5
)/5 + (c*C*x^6)/6 + a*A*Log[x]
```


Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

method	result	size
norman	$\left(\frac{Ab}{2}+\frac{aC}{2}\right)x^2+\left(\frac{Ac}{4}+\frac{Cb}{4}\right)x^4+Bax+\frac{Bb\,x^3}{3}+\frac{Bc\,x^5}{5}+\frac{cC\,x^6}{6}+aA\ln\left(x\right)$	58
default	$\frac{cC\,x^6}{6}+\frac{Bc\,x^5}{5}+\frac{Ac\,x^4}{4}+\frac{Cb\,x^4}{4}+\frac{Bb\,x^3}{3}+\frac{Ab\,x^2}{2}+\frac{Ca\,x^2}{2}+Bax+aA\ln\left(x\right)$	60
risch	$\frac{cC\,x^6}{6}+\frac{Bc\,x^5}{5}+\frac{Ac\,x^4}{4}+\frac{Cb\,x^4}{4}+\frac{Bb\,x^3}{3}+\frac{Ab\,x^2}{2}+\frac{Ca\,x^2}{2}+Bax+aA\ln\left(x\right)$	60
parallelrisch	$\frac{cC\,x^6}{6}+\frac{Bc\,x^5}{5}+\frac{Ac\,x^4}{4}+\frac{Cb\,x^4}{4}+\frac{Bb\,x^3}{3}+\frac{Ab\,x^2}{2}+\frac{Ca\,x^2}{2}+Bax+aA\ln\left(x\right)$	60

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/2*A*b+1/2*a*C)*x^2+(1/4*A*c+1/4*C*b)*x^4+B*a*x+1/3*B*b*x^3+1/5*B*c*x^5+
1/6*c*C*x^6+a*A*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)}{x}\,dx=\frac{1}{6}\,Ccx^6+\frac{1}{5}\,Bcx^5+\frac{1}{3}\,Bbx^3+\frac{1}{4}\,(Cb+Ac)x^4+Bax+\frac{1}{2}\,(Ca+Ab)x^2+Aa\log\left(x\right)$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

output
```
1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/
2*(C*a + A*b)*x^2 + A*a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = Aa\log(x)+Bax+\frac{Bbx^3}{3}+\frac{Bcx^5}{5}+\frac{Ccx^6}{6}+x^4\left(\frac{Ac}{4}+\frac{Cb}{4}\right)+x^2\left(\frac{Ab}{2}+\frac{Ca}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x,x)
```

output
```
A*a*log(x) + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*c*x**6/6 + x**4*(A*c/4 +
C*b/4) + x**2*(A*b/2 + C*a/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = \frac{1}{6}Ccx^6+\frac{1}{5}Bcx^5+\frac{1}{3}Bbx^3+\frac{1}{4}(Cb+Ac)x^4+Bax+\frac{1}{2}(Ca+Ab)x^2+Aa\log(x)$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

output
```
1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/
2*(C*a + A*b)*x^2 + A*a*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = \frac{1}{6}Ccx^6+\frac{1}{5}Bcx^5+\frac{1}{4}Cbx^4+\frac{1}{4}Acx^4+\frac{1}{3}Bbx^3+\frac{1}{2}Cax^2+\frac{1}{2}Abx^2+Bax+Aa\log(|x|)$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

output
```
1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/4*C*b*x^4 + 1/4*A*c*x^4 + 1/3*B*b*x^3 + 1/2*
C*a*x^2 + 1/2*A*b*x^2 + B*a*x + A*a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = x^2\left(\frac{A\,b}{2}+\frac{C\,a}{2}\right)+x^4\left(\frac{A\,c}{4}+\frac{C\,b}{4}\right)+B\,a\,x+\frac{B\,b\,x^3}{3}+\frac{B\,c\,x^5}{5}+\frac{C\,c\,x^6}{6}+A\,a\,\ln(x)$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x)
```

output
```
x^2*((A*b)/2 + (C*a)/2) + x^4*((A*c)/4 + (C*b)/4) + B*a*x + (B*b*x^3)/3 +
(B*c*x^5)/5 + (C*c*x^6)/6 + A*a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx = \log(x)\,a^2+\frac{ab\,x^2}{2}+abx+\frac{ac\,x^4}{4}+\frac{ac\,x^2}{2}+\frac{b^2x^3}{3}+\frac{bc\,x^5}{5}+\frac{bc\,x^4}{4}+\frac{c^2x^6}{6}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x)
```

output
```
(60*log(x)*a**2 + 30*a*b*x**2 + 60*a*b*x + 15*a*c*x**4 + 30*a*c*x**2 + 20*
b**2*x**3 + 12*b*c*x**5 + 15*b*c*x**4 + 10*c**2*x**6)/60
```

3.5 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2} dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}bBx^2+\frac{1}{3}(Ac+bC)x^3 +\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5+aB\log(x)$$

output
```
-a*A/x+(A*b+C*a)*x+1/2*b*B*x^2+1/3*(A*c+C*b)*x^3+1/4*B*c*x^4+1/5*c*C*x^5+a
*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2} dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}bBx^2+\frac{1}{3}(Ac+bC)x^3 +\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5+aB\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

output
```
-((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)
/4 + (c*C*x^5)/5 + a*B*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^2}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(Ab\left(\frac{aC}{Ab}+1\right)+\frac{aA}{x^2}+\frac{aB}{x}+x^2(Ac+bC)+bBx+Bcx^3+cCx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x(aC+Ab)-\frac{aA}{x}+aB\log(x)+\frac{1}{3}x^3(Ac+bC)+\frac{1}{2}bBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

output

```
-((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)
/4 + (c*C*x^5)/5 + a*B*Log[x]
```


Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{cC\,x^{5}}{5}+\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{3}}{3}+\frac{Cb\,x^{3}}{3}+\frac{bB\,x^{2}}{2}+Abx+Cax-\frac{aA}{x}+aB\ln\left(x\right)$	57
risch	$\frac{cC\,x^{5}}{5}+\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{3}}{3}+\frac{Cb\,x^{3}}{3}+\frac{bB\,x^{2}}{2}+Abx+Cax-\frac{aA}{x}+aB\ln\left(x\right)$	57
norman	$\frac{\left(\frac{Ac}{3}+\frac{Cb}{3}\right)x^{4}+(Ab+aC)x^{2}-Aa+\frac{Bb\,x^{3}}{2}+\frac{Bc\,x^{5}}{4}+\frac{cC\,x^{6}}{5}}{x}+aB\ln\left(x\right)$	61
parallelrisch	$\frac{12cC\,x^{6}+15Bc\,x^{5}+20Ac\,x^{4}+20Cb\,x^{4}+30Bb\,x^{3}+60Ab\,x^{2}+60Ba\ln(x)x+60Ca\,x^{2}-60Aa}{60x}$	67

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*c*C*x^5+1/4*B*c*x^4+1/3*A*c*x^3+1/3*C*b*x^3+1/2*b*B*x^2+A*b*x+C*a*x-a*
A/x+a*B*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^2}\,dx = \frac{12\,Ccx^{6}+15\,Bcx^{5}+30\,Bbx^{3}+20\,(Cb+Ac)x^{4}+60\,Bax\log\left(x\right)+60\,(Ca+Ab)x^{2}-60\,Aa}{60\,x}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

output
```
1/60*(12*C*c*x^6 + 15*B*c*x^5 + 30*B*b*x^3 + 20*(C*b + A*c)*x^4 + 60*B*a*x
*log(x) + 60*(C*a + A*b)*x^2 - 60*A*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = -\frac{Aa}{x} + Ba\log(x) + \frac{Bbx^2}{2} + \frac{Bcx^4}{4} + \frac{Ccx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Cb}{3}\right) + x(Ab + Ca)$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**2,x)
```

output
```
-A*a/x + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 +
 C*b/3) + x*(A*b + C*a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = \frac{1}{5}Ccx^5 + \frac{1}{4}Bcx^4 + \frac{1}{2}Bbx^2 + \frac{1}{3}(Cb + Ac)x^3 + Ba\log(x) + (Ca + Ab)x - \frac{Aa}{x}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

output
```
1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/3*(C*b + A*c)*x^3 + B*a*log(x)
 + (C*a + A*b)*x - A*a/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = \frac{1}{5}Ccx^5+\frac{1}{4}Bcx^4+\frac{1}{3}Cbx^3+\frac{1}{3}Acx^3 + \frac{1}{2}Bbx^2+Cax+Abx+Ba\log(|x|)-\frac{Aa}{x}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

output
```
1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*b*x^3 + 1/3*A*c*x^3 + 1/2*B*b*x^2 + C*a*
x + A*b*x + B*a*log(abs(x)) - A*a/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2}\,dx = x\,(A\,b+C\,a)+x^3\left(\frac{A\,c}{3}+\frac{C\,b}{3}\right)-\frac{A\,a}{x} + \frac{B\,b\,x^2}{2}+\frac{B\,c\,x^4}{4}+\frac{C\,c\,x^5}{5}+B\,a\,\ln(x)$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x)
```

output
```
x*(A*b + C*a) + x^3*((A*c)/3 + (C*b)/3) - (A*a)/x + (B*b*x^2)/2 + (B*c*x^4
)/4 + (C*c*x^5)/5 + B*a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^2}\,dx$$
$$= \frac{60\log(x)\,abx-60a^2+60ab\,x^2+20ac\,x^4+60ac\,x^2+30b^2x^3+15bc\,x^5+20bc\,x^4+12c^2x^6}{60x}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x)
```

output
```
(60*log(x)*a*b*x - 60*a**2 + 60*a*b*x**2 + 20*a*c*x**4 + 60*a*c*x**2 + 30*
b**2*x**3 + 15*b*c*x**5 + 20*b*c*x**4 + 12*c**2*x**6)/(60*x)
```

3.6 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = -\frac{aA}{2x^2}-\frac{aB}{x}+bBx+\frac{1}{2}(Ac+bC)x^2+\frac{1}{3}Bcx^3+\frac{1}{4}cCx^4+(Ab+aC)\log(x)$$

output
```
-1/2*a*A/x^2-a*B/x+b*B*x+1/2*(A*c+C*b)*x^2+1/3*B*c*x^3+1/4*c*C*x^4+(A*b+C*
a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = -\frac{a(A+2Bx)}{2x^2}+\frac{1}{12}x\left(6b(2B+Cx)+cx\left(6A+4Bx+3Cx^2\right)\right)+(Ab+aC)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

output
```
-1/2*(a*(A + 2*B*x))/x^2 + (x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C*x^
2)))/12 + (A*b + a*C)*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^3}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{aC+Ab}{x}+\frac{aA}{x^3}+\frac{aB}{x^2}+x(Ac+bC)+bB+Bcx^2+cCx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}-\frac{aB}{x}+\frac{1}{2}x^2(Ac+bC)+bBx+\frac{1}{3}Bcx^3+\frac{1}{4}cCx^4$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 - (a*B)/x + b*B*x + ((A*c + b*C)*x^2)/2 + (B*c*x^3)/3 + (c*
C*x^4)/4 + (A*b + a*C)*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{Cc\,x^{4}}{4}+\frac{Bc\,x^{3}}{3}+\frac{Ac\,x^{2}}{2}+\frac{Cb\,x^{2}}{2}+bBx-\frac{aA}{2x^{2}}-\frac{aB}{x}+\left(Ab+aC\right)\ln\left(x\right)$	58
risch	$\frac{Cc\,x^{4}}{4}+\frac{Bc\,x^{3}}{3}+\frac{Ac\,x^{2}}{2}+\frac{Cb\,x^{2}}{2}+bBx+\frac{-Bax-\frac{1}{2}Aa}{x^{2}}+A\ln\left(x\right)b+C\ln\left(x\right)a$	58
norman	$\frac{\left(\frac{Ac}{2}+\frac{Cb}{2}\right)x^{4}+Bb\,x^{3}-\frac{Aa}{2}-Bax+\frac{Bc\,x^{5}}{3}+\frac{cC\,x^{6}}{4}}{x^{2}}+\left(Ab+aC\right)\ln\left(x\right)$	59
parallelrisch	$\frac{3cC\,x^{6}+4Bc\,x^{5}+6Ac\,x^{4}+6Cb\,x^{4}+12A\ln\left(x\right)x^{2}b+12Bb\,x^{3}+12C\ln\left(x\right)x^{2}a-12Bax-6Aa}{12x^{2}}$	69

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*C*c*x^4+1/3*B*c*x^3+1/2*A*c*x^2+1/2*C*b*x^2+b*B*x-1/2*a*A/x^2-a*B/x+(A
*b+C*a)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int\frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^3}\,dx$$
$$=\frac{3\,Ccx^6+4\,Bcx^5+12\,Bbx^3+6\,(Cb+Ac)x^4+12\,(Ca+Ab)x^2\log\left(x\right)-12\,Bax-6\,Aa}{12\,x^2}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

output
```
1/12*(3*C*c*x^6 + 4*B*c*x^5 + 12*B*b*x^3 + 6*(C*b + A*c)*x^4 + 12*(C*a + A
*b)*x^2*log(x) - 12*B*a*x - 6*A*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = Bbx+\frac{Bcx^3}{3}+\frac{Ccx^4}{4}+x^2\left(\frac{Ac}{2}+\frac{Cb}{2}\right)+(Ab+Ca)\log(x)+\frac{-Aa-2Bax}{2x^2}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**3,x)
```

output
```
B*b*x + B*c*x**3/3 + C*c*x**4/4 + x**2*(A*c/2 + C*b/2) + (A*b + C*a)*log(x
) + (-A*a - 2*B*a*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = \frac{1}{4}Ccx^4+\frac{1}{3}Bcx^3+Bbx+\frac{1}{2}(Cb+Ac)x^2+(Ca+Ab)\log(x)-\frac{2Bax+Aa}{2x^2}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

output
```
1/4*C*c*x^4 + 1/3*B*c*x^3 + B*b*x + 1/2*(C*b + A*c)*x^2 + (C*a + A*b)*log(
x) - 1/2*(2*B*a*x + A*a)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = \frac{1}{4}Ccx^4 + \frac{1}{3}Bcx^3 + \frac{1}{2}Cbx^2 + \frac{1}{2}Acx^2 + Bbx + (Ca+Ab)\log(|x|) - \frac{2\,Bax + Aa}{2\,x^2}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

output
```
1/4*C*c*x^4 + 1/3*B*c*x^3 + 1/2*C*b*x^2 + 1/2*A*c*x^2 + B*b*x + (C*a + A*b
)*log(abs(x)) - 1/2*(2*B*a*x + A*a)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx = x^2\left(\frac{A\,c}{2}+\frac{C\,b}{2}\right) - \frac{\frac{A\,a}{2}+B\,a\,x}{x^2} + \ln(x)\,(A\,b+C\,a) + B\,b\,x + \frac{B\,c\,x^3}{3} + \frac{C\,c\,x^4}{4}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x)
```

output
```
x^2*((A*c)/2 + (C*b)/2) - ((A*a)/2 + B*a*x)/x^2 + log(x)*(A*b + C*a) + B*b
*x + (B*c*x^3)/3 + (C*c*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx$$
$$=\frac{12\log(x)\,ab\,x^2+12\log(x)\,ac\,x^2-6a^2-12abx+6ac\,x^4+12b^2x^3+4bc\,x^5+6bc\,x^4+3c^2x^6}{12x^2}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x)
```

output
```
(12*log(x)*a*b*x**2 + 12*log(x)*a*c*x**2 - 6*a**2 - 12*a*b*x + 6*a*c*x**4
+ 12*b**2*x**3 + 4*b*c*x**5 + 6*b*c*x**4 + 3*c**2*x**6)/(12*x**2)
```

3.7 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4} dx = -\frac{aA}{3x^3} - \frac{aB}{2x^2} - \frac{Ab+aC}{x} + (Ac+bC)x + \frac{1}{2}Bcx^2 + \frac{1}{3}cCx^3 + bB\log(x)$$

output
```
-1/3*a*A/x^3-1/2*a*B/x^2-(A*b+C*a)/x+(A*c+C*b)*x+1/2*B*c*x^2+1/3*c*C*x^3+b
*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4} dx = -\frac{Ab}{x} + Acx + bCx + \frac{1}{2}Bcx^2 + \frac{1}{3}cCx^3 - \frac{a(2A+3x(B+2Cx))}{6x^3} + bB\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

output
```
-((A*b)/x) + A*c*x + b*C*x + (B*c*x^2)/2 + (c*C*x^3)/3 - (a*(2*A + 3*x*(B
+ 2*C*x)))/(6*x^3) + b*B*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^4}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{aC+Ab}{x^2}+\frac{aA}{x^4}+\frac{aB}{x^3}+Ac\left(\frac{bC}{Ac}+1\right)+\frac{bB}{x}+Bcx+cCx^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}-\frac{aB}{2x^2}+x(Ac+bC)+bB\log(x)+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 - (a*B)/(2*x^2) - (A*b + a*C)/x + (A*c + b*C)*x + (B*c*x^2)
/2 + (c*C*x^3)/3 + b*B*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{cC\,x^{3}}{3}+\frac{Bc\,x^{2}}{2}+Acx+Cbx-\frac{aA}{3x^{3}}-\frac{aB}{2x^{2}}-\frac{Ab+aC}{x}+bB\ln\left(x\right)$	55
risch	$\frac{cC\,x^{3}}{3}+\frac{Bc\,x^{2}}{2}+Acx+Cbx+\frac{(-Ab-aC)x^{2}-\frac{Bax}{2}-\frac{Aa}{3}}{x^{3}}+bB\ln\left(x\right)$	56
norman	$\frac{(-Ab-aC)x^{2}+(Ac+Cb)x^{4}-\frac{Aa}{3}-\frac{Bax}{2}+\frac{Bc\,x^{5}}{2}+\frac{cC\,x^{6}}{3}}{x^{3}}+bB\ln\left(x\right)$	59
parallelrisch	$\frac{2cC\,x^{6}+3Bc\,x^{5}+6Ac\,x^{4}+6Bb\ln(x)x^{3}+6Cb\,x^{4}-6Ab\,x^{2}-6Ca\,x^{2}-3Bax-2Aa}{6x^{3}}$	67

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*c*C*x^3+1/2*B*c*x^2+A*c*x+C*b*x-1/3*a*A/x^3-1/2*a*B/x^2-(A*b+C*a)/x+b*
B*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = \frac{2\,Ccx^6+3\,Bcx^5+6\,Bbx^3\log\left(x\right)+6\,(Cb+Ac)x^4-3\,Bax-6\,(Ca+Ab)x^2-2\,Aa}{6\,x^3}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*C*c*x^6 + 3*B*c*x^5 + 6*B*b*x^3*log(x) + 6*(C*b + A*c)*x^4 - 3*B*a*
x - 6*(C*a + A*b)*x^2 - 2*A*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = Bb\log(x) + \frac{Bcx^2}{2} + \frac{Ccx^3}{3} + x(Ac+Cb) + \frac{-2Aa-3Bax+x^2(-6Ab-6Ca)}{6x^3}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**4,x)
```

output
```
B*b*log(x) + B*c*x**2/2 + C*c*x**3/3 + x*(A*c + C*b) + (-2*A*a - 3*B*a*x +
 x**2*(-6*A*b - 6*C*a))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = \frac{1}{3}Ccx^3 + \frac{1}{2}Bcx^2 + Bb\log(x) + (Cb+Ac)x - \frac{3\,Bax + 6\,(Ca+Ab)x^2 + 2\,Aa}{6\,x^3}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")
```

output
```
1/3*C*c*x^3 + 1/2*B*c*x^2 + B*b*log(x) + (C*b + A*c)*x - 1/6*(3*B*a*x + 6*
(C*a + A*b)*x^2 + 2*A*a)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = \frac{1}{3}Ccx^3 + \frac{1}{2}Bcx^2 + Cbx + Acx + Bb\log(|x|) - \frac{3\,Bax + 6\,(Ca+Ab)x^2 + 2\,Aa}{6\,x^3}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="giac")
```

output
```
1/3*C*c*x^3 + 1/2*B*c*x^2 + C*b*x + A*c*x + B*b*log(abs(x)) - 1/6*(3*B*a*x
 + 6*(C*a + A*b)*x^2 + 2*A*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = x\,(A\,c+C\,b) - \frac{(A\,b+C\,a)\,x^2 + \frac{B\,a\,x}{2} + \frac{A\,a}{3}}{x^3} + \frac{B\,c\,x^2}{2} + \frac{C\,c\,x^3}{3} + B\,b\,\ln(x)$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x)
```

output
```
x*(A*c + C*b) - ((A*a)/3 + x^2*(A*b + C*a) + (B*a*x)/2)/x^3 + (B*c*x^2)/2
+ (C*c*x^3)/3 + B*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx = \frac{6\log(x)\,b^2x^3-2a^2-6ab\,x^2-3abx+6ac\,x^4-6ac\,x^2+3bc\,x^5+6bc\,x^4+2c^2x^6}{6x^3}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x)
```

output
```
(6*log(x)*b**2*x**3 - 2*a**2 - 6*a*b*x**2 - 3*a*b*x + 6*a*c*x**4 - 6*a*c*x
**2 + 3*b*c*x**5 + 6*b*c*x**4 + 2*c**2*x**6)/(6*x**3)
```

3.8 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = -\frac{aA}{4x^4}-\frac{aB}{3x^3}-\frac{Ab+aC}{2x^2}-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2+(Ac+bC)\log(x)$$

output
```
-1/4*a*A/x^4-1/3*a*B/x^3-1/2*(A*b+C*a)/x^2-b*B/x+B*c*x+1/2*c*C*x^2+(A*c+C*
b)*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = -\frac{a(3A+4Bx+6Cx^2)}{12x^4}+\frac{-Ab-2bBx+cx^3(2B+Cx)}{2x^2}+(Ac+bC)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

output
```
-1/12*(a*(3*A + 4*B*x + 6*C*x^2))/x^4 + (-(A*b) - 2*b*B*x + c*x^3*(2*B + C
*x))/(2*x^2) + (A*c + b*C)*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^5}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{aC+Ab}{x^3}+\frac{aA}{x^5}+\frac{aB}{x^4}+\frac{Ac+bC}{x}+\frac{bB}{x^2}+Bc+cCx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aC+Ab}{2x^2}-\frac{aA}{4x^4}-\frac{aB}{3x^3}+\log(x)(Ac+bC)-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

output
```
-1/4*(a*A)/x^4 - (a*B)/(3*x^3) - (A*b + a*C)/(2*x^2) - (b*B)/x + B*c*x + (
c*C*x^2)/2 + (A*c + b*C)*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{aA}{4x^4}-\frac{aB}{3x^3}-\frac{Ab+aC}{2x^2}-\frac{bB}{x}+Bcx+\frac{cC\,x^2}{2}+(Ac+Cb)\ln(x)$	56
risch	$\frac{cC\,x^2}{2}+Bcx+\frac{-Bb\,x^3+\left(-\frac{Ab}{2}-\frac{aC}{2}\right)x^2-\frac{Bax}{3}-\frac{Aa}{4}}{x^4}+A\ln(x)\,c+C\ln(x)\,b$	57
norman	$\frac{\left(-\frac{Ab}{2}-\frac{aC}{2}\right)x^2+Bc\,x^5-\frac{Aa}{4}-\frac{Bax}{3}-Bb\,x^3+\frac{cC\,x^6}{2}}{x^4}+(Ac+Cb)\ln(x)$	59
parallelrisch	$\frac{6cC\,x^6+12A\ln(x)x^4c+12Bc\,x^5+12C\ln(x)x^4b-12Bb\,x^3-6Ab\,x^2-6Ca\,x^2-4Bax-3Aa}{12x^4}$	69

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*A/x^4-1/3*a*B/x^3-1/2*(A*b+C*a)/x^2-b*B/x+B*c*x+1/2*c*C*x^2+(A*c+C*
b)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^5}\,dx$$
$$=\frac{6\,Ccx^6+12\,Bcx^5+12\,(Cb+Ac)x^4\log(x)-12\,Bbx^3-4\,Bax-6\,(Ca+Ab)x^2-3\,Aa}{12\,x^4}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")
```

output
```
1/12*(6*C*c*x^6 + 12*B*c*x^5 + 12*(C*b + A*c)*x^4*log(x) - 12*B*b*x^3 - 4*
B*a*x - 6*(C*a + A*b)*x^2 - 3*A*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx$$
$$= Bcx+\frac{Ccx^2}{2}+(Ac+Cb)\log(x)+\frac{-3Aa-4Bax-12Bbx^3+x^2(-6Ab-6Ca)}{12x^4}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**5,x)
```

output
```
B*c*x + C*c*x**2/2 + (A*c + C*b)*log(x) + (-3*A*a - 4*B*a*x - 12*B*b*x**3
+ x**2*(-6*A*b - 6*C*a))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = \frac{1}{2}\,Ccx^2+Bcx+(Cb+Ac)\log(x)$$
$$-\frac{12\,Bbx^3+4\,Bax+6\,(Ca+Ab)x^2+3\,Aa}{12\,x^4}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")
```

output
```
1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(x) - 1/12*(12*B*b*x^3 + 4*B*a*x + 6*
(C*a + A*b)*x^2 + 3*A*a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = \frac{1}{2}Ccx^2+Bcx+(Cb+Ac)\log(|x|) - \frac{12\,Bbx^3+4\,Bax+6\,(Ca+Ab)x^2+3\,Aa}{12\,x^4}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="giac")
```

output
```
1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(abs(x)) - 1/12*(12*B*b*x^3 + 4*B*a*x
 + 6*(C*a + A*b)*x^2 + 3*A*a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx = \ln(x)\,(A\,c+C\,b) - \frac{B\,b\,x^3+\left(\frac{A\,b}{2}+\frac{C\,a}{2}\right)\,x^2+\frac{B\,a\,x}{3}+\frac{A\,a}{4}}{x^4} + B\,c\,x+\frac{C\,c\,x^2}{2}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x)
```

output
```
log(x)*(A*c + C*b) - ((A*a)/4 + x^2*((A*b)/2 + (C*a)/2) + (B*a*x)/3 + B*b*
x^3)/x^4 + B*c*x + (C*c*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx$$
$$= \frac{12\log(x)\,ac\,x^4+12\log(x)\,bc\,x^4-3a^2-6ab\,x^2-4abx-6ac\,x^2-12b^2x^3+12bc\,x^5+6c^2x^6}{12x^4}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x)
```

output
```
(12*log(x)*a*c*x**4 + 12*log(x)*b*c*x**4 - 3*a**2 - 6*a*b*x**2 - 4*a*b*x -
 6*a*c*x**2 - 12*b**2*x**3 + 12*b*c*x**5 + 6*c**2*x**6)/(12*x**4)
```

3.9 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6} dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx = -\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ab+aC}{3x^3}-\frac{bB}{2x^2}-\frac{Ac+bC}{x}+cCx+Bc\log(x)$$

output
```
-1/5*a*A/x^5-1/4*a*B/x^4-1/3*(A*b+C*a)/x^3-1/2*b*B/x^2-(A*c+C*b)/x+c*C*x+B
*c*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx = -\frac{12aA-60cCx^6+30bx^3(B+2Cx)+5ax(3B+4Cx)+20Ax^2(b+3cx^2)}{60x^5} + Bc\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

output
```
-1/60*(12*a*A - 60*c*C*x^6 + 30*b*x^3*(B + 2*C*x) + 5*a*x*(3*B + 4*C*x) +
20*A*x^2*(b + 3*c*x^2))/x^5 + B*c*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^6}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{aC+Ab}{x^4}+\frac{aA}{x^6}+\frac{aB}{x^5}+\frac{Ac+bC}{x^2}+\frac{bB}{x^3}+\frac{Bc}{x}+cC\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aC+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ac+bC}{x}-\frac{bB}{2x^2}+Bc\log(x)+cCx$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 - (a*B)/(4*x^4) - (A*b + a*C)/(3*x^3) - (b*B)/(2*x^2) - (A*
c + b*C)/x + c*C*x + B*c*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ab+aC}{3x^3}-\frac{bB}{2x^2}-\frac{Ac+Cb}{x}+cCx+Bc\ln\left(x\right)$	56
risch	$cCx+\frac{(-Ac-Cb)x^4-\frac{Bb\,x^3}{2}+\left(-\frac{Ab}{3}-\frac{aC}{3}\right)x^2-\frac{Bax}{4}-\frac{Aa}{5}}{x^5}+Bc\ln\left(x\right)$	58
norman	$\frac{\left(-\frac{Ab}{3}-\frac{aC}{3}\right)x^2+(-Ac-Cb)x^4+cC\,x^6-\frac{Aa}{5}-\frac{Bax}{4}-\frac{Bb\,x^3}{2}}{x^5}+Bc\ln\left(x\right)$	60
parallelrisch	$-\frac{-60Bc\ln(x)x^5-60cC\,x^6+60Ac\,x^4+60Cb\,x^4+30Bb\,x^3+20Ab\,x^2+20Ca\,x^2+15Bax+12Aa}{60x^5}$	67

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a*A/x^5-1/4*a*B/x^4-1/3*(A*b+C*a)/x^3-1/2*b*B/x^2-(A*c+C*b)/x+c*C*x+B
*c*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int\frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^6}\,dx$$
$$=\frac{60\,Ccx^6+60\,Bcx^5\log\left(x\right)-30\,Bbx^3-60\,(Cb+Ac)x^4-15\,Bax-20\,(Ca+Ab)x^2-12\,Aa}{60\,x^5}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")
```

output
```
1/60*(60*C*c*x^6 + 60*B*c*x^5*log(x) - 30*B*b*x^3 - 60*(C*b + A*c)*x^4 - 1
5*B*a*x - 20*(C*a + A*b)*x^2 - 12*A*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx$$
$$= Bc\log(x) + Ccx + \frac{-12Aa-15Bax-30Bbx^3+x^4(-60Ac-60Cb)+x^2(-20Ab-20Ca)}{60x^5}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**6,x)
```

output
```
B*c*log(x) + C*c*x + (-12*A*a - 15*B*a*x - 30*B*b*x**3 + x**4*(-60*A*c - 6
0*C*b) + x**2*(-20*A*b - 20*C*a))/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx$$
$$= Ccx + Bc\log(x) - \frac{30\,Bbx^3+60\,(Cb+Ac)x^4+15\,Bax+20\,(Ca+Ab)x^2+12\,Aa}{60\,x^5}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")
```

output
```
C*c*x + B*c*log(x) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x + 20
*(C*a + A*b)*x^2 + 12*A*a)/x^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^6}\,dx$$
$$= Ccx + Bc\log(|x|) - \frac{30\,Bbx^3 + 60\,(Cb+Ac)x^4 + 15\,Bax + 20\,(Ca+Ab)x^2 + 12\,Aa}{60\,x^5}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="giac")
```

output
```
C*c*x + B*c*log(abs(x)) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x
 + 20*(C*a + A*b)*x^2 + 12*A*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 18.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)}{x^6}\,dx$$
$$= C\,c\,x - \frac{(A\,c+C\,b)\;x^4 + \frac{B\,b\,x^3}{2} + \left(\frac{A\,b}{3}+\frac{C\,a}{3}\right)\;x^2 + \frac{B\,a\,x}{4} + \frac{A\,a}{5}}{x^5} + B\,c\,\ln(x)$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x)
```

output
```
C*c*x - ((A*a)/5 + x^2*((A*b)/3 + (C*a)/3) + x^4*(A*c + C*b) + (B*a*x)/4 +
 (B*b*x^3)/2)/x^5 + B*c*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx$$
$$=\frac{60\log(x)\,bc\,x^5-12a^2-20ab\,x^2-15abx-60ac\,x^4-20ac\,x^2-30b^2x^3-60bc\,x^4+60c^2x^6}{60x^5}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x)
```

output
```
(60*log(x)*b*c*x**5 - 12*a**2 - 20*a*b*x**2 - 15*a*b*x - 60*a*c*x**4 - 20*
a*c*x**2 - 30*b**2*x**3 - 60*b*c*x**4 + 60*c**2*x**6)/(60*x**5)
```

3.10 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 68

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx = -\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ab+aC}{4x^4}-\frac{bB}{3x^3}-\frac{Ac+bC}{2x^2}-\frac{Bc}{x}+cC\log(x)$$

output
```
-1/6*a*A/x^6-1/5*a*B/x^5-1/4*(A*b+C*a)/x^4-1/3*b*B/x^3-1/2*(A*c+C*b)/x^2-B
*c/x+c*C*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx = -\frac{a(10A+3x(4B+5Cx))+5x^2(3A(b+2cx^2)+2x(2bB+3bCx+6Bcx^2))}{60x^6}+cC\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

output
```
-1/60*(a*(10*A + 3*x*(4*B + 5*C*x)) + 5*x^2*(3*A*(b + 2*c*x^2) + 2*x*(2*b*
B + 3*b*C*x + 6*B*c*x^2)))/x^6 + c*C*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)(A+Bx+Cx^2)}{x^7}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{aC+Ab}{x^5}+\frac{aA}{x^7}+\frac{aB}{x^6}+\frac{Ac+bC}{x^3}+\frac{bB}{x^4}+\frac{Bc}{x^2}+\frac{cC}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aC+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ac+bC}{2x^2}-\frac{bB}{3x^3}-\frac{Bc}{x}+cC\log(x)$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

output
```
-1/6*(a*A)/x^6 - (a*B)/(5*x^5) - (A*b + a*C)/(4*x^4) - (b*B)/(3*x^3) - (A*
c + b*C)/(2*x^2) - (B*c)/x + c*C*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ab+aC}{4x^4}-\frac{bB}{3x^3}-\frac{Ac+Cb}{2x^2}-\frac{Bc}{x}+cC\ln\left(x\right)$	59
norman	$\frac{\left(-\frac{Ab}{4}-\frac{aC}{4}\right)x^2+\left(-\frac{Ac}{2}-\frac{Cb}{2}\right)x^4-\frac{Aa}{6}-\frac{Bax}{5}-\frac{Bb\,x^3}{3}-Bc\,x^5}{x^6}+cC\ln\left(x\right)$	61
risch	$\frac{\left(-\frac{Ab}{4}-\frac{aC}{4}\right)x^2+\left(-\frac{Ac}{2}-\frac{Cb}{2}\right)x^4-\frac{Aa}{6}-\frac{Bax}{5}-\frac{Bb\,x^3}{3}-Bc\,x^5}{x^6}+cC\ln\left(x\right)$	61
parallelrisch	$-\frac{-60Cc\ln(x)x^6+60Bc\,x^5+30Ac\,x^4+30Cb\,x^4+20Bb\,x^3+15Ab\,x^2+15Ca\,x^2+12Bax+10Aa}{60x^6}$	67

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*A/x^6-1/5*a*B/x^5-1/4*(A*b+C*a)/x^4-1/3*b*B/x^3-1/2*(A*c+C*b)/x^2-B
*c/x+c*C*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)}{x^7}\,dx = \frac{60\,Ccx^6\log\left(x\right)-60\,Bcx^5-20\,Bbx^3-30\,(Cb+Ac)x^4-12\,Bax-15\,(Ca+Ab)x^2-10\,Aa}{60\,x^6}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")
```

output
```
1/60*(60*C*c*x^6*log(x) - 60*B*c*x^5 - 20*B*b*x^3 - 30*(C*b + A*c)*x^4 - 1
2*B*a*x - 15*(C*a + A*b)*x^2 - 10*A*a)/x^6
```

Sympy [A] (verification not implemented)

Time = 7.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx \\ = Cc\log(x) \\ + \frac{-10Aa-12Bax-20Bbx^3-60Bcx^5+x^4(-30Ac-30Cb)+x^2(-15Ab-15Ca)}{60x^6}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**7,x)
```

output
```
C*c*log(x) + (-10*A*a - 12*B*a*x - 20*B*b*x**3 - 60*B*c*x**5 + x**4*(-30*A
*c - 30*C*b) + x**2*(-15*A*b - 15*C*a))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx \\ = Cc\log(x) \\ - \frac{60\,Bcx^5+20\,Bbx^3+30\,(Cb+Ac)x^4+12\,Bax+15\,(Ca+Ab)x^2+10\,Aa}{60\,x^6}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")
```

output
```
C*c*log(x) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B*a*x
 + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

Giac [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx = Cc\log(|x|) - \frac{60\,Bcx^5+20\,Bbx^3+30\,(Cb+Ac)x^4+12\,Bax+15\,(Ca+Ab)x^2+10\,Aa}{60\,x^6}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="giac")
```

output
```
C*c*log(abs(x)) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*
B*a*x + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx = C\,c\,\ln(x) - \frac{B\,c\,x^5+\left(\frac{A\,c}{2}+\frac{C\,b}{2}\right)\,x^4+\frac{B\,b\,x^3}{3}+\left(\frac{A\,b}{4}+\frac{C\,a}{4}\right)\,x^2+\frac{B\,a\,x}{5}+\frac{A\,a}{6}}{x^6}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x)
```

output
```
C*c*log(x) - ((A*a)/6 + x^2*((A*b)/4 + (C*a)/4) + x^4*((A*c)/2 + (C*b)/2)
+ (B*a*x)/5 + (B*b*x^3)/3 + B*c*x^5)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$$
$$= \frac{60\log(x)\,c^2x^6-10a^2-15ab\,x^2-12abx-30ac\,x^4-15ac\,x^2-20b^2x^3-60bc\,x^5-30bc\,x^4}{60x^6}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x)
```

output
```
(60*log(x)*c**2*x**6 - 10*a**2 - 15*a*b*x**2 - 12*a*b*x - 30*a*c*x**4 - 15
*a*c*x**2 - 20*b**2*x**3 - 60*b*c*x**5 - 30*b*c*x**4)/(60*x**6)
```

3.11 $\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 159

$$\begin{aligned}
&\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx\\
&=\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{3}abBx^6\\
&\quad+\frac{1}{7}\left(A\left(b^2+2ac\right)+2abC\right)x^7+\frac{1}{8}B\left(b^2+2ac\right)x^8+\frac{1}{9}\left(2Abc+\left(b^2+2ac\right)C\right)x^9\\
&\quad+\frac{1}{5}bBcx^{10}+\frac{1}{11}c(Ac+2bC)x^{11}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}
\end{aligned}$$

output
```
1/3*a^2*A*x^3+1/4*a^2*B*x^4+1/5*a*(2*A*b+C*a)*x^5+1/3*a*b*B*x^6+1/7*(A*(2*
a*c+b^2)+2*a*b*C)*x^7+1/8*B*(2*a*c+b^2)*x^8+1/9*(2*A*b*c+(2*a*c+b^2)*C)*x^
9+1/5*b*B*c*x^10+1/11*c*(A*c+2*C*b)*x^11+1/12*B*c^2*x^12+1/13*c^2*C*x^13
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx$$
$$=\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{3}abBx^6+\frac{1}{7}\left(Ab^2+2aAc+2abC\right)x^7+\frac{1}{8}B\left(b^2+2ac\right)x^8+\frac{1}{9}\left(2Abc+b^2C+2acC\right)x^9+\frac{1}{5}bBcx^{10}+\frac{1}{11}c(Ac+2bC)x^{11}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

input
```
Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 +
((A*b^2 + 2*a*A*c + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c
+ b^2*C + 2*a*c*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (
B*c^2*x^12)/12 + (c^2*C*x^13)/13
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2+cx^4\right)^2\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(a^2Ax^2+a^2Bx^3+x^8\left(C\left(2ac+b^2\right)+2Abc\right)+x^6\left(A\left(2ac+b^2\right)+2abC\right)+ax^4(aC+2Ab)+Bx^7\left(2ac+b^2\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{7}x^7\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)+\frac{1}{3}abBx^6+\frac{1}{11}cx^{11}(Ac+2bC)+\frac{1}{5}bBcx^{10}+\frac{1}{12}Bc^2x^{12}+\frac{1}{13}c^2Cx^{13}$$

input
```
Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 +
((A*(b^2 + 2*a*c) + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c
+ (b^2 + 2*a*c)*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (
B*c^2*x^12)/12 + (c^2*C*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

method	result
default	$\frac{c^2C\,x^{13}}{13}+\frac{B\,c^2x^{12}}{12}+\frac{(A\,c^2+2Ccb)x^{11}}{11}+\frac{bBc\,x^{10}}{5}+\frac{(2Abc+(2ac+b^2)C)x^9}{9}+\frac{B(2ac+b^2)x^8}{8}+\frac{(A(2ac+b^2)+2Cb}{7}$
norman	$\frac{c^2C\,x^{13}}{13}+\frac{B\,c^2x^{12}}{12}+\left(\frac{1}{11}A\,c^2+\frac{2}{11}Ccb\right)x^{11}+\frac{bBc\,x^{10}}{5}+\left(\frac{2}{9}Abc+\frac{2}{9}Cac+\frac{1}{9}C\,b^2\right)x^9+\left(\frac{1}{4}Bac+\right.$
gosper	$\frac{1}{13}c^2C\,x^{13}+\frac{1}{12}B\,c^2x^{12}+\frac{1}{11}x^{11}A\,c^2+\frac{2}{11}x^{11}Ccb+\frac{1}{5}bBc\,x^{10}+\frac{2}{9}x^9Abc+\frac{2}{9}x^9Cac+\frac{1}{9}x^9C\,b^2$
risch	$\frac{1}{13}c^2C\,x^{13}+\frac{1}{12}B\,c^2x^{12}+\frac{1}{11}x^{11}A\,c^2+\frac{2}{11}x^{11}Ccb+\frac{1}{5}bBc\,x^{10}+\frac{2}{9}x^9Abc+\frac{2}{9}x^9Cac+\frac{1}{9}x^9C\,b^2$
parallelrisch	$\frac{1}{13}c^2C\,x^{13}+\frac{1}{12}B\,c^2x^{12}+\frac{1}{11}x^{11}A\,c^2+\frac{2}{11}x^{11}Ccb+\frac{1}{5}bBc\,x^{10}+\frac{2}{9}x^9Abc+\frac{2}{9}x^9Cac+\frac{1}{9}x^9C\,b^2$
orering	$x^3\left(27720c^2C\,x^{10}+30030B\,c^2x^9+32760A\,c^2x^8+65520Cbc\,x^8+72072bBc\,x^7+80080Abc\,x^6+80080Cac\,x^6+40040C\,b^2x^6+90\right.$

input
```
int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*c^2*C*x^13+1/12*B*c^2*x^12+1/11*(A*c^2+2*C*b*c)*x^11+1/5*b*B*c*x^10+1
/9*(2*A*b*c+(2*a*c+b^2)*C)*x^9+1/8*B*(2*a*c+b^2)*x^8+1/7*(A*(2*a*c+b^2)+2*
C*b*a)*x^7+1/3*a*b*B*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*A*x
^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{13}\,Cc^2x^{13} + \frac{1}{12}\,Bc^2x^{12} + \frac{1}{5}\,Bbcx^{10} + \frac{1}{11}\,\left(2\,Cbc + Ac^2\right)x^{11} + \frac{1}{9}\,\left(Cb^2 + 2\,(Ca + Ab)c\right)x^9 + \frac{1}{3}\,Babx^6 + \frac{1}{8}\,\left(Bb^2 + 2\,Bac\right)x^8 + \frac{1}{7}\,\left(2\,Cab + Ab^2 + 2\,Aac\right)x^7 + \frac{1}{4}\,Ba^2x^4 + \frac{1}{3}\,Aa^2x^3 + \frac{1}{5}\,\left(Ca^2 + 2\,Aab\right)x^5$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*c + A*c^2
)*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x^6 + 1/8*(B*b^2 +
2*B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + 1/3*A
*a^2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{Aa^2x^3}{3}+\frac{Ba^2x^4}{4}+\frac{Babx^6}{3}+\frac{Bbcx^{10}}{5}+\frac{Bc^2x^{12}}{12}+\frac{Cc^2x^{13}}{13}+x^{11}\left(\frac{Ac^2}{11}+\frac{2Cbc}{11}\right)+x^9\cdot\left(\frac{2Abc}{9}+\frac{2Cac}{9}+\frac{Cb^2}{9}\right)+x^8\left(\frac{Bac}{4}+\frac{Bb^2}{8}\right)+x^7\cdot\left(\frac{2Aac}{7}+\frac{Ab^2}{7}+\frac{2Cab}{7}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ca^2}{5}\right)$$

input
```
integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

output
```
A*a**2*x**3/3 + B*a**2*x**4/4 + B*a*b*x**6/3 + B*b*c*x**10/5 + B*c**2*x**1
2/12 + C*c**2*x**13/13 + x**11*(A*c**2/11 + 2*C*b*c/11) + x**9*(2*A*b*c/9
+ 2*C*a*c/9 + C*b**2/9) + x**8*(B*a*c/4 + B*b**2/8) + x**7*(2*A*a*c/7 + A*
b**2/7 + 2*C*a*b/7) + x**5*(2*A*a*b/5 + C*a**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{13}\,Cc^2x^{13}+\frac{1}{12}\,Bc^2x^{12}+\frac{1}{5}\,Bbcx^{10}+\frac{1}{11}\,(2\,Cbc+Ac^2)x^{11}+\frac{1}{9}\,(Cb^2+2\,(Ca+Ab)c)x^9+\frac{1}{3}\,Babx^6+\frac{1}{8}\,(Bb^2+2\,Bac)x^8+\frac{1}{7}\,(2\,Cab+Ab^2+2\,Aac)x^7+\frac{1}{4}\,Ba^2x^4+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,(Ca^2+2\,Aab)x^5$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*c + A*c^2
)*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x^6 + 1/8*(B*b^2 +
2*B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + 1/3*A
*a^2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{13}Cc^2x^{13}+\frac{1}{12}Bc^2x^{12}+\frac{2}{11}Cbcx^{11}+\frac{1}{11}Ac^2x^{11}+\frac{1}{5}Bbcx^{10}+\frac{1}{9}Cb^2x^9+\frac{2}{9}Cacx^9+\frac{2}{9}Abcx^9+\frac{1}{8}Bb^2x^8+\frac{1}{4}Bacx^8+\frac{2}{7}Cabx^7+\frac{1}{7}Ab^2x^7+\frac{2}{7}Aacx^7+\frac{1}{3}Babx^6+\frac{1}{5}Ca^2x^5+\frac{2}{5}Aabx^5+\frac{1}{4}Ba^2x^4+\frac{1}{3}Aa^2x^3$$

input
```
integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 2/11*C*b*c*x^11 + 1/11*A*c^2*x^11 + 1/
5*B*b*c*x^10 + 1/9*C*b^2*x^9 + 2/9*C*a*c*x^9 + 2/9*A*b*c*x^9 + 1/8*B*b^2*x
^8 + 1/4*B*a*c*x^8 + 2/7*C*a*b*x^7 + 1/7*A*b^2*x^7 + 2/7*A*a*c*x^7 + 1/3*B
*a*b*x^6 + 1/5*C*a^2*x^5 + 2/5*A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 15.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = x^5\left(\frac{C\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^{11}\left(\frac{A\,c^2}{11}+\frac{2\,C\,b\,c}{11}\right)+x^7\left(\frac{A\,b^2}{7}+\frac{2\,C\,a\,b}{7}+\frac{2\,A\,a\,c}{7}\right)+x^9\left(\frac{C\,b^2}{9}+\frac{2\,A\,c\,b}{9}+\frac{2\,C\,a\,c}{9}\right)+\frac{A\,a^2\,x^3}{3}+\frac{B\,a^2\,x^4}{4}+\frac{B\,c^2\,x^{12}}{12}+\frac{C\,c^2\,x^{13}}{13}+\frac{B\,x^8\,(b^2+2\,a\,c)}{8}+\frac{B\,a\,b\,x^6}{3}+\frac{B\,b\,c\,x^{10}}{5}$$

input
```
int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^5*((C*a^2)/5 + (2*A*a*b)/5) + x^11*((A*c^2)/11 + (2*C*b*c)/11) + x^7*((A
*b^2)/7 + (2*A*a*c)/7 + (2*C*a*b)/7) + x^9*((C*b^2)/9 + (2*A*b*c)/9 + (2*C
*a*c)/9) + (A*a^2*x^3)/3 + (B*a^2*x^4)/4 + (B*c^2*x^12)/12 + (C*c^2*x^13)/
13 + (B*x^8*(2*a*c + b^2))/8 + (B*a*b*x^6)/3 + (B*b*c*x^10)/5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int x^2(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{x^3(27720c^3x^{10}+30030b\,c^2x^9+32760a\,c^2x^8+65520b\,c^2x^8+72072b^2c\,x^7+80080abc\,x^6+80080a\,c^2x^6+$$

input
```
int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

output
```
(x**3*(120120*a**3 + 144144*a**2*b*x**2 + 90090*a**2*b*x + 102960*a**2*c*x
**4 + 72072*a**2*c*x**2 + 51480*a*b**2*x**4 + 120120*a*b**2*x**3 + 80080*a
*b*c*x**6 + 90090*a*b*c*x**5 + 102960*a*b*c*x**4 + 32760*a*c**2*x**8 + 800
80*a*c**2*x**6 + 45045*b**3*x**5 + 72072*b**2*c*x**7 + 40040*b**2*c*x**6 +
 30030*b*c**2*x**9 + 65520*b*c**2*x**8 + 27720*c**3*x**10))/360360
```

3.12 $\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 159

$$\begin{aligned}\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx &= \frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{4}a(2Ab+aC)x^4\\&+\frac{2}{5}abBx^5+\frac{1}{6}\left(A\left(b^2+2ac\right)+2abC\right)x^6\\&+\frac{1}{7}B\left(b^2+2ac\right)x^7\\&+\frac{1}{8}\left(2Abc+\left(b^2+2ac\right)C\right)x^8\\&+\frac{2}{9}bBcx^9+\frac{1}{10}c(Ac+2bC)x^{10}\\&+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}\end{aligned}$$

output

```
1/2*a^2*A*x^2+1/3*a^2*B*x^3+1/4*a*(2*A*b+C*a)*x^4+2/5*a*b*B*x^5+1/6*(A*(2*
a*c+b^2)+2*a*b*C)*x^6+1/7*B*(2*a*c+b^2)*x^7+1/8*(2*A*b*c+(2*a*c+b^2)*C)*x^
8+2/9*b*B*c*x^9+1/10*c*(A*c+2*C*b)*x^10+1/11*B*c^2*x^11+1/12*c^2*C*x^12
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{4}a(2Ab+aC)x^4+\frac{2}{5}abBx^5+\frac{1}{6}\left(Ab^2+2aAc+2abC\right)x^6+\frac{1}{7}B\left(b^2+2ac\right)x^7+\frac{1}{8}\left(2Abc+b^2C+2acC\right)x^8+\frac{2}{9}bBcx^9+\frac{1}{10}c(Ac+2bC)x^{10}+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

input
```
Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5
+ ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*
c + b^2*C + 2*a*c*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10
+ (B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2+cx^4\right)^2\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(a^2Ax+a^2Bx^2+x^7\left(C\left(2ac+b^2\right)+2Abc\right)+x^5\left(A\left(2ac+b^2\right)+2abC\right)+ax^3(aC+2Ab)+Bx^6\left(2ac+b^2\right)+\right.$$

$$\downarrow \text{2009}$$

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{6}x^6\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)+\frac{2}{5}abBx^5+\frac{1}{10}cx^{10}(Ac+2bC)+\frac{2}{9}bBcx^9+\frac{1}{11}Bc^2x^{11}+\frac{1}{12}c^2Cx^{12}$$

input
```
Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5
+ ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*
c + (b^2 + 2*a*c)*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10
+ (B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

method	result
default	$\frac{c^2C\,x^{12}}{12}+\frac{B\,c^2x^{11}}{11}+\frac{(A\,c^2+2Ccb)x^{10}}{10}+\frac{2bBc\,x^9}{9}+\frac{(2Abc+(2ac+b^2)C)x^8}{8}+\frac{B(2ac+b^2)x^7}{7}+\frac{(A(2ac+b^2)+2Cb}{6}$
norman	$\frac{c^2C\,x^{12}}{12}+\frac{B\,c^2x^{11}}{11}+\left(\frac{1}{10}A\,c^2+\frac{1}{5}Ccb\right)x^{10}+\frac{2bBc\,x^9}{9}+\left(\frac{1}{4}Abc+\frac{1}{4}Cac+\frac{1}{8}C\,b^2\right)x^8+\left(\frac{2}{7}Bac+\right.$
gosper	$\frac{1}{12}c^2C\,x^{12}+\frac{1}{11}B\,c^2x^{11}+\frac{1}{10}x^{10}A\,c^2+\frac{1}{5}x^{10}Ccb+\frac{2}{9}bBc\,x^9+\frac{1}{4}x^8Abc+\frac{1}{4}x^8Cac+\frac{1}{8}x^8C\,b^2+$
risch	$\frac{1}{12}c^2C\,x^{12}+\frac{1}{11}B\,c^2x^{11}+\frac{1}{10}x^{10}A\,c^2+\frac{1}{5}x^{10}Ccb+\frac{2}{9}bBc\,x^9+\frac{1}{4}x^8Abc+\frac{1}{4}x^8Cac+\frac{1}{8}x^8C\,b^2+$
parallelrisch	$\frac{1}{12}c^2C\,x^{12}+\frac{1}{11}B\,c^2x^{11}+\frac{1}{10}x^{10}A\,c^2+\frac{1}{5}x^{10}Ccb+\frac{2}{9}bBc\,x^9+\frac{1}{4}x^8Abc+\frac{1}{4}x^8Cac+\frac{1}{8}x^8C\,b^2+$
orering	$\frac{x^2(2310c^2C\,x^{10}+2520B\,c^2x^9+2772A\,c^2x^8+5544Cbc\,x^8+6160bBc\,x^7+6930Abc\,x^6+6930Cac\,x^6+3465C\,b^2x^6+7920Bac\,x^5}{277}$

input
```
int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*c^2*C*x^12+1/11*B*c^2*x^11+1/10*(A*c^2+2*C*b*c)*x^10+2/9*b*B*c*x^9+1/
8*(2*A*b*c+(2*a*c+b^2)*C)*x^8+1/7*B*(2*a*c+b^2)*x^7+1/6*(A*(2*a*c+b^2)+2*C
*b*a)*x^6+2/5*a*b*B*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*x^
2
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{12}\,Cc^2x^{12}+\frac{1}{11}\,Bc^2x^{11} + \frac{2}{9}\,Bbcx^9+\frac{1}{10}\,(2\,Cbc+Ac^2)x^{10} + \frac{1}{8}\,(Cb^2+2\,(Ca+Ab)c)x^8 + \frac{2}{5}\,Babx^5+\frac{1}{7}\,(Bb^2+2\,Bac)x^7 + \frac{1}{6}\,(2\,Cab+Ab^2+2\,Aac)x^6+\frac{1}{3}\,Ba^2x^3 + \frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,(Ca^2+2\,Aab)x^4$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/10*(2*C*b*c + A*c^2)
*x^10 + 1/8*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + 2/5*B*a*b*x^5 + 1/7*(B*b^2 + 2
*B*a*c)*x^7 + 1/6*(2*C*a*b + A*b^2 + 2*A*a*c)*x^6 + 1/3*B*a^2*x^3 + 1/2*A*
a^2*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{2Babx^5}{5}+\frac{2Bbcx^9}{9}+\frac{Bc^2x^{11}}{11}+\frac{Cc^2x^{12}}{12}+x^{10}\left(\frac{Ac^2}{10}+\frac{Cbc}{5}\right)+x^8\left(\frac{Abc}{4}+\frac{Cac}{4}+\frac{Cb^2}{8}\right)+x^7\cdot\left(\frac{2Bac}{7}+\frac{Bb^2}{7}\right)+x^6\left(\frac{Aac}{3}+\frac{Ab^2}{6}+\frac{Cab}{3}\right)+x^4\left(\frac{Aab}{2}+\frac{Ca^2}{4}\right)$$

input
```
integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

output
```
A*a**2*x**2/2 + B*a**2*x**3/3 + 2*B*a*b*x**5/5 + 2*B*b*c*x**9/9 + B*c**2*x
**11/11 + C*c**2*x**12/12 + x**10*(A*c**2/10 + C*b*c/5) + x**8*(A*b*c/4 +
C*a*c/4 + C*b**2/8) + x**7*(2*B*a*c/7 + B*b**2/7) + x**6*(A*a*c/3 + A*b**2
/6 + C*a*b/3) + x**4*(A*a*b/2 + C*a**2/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{12}\,Cc^2x^{12}+\frac{1}{11}\,Bc^2x^{11}+\frac{2}{9}\,Bbcx^9+\frac{1}{10}\,(2\,Cbc+Ac^2)x^{10}+\frac{1}{8}\,(Cb^2+2\,(Ca+Ab)c)x^8+\frac{2}{5}\,Babx^5+\frac{1}{7}\,(Bb^2+2\,Bac)x^7+\frac{1}{6}\,(2\,Cab+Ab^2+2\,Aac)x^6+\frac{1}{3}\,Ba^2x^3+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,(Ca^2+2\,Aab)x^4$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/10*(2*C*b*c + A*c^2)
*x^10 + 1/8*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + 2/5*B*a*b*x^5 + 1/7*(B*b^2 + 2
*B*a*c)*x^7 + 1/6*(2*C*a*b + A*b^2 + 2*A*a*c)*x^6 + 1/3*B*a^2*x^3 + 1/2*A*
a^2*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

Giac [A] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{12}Cc^2x^{12} + \frac{1}{11}Bc^2x^{11} + \frac{1}{5}Cbcx^{10} + \frac{1}{10}Ac^2x^{10} + \frac{2}{9}Bbcx^9 + \frac{1}{8}Cb^2x^8 + \frac{1}{4}Cacx^8 + \frac{1}{4}Abcx^8 + \frac{1}{7}Bb^2x^7 + \frac{2}{7}Bacx^7 + \frac{1}{3}Cabx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{3}Aacx^6 + \frac{2}{5}Babx^5 + \frac{1}{4}Ca^2x^4 + \frac{1}{2}Aabx^4 + \frac{1}{3}Ba^2x^3 + \frac{1}{2}Aa^2x^2$$

input
```
integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 1/5*C*b*c*x^10 + 1/10*A*c^2*x^10 + 2/9
*B*b*c*x^9 + 1/8*C*b^2*x^8 + 1/4*C*a*c*x^8 + 1/4*A*b*c*x^8 + 1/7*B*b^2*x^7
 + 2/7*B*a*c*x^7 + 1/3*C*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*A*a*c*x^6 + 2/5*B*a
*b*x^5 + 1/4*C*a^2*x^4 + 1/2*A*a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = x^4\left(\frac{C\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^{10}\left(\frac{A\,c^2}{10}+\frac{C\,b\,c}{5}\right)+x^6\left(\frac{A\,b^2}{6}+\frac{C\,a\,b}{3}+\frac{A\,a\,c}{3}\right)+x^8\left(\frac{C\,b^2}{8}+\frac{A\,c\,b}{4}+\frac{C\,a\,c}{4}\right)+\frac{A\,a^2\,x^2}{2}+\frac{B\,a^2\,x^3}{3}+\frac{B\,c^2\,x^{11}}{11}+\frac{C\,c^2\,x^{12}}{12}+\frac{B\,x^7\,(b^2+2\,a\,c)}{7}+\frac{2\,B\,a\,b\,x^5}{5}+\frac{2\,B\,b\,c\,x^9}{9}$$

input
```
int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^4*((C*a^2)/4 + (A*a*b)/2) + x^10*((A*c^2)/10 + (C*b*c)/5) + x^6*((A*b^2)
/6 + (A*a*c)/3 + (C*a*b)/3) + x^8*((C*b^2)/8 + (A*b*c)/4 + (C*a*c)/4) + (A
*a^2*x^2)/2 + (B*a^2*x^3)/3 + (B*c^2*x^11)/11 + (C*c^2*x^12)/12 + (B*x^7*(
2*a*c + b^2))/7 + (2*B*a*b*x^5)/5 + (2*B*b*c*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int x(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{x^2(2310c^3x^{10}+2520b\,c^2x^9+2772a\,c^2x^8+5544b\,c^2x^8+6160b^2c\,x^7+6930abc\,x^6+6930a\,c^2x^6+3465b^2c$$

input
```
int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

output
```
(x**2*(13860*a**3 + 13860*a**2*b*x**2 + 9240*a**2*b*x + 9240*a**2*c*x**4 +
 6930*a**2*c*x**2 + 4620*a*b**2*x**4 + 11088*a*b**2*x**3 + 6930*a*b*c*x**6
 + 7920*a*b*c*x**5 + 9240*a*b*c*x**4 + 2772*a*c**2*x**8 + 6930*a*c**2*x**6
 + 3960*b**3*x**5 + 6160*b**2*c*x**7 + 3465*b**2*c*x**6 + 2520*b*c**2*x**9
 + 5544*b*c**2*x**8 + 2310*c**3*x**10))/27720
```

3.13 $\int (A+Bx+Cx^2)(a+bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 154

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{2}abBx^4+\frac{1}{5}\left(A\left(b^2+2ac\right)+2abC\right)x^5+\frac{1}{6}B\left(b^2+2ac\right)x^6+\frac{1}{7}\left(2Abc+\left(b^2+2ac\right)C\right)x^7+\frac{1}{4}bBcx^8+\frac{1}{9}c(Ac+2bC)x^9+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

output
```
a^2*A*x+1/2*a^2*B*x^2+1/3*a*(2*A*b+C*a)*x^3+1/2*a*b*B*x^4+1/5*(A*(2*a*c+b^
2)+2*a*b*C)*x^5+1/6*B*(2*a*c+b^2)*x^6+1/7*(2*A*b*c+(2*a*c+b^2)*C)*x^7+1/4*
b*B*c*x^8+1/9*c*(A*c+2*C*b)*x^9+1/10*B*c^2*x^10+1/11*c^2*C*x^11
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx = a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{2}abBx^4+\frac{1}{5}\left(Ab^2+2aAc+2abC\right)x^5+\frac{1}{6}B\left(b^2+2ac\right)x^6+\frac{1}{7}\left(2Abc+b^2C+2acC\right)x^7+\frac{1}{4}bBcx^8+\frac{1}{9}c(Ac+2bC)x^9+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

input

```
Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output

```
a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*b^
2 + 2*a*A*c + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + b^2*
C + 2*a*c*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^1
0)/10 + (c^2*C*x^11)/11
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^2\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(a^2A+a^2Bx+x^6\left(C\left(2ac+b^2\right)+2Abc\right)+x^4\left(A\left(2ac+b^2\right)+2abC\right)+ax^2(aC+2Ab)+Bx^5\left(2ac+b^2\right)+2a\right.$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{5}x^5\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+\frac{1}{2}abBx^4+\frac{1}{9}cx^9(Ac+2bC)+\frac{1}{4}bBcx^8+\frac{1}{10}Bc^2x^{10}+\frac{1}{11}c^2Cx^{11}$$

input
```
Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*(b
^2 + 2*a*c) + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + (b^2
 + 2*a*c)*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^1
0)/10 + (c^2*C*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^2Cx^{11}}{11}+\frac{Bc^2x^{10}}{10}+\frac{(Ac^2+2Ccb)x^9}{9}+\frac{bBcx^8}{4}+\frac{(2Abc+(2ac+b^2)C)x^7}{7}+\frac{B(2ac+b^2)x^6}{6}+\frac{(A(2ac+b^2)+2Cba)}{5}$
norman	$\frac{c^2Cx^{11}}{11}+\frac{Bc^2x^{10}}{10}+\left(\frac{1}{9}Ac^2+\frac{2}{9}Ccb\right)x^9+\frac{bBcx^8}{4}+\left(\frac{2}{7}Abc+\frac{2}{7}Cac+\frac{1}{7}Cb^2\right)x^7+\left(\frac{1}{3}Bac+\frac{1}{6}B\right.$
gosper	$\frac{1}{11}c^2Cx^{11}+\frac{1}{10}Bc^2x^{10}+\frac{1}{9}x^9Ac^2+\frac{2}{9}x^9Ccb+\frac{1}{4}bBcx^8+\frac{2}{7}x^7Abc+\frac{2}{7}x^7Cac+\frac{1}{7}x^7Cb^2+\frac{1}{3}$
risch	$\frac{1}{11}c^2Cx^{11}+\frac{1}{10}Bc^2x^{10}+\frac{1}{9}x^9Ac^2+\frac{2}{9}x^9Ccb+\frac{1}{4}bBcx^8+\frac{2}{7}x^7Abc+\frac{2}{7}x^7Cac+\frac{1}{7}x^7Cb^2+\frac{1}{3}$
parallelrisch	$\frac{1}{11}c^2Cx^{11}+\frac{1}{10}Bc^2x^{10}+\frac{1}{9}x^9Ac^2+\frac{2}{9}x^9Ccb+\frac{1}{4}bBcx^8+\frac{2}{7}x^7Abc+\frac{2}{7}x^7Cac+\frac{1}{7}x^7Cb^2+\frac{1}{3}$
orering	$\frac{x(1260c^2Cx^{10}+1386Bc^2x^9+1540Ac^2x^8+3080Cbcx^8+3465bBcx^7+3960Abcx^6+3960Cacx^6+1980Cb^2x^6+4620Bacx^5+}{13860}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*c^2*C*x^11+1/10*B*c^2*x^10+1/9*(A*c^2+2*C*b*c)*x^9+1/4*b*B*c*x^8+1/7*
(2*A*b*c+(2*a*c+b^2)*C)*x^7+1/6*B*(2*a*c+b^2)*x^6+1/5*(A*(2*a*c+b^2)+2*C*b
*a)*x^5+1/2*B*a*b*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{11}Cc^2x^{11}+\frac{1}{10}Bc^2x^{10} + \frac{1}{4}Bbcx^8+\frac{1}{9}\left(2\,Cbc+Ac^2\right)x^9 + \frac{1}{7}\left(Cb^2+2\,(Ca+Ab)c\right)x^7 + \frac{1}{2}Babx^4+\frac{1}{6}\left(Bb^2+2\,Bac\right)x^6 + \frac{1}{5}\left(2\,Cab+Ab^2+2\,Aac\right)x^5+\frac{1}{2}Ba^2x^2 + Aa^2x+\frac{1}{3}\left(Ca^2+2\,Aab\right)x^3$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/9*(2*C*b*c + A*c^2)*
x^9 + 1/7*(C*b^2 + 2*(C*a + A*b)*c)*x^7 + 1/2*B*a*b*x^4 + 1/6*(B*b^2 + 2*B
*a*c)*x^6 + 1/5*(2*C*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/2*B*a^2*x^2 + A*a^2*x
+ 1/3*(C*a^2 + 2*A*a*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.07

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = Aa^2x+\frac{Ba^2x^2}{2}+\frac{Babx^4}{2}+\frac{Bbcx^8}{4}+\frac{Bc^2x^{10}}{10}+\frac{Cc^2x^{11}}{11}+x^9\left(\frac{Ac^2}{9}+\frac{2Cbc}{9}\right)+x^7\cdot\left(\frac{2Abc}{7}+\frac{2Cac}{7}+\frac{Cb^2}{7}\right)+x^6\left(\frac{Bac}{3}+\frac{Bb^2}{6}\right)+x^5\cdot\left(\frac{2Aac}{5}+\frac{Ab^2}{5}+\frac{2Cab}{5}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ca^2}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

output
```
A*a**2*x + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b*c*x**8/4 + B*c**2*x**10/10 +
 C*c**2*x**11/11 + x**9*(A*c**2/9 + 2*C*b*c/9) + x**7*(2*A*b*c/7 + 2*C*a*c
/7 + C*b**2/7) + x**6*(B*a*c/3 + B*b**2/6) + x**5*(2*A*a*c/5 + A*b**2/5 +
2*C*a*b/5) + x**3*(2*A*a*b/3 + C*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{11}\,Cc^2x^{11}+\frac{1}{10}\,Bc^2x^{10}+\frac{1}{4}\,Bbcx^8+\frac{1}{9}\,(2\,Cbc+Ac^2)x^9+\frac{1}{7}\,(Cb^2+2\,(Ca+Ab)c)x^7+\frac{1}{2}\,Babx^4+\frac{1}{6}\,(Bb^2+2\,Bac)x^6+\frac{1}{5}\,(2\,Cab+Ab^2+2\,Aac)x^5+\frac{1}{2}\,Ba^2x^2+Aa^2x+\frac{1}{3}\,(Ca^2+2\,Aab)x^3$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/9*(2*C*b*c + A*c^2)*
x^9 + 1/7*(C*b^2 + 2*(C*a + A*b)*c)*x^7 + 1/2*B*a*b*x^4 + 1/6*(B*b^2 + 2*B
*a*c)*x^6 + 1/5*(2*C*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/2*B*a^2*x^2 + A*a^2*x
+ 1/3*(C*a^2 + 2*A*a*b)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx = {} & \frac{1}{11}Cc^2x^{11}+\frac{1}{10}Bc^2x^{10}+\frac{2}{9}Cbcx^9 \\
& +\frac{1}{9}Ac^2x^9+\frac{1}{4}Bbcx^8+\frac{1}{7}Cb^2x^7+\frac{2}{7}Cacx^7 \\
& +\frac{2}{7}Abcx^7+\frac{1}{6}Bb^2x^6+\frac{1}{3}Bacx^6+\frac{2}{5}Cabx^5 \\
& +\frac{1}{5}Ab^2x^5+\frac{2}{5}Aacx^5+\frac{1}{2}Babx^4 \\
& +\frac{1}{3}Ca^2x^3+\frac{2}{3}Aabx^3+\frac{1}{2}Ba^2x^2+Aa^2x
\end{aligned}
$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 2/9*C*b*c*x^9 + 1/9*A*c^2*x^9 + 1/4*B*
b*c*x^8 + 1/7*C*b^2*x^7 + 2/7*C*a*c*x^7 + 2/7*A*b*c*x^7 + 1/6*B*b^2*x^6 +
1/3*B*a*c*x^6 + 2/5*C*a*b*x^5 + 1/5*A*b^2*x^5 + 2/5*A*a*c*x^5 + 1/2*B*a*b*
x^4 + 1/3*C*a^2*x^3 + 2/3*A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = x^3\left(\frac{C\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^9\left(\frac{A\,c^2}{9}+\frac{2\,C\,b\,c}{9}\right)+x^5\left(\frac{A\,b^2}{5}+\frac{2\,C\,a\,b}{5}+\frac{2\,A\,a\,c}{5}\right)+x^7\left(\frac{C\,b^2}{7}+\frac{2\,A\,c\,b}{7}+\frac{2\,C\,a\,c}{7}\right)+\frac{B\,a^2\,x^2}{2}+\frac{B\,c^2\,x^{10}}{10}+\frac{C\,c^2\,x^{11}}{11}+\frac{B\,x^6\,(b^2+2\,a\,c)}{6}+A\,a^2\,x+\frac{B\,a\,b\,x^4}{2}+\frac{B\,b\,c\,x^8}{4}$$

input
```
int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^3*((C*a^2)/3 + (2*A*a*b)/3) + x^9*((A*c^2)/9 + (2*C*b*c)/9) + x^5*((A*b^
2)/5 + (2*A*a*c)/5 + (2*C*a*b)/5) + x^7*((C*b^2)/7 + (2*A*b*c)/7 + (2*C*a*
c)/7) + (B*a^2*x^2)/2 + (B*c^2*x^10)/10 + (C*c^2*x^11)/11 + (B*x^6*(2*a*c
+ b^2))/6 + A*a^2*x + (B*a*b*x^4)/2 + (B*b*c*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01

$$\int (A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2\,dx = \frac{x(1260c^3x^{10}+1386b\,c^2x^9+1540a\,c^2x^8+3080b\,c^2x^8+3465b^2c\,x^7+3960abc\,x^6+3960a\,c^2x^6+1980b^2c}{}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

output
```
(x*(13860*a**3 + 9240*a**2*b*x**2 + 6930*a**2*b*x + 5544*a**2*c*x**4 + 462
0*a**2*c*x**2 + 2772*a*b**2*x**4 + 6930*a*b**2*x**3 + 3960*a*b*c*x**6 + 46
20*a*b*c*x**5 + 5544*a*b*c*x**4 + 1540*a*c**2*x**8 + 3960*a*c**2*x**6 + 23
10*b**3*x**5 + 3465*b**2*c*x**7 + 1980*b**2*c*x**6 + 1386*b*c**2*x**9 + 30
80*b*c**2*x**8 + 1260*c**3*x**10))/13860
```

3.14 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 150

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x} \, dx = {} & a^2Bx+\frac{1}{2}a(2Ab+aC)x^2+\frac{2}{3}abBx^3 \\ & +\frac{1}{4}\left(A\left(b^2+2ac\right)+2abC\right)x^4 \\ & +\frac{1}{5}B\left(b^2+2ac\right)x^5 \\ & +\frac{1}{6}\left(2Abc+\left(b^2+2ac\right)C\right)x^6 \\ & +\frac{2}{7}bBcx^7+\frac{1}{8}c(Ac+2bC)x^8 \\ & +\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}+a^2A\log(x)\end{aligned}$$

output
```
a^2*B*x+1/2*a*(2*A*b+C*a)*x^2+2/3*a*b*B*x^3+1/4*(A*(2*a*c+b^2)+2*a*b*C)*x^
4+1/5*B*(2*a*c+b^2)*x^5+1/6*(2*A*b*c+(2*a*c+b^2)*C)*x^6+2/7*b*B*c*x^7+1/8*
c*(A*c+2*C*b)*x^8+1/9*B*c^2*x^9+1/10*c^2*C*x^10+a^2*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x}\,dx = a^2Bx+\frac{1}{2}a(2Ab+aC)x^2+\frac{2}{3}abBx^3 +\frac{1}{4}\left(Ab^2+2aAc+2abC\right)x^4+\frac{1}{5}B\left(b^2+2ac\right)x^5 +\frac{1}{6}\left(2Abc+b^2C+2acC\right)x^6 +\frac{2}{7}bBcx^7+\frac{1}{8}c(Ac+2bC)x^8 +\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}+a^2A\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

output
```
a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*b^2 + 2*a*A*c +
2*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + b^2*C + 2*a*c*C)*x
^6)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C
*x^10)/10 + a^2*A*Log[x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^2\left(A+Bx+Cx^2\right)}{x}\,dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(\frac{a^2A}{x}+a^2B+x^5\left(C\left(2ac+b^2\right)+2Abc\right)+x^3\left(A\left(2ac+b^2\right)+2abC\right)+ax(aC+2Ab)+Bx^4\left(2ac+b^2\right)+2a\right.$$

$$\Big\downarrow \text{ 2009}$$

$$a^2A\log(x)+a^2Bx+\frac{1}{6}x^6\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{4}x^4\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)+\frac{2}{3}abBx^3+\frac{1}{8}cx^8(Ac+2bC)+\frac{2}{7}bBcx^7+\frac{1}{9}Bc^2x^9+\frac{1}{10}c^2Cx^{10}$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

output
```
a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*(b^2 + 2*a*c) +
2*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x
^6)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C
*x^10)/10 + a^2*A*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

method	result
norman	$\left(\frac{1}{8}A\,c^2+\frac{1}{4}Ccb\right)x^8+\left(Aab+\frac{1}{2}a^2C\right)x^2+\left(\frac{2}{5}Bac+\frac{1}{5}B\,b^2\right)x^5+\left(\frac{1}{2}Aac+\frac{1}{4}A\,b^2+\frac{1}{2}Cba\right)x^4$
default	$\frac{c^2C\,x^{10}}{10}+\frac{B\,c^2x^9}{9}+\frac{A\,c^2x^8}{8}+\frac{Cbc\,x^8}{4}+\frac{2bBc\,x^7}{7}+\frac{Abc\,x^6}{3}+\frac{Cac\,x^6}{3}+\frac{C\,b^2x^6}{6}+\frac{2Bac\,x^5}{5}+\frac{B\,b^2x^5}{5}+\frac{Aac}{2}$
risch	$\frac{c^2C\,x^{10}}{10}+\frac{B\,c^2x^9}{9}+\frac{A\,c^2x^8}{8}+\frac{Cbc\,x^8}{4}+\frac{2bBc\,x^7}{7}+\frac{Abc\,x^6}{3}+\frac{Cac\,x^6}{3}+\frac{C\,b^2x^6}{6}+\frac{2Bac\,x^5}{5}+\frac{B\,b^2x^5}{5}+\frac{Aac}{2}$
parallelrisch	$\frac{c^2C\,x^{10}}{10}+\frac{B\,c^2x^9}{9}+\frac{A\,c^2x^8}{8}+\frac{Cbc\,x^8}{4}+\frac{2bBc\,x^7}{7}+\frac{Abc\,x^6}{3}+\frac{Cac\,x^6}{3}+\frac{C\,b^2x^6}{6}+\frac{2Bac\,x^5}{5}+\frac{B\,b^2x^5}{5}+\frac{Aac}{2}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(1/8*A*c^2+1/4*C*c*b)*x^8+(A*a*b+1/2*a^2*C)*x^2+(2/5*B*a*c+1/5*B*b^2)*x^5+
(1/2*A*a*c+1/4*A*b^2+1/2*C*b*a)*x^4+(1/3*A*b*c+1/3*C*a*c+1/6*C*b^2)*x^6+B*
a^2*x+1/9*B*c^2*x^9+1/10*c^2*C*x^10+2/3*B*a*b*x^3+2/7*b*B*c*x^7+a^2*A*ln(x
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}\,Cc^2x^{10} + \frac{1}{9}\,Bc^2x^9 + \frac{2}{7}\,Bbcx^7 + \frac{1}{8}\,(2\,Cbc + Ac^2)x^8 + \frac{1}{6}\,(Cb^2 + 2\,(Ca + Ab)c)x^6 + \frac{2}{3}\,Babx^3 + \frac{1}{5}\,(Bb^2 + 2\,Bac)x^5 + \frac{1}{4}\,(2\,Cab + Ab^2 + 2\,Aac)x^4 + Ba^2x + Aa^2\log(x) + \frac{1}{2}\,(Ca^2 + 2\,Aab)x^2$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")
```

output
```
1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^
8 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a
*c)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1
/2*(C*a^2 + 2*A*a*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx = Aa^2\log(x)+Ba^2x+\frac{2Babx^3}{3}+\frac{2Bbcx^7}{7}+\frac{Bc^2x^9}{9}+\frac{Cc^2x^{10}}{10}+x^8\left(\frac{Ac^2}{8}+\frac{Cbc}{4}\right)+x^6\left(\frac{Abc}{3}+\frac{Cac}{3}+\frac{Cb^2}{6}\right)+x^5\cdot\left(\frac{2Bac}{5}+\frac{Bb^2}{5}\right)+x^4\left(\frac{Aac}{2}+\frac{Ab^2}{4}+\frac{Cab}{2}\right)+x^2\left(Aab+\frac{Ca^2}{2}\right)$$

input

```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x,x)
```

output

```
A*a**2*log(x) + B*a**2*x + 2*B*a*b*x**3/3 + 2*B*b*c*x**7/7 + B*c**2*x**9/9
 + C*c**2*x**10/10 + x**8*(A*c**2/8 + C*b*c/4) + x**6*(A*b*c/3 + C*a*c/3 +
 C*b**2/6) + x**5*(2*B*a*c/5 + B*b**2/5) + x**4*(A*a*c/2 + A*b**2/4 + C*a*
b/2) + x**2*(A*a*b + C*a**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}Cc^2x^{10}+\frac{1}{9}Bc^2x^9+\frac{2}{7}Bbcx^7+\frac{1}{8}\left(2\,Cbc+Ac^2\right)x^8+\frac{1}{6}\left(Cb^2+2\,(Ca+Ab)c\right)x^6+\frac{2}{3}Babx^3+\frac{1}{5}\left(Bb^2+2\,Bac\right)x^5+\frac{1}{4}\left(2\,Cab+Ab^2+2\,Aac\right)x^4+Ba^2x+Aa^2\log(x)+\frac{1}{2}\left(Ca^2+2\,Aab\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")
```

output
```
1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^
8 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a
*c)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1
/2*(C*a^2 + 2*A*a*b)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}\,Cc^2x^{10} + \frac{1}{9}\,Bc^2x^9 + \frac{1}{4}\,Cbcx^8 + \frac{1}{8}\,Ac^2x^8 + \frac{2}{7}\,Bbcx^7 + \frac{1}{6}\,Cb^2x^6 + \frac{1}{3}\,Cacx^6 + \frac{1}{3}\,Abcx^6 + \frac{1}{5}\,Bb^2x^5 + \frac{2}{5}\,Bacx^5 + \frac{1}{2}\,Cabx^4 + \frac{1}{4}\,Ab^2x^4 + \frac{1}{2}\,Aacx^4 + \frac{2}{3}\,Babx^3 + \frac{1}{2}\,Ca^2x^2 + Aabx^2 + Ba^2x + Aa^2\log(|x|)$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="giac")
```

output
```
1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 1/4*C*b*c*x^8 + 1/8*A*c^2*x^8 + 2/7*B*b*
c*x^7 + 1/6*C*b^2*x^6 + 1/3*C*a*c*x^6 + 1/3*A*b*c*x^6 + 1/5*B*b^2*x^5 + 2/
5*B*a*c*x^5 + 1/2*C*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*A*a*c*x^4 + 2/3*B*a*b*x^
3 + 1/2*C*a^2*x^2 + A*a*b*x^2 + B*a^2*x + A*a^2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 11.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x}\,dx = x^2\left(\frac{C\,a^2}{2}+A\,b\,a\right)+x^8\left(\frac{A\,c^2}{8}+\frac{C\,b\,c}{4}\right)+x^4\left(\frac{A\,b^2}{4}+\frac{C\,a\,b}{2}+\frac{A\,a\,c}{2}\right)+x^6\left(\frac{C\,b^2}{6}+\frac{A\,c\,b}{3}+\frac{C\,a\,c}{3}\right)+\frac{B\,c^2\,x^9}{9}+\frac{C\,c^2\,x^{10}}{10}+A\,a^2\,\ln(x)+\frac{B\,x^5\,(b^2+2\,a\,c)}{5}+B\,a^2\,x+\frac{2\,B\,a\,b\,x^3}{3}+\frac{2\,B\,b\,c\,x^7}{7}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x)
```

output
```
x^2*((C*a^2)/2 + A*a*b) + x^8*((A*c^2)/8 + (C*b*c)/4) + x^4*((A*b^2)/4 + (
A*a*c)/2 + (C*a*b)/2) + x^6*((C*b^2)/6 + (A*b*c)/3 + (C*a*c)/3) + (B*c^2*x
^9)/9 + (C*c^2*x^10)/10 + A*a^2*log(x) + (B*x^5*(2*a*c + b^2))/5 + B*a^2*x
 + (2*B*a*b*x^3)/3 + (2*B*b*c*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x}\,dx = \log(x)\,a^3+a^2b\,x^2+a^2bx+\frac{a^2c\,x^4}{2}+\frac{a^2c\,x^2}{2}+\frac{a\,b^2x^4}{4}+\frac{2a\,b^2x^3}{3}+\frac{abc\,x^6}{3}+\frac{2abc\,x^5}{5}+\frac{abc\,x^4}{2}+\frac{a\,c^2x^8}{8}+\frac{a\,c^2x^6}{3}+\frac{b^3x^5}{5}+\frac{2b^2c\,x^7}{7}+\frac{b^2c\,x^6}{6}+\frac{b\,c^2x^9}{9}+\frac{b\,c^2x^8}{4}+\frac{c^3x^{10}}{10}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x)
```

output
```
(2520*log(x)*a**3 + 2520*a**2*b*x**2 + 2520*a**2*b*x + 1260*a**2*c*x**4 +
1260*a**2*c*x**2 + 630*a*b**2*x**4 + 1680*a*b**2*x**3 + 840*a*b*c*x**6 + 1
008*a*b*c*x**5 + 1260*a*b*c*x**4 + 315*a*c**2*x**8 + 840*a*c**2*x**6 + 504
*b**3*x**5 + 720*b**2*c*x**7 + 420*b**2*c*x**6 + 280*b*c**2*x**9 + 630*b*c
**2*x**8 + 252*c**3*x**10)/2520
```

3.15 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 145

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = & -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2 \\ & +\frac{1}{3}\left(A\left(b^2+2ac\right)+2abC\right)x^3 \\ & +\frac{1}{4}B\left(b^2+2ac\right)x^4 \\ & +\frac{1}{5}\left(2Abc+\left(b^2+2ac\right)C\right)x^5 \\ & +\frac{1}{3}bBcx^6+\frac{1}{7}c(Ac+2bC)x^7 \\ & +\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9+a^2B\log(x)\end{aligned}$$

output
```
-a^2*A/x+a*(2*A*b+C*a)*x+a*b*B*x^2+1/3*(A*(2*a*c+b^2)+2*a*b*C)*x^3+1/4*B*(
2*a*c+b^2)*x^4+1/5*(2*A*b*c+(2*a*c+b^2)*C)*x^5+1/3*b*B*c*x^6+1/7*c*(A*c+2*
C*b)*x^7+1/8*B*c^2*x^8+1/9*c^2*C*x^9+a^2*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^2}\,dx = -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2 +\frac{1}{3}\left(Ab^2+2aAc+2abC\right)x^3 +\frac{1}{4}B\left(b^2+2ac\right)x^4+\frac{1}{5}\left(2Abc+b^2C+2acC\right)x^5 +\frac{1}{3}bBcx^6+\frac{1}{7}c(Ac+2bC)x^7 +\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9+a^2B\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

output
```
-((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*b^2 + 2*a*A*c + 2*a*b*C
)*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^5)/5 +
 (b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 +
 a^2*B*Log[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^2(A+Bx+Cx^2)}{x^2}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^2A}{x^2}+\frac{a^2B}{x}+x^4\left(C\left(2ac+b^2\right)+2Abc\right)+x^2\left(A\left(2ac+b^2\right)+2abC\right)+a(aC+2Ab)+Bx^3\left(2ac+b^2\right)+2ab\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{3}x^3\left(A\left(2ac+b^2\right)+2abC\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+abBx^2+\frac{1}{7}cx^7(Ac+2bC)+\frac{1}{3}bBcx^6+\frac{1}{8}Bc^2x^8+\frac{1}{9}c^2Cx^9$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

output
```
-((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*(b^2 + 2*a*c) + 2*a*b*C
)*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^5)/5 +
 (b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 +
 a^2*B*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

method	result
norman	$\frac{\left(\frac{1}{7}A\,c^2+\frac{2}{7}Ccb\right)x^8+\left(\frac{1}{2}Bac+\frac{1}{4}B\,b^2\right)x^5+\left(\frac{2}{3}Aac+\frac{1}{3}A\,b^2+\frac{2}{3}Cba\right)x^4+\left(\frac{2}{5}Abc+\frac{2}{5}Cac+\frac{1}{5}C\,b^2\right)x^6+\left(2Aab+a^2C\right)x^2+Bab\,x^3-A}{x}$
default	$\frac{c^2C\,x^9}{9}+\frac{B\,c^2x^8}{8}+\frac{A\,c^2x^7}{7}+\frac{2Cbc\,x^7}{7}+\frac{bBc\,x^6}{3}+\frac{2Abc\,x^5}{5}+\frac{2Cac\,x^5}{5}+\frac{C\,b^2x^5}{5}+\frac{Bac\,x^4}{2}+\frac{B\,b^2x^4}{4}+\frac{2Aa}{3}$
risch	$\frac{c^2C\,x^9}{9}+\frac{B\,c^2x^8}{8}+\frac{A\,c^2x^7}{7}+\frac{2Cbc\,x^7}{7}+\frac{bBc\,x^6}{3}+\frac{2Abc\,x^5}{5}+\frac{2Cac\,x^5}{5}+\frac{C\,b^2x^5}{5}+\frac{Bac\,x^4}{2}+\frac{B\,b^2x^4}{4}+\frac{2Aa}{3}$
parallelrisch	$\frac{280c^2C\,x^{10}+315B\,c^2x^9+360A\,c^2x^8+720Cbc\,x^8+840bBc\,x^7+1008Abc\,x^6+1008Cac\,x^6+504C\,b^2x^6+1260Bac\,x^5+630B\,b^2x}{2520x}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
((1/7*A*c^2+2/7*C*c*b)*x^8+(1/2*B*a*c+1/4*B*b^2)*x^5+(2/3*A*a*c+1/3*A*b^2+
2/3*C*b*a)*x^4+(2/5*A*b*c+2/5*C*a*c+1/5*C*b^2)*x^6+(2*A*a*b+C*a^2)*x^2+B*a
*b*x^3-A*a^2+1/8*B*c^2*x^9+1/9*c^2*C*x^10+1/3*b*B*c*x^7)/x+a^2*B*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = \frac{280\,Cc^2x^{10}+315\,Bc^2x^9+840\,Bbcx^7+360\,(2\,Cbc+Ac^2)x^8+504\,(Cb^2+2\,(Ca+Ab)c)x^6+2520\,Bab}{}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/2520*(280*C*c^2*x^10 + 315*B*c^2*x^9 + 840*B*b*c*x^7 + 360*(2*C*b*c + A*
c^2)*x^8 + 504*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2520*B*a*b*x^3 + 630*(B*b^2
 + 2*B*a*c)*x^5 + 840*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 2520*B*a^2*x*log(x
) - 2520*A*a^2 + 2520*(C*a^2 + 2*A*a*b)*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = -\frac{Aa^2}{x}+Ba^2\log{(x)}+Babx^2+\frac{Bbcx^6}{3}+\frac{Bc^2x^8}{8}+\frac{Cc^2x^9}{9}+x^7\left(\frac{Ac^2}{7}+\frac{2Cbc}{7}\right)+x^5\cdot\left(\frac{2Abc}{5}+\frac{2Cac}{5}+\frac{Cb^2}{5}\right)+x^4\left(\frac{Bac}{2}+\frac{Bb^2}{4}\right)+x^3\cdot\left(\frac{2Aac}{3}+\frac{Ab^2}{3}+\frac{2Cab}{3}\right)+x\left(2Aab+Ca^2\right)$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**2,x)
```

output
```
-A*a**2/x + B*a**2*log(x) + B*a*b*x**2 + B*b*c*x**6/3 + B*c**2*x**8/8 + C*
c**2*x**9/9 + x**7*(A*c**2/7 + 2*C*b*c/7) + x**5*(2*A*b*c/5 + 2*C*a*c/5 +
C*b**2/5) + x**4*(B*a*c/2 + B*b**2/4) + x**3*(2*A*a*c/3 + A*b**2/3 + 2*C*a
*b/3) + x*(2*A*a*b + C*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{9}\,Cc^2x^9+\frac{1}{8}\,Bc^2x^8+\frac{1}{3}\,Bbcx^6+\frac{1}{7}\,(2\,Cbc+Ac^2)x^7+\frac{1}{5}\,(Cb^2+2\,(Ca+Ab)c)x^5+Babx^2+\frac{1}{4}\,(Bb^2+2\,Bac)x^4+\frac{1}{3}\,(2\,Cab+Ab^2+2\,Aac)x^3+Ba^2\log(x)-\frac{Aa^2}{x}+(Ca^2+2\,Aab)x$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")
```

output
```
1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/7*(2*C*b*c + A*c^2)*x^7
+ 1/5*(C*b^2 + 2*(C*a + A*b)*c)*x^5 + B*a*b*x^2 + 1/4*(B*b^2 + 2*B*a*c)*x^
4 + 1/3*(2*C*a*b + A*b^2 + 2*A*a*c)*x^3 + B*a^2*log(x) - A*a^2/x + (C*a^2
+ 2*A*a*b)*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{9}\,Cc^2x^9+\frac{1}{8}\,Bc^2x^8+\frac{2}{7}\,Cbcx^7+\frac{1}{7}\,Ac^2x^7+\frac{1}{3}\,Bbcx^6+\frac{1}{5}\,Cb^2x^5+\frac{2}{5}\,Cacx^5+\frac{2}{5}\,Abcx^5+\frac{1}{4}\,Bb^2x^4+\frac{1}{2}\,Bacx^4+\frac{2}{3}\,Cabx^3+\frac{1}{3}\,Ab^2x^3+\frac{2}{3}\,Aacx^3+Babx^2+Ca^2x+2\,Aabx+Ba^2\log(|x|)-\frac{Aa^2}{x}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")
```

output
```
1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 2/7*C*b*c*x^7 + 1/7*A*c^2*x^7 + 1/3*B*b*c*
x^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 2/5*A*b*c*x^5 + 1/4*B*b^2*x^4 + 1/2*
B*a*c*x^4 + 2/3*C*a*b*x^3 + 1/3*A*b^2*x^3 + 2/3*A*a*c*x^3 + B*a*b*x^2 + C*
a^2*x + 2*A*a*b*x + B*a^2*log(abs(x)) - A*a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^2}\,dx = x^7\left(\frac{A\,c^2}{7}+\frac{2\,C\,b\,c}{7}\right) + x^3\left(\frac{A\,b^2}{3}+\frac{2\,C\,a\,b}{3}+\frac{2\,A\,a\,c}{3}\right) + x^5\left(\frac{C\,b^2}{5}+\frac{2\,A\,c\,b}{5}+\frac{2\,C\,a\,c}{5}\right) + x\left(C\,a^2+2\,A\,b\,a\right) - \frac{A\,a^2}{x} + \frac{B\,c^2\,x^8}{8} + \frac{C\,c^2\,x^9}{9} + B\,a^2\,\ln(x) + \frac{B\,x^4\,(b^2+2\,a\,c)}{4} + B\,a\,b\,x^2 + \frac{B\,b\,c\,x^6}{3}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x)
```

output
```
x^7*((A*c^2)/7 + (2*C*b*c)/7) + x^3*((A*b^2)/3 + (2*A*a*c)/3 + (2*C*a*b)/3
) + x^5*((C*b^2)/5 + (2*A*b*c)/5 + (2*C*a*c)/5) + x*(C*a^2 + 2*A*a*b) - (A
*a^2)/x + (B*c^2*x^8)/8 + (C*c^2*x^9)/9 + B*a^2*log(x) + (B*x^4*(2*a*c + b
^2))/4 + B*a*b*x^2 + (B*b*c*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^2}\,dx$$
$$= \frac{2520\log(x)\,a^2bx-2520a^3+5040a^2b\,x^2+1680a^2c\,x^4+2520a^2c\,x^2+840a\,b^2x^4+2520a\,b^2x^3+1008abc$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x)
```

output
```
(2520*log(x)*a**2*b*x - 2520*a**3 + 5040*a**2*b*x**2 + 1680*a**2*c*x**4 +
2520*a**2*c*x**2 + 840*a*b**2*x**4 + 2520*a*b**2*x**3 + 1008*a*b*c*x**6 +
1260*a*b*c*x**5 + 1680*a*b*c*x**4 + 360*a*c**2*x**8 + 1008*a*c**2*x**6 + 6
30*b**3*x**5 + 840*b**2*c*x**7 + 504*b**2*c*x**6 + 315*b*c**2*x**9 + 720*b
*c**2*x**8 + 280*c**3*x**10)/(2520*x)
```

3.16 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 149

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx = &-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+2abBx\\&+\frac{1}{2}\left(A\left(b^2+2ac\right)+2abC\right)x^2\\&+\frac{1}{3}B\left(b^2+2ac\right)x^3\\&+\frac{1}{4}\left(2Abc+\left(b^2+2ac\right)C\right)x^4\\&+\frac{2}{5}bBcx^5+\frac{1}{6}c(Ac+2bC)x^6+\frac{1}{7}Bc^2x^7\\&+\frac{1}{8}c^2Cx^8+a(2Ab+aC)\log(x)\end{aligned}$$

output
```
-1/2*a^2*A/x^2-a^2*B/x+2*a*b*B*x+1/2*(A*(2*a*c+b^2)+2*a*b*C)*x^2+1/3*B*(2*
a*c+b^2)*x^3+1/4*(2*A*b*c+(2*a*c+b^2)*C)*x^4+2/5*b*B*c*x^5+1/6*c*(A*c+2*C*
b)*x^6+1/7*B*c^2*x^7+1/8*c^2*C*x^8+a*(2*A*b+C*a)*ln(x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^3}\,dx$$
$$= -\frac{a^2(A+2Bx)}{2x^2}+\frac{1}{6}ax\left(6b(2B+Cx)+cx\left(6A+4Bx+3Cx^2\right)\right) + \frac{1}{840}x^2\left(70b^2x(4B+3Cx)+56bcx^3(6B+5Cx)+15c^2x^5(8B+7Cx) + 140A\left(3b^2+3bcx^2+c^2x^4\right)\right)+a(2Ab+aC)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

output
```
-1/2*(a^2*(A + 2*B*x))/x^2 + (a*x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*
C*x^2)))/6 + (x^2*(70*b^2*x*(4*B + 3*C*x) + 56*b*c*x^3*(6*B + 5*C*x) + 15*
c^2*x^5*(8*B + 7*C*x) + 140*A*(3*b^2 + 3*b*c*x^2 + c^2*x^4)))/840 + a*(2*A
*b + a*C)*Log[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2\,(A+Bx+Cx^2)}{x^3}\,dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+x^3\left(C\left(2ac+b^2\right)+2Abc\right)+x\left(A\left(2ac+b^2\right)+2abC\right)+\frac{a(aC+2Ab)}{x}+Bx^2\left(2ac+b^2\right)+2abI\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{4}x^4\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{2}x^2\left(A\left(2ac+b^2\right)+2abC\right)+a\log(x)(aC+2Ab)+\frac{1}{3}Bx^3\left(2ac+b^2\right)+2abBx+\frac{1}{6}cx^6(Ac+2bC)+\frac{2}{5}bBcx^5+\frac{1}{7}Bc^2x^7+\frac{1}{8}c^2Cx^8$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

output
```
-1/2*(a^2*A)/x^2 - (a^2*B)/x + 2*a*b*B*x + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^
2)/2 + (B*(b^2 + 2*a*c)*x^3)/3 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^4)/4 + (2*
b*B*c*x^5)/5 + (c*(A*c + 2*b*C)*x^6)/6 + (B*c^2*x^7)/7 + (c^2*C*x^8)/8 + a
*(2*A*b + a*C)*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

method	result
norman	$\frac{\left(\frac{1}{6}Ac^2+\frac{1}{3}Ccb\right)x^8+\left(\frac{2}{3}Bac+\frac{1}{3}Bb^2\right)x^5+\left(Aac+\frac{1}{2}Ab^2+Cba\right)x^4+\left(\frac{1}{2}Abc+\frac{1}{2}Cac+\frac{1}{4}Cb^2\right)x^6-\frac{Aa^2}{2}-Ba^2x+\frac{Bc^2x^9}{7}+\frac{c^2Cx^{10}}{8}}{x^2}$
default	$\frac{c^2Cx^8}{8}+\frac{Bc^2x^7}{7}+\frac{Ac^2x^6}{6}+\frac{Cbcx^6}{3}+\frac{2bBcx^5}{5}+\frac{Abcx^4}{2}+\frac{Cacx^4}{2}+\frac{Cb^2x^4}{4}+\frac{2Bacx^3}{3}+\frac{Bb^2x^3}{3}+Aac$
risch	$\frac{c^2Cx^8}{8}+\frac{Bc^2x^7}{7}+\frac{Ac^2x^6}{6}+\frac{Cbcx^6}{3}+\frac{2bBcx^5}{5}+\frac{Abcx^4}{2}+\frac{Cacx^4}{2}+\frac{Cb^2x^4}{4}+\frac{2Bacx^3}{3}+\frac{Bb^2x^3}{3}+Aac$
parallelrisch	$\frac{105c^2Cx^{10}+120Bc^2x^9+140Ac^2x^8+280Cbcx^8+336bBcx^7+420Abcx^6+420Cacx^6+210Cb^2x^6+560Bacx^5+280Bb^2x^5+}{840x^2}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
((1/6*A*c^2+1/3*C*c*b)*x^8+(2/3*B*a*c+1/3*B*b^2)*x^5+(A*a*c+1/2*A*b^2+C*b*
a)*x^4+(1/2*A*b*c+1/2*C*a*c+1/4*C*b^2)*x^6-1/2*A*a^2-B*a^2*x+1/7*B*c^2*x^9
+1/8*c^2*C*x^10+2*B*a*b*x^3+2/5*b*B*c*x^7)/x^2+(2*A*a*b+C*a^2)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx$$
$$= \frac{105\,Cc^2x^{10}+120\,Bc^2x^9+336\,Bbcx^7+140\,(2\,Cbc+Ac^2)x^8+210\,(Cb^2+2\,(Ca+Ab)c)x^6+1680\,Bab}{}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/840*(105*C*c^2*x^10 + 120*B*c^2*x^9 + 336*B*b*c*x^7 + 140*(2*C*b*c + A*c
^2)*x^8 + 210*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 1680*B*a*b*x^3 + 280*(B*b^2
+ 2*B*a*c)*x^5 + 420*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 840*B*a^2*x + 840*(
C*a^2 + 2*A*a*b)*x^2*log(x) - 420*A*a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx = 2Babx+\frac{2Bbcx^5}{5}+\frac{Bc^2x^7}{7}+\frac{Cc^2x^8}{8} + a(2Ab+Ca)\log(x)+x^6\left(\frac{Ac^2}{6}+\frac{Cbc}{3}\right) + x^4\left(\frac{Abc}{2}+\frac{Cac}{2}+\frac{Cb^2}{4}\right) + x^3\cdot\left(\frac{2Bac}{3}+\frac{Bb^2}{3}\right) + x^2\left(Aac+\frac{Ab^2}{2}+Cab\right)+\frac{-Aa^2-2Ba^2x}{2x^2}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**3,x)
```

output
```
2*B*a*b*x + 2*B*b*c*x**5/5 + B*c**2*x**7/7 + C*c**2*x**8/8 + a*(2*A*b + C*
a)*log(x) + x**6*(A*c**2/6 + C*b*c/3) + x**4*(A*b*c/2 + C*a*c/2 + C*b**2/4
) + x**3*(2*B*a*c/3 + B*b**2/3) + x**2*(A*a*c + A*b**2/2 + C*a*b) + (-A*a*
*2 - 2*B*a**2*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx &= \frac{1}{8}\,Cc^2x^8+\frac{1}{7}\,Bc^2x^7+\frac{2}{5}\,Bbcx^5+\frac{1}{6}\left(2\,Cbc+Ac^2\right)x^6 \\ &\quad+\frac{1}{4}\left(Cb^2+2\,(Ca+Ab)c\right)x^4+2\,Babx+\frac{1}{3}\left(Bb^2+2\,Bac\right)x^3 \\ &\quad+\frac{1}{2}\left(2\,Cab+Ab^2+2\,Aac\right)x^2+\left(Ca^2+2\,Aab\right)\log\left(x\right)-\frac{2\,Ba^2x+Aa^2}{2\,x^2}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")
```

output
```
1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/6*(2*C*b*c + A*c^2)*x^6
+ 1/4*(C*b^2 + 2*(C*a + A*b)*c)*x^4 + 2*B*a*b*x + 1/3*(B*b^2 + 2*B*a*c)*x^
3 + 1/2*(2*C*a*b + A*b^2 + 2*A*a*c)*x^2 + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(
2*B*a^2*x + A*a^2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx &= \frac{1}{8}\,Cc^2x^8+\frac{1}{7}\,Bc^2x^7+\frac{1}{3}\,Cbcx^6+\frac{1}{6}\,Ac^2x^6 \\ &\quad+\frac{2}{5}\,Bbcx^5+\frac{1}{4}\,Cb^2x^4+\frac{1}{2}\,Cacx^4 \\ &\quad+\frac{1}{2}\,Abcx^4+\frac{1}{3}\,Bb^2x^3+\frac{2}{3}\,Bacx^3 \\ &\quad+Cabx^2+\frac{1}{2}\,Ab^2x^2+Aacx^2+2\,Babx \\ &\quad+\left(Ca^2+2\,Aab\right)\log\left(|x|\right)-\frac{2\,Ba^2x+Aa^2}{2\,x^2}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")
```

output
```
1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 1/3*C*b*c*x^6 + 1/6*A*c^2*x^6 + 2/5*B*b*c*
x^5 + 1/4*C*b^2*x^4 + 1/2*C*a*c*x^4 + 1/2*A*b*c*x^4 + 1/3*B*b^2*x^3 + 2/3*
B*a*c*x^3 + C*a*b*x^2 + 1/2*A*b^2*x^2 + A*a*c*x^2 + 2*B*a*b*x + (C*a^2 + 2
*A*a*b)*log(abs(x)) - 1/2*(2*B*a^2*x + A*a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx = x^6\left(\frac{A\,c^2}{6}+\frac{C\,b\,c}{3}\right)+\ln(x)\,\left(C\,a^2+2\,A\,b\,a\right)+x^2\left(\frac{A\,b^2}{2}+C\,a\,b+A\,a\,c\right)+x^4\left(\frac{C\,b^2}{4}+\frac{A\,c\,b}{2}+\frac{C\,a\,c}{2}\right)-\frac{\frac{A\,a^2}{2}+B\,a^2\,x}{x^2}+\frac{B\,c^2\,x^7}{7}+\frac{C\,c^2\,x^8}{8}+\frac{B\,x^3\,(b^2+2\,a\,c)}{3}+\frac{2\,B\,b\,c\,x^5}{5}+2\,B\,a\,b\,x$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x)
```

output
```
x^6*((A*c^2)/6 + (C*b*c)/3) + log(x)*(C*a^2 + 2*A*a*b) + x^2*((A*b^2)/2 +
A*a*c + C*a*b) + x^4*((C*b^2)/4 + (A*b*c)/2 + (C*a*c)/2) - ((A*a^2)/2 + B*
a^2*x)/x^2 + (B*c^2*x^7)/7 + (C*c^2*x^8)/8 + (B*x^3*(2*a*c + b^2))/3 + (2*
B*b*c*x^5)/5 + 2*B*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3}\,dx$$
$$= \frac{1680\log(x)\,a^2b\,x^2+840\log(x)\,a^2c\,x^2-420a^3-840a^2bx+840a^2c\,x^4+420a\,b^2x^4+1680a\,b^2x^3+420ab}{}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x)
```

output
```
(1680*log(x)*a**2*b*x**2 + 840*log(x)*a**2*c*x**2 - 420*a**3 - 840*a**2*b*
x + 840*a**2*c*x**4 + 420*a*b**2*x**4 + 1680*a*b**2*x**3 + 420*a*b*c*x**6
+ 560*a*b*c*x**5 + 840*a*b*c*x**4 + 140*a*c**2*x**8 + 420*a*c**2*x**6 + 28
0*b**3*x**5 + 336*b**2*c*x**7 + 210*b**2*c*x**6 + 120*b*c**2*x**9 + 280*b*
c**2*x**8 + 105*c**3*x**10)/(840*x**2)
```

3.17 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 149

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x} + \left(A\left(b^2+2ac\right)+2abC\right)x+\frac{1}{2}B\left(b^2+2ac\right)x^2 + \frac{1}{3}\left(2Abc+\left(b^2+2ac\right)C\right)x^3 + \frac{1}{2}bBcx^4+\frac{1}{5}c(Ac+2bC)x^5 + \frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7+2abB\log(x)$$

output
```
-1/3*a^2*A/x^3-1/2*a^2*B/x^2-a*(2*A*b+C*a)/x+(A*(2*a*c+b^2)+2*a*b*C)*x+1/2
*B*(2*a*c+b^2)*x^2+1/3*(2*A*b*c+(2*a*c+b^2)*C)*x^3+1/2*b*B*c*x^4+1/5*c*(A*
c+2*C*b)*x^5+1/6*B*c^2*x^6+1/7*c^2*C*x^7+2*a*b*B*ln(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{-2aAb-a^2C}{x} + (Ab^2+2aAc+2abC)\,x+\frac{1}{2}B(b^2+2ac)\,x^2 + \frac{1}{3}(2Abc+b^2C+2acC)\,x^3 + \frac{1}{2}bBcx^4+\frac{1}{5}c(Ac+2bC)x^5 + \frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7+2abB\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) + (-2*a*A*b - a^2*C)/x + (A*b^2 + 2*a*A
*c + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + b^2*C + 2*a*c*C)*x
^3)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x
^7)/7 + 2*a*b*B*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2(A+Bx+Cx^2)}{x^4}\,dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+x^2(C(2ac+b^2)+2Abc)+Ab^2\left(\frac{2a(Ac+bC)}{Ab^2}+1\right)+\frac{a(aC+2Ab)}{x^2}+Bx(2ac+b^2)+\frac{2abB}{x}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(C\left(2ac+b^2\right)+2Abc\right)+x\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log(x)+\frac{1}{5}cx^5(Ac+2bC)+\frac{1}{2}bBcx^4+\frac{1}{6}Bc^2x^6+\frac{1}{7}c^2Cx^7$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + (A*(b^2 + 2*a*c
) + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^
3)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^
7)/7 + 2*a*b*B*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.95

method	result
norman	$\frac{\left(\frac{1}{5}A\,c^2+\frac{2}{5}Ccb\right)x^8+\left(Bac+\frac{1}{2}B\,b^2\right)x^5+\left(\frac{2}{3}Abc+\frac{2}{3}Cac+\frac{1}{3}C\,b^2\right)x^6+\left(-2Aab-a^2C\right)x^2+\left(2Aac+A\,b^2+2Cba\right)x^4-\frac{A\,a^2}{3}-\frac{B\,a^2x}{2}}{x^3}$
default	$\frac{c^2C\,x^7}{7}+\frac{B\,c^2x^6}{6}+\frac{A\,c^2x^5}{5}+\frac{2Cbc\,x^5}{5}+\frac{bBc\,x^4}{2}+\frac{2Abc\,x^3}{3}+\frac{2Cac\,x^3}{3}+\frac{C\,b^2x^3}{3}+Bac\,x^2+\frac{B\,b^2x^2}{2}+2x$
risch	$\frac{c^2C\,x^7}{7}+\frac{B\,c^2x^6}{6}+\frac{A\,c^2x^5}{5}+\frac{2Cbc\,x^5}{5}+\frac{bBc\,x^4}{2}+\frac{2Abc\,x^3}{3}+\frac{2Cac\,x^3}{3}+\frac{C\,b^2x^3}{3}+Bac\,x^2+\frac{B\,b^2x^2}{2}+2x$
parallelrisch	$\frac{30c^2C\,x^{10}+35B\,c^2x^9+42A\,c^2x^8+84Cbc\,x^8+105bBc\,x^7+140Abc\,x^6+140Cac\,x^6+70C\,b^2x^6+210Bac\,x^5+105B\,b^2x^5+420Aa}{210x^3}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
((1/5*A*c^2+2/5*C*c*b)*x^8+(B*a*c+1/2*B*b^2)*x^5+(2/3*A*b*c+2/3*C*a*c+1/3*
C*b^2)*x^6+(-2*A*a*b-C*a^2)*x^2+(2*A*a*c+A*b^2+2*C*a*b)*x^4-1/3*A*a^2-1/2*
B*a^2*x+1/6*B*c^2*x^9+1/7*c^2*C*x^10+1/2*b*B*c*x^7)/x^3+2*a*b*B*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx$$
$$=\frac{30\,Cc^2x^{10}+35\,Bc^2x^9+105\,Bbcx^7+42\,(2\,Cbc+Ac^2)x^8+70\,(Cb^2+2\,(Ca+Ab)c)x^6+420\,Babx^3\log}{210}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")
```

output
```
1/210*(30*C*c^2*x^10 + 35*B*c^2*x^9 + 105*B*b*c*x^7 + 42*(2*C*b*c + A*c^2)
*x^8 + 70*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 420*B*a*b*x^3*log(x) + 105*(B*b^
2 + 2*B*a*c)*x^5 + 210*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 105*B*a^2*x - 70*
A*a^2 - 210*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = 2Bab\log(x)+\frac{Bbcx^4}{2}+\frac{Bc^2x^6}{6}+\frac{Cc^2x^7}{7}+x^5\left(\frac{Ac^2}{5}+\frac{2Cbc}{5}\right)+x^3\cdot\left(\frac{2Abc}{3}+\frac{2Cac}{3}+\frac{Cb^2}{3}\right)+x^2\left(Bac+\frac{Bb^2}{2}\right)+x\left(2Aac+Ab^2+2Cab\right)+\frac{-2Aa^2-3Ba^2x+x^2(-12Aab-6Ca^2)}{6x^3}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**4,x)
```

output
```
2*B*a*b*log(x) + B*b*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x**5*(A*c*
*2/5 + 2*C*b*c/5) + x**3*(2*A*b*c/3 + 2*C*a*c/3 + C*b**2/3) + x**2*(B*a*c
+ B*b**2/2) + x*(2*A*a*c + A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x
**2*(-12*A*a*b - 6*C*a**2))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

$$
\begin{aligned}
\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^4}\,dx = {} & \frac{1}{7}\,Cc^2x^7+\frac{1}{6}\,Bc^2x^6+\frac{1}{2}\,Bbcx^4 \\
& +\frac{1}{5}\,\left(2\,Cbc+Ac^2\right)x^5 \\
& +\frac{1}{3}\,\left(Cb^2+2\,(Ca+Ab)c\right)x^3 \\
& +2\,Bab\log\left(x\right)+\frac{1}{2}\,\left(Bb^2+2\,Bac\right)x^2 \\
& +\left(2\,Cab+Ab^2+2\,Aac\right)x \\
& -\frac{3\,Ba^2x+2\,Aa^2+6\,(Ca^2+2\,Aab)x^2}{6\,x^3}
\end{aligned}
$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")
```

output
```
1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/5*(2*C*b*c + A*c^2)*x^5
+ 1/3*(C*b^2 + 2*(C*a + A*b)*c)*x^3 + 2*B*a*b*log(x) + 1/2*(B*b^2 + 2*B*a*
c)*x^2 + (2*C*a*b + A*b^2 + 2*A*a*c)*x - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a
^2 + 2*A*a*b)*x^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{7}Cc^2x^7 + \frac{1}{6}Bc^2x^6 + \frac{2}{5}Cbcx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{2}Bbcx^4 + \frac{1}{3}Cb^2x^3 + \frac{2}{3}Cacx^3 + \frac{2}{3}Abcx^3 + \frac{1}{2}Bb^2x^2 + Bacx^2 + 2Cabx + Ab^2x + 2Aacx + 2Bab\log(|x|) - \frac{3Ba^2x + 2Aa^2 + 6(Ca^2 + 2Aab)x^2}{6x^3}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")
```

output
```
1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*b*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*b*c*
x^4 + 1/3*C*b^2*x^3 + 2/3*C*a*c*x^3 + 2/3*A*b*c*x^3 + 1/2*B*b^2*x^2 + B*a*
c*x^2 + 2*C*a*b*x + A*b^2*x + 2*A*a*c*x + 2*B*a*b*log(abs(x)) - 1/6*(3*B*a
^2*x + 2*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx = x^5\left(\frac{A\,c^2}{5} + \frac{2\,C\,b\,c}{5}\right) - \frac{x^2\,(C\,a^2 + 2\,A\,b\,a) + \frac{A\,a^2}{3} + \frac{B\,a^2\,x}{2}}{x^3} + x\,(A\,b^2 + 2\,C\,a\,b + 2\,A\,a\,c) + x^3\left(\frac{C\,b^2}{3} + \frac{2\,A\,c\,b}{3} + \frac{2\,C\,a\,c}{3}\right) + \frac{B\,c^2\,x^6}{6} + \frac{C\,c^2\,x^7}{7} + \frac{B\,x^2\,(b^2 + 2\,a\,c)}{2} + \frac{B\,b\,c\,x^4}{2} + 2\,B\,a\,b\,\ln(x)$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x)
```

output
```
x^5*((A*c^2)/5 + (2*C*b*c)/5) - (x^2*(C*a^2 + 2*A*a*b) + (A*a^2)/3 + (B*a^
2*x)/2)/x^3 + x*(A*b^2 + 2*A*a*c + 2*C*a*b) + x^3*((C*b^2)/3 + (2*A*b*c)/3
 + (2*C*a*c)/3) + (B*c^2*x^6)/6 + (C*c^2*x^7)/7 + (B*x^2*(2*a*c + b^2))/2
+ (B*b*c*x^4)/2 + 2*B*a*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^4}\,dx$$
$$= \frac{420\log(x)\,a\,b^2x^3-70a^3-420a^2b\,x^2-105a^2bx+420a^2c\,x^4-210a^2c\,x^2+210a\,b^2x^4+140abc\,x^6+210a}{2}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x)
```

output
```
(420*log(x)*a*b**2*x**3 - 70*a**3 - 420*a**2*b*x**2 - 105*a**2*b*x + 420*a
**2*c*x**4 - 210*a**2*c*x**2 + 210*a*b**2*x**4 + 140*a*b*c*x**6 + 210*a*b*
c*x**5 + 420*a*b*c*x**4 + 42*a*c**2*x**8 + 140*a*c**2*x**6 + 105*b**3*x**5
 + 105*b**2*c*x**7 + 70*b**2*c*x**6 + 35*b*c**2*x**9 + 84*b*c**2*x**8 + 30
*c**3*x**10)/(210*x**3)
```

3.18 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} dx$

Optimal result

Integrand size = 28, antiderivative size = 148

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} dx = -\frac{a^2A}{4x^4} - \frac{a^2B}{3x^3} - \frac{a(2Ab+aC)}{2x^2} - \frac{2abB}{x} + B(b^2+2ac)x + \frac{1}{2}(2Abc+(b^2+2ac)C)x^2 + \frac{2}{3}bBcx^3 + \frac{1}{4}c(Ac+2bC)x^4 + \frac{1}{5}Bc^2x^5 + \frac{1}{6}c^2Cx^6 + (A(b^2+2ac)+2abC)\log(x)$$

output
```
-1/4*a^2*A/x^4-1/3*a^2*B/x^3-1/2*a*(2*A*b+C*a)/x^2-2*a*b*B/x+B*(2*a*c+b^2)
*x+1/2*(2*A*b*c+(2*a*c+b^2)*C)*x^2+2/3*b*B*c*x^3+1/4*c*(A*c+2*C*b)*x^4+1/5
*B*c^2*x^5+1/6*c^2*C*x^6+(A*(2*a*c+b^2)+2*a*b*C)*ln(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx = -\frac{a^2(3A+4Bx+6Cx^2)}{12x^4} + \frac{a(-Ab-2bBx+cx^3(2B+Cx))}{x^2} + \frac{1}{60}x\left(30b^2(2B+Cx) + 10bcx(6A+x(4B+3Cx)) + c^2x^3(15A+2x(6B+5Cx))\right) + \left(A\left(b^2+2ac\right)+2abC\right)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

output
```
-1/12*(a^2*(3*A + 4*B*x + 6*C*x^2))/x^4 + (a*(-(A*b) - 2*b*B*x + c*x^3*(2*
B + C*x)))/x^2 + (x*(30*b^2*(2*B + C*x) + 10*b*c*x*(6*A + x*(4*B + 3*C*x))
 + c^2*x^3*(15*A + 2*x*(6*B + 5*C*x))))/60 + (A*(b^2 + 2*a*c) + 2*a*b*C)*L
og[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2(A+Bx+Cx^2)}{x^5}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^2A}{x^5}+\frac{a^2B}{x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{A\left(2ac+b^2\right)+2abC}{x}+\frac{a(aC+2Ab)}{x^3}+B\left(2ac+b^2\right)+\frac{2abB}{x^2}+cx\right.$$

$$\downarrow 2009$$

$$-\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}+\frac{1}{2}x^2\left(C\left(2ac+b^2\right)+2Abc\right)+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{2x^2}+Bx\left(2ac+b^2\right)-\frac{2abB}{x}+\frac{1}{4}cx^4(Ac+2bC)+\frac{2}{3}bBcx^3+\frac{1}{5}Bc^2x^5+\frac{1}{6}c^2Cx^6$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

output
```
-1/4*(a^2*A)/x^4 - (a^2*B)/(3*x^3) - (a*(2*A*b + a*C))/(2*x^2) - (2*a*b*B)
/x + B*(b^2 + 2*a*c)*x + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^2)/2 + (2*b*B*c*x^
3)/3 + (c*(A*c + 2*b*C)*x^4)/4 + (B*c^2*x^5)/5 + (c^2*C*x^6)/6 + (A*(b^2 +
 2*a*c) + 2*a*b*C)*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^2C\,x^6}{6}+\frac{B\,c^2x^5}{5}+\frac{A\,c^2x^4}{4}+\frac{Cbc\,x^4}{2}+\frac{2Bbc\,x^3}{3}+Abc\,x^2+Cac\,x^2+\frac{C\,b^2x^2}{2}+2Bacx+xB\,b^2-$
norman	$\frac{\left(\frac{1}{4}A\,c^2+\frac{1}{2}Ccb\right)x^8+\left(-Aab-\frac{1}{2}a^2C\right)x^2+\left(Abc+Cac+\frac{1}{2}C\,b^2\right)x^6+\left(2Bac+B\,b^2\right)x^5-\frac{A\,a^2}{4}-\frac{B\,a^2x}{3}+\frac{B\,c^2x^9}{5}+\frac{c^2C\,x^{10}}{6}-2Bab\,x^3}{x^4}$
risch	$\frac{c^2C\,x^6}{6}+\frac{B\,c^2x^5}{5}+\frac{A\,c^2x^4}{4}+\frac{Cbc\,x^4}{2}+\frac{2Bbc\,x^3}{3}+Abc\,x^2+Cac\,x^2+\frac{C\,b^2x^2}{2}+2Bacx+xB\,b^2+$
parallelrisch	$\frac{10c^2C\,x^{10}+12B\,c^2x^9+15A\,c^2x^8+30Cbc\,x^8+40bBc\,x^7+60Abc\,x^6+60Cac\,x^6+30C\,b^2x^6+120A\ln(x)x^4ac+60A\ln(x)x^4b^2+}{60x^4}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*C*x^6+1/5*B*c^2*x^5+1/4*A*c^2*x^4+1/2*C*b*c*x^4+2/3*B*b*c*x^3+A*b*
c*x^2+C*a*c*x^2+1/2*C*b^2*x^2+2*B*a*c*x+x*B*b^2-1/4*a^2*A/x^4-1/3*a^2*B/x^
3-1/2*a*(2*A*b+C*a)/x^2-2*a*b*B/x+(2*A*a*c+A*b^2+2*C*a*b)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx = \frac{10\,Cc^2x^{10}+12\,Bc^2x^9+40\,Bbcx^7+15\,(2\,Cbc+Ac^2)x^8+30\,(Cb^2+2\,(Ca+Ab)c)x^6-120\,Babx^3+60}{60\,x^4}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")
```

output
```
1/60*(10*C*c^2*x^10 + 12*B*c^2*x^9 + 40*B*b*c*x^7 + 15*(2*C*b*c + A*c^2)*x
^8 + 30*(C*b^2 + 2*(C*a + A*b)*c)*x^6 - 120*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*
c)*x^5 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4*log(x) - 20*B*a^2*x - 15*A*a^2
 - 30*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\begin{aligned}
\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx &= \frac{2Bbcx^3}{3}+\frac{Bc^2x^5}{5}+\frac{Cc^2x^6}{6}+x^4\left(\frac{Ac^2}{4}+\frac{Cbc}{2}\right)+x^2\left(Abc+Cac+\frac{Cb^2}{2}\right)\\
&\quad+x\left(2Bac+Bb^2\right)+\left(2Aac+Ab^2+2Cab\right)\log{(x)}\\
&\quad+\frac{-3Aa^2-4Ba^2x-24Babx^3+x^2(-12Aab-6Ca^2)}{12x^4}
\end{aligned}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**5,x)
```

output
```
2*B*b*c*x**3/3 + B*c**2*x**5/5 + C*c**2*x**6/6 + x**4*(A*c**2/4 + C*b*c/2)
 + x**2*(A*b*c + C*a*c + C*b**2/2) + x*(2*B*a*c + B*b**2) + (2*A*a*c + A*b
**2 + 2*C*a*b)*log(x) + (-3*A*a**2 - 4*B*a**2*x - 24*B*a*b*x**3 + x**2*(-1
2*A*a*b - 6*C*a**2))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{6}Cc^2x^6 + \frac{1}{5}Bc^2x^5 + \frac{2}{3}Bbcx^3 + \frac{1}{4}\left(2\,Cbc + Ac^2\right)x^4 + \frac{1}{2}\left(Cb^2 + 2\,(Ca + Ab)c\right)x^2 + \left(Bb^2 + 2\,Bac\right)x + \left(2\,Cab + Ab^2 + 2\,Aac\right)\log(x) - \frac{24\,Babx^3 + 4\,Ba^2x + 3\,Aa^2 + 6\,(Ca^2 + 2\,Aab)x^2}{12\,x^4}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")
```

output
```
1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/4*(2*C*b*c + A*c^2)*x^4
+ 1/2*(C*b^2 + 2*(C*a + A*b)*c)*x^2 + (B*b^2 + 2*B*a*c)*x + (2*C*a*b + A*b
^2 + 2*A*a*c)*log(x) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2 + 6*(C*a^2
 + 2*A*a*b)*x^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{6}Cc^2x^6 + \frac{1}{5}Bc^2x^5 + \frac{1}{2}Cbcx^4 + \frac{1}{4}Ac^2x^4 + \frac{2}{3}Bbcx^3 + \frac{1}{2}Cb^2x^2 + Cacx^2 + Abcx^2 + Bb^2x + 2\,Bacx + \left(2\,Cab + Ab^2 + 2\,Aac\right)\log(|x|) - \frac{24\,Babx^3 + 4\,Ba^2x + 3\,Aa^2 + 6\,(Ca^2 + 2\,Aab)x^2}{12\,x^4}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")
```

output
```
1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 1/2*C*b*c*x^4 + 1/4*A*c^2*x^4 + 2/3*B*b*c*
x^3 + 1/2*C*b^2*x^2 + C*a*c*x^2 + A*b*c*x^2 + B*b^2*x + 2*B*a*c*x + (2*C*a
*b + A*b^2 + 2*A*a*c)*log(abs(x)) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a
^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 18.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5}\,dx = x^4\left(\frac{A\,c^2}{4}+\frac{C\,b\,c}{2}\right)-\frac{x^2\left(\frac{C\,a^2}{2}+A\,b\,a\right)+\frac{A\,a^2}{4}+\frac{B\,a^2\,x}{3}+2\,B\,a\,b\,x^3}{x^4} + x^2\left(\frac{C\,b^2}{2}+A\,c\,b+C\,a\,c\right)+\ln(x)\,\left(A\,b^2+2\,C\,a\,b+2\,A\,a\,c\right) + \frac{B\,c^2\,x^5}{5}+\frac{C\,c^2\,x^6}{6}+B\,x\,\left(b^2+2\,a\,c\right)+\frac{2\,B\,b\,c\,x^3}{3}$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x)
```

output
```
x^4*((A*c^2)/4 + (C*b*c)/2) - (x^2*((C*a^2)/2 + A*a*b) + (A*a^2)/4 + (B*a^
2*x)/3 + 2*B*a*b*x^3)/x^4 + x^2*((C*b^2)/2 + A*b*c + C*a*c) + log(x)*(A*b^
2 + 2*A*a*c + 2*C*a*b) + (B*c^2*x^5)/5 + (C*c^2*x^6)/6 + B*x*(2*a*c + b^2)
 + (2*B*b*c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^5}\,dx$$
$$= \frac{120\log(x)\,a^2c\,x^4+60\log(x)\,a\,b^2x^4+120\log(x)\,abc\,x^4-15a^3-60a^2b\,x^2-20a^2bx-30a^2c\,x^2-120a\,b^2}{}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x)
```

output
```
(120*log(x)*a**2*c*x**4 + 60*log(x)*a*b**2*x**4 + 120*log(x)*a*b*c*x**4 -
15*a**3 - 60*a**2*b*x**2 - 20*a**2*b*x - 30*a**2*c*x**2 - 120*a*b**2*x**3
+ 60*a*b*c*x**6 + 120*a*b*c*x**5 + 15*a*c**2*x**8 + 60*a*c**2*x**6 + 60*b*
*3*x**5 + 40*b**2*c*x**7 + 30*b**2*c*x**6 + 12*b*c**2*x**9 + 30*b*c**2*x**
8 + 10*c**3*x**10)/(60*x**4)
```

3.19 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 143

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx = -\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3} -\frac{abB}{x^2}-\frac{A(b^2+2ac)+2abC}{x} +\left(2Abc+\left(b^2+2ac\right)C\right)x+bBcx^2 +\frac{1}{3}c(Ac+2bC)x^3+\frac{1}{4}Bc^2x^4 +\frac{1}{5}c^2Cx^5+B\left(b^2+2ac\right)\log(x)$$

output
```
-1/5*a^2*A/x^5-1/4*a^2*B/x^4-1/3*a*(2*A*b+C*a)/x^3-a*b*B/x^2-(A*(2*a*c+b^2
)+2*a*b*C)/x+(2*A*b*c+(2*a*c+b^2)*C)*x+b*B*c*x^2+1/3*c*(A*c+2*C*b)*x^3+1/4
*B*c^2*x^4+1/5*c^2*C*x^5+B*(2*a*c+b^2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^6}\,dx = -\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-\frac{abB}{x^2} - \frac{Ab^2+2aAc+2abC}{x}+2Abcx + \left(b^2+2ac\right)Cx+bBcx^2+\frac{1}{3}c(Ac+2bC)x^3 + \frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5+B\left(b^2+2ac\right)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x
^2 - (A*b^2 + 2*a*A*c + 2*a*b*C)/x + 2*A*b*c*x + (b^2 + 2*a*c)*C*x + b*B*c
*x^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 +
2*a*c)*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^2\left(A+Bx+Cx^2\right)}{x^6}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{a^2A}{x^6}+\frac{a^2B}{x^5}+\frac{A\left(2ac+b^2\right)+2abC}{x^2}+2Abc\left(\frac{bC\left(\frac{2ac}{b^2}+1\right)}{2Ac}+1\right)+\frac{a(aC+2Ab)}{x^4}+\frac{B\left(2ac+b^2\right)}{x}+\frac{2abB}{x^3}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{A\left(2ac+b^2\right)+2abC}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(Ac+2bC)+bBcx^2+\frac{1}{4}Bc^2x^4+\frac{1}{5}c^2Cx^5$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x
^2 - (A*(b^2 + 2*a*c) + 2*a*b*C)/x + (2*A*b*c + (b^2 + 2*a*c)*C)*x + b*B*c
*x^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 +
2*a*c)*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^2C\,x^5}{5}+\frac{B\,c^2x^4}{4}+\frac{A\,c^2x^3}{3}+\frac{2Cbc\,x^3}{3}+Bbc\,x^2+2Abcx+2Cacx+C\,b^2x-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-$
risch	$\frac{c^2C\,x^5}{5}+\frac{B\,c^2x^4}{4}+\frac{A\,c^2x^3}{3}+\frac{2Cbc\,x^3}{3}+Bbc\,x^2+2Abcx+2Cacx+C\,b^2x+\frac{(-2Aac-A\,b^2-2Cba)x^4-}{}$
norman	$\frac{(\frac{1}{3}A\,c^2+\frac{2}{3}Ccb)x^8+(-\frac{2}{3}Aab-\frac{1}{3}a^2C)x^2+(2Abc+2Cac+C\,b^2)x^6+(-2Aac-A\,b^2-2Cba)x^4+bBc\,x^7-\frac{A\,a^2}{5}-\frac{B\,a^2x}{4}+\frac{B\,c^2x^9}{4}}{x^5}$
parallelrisch	$\frac{12c^2C\,x^{10}+15B\,c^2x^9+20A\,c^2x^8+40Cbc\,x^8+60bBc\,x^7+120Abc\,x^6+120B\ln(x)x^5ac+60B\ln(x)x^5b^2+120Cac\,x^6+60C\,b^2x^6}{60x^5}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/5*c^2*C*x^5+1/4*B*c^2*x^4+1/3*A*c^2*x^3+2/3*C*b*c*x^3+B*b*c*x^2+2*A*b*c*
x+2*C*a*c*x+C*b^2*x-1/4*a^2*B/x^4-1/3*a*(2*A*b+C*a)/x^3-a*b*B/x^2-(2*A*a*c
+A*b^2+2*C*a*b)/x+B*(2*a*c+b^2)*ln(x)-1/5*a^2*A/x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$$
$$= \frac{12\,Cc^2x^{10}+15\,Bc^2x^9+60\,Bbcx^7+20\,(2\,Cbc+Ac^2)x^8+60\,(Cb^2+2\,(Ca+Ab)c)x^6+60\,(Bb^2+2\,Bac}{60\,x^5}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")
```

output
```
1/60*(12*C*c^2*x^10 + 15*B*c^2*x^9 + 60*B*b*c*x^7 + 20*(2*C*b*c + A*c^2)*x
^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 60*(B*b^2 + 2*B*a*c)*x^5*log(x) -
60*B*a*b*x^3 - 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 15*B*a^2*x - 12*A*a^2
- 20*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx = Bbcx^2+\frac{Bc^2x^4}{4}+B\left(2ac+b^2\right)\log\left(x\right)$$
$$+\frac{Cc^2x^5}{5}+x^3\left(\frac{Ac^2}{3}+\frac{2Cbc}{3}\right)+x\left(2Abc+2Cac+Cb^2\right)$$
$$+\frac{-12Aa^2-15Ba^2x-60Babx^3+x^4(-120Aac-60Ab^2-120Cab)+x^2(-40Aab-20Ca^2)}{60x^5}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**6,x)
```

output
```
B*b*c*x**2 + B*c**2*x**4/4 + B*(2*a*c + b**2)*log(x) + C*c**2*x**5/5 + x**
3*(A*c**2/3 + 2*C*b*c/3) + x*(2*A*b*c + 2*C*a*c + C*b**2) + (-12*A*a**2 -
15*B*a**2*x - 60*B*a*b*x**3 + x**4*(-120*A*a*c - 60*A*b**2 - 120*C*a*b) +
x**2*(-40*A*a*b - 20*C*a**2))/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^6}\,dx = \frac{1}{5}\,Cc^2x^5 + \frac{1}{4}\,Bc^2x^4 + Bbcx^2 + \frac{1}{3}\,\left(2\,Cbc + Ac^2\right)x^3 + \left(Cb^2 + 2\,(Ca + Ab)c\right)x + \left(Bb^2 + 2\,Bac\right)\log\left(x\right) - \frac{60\,Babx^3 + 60\,(2\,Cab + Ab^2 + 2\,Aac)x^4 + 15\,Ba^2x + 12\,Aa^2 + 20\,(Ca^2 + 2\,Aab)x^2}{60\,x^5}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")
```

output
```
1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/3*(2*C*b*c + A*c^2)*x^3 + (C
*b^2 + 2*(C*a + A*b)*c)*x + (B*b^2 + 2*B*a*c)*log(x) - 1/60*(60*B*a*b*x^3
+ 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 20*(C*a^2 +
 2*A*a*b)*x^2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx+Cx^2)\,(a+bx^2+cx^4)^2}{x^6}\,dx = \frac{1}{5}\,Cc^2x^5 + \frac{1}{4}\,Bc^2x^4 + \frac{2}{3}\,Cbcx^3 + \frac{1}{3}\,Ac^2x^3 + Bbcx^2 + Cb^2x + 2\,Cacx + 2\,Abcx + \left(Bb^2 + 2\,Bac\right)\log\left(|x|\right) - \frac{60\,Babx^3 + 60\,(2\,Cab + Ab^2 + 2\,Aac)x^4 + 15\,Ba^2x + 12\,Aa^2 + 20\,(Ca^2 + 2\,Aab)x^2}{60\,x^5}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")
```

output
```
1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + 2/3*C*b*c*x^3 + 1/3*A*c^2*x^3 + B*b*c*x^2
+ C*b^2*x + 2*C*a*c*x + 2*A*b*c*x + (B*b^2 + 2*B*a*c)*log(abs(x)) - 1/60*(
60*B*a*b*x^3 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2
+ 20*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$$
$$= x^3\left(\frac{A\,c^2}{3}+\frac{2\,C\,b\,c}{3}\right) - \frac{x^2\left(\frac{C\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+\frac{A\,a^2}{5}+x^4\,(A\,b^2+2\,C\,a\,b+2\,A\,a\,c)+\frac{B\,a^2\,x}{4}+B\,a\,b\,x^3}{x^5} + x\,(C\,b^2+2\,A\,c\,b+2\,C\,a\,c)+\ln(x)\,\left(B\,b^2+2\,B\,a\,c\right)+\frac{B\,c^2\,x^4}{4}+\frac{C\,c^2\,x^5}{5}+B\,b\,c\,x^2$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x)
```

output
```
x^3*((A*c^2)/3 + (2*C*b*c)/3) - (x^2*((C*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/5
 + x^4*(A*b^2 + 2*A*a*c + 2*C*a*b) + (B*a^2*x)/4 + B*a*b*x^3)/x^5 + x*(C*b
^2 + 2*A*b*c + 2*C*a*c) + log(x)*(B*b^2 + 2*B*a*c) + (B*c^2*x^4)/4 + (C*c^
2*x^5)/5 + B*b*c*x^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$$
$$= \frac{120\log(x)\,abc\,x^5+60\log(x)\,b^3x^5-12a^3-40a^2b\,x^2-15a^2bx-120a^2c\,x^4-20a^2c\,x^2-60a\,b^2x^4-60a\,\cdots}{6\cdots}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x)
```

output
```
(120*log(x)*a*b*c*x**5 + 60*log(x)*b**3*x**5 - 12*a**3 - 40*a**2*b*x**2 -
15*a**2*b*x - 120*a**2*c*x**4 - 20*a**2*c*x**2 - 60*a*b**2*x**4 - 60*a*b**
2*x**3 + 120*a*b*c*x**6 - 120*a*b*c*x**4 + 20*a*c**2*x**8 + 120*a*c**2*x**
6 + 60*b**2*c*x**7 + 60*b**2*c*x**6 + 15*b*c**2*x**9 + 40*b*c**2*x**8 + 12
*c**3*x**10)/(60*x**5)
```

3.20 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7} dx$

Optimal result

Integrand size = 28, antiderivative size = 149

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7} dx = -\frac{a^2A}{6x^6} - \frac{a^2B}{5x^5} - \frac{a(2Ab+aC)}{4x^4} - \frac{2abB}{3x^3} - \frac{A(b^2+2ac)+2abC}{2x^2} - \frac{B(b^2+2ac)}{x} + 2bBcx + \frac{1}{2}c(Ac+2bC)x^2 + \frac{1}{3}Bc^2x^3 + \frac{1}{4}c^2Cx^4 + \left(2Abc+\left(b^2+2ac\right)C\right)\log(x)$$

output
```
-1/6*a^2*A/x^6-1/5*a^2*B/x^5-1/4*a*(2*A*b+C*a)/x^4-2/3*a*b*B/x^3-1/2*(A*(2
*a*c+b^2)+2*a*b*C)/x^2-B*(2*a*c+b^2)/x+2*b*B*c*x+1/2*c*(A*c+2*C*b)*x^2+1/3
*B*c^2*x^3+1/4*c^2*C*x^4+(2*A*b*c+(2*a*c+b^2)*C)*ln(x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)\left(a+bx^2+cx^4\right)^2}{x^7}\,dx$$
$$= -\frac{b^2B}{x}+bcx(2B+Cx)+\frac{1}{12}c^2x^3(4B+3Cx)$$
$$+\frac{A(-b^2+c^2x^4)}{2x^2}-\frac{a^2(10A+3x(4B+5Cx))}{60x^6}$$
$$-\frac{a(3A(b+2cx^2)+2x(2bB+3bCx+6Bcx^2))}{6x^4}+\left(2Abc+\left(b^2+2ac\right)C\right)\log(x)$$

input
```
Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

output
```
-((b^2*B)/x) + b*c*x*(2*B + C*x) + (c^2*x^3*(4*B + 3*C*x))/12 + (A*(-b^2 +
 c^2*x^4))/(2*x^2) - (a^2*(10*A + 3*x*(4*B + 5*C*x)))/(60*x^6) - (a*(3*A*(
b + 2*c*x^2) + 2*x*(2*b*B + 3*b*C*x + 6*B*c*x^2)))/(6*x^4) + (2*A*b*c + (b
^2 + 2*a*c)*C)*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^2(A+Bx+Cx^2)}{x^7}\,dx$$
$$\Big\downarrow \ 2159$$
$$\int \left(\frac{a^2A}{x^7}+\frac{a^2B}{x^6}+\frac{A\left(2ac+b^2\right)+2abC}{x^3}+\frac{C\left(2ac+b^2\right)+2Abc}{x}+\frac{a(aC+2Ab)}{x^5}+\frac{B\left(2ac+b^2\right)}{x^2}+\frac{2abB}{x^4}+cx(A$$
$$\Big\downarrow \ 2009$$

$$-\frac{a^2 A}{6x^6}-\frac{a^2 B}{5x^5}-\frac{A(2ac+b^2)+2abC}{2x^2}+\log(x)\left(C(2ac+b^2)+2Abc\right)-\frac{a(aC+2Ab)}{4x^4}-\frac{B(2ac+b^2)}{x}-\frac{2abB}{3x^3}+\frac{1}{2}cx^2(Ac+2bC)+2bBcx+\frac{1}{3}Bc^2x^3+\frac{1}{4}c^2Cx^4$$

input
```
Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

output
```
-1/6*(a^2*A)/x^6 - (a^2*B)/(5*x^5) - (a*(2*A*b + a*C))/(4*x^4) - (2*a*b*B)
/(3*x^3) - (A*(b^2 + 2*a*c) + 2*a*b*C)/(2*x^2) - (B*(b^2 + 2*a*c))/x + 2*b
*B*c*x + (c*(A*c + 2*b*C)*x^2)/2 + (B*c^2*x^3)/3 + (c^2*C*x^4)/4 + (2*A*b*
c + (b^2 + 2*a*c)*C)*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

method	result
default	$\frac{c^2 C x^4}{4}+\frac{B c^2 x^3}{3}+\frac{A c^2 x^2}{2}+Ccb\,x^2+2Bbcx-\frac{a(2Ab+aC)}{4x^4}-\frac{2abB}{3x^3}-\frac{a^2 A}{6x^6}-\frac{2Aac+A b^2+2Cba}{2x^2}-\frac{B(2$
norman	$\frac{\left(\frac{1}{2}A c^2+Ccb\right)x^8+\left(-\frac{1}{2}Aab-\frac{1}{4}a^2 C\right)x^2+\left(-Aac-\frac{1}{2}A b^2-Cba\right)x^4+\left(-2Bac-B b^2\right)x^5-\frac{A a^2}{6}-\frac{B a^2 x}{5}+\frac{B c^2 x^9}{3}+\frac{c^2 C x^{10}}{4}-\frac{2Ba}{3}}{x^6}$
risch	$\frac{c^2 C x^4}{4}+\frac{B c^2 x^3}{3}+\frac{A c^2 x^2}{2}+Ccb\,x^2+2Bbcx+\frac{\left(-2Bac-B b^2\right)x^5+\left(-Aac-\frac{1}{2}A b^2-Cba\right)x^4-\frac{2Bab\,x^3}{3}+\left(-\frac{1}{2}Ac\right.}{x^6}$
parallelrisch	$\frac{15c^2 C x^{10}+20B c^2 x^9+30A c^2 x^8+60Cbc\,x^8+120A\ln(x)x^6 bc+120bBc\,x^7+120C\ln(x)x^6 ac+60C\ln(x)x^6 b^2-120Bac\,x^5-60}{60x^6}$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/4*c^2*C*x^4+1/3*B*c^2*x^3+1/2*A*c^2*x^2+C*c*b*x^2+2*B*b*c*x-1/4*a*(2*A*b
+C*a)/x^4-2/3*a*b*B/x^3-1/6*a^2*A/x^6-1/2*(2*A*a*c+A*b^2+2*C*a*b)/x^2-B*(2
*a*c+b^2)/x+(2*A*b*c+2*C*a*c+C*b^2)*ln(x)-1/5*a^2*B/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$$
$$= \frac{15\,Cc^2x^{10}+20\,Bc^2x^9+120\,Bbcx^7+30\,(2\,Cbc+Ac^2)x^8+60\,(Cb^2+2\,(Ca+Ab)c)x^6\log(x)-40\,Bab}{60\,x^6}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="fricas")
```

output
```
1/60*(15*C*c^2*x^10 + 20*B*c^2*x^9 + 120*B*b*c*x^7 + 30*(2*C*b*c + A*c^2)*
x^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6*log(x) - 40*B*a*b*x^3 - 60*(B*b^2 +
 2*B*a*c)*x^5 - 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 12*B*a^2*x - 10*A*a^2
 - 15*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$$
$$= 2Bbcx+\frac{Bc^2x^3}{3}+\frac{Cc^2x^4}{4}+x^2\left(\frac{Ac^2}{2}+Cbc\right)+(2Abc+2Cac+Cb^2)\log(x)$$
$$+\frac{-10Aa^2-12Ba^2x-40Babx^3+x^5(-120Bac-60Bb^2)+x^4(-60Aac-30Ab^2-60Cab)+x^2(-30A}{60x^6}$$

input
```
integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**7,x)
```

output

```
2*B*b*c*x + B*c**2*x**3/3 + C*c**2*x**4/4 + x**2*(A*c**2/2 + C*b*c) + (2*A
*b*c + 2*C*a*c + C*b**2)*log(x) + (-10*A*a**2 - 12*B*a**2*x - 40*B*a*b*x**
3 + x**5*(-120*B*a*c - 60*B*b**2) + x**4*(-60*A*a*c - 30*A*b**2 - 60*C*a*b
) + x**2*(-30*A*a*b - 15*C*a**2))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$$
$$= \frac{1}{4}Cc^2x^4 + \frac{1}{3}Bc^2x^3 + 2Bbcx + \frac{1}{2}(2Cbc + Ac^2)x^2 + (Cb^2 + 2(Ca + Ab)c)\log(x)$$
$$- \frac{40Babx^3 + 60(Bb^2 + 2Bac)x^5 + 30(2Cab + Ab^2 + 2Aac)x^4 + 12Ba^2x + 10Aa^2 + 15(Ca^2 + 2Aa}{60x^6}$$

input

```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="maxima")
```

output

```
1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + 2*B*b*c*x + 1/2*(2*C*b*c + A*c^2)*x^2 + (C
*b^2 + 2*(C*a + A*b)*c)*log(x) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)
*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15*(C*
a^2 + 2*A*a*b)*x^2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$$
$$= \frac{1}{4}Cc^2x^4 + \frac{1}{3}Bc^2x^3 + Cbcx^2 + \frac{1}{2}Ac^2x^2 + 2Bbcx + (Cb^2 + 2Cac + 2Abc)\log(|x|)$$
$$- \frac{40Babx^3 + 60(Bb^2 + 2Bac)x^5 + 30(2Cab + Ab^2 + 2Aac)x^4 + 12Ba^2x + 10Aa^2 + 15(Ca^2 + 2Aa}{60x^6}$$

input

```
integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="giac")
```

output
```
1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + C*b*c*x^2 + 1/2*A*c^2*x^2 + 2*B*b*c*x + (C
*b^2 + 2*C*a*c + 2*A*b*c)*log(abs(x)) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2
*B*a*c)*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 +
 15*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx = x^2\left(\frac{A\,c^2}{2}+C\,b\,c\right) - \frac{x^2\left(\frac{C\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^5\,(B\,b^2+2\,B\,a\,c)+\frac{A\,a^2}{6}+x^4\left(\frac{A\,b^2}{2}+C\,a\,b+A\,a\,c\right)+\frac{B\,a^2\,x}{5}+\frac{2\,B\,a\,b\,x^3}{3}}{x^6} + \ln(x)\,\left(C\,b^2+2\,A\,c\,b+2\,C\,a\,c\right)+\frac{B\,c^2\,x^3}{3}+\frac{C\,c^2\,x^4}{4}+2\,B\,b\,c\,x$$

input
```
int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x)
```

output
```
x^2*((A*c^2)/2 + C*b*c) - (x^2*((C*a^2)/4 + (A*a*b)/2) + x^5*(B*b^2 + 2*B*
a*c) + (A*a^2)/6 + x^4*((A*b^2)/2 + A*a*c + C*a*b) + (B*a^2*x)/5 + (2*B*a*
b*x^3)/3)/x^6 + log(x)*(C*b^2 + 2*A*b*c + 2*C*a*c) + (B*c^2*x^3)/3 + (C*c^
2*x^4)/4 + 2*B*b*c*x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx = \frac{120\log(x)\,abc\,x^6+120\log(x)\,a\,c^2x^6+60\log(x)\,b^2c\,x^6-10a^3-30a^2b\,x^2-12a^2bx-60a^2c\,x^4-15a^2c\,x}{}$$

input
```
int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x)
```

output
```
(120*log(x)*a*b*c*x**6 + 120*log(x)*a*c**2*x**6 + 60*log(x)*b**2*c*x**6 -
10*a**3 - 30*a**2*b*x**2 - 12*a**2*b*x - 60*a**2*c*x**4 - 15*a**2*c*x**2 -
 30*a*b**2*x**4 - 40*a*b**2*x**3 - 120*a*b*c*x**5 - 60*a*b*c*x**4 + 30*a*c
**2*x**8 - 60*b**3*x**5 + 120*b**2*c*x**7 + 20*b*c**2*x**9 + 60*b*c**2*x**
8 + 15*c**3*x**10)/(60*x**6)
```

3.21 $\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 339

$$\begin{aligned}\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx &= \frac{(Ac-bC)x}{c^2}+\frac{Bx^2}{2c}+\frac{Cx^3}{3c}\\ &-\frac{\left(Abc-b^2C+acC-\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\\ &-\frac{\left(Abc-b^2C+acC+\frac{Ac(b^2-2ac)-b(b^2-3ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\\ &-\frac{B(b^2-2ac)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{bB\log(a+bx^2+cx^4)}{4c^2}\end{aligned}$$

output
```
(A*c-C*b)*x/c^2+1/2*B*x^2/c+1/3*C*x^3/c-1/2*(A*b*c-b^2*C+a*c*C-(A*c*(-2*a*
c+b^2)-b*(-3*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2
*(A*b*c-b^2*C+a*c*C+(A*c*(-2*a*c+b^2)-b*(-3*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2)
)*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(
b+(-4*a*c+b^2)^(1/2))^(1/2)-1/2*B*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c
+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)-1/4*b*B*ln(c*x^4+b*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.36

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$
$$= \frac{12\sqrt{c}(Ac-bC)x+6Bc^{3/2}x^2+4c^{3/2}Cx^3+\frac{6\sqrt{2}\left(Ac\left(b^2-2ac-b\sqrt{b^2-4ac}\right)+\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)C\right)\text{arcta}}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(12*Sqrt[c]*(A*c - b*C)*x + 6*B*c^(3/2)*x^2 + 4*c^(3/2)*C*x^3 + (6*Sqrt[2]
*(A*c*(b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c]) + (-b^3 + 3*a*b*c + b^2*Sqrt[b^2
 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (6*
Sqrt[2]*(-(A*c*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])) + (b^3 - 3*a*b*c + b^2
*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]) - (3*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2
- 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - (3*B*Sqrt[c]*(b^2 - 2*a*c + b*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(
12*c^(5/2))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2193, 27, 1434, 1143, 1602, 27, 1602, 25, 1480, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{ 2193}$$

$$\int \frac{x^4\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\int \frac{Bx^5}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \; \color{blue}{27}$$

$$\int \frac{x^4\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+B\int \frac{x^5}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \; \color{blue}{1434}$$

$$\int \frac{x^4\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\int \frac{x^4}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \; \color{blue}{1143}$$

$$\int \frac{x^4\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \; \color{blue}{1602}$$

$$-\frac{\int \frac{3x^2\left(aC-(Ac-bC)x^2\right)}{cx^4+bx^2+a}dx}{3c}+\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2+\frac{Cx^3}{3c}$$

$$\Big\downarrow \; \color{blue}{27}$$

$$-\frac{\int \frac{x^2\left(aC-(Ac-bC)x^2\right)}{cx^4+bx^2+a}dx}{c}+\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2+\frac{Cx^3}{3c}$$

$$\Big\downarrow \; \color{blue}{1602}$$

$$-\frac{-\frac{\int -\frac{\left(-Cb^2+Acb+acC\right)x^2+a(Ac-bC)}{cx^4+bx^2+a}dx}{c}-\frac{x(Ac-bC)}{c}}{c}+\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2+\frac{Cx^3}{3c}$$

$$\Big\downarrow \; \color{blue}{25}$$

$$-\frac{\frac{\int \frac{\left(-Cb^2+Acb+acC\right)x^2+a(Ac-bC)}{cx^4+bx^2+a}dx}{c}-\frac{x(Ac-bC)}{c}}{c}+\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2+\frac{Cx^3}{3c}$$

$$\Big\downarrow \; \color{blue}{1480}$$

$$-\frac{\frac{\frac{1}{2}\left(-\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\int \frac{}{cx^2+\frac{1}{2}}}{c}}{c}$$

$$\frac{1}{2}B\int \left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2+\frac{Cx^3}{3c}$$

$$\Big\downarrow \; \color{blue}{218}$$

$$\frac{1}{2}B\int\left(\frac{1}{c}-\frac{bx^2+a}{c\left(cx^4+bx^2+a\right)}\right)dx^2-\frac{\frac{\frac{\left(-\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{c}$$

$$\frac{Cx^3}{3c}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{\frac{\left(-\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{Ac\left(b^2-2ac\right)-bC\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{c}$$

$$\frac{1}{2}B\left(-\frac{\left(b^2-2ac\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{2c^2}+\frac{x^2}{c}\right)+\frac{Cx^3}{3c}$$

input
```
Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(C*x^3)/(3*c) - (-(((A*c - b*C)*x)/c) + (((A*b*c - b^2*C + a*c*C - (A*c*(b
^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) + ((A*b*c - b^2*C + a*c*C + (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/c + (B*(x^2/c - ((b^2
 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]
) - (b*Log[a + b*x^2 + c*x^4])/(2*c^2)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.35

method	result
risch	$\frac{C x^3}{3c}+\frac{B x^2}{2c}+\frac{Ax}{c}-\frac{Cbx}{c^2}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-Bbc\_R^3+\left(-Abc-Cac+Cb^2\right)\_R^2-Bac\_R-Aac+Cba\right)\ln\left(x\right.}{2\_R^3c+\_Rb}}{2c^2}$
default	$\frac{\frac{1}{3}cCx^3+\frac{1}{2}Bcx^2+Acx-Cbx}{c^2}+\frac{-\frac{\left(2ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}+4abc-b^3\right)\left(\frac{B\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)}{2}+\frac{\left(2Ac-C\sqrt{-4ac+b^2}-Cb\right)\sqrt{2}}{2\sqrt{\left(b+\sqrt{-}\right.}}\right.}{2c\left(4ac-b^2\right)}}{}$

input
```
int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*C*x^3/c+1/2*B*x^2/c+1/c*A*x-1/c^2*C*b*x+1/2/c^2*sum((-B*b*c*_R^3+(-A*b
*c-C*a*c+C*b^2)*_R^2-B*a*c*_R-A*a*c+C*b*a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=Roo
tOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx=\text{Timed out}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/6*(2*C*c*x^3 + 3*B*c*x^2 - 6*(C*b - A*c)*x)/c^2 - integrate((B*b*c*x^3 +
 B*a*c*x - C*a*b + A*a*c - (C*b^2 - (C*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a
), x)/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5307 vs. 2(294) = 588.

Time = 0.95 (sec) , antiderivative size = 5307, normalized size of antiderivative = 15.65

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*B*b*log(abs(c*x^4 + b*x^2 + a))/c^2 - 1/8*((2*b^5*c^3 - 16*a*b^3*c^4
+ 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2
 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3
- 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4
*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3
 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4
*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^
3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3
+ 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 -
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b
^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*...
```

Mupad [B] (verification not implemented)

Time = 18.26 (sec) , antiderivative size = 2588, normalized size of antiderivative = 7.63

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
x*(A/c - (C*b)/c^2) + symsum(log((C^3*a^4*c - C^3*a^3*b^2 - A*B^2*a^3*c^2
+ A*C^2*a^2*b^3 + A^2*C*a^3*c^2 + A^3*a^2*b*c^2 + A*B^2*a^2*b^2*c - 2*A^2*
C*a^2*b^2*c - B^2*C*a^3*b*c)/c^3 - root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4
 - 256*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*
c^3*z^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2
 + 80*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28
*A^2*a*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2
*b^3*c^2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z
^2 - 4*A^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^
2*b^3*c*z - 20*B*C^2*a^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^
2*z + 4*B^3*a^2*b^3*c*z + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*
B*a^3*c^3*z + 2*A^3*C*a^3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^
3*a^4*b - A^2*B^2*a^3*b*c - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2
- B^4*a^4*c - C^4*a^5, z, k)*(root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 25
6*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z
^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80
*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*
a*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*
c^2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 -
4*A^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1668, normalized size of antiderivative = 4.92

$$\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

output
```
(12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c - 6*
sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3 + 6*sqrt(
a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(
c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c + 12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(
a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*a*c**2 - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c + 12
*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2
*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c - 6*sqrt(c)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*a*b**2 - 18*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*
c + 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b
) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3 + 12*sqrt(2*sqrt(c)*sqr
t(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +
2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c - 6*sqrt(2*sqrt(c)*sqrt(a)
 + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3 - 6*sqrt(a)*sqrt(2*sqrt(c)*s...
```

3.22 $\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 28, antiderivative size = 278

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4} dx = \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(Abc-b^2C+2acC)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} + \frac{(Ac-bC)\log(a+bx^2+cx^4)}{4c^2}$$

output
```
B*x/c+1/2*C*x^2/c-1/2*B*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)
*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)-1/2*B*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/
2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)+1/2*(A*b*c+2*C*a*c-C*b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^
2/(-4*a*c+b^2)^(1/2)+1/4*(A*c-C*b)*ln(c*x^4+b*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.36

$$\int \frac{x^3(A + Bx + Cx^2)}{a + bx^2 + cx^4} \, dx$$
$$= \frac{4Bcx + 2cCx^2 - \frac{2\sqrt{2}B\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}B\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(4*B*c*x + 2*c*C*x^2 - (2*Sqrt[2]*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
- 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*B*Sqrt[c]*(b^2 - 2*a*c
+ b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*c*(-b + Sqrt[b
^2 - 4*a*c]) + (b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 -
4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((-(A*c*(b + Sqrt[b^2 - 4*a*c])) +
(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/Sqrt[b^2 - 4*a*c])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2193, 27, 1442, 1480, 218, 1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A + Bx + Cx^2)}{a + bx^2 + cx^4} \, dx$$
$$\Big\downarrow \text{2193}$$
$$\int \frac{x^3(Cx^2 + A)}{cx^4 + bx^2 + a}dx + \int \frac{Bx^4}{cx^4 + bx^2 + a}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x^3\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+B\int \frac{x^4}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \; 1442$$

$$\int \frac{x^3\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+B\left(\frac{x}{c}-\frac{\int \frac{bx^2+a}{cx^4+bx^2+a}dx}{c}\right)$$

$$\Big\downarrow \; 1480$$

$$\int \frac{x^3\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+ B\left(\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}\right)$$

$$\Big\downarrow \; 218$$

$$\int \frac{x^3\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+ B\left(\frac{x}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)$$

$$\Big\downarrow \; 1578$$

$$\frac{1}{2}\int \frac{x^2\left(Cx^2+A\right)}{cx^4+bx^2+a}dx^2+ B\left(\frac{x}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2}\int \left(\frac{C}{c}-\frac{aC-(Ac-bC)x^2}{c\,(cx^4+bx^2+a)}\right)dx^2+$$
$$B\left(\frac{x}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{(2acC+Abc+b^2(-C))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{2c^2}+\frac{Cx^2}{c}\right)+$$
$$B\left(\frac{x}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}\right)$$

input
```
Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
B*(x/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))
/c) + ((C*x^2)/c + ((A*b*c - b^2*C + 2*a*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b
^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((A*c - b*C)*Log[a + b*x^2 + c*x^4
])/(2*c^2))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.31

method	result
risch	$\frac{C x^2}{2c}+\frac{Bx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left((Ac-Cb)\_R^3-bB\_R^2-Ca\_R-Ba\right)\ln\left(x-\_R\right)}{2\_R^3 c+\_R b}}{2c}$
default	$\frac{\frac{1}{2}C x^2+Bx}{c}+\frac{\frac{\left(-A\sqrt{-4ac+b^2}\,bc+4A\,c^2 a-A\,b^2 c-2C\sqrt{-4ac+b^2}\,ac+C\sqrt{-4ac+b^2}\,b^2-4Cacb+b^3 C\right)\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{\left(-2B\sqrt{-4a\right.}{}}{c(4ac-b^2)}$

input
```
int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*C*x^2/c+B*x/c+1/2/c*sum(((A*c-C*b)*_R^3-b*B*_R^2-C*a*_R-B*a)/(2*_R^3*c
+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 61.04 (sec) , antiderivative size = 1329593, normalized size of antiderivative = 4782.71

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(C*x^2 + 2*B*x)/c + integrate(-(B*b*x^2 + (C*b - A*c)*x^3 + C*a*x + B*
a)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3521 vs. $2(232) = 464$.

Time = 0.86 (sec) , antiderivative size = 3521, normalized size of antiderivative = 12.67

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(C*b - A*c)*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/2*(C*c*x^2 + 2*B*c*x)
/c^2 + 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B
*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a
*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^5*c^4 - 12*a*b^3*c^5 +
 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^
3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c...
```

Mupad [B] (verification not implemented)

Time = 18.71 (sec) , antiderivative size = 2696, normalized size of antiderivative = 9.70

$$\int \frac{x^3(A + Bx + Cx^2)}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
int((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log((B^3*a^2*b*c - B*C^2*a^3*c + A^2*B*a^2*c^2 + B*C^2*a^2*b^2 - 2*
A*B*C*a^2*b*c)/c^2 - root(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6
*z^4 - 256*C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 1
6*C*b^5*c^2*z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3
*z^2 - 72*A*C*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*
B^2*a*b^3*c^2*z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2
*b^2*c^2*z^2 - 4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 -
96*A^2*a^2*c^4*z^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*
c^2*z + 12*A*C^2*a^2*b^2*c*z + 16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z -
4*A*B^2*a*b^3*c*z - 4*A*C^2*a*b^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2
*z + 16*A*C^2*a^3*c^2*z - 16*C^3*a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*
c^3*z + 2*A^3*C*a^2*b*c + 4*A*B^2*C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*
b - A^2*B^2*a^2*b*c - B^2*C^2*a^3*b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*
a^3*c - C^4*a^4, z, k)*(root(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*
c^6*z^4 - 256*C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3
- 16*C*b^5*c^2*z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*
c^3*z^2 - 72*A*C*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 +
28*B^2*a*b^3*c^2*z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*
a^2*b^2*c^2*z^2 - 4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2
 - 96*A^2*a^2*c^4*z^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1156, normalized size of antiderivative = 4.16

$$\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c + 4*s
qrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2 - 2*sqrt
(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c + 2*sqrt(a)
*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c + 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
 + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*a*b*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3 + 2*sqrt(
2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c + 4*sqrt(2*sq
rt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2 - 2*sqrt(2*sqrt(
c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c - 2*sqrt(a)*sqrt(2*
sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt
(2*sqrt(c)*sqrt(a) + b))*b**2*c - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +...
```

3.23 $\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 270

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-(b^2-2ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{bB\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B\log(a+bx^2+cx^4)}{4c}$$

output

```
C*x/c+1/2*(A*c-C*b-(A*b*c-(-2*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/
2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2)+1/2*(A*c-C*b+(A*b*c+2*C*a*c-C*b^2)/(-4*a*c+b^2)^(1/2))*arcta
n(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)+1/2*b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)+1/4*B*ln(c*x^4+b*x^2+a)/c
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.33

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$= \frac{4\sqrt{c}Cx - \frac{2\sqrt{2}\left(Ac\left(b-\sqrt{b^2-4ac}\right)+\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\left(-Ac\left(b+\sqrt{b^2-4ac}\right)+\left(b^2-2ac+b\sqrt{b^2-4\right.\right.}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-}}}}{4c^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(4*Sqrt[c]*C*x - (2*Sqrt[2]*(A*c*(b - Sqrt[b^2 - 4*a*c]) + (-b^2 + 2*a*c +
 b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*(-(A*
c*(b + Sqrt[b^2 - 4*a*c])) + (b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + (B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*Log[-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*Sqrt[c]*(b + Sqrt[b^2 -
 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^(3/
2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2193, 27, 1434, 1142, 1083, 219, 1103, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 2193$$

$$\int \frac{x^2(Cx^2+A)}{cx^4+bx^2+a}dx + \int \frac{Bx^3}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+B\int \frac{x^3}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\int \frac{x^2}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 1142}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\left(\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{c}+\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\left(\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}+\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\int \frac{x^2\left(Cx^2+A\right)}{cx^4+bx^2+a}dx+\frac{1}{2}B\left(\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{2c}\right)$$

$$\Big\downarrow \text{ 1602}$$

$$-\frac{\int \frac{aC-(Ac-bC)x^2}{cx^4+bx^2+a}dx}{c}+\frac{1}{2}B\left(\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{2c}\right)+\frac{Cx}{c}$$

$$\Big\downarrow \text{ 1480}$$

$$-\frac{-\frac{1}{2}\left(-\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{Abc-C\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}+Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}}{c}$$

$$\frac{1}{2}B\left(\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{2c}\right)+\frac{Cx}{c}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{-\frac{\left(-\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{Abc-C\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}+Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}+\frac{1}{2}B\left(\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{2c}\right)+\frac{Cx}{c}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(C*x)/c - (-(((A*c - b*C - (A*b*c - b^2*C + 2*a*c*C)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]])) - ((A*c - b*C + (A*b*c - (b^2 - 2*a*c)*C)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c + (B*((b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a + b*x^2 + c*x^4]/(2*
c)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.26

method	result
risch	$\frac{Cx}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(Bc\_R^3+(Ac-Cb)\_R^2-aC\right)\ln\left(x-\_R\right)}{2\_R^3c+\_Rb}}{2c}$
default	$\frac{Cx}{c}-\frac{\left(b^2-4ac+b\sqrt{-4ac+b^2}\right)\left(\frac{B\ln\left(2cx^2+\sqrt{-4ac+b^2}+b\right)}{2}+\frac{\left(2Ac-C\sqrt{-4ac+b^2}-Cb\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c(4ac-b^2)}+\frac{\left(b\sqrt{-4\cdots\right.}{\cdots}$

input
```
int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
C*x/c+1/2/c*sum((B*c*_R^3+(A*c-C*b)*_R^2-a*C)/(2*_R^3*c+_R*b)*ln(x-_R),_R=
RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 57.84 (sec) , antiderivative size = 861800, normalized size of antiderivative = 3191.85

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
C*x/c + integrate((B*c*x^3 - (C*b - A*c)*x^2 - C*a)/(c*x^4 + b*x^2 + a), x
)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3845 vs. $2(227) = 454$.

Time = 0.96 (sec) , antiderivative size = 3845, normalized size of antiderivative = 14.24

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
C*x/c + 1/4*B*log(abs(c*x^4 + b*x^2 + a))/c - 1/8*((2*b^4*c^3 - 16*a*b^2*c
^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 -
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)
*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a
^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 +
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^
2 + 8*(b^2 - 4*a*c)*a*b*c^3)*C*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqr...
```

Mupad [B] (verification not implemented)

Time = 19.48 (sec) , antiderivative size = 1890, normalized size of antiderivative = 7.00

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log(- root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 1
28*B*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c
^2*z^2 + 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 2
8*C^2*a*b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*
z^2 - 96*B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b
^2*c*z + 16*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3
*a*b^2*c*z + 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*
a*b*c - 2*A^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 -
B^4*a^2*c - A^4*a*c^2 - C^4*a^3, z, k)*((8*B*C*a^2*c^2 - 4*A*B*a*b*c^2)/c
- root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B*a*b^2*
c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^2 + 8*
A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*a*b^3
*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 96*B^
2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z + 16
*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*z
+ 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*A
^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c -
 A^4*a*c^2 - C^4*a^3, z, k)*((16*C*a^2*c^3 - 4*C*a*b^2*c^2)/c + (x*(8*B*b^
3*c^2 - 32*B*a*b*c^3))/c - (root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*
a^2*c^5*z^4 - 128*B*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1112, normalized size of antiderivative = 4.12

$$\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

output

```
(2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2 - 4*sq
rt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(
a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*b*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b + 4*sqrt(c
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*b**2 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b
))*b**2 + 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c - 2*sqrt(a)*sqrt(2
*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqr
t(2*sqrt(c)*sqrt(a) + b))*b*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a*b - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c + 2*sqrt(c)*sqrt(2*sqr
t(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt...
```

3.24 $\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 223

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = -\frac{B\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{B\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{(2Ac-bC)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{C\log(a+bx^2+cx^4)}{4c}$$

output
```
-1/2*B*(b-(-4*a*c+b^2)^(1/2))^(1/2)*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^
2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*B*(b+(-4*a*c+b^2)^
(1/2))^(1/2)*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2
)/c^(1/2)/(-4*a*c+b^2)^(1/2)-1/2*(2*A*c-C*b)*arctanh((2*c*x^2+b)/(-4*a*c+b
^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)+1/4*C*ln(c*x^4+b*x^2+a)/c
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \frac{-2\sqrt{2}B\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+2\sqrt{2}B\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)+(}{}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
(-2*Sqrt[2]*B*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]] + 2*Sqrt[2]*B*Sqrt[c]*Sqrt[b + Sqrt[b^2 -
4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]] + (2*A*c +
 (-b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - (2*A*
c - (b + Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c*
Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2193, 27, 1450, 218, 1576, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 2193}$$

$$\int \frac{x(Cx^2+A)}{cx^4+bx^2+a}dx+\int \frac{Bx^2}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x(Cx^2+A)}{cx^4+bx^2+a}dx+B\int \frac{x^2}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \ 1450$$

$$\int \frac{x(Cx^2+A)}{cx^4+bx^2+a}dx+ B\left(\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx\right)$$

$$\Big\downarrow \ 218$$

$$\int \frac{x(Cx^2+A)}{cx^4+bx^2+a}dx+ B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \ 1576$$

$$\frac{1}{2}\int \frac{Cx^2+A}{cx^4+bx^2+a}dx^2+ B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{2}\left(\frac{(2Ac-bC)\int \frac{1}{cx^4+bx^2+a}dx^2}{2c}+\frac{C\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}\right)+ B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{2}\left(\frac{C\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{(2Ac-bC)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{c}\right)+ B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left(\frac{C\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)+$$
$$B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$\Big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{C\log\left(a+bx^2+cx^4\right)}{2c}-\frac{(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)+$$
$$B\left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

input
```
Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
B*(((1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((1 + b/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])) + (-(((2*A*c - b*C)*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c])) + (C*Log[a + b*x^2
+ c*x^4])/(2*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(C\_R^3+B\_R^2+A\_R\right)\ln\left(x-\_R\right)}{2\_R^3 c+\_R b}\right)}{2}$
default	$4c\left(\frac{\frac{\left(2A\sqrt{-4ac+b^2}\,c-C\sqrt{-4ac+b^2}\,b+4Cac-C\,b^2\right)\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{\left(-B\sqrt{-4ac+b^2}\,b+4Bac-B\,b^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right.}}\right.}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{4c(4ac-b^2)}\right.$

input
```
int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((C*_R^3+B*_R^2+A*_R)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^
2*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.63 (sec) , antiderivative size = 845032, normalized size of antiderivative = 3789.38

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^2+Bx+A)x}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2370 vs. $2(179) = 358$.

Time $= 0.83$ (sec) , antiderivative size $= 2370$, normalized size of antiderivative $= 10.63$

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*C*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*
a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(
b^2 - 4*a*c)*a*c^3)*B*c^2 - (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*
sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^2 - 8*a^2
*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)
*c^2) - 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s...
```

Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 5594, normalized size of antiderivative = 25.09

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log(A^3*c^2*x - B^3*a*c - B*C^2*a*b - 8*root(128*a*b^2*c^3*z^4 - 16
*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 +
 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^
2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c
^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^
2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2
*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A
^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2
 - C^4*a^2 - A^4*c^2, z, k)^3*b^3*c^2*x - C^3*a*b*x + A*C^2*b^2*x - 2*C^2*
root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^
2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^
3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A
^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16
*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16
*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*
C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*
b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b^3*x + 32*root(128
*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 +
256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2
+ 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 697, normalized size of antiderivative = 3.13

$$\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

output

```
( - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c + 2*
sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c - 4*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*b**2 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b
))*a*c + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((s
qrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c
 + 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c - 2*sqrt(c)*sqrt(2*sqrt(c
)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqr
t(c)*sqrt(a) + b))*b**2 + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqr
t(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b*c - 2*sqrt(a)*sqrt(
2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(
c)*x**2)*b*c + sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*s
qrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**2 - sqrt(c)*sqrt(2*sqrt(c)*sqrt
(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b*...
```

3.25 $\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{B\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output

```
1/2*(C+(2*A*c-C*b)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*(C-(2
*A*c-C*b)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/
2))^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-B*arctanh((2*c*x^2
+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx$$
$$= \frac{\frac{\sqrt{2}\left(2Ac+\left(-b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2Ac+\left(b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}+B\log\left(-b+\sqrt{b^2}\right.}{2\sqrt{b^2-4ac}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

output
```
((Sqrt[2]*(2*A*c + (-b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*(-2*A*c + (b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + B*Log[-b
+ Sqrt[b^2 - 4*a*c] - 2*c*x^2] - B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
2*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2202, 27, 1432, 1083, 219, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}dx$$
$$\Big\downarrow \text{ 2202}$$
$$\int \frac{Cx^2+A}{cx^4+bx^2+a}dx+\int \frac{Bx}{cx^4+bx^2+a}dx$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{Cx^2+A}{cx^4+bx^2+a}dx+B\int \frac{x}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \; 1432$$

$$\int \frac{Cx^2+A}{cx^4+bx^2+a}dx+\frac{1}{2}B\int \frac{1}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \; 1083$$

$$\int \frac{Cx^2+A}{cx^4+bx^2+a}dx-B\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)$$

$$\Big\downarrow \; 219$$

$$\int \frac{Cx^2+A}{cx^4+bx^2+a}dx-\frac{B\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 1480$$

$$\frac{1}{2}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx-\frac{B\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 218$$

$$\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

output
```
((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((C
- (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(C\_R^2+B\_R+A\right)\ln\left(x-\_R\right)}{2\_R^3 c+\_R b}\right)}{2}$
default	$4c\left(\frac{\sqrt{-4ac+b^2}\left(\frac{B\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{2}+\frac{\left(2Ac-C\sqrt{-4ac+b^2}-Cb\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4c(4ac-b^2)}-\frac{\sqrt{-4ac+b^2}\left(\frac{B\ln\left(-2\right.}{}\right.}{}\right.$

input
```
int((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((C*_R^2+B*_R+A)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.82 (sec) , antiderivative size = 578003, normalized size of antiderivative = 2739.35

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx=\int \frac{Cx^2+Bx+A}{cx^4+bx^2+a}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1712 vs. $2(171) = 342$.

Time = 1.72 (sec) , antiderivative size = 1712, normalized size of antiderivative = 8.11

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*B*log(x^2 + 1/2*
(b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*
b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*
b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8
*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2
*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*A - 2*(2*a*b^2*c^2 - 8
*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a
*c^2)*C)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8
*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)
*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2...
```

Mupad [B] (verification not implemented)

Time = 19.97 (sec) , antiderivative size = 3942, normalized size of antiderivative = 18.68

$$\int \frac{A + Bx + Cx^2}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log(A*B^2*c^2 - A^2*C*c^2 + B^3*c^2*x - C^3*a*c + A*C^2*b*c - 8*roo
t(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*
z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C
*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16
*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*
A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^
2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^3*b^3*c^2*x -
 16*A*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C
*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2
 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2
*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c
^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2
*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*a*
c^3 - 4*A^2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 -
16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c
^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a
^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2
*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B
^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k
)*c^3*x + 4*A*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 786, normalized size of antiderivative = 3.73

$$\int \frac{A + Bx + Cx^2}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

output
```
( - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b + 2*sq
rt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a)
 + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*c - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a + 2*sqrt(c)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*b - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(
c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sq
rt(c)*sqrt(a) + b))*b - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b + 4*sq
rt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*c + 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
 + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*a - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b - sqrt(a)*sqrt(2
*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqr
t(c)*x**2)*b + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)
*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*c + sqrt(a)*sqrt(2*sqrt(c)*sq...
```

3.26 $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 229

$$\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx = \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(Ab-2aC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{A\log(x)}{a} - \frac{A\log(a+bx^2+cx^4)}{4a}$$

output
```
2^(1/2)*B*c^(1/2)*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))/(
-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2^(1/2)*B*c^(1/2)*arctan(2^
(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2)+1/2*(A*b-2*C*a)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2)
)/a/(-4*a*c+b^2)^(1/2)+A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx = \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a} - \frac{\left(A\left(b+\sqrt{b^2-4ac}\right)-2aC\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{4a\sqrt{b^2-4ac}} - \frac{\left(A\left(-b+\sqrt{b^2-4ac}\right)+2aC\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{4a\sqrt{b^2-4ac}}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[b + Sqrt[b^2 - 4*a*c]]) + (A*Log[x])/a - ((A*(b + Sqrt[b^2 - 4*a*c]) -
 2*a*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c]) - (
(A*(-b + Sqrt[b^2 - 4*a*c]) + 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])
/(4*a*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2193, 27, 1406, 218, 1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \; 2193$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)}dx+\int \frac{B}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)}dx+B\int \frac{1}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \; 1406$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)}dx+B\left(\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \; 218$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)}dx+B\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \; 1578$$

$$\frac{1}{2}\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx^2+ B\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2}\int \left(\frac{A}{ax^2}+\frac{-Acx^2-Ab+aC}{a\left(cx^4+bx^2+a\right)}\right)dx^2+ B\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{(Ab-2aC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{2a}+\frac{A\log\left(x^2\right)}{a}\right)+ B\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

input
```
Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
B*((Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])) + (((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b
^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + (A*Log[x^2])/a - (A*Log[a + b*x^2 +
c*x^4])/(2*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

method	result
default	$\frac{A\ln(x)}{a}+\frac{4c\left(\frac{\sqrt{-4ac+b^2}\left(\frac{\left(\sqrt{-4ac+b^2}\,A-Ab+2aC\right)\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{Ba\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{16ac-4b^2}+\frac{\sqrt{-4ac+b^2}\left(-\right.}{}\right.}{a}$
risch	$\frac{A\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^4c^2-8a^3b^2c+a^2b^4\right)\_Z^4+\left(32A\,a^3c^2-16A\,a^2b^2c+2Aa\,b^4\right)\_Z^3+\left(24a^2c^2A^2-10a\,b^2c\,A^2+b^4A^2-8AC\,a^2bc+\right.\right.}\right.}{}$

input
```
int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a+4/a*c*((-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*(1/4*((-4*a*c+b^2)^(1/2
)*A-A*b+2*a*C)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+B*a*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
+(-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*(-1/4*(-(-4*a*c+b^2)^(1/2)*A-A*b+2*a*C)
/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx=\int \frac{Cx^2+Bx+A}{(cx^4+bx^2+a)x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
A*log(x)/a - integrate((A*c*x^3 - B*a - (C*a - A*b)*x)/(c*x^4 + b*x^2 + a)
, x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2338 vs. $2(186) = 372$.

Time = 0.82 (sec) , antiderivative size = 2338, normalized size of antiderivative = 10.21

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*A*log(abs(c*x^4 + b*x^2 + a))/a + A*log(abs(x))/a + 1/4*((sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 1
6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^2 + 16*a*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*
a^2*c^3 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2)*B*abs(c) + (2*b^3
*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^
2 - 4*a*c)*b*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c + sqrt(a^4*b^2*c^2
 - 4*a^5*c^3))/(a^2*c^2)))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2
+ 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
- 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*
b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 - 2
*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2)*B*abs(c) - (2*b^3*c^3 - 8...
```

Mupad [B] (verification not implemented)

Time = 18.68 (sec) , antiderivative size = 2258, normalized size of antiderivative = 9.86

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x)
```

output
```
symsum(log(x*(B^4*c^3 + C^4*a*c^2 + A^2*C^2*c^3 - 3*A*B^2*C*c^3 - A*C^3*b*
c^2 + B^2*C^2*b*c^2) - root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b
^4*z^4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C
*a^2*b*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 -
 32*C^2*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z
^2 - 4*A^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*
z - 4*B^2*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4
*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b -
 B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z,
 k)*(x*(A*B^2*b*c^3 - 5*A^3*c^4 - 13*A*C^2*a*c^3 + 6*A^2*C*b*c^3 + 17*B^2*
C*a*c^3 + C^3*a*b*c^2 + A*C^2*b^2*c^2 - 4*B^2*C*b^2*c^2) - root(128*a^3*b^
2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*A*a^2*b^2*c*z^3 - 256*A*a
^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^
2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z^2 - 4*B^2*a*b^3*z^2 +
8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z^2 + 16*A^2*C*a*b*c*z
+ 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2*z + 4*A*C^2*a*b^2*z
+ 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C
^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c -
 A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*(x*(60*A^2*a*c^4 - 16*A^2*b^2*c^3
+ 4*B^2*b^3*c^2 + 36*C^2*a^2*c^3 + 8*A*C*b^3*c^2 - 14*B^2*a*b*c^3 - 10*...
```

Reduce [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 711, normalized size of antiderivative = 3.10

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b - 4*sqr
t(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c + 2*sqrt(a)*s
qrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x
)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2 - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a*b + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a*b - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c -
2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +
2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2 + 4*sqrt(c)*sqrt(2*sqrt(c)*
sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a*b - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*s
qrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**2 + sqrt(a)*sqrt(2*sqrt
(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**
2)*b**2 - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt
(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a
) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b ...
```

3.27 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 260

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)}\,dx = -\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}} + \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{B\log(x)}{a}-\frac{B\log\left(a+bx^2+cx^4\right)}{4a}$$

output
```
-A/a/x-1/2*c^(1/2)*(A+(A*b-2*C*a)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/
2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b-(-4*a*c+b^2)^(1/2))^(1/2)-
1/2*c^(1/2)*(A-(A*b-2*C*a)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b
+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)+1/2*b*B
*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+B*ln(x)/a-1/
4*B*ln(c*x^4+b*x^2+a)/a
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx =$$

$$-\frac{\frac{4A}{x}+\frac{2\sqrt{2}\sqrt{c}\left(A\left(b+\sqrt{b^2-4ac}\right)-2aC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{2\sqrt{2}\sqrt{c}\left(A\left(-b+\sqrt{b^2-4ac}\right)+2aC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}-4B\,\mathrm{l}}{4a}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/4*((4*A)/x + (2*Sqrt[2]*Sqrt[c]*(A*(b + Sqrt[b^2 - 4*a*c]) - 2*a*C)*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (2*Sqrt[2]*Sqrt[c]*(A*(-b + Sqrt[b^2 - 4*a*c
]) + 2*a*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - 4*B*Log[x] + (B*(b + Sqrt[b^2
 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*(
-b + Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4
*a*c])/a
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2193, 27, 1434, 1144, 25, 1142, 1083, 219, 1103, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\int \frac{B}{x\left(cx^4+bx^2+a\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+B\int \frac{1}{x\left(cx^4+bx^2+a\right)}dx$$

$$\Big\downarrow \ 1434$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\int \frac{1}{x^2\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \ 1144$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\int -\frac{cx^2+b}{cx^4+bx^2+a}dx^2}{a}+\frac{\log\left(x^2\right)}{a}\right)$$

$$\Big\downarrow \ 25$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\int \frac{cx^2+b}{cx^4+bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \ 1142$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^4+bx^2+a}dx^2+\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \ 1083$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2-b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{a}\right)$$

$$\Big\downarrow \ 219$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2-\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \ 1103$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)}dx+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^2+cx^4\right)-\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \ 1604$$

$$-\frac{\int \frac{Acx^2+Ab-aC}{cx^4+bx^2+a}dx}{a}-\frac{A}{ax}+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^2+cx^4\right)-\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$\Big\downarrow$ 1480

$$-\frac{\frac{1}{2}c\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{A}{ax}+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^2+cx^4\right)-\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$\Big\downarrow$ 218

$$-\frac{\frac{\sqrt{c}\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{A}{ax}+\frac{1}{2}B\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^2+cx^4\right)-\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-(A/(a*x)) - ((Sqrt[c]*(A + (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[c]*(A - (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/a + (B*(Log[x^2]/a - (-((b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
Sqrt[b^2 - 4*a*c]) + Log[a + b*x^2 + c*x^4]/2)/a))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.25

method	result
default	$-\frac{A}{ax}+\frac{B\ln(x)}{a}+\frac{4c\left(\frac{\frac{\left(-B\sqrt{-4ac+b^2}\,b-4Bac+B\,b^2\right)\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{\left(-A\sqrt{-4ac+b^2}\,b-4Aac+A\,b^2+2C\sqrt{-4ac+b^2}\,a\right)\sqrt{2}\,\mathrm{a}}{2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{16ac-4b^2}\right.}{}$
risch	$-\frac{A}{ax}+\frac{B\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^5c^2-8a^4b^2c+b^4a^3\right)\_Z^4+\left(32B\,a^4c^2-16B\,a^3b^2c+2B\,a^2b^4\right)\_Z^3+\left(12A^2a^2b\,c^2-7A^2a\,b^3c+A^2b^5\right.\right.}\right.}{}$

input
```
int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-A/a/x+B*ln(x)/a+4/a*c*(1/(16*a*c-4*b^2)*(1/4*(-B*(-4*a*c+b^2)^(1/2)*b-4*B
*a*c+B*b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+1/2*(-A*(-4*a*c+b^2)^(1/2)*
b-4*A*a*c+A*b^2+2*C*(-4*a*c+b^2)^(1/2)*a)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4
*b^2)*(-1/4*(-B*(-4*a*c+b^2)^(1/2)*b+4*B*a*c-B*b^2)/c*ln(-2*c*x^2+(-4*a*c+
b^2)^(1/2)-b)+1/2*(-A*(-4*a*c+b^2)^(1/2)*b+4*A*a*c-A*b^2+2*C*(-4*a*c+b^2)^
(1/2)*a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\int \frac{Cx^2+Bx+A}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
B*log(x)/a - integrate((B*c*x^3 + A*c*x^2 + B*b*x - C*a + A*b)/(c*x^4 + b*
x^2 + a), x)/a - A/(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3510 vs. $2(218)=436$.

Time = 1.02 (sec) , antiderivative size = 3510, normalized size of antiderivative = 13.50

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*B*log(abs(c*x^4 + b*x^2 + a))/a + B*log(abs(x))/a - A/(a*x) - 1/8*((2
*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 -
2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*c^2 + 2*(sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^
2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*
A*abs(c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*s...
```

Mupad [B] (verification not implemented)

Time = 18.60 (sec) , antiderivative size = 2588, normalized size of antiderivative = 9.95

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

output
```
symsum(log(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128
*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c
*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*
C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^
2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2
*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a
*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c - B^2*C^2*a*
b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2*b*c^2 - C^
4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2
*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2
*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 -
 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a
^3*c^2*z^2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*
A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z
+ 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 +
 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2
*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*(root(128*
a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 2
56*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z
^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 +...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{C\,x^2+Bx+A}{x^2\,(c\,x^4+b\,x^2+a)}dx$$

input
```
int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x)
```

output
```
int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x)
```

3.28 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 288

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx = -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(A(b^2-2ac)-abC)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} - \frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\log(a+bx^2+cx^4)}{4a^2}$$

output
```
-1/2*A/a/x^2-B/a/x-1/2*B*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c
^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b-(-4*a*c+b^2)^(1/2))^(1
/2)-1/2*B*c^(1/2)*(1-b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)-1/2*(A*(-2*
a*c+b^2)-a*b*C)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(
1/2)-(A*b-C*a)*ln(x)/a^2+1/4*(A*b-C*a)*ln(c*x^4+b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx$$
$$= \frac{-\frac{2aA}{x^2}-\frac{4aB}{x}-\frac{2\sqrt{2}aB\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}aB\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}+4(-Ab+$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a*A)/x^2 - (4*a*B)/x - (2*Sqrt[2]*a*B*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*
c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*a*B*Sqrt[c]*(-b + Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*(-(A*b) + a*C)*Log[x] + ((A*(b^
2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]) - a*(b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + S
qrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((A*(-b^2 + 2*a*c + b*Sqr
t[b^2 - 4*a*c]) + a*(b - Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {2193, 27, 1443, 25, 1480, 218, 1578, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)}\,dx$$
$$\Big\downarrow \; 2193$$
$$\int \frac{Cx^2+A}{x^3(cx^4+bx^2+a)}dx+\int \frac{B}{x^2(cx^4+bx^2+a)}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)}dx+B\int \frac{1}{x^2\left(cx^4+bx^2+a\right)}dx$$

$$\Big\downarrow \text{1443}$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)}dx+B\left(\frac{\int -\frac{cx^2+b}{cx^4+bx^2+a}dx}{a}-\frac{1}{ax}\right)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)}dx+B\left(-\frac{\int \frac{cx^2+b}{cx^4+bx^2+a}dx}{a}-\frac{1}{ax}\right)$$

$$\Big\downarrow \text{1480}$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)}dx+$$
$$B\left(-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{1}{ax}\right)$$

$$\Big\downarrow \text{218}$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)}dx+$$
$$B\left(-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax}\right)$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{Cx^2+A}{x^4\left(cx^4+bx^2+a\right)}dx^2+$$
$$B\left(-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\int \left(\frac{A}{ax^4}+\frac{c(Ab-aC)x^2+A(b^2-ac)-abC}{a^2(cx^4+bx^2+a)}+\frac{aC-Ab}{a^2x^2}\right)dx^2+$$
$$B\left(-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax}\right)$$

$\downarrow$ 2009

$$\frac{1}{2}\left(-\frac{(A(b^2-2ac)-abC)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{(Ab-aC)\log(a+bx^2+cx^4)}{2a^2}-\frac{\log(x^2)(Ab-aC)}{a^2}-\frac{A}{ax^2}\right)+$$
$$B\left(-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax}\right)$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
B*(-(1/(a*x)) - ((Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
 (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a) + (-(A/(a*x^2
)) - ((A*(b^2 - 2*a*c) - a*b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*C)*Log[x^2])/a^2 + ((A*b - a*C)*Log[a
+ b*x^2 + c*x^4])/(2*a^2))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.04

method	result
default	$-\frac{B}{ax}-\frac{A}{2a\,x^{2}}+\frac{(-Ab+aC)\ln(x)}{a^{2}}+\frac{4c\left(-\frac{\left(b\sqrt{-4ac+b^{2}}+4ac-b^{2}\right)\left(\frac{\left(\sqrt{-4ac+b^{2}}\,A-Ab+2aC\right)\ln\left(2c\,x^{2}+\sqrt{-4ac+b^{2}}+b\right)}{4c}+\frac{Ba\sqrt{2}\,\mathrm{arct}}{\sqrt{}}\right.}{32ac-8b^{2}}\right.}{}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-B/a/x-1/2*A/a/x^2+1/a^2*(-A*b+C*a)*ln(x)+4/a^2*c*(-(b*(-4*a*c+b^2)^(1/2)+
4*a*c-b^2)/(32*a*c-8*b^2)*(1/4*((-4*a*c+b^2)^(1/2)*A-A*b+2*a*C)/c*ln(2*c*x
^2+(-4*a*c+b^2)^(1/2)+b)+B*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-(b^2-4*a*c+b*(-4*a*c+b^2
)^(1/2))/(32*a*c-8*b^2)*(-1/4*(-(-4*a*c+b^2)^(1/2)*A-A*b+2*a*C)/c*ln(-2*c*
x^2+(-4*a*c+b^2)^(1/2)-b)+B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \int \frac{Cx^2+Bx+A}{(cx^4+bx^2+a)x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
(C*a - A*b)*log(x)/a^2 + integrate(-(B*a*c*x^2 + (C*a - A*b)*c*x^3 + B*a*b
 + (C*a*b - A*b^2 + A*a*c)*x)/(c*x^4 + b*x^2 + a), x)/a^2 - 1/2*(2*B*x + A
)/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3355 vs. $2(240) = 480$.

Time = 0.97 (sec) , antiderivative size = 3355, normalized size of antiderivative = 11.65

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(C*a - A*b)*log(abs(c*x^4 + b*x^2 + a))/a^2 + (C*a - A*b)*log(abs(x))
/a^2 - 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 + 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*
a*c)*a*b*c^3)*B*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*...
```

Mupad [B] (verification not implemented)

Time = 18.32 (sec) , antiderivative size = 3563, normalized size of antiderivative = 12.37

$$\int \frac{A + Bx + Cx^2}{x^3 (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

output
```
symsum(log(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128
*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c
^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*
A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*
c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z
^2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3
*c^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 1
2*A^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*
b^3*c*z + 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*
C*a^2*c^3*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*
A*C^3*a*b*c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^
2*a*b*c^2 - A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^
4*c^4, z, k)*(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 +
128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^
5*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 -
72*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3
*b*c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^
2*z^2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*
a^3*c^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z
- 12*A^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1279, normalized size of antiderivative = 4.44

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x)
```

output

```
(4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c*x**2
 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*x*
*2 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*x
**2 + 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*x**2 - 2*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*x**2 + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
+ b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a*b**2*x**2 + 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a
) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*a**2*c*x**2 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt
(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*s
qrt(a) + b))*a*b**2*x**2 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sq
rt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)
*sqrt(a) + b))*a*b*c*x**2 - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*
c*x**2 + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqr...
```

3.29 $\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 424

$$\begin{aligned}\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = &-\frac{Bx(ab+(b^2-2ac)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\ &+\frac{x^2(a(2Ac-bC)+(Abc-b^2C+2acC)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\ &-\frac{B\left(6a-\frac{b^2}{c}+\frac{b(b^2-8ac)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}} \\ &-\frac{B\left(6a-\frac{b^2}{c}-\frac{b(b^2-8ac)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}} \\ &+\frac{(b^3C+2ac(2Ac-3bC))\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\,(b^2-4ac)^{3/2}} \\ &+\frac{C\log\left(a+bx^2+cx^4\right)}{4c^2}\end{aligned}$$

output
```
-1/2*B*x*(a*b+(-2*a*c+b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x^2*(a*
(2*A*c-C*b)+(A*b*c+2*C*a*c-C*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*
B*(6*a-b^2/c+b*(-8*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x
/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2
)^(1/2))^(1/2)-1/4*B*(6*a-b^2/c-b*(-8*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))*arcta
n(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+
b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+1/2*(b^3*C+2*a*c*(2*A*c-3*C*b))*arctanh(
(2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*C*ln(c*x^4+b*x^
2+a)/c^2
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.18

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{-\frac{2(2a^2cC+b^2x^2(Ac-bC+Bcx)+a(-b^2C-2Bc^2x^3+bcx(B+3Cx)+Ac(b-2cx^2)))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}B\sqrt{c}\left(-b^3+8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)\text{arct}}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(x^5*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((-2*(2*a^2*c*C + b^2*x^2*(A*c - b*C + B*c*x) + a*(-(b^2*C) - 2*B*c^2*x^3
+ b*c*x*(B + 3*C*x) + A*c*(b - 2*c*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (Sqrt[2]*B*Sqrt[c]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*B*Sqrt[c]*(
b^3 - 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]) + ((b^2*(-b + Sqrt[b^2 - 4*a*c])*C - 2*a*c*(2*A*c
- 3*b*C + 2*Sqrt[b^2 - 4*a*c]*C))*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(
b^2 - 4*a*c)^(3/2) + ((b^2*(b + Sqrt[b^2 - 4*a*c])*C + 2*a*c*(2*A*c - 3*b*
C - 2*Sqrt[b^2 - 4*a*c]*C))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4
*a*c)^(3/2))/(4*c^2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {2193, 27, 1440, 1578, 1233, 25, 1142, 1083, 219, 1103, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{x^5 (Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\int \frac{Bx^6}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^5 (Cx^2+A)}{(cx^4+bx^2+a)^2}dx+B\int \frac{x^6}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{1440}$$

$$\int \frac{x^5 (Cx^2+A)}{(cx^4+bx^2+a)^2}dx+B\left(\frac{x^3 (2a+bx^2)}{2 (b^2-4ac) (a+bx^2+cx^4)}-\frac{\int \frac{x^2 (bx^2+6a)}{cx^4+bx^2+a}dx}{2 (b^2-4ac)}\right)$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^4 (Cx^2+A)}{(cx^4+bx^2+a)^2}dx^2+B\left(\frac{x^3 (2a+bx^2)}{2 (b^2-4ac) (a+bx^2+cx^4)}-\frac{\int \frac{x^2 (bx^2+6a)}{cx^4+bx^2+a}dx}{2 (b^2-4ac)}\right)$$

$$\Big\downarrow \text{1233}$$

$$\frac{1}{2}\left(\frac{\int -\frac{a(2Ac-bC)-(b^2-4ac)Cx^2}{cx^4+bx^2+a}dx^2}{c (b^2-4ac)}+\frac{x^2 (x^2 (2acC+Abc+b^2(-C))+a(2Ac-bC))}{c (b^2-4ac) (a+bx^2+cx^4)}\right)+B\left(\frac{x^3 (2a+bx^2)}{2 (b^2-4ac) (a+bx^2+cx^4)}-\frac{\int \frac{x^2 (bx^2+6a)}{cx^4+bx^2+a}dx}{2 (b^2-4ac)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{x^2\left(x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{a(2Ac-bC)-\left(b^2-4ac\right)Cx^2}{cx^4+bx^2+a}dx^2}{c\left(b^2-4ac\right)}\right)+$$
$$B\left(\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{x^2\left(bx^2+6a\right)}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}\right)$$

$\downarrow$ 1142

$$\frac{1}{2}\left(\frac{x^2\left(x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{\left(2ac(2Ac-3bC)+b^3C\right)\int\frac{1}{cx^4+bx^2+a}dx^2}{2c}-\frac{C\left(b^2-4ac\right)\int\frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}}{c\left(b^2-4ac\right)}\right.$$
$$B\left(\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{x^2\left(bx^2+6a\right)}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}\right)$$

$\downarrow$ 1083

$$\frac{1}{2}\left(\frac{x^2\left(x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{\left(2ac(2Ac-3bC)+b^3C\right)\int\frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{c}-\frac{C\left(b^2-4ac\right)\int\frac{2c}{cx^4}}{2c}}{c\left(b^2-4ac\right)}\right.$$
$$B\left(\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{x^2\left(bx^2+6a\right)}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}\right)$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{x^2\left(x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{C\left(b^2-4ac\right)\int\frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{\left(2ac(2Ac-3bC)+b^3C\right)\text{arctanh}\left(\frac{b+}{\sqrt{b}}\right.}{c\sqrt{b^2-4ac}}}{c\left(b^2-4ac\right)}\right.$$
$$B\left(\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{x^2\left(bx^2+6a\right)}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}\right)$$

$\downarrow$ 1103

$$B\left(\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{x^2\left(bx^2+6a\right)}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}\right)+$$
$$\frac{1}{2}\left(\frac{x^2\left(x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{\left(2ac(2Ac-3bC)+b^3C\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}-\frac{C\left(b^2-4ac\right)\log\left(a+bx\right.}{2c}}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{1602}$$

$$B\left(\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx}{c}-\frac{\int\frac{\left(b^2-6ac\right)x^2+ab}{cx^4+bx^2+a}dx}{c}}{2\left(b^2-4ac\right)}\right)+$$

$$\frac{1}{2}\left(\frac{x^2\left(x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{(2ac(2Ac-3bC)+b^3C)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}-\frac{C\left(b^2-4ac\right)\log(a+bx}{2c}}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{1480}$$

$$B\left(\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx}{c}-\frac{\frac{1}{2}\left(-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\int\frac{}{cx^2+\frac{1}{2}\left(b+\right.}}{c}}{2\left(b^2-4ac\right)}\right.$$

$$\frac{1}{2}\left(\frac{x^2\left(x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{(2ac(2Ac-3bC)+b^3C)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}-\frac{C\left(b^2-4ac\right)\log(a+bx}{2c}}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{x^2\left(x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{(2ac(2Ac-3bC)+b^3C)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}-\frac{C\left(b^2-4ac\right)\log(a+bx}{2c}}{c\left(b^2-4ac\right)}\right.$$

$$B\left(\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx}{c}-\frac{\frac{\left(-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+}}\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{2\left(b^2-4ac\right)}\right.$$

input
```
Int[(x^5*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
B*((x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b*x)/c -
(((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 -
4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/c)/(2*(b^2 - 4*a*c))) + ((x^2*(a*(2*A*c - b*C) + (A*b*
c - b^2*C + 2*a*c*C)*x^2))/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (-(((b^
3*C + 2*a*c*(2*A*c - 3*b*C))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c*
Sqrt[b^2 - 4*a*c])) - ((b^2 - 4*a*c)*C*Log[a + b*x^2 + c*x^4])/(2*c))/(c*(
b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.63

method	result
risch	$\frac{-\frac{\left(2ac-b^{2}\right)B\,x^{3}}{2c\left(4ac-b^{2}\right)}-\frac{\left(2A\,c^{2}a-A\,b^{2}c-3Cacb+b^{3}C\right)x^{2}}{2c^{2}\left(4ac-b^{2}\right)}+\frac{axBb}{2c\left(4ac-b^{2}\right)}+\frac{a\left(Abc+2Cac-C\,b^{2}\right)}{2\left(4ac-b^{2}\right)c^{2}}}{c\,x^{4}+b\,x^{2}+a}+\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(2C\_R^{3}+\cdots\right.}{\cdots}$
default	$\frac{-\frac{\left(2ac-b^{2}\right)B\,x^{3}}{2c\left(4ac-b^{2}\right)}-\frac{\left(2A\,c^{2}a-A\,b^{2}c-3Cacb+b^{3}C\right)x^{2}}{2c^{2}\left(4ac-b^{2}\right)}+\frac{axBb}{2c\left(4ac-b^{2}\right)}+\frac{a\left(Abc+2Cac-C\,b^{2}\right)}{2\left(4ac-b^{2}\right)c^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\left(8A\sqrt{-4ac+b^{2}}\,a\,c^{2}-12C\sqrt{-4ac+b^{2}}\,abc+2C\sqrt{\cdots}\right.}{\cdots}$

input
```
int(x^5*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(2*a*c-b^2)*B/c/(4*a*c-b^2)*x^3-1/2/c^2*(2*A*a*c^2-A*b^2*c-3*C*a*b*c
+C*b^3)/(4*a*c-b^2)*x^2+1/2*a/c/(4*a*c-b^2)*x*B*b+1/2*a*(A*b*c+2*C*a*c-C*b
^2)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+a)+1/4/c*sum((2*C*_R^3+B*(6*a*c-b^2)/(4*
a*c-b^2)*_R^2+2*a*(2*A*c-C*b)/(4*a*c-b^2)*_R-B*a*b/(4*a*c-b^2))/(2*_R^3*c+
_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x^5}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a*b*c*x - C*a*b^2 + (B*b^2*c - 2*B*a*c^2)*x^3 - (C*b^3 + 2*A*a*c^2
 - (3*C*a*b + A*b^2)*c)*x^2 + (2*C*a^2 + A*a*b)*c)/(a*b^2*c^2 - 4*a^2*c^3
+ (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) - 1/2*integrate(-(2
*(C*b^2 - 4*C*a*c)*x^3 + B*a*b + (B*b^2 - 6*B*a*c)*x^2 + 2*(C*a*b - 2*A*a*
c)*x)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5784 vs. $2(371) = 742$.

Time = 1.03 (sec) , antiderivative size = 5784, normalized size of antiderivative = 13.64

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*C*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/16*((b^4*c^3 - 8*a*b^2*c^4 + 16*
a^2*c^5)^2*(2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*B + 2*(sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^4 - 12*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^5*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^6*c^5 - 2*a*b^7*c^5 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b^3*c^6 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 + sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 + 24*a^2*b^5*c^6 - 64*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^7 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^2*c^7 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
3*c^7 - 96*a^3*b^3*c^7 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^8 + 128*a^4*b*c^8 + 2*(b^2 - 4*a*c)*a*b^5*c^5 - 16*(b^2 - 4*a*c)*a^2*b^3
*c^6 + 32*(b^2 - 4*a*c)*a^3*b*c^7)*B*abs(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5) - (2*b^12*c^8 - 48*a*b^10*c^9 + 448*a^2*b^8*c^10 - 2048*a^3*b^6*c^11...
```

Mupad [B] (verification not implemented)

Time = 19.08 (sec) , antiderivative size = 6033, normalized size of antiderivative = 14.23

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^5*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((x*(32*A^3*a^3*c^4 - 12*C^3*a^2*b^5 + 8*A*C^2*a*b^6 + 128*A*C^2
*a^4*c^3 - 144*B^2*C*a^4*c^3 + 124*C^3*a^3*b^3*c - 320*C^3*a^4*b*c^2 + 96*
A*B^2*a^3*b*c^3 - 88*A*C^2*a^2*b^4*c - 176*A^2*C*a^3*b*c^3 - 3*B^2*C*a^2*b
^4*c - 28*A*B^2*a^2*b^3*c^2 + 216*A*C^2*a^3*b^2*c^2 + 32*A^2*C*a^2*b^3*c^2
 + 40*B^2*C*a^3*b^2*c^2 + 2*A*B^2*a*b^5*c))/(4*(64*a^3*c^5 - b^6*c^2 + 12*
a*b^4*c^3 - 48*a^2*b^2*c^4)) - root(1572864*a^5*b^2*c^9*z^4 - 983040*a^4*b
^4*c^8*z^4 + 327680*a^3*b^6*c^7*z^4 - 61440*a^2*b^8*c^6*z^4 + 6144*a*b^10*
c^5*z^4 - 256*b^12*c^4*z^4 - 1048576*a^6*c^10*z^4 - 1572864*C*a^5*b^2*c^7*
z^3 + 983040*C*a^4*b^4*c^6*z^3 - 327680*C*a^3*b^6*c^5*z^3 + 61440*C*a^2*b^
8*c^4*z^3 - 6144*C*a*b^10*c^3*z^3 + 256*C*b^12*c^2*z^3 + 1048576*C*a^6*c^8
*z^3 + 98304*A*C*a^5*b*c^6*z^2 + 256*A*C*a*b^9*c^2*z^2 - 90112*A*C*a^4*b^3
*c^5*z^2 + 30720*A*C*a^3*b^5*c^4*z^2 - 4608*A*C*a^2*b^7*c^3*z^2 + 61440*B^
2*a^5*b*c^6*z^2 + 432*B^2*a*b^9*c^2*z^2 + 1536*C^2*a*b^10*c*z^2 + 516096*C
^2*a^5*b^2*c^5*z^2 - 288768*C^2*a^4*b^4*c^4*z^2 + 88576*C^2*a^3*b^6*c^3*z^
2 - 15744*C^2*a^2*b^8*c^2*z^2 - 61440*B^2*a^4*b^3*c^5*z^2 + 24064*B^2*a^3*
b^5*c^4*z^2 - 4608*B^2*a^2*b^7*c^3*z^2 + 24576*A^2*a^4*b^2*c^6*z^2 - 6144*
A^2*a^3*b^4*c^5*z^2 + 512*A^2*a^2*b^6*c^4*z^2 - 16*B^2*b^11*c*z^2 - 393216
*C^2*a^6*c^6*z^2 - 32768*A^2*a^5*c^7*z^2 - 64*C^2*b^12*z^2 - 3072*B^2*C*a^
5*b*c^4*z + 48*B^2*C*a^2*b^7*c*z - 49152*A*C^2*a^5*b*c^4*z + 2304*A*C^2*a^
2*b^7*c*z - 32*A*B^2*a*b^8*c*z + 2304*B^2*C*a^4*b^3*c^3*z - 576*B^2*C*a...
```

Reduce [F]

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^5(C\,x^2+Bx+A)}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int(x^5*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(x^5*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

3.30 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 405

$$
\begin{aligned}
\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx
&= \frac{Bx^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{x(a(2Ac-bC)+(Abc-b^2C+2acC)x^2)}{2c(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad + \frac{\left(Ab-6aC+\frac{b^2C}{c}-\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad + \frac{\left(Ab-6aC+\frac{b^2C}{c}+\frac{Ac(b^2+4ac)+b(b^2-8ac)C}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} \\
&\quad + \frac{2aB\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}
$$

output
```
1/2*B*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*(a*(2*A*c-C*b)+(A
*b*c+2*C*a*c-C*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*(A*b-6*a*C+b^2
*C/c-(A*c*(4*a*c+b^2)+b*(-8*a*c+b^2)*C)/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/
2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b
-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*(A*b-6*a*C+b^2*C/c+(A*c*(4*a*c+b^2)+b*(-8*a
*c+b^2)*C)/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^
(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+2*
a*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.10

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{1}{4}\left(\frac{2(bx^2(-Acx+b(B+Cx))+a(b(B+Cx)-2cx(A+x(B+Cx))))}{c\,(-b^2+4ac)\,(a+bx^2+cx^4)}\right.$$

$$+\frac{\sqrt{2}\left(-Ac\left(b^2+4ac-b\sqrt{b^2-4ac}\right)+\left(-b^3+8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}\ldots}{\sqrt{b-\sqrt{b^2\ldots}}}\right.}{c^{3/2}\,(b^2-4ac)^{3/2}\,\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\sqrt{2}\left(Ac\left(b^2+4ac+b\sqrt{b^2-4ac}\right)+\left(b^3-8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right.}{c^{3/2}\,(b^2-4ac)^{3/2}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

$$\left.-\frac{4aB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{4aB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*(b*x^2*(-(A*c*x) + b*(B + C*x)) + a*(b*(B + C*x) - 2*c*x*(A + x*(B + C
*x)))))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(A*c*(b^2 + 4*
a*c - b*Sqrt[b^2 - 4*a*c])) + (-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*
a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt
[2]*(A*c*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (b^3 - 8*a*b*c + b^2*Sqrt[b
^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]) - (4*a*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/
2) + (4*a*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2193, 27, 1434, 1153, 1083, 219, 1598, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{x^4(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\int \frac{Bx^5}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^4(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+B\int \frac{x^5}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{1434}$$

$$\int \frac{x^4(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\int \frac{x^4}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{1153}$$

$$\int \frac{x^4(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\left(\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2a\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{x^4\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{4a\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}+\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{x^4\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{4a\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1598}$$

$$\frac{\int \frac{x^2\left(\left(2Ac-bC\right)x^2+3\left(Ab-2aC\right)\right)}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x^3\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+ \frac{1}{2}B\left(\frac{4a\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1602}$$

$$\frac{\frac{x(2Ac-bC)}{c}-\frac{\int \frac{a(2Ac-bC)-\left(Abc+\left(b^2-6ac\right)C\right)x^2}{cx^4+bx^2+a}dx}{c}}{2\left(b^2-4ac\right)}-\frac{x^3\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+ \frac{1}{2}B\left(\frac{4a\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{x(2Ac-bC)}{c}-\frac{-\frac{1}{2}\left(-\frac{Ac\left(4ac+b^2\right)+bC\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{Ac\left(4ac+b^2\right)+bC\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right.}{c}}{2\left(b^2-4ac\right)}$$

$$\frac{x^3\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{4a\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{x(2Ac-bC)}{c}-\frac{-\frac{\left(-\frac{Ac\left(4ac+b^2\right)+bC\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{Ac\left(4ac+b^2\right)+bC\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\arctan\left(\cdots}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{2\left(b^2-4ac\right)}$$

$$\frac{x^3\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{4a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

input
```
Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x^3*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (((2*A*c - b*C)*x)/c - (-(((A*b*c + (b^2 - 6*a*c)*C - (A*c*(b^2 +
 4*a*c) + b*(b^2 - 8*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]
])) - ((A*b*c + (b^2 - 6*a*c)*C + (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*a*c)*C)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/(2*(b^2 - 4*a*c)) + (B
*((x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (4*a*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.61

method	result
risch	$\frac{-\frac{\left(Abc+2Cac-C\,b^{2}\right)x^{3}}{2c\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)B\,x^{2}}{2c\left(4ac-b^{2}\right)}-\frac{a\left(2Ac-Cb\right)x}{2\left(4ac-b^{2}\right)c}+\frac{Bab}{2c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(-\frac{\left(Abc-6Cac+C\,b^{2}\right)\_R^{2}}{c\left(4ac-b^{2}\right)}\cdots\right.}{2\_\cdots}\right)}{4}\cdots$
default	$\frac{-\frac{\left(Abc+2Cac-C\,b^{2}\right)x^{3}}{2c\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)B\,x^{2}}{2c\left(4ac-b^{2}\right)}-\frac{a\left(2Ac-Cb\right)x}{2\left(4ac-b^{2}\right)c}+\frac{Bab}{2c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{2B\sqrt{-4ac+b^{2}}\,ac\ln\left(2c\,x^{2}+\sqrt{-4ac+b^{2}}+b\right)+\frac{\left(4A\sqrt{-4ac+b^{2}}\,a\,c^{2}+\cdots\right.}{\cdots}}{\cdots}\cdots$

input
```
int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(A*b*c+2*C*a*c-C*b^2)/c/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)*B/c/(4*a*c-b
^2)*x^2-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c*x+1/2*B*a*b/c/(4*a*c-b^2))/(c*x^4+
b*x^2+a)+1/4*sum((-(A*b*c-6*C*a*c+C*b^2)/c/(4*a*c-b^2)*_R^2+4/(4*a*c-b^2)*
_R*B*a+a*(2*A*c-C*b)/(4*a*c-b^2)/c)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^
4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx=\int \frac{(Cx^2+Bx+A)x^4}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((C*b^2 - (2*C*a + A*b)*c)*x^3 + B*a*b + (B*b^2 - 2*B*a*c)*x^2 + (C*a
*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c -
 4*a*b*c^2)*x^2) + 1/2*integrate(-(4*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 -
(6*C*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5221 vs. $2(360) = 720$.

Time = 1.20 (sec) , antiderivative size = 5221, normalized size of antiderivative = 12.89

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(C*b^2*x^3 - 2*C*a*c*x^3 - A*b*c*x^3 + B*b^2*x^2 - 2*B*a*c*x^2 + C*a*
b*x - 2*A*a*c*x + B*a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((
2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 -
2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 +
 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2
 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*C - 4*(sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^
4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a...
```

Mupad [B] (verification not implemented)

Time = 19.26 (sec) , antiderivative size = 4754, normalized size of antiderivative = 11.74

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log(- root(1572864*a^5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 32768
0*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z^4 + 6144*a*b^10*c^4*z^4 - 256*b^12
*c^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2
*c^5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^
10*c*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b
^9*c*z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a
^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 61
44*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^
2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2
- 16*C^2*b^11*z^2 - 3072*A*B*C*a^3*b^3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4
096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^
2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1
536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z - 32*B*C^2*a*b^8*z - 184
32*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^3*b^2*c^2 - 32*A
*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A
^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^3*b^3*c + 198*A^2*C^2
*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b*c^3 + 2016*A*C^3*a^
4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 768*A*B^2*C*a^4*c^3
+ 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5 - 9*A^2*C^2*a*b^6
 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*a^2*b^5 - 1296...
```

Reduce [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx=\int \frac{x^4(C\,x^2+Bx+A)}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

3.31 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 347

$$\begin{aligned}\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx &= \frac{Bx(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\ &+ \frac{a(2Ac-bC)+(Abc-b^2C+2acC)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} \\ &+ \frac{B\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\ &+ \frac{B\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \\ &- \frac{(Ab-2aC)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

output
```
1/2*B*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(a*(2*A*c-C*b)+(A*b*c
+2*C*a*c-C*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*B*(b-(4*a*c+b^2)/(
-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*
2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*B*(b^2+4*a*c
+b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/
2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-(A*b-2
*C*a)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.03

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{1}{4}\left(-\frac{2(bx^2(Ac-bC+Bcx)+a(2Ac-bC+2cx(B+Cx)))}{c\,(-b^2+4ac)\,(a+bx^2+cx^4)} + \frac{\sqrt{2}B\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}B\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2(Ab-2aC)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2(Ab-2aC)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((-2*(b*x^2*(A*c - b*C + B*c*x) + a*(2*A*c - b*C + 2*c*x*(B + C*x))))/(c*(
-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*B*(-b^2 - 4*a*c + b*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[
c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*B*(b^2 + 4*
a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(
A*b - 2*a*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) -
(2*(A*b - 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2)
)/4
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {2193, 27, 1440, 1480, 218, 1578, 1224, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \ 2193$$

$$\int \frac{x^3\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^4}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{x^3\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^4}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 1440$$

$$\int \frac{x^3\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{2a-bx^2}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \ 1480$$

$$\int \frac{x^3\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+$$
$$B\left(\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{1}{2}\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4a}\right.}}{2\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \ 218$$

$$\int \frac{x^3\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+$$
$$B\left(\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx^2+B\left(\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}\right)$$

$$\Big\downarrow \text{1224}$$

$$\frac{1}{2}\left(\frac{(Ab-2aC)\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{x^2(Abc-C(b^2-2ac))+a(2Ac-bC)}{c(b^2-4ac)(a+bx^2+cx^4)}\right)+B\left(\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{x^2(Abc-C(b^2-2ac))+a(2Ac-bC)}{c(b^2-4ac)(a+bx^2+cx^4)}-\frac{2(Ab-2aC)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}\right)+B\left(\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^2(Abc-C(b^2-2ac))+a(2Ac-bC)}{c(b^2-4ac)(a+bx^2+cx^4)}-\frac{2(Ab-2aC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)+B\left(\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}\right)$$

input

```
Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
B*((x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (-(((b - (b^2
 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - ((b + (b^2 +
4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)))
 + ((a*(2*A*c - b*C) + (A*b*c - (b^2 - 2*a*c)*C)*x^2)/(c*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) - (2*(A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*
c]])/(b^2 - 4*a*c)^(3/2))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1224
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c))   Int[(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -
1] &&  !(IntegerQ[p] && NeQ[a, 0] && NiceSqrtQ[b^2 - 4*a*c])
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.61

method	result
risch	$\frac{-\frac{Bb\,x^3}{2(4ac-b^2)}-\frac{(Abc+2Cac-C\,b^2)x^2}{2c(4ac-b^2)}-\frac{xBa}{4ac-b^2}-\frac{a(2Ac-Cb)}{2(4ac-b^2)c}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{Bb\_R^2}{4ac-b^2}-\frac{2(Ab-2aC)\_R}{4ac-b^2}+\frac{2B}{4ac-}\right.}{2\_R^3c+\_Rb}\right.}{4}$
default	$\frac{-\frac{Bb\,x^3}{2(4ac-b^2)}-\frac{(Abc+2Cac-C\,b^2)x^2}{2c(4ac-b^2)}-\frac{xBa}{4ac-b^2}-\frac{a(2Ac-Cb)}{2(4ac-b^2)c}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\frac{\left(-4A\sqrt{-4ac+b^2}\,bc+8C\sqrt{-4ac+b^2}\,ac\right)\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{4c}+\frac{(4}{}}{}\right.}{}$

input
```
int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*B*b/(4*a*c-b^2)*x^3-1/2*(A*b*c+2*C*a*c-C*b^2)/c/(4*a*c-b^2)*x^2-1/(4
*a*c-b^2)*x*B*a-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c)/(c*x^4+b*x^2+a)+1/4*sum((
-B*b/(4*a*c-b^2)*_R^2-2*(A*b-2*C*a)/(4*a*c-b^2)*_R+2*B*a/(4*a*c-b^2))/(2*_
R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*c*x^3 + 2*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (2*C*a + A*b)*c)*x
^2)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x
^2) + 1/2*integrate((B*b*x^2 - 2*B*a - 2*(2*C*a - A*b)*x)/(c*x^4 + b*x^2 +
 a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3228 vs. $2(296) = 592$.

Time $= 0.83$ (sec) , antiderivative size $= 3228$, normalized size of antiderivative $= 9.30$

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(B*b*c*x^3 - C*b^2*x^2 + 2*C*a*c*x^2 + A*b*c*x^2 + 2*B*a*c*x - C*a*b +
 2*A*a*c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) - 1/16*((2*b^3*c^2 - 8*a
*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*B + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b
^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c) - (2*b
^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s...
```

Mupad [B] (verification not implemented)

Time = 19.45 (sec) , antiderivative size = 3278, normalized size of antiderivative = 9.45

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log(root(1572864*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*
a^3*b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^
6*c^7*z^4 - 256*b^12*c*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2
 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*
c*z^2 + 12288*B^2*a^4*b*c^4*z^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b
^2*c^3*z^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^
2*a^2*b^5*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 +
128*A^2*b^8*c*z^2 - 32768*C^2*a^5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2
*b^4*c*z - 1024*A*B^2*a^3*b*c^3*z - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b
^2*c^2*z + 768*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*a*b^6*z + 2048*B^2*C*a^4*c^3
*z + 16*A*B^2*b^7*z + 192*A*B^2*C*a^2*b^2*c + 512*A*C^3*a^3*b*c + 128*A^3*
C*a*b^3*c + 16*A*B^2*C*a*b^4 - 384*A^2*C^2*a^2*b^2*c - 192*B^2*C^2*a^3*b*c
 - 48*A^2*B^2*a*b^3*c - 24*B^4*a^2*b^2*c - 16*B^2*C^2*a^2*b^3 - 16*B^4*a^3
*c^2 - 4*A^2*B^2*b^5 - 256*C^4*a^4*c - 16*A^4*b^4*c - 9*B^4*a*b^4, z, k)*(
(256*A*B*a^2*b^2*c^3 + 128*B*C*a^2*b^3*c^2 - 64*A*B*a*b^4*c^2 - 512*B*C*a^
3*b*c^3)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(15728
64*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*b^6*c^4*z^4 - 614
40*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7*z^4 - 256*b^12*
c*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*
c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 + 12288*B^2*...
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 2670, normalized size of antiderivative = 7.69

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

output

```
(8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c
 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c
**2 + 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3
*c*x**2 + 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
b**2*c**2*x**4 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a*b**2*c**2*x**2 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqr
t(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a*b*c**3*x**4 + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
b**3*c + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*c*x**2 + 8*sqrt(a)
*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c**2*x**4 - 8*sqrt(c)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*a**2*b**2*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*a...
```

3.32 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 356

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*(2*A*c-C*b-(4*A*b*c-(4*a*c+b^2)*C)
/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2)
)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*(2*A*c-C*b
+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 2193}$$

$$\int \frac{x^2 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^2 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\int \frac{x^2 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 1159}$$

$$\int \frac{x^2 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{x^2 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\int \frac{x^2 \left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 1598}$$

$$\frac{\int \frac{-\left(\left(2Ac-bC\right)x^2\right)+Ab-2aC}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}}{2\left(b^2-4ac\right)}$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$\downarrow$ 218

$$\frac{-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2(4ac-b^2)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln(x-\_}{2\_R^3c+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2(4ac-b^2)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4A\sqrt{-4ac+b^2}\,bc+8A\,c^2a-2A\,b^2c+\right.}{4c(4}\right.}{}$

input
```
int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-
b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln
(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4439 vs. $2(306) = 612$.

Time = 1.04 (sec) , antiderivative size = 4439, normalized size of antiderivative = 12.47

$$\int \frac{x^2(A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^2(C\,x^2+Bx+A)}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

output

```
int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

3.33 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 317

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx = -\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{B\sqrt{c}(2b-\sqrt{b^2-4ac})\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}(2b+\sqrt{b^2-4ac})\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
-1/2*B*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(A*b-2*a*C+(2*A*c-C*
b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*B*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2)
)*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(-4*a*c+b
^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*B*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/
2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(-4*a*c
+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+(2*A*c-C*b)*arctanh((2*c*x^2+b)/(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.06

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\left(\frac{2aC-A(b+2cx^2)+x(-bB+bCx-2Bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}B\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(-2Ac+bC)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{(2Ac-bC)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*a*C - A*(b + 2*c*x^2) + x*(-(b*B) + b*C*x - 2*B*c*x^2))/((b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) - (Sqrt[2]*B*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((-2*A*c + b*C)*Log[-b + Sqrt[b^2 - 4
*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*c - b*C)*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/2
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2193, 27, 1439, 1480, 218, 1576, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \; 2193$$

$$\int \frac{x\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^2}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 1439$$

$$\int \frac{x\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{\int \frac{b-2cx^2}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 1480$$

$$\int \frac{x\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+$$
$$B\left(\frac{-c\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-c\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+\right.}\right.$$

$$\Big\downarrow \; 218$$

$$\int \frac{x\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+$$
$$B\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right.}\right.$$

$$\Big\downarrow \; 1576$$

$$\frac{1}{2}\int \frac{Cx^2+A}{\left(cx^4+bx^2+a\right)^2}dx^2+$$
$$B\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right.}\right.$$

$$\Big\downarrow \; 1159$$

$$\frac{1}{2}\left(-\frac{(2Ac-bC)\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{-2aC+x^2(2Ac-bC)+Ab}{(b^2-4ac)(a+bx^2+cx^4)}\right)+ B\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\right.$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{2}\left(\frac{2(2Ac-bC)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{-2aC+x^2(2Ac-bC)+Ab}{(b^2-4ac)(a+bx^2+cx^4)}\right)+ B\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\right.$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{2(2Ac-bC)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aC+x^2(2Ac-bC)+Ab}{(b^2-4ac)(a+bx^2+cx^4)}\right)+ B\left(\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\right.$$

input
```
Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
B*(-1/2*(x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-((Sqrt[2
]*Sqrt[c]*(1 - (2*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(1
+ (2*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*(b^2 - 4*a*c))) + (-((A*b - 2*a*
C + (2*A*c - b*C)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (2*(2*A*c -
b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.62

method	result
risch	$\frac{\frac{B c x^3}{4ac-b^2}+\frac{(2Ac-Cb)x^2}{8ac-2b^2}+\frac{bBx}{8ac-2b^2}+\frac{Ab-2aC}{8ac-2b^2}}{c x^4+b x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(\frac{2c\_R^2 B}{4ac-b^2}+\frac{2(2Ac-Cb)\_R}{4ac-b^2}-\frac{Bb}{4ac-b^2}\right)\ln\left(x-\_R\right.}{2\_R^3 c+\_R b}\right.}{4}$
default	$16c^2\left(\frac{\frac{\frac{B\left(4ac-b^2\right)x}{8c}+\frac{8A c^2 a-2A b^2 c-4C\sqrt{-4ac+b^2}\,ac+C\sqrt{-4ac+b^2}\,b^2-4Cacb+b^3C}{16c^2}}{x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}}+\frac{\left(4A\sqrt{-4ac+b^2}\,c-2C\sqrt{-4ac+b^2}\,b\right)\ln\left(2c x^2+\sqrt{-\right.}{16c}}{4c\left(4ac-b^2\right)^2}\right.$

input
```
int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(B*c/(4*a*c-b^2)*x^3+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^2+1/2/(4*a*c-b^2)*x*B*b
+1/2*(A*b-2*C*a)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum((2*c/(4*a*c-b^2)*_R^
2*B+2*(2*A*c-C*b)/(4*a*c-b^2)*_R-B*b/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R)
,_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*B*c*x^3 + B*b*x - (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate((2*B*c*x^
2 - B*b - 2*(C*b - 2*A*c)*x)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3014 vs. $2(270) = 540$.

Time = 0.81 (sec) , antiderivative size = 3014, normalized size of antiderivative = 9.51

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*B*c*x^3 - C*b*x^2 + 2*A*c*x^2 + B*b*x - 2*C*a + A*b)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) + 1/8*((2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2*B + (sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 3
2*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*B*abs(b^2 -
 4*a*c) - 2*(2*b^6*c^2 - 16*a*b^4*c^3 + 32*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 + 8*(b^2 - 4*a*c)*a...
```

Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 3198, normalized size of antiderivative = 10.09

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((4*B^3*a*c^4 + 3*B^3*b^2*c^3 + 8*A^2*B*b*c^4 + 2*B*C^2*b^3*c^2
- 8*A*B*C*b^2*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) -
root(1572864*a^6*b^2*c^5*z^4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3
*z^4 - 61440*a^3*b^8*c^2*z^4 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 -
 256*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*
C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12
288*B^2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 +
 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^
2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2 - 6144*A^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b
^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9*z^2 + 1024*B^2*C*a^3*b*c^3*z -
 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z - 768*B^2*C*a^2*b^3*c^2*z + 1
536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b^6*c*z - 2048*A*B^2*a^3*c^4*z - 16*B^2
*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a*b*c^3 + 128*A*C^3*a*b^3*c +
 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*B^2*C^2*a*b^3*c - 192*A^2*B
^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c^2 - 16*B^4*a^2*c^3 - 4*B^
2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*a*c^4, z, k)*(root(157286
4*a^6*b^2*c^5*z^4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 6144
0*a^3*b^8*c^2*z^4 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12
*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c
^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*...
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 2667, normalized size of antiderivative = 8.41

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
( - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*
c - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*
*2*c*x**2 + 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**2*b**2*c - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b
))*a**2*b*c**2*x**4 + 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
 - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*a*b**3*c*x**2 + 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqr
t(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a*b**2*c**2*x**4 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a**2*b**2*c - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4 - 8*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c*x**2 - 8*sqrt(a)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*a*b**2*c**2*x**4 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +...
```

3.34 $\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 368

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2} \, dx = & -\frac{B(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\
& + \frac{x(A(b^2-2ac)-abC+c(Ab-2aC)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} \\
& + \frac{\sqrt{c}\left(Ab-2aC+\frac{A(b^2-12ac)+4abC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\
& + \frac{\sqrt{c}\left(Ab-2aC-\frac{Ab^2-12aAc+4abC}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} \\
& + \frac{2Bc\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}
$$

output
```
-1/2*B*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*(A*(-2*a*c+b^2)-a*b*
C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/2)*(A*b-2*a*C
+(A*(-12*a*c+b^2)+4*a*b*C)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b
-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2)+1/4*c^(1/2)*(A*b-2*a*C-(-12*A*a*c+A*b^2+4*C*a*b)/(-4*a*c+b^2)^(1/2))
*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+
b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+2*B*c*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{2ab(B+Cx)-2Abx(b+cx^2)+4acx(A+x(B+Cx))}{a(-b^2+4ac)(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\sqrt{c}\left(A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)-2a\left(-2b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt{2}\sqrt{c}\left(A\left(b^2-12ac-b\sqrt{b^2-4ac}\right)+2a\left(2b+\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.-\frac{4Bc\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{4Bc\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*a*b*(B + C*x) - 2*A*b*x*(b + c*x^2) + 4*a*c*x*(A + x*(B + C*x)))/(a*(-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c + b*
Sqrt[b^2 - 4*a*c]) - 2*a*(-2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])
 + 2*a*(2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (
4*B*c*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*B*c*
Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2202, 27, 1432, 1086, 1083, 219, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{Cx^2+A}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{Cx^2+A}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{Cx^2+A}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{1}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{1086}$$

$$\int \frac{Cx^2+A}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{Cx^2+A}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{Cx^2+A}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1492}$$

$$-\frac{\int -\frac{Ab^2+aCb+c(Ab-2aC)x^2-6aAc}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x(cx^2(Ab-2aC)-2aAc-abC+Ab^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{1}{2}B\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$\downarrow$ 25

$$\frac{\int \frac{c(Ab-2aC)x^2+A(b^2-6ac)+abC}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x(cx^2(Ab-2aC)-2aAc-abC+Ab^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{1}{2}B\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$\downarrow$ 1480

$$\frac{\frac{1}{2}c\left(\frac{A(b^2-12ac)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{-12aAc+4abC+Ab^2}{\sqrt{b^2-4ac}}-2aC+Ab\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-\right.}}}{2a(b^2-4ac)}$$
$$\frac{x(cx^2(Ab-2aC)-2aAc-abC+Ab^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{1}{2}B\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$\downarrow$ 218

$$\frac{\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abC+Ab^2}{\sqrt{b^2-4ac}}-2aC+Ab\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a(b^2-4ac)}+$$
$$\frac{x(cx^2(Ab-2aC)-2aAc-abC+Ab^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{1}{2}B\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(A*b^2 - 2*a*A*c - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + ((Sqrt[c]*(A*b - 2*a*C + (A*(b^2 - 12*a*c) + 4*a*b*C)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*C - (A*b^2 -
 12*a*A*c + 4*a*b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 -
4*a*c)) + (B*(-((b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (4*c*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.63

method	result
risch	$\frac{-\frac{c(Ab-2aC)x^3}{2a(4ac-b^2)}+\frac{c x^2 B}{4ac-b^2}+\frac{(2Aac-A b^2+Cba)x}{2a(4ac-b^2)}+\frac{Bb}{8ac-2b^2}}{c x^4+b x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2 b+a)}\frac{\left(-\frac{c(Ab-2aC)\_R^2}{a(4ac-b^2)}+\frac{4Bc\_R}{4ac-b^2}+\frac{6Aac}{a(\cdots}\right.}{2\_R^3 c+\_R\cdots}\right.}{4}$
default	$16c^2\left(\frac{\frac{\frac{(4A\sqrt{-4ac+b^2}\,ac-A\sqrt{-4ac+b^2}\,b^2-4Aabc+A b^3+8C a^2c-2Ca b^2)x}{16ac}+\frac{B(4ac-b^2)}{8c}}{x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}}+\frac{2B\sqrt{-4ac+b^2}\,a\ln(2c x^2+\sqrt{-4ac+b^2}+b)+\frac{(12\cdots}{\cdots}}{\cdots}}{4(4ac-b^2)^2 c\cdots}\right.$

input
```
int((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*c*(A*b-2*C*a)/a/(4*a*c-b^2)*x^3+c/(4*a*c-b^2)*x^2*B+1/2*(2*A*a*c-A*b
^2+C*a*b)/a/(4*a*c-b^2)*x+1/2*B*b/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum((-c
*(A*b-2*C*a)/a/(4*a*c-b^2)*_R^2+4*B*c/(4*a*c-b^2)*_R+(6*A*a*c-A*b^2-C*a*b)
/a/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx = \int \frac{Cx^2+Bx+A}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*B*a*c*x^2 + (2*C*a - A*b)*c*x^3 + B*a*b + (C*a*b - A*b^2 + 2*A*a*c
)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*
x^2) + 1/2*integrate(-(4*B*a*c*x + (2*C*a - A*b)*c*x^2 - C*a*b - A*b^2 + 6
*A*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5159 vs. $2(323) = 646$.

Time = 1.31 (sec) , antiderivative size = 5159, normalized size of antiderivative = 14.02

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*C*a*c*x^3 - A*b*c*x^3 + 2*B*a*c*x^2 + C*a*b*x - A*b^2*x + 2*A*a*c*
x + B*a*b)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*
a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^
2)*(a*b^2 - 4*a^2*c)^2*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*C +
 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*
c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 -
 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 19
2*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48...
```

Mupad [B] (verification not implemented)

Time = 19.11 (sec) , antiderivative size = 4707, normalized size of antiderivative = 12.79

$$\int \frac{A+Bx+Cx^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((B*b)/(2*(4*a*c - b^2)) + (x*(2*A*a*c - A*b^2 + C*a*b))/(2*a*(4*a*c - b^2
)) + (B*c*x^2)/(4*a*c - b^2) - (c*x^3*(A*b - 2*C*a))/(2*a*(4*a*c - b^2)))/
(a + b*x^2 + c*x^4) + symsum(log((5*A^3*b^3*c^4 + 8*C^3*a^3*c^4 + 6*C^3*a^
2*b^2*c^3 - 36*A^3*a*b*c^5 - 96*A*B^2*a^2*c^5 + 72*A^2*C*a^2*c^5 - 3*A^2*C
*b^4*c^3 + 16*A*B^2*a*b^2*c^4 + 3*A*C^2*a*b^3*c^3 - 60*A*C^2*a^2*b*c^4 + 1
8*A^2*C*a*b^2*c^4 + 16*B^2*C*a^2*b*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2)) - root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c
^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*
z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*A*C*a^2*b^8*c*z^2 + 245
76*A*C*a^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z
^2 - 32*A*C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2
 + 432*A^2*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2
+ 1536*C^2*a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*
c^3*z^2 + 512*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*
a^3*b^5*c^3*z^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^
2*a^2*b^9*z^2 - 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2
*b^5*c^2*z - 4096*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^
6*c*z - 672*A^2*B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b
^4*c^2*z - 15872*A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B
*b^8*c*z - 2048*B*C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{C\,x^2+Bx+A}{\left(c\,x^4+b\,x^2+a\right)^2}dx$$

input
```
int((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

3.35 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 403

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{Bx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\ &\quad + \frac{A(b^2-2ac)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\ &\quad + \frac{B\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\ &\quad - \frac{B\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \\ &\quad + \frac{\left(A(b^3-6abc)+4a^2cC\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} \\ &\quad + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}\end{aligned}$$

output
```
1/2*B*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-2*a*c+
b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*B*c^(1/2)
*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2
)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-
1/4*B*c^(1/2)*(b^2-12*a*c-b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(
b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)+1/2*(A*(-6*a*b*c+b^3)+4*a^2*c*C)*arctanh((2*c*x^2+b)/(-4*a*c+
b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+A*ln(x)/a^2-1/4*A*ln(c*x^4+b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{2a\left(abC+2acx(B+Cx)-bBx\left(b+cx^2\right)-A\left(b^2-2ac+bcx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}aB\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}aB\sqrt{c}\left(-b^2\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-2*a*(a*b*C + 2*a*c*x*(B + C*x) - b*B*x*(b + c*x^2) - A*(b^2 - 2*a*c + b
*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(b^2
- 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2
]*a*B*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + 4*A*Log[x] - ((A*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a
*c*Sqrt[b^2 - 4*a*c]) + 4*a^2*c*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/
(b^2 - 4*a*c)^(3/2) - ((A*(-b^3 + 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*
Sqrt[b^2 - 4*a*c]) - 4*a^2*c*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2
 - 4*a*c)^(3/2))/(4*a^2)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {2193, 27, 1405, 25, 1480, 218, 1578, 1235, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+\int \frac{B}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{1}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{1405}$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1480}$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+$$
$$B\left(\frac{\frac{1}{2}c\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx\right.}{2a\left(b^2-4ac\right)\left(a+bx^2\right.}\right.$$

$$\Big\downarrow \text{218}$$

$$\int \frac{Cx^2+A}{x\left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{2}\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)^2}dx^2+B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aC)-2aAc-abC+Ab^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{c(Ab-2aC)x^2+A\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)+B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{c(Ab-2aC)x^2+A\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aC)-2aAc-abC+Ab^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{-Ab^3+5aAcb-Ac(b^2-4ac)x^2-2a^2cC}{a(cx^4+bx^2+a)}-\frac{A(4ac-b^2)}{ax^2}\right)dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab-2aC)-2aAc-abC+Ab^2}{a(b^2-4ac)(a+bx^2+cx^4)}\right)+B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a(b^2-4ac)}+\frac{x(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{\frac{(4a^2cC+A(b^3-6abc))\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{A\log(x^2)(b^2-4ac)}{a}-\frac{A(b^2-4ac)\log(a+bx^2+cx^4)}{2a}}{a(b^2-4ac)}+\frac{cx^2(Ab-2aC)-2aAc\ \cdots}{a(b^2-4ac)(a+bx^2\ \cdots}\right.$$

$$B\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a(b^2-4ac)}+\frac{x(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
B*((x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (
(Sqrt[c]*(b + (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[c]*(b - (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b
^2 - 4*a*c))) + ((A*b^2 - 2*a*A*c - a*b*C + c*(A*b - 2*a*C)*x^2)/(a*(b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) + (((A*(b^3 - 6*a*b*c) + 4*a^2*c*C)*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + (A*(b^2 - 4*a*c)*
Log[x^2])/a - (A*(b^2 - 4*a*c)*Log[a + b*x^2 + c*x^4])/(2*a))/(a*(b^2 - 4*
a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.40

method	result
default	$-\frac{\frac{Bbc\,x^{3}a}{8ac-2b^{2}}+\frac{ac(Ab-2aC)x^{2}}{8ac-2b^{2}}-\frac{Ba\left(2ac-b^{2}\right)x}{2\left(4ac-b^{2}\right)}-\frac{a\left(2Aac-A\,b^{2}+Cba\right)}{2\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{2c\left(\frac{\frac{\left(12A\sqrt{-4ac+b^{2}}\,abc-2A\sqrt{-4ac+b^{2}}\,b^{3}+32A\,a^{2}c^{2}-16Aa\,b^{2}c+2A\,b\right.}{4c}}{}\right.}{}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*((1/2*B*a*b*c/(4*a*c-b^2)*x^3+1/2*a*c*(A*b-2*C*a)/(4*a*c-b^2)*x^2-1
/2*B*a*(2*a*c-b^2)/(4*a*c-b^2)*x-1/2*a*(2*A*a*c-A*b^2+C*a*b)/(4*a*c-b^2))/
(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/(16*a*c-4*b^2)*(1/4*(12*A*(-4*a*c+b^2)^
(1/2)*a*b*c-2*A*(-4*a*c+b^2)^(1/2)*b^3+32*A*a^2*c^2-16*A*a*b^2*c+2*A*b^4-8
*C*(-4*a*c+b^2)^(1/2)*a^2*c)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+1/2*(-12*B
*(-4*a*c+b^2)^(1/2)*a^2*c+B*(-4*a*c+b^2)^(1/2)*a*b^2+4*B*a^2*b*c-B*a*b^3)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2)*(-1/4*(12*A*(-4*a*c+b^2)^(1/2)*a*b*c
-2*A*(-4*a*c+b^2)^(1/2)*b^3-32*A*a^2*c^2+16*A*a*b^2*c-2*A*b^4-8*C*(-4*a*c+
b^2)^(1/2)*a^2*c)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/2*(-12*B*(-4*a*c+b
^2)^(1/2)*a^2*c+B*(-4*a*c+b^2)^(1/2)*a*b^2-4*B*a^2*b*c+B*a*b^3)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)))))+A*ln(x)/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{x\left(a + bx^2 + cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{x\left(a + bx^2 + cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{Cx^2+Bx+A}{\left(cx^4+bx^2+a\right)^2x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*c*x^3 - (2*C*a - A*b)*c*x^2 - C*a*b + A*b^2 - 2*A*a*c + (B*b^2 -
2*B*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^
2*b*c)*x^2) + 1/2*integrate((B*a*b*c*x^2 + B*a*b^2 - 6*B*a^2*c - 2*(A*b^2*
c - 4*A*a*c^2)*x^3 - 2*(A*b^3 + (2*C*a^2 - 5*A*a*b)*c)*x)/(c*x^4 + b*x^2 +
 a), x)/(a^2*b^2 - 4*a^3*c) + A*log(x)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6023 vs. $2(348) = 696$.

Time = 1.00 (sec) , antiderivative size = 6023, normalized size of antiderivative = 14.95

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/4*A*log(abs(c*x^4 + b*x^2 + a))/a^2 + A*log(abs(x))/a^2 - 1/16*((a^4*b^
4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*B - 2*(sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c - 18*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^5*b^6*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^
2 - 2*a^4*b^8*c^2 + 120*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^
3 + 28*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 36*a^5*b^6*c^3 - 352*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^6*b^3*c^4 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b
^4*c^4 - 240*a^6*b^4*c^4 + 384*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8
*c^5 + 192*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^5 + 64*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 704*a^7*b^2*c^5 - 96*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^6 - 768*a^8*c^6 + 2*(b^2 - 4*a*c)*a
^4*b^6*c^2 - 28*(b^2 - 4*a*c)*a^5*b^4*c^3 + 128*(b^2 - 4*a*c)*a^6*b^2*c^4
- 192*(b^2 - 4*a*c)*a^7*c^5)*B*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)
 + (2*a^8*b^11*c^4 - 56*a^9*b^9*c^5 + 576*a^10*b^7*c^6 - 2816*a^11*b^5*...
```

Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 8129, normalized size of antiderivative = 20.17

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)
```

output
```
((2*A*a*c - A*b^2 + C*a*b)/(2*a*(4*a*c - b^2)) + (B*x*(2*a*c - b^2))/(2*a*
(4*a*c - b^2)) - (c*x^2*(A*b - 2*C*a))/(2*a*(4*a*c - b^2)) - (B*b*c*x^3)/(
2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + symsum(log(root(1572864*a^9*b^2*
c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 327680*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*
c^2*z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a^10*c^6*z^4 - 256*a^4*b^12*z^4 +
1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b^4*c^4*z^3 + 327680*A*a^5*b^6*c^
3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^3*b^10*c*z^3 - 1048576*A*a^8*c^
6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b*c^5*z^2 + 256*A*C*a^2*b^9*c*z
^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^4*b^5*c^3*z^2 - 4608*A*C*a^3*
b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^2*a^2*b^9*c*z^2 + 1536*A^2*a
*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*C^2*a^5*b^4*c^3*z^2 + 512*C
^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 + 24064*B^2*a^4*b^5*c^3*z^2
 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*c^5*z^2 - 288768*A^2*a^4*
b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^2*a^2*b^8*c^2*z^2 - 16*B
^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216*A^2*a^6*c^6*z^2 - 64*A^2*b^
12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*c^2*z + 3072*A*B^2*a^4
*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z - 15872*B^2*C*a^4*b^2
*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*c^2*z - 45056*A^2*C*
a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2*a^4*b^2*c^4*z - 30
72*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*A*B^2*a^3*b^3*c...
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 4396, normalized size of antiderivative = 10.91

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x)
```

output
```
(24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*
c - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*
c**2 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*
b**4 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2
*b**3*c*x**2 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*
atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
b))*a**2*b**2*c**2*x**4 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sq
rt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)
*sqrt(a) + b))*a**2*b**2*c**2*x**2 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2
*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqr
t(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**3*x**4 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)
*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**5*x**2 - 4*sqrt(2*sqrt(c)*sqrt(a) +
b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(
c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*c*x**4 + 16*sqrt(a)*sqrt(2*sqrt(
c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2...
```

3.36 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 519

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2}\,dx=-\frac{A}{a^2x}+\frac{B(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$
$$-\frac{x(A(b^3-3abc)-a(b^2-2ac)C+c(A(b^2-2ac)-abC)x^2)}{2a^2(b^2-4ac)(a+bx^2+cx^4)}$$
$$-\frac{\sqrt{c}\left(A\left(3b^3-16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)-a\left(b^2-12ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}}{\sqrt{b-\sqrt{}}}\right.}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{c}\left(A\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)-a\left(b^2-12ac-b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}}{\sqrt{b+\sqrt{}}}\right.}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$+\frac{bB(b^2-6ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}}+\frac{B\log(x)}{a^2}-\frac{B\log(a+bx^2+cx^4)}{4a^2}$$

output
```
-A/a^2/x+1/2*B*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A
*(-3*a*b*c+b^3)-a*(-2*a*c+b^2)*C+c*(A*(-2*a*c+b^2)-a*b*C)*x^2)/a^2/(-4*a*c
+b^2)/(c*x^4+b*x^2+a)-1/4*c^(1/2)*(A*(3*b^3-16*a*b*c+3*b^2*(-4*a*c+b^2)^(1
/2)-10*a*c*(-4*a*c+b^2)^(1/2))-a*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))*C)*arct
an(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2
)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*c^(1/2)*(A*(3*b^3-16*a*b*c-3*b^2*
(-4*a*c+b^2)^(1/2)+10*a*c*(-4*a*c+b^2)^(1/2))-a*(b^2-12*a*c-b*(-4*a*c+b^2)
^(1/2))*C)*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/
a^2/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+1/2*b*B*(-6*a*c+b^2)*a
rctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+B*ln(x)/a^2-
1/4*B*ln(c*x^4+b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{4A}{x}+\frac{-4a^2c(B+Cx)-2Ab^2x(b+cx^2)+2a\left(2Ac^2x^3+b^2(B+Cx)+bcx(3A+x(B+Cx))\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b\cdots}\right.\right.}{(b^2-4\cdots}}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-4*A)/x + (-4*a^2*c*(B + C*x) - 2*A*b^2*x*(b + c*x^2) + 2*a*(2*A*c^2*x^3
 + b^2*(B + C*x) + b*c*x*(3*A + x*(B + C*x))))/((b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c
] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(A*(3*b^3 - 16*a*b*c -
 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(-b^2 + 12*a*c +
b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*B*Log[x] - (B
*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[-b
+ Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (B*(-b^3 + 6*a*b*c +
 b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c
] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2193, 27, 1434, 1165, 25, 1200, 1600, 25, 1604, 1480, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)^2}dx+\int \frac{B}{x\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{1}{x\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{1434}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{1}{x^2\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{1165}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{b^2+cx^2b-4ac}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{\int \frac{b^2+cx^2b-4ac}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\int \frac{Cx^2+A}{x^2\left(cx^4+bx^2+a\right)^2}dx+$$
$$\frac{1}{2}B\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c\left(b^2-4ac\right)x^2-b\left(b^2-5ac\right)}{a\left(cx^4+bx^2+a\right)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1600}$$

$$-\frac{\int -\frac{3Ab^2-aCb+3c(Ab-2aC)x^2-10aAc}{x^2(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}+\frac{1}{2}B\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3Ab^2-aCb+3c(Ab-2aC)x^2-10aAc}{x^2(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}+\frac{1}{2}B\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 1604}$$

$$\frac{-\frac{\int \frac{c\left(3Ab^2-aCb-10aAc\right)x^2+A\left(3b^3-13abc\right)-a\left(b^2-6ac\right)C}{cx^4+bx^2+a}dx}{a}-\frac{-10aAc-abC+3Ab^2}{ax}}{2a\left(b^2-4ac\right)}+\frac{1}{2}B\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 1480}$$

$$-\frac{\frac{1}{2}c\left(\frac{A\left(3b^3-16abc\right)-aC\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10aAc-abC+3Ab^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{c\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(-b\sqrt{b^2-\cdots\right.\right.}{2\sqrt{b^2-4ac}}\cdots}{a}\cdots$$

$$2a\left(b^2-4ac\right)$$

$$\frac{1}{2}B\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}B\left(\frac{\int\left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+$$

$$-\frac{\frac{\sqrt{c}\left(\frac{A\left(3b^3-16abc\right)-aC\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10aAc-abC+3Ab^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(-b\sqrt{b^2-4ac}\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}}{2a\left(b^2-4ac\right)}$$

$$\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{\sqrt{c}\left(\frac{A\left(3b^3-16abc\right)-aC\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10aAc-abC+3Ab^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(-b\sqrt{b^2-4ac}\right.\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}}{2a\left(b^2-4ac\right)}$$

$$\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+$$

$$\frac{1}{2}B\left(\frac{\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log\left(x^2\right)\left(b^2-4ac\right)}{a}-\frac{\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2a}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x*(a +
b*x^2 + c*x^4)) + (-((3*A*b^2 - 10*a*A*c - a*b*C)/(a*x)) - ((Sqrt[c]*(3*A*
b^2 - 10*a*A*c - a*b*C + (A*(3*b^3 - 16*a*b*c) - a*(b^2 - 12*a*c)*C)/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(A*(3*b^3 - 16*a*b*c - 3*b^2
*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) - a*(b^2 - 12*a*c - b*Sqrt[
b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a)/(2*a*(b^2 - 4*
a*c)) + (B*((b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
+ ((b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2
- 4*a*c]) + ((b^2 - 4*a*c)*Log[x^2])/a - ((b^2 - 4*a*c)*Log[a + b*x^2 + c*
x^4])/(2*a))/(a*(b^2 - 4*a*c))))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.29

method	result
default	$-\frac{A}{a^{2}x}+\frac{B\ln\left(x\right)}{a^{2}}-\frac{\frac{c\left(2Aac-A\,b^{2}+Cba\right)x^{3}}{8ac-2b^{2}}+\frac{Bbc\,x^{2}a}{8ac-2b^{2}}+\frac{\left(3Aabc-A\,b^{3}-2C\,a^{2}c+Ca\,b^{2}\right)x}{8ac-2b^{2}}-\frac{Ba\left(2ac-b^{2}\right)}{2\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+ \cdots 2c\left(\frac{\left(12B\sqrt{-4ac+b^{2}}\,abc-2B \cdots\right.}{\cdots}\right.$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/a^2/x+B*ln(x)/a^2-1/a^2*((1/2*c*(2*A*a*c-A*b^2+C*a*b)/(4*a*c-b^2)*x^3+1
/2*B*a*b*c/(4*a*c-b^2)*x^2+1/2*(3*A*a*b*c-A*b^3-2*C*a^2*c+C*a*b^2)/(4*a*c-
b^2)*x-1/2*B*a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1
/(16*a*c-4*b^2)*(1/4*(12*B*(-4*a*c+b^2)^(1/2)*a*b*c-2*B*(-4*a*c+b^2)^(1/2)
*b^3+32*B*a^2*c^2-16*B*a*b^2*c+2*B*b^4)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)
+1/2*(16*A*(-4*a*c+b^2)^(1/2)*a*b*c-3*A*(-4*a*c+b^2)^(1/2)*b^3+40*A*a^2*c^
2-22*A*a*b^2*c+3*A*b^4-12*C*(-4*a*c+b^2)^(1/2)*a^2*c+C*(-4*a*c+b^2)^(1/2)*
a*b^2+4*C*a^2*b*c-C*a*b^3)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2)*(-1/4*(12
*B*(-4*a*c+b^2)^(1/2)*a*b*c-2*B*(-4*a*c+b^2)^(1/2)*b^3-32*B*a^2*c^2+16*B*a
*b^2*c-2*B*b^4)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/2*(16*A*(-4*a*c+b^2)
^(1/2)*a*b*c-3*A*(-4*a*c+b^2)^(1/2)*b^3-40*A*a^2*c^2+22*A*a*b^2*c-3*A*b^4-
12*C*(-4*a*c+b^2)^(1/2)*a^2*c+C*(-4*a*c+b^2)^(1/2)*a*b^2-4*C*a^2*b*c+C*a*b
^3)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Cx^2+Bx+A}{\left(cx^4+bx^2+a\right)^2x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*a*b*c*x^3 + (10*A*a*c^2 + (C*a*b - 3*A*b^2)*c)*x^4 - 2*A*a*b^2 + 8*
A*a^2*c + (C*a*b^2 - 3*A*b^3 - (2*C*a^2 - 11*A*a*b)*c)*x^2 + (B*a*b^2 - 2*
B*a^2*c)*x)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^
3*b^2 - 4*a^4*c)*x) + 1/2*integrate((C*a*b^2 - 3*A*b^3 - 2*(B*b^2*c - 4*B*
a*c^2)*x^3 + (10*A*a*c^2 + (C*a*b - 3*A*b^2)*c)*x^2 - (6*C*a^2 - 13*A*a*b)
*c - 2*(B*b^3 - 5*B*a*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
+ B*log(x)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9015 vs. $2(455) = 910$.

Time = 1.78 (sec) , antiderivative size = 9015, normalized size of antiderivative = 17.37

$$\int \frac{A + Bx + Cx^2}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/4*B*log(abs(c*x^4 + b*x^2 + a))/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a*b*c*
x^4 - 3*A*b^2*c*x^4 + 10*A*a*c^2*x^4 + B*a*b*c*x^3 + C*a*b^2*x^2 - 3*A*b^3
*x^2 - 2*C*a^2*c*x^2 + 11*A*a*b*c*x^2 + B*a*b^2*x - 2*B*a^2*c*x - 2*A*a*b^
2 + 8*A*a^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*((a^4*b^
4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*
(b^2 - 4*a*c)*a*c^3)*A - (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*a*b
^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 -
 2*(b^2 - 4*a*c)*a*b*c^2)*C + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^4*b^9*c - 49*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 - 6*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 6*a^4*b^9*c^2 + 300...
```

Mupad [B] (verification not implemented)

Time = 20.82 (sec) , antiderivative size = 8684, normalized size of antiderivative = 16.73

$$\int \frac{A+Bx+Cx^2}{x^2 (a+bx^2+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

output
```
symsum(log(root(1572864*a^10*b^2*c^5*z^4 - 983040*a^9*b^4*c^4*z^4 + 327680
*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 6144*a^6*b^10*c*z^4 - 1048576*a
^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8*b^2*c^5*z^3 - 983040*B*a^7*
b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*z^3 + 6144*B*
a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a^3*b^12*z^3 - 2432*A*C*a^2
*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400*A*C*a^5*b^4*c^4*z^2 - 12
9024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*b^12*z^2 +
 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b^10*c*z^2
 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2 + 245760*A*C*a^7*c^6*z
^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^3*z^2 - 4608*C^2*a^4*
b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*a^5*b^4*c^4*z^2 + 88
576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 716800*A^2*a^5*b^3*c
^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7*c^3*z^2 - 33232*A
^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c^6*z^2 - 16*C^2*a
^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*z + 36864*A*B*C*
a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*c^2*z + 3072*B*
C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^7*c^3*z + 1228
80*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3*b^5*c^3*z -
48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^2*B*a^2*b^5*
c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 3328*B^3*a^...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2 (a+bx^2+cx^4)^2} \, dx = \int \frac{C\,x^2+Bx+A}{x^2\,(c\,x^4+b\,x^2+a)^2} dx$$

input
```
int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x)
```

3.37 $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 507

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx &= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{Bx(b(b^2-3ac)+c(b^2-2ac)\,x^2)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\ &\quad -\frac{A(b^3-3abc)-a(b^2-2ac)\,C+c(A(b^2-2ac)-abC)\,x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\ &\quad -\frac{B\sqrt{c}\left(3b^3-16abc+(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\ &\quad +\frac{B\sqrt{c}\left(3b^3-16abc-(3b^2-10ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \\ &\quad -\frac{(2A(b^4-6ab^2c+6a^2c^2)-ab(b^2-6ac)\,C)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}} \\ &\quad -\frac{(2Ab-aC)\log(x)}{a^3}+\frac{(2Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^3}\end{aligned}$$

output
```
-1/2*A/a^2/x^2-B/a^2/x-1/2*B*x*(b*(-3*a*c+b^2)+c*(-2*a*c+b^2)*x^2)/a^2/(-4
*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(A*(-3*a*b*c+b^3)-a*(-2*a*c+b^2)*C+c*(A*(-2*
a*c+b^2)-a*b*C)*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*B*c^(1/2)*(3*b^3
-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-
(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)+1/4*B*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1
/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-
4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-1/2*(2*A*(6*a^2*c^2-6*a*b^2*
c+b^4)-a*b*(-6*a*c+b^2)*C)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4
*a*c+b^2)^(3/2)-(2*A*b-C*a)*ln(x)/a^3+1/4*(2*A*b-C*a)*ln(c*x^4+b*x^2+a)/a^
3
```

Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{2aA}{x^2}-\frac{4aB}{x}-\frac{2a\left(2a^2cC+b^2Bx\left(b+cx^2\right)+A\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)-a\left(b^2C+2Bc^2x^3+bcx(3B+Cx)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}aB\sqrt{c}\left(-3b^3+16abc-3\right.}{}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-2*a*A)/x^2 - (4*a*B)/x - (2*a*(2*a^2*c*C + b^2*B*x*(b + c*x^2) + A*(b^3
 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2) - a*(b^2*C + 2*B*c^2*x^3 + b*c*x*(3*
B + C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(-3
*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*B*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*
b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]) + 4*(-2*A*b + a*C)*Log[x] + ((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 +
b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(-b^3 + 6*a*b*c - b
^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a
*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2
+ b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(b^3 - 6*a*b*c -
b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2193, 27, 1441, 25, 1578, 1235, 25, 1200, 1604, 1480, 218, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 2193}$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)^2}dx+\int \frac{B}{x^2\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{1}{x^2\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 1441}$$

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{3b^2+3cx^2b-10ac}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}\right)$$

$\big\downarrow$ 25

$$\int \frac{Cx^2+A}{x^3\left(cx^4+bx^2+a\right)^2}dx+B\left(\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\big\downarrow$ 1578

$$\frac{1}{2}\int \frac{Cx^2+A}{x^4\left(cx^4+bx^2+a\right)^2}dx^2+B\left(\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\big\downarrow$ 1235

$$\frac{1}{2}\left(\frac{cx^2(Ab-2aC)-2aAc-abC+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{2Ab^2-aCb+2c(Ab-2aC)x^2-6aAc}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)+$$
$$B\left(\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\big\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{2c(Ab-2aC)x^2+2A\left(b^2-3ac\right)-abC}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab-2aC)-2aAc-abC+Ab^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+$$
$$B\left(\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\big\downarrow$ 1200

$$\frac{1}{2}\left(\frac{\int \left(-\frac{\left(4ac-b^2\right)(aC-2Ab)}{a^2x^2}+\frac{c\left(b^2-4ac\right)(2Ab-aC)x^2+2A\left(b^4-5acb^2+3a^2c^2\right)-ab\left(b^2-5ac\right)C}{a^2\left(cx^4+bx^2+a\right)}+\frac{2A\left(b^2-3ac\right)-abC}{ax^4}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{cx^2(Ab}{ax^2}\right.$$
$$B\left(\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\big\downarrow$ 1604

$$\frac{1}{2}\left(\frac{\int\left(-\frac{(4ac-b^2)(aC-2Ab)}{a^2x^2}+\frac{c(b^2-4ac)(2Ab-aC)x^2+2A(b^4-5acb^2+3a^2c^2)-ab(b^2-5ac)C}{a^2(cx^4+bx^2+a)}+\frac{2A(b^2-3ac)-abC}{ax^4}\right)dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab}{ax^2}\right.$$

$$B\left(\frac{-\frac{\int\frac{c(3b^2-10ac)x^2+b(3b^2-13ac)}{cx^4+bx^2+a}dx}{a}-\frac{3b^2-10ac}{ax}}{2a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{2ax(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$\downarrow$ 1480

$$\frac{1}{2}\left(\frac{\int\left(-\frac{(4ac-b^2)(aC-2Ab)}{a^2x^2}+\frac{c(b^2-4ac)(2Ab-aC)x^2+2A(b^4-5acb^2+3a^2c^2)-ab(b^2-5ac)C}{a^2(cx^4+bx^2+a)}+\frac{2A(b^2-3ac)-abC}{ax^4}\right)dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab}{ax^2}\right.$$

$$B\left(\frac{-\frac{\frac{1}{2}c\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{c\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2\sqrt{b^2-4ac}}}{a}-\frac{3b^2}{}}{2a(b^2-4ac)}\right.$$

$\downarrow$ 218

$$\frac{1}{2}\left(\frac{\int\left(-\frac{(4ac-b^2)(aC-2Ab)}{a^2x^2}+\frac{c(b^2-4ac)(2Ab-aC)x^2+2A(b^4-5acb^2+3a^2c^2)-ab(b^2-5ac)C}{a^2(cx^4+bx^2+a)}+\frac{2A(b^2-3ac)-abC}{ax^4}\right)dx^2}{a(b^2-4ac)}+\frac{cx^2(Ab}{ax^2}\right.$$

$$B\left(\frac{-\frac{\frac{\sqrt{c}\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{3b^2-10a}{ax}}{2a(b^2-4ac)}\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{-\frac{(2A(6a^2c^2-6ab^2c+b^4)-abC(b^2-6ac))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{\log(x^2)(b^2-4ac)(2Ab-aC)}{a^2}+\frac{(b^2-4ac)(2Ab-aC)\log(a+bx^2+cx^4)}{2a^2}}{a(b^2-4ac)}\right.$$

$$B\left(\frac{-\frac{\frac{\sqrt{c}\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{3b^2-10ac}{ax}}{2a(b^2-4ac)}\right.$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
B*((b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (-(
(3*b^2 - 10*a*c)/(a*x)) - ((Sqrt[c]*(3*b^2 - 10*a*c + (3*b^3)/Sqrt[b^2 - 4
*a*c] - (16*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b
^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqr
t[b^2 - 4*a*c]]))/a)/(2*a*(b^2 - 4*a*c))) + ((A*b^2 - 2*a*A*c - a*b*C + c*
(A*b - 2*a*C)*x^2)/(a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) + (-((2*A*b^2
 - 6*a*A*c - a*b*C)/(a*x^2)) - ((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2) - a*b*(
b^2 - 6*a*c)*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 -
4*a*c]) - ((b^2 - 4*a*c)*(2*A*b - a*C)*Log[x^2])/a^2 + ((b^2 - 4*a*c)*(2*A
*b - a*C)*Log[a + b*x^2 + c*x^4])/(2*a^2))/(a*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 778, normalized size of antiderivative = 1.53

method	result
default	$-\frac{A}{2a^2x^2}-\frac{B}{a^2x}+\frac{(-2Ab+aC)\ln(x)}{a^3}-\frac{\frac{Bac(2ac-b^2)x^3}{8ac-2b^2}+\frac{ac(2Aac-Ab^2+Cba)x^2}{8ac-2b^2}+\frac{Bab(3ac-b^2)x}{8ac-2b^2}+\frac{a(3Aabc-Ab^3-2Ca^2c+Cab^2}{8ac-2b^2}}{cx^4+bx^2+a}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^2/x^2-B/a^2/x+(-2*A*b+C*a)/a^3*ln(x)-1/a^3*((1/2*B*a*c*(2*a*c-b^2
)/(4*a*c-b^2)*x^3+1/2*a*c*(2*A*a*c-A*b^2+C*a*b)/(4*a*c-b^2)*x^2+1/2*B*a*b*
(3*a*c-b^2)/(4*a*c-b^2)*x+1/2*a*(3*A*a*b*c-A*b^3-2*C*a^2*c+C*a*b^2)/(4*a*c
-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/(16*a*c-4*b^2)*(1/4*(24*A*(-4*a*
c+b^2)^(1/2)*a^2*c^2-24*A*(-4*a*c+b^2)^(1/2)*a*b^2*c+4*A*(-4*a*c+b^2)^(1/2
)*b^4-64*A*a^2*b*c^2+32*A*a*b^3*c-4*A*b^5+12*C*(-4*a*c+b^2)^(1/2)*a^2*b*c-
2*C*(-4*a*c+b^2)^(1/2)*a*b^3+32*C*a^3*c^2-16*C*a^2*b^2*c+2*C*a*b^4)/c*ln(2
*c*x^2+(-4*a*c+b^2)^(1/2)+b)+1/2*(16*B*(-4*a*c+b^2)^(1/2)*a^2*b*c-3*B*(-4*
a*c+b^2)^(1/2)*a*b^3+40*B*a^3*c^2-22*B*a^2*b^2*c+3*B*a*b^4)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)))+1/(16*a*c-4*b^2)*(-1/4*(24*A*(-4*a*c+b^2)^(1/2)*a^2*c^2-24*A*(-4*a*
c+b^2)^(1/2)*a*b^2*c+4*A*(-4*a*c+b^2)^(1/2)*b^4+64*A*a^2*b*c^2-32*A*a*b^3*
c+4*A*b^5+12*C*(-4*a*c+b^2)^(1/2)*a^2*b*c-2*C*(-4*a*c+b^2)^(1/2)*a*b^3-32*
C*a^3*c^2+16*C*a^2*b^2*c-2*C*a*b^4)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/
2*(16*B*(-4*a*c+b^2)^(1/2)*a^2*b*c-3*B*(-4*a*c+b^2)^(1/2)*a*b^3-40*B*a^3*c
^2+22*B*a^2*b^2*c-3*B*a*b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{Cx^2+Bx+A}{\left(cx^4+bx^2+a\right)^2x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*B*b^2*c - 10*B*a*c^2)*x^5 - (6*A*a*c^2 + (C*a*b - 2*A*b^2)*c)*x^4
 + A*a*b^2 - 4*A*a^2*c + (3*B*b^3 - 11*B*a*b*c)*x^3 - (C*a*b^2 - 2*A*b^3 -
 (2*C*a^2 - 7*A*a*b)*c)*x^2 + 2*(B*a*b^2 - 4*B*a^2*c)*x)/((a^2*b^2*c - 4*a
^3*c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4*c)*x^2) - 1/2*i
ntegrate((3*B*a*b^3 - 13*B*a^2*b*c - 2*(4*(C*a^2 - 2*A*a*b)*c^2 - (C*a*b^2
 - 2*A*b^3)*c)*x^3 + (3*B*a*b^2*c - 10*B*a^2*c^2)*x^2 + 2*(C*a*b^3 - 2*A*b
^4 - 6*A*a^2*c^2 - 5*(C*a^2*b - 2*A*a*b^2)*c)*x)/(c*x^4 + b*x^2 + a), x)/(
a^3*b^2 - 4*a^4*c) + (C*a - 2*A*b)*log(x)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6939 vs. $2(445) = 890$.

Time = 1.16 (sec) , antiderivative size = 6939, normalized size of antiderivative = 13.69

$$\int \frac{A + Bx + Cx^2}{x^3 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/16*((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3
 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2
*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 4
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*
b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*B - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^6*b^9*c - 49*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c^
2 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^2 - 6*a^6*b^9*c^2
+ 300*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^3 + 74*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^3 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^6*b^7*c^3 + 98*a^7*b^7*c^3 - 816*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^9*b^3*c^4 - 304*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*
b^4*c^4 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^4 - 600*a^8
*b^5*c^4 + 832*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^5 + 416*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^5 + 152*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^5 + 1632*a^9*b^3*c^5 - 208*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^6 - 1664*a^10*b*c^6 + 6*(b^2 - 4*a*c)*...
```

Mupad [B] (verification not implemented)

Time = 21.03 (sec) , antiderivative size = 10595, normalized size of antiderivative = 20.90

$$\int \frac{A + Bx + Cx^2}{x^3 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

output
```
symsum(log(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 32768
0*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 1048576*
a^12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8
*b^4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 314572
8*A*a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3
 + 122880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*
z^3 - 12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3
 + 512*A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^
2 - 1794048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C
*a^4*b^7*c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C
^2*a^3*b^10*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6
144*A^2*a*b^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4
*z^2 + 88576*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*
a^6*b^3*c^5*z^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2
- 33232*B^2*a^3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4
*b^6*c^4*z^2 - 761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 6
1440*A^2*a^2*b^10*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 -
64*C^2*a^2*b^12*z^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B
^2*C*a^5*b*c^6*z - 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 1228
8*A^2*C*a*b^8*c^3*z + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 1...
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 6801, normalized size of antiderivative = 13.41

$$\int \frac{A + Bx + Cx^2}{x^3 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x)
```

output

```
(48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*c**
2*x**2 - 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*3*b**3*c*x**2 + 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a**3*b**2*c**2*x**4 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*
sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a**3*b**2*c**2*x**2 + 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt
(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**3*x**6 + 8*sqrt(2*sqrt(c)*sqrt(a) +
b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(
c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**5*x**2 - 48*sqrt(2*sqrt(c)*sqrt
(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2
*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**4*c*x**4 - 4*sqrt(2*sqrt(
c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**4*c*x**2 - 48*sqrt
(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c**2*x**
6 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq...
```

3.38 $\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 dx$

Optimal result

Integrand size = 30, antiderivative size = 399

$$\begin{aligned}
&\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 dx \\
&= \frac{a^3A(dx)^{1+m}}{d(1+m)}+\frac{a^3B(dx)^{2+m}}{d^2(2+m)}+\frac{a^2(3Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{3a^2bB(dx)^{4+m}}{d^4(4+m)} \\
&\quad+\frac{3a\left(A\left(b^2+ac\right)+abC\right)(dx)^{5+m}}{d^5(5+m)}+\frac{3aB\left(b^2+ac\right)(dx)^{6+m}}{d^6(6+m)} \\
&\quad+\frac{\left(A\left(b^3+6abc\right)+3a\left(b^2+ac\right)C\right)(dx)^{7+m}}{d^7(7+m)}+\frac{bB\left(b^2+6ac\right)(dx)^{8+m}}{d^8(8+m)} \\
&\quad+\frac{\left(3Ac\left(b^2+ac\right)+b\left(b^2+6ac\right)C\right)(dx)^{9+m}}{d^9(9+m)}+\frac{3Bc\left(b^2+ac\right)(dx)^{10+m}}{d^{10}(10+m)} \\
&\quad+\frac{3c\left(Abc+\left(b^2+ac\right)C\right)(dx)^{11+m}}{d^{11}(11+m)}+\frac{3bBc^2(dx)^{12+m}}{d^{12}(12+m)} \\
&\quad+\frac{c^2(Ac+3bC)(dx)^{13+m}}{d^{13}(13+m)}+\frac{Bc^3(dx)^{14+m}}{d^{14}(14+m)}+\frac{c^3C(dx)^{15+m}}{d^{15}(15+m)}
\end{aligned}$$

output
```
a^3*A*(d*x)^(1+m)/d/(1+m)+a^3*B*(d*x)^(2+m)/d^2/(2+m)+a^2*(3*A*b+C*a)*(d*x
)^(3+m)/d^3/(3+m)+3*a^2*b*B*(d*x)^(4+m)/d^4/(4+m)+3*a*(A*(a*c+b^2)+a*b*C)*
(d*x)^(5+m)/d^5/(5+m)+3*a*B*(a*c+b^2)*(d*x)^(6+m)/d^6/(6+m)+(A*(6*a*b*c+b^
3)+3*a*(a*c+b^2)*C)*(d*x)^(7+m)/d^7/(7+m)+b*B*(6*a*c+b^2)*(d*x)^(8+m)/d^8/
(8+m)+(3*A*c*(a*c+b^2)+b*(6*a*c+b^2)*C)*(d*x)^(9+m)/d^9/(9+m)+3*B*c*(a*c+b
^2)*(d*x)^(10+m)/d^10/(10+m)+3*c*(A*b*c+(a*c+b^2)*C)*(d*x)^(11+m)/d^11/(11
+m)+3*b*B*c^2*(d*x)^(12+m)/d^12/(12+m)+c^2*(A*c+3*C*b)*(d*x)^(13+m)/d^13/(
13+m)+B*c^3*(d*x)^(14+m)/d^14/(14+m)+c^3*C*(d*x)^(15+m)/d^15/(15+m)
```

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.74

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = x(dx)^m \left(\frac{a^3A}{1+m}+\frac{a^3Bx}{2+m}+\frac{a^2(3Ab+aC)x^2}{3+m}+\frac{3a^2bBx^3}{4+m}+\frac{3a(A(b^2+ac)+abC)\,x^4}{5+m}+\frac{3aB(b^2+ac)\,x^5}{6+m}+\frac{(A(b^3+6abc)+3a(b^2+ac)\,C)\,x^6}{7+m}+\frac{bB(b^2+6ac)\,x^7}{8+m}+\frac{(3Ac(b^2+ac)+b(b^2+6ac)\,C)\,x^8}{9+m}+\frac{3Bc(b^2+ac)\,x^9}{10+m}+\frac{3c(Abc+(b^2+ac)\,C)\,x^{10}}{11+m}+\frac{3bBc^2x^{11}}{12+m}+\frac{c^2(Ac+3bC)x^{12}}{13+m}+\frac{Bc^3x^{13}}{14+m}+\frac{c^3Cx^{14}}{15+m}\right)$$

input
```
Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
x*(d*x)^m*((a^3*A)/(1 + m) + (a^3*B*x)/(2 + m) + (a^2*(3*A*b + a*C)*x^2)/(
3 + m) + (3*a^2*b*B*x^3)/(4 + m) + (3*a*(A*(b^2 + a*c) + a*b*C)*x^4)/(5 +
m) + (3*a*B*(b^2 + a*c)*x^5)/(6 + m) + ((A*(b^3 + 6*a*b*c) + 3*a*(b^2 + a*
c)*C)*x^6)/(7 + m) + (b*B*(b^2 + 6*a*c)*x^7)/(8 + m) + ((3*A*c*(b^2 + a*c)
 + b*(b^2 + 6*a*c)*C)*x^8)/(9 + m) + (3*B*c*(b^2 + a*c)*x^9)/(10 + m) + (3
*c*(A*b*c + (b^2 + a*c)*C)*x^10)/(11 + m) + (3*b*B*c^2*x^11)/(12 + m) + (c
^2*(A*c + 3*b*C)*x^12)/(13 + m) + (B*c^3*x^13)/(14 + m) + (c^3*C*x^14)/(15
 + m))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(a^3A(dx)^m+\frac{a^3B(dx)^{m+1}}{d}+\frac{a^2(dx)^{m+2}(aC+3Ab)}{d^2}+\frac{3a^2bB(dx)^{m+3}}{d^3}+\frac{3c(dx)^{m+10}\left(C\left(ac+b^2\right)+Abc\right)}{d^{10}}+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3A(dx)^{m+1}}{d(m+1)}+\frac{a^3B(dx)^{m+2}}{d^2(m+2)}+\frac{a^2(dx)^{m+3}(aC+3Ab)}{d^3(m+3)}+\frac{3a^2bB(dx)^{m+4}}{d^4(m+4)}+\frac{3c(dx)^{m+11}\left(C\left(ac+b^2\right)+Abc\right)}{d^{11}(m+11)}+\frac{(dx)^{m+9}\left(3Ac\left(ac+b^2\right)+bC\left(6ac+b^2\right)\right)}{d^9(m+9)}+\frac{3a(dx)^{m+5}\left(A\left(ac+b^2\right)+abC\right)}{d^5(m+5)}+\frac{(dx)^{m+7}\left(A\left(6abc+b^3\right)+3aC\left(ac+b^2\right)\right)}{d^7(m+7)}+\frac{3Bc\left(ac+b^2\right)(dx)^{m+10}}{d^{10}(m+10)}+\frac{bB\left(6ac+b^2\right)(dx)^{m+8}}{d^8(m+8)}+\frac{3aB\left(ac+b^2\right)(dx)^{m+6}}{d^6(m+6)}+\frac{c^2(dx)^{m+13}(Ac+3bC)}{d^{13}(m+13)}+\frac{3bBc^2(dx)^{m+12}}{d^{12}(m+12)}+\frac{Bc^3(dx)^{m+14}}{d^{14}(m+14)}+\frac{c^3C(dx)^{m+15}}{d^{15}(m+15)}$$

input
```
Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a^3*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^3*B*(d*x)^(2 + m))/(d^2*(2 + m)) +
(a^2*(3*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (3*a^2*b*B*(d*x)^(4 + m)
)/(d^4*(4 + m)) + (3*a*(A*(b^2 + a*c) + a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)
) + (3*a*B*(b^2 + a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((A*(b^3 + 6*a*b*c)
+ 3*a*(b^2 + a*c)*C)*(d*x)^(7 + m))/(d^7*(7 + m)) + (b*B*(b^2 + 6*a*c)*(d*
x)^(8 + m))/(d^8*(8 + m)) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*(d*x)
^(9 + m))/(d^9*(9 + m)) + (3*B*c*(b^2 + a*c)*(d*x)^(10 + m))/(d^10*(10 + m
)) + (3*c*(A*b*c + (b^2 + a*c)*C)*(d*x)^(11 + m))/(d^11*(11 + m)) + (3*b*B
*c^2*(d*x)^(12 + m))/(d^12*(12 + m)) + (c^2*(A*c + 3*b*C)*(d*x)^(13 + m))/
(d^13*(13 + m)) + (B*c^3*(d*x)^(14 + m))/(d^14*(14 + m)) + (c^3*C*(d*x)^(1
5 + m))/(d^15*(15 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5519 vs. $2(399) = 798$.

Time = 0.30 (sec) , antiderivative size = 5520, normalized size of antiderivative = 13.83

method	result	size
gosper	Expression too large to display	5520
risch	Expression too large to display	5520
orering	Expression too large to display	5520
parallelrisch	Expression too large to display	7809

input
```
int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3898 vs. $2(399) = 798$.

Time = 0.18 (sec) , antiderivative size = 3898, normalized size of antiderivative = 9.77

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47658 vs. $2(379) = 758$.

Time = 3.18 (sec) , antiderivative size = 47658, normalized size of antiderivative = 119.44

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**3,x)
```

output

```
Piecewise(((-A*a**3/(14*x**14) - A*a**2*b/(4*x**12) - 3*A*a**2*c/(10*x**10
) - 3*A*a*b**2/(10*x**10) - 3*A*a*b*c/(4*x**8) - A*a*c**2/(2*x**6) - A*b**
3/(8*x**8) - A*b**2*c/(2*x**6) - 3*A*b*c**2/(4*x**4) - A*c**3/(2*x**2) - B
*a**3/(13*x**13) - 3*B*a**2*b/(11*x**11) - B*a**2*c/(3*x**9) - B*a*b**2/(3
*x**9) - 6*B*a*b*c/(7*x**7) - 3*B*a*c**2/(5*x**5) - B*b**3/(7*x**7) - 3*B*
b**2*c/(5*x**5) - B*b*c**2/x**3 - B*c**3/x - C*a**3/(12*x**12) - 3*C*a**2*
b/(10*x**10) - 3*C*a**2*c/(8*x**8) - 3*C*a*b**2/(8*x**8) - C*a*b*c/x**6 -
3*C*a*c**2/(4*x**4) - C*b**3/(6*x**6) - 3*C*b**2*c/(4*x**4) - 3*C*b*c**2/(
2*x**2) + C*c**3*log(x))/d**15, Eq(m, -15)), ((-A*a**3/(13*x**13) - 3*A*a*
*2*b/(11*x**11) - A*a**2*c/(3*x**9) - A*a*b**2/(3*x**9) - 6*A*a*b*c/(7*x**
7) - 3*A*a*c**2/(5*x**5) - A*b**3/(7*x**7) - 3*A*b**2*c/(5*x**5) - A*b*c**
2/x**3 - A*c**3/x - B*a**3/(12*x**12) - 3*B*a**2*b/(10*x**10) - 3*B*a**2*c
/(8*x**8) - 3*B*a*b**2/(8*x**8) - B*a*b*c/x**6 - 3*B*a*c**2/(4*x**4) - B*b
**3/(6*x**6) - 3*B*b**2*c/(4*x**4) - 3*B*b*c**2/(2*x**2) + B*c**3*log(x) -
 C*a**3/(11*x**11) - C*a**2*b/(3*x**9) - 3*C*a**2*c/(7*x**7) - 3*C*a*b**2/
(7*x**7) - 6*C*a*b*c/(5*x**5) - C*a*c**2/x**3 - C*b**3/(5*x**5) - C*b**2*c
/x**3 - 3*C*b*c**2/x + C*c**3*x)/d**14, Eq(m, -14)), ((-A*a**3/(12*x**12)
- 3*A*a**2*b/(10*x**10) - 3*A*a**2*c/(8*x**8) - 3*A*a*b**2/(8*x**8) - A*a*
b*c/x**6 - 3*A*a*c**2/(4*x**4) - A*b**3/(6*x**6) - 3*A*b**2*c/(4*x**4) - 3
*A*b*c**2/(2*x**2) + A*c**3*log(x) - B*a**3/(11*x**11) - B*a**2*b/(3*x*...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.53

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \frac{Cc^3d^mx^{15}x^m}{m+15} + \frac{Bc^3d^mx^{14}x^m}{m+14} + \frac{3\,Cbc^2d^mx^{13}x^m}{m+13} + \frac{Ac^3d^mx^{13}x^m}{m+13} + \frac{3\,Bbc^2d^mx^{12}x^m}{m+12} + \frac{3\,Cb^2cd^mx^{11}x^m}{m+11} + \frac{3\,Cac^2d^mx^{11}x^m}{m+11} + \frac{3\,Abc^2d^mx^{11}x^m}{m+11} + \frac{3\,Bb^2cd^mx^{10}x^m}{m+10} + \frac{3\,Bac^2d^mx^{10}x^m}{m+10} + \frac{Cb^3d^mx^9x^m}{m+9} + \frac{6\,Cabcd^mx^9x^m}{m+9} + \frac{3\,Ab^2cd^mx^9x^m}{m+9} + \frac{3\,Aac^2d^mx^9x^m}{m+9} + \frac{Bb^3d^mx^8x^m}{m+8} + \frac{6\,Babcd^mx^8x^m}{m+8} + \frac{3\,Cab^2d^mx^7x^m}{m+7} + \frac{Ab^3d^mx^7x^m}{m+7} + \frac{3\,Ca^2cd^mx^7x^m}{m+7} + \frac{6\,Aabcd^mx^7x^m}{m+7} + \frac{3\,Bab^2d^mx^6x^m}{m+6} + \frac{3\,Ba^2cd^mx^6x^m}{m+6} + \frac{3\,Ca^2bd^mx^5x^m}{m+5} + \frac{3\,Aab^2d^mx^5x^m}{m+5} + \frac{3\,Aa^2cd^mx^5x^m}{m+5} + \frac{3\,Ba^2bd^mx^4x^m}{m+4} + \frac{Ca^3d^mx^3x^m}{m+3} + \frac{3\,Aa^2bd^mx^3x^m}{m+3} + \frac{Ba^3d^mx^2x^m}{m+2} + \frac{(dx)^{m+1}\,Aa^3}{d(m+1)}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
C*c^3*d^m*x^15*x^m/(m + 15) + B*c^3*d^m*x^14*x^m/(m + 14) + 3*C*b*c^2*d^m*
x^13*x^m/(m + 13) + A*c^3*d^m*x^13*x^m/(m + 13) + 3*B*b*c^2*d^m*x^12*x^m/(
m + 12) + 3*C*b^2*c*d^m*x^11*x^m/(m + 11) + 3*C*a*c^2*d^m*x^11*x^m/(m + 11
) + 3*A*b*c^2*d^m*x^11*x^m/(m + 11) + 3*B*b^2*c*d^m*x^10*x^m/(m + 10) + 3*
B*a*c^2*d^m*x^10*x^m/(m + 10) + C*b^3*d^m*x^9*x^m/(m + 9) + 6*C*a*b*c*d^m*
x^9*x^m/(m + 9) + 3*A*b^2*c*d^m*x^9*x^m/(m + 9) + 3*A*a*c^2*d^m*x^9*x^m/(m
 + 9) + B*b^3*d^m*x^8*x^m/(m + 8) + 6*B*a*b*c*d^m*x^8*x^m/(m + 8) + 3*C*a*
b^2*d^m*x^7*x^m/(m + 7) + A*b^3*d^m*x^7*x^m/(m + 7) + 3*C*a^2*c*d^m*x^7*x^
m/(m + 7) + 6*A*a*b*c*d^m*x^7*x^m/(m + 7) + 3*B*a*b^2*d^m*x^6*x^m/(m + 6)
+ 3*B*a^2*c*d^m*x^6*x^m/(m + 6) + 3*C*a^2*b*d^m*x^5*x^m/(m + 5) + 3*A*a*b^
2*d^m*x^5*x^m/(m + 5) + 3*A*a^2*c*d^m*x^5*x^m/(m + 5) + 3*B*a^2*b*d^m*x^4*
x^m/(m + 4) + C*a^3*d^m*x^3*x^m/(m + 3) + 3*A*a^2*b*d^m*x^3*x^m/(m + 3) +
B*a^3*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a^3/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7808 vs. 2(399) = 798.

Time = 0.20 (sec) , antiderivative size = 7808, normalized size of antiderivative = 19.57

$$\int (dx)^m \left(A + Bx + Cx^2\right) \left(a + bx^2 + cx^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
((d*x)^m*C*c^3*m^14*x^15 + (d*x)^m*B*c^3*m^14*x^14 + 105*(d*x)^m*C*c^3*m^1
3*x^15 + 3*(d*x)^m*C*b*c^2*m^14*x^13 + (d*x)^m*A*c^3*m^14*x^13 + 106*(d*x)
^m*B*c^3*m^13*x^14 + 5005*(d*x)^m*C*c^3*m^12*x^15 + 3*(d*x)^m*B*b*c^2*m^14
*x^12 + 321*(d*x)^m*C*b*c^2*m^13*x^13 + 107*(d*x)^m*A*c^3*m^13*x^13 + 5096
*(d*x)^m*B*c^3*m^12*x^14 + 143325*(d*x)^m*C*c^3*m^11*x^15 + 3*(d*x)^m*C*b^
2*c*m^14*x^11 + 3*(d*x)^m*C*a*c^2*m^14*x^11 + 3*(d*x)^m*A*b*c^2*m^14*x^11
+ 324*(d*x)^m*B*b*c^2*m^13*x^12 + 15567*(d*x)^m*C*b*c^2*m^12*x^13 + 5189*(
d*x)^m*A*c^3*m^12*x^13 + 147056*(d*x)^m*B*c^3*m^11*x^14 + 2749747*(d*x)^m*
C*c^3*m^10*x^15 + 3*(d*x)^m*B*b^2*c*m^14*x^10 + 3*(d*x)^m*B*a*c^2*m^14*x^1
0 + 327*(d*x)^m*C*b^2*c*m^13*x^11 + 327*(d*x)^m*C*a*c^2*m^13*x^11 + 327*(d
*x)^m*A*b*c^2*m^13*x^11 + 15852*(d*x)^m*B*b*c^2*m^12*x^12 + 452829*(d*x)^m
*C*b*c^2*m^11*x^13 + 150943*(d*x)^m*A*c^3*m^11*x^13 + 2840838*(d*x)^m*B*c^
3*m^10*x^14 + 37312275*(d*x)^m*C*c^3*m^9*x^15 + (d*x)^m*C*b^3*m^14*x^9 + 6
*(d*x)^m*C*a*b*c*m^14*x^9 + 3*(d*x)^m*A*b^2*c*m^14*x^9 + 3*(d*x)^m*A*a*c^2
*m^14*x^9 + 330*(d*x)^m*B*b^2*c*m^13*x^10 + 330*(d*x)^m*B*a*c^2*m^13*x^10
+ 16143*(d*x)^m*C*b^2*c*m^12*x^11 + 16143*(d*x)^m*C*a*c^2*m^12*x^11 + 1614
3*(d*x)^m*A*b*c^2*m^12*x^11 + 464976*(d*x)^m*B*b*c^2*m^11*x^12 + 8812089*(
d*x)^m*C*b*c^2*m^10*x^13 + 2937363*(d*x)^m*A*c^3*m^10*x^13 + 38786748*(d*x
)^m*B*c^3*m^9*x^14 + 368411615*(d*x)^m*C*c^3*m^8*x^15 + (d*x)^m*B*b^3*m^14
*x^8 + 6*(d*x)^m*B*a*b*c*m^14*x^8 + 111*(d*x)^m*C*b^3*m^13*x^9 + 666*(d...
```

Mupad [B] (verification not implemented)

Time = 21.06 (sec) , antiderivative size = 2443, normalized size of antiderivative = 6.12

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 dx = \text{Too large to display}$$

input
```
int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

output
```
(x^7*(d*x)^m*(A*b^3 + 3*C*a*b^2 + 3*C*a^2*c + 6*A*a*b*c)*(593193196800*m +
 796089202560*m^2 + 608700928752*m^3 + 299730345264*m^4 + 101420251688*m^5
 + 24483279856*m^6 + 4306835671*m^7 + 557256047*m^8 + 52977099*m^9 + 36544
83*m^10 + 177877*m^11 + 5789*m^12 + 113*m^13 + m^14 + 186810624000))/(4339
163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 +
1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 +
928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 1
20*m^14 + m^15 + 1307674368000) + (x^9*(d*x)^m*(C*b^3 + 3*A*a*c^2 + 3*A*b^
2*c + 6*C*a*b*c)*(465985094400*m + 633314724480*m^2 + 491520108816*m^3 + 2
46143692976*m^4 + 84836490456*m^5 + 20885191136*m^6 + 3749548713*m^7 + 495
342143*m^8 + 48083733*m^9 + 3386083*m^10 + 168171*m^11 + 5581*m^12 + 111*m
^13 + m^14 + 145297152000))/(4339163001600*m + 6165817614720*m^2 + 5056995
703824*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54
631129553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m
^11 + 218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (B*c^3*
x^14*(d*x)^m*(303268406400*m + 418753514880*m^2 + 331303013496*m^3 + 16967
9309436*m^4 + 59999485546*m^5 + 15200266081*m^6 + 2816490248*m^7 + 3850812
68*m^8 + 38786748*m^9 + 2840838*m^10 + 147056*m^11 + 5096*m^12 + 106*m^13
+ m^14 + 93405312000))/(4339163001600*m + 6165817614720*m^2 + 505699570382
4*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 5463...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5520, normalized size of antiderivative = 13.83

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^3 dx = \text{Too large to display}$$

input
```
int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x)
```

output
```
(x**m*d**m*x*(a**4*m**14 + 119*a**4*m**13 + 6461*a**4*m**12 + 211939*a**4*
m**11 + 4687683*a**4*m**10 + 73870797*a**4*m**9 + 854224943*a**4*m**8 + 73
53403057*a**4*m**7 + 47277726496*a**4*m**6 + 225525484184*a**4*m**5 + 7841
46622896*a**4*m**4 + 1922666722704*a**4*m**3 + 3134328981120*a**4*m**2 + 3
031488633600*a**4*m + 1307674368000*a**4 + 3*a**3*b*m**14*x**2 + a**3*b*m*
*14*x + 351*a**3*b*m**13*x**2 + 118*a**3*b*m**13*x + 18687*a**3*b*m**12*x*
*2 + 6344*a**3*b*m**12*x + 599139*a**3*b*m**11*x**2 + 205712*a**3*b*m**11*
x + 12901449*a**3*b*m**10*x**2 + 4488198*a**3*b*m**10*x + 196971093*a**3*b
*m**9*x**2 + 69582084*a**3*b*m**9*x + 2193373941*a**3*b*m**8*x**2 + 788931
572*a**3*b*m**8*x + 18042762177*a**3*b*m**7*x**2 + 6629764856*a**3*b*m**7*
x + 109765102128*a**3*b*m**6*x**2 + 41371599841*a**3*b*m**6*x + 4891143256
56*a**3*b*m**5*x**2 + 190060010998*a**3*b*m**5*x + 1561673344272*a**3*b*m*
*4*x**2 + 629552085084*a**3*b*m**4*x + 3435420003984*a**3*b*m**3*x**2 + 14
47709175432*a**3*b*m**3*x + 4864727099520*a**3*b*m**2*x**2 + 2161577352960
*a**3*b*m**2*x + 3903271545600*a**3*b*m*x**2 + 1842662908800*a**3*b*m*x +
1307674368000*a**3*b*x**2 + 653837184000*a**3*b*x + 3*a**3*c*m**14*x**4 +
a**3*c*m**14*x**2 + 345*a**3*c*m**13*x**4 + 117*a**3*c*m**13*x**2 + 18015*
a**3*c*m**12*x**4 + 6229*a**3*c*m**12*x**2 + 565125*a**3*c*m**11*x**4 + 19
9713*a**3*c*m**11*x**2 + 11873241*a**3*c*m**10*x**4 + 4300483*a**3*c*m**10
*x**2 + 176309235*a**3*c*m**9*x**4 + 65657031*a**3*c*m**9*x**2 + 190274...
```

3.39 $\int (dx)^m (A+Bx+Cx^2) (a+bx^2+cx^4)^2 \, dx$

Optimal result

Integrand size = 30, antiderivative size = 260

$$\begin{aligned}\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 \, dx &= \frac{a^2A(dx)^{1+m}}{d(1+m)} + \frac{a^2B(dx)^{2+m}}{d^2(2+m)} + \frac{a(2Ab+aC)(dx)^{3+m}}{d^3(3+m)} + \frac{2abB(dx)^{4+m}}{d^4(4+m)} \\ &+ \frac{\left(A(b^2+2ac)+2abC\right)(dx)^{5+m}}{d^5(5+m)} + \frac{B(b^2+2ac)\,(dx)^{6+m}}{d^6(6+m)} \\ &+ \frac{\left(2Abc+(b^2+2ac)\,C\right)(dx)^{7+m}}{d^7(7+m)} + \frac{2bBc(dx)^{8+m}}{d^8(8+m)} \\ &+ \frac{c(Ac+2bC)(dx)^{9+m}}{d^9(9+m)} + \frac{Bc^2(dx)^{10+m}}{d^{10}(10+m)} + \frac{c^2C(dx)^{11+m}}{d^{11}(11+m)}\end{aligned}$$

output
```
a^2*A*(d*x)^(1+m)/d/(1+m)+a^2*B*(d*x)^(2+m)/d^2/(2+m)+a*(2*A*b+C*a)*(d*x)^
(3+m)/d^3/(3+m)+2*a*b*B*(d*x)^(4+m)/d^4/(4+m)+(A*(2*a*c+b^2)+2*a*b*C)*(d*x
)^(5+m)/d^5/(5+m)+B*(2*a*c+b^2)*(d*x)^(6+m)/d^6/(6+m)+(2*A*b*c+(2*a*c+b^2)
*C)*(d*x)^(7+m)/d^7/(7+m)+2*b*B*c*(d*x)^(8+m)/d^8/(8+m)+c*(A*c+2*C*b)*(d*x
)^(9+m)/d^9/(9+m)+B*c^2*(d*x)^(10+m)/d^10/(10+m)+c^2*C*(d*x)^(11+m)/d^11/(
11+m)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.71

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx = x(dx)^m\left(\frac{a^2A}{1+m}+\frac{a^2Bx}{2+m}+\frac{a(2Ab+aC)x^2}{3+m}+\frac{2abBx^3}{4+m}+\frac{(A(b^2+2ac)+2abC)\,x^4}{5+m}+\frac{B(b^2+2ac)\,x^5}{6+m}+\frac{(2Abc+(b^2+2ac)\,C)\,x^6}{7+m}+\frac{2bBcx^7}{8+m}+\frac{c(Ac+2bC)x^8}{9+m}+\frac{Bc^2x^9}{10+m}+\frac{c^2Cx^{10}}{11+m}\right)$$

input
```
Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
x*(d*x)^m*((a^2*A)/(1 + m) + (a^2*B*x)/(2 + m) + (a*(2*A*b + a*C)*x^2)/(3
+ m) + (2*a*b*B*x^3)/(4 + m) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^4)/(5 + m) +
 (B*(b^2 + 2*a*c)*x^5)/(6 + m) + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6)/(7 + m)
 + (2*b*B*c*x^7)/(8 + m) + (c*(A*c + 2*b*C)*x^8)/(9 + m) + (B*c^2*x^9)/(10
 + m) + (c^2*C*x^10)/(11 + m))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^2\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(a^2A(dx)^m+\frac{a^2B(dx)^{m+1}}{d}+\frac{(dx)^{m+6}\left(C\left(2ac+b^2\right)+2Abc\right)}{d^6}+\frac{(dx)^{m+4}\left(A\left(2ac+b^2\right)+2abC\right)}{d^4}+\frac{a(dx)^{m+}}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2A(dx)^{m+1}}{d(m+1)}+\frac{a^2B(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+7}\left(C\left(2ac+b^2\right)+2Abc\right)}{d^7(m+7)}+\frac{(dx)^{m+5}\left(A\left(2ac+b^2\right)+2abC\right)}{d^5(m+5)}+\frac{a(dx)^{m+3}(aC+2Ab)}{d^3(m+3)}+\frac{B\left(2ac+b^2\right)(dx)^{m+6}}{d^6(m+6)}+\frac{2abB(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+9}(Ac+2bC)}{d^9(m+9)}+\frac{2bBc(dx)^{m+8}}{d^8(m+8)}+\frac{Bc^2(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2C(dx)^{m+11}}{d^{11}(m+11)}$$

input
```
Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^2*B*(d*x)^(2 + m))/(d^2*(2 + m)) +
(a*(2*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*a*b*B*(d*x)^(4 + m))/(d
^4*(4 + m)) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) +
(B*(b^2 + 2*a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((2*A*b*c + (b^2 + 2*a*c)*
C)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*B*c*(d*x)^(8 + m))/(d^8*(8 + m)) +
(c*(A*c + 2*b*C)*(d*x)^(9 + m))/(d^9*(9 + m)) + (B*c^2*(d*x)^(10 + m))/(d^
10*(10 + m)) + (c^2*C*(d*x)^(11 + m))/(d^11*(11 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2186 vs. $2(260) = 520$.

Time = 0.13 (sec) , antiderivative size = 2187, normalized size of antiderivative = 8.41

method	result	size
gosper	Expression too large to display	2187
risch	Expression too large to display	2187
orering	Expression too large to display	2187
parallelrisch	Expression too large to display	3204

input
```
int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(C*c^2*m^10*x^10+B*c^2*m^10*x^9+55*C*c^2*m^9*x^10+A*c^2*m^10*x^8+56*B*c^
2*m^9*x^9+2*C*b*c*m^10*x^8+1320*C*c^2*m^8*x^10+57*A*c^2*m^9*x^8+2*B*b*c*m^
10*x^7+1365*B*c^2*m^8*x^9+114*C*b*c*m^9*x^8+18150*C*c^2*m^7*x^10+2*A*b*c*m
^10*x^6+1412*A*c^2*m^8*x^8+116*B*b*c*m^9*x^7+19020*B*c^2*m^7*x^9+2*C*a*c*m
^10*x^6+C*b^2*m^10*x^6+2824*C*b*c*m^8*x^8+157773*C*c^2*m^6*x^10+118*A*b*c*
m^9*x^6+19962*A*c^2*m^7*x^8+2*B*a*c*m^10*x^5+B*b^2*m^10*x^5+2922*B*b*c*m^8
*x^7+167223*B*c^2*m^6*x^9+118*C*a*c*m^9*x^6+59*C*b^2*m^9*x^6+39924*C*b*c*m
^7*x^8+902055*C*c^2*m^5*x^10+2*A*a*c*m^10*x^4+A*b^2*m^10*x^4+3024*A*b*c*m^
8*x^6+177765*A*c^2*m^6*x^8+120*B*a*c*m^9*x^5+60*B*b^2*m^9*x^5+41964*B*b*c*
m^7*x^7+965328*B*c^2*m^5*x^9+2*C*a*b*m^10*x^4+3024*C*a*c*m^8*x^6+1512*C*b^
2*m^8*x^6+355530*C*b*c*m^6*x^8+3416930*C*c^2*m^4*x^10+122*A*a*c*m^9*x^4+61
*A*b^2*m^9*x^4+44172*A*b*c*m^7*x^6+1037673*A*c^2*m^5*x^8+2*B*a*b*m^10*x^3+
3130*B*a*c*m^8*x^5+1565*B*b^2*m^8*x^5+379134*B*b*c*m^6*x^7+3686255*B*c^2*m
^4*x^9+122*C*a*b*m^9*x^4+44172*C*a*c*m^7*x^6+22086*C*b^2*m^7*x^6+2075346*C
*b*c*m^5*x^8+8409500*C*c^2*m^3*x^10+2*A*a*b*m^10*x^2+3240*A*a*c*m^8*x^4+16
20*A*b^2*m^8*x^4+405642*A*b*c*m^6*x^6+4000478*A*c^2*m^4*x^8+124*B*a*b*m^9*
x^3+46560*B*a*c*m^7*x^5+23280*B*b^2*m^7*x^5+2242044*B*b*c*m^5*x^7+9133180*
B*c^2*m^3*x^9+C*a^2*m^10*x^2+3240*C*a*b*m^8*x^4+405642*C*a*c*m^6*x^6+20282
1*C*b^2*m^6*x^6+8000956*C*b*c*m^4*x^8+12753576*C*c^2*m^2*x^10+126*A*a*b*m^
9*x^2+49140*A*a*c*m^7*x^4+24570*A*b^2*m^7*x^4+2435622*A*b*c*m^5*x^6+999...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(260) = 520$.

Time = 0.12 (sec) , antiderivative size = 1603, normalized size of antiderivative = 6.17

$$\int (dx)^m \left(A + Bx + Cx^2\right) \left(a + bx^2 + cx^4\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
((C*c^2*m^10 + 55*C*c^2*m^9 + 1320*C*c^2*m^8 + 18150*C*c^2*m^7 + 157773*C*
c^2*m^6 + 902055*C*c^2*m^5 + 3416930*C*c^2*m^4 + 8409500*C*c^2*m^3 + 12753
576*C*c^2*m^2 + 10628640*C*c^2*m + 3628800*C*c^2)*x^11 + (B*c^2*m^10 + 56*
B*c^2*m^9 + 1365*B*c^2*m^8 + 19020*B*c^2*m^7 + 167223*B*c^2*m^6 + 965328*B
*c^2*m^5 + 3686255*B*c^2*m^4 + 9133180*B*c^2*m^3 + 13926276*B*c^2*m^2 + 11
655216*B*c^2*m + 3991680*B*c^2)*x^10 + ((2*C*b*c + A*c^2)*m^10 + 57*(2*C*b
*c + A*c^2)*m^9 + 1412*(2*C*b*c + A*c^2)*m^8 + 19962*(2*C*b*c + A*c^2)*m^7
 + 177765*(2*C*b*c + A*c^2)*m^6 + 1037673*(2*C*b*c + A*c^2)*m^5 + 4000478*
(2*C*b*c + A*c^2)*m^4 + 9991428*(2*C*b*c + A*c^2)*m^3 + 8870400*C*b*c + 44
35200*A*c^2 + 15335224*(2*C*b*c + A*c^2)*m^2 + 12900960*(2*C*b*c + A*c^2)*
m)*x^9 + 2*(B*b*c*m^10 + 58*B*b*c*m^9 + 1461*B*b*c*m^8 + 20982*B*b*c*m^7 +
 189567*B*b*c*m^6 + 1121022*B*b*c*m^5 + 4371359*B*b*c*m^4 + 11024858*B*b*c
*m^3 + 17059212*B*b*c*m^2 + 14444280*B*b*c*m + 4989600*B*b*c)*x^8 + ((C*b^
2 + 2*(C*a + A*b)*c)*m^10 + 59*(C*b^2 + 2*(C*a + A*b)*c)*m^9 + 1512*(C*b^2
 + 2*(C*a + A*b)*c)*m^8 + 22086*(C*b^2 + 2*(C*a + A*b)*c)*m^7 + 202821*(C*
b^2 + 2*(C*a + A*b)*c)*m^6 + 1217811*(C*b^2 + 2*(C*a + A*b)*c)*m^5 + 48148
58*(C*b^2 + 2*(C*a + A*b)*c)*m^4 + 12291724*(C*b^2 + 2*(C*a + A*b)*c)*m^3
+ 5702400*C*b^2 + 19216008*(C*b^2 + 2*(C*a + A*b)*c)*m^2 + 11404800*(C*a +
 A*b)*c + 16405920*(C*b^2 + 2*(C*a + A*b)*c)*m)*x^7 + ((B*b^2 + 2*B*a*c)*m
^10 + 60*(B*b^2 + 2*B*a*c)*m^9 + 1565*(B*b^2 + 2*B*a*c)*m^8 + 23280*(B*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16323 vs. $2(245) = 490$.

Time = 1.42 (sec) , antiderivative size = 16323, normalized size of antiderivative = 62.78

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

output
```
Piecewise(((-A*a**2/(10*x**10) - A*a*b/(4*x**8) - A*a*c/(3*x**6) - A*b**2/
(6*x**6) - A*b*c/(2*x**4) - A*c**2/(2*x**2) - B*a**2/(9*x**9) - 2*B*a*b/(7
*x**7) - 2*B*a*c/(5*x**5) - B*b**2/(5*x**5) - 2*B*b*c/(3*x**3) - B*c**2/x
- C*a**2/(8*x**8) - C*a*b/(3*x**6) - C*a*c/(2*x**4) - C*b**2/(4*x**4) - C*
b*c/x**2 + C*c**2*log(x))/d**11, Eq(m, -11)), ((-A*a**2/(9*x**9) - 2*A*a*b
/(7*x**7) - 2*A*a*c/(5*x**5) - A*b**2/(5*x**5) - 2*A*b*c/(3*x**3) - A*c**2
/x - B*a**2/(8*x**8) - B*a*b/(3*x**6) - B*a*c/(2*x**4) - B*b**2/(4*x**4) -
 B*b*c/x**2 + B*c**2*log(x) - C*a**2/(7*x**7) - 2*C*a*b/(5*x**5) - 2*C*a*c
/(3*x**3) - C*b**2/(3*x**3) - 2*C*b*c/x + C*c**2*x)/d**10, Eq(m, -10)), ((
-A*a**2/(8*x**8) - A*a*b/(3*x**6) - A*a*c/(2*x**4) - A*b**2/(4*x**4) - A*b
*c/x**2 + A*c**2*log(x) - B*a**2/(7*x**7) - 2*B*a*b/(5*x**5) - 2*B*a*c/(3*
x**3) - B*b**2/(3*x**3) - 2*B*b*c/x + B*c**2*x - C*a**2/(6*x**6) - C*a*b/(
2*x**4) - C*a*c/x**2 - C*b**2/(2*x**2) + 2*C*b*c*log(x) + C*c**2*x**2/2)/d
**9, Eq(m, -9)), ((-A*a**2/(7*x**7) - 2*A*a*b/(5*x**5) - 2*A*a*c/(3*x**3)
- A*b**2/(3*x**3) - 2*A*b*c/x + A*c**2*x - B*a**2/(6*x**6) - B*a*b/(2*x**4
) - B*a*c/x**2 - B*b**2/(2*x**2) + 2*B*b*c*log(x) + B*c**2*x**2/2 - C*a**2
/(5*x**5) - 2*C*a*b/(3*x**3) - 2*C*a*c/x - C*b**2/x + 2*C*b*c*x + C*c**2*x
**3/3)/d**8, Eq(m, -8)), ((-A*a**2/(6*x**6) - A*a*b/(2*x**4) - A*a*c/x**2
- A*b**2/(2*x**2) + 2*A*b*c*log(x) + A*c**2*x**2/2 - B*a**2/(5*x**5) - 2*B
*a*b/(3*x**3) - 2*B*a*c/x - B*b**2/x + 2*B*b*c*x + B*c**2*x**3/3 - C*a*...
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.32

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx = \frac{Cc^2d^mx^{11}x^m}{m+11} + \frac{Bc^2d^mx^{10}x^m}{m+10} + \frac{2\,Cbcd^mx^9x^m}{m+9} + \frac{Ac^2d^mx^9x^m}{m+9} + \frac{2\,Bbcd^mx^8x^m}{m+8} + \frac{Cb^2d^mx^7x^m}{m+7} + \frac{2\,Cacd^mx^7x^m}{m+7} + \frac{2\,Abcd^mx^7x^m}{m+7} + \frac{Bb^2d^mx^6x^m}{m+6} + \frac{2\,Bacd^mx^6x^m}{m+6} + \frac{2\,Cabd^mx^5x^m}{m+5} + \frac{Ab^2d^mx^5x^m}{m+5} + \frac{2\,Aacd^mx^5x^m}{m+5} + \frac{2\,Babd^mx^4x^m}{m+4} + \frac{Ca^2d^mx^3x^m}{m+3} + \frac{2\,Aabd^mx^3x^m}{m+3} + \frac{Ba^2d^mx^2x^m}{m+2} + \frac{(dx)^{m+1}Aa^2}{d(m+1)}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
C*c^2*d^m*x^11*x^m/(m + 11) + B*c^2*d^m*x^10*x^m/(m + 10) + 2*C*b*c*d^m*x^
9*x^m/(m + 9) + A*c^2*d^m*x^9*x^m/(m + 9) + 2*B*b*c*d^m*x^8*x^m/(m + 8) +
C*b^2*d^m*x^7*x^m/(m + 7) + 2*C*a*c*d^m*x^7*x^m/(m + 7) + 2*A*b*c*d^m*x^7*
x^m/(m + 7) + B*b^2*d^m*x^6*x^m/(m + 6) + 2*B*a*c*d^m*x^6*x^m/(m + 6) + 2*
C*a*b*d^m*x^5*x^m/(m + 5) + A*b^2*d^m*x^5*x^m/(m + 5) + 2*A*a*c*d^m*x^5*x^
m/(m + 5) + 2*B*a*b*d^m*x^4*x^m/(m + 4) + C*a^2*d^m*x^3*x^m/(m + 3) + 2*A*
a*b*d^m*x^3*x^m/(m + 3) + B*a^2*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a^2/
(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3203 vs. $2(260) = 520$.

Time $= 0.14$ (sec) , antiderivative size $= 3203$, normalized size of antiderivative $= 12.32$

$$\int (dx)^m \left(A + Bx + Cx^2\right) \left(a + bx^2 + cx^4\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
((d*x)^m*C*c^2*m^10*x^11 + (d*x)^m*B*c^2*m^10*x^10 + 55*(d*x)^m*C*c^2*m^9*
x^11 + 2*(d*x)^m*C*b*c*m^10*x^9 + (d*x)^m*A*c^2*m^10*x^9 + 56*(d*x)^m*B*c^
2*m^9*x^10 + 1320*(d*x)^m*C*c^2*m^8*x^11 + 2*(d*x)^m*B*b*c*m^10*x^8 + 114*
(d*x)^m*C*b*c*m^9*x^9 + 57*(d*x)^m*A*c^2*m^9*x^9 + 1365*(d*x)^m*B*c^2*m^8*
x^10 + 18150*(d*x)^m*C*c^2*m^7*x^11 + (d*x)^m*C*b^2*m^10*x^7 + 2*(d*x)^m*C
*a*c*m^10*x^7 + 2*(d*x)^m*A*b*c*m^10*x^7 + 116*(d*x)^m*B*b*c*m^9*x^8 + 282
4*(d*x)^m*C*b*c*m^8*x^9 + 1412*(d*x)^m*A*c^2*m^8*x^9 + 19020*(d*x)^m*B*c^2
*m^7*x^10 + 157773*(d*x)^m*C*c^2*m^6*x^11 + (d*x)^m*B*b^2*m^10*x^6 + 2*(d*
x)^m*B*a*c*m^10*x^6 + 59*(d*x)^m*C*b^2*m^9*x^7 + 118*(d*x)^m*C*a*c*m^9*x^7
 + 118*(d*x)^m*A*b*c*m^9*x^7 + 2922*(d*x)^m*B*b*c*m^8*x^8 + 39924*(d*x)^m*
C*b*c*m^7*x^9 + 19962*(d*x)^m*A*c^2*m^7*x^9 + 167223*(d*x)^m*B*c^2*m^6*x^1
0 + 902055*(d*x)^m*C*c^2*m^5*x^11 + 2*(d*x)^m*C*a*b*m^10*x^5 + (d*x)^m*A*b
^2*m^10*x^5 + 2*(d*x)^m*A*a*c*m^10*x^5 + 60*(d*x)^m*B*b^2*m^9*x^6 + 120*(d
*x)^m*B*a*c*m^9*x^6 + 1512*(d*x)^m*C*b^2*m^8*x^7 + 3024*(d*x)^m*C*a*c*m^8*
x^7 + 3024*(d*x)^m*A*b*c*m^8*x^7 + 41964*(d*x)^m*B*b*c*m^7*x^8 + 355530*(d
*x)^m*C*b*c*m^6*x^9 + 177765*(d*x)^m*A*c^2*m^6*x^9 + 965328*(d*x)^m*B*c^2*
m^5*x^10 + 3416930*(d*x)^m*C*c^2*m^4*x^11 + 2*(d*x)^m*B*a*b*m^10*x^4 + 122
*(d*x)^m*C*a*b*m^9*x^5 + 61*(d*x)^m*A*b^2*m^9*x^5 + 122*(d*x)^m*A*a*c*m^9*
x^5 + 1565*(d*x)^m*B*b^2*m^8*x^6 + 3130*(d*x)^m*B*a*c*m^8*x^6 + 22086*(d*x
)^m*C*b^2*m^7*x^7 + 44172*(d*x)^m*C*a*c*m^7*x^7 + 44172*(d*x)^m*A*b*c*m...
```

Mupad [B] (verification not implemented)

Time = 19.46 (sec) , antiderivative size = 1314, normalized size of antiderivative = 5.05

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right)^2 dx = \text{Too large to display}$$

input
```
int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
(x^5*(d*x)^m*(A*b^2 + 2*A*a*c + 2*C*a*b)*(22512096*m + 25681176*m^2 + 1591
5380*m^3 + 6016070*m^4 + 1464693*m^5 + 234573*m^6 + 24570*m^7 + 1620*m^8 +
 61*m^9 + m^10 + 7983360))/(120543840*m + 150917976*m^2 + 105258076*m^3 +
45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*
m^9 + 66*m^10 + m^11 + 39916800) + (x^7*(d*x)^m*(C*b^2 + 2*A*b*c + 2*C*a*c
)*(16405920*m + 19216008*m^2 + 12291724*m^3 + 4814858*m^4 + 1217811*m^5 +
202821*m^6 + 22086*m^7 + 1512*m^8 + 59*m^9 + m^10 + 5702400))/(120543840*m
 + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m
^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (B*x
^6*(d*x)^m*(2*a*c + b^2)*(18981840*m + 21989356*m^2 + 13878120*m^3 + 53529
35*m^4 + 1331100*m^5 + 217743*m^6 + 23280*m^7 + 1565*m^8 + 60*m^9 + m^10 +
 6652800))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 1
3339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 +
m^11 + 39916800) + (A*a^2*x*(d*x)^m*(80627040*m + 70290936*m^2 + 34967140*
m^3 + 11028590*m^4 + 2310945*m^5 + 326613*m^6 + 30810*m^7 + 1860*m^8 + 65*
m^9 + m^10 + 39916800))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 459
95730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9
 + 66*m^10 + m^11 + 39916800) + (c*x^9*(d*x)^m*(A*c + 2*C*b)*(12900960*m +
 15335224*m^2 + 9991428*m^3 + 4000478*m^4 + 1037673*m^5 + 177765*m^6 + 199
62*m^7 + 1412*m^8 + 57*m^9 + m^10 + 4435200))/(120543840*m + 150917976*...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2220, normalized size of antiderivative = 8.54

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx = \text{Too large to display}$$

input
```
int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

output

```
(x**m*d**m*x*(a**3*m**10 + 65*a**3*m**9 + 1860*a**3*m**8 + 30810*a**3*m**7
 + 326613*a**3*m**6 + 2310945*a**3*m**5 + 11028590*a**3*m**4 + 34967140*a*
*3*m**3 + 70290936*a**3*m**2 + 80627040*a**3*m + 39916800*a**3 + 2*a**2*b*
m**10*x**2 + a**2*b*m**10*x + 126*a**2*b*m**9*x**2 + 64*a**2*b*m**9*x + 34
72*a**2*b*m**8*x**2 + 1797*a**2*b*m**8*x + 54924*a**2*b*m**7*x**2 + 29076*
a**2*b*m**7*x + 550074*a**2*b*m**6*x**2 + 299271*a**2*b*m**6*x + 3624894*a
**2*b*m**5*x**2 + 2039016*a**2*b*m**5*x + 15804388*a**2*b*m**4*x**2 + 9261
503*a**2*b*m**4*x + 44578296*a**2*b*m**3*x**2 + 27472724*a**2*b*m**3*x + 7
6781264*a**2*b*m**2*x**2 + 50312628*a**2*b*m**2*x + 71492160*a**2*b*m*x**2
 + 50292720*a**2*b*m*x + 26611200*a**2*b*x**2 + 19958400*a**2*b*x + 2*a**2
*c*m**10*x**4 + a**2*c*m**10*x**2 + 122*a**2*c*m**9*x**4 + 63*a**2*c*m**9*
x**2 + 3240*a**2*c*m**8*x**4 + 1736*a**2*c*m**8*x**2 + 49140*a**2*c*m**7*x
**4 + 27462*a**2*c*m**7*x**2 + 469146*a**2*c*m**6*x**4 + 275037*a**2*c*m**
6*x**2 + 2929386*a**2*c*m**5*x**4 + 1812447*a**2*c*m**5*x**2 + 12032140*a*
*2*c*m**4*x**4 + 7902194*a**2*c*m**4*x**2 + 31830760*a**2*c*m**3*x**4 + 22
289148*a**2*c*m**3*x**2 + 51362352*a**2*c*m**2*x**4 + 38390632*a**2*c*m**2
*x**2 + 45024192*a**2*c*m*x**4 + 35746080*a**2*c*m*x**2 + 15966720*a**2*c*
x**4 + 13305600*a**2*c*x**2 + a*b**2*m**10*x**4 + 2*a*b**2*m**10*x**3 + 61
*a*b**2*m**9*x**4 + 124*a*b**2*m**9*x**3 + 1620*a*b**2*m**8*x**4 + 3354*a*
b**2*m**8*x**3 + 24570*a*b**2*m**7*x**4 + 51924*a*b**2*m**7*x**3 + 2345...
```

3.40 $\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)\, dx$

Optimal result

Integrand size = 28, antiderivative size = 137

$$\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)\, dx = \frac{aA(dx)^{1+m}}{d(1+m)}+\frac{aB(dx)^{2+m}}{d^2(2+m)}+\frac{(Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{bB(dx)^{4+m}}{d^4(4+m)} + \frac{(Ac+bC)(dx)^{5+m}}{d^5(5+m)}+\frac{Bc(dx)^{6+m}}{d^6(6+m)}+\frac{cC(dx)^{7+m}}{d^7(7+m)}$$

output
```
a*A*(d*x)^(1+m)/d/(1+m)+a*B*(d*x)^(2+m)/d^2/(2+m)+(A*b+C*a)*(d*x)^(3+m)/d^
3/(3+m)+b*B*(d*x)^(4+m)/d^4/(4+m)+(A*c+C*b)*(d*x)^(5+m)/d^5/(5+m)+B*c*(d*x
)^(6+m)/d^6/(6+m)+c*C*(d*x)^(7+m)/d^7/(7+m)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int (dx)^m (A+Bx+Cx^2)(a+bx^2+cx^4)\, dx = x(dx)^m\left(\frac{aA}{1+m}+\frac{aBx}{2+m}+\frac{(Ab+aC)x^2}{3+m}+\frac{bBx^3}{4+m}+\frac{(Ac+bC)x^4}{5+m}+\frac{Bcx^5}{6+m}+\frac{cCx^6}{7+m}\right)$$

input
```
Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
x*(d*x)^m*((a*A)/(1 + m) + (a*B*x)/(2 + m) + ((A*b + a*C)*x^2)/(3 + m) + (
b*B*x^3)/(4 + m) + ((A*c + b*C)*x^4)/(5 + m) + (B*c*x^5)/(6 + m) + (c*C*x^
6)/(7 + m))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^2+cx^4\right) \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{(dx)^{m+2}(aC+Ab)}{d^2}+aA(dx)^m+\frac{aB(dx)^{m+1}}{d}+\frac{(dx)^{m+4}(Ac+bC)}{d^4}+\frac{bB(dx)^{m+3}}{d^3}+\frac{Bc(dx)^{m+5}}{d^5}+\frac{cC(dx}{d^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(dx)^{m+3}(aC+Ab)}{d^3(m+3)}+\frac{aA(dx)^{m+1}}{d(m+1)}+\frac{aB(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}(Ac+bC)}{d^5(m+5)}+\frac{bB(dx)^{m+4}}{d^4(m+4)}+\frac{Bc(dx)^{m+6}}{d^6(m+6)}+\frac{cC(dx)^{m+7}}{d^7(m+7)}$$

input
```
Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

output
```
(a*A*(d*x)^(1 + m))/(d*(1 + m)) + (a*B*(d*x)^(2 + m))/(d^2*(2 + m)) + ((A*
b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (b*B*(d*x)^(4 + m))/(d^4*(4 + m))
+ ((A*c + b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*c*(d*x)^(6 + m))/(d^6*(6
+ m)) + (c*C*(d*x)^(7 + m))/(d^7*(7 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

method	result
norman	$\frac{(Ab+aC)x^3 e^{m\ln(dx)}}{3+m}+\frac{(Ac+Cb)x^5 e^{m\ln(dx)}}{5+m}+\frac{Aax\,e^{m\ln(dx)}}{1+m}+\frac{Ba\,x^2 e^{m\ln(dx)}}{2+m}+\frac{Bb\,x^4 e^{m\ln(dx)}}{4+m}+\frac{Bc\,x^6 e^{m\ln(dx)}}{6+m}$
gosper	$\frac{x(Cc\,m^6x^6+Bc\,m^6x^5+21Cc\,m^5x^6+Ac\,m^6x^4+22Bc\,m^5x^5+Cb\,m^6x^4+175Cc\,m^4x^6+23Ac\,m^5x^4+Bb\,m^6x^3+190Bc\,m^4x^5}{}$
risch	$\frac{x(Cc\,m^6x^6+Bc\,m^6x^5+21Cc\,m^5x^6+Ac\,m^6x^4+22Bc\,m^5x^5+Cb\,m^6x^4+175Cc\,m^4x^6+23Ac\,m^5x^4+Bb\,m^6x^3+190Bc\,m^4x^5}{}$
orering	$\frac{x(Cc\,m^6x^6+Bc\,m^6x^5+21Cc\,m^5x^6+Ac\,m^6x^4+22Bc\,m^5x^5+Cb\,m^6x^4+175Cc\,m^4x^6+23Ac\,m^5x^4+Bb\,m^6x^3+190Bc\,m^4x^5}{}$
parallelrisch	$\frac{720C\,x^7(dx)^m c+840B\,x^6(dx)^m c+1008A\,x^5(dx)^m c+1008C\,x^5(dx)^m b+1260B\,x^4(dx)^m b+1680A\,x^3(dx)^m b+1680C\,x^3(dx)^m}{}$

input
```
int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(A*b+C*a)/(3+m)*x^3*exp(m*ln(d*x))+(A*c+C*b)/(5+m)*x^5*exp(m*ln(d*x))+A*a/
(1+m)*x*exp(m*ln(d*x))+B*a/(2+m)*x^2*exp(m*ln(d*x))+B*b/(4+m)*x^4*exp(m*ln
(d*x))+B*c/(6+m)*x^6*exp(m*ln(d*x))+C*c/(7+m)*x^7*exp(m*ln(d*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(137) = 274$.

Time = 0.10 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.24

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4) \, dx = \frac{((Ccm^6 + 21\,Ccm^5 + 175\,Ccm^4 + 735\,Ccm^3 + 1624\,Ccm^2 + 1764\,Ccm + 720\,Cc)x^7 + (Bcm^6 + 22\,Bcm}{}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
((C*c*m^6 + 21*C*c*m^5 + 175*C*c*m^4 + 735*C*c*m^3 + 1624*C*c*m^2 + 1764*C
*c*m + 720*C*c)*x^7 + (B*c*m^6 + 22*B*c*m^5 + 190*B*c*m^4 + 820*B*c*m^3 +
1849*B*c*m^2 + 2038*B*c*m + 840*B*c)*x^6 + ((C*b + A*c)*m^6 + 23*(C*b + A*
c)*m^5 + 207*(C*b + A*c)*m^4 + 925*(C*b + A*c)*m^3 + 2144*(C*b + A*c)*m^2
+ 1008*C*b + 1008*A*c + 2412*(C*b + A*c)*m)*x^5 + (B*b*m^6 + 24*B*b*m^5 +
226*B*b*m^4 + 1056*B*b*m^3 + 2545*B*b*m^2 + 2952*B*b*m + 1260*B*b)*x^4 + (
(C*a + A*b)*m^6 + 25*(C*a + A*b)*m^5 + 247*(C*a + A*b)*m^4 + 1219*(C*a + A
*b)*m^3 + 3112*(C*a + A*b)*m^2 + 1680*C*a + 1680*A*b + 3796*(C*a + A*b)*m)
*x^3 + (B*a*m^6 + 26*B*a*m^5 + 270*B*a*m^4 + 1420*B*a*m^3 + 3929*B*a*m^2 +
 5274*B*a*m + 2520*B*a)*x^2 + (A*a*m^6 + 27*A*a*m^5 + 295*A*a*m^4 + 1665*A
*a*m^3 + 5104*A*a*m^2 + 8028*A*a*m + 5040*A*a)*x)*(d*x)^m/(m^7 + 28*m^6 +
322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3628 vs. $2(122) = 244$.

Time = 0.77 (sec) , antiderivative size = 3628, normalized size of antiderivative = 26.48

$$\int (dx)^m (A + Bx + Cx^2) (a + bx^2 + cx^4) \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

output
```
Piecewise(((-A*a/(6*x**6) - A*b/(4*x**4) - A*c/(2*x**2) - B*a/(5*x**5) - B
*b/(3*x**3) - B*c/x - C*a/(4*x**4) - C*b/(2*x**2) + C*c*log(x))/d**7, Eq(m
, -7)), ((-A*a/(5*x**5) - A*b/(3*x**3) - A*c/x - B*a/(4*x**4) - B*b/(2*x**
2) + B*c*log(x) - C*a/(3*x**3) - C*b/x + C*c*x)/d**6, Eq(m, -6)), ((-A*a/(
4*x**4) - A*b/(2*x**2) + A*c*log(x) - B*a/(3*x**3) - B*b/x + B*c*x - C*a/(
2*x**2) + C*b*log(x) + C*c*x**2/2)/d**5, Eq(m, -5)), ((-A*a/(3*x**3) - A*b
/x + A*c*x - B*a/(2*x**2) + B*b*log(x) + B*c*x**2/2 - C*a/x + C*b*x + C*c*
x**3/3)/d**4, Eq(m, -4)), ((-A*a/(2*x**2) + A*b*log(x) + A*c*x**2/2 - B*a/
x + B*b*x + B*c*x**3/3 + C*a*log(x) + C*b*x**2/2 + C*c*x**4/4)/d**3, Eq(m,
 -3)), ((-A*a/x + A*b*x + A*c*x**3/3 + B*a*log(x) + B*b*x**2/2 + B*c*x**4/
4 + C*a*x + C*b*x**3/3 + C*c*x**5/5)/d**2, Eq(m, -2)), ((A*a*log(x) + A*b*
x**2/2 + A*c*x**4/4 + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*a*x**2/2 + C*b*x
**4/4 + C*c*x**6/6)/d, Eq(m, -1)), (A*a*m**6*x*(d*x)**m/(m**7 + 28*m**6 +
322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 27*A*a*m
**5*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*
m**2 + 13068*m + 5040) + 295*A*a*m**4*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1665*A*a*m**3*x
*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
+ 13068*m + 5040) + 5104*A*a*m**2*x*(d*x)**m/(m**7 + 28*m**6 + 322*m**5 +
1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 8028*A*a*m*x*(d*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.13

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right) \, dx = \frac{Ccd^m x^7 x^m}{m+7} + \frac{Bcd^m x^6 x^m}{m+6} + \frac{Cbd^m x^5 x^m}{m+5} + \frac{Acd^m x^5 x^m}{m+5} + \frac{Bbd^m x^4 x^m}{m+4} + \frac{Cad^m x^3 x^m}{m+3} + \frac{Abd^m x^3 x^m}{m+3} + \frac{Bad^m x^2 x^m}{m+2} + \frac{(dx)^{m+1} Aa}{d(m+1)}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
C*c*d^m*x^7*x^m/(m + 7) + B*c*d^m*x^6*x^m/(m + 6) + C*b*d^m*x^5*x^m/(m + 5
) + A*c*d^m*x^5*x^m/(m + 5) + B*b*d^m*x^4*x^m/(m + 4) + C*a*d^m*x^3*x^m/(m
 + 3) + A*b*d^m*x^3*x^m/(m + 3) + B*a*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*
A*a/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(137) = 274$.

Time = 0.15 (sec) , antiderivative size = 914, normalized size of antiderivative = 6.67

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
((d*x)^m*C*c*m^6*x^7 + (d*x)^m*B*c*m^6*x^6 + 21*(d*x)^m*C*c*m^5*x^7 + (d*x
)^m*C*b*m^6*x^5 + (d*x)^m*A*c*m^6*x^5 + 22*(d*x)^m*B*c*m^5*x^6 + 175*(d*x)
^m*C*c*m^4*x^7 + (d*x)^m*B*b*m^6*x^4 + 23*(d*x)^m*C*b*m^5*x^5 + 23*(d*x)^m
*A*c*m^5*x^5 + 190*(d*x)^m*B*c*m^4*x^6 + 735*(d*x)^m*C*c*m^3*x^7 + (d*x)^m
*C*a*m^6*x^3 + (d*x)^m*A*b*m^6*x^3 + 24*(d*x)^m*B*b*m^5*x^4 + 207*(d*x)^m*
C*b*m^4*x^5 + 207*(d*x)^m*A*c*m^4*x^5 + 820*(d*x)^m*B*c*m^3*x^6 + 1624*(d*
x)^m*C*c*m^2*x^7 + (d*x)^m*B*a*m^6*x^2 + 25*(d*x)^m*C*a*m^5*x^3 + 25*(d*x)
^m*A*b*m^5*x^3 + 226*(d*x)^m*B*b*m^4*x^4 + 925*(d*x)^m*C*b*m^3*x^5 + 925*(
d*x)^m*A*c*m^3*x^5 + 1849*(d*x)^m*B*c*m^2*x^6 + 1764*(d*x)^m*C*c*m*x^7 + (
d*x)^m*A*a*m^6*x + 26*(d*x)^m*B*a*m^5*x^2 + 247*(d*x)^m*C*a*m^4*x^3 + 247*
(d*x)^m*A*b*m^4*x^3 + 1056*(d*x)^m*B*b*m^3*x^4 + 2144*(d*x)^m*C*b*m^2*x^5
+ 2144*(d*x)^m*A*c*m^2*x^5 + 2038*(d*x)^m*B*c*m*x^6 + 720*(d*x)^m*C*c*x^7
+ 27*(d*x)^m*A*a*m^5*x + 270*(d*x)^m*B*a*m^4*x^2 + 1219*(d*x)^m*C*a*m^3*x^
3 + 1219*(d*x)^m*A*b*m^3*x^3 + 2545*(d*x)^m*B*b*m^2*x^4 + 2412*(d*x)^m*C*b
*m*x^5 + 2412*(d*x)^m*A*c*m*x^5 + 840*(d*x)^m*B*c*x^6 + 295*(d*x)^m*A*a*m^
4*x + 1420*(d*x)^m*B*a*m^3*x^2 + 3112*(d*x)^m*C*a*m^2*x^3 + 3112*(d*x)^m*A
*b*m^2*x^3 + 2952*(d*x)^m*B*b*m*x^4 + 1008*(d*x)^m*C*b*x^5 + 1008*(d*x)^m*
A*c*x^5 + 1665*(d*x)^m*A*a*m^3*x + 3929*(d*x)^m*B*a*m^2*x^2 + 3796*(d*x)^m
*C*a*m*x^3 + 3796*(d*x)^m*A*b*m*x^3 + 1260*(d*x)^m*B*b*x^4 + 5104*(d*x)^m*
A*a*m^2*x + 5274*(d*x)^m*B*a*m*x^2 + 1680*(d*x)^m*C*a*x^3 + 1680*(d*x)^...
```

Mupad [B] (verification not implemented)

Time = 18.65 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.85

$$\int (dx)^m \left(A+Bx+Cx^2\right) \left(a+bx^2+cx^4\right) \, dx$$
$$= \frac{x^3\,{\left(d\,x\right)}^m\,\left(A\,b+C\,a\right)\,\left(m^6+25\,m^5+247\,m^4+1219\,m^3+3112\,m^2+3796\,m+1680\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+ \frac{x^5\,{\left(d\,x\right)}^m\,\left(A\,c+C\,b\right)\,\left(m^6+23\,m^5+207\,m^4+925\,m^3+2144\,m^2+2412\,m+1008\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+ \frac{A\,a\,x\,{\left(d\,x\right)}^m\,\left(m^6+27\,m^5+295\,m^4+1665\,m^3+5104\,m^2+8028\,m+5040\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+ \frac{B\,a\,x^2\,{\left(d\,x\right)}^m\,\left(m^6+26\,m^5+270\,m^4+1420\,m^3+3929\,m^2+5274\,m+2520\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+ \frac{B\,b\,x^4\,{\left(d\,x\right)}^m\,\left(m^6+24\,m^5+226\,m^4+1056\,m^3+2545\,m^2+2952\,m+1260\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+ \frac{B\,c\,x^6\,{\left(d\,x\right)}^m\,\left(m^6+22\,m^5+190\,m^4+820\,m^3+1849\,m^2+2038\,m+840\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$
$$+ \frac{C\,c\,x^7\,{\left(d\,x\right)}^m\,\left(m^6+21\,m^5+175\,m^4+735\,m^3+1624\,m^2+1764\,m+720\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}$$

input
```
int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

output
```
(x^3*(d*x)^m*(A*b + C*a)*(3796*m + 3112*m^2 + 1219*m^3 + 247*m^4 + 25*m^5
+ m^6 + 1680))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m
^6 + m^7 + 5040) + (x^5*(d*x)^m*(A*c + C*b)*(2412*m + 2144*m^2 + 925*m^3 +
 207*m^4 + 23*m^5 + m^6 + 1008))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^
4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (A*a*x*(d*x)^m*(8028*m + 5104*m^2 + 1
665*m^3 + 295*m^4 + 27*m^5 + m^6 + 5040))/(13068*m + 13132*m^2 + 6769*m^3
+ 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*a*x^2*(d*x)^m*(5274*m + 3
929*m^2 + 1420*m^3 + 270*m^4 + 26*m^5 + m^6 + 2520))/(13068*m + 13132*m^2
+ 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*b*x^4*(d*x)^m*
(2952*m + 2545*m^2 + 1056*m^3 + 226*m^4 + 24*m^5 + m^6 + 1260))/(13068*m +
 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*c*x
^6*(d*x)^m*(2038*m + 1849*m^2 + 820*m^3 + 190*m^4 + 22*m^5 + m^6 + 840))/(
13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)
 + (C*c*x^7*(d*x)^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6
+ 720))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^
7 + 5040)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.42

$$\int (dx)^m (A+Bx+Cx^2) (a+bx^2+cx^4) \, dx$$
$$= \frac{x^m d^m x(c^2m^6x^6 + bc\,m^6x^5 + 21c^2m^5x^6 + ac\,m^6x^4 + bc\,m^6x^4 + 22bc\,m^5x^5 + 175c^2m^4x^6 + 23ac\,m^5x^4 + b}{}$$

input
```
int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

output
```
(x**m*d**m*x*(a**2*m**6 + 27*a**2*m**5 + 295*a**2*m**4 + 1665*a**2*m**3 +
5104*a**2*m**2 + 8028*a**2*m + 5040*a**2 + a*b*m**6*x**2 + a*b*m**6*x + 25
*a*b*m**5*x**2 + 26*a*b*m**5*x + 247*a*b*m**4*x**2 + 270*a*b*m**4*x + 1219
*a*b*m**3*x**2 + 1420*a*b*m**3*x + 3112*a*b*m**2*x**2 + 3929*a*b*m**2*x +
3796*a*b*m*x**2 + 5274*a*b*m*x + 1680*a*b*x**2 + 2520*a*b*x + a*c*m**6*x**
4 + a*c*m**6*x**2 + 23*a*c*m**5*x**4 + 25*a*c*m**5*x**2 + 207*a*c*m**4*x**
4 + 247*a*c*m**4*x**2 + 925*a*c*m**3*x**4 + 1219*a*c*m**3*x**2 + 2144*a*c*
m**2*x**4 + 3112*a*c*m**2*x**2 + 2412*a*c*m*x**4 + 3796*a*c*m*x**2 + 1008*
a*c*x**4 + 1680*a*c*x**2 + b**2*m**6*x**3 + 24*b**2*m**5*x**3 + 226*b**2*m
**4*x**3 + 1056*b**2*m**3*x**3 + 2545*b**2*m**2*x**3 + 2952*b**2*m*x**3 +
1260*b**2*x**3 + b*c*m**6*x**5 + b*c*m**6*x**4 + 22*b*c*m**5*x**5 + 23*b*c
*m**5*x**4 + 190*b*c*m**4*x**5 + 207*b*c*m**4*x**4 + 820*b*c*m**3*x**5 + 9
25*b*c*m**3*x**4 + 1849*b*c*m**2*x**5 + 2144*b*c*m**2*x**4 + 2038*b*c*m*x*
*5 + 2412*b*c*m*x**4 + 840*b*c*x**5 + 1008*b*c*x**4 + c**2*m**6*x**6 + 21*
c**2*m**5*x**6 + 175*c**2*m**4*x**6 + 735*c**2*m**3*x**6 + 1624*c**2*m**2*
x**6 + 1764*c**2*m*x**6 + 720*c**2*x**6))/(m**7 + 28*m**6 + 322*m**5 + 196
0*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
```

3.41 $\int \frac{(dx)^m(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 368

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{a+bx^2+cx^4} dx = \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m} \text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m} \text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)d(1+m)} + \frac{2Bc(dx)^{2+m} \text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d^2(2+m)} - \frac{2Bc(dx)^{2+m} \text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d^2(2+m)}$$

output
```
(C+(2*A*c-C*b)/(-4*a*c+b^2)^(1/2))*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3
/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))/(b-(-4*a*c+b^2)^(1/2))/d/(1+m)+
(C-(2*A*c-C*b)/(-4*a*c+b^2)^(1/2))*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3
/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/(b+(-4*a*c+b^2)^(1/2))/d/(1+m)+
2*B*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2
)^(1/2)))/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))/d^2/(2+m)-2*B*c*(d*x)^
(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/(-
4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/d^2/(2+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 2.36 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.19

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$$

$$= \frac{(dx)^m \left(A(2+3m+m^2)\,\text{RootSum}\left[a+b\#1^2+c\#1^4\&, \frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1+2c\#1^3}\right.\right.}{}$$

input
```
Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
((d*x)^m*(A*(2 + 3*m + m^2)*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric
2F1[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(b*#1 + 2*c*#1^3)) & ]
 + B*(2 + m)*RootSum[a + b*#1^2 + c*#1^4 & , (m*x + (Hypergeometric2F1[-m,
 -m, 1 - m, -(#1/(x - #1))]*#1)/(x/(x - #1))^m + (m*Hypergeometric2F1[-m, -
m, 1 - m, -(#1/(x - #1))]*#1)/(x/(x - #1))^m)/(b*#1 + 2*c*#1^3) & ] + C*Ro
otSum[a + b*#1^2 + c*#1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*
Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3
*m*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m +
 (m^2*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^
m + (m*#1^2)/(x/#1)^m)/(b*#1 + 2*c*#1^3) & ]))/(2*m*(1 + m)*(2 + m))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2193, 27, 1451, 27, 278, 1608, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{(dx)^m (Cx^2+A)}{cx^4+bx^2+a}dx+\frac{\int \frac{B(dx)^{m+1}}{cx^4+bx^2+a}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^m (Cx^2+A)}{cx^4+bx^2+a}dx+\frac{B\int \frac{(dx)^{m+1}}{cx^4+bx^2+a}dx}{d}$$

$$\Big\downarrow \text{1451}$$

$$\int \frac{(dx)^m (Cx^2+A)}{cx^4+bx^2+a}dx+\frac{B\left(\frac{c\int \frac{2(dx)^{m+1}}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2(dx)^{m+1}}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^m (Cx^2+A)}{cx^4+bx^2+a}dx+\frac{B\left(\frac{2c\int \frac{(dx)^{m+1}}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(dx)^{m+1}}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}\right)}{d}$$

$$\Big\downarrow \text{278}$$

$$\int \frac{(dx)^m (Cx^2+A)}{cx^4+bx^2+a}dx+\frac{B\left(\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)}{d}$$

$$\Big\downarrow \text{1608}$$

$$\frac{1}{2}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\int \frac{2(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx+$$
$$\frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{2(dx)^m}{2cx^2+b+\sqrt{b^2-4ac}}dx+$$
$$\frac{B\left(\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\int \frac{(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx+$$
$$\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{(dx)^m}{2cx^2+b+\sqrt{b^2-4ac}}dx+$$
$$\frac{B\left(\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{(dx)^{m+1}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}+$$
$$\frac{(dx)^{m+1}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\left(\sqrt{b^2-4ac}+b\right)}+$$
$$\frac{B\left(\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}\right)}{d}$$

input
```
Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

output
```
((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2
- 4*a*c])*d*(1 + m)) + ((C - (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m
)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])/((b + Sqrt[b^2 - 4*a*c])*d*(1 + m)) + (B*((2*c*(d*x)^(2 + m)*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])
/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(2 + m)) - (2*c*(d*x)^(2 + m
)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(2 + m))))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1451
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c*x^2), x]
, x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b,
c, d, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 1608
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d -
b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d
- b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [F]

$$\int \frac{(dx)^m \left(C\,x^2+Bx+A\right)}{c\,x^4+b\,x^2+a}dx$$

input
```
int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

output
```
int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(dx)^m \left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx=\int \frac{(Cx^2+Bx+A)(dx)^m}{cx^4+bx^2+a}\,dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(dx)^m \left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx=\int \frac{(dx)^m \left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx$$

input
```
integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

output
```
Integral((d*x)**m*(A + B*x + C*x**2)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{a + bx^2 + cx^4} \, dx = \int \frac{(Cx^2 + Bx + A)(dx)^m}{cx^4 + bx^2 + a} \, dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{a + bx^2 + cx^4} \, dx = \int \frac{(Cx^2 + Bx + A)(dx)^m}{cx^4 + bx^2 + a} \, dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{a + bx^2 + cx^4} \, dx = \int \frac{(d\,x)^m\,(C\,x^2 + B\,x + A)}{c\,x^4 + b\,x^2 + a} \, dx$$

input
```
int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

output
```
int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x)
```

Reduce [F]

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{a + bx^2 + cx^4} \, dx = d^m \left(\left(\int \frac{x^m}{c\,x^4 + b\,x^2 + a} dx \right) a + \left(\int \frac{x^m x^2}{c\,x^4 + b\,x^2 + a} dx \right) c + \left(\int \frac{x^m x}{c\,x^4 + b\,x^2 + a} dx \right) b \right)$$

input
```
int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

output
```
d**m*(int(x**m/(a + b*x**2 + c*x**4),x)*a + int((x**m*x**2)/(a + b*x**2 +
c*x**4),x)*c + int((x**m*x)/(a + b*x**2 + c*x**4),x)*b)
```

3.42 $\int \frac{(dx)^m(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 685

$$\begin{aligned}
&\int \frac{(dx)^m\,(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{B(dx)^{2+m}\,(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,d^2\,(a+bx^2+cx^4)} + \frac{(dx)^{1+m}\,(A(b^2-2ac)-abC+c(Ab-2aC)x^2)}{2a\,(b^2-4ac)\,d\,(a+bx^2+cx^4)} \\
&+ \frac{c\left(2aC\left(2b-\sqrt{b^2-4ac}(1-m)\right)+A\left(b^2(1-m)+b\sqrt{b^2-4ac}(1-m)-4ac(3-m)\right)\right)(dx)^{1+m}\,\text{Hype}}{2a\,(b^2-4ac)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} \\
&- \frac{c\left(2aC\left(2b+\sqrt{b^2-4ac}(1-m)\right)+A\left(b^2(1-m)-b\sqrt{b^2-4ac}(1-m)-4ac(3-m)\right)\right)(dx)^{1+m}\,\text{Hype}}{2a\,(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)} \\
&- \frac{Bc\left(4ac(2-m)+b\left(b+\sqrt{b^2-4ac}\right)m\right)(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2a\,(b^2-4ac)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d^2(2+m)} \\
&+ \frac{Bc\left(4ac(2-m)+b\left(b-\sqrt{b^2-4ac}\right)m\right)(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2a\,(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d^2(2+m)}
\end{aligned}$$

output
```
1/2*B*(d*x)^(2+m)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d^2/(c*x^4+b*x^2+a)+1
/2*(d*x)^(1+m)*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/d/(
c*x^4+b*x^2+a)+1/2*c*(2*a*C*(2*b-(-4*a*c+b^2)^(1/2)*(1-m))+A*(b^2*(1-m)+b*
(-4*a*c+b^2)^(1/2)*(1-m)-4*a*c*(3-m)))*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m
],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/(b-(-4
*a*c+b^2)^(1/2))/d/(1+m)-1/2*c*(2*a*C*(2*b+(-4*a*c+b^2)^(1/2)*(1-m))+A*(b^
2*(1-m)-b*(-4*a*c+b^2)^(1/2)*(1-m)-4*a*c*(3-m)))*(d*x)^(1+m)*hypergeom([1,
 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3
/2)/(b+(-4*a*c+b^2)^(1/2))/d/(1+m)-1/2*B*c*(4*a*c*(2-m)+b*(b+(-4*a*c+b^2)^
(1/2))*m)*(d*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b-(-4*a*c
+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))/d^2/(2+m)+1/2*B*
c*(4*a*c*(2-m)+b*(b-(-4*a*c+b^2)^(1/2))*m)*(d*x)^(2+m)*hypergeom([1, 1+1/2
*m],[2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/(b+(-4
*a*c+b^2)^(1/2))/d^2/(2+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.56 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.35

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} \, dx$$
$$= \frac{x(dx)^m \left(A(6 + 5m + m^2) \operatorname{AppellF1}\left(\frac{1+m}{2}, 2, 2, \frac{3+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right) + (1+m)x\Big(B(3+m)\operatorname{Ap}\right.}{a^2(1+}$$

input
```
Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(d*x)^m*(A*(6 + 5*m + m^2)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)
*x*(B*(3 + m)*AppellF1[(2 + m)/2, 2, 2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + C*(2 + m)*x*AppellF1[(3
 + m)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])])))/(a^2*(1 + m)*(2 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2193, 27, 1441, 1600, 25, 1608, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{(dx)^m (Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{\int \frac{B(dx)^{m+1}}{(cx^4+bx^2+a)^2}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^m (Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{B\int \frac{(dx)^{m+1}}{(cx^4+bx^2+a)^2}dx}{d}$$

$$\Big\downarrow \text{1441}$$

$$\int \frac{(dx)^m (Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{B\left(\frac{(dx)^{m+2}(-2ac+b^2+bcx^2)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{(dx)^{m+1}(mb^2+cmx^2b+2ac(2-m))}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}\right)}{d}$$

$$\Big\downarrow \text{1600}$$

$$-\frac{\int -\frac{(dx)^m(A(1-m)b^2+aC(m+1)b+c(Ab-2aC)(1-m)x^2-2aAc(3-m))}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{B\left(\frac{(dx)^{m+2}(-2ac+b^2+bcx^2)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{(dx)^{m+1}(mb^2+cmx^2b+2ac(2-m))}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}\right)}{d}+\frac{(dx)^{m+1}\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx)^m \left(A(1-m)b^2+aC(m+1)b+c(Ab-2aC)(1-m)x^2-2aAc(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{B\left(\frac{(dx)^{m+2}\left(-2ac+b^2+bcx^2\right)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{(dx)^{m+1}\left(mb^2+cmx^2b+2ac(2-m)\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}\right)}{d}+\frac{(dx)^{m+1}\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 1608

$$\frac{\frac{c\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right)\int \frac{2(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}-\frac{c\left(-(1-m)\sqrt{b^2-4ac}(Ab-2aC)-4aAc(\cdots\right.}{\cdots}}{2a\left(b^2-4ac\right)}+$$

$$\frac{B\left(\frac{(dx)^{m+2}\left(-2ac+b^2+bcx^2\right)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{c\left(bm\left(\sqrt{b^2-4ac}+b\right)+4ac(2-m)\right)\int \frac{2(dx)^{m+1}}{2cx^2+b-\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}-\frac{c\left(bm\left(b-\sqrt{b^2-4ac}\right)+4ac(2-m)\right)\int \frac{2(dx)^{m+1}}{2cx^2+b+\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}}{2a(b^2-4ac)}\cdots\right.}{d}+\frac{(dx)^{m+1}\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 27

$$\frac{\frac{c\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right)\int \frac{(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\left(-(1-m)\sqrt{b^2-4ac}(Ab-2aC)-4aAc(\cdots\right.}{\cdots}}{2a\left(b^2-4ac\right)}+$$

$$\frac{B\left(\frac{(dx)^{m+2}\left(-2ac+b^2+bcx^2\right)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{c\left(bm\left(\sqrt{b^2-4ac}+b\right)+4ac(2-m)\right)\int \frac{(dx)^{m+1}}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\left(bm\left(b-\sqrt{b^2-4ac}\right)+4ac(2-m)\right)\int \frac{(dx)^{m+1}}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}}{2a(b^2-4ac)}\cdots\right.}{d}+\frac{(dx)^{m+1}\left(A\left(b^2-2ac\right)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\downarrow$ 278

$$\frac{\frac{c(dx)^{m+1}\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(\cdots}{\cdots}}{2a\left(b^2-4ac\right)}$$

$$\frac{(dx)^{m+1}\left(A(b^2-2ac)+cx^2(Ab-2aC)-abC\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+$$

$$\frac{B\left(\frac{(dx)^{m+2}(-2ac+b^2+bcx^2)}{2ad(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{c(dx)^{m+2}\left(bm\left(\sqrt{b^2-4ac}+b\right)+4ac(2-m)\right)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+2)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+2}\left(bm\left(b-\sqrt{b\cdots}\right.\right.}{\cdots}}{2a(b^2-4ac)}\cdots\right.}{d}$$

input
```
Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((d*x)^(1 + m)*(A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2
 - 4*a*c)*d*(a + b*x^2 + c*x^4)) + ((c*(2*a*C*(2*b - Sqrt[b^2 - 4*a*c]*(1 -
 m)) + A*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m)))*(d*x
)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)) - (c
*(4*a*b*C + A*b^2*(1 - m) - Sqrt[b^2 - 4*a*c]*(A*b - 2*a*C)*(1 - m) - 4*a*
A*c*(3 - m))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]
)*d*(1 + m)))/(2*a*(b^2 - 4*a*c)) + (B*(((d*x)^(2 + m)*(b^2 - 2*a*c + b*c*
x^2))/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c*x^4)) - ((c*(4*a*c*(2 - m) + b*(
b + Sqrt[b^2 - 4*a*c])*m)*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt
[b^2 - 4*a*c])*d*(2 + m)) - (c*(4*a*c*(2 - m) + b*(b - Sqrt[b^2 - 4*a*c])*
m)*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(2 + m
)))/(2*a*(b^2 - 4*a*c))))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1608
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d -
b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d
- b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [F]

$$\int \frac{(dx)^m (C\,x^2+Bx+A)}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(dx)^m (A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)(dx)^m}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^
4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} \, dx = \int \frac{(Cx^2 + Bx + A)(dx)^m}{(cx^4 + bx^2 + a)^2} \, dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} \, dx = \int \frac{(Cx^2 + Bx + A)(dx)^m}{(cx^4 + bx^2 + a)^2} \, dx$$

input
```
integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2}\, dx = \int \frac{(d\,x)^m\,(C\,x^2 + B\,x + A)}{(c\,x^4 + b\,x^2 + a)^2}\, dx$$

input
```
int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2}\, dx = d^m \left(\left(\int \frac{x^m}{c^2x^8 + 2bc\,x^6 + 2ac\,x^4 + b^2x^4 + 2ab\,x^2 + a^2} dx \right) a + \left(\int \frac{x^m x^2}{c^2x^8 + 2bc\,x^6 + 2ac\,x^4 + b^2x^4 + 2ab\,x^2 + a^2} dx \right) c + \left(\int \frac{x^m x}{c^2x^8 + 2bc\,x^6 + 2ac\,x^4 + b^2x^4 + 2ab\,x^2 + a^2} dx \right) b \right)$$

input
```
int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
d**m*(int(x**m/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 +
c**2*x**8),x)*a + int((x**m*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x
**4 + 2*b*c*x**6 + c**2*x**8),x)*c + int((x**m*x)/(a**2 + 2*a*b*x**2 + 2*a
*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*b)
```

3.43 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 356

$$\begin{aligned}\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx &= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\ &\quad - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\ &\quad - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\ &\quad - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} \\ &\quad - \frac{bB\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

output
```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*(2*A*c-C*b-(4*A*b*c-(4*a*c+b^2)*C)
/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2)
)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*(2*A*c-C*b
+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \; 2193$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 1434$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \; 1159$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 1083$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \; 219$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \; 1598$$

$$\frac{\int \frac{-\left(\left(2Ac-bC\right)x^2\right)+Ab-2aC}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \; 1480$$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}}{2\left(b^2-4ac\right)}$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \ 218$$

$$\frac{-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_R\,Bb}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln\left(x-\right.}{2\_R^3c+\_R b}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4A\sqrt{-4ac+b^2}\,bc+8A\,c^2a-2A\,b^2c+\right.}{}}{4c\left(4\right.}\right.}{}$

input
```
int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-
b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln
(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4439 vs. $2(306) = 612$.

Time = 1.04 (sec) , antiderivative size = 4439, normalized size of antiderivative = 12.47

$$\int \frac{x^2(A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^2(C\,x^2+Bx+A)}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

3.44 $\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 356

$$\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx = \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*(2*A*c-C*b-(4*A*b*c-(4*a*c+b^2)*C)
/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2)
)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*(2*A*c-C*b
+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {9, 2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(Ax+Bx^2+Cx^3\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \; 2193$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 1434$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \; 1159$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 1083$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \; 219$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \; 1598$$

$$\frac{\int \frac{-\left((2Ac-bC)x^2\right)+Ab-2aC}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \; 1480$$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}}{2\left(b^2-4ac\right)}$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Int[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_R Bb}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln\left(x-\right.}{2\_R^3c+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4A\sqrt{-4ac+b^2}\,bc+8A\,c^2a-2A\,b^2c+\right.}{}}{4c\left(4\right.}\right.}{}$

input
```
int(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-
b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln
(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx = \text{Timed out}$$

input
```
integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx = \text{Timed out}$$

input
```
integrate(x*(C*x**3+B*x**2+A*x)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx = \int \frac{(Cx^3 + Bx^2 + Ax)x}{(cx^4 + bx^2 + a)^2} dx$$

input
```
integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4439 vs. $2(306) = 612$.

Time = 1.12 (sec) , antiderivative size = 4439, normalized size of antiderivative = 12.47

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

Mupad [B] (verification not implemented)

Time = 19.63 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx = \text{Too large to display}$$

input
```
int((x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

Reduce [F]

$$\int \frac{x(Ax + Bx^2 + Cx^3)}{(a + bx^2 + cx^4)^2} dx = \int \frac{x(C\,x^3 + B\,x^2 + Ax)}{(c\,x^4 + b\,x^2 + a)^2} dx$$

input
```
int(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x)
```

3.45 $\int \frac{Ax^2+Bx^3+Cx^4}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 356

$$\int \frac{Ax^2+Bx^3+Cx^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{B(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

output

```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*(2*A*c-C*b-(4*A*b*c-(4*a*c+b^2)*C)
/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2)
)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*(2*A*c-C*b
+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}\,dx=\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)}\right.$$
$$+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$\left.+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.323$, Rules used = {2028, 2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax^2+Bx^3+Cx^4}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{2028}$$

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{2193}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\int \frac{Bx^3}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+B\int \frac{x^3}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{1434}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\int \frac{x^2}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{1159}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{x^2(Cx^2+A)}{(cx^4+bx^2+a)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)$$

$$\Big\downarrow \text{1598}$$

$$\frac{\int \frac{-((2Ac-bC)x^2)+Ab-2aC}{cx^4+bx^2+a}dx}{2(b^2-4ac)}-\frac{x(-2aC+x^2(2Ac-bC)+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{1}{2}B\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}}{2\left(b^2-4ac\right)}$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow 218$$

$$\frac{-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Int[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] && !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_R Bb}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln\left(x-\right.}{2\_R^3c+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4A\sqrt{-4ac+b^2}\,bc+8A\,c^2a-2A\,b^2c+\right.}{}}{4c\left(4\right.}\right.}{}$

input
```
int((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-
b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln
(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2}\, dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2}\, dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**3+A*x**2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2}\, dx = \int \frac{Cx^4 + Bx^3 + Ax^2}{(cx^4 + bx^2 + a)^2}\, dx$$

input
```
integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4439 vs. $2(306) = 612$.

Time = 1.05 (sec) , antiderivative size = 4439, normalized size of antiderivative = 12.47

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

Mupad [B] (verification not implemented)

Time = 19.55 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

Reduce [F]

$$\int \frac{Ax^2 + Bx^3 + Cx^4}{(a + bx^2 + cx^4)^2} \, dx = \int \frac{C\,x^4 + B\,x^3 + A\,x^2}{(c\,x^4 + b\,x^2 + a)^2} dx$$

input
```
int((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x)
```

3.46 $\int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 356

$$\int \frac{Ax^3+Bx^4+Cx^5}{x(a+bx^2+cx^4)^2}\,dx = \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} - \frac{bB\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output

```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*(2*A*c-C*b-(4*A*b*c-(4*a*c+b^2)*C)
/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2)
)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*(2*A*c-C*b
+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx=\frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Integrate[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.294$, Rules used = {9, 2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \ 2193$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 1434$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \ 1159$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \ 1083$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \ 219$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \ 1598$$

$$\frac{\int \frac{-\left(\left(2Ac-bC\right)x^2\right)+Ab-2aC}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \ 1480$$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}}{2\left(b^2-4ac\right)}$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Int[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln\left(x-\right.}{2\_R^3c+\_Rb}\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2\left(4ac-b^2\right)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4A\sqrt{-4ac+b^2}\,bc+8A\,c^2a-2A\,b^2c+\right.}{}}{4c\left(4\right.}\right.}{}$

input
```
int((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-
b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln
(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x**5+B*x**4+A*x**3)/x/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{Cx^5+Bx^4+Ax^3}{\left(cx^4+bx^2+a\right)^2x}\,dx$$

input
```
integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4439 vs. $2(306) = 612$.

Time = 1.03 (sec) , antiderivative size = 4439, normalized size of antiderivative = 12.47

$$\int \frac{Ax^3 + Bx^4 + Cx^5}{x(a + bx^2 + cx^4)^2} dx = \text{Too large to display}$$

input
```
integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
int((A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x)
```

output
```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

Reduce [F]

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{C\,x^5+B\,x^4+A\,x^3}{x\left(c\,x^4+b\,x^2+a\right)^2}dx$$

input
```
int((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x)
```

3.47 $\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 356

$$\begin{aligned}\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx &= \frac{B(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\ &\quad - \frac{x(Ab-2aC+(2Ac-bC)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\ &\quad - \frac{\left(2Ac-bC-\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\ &\quad - \frac{\left(2Ac-bC+\frac{4Abc-(b^2+4ac)C}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} \\ &\quad - \frac{bB\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

output

```
1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b
)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*(2*A*c-C*b-(4*A*b*c-(4*a*c+b^2)*C)
/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2)
)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*(2*A*c-C*b
+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{4a(B+Cx)+2x(bx(B+Cx)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-2Ac\left(-2b+\sqrt{b^2-4ac}\right)+\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2Ac\left(2b+\sqrt{b^2-4ac}\right)+\left(b^2+4ac+b\sqrt{b^2-4ac}\right)C\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2bB\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2bB\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Integrate[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqr
t[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {9, 2193, 27, 1434, 1159, 1083, 219, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \ 2193$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\int \frac{Bx^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+B\int \frac{x^3}{\left(cx^4+bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 1434$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\int \frac{x^2}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \ 1159$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \ 1083$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \ 219$$

$$\int \frac{x^2\left(Cx^2+A\right)}{\left(cx^4+bx^2+a\right)^2}dx+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \ 1598$$

$$\frac{\int \frac{-\left(\left(2Ac-bC\right)x^2\right)+Ab-2aC}{cx^4+bx^2+a}dx}{2\left(b^2-4ac\right)}-\frac{x\left(-2aC+x^2\left(2Ac-bC\right)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \ 1480$$

$$\frac{-\frac{1}{2}\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}}{2\left(b^2-4ac\right)}$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-$$

$$\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\left(\frac{2a+bx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Int[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/2*(x*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-(((2*A*c - b*C - (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*S
qrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c)) + (B*(
(2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2193
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k),
{k, 0, q/2 + 1}]*(d*x)^m*(a + b*x^2 + c*x^4)^p, x] + Simp[1/d   Int[Sum[Coe
ff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q + 1)/2}]*(d*x)^(m + 1)*(a + b*x^2 + c
*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !PolyQ
[Pq, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2(4ac-b^2)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(2Ac-Cb)\_R^2}{4ac-b^2}-\frac{2\_RBb}{4ac-b^2}-\frac{Ab-2aC}{4ac-b^2}\right)\ln(x-\cdots}{2\_R^3c+\_Rb}\cdots\right.}{4}$
default	$\frac{\frac{(2Ac-Cb)x^3}{8ac-2b^2}-\frac{Bb\,x^2}{2(4ac-b^2)}+\frac{(Ab-2aC)x}{8ac-2b^2}-\frac{Ba}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{-B\sqrt{-4ac+b^2}\,b\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)+\frac{\left(-4A\sqrt{-4ac+b^2}\,bc+8A\,c^2a-2A\,b^2c+\cdots\right.}{\cdots}}{4c(4\cdots}\right.\cdots}{\cdots}$

input
```
int((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*B*b/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(
4*a*c-b^2)*x-B*a/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum(((2*A*c-C*b)/(4*a*c-
b^2)*_R^2-2/(4*a*c-b^2)*_R*B*b-(A*b-2*C*a)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln
(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((C*x**6+B*x**5+A*x**4)/x**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \int \frac{Cx^6 + Bx^5 + Ax^4}{(cx^4 + bx^2 + a)^2 x^2} \, dx$$

input
```
integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*
x + (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4439 vs. $2(306) = 612$.

Time = 1.11 (sec) , antiderivative size = 4439, normalized size of antiderivative = 12.47

$$\int \frac{Ax^4 + Bx^5 + Cx^6}{x^2 (a + bx^2 + cx^4)^2} dx = \text{Too large to display}$$

input
```
integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*
x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*
a*c)*b*c^2)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c
^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*
(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3...
```

Mupad [B] (verification not implemented)

Time = 19.15 (sec) , antiderivative size = 3835, normalized size of antiderivative = 10.77

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

output
```
symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*
A*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A
*C^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^
2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^
8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*
z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*
b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*
c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 819
2*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A
^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B
*a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*
c^2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c
+ 48*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*
C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c +
 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6
*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4
*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(25...
```

Reduce [F]

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{C\,x^6+B\,x^5+A\,x^4}{x^2\left(c\,x^4+b\,x^2+a\right)^2}dx$$

input
```
int((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x)
```

3.48 $\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 273

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\,x^2}{2c^4} + \frac{(c^2d+b^2f-c(be+af))\,x^4}{4c^3} + \frac{(ce-bf)x^6}{6c^2} + \frac{fx^8}{8c} - \frac{(b^4ce-4ab^2c^2e+2a^2c^3e-b^5f-b^3c(cd-5af)+abc^2(3cd-5af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^5\sqrt{b^2-4ac}} - \frac{(b^3ce-2abc^2e-b^4f-b^2c(cd-3af)+ac^2(cd-af))\log(a+bx^2+cx^4)}{4c^5}$$

output
```
1/2*(b^2*c*e-a*c^2*e-b^3*f-b*c*(-2*a*f+c*d))*x^2/c^4+1/4*(c^2*d+b^2*f-c*(a
*f+b*e))*x^4/c^3+1/6*(-b*f+c*e)*x^6/c^2+1/8*f*x^8/c-1/2*(b^4*c*e-4*a*b^2*c
^2*e+2*a^2*c^3*e-b^5*f-b^3*c*(-5*a*f+c*d)+a*b*c^2*(-5*a*f+3*c*d))*arctanh(
(2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)-1/4*(b^3*c*e-2*a*b*
c^2*e-b^4*f-b^2*c*(-3*a*f+c*d)+a*c^2*(-a*f+c*d))*ln(c*x^4+b*x^2+a)/c^5
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.95

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$

$$= \frac{-12c(-b^2ce+ac^2e+b^3f+bc(cd-2af))\,x^2+6c^2(c^2d+b^2f-c(be+af))\,x^4+4c^3(ce-bf)x^6+3c^4f}{}$$

input
```
Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(-12*c*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*x^2 + 6*c^2*(c^2
*d + b^2*f - c*(b*e + a*f))*x^4 + 4*c^3*(c*e - b*f)*x^6 + 3*c^4*f*x^8 - (1
2*(-(b^4*c*e) + 4*a*b^2*c^2*e - 2*a^2*c^3*e + b^5*f + b^3*c*(c*d - 5*a*f)
+ a*b*c^2*(-3*c*d + 5*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + 6*(-(b^3*c*e) + 2*a*b*c^2*e + b^4*f + b^2*c*(c*d - 3*a*f)
 + a*c^2*(-(c*d) + a*f))*Log[a + b*x^2 + c*x^4])/(24*c^5)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{x^6\left(fx^4+ex^2+d\right)}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 2159}$$

$$\frac{1}{2}\int \left(\frac{fx^6}{c}+\frac{(ce-bf)x^4}{c^2}+\frac{\left(fb^2+c^2d-c(be+af)\right)x^2}{c^3}+\frac{-fb^3+ceb^2-c(cd-2af)b-ac^2e}{c^4}+\frac{-((-fb^4+c$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2c^3e-b^3c(cd-5af)-4ab^2c^2e+abc^2(3cd-5af)+b^5(-f)+b^4ce\right)}{c^5\sqrt{b^2-4ac}}+\frac{x^4(-c(af+$$

input

```
Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output

```
(((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*x^2)/c^4 + ((c^2*d + b^2
*f - c*(b*e + a*f))*x^4)/(2*c^3) + ((c*e - b*f)*x^6)/(3*c^2) + (f*x^8)/(4*
c) - ((b^4*c*e - 4*a*b^2*c^2*e + 2*a^2*c^3*e - b^5*f - b^3*c*(c*d - 5*a*f)
 + a*b*c^2*(3*c*d - 5*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^5
*Sqrt[b^2 - 4*a*c]) - ((b^3*c*e - 2*a*b*c^2*e - b^4*f - b^2*c*(c*d - 3*a*f
) + a*c^2*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(2*c^5))/2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.21

method	result
default	$\frac{\frac{1}{4} f\, x^{8} c^{3}-\frac{1}{3} b\, c^{2} f\, x^{6}+\frac{1}{3} e\, c^{3} x^{6}-\frac{1}{2} a\, c^{2} f\, x^{4}+\frac{1}{2} b^{2} c f\, x^{4}-\frac{1}{2} b\, c^{2} e\, x^{4}+\frac{1}{2} c^{3} d\, x^{4}+2 a b c f\, x^{2}-a\, c^{2} e\, x^{2}-b^{3} f\, x^{2}+b^{2} c e\, x^{2}-b\, c^{2} d\, x^{2}}{2 c^{4}}+\frac{\left(a^{2} c^{2}\right.}{}$
risch	Expression too large to display

input
```
int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^4*(1/4*f*x^8*c^3-1/3*b*c^2*f*x^6+1/3*e*c^3*x^6-1/2*a*c^2*f*x^4+1/2*b
^2*c*f*x^4-1/2*b*c^2*e*x^4+1/2*c^3*d*x^4+2*a*b*c*f*x^2-a*c^2*e*x^2-b^3*f*x
^2+b^2*c*e*x^2-b*c^2*d*x^2)+1/2/c^4*(1/2*(a^2*c^2*f-3*a*b^2*c*f+2*a*b*c^2*
e-a*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)/c*ln(c*x^4+b*x^2+a)+2*(-2*a^2*b*c*f+a^2
*c^2*e+a*b^3*f-a*b^2*c*e+a*b*c^2*d-1/2*(a^2*c^2*f-3*a*b^2*c*f+2*a*b*c^2*e-
a*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.30

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c^4 - 4*a*c^5)*e - (b^3*c^3 -
 4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e +
(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*((b^3*c^3 - 4*a*b*c^4)*d -
 (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^
3)*f)*x^2 + 6*sqrt(b^2 - 4*a*c)*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^
2*c^2 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f)*log((2*c^2*x^4 +
 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
 + a)) + 6*((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 +
 8*a^2*b*c^3)*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*
x^4 + b*x^2 + a))/(b^2*c^5 - 4*a*c^6), 1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 +
 4*((b^2*c^4 - 4*a*c^5)*e - (b^3*c^3 - 4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4
*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*
f)*x^4 - 12*((b^3*c^3 - 4*a*b*c^4)*d - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)
*e + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*f)*x^2 + 12*sqrt(-b^2 + 4*a*c)*((
b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e + (b^5 - 5*a*
b^3*c + 5*a^2*b*c^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)) + 6*((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 +
8*a^2*b*c^3)*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x
^4 + b*x^2 + a))/(b^2*c^5 - 4*a*c^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.09

$$\begin{aligned}
&\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx \\
&= \frac{3\,c^3fx^8+4\,c^3ex^6-4\,bc^2fx^6+6\,c^3dx^4-6\,bc^2ex^4+6\,b^2cfx^4-6\,ac^2fx^4-12\,bc^2dx^2+12\,b^2cex^2-12\,ac}{24\,c^4} \\
&\quad+\frac{(b^2c^2d-ac^3d-b^3ce+2\,abc^2e+b^4f-3\,ab^2cf+a^2c^2f)\log(cx^4+bx^2+a)}{4\,c^5} \\
&\quad-\frac{(b^3c^2d-3\,abc^3d-b^4ce+4\,ab^2c^2e-2\,a^2c^3e+b^5f-5\,ab^3cf+5\,a^2bc^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^5}
\end{aligned}$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/24*(3*c^3*f*x^8 + 4*c^3*e*x^6 - 4*b*c^2*f*x^6 + 6*c^3*d*x^4 - 6*b*c^2*e*
x^4 + 6*b^2*c*f*x^4 - 6*a*c^2*f*x^4 - 12*b*c^2*d*x^2 + 12*b^2*c*e*x^2 - 12
*a*c^2*e*x^2 - 12*b^3*f*x^2 + 24*a*b*c*f*x^2)/c^4 + 1/4*(b^2*c^2*d - a*c^3
*d - b^3*c*e + 2*a*b*c^2*e + b^4*f - 3*a*b^2*c*f + a^2*c^2*f)*log(c*x^4 +
b*x^2 + a)/c^5 - 1/2*(b^3*c^2*d - 3*a*b*c^3*d - b^4*c*e + 4*a*b^2*c^2*e -
2*a^2*c^3*e + b^5*f - 5*a*b^3*c*f + 5*a^2*b*c^2*f)*arctan((2*c*x^2 + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

Mupad [B] (verification not implemented)

Time = 18.89 (sec) , antiderivative size = 2972, normalized size of antiderivative = 10.89

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
x^6*(e/(6*c) - (b*f)/(6*c^2)) - x^4*((b*(e/c - (b*f)/c^2))/(4*c) - d/(4*c)
 + (a*f)/(4*c^2)) - x^2*((a*(e/c - (b*f)/c^2))/(2*c) - (b*((b*(e/c - (b*f)
/c^2))/c - d/c + (a*f)/c^2))/(2*c)) + (f*x^8)/(8*c) - (log(a + b*x^2 + c*x
^4)*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^
2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^
3*e))/(2*(16*a*c^6 - 4*b^2*c^5)) + (atan((2*c^8*(4*a*c - b^2)*(x^2*(((((4*
a^2*c^8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5*f + 10*a*b*c^8*d - 16*a*
b^2*c^7*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8 - (4*b*c^2*(2*b^6*f + 8*a
^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a
*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 -
4*b^2*c^5))*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a
*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(8*c^5*(4*a*c - b^2)^(1/2)) - (
b*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f +
 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a
^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d +
12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(2*c^3*(4*a*c - b^2)^(1/2)*(16*a*c^6 - 4
*b^2*c^5)))/a - (b*((((4*a^2*c^8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5
*f + 10*a*b*c^8*d - 16*a*b^2*c^7*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8
- (4*b*c^2*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e
+ 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 1...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1847, normalized size of antiderivative = 6.77

$$\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

output

```
(60*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**
2*f - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*
c**3*e - 60*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
b**3*c*f + 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a*b**2*c**2*e - 36*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*a*b*c**3*d + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) -
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
 + b))*b**5*f - 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*b**4*c*e + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*b**3*c**2*d + 60*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) -
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*a**2*b*c**2*f - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt
(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c...
```

3.49 $\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 203

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{(c^2d+b^2f-c(be+af))\,x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c} + \frac{(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\sqrt{b^2-4ac}} + \frac{(b^2ce-ac^2e-b^3f-bc(cd-2af))\log(a+bx^2+cx^4)}{4c^4}$$

output

```
1/2*(c^2*d+b^2*f-c*(a*f+b*e))*x^2/c^3+1/4*(-b*f+c*e)*x^4/c^2+1/6*f*x^6/c+1
/2*(b^3*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d))*arcta
nh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)+1/4*(b^2*c*e-a*c
^2*e-b^3*f-b*c*(-2*a*f+c*d))*ln(c*x^4+b*x^2+a)/c^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$= \frac{6c(c^2d+b^2f-c(be+af))\,x^2+3c^2(ce-bf)x^4+2c^3fx^6+\frac{6(-b^3ce+3abc^2e+b^4f+b^2c(cd-4af)+2ac^2(-cd+af))\arctan}{\sqrt{-b^2+4ac}}}{12c^4}$$

input
```
Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(6*c*(c^2*d + b^2*f - c*(b*e + a*f))*x^2 + 3*c^2*(c*e - b*f)*x^4 + 2*c^3*f
*x^6 + (6*(-(b^3*c*e) + 3*a*b*c^2*e + b^4*f + b^2*c*(c*d - 4*a*f) + 2*a*c^
2*(-(c*d) + a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c] - 3*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*Log[a + b*x^2
+ c*x^4])/(12*c^4)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{2194}$$
$$\frac{1}{2}\int \frac{x^4\left(fx^4+ex^2+d\right)}{cx^4+bx^2+a}dx^2$$
$$\Big\downarrow \text{2159}$$
$$\frac{1}{2}\int \left(\frac{fx^4}{c}+\frac{(ce-bf)x^2}{c^2}+\frac{fb^2+c^2d-c(be+af)}{c^3}-\frac{a\left(fb^2+c^2d-c(be+af)\right)-\left(-fb^3+ceb^2-c(cd-2af\right.}{c^3\left(cx^4+bx^2+a\right)}\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)}{c^4\sqrt{b^2-4ac}}+\frac{x^2\left(-c(af+be)+b^2f+\right.}{c^3}\right.$$

input
```
Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(((c^2*d + b^2*f - c*(b*e + a*f))*x^2)/c^3 + ((c*e - b*f)*x^4)/(2*c^2) + (
f*x^6)/(3*c) + ((b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a
*c^2*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2
- 4*a*c]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*Log[a + b*x^2
 + c*x^4])/(2*c^4))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.10

method	result
default	$-\frac{-\frac{1}{3}f\,x^{6}c^{2}+\frac{1}{2}bcf\,x^{4}-\frac{1}{2}e\,c^{2}x^{4}+acf\,x^{2}-b^{2}f\,x^{2}+bce\,x^{2}-c^{2}d\,x^{2}}{2c^{3}}+\frac{\frac{\left(2abcf-a\,c^{2}e-b^{3}f+b^{2}ce-b\,c^{2}d\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(a^{2}cf-a\,b^{2}\right.}{}}{}$
risch	Expression too large to display

input
```
int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^3*(-1/3*f*x^6*c^2+1/2*b*c*f*x^4-1/2*e*c^2*x^4+a*c*f*x^2-b^2*f*x^2+b
*c*e*x^2-c^2*d*x^2)+1/2/c^3*(1/2*(2*a*b*c*f-a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)
/c*ln(c*x^4+b*x^2+a)+2*(a^2*c*f-a*b^2*f+a*b*c*e-a*c^2*d-1/2*(2*a*b*c*f-a*c
^2*e-b^3*f+b^2*c*e-b*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a
*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.33

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$=\left[\frac{2\,(b^2c^3-4\,ac^4)fx^6+3\,((b^2c^3-4\,ac^4)e-(b^3c^2-4\,abc^3)f)x^4+6\,((b^2c^3-4\,ac^4)d-(b^3c^2-4\,abc^3)e+}{}\right.$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e - (b^3*c^2 -
 4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3)*e +
(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 + 3*sqrt(b^2 - 4*a*c)*((b^2*c^2 -
 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*f)*log
((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(
c*x^4 + b*x^2 + a)) - 3*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 +
4*a^2*c^3)*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/
(b^2*c^4 - 4*a*c^5), 1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a
*c^4)*e - (b^3*c^2 - 4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c
^2 - 4*a*b*c^3)*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 - 6*sqrt(-b^2
 + 4*a*c)*((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*
c + 2*a^2*c^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))
- 3*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e + (b^5
- 6*a*b^3*c + 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^4 - 4*a*c^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{2\,c^2fx^6+3\,c^2ex^4-3\,bcfx^4+6\,c^2dx^2-6\,bcex^2+6\,b^2fx^2-6\,acfx^2}{12\,c^3} - \frac{(bc^2d-b^2ce+ac^2e+b^3f-2\,abcf)\log\left(cx^4+bx^2+a\right)}{4\,c^4} + \frac{(b^2c^2d-2\,ac^3d-b^3ce+3\,abc^2e+b^4f-4\,ab^2cf+2\,a^2c^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^4}$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/12*(2*c^2*f*x^6 + 3*c^2*e*x^4 - 3*b*c*f*x^4 + 6*c^2*d*x^2 - 6*b*c*e*x^2
+ 6*b^2*f*x^2 - 6*a*c*f*x^2)/c^3 - 1/4*(b*c^2*d - b^2*c*e + a*c^2*e + b^3*
f - 2*a*b*c*f)*log(c*x^4 + b*x^2 + a)/c^4 + 1/2*(b^2*c^2*d - 2*a*c^3*d - b
^3*c*e + 3*a*b*c^2*e + b^4*f - 4*a*b^2*c*f + 2*a^2*c^2*f)*arctan((2*c*x^2
+ b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 18.69 (sec) , antiderivative size = 2295, normalized size of antiderivative = 11.31

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
x^4*(e/(4*c) - (b*f)/(4*c^2)) - x^2*((b*(e/c - (b*f)/c^2))/(2*c) - d/(2*c)
 + (a*f)/(2*c^2)) + (log(a + b*x^2 + c*x^4)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3
*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*
b*c^2*f))/(2*(16*a*c^5 - 4*b^2*c^4)) + (f*x^6)/(6*c) + (atan((2*c^6*(4*a*c
 - b^2)*(x^2*(((((6*b^2*c^6*d + 4*a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f -
4*a*c^7*d + 10*a*b*c^6*e - 16*a*b^2*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2
*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c
^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(b^4*f + b^2*c^2*d + 2*a^2
*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(8*c^4*(4*a*c -
 b^2)^(1/2)) + (b*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e +
 3*a*b*c^2*e - 4*a*b^2*c*f)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c
*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*c^2
*(4*a*c - b^2)^(1/2)*(16*a*c^5 - 4*b^2*c^4)))/a - (b*((b^7*f^2 + b^3*c^4*d
^2 + b^5*c^2*e^2 - 3*a*b^3*c^3*e^2 + 2*a^2*b*c^4*e^2 - 2*a^3*b*c^3*f^2 - 2
*b^6*c*e*f + 7*a^2*b^3*c^2*f^2 - a*b*c^5*d^2 - 5*a*b^5*c*f^2 - a^2*c^5*d*e
 - 2*b^4*c^3*d*e + a^3*c^4*e*f + 2*b^5*c^2*d*f + 4*a*b^2*c^4*d*e - 6*a*b^3
*c^3*d*f + 3*a^2*b*c^4*d*f + 8*a*b^4*c^2*e*f - 8*a^2*b^2*c^3*e*f)/c^6 + ((
(6*b^2*c^6*d + 4*a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f - 4*a*c^7*d + 10*a*
b*c^6*e - 16*a*b^2*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^
2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1483, normalized size of antiderivative = 7.31

$$\int \frac{x^5(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

output
```
( - 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c*
*2*f + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*
*2*c*f - 18*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
b*c**2*e + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a*c**3*d - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b
**4*f + 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3
*c*e - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*
c**2*d - 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*2*c**2*f + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a*b**2*c*f - 18*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) ...
```

3.50 $\int \frac{x^3\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx=\frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(c^2d+b^2f-c(be+af))\log\left(a+bx^2+cx^4\right)}{4c^3}$$

output

```
1/2*(-b*f+c*e)*x^2/c^2+1/4*f*x^4/c-1/2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*
f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)+1/4
*(c^2*d+b^2*f-c*(a*f+b*e))*ln(c*x^4+b*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$

$$=\frac{2c(ce-bf)x^2+c^2fx^4-\frac{2(-b^2ce+2ac^2e+b^3f+bc(cd-3af))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(c^2d+b^2f-c(be+af))\log\left(a+\right.}{4c^3}$$

input
```
Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*(c*e - b*f)*x^2 + c^2*f*x^4 - (2*(-(b^2*c*e) + 2*a*c^2*e + b^3*f + b*
c*(c*d - 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a
*c] + (c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{x^2(fx^4+ex^2+d)}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 2159}$$

$$\frac{1}{2}\int \left(\frac{fx^2}{c}+\frac{ce-bf}{c^2}-\frac{a(ce-bf)-(fb^2-ceb+c^2d-acf)x^2}{c^2(cx^4+bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)}{c^3\sqrt{b^2-4ac}}+\frac{\log(a+bx^2+cx^4)(-c(af+be)+b^2f+\right.$$

input
```
Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(((c*e - b*f)*x^2)/c^2 + (f*x^4)/(2*c) - ((b^2*c*e - 2*a*c^2*e - b^3*f - b
*c*(c*d - 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2
- 4*a*c]) + ((c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(2*c^
3))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

method	result	s
default	$-\frac{-\frac{1}{2}cf\,x^{4}+bf\,x^{2}-ce\,x^{2}}{2c^{2}}+\frac{\frac{\left(-acf+b^{2}f-ceb+d\,c^{2}\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(abf-ace-\frac{\left(-acf+b^{2}f-ceb+d\,c^{2}\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2c^{2}}$	1
risch	Expression too large to display	3

input
```
int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2*(-1/2*c*f*x^4+b*f*x^2-c*e*x^2)+1/2/c^2*(1/2*(-a*c*f+b^2*f-b*c*e+c
^2*d)/c*ln(c*x^4+b*x^2+a)+2*(a*b*f-a*c*e-1/2*(-a*c*f+b^2*f-b*c*e+c^2*d)*b/
c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.28

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)fx^4+2\,((b^2c^2-4\,ac^3)e-(b^3c-4\,abc^2)f)x^2-(bc^2d-(b^2c-2\,ac^2)e+(b^3-3\,abc)f)\sqrt{}}{}\right.$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*
b*c^2)*f)*x^2 - (b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a
*b*c^2)*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*
c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e
- (b^3*c - 4*a*b*c^2)*f)*x^2 + 2*(b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3
*a*b*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^
2 - 4*a*c)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 5*a*
b^2*c + 4*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{cfx^4+2\,cex^2-2\,bfx^2}{4\,c^2} + \frac{(c^2d-bce+b^2f-acf)\log(cx^4+bx^2+a)}{4\,c^3} - \frac{(bc^2d-b^2ce+2\,ac^2e+b^3f-3\,abcf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(c*f*x^4 + 2*c*e*x^2 - 2*b*f*x^2)/c^2 + 1/4*(c^2*d - b*c*e + b^2*f - a
*c*f)*log(c*x^4 + b*x^2 + a)/c^3 - 1/2*(b*c^2*d - b^2*c*e + 2*a*c^2*e + b^
3*f - 3*a*b*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 18.45 (sec) , antiderivative size = 1689, normalized size of antiderivative = 11.73

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
x^2*(e/(2*c) - (b*f)/(2*c^2)) + (f*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2
*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e -
 10*a*b^2*c*f))/(2*(16*a*c^4 - 4*b^2*c^3)) - (atan((2*c^4*(4*a*c - b^2)*(x
^2*(((((6*b^3*c^3*f - 6*b^2*c^4*e + 4*a*c^5*e + 6*b*c^5*d - 10*a*b*c^4*f)/
c^4 + (4*b*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*
e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(b^3*f + 2*a*c^2*
e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(8*c^3*(4*a*c - b^2)^(1/2)) + (b*(b^3*
f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f)*(2*b^4*f + 2*b^2*c^2*d + 8*
a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(2*c*(4*a
*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a - (b*((b^5*f^2 + b*c^4*d^2 + b^
3*c^2*e^2 + 2*a^2*b*c^2*f^2 + a*c^4*d*e - 2*b^4*c*e*f - a*b*c^3*e^2 - 3*a*
b^3*c*f^2 - 2*b^2*c^3*d*e - a^2*c^3*e*f + 2*b^3*c^2*d*f + 4*a*b^2*c^2*e*f
- 3*a*b*c^3*d*f)/c^4 + (((6*b^3*c^3*f - 6*b^2*c^4*e + 4*a*c^5*e + 6*b*c^5*
d - 10*a*b*c^4*f)/c^4 + (4*b*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*
a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))
*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*
e - 10*a*b^2*c*f))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b*(b^3*f + 2*a*c^2*e + b*
c^2*d - b^2*c*e - 3*a*b*c*f)^2)/(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)
^(1/2))) - ((((8*a^2*c^4*f - 8*a*c^5*d + 8*a*b*c^4*e - 8*a*b^2*c^3*f)/c^4
- (8*a*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1084, normalized size of antiderivative = 7.53

$$\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

output
```
( - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*f
+ 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*e +
 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*f - 2*
sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*e + 2*s
qrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c**2*d - 6*sq
rt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*f + 4*sqrt
(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*e + 2*sqrt(
2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*f - 2*sqrt(2*s
qrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*e + 2*sqrt(2*sq
rt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c**2*d - 4*log( - ...
```

3.51 $\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 103

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{fx^2}{2c} - \frac{(2c^2d-bce+b^2f-2acf)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} + \frac{(ce-bf)\log(a+bx^2+cx^4)}{4c^2}$$

output
```
1/2*f*x^2/c-1/2*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*arctanh((2*c*x^2+b)/(-4*a*c
+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)+1/4*(-b*f+c*e)*ln(c*x^4+b*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{2cfx^2 + \frac{2(2c^2d+b^2f-c(be+2af))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (ce-bf)\log(a+bx^2+cx^4)}{4c^2}$$

input
```
Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*f*x^2 + (2*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*ArcTan[(b + 2*c*x^2)/S
qrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*e - b*f)*Log[a + b*x^2 + c*x^4
])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2194, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\int \left(\frac{f}{c}+\frac{(ce-bf)x^2+cd-af}{c\,(cx^4+bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-2acf+b^2f-bce+2c^2d)}{c^2\sqrt{b^2-4ac}}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{2c^2}+\frac{fx^2}{c}\right)$$

input
```
Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
((f*x^2)/c - ((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*ArcTanh[(b + 2*c*x^2)/Sq
rt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*e - b*f)*Log[a + b*x^2 + c
*x^4])/(2*c^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{f\,x^{2}}{2c}+\frac{\frac{(-bf+ce)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(-af+cd-\frac{(-bf+ce)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2c}$	101
risch	Expression too large to display	1690

input
```
int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*f*x^2/c+1/2/c*(1/2*(-b*f+c*e)/c*ln(c*x^4+b*x^2+a)+2*(-a*f+c*d-1/2*(-b*
f+c*e)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.09

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{2\,(b^2c-4\,ac^2)fx^2-(2\,c^2d-bce+(b^2-2\,ac)f)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+}{4\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*(b^2*c - 4*a*c^2)*f*x^2 - (2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt
(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqr
t(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c - 4*a*c^2)*e - (b^3 - 4*a*b
*c)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(b^2*c - 4*a*c^
2)*f*x^2 - 2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*e -
 (b^3 - 4*a*b*c)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{fx^2}{2\,c} + \frac{(ce-bf)\log\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{(2\,c^2d-bce+b^2f-2\,acf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*f*x^2/c + 1/4*(c*e - b*f)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(2*c^2*d -
b*c*e + b^2*f - 2*a*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b
^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 19.15 (sec) , antiderivative size = 1081, normalized size of antiderivative = 10.50

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
(f*x^2)/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e -
 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) + (atan((2*c^2*(4*a*c - b^2)*(x^2*
(((((4*c^4*d + 6*b^2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*
f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d +
 b^2*f - 2*a*c*f - b*c*e))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b*(2*c^2*d + b^2
*f - 2*a*c*f - b*c*e)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(2*(4
*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a - (b*((b^3*f^2 + b*c^2*e^2 -
c^3*d*e - a*b*c*f^2 + a*c^2*e*f + b*c^2*d*f - 2*b^2*c*e*f)/c^2 + (((4*c^4*
d + 6*b^2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*f + 8*a*c^2
*e - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3*f + 8*a*c^2*e
- 2*b^2*c*e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b*(2*c^2*d + b^2*f
 - 2*a*c*f - b*c*e)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2)))
- ((((8*a*c^3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*b^3*f + 8*a*c^2*e - 2*b^2
*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d + b^2*f - 2*a*c*f - b*
c*e))/(8*c^2*(4*a*c - b^2)^(1/2)) - (a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)
*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/((4*a*c - b^2)^(1/2)*(16*a
*c^3 - 4*b^2*c^2)))/a + (b*((((8*a*c^3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*
b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3
*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*b
^2*f^2 + a*c^2*e^2 - 2*a*b*c*e*f)/c^2 + (a*(2*c^2*d + b^2*f - 2*a*c*f -...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 720, normalized size of antiderivative = 6.99

$$\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

output
```
(4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*f - 2*s
qrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*f + 2*sqrt
(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c*e - 4*sqrt(2*s
qrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*c**2*d + 4*sqrt(2*sqrt
(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*f - 2*sqrt(2*sqrt(c)*
sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*f + 2*sqrt(2*sqrt(c)*sqr
t(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +
2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c*e - 4*sqrt(2*sqrt(c)*sqrt(a)
 + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*c**2*d - 4*log( - sqrt(2*sqrt(c)*sqr
t(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*c*f + 4*log( - sqrt(2*sqrt(c)*sq
rt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c**2*e + log( - sqrt(2*sqrt(c)*sq
rt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**3*f - log( - sqrt(2*sqrt(c)*sqrt
(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**2*c*e - 4*log(sqrt(2*sqrt(c)*sq...
```

3.52 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 97

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx = \frac{(bcd-2ace+abf)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2ac\sqrt{b^2-4ac}} + \frac{d\log(x)}{a} - \frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}$$

output
```
1/2*(a*b*f-2*a*c*e+b*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a/c/(-4*
a*c+b^2)^(1/2)+d*ln(x)/a-1/4*(-a*f+c*d)*ln(c*x^4+b*x^2+a)/a/c
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.84

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx = \frac{4c\sqrt{b^2-4ac}d\log(x) - \left(bcd+c\sqrt{b^2-4ac}d-2ace+abf-a\sqrt{b^2-4ac}f\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right) + \cdots}{4ac\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(4*c*Sqrt[b^2 - 4*a*c]*d*Log[x] - (b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e
 + a*b*f - a*Sqrt[b^2 - 4*a*c]*f)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (
b*c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*L
og[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*c*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^2\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\int \left(\frac{d}{ax^2}+\frac{-\left((cd-af)x^2\right)-bd+ae}{a\left(cx^4+bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{ac\sqrt{b^2-4ac}}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{2ac}+\frac{d\log\left(x^2\right)}{a}\right)$$

input
```
Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(((b*c*d - 2*a*c*e + a*b*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*c
*Sqrt[b^2 - 4*a*c]) + (d*Log[x^2])/a - ((c*d - a*f)*Log[a + b*x^2 + c*x^4]
)/(2*a*c))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

method	result
default	$\frac{d\ln(x)}{a}+\frac{\frac{(af-cd)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(ae-bd-\frac{(af-cd)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a}$
risch	$\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^2c^2-a\,b^2c\right)\_Z^2+\left(-4a^2cf+a\,b^2f+4a\,c^2d-b^2cd\right)\_Z+a^2f^2-abef-2dacf+e^2ac+b^2df-bcde+d^2c^2\right)}\_R\ln\left(\right.\right.}{}$

input
```
int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d*ln(x)/a+1/2/a*(1/2*(a*f-c*d)/c*ln(c*x^4+b*x^2+a)+2*(a*e-b*d-1/2*(a*f-c*d
)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.19

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx$$
$$= \left[\frac{4\left(b^2c-4\,ac^2\right)d\log\left(x\right)+\left(bcd-2\,ace+abf\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-((b^2}{4\left(ab^2c-4\,a^2c^2\right)}\right.$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(4*(b^2*c - 4*a*c^2)*d*log(x) + (b*c*d - 2*a*c*e + a*b*f)*sqrt(b^2 -
4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 -
 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*c)*f
)*log(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2), 1/4*(4*(b^2*c - 4*a*c^2)*
d*log(x) + 2*(b*c*d - 2*a*c*e + a*b*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2
 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 -
4*a^2*c)*f)*log(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx = \frac{d\log(x^2)}{2a} - \frac{(cd-af)\log(cx^4+bx^2+a)}{4ac} - \frac{(bcd-2ace+abf)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}ac}$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*d*log(x^2)/a - 1/4*(c*d - a*f)*log(c*x^4 + b*x^2 + a)/(a*c) - 1/2*(b*c
*d - 2*a*c*e + a*b*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*a*c)
```

Mupad [B] (verification not implemented)

Time = 26.88 (sec) , antiderivative size = 3927, normalized size of antiderivative = 40.48

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x)
```

output
```
(d*log(x))/a - (log((b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e^
2 - b*e*f - c*d*f) + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*
c^2*(4*a*c - b^2)))^(1/2))*(a*b^2*f^2 - x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3
*d*e + 11*a*b*c*f^2 - 9*a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) + a*c^2*e^2
 - 4*b*c^2*d*e + 4*b^2*c*d*f + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*
d)^2/(a^2*c^2*(4*a*c - b^2)))^(1/2))*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b*
c^2*d - 4*b^2*c*e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f
+ (b*c*(c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^
2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) - 2*a*b*c*e*f))/(4
*a*c) - 2*b*c*d*e*f)*(b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e
^2 - b*e*f - c*d*f) + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2
*c^2*(4*a*c - b^2)))^(1/2))*(x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3*d*e + 11*a
*b*c*f^2 - 9*a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) - a*b^2*f^2 - a*c^2*e^
2 + 4*b*c^2*d*e - 4*b^2*c*d*f + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c
*d)^2/(a^2*c^2*(4*a*c - b^2)))^(1/2))*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b
*c^2*d - 4*b^2*c*e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f
 - (b*c*(a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b
^2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) + 2*a*b*c*e*f))/(
4*a*c) - 2*b*c*d*e*f))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2
*(16*a^2*c^2 - 4*a*b^2*c)) + (atan(((4*a*c - b^2)*(((((a*b*f - 2*a*c*e ...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 611, normalized size of antiderivative = 6.30

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x)
```

output

```
(2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*f - 4*s
qrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*e + 2*sqrt(
2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c*d + 2*sqrt(2*sq
rt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*f - 4*sqrt(2*sqrt(c
)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*e + 2*sqrt(2*sqrt(c)*sq
rt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c*d + 4*log( - sqrt(2*sqrt(c)
*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a**2*c*f - log( - sqrt(2*sqrt(c)
*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b**2*f - 4*log( - sqrt(2*sqrt(
c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c**2*d + log( - sqrt(2*sqrt(
c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**2*c*d + 4*log(sqrt(2*sqrt(c
)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a**2*c*f - log(sqrt(2*sqrt(c)*s
qrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b**2*f - 4*log(sqrt(2*sqrt(c)*sq
rt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c**2*d + log(sqrt(2*sqrt(c)*sqrt(
a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**2*c*d + 16*log(x)*a*c**2*d - 4*l...
```

3.53 $\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 118

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx=-\frac{d}{2ax^2}-\frac{\left(b^2d-abe-2a(cd-af)\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}$$

output
```
-1/2*d/a/x^2-1/2*(b^2*d-a*b*e-2*a*(-a*f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+
b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)-(-a*e+b*d)*ln(x)/a^2+1/4*(-a*e+b*d)*ln(
c*x^4+b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.72

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx$$

$$=\frac{-\frac{2ad}{x^2}+4(-bd+ae)\log(x)+\frac{\left(b^2d+b\left(\sqrt{b^2-4ac}d-ae\right)+a\left(-2cd-\sqrt{b^2-4ac}e+2af\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2d+b\left(\sqrt{b^2-\right.\right.}{}}{4a^2}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a*d)/x^2 + 4*(-(b*d) + a*e)*Log[x] + ((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d
 - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d +
a*e) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^4\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \text{ 2159}$$

$$\frac{1}{2}\int \left(\frac{d}{ax^4}+\frac{db^2-aeb+c(bd-ae)x^2-a(cd-af)}{a^2\left(cx^4+bx^2+a\right)}+\frac{ae-bd}{a^2x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-abe-2a(cd-af)+b^2d\right)}{a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{2a^2}-\frac{\log\left(x^2\right)(bd-ae)}{a^2}-\frac{d}{ax}\right.$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
(-(d/(a*x^2)) - ((b^2*d - a*b*e - 2*a*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/S
qrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x^2])/a^2 +
((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(2*a^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

method	result
default	$-\frac{d}{2a\,x^{2}}+\frac{(ae-bd)\ln(x)}{a^{2}}+\frac{\frac{(-ace+bcd)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(f\,a^{2}-abe-dac+b^{2}d-\frac{(-ace+bcd)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}}$
risch	$-\frac{d}{2a\,x^{2}}+\frac{\ln(x)e}{a}-\frac{\ln(x)bd}{a^{2}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(4a^{2}ce-a\,b^{2}e-4abcd+b^{3}d\right)\_Z+a^{2}f^{2}-abef-2dacf+e^{2}ac+b^{2}df-b\right.}\right.}{}$

input
```
int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a/x^2+(a*e-b*d)/a^2*ln(x)+1/2/a^2*(1/2*(-a*c*e+b*c*d)/c*ln(c*x^4+b*
x^2+a)+2*(f*a^2-a*b*e-d*a*c+b^2*d-1/2*(-a*c*e+b*c*d)*b/c)/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.38

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$= \left[-\frac{(abe-2\,a^2f-(b^2-2\,ac)d)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-((b^3-4\,abc)d-}{4\,(a^2b^2-}\right.$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*((a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^
2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4
+ b*x^2 + a)) - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*x^4 +
b*x^2 + a) + 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x) + 2*(a
*b^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(a*b*e - 2*a^2*f - (b
^2 - 2*a*c)*d)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*
a*c)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*
x^4 + b*x^2 + a) - 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x)
- 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex^2 + fx^4}{x^3 (a + bx^2 + cx^4)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.11

$$\int \frac{d + ex^2 + fx^4}{x^3 (a + bx^2 + cx^4)} \, dx = \frac{(bd - ae) \log (cx^4 + bx^2 + a)}{4 a^2} - \frac{(bd - ae) \log (x^2)}{2 a^2} + \frac{(b^2 d - 2 acd - abe + 2 a^2 f) \arctan \left(\frac{2 cx^2 + b}{\sqrt{-b^2 + 4 ac}}\right)}{2 \sqrt{-b^2 + 4 ac} a^2} + \frac{bdx^2 - aex^2 - ad}{2 a^2 x^2}$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2
+ 1/2*(b^2*d - 2*a*c*d - a*b*e + 2*a^2*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*d*x^2 - a*e*x^2 - a*d)/(a^2*x^2
)
```

Mupad [B] (verification not implemented)

Time = 26.39 (sec) , antiderivative size = 4437, normalized size of antiderivative = 37.60

$$\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x)
```

output
```
(log(x)*(a*e - b*d))/a^2 - d/(2*a*x^2) - (log(((c^2*(a*e - b*d)*(a*f - c*d
)^2)/a^3 - ((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*
(4*a*c - b^2)))^(1/2))*(((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a
*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((2*c^2*x^2*(10*a*c^2*d + 4*a*b^2*f +
b^2*c*d - 10*a^2*c*f - 5*a*b*c*e))/a + (4*b*c^2*(b^2*d + a^2*f - a*b*e - a
*c*d))/a + (b*c^2*(b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2
/(a^4*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2))/(4*a^2)
 + (c^2*(a*f - c*d)*(4*b^2*d + a^2*f - 4*a*b*e - a*c*d))/a^2 - (c^2*x^2*(a
*f - c*d)*(a*b*f + 5*a*c*e - 6*b*c*d))/a^2))/(4*a^2) + (c^2*x^2*(a*f - c*d
)^3)/a^3)*((c^2*(a*e - b*d)*(a*f - c*d)^2)/a^3 - ((a*e - b*d + a^2*(-(b^2*
d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*(((a*e - b*d
+ a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*
((2*c^2*x^2*(10*a*c^2*d + 4*a*b^2*f + b^2*c*d - 10*a^2*c*f - 5*a*b*c*e))/a
 + (4*b*c^2*(b^2*d + a^2*f - a*b*e - a*c*d))/a - (b*c^2*(a*e - b*d + a^2*(
-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*(a*b +
3*b^2*x^2 - 10*a*c*x^2))/a^2))/(4*a^2) - (c^2*(a*f - c*d)*(4*b^2*d + a^2*f
 - 4*a*b*e - a*c*d))/a^2 + (c^2*x^2*(a*f - c*d)*(a*b*f + 5*a*c*e - 6*b*c*d
))/a^2))/(4*a^2) + (c^2*x^2*(a*f - c*d)^3)/a^3))*(2*b^3*d - 2*a*b^2*e + 8*
a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)) - (atan((16*a^6*(4*a*c -
b^2)^(3/2)*(x^2*((((c^5*d^3 - a^3*c^2*f^3 + 3*a^2*c^3*d*f^2 - 3*a*c^4*d...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 811, normalized size of antiderivative = 6.87

$$\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x)
```

output

```
( - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*f*x
**2 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*e*
x**2 + 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d
*x**2 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2
*d*x**2 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*2*f*x**2 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a*b*e*x**2 + 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a*c*d*x**2 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b)
)*b**2*d*x**2 - 4*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)
*x**2)*a**2*c*e*x**2 + log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sq
rt(c)*x**2)*a*b**2*e*x**2 + 4*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(
a) + sqrt(c)*x**2)*a*b*c*d*x**2 - log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x ...
```

3.54 $\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 174

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx = &-\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2} \\ &+\frac{(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}} \\ &+\frac{(b^2d-abe-a(cd-af))\log(x)}{a^3} \\ &-\frac{(b^2d-abe-a(cd-af))\log(a+bx^2+cx^4)}{4a^3}\end{aligned}$$

output
```
-1/4*d/a/x^4+1/2*(-a*e+b*d)/a^2/x^2+1/2*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f
+3*c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)+(b
^2*d-a*b*e-a*(-a*f+c*d))*ln(x)/a^3-1/4*(b^2*d-a*b*e-a*(-a*f+c*d))*ln(c*x^4
+b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.80

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)}\,dx =$$

$$-\frac{\frac{a^2d}{x^4}+\frac{2a(-bd+ae)}{x^2}-4(b^2d-abe+a(-cd+af))\log(x)+\frac{\left(b^3d+b^2\left(\sqrt{b^2-4ac}d-ae\right)+ab\left(-3cd-\sqrt{b^2-4ac}e+af\right)+a\left(-\right.\right.}{\sqrt{b^2-}}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/4*((a^2*d)/x^4 + (2*a*(-(b*d) + a*e))/x^2 - 4*(b^2*d - a*b*e + a*(-(c*d
) + a*f))*Log[x] + ((b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*b*(-3*c*d
 - Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*a*c*e + a*
Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a
*c] + ((-(b^3*d) + b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*b*(-3*c*d + Sqrt[b^
2 - 4*a*c]*e + a*f) + a*(-(c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + a*Sqrt[b^2 -
 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/a^3
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)}dx$$

$$\Big\downarrow \ 2194$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^6\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \ 2159$$

$$\frac{1}{2}\int \left(\frac{d}{ax^6}+\frac{-db^3+aeb^2+a(2cd-af)b-c\left(db^2-aeb-a(cd-af)\right)x^2-a^2ce}{a^3\left(cx^4+bx^2+a\right)}+\frac{db^2-aeb-a(cd-af)}{a^3x^2}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)\left(-abe-a(cd-af)+b^2d\right)}{a^3}-\frac{\log\left(a+bx^2+cx^4\right)\left(-abe-a(cd-af)+b^2d\right)}{2a^3}+\frac{bd-ae}{a^2x^2}+\frac{\text{arctanh}}{}\right.$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

output
```
(-1/2*d/(a*x^4) + (b*d - a*e)/(a^2*x^2) + ((b^3*d - a*b^2*e + 2*a^2*c*e -
a*b*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2
 - 4*a*c]) + ((b^2*d - a*b*e - a*(c*d - a*f))*Log[x^2])/a^3 - ((b^2*d - a*
b*e - a*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(2*a^3))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.17

method	result
default	$-\frac{d}{4a\,x^4}-\frac{ae-bd}{2a^2x^2}+\frac{(f\,a^2-abe-dac+b^2d)\ln(x)}{a^3}-\frac{\frac{(a^2cf-abce-a\,c^2d+b^2cd)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\left(a^2bf+a^2ce-a\,b^2e-2abcd+b^3d-\right.}{}}{2a^3}$
risch	$\frac{-\frac{(ae-bd)x^2}{2a^2}-\frac{d}{4a}}{x^4}+\frac{\ln(x)f}{a}-\frac{\ln(x)be}{a^2}-\frac{\ln(x)dc}{a^2}+\frac{\ln(x)b^2d}{a^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((4a^4c-a^3b^2)\_Z^2+(4a^3cf-a^2b^2f-4a^2bce-4a^2c\right.}\right.}{}$

input
```
int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*d/a/x^4-1/2*(a*e-b*d)/a^2/x^2+(a^2*f-a*b*e-a*c*d+b^2*d)/a^3*ln(x)-1/2
/a^3*(1/2*(a^2*c*f-a*b*c*e-a*c^2*d+b^2*c*d)/c*ln(c*x^4+b*x^2+a)+2*(a^2*b*f
+a^2*c*e-a*b^2*e-2*a*b*c*d+b^3*d-1/2*(a^2*c*f-a*b*c*e-a*c^2*d+b^2*c*d)*b/c
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.50

$$\int \frac{d+ex^2+fx^4}{x^5\,(a+bx^2+cx^4)}\,dx$$
$$=\left[\frac{(a^2bf+(b^3-3\,abc)d-(ab^2-2\,a^2c)e)\sqrt{b^2-4\,ac}x^4\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-((b^4-\right.}{}\right.$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((a^2*b*f + (b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(b^2 - 4*a*c
)*x^4*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 -
4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 -
 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^
4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)
*f)*x^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (
a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4), 1/4*(2*(a^2*b*f + (b^3 -
3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(-b^2 + 4*a*c)*x^4*arctan(-(2*c*x^2
+ b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d
- (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a
) + 4*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2
- 4*a^3*c)*f)*x^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*
e)*x^2 - (a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4}{x^5 (a + bx^2 + cx^4)} \, dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex^2 + fx^4}{x^5 (a + bx^2 + cx^4)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx = -\frac{(b^2d-acd-abe+a^2f)\log(cx^4+bx^2+a)}{4a^3} + \frac{(b^2d-acd-abe+a^2f)\log(x^2)}{2a^3} - \frac{(b^3d-3abcd-ab^2e+2a^2ce+a^2bf)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}a^3} - \frac{3b^2dx^4-3acdx^4-3abex^4+3a^2fx^4-2abdx^2+2a^2ex^2+a^2d}{4a^3x^4}$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(b^2*d - a*c*d - a*b*e + a^2*f)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2
*d - a*c*d - a*b*e + a^2*f)*log(x^2)/a^3 - 1/2*(b^3*d - 3*a*b*c*d - a*b^2*
e + 2*a^2*c*e + a^2*b*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b
^2 + 4*a*c)*a^3) - 1/4*(3*b^2*d*x^4 - 3*a*c*d*x^4 - 3*a*b*e*x^4 + 3*a^2*f*
x^4 - 2*a*b*d*x^2 + 2*a^2*e*x^2 + a^2*d)/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 29.61 (sec) , antiderivative size = 6187, normalized size of antiderivative = 35.56

$$\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x)
```

output
```
(log(x)*(b^2*d + a^2*f - a*b*e - a*c*d))/a^3 - (d/(4*a) + (x^2*(a*e - b*d)
)/(2*a^2))/x^4 + (log((((((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^
2*c*e + 5*a*b*c*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e -
2*a*b*c*d))/a^2 + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^2*d + a^2*f + a
^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b
^2)))^(1/2) - a*b*e - a*c*d))/a^3)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e
 + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a*b*e -
 a*c*d))/(4*a^3) + (c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2
*c*e - 5*a*b*c*d))/a^4 + (c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e
 - 5*a*c*d))/a^4)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^
2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a*b*e - a*c*d))/(4*a^3)
+ (c^4*(a*e - b*d)^2*(b^2*d + a^2*f - a*b*e - a*c*d))/a^6 - (c^5*x^2*(a*e
- b*d)^3)/a^6)*((((c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2*
c*e - 5*a*b*c*d))/a^4 - (((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^2
*c*e + 5*a*b*c*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2
*a*b*c*d))/a^2 - (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(a^3*(-(b^3*d - a*b
^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a^2
*f - b^2*d + a*b*e + a*c*d))/a^3)*(a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^
2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a^2*f - b^2*d + a*b*e +
a*c*d))/(4*a^3) + (c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e - 5...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1239, normalized size of antiderivative = 7.12

$$\int \frac{d + ex^2 + fx^4}{x^5 (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x)
```

output

```
(2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*f*x*
*4 + 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c*
e*x**4 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((s
qrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b
**2*e*x**4 - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a*b*c*d*x**4 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*
atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
b))*b**3*d*x**4 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a**2*b*f*x**4 + 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
 - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*a**2*c*e*x**4 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqr
t(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a*b**2*e*x**4 - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)
*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*a*b*c*d*x**4 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt
(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(...
```

3.55 $\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)} dx$

Optimal result

Integrand size = 30, antiderivative size = 244

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^7(a+bx^2+cx^4)}\,dx &= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}\\ &\quad-\frac{(b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\sqrt{b^2-4ac}}\\ &\quad-\frac{(b^3d-ab^2e+a^2ce-ab(2cd-af))\log(x)}{a^4}\\ &\quad+\frac{(b^3d-ab^2e+a^2ce-ab(2cd-af))\log(a+bx^2+cx^4)}{4a^4}\end{aligned}$$

output
```
-1/6*d/a/x^6+1/4*(-a*e+b*d)/a^2/x^4-1/2*(b^2*d-a*b*e-a*(-a*f+c*d))/a^3/x^2
-1/2*(b^4*d-a*b^3*e+3*a^2*b*c*e+2*a^2*c*(-a*f+c*d)-a*b^2*(-a*f+4*c*d))*arc
tanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(1/2)-(b^3*d-a*b^2*e
+a^2*c*e-a*b*(-a*f+2*c*d))*ln(x)/a^4+1/4*(b^3*d-a*b^2*e+a^2*c*e-a*b*(-a*f+
2*c*d))*ln(c*x^4+b*x^2+a)/a^4
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.70

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx$$
$$= \frac{-\frac{2a^3d}{x^6}+\frac{3a^2(bd-ae)}{x^4}+\frac{6a\left(-b^2d+abe+a(cd-af)\right)}{x^2}-12\left(b^3d-ab^2e+a^2ce+ab(-2cd+af)\right)\log(x)+\frac{3\left(b^4d+b^3\left(\sqrt{b^2\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a^3*d)/x^6 + (3*a^2*(b*d - a*e))/x^4 + (6*a*(-(b^2*d) + a*b*e + a*(c*
d - a*f)))/x^2 - 12*(b^3*d - a*b^2*e + a^2*c*e + a*b*(-2*c*d + a*f))*Log[x
] + (3*(b^4*d + b^3*(Sqrt[b^2 - 4*a*c]*d - a*e) + a^2*c*(2*c*d + Sqrt[b^2
- 4*a*c]*e - 2*a*f) + a*b^2*(-4*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-2
*c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (3*(-(b^4*d) + b^3*(Sqrt[b^2 -
4*a*c]*d + a*e) - a*b^2*(-4*c*d + Sqrt[b^2 - 4*a*c]*e + a*f) + a^2*c*(-2*c
*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + a*b*(-2*c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*
e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2
 - 4*a*c])/(12*a^4)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2194, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx$$
$$\Big\downarrow \; 2194$$
$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^8\left(cx^4+bx^2+a\right)}dx^2$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\int \left(\frac{d}{ax^8}+\frac{db^4-aeb^3-a(3cd-af)b^2+2a^2ceb+c\left(db^3-aeb^2-a(2cd-af)b+a^2ce\right)x^2+a^2c(cd-af)}{a^4\left(cx^4+bx^2+a\right)}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{-abe-a(cd-af)+b^2d}{a^3x^2}+\frac{bd-ae}{2a^2x^4}-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af\right.}{a^4\sqrt{b^2-4ac}}\right.$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

output
```
(-1/3*d/(a*x^6) + (b*d - a*e)/(2*a^2*x^4) - (b^2*d - a*b*e - a*(c*d - a*f)
)/(a^3*x^2) - ((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^
2*(4*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 -
 4*a*c]) - ((b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[x^2])/a^4
+ ((b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])
/(2*a^4))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.20

method	result
default	$-\frac{d}{6a\,x^{6}}-\frac{ae-bd}{4a^{2}x^{4}}-\frac{f\,a^{2}-abe-dac+b^{2}d}{2a^{3}x^{2}}+\frac{\left(-a^{2}bf-a^{2}ce+a\,b^{2}e+2abcd-b^{3}d\right)\ln\left(x\right)}{a^{4}}-\frac{\frac{\left(-a^{2}bcf-a^{2}c^{2}e+a\,b^{2}ce+2ab\,c^{2}d-b^{3}cd\right)}{2c}}{}$
risch	$\frac{-\frac{\left(f\,a^{2}-abe-dac+b^{2}d\right)x^{4}}{2a^{3}}-\frac{\left(ae-bd\right)x^{2}}{4a^{2}}-\frac{d}{6a}}{x^{6}}-\frac{\ln\left(x\right)bf}{a^{2}}-\frac{\ln\left(x\right)ce}{a^{2}}+\frac{\ln\left(x\right)b^{2}e}{a^{3}}+\frac{2\ln\left(x\right)bcd}{a^{3}}-\frac{\ln\left(x\right)b^{3}d}{a^{4}}+\frac{\left(\_R=\mathrm{RootOf}\left(\left(4c\right.\right.\right.}{}$

input
```
int((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*d/a/x^6-1/4*(a*e-b*d)/a^2/x^4-1/2*(a^2*f-a*b*e-a*c*d+b^2*d)/a^3/x^2+1
/a^4*(-a^2*b*f-a^2*c*e+a*b^2*e+2*a*b*c*d-b^3*d)*ln(x)-1/2/a^4*(1/2*(-a^2*b
*c*f-a^2*c^2*e+a*b^2*c*e+2*a*b*c^2*d-b^3*c*d)/c*ln(c*x^4+b*x^2+a)+2*(a^3*c
*f-a^2*b^2*f-2*a^2*b*c*e-a^2*c^2*d+a*b^3*e+3*a*b^2*c*d-b^4*d-1/2*(-a^2*b*c
*f-a^2*c^2*e+a*b^2*c*e+2*a*b*c^2*d-b^3*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 834, normalized size of antiderivative = 3.42

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/12*(3*sqrt(b^2 - 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*
a^2*b*c)*e + (a^2*b^2 - 2*a^3*c)*f)*x^6*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^5 -
 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b
^3 - 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b^5 - 6*a*b^3*c + 8*a
^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*
f)*x^6*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*
b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^
2 - 4*a^4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 - 4*a^5*c)*x^6),
-1/12*(6*sqrt(-b^2 + 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*
a^2*b*c)*e + (a^2*b^2 - 2*a^3*c)*f)*x^6*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) - 3*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*
a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2
+ a) + 12*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^
3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(x) + 6*((a*b^4 - 5*a^2*b^2*c +
 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*(
(a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^2 - 4*a^4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*
c)*d)/((a^4*b^2 - 4*a^5*c)*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4}{x^7 (a + bx^2 + cx^4)} \, dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**7/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.24

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx = \frac{\left(b^3d-2\,abcd-ab^2e+a^2ce+a^2bf\right)\log\left(cx^4+bx^2+a\right)}{4\,a^4} - \frac{\left(b^3d-2\,abcd-ab^2e+a^2ce+a^2bf\right)\log\left(x^2\right)}{2\,a^4} + \frac{\left(b^4d-4\,ab^2cd+2\,a^2c^2d-ab^3e+3\,a^2bce+a^2b^2f-2\,a^3cf\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\sqrt{-b^2+4\,ac}a^4} + \frac{11\,b^3dx^6-22\,abcdx^6-11\,ab^2ex^6+11\,a^2cex^6+11\,a^2bfx^6-6\,ab^2dx^4+6\,a^2cdx^4+6\,a^2bex^4-6\,a^3fx^4}{12\,a^4x^6}$$

input
```
integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + a^2*b*f)*log(c*x^4 + b*x^2 +
a)/a^4 - 1/2*(b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + a^2*b*f)*log(x^2)/a^
4 + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e + a^2*b
^2*f - 2*a^3*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*
a*c)*a^4) + 1/12*(11*b^3*d*x^6 - 22*a*b*c*d*x^6 - 11*a*b^2*e*x^6 + 11*a^2*
c*e*x^6 + 11*a^2*b*f*x^6 - 6*a*b^2*d*x^4 + 6*a^2*c*d*x^4 + 6*a^2*b*e*x^4 -
 6*a^3*f*x^4 + 3*a^2*b*d*x^2 - 3*a^3*e*x^2 - 2*a^3*d)/(a^4*x^6)
```

Mupad [B] (verification not implemented)

Time = 35.31 (sec) , antiderivative size = 9141, normalized size of antiderivative = 37.46

$$\int \frac{d+ex^2+fx^4}{x^7 (a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x)
```

output
```
(atan((16*a^12*(4*a*c - b^2)^(3/2)*(x^2*((((a^3*c^8*d^3 - b^6*c^5*d^3 - a^
6*c^5*f^3 + 3*a*b^4*c^6*d^3 - 3*a^4*c^7*d^2*f + 3*a^5*c^6*d*f^2 - 3*a^2*b^
2*c^7*d^3 + a^3*b^3*c^5*e^3 + 3*a*b^5*c^5*d^2*e + 3*a^3*b*c^7*d^2*e + 3*a^
5*b*c^5*e*f^2 - 6*a^2*b^3*c^6*d^2*e - 3*a^2*b^4*c^5*d*e^2 + 3*a^3*b^2*c^6*
d*e^2 - 3*a^2*b^4*c^5*d^2*f + 6*a^3*b^2*c^6*d^2*f - 3*a^4*b^2*c^5*d*f^2 -
3*a^4*b^2*c^5*e^2*f - 6*a^4*b*c^6*d*e*f + 6*a^3*b^3*c^5*d*e*f)/a^9 - (((11
*a^5*b*c^6*d^2 - 5*a^6*b*c^5*e^2 + 6*a^7*b*c^4*f^2 + 6*a^3*b^5*c^4*d^2 - 1
7*a^4*b^3*c^5*d^2 + 6*a^5*b^3*c^4*e^2 - 5*a^6*c^6*d*e + 5*a^7*c^5*e*f - 17
*a^6*b*c^5*d*f - 12*a^4*b^4*c^4*d*e + 22*a^5*b^2*c^5*d*e + 12*a^5*b^3*c^4*
d*f - 12*a^6*b^2*c^4*e*f)/a^9 + (((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4
*c^3*d + 8*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^
4*e)/a^9 + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^
3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2
*b^2*c*e))/(2*a^9*(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*
c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b
^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e
- 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e
))/(2*(16*a^5*c - 4*a^4*b^2)) + (((((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b
^4*c^3*d + 8*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*
c^4*e)/a^9 + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f -...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1677, normalized size of antiderivative = 6.87

$$\int \frac{d+ex^2+fx^4}{x^7 (a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x)
```

output

```
(12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c*f*x
**6 - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b
**2*f*x**6 - 18*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b)
)*a**2*b*c*e*x**6 - 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
- b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a**2*c**2*d*x**6 + 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*s
qrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a*b**3*e*x**6 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt
(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*s
qrt(c)*sqrt(a) + b))*a*b**2*c*d*x**6 - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(
2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*b**4*d*x**6 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sq
rt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)
/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c*f*x**6 - 6*sqrt(2*sqrt(c)*sqrt(a) + b
)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*f*x**6 - 18*sqrt(2*sqrt(c)*sqr
t(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)...
```

3.56 $\int \frac{x^4\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 369

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)-\frac{b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)+\frac{b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{7/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
(c^2*d+b^2*f-c*(a*f+b*e))*x/c^3+1/3*(-b*f+c*e)*x^3/c^2+1/5*f*x^5/c+1/2*(b^
2*c*e-a*c^2*e-b^3*f-b*c*(-2*a*f+c*d)-(b^3*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*
a*f+c*d)+2*a*c^2*(-a*f+c*d))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/
(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(7/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
+1/2*(b^2*c*e-a*c^2*e-b^3*f-b*c*(-2*a*f+c*d)+(b^3*c*e-3*a*b*c^2*e-b^4*f-b^
2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^
(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(7/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.24

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \frac{(c^2d+b^2f-c(be+af))\,x}{c^3}+\frac{(ce-bf)x^3}{3c^2}+\frac{fx^5}{5c}$$
$$-\frac{\left(-b^4f-b^2c\left(cd+\sqrt{b^2-4ac}e-4af\right)+ac^2\left(2cd+\sqrt{b^2-4ac}e-2af\right)+b^3\left(ce+\sqrt{b^2-4ac}f\right)+bc(\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(b^4f+b^2c\left(cd-\sqrt{b^2-4ac}e-4af\right)+ac^2\left(-2cd+\sqrt{b^2-4ac}e+2af\right)+b^3\left(-ce+\sqrt{b^2-4ac}f\right)+bc\right.}{\sqrt{2}c^{7/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x
^5)/(5*c) - ((-(b^4*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*e - 4*a*f) + a*c^2
*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f) + b^3*(c*e + Sqrt[b^2 - 4*a*c]*f) +
 b*c*(c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e - 2*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2
 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^4*f + b^2*c*(c*d - Sqrt[b^2 -
 4*a*c]*e - 4*a*f) + a*c^2*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b^3*(-
(c*e) + Sqrt[b^2 - 4*a*c]*f) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e - 2*a*
Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] (verified)

Time = 3.07 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2195}$$

$$\int \left(\frac{-c(af+be)+b^2f+c^2d}{c^3}-\frac{a(-c(af+be)+b^2f+c^2d)-x^2\left(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{c^3\left(a+bx^2+cx^4\right)}+\frac{x}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{\sqrt{2}c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(-c(af+be)+b^2f+c^2d\right)}{c^3}+\frac{x^3(ce-bf)}{3c^2}+\frac{fx^5}{5c}$$

input
```
Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x
^5)/(5*c) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f) - (b^3*c*e - 3
*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*
(c*d - 2*a*f) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a
*c^2*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.44

method	result
risch	$\frac{f x^5}{5c} - \frac{bf x^3}{3c^2} + \frac{e x^3}{3c} - \frac{xaf}{c^2} + \frac{x b^2 f}{c^3} - \frac{bex}{c^2} + \frac{xd}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(\left(2abcf-a c^2 e-b^3 f+b^2 ce-b c^2 d\right)\_R^2+ \right.}{2\_R^3 c-}}{2c^3}$
default	$-\frac{-\frac{1}{5} f x^5 c^2+\frac{1}{3} bcf x^3-\frac{1}{3} c^2 e x^3+xacf-x b^2 f+bcex-c^2 xd}{c^3} + \frac{\left(2abcf \sqrt{-4ac+b^2}-a c^2 e \sqrt{-4ac+b^2}-b^3 f \sqrt{-4ac+b^2}+b^2 ce \sqrt{-4ac+b^2}-b \right.}{2}$

input
```
int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*f*x^5/c-1/3/c^2*b*f*x^3+1/3*e*x^3/c-1/c^2*x*a*f+1/c^3*x*b^2*f-1/c^2*b*
e*x+1/c*x*d+1/2/c^3*sum(((2*a*b*c*f-a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)*_R^2+a^
2*c*f-a*b^2*f+a*b*c*e-a*c^2*d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_
Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15467 vs. 2(331) = 662.

Time = 35.92 (sec) , antiderivative size = 15467, normalized size of antiderivative = 41.92

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx=\int \frac{(fx^4+ex^2+d)x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/15*(3*c^2*f*x^5 + 5*(c^2*e - b*c*f)*x^3 + 15*(c^2*d - b*c*e + (b^2 - a*c
)*f)*x)/c^3 + integrate(-(a*c^2*d - a*b*c*e + (b*c^2*d - (b^2*c - a*c^2)*e
 + (b^3 - 2*a*b*c)*f)*x^2 + (a*b^2 - a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7235 vs. $2(331) = 662$.

Time = 0.95 (sec) , antiderivative size = 7235, normalized size of antiderivative = 19.61

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/8*((2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)
*c^2*d - (2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 9*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 24*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 10*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^3 + 10*(b^2 - 4*
a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*e + (2*b^7*c^2 - 20*a*b^5...
```

Mupad [B] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 23332, normalized size of antiderivative = 63.23

$$\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
x^3*(e/(3*c) - (b*f)/(3*c^2)) - x*((b*(e/c - (b*f)/c^2))/c - d/c + (a*f)/c
^2) + atan(((((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^
6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 - (2*x*(4*b^3*c^
7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c
- b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 -
2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) +
b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3
*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*
f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f
 + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2
*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 3
6*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a
^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^
2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1
/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 -
 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2379, normalized size of antiderivative = 6.45

$$\int \frac{x^4(d + ex^2 + fx^4)}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

output

```
( - 90*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2*f + 60*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**3*e + 30*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a)
+ b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*b**3*c*f - 30*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c**2*e
 + 30*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b
) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c**3*d - 60*sqrt(c)*sqrt(2
*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqr
t(2*sqrt(c)*sqrt(a) + b))*a**2*c**2*f + 120*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
 + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*a*b**2*c*f - 90*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2
*e + 60*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**3*d - 30*sqrt(c)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*b**4*f + 30*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*b**3*c*e - 30*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sq...
```

3.57 $\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 282

$$\begin{aligned}\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx &= \frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}\\&+\frac{\left(c^2d+b^2f-c(be+af)+\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\\&+\frac{\left(c^2d+b^2f-c(be+af)-\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
(-b*f+c*e)*x/c^2+1/3*f*x^3/c+1/2*(c^2*d+b^2*f-c*(a*f+b*e)+(b^2*c*e-2*a*c^2
*e-b^3*f-b*c*(-3*a*f+c*d))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b
-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1
/2*(c^2*d+b^2*f-c*(a*f+b*e)-(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d))/(-4
*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^
(1/2)/c^(5/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.29

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$= \frac{6\sqrt{c}(ce-bf)x+2c^{3/2}fx^3+\frac{3\sqrt{2}\left(-b^3f-bc\left(cd+\sqrt{b^2-4ac}e-3af\right)+b^2\left(ce+\sqrt{b^2-4ac}f\right)+c\left(c\sqrt{b^2-4ac}d-2ace-a\sqrt{b^2-4ac}f\right)\right)\text{arc}}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{6}$$

input
```
Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(6*Sqrt[c]*(c*e - b*f)*x + 2*c^(3/2)*f*x^3 + (3*Sqrt[2]*(-(b^3*f) - b*c*(c
*d + Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) + c*(c
*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + (3*Sqrt[2]*(b^3*f + b*c*(c*d - Sqrt[b^2 - 4*a*c]*e - 3*a*f
) + b^2*(-(c*e) + Sqrt[b^2 - 4*a*c]*f) + c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*
e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 2195$$
$$\int \left(-\frac{a(ce-bf)-x^2(-acf+b^2f-bce+c^2d)}{c^2(a+bx^2+cx^4)}+\frac{ce-bf}{c^2}+\frac{fx^2}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(ce-bf)}{c^2}+\frac{fx^3}{3c}$$

input
```
Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
((c*e - b*f)*x)/c^2 + (f*x^3)/(3*c) + ((c^2*d - b*c*e + b^2*f - a*c*f + (b
^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) + ((c^2*d - b*c*e + b^2*f - a*c*f - (b^2*c*e - 2*a*c
^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4
*a*c]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.35

method	result
risch	$\frac{f x^3}{3c} - \frac{bfx}{c^2} + \frac{ex}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(\left(-acf+b^2 f-ceb+d\,c^2\right)\_R^2+abf-ace\right)\ln\left(x-\_R\right)}{2\_R^3 c+\_R b}}{2c^2}$
default	$-\frac{-\frac{1}{3} f x^3 c+bfx-cex}{c^2} + \frac{\left(-acf\sqrt{-4ac+b^2}+b^2 f\sqrt{-4ac+b^2}-ceb\sqrt{-4ac+b^2}+d\,c^2\sqrt{-4ac+b^2}-3abcf+2a\,c^2 e+b^3 f-b^2 ce+b\,c^2 d\right)\sqrt{2}\arctan\left(\right.}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$

input
```
int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*f/c*x^3-1/c^2*b*f*x+e*x/c+1/2/c^2*sum(((-a*c*f+b^2*f-b*c*e+c^2*d)*_R^2
+a*b*f-a*c*e)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9364 vs. 2(246) = 492.

Time = 7.89 (sec) , antiderivative size = 9364, normalized size of antiderivative = 33.21

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(fx^4+ex^2+d)x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/3*(c*f*x^3 + 3*(c*e - b*f)*x)/c^2 - integrate((a*c*e - a*b*f - (c^2*d -
b*c*e + (b^2 - a*c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5454 vs. $2(246) = 492$.

Time = 0.78 (sec) , antiderivative size = 5454, normalized size of antiderivative = 19.34

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/8*((2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d - (
2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2*e +
(2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt...
```

Mupad [B] (verification not implemented)

Time = 20.59 (sec) , antiderivative size = 15674, normalized size of antiderivative = 55.58

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
x*(e/c - (b*f)/c^2) - atan(((((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*
f - 16*a^2*b*c^4*f)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c
^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a
^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3
*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*
c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*
a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*
(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 -
c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3
)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^
2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*
c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f
+ 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f +
 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1710, normalized size of antiderivative = 6.06

$$\int \frac{x^2(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

output
```
(12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*f - 6*sqrt(a)*sqrt(2*sq
rt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2
*sqrt(c)*sqrt(a) + b))*b**2*c*f + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b)
)*b*c**2*e - 12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*c**3*d - 18*sqrt(c
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*f + 12*sqrt(c)*sqrt(2*sqrt(c)*sqrt
(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*s
qrt(a) + b))*a*c**2*e + 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*f -
 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*e + 6*sqrt(c)*sqrt(2*sqr
t(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*
sqrt(c)*sqrt(a) + b))*b*c**2*d - 12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b)
)*a*c**2*f + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*f - 6*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqr...
```

3.58 $\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \frac{fx}{c} + \frac{\left(ce-bf+\frac{2c^2d+b^2f-c(be+2af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(ce-bf-\frac{2c^2d-bce+b^2f-2acf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
f*x/c+1/2*(c*e-b*f+(2*c^2*d+b^2*f-c*(2*a*f+b*e))/(-4*a*c+b^2)^(1/2))*arcta
n(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2)+1/2*(c*e-b*f-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)/(-4*a*c+b
^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/
c^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.18

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx$$
$$= \frac{2\sqrt{c}fx + \frac{\sqrt{2}\left(2c^2d+b\left(b-\sqrt{b^2-4ac}\right)f+c\left(-be+\sqrt{b^2-4ac}e-2af\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(2c^2d+b\left(b+\sqrt{b^2-4ac}\right)f-c\left(be+\sqrt{b^2-\right.\right.}{\sqrt{b^2-4ac}\sqrt{b+}}}{2c^{3/2}}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

output
```
(2*Sqrt[c]*f*x + (Sqrt[2]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*
e) + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[
2]*(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e +
 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 2205$$
$$\int \left(\frac{-af+x^2(ce-bf)+cd}{c\left(a+bx^2+cx^4\right)}+\frac{f}{c}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c}$$

input
```
Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

output
```
(f*x)/c + ((c*e - b*f + (2*c^2*d + b^2*f - c*(b*e + 2*a*f))/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(
3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*e - b*f - (2*c^2*d - b*c*e + b^2*f
 - 2*a*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.31

method	result
risch	$\frac{fx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left((-bf+ce)\_R^2-af+cd\right)\ln\left(x-\_R\right)}{2\_R^3 c+\_R b}}{2c}$
default	$\frac{fx}{c}+\frac{\left(-bf\sqrt{-4ac+b^2}+ce\sqrt{-4ac+b^2}+2acf-b^2f+ceb-2d\,c^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-bf\sqrt{-4ac+b^2}+ce\sqrt{-4ac\right.}{}$

input
```
int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
f*x/c+1/2/c*sum(((-b*f+c*e)*_R^2-a*f+c*d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=Root
Of(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5788 vs. 2(185) = 370.

Time = 3.74 (sec) , antiderivative size = 5788, normalized size of antiderivative = 26.43

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \int \frac{fx^4+ex^2+d}{cx^4+bx^2+a}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
f*x/c - integrate(-((c*e - b*f)*x^2 + c*d - a*f)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4082 vs. $2(185) = 370$.

Time = 0.62 (sec) , antiderivative size = 4082, normalized size of antiderivative = 18.64

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
f*x/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*
e - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*f + 2
*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c^4 - 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 +
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c +...
```

Mupad [B] (verification not implemented)

Time = 20.49 (sec) , antiderivative size = 10209, normalized size of antiderivative = 46.62

$$\int \frac{d + ex^2 + fx^4}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x)
```

output
```
(f*x)/c - atan(((((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f
)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(
-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4
*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^
2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2
+ 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*
a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f
- 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a
*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2
*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-
(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*
f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^
2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f -
8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*
f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 +
 a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*
e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*
f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*
c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e
^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1309, normalized size of antiderivative = 5.98

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*f - 4*sqrt(a)*sqrt(2*sqrt
(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*s
qrt(c)*sqrt(a) + b))*a*c**2*e + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
b*c**2*d + 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c*f - 2*sqrt(c)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*f + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a
) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*a*b*c*e - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*d -
2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +
2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*f + 4*sqrt(a)*sqrt(2*sqrt(
c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sq
rt(c)*sqrt(a) + b))*a*c**2*e - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b
*c**2*d - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c*f + 2*sqrt(c)*s
qrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c...
```

3.59 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 213

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx = -\frac{d}{ax} - \frac{\left(cd-af+\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-af-\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
-d/a/x-1/2*(c*d-a*f+(a*b*f-2*a*c*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/
2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/c^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2)-1/2*(c*d-a*f-(a*b*f-2*a*c*e+b*c*d)/(-4*a*c+b^2)^(1/2))*arc
tan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/c^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.19

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx = \frac{-\frac{2d}{x} - \frac{\sqrt{2}\left(bcd+c\sqrt{b^2-4ac}d-2ace+abf-a\sqrt{b^2-4ac}f\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(bcd-c\sqrt{b^2-4ac}d-2ace+abf+a\sqrt{b^2-4ac}f\right)\arctan}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*d)/x - (Sqrt[2]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f - a*
Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b*
c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \; 2195$$

$$\int \left(\frac{-(x^2(cd-af))+ae-bd}{a(a+bx^2+cx^4)}+\frac{d}{ax^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-(d/(a*x)) - ((c*d - a*f + (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((c*d - a*f - (b*c*d - 2*a*c*e + a*b*f)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*a*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03

method	result
default	$-\frac{d}{ax}+\frac{4c\left(\frac{\left(a\sqrt{-4ac+b^2}\,f-c\sqrt{-4ac+b^2}\,d+abf-2ace+bcd\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(a\sqrt{-4ac+b^2}\,f-c\sqrt{-4ac+b^2}\,d-abf+2ace \cdots\right.}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(\cdots\right.}}\right.}{a}$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-d/a/x+4/a*c*(1/8*(a*(-4*a*c+b^2)^(1/2)*f-c*(-4*a*c+b^2)^(1/2)*d+a*b*f-2*a
*c*e+b*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(a*(-4*a*c+b^2)^(
1/2)*f-c*(-4*a*c+b^2)^(1/2)*d-a*b*f+2*a*c*e-b*c*d)/c/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5930 vs. $2(177) = 354$.

Time = 1.45 (sec) , antiderivative size = 5930, normalized size of antiderivative = 27.84

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx=\int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(-((c*d - a*f)*x^2 + b*d - a*e)/(c*x^4 + b*x^2 + a), x)/a - d/(a*
x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3984 vs. $2(177) = 354$.

Time = 0.94 (sec) , antiderivative size = 3984, normalized size of antiderivative = 18.70

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-d/(a*x) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*a
^2*d - (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c
^3)*a^2*f + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^4*c^2 - 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2...
```

Mupad [B] (verification not implemented)

Time = 21.42 (sec) , antiderivative size = 10170, normalized size of antiderivative = 47.75

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x)
```

output
```
- atan(((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^
2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f)
 + (-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3
*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3
*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f
 + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2
) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)
*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-
(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*
a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 -
 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a
^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e -
 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^
3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2*d + 4*a^5*
b^2*c^2*e + 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c...
```

Reduce [F]

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{f\,x^4+e\,x^2+d}{x^2\,(c\,x^4+b\,x^2+a)}dx$$

input
```
int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x)
```

output
```
int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x)
```

3.60 $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 267

$$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx = -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\sqrt{c}\left(bd-ae+\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2d-b\left(\sqrt{b^2-4ac}d+ae\right)-a\left(2cd-\sqrt{b^2-4ac}e-2af\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
-1/3*d/a/x^3+(-a*e+b*d)/a^2/x+1/2*c^(1/2)*(b*d-a*e+(b^2*d-a*b*e-2*a*(-a*f+
c*d))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*2^(1/2)/a^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*c^(1/2)*(b^2*d-b*((-4*
a*c+b^2)^(1/2)*d+a*e)-a*(2*c*d-(-4*a*c+b^2)^(1/2)*e-2*a*f))*arctan(2^(1/2)
*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(1/2)/(b
+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.06

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx$$
$$=\frac{-\frac{2ad}{x^3}+\frac{6bd-6ae}{x}+\frac{3\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}d-ae\right)+a\left(-2cd-\sqrt{b^2-4ac}e+2af\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-b^2d+b\left(\sqrt{b^2-4a\right.\right.}{}}{6a^2}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a*d)/x^3 + (6*b*d - 6*a*e)/x + (3*Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^
2 - 4*a*c]*d - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]
*d + a*e) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/(6*a^2)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx$$
$$\Big\downarrow \ 2195$$
$$\int \left(\frac{cx^2(bd-ae)-abe-a(cd-af)+b^2d}{a^2\left(a+bx^2+cx^4\right)}+\frac{ae-bd}{a^2x^2}+\frac{d}{ax^4}\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac}-2af+2cd\right)-b\left(d\sqrt{b^2-4ac}+ae\right)+b^2d\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bd-ae}{a^2x}-\frac{d}{3ax^3}$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/3*d/(a*x^3) + (b*d - a*e)/(a^2*x) + (Sqrt[c]*(b*d - a*e + (b^2*d - a*b*
e - 2*a*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c
]*(b^2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e
- 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.91

method	result
default	$-\frac{d}{3a\,x^3}-\frac{ae-bd}{a^2x}+\frac{4c\left(\frac{\left(-ae\sqrt{-4ac+b^2}+bd\sqrt{-4ac+b^2}-2f\,a^2+abe+2dac-b^2d\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-ae\sqrt{-4ac+b^2}+b\right.}{}\right.}{a^2}$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/a/x^3-(a*e-b*d)/a^2/x+4/a^2*c*(1/8*(-a*e*(-4*a*c+b^2)^(1/2)+b*d*(-4
*a*c+b^2)^(1/2)-2*f*a^2+a*b*e+2*d*a*c-b^2*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))-1/8*(-a*e*(-4*a*c+b^2)^(1/2)+b*d*(-4*a*c+b^2)^(1/2)+2*f*a^2-a*b*
e-2*d*a*c+b^2*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9850 vs. 2(226) = 452.

Time = 10.68 (sec) , antiderivative size = 9850, normalized size of antiderivative = 36.89

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx = \int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)x^4}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((a*b*e - a^2*f - (b*c*d - a*c*e)*x^2 - (b^2 - a*c)*d)/(c*x^4 +
b*x^2 + a), x)/a^2 + 1/3*(3*(b*d - a*e)*x^2 - a*d)/(a^2*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3810 vs. 2(226) = 452.

Time = 0.58 (sec) , antiderivative size = 3810, normalized size of antiderivative = 14.27

$$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
5*c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 1
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a
^2*b^2*c^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
c^4 + 32*a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^4*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 +
 2*(b^2 - 4*a*c)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*
c^2 + 8*(b^2 - 4*a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*d - (sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^
5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*...
```

Mupad [B] (verification not implemented)

Time = 23.04 (sec) , antiderivative size = 15505, normalized size of antiderivative = 58.07

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x)
```

output
```
atan(((x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d
^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*
d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^
2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2
*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*
c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^
2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e
 - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) +
 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4
*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c
*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2
*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^
10*b^3*c^2)*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*
b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^
6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^
2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5
*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-...
```

Reduce [F]

$$\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx = \int \frac{f\,x^4+e\,x^2+d}{x^4(c\,x^4+b\,x^2+a)}dx$$

input
```
int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x)
```

output
```
int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x)
```

3.61 $\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 329

$$\int \frac{d+ex^2+fx^4}{x^6(a+bx^2+cx^4)}\,dx = -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x} - \frac{\sqrt{c}\left(b^2d-abe-a(cd-af)+\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2d-abe-a(cd-af)-\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^3\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/5*d/a/x^5+1/3*(-a*e+b*d)/a^2/x^3-(b^2*d-a*b*e-a*(-a*f+c*d))/a^3/x-1/2*c
^(1/2)*(b^2*d-a*b*e-a*(-a*f+c*d)+(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d)
)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*2^(1/2)/a^3/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*c^(1/2)*(b^2*d-a*b*e-a*(-a
*f+c*d)-(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d))/(-4*a*c+b^2)^(1/2))*arc
tan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/(b+(-4*a*c
+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.20

$$\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}\,dx$$
$$= \frac{-\frac{6a^2d}{x^5}+\frac{10a(bd-ae)}{x^3}+\frac{30(-b^2d+abe+a(cd-af))}{x}-\frac{15\sqrt{2}\sqrt{c}\left(b^3d+b^2\left(\sqrt{b^2-4ac}d-ae\right)+ab\left(-3cd-\sqrt{b^2-4ac}e+af\right)+a\left(-c\sqrt{b^2-4ac}d+\right.\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

output
```
((-6*a^2*d)/x^5 + (10*a*(b*d - a*e))/x^3 + (30*(-(b^2*d) + a*b*e + a*(c*d
- a*f)))/x - (15*Sqrt[2]*Sqrt[c]*(b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e)
+ a*b*(-3*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) +
 2*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (15*Sq
rt[2]*Sqrt[c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-3*c*d + Sqr
t[b^2 - 4*a*c]*e + a*f) + a*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*Sqrt[b^2
- 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(30*a^3)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}\,dx$$
$$\Big\downarrow \; 2195$$

$$\int \left(\frac{-abe-a(cd-af)+b^2d}{a^3x^2}+\frac{ae-bd}{a^2x^4}+\frac{-a^2ce-cx^2\left(-abe-a(cd-af)+b^2d\right)+ab^2e+ab(2cd-af)+b^3}{a^3\left(a+bx^2+cx^4\right)}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-abe-a(cd-af)+b^2d}{a^3x}+\frac{bd-ae}{3a^2x^3}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{5ax^5}$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/5*d/(a*x^5) + (b*d - a*e)/(3*a^2*x^3) - (b^2*d - a*b*e - a*(c*d - a*f))
/(a^3*x) - (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) + (b^3*d - a*b^2*e + 2*
a^2*c*e - a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) - (b^3*d - a*b^2*e + 2*a^2*c*e -
a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.09

method	result
default	$-\frac{d}{5a\,x^5}-\frac{ae-bd}{3a^2x^3}-\frac{f\,a^2-abe-dac+b^2d}{a^3x}+\frac{4c\left(\frac{\left(-f\,a^2\sqrt{-4ac+b^2}+abe\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,acd-\sqrt{-4ac+b^2}\,b^2d+a^2bf+2a^2ce-a\right.}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.}{}$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/5*d/a/x^5-1/3*(a*e-b*d)/a^2/x^3-(a^2*f-a*b*e-a*c*d+b^2*d)/a^3/x+4/a^3*c
*(1/8*(-f*a^2*(-4*a*c+b^2)^(1/2)+a*b*e*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/
2)*a*c*d-(-4*a*c+b^2)^(1/2)*b^2*d+a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*
d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-f*a^2*(-4*a*c+b^2)^(1/2)+a
*b*e*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*a*c*d-(-4*a*c+b^2)^(1/2)*b^2*d-
a^2*b*f-2*a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15830 vs. 2(289) = 578.

Time = 42.01 (sec) , antiderivative size = 15830, normalized size of antiderivative = 48.12

$$\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4}{x^6 (a + bx^2 + cx^4)} \, dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**6/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d + ex^2 + fx^4}{x^6 (a + bx^2 + cx^4)} \, dx = \int \frac{fx^4 + ex^2 + d}{(cx^4 + bx^2 + a)x^6} \, dx$$

input
```
integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((a^2*b*f - (a*b*c*e - a^2*c*f - (b^2*c - a*c^2)*d)*x^2 + (b^3 -
 2*a*b*c)*d - (a*b^2 - a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/a^3 + 1/15*(15*(a
*b*e - a^2*f - (b^2 - a*c)*d)*x^4 - 3*a^2*d + 5*(a*b*d - a^2*e)*x^2)/(a^3*
x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6710 vs. $2(289) = 578$.

Time = 1.13 (sec) , antiderivative size = 6710, normalized size of antiderivative = 20.40

$$\int \frac{d + ex^2 + fx^4}{x^6 (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/8*((2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^
2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*a^2*d - (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 3
2*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2
- 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*a^2*e + (2*a^2*b^4*c^2 ...
```

Mupad [B] (verification not implemented)

Time = 24.43 (sec) , antiderivative size = 23019, normalized size of antiderivative = 69.97

$$\int \frac{d + ex^2 + fx^4}{x^6 (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x)
```

output
```
atan(((x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3
*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*
a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e
 + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b
^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*
b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^
2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^
2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2
- 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*
(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2
)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2
*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(
-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c
^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5
*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^
3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^
3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2
) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*
b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2
) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c...
```

Reduce [F]

$$\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}\,dx=\int \frac{f\,x^4+e\,x^2+d}{x^6\left(c\,x^4+b\,x^2+a\right)}dx$$

input
```
int((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x)
```

output
```
int((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x)
```

3.62 $\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 323

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{(ce-2bf)x^2}{2c^3}+\frac{fx^4}{4c^2}$$
$$-\frac{a(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))+(b^4ce-4ab^2c^2e+2a^2c^3e-b^5f-b^3c(cd-}{2c^4(b^2-4ac)(a+bx^2+cx^4)}$$
$$-\frac{(2b^4ce-12ab^2c^2e+12a^2c^3e-3b^5f-b^3c(cd-20af)+6abc^2(cd-5af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4(b^2-4ac)^{3/2}}$$
$$+\frac{(c^2d+3b^2f-2c(be+af))\log(a+bx^2+cx^4)}{4c^4}$$

output
```
1/2*(-2*b*f+c*e)*x^2/c^3+1/4*f*x^4/c^2-1/2*(a*(b^3*c*e-3*a*b*c^2*e-b^4*f-b
^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d))+(b^4*c*e-4*a*b^2*c^2*e+2*a^2*c^3*e-b
^5*f-b^3*c*(-5*a*f+c*d)+a*b*c^2*(-5*a*f+3*c*d))*x^2)/c^4/(-4*a*c+b^2)/(c*x
^4+b*x^2+a)-1/2*(2*b^4*c*e-12*a*b^2*c^2*e+12*a^2*c^3*e-3*b^5*f-b^3*c*(-20*
a*f+c*d)+6*a*b*c^2*(-5*a*f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c
^4/(-4*a*c+b^2)^(3/2)+1/4*(c^2*d+3*b^2*f-2*c*(a*f+b*e))*ln(c*x^4+b*x^2+a)/
c^4
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.96

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{2c(ce-2bf)x^2+c^2fx^4+\frac{2\left(2a^3c^2f+b^3\left(c^2d-bce+b^2f\right)x^2+ab\left(b^3f-3c^3dx^2+bc^2\left(d+4ex^2\right)-b^2c\left(e+5fx^2\right)\right)+a^2c\left(-4b^2f-2c^2\left(d+ex^2\right.\right.\right.}{(b^2-4ac)(a+bx^2+cx^4)}}{}$$

input
```
Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*c*(c*e - 2*b*f)*x^2 + c^2*f*x^4 + (2*(2*a^3*c^2*f + b^3*(c^2*d - b*c*e
+ b^2*f)*x^2 + a*b*(b^3*f - 3*c^3*d*x^2 + b*c^2*(d + 4*e*x^2) - b^2*c*(e +
 5*f*x^2)) + a^2*c*(-4*b^2*f - 2*c^2*(d + e*x^2) + b*c*(3*e + 5*f*x^2))))/
((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(-2*b^4*c*e + 12*a*b^2*c^2*e - 12
*a^2*c^3*e + 3*b^5*f + b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(-(c*d) + 5*a*f))*
ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c^2*d +
3*b^2*f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2194, 2175, 27, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{x^6\left(fx^4+ex^2+d\right)}{\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 2175}$$

$$\frac{1}{2}\left(\frac{x^6\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{x^4\left(3c\left(2ae-\frac{b(cd+af)}{c}\right)-\left(3fb^2-2ceb+4c^2d-8acf\right)x^2\right)}{c(cx^4+bx^2+a)}}{b^2-4ac}\right.$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{2}\left(\frac{x^6\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int-\frac{x^4\left(\left(3fb^2-2ceb+4c^2d-8acf\right)x^2+3(bcd-2ace+abf)\right)}{cx^4+bx^2+a}}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \textbf{25}$$

$$\frac{1}{2}\left(\frac{\int\frac{x^4\left(\left(3fb^2-2ceb+4c^2d-8acf\right)x^2+3(bcd-2ace+abf)\right)}{cx^4+bx^2+a}dx^2}{c\left(b^2-4ac\right)}+\frac{x^6\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right.}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right.$$

$$\Big\downarrow \textbf{1200}$$

$$\frac{1}{2}\left(\frac{\int\left(-\frac{3fb^3}{c^2}+\frac{2eb^2}{c}-db+\frac{11afb}{c}+\frac{\left(3fb^2-2ceb+4c^2d-8acf\right)x^2}{c}-6ae+\frac{\left(b^2-4ac\right)\left(3fb^2+c^2d-2c(be+af)\right)x^2-a\left(-3fb^3+2ceb^2-\right.}{c^2\left(cx^4+bx^2+a\right)}\right.}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{1}{2}\left(\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^3e-b^3c(cd-20af)-12ab^2c^2e+6abc^2(cd-5af)-3b^5f+2b^4ce\right)}{c^3\sqrt{b^2-4ac}}+\frac{x^4\left(-8acf+3b^2f-2bce+4c^2d\right)}{2c}+\frac{\left(b^2-4ac\right.}{}}{c\left(b^2-4ac\right)}\right.$$

input
```
Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((x^6*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x
^2))/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((((2*b^2*c*e - 6*a*c^2*e - 3*
b^3*f - b*c*(c*d - 11*a*f))*x^2)/c^2 + ((4*c^2*d - 2*b*c*e + 3*b^2*f - 8*a
*c*f)*x^4)/(2*c) - ((2*b^4*c*e - 12*a*b^2*c^2*e + 12*a^2*c^3*e - 3*b^5*f -
 b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(c*d - 5*a*f))*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*(c^2*d + 3*b^2*f
 - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(2*c^3))/(c*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.34

method	result
default	$\frac{\left(-cf\,x^{2}+2bf-ce\right)^{2}}{4c^{4}f}+\frac{-\frac{\left(5a^{2}b\,c^{2}f-2e\,a^{2}c^{3}-5a\,b^{3}cf+4a\,b^{2}c^{2}e-3ab\,c^{3}d+b^{5}f-b^{4}ec+b^{3}c^{2}d\right)x^{2}}{c\left(4ac-b^{2}\right)}-\frac{a\left(2a^{2}c^{2}f-4a\,b^{2}cf+3ab\,c^{2}e-2a\,c^{3}d+b^{4}f\right.}{c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}$
risch	Expression too large to display

input
```
int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c*f*x^2+2*b*f-c*e)^2/c^4/f+1/2/c^3*((-(5*a^2*b*c^2*f-2*a^2*c^3*e-5*a
*b^3*c*f+4*a*b^2*c^2*e-3*a*b*c^3*d+b^5*f-b^4*c*e+b^3*c^2*d)/c/(4*a*c-b^2)*
x^2-a*(2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^2*e-2*a*c^3*d+b^4*f-b^3*c*e+b^2*c^2
*d)/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-8*a^2*c^2*f+14*a*b
^2*c*f-8*a*b*c^2*e+4*a*c^3*d-3*b^4*f+2*b^3*c*e-b^2*c^2*d)/c*ln(c*x^4+b*x^2
+a)+2*(11*a^2*b*c*f-6*a^2*c^2*e-3*a*b^3*f+2*a*b^2*c*e-a*b*c^2*d-1/2*(-8*a^
2*c^2*f+14*a*b^2*c*f-8*a*b*c^2*e+4*a*c^3*d-3*b^4*f+2*b^3*c*e-b^2*c^2*d)*b/
c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. $2(309) = 618$.

Time = 0.34 (sec) , antiderivative size = 2111, normalized size of antiderivative = 6.54

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*
(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^
2*b^2*c^3 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)
*d - (b^6*c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b
^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 - (((b^3*c^3 - 6*a*b*c^4)*d -
 2*(b^4*c^2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^
2*b*c^3)*f)*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*
a^2*b*c^3)*e + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 -
 6*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 2
0*a^2*b^3*c + 30*a^3*b*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) +
2*(a*b^4*c^2 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c - 7*a^2*b^3*c^2 +
 12*a^3*b*c^3)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*f
+ (((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 - 8*a*b^3*c^3 + 16
*a^2*b*c^4)*e + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*f)*
x^4 + ((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c - 8*a*b^4*c^2 +
 16*a^2*b^2*c^3)*e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*
f)*x^2 + (a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b
^3*c^2 + 16*a^3*b*c^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.28

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(b^3c^2d-6\,abc^3d-2\,b^4ce+12\,ab^2c^2e-12\,a^2c^3e+3\,b^5f-20\,ab^3cf+30\,a^2bc^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^4-4\,ac^5)\sqrt{-b^2+4\,ac}} - \frac{b^2c^3dx^4-4\,ac^4dx^4-2\,b^3c^2ex^4+8\,abc^3ex^4+3\,b^4cfx^4-14\,ab^2c^2fx^4+8\,a^2c^3fx^4-b^3c^2dx^2+2\,abc^3d}{4\,(b^2c^4-4\,ac^5)(c} + \frac{(c^2d-2\,bce+3\,b^2f-2\,acf)\log(cx^4+bx^2+a)}{4\,c^4} + \frac{c^2fx^4+2\,c^2ex^2-4\,bcfx^2}{4\,c^4}$$

input
```
integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^3*c^2*d - 6*a*b*c^3*d - 2*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2*c^3*e
+ 3*b^5*f - 20*a*b^3*c*f + 30*a^2*b*c^2*f)*arctan((2*c*x^2 + b)/sqrt(-b^2
+ 4*a*c))/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c^3*d*x^4 -
4*a*c^4*d*x^4 - 2*b^3*c^2*e*x^4 + 8*a*b*c^3*e*x^4 + 3*b^4*c*f*x^4 - 14*a*b
^2*c^2*f*x^4 + 8*a^2*c^3*f*x^4 - b^3*c^2*d*x^2 + 2*a*b*c^3*d*x^2 + 4*a^2*c
^3*e*x^2 + b^5*f*x^2 - 4*a*b^3*c*f*x^2 - 2*a^2*b*c^2*f*x^2 - a*b^2*c^2*d +
 2*a^2*b*c^2*e + a*b^4*f - 6*a^2*b^2*c*f + 4*a^3*c^2*f)/((b^2*c^4 - 4*a*c^
5)*(c*x^4 + b*x^2 + a)) + 1/4*(c^2*d - 2*b*c*e + 3*b^2*f - 2*a*c*f)*log(c*
x^4 + b*x^2 + a)/c^4 + 1/4*(c^2*f*x^4 + 2*c^2*e*x^2 - 4*b*c*f*x^2)/c^4
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 3499, normalized size of antiderivative = 10.83

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^2*(e/(2*c^2) - (b*f)/c^3) - ((2*a^3*c^2*f - 2*a^2*c^3*d + a*b^4*f - a*b^
3*c*e + a*b^2*c^2*d + 3*a^2*b*c^2*e - 4*a^2*b^2*c*f)/(2*c*(4*a*c - b^2)) +
 (x^2*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c
*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(2*c*(4*a*c - b^2)))/(a*c^3 + c^4*x^4
 + b*c^3*x^2) - (log(a + b*x^2 + c*x^4)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c
^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e +
336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48
*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^
5 - 192*a^2*b^2*c^6)) + (f*x^4)/(4*c^2) + (atan(((8*a*c^7*(4*a*c - b^2)^3
- 2*b^2*c^6*(4*a*c - b^2)^3)*(((((16*a^2*c^5*f - 8*a*c^6*d + 16*a*b*c^5*e
- 24*a*b^2*c^4*f)/c^6 - (8*a*c^2*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d +
256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2
*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*
c^2*e + 256*a^3*b*c^4*e))/(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^
2*b^2*c^6))*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d
- 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f))/(8*c^4*(4*a*c - b^2)^(3
/2)) - (a*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d -
20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f)*(6*b^8*f - 128*a^3*c^5*d +
 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*
c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5521, normalized size of antiderivative = 17.09

$$\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
( - 60*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*
*2*c**2*f + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a**3*b*c**3*e + 40*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a**2*b**4*c*f - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a
) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*a**2*b**3*c**2*e - 60*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c
)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqr
t(c)*sqrt(a) + b))*a**2*b**3*c**2*f*x**2 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*
sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**3*d + 24*sqrt(2*sqrt(c)*sqrt(
a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**3*e*x**2 - 60*sqrt(2*
sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**3*f*x**4
 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c
**4*e*x**4 - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*...
```

3.63 $\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 244

$$\begin{aligned}\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx &= \frac{fx^2}{2c^2}\\
&-\frac{ac\left(bcd-b^2e+2ace-3abf+\frac{b^3f}{c}\right)-(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))\,x^2}{2c^3\,(b^2-4ac)\,(a+bx^2+cx^4)}\\
&-\frac{(12a^2c^2f-b^3(ce-2bf)-2ac(2c^2d-3bce+6b^2f))\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2-4ac)^{3/2}}\\
&+\frac{(ce-2bf)\log\left(a+bx^2+cx^4\right)}{4c^3}\end{aligned}$$

output
```
1/2*f*x^2/c^2-1/2*(a*c*(b*c*d-b^2*e+2*a*c*e-3*a*b*f+b^3*f/c)-(b^3*c*e-3*a*
b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d))*x^2)/c^3/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)-1/2*(12*a^2*c^2*f-b^3*(-2*b*f+c*e)-2*a*c*(6*b^2*f-3*b*c*e
+2*c^2*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)+
1/4*(-2*b*f+c*e)*ln(c*x^4+b*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.97

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{2cfx^2 - \frac{2(b^2(c^2d-bce+b^2f)x^2+a^2c(-3bf+2c(e+fx^2))+a(b^3f-2c^3dx^2+bc^2(d+3ex^2)-b^2c(e+4fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{2(12a^2c^2f+b^3(-ce+2bf)-2}{}}{4c^3}$$

input
```
Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*c*f*x^2 - (2*(b^2*(c^2*d - b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e
+ f*x^2)) + a*(b^3*f - 2*c^3*d*x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*
x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(12*a^2*c^2*f + b^3*(-(c*
e) + 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTan[(b + 2*c*x^2)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c*e - 2*b*f)*Log[a + b*x^2 + c*
x^4])/(4*c^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2194, 2175, 27, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 2194$$

$$\frac{1}{2}\int \frac{x^4(fx^4+ex^2+d)}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 2175$$

$$\frac{1}{2}\left(\frac{x^4\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{x^2\left(2c\left(2ae-\frac{b(cd+af)}{c}\right)-\left(2fb^2-ceb+2c^2d-6acf\right)x^2\right)}{c(cx^4+bx^2+a)}d}{b^2-4ac}\right.$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\left(\frac{x^4\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int-\frac{x^2\left(\left(2fb^2-ceb+2c^2d-6acf\right)x^2+2(bcd-2ace+abf)\right)}{cx^4+bx^2+a}d}{c\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow 25$$

$$\frac{1}{2}\left(\frac{\int\frac{x^2\left(\left(2fb^2-ceb+2c^2d-6acf\right)x^2+2(bcd-2ace+abf)\right)}{cx^4+bx^2+a}dx^2}{c\left(b^2-4ac\right)}+\frac{x^4\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right.$$

$$\Big\downarrow 1200$$

$$\frac{1}{2}\left(\frac{\int\left(\frac{2fb^2}{c}-eb+2cd-6af-\frac{a\left(2fb^2-ceb+2c^2d-6acf\right)-\left(b^2-4ac\right)(ce-2bf)x^2}{c(cx^4+bx^2+a)}\right)dx^2}{c\left(b^2-4ac\right)}+\frac{x^4\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-\right.\right.}{c\left(b^2-4ac\right)\left(a+\right.}\right.$$

$$\Big\downarrow 2009$$

$$\frac{1}{2}\left(\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^2f-2ac\left(6b^2f-3bce+2c^2d\right)-\left(b^3(ce-2bf)\right)\right)}{c^2\sqrt{b^2-4ac}}+\frac{\left(b^2-4ac\right)(ce-2bf)\log\left(a+bx^2+cx^4\right)}{2c^2}+x^2\left(-6af+\frac{2b}{\ }\right.}{c\left(b^2-4ac\right)}\right.$$

input
```
Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((x^4*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x
^2))/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((2*c*d - b*e - 6*a*f + (2*b^
2*f)/c)*x^2 - ((12*a^2*c^2*f - b^3*(c*e - 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*
e + 6*b^2*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*
a*c]) + ((b^2 - 4*a*c)*(c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(2*c^2))/(c*(
b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.27

method	result
default	$\frac{f\,x^2}{2c^2}-\frac{\frac{-\frac{\left(2a^2c^2f-4a\,b^2cf+3ab\,c^2e-2a\,c^3d+b^4f-b^3ce+b^2c^2d\right)x^2}{c\left(4ac-b^2\right)}+\frac{a\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)}{c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(8abcf-4a\,c^2e-2b^3f+b^2ce\right)\ln(}{2c}}{}}{2c^2}$
risch	Expression too large to display

input
```
int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*f*x^2/c^2-1/2/c^2*((-(2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^2*e-2*a*c^3*d+b^
4*f-b^3*c*e+b^2*c^2*d)/c/(4*a*c-b^2)*x^2+a*(3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*
c*e-b*c^2*d)/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(8*a*b*c*f-
4*a*c^2*e-2*b^3*f+b^2*c*e)/c*ln(c*x^4+b*x^2+a)+2*(6*a^2*c*f-2*a*b^2*f+a*b*
c*e-2*a*c^2*d-1/2*(8*a*b*c*f-4*a*c^2*e-2*b^3*f+b^2*c*e)*b/c)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. 2(232) = 464.

Time = 0.18 (sec) , antiderivative size = 1455, normalized size of antiderivative = 5.96

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d - (b^5*
c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24
*a^3*c^3)*f)*x^2 + (4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2
*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d + (b^4*c - 6*a*b^
2*c^2)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^3*c - 6*a^2*b*c
^2)*e - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4
+ b*x^2 + a)) - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2
 + 8*a^3*c^3)*e - 2*(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*e - 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^
4 + ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*
b^2*c^2)*f)*x^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^
2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 -
 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2), 1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*f*x^6 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 -
6*a*b^2*c^3 + 8*a^2*c^4)*d - (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6
 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*f)*x^2 + 2*(4*a^2*c^3*d + (4*a
*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.11

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{fx^2}{2\,c^2} - \frac{(4\,ac^3d+b^3ce-6\,abc^2e-2\,b^4f+12\,ab^2cf-12\,a^2c^2f)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}}$$
$$- \frac{b^2cex^4-4\,ac^2ex^4-2\,b^3fx^4+8\,abcfx^4+2\,b^2cdx^2-4\,ac^2dx^2-b^3ex^2+2\,abcex^2+4\,a^2cfx^2+2\,abcd}{4\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)}$$
$$+ \frac{(ce-2\,bf)\log\left(cx^4+bx^2+a\right)}{4\,c^3}$$

input
```
integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*f*x^2/c^2 - 1/2*(4*a*c^3*d + b^3*c*e - 6*a*b*c^2*e - 2*b^4*f + 12*a*b^
2*c*f - 12*a^2*c^2*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 -
 4*a*c^4)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*e*x^4 - 4*a*c^2*e*x^4 - 2*b^3*f
*x^4 + 8*a*b*c*f*x^4 + 2*b^2*c*d*x^2 - 4*a*c^2*d*x^2 - b^3*e*x^2 + 2*a*b*c
*e*x^2 + 4*a^2*c*f*x^2 + 2*a*b*c*d - a*b^2*e + 2*a^2*b*f)/((c*x^4 + b*x^2
+ a)*(b^2*c^2 - 4*a*c^3)) + 1/4*(c*e - 2*b*f)*log(c*x^4 + b*x^2 + a)/c^3
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 2450, normalized size of antiderivative = 10.04

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input

```
int((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output

```
((a*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c*(4*a*c - b^2
)) + (x^2*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c
^2*e - 4*a*b^2*c*f))/(2*c*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) +
(f*x^2)/(2*c^2) + (log(a + b*x^2 + c*x^4)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6
*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*
e - 256*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2
*b^2*c^5)) - (atan(((8*a*c^5*(4*a*c - b^2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*
(x^2*(((((24*a^2*c^5*f - 6*b^3*c^4*e + 12*b^4*c^3*f - 8*a*c^6*d + 28*a*b*c
^5*e - 56*a*b^2*c^4*f)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 - 32*a*b*c^7)*(4*
b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f -
 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(4*a*c^5 - b^2*c^4)*
(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(2*b^4*f + 12
*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f))/(8*c^3*(4*
a*c - b^2)^(3/2)) + ((8*b^3*c^6 - 32*a*b*c^7)*(2*b^4*f + 12*a^2*c^2*f - 4*
a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)*(4*b^7*f + 128*a^3*c^4*e -
 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^
4*c^2*e - 256*a^3*b*c^3*f))/(16*c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4
)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c -
 b^2)) + (b*((4*b^5*f^2 + b^3*c^2*e^2 + 12*a^2*b*c^2*f^2 + 2*a*c^4*d*e - 4
*b^4*c*e*f - 5*a*b*c^3*e^2 - 20*a*b^3*c*f^2 - 6*a^2*c^3*e*f + 20*a*b^2*...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3787, normalized size of antiderivative = 15.52

$$\int \frac{x^5(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input

```
int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output

```
(24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**
2*f - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*
b**3*c*f + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**2*b**2*c**2*e + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) -
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*a**2*b**2*c**2*f*x**2 - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(
c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sq
rt(c)*sqrt(a) + b))*a**2*b*c**3*d + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*
sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt
(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**3*f*x**4 + 4*sqrt(2*sqrt(c)*sqrt(a) + b
)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**5*f - 2*sqrt(2*sqrt(c)*sqrt(a) + b)
*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*c*e - 24*sqrt(2*sqrt(c)*sqrt(a) +
b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(
c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*c*f*x**2 + 12*sqrt(2*sqrt(c)*sqr
t(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)...
```

3.64 $\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 182

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{a(2c^2d+b^2f-c(be+2af))-(b^2ce-2ac^2e-b^3f-bc(cd-3af))\,x^2}{2c^2(b^2-4ac)(a+bx^2+cx^4)} + \frac{(4ac^2e+b^3f-2bc(cd+3af))\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}} + \frac{f\log(a+bx^2+cx^4)}{4c^2}$$

output
```
1/2*(a*(2*c^2*d+b^2*f-c*(2*a*f+b*e))-(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+
c*d))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(4*a*c^2*e+b^3*f-2*b*c*(3*
a*f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1
/4*f*ln(c*x^4+b*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{\frac{2(-2a^2cf+b(c^2d-bce+b^2f)x^2+a(b^2f+2c^2(d+ex^2)-bc(e+3fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{2(4ac^2e+b^3f-2bc(cd+3af))\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+f\log(a+}{4c^2}$$

input
```
Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 + a*(b^2*f + 2*c^2*(d + e*
x^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(4*a*
c^2*e + b^3*f - 2*b*c*(c*d + 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*
c]])/(-b^2 + 4*a*c)^(3/2) + f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2194, 2175, 27, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$
$$\Big\downarrow \text{2194}$$
$$\frac{1}{2}\int \frac{x^2(fx^4+ex^2+d)}{(cx^4+bx^2+a)^2}dx^2$$
$$\Big\downarrow \text{2175}$$

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{c\left(2ae-\frac{b(cd+af)}{c}\right)-\left(b^2-4ac\right)fx^2}{c\left(cx^4+bx^2+a\right)}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-\left(\left(b^2-4ac\right)fx^2\right)+2ace-b(cd+af)}{cx^4+bx^2+a}dx^2}{c\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{\left(-2bc(3af+cd)+4ac^2e+b^3f\right)\int \frac{1}{cx^4+bx^2+a}dx^2}{2c}-\frac{f\left(b^2-\cdots\right.}{\cdots}}{c\left(b^2-4ac\right)}\cdots\right.$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{f\left(b^2-4ac\right)\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{\left(-2bc(3af+cd)+4ac^2e\cdots\right.}{\cdots}}{c\left(b^2-4ac\right)}\cdots\right.$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{f\left(b^2-4ac\right)\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\cdots}{c\cdots}}{c\left(b^2-4ac\right)}\cdots\right.$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2}\left(\frac{x^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2bc(3af+cd)+4ac^2e+b^3f\right)}{c\sqrt{b^2-4ac}}-\cdots}{c\left(b^2-4ac\right)}\cdots\right.$$

input
```
Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((x^2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x
^2))/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (-(((4*a*c^2*e + b^3*f - 2*b*
c*(c*d + 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4
*a*c])) - ((b^2 - 4*a*c)*f*Log[a + b*x^2 + c*x^4])/(2*c))/(c*(b^2 - 4*a*c)
))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.25

method	result
default	$\frac{\frac{\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x^2}{c^2\left(4ac-b^2\right)}+\frac{a\left(2acf-b^2f+ceb-2d\,c^2\right)}{\left(4ac-b^2\right)c^2}}{2c\,x^4+2b\,x^2+2a}+\frac{\frac{\left(4acf-b^2f\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-abf+2ace-bcd-\frac{\left(4acf-b^2f\right)b}{2c}\right)\mathrm{ar}}{\sqrt{4ac-b^2}}}{2c(4ac-b^2)}$
risch	Expression too large to display

input
```
int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*((3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/c^2/(4*a*c-b^2)*x^2+a*(2*
a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+a)+1/2/c/(4*a*c-b
^2)*(1/2*(4*a*c*f-b^2*f)/c*ln(c*x^4+b*x^2+a)+2*(-a*b*f+2*a*c*e-b*c*d-1/2*(
4*a*c*f-b^2*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(172) = 344$.

Time = 0.12 (sec) , antiderivative size = 970, normalized size of antiderivative = 5.33

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e + (
b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - (2*a*b*c^2*d - 4*a^2*c^2*e + (2*b
*c^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^2*c^2*d - 4*a*b*c^2
*e - (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))
/(c*x^4 + b*x^2 + a)) + 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b
*c^2)*e + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*f*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^
3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a
*b^3*c^3 + 16*a^2*b*c^4)*x^2), 1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c -
6*a*b^2*c^2 + 8*a^2*c^3)*e + (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - 2*(
2*a*b*c^2*d - 4*a^2*c^2*e + (2*b*c^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f
)*x^4 + (2*b^2*c^2*d - 4*a*b*c^2*e - (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6
*a^2*b*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)) + 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*e
+ 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*f*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.05

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{(2\,bc^2d-4\,ac^2e-b^3f+6\,abcf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{f\log\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{2\,ac^2d-abce+ab^2f-2\,a^2cf+(bc^2d-b^2ce+2\,ac^2e+b^3f-3\,abcf)x^2}{2\,(cx^4+bx^2+a)(b^2-4\,ac)c^2}$$

input
```
integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(2*b*c^2*d - 4*a*c^2*e - b^3*f + 6*a*b*c*f)*arctan((2*c*x^2 + b)/sqrt(
-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*f*log(c*x^4
+ b*x^2 + a)/c^2 + 1/2*(2*a*c^2*d - a*b*c*e + a*b^2*f - 2*a^2*c*f + (b*c^2
*d - b^2*c*e + 2*a*c^2*e + b^3*f - 3*a*b*c*f)*x^2)/((c*x^4 + b*x^2 + a)*(b
^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 21.09 (sec) , antiderivative size = 1651, normalized size of antiderivative = 9.07

$$\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
- ((a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c^2*(4*a*c - b^2)) + (x^2*(b
^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c^2*(4*a*c - b^2)))/
(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6*f - 128*a^3*c^3*f + 9
6*a^2*b^2*c^2*f - 24*a*b^4*c*f))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^
3 - 192*a^2*b^2*c^4)) - (atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c
 - b^2)^3)*((((8*a*f + (8*a*c^2*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*
f - 24*a*b^4*c*f))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c
^4))*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(8*c^2*(4*a*c - b^2)^(3/
2)) + (a*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f)*(b^3*
f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/((4*a*c - b^2)^(3/2)*(256*a^3*c^5
- 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - x^2*((
(((6*b^3*c^2*f + 8*a*c^4*e - 4*b*c^4*d - 28*a*b*c^3*f)/(4*a*c^3 - b^2*c^2)
 + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f -
 24*a*b^4*c*f))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4
*c^3 - 192*a^2*b^2*c^4)))*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(8*
c^2*(4*a*c - b^2)^(3/2)) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6*f - 128*a^3*c^
3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f)*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*
a*b*c*f))/(16*c^2*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4
*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) + (b*((b^3*
f^2 - 5*a*b*c*f^2 + 2*a*c^2*e*f - b*c^2*d*f)/(4*a*c^3 - b^2*c^2) + (((6...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2320, normalized size of antiderivative = 12.75

$$\int \frac{x^3(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input

```
int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output

```
(12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*
c*f - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b
*c**2*e - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
b**4*f + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
b**3*c*f*x**2 + 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*
atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
b))*a*b**2*c**2*d - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) -
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*a*b**2*c**2*e*x**2 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)
*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*a*b**2*c**2*f*x**4 - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(
2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*a*b*c**3*e*x**4 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)
*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**5*f*x**2 - 2*sqrt(2*sqrt(c)*sqrt(a) +
b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sq...
```

3.65 $\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 123

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(2cd-be+2af)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2
)/(c*x^4+b*x^2+a)+(2*a*f-b*e+2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{abf+2c^2dx^2+b^2fx^2+bc(d-ex^2)-2ac(e+fx^2)}{2c\,(-b^2+4ac)\,(a+bx^2+cx^4)} - \frac{(-2cd+be-2af)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(e + f*x^2))/(2
*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*c*d + b*e - 2*a*f)*ArcTan[(b
 + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2194, 2191, 27, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2(-2acf+b^2f-bce+2c^2d)}{c(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{2cd-be+2af}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2(-2acf+b^2f-bce+2c^2d)}{c(b^2-4ac)(a+bx^2+cx^4)}-\frac{(2af-be+2cd)\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{2(2af-be+2cd)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}+\frac{c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2(-2acf+b^2f-bce+2c^2d)}{c(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\downarrow 219$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af-be+2cd)}{(b^2-4ac)^{3/2}}+\frac{c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x^2(-2acf+b^2f-bce+2c^2d)}{c(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2)/(
c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(2*c*d - b*e + 2*a*f)*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.13

method	result
default	$\frac{-\frac{\left(2acf-b^2f+ceb-2d\,c^2\right)x^2}{c\left(4ac-b^2\right)}+\frac{abf-2ace+bcd}{c\left(4ac-b^2\right)}}{2c\,x^4+2b\,x^2+2a}+\frac{(2af-be+2cd)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2acf-b^2f+ceb-2d\,c^2\right)x^2}{2c\left(4ac-b^2\right)}+\frac{abf-2ace+bcd}{2c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^2+8c\,a^2-2b^2a\right)af}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-\right.\right.}{2(-4ac+}$

input
```
int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c/(4*a*c-b^2)*x^2+1/c*(a*b*f-2*a*c*e+b
*c*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+(2*a*f-b*e+2*c*d)/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(117) = 234.

Time = 0.08 (sec) , antiderivative size = 650, normalized size of antiderivative = 5.28

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \left[-\frac{(2\,(b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e+(b^4-6\,ab^2c+8\,a^2c^2)f)x^2+((2\,c^3d-bc^2e+2\,ac^2f)x^4+2\,ac^2}{2\,(ab^4c-8\,a^2b^2c^2}\right.$$

$$-\frac{(2\,(b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e+(b^4-6\,ab^2c+8\,a^2c^2)f)x^2-2\,((2\,c^3d-bc^2e+2\,ac^2f)x^4+2\,ac}{2\,(ab^4c-8\,a^2b^2c^2+16}$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c
 + 8*a^2*c^2)*f)*x^2 + ((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d -
a*b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a)) + (b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c
^2)*e + (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*x^2), -1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6
*a*b^2*c + 8*a^2*c^2)*f)*x^2 - 2*((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*
a*c^2*d - a*b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqr
t(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
(b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e + (a*b^3 - 4*a^2*b*c)*f)
/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(112) = 224$.

Time = 18.15 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.85

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx =$$

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2af-be+2cd)\log\left(x^2+\frac{-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\cdot(2af-be+2cd)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\cdot(2af-be+2cd)+2a}{4acf-2bce+4c^2d}\right.}{2}$$

$$+\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2af-be+2cd)\log\left(x^2+\frac{16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\cdot(2af-be+2cd)-8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\cdot(2af-be+2cd)+2ab}{4acf-2bce+4c^2d}\right.}{2}$$

$$+\frac{abf-2ace+bcd+x^2(-2acf+b^2f-bce+2c^2d)}{8a^2c^2-2ab^2c+x^4\cdot(8ac^3-2b^2c^2)+x^2\cdot(8abc^2-2b^3c)}$$

input
```
integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d)*log(x**2 + (-16*a**2*c**
2*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 8*a*b**2*c*sqrt(-1/(4
*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 2*a*b*f - b**4*sqrt(-1/(4*a*c - b
**2)**3)*(2*a*f - b*e + 2*c*d) - b**2*e + 2*b*c*d)/(4*a*c*f - 2*b*c*e + 4*
c**2*d))/2 + sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d)*log(x**2 + (
16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) - 8*a*b**2*c
*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 2*a*b*f + b**4*sqrt(-1
/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) - b**2*e + 2*b*c*d)/(4*a*c*f - 2
*b*c*e + 4*c**2*d))/2 + (a*b*f - 2*a*c*e + b*c*d + x**2*(-2*a*c*f + b**2*f
 - b*c*e + 2*c**2*d))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*
c**2) + x**2*(8*a*b*c**2 - 2*b**3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(2\,cd-be+2\,af)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,c^2dx^2-bcex^2+b^2fx^2-2\,acfx^2+bcd-2\,ace+abf}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

input
```
integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-(2*c*d - b*e + 2*a*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*
a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c^2*d*x^2 - b*c*e*x^2 + b^2*f*x^2 - 2*a*
c*f*x^2 + b*c*d - 2*a*c*e + a*b*f)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.78

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{a\,b\,f-2\,a\,c\,e+b\,c\,d}{2\,c\,(4\,a\,c-b^2)}+\frac{x^2\,(f\,b^2-e\,b\,c+2\,d\,c^2-2\,a\,f\,c)}{2\,c\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a} + \frac{\mathrm{atan}\left(\frac{(4\,a\,c-b^2)^4\,\left(x^2\,\left(\frac{\left(2\,c^3\,d+2\,a\,c^2\,f-b\,c^2\,e\right)\,(2\,a\,f-b\,e+2\,c\,d)}{a\,\left(4\,a\,c-b^2\right)^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)\,(2\,a\,f-b\,e+2\,c\,d)^2}{2\,a\,\left(4\,a\,c-b^2\right)^{13/2}}\right)-\frac{2\,c^2\,\left(b^3-4\,a\,b\,c\right)\,(2\,a\,f-b\,e}{\left(4\,a\,c-b^2\right)^{11/2}}\right)}{8\,a^2\,c^2\,f^2-8\,a\,b\,c^2\,e\,f+16\,a\,c^3\,d\,f+2\,b^2\,c^2\,e^2-8\,b\,c^3\,d\,e+8\,c^4\,d^2}\right)}{(4\,a\,c-b^2)^{3/2}}$$

input
```
int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((a*b*f - 2*a*c*e + b*c*d)/(2*c*(4*a*c - b^2)) + (x^2*(2*c^2*d + b^2*f - 2
*a*c*f - b*c*e))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (atan(((4*a*c
- b^2)^4*(x^2*(((2*c^3*d + 2*a*c^2*f - b*c^2*e)*(2*a*f - b*e + 2*c*d))/(a*
(4*a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c)*(2*a*f - b
*e + 2*c*d)^2)/(2*a*(4*a*c - b^2)^(13/2))) - (2*c^2*(b^3 - 4*a*b*c)*(2*a*f
 - b*e + 2*c*d)^2)/(4*a*c - b^2)^(11/2)))/(8*c^4*d^2 + 8*a^2*c^2*f^2 + 2*b
^2*c^2*e^2 + 16*a*c^3*d*f - 8*b*c^3*d*e - 8*a*b*c^2*e*f))*(2*a*f - b*e + 2
*c*d))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1274, normalized size of antiderivative = 10.36

$$\int \frac{x(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
( - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*f
 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*e
- 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*f*x
**2 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*
d - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*f*
x**4 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*
e*x**2 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((s
qrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**
2*c*d*x**2 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*b**2*c*e*x**4 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*b*c**2*d*x**4 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
- b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq...
```

3.66 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 166

$$\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)^2} \, dx = \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{(b^3d+4a^2ce-2ab(3cd+af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}} + \frac{d\log(x)}{a^2} - \frac{d\log(a+bx^2+cx^4)}{4a^2}$$

output
```
1/2*(b^2*d-a*b*e-2*a*(-a*f+c*d)+(a*b*f-2*a*c*e+b*c*d)*x^2)/a/(-4*a*c+b^2)/
(c*x^4+b*x^2+a)+1/2*(b^3*d+4*a^2*c*e-2*a*b*(a*f+3*c*d))*arctanh((2*c*x^2+b
)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+d*ln(x)/a^2-1/4*d*ln(c*x^4+b*
x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.61

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx =$$

$$-\frac{-\frac{2a\left(b^2d+b\left(-ae+cdx^2+afx^2\right)+2a\left(af-c\left(d+ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-4d\log(x)+\frac{\left(b^3d+b^2\sqrt{b^2-4ac}d+4ac\left(-\sqrt{b^2-4ac}d+ae\right)-2ab(3cd+af)\right)\log}{\left(b^2-4ac\right)^{3/2}}}{4a^2}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/4*((-2*a*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*
x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - 4*d*Log[x] + ((b^3*d + b^2*S
qrt[b^2 - 4*a*c]*d + 4*a*c*(-(Sqrt[b^2 - 4*a*c]*d) + a*e) - 2*a*b*(3*c*d +
 a*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-(b^3
*d) + b^2*Sqrt[b^2 - 4*a*c]*d - 4*a*c*(Sqrt[b^2 - 4*a*c]*d + a*e) + 2*a*b*
(3*c*d + a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/
a^2
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2194, 2177, 25, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \ 2194$$

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^2\left(cx^4+bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \ 2177$$

$$\frac{1}{2}\left(\frac{a\left(\frac{b^2d}{a}+2af-be-2cd\right)+x^2(abf-2ace+bcd)}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(bcd-2ace+abf)x^2+a\left(\frac{b^2}{a}-4c\right)d}{ax^2(cx^4+bx^2+a)}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int \frac{(bcd-2ace+abf)x^2+(b^2-4ac)d}{ax^2(cx^4+bx^2+a)}dx^2}{b^2-4ac}+\frac{a\left(\frac{b^2d}{a}+2af-be-2cd\right)+x^2(abf-2ace+bcd)}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\int \frac{(bcd-2ace+abf)x^2+(b^2-4ac)d}{x^2(cx^4+bx^2+a)}dx^2}{a\left(b^2-4ac\right)}+\frac{a\left(\frac{b^2d}{a}+2af-be-2cd\right)+x^2(abf-2ace+bcd)}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{-db^3+a(5cd+af)b-c(b^2-4ac)dx^2-2a^2ce}{a(cx^4+bx^2+a)}-\frac{(4ac-b^2)d}{ax^2}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{a\left(\frac{b^2d}{a}+2af-be-2cd\right)+x^2(abf-2ace+bcd)}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(4a^2ce-2ab(af+3cd)+b^3d)}{a\sqrt{b^2-4ac}}+\frac{d\log\left(x^2\right)(b^2-4ac)}{a}-\frac{d(b^2-4ac)\log\left(a+bx^2+cx^4\right)}{2a}}{a\left(b^2-4ac\right)}+\frac{a\left(\frac{b^2d}{a}+2af-be-2\right.}{a\left(b^2-4ac\right.}\right.$$

input
```
Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((a*((b^2*d)/a - 2*c*d - b*e + 2*a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2)/(a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (((b^3*d + 4*a^2*c*e - 2*a*b*(3*c*d +
 a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + (
(b^2 - 4*a*c)*d*Log[x^2])/a - ((b^2 - 4*a*c)*d*Log[a + b*x^2 + c*x^4])/(2*
a))/(a*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37

method	result
default	$\frac{\frac{-\frac{a(abf-2ace+bcd)x^2}{4ac-b^2}-\frac{a\left(2f\,a^2-abe-2dac+b^2d\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(-4a\,c^2d+b^2cd\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-a^2bf+2a^2ce-5abcd+b^3d-\frac{\left(-4a\,c^2d+b^2cd\right)b}{2c}\right)\cdots}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}\cdots$
risch	$\frac{-\frac{(abf-2ace+bcd)x^2}{2a\left(4ac-b^2\right)}-\frac{2f\,a^2-abe-2dac+b^2d}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{d\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3-48b^2c^2a^4+12b^4c\,a^3-b^6a^2\right)\_Z^2+\left(64a^3c^3d-48a^2b^2\cdots\right.\right.}\cdots\right.}{\cdots}$

input
```
int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*((-a*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)*x^2-a*(2*a^2*f-a*b*e-2*a*c*
d+b^2*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-4*a*c^2*d+b^2*c
*d)/c*ln(c*x^4+b*x^2+a)+2*(-a^2*b*f+2*a^2*c*e-5*a*b*c*d+b^3*d-1/2*(-4*a*c^
2*d+b^2*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
)+d*ln(x)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(156) = 312.

Time = 0.78 (sec) , antiderivative size = 1103, normalized size of antiderivative = 6.64

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^
3 - 4*a^3*b*c)*f)*x^2 + (4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*
f + (b^3*c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b
^2*c)*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2
*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 +
 a)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*b*c)*e +
 4*(a^3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d
)*log(c*x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (
b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
*d)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c
^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2
*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3 - 4*a
^3*b*c)*f)*x^2 + 2*(4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f + (
b^3*c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*c)
*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d
- 2*(a^2*b^3 - 4*a^3*b*c)*e + 4*(a^3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4
- 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(c*x^4 + b*x^2 + a) + 4*((b^4*c - 8*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4}{x\left(a + bx^2 + cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.35

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{\left(b^3d-6\,abcd+4\,a^2ce-2\,a^2bf\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}} - \frac{d\log\left(cx^4+bx^2+a\right)}{4\,a^2} + \frac{d\log\left(x^2\right)}{2\,a^2} + \frac{b^2cdx^4-4\,ac^2dx^4+b^3dx^2-2\,abcdx^2-4\,a^2cex^2+2\,a^2bfx^2+3\,ab^2d-8\,a^2cd-2\,a^2be+4\,a^3f}{4\,\left(cx^4+bx^2+a\right)\left(a^2b^2-4\,a^3c\right)}$$

input
```
integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^3*d - 6*a*b*c*d + 4*a^2*c*e - 2*a^2*b*f)*arctan((2*c*x^2 + b)/sqrt
(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*d*log(c*x^4
 + b*x^2 + a)/a^2 + 1/2*d*log(x^2)/a^2 + 1/4*(b^2*c*d*x^4 - 4*a*c^2*d*x^4
+ b^3*d*x^2 - 2*a*b*c*d*x^2 - 4*a^2*c*e*x^2 + 2*a^2*b*f*x^2 + 3*a*b^2*d -
8*a^2*c*d - 2*a^2*b*e + 4*a^3*f)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

Mupad [B] (verification not implemented)

Time = 32.69 (sec) , antiderivative size = 8706, normalized size of antiderivative = 52.45

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x)
```

output
```
(d*log(x))/a^2 - ((b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)/(2*a*(4*a*c - b^2))
+ (x^2*(a*b*f - 2*a*c*e + b*c*d))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
 - (log((((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4
*a*c - b^2)^3))^(1/2))*(((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b
*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((2*c^2*x^2*(20*a^2*c^2*e + 4*a*b^3*
f - b^3*c*d + 10*a*b*c^2*d - 8*a*b^2*c*e - 10*a^2*b*c*f))/(a*(4*a*c - b^2)
) + (b*c^2*(d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(
4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 - (4*b*c^2*(b^
3*d - a^2*b*f + 2*a^2*c*e - 5*a*b*c*d))/(a*(4*a*c - b^2))))/(4*a^2) + (c^2
*(a^3*b^2*f^2 - 4*b^4*c*d^2 + 4*a^3*c^2*e^2 + 17*a*b^2*c^2*d^2 - 4*a*b^4*d
*f - 36*a^2*b*c^2*d*e + 18*a^2*b^2*c*d*f + 8*a*b^3*c*d*e - 4*a^3*b*c*e*f))
/(a^2*(4*a*c - b^2)^2) - (c^2*x^2*(a^2*b^3*f^2 + 6*b^3*c^2*d^2 + 4*a^2*b*c
^2*e^2 - 20*a*b*c^3*d^2 + 40*a^2*c^3*d*e - 14*a*b^2*c^2*d*e - 20*a^2*b*c^2
*d*f - 4*a^2*b^2*c*e*f + 7*a*b^3*c*d*f))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) -
 (c^2*x^2*(a*b*f - 2*a*c*e + b*c*d)^3)/(a^3*(4*a*c - b^2)^3) + (c^2*d*(a*b
*f - 2*a*c*e + b*c*d)^2)/(a^3*(4*a*c - b^2)^2))*(((d - a^2*(-(b^3*d - 2*a^
2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((d - a^2*
(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/
2))*((2*c^2*x^2*(20*a^2*c^2*e + 4*a*b^3*f - b^3*c*d + 10*a*b*c^2*d - 8*a*b
^2*c*e - 10*a^2*b*c*f))/(a*(4*a*c - b^2)) + (b*c^2*(d - a^2*(-(b^3*d - ...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2436, normalized size of antiderivative = 14.67

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x)
```

output

```
(4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*f
 - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c*
e + 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**
3*f*x**2 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**2*b**2*c*d - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*
atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
b))*a**2*b**2*c*e*x**2 + 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt
(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*s
qrt(a) + b))*a**2*b**2*c*f*x**4 - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqr
t(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*
sqrt(c)*sqrt(a) + b))*a**2*b*c**2*e*x**4 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*s
qrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x
)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*d + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*s
qrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x
)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c*d*x**2 + 12*sqrt(2*sqrt(c)*sqrt(a)
 + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2...
```

3.67 $\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 234

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx &= -\frac{d}{2a^2x^2}-\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c(b^2d-abe-2a(cd-af))x^2}{2a^2(b^2-4ac)(a+bx^2+cx^4)}\\ &\quad-\frac{(2b^4d-12ab^2cd-ab^3e+6a^2bce+4a^2c(3cd-af))\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3(b^2-4ac)^{3/2}}\\ &\quad-\frac{(2bd-ae)\log(x)}{a^3}+\frac{(2bd-ae)\log(a+bx^2+cx^4)}{4a^3}\end{aligned}$$

output
```
-1/2*d/a^2/x^2-1/2*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d)+c*(b^2*d-a*b*
e-2*a*(-a*f+c*d))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(2*b^4*d-12*a*
b^2*c*d-a*b^3*e+6*a^2*b*c*e+4*a^2*c*(-a*f+3*c*d))*arctanh((2*c*x^2+b)/(-4*
a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-(-a*e+2*b*d)*ln(x)/a^3+1/4*(-a*e+2*
b*d)*ln(c*x^4+b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.72

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{-\frac{2ad}{x^2}-\frac{2a\left(b^3d+b^2\left(-ae+cdx^2\right)+ab\left(af-c\left(3d+ex^2\right)\right)+2ac\left(-cdx^2+a\left(e+fx^2\right)\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+4(-2bd+ae)\log(x)+\frac{\left(2b^4d+b^3\left(2\sqrt{b^2-4acd}\right.\right.}{}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-2*a*d)/x^2 - (2*a*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d +
 e*x^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2
+ c*x^4)) + 4*(-2*b*d + a*e)*Log[x] + ((2*b^4*d + b^3*(2*Sqrt[b^2 - 4*a*c]
*d - a*e) + 2*a*b*c*(-4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(12*c*d + Sqr
t[b^2 - 4*a*c]*e) + 4*a^2*c*(3*c*d + Sqrt[b^2 - 4*a*c]*e - a*f))*Log[b - S
qrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-2*b^4*d + b^3*(2*Sqr
t[b^2 - 4*a*c]*d + a*e) - 2*a*b*c*(4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*
(12*c*d - Sqrt[b^2 - 4*a*c]*e) + 4*a^2*c*(-3*c*d + Sqrt[b^2 - 4*a*c]*e + a
*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2194, 2177, 25, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$
$$\Big\downarrow \; 2194$$
$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^4\left(cx^4+bx^2+a\right)^2}dx^2$$

$\Big\downarrow$ 2177

$$\frac{1}{2}\left(-\frac{\int -\frac{-\frac{c\left(db^2-aeb-2a(cd-af)\right)x^4}{a^2}-\frac{\left(b^2-4ac\right)(bd-ae)x^2}{a^2}+\left(\frac{b^2}{a}-4c\right)d}{x^4(cx^4+bx^2+a)}dx^2}{b^2-4ac}-\frac{a^2\left(\frac{b^3d}{a^2}-\frac{b(be+3cd)}{a}+bf+2ce\right)+cx^2(-abe-2a\right.$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{-\frac{c\left(db^2-aeb-2a(cd-af)\right)x^4}{a^2}-\frac{\left(b^2-4ac\right)(bd-ae)x^2}{a^2}+\left(\frac{b^2}{a}-4c\right)d}{x^4(cx^4+bx^2+a)}dx^2}{b^2-4ac}-\frac{a^2\left(\frac{b^3d}{a^2}-\frac{b(be+3cd)}{a}+bf+2ce\right)+cx^2(-abe-2a(cd\right.$$

$\Big\downarrow$ 2159

$$\frac{1}{2}\left(\frac{\int\left(-\frac{(4ac-b^2)d}{a^2x^4}+\frac{2db^4-aeb^3-10acdb^2+5a^2ceb+c(b^2-4ac)(2bd-ae)x^2+2a^2c(3cd-af)}{a^3(cx^4+bx^2+a)}-\frac{(4ac-b^2)(ae-2bd)}{a^3x^2}\right)dx^2}{b^2-4ac}-\frac{a^2\left(\frac{b^3d}{a^2}-\right.}{}\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{-\frac{\log(x^2)(b^2-4ac)(2bd-ae)}{a^3}+\frac{(b^2-4ac)(2bd-ae)\log(a+bx^2+cx^4)}{2a^3}-\frac{d(b^2-4ac)}{a^2x^2}-\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(6a^2bce+4a^2c(3cd-af)-a}{a^3\sqrt{b^2-4ac}}}{b^2-4ac}\right.$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(-((a^2*((b^3*d)/a^2 + 2*c*e - (b*(3*c*d + b*e))/a + b*f) + c*(b^2*d - a*b
*e - 2*a*(c*d - a*f))*x^2)/(a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (-((
(b^2 - 4*a*c)*d)/(a^2*x^2)) - ((2*b^4*d - 12*a*b^2*c*d - a*b^3*e + 6*a^2*b
*c*e + 4*a^2*c*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a
^3*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)*(2*b*d - a*e)*Log[x^2])/a^3 + ((b^2
 - 4*a*c)*(2*b*d - a*e)*Log[a + b*x^2 + c*x^4])/(2*a^3))/(b^2 - 4*a*c))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.35

method	result
default	$-\frac{d}{2a^{2}x^{2}}+\frac{(ae-2bd)\ln(x)}{a^{3}}+\frac{\frac{ac\left(2f\,a^{2}-abe-2dac+b^{2}d\right)x^{2}}{4ac-b^{2}}+\frac{a\left(a^{2}bf+2a^{2}ce-a\,b^{2}e-3abcd+b^{3}d\right)}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\frac{\left(-4a^{2}c^{2}e+a\,b^{2}ce+8ab\,c^{2}d-2b^{3}cd\right)}{2c}}{}$
risch	$\frac{\frac{c\left(2f\,a^{2}-abe-6dac+2b^{2}d\right)x^{4}}{2a^{2}\left(4ac-b^{2}\right)}+\frac{\left(a^{2}bf+2a^{2}ce-a\,b^{2}e-7abcd+2b^{3}d\right)x^{2}}{2\left(4ac-b^{2}\right)a^{2}}-\frac{d}{2a}}{x^{2}(c\,x^{4}+b\,x^{2}+a)}+\frac{\ln(x)e}{a^{2}}-\frac{2\ln(x)bd}{a^{3}}+\frac{\left(\_R=\mathrm{RootOf}\left(\left(64a^{6}c^{3}-48a^{5}b^{2}c^{2}\right.\right.\right.}{}$

input
```
int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/a^2/x^2+(a*e-2*b*d)/a^3*ln(x)+1/2/a^3*((a*c*(2*a^2*f-a*b*e-2*a*c*d+
b^2*d)/(4*a*c-b^2)*x^2+a*(a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*
c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-4*a^2*c^2*e+a*b^2*c*e+8*a*b*c
^2*d-2*b^3*c*d)/c*ln(c*x^4+b*x^2+a)+2*(2*a^3*c*f-5*a^2*b*c*e-6*a^2*c^2*d+a
*b^3*e+10*a*b^2*c*d-2*b^4*d-1/2*(-4*a^2*c^2*e+a*b^2*c*e+8*a*b*c^2*d-2*b^3*
c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 870 vs. 2(222) = 444.

Time = 1.76 (sec) , antiderivative size = 1764, normalized size of antiderivative = 7.54

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b^3*c - 4*a^3*
b*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15*a^2*b^3*c +
 28*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3*b^3 - 4*a^
4*b*c)*f)*x^2 + ((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d + (a
*b^3*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*c + 6*a^2*b
*c^2)*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 - 6*a^2*b^2
*c + 6*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(b^2 - 4*a*c)*log((2
*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x
^4 + b*x^2 + a)) + 2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)
*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16
*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4
- 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((2*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e
)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4
 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 +
 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 -
 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*
a^3*c^3)*d - (a^2*b^3*c - 4*a^3*b*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.20

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{(2\,b^4d-12\,ab^2cd+12\,a^2c^2d-ab^3e+6\,a^2bce-4\,a^3cf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}}$$
$$-\frac{2\,b^2cdx^4-6\,ac^2dx^4-abcex^4+2\,a^2cfx^4+2\,b^3dx^2-7\,abcdx^2-ab^2ex^2+2\,a^2cex^2+a^2bfx^2+ab^2d-}{2\,(cx^6+bx^4+ax^2)(a^2b^2-4\,a^3c)}$$
$$+\frac{(2\,bd-ae)\log\left(cx^4+bx^2+a\right)}{4\,a^3}-\frac{(2\,bd-ae)\log\left(x^2\right)}{2\,a^3}$$

input
```
integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(2*b^4*d - 12*a*b^2*c*d + 12*a^2*c^2*d - a*b^3*e + 6*a^2*b*c*e - 4*a^3
*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-
b^2 + 4*a*c)) - 1/2*(2*b^2*c*d*x^4 - 6*a*c^2*d*x^4 - a*b*c*e*x^4 + 2*a^2*c
*f*x^4 + 2*b^3*d*x^2 - 7*a*b*c*d*x^2 - a*b^2*e*x^2 + 2*a^2*c*e*x^2 + a^2*b
*f*x^2 + a*b^2*d - 4*a^2*c*d)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 - 4*a^3*c)
) + 1/4*(2*b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^3 - 1/2*(2*b*d - a*e)*log(x
^2)/a^3
```

Mupad [B] (verification not implemented)

Time = 33.42 (sec) , antiderivative size = 11879, normalized size of antiderivative = 50.76

$$\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

output
```
((x^2*(2*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 7*a*b*c*d))/(2*a^2*(4*a*c
 - b^2)) - d/(2*a) + (c*x^4*(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d))/(2*a^2*
(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) + (log(x)*(a*e - 2*b*d))/a^3 + (lo
g((((((a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f -
 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((2*c^3*x^2*(
2*b^4*d - 60*a^2*c^2*d - 8*a^2*b^2*f - a*b^3*e + 20*a^3*c*f + 4*a*b^2*c*d
+ 10*a^2*b*c*e))/(a^2*(4*a*c - b^2)) + (4*b*c^2*(2*b^4*d + 6*a^2*c^2*d - a
*b^3*e - 2*a^3*c*f - 10*a*b^2*c*d + 5*a^2*b*c*e))/(a^2*(4*a*c - b^2)) + (b
*c^2*(a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f -
12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x
^2 - 10*a*c*x^2))/a^3))/(4*a^3) + (c^3*(4*a^5*c*f^2 - 16*b^6*d^2 - 4*a^2*b
^4*e^2 + 36*a^3*c^3*d^2 + 17*a^3*b^2*c*e^2 + 16*a*b^5*d*e - 216*a^2*b^2*c^
2*d^2 + 116*a*b^4*c*d^2 - 16*a^2*b^4*d*f + 8*a^3*b^3*e*f - 24*a^4*c^2*d*f
- 92*a^2*b^3*c*d*e + 108*a^3*b*c^2*d*e + 72*a^3*b^2*c*d*f - 36*a^4*b*c*e*f
))/(a^4*(4*a*c - b^2)^2) - (2*c^4*x^2*(12*b^5*d^2 + 2*a^4*b*f^2 + 3*a^2*b^
3*e^2 + 138*a^2*b*c^2*d^2 - 12*a*b^4*d*e + 20*a^4*c*e*f - 82*a*b^3*c*d^2 -
 10*a^3*b*c*e^2 + 14*a^2*b^3*d*f - 60*a^3*c^2*d*e - 7*a^3*b^2*e*f + 61*a^2
*b^2*c*d*e - 52*a^3*b*c*d*f))/(a^4*(4*a*c - b^2)^2))*(a*e - 2*b*d + a^3*(-
(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e
)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3) + (c^4*(a*e - 2*b*d)*(2*b^2*...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 4142, normalized size of antiderivative = 17.70

$$\int \frac{d+ex^2+fx^4}{x^3(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x)
```

output

```
( - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*c
*f*x**2 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a
**3*b**2*c*e*x**2 - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) -
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*a**3*b**2*c*f*x**4 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)
*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*a**3*b*c**2*d*x**2 - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(
2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**2*f*x**6 - 2*sqrt(2*sqrt(c)*sqrt(a) +
 b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt
(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**4*e*x**2 - 24*sqrt(2*sqrt(c)*s
qrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c*d*x**2 + 12*sqrt(2
*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqr
t(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c*e*x**4 +
 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*
c**2*d*x**4 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b...
```

3.68 $\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 329

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{x^5(a+bx^2+cx^4)^2}\,dx &= -\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}\\ &+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c(b^3d-ab^2e+2a^2ce-ab(3cd-af))x^2}{2a^3(b^2-4ac)(a+bx^2+cx^4)}\\ &+\frac{(3b^5d-2ab^4e+12a^2b^2ce-12a^3c^2e+6a^2bc(5cd-af)-ab^3(20cd-af))\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{3/2}}\\ &+\frac{(3b^2d-2abe-a(2cd-af))\log(x)}{a^4}\\ &-\frac{(3b^2d-2abe-a(2cd-af))\log(a+bx^2+cx^4)}{4a^4}\end{aligned}$$

output
```
-1/4*d/a^2/x^4+1/2*(-a*e+2*b*d)/a^3/x^2+1/2*(b^4*d-a*b^3*e+3*a^2*b*c*e+2*a
^2*c*(-a*f+c*d)-a*b^2*(-a*f+4*c*d)+c*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*
c*d))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(3*b^5*d-2*a*b^4*e+12*a^2*
b^2*c*e-12*a^3*c^2*e+6*a^2*b*c*(-a*f+5*c*d)-a*b^3*(-a*f+20*c*d))*arctanh((
2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(3*b^2*d-2*a*b*e-a*(
-a*f+2*c*d))*ln(x)/a^4-1/4*(3*b^2*d-2*a*b*e-a*(-a*f+2*c*d))*ln(c*x^4+b*x^2
+a)/a^4
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.80

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx =$$

$$-\frac{\frac{a^2d}{x^4}+\frac{2a(-2bd+ae)}{x^2}+\frac{2a\left(-b^4d+b^3\left(ae-cdx^2\right)+ab^2\left(4cd-af+cex^2\right)-abc\left(3ae-3cdx^2+afx^2\right)+2a^2c\left(af-c\left(d+ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-4(3b^2d-\ldots}{\ldots}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/4*((a^2*d)/x^4 + (2*a*(-2*b*d + a*e))/x^2 + (2*a*(-(b^4*d) + b^3*(a*e -
 c*d*x^2) + a*b^2*(4*c*d - a*f + c*e*x^2) - a*b*c*(3*a*e - 3*c*d*x^2 + a*f
*x^2) + 2*a^2*c*(a*f - c*(d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) - 4*(3*b^2*d - 2*a*b*e + a*(-2*c*d + a*f))*Log[x] + ((3*b^5*d + b^4*(3*S
qrt[b^2 - 4*a*c]*d - 2*a*e) + 2*a^2*b*c*(15*c*d + 4*Sqrt[b^2 - 4*a*c]*e -
3*a*f) + a*b^3*(-20*c*d - 2*Sqrt[b^2 - 4*a*c]*e + a*f) - 4*a^2*c*(-2*c*Sqr
t[b^2 - 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-14*c*Sqrt[b^
2 - 4*a*c]*d + 12*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-3*b^5*d + b^4*(3*Sqrt[b^2 - 4*a*c]*
d + 2*a*e) - a*b^3*(-20*c*d + 2*Sqrt[b^2 - 4*a*c]*e + a*f) + 2*a^2*b*c*(-1
5*c*d + 4*Sqrt[b^2 - 4*a*c]*e + 3*a*f) + 4*a^2*c*(2*c*Sqrt[b^2 - 4*a*c]*d
+ 3*a*c*e - a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-2*c*(7*Sqrt[b^2 - 4*a*c]*d +
6*a*e) + a*Sqrt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^
2 - 4*a*c)^(3/2))/a^4
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2194, 2177, 25, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx$$

$\Big\downarrow$ 2194

$$\frac{1}{2}\int \frac{fx^4+ex^2+d}{x^6\left(cx^4+bx^2+a\right)^2}dx^2$$

$\Big\downarrow$ 2177

$$\frac{1}{2}\left(\frac{cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)+3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d}{a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{c\left(db^3\right.}{}}{}\right.$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{\frac{c\left(db^3-aeb^2-a(3cd-af)b+2a^2ce\right)x^6}{a^3}+\frac{\left(b^2-4ac\right)\left(db^2-aeb-a(cd-af)\right)x^4}{a^3}-\frac{\left(b^2-4ac\right)(bd-ae)x^2}{a^2}+\left(\frac{b^2}{a}-4c\right)d}{x^6\left(cx^4+bx^2+a\right)}dx^2}{b^2-4ac}+\frac{cx^2\left(2a^2ce-ab^2e-\right.}{}\right.$$

$\Big\downarrow$ 2159

$$\frac{1}{2}\left(\frac{\int \left(-\frac{\left(4ac-b^2\right)d}{a^2x^6}+\frac{-3db^5+2aeb^4+a(17cd-af)b^3-10a^2ceb^2-a^2c(19cd-5af)b-c\left(b^2-4ac\right)\left(3db^2-2aeb-a(2cd-af)\right)x^2+6a^3c^2e}{a^4\left(cx^4+bx^2+a\right)}+\frac{\left(b^2-\right.}{}\right.}{b^2-4ac}\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)+3a^2bce+2a^2c(cd-af)-ab^3e-ab^2(4cd-af)+b^4d}{a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(x^2\right)\left(b\right.}{}\right.$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^2*(4*c*d - a*f
) + c*(b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))*x^2)/(a^3*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (-1/2*((b^2 - 4*a*c)*d)/(a^2*x^4) + ((b^2 - 4
*a*c)*(2*b*d - a*e))/(a^3*x^2) + ((3*b^5*d - 2*a*b^4*e + 12*a^2*b^2*c*e -
12*a^3*c^2*e + 6*a^2*b*c*(5*c*d - a*f) - a*b^3*(20*c*d - a*f))*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*(3
*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[x^2])/a^4 - ((b^2 - 4*a*c)*(3*b^2*
d - 2*a*b*e - a*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(2*a^4))/(b^2 - 4*a
*c))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.42

method	result
default	$-\frac{d}{4a^2x^4}-\frac{ae-2bd}{2a^3x^2}+\frac{(f\,a^2-2abe-2dac+3b^2d)\ln(x)}{a^4}-\frac{\frac{ac(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d)x^2}{4ac-b^2}-\frac{a(2a^3cf-a^2b^2f-3a^2bce-2a^2c^2d}{4ac-b^2}}{c\,x^4+b\,x^2+a}$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*d/a^2/x^4-1/2*(a*e-2*b*d)/a^3/x^2+(a^2*f-2*a*b*e-2*a*c*d+3*b^2*d)/a^4
*ln(x)-1/2/a^4*((a*c*(a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^
2)*x^2-a*(2*a^3*c*f-a^2*b^2*f-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*b^2*c*d-
b^4*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*a^3*c^2*f-a^2*b^
2*c*f-8*a^2*b*c^2*e-8*a^2*c^3*d+2*a*b^3*c*e+14*a*b^2*c^2*d-3*b^4*c*d)/c*ln
(c*x^4+b*x^2+a)+2*(5*a^3*b*c*f+6*e*a^3*c^2-a^2*b^3*f-10*a^2*b^2*c*e-19*a^2
*c^2*b*d+2*a*b^4*e+17*a*b^3*c*d-3*b^5*d-1/2*(4*a^3*c^2*f-a^2*b^2*c*f-8*a^2
*b*c^2*e-8*a^2*c^3*d+2*a*b^3*c*e+14*a*b^2*c^2*d-3*b^4*c*d)*b/c)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1272 vs. 2(315) = 630.

Time = 3.97 (sec) , antiderivative size = 2567, normalized size of antiderivative = 7.80

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*((3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*
a^3*b^2*c^2 + 12*a^4*c^3)*e + (a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6
 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*
b^3*c + 28*a^4*b*c^2)*e + 2*(a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f)*x^4 + (
3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2)*e)*x^2 + (((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*
c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*
b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^
2)*e + (a^2*b^4 - 6*a^3*b^2*c)*f)*x^6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*
b*c^2)*d - 2*(a^2*b^4 - 6*a^3*b^2*c + 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)
*f)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c
*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2)*d - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4
)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^
2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^
3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3
*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2
- 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 -
8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*log(c*x^4 + b*x^2 + a) + 4*(((3*b^6*c -
26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.58

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx =$$
$$-\frac{(3\,b^5d-20\,ab^3cd+30\,a^2bc^2d-2\,ab^4e+12\,a^2b^2ce-12\,a^3c^2e+a^2b^3f-6\,a^3bcf)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^4b^2-4\,a^5c)\sqrt{-b^2+4\,ac}}$$
$$+\frac{3\,b^4cdx^4-14\,ab^2c^2dx^4+8\,a^2c^3dx^4-2\,ab^3cex^4+8\,a^2bc^2ex^4+a^2b^2cfx^4-4\,a^3c^2fx^4+3\,b^5dx^2-12\,a}{}$$
$$-\frac{(3\,b^2d-2\,acd-2\,abe+a^2f)\log\left(cx^4+bx^2+a\right)}{4\,a^4}$$
$$+\frac{(3\,b^2d-2\,acd-2\,abe+a^2f)\log\left(x^2\right)}{2\,a^4}$$
$$-\frac{9\,b^2dx^4-6\,acdx^4-6\,abex^4+3\,a^2fx^4-4\,abdx^2+2\,a^2ex^2+a^2d}{4\,a^4x^4}$$

input
```
integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^5*d - 20*a*b^3*c*d + 30*a^2*b*c^2*d - 2*a*b^4*e + 12*a^2*b^2*c*e
 - 12*a^3*c^2*e + a^2*b^3*f - 6*a^3*b*c*f)*arctan((2*c*x^2 + b)/sqrt(-b^2
+ 4*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(3*b^4*c*d*x^4 -
14*a*b^2*c^2*d*x^4 + 8*a^2*c^3*d*x^4 - 2*a*b^3*c*e*x^4 + 8*a^2*b*c^2*e*x^4
 + a^2*b^2*c*f*x^4 - 4*a^3*c^2*f*x^4 + 3*b^5*d*x^2 - 12*a*b^3*c*d*x^2 + 2*
a^2*b*c^2*d*x^2 - 2*a*b^4*e*x^2 + 6*a^2*b^2*c*e*x^2 + 4*a^3*c^2*e*x^2 + a^
2*b^3*f*x^2 - 2*a^3*b*c*f*x^2 + 5*a*b^4*d - 22*a^2*b^2*c*d + 12*a^3*c^2*d
- 4*a^2*b^3*e + 14*a^3*b*c*e + 3*a^3*b^2*f - 8*a^4*c*f)/((a^4*b^2 - 4*a^5*
c)*(c*x^4 + b*x^2 + a)) - 1/4*(3*b^2*d - 2*a*c*d - 2*a*b*e + a^2*f)*log(c*
x^4 + b*x^2 + a)/a^4 + 1/2*(3*b^2*d - 2*a*c*d - 2*a*b*e + a^2*f)*log(x^2)/
a^4 - 1/4*(9*b^2*d*x^4 - 6*a*c*d*x^4 - 6*a*b*e*x^4 + 3*a^2*f*x^4 - 4*a*b*d
*x^2 + 2*a^2*e*x^2 + a^2*d)/(a^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 46.05 (sec) , antiderivative size = 15905, normalized size of antiderivative = 48.34

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x)
```

output
```
(log(x)*(3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 - (log((((((4*b*c^2*(3
*b^5*d + a^2*b^3*f - 6*a^3*c^2*e - 2*a*b^4*e - 17*a*b^3*c*d - 5*a^3*b*c*f
+ 19*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(a^3*(4*a*c - b^2)) - (b*c^2*(a*b + 3*
b^2*x^2 - 10*a*c*x^2)*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4
*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*
(4*a*c - b^2)^3))^(1/2) + 3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 + (2*c
^3*x^2*(3*b^5*d + a^2*b^3*f + 60*a^3*c^2*e - 2*a*b^4*e + 4*a*b^3*c*d - 10*
a^3*b*c*f - 70*a^2*b*c^2*d - 4*a^2*b^2*c*e))/(a^3*(4*a*c - b^2)))*(a^4*(-(
3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*
f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2) + 3*b^
2*d + a^2*f - 2*a*b*e - 2*a*c*d))/(4*a^4) + (c^3*(36*b^8*d^2 + 16*a^2*b^6*
e^2 + 4*a^4*b^4*f^2 - 36*a^5*c^3*e^2 - 116*a^3*b^4*c*e^2 - 17*a^5*b^2*c*f^
2 - 48*a*b^7*d*e + 778*a^2*b^4*c^2*d^2 - 473*a^3*b^2*c^3*d^2 + 216*a^4*b^2
*c^2*e^2 - 309*a*b^6*c*d^2 + 24*a^2*b^6*d*f - 16*a^3*b^5*e*f + 380*a^2*b^5
*c*d*e + 324*a^4*b*c^3*d*e - 154*a^3*b^4*c*d*f + 92*a^4*b^3*c*e*f - 108*a^
5*b*c^2*e*f - 832*a^3*b^3*c^2*d*e + 230*a^4*b^2*c^2*d*f))/(a^6*(4*a*c - b^
2)^2) + (c^4*x^2*(54*b^7*d^2 + 24*a^2*b^5*e^2 + 6*a^4*b^3*f^2 - 440*a^3*b*
c^3*d^2 - 164*a^3*b^3*c*e^2 + 276*a^4*b*c^2*e^2 - 72*a*b^6*d*e + 1011*a^2*
b^3*c^2*d^2 - 441*a*b^5*c*d^2 - 20*a^5*b*c*f^2 + 36*a^2*b^5*d*f + 240*a^4*
c^3*d*e - 24*a^3*b^4*e*f - 120*a^5*c^2*e*f + 540*a^2*b^4*c*d*e - 207*a^...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6104, normalized size of antiderivative = 18.55

$$\int \frac{d + ex^2 + fx^4}{x^5 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x)
```

output
```
(12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**2*
c*f*x**4 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**4*b*c**2*e*x**4 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
- b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a**3*b**4*f*x**4 - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*
sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a**3*b**3*c*e*x**4 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(
2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*a**3*b**3*c*f*x**6 - 60*sqrt(2*sqrt(c)*sqrt(a)
+ b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqr
t(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**2*d*x**4 + 24*sqrt(2*sqr
t(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a)
 - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**2*e*x**6 +
12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c
**2*f*x**8 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b)
)*a**3*b*c**3*e*x**8 + 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqr...
```

3.69 $\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 550

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2}$$

$$+\frac{x(a(b^2ce-2ac^2e-b^3f-bc(cd-3af))+(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2ac^2(cd-af))\,x^2}{2c^3\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$-\frac{\left(3b^3ce-13abc^2e-5b^4f-b^2c(cd-24af)+2ac^2(3cd-7af)-\frac{3b^4ce-19ab^2c^2e+20a^2c^3e-5b^5f-b^3c(cd-34af)+}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}c^{7/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}$$

$$-\frac{\left(3b^3ce-13abc^2e-5b^4f-b^2c(cd-24af)+2ac^2(3cd-7af)+\frac{3b^4ce-19ab^2c^2e+20a^2c^3e-5b^5f-b^3c(cd-34af)+}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}c^{7/2}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
(-2*b*f+c*e)*x/c^3+1/3*f*x^3/c^2+1/2*x*(a*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3
*a*f+c*d))+(b^3*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d
))*x^2)/c^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*(3*b^3*c*e-13*a*b*c^2*e-5*b^4
*f-b^2*c*(-24*a*f+c*d)+2*a*c^2*(-7*a*f+3*c*d)-(3*b^4*c*e-19*a*b^2*c^2*e+20
*a^2*c^3*e-5*b^5*f-b^3*c*(-34*a*f+c*d)+4*a*b*c^2*(-13*a*f+2*c*d))/(-4*a*c+
b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)
/c^(7/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*(3*b^3*c*e-13*a*b*c
^2*e-5*b^4*f-b^2*c*(-24*a*f+c*d)+2*a*c^2*(-7*a*f+3*c*d)+(3*b^4*c*e-19*a*b^
2*c^2*e+20*a^2*c^3*e-5*b^5*f-b^3*c*(-34*a*f+c*d)+4*a*b*c^2*(-13*a*f+2*c*d)
)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2
))*2^(1/2)/c^(7/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.18

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{12\sqrt{c}(ce-2bf)x+4c^{3/2}fx^3-\frac{6\sqrt{c}x(b^2(c^2d-bce+b^2f)x^2+a^2c(-3bf+2c(e+fx^2))+a(b^3f-2c^3dx^2+bc^2(d+3ex^2)-b^2c(e+4fx^2)}{(b^2-4ac)(a+bx^2+cx^4)}}{}$$

input
```
Integrate[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(12*Sqrt[c]*(c*e - 2*b*f)*x + 4*c^(3/2)*f*x^3 - (6*Sqrt[c]*x*(b^2*(c^2*d -
 b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e + f*x^2)) + a*(b^3*f - 2*c^3*
d*x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^2))))/((b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*f + a*b*c^2*(8*c*d + 13*Sqrt[b^2 - 4*a
*c]*e - 52*a*f) - b^3*c*(c*d + 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + b^4*(3*c*
e + 5*Sqrt[b^2 - 4*a*c]*f) + b^2*c*(c*Sqrt[b^2 - 4*a*c]*d - 19*a*c*e - 24*
a*Sqrt[b^2 - 4*a*c]*f) + 2*a*c^2*(-3*c*Sqrt[b^2 - 4*a*c]*d + 10*a*c*e + 7*
a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b
^5*f + b^3*c*(c*d - 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + a*b*c^2*(-8*c*d + 13
*Sqrt[b^2 - 4*a*c]*e + 52*a*f) + b^4*(-3*c*e + 5*Sqrt[b^2 - 4*a*c]*f) + b^
2*c*(c*Sqrt[b^2 - 4*a*c]*d + 19*a*c*e - 24*a*Sqrt[b^2 - 4*a*c]*f) - 2*a*c^
2*(3*c*Sqrt[b^2 - 4*a*c]*d + 10*a*c*e - 7*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]))/(12*c^(7/2))
```

Rubi [A] (verified)

Time = 6.90 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2197, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \left(d+e x^2+f x^4\right)}{\left(a+b x^2+c x^4\right)^2} \, dx$$

$$\Big\downarrow \ 2197$$

$$\frac{x\left(a\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)+x^2\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{\int \frac{2a\left(4a-\frac{b^2}{c}\right)fx^6-\frac{2a\left(b^2-4ac\right)(ce-bf)x^4}{c^2}+\frac{a\left(-fb^4+ceb^3-c(cd-6af)b^2-5ac^2eb+6ac^2(cd-af)\right)x^2}{c^3}+\frac{a^2\left(-fb^3+ceb^2-c(cd-3af)b-2ac^2e\right)}{c^3}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 2205$$

$$\frac{x\left(a\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)+x^2\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$\frac{\int\left(-\frac{2a(b^2-4ac)fx^2}{c^2}-\frac{2a(b^2-4ac)(ce-2bf)}{c^3}-\frac{-((-5fb^3+3ceb^2-c(cd-19af)b-10ac^2e)a^2)-(-5fb^4+3ceb^3-c(cd-24af)b^2-13ac^2eb+2}{c^3(cx^4+bx^2+a)}\right.}{2a\left(b^2-4ac\right)}$$

$\downarrow$ 2009

$$\frac{x\left(a\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)+x^2\left(-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce\right)\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(3cd-7af)-5b^4f+3b\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

input
```
Int[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(a*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f)) + (b^3*c*e - 3*a*b
*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))*x^2))/(2*c^3*(
b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*a*(b^2 - 4*a*c)*(c*e - 2*b*f)*x)/
c^3 - (2*a*(b^2 - 4*a*c)*f*x^3)/(3*c^2) + (a*(3*b^3*c*e - 13*a*b*c^2*e - 5
*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) - (3*b^4*c*e - 19*
a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d - 34*a*f) + 4*a*b*c^2*(2
*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (a*(3*b^
3*c*e - 13*a*b*c^2*e - 5*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7
*a*f) + (3*b^4*c*e - 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d
- 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.59

method	result
risch	$\frac{f\,x^{3}}{3c^{2}}-\frac{2bfx}{c^{3}}+\frac{ex}{c^{2}}+\frac{\frac{\left(2a^{2}c^{2}f-4a\,b^{2}cf+3ab\,c^{2}e-2a\,c^{3}d+b^{4}f-b^{3}ce+b^{2}c^{2}d\right)x^{3}}{8ac-2b^{2}}-\frac{a\left(3abcf-2a\,c^{2}e-b^{3}f+b^{2}ce-b\,c^{2}d\right)x}{2\left(4ac-b^{2}\right)}}{c^{3}\left(c\,x^{4}+b\,x^{2}+a\right)}+\frac{\_R=\text{RootOf}\ldots}{\ldots}$
default	$-\frac{-\frac{1}{3}f\,x^{3}c+2bfx-cex}{c^{3}}+\frac{\frac{\frac{\left(2a^{2}c^{2}f-4a\,b^{2}cf+3ab\,c^{2}e-2a\,c^{3}d+b^{4}f-b^{3}ce+b^{2}c^{2}d\right)x^{3}}{8ac-2b^{2}}-\frac{a\left(3abcf-2a\,c^{2}e-b^{3}f+b^{2}ce-b\,c^{2}d\right)x}{2\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{2c\left(\frac{\left(-14a^{2}\ldots\right.}{\ldots}\right.}{\ldots}}{\ldots}$

input
```
int(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*f*x^3/c^2-2/c^3*b*f*x+e*x/c^2+(1/2*(2*a^2*c^2*f-4*a*b^2*c*f+3*a*b*c^2*
e-2*a*c^3*d+b^4*f-b^3*c*e+b^2*c^2*d)/(4*a*c-b^2)*x^3-1/2*a*(3*a*b*c*f-2*a*
c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)*x)/c^3/(c*x^4+b*x^2+a)+1/4/c^3*su
m((-(14*a^2*c^2*f-24*a*b^2*c*f+13*a*b*c^2*e-6*a*c^3*d+5*b^4*f-3*b^3*c*e+b^
2*c^2*d)/(4*a*c-b^2)*_R^2+a*(19*a*b*c*f-10*a*c^2*e-5*b^3*f+3*b^2*c*e-b*c^2
*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18909 vs. 2(506) = 1012.

Time = 75.02 (sec) , antiderivative size = 18909, normalized size of antiderivative = 34.38

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(fx^4+ex^2+d)x^6}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c +
2*a^2*c^2)*f)*x^3 + (a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e + (a*b^3 - 3*a^2*
b*c)*f)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4
*a*b*c^4)*x^2) + 1/2*integrate((a*b*c^2*d + ((b^2*c^2 - 6*a*c^3)*d - (3*b^
3*c - 13*a*b*c^2)*e + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*f)*x^2 - (3*a*b^2*
c - 10*a^2*c^2)*e + (5*a*b^3 - 19*a^2*b*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2
*c^3 - 4*a*c^4) + 1/3*(c*f*x^3 + 3*(c*e - 2*b*f)*x)/c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8946 vs. $2(506) = 1012$.

Time = 1.32 (sec) , antiderivative size = 8946, normalized size of antiderivative = 16.27

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*c^2*d*x^3 - 2*a*c^3*d*x^3 - b^3*c*e*x^3 + 3*a*b*c^2*e*x^3 + b^4*
f*x^3 - 4*a*b^2*c*f*x^3 + 2*a^2*c^2*f*x^3 + a*b*c^2*d*x - a*b^2*c*e*x + 2*
a^2*c^2*e*x + a*b^3*f*x - 3*a^2*b*c*f*x)/((b^2*c^3 - 4*a*c^4)*(c*x^4 + b*x
^2 + a)) - 1/16*((2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*
c^5)*(b^2*c^3 - 4*a*c^4)^2*d - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 -
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3
*c^3 + 26*(b^2 - 4*a*c)*a*b*c^4)*(b^2*c^3 - 4*a*c^4)^2*e + (10*b^6*c^2 ...
```

Mupad [B] (verification not implemented)

Time = 21.53 (sec) , antiderivative size = 33799, normalized size of antiderivative = 61.45

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
x*(e/c^2 - (2*b*f)/c^3) + ((x^3*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3
*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(2*(4*a*c - b^2)) + (x*(2*a^2*c
^2*e + a*b^3*f + a*b*c^2*d - a*b^2*c*e - 3*a^2*b*c*f))/(2*(4*a*c - b^2)))/
(a*c^3 + c^4*x^4 + b*c^3*x^2) - atan(((((10240*a^5*c^9*e + 192*a^2*b^5*c^7
*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^
4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*
f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f -
19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^
7)) - (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*
(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 8064
0*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d
^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 +
 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c -
 b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*
f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*
f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9
*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d
*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4
*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - ...
```

Reduce [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 9400, normalized size of antiderivative = 17.09

$$\int \frac{x^6(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
(168*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**3*f - 174*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**2*f + 96*sqrt(a)*sqrt(2*sqrt(c)*
sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a**2*b*c**3*e + 168*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b
))*a**2*b*c**3*f*x**2 - 72*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c*
*4*d + 168*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**4*f*x**4 + 30*s
qrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*c*f - 18*sqrt(a)*sqrt(2*sqrt
(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*s
qrt(c)*sqrt(a) + b))*a*b**3*c**2*e - 174*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a*b**3*c**2*f*x**2 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*
*2*c**3*d + 96*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**3*e*x*...
```

3.70 $\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 437

$$\begin{aligned}
&\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{fx}{c^2}+\frac{x(a(2c^2d+b^2f-c(be+2af))-(b^2ce-2ac^2e-b^3f-bc(cd-3af))x^2)}{2c^2(b^2-4ac)(a+bx^2+cx^4)} \\
&\quad+\frac{\left(b^2ce-6ac^2e-3b^3f+bc(cd+13af)-\frac{b^3ce-8abc^2e-3b^4f+4ac^2(cd-5af)+b^2c(cd+19af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad+\frac{\left(b^2ce-6ac^2e-3b^3f+bc(cd+13af)+\frac{b^3ce-8abc^2e-3b^4f+4ac^2(cd-5af)+b^2c(cd+19af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

output
```
f*x/c^2+1/2*x*(a*(2*c^2*d+b^2*f-c*(2*a*f+b*e))-(b^2*c*e-2*a*c^2*e-b^3*f-b*
c*(-3*a*f+c*d))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*(b^2*c*e-6*a*c^2
*e-3*b^3*f+b*c*(13*a*f+c*d)-(b^3*c*e-8*a*b*c^2*e-3*b^4*f+4*a*c^2*(-5*a*f+c
*d)+b^2*c*(19*a*f+c*d))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)+1/4*(b^2*c*e-6*a*c^2*e-3*b^3*f+b*c*(13*a*f+c*d)+(b^3*c*e-8*a*b*c^
2*e-3*b^4*f+4*a*c^2*(-5*a*f+c*d)+b^2*c*(19*a*f+c*d))/(-4*a*c+b^2)^(1/2))*a
rctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(-4*
a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.17

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{4\sqrt{c}fx + \frac{2\sqrt{c}x(-2a^2cf+b(c^2d-bce+b^2f)x^2+a(b^2f+2c^2(d+ex^2)-bc(e+3fx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4f+2ac^2\left(2cd+3\sqrt{b^2-4ac}e-10af\right)+b^2c\left(\right.\right.}{}}{}$$

input
```
Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(4*Sqrt[c]*f*x + (2*Sqrt[c]*x*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2
+ a*(b^2*f + 2*c^2*(d + e*x^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*f + 2*a*c^2*(2*c*d + 3*Sqrt[b^2 - 4*a*c
]*e - 10*a*f) + b^2*c*(c*d - Sqrt[b^2 - 4*a*c]*e + 19*a*f) + b^3*(c*e + 3*
Sqrt[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2 - 4*a*c]*d + 8*a*c*e + 13*a*Sqrt[b^
2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*f + 2*a*
c^2*(-2*c*d + 3*Sqrt[b^2 - 4*a*c]*e + 10*a*f) - b^2*c*(c*d + Sqrt[b^2 - 4*
a*c]*e + 19*a*f) + b^3*(-(c*e) + 3*Sqrt[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2
- 4*a*c]*d - 8*a*c*e + 13*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]))/(4*c^(5/2))
```

Rubi [A] (verified)

Time = 3.16 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2197, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{2a\left(4a-\frac{b^2}{c}\right)fx^4-\frac{a\left(-fb^3+ceb^2+c(cd+5af)b-6ac^2e\right)x^2}{c^2}+\frac{a^2\left(fb^2+2c^2d-c(be+2af)\right)}{c^2}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{2205}$$

$$\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\left(\frac{a^2\left(3fb^2-ceb+2c^2d-10acf\right)-a\left(-3fb^3+ceb^2+c(cd+13af)b-6ac^2e\right)x^2}{c^2\left(cx^4+bx^2+a\right)}-\frac{2a\left(b^2-4ac\right)f}{c^2}\right)dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{b^2c(19af+cd)-8abc^2e+4ac^2(cd-5af)-3b^4f+b^3ce}{\sqrt{b^2-4ac}}+bc(13af+cd)-6ac^2e-3b^3f+b^2ce\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\cdots}{\cdots}}{2a\left(b^2-4ac\right)}$$

input

```
Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output

```
(x*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) - (b^2*c*e - 2*a*c^2*e - b^3*f -
 b*c*(c*d - 3*a*f))*x^2))/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2
*a*(b^2 - 4*a*c)*f*x)/c^2 - (a*(b^2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(c*d +
 13*a*f) - (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f + 4*a*c^2*(c*d - 5*a*f) + b^2*
c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (a*(b^
2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(c*d + 13*a*f) + (b^3*c*e - 8*a*b*c^2*e
- 3*b^4*f + 4*a*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.55

method	result
risch	$\frac{fx}{c^2}+\frac{\frac{\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x^3}{8ac-2b^2}+\frac{a\left(2acf-b^2f+ceb-2d\,c^2\right)x}{8ac-2b^2}}{c^2\left(c\,x^4+b\,x^2+a\right)}+\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{\left(13abcf-6a\,c^2e-3b^3f+b^2\right.}{4ac-b^2}\right.}{}}{4}$
default	$\frac{fx}{c^2}-\frac{-\frac{\left(3abcf-2a\,c^2e-b^3f+b^2ce-b\,c^2d\right)x^3}{2\left(4ac-b^2\right)}-\frac{a\left(2acf-b^2f+ceb-2d\,c^2\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(13abcf\sqrt{-4ac+b^2}-6a\,c^2e\sqrt{-4ac+b^2}-3b^3f\sqrt{-4ac+b^2}+b^2\right.}{}\right.}{}$

input
```
int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
f*x/c^2+(1/2*(3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)*x^3+1
/2*a*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)*x)/c^2/(c*x^4+b*x^2+a)+1/4/
c^2*sum((-(13*a*b*c*f-6*a*c^2*e-3*b^3*f+b^2*c*e+b*c^2*d)/(4*a*c-b^2)*_R^2-
a*(10*a*c*f-3*b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_
R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12597 vs. $2(395) = 790$.

Time = 21.86 (sec) , antiderivative size = 12597, normalized size of antiderivative = 28.83

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(fx^4+ex^2+d)x^4}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*x^3 + (2*a*c^2*d
- a*b*c*e + (a*b^2 - 2*a^2*c)*f)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*
a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + f*x/c^2 + 1/2*integrate(-(2*a*c^
2*d - a*b*c*e - (b*c^2*d + (b^2*c - 6*a*c^2)*e - (3*b^3 - 13*a*b*c)*f)*x^2
 + (3*a*b^2 - 10*a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7479 vs. 2(395) = 790.

Time = 1.21 (sec) , antiderivative size = 7479, normalized size of antiderivative = 17.11

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
f*x/c^2 + 1/2*(b*c^2*d*x^3 - b^2*c*e*x^3 + 2*a*c^2*e*x^3 + b^3*f*x^3 - 3*a
*b*c*f*x^3 + 2*a*c^2*d*x - a*b*c*e*x + a*b^2*f*x - 2*a^2*c*f*x)/((c*x^4 +
b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2
 - 4*a*c^3)^2*d + (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c
^4)*(b^2*c^2 - 4*a*c^3)^2*e - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt...
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 25862, normalized size of antiderivative = 59.18

$$\int \frac{x^4(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
(f*x)/c^2 - atan(((((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d
+ 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6
*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*
b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 +
12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c
^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)
^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*
b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5
*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2
 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c
 - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2
 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f
+ 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*
(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^
10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*
c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b
^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*
d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f
- 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a...
```

Reduce [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 7390, normalized size of antiderivative = 16.91

$$\int \frac{x^4(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
(32*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2*f - 24*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**3*e - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a
) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*a*b**3*c*f + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c
**2*e + 32*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**2*f*x**2 + 8*
sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**3*d - 24*sqrt(a)*sqrt(2*sqr
t(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*
sqrt(c)*sqrt(a) + b))*a*b*c**3*e*x**2 + 32*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a)
+ b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a*b*c**3*f*x**4 - 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*
*4*e*x**4 - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqr
t(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*c*f*x**2 + 2*sq
rt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c**2*e*x**2 - 6*sqrt(a)*sqrt...
```

3.71 $\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 362

$$\begin{aligned}\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx &= -\frac{x(bcd-2ace+abf+(2c^2d-bce+b^2f-2acf)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}\\
&\quad-\frac{\left(2cd-be+6af-\frac{b^2f}{c}+\frac{b^2ce+4ac^2e+b^3f-4bc(cd+2af)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad-\frac{\left(2cd-be+6af-\frac{b^2f}{c}-\frac{b^2ce+4ac^2e+b^3f-4bc(cd+2af)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\,\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/2*x*(b*c*d-2*a*c*e+a*b*f+(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+
b^2)/(c*x^4+b*x^2+a)-1/4*(2*c*d-b*e+6*a*f-b^2*f/c+(b^2*c*e+4*a*c^2*e+b^3*f
-4*b*c*(2*a*f+c*d))/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*
a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))
^(1/2)-1/4*(2*c*d-b*e+6*a*f-b^2*f/c-(b^2*c*e+4*a*c^2*e+b^3*f-4*b*c*(2*a*f+
c*d))/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2)
)^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.14

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{-\frac{2\sqrt{c}x(abf+2c^2dx^2+b^2fx^2+bc(d-ex^2)-2ac(e+fx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^3f+bc\left(4cd+\sqrt{b^2-4ac}e+8af\right)+b^2\left(-ce+\sqrt{b^2-4ac}f\right)-2c\left(c\sqrt{b^2-4ac}d+\right.\right.}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((-2*Sqrt[c]*x*(a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*
(e + f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*f) + b
*c*(4*c*d + Sqrt[b^2 - 4*a*c]*e + 8*a*f) + b^2*(-(c*e) + Sqrt[b^2 - 4*a*c]
*f) - 2*c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*f + b*c*(-4*c*d + Sqrt[b^2 -
 4*a*c]*e - 8*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) - 2*c*(c*Sqrt[b^2 - 4
*a*c]*d - 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/(4*c^(3/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2197, 25, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(d+ex^2+fx^4\right)}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 2197$$

$$-\frac{\int -\frac{a\left(-c\left(-\frac{fb^2}{c}-eb+2cd+6af\right)x^2+bcd-2ace+abf\right)}{c(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{a\left(-\left(\left(-fb^2-ceb+2c^2d+6acf\right)x^2\right)+bcd-2ace+abf\right)}{c(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{-\left(\left(-fb^2-ceb+2c^2d+6acf\right)x^2\right)+bcd-2ace+abf}{cx^4+bx^2+a}dx}{2c\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 1480

$$\frac{-\frac{1}{2}\left(\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{\sqrt{b^2-4ac}}+6acf+b^2(-f)-bce+2c^2d\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(-\frac{-4bc(2af+cd)+4ac^2e\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{2c\left(b^2-4ac\right)}$$
$$\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 218

$$\frac{-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{\sqrt{b^2-4ac}}+6acf+b^2(-f)-bce+2c^2d\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-4bc(2af+cd)+4ac^2e+b^3f\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+\cdots}}}{2c\left(b^2-4ac\right)}$$
$$\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(b*c*d - 2*a*c*e + a*b*f + (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2
))/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-(((2*c^2*d - b*c*e - b^2*f +
6*a*c*f + (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - ((2*c^2*d - b*c*e - b^2*f + 6*a*c*f
 - (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.55

method	result
risch	$\frac{-\frac{\left(2acf-b^{2}f+ceb-2d\,c^{2}\right)x^{3}}{2c\left(4ac-b^{2}\right)}+\frac{(abf-2ace+bcd)x}{2c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(\frac{\left(6acf-b^{2}f-ceb+2d\,c^{2}\right)\_R^{2}}{4ac-b^{2}}-\frac{abf-2ace+bcd}{4ac-b^{2}}\right)\ln(}{2\_R^{3}c+\_Rb}}{4c}$
default	$\frac{-\frac{\left(2acf-b^{2}f+ceb-2d\,c^{2}\right)x^{3}}{2c\left(4ac-b^{2}\right)}+\frac{(abf-2ace+bcd)x}{2c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\left(6acf\sqrt{-4ac+b^{2}}-b^{2}f\sqrt{-4ac+b^{2}}-ceb\sqrt{-4ac+b^{2}}+2d\,c^{2}\sqrt{-4ac+b^{2}}+8abcf-4a\,c^{2}e-b^{3}f}{4c\sqrt{-4ac+b^{2}}\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}$

input
```
int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c/(4*a*c-b^2)*x^3+1/2/c*(a*b*f-2*a*c*e
+b*c*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/c*sum(((6*a*c*f-b^2*f-b*c*e+2*c
^2*d)/(4*a*c-b^2)*_R^2-(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*
ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8951 vs. 2(320) = 640.

Time = 11.98 (sec) , antiderivative size = 8951, normalized size of antiderivative = 24.73

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(fx^4+ex^2+d)x^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + (b*c*d - 2*a*c*e + a*b*f)*
x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^
2) - 1/2*integrate(-(b*c*d - 2*a*c*e + a*b*f - (2*c^2*d - b*c*e - (b^2 - 6
*a*c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6200 vs. $2(320) = 640$.

Time = 1.07 (sec) , antiderivative size = 6200, normalized size of antiderivative = 17.13

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*c^2*d*x^3 - b*c*e*x^3 + b^2*f*x^3 - 2*a*c*f*x^3 + b*c*d*x - 2*a*c*
e*x + a*b*f*x)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) - 1/16*(2*(2*b^2*c^
4 - 8*a*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^4 - 2*(b^2 - 4*a*c
)*c^4)*(b^2*c - 4*a*c^2)^2*d - (2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2
*e - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^
2)^2*f - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 8*sqrt(2)...
```

Mupad [B] (verification not implemented)

Time = 24.80 (sec) , antiderivative size = 19494, normalized size of antiderivative = 53.85

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x^3*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c*(4*a*c - b^2)) + (x*(a*b*f
 - 2*a*c*e + b*c*d))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((2
048*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 153
6*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d
- 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(
8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((768*a^4*b*c
^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b
^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^
2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9
*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6
*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 15
04*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^
6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*
d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e -
 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2
*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^
5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a
^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6
 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^
4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2...
```

Reduce [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 6216, normalized size of antiderivative = 17.17

$$\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
( - 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**2*f + 2*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c*f + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt
(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*s
qrt(a) + b))*a**2*b*c**2*e - 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*2*b*c**2*f*x**2 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**3*d -
 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**3*f*x**4 + 2*sqrt(a)*s
qrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x
)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c*f*x**2 - 2*sqrt(a)*sqrt(2*sqrt(c)*
sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a*b**2*c**2*d + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a*b**2*c**2*e*x**2 + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**
2*f*x**4 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**3*d*x**2 + ...
```

3.72 $\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 346

$$\begin{aligned}\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2} \, dx &= \frac{x(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} \\ &+ \frac{\left(bcd-2ace+abf+\frac{4abce+b^2(cd-af)-4ac(3cd+af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} \\ &+ \frac{\left(bcd-2ace+abf-\frac{4abce+b^2(cd-af)-4ac(3cd+af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output

```
1/2*x*(b^2*d-a*b*e-2*a*(-a*f+c*d)+(a*b*f-2*a*c*e+b*c*d)*x^2)/a/(-4*a*c+b^2
)/(c*x^4+b*x^2+a)+1/4*(b*c*d-2*a*c*e+a*b*f+(4*a*b*c*e+b^2*(-a*f+c*d)-4*a*c
*(a*f+3*c*d))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)
^(1/2))^(1/2))*2^(1/2)/a/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
+1/4*(b*c*d-2*a*c*e+a*b*f-(4*a*b*c*e+b^2*(-a*f+c*d)-4*a*c*(a*f+3*c*d))/(-4
*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^
(1/2)/a/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.10

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2x(b^2d+b(-ae+cdx^2+afx^2)+2a(af-c(d+ex^2)))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(b^2(cd-af)-2ac\left(6cd+\sqrt{b^2-4ac}e+2af\right)+b\left(c\sqrt{b^2-4ac}d+4ace+a\sqrt{b^2-4ac}f\right)\right)\text{ar}}{\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4a}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*x*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x^2)))
)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d - a*f) - 2*a*c*
(6*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e
 + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*(b^2*(-(c*d) + a*f) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b
*(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2206, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{x\left(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{db^2+aeb+(bcd-2ace+abf)x^2-2a(3cd+af)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{db^2+aeb+(bcd-2ace+abf)x^2-2a(3cd+af)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 1480

$$\frac{\frac{1}{2}\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(-\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+ab\right.}{2a\left(b^2-4ac\right)}$$

$$\frac{x\left(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 218

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2ace+bcd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}$$

$$\frac{x\left(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input

```
Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

output

```
(x*(b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2))/(2*a
*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (((b*c*d - 2*a*c*e + a*b*f + (4*a*b*
c*e + b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*e + a*b*f - (4*a*b*c*e + b^2*(c*d - a
*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
)/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.58

method	result
risch	$\frac{-\frac{(abf-2ace+bcd)x^3}{2a(4ac-b^2)}-\frac{(2fa^2-abe-2dac+b^2d)x}{2a(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{(abf-2ace+bcd)\_R^2}{4ac-b^2}+\frac{2fa^2-abe+6dac-b^2d}{4ac-b^2}\right)\ln}{2\_R^3c+\_Rb}}{4a}$
default	$\frac{-\frac{(abf-2ace+bcd)x^3}{2a(4ac-b^2)}-\frac{(2fa^2-abe-2dac+b^2d)x}{2a(4ac-b^2)}}{cx^4+bx^2+a}+\frac{2c\left(\frac{\left(-\sqrt{-4ac+b^2}\,abf+2ace\sqrt{-4ac+b^2}-bcd\sqrt{-4ac+b^2}-4a^2cf-ab^2f+4abce-12ac^2d\right.}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right.}{}$

input
```
int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)*x^3-1/2*(2*a^2*f-a*b*e-2*a*c*d+b
^2*d)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/a*sum((-(a*b*f-2*a*c*e+b*c*d)/(
4*a*c-b^2)*_R^2+(2*a^2*f-a*b*e+6*a*c*d-b^2*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)
*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8991 vs. $2(304) = 608$.

Time = 10.66 (sec) , antiderivative size = 8991, normalized size of antiderivative = 25.99

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b*c*d - 2*a*c*e + a*b*f)*x^3 - (a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*x
)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2
) + 1/2*integrate((a*b*e - 2*a^2*f + (b*c*d - 2*a*c*e + a*b*f)*x^2 + (b^2
- 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6348 vs. $2(304) = 608$.

Time = 1.05 (sec) , antiderivative size = 6348, normalized size of antiderivative = 18.35

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*d*x^3 - 2*a*c*e*x^3 + a*b*f*x^3 + b^2*d*x - 2*a*c*d*x - a*b*e*x +
 2*a^2*f*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^3 - 8
*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*
c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^2*c^3 - 8*a^2*c^4 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4
*a^2*c)^2*e + (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2*f + 2*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 14*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^5*c^2 - 2*a*b^6*c^2 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^2*c^3 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + sqrt(...
```

Mupad [B] (verification not implemented)

Time = 24.28 (sec) , antiderivative size = 19589, normalized size of antiderivative = 56.62

$$\int \frac{d + ex^2 + fx^4}{(a + bx^2 + cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
atan(((((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^
4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*
a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f
+ 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2)) - (x*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4
*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*
a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a
*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^
2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96
*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*
c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6
*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*
a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2
*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*
f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d
*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*
c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2
*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4
*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 -
a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*...
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 6214, normalized size of antiderivative = 17.96

$$\int \frac{d+ex^2+fx^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output

```
(8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c*f - 8*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**3*c**2*e - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a**2*b**2*c*e + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*
c*f*x**2 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqr
t(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2*d - 8*sq
rt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2*e*x**2 + 8*sqrt(a)*sqrt(
2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2*f*x**4 - 8*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a**2*c**3*e*x**4 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b
))*a*b**3*c*d - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c*e*x**2
+ 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**2*d*x**2 - 2*sqr...
```

3.73 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 430

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx$$
$$= -\frac{d}{a^2x} - \frac{x(b^3d - ab^2e + 2a^2ce - ab(3cd - af) + c(b^2d - abe - 2a(cd - af))x^2)}{2a^2(b^2-4ac)(a+bx^2+cx^4)}$$
$$- \frac{\sqrt{c}\left(3b^2d - abe - 2a(5cd - af) + \frac{3b^3d - ab^2e + 12a^2ce - 4ab(4cd + af)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\sqrt{c}(3b^3d - b^2(3\sqrt{b^2-4ac}d + ae) - ab(16cd - \sqrt{b^2-4ac}e + 4af) + 2a(5c\sqrt{b^2-4ac}d + 6ace - a\sqrt{b}}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
-d/a^2/x-1/2*x*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d)+c*(b^2*d-a*b*e-2*
a*(-a*f+c*d))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*c^(1/2)*(3*b^2*d-a
*b*e-2*a*(-a*f+5*c*d)+(3*b^3*d-a*b^2*e+12*a^2*c*e-4*a*b*(a*f+4*c*d))/(-4*a
*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1
/2)/a^2/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*c^(1/2)*(3*b^3*d-b^2
*(3*(-4*a*c+b^2)^(1/2)*d+a*e)-a*b*(16*c*d-(-4*a*c+b^2)^(1/2)*e+4*a*f)+2*a*
(5*c*d*(-4*a*c+b^2)^(1/2)+6*a*c*e-a*f*(-4*a*c+b^2)^(1/2)))*arctan(2^(1/2)*
c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(3/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.03

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$=\frac{-\frac{4d}{x}-\frac{2x\left(b^3d+b^2\left(-ae+cdx^2\right)+ab\left(af-c\left(3d+ex^2\right)\right)+2ac\left(-cdx^2+a\left(e+fx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3d+b^2\left(-3\sqrt{b^2-4ac}d+ae\right)+ab\left(16cd+\sqrt{b^2-\right.\right.}{(}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-4*d)/x - (2*x*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d + e*x
^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3*d + b^2*(-3*Sqrt[b^2 - 4*a*c]*d + a*e) +
a*b*(16*c*d + Sqrt[b^2 - 4*a*c]*e + 4*a*f) - 2*a*(-5*c*Sqrt[b^2 - 4*a*c]*d
 + 6*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*Sqrt[c]*(3*b^3*d - b^2*(3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-16*c*d
 + Sqrt[b^2 - 4*a*c]*e - 4*a*f) + 2*a*(5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e -
 a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \ 2198$$

$$-\frac{\int -\frac{-\frac{c\left(db^2-aeb-2a(cd-af)\right)x^4}{a}-\frac{\left(db^3-aeb^2-a(5cd+af)b+6a^2ce\right)x^2}{a}+2\left(b^2-4ac\right)d}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2\left(-abe-2a(cd-af)+b^2d\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{c\left(db^2-aeb-2a(cd-af)\right)x^4}{a}-\frac{\left(db^3-aeb^2-a(5cd+af)b+6a^2ce\right)x^2}{a}+2\left(b^2-4ac\right)d}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2\left(-abe-2a(cd-af)+b^2d\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 2195$$

$$\frac{\int \left(\frac{-3db^3+aeb^2+a(13cd+af)b-c\left(3db^2-aeb-2a(5cd-af)\right)x^2-6a^2ce}{a\left(cx^4+bx^2+a\right)}-\frac{2\left(4ac-b^2\right)d}{ax^2}\right)dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2\left(-abe-2a(cd-af)+b^2d\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12a^2ce-ab^2e-4ab(af+4cd)+3b^3d}{\sqrt{b^2-4ac}}-abe-2a(5cd-af)+3b^2d\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12a^2ce-ab^2e-4ab(a}{\sqrt{b^2-4a}}\right.}{\sqrt{2}a\sqrt{\sqrt{b^2-4}}}}{2a\left(b^2-4ac\right)}$$

$$\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2\left(-abe-2a(cd-af)+b^2d\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input

```
Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/2*(x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f)) + c*(b^2*d - a*
b*e - 2*a*(c*d - a*f))*x^2))/(a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((-
2*(b^2 - 4*a*c)*d)/(a*x) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) +
 (3*b^3*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) -
(3*b^3*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.02

method	result
default	$-\frac{d}{a^{2} x}+\frac{\frac{c\left(2 f a^{2}-a b e-2 d a c+b^{2} d\right) x^{3}}{8 a c-2 b^{2}}+\frac{\left(a^{2} b f+2 a^{2} c e-a b^{2} e-3 a b c d+b^{3} d\right) x}{8 a c-2 b^{2}}}{c x^{4}+b x^{2}+a}+\frac{2 c\left(\frac{\left(2 f a^{2} \sqrt{-4 a c+b^{2}}-a b e \sqrt{-4 a c+b^{2}}-10 \sqrt{-4 a c+b^{2}}\, a c d+3 \cdots\right.}{8 \cdots}\right.}{\cdots}$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-d/a^2/x+1/a^2*((1/2*c*(2*a^2*f-a*b*e-2*a*c*d+b^2*d)/(4*a*c-b^2)*x^3+1/2*(
a^2*b*f+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+
2/(4*a*c-b^2)*c*(1/8*(2*f*a^2*(-4*a*c+b^2)^(1/2)-a*b*e*(-4*a*c+b^2)^(1/2)-
10*(-4*a*c+b^2)^(1/2)*a*c*d+3*(-4*a*c+b^2)^(1/2)*b^2*d+4*a^2*b*f-12*a^2*c*
e+a*b^2*e+16*a*b*c*d-3*b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(
2*f*a^2*(-4*a*c+b^2)^(1/2)-a*b*e*(-4*a*c+b^2)^(1/2)-10*(-4*a*c+b^2)^(1/2)*
a*c*d+3*(-4*a*c+b^2)^(1/2)*b^2*d-4*a^2*b*f+12*a^2*c*e-a*b^2*e-16*a*b*c*d+3
*b^3*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13111 vs. 2(380) = 760.

Time = 25.49 (sec) , antiderivative size = 13111, normalized size of antiderivative = 30.49

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx = \int \frac{fx^4+ex^2+d}{(cx^4+bx^2+a)^2x^2}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b*f + (3*b^
3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*
b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x
) - 1/2*integrate(-(a^2*b*f + (a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*
d)*x^2 - (3*b^3 - 13*a*b*c)*d + (a*b^2 - 6*a^2*c)*e)/(c*x^4 + b*x^2 + a),
x)/(a^2*b^2 - 4*a^3*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7173 vs. $2(380) = 760$.

Time = 0.99 (sec) , antiderivative size = 7173, normalized size of antiderivative = 16.68

$$\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^2*c*d*x^4 - 10*a*c^2*d*x^4 - a*b*c*e*x^4 + 2*a^2*c*f*x^4 + 3*b^3
*d*x^2 - 11*a*b*c*d*x^2 - a*b^2*e*x^2 + 2*a^2*c*e*x^2 + a^2*b*f*x^2 + 2*a*
b^2*d - 8*a^2*c*d)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) + 1/16*((6*
b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c
)^2*d - (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*e + 2*(2*a^2
*b^2*c^2 - 8*a^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^...
```

Mupad [B] (verification not implemented)

Time = 24.74 (sec) , antiderivative size = 28164, normalized size of antiderivative = 65.50

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

output
```
((x^2*(3*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 11*a*b*c*d))/(2*a^2*(4*a*
c - b^2)) - d/a + (c*x^4*(3*b^2*d + 2*a^2*f - a*b*e - 10*a*c*d))/(2*a^2*(4
*a*c - b^2)))/(a*x + b*x^3 + c*x^5) - atan(((x*(204800*a^12*c^9*d^2 - 7372
8*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*
c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4
*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4
*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7
*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^
2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e +
40960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*
a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a
^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11
*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7
*c^4*e*f + 9216*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((27*a^3*b^9*
c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 -
a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840
*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2
+ 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*
b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
 - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504...
```

Reduce [F]

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{f\,x^4+e\,x^2+d}{x^2\left(c\,x^4+b\,x^2+a\right)^2}dx$$

input
```
int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x)
```

3.74 $\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 570

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}$$
$$-\frac{x(abc(bd-ae)-(b^2-2ac)\left(b^2d-abe-a(cd-af)\right)-c(b^3d-ab^2e+2a^2ce-ab(3cd-af))\,x^2)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$
$$+\frac{\sqrt{c}\left(5b^4d+b^3\left(5\sqrt{b^2-4ac}d-3ae\right)+2a^2c\left(14cd+5\sqrt{b^2-4ac}e-6af\right)-ab^2\left(29cd+3\sqrt{b^2-4ac}e-\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-}}}$$
$$-\frac{\sqrt{c}\left(5b^4d-b^3\left(5\sqrt{b^2-4ac}d+3ae\right)+2a^2c\left(14cd-5\sqrt{b^2-4ac}e-6af\right)-ab^2\left(29cd-3\sqrt{b^2-4ac}e-\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-}}}$$

output
```
-1/3*d/a^2/x^3+(-a*e+2*b*d)/a^3/x-1/2*x*(a*b*c*(-a*e+b*d)-(-2*a*c+b^2)*(b^
2*d-a*b*e-a*(-a*f+c*d))-c*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d))*x^2)/
a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/2)*(5*b^4*d+b^3*(5*(-4*a*c+b^2)^
(1/2)*d-3*a*e)+2*a^2*c*(14*c*d+5*(-4*a*c+b^2)^(1/2)*e-6*a*f)-a*b^2*(29*c*d
+3*(-4*a*c+b^2)^(1/2)*e-a*f)-a*b*(19*c*d*(-4*a*c+b^2)^(1/2)-16*a*c*e-a*f*(
-4*a*c+b^2)^(1/2)))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))
*2^(1/2)/a^3/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*c^(1/2)*(
5*b^4*d-b^3*(5*(-4*a*c+b^2)^(1/2)*d+3*a*e)+2*a^2*c*(14*c*d-5*(-4*a*c+b^2)^
(1/2)*e-6*a*f)-a*b^2*(29*c*d-3*(-4*a*c+b^2)^(1/2)*e-a*f)+a*b*(19*c*d*(-4*a
*c+b^2)^(1/2)+16*a*c*e-a*f*(-4*a*c+b^2)^(1/2)))*arctan(2^(1/2)*c^(1/2)*x/(
b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.96

$$\int \frac{d+ex^2+fx^4}{x^4 (a+bx^2+cx^4)^2}\,dx$$

$$= \frac{-\frac{4ad}{x^3}+\frac{24bd-12ae}{x}+\frac{6x(b^4d+b^3(-ae+cdx^2)+abc(3ae-3cdx^2+afx^2)+2a^2c(-af+c(d+ex^2))+ab^2(af-c(4d+ex^2)))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^4d\right.}{}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
 + a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x^2) + 2*a^2*c*(-(a*f) + c*(d + e*x^2))
+ a*b^2*(a*f - c*(4*d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3
*Sqrt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*
(14*c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4
*a*c]*e + a*f) + a*b*(-19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 -
4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2
- 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4*d
 + b^3*(5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a
*c]*e + a*f) + 2*a^2*c*(-14*c*d + 5*Sqrt[b^2 - 4*a*c]*e + 6*a*f) + a*b*(-1
9*c*Sqrt[b^2 - 4*a*c]*d - 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b +
Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Rubi [A] (verified)

Time = 4.85 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 2198}$$

$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{\frac{c\left(db^3-aeb^2-a(3cd-af)b+2a^2ce\right)x^6}{a^2}+\left(\frac{db^4}{a^2}-\frac{(6cd+be)b^2}{a}+fb^2+5ceb+6c^2d-6acf\right)x^4-\frac{2\left(b^2-4ac\right)(bd-ae)x^2}{a}+2\left(b^2-4ac\right)d}{x^4(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\frac{c\left(db^3-aeb^2-a(3cd-af)b+2a^2ce\right)x^6}{a^2}+\left(\frac{db^4}{a^2}-\frac{(6cd+be)b^2}{a}+fb^2+5ceb+6c^2d-6acf\right)x^4-\frac{2\left(b^2-4ac\right)(bd-ae)x^2}{a}+2\left(b^2-4ac\right)d}{x^4(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}+$$
$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 2195}$$

$$\frac{\int \left(-\frac{2(4ac-b^2)d}{ax^4}+\frac{5db^4-3aeb^3-a(24cd-af)b^2+13a^2ceb+c(5db^3-3aeb^2-a(19cd-af)b+10a^2ce)x^2+2a^2c(7cd-3af)}{a^2(cx^4+bx^2+a)}-\frac{2\left(4ac-b^2\right)(ae-2b}{a^2x^2}\right.}{2a\left(b^2-4ac\right)}$$
$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2a^2c\left(5e\sqrt{b^2-4ac}-6af+14cd\right)-ab^2\left(3e\sqrt{b^2-4ac}-af+29cd\right)-ab\left(19cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}-16ace\right)+b^3\left(5d\sqrt{b^2}\right.\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f -
 2*a*c*f) + c*(b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f))*x^2))/(2*a
^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((-2*(b^2 - 4*a*c)*d)/(3*a*x^3) +
(2*(b^2 - 4*a*c)*(2*b*d - a*e))/(a^2*x) + (Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[
b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f)
 - a*b^2*(29*c*d + 3*Sqrt[b^2 - 4*a*c]*e - a*f) - a*b*(19*c*Sqrt[b^2 - 4*a
*c]*d - 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) - (Sqrt[c]*(5*b^4*d - b^3*(5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + 2*
a^2*c*(14*c*d - 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) - a*b^2*(29*c*d - 3*Sqrt[b^
2 - 4*a*c]*e - a*f) + a*b*(19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e - a*Sqrt[b^
2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a
*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.00

method	result
default	$-\frac{d}{3a^2x^3}-\frac{ae-2bd}{a^3x}+\frac{-\frac{c\left(a^2bf+2a^2ce-a\,b^2e-3abcd+b^3d\right)x^3}{2\left(4ac-b^2\right)}+\frac{\left(2a^3cf-a^2b^2f-3a^2bce-2a^2c^2d+a\,b^3e+4a\,b^2cd-b^4d\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(-a^2bf\sqrt{\ }\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/a^2/x^3-(a*e-2*b*d)/a^3/x+1/a^3*((-1/2*c*(a^2*b*f+2*a^2*c*e-a*b^2*e
-3*a*b*c*d+b^3*d)/(4*a*c-b^2)*x^3+1/2*(2*a^3*c*f-a^2*b^2*f-3*a^2*b*c*e-2*a
^2*c^2*d+a*b^3*e+4*a*b^2*c*d-b^4*d)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*
c-b^2)*c*(1/8*(-a^2*b*f*(-4*a*c+b^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3
*a*b^2*e*(-4*a*c+b^2)^(1/2)+19*a*b*c*d*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+
b^2)^(1/2)-12*a^3*c*f+a^2*b^2*f+16*a^2*b*c*e+28*a^2*c^2*d-3*a*b^3*e-29*a*b
^2*c*d+5*b^4*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(-a^2*b*f*(-4*a
*c+b^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2)+1
9*a*b*c*d*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+b^2)^(1/2)+12*a^3*c*f-a^2*b^2
*f-16*a^2*b*c*e-28*a^2*c^2*d+3*a*b^3*e+29*a*b^2*c*d-5*b^4*d)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19333 vs. 2(505) = 1010.

Time = 80.65 (sec) , antiderivative size = 19333, normalized size of antiderivative = 33.92

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{fx^4+ex^2+d}{\left(cx^4+bx^2+a\right)^2x^4}\,dx$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*(3*(a^2*b*c*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)
*x^6 + ((15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d - 3*(3*a*b^3 - 11*a^2*b*c)*e
+ 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 - 4*a^2*b*c)*d - 3*(a^2*b^2 -
 4*a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*c - 4*a^4*c^2)*x^7 +
 (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) + 1/2*integrate(((a^
2*b*c*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*
b^4 - 24*a*b^2*c + 14*a^2*c^2)*d - (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6
*a^3*c)*f)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8649 vs. $2(505) = 1010$.

Time = 1.29 (sec) , antiderivative size = 8649, normalized size of antiderivative = 15.17

$$\int \frac{d + ex^2 + fx^4}{x^4 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 - a*b^2*c*e*x^3 + 2*a^2*c^2*e*x^3 + a^2
*b*c*f*x^3 + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x - a*b^3*e*x + 3*a^2*b
*c*e*x + a^2*b^2*f*x - 2*a^3*c*f*x)/((a^3*b^2 - 4*a^4*c)*(c*x^4 + b*x^2 +
a)) - 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c
)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^
3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4
+ 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c +
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a
*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e + (2*...
```

Mupad [B] (verification not implemented)

Time = 26.59 (sec) , antiderivative size = 36097, normalized size of antiderivative = 63.33

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

output
```
atan(((x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2
+ 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2
- 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^
8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*
c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b
^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8
*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2
*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^
8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*
c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^1
5*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^1
3*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896
*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a
^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) + (-(
25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b
^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 -
 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1
/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 1169
28*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a
^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)...
```

Reduce [F]

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{f\,x^4+e\,x^2+d}{x^4\left(c\,x^4+b\,x^2+a\right)^2}dx$$

input
```
int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x)
```

3.75 $\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 65

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{293x^2}{2}+\frac{49x^4}{2}-\frac{9x^6}{2}+\frac{5x^8}{8}-\frac{1}{2(1+x^2)}+\frac{208}{2+x^2}+2\log(1+x^2)+392\log(2+x^2)$$

output
```
-293/2*x^2+49/2*x^4-9/2*x^6+5/8*x^8-1/(2*x^2+2)+208/(x^2+2)+2*ln(x^2+1)+39
2*ln(x^2+2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{1}{8}\left(-1172x^2+196x^4-36x^6+5x^8+\frac{4(414+415x^2)}{2+3x^2+x^4}+16\log(1+x^2)+3136\log(2+x^2)\right)$$

input
```
Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(-1172*x^2 + 196*x^4 - 36*x^6 + 5*x^8 + (4*(414 + 415*x^2))/(2 + 3*x^2 + x
^4) + 16*Log[1 + x^2] + 3136*Log[2 + x^2])/8
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2191, 25, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2} dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^8 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{415x^2+414}{x^4+3x^2+2}-\int -\frac{5x^{10}-12x^8+27x^6-53x^4+105x^2+206}{x^4+3x^2+2}dx^2\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\int \frac{5x^{10}-12x^8+27x^6-53x^4+105x^2+206}{x^4+3x^2+2}dx^2+\frac{415x^2+414}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(\int \left(5x^6-27x^4+98x^2+\frac{4(197x^2+198)}{x^4+3x^2+2}-293\right)dx^2+\frac{415x^2+414}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{5x^8}{4}-9x^6+49x^4-293x^2+4\log(x^2+1)+784\log(x^2+2)+\frac{415x^2+414}{x^4+3x^2+2}\right)$$

input
```
Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(-293*x^2 + 49*x^4 - 9*x^6 + (5*x^8)/4 + (414 + 415*x^2)/(2 + 3*x^2 + x^4)
 + 4*Log[1 + x^2] + 784*Log[2 + x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result
default	$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+2\ln\left(x^2+1\right)-\frac{1}{2(x^2+1)}+392\ln\left(x^2+2\right)+\frac{208}{x^2+2}$
norman	$\frac{1086x^2-82x^6+\frac{49}{4}x^8-\frac{21}{8}x^{10}+\frac{5}{8}x^{12}+988}{x^4+3x^2+2}+2\ln\left(x^2+1\right)+392\ln\left(x^2+2\right)$
risch	$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+\frac{\frac{415x^2}{2}+207}{x^4+3x^2+2}+2\ln\left(x^2+1\right)+392\ln\left(x^2+2\right)$
parallelrisch	$\frac{5x^{12}-21x^{10}+98x^8-656x^6+16\ln(x^2+1)x^4+3136\ln(x^2+2)x^4+7904+48\ln(x^2+1)x^2+9408\ln(x^2+2)x^2+8688x^2+32\ln(x^2}{8x^4+24x^2+16}$

input
```
int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
5/8*x^8-9/2*x^6+49/2*x^4-293/2*x^2+2*ln(x^2+1)-1/2/(x^2+1)+392*ln(x^2+2)+2
08/(x^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$$
$$=\frac{5\,x^{12}-21\,x^{10}+98\,x^8-656\,x^6-3124\,x^4-684\,x^2+3136\,(x^4+3\,x^2+2)\log\left(x^2+2\right)+16\,(x^4+3\,x^2+}{8\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/8*(5*x^12 - 21*x^10 + 98*x^8 - 656*x^6 - 3124*x^4 - 684*x^2 + 3136*(x^4
+ 3*x^2 + 2)*log(x^2 + 2) + 16*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 1656)/(x^4
 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^8}{8} - \frac{9x^6}{2} + \frac{49x^4}{2} - \frac{293x^2}{2} + \frac{415x^2+414}{2x^4+6x^2+4} + 2\log\left(x^2+1\right) + 392\log\left(x^2+2\right)$$

input
```
integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**8/8 - 9*x**6/2 + 49*x**4/2 - 293*x**2/2 + (415*x**2 + 414)/(2*x**4 +
6*x**2 + 4) + 2*log(x**2 + 1) + 392*log(x**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{8}\,x^8 - \frac{9}{2}\,x^6 + \frac{49}{2}\,x^4 - \frac{293}{2}\,x^2 + \frac{415\,x^2+414}{2\,(x^4+3\,x^2+2)} + 392\,\log\left(x^2+2\right) + 2\,\log\left(x^2+1\right)$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 + 1/2*(415*x^2 + 414)/(x^4 + 3*x^
2 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{8}\,x^8 - \frac{9}{2}\,x^6 + \frac{49}{2}\,x^4 - \frac{293}{2}\,x^2 - \frac{394\,x^4+767\,x^2+374}{2\,(x^4+3\,x^2+2)} + 392\,\log\left(x^2+2\right) + 2\,\log\left(x^2+1\right)$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 - 1/2*(394*x^4 + 767*x^2 + 374)/(
x^4 + 3*x^2 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = 2 \ln\left(x^2+1\right) + 392 \ln\left(x^2+2\right) + \frac{\frac{415 x^2}{2}+207}{x^4+3 x^2+2} - \frac{293 x^2}{2} + \frac{49 x^4}{2} - \frac{9 x^6}{2} + \frac{5 x^8}{8}$$

input
```
int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
2*log(x^2 + 1) + 392*log(x^2 + 2) + ((415*x^2)/2 + 207)/(3*x^2 + x^4 + 2)
- (293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.57

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = \frac{3136 \log(x^2+2) x^4 + 9408 \log(x^2+2) x^2 + 6272 \log(x^2+2) + 16 \log(x^2+1) x^4 + 48 \log(x^2+1) x^2 + 3}{8x^4+24x^2+16}$$

input
```
int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

output
```
(3136*log(x**2 + 2)*x**4 + 9408*log(x**2 + 2)*x**2 + 6272*log(x**2 + 2) +
16*log(x**2 + 1)*x**4 + 48*log(x**2 + 1)*x**2 + 32*log(x**2 + 1) + 5*x**12
 - 21*x**10 + 98*x**8 - 656*x**6 - 2896*x**4 + 2112)/(8*(x**4 + 3*x**2 + 2
))
```

3.76 $\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 58

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}+\frac{1}{2(1+x^2)}-\frac{104}{2+x^2}-\frac{5}{2}\log(1+x^2)-144\log(2+x^2)$$

output
```
49*x^2-27/4*x^4+5/6*x^6+1/(2*x^2+2)-104/(x^2+2)-5/2*ln(x^2+1)-144*ln(x^2+2
)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}+\frac{-206-207x^2}{2(2+3x^2+x^4)}-\frac{5}{2}\log(1+x^2)-144\log(2+x^2)$$

input
```
Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
49*x^2 - (27*x^4)/4 + (5*x^6)/6 + (-206 - 207*x^2)/(2*(2 + 3*x^2 + x^4)) -
 (5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2194, 2191, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^6 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(-\int \frac{-5x^8+12x^6-27x^4+53x^2+102}{x^4+3x^2+2}dx^2-\frac{207x^2+206}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(-\int \left(-5x^4+27x^2+\frac{293x^2+298}{x^4+3x^2+2}-98\right)dx^2-\frac{207x^2+206}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{5x^6}{3}-\frac{27x^4}{2}+98x^2-5\log\left(x^2+1\right)-288\log\left(x^2+2\right)-\frac{207x^2+206}{x^4+3x^2+2}\right)$$

input
```
Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(98*x^2 - (27*x^4)/2 + (5*x^6)/3 - (206 + 207*x^2)/(2 + 3*x^2 + x^4) - 5*L
og[1 + x^2] - 288*Log[2 + x^2])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

method	result
default	$49x^2 - \frac{27x^4}{4} + \frac{5x^6}{6} + \frac{1}{2x^2+2} - \frac{104}{x^2+2} - \frac{5\ln(x^2+1)}{2} - 144\ln(x^2+2)$
norman	$\frac{-406x^2+\frac{365}{12}x^6-\frac{17}{4}x^8+\frac{5}{6}x^{10}-370}{x^4+3x^2+2} - \frac{5\ln(x^2+1)}{2} - 144\ln(x^2+2)$
risch	$\frac{5x^6}{6} - \frac{27x^4}{4} + 49x^2 + \frac{-\frac{207x^2}{2}-103}{x^4+3x^2+2} - \frac{5\ln(x^2+1)}{2} - 144\ln(x^2+2)$
parallelrisch	$-\frac{-10x^{10}+51x^8-365x^6+30\ln(x^2+1)x^4+1728\ln(x^2+2)x^4+4440+90\ln(x^2+1)x^2+5184\ln(x^2+2)x^2+4872x^2+60\ln(x^2+}{12(x^4+3x^2+2)}$

input
```
int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
5/6*x^6-27/4*x^4+49*x^2-5/2*ln(x^2+1)+1/2/(x^2+1)-144*ln(x^2+2)-104/(x^2+2
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{10\,x^{10}-51\,x^8+365\,x^6+1602\,x^4-66\,x^2-1728\,(x^4+3\,x^2+2)\log(x^2+2)-30\,(x^4+3\,x^2+2)\log(x^2}{12\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/12*(10*x^10 - 51*x^8 + 365*x^6 + 1602*x^4 - 66*x^2 - 1728*(x^4 + 3*x^2 +
 2)*log(x^2 + 2) - 30*(x^4 + 3*x^2 + 2)*log(x^2 + 1) - 1236)/(x^4 + 3*x^2
+ 2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^6}{6}-\frac{27x^4}{4}+49x^2+\frac{-207x^2-206}{2x^4+6x^2+4} - \frac{5\log(x^2+1)}{2}-144\log\left(x^2+2\right)$$

input
```
integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**6/6 - 27*x**4/4 + 49*x**2 + (-207*x**2 - 206)/(2*x**4 + 6*x**2 + 4) -
 5*log(x**2 + 1)/2 - 144*log(x**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{27}{4}\,x^4 + 49\,x^2 - \frac{207\,x^2+206}{2\,(x^4+3\,x^2+2)} - 144\,\log\left(x^2+2\right) - \frac{5}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/6*x^6 - 27/4*x^4 + 49*x^2 - 1/2*(207*x^2 + 206)/(x^4 + 3*x^2 + 2) - 144*
log(x^2 + 2) - 5/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{27}{4}\,x^4 + 49\,x^2 + \frac{293\,x^4+465\,x^2+174}{4\,(x^4+3\,x^2+2)} - 144\,\log\left(x^2+2\right) - \frac{5}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/6*x^6 - 27/4*x^4 + 49*x^2 + 1/4*(293*x^4 + 465*x^2 + 174)/(x^4 + 3*x^2 +
 2) - 144*log(x^2 + 2) - 5/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 49\,x^2-144\,\ln\left(x^2+2\right)-\frac{\frac{207\,x^2}{2}+103}{x^4+3\,x^2+2}-\frac{5\,\ln\left(x^2+1\right)}{2}-\frac{27\,x^4}{4}+\frac{5\,x^6}{6}$$

input
```
int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
49*x^2 - 144*log(x^2 + 2) - ((207*x^2)/2 + 103)/(3*x^2 + x^4 + 2) - (5*log
(x^2 + 1))/2 - (27*x^4)/4 + (5*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{-1728\log(x^2+2)\,x^4-5184\log(x^2+2)\,x^2-3456\log(x^2+2)-30\log(x^2+1)\,x^4-90\log(x^2+1)\,x^2-}{12x^4+36x^2+24}$$

input
```
int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

output
```
( - 1728*log(x**2 + 2)*x**4 - 5184*log(x**2 + 2)*x**2 - 3456*log(x**2 + 2)
 - 30*log(x**2 + 1)*x**4 - 90*log(x**2 + 1)*x**2 - 60*log(x**2 + 1) + 10*x
**10 - 51*x**8 + 365*x**6 + 1624*x**4 - 1192)/(12*(x**4 + 3*x**2 + 2))
```

3.77 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 51

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{27x^2}{2}+\frac{5x^4}{4}-\frac{1}{2(1+x^2)}+\frac{52}{2+x^2} +3\log\left(1+x^2\right)+46\log\left(2+x^2\right)$$

output
```
-27/2*x^2+5/4*x^4-1/(2*x^2+2)+52/(x^2+2)+3*ln(x^2+1)+46*ln(x^2+2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2(2+3x^2+x^4)} +3\log\left(1+x^2\right)+46\log\left(2+x^2\right)$$

input
```
Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1
+ x^2] + 46*Log[2 + x^2]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2191, 25, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2} dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^4 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{103x^2+102}{x^4+3x^2+2}-\int -\frac{5x^6-12x^4+27x^2+50}{x^4+3x^2+2}dx^2\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\int \frac{5x^6-12x^4+27x^2+50}{x^4+3x^2+2}dx^2+\frac{103x^2+102}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(\int \left(5x^2+\frac{2(49x^2+52)}{x^4+3x^2+2}-27\right)dx^2+\frac{103x^2+102}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{5x^4}{2}-27x^2+6\log(x^2+1)+92\log(x^2+2)+\frac{103x^2+102}{x^4+3x^2+2}\right)$$

input
```
Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(-27*x^2 + (5*x^4)/2 + (102 + 103*x^2)/(2 + 3*x^2 + x^4) + 6*Log[1 + x^2]
+ 92*Log[2 + x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result
default	$\frac{5x^4}{4}-\frac{27x^2}{2}+3\ln\left(x^2+1\right)-\frac{1}{2(x^2+1)}+46\ln\left(x^2+2\right)+\frac{52}{x^2+2}$
norman	$\frac{\frac{277}{2}x^2-\frac{39}{4}x^6+\frac{5}{4}x^8+127}{x^4+3x^2+2}+3\ln\left(x^2+1\right)+46\ln\left(x^2+2\right)$
risch	$\frac{5x^4}{4}-\frac{27x^2}{2}+\frac{729}{20}+\frac{\frac{103x^2}{2}+51}{x^4+3x^2+2}+3\ln\left(x^2+1\right)+46\ln\left(x^2+2\right)$
parallelrisch	$\frac{5x^8-39x^6+12\ln(x^2+1)x^4+184\ln(x^2+2)x^4+508+36\ln(x^2+1)x^2+552\ln(x^2+2)x^2+554x^2+24\ln(x^2+1)+368\ln(x^2+2)}{4x^4+12x^2+8}$

input
```
int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
5/4*x^4-27/2*x^2+3*ln(x^2+1)-1/2/(x^2+1)+46*ln(x^2+2)+52/(x^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5\,x^8-39\,x^6-152\,x^4+98\,x^2+184\,(x^4+3\,x^2+2)\log(x^2+2)+12\,(x^4+3\,x^2+2)\log(x^2+1)+204}{4\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/4*(5*x^8 - 39*x^6 - 152*x^4 + 98*x^2 + 184*(x^4 + 3*x^2 + 2)*log(x^2 + 2
) + 12*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 204)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^4}{4}-\frac{27x^2}{2}+\frac{103x^2+102}{2x^4+6x^2+4}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)$$

input
```
integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**4/4 - 27*x**2/2 + (103*x**2 + 102)/(2*x**4 + 6*x**2 + 4) + 3*log(x**2
 + 1) + 46*log(x**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{4}\,x^4 - \frac{27}{2}\,x^2 + \frac{103\,x^2+102}{2\,(x^4+3\,x^2+2)} + 46\,\log\left(x^2+2\right) + 3\,\log\left(x^2+1\right)$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/4*x^4 - 27/2*x^2 + 1/2*(103*x^2 + 102)/(x^4 + 3*x^2 + 2) + 46*log(x^2 +
2) + 3*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{4}\,x^4 - \frac{27}{2}\,x^2 - \frac{49\,x^4+44\,x^2-4}{2\,(x^4+3\,x^2+2)} + 46\,\log\left(x^2+2\right) + 3\,\log\left(x^2+1\right)$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/4*x^4 - 27/2*x^2 - 1/2*(49*x^4 + 44*x^2 - 4)/(x^4 + 3*x^2 + 2) + 46*log(
x^2 + 2) + 3*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 18.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 3\,\ln\left(x^2+1\right) + 46\,\ln\left(x^2+2\right) + \frac{\frac{103\,x^2}{2}+51}{x^4+3\,x^2+2} - \frac{27\,x^2}{2} + \frac{5\,x^4}{4}$$

input
```
int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
3*log(x^2 + 1) + 46*log(x^2 + 2) + ((103*x^2)/2 + 51)/(3*x^2 + x^4 + 2) -
(27*x^2)/2 + (5*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$$
$$= \frac{552\log(x^2+2)\,x^4+1656\log(x^2+2)\,x^2+1104\log(x^2+2)+36\log(x^2+1)\,x^4+108\log(x^2+1)\,x^2+7}{12x^4+36x^2+24}$$

input
```
int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

output
```
(552*log(x**2 + 2)*x**4 + 1656*log(x**2 + 2)*x**2 + 1104*log(x**2 + 2) + 3
6*log(x**2 + 1)*x**4 + 108*log(x**2 + 1)*x**2 + 72*log(x**2 + 1) + 15*x**8
 - 117*x**6 - 554*x**4 + 416)/(12*(x**4 + 3*x**2 + 2))
```

3.78 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 46

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^2}{2}+\frac{1}{2(1+x^2)}-\frac{26}{2+x^2}-\frac{7}{2}\log(1+x^2)-10\log(2+x^2)$$

output
```
5/2*x^2+1/(2*x^2+2)-26/(x^2+2)-7/2*ln(x^2+1)-10*ln(x^2+2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^2}{2}+\frac{-50-51x^2}{2(2+3x^2+x^4)}-\frac{7}{2}\log(1+x^2)-10\log(2+x^2)$$

input
```
Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(5*x^2)/2 + (-50 - 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10
*Log[2 + x^2]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2194, 2191, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{x^2(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx^2$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{2}\left(-\int \frac{-5x^4+12x^2+24}{x^4+3x^2+2}dx^2-\frac{51x^2+50}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{ 2188}$$

$$\frac{1}{2}\left(-\int \left(\frac{27x^2+34}{x^4+3x^2+2}-5\right)dx^2-\frac{51x^2+50}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(5x^2-7\log\left(x^2+1\right)-20\log\left(x^2+2\right)-\frac{51x^2+50}{x^4+3x^2+2}\right)$$

input
```
Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(5*x^2 - (50 + 51*x^2)/(2 + 3*x^2 + x^4) - 7*Log[1 + x^2] - 20*Log[2 + x^2
])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{5x^2}{2}+\frac{1}{2x^2+2}-\frac{26}{x^2+2}-\frac{7\ln(x^2+1)}{2}-10\ln\left(x^2+2\right)$	41
norman	$\frac{-43x^2+\frac{5}{2}x^6-40}{x^4+3x^2+2}-\frac{7\ln(x^2+1)}{2}-10\ln\left(x^2+2\right)$	43
risch	$\frac{5x^2}{2}+\frac{-\frac{51x^2}{2}-25}{x^4+3x^2+2}-\frac{7\ln(x^2+1)}{2}-10\ln\left(x^2+2\right)$	43
parallelrisch	$-\frac{-5x^6+7\ln(x^2+1)x^4+20\ln(x^2+2)x^4+80+21\ln(x^2+1)x^2+60\ln(x^2+2)x^2+86x^2+14\ln(x^2+1)+40\ln(x^2+2)}{2(x^4+3x^2+2)}$	87

input
```
int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
5/2*x^2-7/2*ln(x^2+1)+1/2/(x^2+1)-10*ln(x^2+2)-26/(x^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5\,x^6+15\,x^4-41\,x^2-20\,(x^4+3\,x^2+2)\log\left(x^2+2\right)-7\,(x^4+3\,x^2+2)\log\left(x^2+1\right)-50}{2\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/2*(5*x^6 + 15*x^4 - 41*x^2 - 20*(x^4 + 3*x^2 + 2)*log(x^2 + 2) - 7*(x^4
+ 3*x^2 + 2)*log(x^2 + 1) - 50)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^2}{2}+\frac{-51x^2-50}{2x^4+6x^2+4}-\frac{7\log\left(x^2+1\right)}{2}-10\log\left(x^2+2\right)$$

input
```
integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**2/2 + (-51*x**2 - 50)/(2*x**4 + 6*x**2 + 4) - 7*log(x**2 + 1)/2 - 10*
log(x**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{2}\,x^2 - \frac{51\,x^2+50}{2\,(x^4+3\,x^2+2)} - 10\,\log\left(x^2+2\right) - \frac{7}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/2*x^2 - 1/2*(51*x^2 + 50)/(x^4 + 3*x^2 + 2) - 10*log(x^2 + 2) - 7/2*log(
x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{2}\,x^2 - \frac{51\,x^2+50}{2\,(x^2+2)(x^2+1)} - 10\,\log\left(x^2+2\right) - \frac{7}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/2*x^2 - 1/2*(51*x^2 + 50)/((x^2 + 2)*(x^2 + 1)) - 10*log(x^2 + 2) - 7/2*
log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 18.56 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5\,x^2}{2} - 10\,\ln\left(x^2+2\right) - \frac{\frac{51\,x^2}{2}+25}{x^4+3\,x^2+2} - \frac{7\,\ln\left(x^2+1\right)}{2}$$

input
```
int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
(5*x^2)/2 - 10*log(x^2 + 2) - ((51*x^2)/2 + 25)/(3*x^2 + x^4 + 2) - (7*log
(x^2 + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.89

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$$
$$= \frac{-60\log(x^2+2)\,x^4-180\log(x^2+2)\,x^2-120\log(x^2+2)-21\log(x^2+1)\,x^4-63\log(x^2+1)\,x^2-42\,\mathrm{l}}{6x^4+18x^2+12}$$

input
```
int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

output
```
( - 60*log(x**2 + 2)*x**4 - 180*log(x**2 + 2)*x**2 - 120*log(x**2 + 2) - 2
1*log(x**2 + 1)*x**4 - 63*log(x**2 + 1)*x**2 - 42*log(x**2 + 1) + 15*x**6
+ 86*x**4 - 68)/(6*(x**4 + 3*x**2 + 2))
```

3.79 $\int \frac{x\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 39

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx=-\frac{1}{2(1+x^2)}+\frac{13}{2+x^2}+4\log\left(1+x^2\right)-\frac{3}{2}\log\left(2+x^2\right)$$

output
```
-1/2/(x^2+1)+13/(x^2+2)+4*ln(x^2+1)-3/2*ln(x^2+2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx=\frac{24+25x^2}{2(2+3x^2+x^4)}+4\log\left(1+x^2\right)-\frac{3}{2}\log\left(2+x^2\right)$$

input
```
Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2194, 2191, 25, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}\,dx$$

$$\Big\downarrow \; 2194$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{(x^4+3x^2+2)^2}dx^2$$

$$\Big\downarrow \; 2191$$

$$\frac{1}{2}\left(\frac{25x^2+24}{x^4+3x^2+2}-\int -\frac{5x^2+13}{x^4+3x^2+2}dx^2\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\int \frac{5x^2+13}{x^4+3x^2+2}dx^2+\frac{25x^2+24}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \; 1141$$

$$\frac{1}{2}\left(\int \left(\frac{8}{x^2+1}-\frac{3}{x^2+2}\right)dx^2+\frac{25x^2+24}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(8\log(x^2+1)-3\log(x^2+2)+\frac{25x^2+24}{x^4+3x^2+2}\right)$$

input
```
Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
((24 + 25*x^2)/(2 + 3*x^2 + x^4) + 8*Log[1 + x^2] - 3*Log[2 + x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{1}{2(x^2+1)}+\frac{13}{x^2+2}+4\ln\left(x^2+1\right)-\frac{3\ln\left(x^2+2\right)}{2}$	36
norman	$\frac{\frac{25x^2}{2}+12}{x^4+3x^2+2}+4\ln\left(x^2+1\right)-\frac{3\ln\left(x^2+2\right)}{2}$	38
risch	$\frac{\frac{25x^2}{2}+12}{x^4+3x^2+2}+4\ln\left(x^2+1\right)-\frac{3\ln\left(x^2+2\right)}{2}$	38
parallelrisch	$\frac{8\ln\left(x^2+1\right)x^4-3\ln\left(x^2+2\right)x^4+24+24\ln\left(x^2+1\right)x^2-9\ln\left(x^2+2\right)x^2+25x^2+16\ln\left(x^2+1\right)-6\ln\left(x^2+2\right)}{2x^4+6x^2+4}$	82

input
```
int(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^2+1)+13/(x^2+2)+4*ln(x^2+1)-3/2*ln(x^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25\,x^2-3\,(x^4+3\,x^2+2)\log\left(x^2+2\right)+8\,(x^4+3\,x^2+2)\log\left(x^2+1\right)+24}{2\,(x^4+3\,x^2+2)}$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/2*(25*x^2 - 3*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 8*(x^4 + 3*x^2 + 2)*log(x
^2 + 1) + 24)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25x^2+24}{2x^4+6x^2+4} + 4\log\left(x^2+1\right) - \frac{3\log\left(x^2+2\right)}{2}$$

input
```
integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
(25*x**2 + 24)/(2*x**4 + 6*x**2 + 4) + 4*log(x**2 + 1) - 3*log(x**2 + 2)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25\,x^2+24}{2\,(x^4+3\,x^2+2)} - \frac{3}{2}\,\log\left(x^2+2\right) + 4\,\log\left(x^2+1\right)$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
1/2*(25*x^2 + 24)/(x^4 + 3*x^2 + 2) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{25\,x^2+24}{2\,(x^2+2)(x^2+1)} - \frac{3}{2}\,\log\left(x^2+2\right) + 4\,\log\left(x^2+1\right)$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
1/2*(25*x^2 + 24)/((x^2 + 2)*(x^2 + 1)) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1
)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 4\,\ln\left(x^2+1\right) - \frac{3\,\ln\left(x^2+2\right)}{2} + \frac{\frac{25\,x^2}{2}+12}{x^4+3\,x^2+2}$$

input
```
int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
4*log(x^2 + 1) - (3*log(x^2 + 2))/2 + ((25*x^2)/2 + 12)/(3*x^2 + x^4 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$$
$$= \frac{-9\log(x^2+2)\,x^4 - 27\log(x^2+2)\,x^2 - 18\log(x^2+2) + 24\log(x^2+1)\,x^4 + 72\log(x^2+1)\,x^2 + 48\log(x}{6x^4+18x^2+12}$$

input
```
int(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

output
```
( - 9*log(x**2 + 2)*x***4 - 27*log(x**2 + 2)*x***2 - 18*log(x**2 + 2) + 24*l
og(x**2 + 1)*x***4 + 72*log(x**2 + 1)*x***2 + 48*log(x**2 + 1) - 25*x***4 + 2
2)/(6*(x**4 + 3*x**2 + 2))
```

3.80 $\int \frac{4+x^2+3x^4+5x^6}{x(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 43

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx=\frac{1}{2\left(1+x^2\right)}-\frac{13}{2\left(2+x^2\right)}+\log(x)-\frac{9}{2}\log\left(1+x^2\right)+4\log\left(2+x^2\right)$$

output
```
1/(2*x^2+2)-13/(2*x^2+4)+ln(x)-9/2*ln(x^2+1)+4*ln(x^2+2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx=\frac{-11-12x^2}{2\left(2+3x^2+x^4\right)}+\log(x)-\frac{9}{2}\log\left(1+x^2\right)+4\log\left(2+x^2\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-11 - 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log
[2 + x^2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2177, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x\left(x^4+3x^2+2\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{x^2\left(x^4+3x^2+2\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(-\int -\frac{2-7x^2}{x^2\left(x^4+3x^2+2\right)}dx^2-\frac{12x^2+11}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\int \frac{2-7x^2}{x^2\left(x^4+3x^2+2\right)}dx^2-\frac{12x^2+11}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(\int \left(\frac{8}{x^2+2}+\frac{1}{x^2}-\frac{9}{x^2+1}\right)dx^2-\frac{12x^2+11}{x^4+3x^2+2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\log\left(x^2\right)-9\log\left(x^2+1\right)+8\log\left(x^2+2\right)-\frac{12x^2+11}{x^4+3x^2+2}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-((11 + 12*x^2)/(2 + 3*x^2 + x^4)) + Log[x^2] - 9*Log[1 + x^2] + 8*Log[2
+ x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result
default	$-\frac{9\ln(x^2+1)}{2}+\frac{1}{2x^2+2}+4\ln\left(x^2+2\right)-\frac{13}{2(x^2+2)}+\ln\left(x\right)$
norman	$\frac{-6x^2-\frac{11}{2}}{x^4+3x^2+2}-\frac{9\ln(x^2+1)}{2}+4\ln\left(x^2+2\right)+\ln\left(x\right)$
risch	$\frac{-6x^2-\frac{11}{2}}{x^4+3x^2+2}-\frac{9\ln(x^2+1)}{2}+4\ln\left(x^2+2\right)+\ln\left(x\right)$
parallelrisch	$\frac{2\ln(x)x^4-9\ln(x^2+1)x^4+8\ln(x^2+2)x^4-11+6\ln(x)x^2-27\ln(x^2+1)x^2+24\ln(x^2+2)x^2-12x^2+4\ln(x)-18\ln(x^2+1)+16\,\mathrm{l}}{2x^4+6x^2+4}$

input
```
int((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-9/2*ln(x^2+1)+1/2/(x^2+1)+4*ln(x^2+2)-13/2/(x^2+2)+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{4+x^2+3x^4+5x^6}{x(2+3x^2+x^4)^2}\,dx = -\frac{12\,x^2-8\left(x^4+3\,x^2+2\right)\log\left(x^2+2\right)+9\left(x^4+3\,x^2+2\right)\log\left(x^2+1\right)-2\left(x^4+3\,x^2+2\right)\log\left(x\right)+11}{2\left(x^4+3\,x^2+2\right)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
-1/2*(12*x^2 - 8*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 9*(x^4 + 3*x^2 + 2)*log(
x^2 + 1) - 2*(x^4 + 3*x^2 + 2)*log(x) + 11)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx=\frac{-12x^2-11}{2x^4+6x^2+4}+\log\left(x\right)-\frac{9\log\left(x^2+1\right)}{2}+4\log\left(x^2+2\right)$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+3*x**2+2)**2,x)
```

output
```
(-12*x**2 - 11)/(2*x**4 + 6*x**2 + 4) + log(x) - 9*log(x**2 + 1)/2 + 4*log
(x**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx=-\frac{12\,x^2+11}{2\left(x^4+3\,x^2+2\right)}+4\,\log\left(x^2+2\right)-\frac{9}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
-1/2*(12*x^2 + 11)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 + 1) +
 1/2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx=\frac{x^4-21\,x^2-20}{4\left(x^4+3\,x^2+2\right)}+4\,\log\left(x^2+2\right)-\frac{9}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
1/4*(x^4 - 21*x^2 - 20)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 +
 1) + 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = 4\,\ln\left(x^2+2\right) - \frac{9\,\ln\left(x^2+1\right)}{2} + \ln\left(x\right) - \frac{6\,x^2+\frac{11}{2}}{x^4+3\,x^2+2}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(3*x^2 + x^4 + 2)^2),x)
```

output
```
4*log(x^2 + 2) - (9*log(x^2 + 1))/2 + log(x) - (6*x^2 + 11/2)/(3*x^2 + x^4
 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.33

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx$$
$$= \frac{8\log(x^2+2)\,x^4+24\log(x^2+2)\,x^2+16\log(x^2+2)-9\log(x^2+1)\,x^4-27\log(x^2+1)\,x^2-18\log(x^2}{2x^4+6x^2+4}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x)
```

output
```
(8*log(x**2 + 2)*x**4 + 24*log(x**2 + 2)*x**2 + 16*log(x**2 + 2) - 9*log(x
**2 + 1)*x**4 - 27*log(x**2 + 1)*x**2 - 18*log(x**2 + 1) + 2*log(x)*x**4 +
 6*log(x)*x**2 + 4*log(x) + 4*x**4 - 3)/(2*(x**4 + 3*x**2 + 2))
```

3.81 $\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 54

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2} dx = -\frac{1}{2x^2} - \frac{1}{2(1+x^2)} + \frac{13}{4(2+x^2)} - \frac{11\log(x)}{4} + 5\log(1+x^2) - \frac{29}{8}\log(2+x^2)$$

output
```
-1/2/x^2-1/(2*x^2+2)+13/(4*x^2+8)-11/4*ln(x)+5*ln(x^2+1)-29/8*ln(x^2+2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2} dx = \frac{1}{8}\left(-\frac{4}{x^2} + \frac{18+22x^2}{2+3x^2+x^4} - 22\log(x) + 40\log(1+x^2) - 29\log(2+x^2)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-4/x^2 + (18 + 22*x^2)/(2 + 3*x^2 + x^4) - 22*Log[x] + 40*Log[1 + x^2] -
29*Log[2 + x^2])/8
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^3 \left(x^4+3x^2+2\right)^2} \, dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2} \int \frac{5x^6+3x^4+x^2+4}{x^4 \left(x^4+3x^2+2\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2} \left(\frac{11x^2+9}{2 \left(x^4+3x^2+2\right)}-\int -\frac{11x^4-5x^2+4}{2x^4 \left(x^4+3x^2+2\right)}dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{1}{2} \int \frac{11x^4-5x^2+4}{x^4 \left(x^4+3x^2+2\right)}dx^2+\frac{11x^2+9}{2 \left(x^4+3x^2+2\right)}\right)$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2} \left(\frac{1}{2} \int \left(-\frac{29}{2 \left(x^2+2\right)}-\frac{11}{2x^2}+\frac{2}{x^4}+\frac{20}{x^2+1}\right)dx^2+\frac{11x^2+9}{2 \left(x^4+3x^2+2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{1}{2} \left(-\frac{2}{x^2}-\frac{11}{2} \log \left(x^2\right)+20 \log \left(x^2+1\right)-\frac{29}{2} \log \left(x^2+2\right)\right)+\frac{11x^2+9}{2 \left(x^4+3x^2+2\right)}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

output
```
((9 + 11*x^2)/(2*(2 + 3*x^2 + x^4)) + (-2/x^2 - (11*Log[x^2])/2 + 20*Log[1
 + x^2] - (29*Log[2 + x^2])/2)/2)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result
default	$5\ln\left(x^{2}+1\right)-\frac{1}{2\left(x^{2}+1\right)}-\frac{29\ln\left(x^{2}+2\right)}{8}+\frac{13}{4\left(x^{2}+2\right)}-\frac{1}{2x^{2}}-\frac{11\ln\left(x\right)}{4}$
norman	$\frac{-1+\frac{3}{4}x^{2}+\frac{9}{4}x^{4}}{x^{2}\left(x^{4}+3x^{2}+2\right)}-\frac{11\ln\left(x\right)}{4}+5\ln\left(x^{2}+1\right)-\frac{29\ln\left(x^{2}+2\right)}{8}$
risch	$\frac{-1+\frac{3}{4}x^{2}+\frac{9}{4}x^{4}}{x^{2}\left(x^{4}+3x^{2}+2\right)}-\frac{11\ln\left(x\right)}{4}+5\ln\left(x^{2}+1\right)-\frac{29\ln\left(x^{2}+2\right)}{8}$
parallelrisch	$-\frac{22\ln\left(x\right)x^{6}-40\ln\left(x^{2}+1\right)x^{6}+29\ln\left(x^{2}+2\right)x^{6}+8+66\ln\left(x\right)x^{4}-120\ln\left(x^{2}+1\right)x^{4}+87\ln\left(x^{2}+2\right)x^{4}-18x^{4}+44\ln\left(x\right)x^{2}-80\ln\left(x\right.}{8x^{2}\left(x^{4}+3x^{2}+2\right)}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
5*ln(x^2+1)-1/2/(x^2+1)-29/8*ln(x^2+2)+13/4/(x^2+2)-1/2/x^2-11/4*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(44) = 88.

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \frac{4+x^{2}+3x^{4}+5x^{6}}{x^{3}\left(2+3x^{2}+x^{4}\right)^{2}}\,dx = \frac{18\,x^{4}+6\,x^{2}-29\,\left(x^{6}+3\,x^{4}+2\,x^{2}\right)\log\left(x^{2}+2\right)+40\,\left(x^{6}+3\,x^{4}+2\,x^{2}\right)\log\left(x^{2}+1\right)-22\,\left(x^{6}+3\,x^{4}+2\,x\right.}{8\,\left(x^{6}+3\,x^{4}+2\,x^{2}\right)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/8*(18*x^4 + 6*x^2 - 29*(x^6 + 3*x^4 + 2*x^2)*log(x^2 + 2) + 40*(x^6 + 3*
x^4 + 2*x^2)*log(x^2 + 1) - 22*(x^6 + 3*x^4 + 2*x^2)*log(x) - 8)/(x^6 + 3*
x^4 + 2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = \frac{9x^4+3x^2-4}{4x^6+12x^4+8x^2} - \frac{11\log\left(x\right)}{4} + 5\log\left(x^2+1\right) - \frac{29\log\left(x^2+2\right)}{8}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+3*x**2+2)**2,x)
```

output
```
(9*x**4 + 3*x**2 - 4)/(4*x**6 + 12*x**4 + 8*x**2) - 11*log(x)/4 + 5*log(x*
*2 + 1) - 29*log(x**2 + 2)/8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = \frac{9\,x^4+3\,x^2-4}{4\left(x^6+3\,x^4+2\,x^2\right)} - \frac{29}{8}\,\log\left(x^2+2\right) + 5\,\log\left(x^2+1\right) - \frac{11}{8}\,\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(
x^2 + 1) - 11/8*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = \frac{9\,x^4+3\,x^2-4}{4\,(x^6+3\,x^4+2\,x^2)} - \frac{29}{8}\,\log\left(x^2+2\right) + 5\,\log\left(x^2+1\right) - \frac{11}{8}\,\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(
x^2 + 1) - 11/8*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 18.63 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = 5\,\ln\left(x^2+1\right) - \frac{29\,\ln\left(x^2+2\right)}{8} - \frac{11\,\ln\left(x\right)}{4} + \frac{\frac{9\,x^4}{4}+\frac{3\,x^2}{4}-1}{x^6+3\,x^4+2\,x^2}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(3*x^2 + x^4 + 2)^2),x)
```

output
```
5*log(x^2 + 1) - (29*log(x^2 + 2))/8 - (11*log(x))/4 + ((3*x^2)/4 + (9*x^4
)/4 - 1)/(2*x^2 + 3*x^4 + x^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.15

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = \frac{-29\log(x^2+2)\,x^6-87\log(x^2+2)\,x^4-58\log(x^2+2)\,x^2+40\log(x^2+1)\,x^6+120\log(x^2+1)\,x^4+80}{8x^2\left(x^4+3x^2+2\right)}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x)
```

output
```
( - 29*log(x**2 + 2)*x***6 - 87*log(x**2 + 2)*x***4 - 58*log(x**2 + 2)*x***2
+ 40*log(x**2 + 1)*x***6 + 120*log(x**2 + 1)*x***4 + 80*log(x**2 + 1)*x***2 -
 22*log(x)*x***6 - 66*log(x)*x***4 - 44*log(x)*x***2 - 6*x**6 - 6*x**2 - 8)/(
8*x**2*(x**4 + 3*x**2 + 2))
```

3.82 $\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 63

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = -\frac{1}{4x^4}+\frac{11}{8x^2}+\frac{1}{2\left(1+x^2\right)}-\frac{13}{8\left(2+x^2\right)} +\frac{23\log(x)}{4}-\frac{11}{2}\log\left(1+x^2\right)+\frac{21}{8}\log\left(2+x^2\right)$$

output
```
-1/4/x^4+11/8/x^2+1/(2*x^2+2)-13/(8*x^2+16)+23/4*ln(x)-11/2*ln(x^2+1)+21/8
*ln(x^2+2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{8}\left(-\frac{2}{x^4}+\frac{11}{x^2}-\frac{5+9x^2}{2+3x^2+x^4}+46\log(x)-44\log\left(1+x^2\right) +21\log\left(2+x^2\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-2/x^4 + 11/x^2 - (5 + 9*x^2)/(2 + 3*x^2 + x^4) + 46*Log[x] - 44*Log[1 +
x^2] + 21*Log[2 + x^2])/8
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^5 \left(x^4+3x^2+2\right)^2} \, dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2} \int \frac{5x^6+3x^4+x^2+4}{x^6 \left(x^4+3x^2+2\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2} \left(-\int -\frac{-9x^6+17x^4-10x^2+8}{4x^6 \left(x^4+3x^2+2\right)}dx^2-\frac{9x^2+5}{4 \left(x^4+3x^2+2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{1}{4} \int \frac{-9x^6+17x^4-10x^2+8}{x^6 \left(x^4+3x^2+2\right)}dx^2-\frac{9x^2+5}{4 \left(x^4+3x^2+2\right)}\right)$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2} \left(\frac{1}{4} \int \left(\frac{21}{x^2+2}+\frac{23}{x^2}-\frac{11}{x^4}+\frac{4}{x^6}-\frac{44}{x^2+1}\right)dx^2-\frac{9x^2+5}{4 \left(x^4+3x^2+2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{1}{4} \left(-\frac{2}{x^4}+\frac{11}{x^2}+23 \log \left(x^2\right)-44 \log \left(x^2+1\right)+21 \log \left(x^2+2\right)\right)-\frac{9x^2+5}{4 \left(x^4+3x^2+2\right)}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-1/4*(5 + 9*x^2)/(2 + 3*x^2 + x^4) + (-2/x^4 + 11/x^2 + 23*Log[x^2] - 44*
Log[1 + x^2] + 21*Log[2 + x^2])/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

method	result
default	$-\frac{11 \ln(x^2+1)}{2}+\frac{1}{2x^2+2}+\frac{21 \ln(x^2+2)}{8}-\frac{13}{8(x^2+2)}-\frac{1}{4x^4}+\frac{11}{8x^2}+\frac{23 \ln(x)}{4}$
norman	$\frac{-\frac{1}{2}+\frac{1}{4}x^6+\frac{13}{4}x^4+2x^2}{x^4(x^4+3x^2+2)}+\frac{23 \ln(x)}{4}-\frac{11 \ln(x^2+1)}{2}+\frac{21 \ln(x^2+2)}{8}$
risch	$\frac{-\frac{1}{2}+\frac{1}{4}x^6+\frac{13}{4}x^4+2x^2}{x^4(x^4+3x^2+2)}+\frac{23 \ln(x)}{4}-\frac{11 \ln(x^2+1)}{2}+\frac{21 \ln(x^2+2)}{8}$
parallelrisch	$\frac{46 \ln(x)x^8-44 \ln(x^2+1)x^8+21 \ln(x^2+2)x^8-4+138 \ln(x)x^6-132 \ln(x^2+1)x^6+63 \ln(x^2+2)x^6+2x^6+92 \ln(x)x^4-88 \ln(x^2+}{8x^4(x^4+3x^2+2)}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-11/2*ln(x^2+1)+1/2/(x^2+1)+21/8*ln(x^2+2)-13/8/(x^2+2)-1/4/x^4+11/8/x^2+2
3/4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx$$
$$=\frac{2\,x^6+26\,x^4+16\,x^2+21\,(x^8+3\,x^6+2\,x^4)\log\left(x^2+2\right)-44\,(x^8+3\,x^6+2\,x^4)\log\left(x^2+1\right)+46\,(x^8+3}{8\,(x^8+3\,x^6+2\,x^4)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/8*(2*x^6 + 26*x^4 + 16*x^2 + 21*(x^8 + 3*x^6 + 2*x^4)*log(x^2 + 2) - 44*
(x^8 + 3*x^6 + 2*x^4)*log(x^2 + 1) + 46*(x^8 + 3*x^6 + 2*x^4)*log(x) - 4)/
(x^8 + 3*x^6 + 2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(2+3x^2+x^4)^2}\,dx = \frac{23\log(x)}{4} - \frac{11\log(x^2+1)}{2} + \frac{21\log(x^2+2)}{8} + \frac{x^6+13x^4+8x^2-2}{4x^8+12x^6+8x^4}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+3*x**2+2)**2,x)
```

output
```
23*log(x)/4 - 11*log(x**2 + 1)/2 + 21*log(x**2 + 2)/8 + (x**6 + 13*x**4 +
8*x**2 - 2)/(4*x**8 + 12*x**6 + 8*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(2+3x^2+x^4)^2}\,dx = \frac{x^6+13\,x^4+8\,x^2-2}{4\,(x^8+3\,x^6+2\,x^4)} + \frac{21}{8}\log\left(x^2+2\right) - \frac{11}{2}\log\left(x^2+1\right) + \frac{23}{8}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
1/4*(x^6 + 13*x^4 + 8*x^2 - 2)/(x^8 + 3*x^6 + 2*x^4) + 21/8*log(x^2 + 2) -
 11/2*log(x^2 + 1) + 23/8*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = \frac{23\,x^4+51\,x^2+36}{16\,(x^4+3\,x^2+2)} - \frac{69\,x^4-22\,x^2+4}{16\,x^4} + \frac{21}{8}\,\log\left(x^2+2\right) - \frac{11}{2}\,\log\left(x^2+1\right) + \frac{23}{8}\,\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
1/16*(23*x^4 + 51*x^2 + 36)/(x^4 + 3*x^2 + 2) - 1/16*(69*x^4 - 22*x^2 + 4)
/x^4 + 21/8*log(x^2 + 2) - 11/2*log(x^2 + 1) + 23/8*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 18.61 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = \frac{21\,\ln\left(x^2+2\right)}{8} - \frac{11\,\ln\left(x^2+1\right)}{2} + \frac{23\,\ln\left(x\right)}{4} + \frac{\frac{x^6}{4}+\frac{13\,x^4}{4}+2\,x^2-\frac{1}{2}}{x^8+3\,x^6+2\,x^4}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(3*x^2 + x^4 + 2)^2),x)
```

output
```
(21*log(x^2 + 2))/8 - (11*log(x^2 + 1))/2 + (23*log(x))/4 + (2*x^2 + (13*x
^4)/4 + x^6/4 - 1/2)/(2*x^4 + 3*x^6 + x^8)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.92

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx$$
$$= \frac{63\log(x^2+2)\,x^8+189\log(x^2+2)\,x^6+126\log(x^2+2)\,x^4-132\log(x^2+1)\,x^8-396\log(x^2+1)\,x^6-2}{24x^4\,(x^4+3x^2-}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x)
```

output
```
(63*log(x**2 + 2)*x**8 + 189*log(x**2 + 2)*x**6 + 126*log(x**2 + 2)*x**4 -
 132*log(x**2 + 1)*x**8 - 396*log(x**2 + 1)*x**6 - 264*log(x**2 + 1)*x**4
+ 138*log(x)*x**8 + 414*log(x)*x**6 + 276*log(x)*x**4 - 2*x**8 + 74*x**4 +
 48*x**2 - 12)/(24*x**4*(x**4 + 3*x**2 + 2))
```

3.83 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 70

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x(206+207x^2)}{2(2+3x^2+x^4)}+\frac{9\arctan(x)}{2}+340\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

output
```
-293*x+98/3*x^3-27/5*x^5+5/7*x^7-x*(207*x^2+206)/(2*x^4+6*x^2+4)+9/2*arcta
n(x)+340*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}+\frac{-206x-207x^3}{2(2+3x^2+x^4)}+\frac{9\arctan(x)}{2}+340\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

input
```
Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
-293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 + (-206*x - 207*x^3)/(2*(2 +
3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{1}{4}\int -\frac{2(10x^{10}-24x^8+54x^6-106x^4+3x^2+206)}{x^4+3x^2+2}dx-\frac{x(207x^2+206)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{10x^{10}-24x^8+54x^6-106x^4+3x^2+206}{x^4+3x^2+2}dx-\frac{x(207x^2+206)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{2}\int \left(10x^6-54x^4+196x^2+\frac{1369x^2+1378}{x^4+3x^2+2}-586\right)dx-\frac{x(207x^2+206)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(9\arctan(x)+680\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{10x^7}{7}-\frac{54x^5}{5}+\frac{196x^3}{3}-586x\right)-\frac{x(207x^2+206)}{2(x^4+3x^2+2)}$$

input
```
Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
-1/2*(x*(206 + 207*x^2))/(2 + 3*x^2 + x^4) + (-586*x + (196*x^3)/3 - (54*x
^5)/5 + (10*x^7)/7 + 9*ArcTan[x] + 680*Sqrt[2]*ArcTan[x/Sqrt[2]])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x+\frac{x}{2x^2+2}+\frac{9\arctan(x)}{2}-\frac{104x}{x^2+2}+340\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)$	56
risch	$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x+\frac{-\frac{207}{2}x^3-103x}{x^4+3x^2+2}+\frac{9\arctan(x)}{2}+340\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)$	58

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
5/7*x^7-27/5*x^5+98/3*x^3-293*x+1/2*x/(x^2+1)+9/2*arctan(x)-104*x/(x^2+2)+
340*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{150\,x^{11} - 684\,x^9 + 3758\,x^7 - 43218\,x^5 - 192605\,x^3 + 71400\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 945\,(x^4 + 3\,x^2 + 2) \arctan(x) - 144690\,x}{210\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/210*(150*x^11 - 684*x^9 + 3758*x^7 - 43218*x^5 - 192605*x^3 + 71400*sqrt
(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) + 945*(x^4 + 3*x^2 + 2)*arctan
(x) - 144690*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^7}{7} - \frac{27x^5}{5} + \frac{98x^3}{3} - 293x + \frac{-207x^3 - 206x}{2x^4+6x^2+4} + \frac{9\,\text{atan}\,(x)}{2} + 340\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

input
```
integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**7/7 - 27*x**5/5 + 98*x**3/3 - 293*x + (-207*x**3 - 206*x)/(2*x**4 + 6
*x**2 + 4) + 9*atan(x)/2 + 340*sqrt(2)*atan(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{7}\,x^7 - \frac{27}{5}\,x^5 + \frac{98}{3}\,x^3 + 340\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 293\,x - \frac{207\,x^3 + 206\,x}{2\,(x^4 + 3\,x^2 + 2)} + \frac{9}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x
- 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{7}\,x^7 - \frac{27}{5}\,x^5 + \frac{98}{3}\,x^3 + 340\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 293\,x - \frac{207\,x^3 + 206\,x}{2\,(x^4 + 3\,x^2 + 2)} + \frac{9}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x
- 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{9\,\mathrm{atan}(x)}{2} - 293\,x + 340\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) - \frac{\frac{207\,x^3}{2}+103\,x}{x^4+3\,x^2+2} + \frac{98\,x^3}{3} - \frac{27\,x^5}{5} + \frac{5\,x^7}{7}$$

input
```
int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
(9*atan(x))/2 - 293*x + 340*2^(1/2)*atan((2^(1/2)*x)/2) - (103*x + (207*x^
3)/2)/(3*x^2 + x^4 + 2) + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{71400\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^4 + 214200\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^2 + 142800\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right) + 945 atan(x)\,x^4 + 2835 atan(x}{210x^4+630x^2+420}$$

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

output
```
(71400*sqrt(2)*atan(x/sqrt(2))*x**4 + 214200*sqrt(2)*atan(x/sqrt(2))*x**2
+ 142800*sqrt(2)*atan(x/sqrt(2)) + 945*atan(x)*x**4 + 2835*atan(x)*x**2 +
1890*atan(x) + 150*x**11 - 684*x**9 + 3758*x**7 - 43218*x**5 - 192605*x**3
 - 144690*x)/(210*(x**4 + 3*x**2 + 2))
```

3.84 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 57

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 98x-9x^3+x^5+\frac{x(102+103x^2)}{2(2+3x^2+x^4)} -\frac{11\arctan(x)}{2}-118\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

output
```
98*x-9*x^3+x^5+x*(103*x^2+102)/(2*x^4+6*x^2+4)-11/2*arctan(x)-118*2^(1/2)*
arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 98x-9x^3+x^5+\frac{102x+103x^3}{2(2+3x^2+x^4)} -\frac{11\arctan(x)}{2}-118\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

input
```
Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
98*x - 9*x^3 + x^5 + (102*x + 103*x^3)/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan[
x])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{ 2197}$$

$$\frac{x (103x^2+102)}{2 (x^4+3x^2+2)}-\frac{1}{4}\int \frac{2 (-10x^8+24x^6-54x^4+3x^2+102)}{x^4+3x^2+2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x (103x^2+102)}{2 (x^4+3x^2+2)}-\frac{1}{2}\int \frac{-10x^8+24x^6-54x^4+3x^2+102}{x^4+3x^2+2}dx$$

$$\Big\downarrow \text{ 2205}$$

$$\frac{x (103x^2+102)}{2 (x^4+3x^2+2)}-\frac{1}{2}\int \left(-10x^4+54x^2+\frac{483x^2+494}{x^4+3x^2+2}-196\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-11\arctan(x)-236\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+2x^5-18x^3+196x\right)+\frac{x (103x^2+102)}{2 (x^4+3x^2+2)}$$

input
```
Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(x*(102 + 103*x^2))/(2*(2 + 3*x^2 + x^4)) + (196*x - 18*x^3 + 2*x^5 - 11*A
rcTan[x] - 236*Sqrt[2]*ArcTan[x/Sqrt[2]])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

method	result	size
default	$x^{5}-9x^{3}+98x-\frac{x}{2(x^{2}+1)}-\frac{11\arctan\left(x\right)}{2}+\frac{52x}{x^{2}+2}-118\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)$	49
risch	$x^{5}-9x^{3}+98x+\frac{\frac{103}{2}x^{3}+51x}{x^{4}+3x^{2}+2}-118\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)-\frac{11\arctan\left(x\right)}{2}$	51

input
```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
x^5-9*x^3+98*x-1/2*x/(x^2+1)-11/2*arctan(x)+52*x/(x^2+2)-118*2^(1/2)*arcta
n(1/2*x*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{2\,x^9-12\,x^7+146\,x^5+655\,x^3-236\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-11\,(x^4+3\,x^2+2)\arctan\left(x\right)}{2\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/2*(2*x^9 - 12*x^7 + 146*x^5 + 655*x^3 - 236*sqrt(2)*(x^4 + 3*x^2 + 2)*ar
ctan(1/2*sqrt(2)*x) - 11*(x^4 + 3*x^2 + 2)*arctan(x) + 494*x)/(x^4 + 3*x^2
 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = x^5-9x^3+98x+\frac{103x^3+102x}{2x^4+6x^2+4} - \frac{11\,\mathrm{atan}\,(x)}{2}-118\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

input
```
integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
x**5 - 9*x**3 + 98*x + (103*x**3 + 102*x)/(2*x**4 + 6*x**2 + 4) - 11*atan(
x)/2 - 118*sqrt(2)*atan(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = x^5 - 9\,x^3 - 118\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 98\,x + \frac{103\,x^3+102\,x}{2\,(x^4+3\,x^2+2)} - \frac{11}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 10
2*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = x^5 - 9\,x^3 - 118\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 98\,x + \frac{103\,x^3+102\,x}{2\,(x^4+3\,x^2+2)} - \frac{11}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 10
2*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 98\,x-\frac{11\,\mathrm{atan}(x)}{2}-118\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)+\frac{\frac{103\,x^3}{2}+51\,x}{x^4+3\,x^2+2}-9\,x^3+x^5$$

input
```
int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
98*x - (11*atan(x))/2 - 118*2^(1/2)*atan((2^(1/2)*x)/2) + (51*x + (103*x^3
)/2)/(3*x^2 + x^4 + 2) - 9*x^3 + x^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{-236\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^4-708\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^2-472\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)-11atan(x)\,x^4-33atan(x)\,x^2-22a}{2x^4+6x^2+4}$$

input
```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

output
```
( - 236*sqrt(2)*atan(x/sqrt(2))*x**4 - 708*sqrt(2)*atan(x/sqrt(2))*x**2 -
472*sqrt(2)*atan(x/sqrt(2)) - 11*atan(x)*x**4 - 33*atan(x)*x**2 - 22*atan(
x) + 2*x**9 - 12*x**7 + 146*x**5 + 655*x**3 + 494*x)/(2*(x**4 + 3*x**2 + 2
))
```

3.85 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 56

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -27x+\frac{5x^3}{3}-\frac{x(50+51x^2)}{2(2+3x^2+x^4)}+\frac{13\arctan(x)}{2}+33\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

output
```
-27*x+5/3*x^3-x*(51*x^2+50)/(2*x^4+6*x^2+4)+13/2*arctan(x)+33*2^(1/2)*arct
an(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -27x+\frac{5x^3}{3}+\frac{-50x-51x^3}{2(2+3x^2+x^4)}+\frac{13\arctan(x)}{2}+33\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

input
```
Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
-27*x + (5*x^3)/3 + (-50*x - 51*x^3)/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x]
)/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (5x^6 + 3x^4 + x^2 + 4)}{(x^4 + 3x^2 + 2)^2} \, dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{1}{4} \int -\frac{2(10x^6 - 24x^4 + 3x^2 + 50)}{x^4 + 3x^2 + 2} dx - \frac{x(51x^2 + 50)}{2(x^4 + 3x^2 + 2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \int \frac{10x^6 - 24x^4 + 3x^2 + 50}{x^4 + 3x^2 + 2} dx - \frac{x(51x^2 + 50)}{2(x^4 + 3x^2 + 2)}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{2} \int \left(10x^2 + \frac{145x^2 + 158}{x^4 + 3x^2 + 2} - 54\right) dx - \frac{x(51x^2 + 50)}{2(x^4 + 3x^2 + 2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(13 \arctan(x) + 66\sqrt{2} \arctan\left(\frac{x}{\sqrt{2}}\right) + \frac{10x^3}{3} - 54x\right) - \frac{x(51x^2 + 50)}{2(x^4 + 3x^2 + 2)}$$

input
```
Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
-1/2*(x*(50 + 51*x^2))/(2 + 3*x^2 + x^4) + (-54*x + (10*x^3)/3 + 13*ArcTan
[x] + 66*Sqrt[2]*ArcTan[x/Sqrt[2]])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{5x^3}{3}-27x+\frac{x}{2x^2+2}+\frac{13\arctan(x)}{2}-\frac{26x}{x^2+2}+33\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)$	46
risch	$\frac{5x^3}{3}-27x+\frac{-\frac{51}{2}x^3-25x}{x^4+3x^2+2}+33\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)+\frac{13\arctan(x)}{2}$	48

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
5/3*x^3-27*x+1/2*x/(x^2+1)+13/2*arctan(x)-26*x/(x^2+2)+33*2^(1/2)*arctan(1
/2*x*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{10\,x^7-132\,x^5-619\,x^3+198\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+39\,(x^4+3\,x^2+2)\arctan\left(x\right)-474\,x}{6\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/6*(10*x^7 - 132*x^5 - 619*x^3 + 198*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2
*sqrt(2)*x) + 39*(x^4 + 3*x^2 + 2)*arctan(x) - 474*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5x^3}{3}-27x+\frac{-51x^3-50x}{2x^4+6x^2+4} + \frac{13\,\mathrm{atan}\,(x)}{2}+33\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

input
```
integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x**3/3 - 27*x + (-51*x**3 - 50*x)/(2*x**4 + 6*x**2 + 4) + 13*atan(x)/2 +
 33*sqrt(2)*atan(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{3}\,x^3+33\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-27\,x-\frac{51\,x^3+50\,x}{2\,(x^4+3\,x^2+2)}+\frac{13}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x
^4 + 3*x^2 + 2) + 13/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{5}{3}\,x^3+33\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-27\,x-\frac{51\,x^3+50\,x}{2\,(x^4+3\,x^2+2)}+\frac{13}{2}\,\arctan\left(x\right)$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x
^4 + 3*x^2 + 2) + 13/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{13\,\mathrm{atan}(x)}{2} - 27\,x + 33\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) - \frac{\frac{51\,x^3}{2}+25\,x}{x^4+3\,x^2+2} + \frac{5\,x^3}{3}$$

input
```
int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
(13*atan(x))/2 - 27*x + 33*2^(1/2)*atan((2^(1/2)*x)/2) - (25*x + (51*x^3)/
2)/(3*x^2 + x^4 + 2) + (5*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.62

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{198\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^4 + 594\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^2 + 396\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right) + 39atan(x)\,x^4 + 117atan(x)\,x^2 + 78at}{6x^4+18x^2+12}$$

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

output
```
(198*sqrt(2)*atan(x/sqrt(2))*x**4 + 594*sqrt(2)*atan(x/sqrt(2))*x**2 + 396
*sqrt(2)*atan(x/sqrt(2)) + 39*atan(x)*x**4 + 117*atan(x)*x**2 + 78*atan(x)
 + 10*x**7 - 132*x**5 - 619*x**3 - 474*x)/(6*(x**4 + 3*x**2 + 2))
```

3.86 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 49

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = 5x + \frac{x(24+25x^2)}{2(2+3x^2+x^4)} - \frac{15\arctan(x)}{2} - \frac{7\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
5*x+x*(25*x^2+24)/(2*x^4+6*x^2+4)-15/2*arctan(x)-7/2*2^(1/2)*arctan(1/2*x*
2^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx = 5x + \frac{24x+25x^3}{2(2+3x^2+x^4)} - \frac{15\arctan(x)}{2} - \frac{7\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
5*x + (24*x + 25*x^3)/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTan
[x/Sqrt[2]])/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^2} \, dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{x (25x^2+24)}{2 (x^4+3x^2+2)}-\frac{1}{4} \int \frac{2 (-10x^4-x^2+24)}{x^4+3x^2+2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x (25x^2+24)}{2 (x^4+3x^2+2)}-\frac{1}{2} \int \frac{-10x^4-x^2+24}{x^4+3x^2+2}dx$$

$$\Big\downarrow \text{2205}$$

$$\frac{x (25x^2+24)}{2 (x^4+3x^2+2)}-\frac{1}{2} \int \left(\frac{29x^2+44}{x^4+3x^2+2}-10\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-15 \arctan(x)-7 \sqrt{2} \arctan\left(\frac{x}{\sqrt{2}}\right)+10x\right)+\frac{x (25x^2+24)}{2 (x^4+3x^2+2)}$$

input
```
Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

output
```
(x*(24 + 25*x^2))/(2*(2 + 3*x^2 + x^4)) + (10*x - 15*ArcTan[x] - 7*Sqrt[2]
*ArcTan[x/Sqrt[2]])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result	size
default	$5x - \frac{x}{2(x^2+1)} - \frac{15\arctan(x)}{2} + \frac{13x}{x^2+2} - \frac{7\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2}$	41
risch	$5x + \frac{\frac{25}{2}x^3+12x}{x^4+3x^2+2} - \frac{7\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2} - \frac{15\arctan(x)}{2}$	43

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
5*x-1/2*x/(x^2+1)-15/2*arctan(x)+13*x/(x^2+2)-7/2*2^(1/2)*arctan(1/2*x*2^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{10\,x^5+55\,x^3-7\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-15\,(x^4+3\,x^2+2)\arctan\left(x\right)+44\,x}{2\,(x^4+3\,x^2+2)}$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/2*(10*x^5 + 55*x^3 - 7*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) -
 15*(x^4 + 3*x^2 + 2)*arctan(x) + 44*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 5x+\frac{25x^3+24x}{2x^4+6x^2+4}-\frac{15\,\mathrm{atan}\,(x)}{2}-\frac{7\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

input
```
integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
5*x + (25*x**3 + 24*x)/(2*x**4 + 6*x**2 + 4) - 15*atan(x)/2 - 7*sqrt(2)*at
an(sqrt(2)*x/2)/2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{7}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5\,x+\frac{25\,x^3+24\,x}{2\,(x^4+3\,x^2+2)}-\frac{15}{2}\arctan\left(x\right)$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
-7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^
2 + 2) - 15/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = -\frac{7}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5\,x+\frac{25\,x^3+24\,x}{2\,(x^4+3\,x^2+2)}-\frac{15}{2}\arctan\left(x\right)$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
-7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^
2 + 2) - 15/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 18.58 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = 5\,x-\frac{15\,\mathrm{atan}(x)}{2}-\frac{7\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}+\frac{\frac{25\,x^3}{2}+12\,x}{x^4+3\,x^2+2}$$

input
```
int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

output
```
5*x - (15*atan(x))/2 - (7*2^(1/2)*atan((2^(1/2)*x)/2))/2 + (12*x + (25*x^3
)/2)/(3*x^2 + x^4 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx = \frac{-7\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^4-21\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^2-14\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)-15atan(x)\,x^4-45atan(x)\,x^2-30atan(}{2x^4+6x^2+4}$$

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

output
```
( - 7*sqrt(2)*atan(x/sqrt(2))*x**4 - 21*sqrt(2)*atan(x/sqrt(2))*x**2 - 14*
sqrt(2)*atan(x/sqrt(2)) - 15*atan(x)*x**4 - 45*atan(x)*x**2 - 30*atan(x) +
 10*x**5 + 55*x**3 + 44*x)/(2*(x**4 + 3*x**2 + 2))
```

3.87 $\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 48

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2} \, dx = -\frac{x(11+12x^2)}{2\left(2+3x^2+x^4\right)}+\frac{17\arctan(x)}{2}-\frac{19\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

output
```
-1/2*x*(12*x^2+11)/(x^4+3*x^2+2)+17/2*arctan(x)-19/4*2^(1/2)*arctan(1/2*x*
2^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2} \, dx = \frac{1}{4}\left(-\frac{2x(11+12x^2)}{2+3x^2+x^4}+34\arctan(x)-19\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

output
```
((-2*x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + 34*ArcTan[x] - 19*Sqrt[2]*ArcTan
[x/Sqrt[2]])/4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2206, 27, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{ 2206}$$

$$-\frac{1}{4}\int -\frac{2(15-2x^2)}{x^4+3x^2+2}dx-\frac{x(12x^2+11)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{15-2x^2}{x^4+3x^2+2}dx-\frac{x(12x^2+11)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{2}\left(17\int \frac{1}{x^2+1}dx-19\int \frac{1}{x^2+2}dx\right)-\frac{x(12x^2+11)}{2(x^4+3x^2+2)}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(17\arctan(x)-\frac{19\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}\right)-\frac{x(12x^2+11)}{2(x^4+3x^2+2)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

output
```
-1/2*(x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + (17*ArcTan[x] - (19*ArcTan[x/Sq
rt[2]])/Sqrt[2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x}{2x^2+2}+\frac{17\arctan(x)}{2}-\frac{13x}{2(x^2+2)}-\frac{19\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{4}$	38
risch	$\frac{-6x^3-\frac{11}{2}x}{x^4+3x^2+2}+\frac{17\arctan(x)}{2}-\frac{19\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{4}$	40

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(x^2+1)+17/2*arctan(x)-13/2*x/(x^2+2)-19/4*2^(1/2)*arctan(1/2*x*2^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = -\frac{24\,x^3+19\,\sqrt{2}(x^4+3\,x^2+2)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-34\,(x^4+3\,x^2+2)\arctan\left(x\right)+22\,x}{4\,(x^4+3\,x^2+2)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
-1/4*(24*x^3 + 19*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) - 34*(x^
4 + 3*x^2 + 2)*arctan(x) + 22*x)/(x^4 + 3*x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = \frac{-12x^3-11x}{2x^4+6x^2+4}+\frac{17\,\mathrm{atan}\left(x\right)}{2}-\frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

output
```
(-12*x**3 - 11*x)/(2*x**4 + 6*x**2 + 4) + 17*atan(x)/2 - 19*sqrt(2)*atan(s
qrt(2)*x/2)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = -\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{12\,x^3+11\,x}{2\,(x^4+3\,x^2+2)} + \frac{17}{2}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
-19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2
) + 17/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = -\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{12\,x^3+11\,x}{2\,(x^4+3\,x^2+2)} + \frac{17}{2}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
-19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2
) + 17/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx = \frac{17\,\mathrm{atan}(x)}{2} - \frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4} - \frac{6\,x^3+\frac{11\,x}{2}}{x^4+3\,x^2+2}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^2,x)
```

output
```
(17*atan(x))/2 - (19*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((11*x)/2 + 6*x^3)/(
3*x^2 + x^4 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.69

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^2}\,dx$$
$$= \frac{-19\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^4-57\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^2-38\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)+34atan(x)\,x^4+102atan(x)\,x^2+68ata}{4x^4+12x^2+8}$$

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

output
```
( - 19*sqrt(2)*atan(x/sqrt(2))*x**4 - 57*sqrt(2)*atan(x/sqrt(2))*x**2 - 38
*sqrt(2)*atan(x/sqrt(2)) + 34*atan(x)*x**4 + 102*atan(x)*x**2 + 68*atan(x)
 - 24*x**3 - 22*x)/(4*(x**4 + 3*x**2 + 2))
```

3.88 $\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 53

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2} dx = -\frac{1}{x}+\frac{x(9+11x^2)}{4(2+3x^2+x^4)}-\frac{19\arctan(x)}{2}+\frac{45\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
-1/x+x*(11*x^2+9)/(4*x^4+12*x^2+8)-19/2*arctan(x)+45/8*2^(1/2)*arctan(1/2*
x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^2} dx = \frac{1}{8}\left(-\frac{8}{x}+\frac{2x(9+11x^2)}{2+3x^2+x^4}-76\arctan(x)+45\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-8/x + (2*x*(9 + 11*x^2))/(2 + 3*x^2 + x^4) - 76*ArcTan[x] + 45*Sqrt[2]*A
rcTan[x/Sqrt[2]])/8
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^2 (x^4+3x^2+2)^2} \, dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{x(11x^2+9)}{4(x^4+3x^2+2)}-\frac{1}{4}\int -\frac{11x^4-19x^2+8}{x^2(x^4+3x^2+2)}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{11x^4-19x^2+8}{x^2(x^4+3x^2+2)}dx+\frac{x(11x^2+9)}{4(x^4+3x^2+2)}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{4}\int \left(\frac{45}{x^2+2}+\frac{4}{x^2}-\frac{38}{x^2+1}\right)dx+\frac{x(11x^2+9)}{4(x^4+3x^2+2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-38\arctan(x)+\frac{45\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{4}{x}\right)+\frac{x(11x^2+9)}{4(x^4+3x^2+2)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(x*(9 + 11*x^2))/(4*(2 + 3*x^2 + x^4)) + (-4/x - 38*ArcTan[x] + (45*ArcTan
[x/Sqrt[2]])/Sqrt[2])/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{x}{2(x^2+1)}-\frac{19\arctan(x)}{2}+\frac{13x}{4(x^2+2)}+\frac{45\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{8}-\frac{1}{x}$	43
risch	$\frac{\frac{7}{4}x^4-\frac{3}{4}x^2-2}{x(x^4+3x^2+2)}+\frac{45\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{8}-\frac{19\arctan(x)}{2}$	46

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^2+1)-19/2*arctan(x)+13/4*x/(x^2+2)+45/8*2^(1/2)*arctan(1/2*x*2^(
1/2))-1/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{14\,x^4+45\,\sqrt{2}(x^5+3\,x^3+2\,x)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-6\,x^2-76\,(x^5+3\,x^3+2\,x)\arctan\left(x\right)-16}{8\,(x^5+3\,x^3+2\,x)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/8*(14*x^4 + 45*sqrt(2)*(x^5 + 3*x^3 + 2*x)*arctan(1/2*sqrt(2)*x) - 6*x^2
 - 76*(x^5 + 3*x^3 + 2*x)*arctan(x) - 16)/(x^5 + 3*x^3 + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{7x^4-3x^2-8}{4x^5+12x^3+8x}-\frac{19\,\mathrm{atan}\left(x\right)}{2}+\frac{45\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**2,x)
```

output
```
(7*x**4 - 3*x**2 - 8)/(4*x**5 + 12*x**3 + 8*x) - 19*atan(x)/2 + 45*sqrt(2)
*atan(sqrt(2)*x/2)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{45}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{7\,x^4-3\,x^2-8}{4\left(x^5+3\,x^3+2\,x\right)}-\frac{19}{2}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3
+ 2*x) - 19/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{45}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{7\,x^4-3\,x^2-8}{4\left(x^5+3\,x^3+2\,x\right)}-\frac{19}{2}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3
+ 2*x) - 19/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{45\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}-\frac{19\,\mathrm{atan}(x)}{2}-\frac{-\frac{7\,x^4}{4}+\frac{3\,x^2}{4}+2}{x^5+3\,x^3+2\,x}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^2),x)
```

output
```
(45*2^(1/2)*atan((2^(1/2)*x)/2))/8 - (19*atan(x))/2 - ((3*x^2)/4 - (7*x^4)
/4 + 2)/(2*x + 3*x^3 + x^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.66

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{45\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^5+135\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^3+90\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x-76atan(x)\,x^5-228atan(x)\,x^3-152a}{8x\left(x^4+3x^2+2\right)}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x)
```

output
```
(45*sqrt(2)*atan(x/sqrt(2))*x**5 + 135*sqrt(2)*atan(x/sqrt(2))*x**3 + 90*s
qrt(2)*atan(x/sqrt(2))*x - 76*atan(x)*x**5 - 228*atan(x)*x**3 - 152*atan(x
)*x + 14*x**4 - 6*x**2 - 16)/(8*x*(x**4 + 3*x**2 + 2))
```

3.89 $\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 62

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx = -\frac{1}{3x^3}+\frac{11}{4x}-\frac{x(5+9x^2)}{8\left(2+3x^2+x^4\right)}+\frac{21\arctan(x)}{2}-\frac{71\arctan\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

output
```
-1/3/x^3+11/4/x-x*(9*x^2+5)/(8*x^4+24*x^2+16)+21/2*arctan(x)-71/16*2^(1/2)
*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{48}\left(-\frac{16}{x^3}+\frac{132}{x}-\frac{6x(5+9x^2)}{2+3x^2+x^4}+504\arctan(x)-213\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-16/x^3 + 132/x - (6*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4) + 504*ArcTan[x] - 2
13*Sqrt[2]*ArcTan[x/Sqrt[2]])/48
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^4(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2198}$$

$$-\frac{1}{4}\int -\frac{-9x^6+39x^4-20x^2+16}{2x^4(x^4+3x^2+2)}dx-\frac{x(9x^2+5)}{8(x^4+3x^2+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \frac{-9x^6+39x^4-20x^2+16}{x^4(x^4+3x^2+2)}dx-\frac{x(9x^2+5)}{8(x^4+3x^2+2)}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{8}\int \left(-\frac{71}{x^2+2}-\frac{22}{x^2}+\frac{8}{x^4}+\frac{84}{x^2+1}\right)dx-\frac{x(9x^2+5)}{8(x^4+3x^2+2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(84\arctan(x)-\frac{71\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{8}{3x^3}+\frac{22}{x}\right)-\frac{x(9x^2+5)}{8(x^4+3x^2+2)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```

output
```
-1/8*(x*(5 + 9*x^2))/(2 + 3*x^2 + x^4) + (-8/(3*x^3) + 22/x + 84*ArcTan[x]
 - (71*ArcTan[x/Sqrt[2]])/Sqrt[2])/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{x}{2x^2+2}+\frac{21\arctan(x)}{2}-\frac{13x}{8(x^2+2)}-\frac{71\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{16}-\frac{1}{3x^3}+\frac{11}{4x}$	48
risch	$\frac{\frac{13}{8}x^6+\frac{175}{24}x^4+\frac{9}{2}x^2-\frac{2}{3}}{x^3(x^4+3x^2+2)}+\frac{21\arctan(x)}{2}-\frac{71\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{16}$	51

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(x^2+1)+21/2*arctan(x)-13/8*x/(x^2+2)-71/16*2^(1/2)*arctan(1/2*x*2^(
1/2))-1/3/x^3+11/4/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx = \frac{78\,x^6+350\,x^4-213\,\sqrt{2}(x^7+3\,x^5+2\,x^3)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+216\,x^2+504\,(x^7+3\,x^5+2\,x^3)\arctan\left(x\right)-}{48\,(x^7+3\,x^5+2\,x^3)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/48*(78*x^6 + 350*x^4 - 213*sqrt(2)*(x^7 + 3*x^5 + 2*x^3)*arctan(1/2*sqrt
(2)*x) + 216*x^2 + 504*(x^7 + 3*x^5 + 2*x^3)*arctan(x) - 32)/(x^7 + 3*x^5
+ 2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx = \frac{21\operatorname{atan}(x)}{2}-\frac{71\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}+\frac{39x^6+175x^4+108x^2-16}{24x^7+72x^5+48x^3}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**2,x)
```

output
```
21*atan(x)/2 - 71*sqrt(2)*atan(sqrt(2)*x/2)/16 + (39*x**6 + 175*x**4 + 108
*x**2 - 16)/(24*x**7 + 72*x**5 + 48*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2}\,dx = -\frac{71}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{39\,x^6+175\,x^4+108\,x^2-16}{24\,(x^7+3\,x^5+2\,x^3)} + \frac{21}{2}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
-71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/24*(39*x^6 + 175*x^4 + 108*x^2 -
16)/(x^7 + 3*x^5 + 2*x^3) + 21/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^2}\,dx = -\frac{71}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{9\,x^3+5\,x}{8\,(x^4+3\,x^2+2)} + \frac{33\,x^2-4}{12\,x^3} + \frac{21}{2}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
-71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(9*x^3 + 5*x)/(x^4 + 3*x^2 + 2)
 + 1/12*(33*x^2 - 4)/x^3 + 21/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 18.50 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx=\frac{21\,\mathrm{atan}(x)}{2}-\frac{71\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16}+\frac{\frac{13\,x^6}{8}+\frac{175\,x^4}{24}+\frac{9\,x^2}{2}-\frac{2}{3}}{x^7+3\,x^5+2\,x^3}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^2),x)
```

output
```
(21*atan(x))/2 - (71*2^(1/2)*atan((2^(1/2)*x)/2))/16 + ((9*x^2)/2 + (175*x
^4)/24 + (13*x^6)/8 - 2/3)/(2*x^3 + 3*x^5 + x^7)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx$$
$$=\frac{-213\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^7-639\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^5-426\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^3+504atan(x)\,x^7+1512atan(x)\,x^5}{48x^3\left(x^4+3x^2+2\right)}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x)
```

output
```
( - 213*sqrt(2)*atan(x/sqrt(2))*x**7 - 639*sqrt(2)*atan(x/sqrt(2))*x**5 -
426*sqrt(2)*atan(x/sqrt(2))*x**3 + 504*atan(x)*x**7 + 1512*atan(x)*x**5 +
1008*atan(x)*x**3 + 78*x**6 + 350*x**4 + 216*x**2 - 32)/(48*x**3*(x**4 + 3
*x**2 + 2))
```

3.90 $\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 69

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2}\,dx = -\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x(3-5x^2)}{16(2+3x^2+x^4)} - \frac{23\arctan(x)}{2}+\frac{97\arctan\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

output
```
-1/5/x^5+11/12/x^3-23/4/x-x*(-5*x^2+3)/(16*x^4+48*x^2+32)-23/2*arctan(x)+9
7/32*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^2}\,dx = \frac{1}{480}\left(-\frac{96}{x^5}+\frac{440}{x^3}-\frac{2760}{x}+\frac{30x(-3+5x^2)}{2+3x^2+x^4} - 5520\arctan(x)+1455\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(-96/x^5 + 440/x^3 - 2760/x + (30*x*(-3 + 5*x^2))/(2 + 3*x^2 + x^4) - 5520
*ArcTan[x] + 1455*Sqrt[2]*ArcTan[x/Sqrt[2]])/480
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^6 (x^4+3x^2+2)^2} \, dx$$

$$\Big\downarrow \text{2198}$$

$$-\frac{1}{4}\int -\frac{5x^8-39x^6+68x^4-40x^2+32}{4x^6 (x^4+3x^2+2)}dx-\frac{x(3-5x^2)}{16 (x^4+3x^2+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{5x^8-39x^6+68x^4-40x^2+32}{x^6 (x^4+3x^2+2)}dx-\frac{x(3-5x^2)}{16 (x^4+3x^2+2)}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{16}\int \left(\frac{97}{x^2+2}+\frac{92}{x^2}-\frac{44}{x^4}+\frac{16}{x^6}-\frac{184}{x^2+1}\right)dx-\frac{x(3-5x^2)}{16 (x^4+3x^2+2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(-184\arctan(x)+\frac{97\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{16}{5x^5}+\frac{44}{3x^3}-\frac{92}{x}\right)-\frac{x(3-5x^2)}{16 (x^4+3x^2+2)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

output
```
-1/16*(x*(3 - 5*x^2))/(2 + 3*x^2 + x^4) + (-16/(5*x^5) + 44/(3*x^3) - 92/x
 - 184*ArcTan[x] + (97*ArcTan[x/Sqrt[2]])/Sqrt[2])/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{x}{2(x^{2}+1)}-\frac{23\arctan \left(x \right)}{2}+\frac{13x}{16(x^{2}+2)}+\frac{97\sqrt{2}\,\arctan \left(\frac{x\sqrt{2}}{2}\right)}{32}-\frac{1}{5x^{5}}+\frac{11}{12x^{3}}-\frac{23}{4x}$	53
risch	$\frac{-\frac{87}{16}x^{8}-\frac{793}{48}x^{6}-\frac{179}{20}x^{4}+\frac{37}{30}x^{2}-\frac{2}{5}}{x^{5}(x^{4}+3x^{2}+2)}+\frac{97\sqrt{2}\,\arctan \left(\frac{x\sqrt{2}}{2}\right)}{32}-\frac{23\arctan \left(x \right)}{2}$	56

input
```
int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^2+1)-23/2*arctan(x)+13/16*x/(x^2+2)+97/32*2^(1/2)*arctan(1/2*x*2
^(1/2))-1/5/x^5+11/12/x^3-23/4/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx = -\frac{2610\,x^{8}+7930\,x^{6}+4296\,x^{4}-1455\,\sqrt{2}(x^{9}+3\,x^{7}+2\,x^{5})\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-592\,x^{2}+5520\,(x^{9}+3\,x^{7}}{480\,(x^{9}+3\,x^{7}+2\,x^{5})}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
-1/480*(2610*x^8 + 7930*x^6 + 4296*x^4 - 1455*sqrt(2)*(x^9 + 3*x^7 + 2*x^5
)*arctan(1/2*sqrt(2)*x) - 592*x^2 + 5520*(x^9 + 3*x^7 + 2*x^5)*arctan(x) +
 192)/(x^9 + 3*x^7 + 2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx = -\frac{23\,\mathrm{atan}\,(x)}{2}+\frac{97\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32} + \frac{-1305x^{8}-3965x^{6}-2148x^{4}+296x^{2}-96}{240x^{9}+720x^{7}+480x^{5}}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**2,x)
```

output
```
-23*atan(x)/2 + 97*sqrt(2)*atan(sqrt(2)*x/2)/32 + (-1305*x**8 - 3965*x**6
- 2148*x**4 + 296*x**2 - 96)/(240*x**9 + 720*x**7 + 480*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx = \frac{97}{32}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - \frac{1305\,x^8+3965\,x^6+2148\,x^4-296\,x^2+96}{240\,(x^9+3\,x^7+2\,x^5)} - \frac{23}{2}\,\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/240*(1305*x^8 + 3965*x^6 + 2148*x^
4 - 296*x^2 + 96)/(x^9 + 3*x^7 + 2*x^5) - 23/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx = \frac{97}{32}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{5\,x^3-3\,x}{16\,(x^4+3\,x^2+2)} - \frac{345\,x^4-55\,x^2+12}{60\,x^5} - \frac{23}{2}\,\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(5*x^3 - 3*x)/(x^4 + 3*x^2 + 2)
 - 1/60*(345*x^4 - 55*x^2 + 12)/x^5 - 23/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6 \left(2+3x^2+x^4\right)^2}\,dx = \frac{97\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32} - \frac{23\,\mathrm{atan}(x)}{2} - \frac{\frac{87\,x^8}{16}+\frac{793\,x^6}{48}+\frac{179\,x^4}{20}-\frac{37\,x^2}{30}+\frac{2}{5}}{x^9+3\,x^7+2\,x^5}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^2),x)
```

output
```
(97*2^(1/2)*atan((2^(1/2)*x)/2))/32 - (23*atan(x))/2 - ((179*x^4)/20 - (37
*x^2)/30 + (793*x^6)/48 + (87*x^8)/16 + 2/5)/(2*x^5 + 3*x^7 + x^9)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

$$\int \frac{4+x^2+3x^4+5x^6}{x^6 \left(2+3x^2+x^4\right)^2}\,dx = \frac{1455\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^9+4365\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^7+2910\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^5-5520atan(x)\,x^9-16560atan(x}{480x^5\left(x^4+3x^2+2\right)}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x)
```

output
```
(1455*sqrt(2)*atan(x/sqrt(2))*x**9 + 4365*sqrt(2)*atan(x/sqrt(2))*x**7 + 2
910*sqrt(2)*atan(x/sqrt(2))*x**5 - 5520*atan(x)*x**9 - 16560*atan(x)*x**7
- 11040*atan(x)*x**5 - 2610*x**8 - 7930*x**6 - 4296*x**4 + 592*x**2 - 192)
/(480*x**5*(x**4 + 3*x**2 + 2))
```

3.91 $\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 76

$$\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2}\,dx = -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x(19+3x^2)}{32(2+3x^2+x^4)}+\frac{25\arctan(x)}{2}-\frac{123\arctan\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

output
```
-1/7/x^7+11/20/x^5-23/12/x^3+137/16/x+x*(3*x^2+19)/(32*x^4+96*x^2+64)+25/2
*arctan(x)-123/64*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{4+x^2+3x^4+5x^6}{x^8(2+3x^2+x^4)^2}\,dx = -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{19x+3x^3}{32(2+3x^2+x^4)}+\frac{25\arctan(x)}{2}-\frac{123\arctan\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

output
```
-1/7*1/x^7 + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (19*x + 3*x^3)/(32*(
2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2]
)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^8 \left(x^4+3x^2+2\right)^2} \, dx$$

$$\Big\downarrow \text{ 2198}$$

$$\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}-\frac{1}{4}\int -\frac{3x^{10}+39x^8-84x^6+136x^4-80x^2+64}{8x^8\left(x^4+3x^2+2\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{32}\int \frac{3x^{10}+39x^8-84x^6+136x^4-80x^2+64}{x^8\left(x^4+3x^2+2\right)}dx+\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{ 2195}$$

$$\frac{1}{32}\int \left(-\frac{123}{x^2+2}-\frac{274}{x^2}+\frac{184}{x^4}-\frac{88}{x^6}+\frac{32}{x^8}+\frac{400}{x^2+1}\right)dx+\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{32}\left(400\arctan(x)-\frac{123\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{32}{7x^7}+\frac{88}{5x^5}-\frac{184}{3x^3}+\frac{274}{x}\right)+\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

output
```
(x*(19 + 3*x^2))/(32*(2 + 3*x^2 + x^4)) + (-32/(7*x^7) + 88/(5*x^5) - 184/
(3*x^3) + 274/x + 400*ArcTan[x] - (123*ArcTan[x/Sqrt[2]])/Sqrt[2])/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x}{2x^2+2}+\frac{25\arctan(x)}{2}-\frac{13x}{32(x^2+2)}-\frac{123\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{64}-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}$	58
risch	$\frac{\frac{277}{32}x^{10}+\frac{2339}{96}x^8+\frac{477}{40}x^6-\frac{977}{420}x^4+\frac{47}{70}x^2-\frac{2}{7}}{x^7(x^4+3x^2+2)}+\frac{25\arctan(x)}{2}-\frac{123\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{64}$	61

input
```
int((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(x^2+1)+25/2*arctan(x)-13/32*x/(x^2+2)-123/64*2^(1/2)*arctan(1/2*x*2
^(1/2))-1/7/x^7+11/20/x^5-23/12/x^3+137/16/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx$$
$$= \frac{58170\,x^{10}+163730\,x^{8}+80136\,x^{6}-15632\,x^{4}-12915\,\sqrt{2}(x^{11}+3\,x^{9}+2\,x^{7})\arctan\left(\frac{1}{2}\sqrt{2}x\right)+4512\,x^{2}-}{6720\left(x^{11}+3\,x^{9}+2\,x^{7}\right)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

output
```
1/6720*(58170*x^10 + 163730*x^8 + 80136*x^6 - 15632*x^4 - 12915*sqrt(2)*(x
^11 + 3*x^9 + 2*x^7)*arctan(1/2*sqrt(2)*x) + 4512*x^2 + 84000*(x^11 + 3*x^
9 + 2*x^7)*arctan(x) - 1920)/(x^11 + 3*x^9 + 2*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx = \frac{25\,\mathrm{atan}\,(x)}{2} - \frac{123\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64} + \frac{29085x^{10}+81865x^{8}+40068x^{6}-7816x^{4}+2256x^{2}-960}{3360x^{11}+10080x^{9}+6720x^{7}}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**8/(x**4+3*x**2+2)**2,x)
```

output
```
25*atan(x)/2 - 123*sqrt(2)*atan(sqrt(2)*x/2)/64 + (29085*x**10 + 81865*x**
8 + 40068*x**6 - 7816*x**4 + 2256*x**2 - 960)/(3360*x**11 + 10080*x**9 + 6
720*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx = -\frac{123}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{29085\,x^{10}+81865\,x^8+40068\,x^6-7816\,x^4+2256\,x^2-960}{3360\,(x^{11}+3\,x^9+2\,x^7)} + \frac{25}{2}\arctan(x)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

output
```
-123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/3360*(29085*x^10 + 81865*x^8 + 4
0068*x^6 - 7816*x^4 + 2256*x^2 - 960)/(x^11 + 3*x^9 + 2*x^7) + 25/2*arctan
(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx = -\frac{123}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{3\,x^3+19\,x}{32\,(x^4+3\,x^2+2)} + \frac{14385\,x^6-3220\,x^4+924\,x^2-240}{1680\,x^7} + \frac{25}{2}\arctan(x)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

output
```
-123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/32*(3*x^3 + 19*x)/(x^4 + 3*x^2 +
 2) + 1/1680*(14385*x^6 - 3220*x^4 + 924*x^2 - 240)/x^7 + 25/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx=\frac{25\,\mathrm{atan}(x)}{2}-\frac{123\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64}+\frac{\frac{277\,x^{10}}{32}+\frac{2339\,x^8}{96}+\frac{477\,x^6}{40}-\frac{977\,x^4}{420}+\frac{47\,x^2}{70}-\frac{2}{7}}{x^{11}+3\,x^9+2\,x^7}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^8*(3*x^2 + x^4 + 2)^2),x)
```

output
```
(25*atan(x))/2 - (123*2^(1/2)*atan((2^(1/2)*x)/2))/64 + ((47*x^2)/70 - (97
7*x^4)/420 + (477*x^6)/40 + (2339*x^8)/96 + (277*x^10)/32 - 2/7)/(2*x^7 +
3*x^9 + x^11)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.41

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx$$
$$=\frac{-12915\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^{11}-38745\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^9-25830\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^7+84000atan(x)\,x^{11}+2520}{6720x^7\left(x^4+3\right.}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x)
```

output
```
( - 12915*sqrt(2)*atan(x/sqrt(2))*x**11 - 38745*sqrt(2)*atan(x/sqrt(2))*x*
*9 - 25830*sqrt(2)*atan(x/sqrt(2))*x**7 + 84000*atan(x)*x**11 + 252000*ata
n(x)*x**9 + 168000*atan(x)*x**7 + 58170*x**10 + 163730*x**8 + 80136*x**6 -
 15632*x**4 + 4512*x**2 - 1920)/(6720*x**7*(x**4 + 3*x**2 + 2))
```

3.92 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 81

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 214x-14x^3+x^5+\frac{x(414+415x^2)}{4(2+3x^2+x^4)^2}+\frac{x(824+1669x^2)}{8(2+3x^2+x^4)} + \frac{477\arctan(x)}{8}-351\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

output
```
214*x-14*x^3+x^5+1/4*x*(415*x^2+414)/(x^4+3*x^2+2)^2+x*(1669*x^2+824)/(8*x
^4+24*x^2+16)+477/8*arctan(x)-351*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{x(9324+26736x^2+26775x^4+10581x^6+1144x^8-64x^{10}+8x^{12})}{8(2+3x^2+x^4)^2} + \frac{477\arctan(x)}{8}-351\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)$$

input
```
Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
(x*(9324 + 26736*x^2 + 26775*x^4 + 10581*x^6 + 1144*x^8 - 64*x^10 + 8*x^12
))/(8*(2 + 3*x^2 + x^4)^2) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcTan[x/Sqrt
[2]]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10} \left(5 x^6+3 x^4+x^2+4\right)}{\left(x^4+3 x^2+2\right)^3} \, dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{x \left(415 x^2+414\right)}{4 \left(x^4+3 x^2+2\right)^2}-\frac{1}{8} \int \frac{2 \left(-20 x^{12}+48 x^{10}-108 x^8+212 x^6-420 x^4-1239 x^2+414\right)}{\left(x^4+3 x^2+2\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x \left(415 x^2+414\right)}{4 \left(x^4+3 x^2+2\right)^2}-\frac{1}{4} \int \frac{-20 x^{12}+48 x^{10}-108 x^8+212 x^6-420 x^4-1239 x^2+414}{\left(x^4+3 x^2+2\right)^2}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4} \left(\frac{1}{4} \int -\frac{2 \left(-40 x^8+216 x^6-784 x^4+675 x^2+1238\right)}{x^4+3 x^2+2}dx+\frac{x \left(1669 x^2+824\right)}{2 \left(x^4+3 x^2+2\right)}\right)+\frac{x \left(415 x^2+414\right)}{4 \left(x^4+3 x^2+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \left(\frac{x \left(1669 x^2+824\right)}{2 \left(x^4+3 x^2+2\right)}-\frac{1}{2} \int \frac{-40 x^8+216 x^6-784 x^4+675 x^2+1238}{x^4+3 x^2+2}dx\right)+\frac{x \left(415 x^2+414\right)}{4 \left(x^4+3 x^2+2\right)^2}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{4}\left(\frac{x(1669x^2+824)}{2(x^4+3x^2+2)}-\frac{1}{2}\int\left(-40x^4+336x^2+\frac{9(571x^2+518)}{x^4+3x^2+2}-1712\right)dx\right)+\frac{x(415x^2+414)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(477\arctan(x)-2808\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+8x^5-112x^3+1712x\right)+\frac{x(1669x^2+824)}{2(x^4+3x^2+2)}\right)+\frac{x(415x^2+414)}{4(x^4+3x^2+2)^2}$$

input
```
Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
(x*(414 + 415*x^2))/(4*(2 + 3*x^2 + x^4)^2) + ((x*(824 + 1669*x^2))/(2*(2
+ 3*x^2 + x^4)) + (1712*x - 112*x^3 + 8*x^5 + 477*ArcTan[x] - 2808*Sqrt[2]
*ArcTan[x/Sqrt[2]])/2)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

method	result	size
risch	$x^5-14x^3+214x+\frac{\frac{1669}{8}x^7+\frac{5831}{8}x^5+830x^3+\frac{619}{2}x}{(x^4+3x^2+2)^2}-351\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)+\frac{477\arctan(x)}{8}$	61
default	$x^5-14x^3+214x+\frac{-\frac{11}{8}x^3-\frac{13}{8}x}{(x^2+1)^2}+\frac{477\arctan(x)}{8}-\frac{16\left(-\frac{105}{8}x^3-\frac{79}{4}x\right)}{(x^2+2)^2}-351\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)$	64

input
```
int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
x^5-14*x^3+214*x+(1669/8*x^7+5831/8*x^5+830*x^3+619/2*x)/(x^4+3*x^2+2)^2-3
51*2^(1/2)*arctan(1/2*x*2^(1/2))+477/8*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$$
$$= \frac{8\,x^{13}-64\,x^{11}+1144\,x^9+10581\,x^7+26775\,x^5+26736\,x^3-2808\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\,\text{arc}}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/8*(8*x^13 - 64*x^11 + 1144*x^9 + 10581*x^7 + 26775*x^5 + 26736*x^3 - 280
8*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 477*
(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) + 9324*x)/(x^8 + 6*x^6 + 13*
x^4 + 12*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = x^5-14x^3+214x+\frac{1669x^7+5831x^5+6640x^3+2476x}{8x^8+48x^6+104x^4+96x^2+32} + \frac{477\,\text{atan}\,(x)}{8} - 351\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

input
```
integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
x**5 - 14*x**3 + 214*x + (1669*x**7 + 5831*x**5 + 6640*x**3 + 2476*x)/(8*x
**8 + 48*x**6 + 104*x**4 + 96*x**2 + 32) + 477*atan(x)/8 - 351*sqrt(2)*ata
n(sqrt(2)*x/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = x^5 - 14\,x^3 - 351\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 214\,x + \frac{1669\,x^7 + 5831\,x^5 + 6640\,x^3 + 2476\,x}{8\,(x^8 + 6\,x^6 + 13\,x^4 + 12\,x^2 + 4)} + \frac{477}{8}\,\arctan\left(x\right)$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
 5831*x^5 + 6640*x^3 + 2476*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 477/8
*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = x^5 - 14\,x^3 - 351\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + 214\,x + \frac{1669\,x^7 + 5831\,x^5 + 6640\,x^3 + 2476\,x}{8\,(x^4 + 3\,x^2 + 2)^2} + \frac{477}{8}\,\arctan\left(x\right)$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
 5831*x^5 + 6640*x^3 + 2476*x)/(x^4 + 3*x^2 + 2)^2 + 477/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 18.60 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 214\,x+\frac{477\,\mathrm{atan}(x)}{8}-351\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) + \frac{\frac{1669\,x^7}{8}+\frac{5831\,x^5}{8}+830\,x^3+\frac{619\,x}{2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}-14\,x^3+x^5$$

input
```
int((x^10*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

output
```
214*x + (477*atan(x))/8 - 351*2^(1/2)*atan((2^(1/2)*x)/2) + ((619*x)/2 + 8
30*x^3 + (5831*x^5)/8 + (1669*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
- 14*x^3 + x^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.95

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{-2808\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^8-16848\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^6-36504\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^4-33696\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^2-1}{}$$

input
```
int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

output
```
( - 2808*sqrt(2)*atan(x/sqrt(2))*x**8 - 16848*sqrt(2)*atan(x/sqrt(2))*x**6
 - 36504*sqrt(2)*atan(x/sqrt(2))*x**4 - 33696*sqrt(2)*atan(x/sqrt(2))*x**2
 - 11232*sqrt(2)*atan(x/sqrt(2)) + 477*atan(x)*x**8 + 2862*atan(x)*x**6 +
6201*atan(x)*x**4 + 5724*atan(x)*x**2 + 1908*atan(x) + 8*x**13 - 64*x**11
+ 1144*x**9 + 10581*x**7 + 26775*x**5 + 26736*x**3 + 9324*x)/(8*(x**8 + 6*
x**6 + 13*x**4 + 12*x**2 + 4))
```

3.93 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -42x+\frac{5x^3}{3}-\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2}+\frac{x(24-409x^2)}{8(2+3x^2+x^4)} -\frac{449\arctan(x)}{8}+\frac{219\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
-42*x+5/3*x^3-1/4*x*(207*x^2+206)/(x^4+3*x^2+2)^2+x*(-409*x^2+24)/(8*x^4+2
4*x^2+16)-449/8*arctan(x)+219/2*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{x(-5124-15416x^2-16233x^4-6755x^6-768x^8+40x^{10})}{24(2+3x^2+x^4)^2} -\frac{449\arctan(x)}{8}+\frac{219\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
(x*(-5124 - 15416*x^2 - 16233*x^4 - 6755*x^6 - 768*x^8 + 40*x^10))/(24*(2
+ 3*x^2 + x^4)^2) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^3} \, dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{1}{8}\int -\frac{2(20x^{10}-48x^8+108x^6-212x^4-615x^2+206)}{(x^4+3x^2+2)^2}dx - \frac{x(207x^2+206)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{20x^{10}-48x^8+108x^6-212x^4-615x^2+206}{(x^4+3x^2+2)^2}dx - \frac{x(207x^2+206)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4}\left(\frac{x(24-409x^2)}{2(x^4+3x^2+2)} - \frac{1}{4}\int -\frac{2(40x^6-216x^4+375x^2+182)}{x^4+3x^2+2}dx\right) - \frac{x(207x^2+206)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{40x^6-216x^4+375x^2+182}{x^4+3x^2+2}dx + \frac{x(24-409x^2)}{2(x^4+3x^2+2)}\right) - \frac{x(207x^2+206)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \left(40x^2+\frac{1303x^2+854}{x^4+3x^2+2}-336\right)dx + \frac{x(24-409x^2)}{2(x^4+3x^2+2)}\right) - \frac{x(207x^2+206)}{4(x^4+3x^2+2)^2}$$

$$\downarrow \textit{2009}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-449\arctan(x)+876\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+\frac{40x^3}{3}-336x\right)+\frac{x(24-409x^2)}{2(x^4+3x^2+2)}\right)-\frac{x(207x^2+206)}{4(x^4+3x^2+2)^2}$$

input
```
Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
-1/4*(x*(206 + 207*x^2))/(2 + 3*x^2 + x^4)^2 + ((x*(24 - 409*x^2))/(2*(2 +
 3*x^2 + x^4)) + (-336*x + (40*x^3)/3 - 449*ArcTan[x] + 876*Sqrt[2]*ArcTan
[x/Sqrt[2]])/2)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{5x^3}{3}-42x+\frac{-\frac{409}{8}x^7-\frac{1203}{8}x^5-145x^3-\frac{91}{2}x}{(x^4+3x^2+2)^2}+\frac{219\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2}-\frac{449\arctan(x)}{8}$	58
default	$\frac{5x^3}{3}-42x-\frac{-\frac{15}{8}x^3-\frac{17}{8}x}{(x^2+1)^2}-\frac{449\arctan(x)}{8}+\frac{-53x^3-54x}{(x^2+2)^2}+\frac{219\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{2}$	62

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
5/3*x^3-42*x+(-409/8*x^7-1203/8*x^5-145*x^3-91/2*x)/(x^4+3*x^2+2)^2+219/2*
2^(1/2)*arctan(1/2*x*2^(1/2))-449/8*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$$
$$=\frac{40\,x^{11}-768\,x^9-6755\,x^7-16233\,x^5-15416\,x^3+2628\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\sqrt{2}\right.}{24\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/24*(40*x^11 - 768*x^9 - 6755*x^7 - 16233*x^5 - 15416*x^3 + 2628*sqrt(2)*
(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1347*(x^8 + 6*
x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) - 5124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*
x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{5x^3}{3} - 42x + \frac{-409x^7-1203x^5-1160x^3-364x}{8x^8+48x^6+104x^4+96x^2+32} - \frac{449\,\mathrm{atan}\,(x)}{8} + \frac{219\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

input
```
integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
5*x**3/3 - 42*x + (-409*x**7 - 1203*x**5 - 1160*x**3 - 364*x)/(8*x**8 + 48
*x**6 + 104*x**4 + 96*x**2 + 32) - 449*atan(x)/8 + 219*sqrt(2)*atan(sqrt(2
)*x/2)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{5}{3}\,x^3 + \frac{219}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - 42\,x - \frac{409\,x^7+1203\,x^5+1160\,x^3+364\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} - \frac{449}{8}\,\arctan\,(x)$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203
*x^5 + 1160*x^3 + 364*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 449/8*arcta
n(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{5}{3}\,x^3+\frac{219}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-42\,x-\frac{409\,x^7+1203\,x^5+1160\,x^3+364\,x}{8\,(x^4+3\,x^2+2)^2}-\frac{449}{8}\,\arctan\left(x\right)$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203
*x^5 + 1160*x^3 + 364*x)/(x^4 + 3*x^2 + 2)^2 - 449/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{219\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}-\frac{449\,\mathrm{atan}(x)}{8}-42\,x-\frac{\frac{409\,x^7}{8}+\frac{1203\,x^5}{8}+145\,x^3+\frac{91\,x}{2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}+\frac{5\,x^3}{3}$$

input
```
int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

output
```
(219*2^(1/2)*atan((2^(1/2)*x)/2))/2 - (449*atan(x))/8 - 42*x - ((91*x)/2 +
 145*x^3 + (1203*x^5)/8 + (409*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
 + (5*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.91

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$$
$$= \frac{2628\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^8+15768\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^6+34164\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^4+31536\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^2+105}{}$$

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

output
```
(2628*sqrt(2)*atan(x/sqrt(2))*x**8 + 15768*sqrt(2)*atan(x/sqrt(2))*x**6 +
34164*sqrt(2)*atan(x/sqrt(2))*x**4 + 31536*sqrt(2)*atan(x/sqrt(2))*x**2 +
10512*sqrt(2)*atan(x/sqrt(2)) - 1347*atan(x)*x**8 - 8082*atan(x)*x**6 - 17
511*atan(x)*x**4 - 16164*atan(x)*x**2 - 5388*atan(x) + 40*x**11 - 768*x**9
 - 6755*x**7 - 16233*x**5 - 15416*x**3 - 5124*x)/(24*(x**8 + 6*x**6 + 13*x
**4 + 12*x**2 + 4))
```

3.94 $\int \frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 75

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} \, dx = 5x + \frac{x(102+103x^2)}{4\left(2+3x^2+x^4\right)^2} - \frac{x(244+15x^2)}{8\left(2+3x^2+x^4\right)} + \frac{413\arctan(x)}{8} - \frac{191\arctan\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

output
```
5*x+1/4*x*(103*x^2+102)/(x^4+3*x^2+2)^2-x*(15*x^2+244)/(8*x^4+24*x^2+16)+4
13/8*arctan(x)-191/4*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} \, dx = \frac{1}{8}\left(\frac{x(-124-76x^2+231x^4+225x^6+40x^8)}{\left(2+3x^2+x^4\right)^2} + 413\arctan(x) - 382\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
((x*(-124 - 76*x^2 + 231*x^4 + 225*x^6 + 40*x^8))/(2 + 3*x^2 + x^4)^2 + 41
3*ArcTan[x] - 382*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^3} \, dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}-\frac{1}{8}\int \frac{2(-20x^8+48x^6-108x^4-303x^2+102)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}-\frac{1}{4}\int \frac{-20x^8+48x^6-108x^4-303x^2+102}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \frac{2(40x^4-231x^2+142)}{x^4+3x^2+2}dx-\frac{x(15x^2+244)}{2(x^4+3x^2+2)}\right)+\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{40x^4-231x^2+142}{x^4+3x^2+2}dx-\frac{x(15x^2+244)}{2(x^4+3x^2+2)}\right)+\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \left(\frac{62-351x^2}{x^4+3x^2+2}+40\right)dx-\frac{x(15x^2+244)}{2(x^4+3x^2+2)}\right)+\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(413\arctan(x)-382\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)+40x\right)-\frac{x(15x^2+244)}{2(x^4+3x^2+2)}\right)+\frac{x(103x^2+102)}{4(x^4+3x^2+2)^2}$$

input
```
Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
(x*(102 + 103*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (-1/2*(x*(244 + 15*x^2))/(2
+ 3*x^2 + x^4) + (40*x + 413*ArcTan[x] - 382*Sqrt[2]*ArcTan[x/Sqrt[2]])/2)
/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

method	result	size
risch	$5x+\frac{-\frac{15}{8}x^{7}-\frac{289}{8}x^{5}-\frac{139}{2}x^{3}-\frac{71}{2}x}{\left(x^{4}+3x^{2}+2\right)^{2}}-\frac{191\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{4}+\frac{413\arctan\left(x\right)}{8}$	53
default	$5x+\frac{-\frac{19}{8}x^{3}-\frac{21}{8}x}{\left(x^{2}+1\right)^{2}}+\frac{413\arctan\left(x\right)}{8}-\frac{16\left(-\frac{1}{32}x^{3}+\frac{25}{16}x\right)}{\left(x^{2}+2\right)^{2}}-\frac{191\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{4}$	56

input
```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
5*x+(-15/8*x^7-289/8*x^5-139/2*x^3-71/2*x)/(x^4+3*x^2+2)^2-191/4*2^(1/2)*a
rctan(1/2*x*2^(1/2))+413/8*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{40\,x^9+225\,x^7+231\,x^5-76\,x^3-382\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+413\,(x^8+6\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/8*(40*x^9 + 225*x^7 + 231*x^5 - 76*x^3 - 382*sqrt(2)*(x^8 + 6*x^6 + 13*x
^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 413*(x^8 + 6*x^6 + 13*x^4 + 12*x^
2 + 4)*arctan(x) - 124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 5x + \frac{-15x^7-289x^5-556x^3-284x}{8x^8+48x^6+104x^4+96x^2+32} + \frac{413\,\text{atan}\,(x)}{8} - \frac{191\sqrt{2}\,\text{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

input
```
integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
5*x + (-15*x**7 - 289*x**5 - 556*x**3 - 284*x)/(8*x**8 + 48*x**6 + 104*x**
4 + 96*x**2 + 32) + 413*atan(x)/8 - 191*sqrt(2)*atan(sqrt(2)*x/2)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 5\,x - \frac{15\,x^7+289\,x^5+556\,x^3+284\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} + \frac{413}{8}\arctan(x)$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
-191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x
^3 + 284*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 413/8*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5\,x - \frac{15\,x^7+289\,x^5+556\,x^3+284\,x}{8\,(x^4+3\,x^2+2)^2}+\frac{413}{8}\arctan\left(x\right)$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
-191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x
^3 + 284*x)/(x^4 + 3*x^2 + 2)^2 + 413/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = 5\,x+\frac{413\,\mathrm{atan}(x)}{8}-\frac{191\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{4} - \frac{\frac{15\,x^7}{8}+\frac{289\,x^5}{8}+\frac{139\,x^3}{2}+\frac{71\,x}{2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

input
```
int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

output
```
5*x + (413*atan(x))/8 - (191*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((71*x)/2 +
(139*x^3)/2 + (289*x^5)/8 + (15*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4
)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.97

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$$
$$= \frac{-382\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^8 - 2292\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^6 - 4966\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^4 - 4584\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^2 - 1528\sqrt{}}{}$$

input

```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

output

```
( - 382*sqrt(2)*atan(x/sqrt(2))*x**8 - 2292*sqrt(2)*atan(x/sqrt(2))*x**6 -
 4966*sqrt(2)*atan(x/sqrt(2))*x**4 - 4584*sqrt(2)*atan(x/sqrt(2))*x**2 - 1
528*sqrt(2)*atan(x/sqrt(2)) + 413*atan(x)*x**8 + 2478*atan(x)*x**6 + 5369*
atan(x)*x**4 + 4956*atan(x)*x**2 + 1652*atan(x) + 40*x**9 + 225*x**7 + 231
*x**5 - 76*x**3 - 124*x)/(8*(x**8 + 6*x**6 + 13*x**4 + 12*x**2 + 4))
```

3.95 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 72

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx = -\frac{x(50+51x^2)}{4(2+3x^2+x^4)^2} + \frac{x(254+125x^2)}{8(2+3x^2+x^4)} - \frac{369\arctan(x)}{8} + \frac{267\arctan\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
-1/4*x*(51*x^2+50)/(x^4+3*x^2+2)^2+x*(125*x^2+254)/(8*x^4+24*x^2+16)-369/8
*arctan(x)+267/8*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx = \frac{1}{8}\left(\frac{x(408+910x^2+629x^4+125x^6)}{(2+3x^2+x^4)^2} - 369\arctan(x) + 267\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
((x*(408 + 910*x^2 + 629*x^4 + 125*x^6))/(2 + 3*x^2 + x^4)^2 - 369*ArcTan[
x] + 267*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2197, 27, 2206, 27, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^3}dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{1}{8}\int -\frac{2(20x^6-48x^4-147x^2+50)}{(x^4+3x^2+2)^2}dx-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{20x^6-48x^4-147x^2+50}{(x^4+3x^2+2)^2}dx-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4}\left(\frac{x(125x^2+254)}{2(x^4+3x^2+2)}-\frac{1}{4}\int \frac{6(68-55x^2)}{x^4+3x^2+2}dx\right)-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{x(125x^2+254)}{2(x^4+3x^2+2)}-\frac{3}{2}\int \frac{68-55x^2}{x^4+3x^2+2}dx\right)-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{4}\left(\frac{x(125x^2+254)}{2(x^4+3x^2+2)}-\frac{3}{2}\left(123\int \frac{1}{x^2+1}dx-178\int \frac{1}{x^2+2}dx\right)\right)-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{x(125x^2+254)}{2(x^4+3x^2+2)}-\frac{3}{2}\left(123\arctan(x)-89\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)\right)-\frac{x(51x^2+50)}{4(x^4+3x^2+2)^2}$$

input
```
Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
-1/4*(x*(50 + 51*x^2))/(2 + 3*x^2 + x^4)^2 + ((x*(254 + 125*x^2))/(2*(2 +
3*x^2 + x^4)) - (3*(123*ArcTan[x] - 89*Sqrt[2]*ArcTan[x/Sqrt[2]]))/2)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\frac{125}{8}x^7+\frac{629}{8}x^5+\frac{455}{4}x^3+51x}{(x^4+3x^2+2)^2}-\frac{369\arctan(x)}{8}+\frac{267\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{8}$	50
default	$-\frac{-\frac{23}{8}x^3-\frac{25}{8}x}{(x^2+1)^2}-\frac{369\arctan(x)}{8}+\frac{\frac{51}{4}x^3+\frac{77}{2}x}{(x^2+2)^2}+\frac{267\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{8}$	54

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(125/8*x^7+629/8*x^5+455/4*x^3+51*x)/(x^4+3*x^2+2)^2-369/8*arctan(x)+267/8
*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$$
$$=\frac{125\,x^7+629\,x^5+910\,x^3+267\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-369\,(x^8+6\,x^6+13}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/8*(125*x^7 + 629*x^5 + 910*x^3 + 267*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*
x^2 + 4)*arctan(1/2*sqrt(2)*x) - 369*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*a
rctan(x) + 408*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{125x^7+629x^5+910x^3+408x}{8x^8+48x^6+104x^4+96x^2+32} - \frac{369\,\mathrm{atan}\,(x)}{8} + \frac{267\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

input
```
integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
(125*x**7 + 629*x**5 + 910*x**3 + 408*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96
*x**2 + 32) - 369*atan(x)/8 + 267*sqrt(2)*atan(sqrt(2)*x/2)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{267}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} - \frac{369}{8}\arctan\left(x\right)$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 4
08*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 369/8*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{267}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\,(x^4+3\,x^2+2)^2} - \frac{369}{8}\arctan\left(x\right)$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 4
08*x)/(x^4 + 3*x^2 + 2)^2 - 369/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 18.60 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{267\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8} - \frac{369\,\mathrm{atan}(x)}{8} + \frac{\frac{125\,x^7}{8}+\frac{629\,x^5}{8}+\frac{455\,x^3}{4}+51\,x}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

input
```
int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

output
```
(267*2^(1/2)*atan((2^(1/2)*x)/2))/8 - (369*atan(x))/8 + (51*x + (455*x^3)/
4 + (629*x^5)/8 + (125*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.99

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$$
$$= \frac{267\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^8+1602\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^6+3471\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^4+3204\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^2+1068\sqrt{2}}{8x}$$

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

output
```
(267*sqrt(2)*atan(x/sqrt(2))*x**8 + 1602*sqrt(2)*atan(x/sqrt(2))*x**6 + 34
71*sqrt(2)*atan(x/sqrt(2))*x**4 + 3204*sqrt(2)*atan(x/sqrt(2))*x**2 + 1068
*sqrt(2)*atan(x/sqrt(2)) - 369*atan(x)*x**8 - 2214*atan(x)*x**6 - 4797*ata
n(x)*x**4 - 4428*atan(x)*x**2 - 1476*atan(x) + 125*x**7 + 629*x**5 + 910*x
**3 + 408*x)/(8*(x**8 + 6*x**6 + 13*x**4 + 12*x**2 + 4))
```

3.96 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 72

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{x(24+25x^2)}{4(2+3x^2+x^4)^2} - \frac{x(211+130x^2)}{8(2+3x^2+x^4)} + \frac{317\arctan(x)}{8} - \frac{447\arctan\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

output
```
1/4*x*(25*x^2+24)/(x^4+3*x^2+2)^2-x*(130*x^2+211)/(8*x^4+24*x^2+16)+317/8*
arctan(x)-447/16*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{1}{16}\left(-\frac{2x(374+843x^2+601x^4+130x^6)}{(2+3x^2+x^4)^2} + 634\arctan(x) - 447\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
((-2*x*(374 + 843*x^2 + 601*x^4 + 130*x^6))/(2 + 3*x^2 + x^4)^2 + 634*ArcT
an[x] - 447*Sqrt[2]*ArcTan[x/Sqrt[2]])/16
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2197, 27, 2206, 27, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (5x^6+3x^4+x^2+4)}{(x^4+3x^2+2)^3} \, dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{x (25x^2+24)}{4 (x^4+3x^2+2)^2}-\frac{1}{8} \int \frac{2 (-20x^4-77x^2+24)}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x (25x^2+24)}{4 (x^4+3x^2+2)^2}-\frac{1}{4} \int \frac{-20x^4-77x^2+24}{(x^4+3x^2+2)^2}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4} \left(\frac{1}{4} \int \frac{2 (187-130x^2)}{x^4+3x^2+2}dx-\frac{x (130x^2+211)}{2 (x^4+3x^2+2)}\right)+\frac{x (25x^2+24)}{4 (x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \left(\frac{1}{2} \int \frac{187-130x^2}{x^4+3x^2+2}dx-\frac{x (130x^2+211)}{2 (x^4+3x^2+2)}\right)+\frac{x (25x^2+24)}{4 (x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{4} \left(\frac{1}{2} \left(317 \int \frac{1}{x^2+1}dx-447 \int \frac{1}{x^2+2}dx\right)-\frac{x (130x^2+211)}{2 (x^4+3x^2+2)}\right)+\frac{x (25x^2+24)}{4 (x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(317\arctan(x)-\frac{447\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}\right)-\frac{x(130x^2+211)}{2(x^4+3x^2+2)}\right)+\frac{x(25x^2+24)}{4(x^4+3x^2+2)^2}$$

input
```
Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

output
```
(x*(24 + 25*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (-1/2*(x*(211 + 130*x^2))/(2 +
 3*x^2 + x^4) + (317*ArcTan[x] - (447*ArcTan[x/Sqrt[2]])/Sqrt[2])/2)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{-\frac{65}{4}x^7-\frac{601}{8}x^5-\frac{843}{8}x^3-\frac{187}{4}x}{\left(x^4+3x^2+2\right)^2}+\frac{317\arctan\left(x\right)}{8}-\frac{447\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{16}$	50
default	$\frac{-\frac{27}{8}x^3-\frac{29}{8}x}{\left(x^2+1\right)^2}+\frac{317\arctan\left(x\right)}{8}-\frac{\frac{103}{8}x^3+\frac{129}{4}x}{\left(x^2+2\right)^2}-\frac{447\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{16}$	53

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-65/4*x^7-601/8*x^5-843/8*x^3-187/4*x)/(x^4+3*x^2+2)^2+317/8*arctan(x)-44
7/16*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{260\,x^7+1202\,x^5+1686\,x^3+447\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-634\,(x^8+6\,x^6}{16\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
-1/16*(260*x^7 + 1202*x^5 + 1686*x^3 + 447*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 +
 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 634*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 +
4)*arctan(x) + 748*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{-130x^7-601x^5-843x^3-374x}{8x^8+48x^6+104x^4+96x^2+32} + \frac{317\,\mathrm{atan}\,(x)}{8} - \frac{447\sqrt{2}\,\mathrm{atan}\,\left(\frac{\sqrt{2}x}{2}\right)}{16}$$

input
```
integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
(-130*x**7 - 601*x**5 - 843*x**3 - 374*x)/(8*x**8 + 48*x**6 + 104*x**4 + 9
6*x**2 + 32) + 317*atan(x)/8 - 447*sqrt(2)*atan(sqrt(2)*x/2)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{447}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) - \frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} + \frac{317}{8}\,\arctan\,(x)$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
-447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 +
 374*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 317/8*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{447}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\,(x^4+3\,x^2+2)^2} + \frac{317}{8}\arctan\left(x\right)$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
-447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 +
 374*x)/(x^4 + 3*x^2 + 2)^2 + 317/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = \frac{317\,\mathrm{atan}(x)}{8} - \frac{447\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16} - \frac{\frac{65\,x^7}{4}+\frac{601\,x^5}{8}+\frac{843\,x^3}{8}+\frac{187\,x}{4}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

input
```
int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

output
```
(317*atan(x))/8 - (447*2^(1/2)*atan((2^(1/2)*x)/2))/16 - ((187*x)/4 + (843
*x^3)/8 + (601*x^5)/8 + (65*x^7)/4)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.99

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx$$
$$= \frac{-447\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^8 - 2682\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^6 - 5811\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^4 - 5364\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^2 - 1788\sqrt{}}{1}$$

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

output
```
( - 447*sqrt(2)*atan(x/sqrt(2))*x**8 - 2682*sqrt(2)*atan(x/sqrt(2))*x**6 -
 5811*sqrt(2)*atan(x/sqrt(2))*x**4 - 5364*sqrt(2)*atan(x/sqrt(2))*x**2 - 1
788*sqrt(2)*atan(x/sqrt(2)) + 634*atan(x)*x**8 + 3804*atan(x)*x**6 + 8242*
atan(x)*x**4 + 7608*atan(x)*x**2 + 2536*atan(x) - 260*x**7 - 1202*x**5 - 1
686*x**3 - 748*x)/(16*(x**8 + 6*x**6 + 13*x**4 + 12*x**2 + 4))
```

3.97 $\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^3} \, dx = -\frac{x(11+12x^2)}{4\left(2+3x^2+x^4\right)^2}+\frac{x(335+217x^2)}{16\left(2+3x^2+x^4\right)}-\frac{257\arctan(x)}{8}+\frac{731\arctan\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

output
```
-1/4*x*(12*x^2+11)/(x^4+3*x^2+2)^2+x*(217*x^2+335)/(16*x^4+48*x^2+32)-257/
8*arctan(x)+731/32*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^3} \, dx = \frac{1}{32}\left(\frac{2x(626+1391x^2+986x^4+217x^6)}{\left(2+3x^2+x^4\right)^2}-1028\arctan(x)+731\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

output
```
((2*x*(626 + 1391*x^2 + 986*x^4 + 217*x^6))/(2 + 3*x^2 + x^4)^2 - 1028*Arc
Tan[x] + 731*Sqrt[2]*ArcTan[x/Sqrt[2]])/32
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2206, 27, 1492, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{(x^4+3x^2+2)^3}dx$$

$$\Big\downarrow \text{2206}$$

$$-\frac{1}{8}\int -\frac{2(19-40x^2)}{(x^4+3x^2+2)^2}dx-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{19-40x^2}{(x^4+3x^2+2)^2}dx-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{4}\left(\frac{x(217x^2+335)}{4(x^4+3x^2+2)}-\frac{1}{4}\int \frac{297-217x^2}{x^4+3x^2+2}dx\right)-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(731\int \frac{1}{x^2+2}dx-514\int \frac{1}{x^2+1}dx\right)+\frac{x(217x^2+335)}{4(x^4+3x^2+2)}\right)-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(\frac{731\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-514\arctan(x)\right)+\frac{x(217x^2+335)}{4(x^4+3x^2+2)}\right)-\frac{x(12x^2+11)}{4(x^4+3x^2+2)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

output
```
-1/4*(x*(11 + 12*x^2))/(2 + 3*x^2 + x^4)^2 + ((x*(335 + 217*x^2))/(4*(2 +
3*x^2 + x^4)) + (-514*ArcTan[x] + (731*ArcTan[x/Sqrt[2]])/Sqrt[2])/4)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{\frac{217}{16}x^7+\frac{493}{8}x^5+\frac{1391}{16}x^3+\frac{313}{8}x}{(x^4+3x^2+2)^2}+\frac{731\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{32}-\frac{257\arctan(x)}{8}$	50
default	$-\frac{-\frac{31}{8}x^3-\frac{33}{8}x}{(x^2+1)^2}-\frac{257\arctan(x)}{8}+\frac{\frac{155}{16}x^3+\frac{181}{8}x}{(x^2+2)^2}+\frac{731\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{32}$	53

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(217/16*x^7+493/8*x^5+1391/16*x^3+313/8*x)/(x^4+3*x^2+2)^2+731/32*2^(1/2)*
arctan(1/2*x*2^(1/2))-257/8*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx$$
$$=\frac{434\,x^7+1972\,x^5+2782\,x^3+731\,\sqrt{2}(x^8+6\,x^6+13\,x^4+12\,x^2+4)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)-1028\,(x^8+6\,x^6+}{32\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/32*(434*x^7 + 1972*x^5 + 2782*x^3 + 731*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 +
12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1028*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 +
4)*arctan(x) + 1252*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = \frac{217x^7+986x^5+1391x^3+626x}{16x^8+96x^6+208x^4+192x^2+64} - \frac{257\,\mathrm{atan}\,(x)}{8} + \frac{731\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

output
```
(217*x**7 + 986*x**5 + 1391*x**3 + 626*x)/(16*x**8 + 96*x**6 + 208*x**4 +
192*x**2 + 64) - 257*atan(x)/8 + 731*sqrt(2)*atan(sqrt(2)*x/2)/32
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = \frac{731}{32}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right) + \frac{217\,x^7+986\,x^5+1391\,x^3+626\,x}{16\,(x^8+6\,x^6+13\,x^4+12\,x^2+4)} - \frac{257}{8}\,\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3
+ 626*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 257/8*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = \frac{731}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{217\,x^7+986\,x^5+1391\,x^3+626\,x}{16\,(x^4+3\,x^2+2)^2} - \frac{257}{8}\arctan(x)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3
+ 626*x)/(x^4 + 3*x^2 + 2)^2 - 257/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx = \frac{731\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32} - \frac{257\,\mathrm{atan}(x)}{8} + \frac{\frac{217\,x^7}{16}+\frac{493\,x^5}{8}+\frac{1391\,x^3}{16}+\frac{313\,x}{8}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^3,x)
```

output
```
(731*2^(1/2)*atan((2^(1/2)*x)/2))/32 - (257*atan(x))/8 + ((313*x)/8 + (139
1*x^3)/16 + (493*x^5)/8 + (217*x^7)/16)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.99

$$\int \frac{4+x^2+3x^4+5x^6}{(2+3x^2+x^4)^3}\,dx$$
$$= \frac{731\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^8+4386\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^6+9503\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^4+8772\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^2+2924\sqrt{2}}{3}$$

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

output
```
(731*sqrt(2)*atan(x/sqrt(2))*x**8 + 4386*sqrt(2)*atan(x/sqrt(2))*x**6 + 95
03*sqrt(2)*atan(x/sqrt(2))*x**4 + 8772*sqrt(2)*atan(x/sqrt(2))*x**2 + 2924
*sqrt(2)*atan(x/sqrt(2)) - 1028*atan(x)*x**8 - 6168*atan(x)*x**6 - 13364*a
tan(x)*x**4 - 12336*atan(x)*x**2 - 4112*atan(x) + 434*x**7 + 1972*x**5 + 2
782*x**3 + 1252*x)/(32*(x**8 + 6*x**6 + 13*x**4 + 12*x**2 + 4))
```

3.98 $\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 79

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3} \, dx = -\frac{1}{2x}+\frac{x(9+11x^2)}{8(2+3x^2+x^4)^2}-\frac{x(547+347x^2)}{32(2+3x^2+x^4)} + \frac{189\arctan(x)}{8}-\frac{1119\arctan\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

output
```
-1/2/x+1/8*x*(11*x^2+9)/(x^4+3*x^2+2)^2-x*(347*x^2+547)/(32*x^4+96*x^2+64)
+189/8*arctan(x)-1119/64*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3} \, dx = \frac{1}{64}\left(-\frac{2(64+1250x^2+2499x^4+1684x^6+363x^8)}{x(2+3x^2+x^4)^2} + 1512\arctan(x)-1119\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

output
```
((-2*(64 + 1250*x^2 + 2499*x^4 + 1684*x^6 + 363*x^8))/(x*(2 + 3*x^2 + x^4)
^2) + 1512*ArcTan[x] - 1119*Sqrt[2]*ArcTan[x/Sqrt[2]])/64
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2198, 25, 2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^2\left(x^4+3x^2+2\right)^3}dx$$

$$\Big\downarrow \text{ 2198}$$

$$\frac{x\left(11x^2+9\right)}{8\left(x^4+3x^2+2\right)^2}-\frac{1}{8}\int -\frac{55x^4-29x^2+16}{x^2\left(x^4+3x^2+2\right)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{8}\int \frac{55x^4-29x^2+16}{x^2\left(x^4+3x^2+2\right)^2}dx+\frac{x\left(11x^2+9\right)}{8\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 2198}$$

$$\frac{1}{8}\left(-\frac{1}{4}\int -\frac{-347x^4+441x^2+32}{x^2\left(x^4+3x^2+2\right)}dx-\frac{x\left(347x^2+547\right)}{4\left(x^4+3x^2+2\right)}\right)+\frac{x\left(11x^2+9\right)}{8\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{8}\left(\frac{1}{4}\int \frac{-347x^4+441x^2+32}{x^2\left(x^4+3x^2+2\right)}dx-\frac{x\left(347x^2+547\right)}{4\left(x^4+3x^2+2\right)}\right)+\frac{x\left(11x^2+9\right)}{8\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 2195}$$

$$\frac{1}{8}\left(\frac{1}{4}\int \left(-\frac{1119}{x^2+2}+\frac{16}{x^2}+\frac{756}{x^2+1}\right)dx-\frac{x\left(347x^2+547\right)}{4\left(x^4+3x^2+2\right)}\right)+\frac{x\left(11x^2+9\right)}{8\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}\left(\frac{1}{4}\left(756\arctan(x)-\frac{1119\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{16}{x}\right)-\frac{x(347x^2+547)}{4(x^4+3x^2+2)}\right)+\frac{x(11x^2+9)}{8(x^4+3x^2+2)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

output
```
(x*(9 + 11*x^2))/(8*(2 + 3*x^2 + x^4)^2) + (-1/4*(x*(547 + 347*x^2))/(2 +
3*x^2 + x^4) + (-16/x + 756*ArcTan[x] - (1119*ArcTan[x/Sqrt[2]])/Sqrt[2])/
4)/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{-\frac{363}{32}x^8-\frac{421}{8}x^6-\frac{2499}{32}x^4-\frac{625}{16}x^2-2}{x(x^4+3x^2+2)^2}-\frac{1119\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{64}+\frac{189\arctan(x)}{8}$	56
default	$\frac{-\frac{35}{8}x^3-\frac{37}{8}x}{(x^2+1)^2}+\frac{189\arctan(x)}{8}-\frac{\frac{207}{16}x^3+\frac{233}{8}x}{2(x^2+2)^2}-\frac{1119\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{64}-\frac{1}{2x}$	58

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-363/32*x^8-421/8*x^6-2499/32*x^4-625/16*x^2-2)/x/(x^4+3*x^2+2)^2-1119/64
*2^(1/2)*arctan(1/2*x*2^(1/2))+189/8*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.37

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = -\frac{726\,x^8+3368\,x^6+4998\,x^4+1119\,\sqrt{2}(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x)\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+2500\,x^2-1}{64\left(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x\right)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
-1/64*(726*x^8 + 3368*x^6 + 4998*x^4 + 1119*sqrt(2)*(x^9 + 6*x^7 + 13*x^5
+ 12*x^3 + 4*x)*arctan(1/2*sqrt(2)*x) + 2500*x^2 - 1512*(x^9 + 6*x^7 + 13*
x^5 + 12*x^3 + 4*x)*arctan(x) + 128)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3}\,dx = \frac{-363x^8-1684x^6-2499x^4-1250x^2-64}{32x^9+192x^7+416x^5+384x^3+128x} + \frac{189\,\mathrm{atan}\,(x)}{8} - \frac{1119\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**3,x)
```

output
```
(-363*x**8 - 1684*x**6 - 2499*x**4 - 1250*x**2 - 64)/(32*x**9 + 192*x**7 +
 416*x**5 + 384*x**3 + 128*x) + 189*atan(x)/8 - 1119*sqrt(2)*atan(sqrt(2)*
x/2)/64
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(2+3x^2+x^4)^3}\,dx = -\frac{1119}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{363\,x^8+1684\,x^6+2499\,x^4+1250\,x^2+64}{32\,(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x)} + \frac{189}{8}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
-1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(363*x^8 + 1684*x^6 + 2499*x
^4 + 1250*x^2 + 64)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x) + 189/8*arctan(x
)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = -\frac{1119}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{347\,x^7+1588\,x^5+2291\,x^3+1058\,x}{32\left(x^4+3\,x^2+2\right)^2} - \frac{1}{2\,x} + \frac{189}{8}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
-1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(347*x^7 + 1588*x^5 + 2291*x
^3 + 1058*x)/(x^4 + 3*x^2 + 2)^2 - 1/2/x + 189/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 18.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = \frac{189\,\mathrm{atan}(x)}{8} - \frac{1119\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64} - \frac{\frac{363\,x^8}{32}+\frac{421\,x^6}{8}+\frac{2499\,x^4}{32}+\frac{625\,x^2}{16}+2}{x^9+6\,x^7+13\,x^5+12\,x^3+4\,x}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^3),x)
```

output
```
(189*atan(x))/8 - (1119*2^(1/2)*atan((2^(1/2)*x)/2))/64 - ((625*x^2)/16 +
(2499*x^4)/32 + (421*x^6)/8 + (363*x^8)/32 + 2)/(4*x + 12*x^3 + 13*x^5 + 6
*x^7 + x^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx$$
$$= \frac{-1119\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^9-6714\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^7-14547\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^5-13428\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^3-44}{}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x)
```

output
```
( - 1119*sqrt(2)*atan(x/sqrt(2))*x**9 - 6714*sqrt(2)*atan(x/sqrt(2))*x**7
- 14547*sqrt(2)*atan(x/sqrt(2))*x**5 - 13428*sqrt(2)*atan(x/sqrt(2))*x**3
- 4476*sqrt(2)*atan(x/sqrt(2))*x + 1512*atan(x)*x**9 + 9072*atan(x)*x**7 +
 19656*atan(x)*x**5 + 18144*atan(x)*x**3 + 6048*atan(x)*x - 726*x**8 - 336
8*x**6 - 4998*x**4 - 2500*x**2 - 128)/(64*x*(x**8 + 6*x**6 + 13*x**4 + 12*
x**2 + 4))
```

3.99 $\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3} \, dx = -\frac{1}{6x^3}+\frac{17}{8x}-\frac{x(5+9x^2)}{16\left(2+3x^2+x^4\right)^2}+\frac{x(951+571x^2)}{64\left(2+3x^2+x^4\right)} - \frac{113\arctan(x)}{8}+\frac{1611\arctan\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

output
```
-1/6/x^3+17/8/x-1/16*x*(9*x^2+5)/(x^4+3*x^2+2)^2+x*(571*x^2+951)/(64*x^4+1
92*x^2+128)-113/8*arctan(x)+1611/128*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3} \, dx = \frac{1}{384}\left(-\frac{64}{x^3}+\frac{816}{x}-\frac{24x(5+9x^2)}{\left(2+3x^2+x^4\right)^2}+\frac{6x(951+571x^2)}{2+3x^2+x^4} - 5424\arctan(x)+4833\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

output
```
(-64/x^3 + 816/x - (24*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4)^2 + (6*x*(951 + 57
1*x^2))/(2 + 3*x^2 + x^4) - 5424*ArcTan[x] + 4833*Sqrt[2]*ArcTan[x/Sqrt[2]
])/384
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2198, 27, 2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^4\left(x^4+3x^2+2\right)^3}dx$$

$$\Big\downarrow \text{2198}$$

$$-\frac{1}{8}\int -\frac{-45x^6+73x^4-40x^2+32}{2x^4\left(x^4+3x^2+2\right)^2}dx-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{-45x^6+73x^4-40x^2+32}{x^4\left(x^4+3x^2+2\right)^2}dx-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{16}\left(\frac{x\left(571x^2+951\right)}{4\left(x^4+3x^2+2\right)}-\frac{1}{4}\int -\frac{571x^6-573x^4-176x^2+64}{x^4\left(x^4+3x^2+2\right)}dx\right)-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{16}\left(\frac{1}{4}\int \frac{571x^6-573x^4-176x^2+64}{x^4\left(x^4+3x^2+2\right)}dx+\frac{x\left(571x^2+951\right)}{4\left(x^4+3x^2+2\right)}\right)-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{16}\left(\frac{1}{4}\int \left(\frac{1611}{x^2+2}-\frac{136}{x^2}+\frac{32}{x^4}-\frac{904}{x^2+1}\right)dx+\frac{x\left(571x^2+951\right)}{4\left(x^4+3x^2+2\right)}\right)-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(\frac{1}{4}\left(-904\arctan(x)+\frac{1611\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{32}{3x^3}+\frac{136}{x}\right)+\frac{x(571x^2+951)}{4(x^4+3x^2+2)}\right)-\frac{x(9x^2+5)}{16(x^4+3x^2+2)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

output
```
-1/16*(x*(5 + 9*x^2))/(2 + 3*x^2 + x^4)^2 + ((x*(951 + 571*x^2))/(4*(2 + 3
*x^2 + x^4)) + (-32/(3*x^3) + 136/x - 904*ArcTan[x] + (1611*ArcTan[x/Sqrt[
2]])/Sqrt[2])/4)/16
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{\frac{707}{64}x^{10}+\frac{1301}{24}x^{8}+\frac{5663}{64}x^{6}+\frac{5063}{96}x^{4}+\frac{13}{2}x^{2}-\frac{2}{3}}{x^{3}\left(x^{4}+3x^{2}+2\right)^{2}}-\frac{113\arctan\left(x\right)}{8}+\frac{1611\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{128}$	61
default	$-\frac{-\frac{39}{8}x^{3}-\frac{41}{8}x}{\left(x^{2}+1\right)^{2}}-\frac{113\arctan\left(x\right)}{8}+\frac{\frac{259}{8}x^{3}+\frac{285}{4}x}{8\left(x^{2}+2\right)^{2}}+\frac{1611\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{128}-\frac{1}{6x^{3}}+\frac{17}{8x}$	64

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(707/64*x^10+1301/24*x^8+5663/64*x^6+5063/96*x^4+13/2*x^2-2/3)/x^3/(x^4+3*
x^2+2)^2-113/8*arctan(x)+1611/128*2^(1/2)*arctan(1/2*x*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.38

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx$$
$$= \frac{4242\,x^{10}+20816\,x^{8}+33978\,x^{6}+20252\,x^{4}+4833\,\sqrt{2}(x^{11}+6\,x^{9}+13\,x^{7}+12\,x^{5}+4\,x^{3})\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)}{384\,(x^{11}+6\,x^{9}+13\,x^{7}+12\,x^{5}+4}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
1/384*(4242*x^10 + 20816*x^8 + 33978*x^6 + 20252*x^4 + 4833*sqrt(2)*(x^11
+ 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(1/2*sqrt(2)*x) + 2496*x^2 - 5424
*(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(x) - 256)/(x^11 + 6*x^9 +
 13*x^7 + 12*x^5 + 4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3}\,dx = -\frac{113\,\mathrm{atan}\,(x)}{8} + \frac{1611\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{128} + \frac{2121x^{10}+10408x^8+16989x^6+10126x^4+1248x^2-128}{192x^{11}+1152x^9+2496x^7+2304x^5+768x^3}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**3,x)
```

output
```
-113*atan(x)/8 + 1611*sqrt(2)*atan(sqrt(2)*x/2)/128 + (2121*x**10 + 10408*
x**8 + 16989*x**6 + 10126*x**4 + 1248*x**2 - 128)/(192*x**11 + 1152*x**9 +
 2496*x**7 + 2304*x**5 + 768*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3}\,dx = \frac{1611}{128}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{2121\,x^{10}+10408\,x^8+16989\,x^6+10126\,x^4+1248\,x^2-128}{192\,(x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3)} - \frac{113}{8}\arctan(x)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/192*(2121*x^10 + 10408*x^8 + 16
989*x^6 + 10126*x^4 + 1248*x^2 - 128)/(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*
x^3) - 113/8*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3}\,dx = \frac{1611}{128}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{571\,x^7+2664\,x^5+3959\,x^3+1882\,x}{64\,(x^4+3\,x^2+2)^2} + \frac{51\,x^2-4}{24\,x^3} - \frac{113}{8}\arctan(x)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/64*(571*x^7 + 2664*x^5 + 3959*x
^3 + 1882*x)/(x^4 + 3*x^2 + 2)^2 + 1/24*(51*x^2 - 4)/x^3 - 113/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3}\,dx = \frac{\frac{707\,x^{10}}{64}+\frac{1301\,x^8}{24}+\frac{5663\,x^6}{64}+\frac{5063\,x^4}{96}+\frac{13\,x^2}{2}-\frac{2}{3}}{x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3} - \frac{113\,\mathrm{atan}(x)}{8} + \frac{1611\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{128}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^3),x)
```

output
```
((13*x^2)/2 + (5063*x^4)/96 + (5663*x^6)/64 + (1301*x^8)/24 + (707*x^10)/6
4 - 2/3)/(4*x^3 + 12*x^5 + 13*x^7 + 6*x^9 + x^11) - (113*atan(x))/8 + (161
1*2^(1/2)*atan((2^(1/2)*x)/2))/128
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(2+3x^2+x^4)^3}\,dx$$
$$= \frac{4833\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^{11}+28998\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^9+62829\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^7+57996\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^5+19}{}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x)
```

output
```
(4833*sqrt(2)*atan(x/sqrt(2))*x**11 + 28998*sqrt(2)*atan(x/sqrt(2))*x**9 +
 62829*sqrt(2)*atan(x/sqrt(2))*x**7 + 57996*sqrt(2)*atan(x/sqrt(2))*x**5 +
 19332*sqrt(2)*atan(x/sqrt(2))*x**3 - 5424*atan(x)*x**11 - 32544*atan(x)*x
**9 - 70512*atan(x)*x**7 - 65088*atan(x)*x**5 - 21696*atan(x)*x**3 + 4242*
x**10 + 20816*x**8 + 33978*x**6 + 20252*x**4 + 2496*x**2 - 256)/(384*x**3*
(x**8 + 6*x**6 + 13*x**4 + 12*x**2 + 4))
```

3.100 $\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 93

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx = -\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{93}{16x}-\frac{x(3-5x^2)}{32\left(2+3x^2+x^4\right)^2} - \frac{x(1771+999x^2)}{128\left(2+3x^2+x^4\right)}+\frac{29\arctan(x)}{8}-\frac{2207\arctan\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

output
```
-1/10/x^5+17/24/x^3-93/16/x-1/32*x*(-5*x^2+3)/(x^4+3*x^2+2)^2-x*(999*x^2+1
771)/(128*x^4+384*x^2+256)+29/8*arctan(x)-2207/256*2^(1/2)*arctan(1/2*x*2^
(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx = \frac{-\frac{2\left(768-3136x^2+30816x^4+170702x^6+246477x^8+137120x^{10}+26145x^{12}\right)}{x^5\left(2+3x^2+x^4\right)^2}+13920\arctan(x)-33105\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)}{3840}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

output
```
((-2*(768 - 3136*x^2 + 30816*x^4 + 170702*x^6 + 246477*x^8 + 137120*x^10 +
 26145*x^12))/(x^5*(2 + 3*x^2 + x^4)^2) + 13920*ArcTan[x] - 33105*Sqrt[2]*
ArcTan[x/Sqrt[2]])/3840
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2198, 27, 2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^6 \left(x^4+3x^2+2\right)^3} \, dx$$

$$\Big\downarrow \text{2198}$$

$$-\frac{1}{8} \int -\frac{25x^8-81x^6+136x^4-80x^2+64}{4x^6 \left(x^4+3x^2+2\right)^2}dx-\frac{x \left(3-5x^2\right)}{32 \left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32} \int \frac{25x^8-81x^6+136x^4-80x^2+64}{x^6 \left(x^4+3x^2+2\right)^2}dx-\frac{x \left(3-5x^2\right)}{32 \left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{32} \left(-\frac{1}{4} \int -\frac{-999x^8+681x^6+736x^4-352x^2+128}{x^6 \left(x^4+3x^2+2\right)}dx-\frac{x \left(999x^2+1771\right)}{4 \left(x^4+3x^2+2\right)}\right)-\frac{x \left(3-5x^2\right)}{32 \left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{32} \left(\frac{1}{4} \int \frac{-999x^8+681x^6+736x^4-352x^2+128}{x^6 \left(x^4+3x^2+2\right)}dx-\frac{x \left(999x^2+1771\right)}{4 \left(x^4+3x^2+2\right)}\right)-\frac{x \left(3-5x^2\right)}{32 \left(x^4+3x^2+2\right)^2}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{32}\left(\frac{1}{4}\int \left(-\frac{2207}{x^2+2}+\frac{744}{x^2}-\frac{272}{x^4}+\frac{64}{x^6}+\frac{464}{x^2+1}\right)dx-\frac{x(999x^2+1771)}{4(x^4+3x^2+2)}\right)-\frac{x(3-5x^2)}{32(x^4+3x^2+2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{32}\left(\frac{1}{4}\left(464\arctan(x)-\frac{2207\arctan\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{64}{5x^5}+\frac{272}{3x^3}-\frac{744}{x}\right)-\frac{x(999x^2+1771)}{4(x^4+3x^2+2)}\right)-\frac{x(3-5x^2)}{32(x^4+3x^2+2)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

output
```
-1/32*(x*(3 - 5*x^2))/(2 + 3*x^2 + x^4)^2 + (-1/4*(x*(1771 + 999*x^2))/(2
+ 3*x^2 + x^4) + (-64/(5*x^5) + 272/(3*x^3) - 744/x + 464*ArcTan[x] - (220
7*ArcTan[x/Sqrt[2]])/Sqrt[2])/4)/32
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{-\frac{1743}{128}x^{12}-\frac{857}{12}x^{10}-\frac{82159}{640}x^{8}-\frac{85351}{960}x^{6}-\frac{321}{20}x^{4}+\frac{49}{30}x^{2}-\frac{2}{5}}{x^{5}\left(x^{4}+3x^{2}+2\right)^{2}}-\frac{2207\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{256}+\frac{29\arctan\left(x\right)}{8}$	66
default	$\frac{-\frac{43}{8}x^{3}-\frac{45}{8}x}{\left(x^{2}+1\right)^{2}}+\frac{29\arctan\left(x\right)}{8}-\frac{\frac{311}{8}x^{3}+\frac{337}{4}x}{16\left(x^{2}+2\right)^{2}}-\frac{2207\sqrt{2}\,\arctan\left(\frac{x\sqrt{2}}{2}\right)}{256}-\frac{1}{10x^{5}}+\frac{17}{24x^{3}}-\frac{93}{16x}$	68

input
```
int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1743/128*x^12-857/12*x^10-82159/640*x^8-85351/960*x^6-321/20*x^4+49/30*x
^2-2/5)/x^5/(x^4+3*x^2+2)^2-2207/256*2^(1/2)*arctan(1/2*x*2^(1/2))+29/8*ar
ctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.33

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}\,dx =$$
$$-\frac{52290\,x^{12}+274240\,x^{10}+492954\,x^{8}+341404\,x^{6}+61632\,x^{4}+33105\,\sqrt{2}(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}}{3840\,(x^{13}+6\,x^{11}+13}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

output
```
-1/3840*(52290*x^12 + 274240*x^10 + 492954*x^8 + 341404*x^6 + 61632*x^4 +
33105*sqrt(2)*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(1/2*sqrt(2)
*x) - 6272*x^2 - 13920*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(x)
 + 1536)/(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3}\,dx = \frac{29\,\mathrm{atan}\,(x)}{8} - \frac{2207\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{256} + \frac{-26145x^{12}-137120x^{10}-246477x^8-170702x^6-30816x^4+3136x^2-768}{1920x^{13}+11520x^{11}+24960x^9+23040x^7+7680x^5}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**3,x)
```

output
```
29*atan(x)/8 - 2207*sqrt(2)*atan(sqrt(2)*x/2)/256 + (-26145*x**12 - 137120
*x**10 - 246477*x**8 - 170702*x**6 - 30816*x**4 + 3136*x**2 - 768)/(1920*x
**13 + 11520*x**11 + 24960*x**9 + 23040*x**7 + 7680*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3}\,dx = -\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{26145\,x^{12}+137120\,x^{10}+246477\,x^8+170702\,x^6+30816\,x^4-3136\,x^2+768}{1920\,(x^{13}+6\,x^{11}+13\,x^9+12\,x^7+4\,x^5)} + \frac{29}{8}\arctan(x)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

output
```
-2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1920*(26145*x^12 + 137120*x^10
 + 246477*x^8 + 170702*x^6 + 30816*x^4 - 3136*x^2 + 768)/(x^13 + 6*x^11 +
13*x^9 + 12*x^7 + 4*x^5) + 29/8*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.72

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3}\,dx = -\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{999\,x^7+4768\,x^5+7291\,x^3+3554\,x}{128\,(x^4+3\,x^2+2)^2} - \frac{1395\,x^4-170\,x^2+24}{240\,x^5} + \frac{29}{8}\arctan\left(x\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

output
```
-2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/128*(999*x^7 + 4768*x^5 + 7291
*x^3 + 3554*x)/(x^4 + 3*x^2 + 2)^2 - 1/240*(1395*x^4 - 170*x^2 + 24)/x^5 +
 29/8*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3}\,dx = \frac{29\,\mathrm{atan}(x)}{8} - \frac{2207\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{256} - \frac{\frac{1743\,x^{12}}{128}+\frac{857\,x^{10}}{12}+\frac{82159\,x^8}{640}+\frac{85351\,x^6}{960}+\frac{321\,x^4}{20}-\frac{49\,x^2}{30}+\frac{2}{5}}{x^{13}+6\,x^{11}+13\,x^9+12\,x^7+4\,x^5}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^3),x)
```

output
```
(29*atan(x))/8 - (2207*2^(1/2)*atan((2^(1/2)*x)/2))/256 - ((321*x^4)/20 -
(49*x^2)/30 + (85351*x^6)/960 + (82159*x^8)/640 + (857*x^10)/12 + (1743*x^
12)/128 + 2/5)/(4*x^5 + 12*x^7 + 13*x^9 + 6*x^11 + x^13)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.76

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3}\,dx$$
$$= \frac{-33105\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^{13} - 198630\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^{11} - 430365\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)x^9 - 397260\sqrt{2}\,atan\left(\frac{x}{\sqrt{2}}\right)}{}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x)
```

output
```
( - 33105*sqrt(2)*atan(x/sqrt(2))*x**13 - 198630*sqrt(2)*atan(x/sqrt(2))*x
**11 - 430365*sqrt(2)*atan(x/sqrt(2))*x**9 - 397260*sqrt(2)*atan(x/sqrt(2)
)*x**7 - 132420*sqrt(2)*atan(x/sqrt(2))*x**5 + 13920*atan(x)*x**13 + 83520
*atan(x)*x**11 + 180960*atan(x)*x**9 + 167040*atan(x)*x**7 + 55680*atan(x)
*x**5 - 52290*x**12 - 274240*x**10 - 492954*x**8 - 341404*x**6 - 61632*x**
4 + 6272*x**2 - 1536)/(3840*x**5*(x**8 + 6*x**6 + 13*x**4 + 12*x**2 + 4))
```

3.101 $\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25(15+7x^2)}{8(3+2x^2+x^4)} + \frac{201\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{183}{4}\log\left(3+2x^2+x^4\right)$$

output
```
19*x^2+19/4*x^4-17/6*x^6+5/8*x^8-25*(7*x^2+15)/(8*x^4+16*x^2+24)+201/16*ar
ctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)-183/4*ln(x^4+2*x^2+3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{48}\left(912x^2+228x^4-136x^6+30x^8-\frac{150(15+7x^2)}{3+2x^2+x^4} + 603\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)-2196\log\left(3+2x^2+x^4\right)\right)$$

input
```
Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(912*x^2 + 228*x^4 - 136*x^6 + 30*x^8 - (150*(15 + 7*x^2))/(3 + 2*x^2 + x^
4) + 603*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] - 2196*Log[3 + 2*x^2 + x^4])/48
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9 (5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2} dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^8 (5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int -\frac{2(-20x^{10}+28x^8-100x^4+200x^2+75)}{x^4+2x^2+3}dx^2-\frac{25(7x^2+15)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{-20x^{10}+28x^8-100x^4+200x^2+75}{x^4+2x^2+3}dx^2-\frac{25(7x^2+15)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \left(-20x^6+68x^4-76x^2+\frac{3(244x^2+177)}{x^4+2x^2+3}-152\right)dx^2-\frac{25(7x^2+15)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{201\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{\sqrt{2}}+5x^8-\frac{68x^6}{3}+38x^4+152x^2-366\log\left(x^4+2x^2+3\right)\right)-\frac{25(7x^2+15)}{4(x^4+2x^2+3)}\right)$$

input
```
Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((-25*(15 + 7*x^2))/(4*(3 + 2*x^2 + x^4)) + (152*x^2 + 38*x^4 - (68*x^6)/3
 + 5*x^8 + (201*ArcTan[(1 + x^2)/Sqrt[2]])/Sqrt[2] - 366*Log[3 + 2*x^2 + x
^4])/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2+\frac{-\frac{175x^2}{8}-\frac{375}{8}}{x^4+2x^2+3}-\frac{183\ln\left(x^4+2x^2+3\right)}{4}+\frac{201\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	71
default	$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2-\frac{\frac{175x^2}{4}+\frac{375}{4}}{2\left(x^4+2x^2+3\right)}-\frac{183\ln\left(x^4+2x^2+3\right)}{4}+\frac{201\sqrt{2}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	74

input
```
int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/8*x^8-17/6*x^6+19/4*x^4+19*x^2+(-175/8*x^2-375/8)/(x^4+2*x^2+3)-183/4*ln
(x^4+2*x^2+3)+201/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$=\frac{30\,x^{12}-76\,x^{10}+46\,x^{8}+960\,x^{6}+2508\,x^{4}+603\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+1686\,x^2-2}{48\left(x^4+2\,x^2+3\right)}$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/48*(30*x^12 - 76*x^10 + 46*x^8 + 960*x^6 + 2508*x^4 + 603*sqrt(2)*(x^4 +
 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1686*x^2 - 2196*(x^4 + 2*x^2 +
 3)*log(x^4 + 2*x^2 + 3) - 2250)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^8}{8} - \frac{17x^6}{6} + \frac{19x^4}{4} + 19x^2 + \frac{-175x^2-375}{8x^4+16x^2+24} - \frac{183\log(x^4+2x^2+3)}{4} + \frac{201\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**8/8 - 17*x**6/6 + 19*x**4/4 + 19*x**2 + (-175*x**2 - 375)/(8*x**4 + 1
6*x**2 + 24) - 183*log(x**4 + 2*x**2 + 3)/4 + 201*sqrt(2)*atan(sqrt(2)*x**
2/2 + sqrt(2)/2)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{8}x^8 - \frac{17}{6}x^6 + \frac{19}{4}x^4 + 19x^2 + \frac{201}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{25(7x^2+15)}{8(x^4+2x^2+3)} - \frac{183}{4}\log(x^4+2x^2+3)$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)
*(x^2 + 1)) - 25/8*(7*x^2 + 15)/(x^4 + 2*x^2 + 3) - 183/4*log(x^4 + 2*x^2
+ 3)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{8}\,x^8 - \frac{17}{6}\,x^6 + \frac{19}{4}\,x^4 + 19\,x^2 + \frac{201}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) + \frac{366\,x^4+557\,x^2+723}{8\,(x^4+2\,x^2+3)} - \frac{183}{4}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)
*(x^2 + 1)) + 1/8*(366*x^4 + 557*x^2 + 723)/(x^4 + 2*x^2 + 3) - 183/4*log(
x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 18.88 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{201\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{\frac{175\,x^2}{8}+\frac{375}{8}}{x^4+2\,x^2+3} - \frac{183\,\ln\left(x^4+2\,x^2+3\right)}{4} + 19\,x^2 + \frac{19\,x^4}{4} - \frac{17\,x^6}{6} + \frac{5\,x^8}{8}$$

input
```
int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(201*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - ((175*x^2)/8 + 375/8)
/(2*x^2 + x^4 + 3) - (183*log(2*x^2 + x^4 + 3))/4 + 19*x^2 + (19*x^4)/4 -
(17*x^6)/6 + (5*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.64

$$\int \frac{x^9(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{-603\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^4-1206\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^2-18}{}$$

input
```
int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

output
```
( - 603*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2
) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 1206*sqrt(sqrt(3) + 1)*sqrt(s
qrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt
(2)))*x**2 - 1809*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) -
 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 603*sqrt(sqrt(3) + 1)*sq
rt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*
sqrt(2)))*x**4 - 1206*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(
3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 1809*sqrt(sqrt(
3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqr
t(3) + 1)*sqrt(2))) - 2196*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) +
x**2)*x**4 - 4392*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**
2 - 6588*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) - 2196*log(s
qrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**4 - 4392*log(sqrt(sqrt(3)
- 1)*sqrt(2)*x + sqrt(3) + x**2)*x**2 - 6588*log(sqrt(sqrt(3) - 1)*sqrt(2)
*x + sqrt(3) + x**2) + 30*x**12 - 76*x**10 + 46*x**8 + 960*x**6 + 1665*x**
4 - 4779)/(48*(x**4 + 2*x**2 + 3))
```

3.102 $\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 81

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx = \frac{19x^2}{2} - \frac{17x^4}{4} + \frac{5x^6}{6} + \frac{25(3+5x^2)}{8(3+2x^2+x^4)} - \frac{455 \arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{19}{2}\log\left(3+2x^2+x^4\right)$$

output
```
19/2*x^2-17/4*x^4+5/6*x^6+25*(5*x^2+3)/(8*x^4+16*x^2+24)-455/16*arctan(1/2
*(x^2+1)*2^(1/2))*2^(1/2)+19/2*ln(x^4+2*x^2+3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx = \frac{1}{48}\left(456x^2 - 204x^4 + 40x^6 + \frac{150(3+5x^2)}{3+2x^2+x^4} - 1365\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right) + 456\log\left(3+2x^2+x^4\right)\right)$$

input
```
Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(456*x^2 - 204*x^4 + 40*x^6 + (150*(3 + 5*x^2))/(3 + 2*x^2 + x^4) - 1365*S
qrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 456*Log[3 + 2*x^2 + x^4])/48
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^6\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int -\frac{2\left(-20x^8+28x^6-100x^2+75\right)}{x^4+2x^2+3}dx^2+\frac{25\left(5x^2+3\right)}{4\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{25\left(5x^2+3\right)}{4\left(x^4+2x^2+3\right)}-\frac{1}{4}\int \frac{-20x^8+28x^6-100x^2+75}{x^4+2x^2+3}dx^2\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(\frac{25\left(5x^2+3\right)}{4\left(x^4+2x^2+3\right)}-\frac{1}{4}\int \left(-20x^4+68x^2+\frac{303-152x^2}{x^4+2x^2+3}-76\right)dx^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{455\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{20x^6}{3}-34x^4+76x^2+76\log\left(x^4+2x^2+3\right)\right)+\frac{25\left(5x^2+3\right)}{4\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((25*(3 + 5*x^2))/(4*(3 + 2*x^2 + x^4)) + (76*x^2 - 34*x^4 + (20*x^6)/3 -
(455*ArcTan[(1 + x^2)/Sqrt[2]])/Sqrt[2] + 76*Log[3 + 2*x^2 + x^4])/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{\frac{125x^2}{8}+\frac{75}{8}}{x^4+2x^2+3}+\frac{19\ln(x^4+2x^2+3)}{2}-\frac{455\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	66
default	$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{\frac{125x^2}{4}+\frac{75}{4}}{2x^4+4x^2+6}+\frac{19\ln(x^4+2x^2+3)}{2}-\frac{455\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	69

input
```
int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/6*x^6-17/4*x^4+19/2*x^2+(125/8*x^2+75/8)/(x^4+2*x^2+3)+19/2*ln(x^4+2*x^2
+3)-455/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$=\frac{40\,x^{10}-124\,x^8+168\,x^6+300\,x^4-1365\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+2118\,x^2+456\,(x^4+}{48\,(x^4+2\,x^2+3)}$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/48*(40*x^10 - 124*x^8 + 168*x^6 + 300*x^4 - 1365*sqrt(2)*(x^4 + 2*x^2 +
3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 2118*x^2 + 456*(x^4 + 2*x^2 + 3)*log(x^
4 + 2*x^2 + 3) + 450)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^6}{6} - \frac{17x^4}{4} + \frac{19x^2}{2} + \frac{125x^2+75}{8x^4+16x^2+24} + \frac{19\log(x^4+2x^2+3)}{2} - \frac{455\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**6/6 - 17*x**4/4 + 19*x**2/2 + (125*x**2 + 75)/(8*x**4 + 16*x**2 + 24)
 + 19*log(x**4 + 2*x**2 + 3)/2 - 455*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)
/2)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{17}{4}\,x^4 + \frac{19}{2}\,x^2 - \frac{455}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) + \frac{25\,(5\,x^2+3)}{8\,(x^4+2\,x^2+3)} + \frac{19}{2}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1
)) + 25/8*(5*x^2 + 3)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 + 3)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{6}\,x^6 - \frac{17}{4}\,x^4 + \frac{19}{2}\,x^2 - \frac{455}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{76\,x^4+27\,x^2+153}{8\,(x^4+2\,x^2+3)} + \frac{19}{2}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1
)) - 1/8*(76*x^4 + 27*x^2 + 153)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2
+ 3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{19\,\ln\left(x^4+2\,x^2+3\right)}{2} + \frac{\frac{125\,x^2}{8}+\frac{75}{8}}{x^4+2\,x^2+3} - \frac{455\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} + \frac{19\,x^2}{2} - \frac{17\,x^4}{4} + \frac{5\,x^6}{6}$$

input
```
int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(19*log(2*x^2 + x^4 + 3))/2 + ((125*x^2)/8 + 75/8)/(2*x^2 + x^4 + 3) - (45
5*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 + (19*x^2)/2 - (17*x^4)/4
+ (5*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 394, normalized size of antiderivative = 4.86

$$\int \frac{x^7(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{1365\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^4+2730\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^2+409}{}$$

input
```
int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

output
```
(1365*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2)
- 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 2730*sqrt(sqrt(3) + 1)*sqrt(sqr
t(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2
)))*x**2 + 4095*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1
)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 1365*sqrt(sqrt(3) + 1)*sqr
t(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*s
qrt(2)))*x**4 + 2730*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3
) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 4095*sqrt(sqrt(3
) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt
(3) + 1)*sqrt(2))) + 456*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x*
*2)*x**4 + 912*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**2 +
 1368*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) + 456*log(sqrt(
sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**4 + 912*log(sqrt(sqrt(3) - 1)*
sqrt(2)*x + sqrt(3) + x**2)*x**2 + 1368*log(sqrt(sqrt(3) - 1)*sqrt(2)*x +
sqrt(3) + x**2) + 40*x**10 - 124*x**8 + 168*x**6 - 759*x**4 - 2727)/(48*(x
**4 + 2*x**2 + 3))
```

3.103 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 74

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25(3-x^2)}{8(3+2x^2+x^4)} + \frac{203\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{4}\log\left(3+2x^2+x^4\right)$$

output
```
-17/2*x^2+5/4*x^4+25*(-x^2+3)/(8*x^4+16*x^2+24)+203/16*arctan(1/2*(x^2+1)*
2^(1/2))*2^(1/2)+19/4*ln(x^4+2*x^2+3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{16}\left(-136x^2+20x^4-\frac{50(-3+x^2)}{3+2x^2+x^4} + 203\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)+76\log\left(3+2x^2+x^4\right)\right)$$

input
```
Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(-136*x^2 + 20*x^4 - (50*(-3 + x^2))/(3 + 2*x^2 + x^4) + 203*Sqrt[2]*ArcTa
n[(1 + x^2)/Sqrt[2]] + 76*Log[3 + 2*x^2 + x^4])/16
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{x^4(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{2(20x^6-28x^4+75)}{x^4+2x^2+3}dx^2+\frac{25(3-x^2)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \frac{20x^6-28x^4+75}{x^4+2x^2+3}dx^2+\frac{25(3-x^2)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \left(20x^2+\frac{76x^2+279}{x^4+2x^2+3}-68\right)dx^2+\frac{25(3-x^2)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{203\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{\sqrt{2}}+10x^4-68x^2+38\log\left(x^4+2x^2+3\right)\right)+\frac{25(3-x^2)}{4(x^4+2x^2+3)}\right)$$

input
```
Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((25*(3 - x^2))/(4*(3 + 2*x^2 + x^4)) + (-68*x^2 + 10*x^4 + (203*ArcTan[(1
 + x^2)/Sqrt[2]])/Sqrt[2] + 38*Log[3 + 2*x^2 + x^4])/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{289}{20}+\frac{-\frac{25x^2}{8}+\frac{75}{8}}{x^4+2x^2+3}+\frac{19\ln\left(x^4+2x^2+3\right)}{4}+\frac{203\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	62
default	$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{-\frac{25x^2}{4}+\frac{75}{4}}{2x^4+4x^2+6}+\frac{19\ln\left(x^4+2x^2+3\right)}{4}+\frac{203\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	64

input
```
int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/4*x^4-17/2*x^2+289/20+(-25/8*x^2+75/8)/(x^4+2*x^2+3)+19/4*ln(x^4+2*x^2+3
)+203/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$=\frac{20\,x^8-96\,x^6-212\,x^4+203\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)-458\,x^2+76\,(x^4+2\,x^2+3)\log}{16\,(x^4+2\,x^2+3)}$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/16*(20*x^8 - 96*x^6 - 212*x^4 + 203*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2
*sqrt(2)*(x^2 + 1)) - 458*x^2 + 76*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3)
+ 150)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^4}{4} - \frac{17x^2}{2} + \frac{75-25x^2}{8x^4+16x^2+24} + \frac{19\log(x^4+2x^2+3)}{4} + \frac{203\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**4/4 - 17*x**2/2 + (75 - 25*x**2)/(8*x**4 + 16*x**2 + 24) + 19*log(x**
4 + 2*x**2 + 3)/4 + 203*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{4}x^4 - \frac{17}{2}x^2 + \frac{203}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{25(x^2-3)}{8(x^4+2x^2+3)} + \frac{19}{4}\log\left(x^4+2x^2+3\right)$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(
x^2 - 3)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{4}\,x^4 - \frac{17}{2}\,x^2 + \frac{203}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{38\,x^4+101\,x^2+39}{8\,(x^4+2\,x^2+3)} + \frac{19}{4}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(3
8*x^4 + 101*x^2 + 39)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{19\,\ln\left(x^4+2\,x^2+3\right)}{4} - \frac{\frac{25\,x^2}{8}-\frac{75}{8}}{x^4+2\,x^2+3} + \frac{203\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{17\,x^2}{2} + \frac{5\,x^4}{4}$$

input
```
int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(19*log(2*x^2 + x^4 + 3))/4 - ((25*x^2)/8 - 75/8)/(2*x^2 + x^4 + 3) + (203
*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - (17*x^2)/2 + (5*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 389, normalized size of antiderivative = 5.26

$$\int \frac{x^5(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{-203\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^4-406\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^2-609}{}$$

input
```
int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

output
```
( - 203*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2
) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 406*sqrt(sqrt(3) + 1)*sqrt(sq
rt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(
2)))*x**2 - 609*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1
)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 203*sqrt(sqrt(3) + 1)*sqrt
(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sq
rt(2)))*x**4 - 406*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3)
- 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 609*sqrt(sqrt(3) +
 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3)
 + 1)*sqrt(2))) + 76*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*
x**4 + 152*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**2 + 228
*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) + 76*log(sqrt(sqrt(3
) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**4 + 152*log(sqrt(sqrt(3) - 1)*sqrt(2
)*x + sqrt(3) + x**2)*x**2 + 228*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3)
 + x**2) + 20*x**8 - 96*x**6 + 17*x**4 + 837)/(16*(x**4 + 2*x**2 + 3))
```

3.104 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 65

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^2}{2} - \frac{25(3+x^2)}{8(3+2x^2+x^4)} - \frac{17\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} - \frac{17}{4}\log\left(3+2x^2+x^4\right)$$

output
```
5/2*x^2-25*(x^2+3)/(8*x^4+16*x^2+24)-17/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(
1/2)-17/4*ln(x^4+2*x^2+3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{1}{16}\left(40x^2 - \frac{50(3+x^2)}{3+2x^2+x^4} - 17\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right) - 68\log\left(3+2x^2+x^4\right)\right)$$

input
```
Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(40*x^2 - (50*(3 + x^2))/(3 + 2*x^2 + x^4) - 17*Sqrt[2]*ArcTan[(1 + x^2)/S
qrt[2]] - 68*Log[3 + 2*x^2 + x^4])/16
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2194, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2} \, dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{x^2 (5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx^2$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int -\frac{2(-20x^4+28x^2+25)}{x^4+2x^2+3}dx^2-\frac{25(x^2+3)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{-20x^4+28x^2+25}{x^4+2x^2+3}dx^2-\frac{25(x^2+3)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{ 2188}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \left(\frac{17(4x^2+5)}{x^4+2x^2+3}-20\right)dx^2-\frac{25(x^2+3)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{17\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{\sqrt{2}}+20x^2-34\log\left(x^4+2x^2+3\right)\right)-\frac{25(x^2+3)}{4(x^4+2x^2+3)}\right)$$

input
```
Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((-25*(3 + x^2))/(4*(3 + 2*x^2 + x^4)) + (20*x^2 - (17*ArcTan[(1 + x^2)/Sq
rt[2]])/Sqrt[2] - 34*Log[3 + 2*x^2 + x^4])/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{5x^2}{2}+\frac{-\frac{25x^2}{8}-\frac{75}{8}}{x^4+2x^2+3}-\frac{17\ln\left(x^4+2x^2+3\right)}{4}-\frac{17\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	56
default	$\frac{5x^2}{2}-\frac{\frac{25x^2}{4}+\frac{75}{4}}{2\left(x^4+2x^2+3\right)}-\frac{17\ln\left(x^4+2x^2+3\right)}{4}-\frac{17\sqrt{2}\ \arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	59

input
```
int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/2*x^2+(-25/8*x^2-75/8)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17/16*arctan(1
/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$=\frac{40\,x^6+80\,x^4-17\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+70\,x^2-68\,(x^4+2\,x^2+3)\log\left(x^4+2\,x^2-\right.}{16\,(x^4+2\,x^2+3)}$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/16*(40*x^6 + 80*x^4 - 17*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x
^2 + 1)) + 70*x^2 - 68*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 150)/(x^4
+ 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^2}{2} + \frac{-25x^2-75}{8x^4+16x^2+24} - \frac{17\log(x^4+2x^2+3)}{4} - \frac{17\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**2/2 + (-25*x**2 - 75)/(8*x**4 + 16*x**2 + 24) - 17*log(x**4 + 2*x**2
+ 3)/4 - 17*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{2}x^2 - \frac{17}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{25(x^2+3)}{8(x^4+2x^2+3)} - \frac{17}{4}\log\left(x^4+2x^2+3\right)$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^
4 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5}{2}\,x^2 - \frac{17}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{25\,(x^2+3)}{8\,(x^4+2\,x^2+3)} - \frac{17}{4}\,\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^
4 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 18.71 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5\,x^2}{2} - \frac{\frac{25\,x^2}{8}+\frac{75}{8}}{x^4+2\,x^2+3} - \frac{17\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{17\,\ln\left(x^4+2\,x^2+3\right)}{4}$$

input
```
int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(5*x^2)/2 - ((25*x^2)/8 + 75/8)/(2*x^2 + x^4 + 3) - (17*2^(1/2)*atan(2^(1/
2)/2 + (2^(1/2)*x^2)/2))/16 - (17*log(2*x^2 + x^4 + 3))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 384, normalized size of antiderivative = 5.91

$$\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{17\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^4+34\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^2+51\sqrt{\sqrt{3}}}{}$$

input
```
int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

output
```
(17*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) -
2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 34*sqrt(sqrt(3) + 1)*sqrt(sqrt(3)
 - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*
x**2 + 51*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt
(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 17*sqrt(sqrt(3) + 1)*sqrt(sqrt(3
) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))
*x**4 + 34*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqr
t(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 51*sqrt(sqrt(3) + 1)*sqrt(
sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqr
t(2))) - 68*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**4 - 13
6*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**2 - 204*log( - s
qrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) - 68*log(sqrt(sqrt(3) - 1)*sq
rt(2)*x + sqrt(3) + x**2)*x**4 - 136*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqr
t(3) + x**2)*x**2 - 204*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)
+ 40*x**6 + 45*x**4 - 255)/(16*(x**4 + 2*x**2 + 3))
```

3.105 $\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 58

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx = \frac{25(1+x^2)}{8(3+2x^2+x^4)} - \frac{23 \arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5}{4} \log\left(3+2x^2+x^4\right)$$

output
```
25*(x^2+1)/(8*x^4+16*x^2+24)-23/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)+5/4
*ln(x^4+2*x^2+3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx = \frac{25(1+x^2)}{8(3+2x^2+x^4)} - \frac{23 \arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5}{4} \log\left(3+2x^2+x^4\right)$$

input
```
Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*S
qrt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2194, 2191, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{(x^4+2x^2+3)^2}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int -\frac{2(3-20x^2)}{x^4+2x^2+3}dx^2+\frac{25(x^2+1)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{25(x^2+1)}{4(x^4+2x^2+3)}-\frac{1}{4}\int \frac{3-20x^2}{x^4+2x^2+3}dx^2\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(10\int \frac{2(x^2+1)}{x^4+2x^2+3}dx^2-23\int \frac{1}{x^4+2x^2+3}dx^2\right)+\frac{25(x^2+1)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(20\int \frac{x^2+1}{x^4+2x^2+3}dx^2-23\int \frac{1}{x^4+2x^2+3}dx^2\right)+\frac{25(x^2+1)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(46\int \frac{1}{-x^4-8}d(2x^2+2)+20\int \frac{x^2+1}{x^4+2x^2+3}dx^2\right)+\frac{25(x^2+1)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(20\int\frac{x^2+1}{x^4+2x^2+3}dx^2-\frac{23\arctan\left(\frac{2x^2+2}{2\sqrt{2}}\right)}{\sqrt{2}}\right)+\frac{25(x^2+1)}{4(x^4+2x^2+3)}\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2}\left(\frac{1}{4}\left(10\log\left(x^4+2x^2+3\right)-\frac{23\arctan\left(\frac{2x^2+2}{2\sqrt{2}}\right)}{\sqrt{2}}\right)+\frac{25(x^2+1)}{4(x^4+2x^2+3)}\right)$$

input
```
Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((25*(1 + x^2))/(4*(3 + 2*x^2 + x^4)) + ((-23*ArcTan[(2 + 2*x^2)/(2*Sqrt[2
])])/Sqrt[2] + 10*Log[3 + 2*x^2 + x^4])/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{\frac{25x^2}{8}+\frac{25}{8}}{x^4+2x^2+3}+\frac{5\ln\left(x^4+2x^2+3\right)}{4}-\frac{23\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	51
default	$\frac{\frac{25x^2}{4}+\frac{25}{4}}{2x^4+4x^2+6}+\frac{5\ln\left(x^4+2x^2+3\right)}{4}-\frac{23\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	54

input
```
int(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(25/8*x^2+25/8)/(x^4+2*x^2+3)+5/4*ln(x^4+2*x^2+3)-23/16*arctan(1/2*(x^2+1)
*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx = -\frac{23 \sqrt{2}(x^4+2 x^2+3) \arctan\left(\frac{1}{2} \sqrt{2}(x^2+1)\right) - 50 x^2 - 20 (x^4+2 x^2+3) \log(x^4+2 x^2+3) - 50}{16 (x^4+2 x^2+3)}$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/16*(23*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 50*x^2
 - 20*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 50)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} \, dx = \frac{25x^2+25}{8x^4+16x^2+24} + \frac{5\log(x^4+2x^2+3)}{4} - \frac{23\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
(25*x**2 + 25)/(8*x**4 + 16*x**2 + 24) + 5*log(x**4 + 2*x**2 + 3)/4 - 23*s
qrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{25\,(x^2+1)}{8\,(x^4+2\,x^2+3)} + \frac{5}{4}\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
 + 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{25\,(x^2+1)}{8\,(x^4+2\,x^2+3)} + \frac{5}{4}\log\left(x^4+2\,x^2+3\right)$$

input
```
integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
-23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
 + 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5\,\ln\left(x^4+2\,x^2+3\right)}{4}+\frac{25\,x^2}{8\,\left(x^4+2\,x^2+3\right)}+\frac{25}{8\,\left(x^4+2\,x^2+3\right)}-\frac{23\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(5*log(2*x^2 + x^4 + 3))/4 + (25*x^2)/(8*(2*x^2 + x^4 + 3)) + 25/(8*(2*x^2
 + x^4 + 3)) - (23*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 379, normalized size of antiderivative = 6.53

$$\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{23\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^4+46\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^2+69\sqrt{\sqrt{3}}}{}$$

input
```
int(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

output
```
(23*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) -
2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 46*sqrt(sqrt(3) + 1)*sqrt(sqrt(3)
 - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*
x**2 + 69*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt
(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 23*sqrt(sqrt(3) + 1)*sqrt(sqrt(3
) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))
*x**4 + 46*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqr
t(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 69*sqrt(sqrt(3) + 1)*sqrt(
sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqr
t(2))) + 20*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**4 + 40
*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**2 + 60*log( - sqr
t(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) + 20*log(sqrt(sqrt(3) - 1)*sqrt
(2)*x + sqrt(3) + x**2)*x**4 + 40*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3
) + x**2)*x**2 + 60*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) - 25
*x**4 - 25)/(16*(x**4 + 2*x**2 + 3))
```

3.106 $\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 66

$$\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2} \, dx = \frac{25(1-x^2)}{24(3+2x^2+x^4)} + \frac{89\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{72\sqrt{2}} + \frac{4\log(x)}{9} - \frac{1}{9}\log\left(3+2x^2+x^4\right)$$

output
```
25*(-x^2+1)/(24*x^4+48*x^2+72)+89/144*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)+
4/9*ln(x)-1/9*ln(x^4+2*x^2+3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{4+x^2+3x^4+5x^6}{x(3+2x^2+x^4)^2} \, dx = \frac{1}{288}\left(178\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right) + 128\log(x) + \frac{4(75-75x^2-8(3+2x^2+x^4)\log(3+2x^2+x^4))}{3+2x^2+x^4}\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(178*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 128*Log[x] + (4*(75 - 75*x^2 - 8*
(3 + 2*x^2 + x^4)*Log[3 + 2*x^2 + x^4]))/(3 + 2*x^2 + x^4))/288
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2177, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{x^2\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{2\left(35x^2+16\right)}{3x^2\left(x^4+2x^2+3\right)}dx^2+\frac{25\left(1-x^2\right)}{12\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{12}\int \frac{35x^2+16}{x^2\left(x^4+2x^2+3\right)}dx^2+\frac{25\left(1-x^2\right)}{12\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(\frac{1}{12}\int \left(\frac{73-16x^2}{3\left(x^4+2x^2+3\right)}+\frac{16}{3x^2}\right)dx^2+\frac{25\left(1-x^2\right)}{12\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(\frac{89\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{16\log\left(x^2\right)}{3}-\frac{8}{3}\log\left(x^4+2x^2+3\right)\right)+\frac{25\left(1-x^2\right)}{12\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((25*(1 - x^2))/(12*(3 + 2*x^2 + x^4)) + ((89*ArcTan[(1 + x^2)/Sqrt[2]])/(
3*Sqrt[2]) + (16*Log[x^2])/3 - (8*Log[3 + 2*x^2 + x^4])/3)/12)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\frac{75x^2}{4}-\frac{75}{4}}{18(x^4+2x^2+3)}-\frac{\ln(x^4+2x^2+3)}{9}+\frac{89\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{144}+\frac{4\ln(x)}{9}$	58
risch	$\frac{-\frac{25x^2}{24}+\frac{25}{24}}{x^4+2x^2+3}+\frac{4\ln(x)}{9}-\frac{\ln(7921x^4+15842x^2+23763)}{9}+\frac{89\sqrt{2}\arctan\left(\frac{(89x^2+89)\sqrt{2}}{178}\right)}{144}$	59

input
```
int((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(75/4*x^2-75/4)/(x^4+2*x^2+3)-1/9*ln(x^4+2*x^2+3)+89/144*2^(1/2)*arc
tan(1/4*(2*x^2+2)*2^(1/2))+4/9*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int \frac{4+x^2+3x^4+5x^6}{x\,(3+2x^2+x^4)^2}\,dx$$
$$=\frac{89\,\sqrt{2}(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)-150\,x^2-16\,(x^4+2\,x^2+3)\log\left(x^4+2\,x^2+3\right)+64\,(x^4+}{144\,(x^4+2\,x^2+3)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/144*(89*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 150*x^
2 - 16*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) + 64*(x^4 + 2*x^2 + 3)*log(x
) + 150)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2}\,dx = \frac{25-25x^2}{24x^4+48x^2+72}+\frac{4\log\left(x\right)}{9} - \frac{\log\left(x^4+2x^2+3\right)}{9}+\frac{89\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+2*x**2+3)**2,x)
```

output
```
(25 - 25*x**2)/(24*x**4 + 48*x**2 + 72) + 4*log(x)/9 - log(x**4 + 2*x**2 +
 3)/9 + 89*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/144
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2}\,dx = \frac{89}{144}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)-\frac{25\left(x^2-1\right)}{24\left(x^4+2\,x^2+3\right)} - \frac{1}{9}\,\log\left(x^4+2\,x^2+3\right)+\frac{2}{9}\,\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/24*(x^2 - 1)/(x^4 + 2*x^
2 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2}\,dx = \frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) + \frac{8\,x^4-59\,x^2+99}{72\left(x^4+2\,x^2+3\right)} - \frac{1}{9}\log\left(x^4+2\,x^2+3\right) + \frac{2}{9}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(8*x^4 - 59*x^2 + 99)/
(x^4 + 2*x^2 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 18.69 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2}\,dx = \frac{4\,\ln(x)}{9} - \frac{\ln\left(x^4+2\,x^2+3\right)}{9} - \frac{\frac{25\,x^2}{24}-\frac{25}{24}}{x^4+2\,x^2+3} + \frac{89\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(2*x^2 + x^4 + 3)^2),x)
```

output
```
(4*log(x))/9 - log(2*x^2 + x^4 + 3)/9 - ((25*x^2)/24 - 25/24)/(2*x^2 + x^4
 + 3) + (89*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/144
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 397, normalized size of antiderivative = 6.02

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2}\,dx$$
$$= \frac{-89\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^4-178\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^2-267}{}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x)
```

output
```
( - 89*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2)
 - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 178*sqrt(sqrt(3) + 1)*sqrt(sqr
t(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2
)))*x**2 - 267*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)
*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 89*sqrt(sqrt(3) + 1)*sqrt(s
qrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt
(2)))*x**4 - 178*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) -
1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 267*sqrt(sqrt(3) + 1
)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) +
 1)*sqrt(2))) - 16*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x*
*4 - 32*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**2 - 48*log
( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2) - 16*log(sqrt(sqrt(3) -
1)*sqrt(2)*x + sqrt(3) + x**2)*x**4 - 32*log(sqrt(sqrt(3) - 1)*sqrt(2)*x +
 sqrt(3) + x**2)*x**2 - 48*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**
2) + 64*log(x)*x**4 + 128*log(x)*x**2 + 192*log(x) + 75*x**4 + 375)/(144*(
x**4 + 2*x**2 + 3))
```

3.107 $\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 71

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx=-\frac{2}{9x^2}-\frac{25(5+x^2)}{72\left(3+2x^2+x^4\right)}-\frac{71\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{13\log(x)}{27}+\frac{13}{108}\log\left(3+2x^2+x^4\right)$$

output
```
-2/9/x^2-25*(x^2+5)/(72*x^4+144*x^2+216)-71/432*arctan(1/2*(x^2+1)*2^(1/2)
)*2^(1/2)-13/27*ln(x)+13/108*ln(x^4+2*x^2+3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx=\frac{1}{864}\left(-\frac{192}{x^2}-\frac{300(5+x^2)}{3+2x^2+x^4}-416\log(x)+\sqrt{2}\left(-71i+52\sqrt{2}\right)\log\left(-i+\sqrt{2}-ix^2\right)+\sqrt{2}\left(71i+52\sqrt{2}\right)\log\left(i+\sqrt{2}+ix^2\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(-192/x^2 - (300*(5 + x^2))/(3 + 2*x^2 + x^4) - 416*Log[x] + Sqrt[2]*(-71*
I + 52*Sqrt[2])*Log[-I + Sqrt[2] - I*x^2] + Sqrt[2]*(71*I + 52*Sqrt[2])*Lo
g[I + Sqrt[2] + I*x^2])/864
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^3 \left(x^4+2x^2+3\right)^2} \, dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2} \int \frac{5x^6+3x^4+x^2+4}{x^4 \left(x^4+2x^2+3\right)^2} dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2} \left(\frac{1}{8} \int \frac{2\left(-25x^4-20x^2+48\right)}{9x^4 \left(x^4+2x^2+3\right)} dx^2 - \frac{25\left(x^2+5\right)}{36 \left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{1}{36} \int \frac{-25x^4-20x^2+48}{x^4 \left(x^4+2x^2+3\right)} dx^2 - \frac{25\left(x^2+5\right)}{36 \left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2} \left(\frac{1}{36} \int \left(\frac{52x^2-19}{3 \left(x^4+2x^2+3\right)} - \frac{52}{3x^2} + \frac{16}{x^4}\right) dx^2 - \frac{25\left(x^2+5\right)}{36 \left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{36}\left(-\frac{71\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{16}{x^2}-\frac{52\log\left(x^2\right)}{3}+\frac{26}{3}\log\left(x^4+2x^2+3\right)\right)-\frac{25\left(x^2+5\right)}{36\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((-25*(5 + x^2))/(36*(3 + 2*x^2 + x^4)) + (-16/x^2 - (71*ArcTan[(1 + x^2)/
Sqrt[2]])/(3*Sqrt[2]) - (52*Log[x^2])/3 + (26*Log[3 + 2*x^2 + x^4])/3)/36)
/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{-\frac{75x^2}{4}-\frac{375}{4}}{54x^4+108x^2+162}+\frac{13\ln(x^4+2x^2+3)}{108}-\frac{71\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{432}-\frac{2}{9x^2}-\frac{13\ln(x)}{27}$	63
risch	$\frac{-\frac{41}{72}x^4-\frac{157}{72}x^2-\frac{2}{3}}{x^2(x^4+2x^2+3)}-\frac{13\ln(x)}{27}+\frac{13\ln(5041x^4+10082x^2+15123)}{108}-\frac{71\sqrt{2}\,\arctan\left(\frac{(71x^2+71)\sqrt{2}}{142}\right)}{432}$	67

input
```
int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/54*(-75/4*x^2-375/4)/(x^4+2*x^2+3)+13/108*ln(x^4+2*x^2+3)-71/432*2^(1/2)
*arctan(1/4*(2*x^2+2)*2^(1/2))-2/9/x^2-13/27*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.48

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx = -\frac{246\,x^4+71\,\sqrt{2}(x^6+2\,x^4+3\,x^2)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+942\,x^2-52\,(x^6+2\,x^4+3\,x^2)\log\left(x^4+2\,x\right.}{432\,(x^6+2\,x^4+3\,x^2)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/432*(246*x^4 + 71*sqrt(2)*(x^6 + 2*x^4 + 3*x^2)*arctan(1/2*sqrt(2)*(x^2
 + 1)) + 942*x^2 - 52*(x^6 + 2*x^4 + 3*x^2)*log(x^4 + 2*x^2 + 3) + 208*(x^
6 + 2*x^4 + 3*x^2)*log(x) + 288)/(x^6 + 2*x^4 + 3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx = \frac{-41x^4-157x^2-48}{72x^6+144x^4+216x^2} - \frac{13\log\left(x\right)}{27} + \frac{13\log\left(x^4+2x^2+3\right)}{108} - \frac{71\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+2*x**2+3)**2,x)
```

output
```
(-41*x**4 - 157*x**2 - 48)/(72*x**6 + 144*x**4 + 216*x**2) - 13*log(x)/27
+ 13*log(x**4 + 2*x**2 + 3)/108 - 71*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)
/2)/432
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx = -\frac{71}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{41\,x^4+157\,x^2+48}{72\,(x^6+2\,x^4+3\,x^2)} + \frac{13}{108}\log\left(x^4+2\,x^2+3\right) - \frac{13}{54}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 4
8)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx = -\frac{71}{432}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{41\,x^4+157\,x^2+48}{72\,(x^6+2\,x^4+3\,x^2)} + \frac{13}{108}\,\log\left(x^4+2\,x^2+3\right) - \frac{13}{54}\,\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
-71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 4
8)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx = \frac{13\,\ln\left(x^4+2\,x^2+3\right)}{108} - \frac{13\,\ln\left(x\right)}{27} - \frac{\frac{41\,x^4}{72}+\frac{157\,x^2}{72}+\frac{2}{3}}{x^6+2\,x^4+3\,x^2} - \frac{71\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(2*x^2 + x^4 + 3)^2),x)
```

output
```
(13*log(2*x^2 + x^4 + 3))/108 - (13*log(x))/27 - ((157*x^2)/72 + (41*x^4)/
72 + 2/3)/(3*x^2 + 2*x^4 + x^6) - (71*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^
2)/2))/432
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 419, normalized size of antiderivative = 5.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx = \frac{71\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^6+142\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^4+213\sqrt{\ }}{}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x)
```

output
```
(71*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) -
2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 + 142*sqrt(sqrt(3) + 1)*sqrt(sqrt(3
) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))
*x**4 + 213*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sq
rt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 71*sqrt(sqrt(3) + 1)*sqrt
(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sq
rt(2)))*x**6 + 142*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3)
- 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 213*sqrt(sqrt(3) +
 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3)
 + 1)*sqrt(2)))*x**2 + 52*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x
**2)*x**6 + 104*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**4
+ 156*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**2 + 52*log(s
qrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**6 + 104*log(sqrt(sqrt(3) -
 1)*sqrt(2)*x + sqrt(3) + x**2)*x**4 + 156*log(sqrt(sqrt(3) - 1)*sqrt(2)*x
 + sqrt(3) + x**2)*x**2 - 208*log(x)*x**6 - 416*log(x)*x**4 - 624*log(x)*x
**2 + 123*x**6 - 573*x**2 - 288)/(432*x**2*(x**4 + 2*x**2 + 3))
```

3.108 $\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx=-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25(7+5x^2)}{216\left(3+2x^2+x^4\right)}+\frac{125\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{13\log(x)}{27}-\frac{13}{108}\log\left(3+2x^2+x^4\right)$$

output
```
-1/9/x^4+13/54/x^2+25*(5*x^2+7)/(216*x^4+432*x^2+648)+125/432*arctan(1/2*(
x^2+1)*2^(1/2))*2^(1/2)+13/27*ln(x)-13/108*ln(x^4+2*x^2+3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx=\frac{1}{864}\left(-\frac{96}{x^4}+\frac{208}{x^2}+\frac{700}{3+2x^2+x^4}+\frac{500x^2}{3+2x^2+x^4}+250\sqrt{2}\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)+416\log(x)-104\log\left(3+2x^2+x^4\right)\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(-96/x^4 + 208/x^2 + 700/(3 + 2*x^2 + x^4) + (500*x^2)/(3 + 2*x^2 + x^4) +
 250*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 416*Log[x] - 104*Log[3 + 2*x^2 +
x^4])/864
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^5\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{x^6\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{2\left(125x^6+100x^4-60x^2+144\right)}{27x^6\left(x^4+2x^2+3\right)}dx^2+\frac{25\left(5x^2+7\right)}{108\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{108}\int \frac{125x^6+100x^4-60x^2+144}{x^6\left(x^4+2x^2+3\right)}dx^2+\frac{25\left(5x^2+7\right)}{108\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\left(\frac{1}{108}\int \left(\frac{73-52x^2}{x^4+2x^2+3}+\frac{52}{x^2}-\frac{52}{x^4}+\frac{48}{x^6}\right)dx^2+\frac{25\left(5x^2+7\right)}{108\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{108}\left(\frac{125\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{24}{x^4}+\frac{52}{x^2}+52\log\left(x^2\right)-26\log\left(x^4+2x^2+3\right)\right)+\frac{25(5x^2+7)}{108\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((25*(7 + 5*x^2))/(108*(3 + 2*x^2 + x^4)) + (-24/x^4 + 52/x^2 + (125*ArcTa
n[(1 + x^2)/Sqrt[2]])/Sqrt[2] + 52*Log[x^2] - 26*Log[3 + 2*x^2 + x^4])/108
)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{-\frac{125x^2}{4}-\frac{175}{4}}{54(x^4+2x^2+3)}-\frac{13\ln(x^4+2x^2+3)}{108}+\frac{125\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{432}-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{13\ln(x)}{27}$	68
risch	$\frac{\frac{59}{72}x^6+\frac{85}{72}x^4+\frac{1}{2}x^2-\frac{1}{3}}{x^4(x^4+2x^2+3)}+\frac{13\ln(x)}{27}-\frac{13\ln(15625x^4+31250x^2+46875)}{108}+\frac{125\sqrt{2}\arctan\left(\frac{(125x^2+125)\sqrt{2}}{250}\right)}{432}$	72

input
```
int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/54*(-125/4*x^2-175/4)/(x^4+2*x^2+3)-13/108*ln(x^4+2*x^2+3)+125/432*2^(1
/2)*arctan(1/4*(2*x^2+2)*2^(1/2))-1/9/x^4+13/54/x^2+13/27*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx$$
$$=\frac{354\,x^6+510\,x^4+125\,\sqrt{2}(x^8+2\,x^6+3\,x^4)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)+216\,x^2-52\,(x^8+2\,x^6+3\,x^4)\log(\cdots}{432\,(x^8+2\,x^6+3\,x^4)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/432*(354*x^6 + 510*x^4 + 125*sqrt(2)*(x^8 + 2*x^6 + 3*x^4)*arctan(1/2*sq
rt(2)*(x^2 + 1)) + 216*x^2 - 52*(x^8 + 2*x^6 + 3*x^4)*log(x^4 + 2*x^2 + 3)
 + 208*(x^8 + 2*x^6 + 3*x^4)*log(x) - 144)/(x^8 + 2*x^6 + 3*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx = \frac{13\log(x)}{27} - \frac{13\log(x^4+2x^2+3)}{108} + \frac{125\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432} + \frac{59x^6+85x^4+36x^2-24}{72x^8+144x^6+216x^4}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+2*x**2+3)**2,x)
```

output
```
13*log(x)/27 - 13*log(x**4 + 2*x**2 + 3)/108 + 125*sqrt(2)*atan(sqrt(2)*x*
*2/2 + sqrt(2)/2)/432 + (59*x**6 + 85*x**4 + 36*x**2 - 24)/(72*x**8 + 144*
x**6 + 216*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx = \frac{125}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{59\,x^6+85\,x^4+36\,x^2-24}{72\,(x^8+2\,x^6+3\,x^4)} - \frac{13}{108}\log\left(x^4+2\,x^2+3\right) + \frac{13}{54}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(59*x^6 + 85*x^4 + 36
*x^2 - 24)/(x^8 + 2*x^6 + 3*x^4) - 13/108*log(x^4 + 2*x^2 + 3) + 13/54*log
(x^2)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx = \frac{125}{432}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)+\frac{26\,x^4+177\,x^2+253}{216\,(x^4+2\,x^2+3)} - \frac{39\,x^4-26\,x^2+12}{108\,x^4} - \frac{13}{108}\log\left(x^4+2\,x^2+3\right)+\frac{13}{54}\log\left(x^2\right)$$

input

```
integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output

```
125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/216*(26*x^4 + 177*x^2 +
253)/(x^4 + 2*x^2 + 3) - 1/108*(39*x^4 - 26*x^2 + 12)/x^4 - 13/108*log(x^4
 + 2*x^2 + 3) + 13/54*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 18.69 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx = \frac{13\ln(x)}{27} - \frac{13\ln\left(x^4+2\,x^2+3\right)}{108} + \frac{\frac{59\,x^6}{72}+\frac{85\,x^4}{72}+\frac{x^2}{2}-\frac{1}{3}}{x^8+2\,x^6+3\,x^4} + \frac{125\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

input

```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(2*x^2 + x^4 + 3)^2),x)
```

output

```
(13*log(x))/27 - (13*log(2*x^2 + x^4 + 3))/108 + (x^2/2 + (85*x^4)/72 + (5
9*x^6)/72 - 1/3)/(3*x^4 + 2*x^6 + x^8) + (125*2^(1/2)*atan(2^(1/2)/2 + (2^
(1/2)*x^2)/2))/432
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 424, normalized size of antiderivative = 5.30

$$\int \frac{4+x^2+3x^4+5x^6}{x^5(3+2x^2+x^4)^2}\,dx$$
$$= \frac{-125\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^8-250\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^6-375}{}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x)
```

output
```
( - 125*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2
) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 - 250*sqrt(sqrt(3) + 1)*sqrt(sq
rt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(
2)))*x**6 - 375*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1
)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 125*sqrt(sqrt(3) + 1)
*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) +
1)*sqrt(2)))*x**8 - 250*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqr
t(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 - 375*sqrt(sqrt
(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sq
rt(3) + 1)*sqrt(2)))*x**4 - 52*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3
) + x**2)*x**8 - 104*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*
x**6 - 156*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**4 - 52*
log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**8 - 104*log(sqrt(sqrt
(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**6 - 156*log(sqrt(sqrt(3) - 1)*sqrt
(2)*x + sqrt(3) + x**2)*x**4 + 208*log(x)*x**8 + 416*log(x)*x**6 + 624*log
(x)*x**4 - 177*x**8 - 21*x**4 + 216*x**2 - 144)/(432*x**4*(x**4 + 2*x**2 +
 3))
```

3.109 $\int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 87

$$\int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2}\,dx = -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25(1-7x^2)}{648(3+2x^2+x^4)} - \frac{1237\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{61\log(x)}{243}-\frac{61}{972}\log\left(3+2x^2+x^4\right)$$

output
```
-2/27/x^6+13/108/x^4-13/54/x^2+25*(-7*x^2+1)/(648*x^4+1296*x^2+1944)-1237/
3888*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)+61/243*ln(x)-61/972*ln(x^4+2*x^2+
3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

$$\int \frac{4+x^2+3x^4+5x^6}{x^7(3+2x^2+x^4)^2}\,dx = \frac{-\frac{576}{x^6}+\frac{936}{x^4}-\frac{1872}{x^2}-\frac{300(-1+7x^2)}{3+2x^2+x^4}+1952\log(x)-\sqrt{2}(1237i+244\sqrt{2})\log\left(-i+\sqrt{2}-ix^2\right)+\sqrt{2}(1237i}{7776}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(-576/x^6 + 936/x^4 - 1872/x^2 - (300*(-1 + 7*x^2))/(3 + 2*x^2 + x^4) + 19
52*Log[x] - Sqrt[2]*(1237*I + 244*Sqrt[2])*Log[-I + Sqrt[2] - I*x^2] + Sqr
t[2]*(1237*I - 244*Sqrt[2])*Log[I + Sqrt[2] + I*x^2])/7776
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2194, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^7 \left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{5x^6+3x^4+x^2+4}{x^8 \left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{2\left(-175x^8+400x^6+300x^4-180x^2+432\right)}{81x^8 \left(x^4+2x^2+3\right)}dx^2+\frac{25\left(1-7x^2\right)}{324\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{324}\int \frac{-175x^8+400x^6+300x^4-180x^2+432}{x^8 \left(x^4+2x^2+3\right)}dx^2+\frac{25\left(1-7x^2\right)}{324\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{2}\left(\frac{1}{324}\int \left(\frac{-244x^2-1481}{3\left(x^4+2x^2+3\right)}+\frac{244}{3x^2}+\frac{156}{x^4}-\frac{156}{x^6}+\frac{144}{x^8}\right)dx^2+\frac{25\left(1-7x^2\right)}{324\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{324}\left(-\frac{1237\arctan\left(\frac{x^2+1}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{48}{x^6}+\frac{78}{x^4}-\frac{156}{x^2}+\frac{244\log\left(x^2\right)}{3}-\frac{122}{3}\log\left(x^4+2x^2+3\right)\right)+\frac{25\left(1-7x^2\right)}{324\left(x^4+2x^2+\right.}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((25*(1 - 7*x^2))/(324*(3 + 2*x^2 + x^4)) + (-48/x^6 + 78/x^4 - 156/x^2 -
(1237*ArcTan[(1 + x^2)/Sqrt[2]])/(3*Sqrt[2]) + (244*Log[x^2])/3 - (122*Log
[3 + 2*x^2 + x^4])/3)/324)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\frac{525x^2}{4}-\frac{75}{4}}{486(x^4+2x^2+3)}-\frac{61\ln(x^4+2x^2+3)}{972}-\frac{1237\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{3888}-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{61\ln(x)}{243}$
risch	$\frac{-\frac{331}{648}x^8-\frac{209}{648}x^6-\frac{5}{9}x^4+\frac{23}{108}x^2-\frac{2}{9}}{x^6(x^4+2x^2+3)}+\frac{61\ln(x)}{243}-\frac{61\ln(1530169x^4+3060338x^2+4590507)}{972}-\frac{1237\sqrt{2}\,\arctan\left(\frac{(1237x^2+1237)\sqrt{2}}{2474}\right.}{3888}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/486*(525/4*x^2-75/4)/(x^4+2*x^2+3)-61/972*ln(x^4+2*x^2+3)-1237/3888*2^(
1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))-2/27/x^6+13/108/x^4-13/54/x^2+61/243*ln
(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.32

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = -\frac{1986\,x^8+1254\,x^6+2160\,x^4+1237\,\sqrt{2}(x^{10}+2\,x^8+3\,x^6)\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right)-828\,x^2+244\,(x^{10}}{3888\,(x^{10}+2\,x^8+3\,x^6)}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/3888*(1986*x^8 + 1254*x^6 + 2160*x^4 + 1237*sqrt(2)*(x^10 + 2*x^8 + 3*x
^6)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 828*x^2 + 244*(x^10 + 2*x^8 + 3*x^6)*l
og(x^4 + 2*x^2 + 3) - 976*(x^10 + 2*x^8 + 3*x^6)*log(x) + 864)/(x^10 + 2*x
^8 + 3*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = \frac{61\log{(x)}}{243} - \frac{61\log{(x^4+2x^2+3)}}{972} - \frac{1237\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{3888} + \frac{-331x^8-209x^6-360x^4+138x^2-144}{648x^{10}+1296x^8+1944x^6}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**7/(x**4+2*x**2+3)**2,x)
```

output
```
61*log(x)/243 - 61*log(x**4 + 2*x**2 + 3)/972 - 1237*sqrt(2)*atan(sqrt(2)*
x**2/2 + sqrt(2)/2)/3888 + (-331*x**8 - 209*x**6 - 360*x**4 + 138*x**2 - 1
44)/(648*x**10 + 1296*x**8 + 1944*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = -\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) - \frac{331\,x^8+209\,x^6+360\,x^4-138\,x^2+144}{648\,(x^{10}+2\,x^8+3\,x^6)} - \frac{61}{972}\log\left(x^4+2\,x^2+3\right) + \frac{61}{486}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/648*(331*x^8 + 209*x^
6 + 360*x^4 - 138*x^2 + 144)/(x^10 + 2*x^8 + 3*x^6) - 61/972*log(x^4 + 2*x
^2 + 3) + 61/486*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = -\frac{1237}{3888}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right) + \frac{122\,x^4-281\,x^2+441}{1944\,(x^4+2\,x^2+3)} - \frac{671\,x^6+702\,x^4-351\,x^2+216}{2916\,x^6} - \frac{61}{972}\log\left(x^4+2\,x^2+3\right) + \frac{61}{486}\log\left(x^2\right)$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
-1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/1944*(122*x^4 - 281*x
^2 + 441)/(x^4 + 2*x^2 + 3) - 1/2916*(671*x^6 + 702*x^4 - 351*x^2 + 216)/x
^6 - 61/972*log(x^4 + 2*x^2 + 3) + 61/486*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 18.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = \frac{61\,\ln(x)}{243} - \frac{61\,\ln\left(x^4+2\,x^2+3\right)}{972} - \frac{\frac{331\,x^8}{648}+\frac{209\,x^6}{648}+\frac{5\,x^4}{9}-\frac{23\,x^2}{108}+\frac{2}{9}}{x^{10}+2\,x^8+3\,x^6} - \frac{1237\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{3888}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^7*(2*x^2 + x^4 + 3)^2),x)
```

output
```
(61*log(x))/243 - (61*log(2*x^2 + x^4 + 3))/972 - ((5*x^4)/9 - (23*x^2)/10
8 + (209*x^6)/648 + (331*x^8)/648 + 2/9)/(3*x^6 + 2*x^8 + x^10) - (1237*2^
(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/3888
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 429, normalized size of antiderivative = 4.93

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = \frac{1237\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^{10}+2474\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^{8}+37}{}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x)
```

output
```
(1237*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2)
- 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**10 + 2474*sqrt(sqrt(3) + 1)*sqrt(sq
rt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(
2)))*x**8 + 3711*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) -
1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 + 1237*sqrt(sqrt(3) +
1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3)
+ 1)*sqrt(2)))*x**10 + 2474*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt
(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 + 3711*sqrt
(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sq
rt(sqrt(3) + 1)*sqrt(2)))*x**6 - 244*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x +
sqrt(3) + x**2)*x**10 - 488*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) +
 x**2)*x**8 - 732*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**
6 - 244*log(sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**10 - 488*log(
sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**8 - 732*log(sqrt(sqrt(3)
- 1)*sqrt(2)*x + sqrt(3) + x**2)*x**6 + 976*log(x)*x**10 + 1952*log(x)*x**
8 + 2928*log(x)*x**6 + 993*x**10 + 1725*x**6 - 2160*x**4 + 828*x**2 - 864)
/(3888*x**6*(x**4 + 2*x**2 + 3))
```

3.110 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 203

$$\begin{aligned}
&\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx \\
&= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8(3+2x^2+x^4)} \\
&\quad+\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad-\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad+\frac{1}{16}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)
\end{aligned}$$

output
```
38*x+19/3*x^3-17/5*x^5+5/7*x^7+25*x*(5*x^2+3)/(8*x^4+16*x^2+24)+1/32*(5255
42+1236582*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))^
(1/2))-1/32*(525542+1236582*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*
x)/(2+2*3^(1/2))^(1/2))+1/32*(-525542+1236582*3^(1/2))^(1/2)*arctanh((-2+2
*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.71

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x(3+5x^2)}{8\,(3+2x^2+x^4)} - \frac{\left(352i+1339\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} - \frac{\left(-352i+1339\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*
x^2 + x^4)) - ((352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*S
qrt[2 - (2*I)*Sqrt[2]]) - ((-352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqr
t[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \; 2197$$

$$\frac{1}{48}\int -\frac{6\left(-40x^{10}+56x^8-200x^4+275x^2+75\right)}{x^4+2x^2+3}dx+\frac{25x\left(5x^2+3\right)}{8\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{25x(5x^2+3)}{8(x^4+2x^2+3)}-\frac{1}{8}\int \frac{-40x^{10}+56x^8-200x^4+275x^2+75}{x^4+2x^2+3}dx$$

$$\Big\downarrow \text{ 2205}$$

$$\frac{25x(5x^2+3)}{8(x^4+2x^2+3)}-\frac{1}{8}\int \left(-40x^6+136x^4-152x^2+\frac{1339x^2+987}{x^4+2x^2+3}-304\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{}}{\sqrt{2}}\right.\right.$$
$$\frac{25x(5x^2+3)}{8(x^4+2x^2+3)}$$

input
```
Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(25*x*(3 + 5*x^2))/(8*(3 + 2*x^2 + x^4)) + (304*x + (152*x^3)/3 - (136*x^5
)/5 + (40*x^7)/7 + (Sqrt[(262771 + 618291*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 +
 Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 - (Sqrt[(262771 + 618291*Sqrt[
3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 - (
Sqrt[(-262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x
+ x^2])/4 + (Sqrt[(-262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 +
 Sqrt[3])]*x + x^2])/4)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.39

method	result
risch	$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x+\frac{\frac{125}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(-1339\_R^2-987\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x-\frac{-\frac{125}{8}x^3-\frac{75}{8}x}{x^4+2x^2+3}-\frac{\left(-505\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-176\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{64}-\frac{\left(6\right.}{}$

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/7*x^7-17/5*x^5+19/3*x^3+38*x+(125/8*x^3+75/8*x)/(x^4+2*x^2+3)+1/32*sum((
-1339*_R^2-987)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.32

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{2400\,x^{11} - 6624\,x^9 + 5632\,x^7 + 135968\,x^5 + 371700\,x^3 + 210\,(x^4 + 2\,x^2 + 3)\sqrt{\frac{618291}{2}\sqrt{3} + \frac{262771}{2}}\arctan}{}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/3360*(2400*x^11 - 6624*x^9 + 5632*x^7 + 135968*x^5 + 371700*x^3 + 210*(x
^4 + 2*x^2 + 3)*sqrt(618291/2*sqrt(3) + 262771/2)*arctan(1/734099*(329*sqr
t(3)*x + sqrt(618291/2*sqrt(3) - 262771/2)*(sqrt(3) - 1) - 1339*x)*sqrt(61
8291/2*sqrt(3) + 262771/2)) - 210*(x^4 + 2*x^2 + 3)*sqrt(618291/2*sqrt(3)
+ 262771/2)*arctan(-1/734099*(329*sqrt(3)*x - sqrt(618291/2*sqrt(3) - 2627
71/2)*(sqrt(3) - 1) - 1339*x)*sqrt(618291/2*sqrt(3) + 262771/2)) + 105*(x^
4 + 2*x^2 + 3)*sqrt(618291/2*sqrt(3) - 262771/2)*log(734099*x^2 + 2*(505*s
qrt(3)*x - 176*x)*sqrt(618291/2*sqrt(3) - 262771/2) + 734099*sqrt(3)) - 10
5*(x^4 + 2*x^2 + 3)*sqrt(618291/2*sqrt(3) - 262771/2)*log(734099*x^2 - 2*(
505*sqrt(3)*x - 176*x)*sqrt(618291/2*sqrt(3) - 262771/2) + 734099*sqrt(3))
 + 414540*x)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.35

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^7}{7} - \frac{17x^5}{5} + \frac{19x^3}{3} + 38x + \frac{125x^3+75x}{8x^4+16x^2+24}$$
$$+\mathrm{RootSum}\left(1048576t^4 + 538155008t^2 + 1146851282043, \left(t \mapsto t\log\left(-\frac{165547840t^3}{453886804809} - \frac{11974973632}{45388680480}\right.\right.\right.$$

input
```
integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**7/7 - 17*x**5/5 + 19*x**3/3 + 38*x + (125*x**3 + 75*x)/(8*x**4 + 16*x
**2 + 24) + RootSum(1048576*_t**4 + 538155008*_t**2 + 1146851282043, Lambd
a(_t, _t*log(-16547840*_t**3/453886804809 - 11974973632*_t/453886804809 +
x)))
```

Maxima [F]

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^8}{(x^4+2\,x^2+3)^2}\,dx$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 38*x + 25/8*(5*x^3 + 3*x)/(x^4 + 2*x^2 + 3
) - 1/8*integrate((1339*x^2 + 987)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(146) = 292$.

Time = 0.44 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.88

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 1/20736*sqrt(2)*(1339*3^(3/4)*sqrt(2)*(6*s
qrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) -
 3) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3^(3/4)*(-6
*sqrt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 35532*
3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*s
qrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/20736*sqrt(2)*(1339*3^(3/4)*sq
rt(2)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*
(sqrt(3) - 3) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3
^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18
) + 35532*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*s
qrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/41472*sqrt(2)*(24102
*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2
)*(-6*sqrt(3) + 18)^(3/2) + 1339*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^
(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*s
qrt(3) + 18) - 35532*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/41472*sqrt(2)*(24102*3^(3/4)*sqrt(2
)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2)*(-6*sqrt(3) +
 18)^(3/2) + 1339*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(6*sq
rt(3) + 18)*(sqrt(3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) -
35532*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqr...
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.84

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 38\,x+\frac{\frac{125\,x^3}{8}+\frac{75\,x}{8}}{x^4+2\,x^2+3}+\frac{19\,x^3}{3}-\frac{17\,x^5}{5}+\frac{5\,x^7}{7}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-262771-\sqrt{2}\,734099\mathrm{i}}\,734099\mathrm{i}}{64\,\left(-\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}+\frac{734099\,\sqrt{2}\,x\,\sqrt{-262771-\sqrt{2}\,734099\mathrm{i}}}{128\,\left(-\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}\right)\,\sqrt{-262771-\sqrt{2}\,734099\mathrm{i}}\,1\mathrm{i}}{16}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-262771+\sqrt{2}\,734099\mathrm{i}}\,734099\mathrm{i}}{64\,\left(\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}-\frac{734099\,\sqrt{2}\,x\,\sqrt{-262771+\sqrt{2}\,734099\mathrm{i}}}{128\,\left(\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}\right)\,\sqrt{-262771+\sqrt{2}\,734099\mathrm{i}}\,1\mathrm{i}}{16}$$

input
```
int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
38*x + (atan((x*(- 2^(1/2)*734099i - 262771)^(1/2)*734099i)/(64*((2^(1/2)*
724555713i)/128 - 1112159985/64)) + (734099*2^(1/2)*x*(- 2^(1/2)*734099i -
 262771)^(1/2))/(128*((2^(1/2)*724555713i)/128 - 1112159985/64)))*(- 2^(1/
2)*734099i - 262771)^(1/2)*1i)/16 - (atan((x*(2^(1/2)*734099i - 262771)^(1
/2)*734099i)/(64*((2^(1/2)*724555713i)/128 + 1112159985/64)) - (734099*2^(
1/2)*x*(2^(1/2)*734099i - 262771)^(1/2))/(128*((2^(1/2)*724555713i)/128 +
1112159985/64)))*(2^(1/2)*734099i - 262771)^(1/2)*1i)/16 + ((75*x)/8 + (12
5*x^3)/8)/(2*x^2 + x^4 + 3) + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 806, normalized size of antiderivative = 3.97

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

output
```
(106050*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(
sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 212100*sqrt(sqrt(3) + 1)*sqrt(6)*atan((
sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 31815
0*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(s
qrt(3) + 1)*sqrt(2))) - 36960*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3)
 - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 73920*sqrt(sqrt(3
) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*s
qrt(2)))*x**2 - 110880*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*s
qrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 106050*sqrt(sqrt(3) + 1)*sqrt
(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x*
*4 - 212100*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*
x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 318150*sqrt(sqrt(3) + 1)*sqrt(6)*at
an((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 36960*
sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqr
t(3) + 1)*sqrt(2)))*x**4 + 73920*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt
(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 110880*sqrt(sq
rt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) +
1)*sqrt(2))) - 53025*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sq
rt(2)*x + sqrt(3) + x**2)*x**4 - 106050*sqrt(sqrt(3) - 1)*sqrt(6)*log( - s
qrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**2 - 159075*sqrt(sqrt(3)...
```

3.111 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 192

$$\begin{aligned}
&\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx \\
&= 19x-\frac{17x^3}{3}+x^5+\frac{25x(3-x^2)}{8(3+2x^2+x^4)} \\
&\quad+\frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad-\frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad-\frac{3}{16}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)
\end{aligned}$$

output

```
19*x-17/3*x^3+x^5+25*x*(-x^2+3)/(8*x^4+16*x^2+24)+3/32*(-52014+30066*3^(1/
2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))^(1/2))-3/32*(-52
014+30066*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(
1/2))-3/32*(52014+30066*3^(1/2))^(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(
1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.69

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 19x-\frac{17x^3}{3}+x^5-\frac{25x(-3+x^2)}{8(3+2x^2+x^4)}+\frac{9(90i+31\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}+\frac{9(-90i+31\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
19*x - (17*x^3)/3 + x^5 - (25*x*(-3 + x^2))/(8*(3 + 2*x^2 + x^4)) + (9*(90
*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]
]) + (9*(-90*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (
2*I)*Sqrt[2]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}\,dx$$

$$\Big\downarrow \; 2197$$

$$\frac{1}{48}\int -\frac{6(-40x^8+56x^6-175x^2+75)}{x^4+2x^2+3}dx+\frac{25x(3-x^2)}{8(x^4+2x^2+3)}$$

$$\Big\downarrow \; 27$$

$$\frac{25x(3-x^2)}{8(x^4+2x^2+3)}-\frac{1}{8}\int\frac{-40x^8+56x^6-175x^2+75}{x^4+2x^2+3}dx$$

$$\Big\downarrow \text{2205}$$

$$\frac{25x(3-x^2)}{8(x^4+2x^2+3)}-\frac{1}{8}\int\left(-40x^4+136x^2+\frac{9(59-31x^2)}{x^4+2x^2+3}-152\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(\frac{3}{2}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{3}{2}\sqrt{\frac{3}{2}\left(5011\sqrt{3}-8669\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-\right.}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right.\right.$$
$$\frac{25x(3-x^2)}{8(x^4+2x^2+3)}$$

input
```
Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(25*x*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (152*x - (136*x^3)/3 + 8*x^5 + (3
*Sqrt[(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/
Sqrt[2*(1 + Sqrt[3])]])/2 - (3*Sqrt[(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(
Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 + (3*Sqrt[(3*(8669
 + 5011*Sqrt[3]))/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4 - (3
*Sqrt[(3*(8669 + 5011*Sqrt[3]))/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x
+ x^2])/4)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.38

method	result
risch	$x^5-\frac{17x^3}{3}+19x+\frac{-\frac{25}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3}+\frac{9\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(31\_R^2-59\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$x^5-\frac{17x^3}{3}+19x+\frac{-\frac{25}{8}x^3+\frac{75}{8}x}{x^4+2x^2+3}+\frac{3\left(-76\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-135\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{64}+\frac{3\left(-118\sqrt{3}-\cdots\right.}{}$

input
```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
x^5-17/3*x^3+19*x+(-25/8*x^3+75/8*x)/(x^4+2*x^2+3)+9/32*sum((31*_R^2-59)/(
_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(137) = 274$.

Time = 0.08 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.60

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{96\,x^9 - 352\,x^7 + 1024\,x^5 + 1716\,x^3 - 18\,\sqrt{\frac{3}{2}}(x^4+2\,x^2+3)\sqrt{5011\,\sqrt{3}-8669}\arctan\left(\frac{1}{897}\,\sqrt{\frac{3}{2}}(59\,\sqrt{3}x - \right.}{}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/96*(96*x^9 - 352*x^7 + 1024*x^5 + 1716*x^3 - 18*sqrt(3/2)*(x^4 + 2*x^2 +
 3)*sqrt(5011*sqrt(3) - 8669)*arctan(1/897*sqrt(3/2)*(59*sqrt(3)*x + 93*x)
*sqrt(5011*sqrt(3) - 8669) + 1/598*sqrt(5011*sqrt(3) + 8669)*sqrt(5011*sqr
t(3) - 8669)*(sqrt(3) - 1)) + 18*sqrt(3/2)*(x^4 + 2*x^2 + 3)*sqrt(5011*sqr
t(3) - 8669)*arctan(-1/897*sqrt(3/2)*(59*sqrt(3)*x + 93*x)*sqrt(5011*sqrt(
3) - 8669) + 1/598*sqrt(5011*sqrt(3) + 8669)*sqrt(5011*sqrt(3) - 8669)*(sq
rt(3) - 1)) + 9*sqrt(3/2)*(x^4 + 2*x^2 + 3)*sqrt(5011*sqrt(3) + 8669)*log(
2691*x^2 + 6*sqrt(3/2)*(76*sqrt(3)*x - 135*x)*sqrt(5011*sqrt(3) + 8669) +
2691*sqrt(3)) - 9*sqrt(3/2)*(x^4 + 2*x^2 + 3)*sqrt(5011*sqrt(3) + 8669)*lo
g(2691*x^2 - 6*sqrt(3/2)*(76*sqrt(3)*x - 135*x)*sqrt(5011*sqrt(3) + 8669)
+ 2691*sqrt(3)) + 6372*x)/(x^4 + 2*x^2 + 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. $2(160) = 320$.

Time = 1.02 (sec) , antiderivative size = 1205, normalized size of antiderivative = 6.28

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
x**5 - 17*x**3/3 + 19*x + (-25*x**3 + 75*x)/(8*x**4 + 16*x**2 + 24) - 3*sq
rt(26007/2048 + 15033*sqrt(3)/2048)*log(x**2 + x*(-304*sqrt(2)*sqrt(8669 +
 5011*sqrt(3))/299 - 433349*sqrt(6)*sqrt(8669 + 5011*sqrt(3))/1498289 + 15
2*sqrt(3)*sqrt(8669 + 5011*sqrt(3))*sqrt(43440359*sqrt(3) + 75240962)/1498
289) - 2882918249387*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/22448699275
21 - 993398584*sqrt(6)*sqrt(43440359*sqrt(3) + 75240962)/1343965233 + 4993
6376949404567/2244869927521 + 17261871038090*sqrt(3)/1343965233) + 3*sqrt(
26007/2048 + 15033*sqrt(3)/2048)*log(x**2 + x*(-152*sqrt(3)*sqrt(8669 + 50
11*sqrt(3))*sqrt(43440359*sqrt(3) + 75240962)/1498289 + 433349*sqrt(6)*sqr
t(8669 + 5011*sqrt(3))/1498289 + 304*sqrt(2)*sqrt(8669 + 5011*sqrt(3))/299
) - 2882918249387*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/2244869927521
- 993398584*sqrt(6)*sqrt(43440359*sqrt(3) + 75240962)/1343965233 + 4993637
6949404567/2244869927521 + 17261871038090*sqrt(3)/1343965233) - 2*sqrt(-27
*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/1024 + 234063/2048 + 405891*sqr
t(3)/2048)*atan(2996578*sqrt(3)*x/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(4344
0359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(
3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 +
15033*sqrt(3))) - 1523344*sqrt(6)*sqrt(8669 + 5011*sqrt(3))/(17641*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))
 + 152*sqrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*...
```

Maxima [F]

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^6}{(x^4+2\,x^2+3)^2}\,dx$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
x^5 - 17/3*x^3 + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3) + 9/8*integrate
((31*x^2 - 59)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(137) = 274$.

Time = 0.43 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.00

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
x^5 - 17/3*x^3 - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2)
 + 558*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(s
qrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 2
124*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2)
+ 558*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(sq
rt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 21
24*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) + 1
8))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqr
t(3) + 1/2)) - 1/4608*sqrt(2)*(558*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*s
qrt(3) + 18) - 31*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 31*3^(3/4)*(6*
sqrt(3) + 18)^(3/2) + 558*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 212
4*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 2124*3^(1/4)*sqrt(6*sqrt(3) + 18
))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/4608*sqrt
(2)*(558*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 31*3^(3/4)*
sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 31*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 558*
3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 2124*3^(1/4)*sqrt(2)*sqrt(-6*
sqrt(3) + 18) + 2124*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x...
```

Mupad [B] (verification not implemented)

Time = 18.71 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$$
$$= 19\,x+\frac{\frac{75\,x}{8}-\frac{25\,x^3}{8}}{x^4+2\,x^2+3}-\frac{17\,x^3}{3}+x^5$$
$$-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}\,24219\mathrm{i}}{64\,\left(-\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}-\frac{24219\,\sqrt{2}\,x\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}}{128\,\left(-\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}\right)\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}\,3\mathrm{i}}{16}$$
$$+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{26007+\sqrt{2}\,897\mathrm{i}}\,24219\mathrm{i}}{64\,\left(\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}+\frac{24219\,\sqrt{2}\,x\,\sqrt{26007+\sqrt{2}\,897\mathrm{i}}}{128\,\left(\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}\right)\,\sqrt{26007+\sqrt{2}\,897\mathrm{i}}\,3\mathrm{i}}{16}$$

input
```
int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
19*x + ((75*x)/8 - (25*x^3)/8)/(2*x^2 + x^4 + 3) - (atan((x*(26007 - 2^(1/
2)*897i)^(1/2)*24219i)/(64*((2^(1/2)*4286763i)/128 - 1380483/16)) - (24219
*2^(1/2)*x*(26007 - 2^(1/2)*897i)^(1/2))/(128*((2^(1/2)*4286763i)/128 - 13
80483/16)))*(26007 - 2^(1/2)*897i)^(1/2)*3i)/16 + (atan((x*(2^(1/2)*897i +
 26007)^(1/2)*24219i)/(64*((2^(1/2)*4286763i)/128 + 1380483/16)) + (24219*
2^(1/2)*x*(2^(1/2)*897i + 26007)^(1/2))/(128*((2^(1/2)*4286763i)/128 + 138
0483/16)))*(2^(1/2)*897i + 26007)^(1/2)*3i)/16 - (17*x^3)/3 + x^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 801, normalized size of antiderivative = 4.17

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

output
```
( - 1368*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/
(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 2736*sqrt(sqrt(3) + 1)*sqrt(6)*atan((s
qrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 4104*s
qrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt
(3) + 1)*sqrt(2))) + 2430*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1
)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 4860*sqrt(sqrt(3) + 1
)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2
)))*x**2 + 7290*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2)
- 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 1368*sqrt(sqrt(3) + 1)*sqrt(6)*atan(
(sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 2736
*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sq
rt(3) + 1)*sqrt(2)))*x**2 + 4104*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt
(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 2430*sqrt(sqrt(3) +
 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt
(2)))*x**4 - 4860*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2
) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 7290*sqrt(sqrt(3) + 1)*sqrt(2
)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 68
4*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) +
 x**2)*x**4 + 1368*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt
(2)*x + sqrt(3) + x**2)*x**2 + 2052*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sq...
```

3.112 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 187

$$\begin{aligned}
\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx &= -17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8\,(3+2x^2+x^4)} \\
&\quad -\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\,(-1+\sqrt{3})}-2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&\quad +\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&\quad +\frac{1}{16}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}x}{\sqrt{3}+x^2}\right)
\end{aligned}$$

output

```
-17*x+5/3*x^3-25*x*(x^2+3)/(8*x^4+16*x^2+24)-1/32*(28790+52998*3^(1/2))^(1
/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))^(1/2))+1/32*(28790+529
98*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))+1
/32*(-28790+52998*3^(1/2))^(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x
^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.69

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = -17x+\frac{5x^3}{3}-\frac{25x(3+x^2)}{8(3+2x^2+x^4)} + \frac{(-356i+127\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} + \frac{(356i+127\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
-17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) + ((-356*I + 12
7*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) + (
(356*I + 127*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sq
rt[2]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}\,dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{48}\int \frac{6(40x^6-56x^4-25x^2+75)}{x^4+2x^2+3}dx-\frac{25x(x^2+3)}{8(x^4+2x^2+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \frac{40x^6-56x^4-25x^2+75}{x^4+2x^2+3}dx-\frac{25x\left(x^2+3\right)}{8\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \text{ 2205}$$

$$\frac{1}{8}\int \left(40x^2+\frac{127x^2+483}{x^4+2x^2+3}-136\right)dx-\frac{25x\left(x^2+3\right)}{8\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}\left(-\frac{1}{2}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\right.}}{\sqrt{2\left(1\right.}}\right.\right.$$
$$\frac{25x\left(x^2+3\right)}{8\left(x^4+2x^2+3\right)}$$

input
```
Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(-25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) + (-136*x + (40*x^3)/3 - (Sqrt[(14
395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 +
Sqrt[3])]])/2 + (Sqrt[(14395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt
[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 - (Sqrt[(-14395 + 26499*Sqrt[3])/2]
*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4 + (Sqrt[(-14395 + 26499*
Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.37

method	result
risch	$\frac{5x^3}{3} - 17x + \frac{-\frac{25}{8}x^3 - \frac{75}{8}x}{x^4+2x^2+3} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)} \frac{\left(127\_R^2+483\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$\frac{5x^3}{3} - 17x + \frac{-\frac{25}{8}x^3 - \frac{75}{8}x}{x^4+2x^2+3} + \frac{\left(17\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+178\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{64} + \frac{\left(322\sqrt{3}-\frac{\left(17\sqrt{-2+2\sqrt{3}}\,\sqrt{\right.}}{}\right.}{}$

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5/3*x^3-17*x+(-25/8*x^3-75/8*x)/(x^4+2*x^2+3)+1/32*sum((127*_R^2+483)/(_R^
3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.38

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{160\,x^7-1312\,x^5-3084\,x^3+6\,(x^4+2\,x^2+3)\sqrt{\frac{26499}{2}\,\sqrt{3}+\frac{14395}{2}}\arctan\left(\frac{1}{30817}\left(161\,\sqrt{3}x+\sqrt{\frac{26499}{2}\,\sqrt{3}\right.\right.}{}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/96*(160*x^7 - 1312*x^5 - 3084*x^3 + 6*(x^4 + 2*x^2 + 3)*sqrt(26499/2*sqr
t(3) + 14395/2)*arctan(1/30817*(161*sqrt(3)*x + sqrt(26499/2*sqrt(3) - 143
95/2)*(sqrt(3) - 1) - 127*x)*sqrt(26499/2*sqrt(3) + 14395/2)) - 6*(x^4 + 2
*x^2 + 3)*sqrt(26499/2*sqrt(3) + 14395/2)*arctan(-1/30817*(161*sqrt(3)*x -
 sqrt(26499/2*sqrt(3) - 14395/2)*(sqrt(3) - 1) - 127*x)*sqrt(26499/2*sqrt(
3) + 14395/2)) - 3*(x^4 + 2*x^2 + 3)*sqrt(26499/2*sqrt(3) - 14395/2)*log(3
0817*x^2 + 2*(17*sqrt(3)*x - 178*x)*sqrt(26499/2*sqrt(3) - 14395/2) + 3081
7*sqrt(3)) + 3*(x^4 + 2*x^2 + 3)*sqrt(26499/2*sqrt(3) - 14395/2)*log(30817
*x^2 - 2*(17*sqrt(3)*x - 178*x)*sqrt(26499/2*sqrt(3) - 14395/2) + 30817*sq
rt(3)) - 5796*x)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.32

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5x^3}{3}-17x+\frac{-25x^3-75x}{8x^4+16x^2+24} + \operatorname{RootSum}\left(1048576t^4+29480960t^2+2106591003,\left(t\mapsto t\log\left(\frac{557056t^3}{816619683}+\frac{166600064t}{816619683}+x\right)\right)\right)$$

input
```
integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
5*x**3/3 - 17*x + (-25*x**3 - 75*x)/(8*x**4 + 16*x**2 + 24) + RootSum(1048
576*_t**4 + 29480960*_t**2 + 2106591003, Lambda(_t, _t*log(557056*_t**3/81
6619683 + 166600064*_t/816619683 + x)))
```

Maxima [F]

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^4}{(x^4+2\,x^2+3)^2}\,dx$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5/3*x^3 - 17*x - 25/8*(x^3 + 3*x)/(x^4 + 2*x^2 + 3) + 1/8*integrate((127*x
^2 + 483)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(134) = 268$.

Time = 0.43 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.06

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
5/3*x^3 - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 22
86*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) + 127*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 173
88*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 17388*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2
) + 2286*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)
*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 127*3^(3/4)*(-6*sqrt(3) + 18)^(3/2)
 - 17388*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 17388*3^(1/4)*sqrt(-6*sqrt
(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(
1/6*sqrt(3) + 1/2)) - 1/41472*sqrt(2)*(2286*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*
sqrt(-6*sqrt(3) + 18) - 127*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 127*
3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 2286*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3
) - 3) - 17388*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 17388*3^(1/4)*sqrt(
6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
 + 1/41472*sqrt(2)*(2286*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 1
8) - 127*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 127*3^(3/4)*(6*sqrt(3)
+ 18)^(3/2) + 2286*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 17388*3^(1
/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 17388*3^(1/4)*sqrt(6*sqrt(3) + 18))*lo
g(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 17*x - 25/8*(...
```

Mupad [B] (verification not implemented)

Time = 19.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.87

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{5\,x^3}{3} - \frac{\frac{25\,x^3}{8}+\frac{75\,x}{8}}{x^4+2\,x^2+3} - 17\,x + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}\,30817\mathrm{i}}{64\,\left(-\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)} - \frac{30817\,\sqrt{2}\,x\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}}{128\,\left(-\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)}\right)\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}\,1\mathrm{i}}{16} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}\,30817\mathrm{i}}{64\,\left(\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)} + \frac{30817\,\sqrt{2}\,x\,\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}}{128\,\left(\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)}\right)\,\sqrt{-14395+\sqrt{2}\,30817\mathrm{i}}\,1\mathrm{i}}{16}$$

input
```
int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(atan((x*(- 2^(1/2)*30817i - 14395)^(1/2)*30817i)/(64*((2^(1/2)*14884611i)
/128 - 1571667/64)) - (30817*2^(1/2)*x*(- 2^(1/2)*30817i - 14395)^(1/2))/(
128*((2^(1/2)*14884611i)/128 - 1571667/64)))*(- 2^(1/2)*30817i - 14395)^(1
/2)*1i)/16 - ((75*x)/8 + (25*x^3)/8)/(2*x^2 + x^4 + 3) - 17*x - (atan((x*(
2^(1/2)*30817i - 14395)^(1/2)*30817i)/(64*((2^(1/2)*14884611i)/128 + 15716
67/64)) + (30817*2^(1/2)*x*(2^(1/2)*30817i - 14395)^(1/2))/(128*((2^(1/2)*
14884611i)/128 + 1571667/64)))*(2^(1/2)*30817i - 14395)^(1/2)*1i)/16 + (5*
x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 796, normalized size of antiderivative = 4.26

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

output
```
(102*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqr
t(sqrt(3) + 1)*sqrt(2)))*x**4 + 204*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(s
qrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 306*sqrt(sq
rt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) +
1)*sqrt(2))) - 1068*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt
(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 2136*sqrt(sqrt(3) + 1)*sqrt
(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x*
*2 - 3204*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)
/(sqrt(sqrt(3) + 1)*sqrt(2))) - 102*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(s
qrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 204*sqrt(sq
rt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) +
1)*sqrt(2)))*x**2 - 306*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*
sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 1068*sqrt(sqrt(3) + 1)*sqrt(
2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**
4 + 2136*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/
(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 3204*sqrt(sqrt(3) + 1)*sqrt(2)*atan((s
qrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 51*sqrt(sqr
t(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**
4 - 102*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqr
t(3) + x**2)*x**2 - 153*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) -...
```

3.113 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 180

$$\begin{aligned}\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx &= 5x+\frac{25x(1+x^2)}{8(3+2x^2+x^4)} \\ &\quad +\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ &\quad -\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\ &\quad +\frac{1}{16}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)\end{aligned}$$

output
```
5*x+25*x*(x^2+1)/(8*x^4+16*x^2+24)+1/96*(115746+77394*3^(1/2))^(1/2)*arcta
n(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))^(1/2))-1/96*(115746+77394*3^(1/
2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))+1/96*(-11
5746+77394*3^(1/2))^(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.67

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 5x+\frac{25(x+x^3)}{8(3+2x^2+x^4)} - \frac{\left(-34i+111\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}} - \frac{\left(34i+111\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
5*x + (25*(x + x^3))/(8*(3 + 2*x^2 + x^4)) - ((-34*I + 111*Sqrt[2])*ArcTan
[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) - ((34*I + 111*Sqrt[
2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2197, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{48}\int -\frac{6(-40x^4+31x^2+25)}{x^4+2x^2+3}dx+\frac{25x(x^2+1)}{8(x^4+2x^2+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{25x(x^2+1)}{8(x^4+2x^2+3)}-\frac{1}{8}\int\frac{-40x^4+31x^2+25}{x^4+2x^2+3}dx$$

$\big\downarrow$ 2205

$$\frac{25x(x^2+1)}{8(x^4+2x^2+3)}-\frac{1}{8}\int\left(\frac{111x^2+145}{x^4+2x^2+3}-40\right)dx$$

$\big\downarrow$ 2009

$$\frac{1}{8}\left(\frac{1}{2}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{\right.}}}{\sqrt{2\left(1+\right.}}\right.\right.$$
$$\frac{25x(x^2+1)}{8(x^4+2x^2+3)}$$

input
```
Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(25*x*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) + (40*x + (Sqrt[(19291 + 12899*Sqrt
[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 -
(Sqrt[(19291 + 12899*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqr
t[2*(1 + Sqrt[3])]])/2 - (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt[3] - S
qrt[2*(-1 + Sqrt[3])]*x + x^2])/4 + (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[
Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.36

method	result
risch	$5x+\frac{\frac{25}{8}x^3+\frac{25}{8}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(-111\_R^2-145\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{32}$
default	$5x-\frac{-\frac{25}{8}x^3-\frac{25}{8}x}{x^4+2x^2+3}-\frac{\left(-94\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+51\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{192}-\frac{\left(290\sqrt{3}-\frac{\left(-94\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+51\sqrt{-\right.}{2}\right.}{4}$

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
5*x+(25/8*x^3+25/8*x)/(x^4+2*x^2+3)+1/32*sum((-111*_R^2-145)/(_R^3+_R)*ln(
x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.41

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \frac{160\,x^5+420\,x^3+2\,(x^4+2\,x^2+3)\sqrt{\frac{12899}{6}\,\sqrt{3}+\frac{19291}{6}}\arctan\left(\frac{1}{7969}\left(145\,\sqrt{3}x+3\,\sqrt{\frac{12899}{6}\,\sqrt{3}-\frac{19291}{6}}(\sqrt{3}\right.\right.}{}$$

input

```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output

```
1/32*(160*x^5 + 420*x^3 + 2*(x^4 + 2*x^2 + 3)*sqrt(12899/6*sqrt(3) + 19291
/6)*arctan(1/7969*(145*sqrt(3)*x + 3*sqrt(12899/6*sqrt(3) - 19291/6)*(sqrt
(3) - 1) - 333*x)*sqrt(12899/6*sqrt(3) + 19291/6)) - 2*(x^4 + 2*x^2 + 3)*s
qrt(12899/6*sqrt(3) + 19291/6)*arctan(-1/7969*(145*sqrt(3)*x - 3*sqrt(1289
9/6*sqrt(3) - 19291/6)*(sqrt(3) - 1) - 333*x)*sqrt(12899/6*sqrt(3) + 19291
/6)) + (x^4 + 2*x^2 + 3)*sqrt(12899/6*sqrt(3) - 19291/6)*log(7969*x^2 + 2*
(94*sqrt(3)*x + 51*x)*sqrt(12899/6*sqrt(3) - 19291/6) + 7969*sqrt(3)) - (x
^4 + 2*x^2 + 3)*sqrt(12899/6*sqrt(3) - 19291/6)*log(7969*x^2 - 2*(94*sqrt(
3)*x + 51*x)*sqrt(12899/6*sqrt(3) - 19291/6) + 7969*sqrt(3)) + 580*x)/(x^4
 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.28

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 5x+\frac{25x^3+25x}{8x^4+16x^2+24} + \operatorname{RootSum}\left(3145728t^4+39507968t^2+166384201,\left(t\mapsto t\log\left(-\frac{9240576t^3}{102792131}-\frac{95003488t}{102792131}+x\right)\right)\right)$$

input

```
integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output

```
5*x + (25*x**3 + 25*x)/(8*x**4 + 16*x**2 + 24) + RootSum(3145728*_t**4 + 3
9507968*_t**2 + 166384201, Lambda(_t, _t*log(-9240576*_t**3/102792131 - 95
003488*_t/102792131 + x)))
```

Maxima [F]

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^2}{(x^4+2\,x^2+3)^2}\,dx$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3) - 1/8*integrate((111*x^2 + 145)/(x^
4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(129) = 258$.

Time = 0.44 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.14

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
1/6912*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3
^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1
/6912*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqr
t(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(-
6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^
(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/
13824*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 3
7*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3
/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1740*3^(1/4)*sqrt(2
)*sqrt(-6*sqrt(3) + 18) - 1740*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3
^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/13824*sqrt(2)*(666*3^(3/4
)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(-6*sqr
t(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(6*
sqrt(3) + 18)*(sqrt(3) - 3) - 1740*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) -
 1740*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3
) + 1/2) + sqrt(3)) + 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.87

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx = 5\,x + \frac{\frac{25\,x^3}{8}+\frac{25\,x}{8}}{x^4+2\,x^2+3} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-57873-\sqrt{2}\,23907\mathrm{i}}\,7969\mathrm{i}}{576\,\left(-\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}+\frac{7969\,\sqrt{2}\,x\,\sqrt{-57873-\sqrt{2}\,23907\mathrm{i}}}{1152\,\left(-\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}\right)\,\sqrt{-57873-\sqrt{2}\,23907\mathrm{i}}\,1\mathrm{i}}{48} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-57873+\sqrt{2}\,23907\mathrm{i}}\,7969\mathrm{i}}{576\,\left(\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}-\frac{7969\,\sqrt{2}\,x\,\sqrt{-57873+\sqrt{2}\,23907\mathrm{i}}}{1152\,\left(\frac{374543}{96}+\frac{\sqrt{2}\,1155505\mathrm{i}}{384}\right)}\right)\,\sqrt{-57873+\sqrt{2}\,23907\mathrm{i}}\,1\mathrm{i}}{48}$$

input
```
int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

output
```
5*x + ((25*x)/8 + (25*x^3)/8)/(2*x^2 + x^4 + 3) + (atan((x*(- 2^(1/2)*2390
7i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 - 374543/96)) + (796
9*2^(1/2)*x*(- 2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505i)/38
4 - 374543/96)))*(- 2^(1/2)*23907i - 57873)^(1/2)*1i)/48 - (atan((x*(2^(1/
2)*23907i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 + 374543/96))
 - (7969*2^(1/2)*x*(2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505
i)/384 + 374543/96)))*(2^(1/2)*23907i - 57873)^(1/2)*1i)/48
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 791, normalized size of antiderivative = 4.39

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx = \text{Too large to display}$$

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

output
```
(188*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqr
t(sqrt(3) + 1)*sqrt(2)))*x**4 + 376*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(s
qrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 564*sqrt(sq
rt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) +
1)*sqrt(2))) + 102*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(
2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 204*sqrt(sqrt(3) + 1)*sqrt(2
)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2
 + 306*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(s
qrt(sqrt(3) + 1)*sqrt(2))) - 188*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt
(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 376*sqrt(sqrt(
3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*
sqrt(2)))*x**2 - 564*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqr
t(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 102*sqrt(sqrt(3) + 1)*sqrt(2)*a
tan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 -
204*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt
(sqrt(3) + 1)*sqrt(2)))*x**2 - 306*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sq
rt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 94*sqrt(sqrt(3) -
 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**4 - 18
8*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) +
 x**2)*x**2 - 282*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sq...
```

3.114 $\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 179

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2} dx = \frac{25x(1-x^2)}{24(3+2x^2+x^4)} - \frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}-2x}{\sqrt{2(1+\sqrt{3})}}\right) + \frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{2(1+\sqrt{3})}}\right) - \frac{1}{48}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\text{arctanh}\left(\frac{\sqrt{2(-1+\sqrt{3})}x}{\sqrt{3}+x^2}\right)$$

output
```
25*x*(-x^2+1)/(24*x^4+48*x^2+72)-1/288*(-69402+77382*3^(1/2))^(1/2)*arctan
(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))^(1/2))+1/288*(-69402+77382*3^(1/
2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))-1/288*(69
402+77382*3^(1/2))^(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.64

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = \frac{1}{48}\left(-\frac{50x(-1+x^2)}{3+2x^2+x^4}+\frac{(95+44i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{(95-44i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((-50*x*(-1 + x^2))/(3 + 2*x^2 + x^4) + ((95 + (44*I)*Sqrt[2])*ArcTan[x/Sq
rt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((95 - (44*I)*Sqrt[2])*ArcTan[x/
Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/48
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.47, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2206, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{ 2206}$$

$$\frac{1}{48}\int \frac{2(95x^2+7)}{x^4+2x^2+3}dx+\frac{25x(1-x^2)}{24(x^4+2x^2+3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{24}\int \frac{95x^2+7}{x^4+2x^2+3}dx+\frac{25x(1-x^2)}{24(x^4+2x^2+3)}$$

$$\Big\downarrow \; 1483$$

$$\frac{1}{24}\left(\frac{\int \frac{7\sqrt{2\left(-1+\sqrt{3}\right)}-\left(7-95\sqrt{3}\right)x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\int \frac{\left(7-95\sqrt{3}\right)x+7\sqrt{2\left(-1+\sqrt{3}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right)+\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{24}\left(\frac{\frac{1}{2}\sqrt{51588\sqrt{3}-46268}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\left(7-95\sqrt{3}\right)\int -\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}\sqrt{51588\sqrt{3}-\right.$$

$$\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{24}\left(\frac{\frac{1}{2}\sqrt{51588\sqrt{3}-46268}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx+\frac{1}{2}\left(7-95\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}\sqrt{51588\sqrt{3}-4\right.$$

$$\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{24}\left(\frac{\frac{1}{2}\left(7-95\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\sqrt{51588\sqrt{3}-46268}\int \frac{1}{-\left(2x-\sqrt{2\left(-1+\sqrt{3}\right)}\right)^2-2\left(1+\sqrt{3}\right)}d\left(2x-\sqrt{2\left(-\right.\right.}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.$$

$$\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{24}\left(\frac{\frac{1}{2}(7-95\sqrt{3})\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx+\sqrt{\frac{51588\sqrt{3}-46268}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}(7-95\sqrt{3})\int\frac{2x+}{x^2+\sqrt{2}}\right.$$

$$\frac{25x(1-x^2)}{24\left(x^4+2x^2+3\right)}$$

$\Big\downarrow$ 1103

$$\frac{1}{24}\left(\frac{\sqrt{\frac{51588\sqrt{3}-46268}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}(7-95\sqrt{3})\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\sqrt{\frac{51588\sqrt{3}-462}{2\left(1+\sqrt{3}\right)}}}{}\right.$$

$$\frac{25x(1-x^2)}{24\left(x^4+2x^2+3\right)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(25*x*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) + ((Sqrt[(-46268 + 51588*Sqrt[3])/
(2*(1 + Sqrt[3]))]*ArcTan[(-Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt
[3])]] - ((7 - 95*Sqrt[3])*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
2)/(2*Sqrt[6*(-1 + Sqrt[3])]) + (Sqrt[(-46268 + 51588*Sqrt[3])/(2*(1 + Sqr
t[3]))]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]] + ((7
 - 95*Sqrt[3])*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6
*(-1 + Sqrt[3])]))/24
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.34

method	result
risch	$\frac{-\frac{25}{24}x^3+\frac{25}{24}x}{x^4+2x^2+3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(95\_R^2+7\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{96}$
default	$\frac{-\frac{25}{24}x^3+\frac{25}{24}x}{x^4+2x^2+3}+\frac{\left(-139\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-132\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{576}+\frac{\left(14\sqrt{3}-\frac{\left(-139\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-132\sqrt{-2+2\right.}{2}\right.}{144\sqrt{2}}$

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-25/24*x^3+25/24*x)/(x^4+2*x^2+3)+1/96*sum((95*_R^2+7)/(_R^3+_R)*ln(x-_R)
,_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(126) = 252.

Time = 0.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.50

$$\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^2}\,dx =$$
$$-\frac{100\,x^3+2\left(x^4+2\,x^2+3\right)\sqrt{\frac{4299}{2}\,\sqrt{3}-\frac{11567}{6}}\arctan\left(\frac{3}{13513}\,\sqrt{\frac{4299}{2}\,\sqrt{3}+\frac{11567}{6}}\,\sqrt{\frac{4299}{2}\,\sqrt{3}-\frac{11567}{6}}\,(\sqrt{3}-1\right.}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/96*(100*x^3 + 2*(x^4 + 2*x^2 + 3)*sqrt(4299/2*sqrt(3) - 11567/6)*arctan
(3/13513*sqrt(4299/2*sqrt(3) + 11567/6)*sqrt(4299/2*sqrt(3) - 11567/6)*(sq
rt(3) - 1) + 1/13513*(7*sqrt(3)*x - 285*x)*sqrt(4299/2*sqrt(3) - 11567/6))
 - 2*(x^4 + 2*x^2 + 3)*sqrt(4299/2*sqrt(3) - 11567/6)*arctan(3/13513*sqrt(
4299/2*sqrt(3) + 11567/6)*sqrt(4299/2*sqrt(3) - 11567/6)*(sqrt(3) - 1) - 1
/13513*(7*sqrt(3)*x - 285*x)*sqrt(4299/2*sqrt(3) - 11567/6)) + (x^4 + 2*x^
2 + 3)*sqrt(4299/2*sqrt(3) + 11567/6)*log(13513*x^2 + 2*(139*sqrt(3)*x - 1
32*x)*sqrt(4299/2*sqrt(3) + 11567/6) + 13513*sqrt(3)) - (x^4 + 2*x^2 + 3)*
sqrt(4299/2*sqrt(3) + 11567/6)*log(13513*x^2 - 2*(139*sqrt(3)*x - 132*x)*s
qrt(4299/2*sqrt(3) + 11567/6) + 13513*sqrt(3)) - 100*x)/(x^4 + 2*x^2 + 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(141) = 282$.

Time = 0.88 (sec) , antiderivative size = 1185, normalized size of antiderivative = 6.62

$$\int \frac{4 + x^2 + 3x^4 + 5x^6}{(3 + 2x^2 + x^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

output
```
(-25*x**3 + 25*x)/(24*x**4 + 48*x**2 + 72) + sqrt(11567/55296 + 1433*sqrt(
3)/6144)*log(x**2 + x*(-556*sqrt(2)*sqrt(11567 + 12897*sqrt(3))/13513 - 10
40345*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/174277161 + 278*sqrt(3)*sqrt(115
67 + 12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/174277161) - 47610
276200401*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)/30372528846219921 -
4390831246*sqrt(6)*sqrt(149179599*sqrt(3) + 316396658)/7065021829779 + 128
1046481635939181/30372528846219921 + 200684595453464*sqrt(3)/7065021829779
) - sqrt(11567/55296 + 1433*sqrt(3)/6144)*log(x**2 + x*(-278*sqrt(3)*sqrt(
11567 + 12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/174277161 + 104
0345*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/174277161 + 556*sqrt(2)*sqrt(1156
7 + 12897*sqrt(3))/13513) - 47610276200401*sqrt(2)*sqrt(149179599*sqrt(3)
+ 316396658)/30372528846219921 - 4390831246*sqrt(6)*sqrt(149179599*sqrt(3)
 + 316396658)/7065021829779 + 1281046481635939181/30372528846219921 + 2006
84595453464*sqrt(3)/7065021829779) + 2*sqrt(-sqrt(2)*sqrt(149179599*sqrt(3
) + 316396658)/27648 + 11567/55296 + 1433*sqrt(3)/2048)*atan(348554322*sqr
t(3)*x/(94591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)
+ 11567 + 38691*sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2
*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3))) - 7
170732*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/(94591*sqrt(2)*sqrt(-2*sqrt(2)*
sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)) + 278*sqrt...
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{(x^4+2\,x^2+3)^2}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-25/24*(x^3 - x)/(x^4 + 2*x^2 + 3) + 1/24*integrate((95*x^2 + 7)/(x^4 + 2*
x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(126) = 252$.

Time = 0.45 (sec) , antiderivative size = 565, normalized size of antiderivative = 3.16

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
-1/62208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3
*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
 1/62208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3
*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
 1/124416*sqrt(2)*(1710*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18
) - 95*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 1
8)^(3/2) + 1710*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 252*3^(1/4)*s
qrt(2)*sqrt(-6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 +
 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/124416*sqrt(2)*(1710*
3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 95*3^(3/4)*sqrt(2)*(
-6*sqrt(3) + 18)^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*
sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) +
 18) - 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*s
qrt(3) + 1/2) + sqrt(3)) - 25/24*(x^3 - x)/(x^4 + 2*x^2 + 3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{\frac{25\,x}{24}-\frac{25\,x^3}{24}}{x^4+2\,x^2+3} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}\,13513\mathrm{i}}{15552\,\left(-\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}+\frac{13513\,\sqrt{2}\,x\,\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}}{31104\,\left(-\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}\right)\,\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}\,1\mathrm{i}}{144} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{34701+\sqrt{2}\,40539\mathrm{i}}\,13513\mathrm{i}}{15552\,\left(\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}-\frac{13513\,\sqrt{2}\,x\,\sqrt{34701+\sqrt{2}\,40539\mathrm{i}}}{31104\,\left(\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}\right)\,\sqrt{34701+\sqrt{2}\,40539\mathrm{i}}\,1\mathrm{i}}{144}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^2,x)
```

output
```
((25*x)/24 - (25*x^3)/24)/(2*x^2 + x^4 + 3) - (atan((x*(34701 - 2^(1/2)*40
539i)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 - 1878307/5184)) + (135
13*2^(1/2)*x*(34701 - 2^(1/2)*40539i)^(1/2))/(31104*((2^(1/2)*94591i)/1036
8 - 1878307/5184)))*(34701 - 2^(1/2)*40539i)^(1/2)*1i)/144 + (atan((x*(2^(
1/2)*40539i + 34701)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 + 187830
7/5184)) - (13513*2^(1/2)*x*(2^(1/2)*40539i + 34701)^(1/2))/(31104*((2^(1/
2)*94591i)/10368 + 1878307/5184)))*(2^(1/2)*40539i + 34701)^(1/2)*1i)/144
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 786, normalized size of antiderivative = 4.39

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

output

```
( - 278*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(
sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 556*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqr
t(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 834*sqrt
(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3)
 + 1)*sqrt(2))) + 264*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sq
rt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 528*sqrt(sqrt(3) + 1)*sqr
t(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x
**2 + 792*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)
/(sqrt(sqrt(3) + 1)*sqrt(2))) + 278*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(s
qrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 556*sqrt(sq
rt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) +
1)*sqrt(2)))*x**2 + 834*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*
sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 264*sqrt(sqrt(3) + 1)*sqrt(2
)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4
 - 528*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(s
qrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 792*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt
(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 139*sqrt(sqrt(
3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**4
+ 278*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(
3) + x**2)*x**2 + 417*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1...
```

3.115 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 184

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx = -\frac{4}{9x}-\frac{25x(5+x^2)}{72\left(3+2x^2+x^4\right)} + \frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) - \frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) + \frac{1}{48}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)$$

output
```
-4/9/x-25*x*(x^2+5)/(72*x^4+144*x^2+216)+1/288*(-5790+4194*3^(1/2))^(1/2)*
arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))^(1/2))-1/288*(-5790+4194*3
^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))+1/288
*(5790+4194*3^(1/2))^(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.68

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx = -\frac{4}{9x}-\frac{25x(5+x^2)}{72(3+2x^2+x^4)}-\frac{(26i+19\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{48\sqrt{2-2i\sqrt{2}}} - \frac{(-26i+19\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{48\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

output
```
-4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - ((26*I + 19*Sqrt[2])*
ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(48*Sqrt[2 - (2*I)*Sqrt[2]]) - ((-26*I + 19
*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(48*Sqrt[2 + (2*I)*Sqrt[2]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^2(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{48}\int \frac{2(-25x^4+85x^2+96)}{3x^2(x^4+2x^2+3)}dx-\frac{25x(x^2+5)}{72(x^4+2x^2+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72}\int \frac{-25x^4+85x^2+96}{x^2(x^4+2x^2+3)}dx-\frac{25x(x^2+5)}{72(x^4+2x^2+3)}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{72}\int \left(\frac{32}{x^2}-\frac{3(19x^2-7)}{x^4+2x^2+3}\right)dx-\frac{25x(x^2+5)}{72(x^4+2x^2+3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{72}\left(\frac{1}{2}\sqrt{\frac{3}{2}\left(699\sqrt{3}-965\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}\sqrt{\frac{3}{2}\left(699\sqrt{3}-965\right)}\arctan\left(\frac{2x+\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right.\right.$$
$$\frac{25x(x^2+5)}{72(x^4+2x^2+3)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(-25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) + (-32/x + (Sqrt[(3*(-965 + 699*S
qrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/
2 - (Sqrt[(3*(-965 + 699*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x
)/Sqrt[2*(1 + Sqrt[3])]])/2 - (Sqrt[(3*(965 + 699*Sqrt[3]))/2]*Log[Sqrt[3]
 - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4 + (Sqrt[(3*(965 + 699*Sqrt[3]))/2]*L
og[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4)/72
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.34

method	result
risch	$\frac{-\frac{19}{24}x^4-\frac{21}{8}x^2-\frac{4}{3}}{x(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4-1930\_Z^2+488601\right)}\_R\ln\left(-96\_R^3+34499\_R+361383x\right)\right)}{96}$
default	$-\frac{\frac{25}{8}x^3+\frac{125}{8}x}{9(x^4+2x^2+3)}-\frac{\left(-32\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-39\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{576}-\frac{\left(-14\sqrt{3}-\frac{\left(-32\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-39\sqrt{-2+\right.}{2}\right.}{144\sqrt{}}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-19/24*x^4-21/8*x^2-4/3)/x/(x^4+2*x^2+3)+1/96*sum(_R*ln(-96*_R^3+34499*_R
+361383*x),_R=RootOf(3*_Z^4-1930*_Z^2+488601))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(131) = 262$.

Time = 0.08 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.53

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx =$$

$$-\frac{76\,x^4+2\,(x^5+2\,x^3+3\,x)\sqrt{\frac{233}{2}\,\sqrt{3}-\frac{965}{6}}\arctan\left(\frac{3}{517}\,\sqrt{\frac{233}{2}\,\sqrt{3}+\frac{965}{6}}\sqrt{\frac{233}{2}\,\sqrt{3}-\frac{965}{6}}(\sqrt{3}-1)+\frac{1}{517}\,(\right.}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/96*(76*x^4 + 2*(x^5 + 2*x^3 + 3*x)*sqrt(233/2*sqrt(3) - 965/6)*arctan(3
/517*sqrt(233/2*sqrt(3) + 965/6)*sqrt(233/2*sqrt(3) - 965/6)*(sqrt(3) - 1)
 + 1/517*(7*sqrt(3)*x + 57*x)*sqrt(233/2*sqrt(3) - 965/6)) - 2*(x^5 + 2*x^
3 + 3*x)*sqrt(233/2*sqrt(3) - 965/6)*arctan(3/517*sqrt(233/2*sqrt(3) + 965
/6)*sqrt(233/2*sqrt(3) - 965/6)*(sqrt(3) - 1) - 1/517*(7*sqrt(3)*x + 57*x)
*sqrt(233/2*sqrt(3) - 965/6)) - (x^5 + 2*x^3 + 3*x)*sqrt(233/2*sqrt(3) + 9
65/6)*log(517*x^2 + 2*(32*sqrt(3)*x - 39*x)*sqrt(233/2*sqrt(3) + 965/6) +
517*sqrt(3)) + (x^5 + 2*x^3 + 3*x)*sqrt(233/2*sqrt(3) + 965/6)*log(517*x^2
 - 2*(32*sqrt(3)*x - 39*x)*sqrt(233/2*sqrt(3) + 965/6) + 517*sqrt(3)) + 25
2*x^2 + 128)/(x^5 + 2*x^3 + 3*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(146) = 292$.

Time = 1.08 (sec) , antiderivative size = 1192, normalized size of antiderivative = 6.48

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**2,x)
```

output
```
(-19*x**4 - 63*x**2 - 32)/(24*x**5 + 48*x**3 + 72*x) - sqrt(965/55296 + 23
3*sqrt(3)/18432)*log(x**2 + x*(-128*sqrt(2)*sqrt(965 + 699*sqrt(3))/517 -
21793*sqrt(6)*sqrt(965 + 699*sqrt(3))/361383 + 64*sqrt(3)*sqrt(965 + 699*s
qrt(3))*sqrt(674535*sqrt(3) + 1198514)/361383) - 8882635459*sqrt(2)*sqrt(6
74535*sqrt(3) + 1198514)/130597672689 - 20458048*sqrt(6)*sqrt(674535*sqrt(
3) + 1198514)/560505033 + 18567565928783/130597672689 + 46950427730*sqrt(3
)/560505033) + sqrt(965/55296 + 233*sqrt(3)/18432)*log(x**2 + x*(-64*sqrt(
3)*sqrt(965 + 699*sqrt(3))*sqrt(674535*sqrt(3) + 1198514)/361383 + 21793*s
qrt(6)*sqrt(965 + 699*sqrt(3))/361383 + 128*sqrt(2)*sqrt(965 + 699*sqrt(3)
)/517) - 8882635459*sqrt(2)*sqrt(674535*sqrt(3) + 1198514)/130597672689 -
20458048*sqrt(6)*sqrt(674535*sqrt(3) + 1198514)/560505033 + 18567565928783
/130597672689 + 46950427730*sqrt(3)/560505033) + 2*sqrt(-sqrt(2)*sqrt(6745
35*sqrt(3) + 1198514)/27648 + 965/55296 + 233*sqrt(3)/6144)*atan(722766*sq
rt(3)*x/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sq
rt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt
(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) + 89472*sqrt(6)*sqrt(965
 + 699*sqrt(3))/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(6
74535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt
(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) + 65379*sqrt(2)*
sqrt(965 + 699*sqrt(3))/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqr...
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx=\int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^2x^2}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-1/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x) - 1/24*integrate((19*x^2
- 7)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(131) = 262$.

Time = 0.43 (sec) , antiderivative size = 572, normalized size of antiderivative = 3.11

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
1/62208*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^
(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/
62208*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqr
t(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt(-
6*sqrt(3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(2)
*sqrt(6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3
/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/12
4416*sqrt(2)*(342*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 19
*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 19*3^(3/4)*(6*sqrt(3) + 18)^(3/
2) + 342*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 252*3^(1/4)*sqrt(2)*
sqrt(-6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1
/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/124416*sqrt(2)*(342*3^(3/4)*
sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 19*3^(3/4)*sqrt(2)*(-6*sqrt(
3) + 18)^(3/2) + 19*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(6*sq
rt(3) + 18)*(sqrt(3) - 3) + 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 25
2*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) +
1/2) + sqrt(3)) - 1/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx$$
$$= -\frac{\frac{19\,x^4}{24}+\frac{21\,x^2}{8}+\frac{4}{3}}{x^5+2\,x^3+3\,x} - \frac{\operatorname{atan}\left(\frac{x\,\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}\,517\mathrm{i}}{15552\left(\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}+\frac{517\,\sqrt{2}\,x\,\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}}{31104\left(\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}\right)\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}\,1\mathrm{i}}{144} + \frac{\operatorname{atan}\left(\frac{x\,\sqrt{2895+\sqrt{2}\,1551\mathrm{i}}\,517\mathrm{i}}{15552\left(-\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}-\frac{517\,\sqrt{2}\,x\,\sqrt{2895+\sqrt{2}\,1551\mathrm{i}}}{31104\left(-\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}\right)\sqrt{2895+\sqrt{2}\,1551\mathrm{i}}\,1\mathrm{i}}{144}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^2),x)
```

output
```
(atan((x*(2^(1/2)*1551i + 2895)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368
- 517/162)) - (517*2^(1/2)*x*(2^(1/2)*1551i + 2895)^(1/2))/(31104*((2^(1/2
)*3619i)/10368 - 517/162)))*(2^(1/2)*1551i + 2895)^(1/2)*1i)/144 - (atan((
x*(2895 - 2^(1/2)*1551i)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368 + 517/1
62)) + (517*2^(1/2)*x*(2895 - 2^(1/2)*1551i)^(1/2))/(31104*((2^(1/2)*3619i
)/10368 + 517/162)))*(2895 - 2^(1/2)*1551i)^(1/2)*1i)/144 - ((21*x^2)/8 +
(19*x^4)/24 + 4/3)/(3*x + 2*x^3 + x^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 799, normalized size of antiderivative = 4.34

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x)
```

output
```
(64*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt
(sqrt(3) + 1)*sqrt(2)))*x**5 + 128*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sq
rt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**3 + 192*sqrt(sqr
t(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1
)*sqrt(2)))*x - 78*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(
2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**5 - 156*sqrt(sqrt(3) + 1)*sqrt(2
)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**3
 - 234*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(s
qrt(sqrt(3) + 1)*sqrt(2)))*x - 64*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqr
t(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**5 - 128*sqrt(sqrt
(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)
*sqrt(2)))*x**3 - 192*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sq
rt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x + 78*sqrt(sqrt(3) + 1)*sqrt(2)
*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**5
+ 156*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sq
rt(sqrt(3) + 1)*sqrt(2)))*x**3 + 234*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(
sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x - 32*sqrt(sqrt(
3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**5
- 64*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3
) + x**2)*x**3 - 96*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*...
```

3.116 $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 193

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx &= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x(7+5x^2)}{216(3+2x^2+x^4)} \\
&\quad -\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad +\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad -\frac{1}{432}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)
\end{aligned}$$

output
```
-4/27/x^3+13/27/x+25*x*(5*x^2+7)/(216*x^4+432*x^2+648)-1/2592*(36438+34003
8*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))^(1/2))+1/
2592*(36438+340038*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3
^(1/2))^(1/2))-1/2592*(-36438+340038*3^(1/2))^(1/2)*arctanh((-2+2*3^(1/2))
^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx=\frac{1}{864}\left(\frac{4(-96+248x^2+351x^4+229x^6)}{x^3\left(3+2x^2+x^4\right)}+\frac{2\left(229+46i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{2\left(229-46i\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((4*(-96 + 248*x^2 + 351*x^4 + 229*x^6))/(x^3*(3 + 2*x^2 + x^4)) + (2*(229
 + (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (2
*(229 - (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]]
)/864
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^4\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{48}\int \frac{2\left(125x^6+25x^4-120x^2+288\right)}{9x^4\left(x^4+2x^2+3\right)}dx+\frac{25x\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{216}\int \frac{125x^6+25x^4-120x^2+288}{x^4(x^4+2x^2+3)}dx+\frac{25x(5x^2+7)}{216(x^4+2x^2+3)}$$

$$\Big\downarrow \; 2195$$

$$\frac{1}{216}\int \left(\frac{229x^2+137}{x^4+2x^2+3}-\frac{104}{x^2}+\frac{96}{x^4}\right)dx+\frac{25x(5x^2+7)}{216(x^4+2x^2+3)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{216}\left(-\frac{1}{2}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2\left(\right.}}{\sqrt{2\left(1\right.}}\right.\right.$$
$$\frac{25x(5x^2+7)}{216(x^4+2x^2+3)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(25*x*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) + (-32/x^3 + 104/x - (Sqrt[(607
3 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sq
rt[3])]])/2 + (Sqrt[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]
)] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 + (Sqrt[(-6073 + 56673*Sqrt[3])/6]*Log
[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4 - (Sqrt[(-6073 + 56673*Sqrt[
3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4)/216
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\frac{229}{216}x^6+\frac{13}{8}x^4+\frac{31}{27}x^2-\frac{4}{9}}{x^3(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+12146\_Z^2+3211828929\right)}\_R\ln\left(825\_R^3+11161024\_R+3926135421x\right)\right)}{864}$
default	$\frac{\frac{125}{8}x^3+\frac{175}{8}x}{27x^4+54x^2+81}+\frac{\left(-275\sqrt{-2+2\sqrt{3}}\sqrt{3}-138\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{5184}+\frac{\left(274\sqrt{3}-\frac{\left(-275\sqrt{-2+2\sqrt{3}}\sqrt{3}-138\sqrt{-\right.}{2}\right.}{1296}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(229/216*x^6+13/8*x^4+31/27*x^2-4/9)/x^3/(x^4+2*x^2+3)+1/864*sum(_R*ln(825
*_R^3+11161024*_R+3926135421*x),_R=RootOf(3*_Z^4+12146*_Z^2+3211828929))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.43

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx$$
$$= \frac{916\,x^6+1404\,x^4-2\left(x^7+2\,x^5+3\,x^3\right)\sqrt{\frac{18891}{2}\,\sqrt{3}+\frac{6073}{6}}\arctan\left(\frac{1}{69277}\left(137\,\sqrt{3}x+3\,\sqrt{\frac{18891}{2}\,\sqrt{3}-\frac{6073}{6}}\right.\right.}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/864*(916*x^6 + 1404*x^4 - 2*(x^7 + 2*x^5 + 3*x^3)*sqrt(18891/2*sqrt(3) +
 6073/6)*arctan(1/69277*(137*sqrt(3)*x + 3*sqrt(18891/2*sqrt(3) - 6073/6)*
(sqrt(3) - 1) - 687*x)*sqrt(18891/2*sqrt(3) + 6073/6)) + 2*(x^7 + 2*x^5 +
3*x^3)*sqrt(18891/2*sqrt(3) + 6073/6)*arctan(-1/69277*(137*sqrt(3)*x - 3*s
qrt(18891/2*sqrt(3) - 6073/6)*(sqrt(3) - 1) - 687*x)*sqrt(18891/2*sqrt(3)
+ 6073/6)) - (x^7 + 2*x^5 + 3*x^3)*sqrt(18891/2*sqrt(3) - 6073/6)*log(6927
7*x^2 + 2*(275*sqrt(3)*x - 138*x)*sqrt(18891/2*sqrt(3) - 6073/6) + 69277*s
qrt(3)) + (x^7 + 2*x^5 + 3*x^3)*sqrt(18891/2*sqrt(3) - 6073/6)*log(69277*x
^2 - 2*(275*sqrt(3)*x - 138*x)*sqrt(18891/2*sqrt(3) - 6073/6) + 69277*sqrt
(3)) + 992*x^2 - 384)/(x^7 + 2*x^5 + 3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.31

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx$$
$$= \operatorname{RootSum}\left(2293235712t^4+12437504t^2+4405801,\left(t\mapsto t\log\left(\frac{19707494400t^3}{145412423}+\frac{357152768t}{145412423}+x\right)\right)\right)$$
$$+\frac{229x^6+351x^4+248x^2-96}{216x^7+432x^5+648x^3}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**2,x)
```

output
```
RootSum(2293235712*_t**4 + 12437504*_t**2 + 4405801, Lambda(_t, _t*log(197
07494400*_t**3/145412423 + 357152768*_t/145412423 + x))) + (229*x**6 + 351
*x**4 + 248*x**2 - 96)/(216*x**7 + 432*x**5 + 648*x**3)
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{(x^4+2\,x^2+3)^2x^4}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
1/216*(229*x^6 + 351*x^4 + 248*x^2 - 96)/(x^7 + 2*x^5 + 3*x^3) + 1/216*int
egrate((229*x^2 + 137)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. 2(138) = 276.

Time = 0.46 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.00

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
-1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4122*3^(3/4)*(sqrt(3) + 3)
*sqrt(-6*sqrt(3) + 18) + 229*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 4932*3^(1/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 4932*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arcta
n(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/
2)) - 1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*
3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4122*3^(3/4)*(sqrt(3)
 + 3)*sqrt(-6*sqrt(3) + 18) + 229*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 4932*3
^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 4932*3^(1/4)*sqrt(-6*sqrt(3) + 18))*
arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 1/1119744*sqrt(2)*(4122*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*s
qrt(3) + 18) - 229*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 229*3^(3/4)*(
6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) -
4932*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 4932*3^(1/4)*sqrt(6*sqrt(3) +
 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/111974
4*sqrt(2)*(4122*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 229*
3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 229*3^(3/4)*(6*sqrt(3) + 18)^(3/
2) + 4122*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4932*3^(1/4)*sqrt(2
)*sqrt(-6*sqrt(3) + 18) - 4932*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3
^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 25/216*(5*x^3 + 7*x)/(x^...
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^2}\,dx = \frac{\frac{229\,x^6}{216}+\frac{13\,x^4}{8}+\frac{31\,x^2}{27}-\frac{4}{9}}{x^7+2\,x^5+3\,x^3} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-18219-\sqrt{2}\,207831\mathrm{i}}\,69277\mathrm{i}}{11337408\,\left(-\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}+\frac{69277\,\sqrt{2}\,x\,\sqrt{-18219-\sqrt{2}\,207831\mathrm{i}}}{22674816\,\left(-\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}\right)\,\sqrt{-18219-\sqrt{2}\,207831\mathrm{i}}\,1\mathrm{i}}{1296} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-18219+\sqrt{2}\,207831\mathrm{i}}\,69277\mathrm{i}}{11337408\,\left(\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}-\frac{69277\,\sqrt{2}\,x\,\sqrt{-18219+\sqrt{2}\,207831\mathrm{i}}}{22674816\,\left(\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}\right)\,\sqrt{-18219+\sqrt{2}\,207831\mathrm{i}}\,1\mathrm{i}}{1296}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^2),x)
```

output
```
((31*x^2)/27 + (13*x^4)/8 + (229*x^6)/216 - 4/9)/(3*x^3 + 2*x^5 + x^7) - (
atan((x*(- 2^(1/2)*207831i - 18219)^(1/2)*69277i)/(11337408*((2^(1/2)*9490
949i)/7558272 - 19051175/3779136)) + (69277*2^(1/2)*x*(- 2^(1/2)*207831i -
 18219)^(1/2))/(22674816*((2^(1/2)*9490949i)/7558272 - 19051175/3779136)))
*(- 2^(1/2)*207831i - 18219)^(1/2)*1i)/1296 + (atan((x*(2^(1/2)*207831i -
18219)^(1/2)*69277i)/(11337408*((2^(1/2)*9490949i)/7558272 + 19051175/3779
136)) - (69277*2^(1/2)*x*(2^(1/2)*207831i - 18219)^(1/2))/(22674816*((2^(1
/2)*9490949i)/7558272 + 19051175/3779136)))*(2^(1/2)*207831i - 18219)^(1/2
)*1i)/1296
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 820, normalized size of antiderivative = 4.25

$$\int \frac{4+x^2+3x^4+5x^6}{x^4 (3+2x^2+x^4)^2} \, dx = \text{Too large to display}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x)
```

output
```
( - 550*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(
sqrt(sqrt(3) + 1)*sqrt(2)))*x**7 - 1100*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sq
rt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**5 - 1650*sq
rt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(
3) + 1)*sqrt(2)))*x**3 + 276*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3)
- 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**7 + 552*sqrt(sqrt(3) +
 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt
(2)))*x**5 + 828*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2)
 - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**3 + 550*sqrt(sqrt(3) + 1)*sqrt(6)*
atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**7 +
 1100*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sq
rt(sqrt(3) + 1)*sqrt(2)))*x**5 + 1650*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt
(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**3 - 276*sqrt(
sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3)
+ 1)*sqrt(2)))*x**7 - 552*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1
)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**5 - 828*sqrt(sqrt(3) + 1)
*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)
))*x**3 + 275*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3) - 1)*sqrt(2)*x
 + sqrt(3) + x**2)*x**7 + 550*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3
) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**5 + 825*sqrt(sqrt(3) - 1)*sqrt(6)...
```

3.117 $\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 200

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx = -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x(1-7x^2)}{648(3+2x^2+x^4)} + \frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296} - \frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{1296} + \frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)}{1296}$$

output
```
-4/45/x^5+13/81/x^3-13/27/x+25*x*(-7*x^2+1)/(648*x^4+1296*x^2+1944)+1/7776
*(-6836286+4130514*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3
^(1/2))^(1/2))-1/7776*(-6836286+4130514*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/
2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))+1/7776*(6836286+4130514*3^(1/2))^(1/2)
*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx = \frac{-\frac{4(864-984x^2+3928x^4+2475x^6+2435x^8)}{x^5(3+2x^2+x^4)} - \frac{10i\left(-487i+475\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{10i\left(487i+475\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{12960}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```

output
```
((-4*(864 - 984*x^2 + 3928*x^4 + 2475*x^6 + 2435*x^8))/(x^5*(3 + 2*x^2 + x
^4)) - ((10*I)*(-487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[
1 - I*Sqrt[2]] + ((10*I)*(487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]
]])/Sqrt[1 + I*Sqrt[2]])/12960
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^6(x^4+2x^2+3)^2}\,dx$$

$$\Big\downarrow \text{ 2198}$$

$$\frac{1}{48}\int \frac{2(-175x^8+775x^6+600x^4-360x^2+864)}{27x^6(x^4+2x^2+3)}dx + \frac{25x(1-7x^2)}{648(x^4+2x^2+3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{648}\int \frac{-175x^8+775x^6+600x^4-360x^2+864}{x^6(x^4+2x^2+3)}dx + \frac{25x(1-7x^2)}{648(x^4+2x^2+3)}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{648}\int\left(\frac{463-487x^2}{x^4+2x^2+3}+\frac{312}{x^2}-\frac{312}{x^4}+\frac{288}{x^6}\right)dx+\frac{25x(1-7x^2)}{648\left(x^4+2x^2+3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{648}\left(\frac{1}{2}\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{6}\left(688419\sqrt{3}-1139381\right)}\arctan\left(\frac{2x}{\right.}\right.$$
$$\frac{25x(1-7x^2)}{648\left(x^4+2x^2+3\right)}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```

output
```
(25*x*(1 - 7*x^2))/(648*(3 + 2*x^2 + x^4)) + (-288/(5*x^5) + 104/x^3 - 312
/x + (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] -
 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 - (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*Arc
Tan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2 - (Sqrt[(1139
381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4
+ (Sqrt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]
*x + x^2])/4)/648
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.36

method	result
risch	$\frac{-\frac{487}{648}x^8-\frac{55}{72}x^6-\frac{491}{405}x^4+\frac{41}{135}x^2-\frac{4}{15}}{x^5(x^4+2x^2+3)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4-2278762\_Z^2+473920719561\right)}\_R\ln\left(-2886\_R^3+1211171969\_R+\right.\right.}{2592}$
default	$-\frac{\frac{175}{24}x^3-\frac{25}{24}x}{27(x^4+2x^2+3)}-\frac{\left(-962\sqrt{-2+2\sqrt{3}}\sqrt{3}-1425\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{15552}-\frac{\left(-926\sqrt{3}-\frac{\left(-962\sqrt{-2+2\sqrt{3}}\sqrt{3}-1\right.}{}\right.}{}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-487/648*x^8-55/72*x^6-491/405*x^4+41/135*x^2-4/15)/x^5/(x^4+2*x^2+3)+1/2
592*sum(_R*ln(-2886*_R^3+1211171969*_R+171119622411*x),_R=RootOf(3*_Z^4-22
78762*_Z^2+473920719561))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(143) = 286.

Time = 0.08 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.51

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx =$$
$$-\frac{9740\,x^8+9900\,x^6+15712\,x^4+10\,(x^9+2\,x^7+3\,x^5)\sqrt{\frac{229473}{2}\sqrt{3}-\frac{1139381}{6}}\arctan\left(\frac{3}{248569}\sqrt{\frac{229473}{2}\sqrt{3}\right.}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/12960*(9740*x^8 + 9900*x^6 + 15712*x^4 + 10*(x^9 + 2*x^7 + 3*x^5)*sqrt(
229473/2*sqrt(3) - 1139381/6)*arctan(3/248569*sqrt(229473/2*sqrt(3) + 1139
381/6)*sqrt(229473/2*sqrt(3) - 1139381/6)*(sqrt(3) - 1) + 1/248569*(463*sq
rt(3)*x + 1461*x)*sqrt(229473/2*sqrt(3) - 1139381/6)) - 10*(x^9 + 2*x^7 +
3*x^5)*sqrt(229473/2*sqrt(3) - 1139381/6)*arctan(3/248569*sqrt(229473/2*sq
rt(3) + 1139381/6)*sqrt(229473/2*sqrt(3) - 1139381/6)*(sqrt(3) - 1) - 1/24
8569*(463*sqrt(3)*x + 1461*x)*sqrt(229473/2*sqrt(3) - 1139381/6)) - 5*(x^9
 + 2*x^7 + 3*x^5)*sqrt(229473/2*sqrt(3) + 1139381/6)*log(248569*x^2 + 2*(9
62*sqrt(3)*x - 1425*x)*sqrt(229473/2*sqrt(3) + 1139381/6) + 248569*sqrt(3)
) + 5*(x^9 + 2*x^7 + 3*x^5)*sqrt(229473/2*sqrt(3) + 1139381/6)*log(248569*
x^2 - 2*(962*sqrt(3)*x - 1425*x)*sqrt(229473/2*sqrt(3) + 1139381/6) + 2485
69*sqrt(3)) - 3936*x^2 + 3456)/(x^9 + 2*x^7 + 3*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(162) = 324.

Time = 1.07 (sec) , antiderivative size = 1202, normalized size of antiderivative = 6.01

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+2*x**2+3)**2,x)
```

output
```
-sqrt(1139381/40310784 + 2833*sqrt(3)/165888)*log(x**2 + x*(-3848*sqrt(2)*
sqrt(1139381 + 688419*sqrt(3))/248569 - 769085497*sqrt(6)*sqrt(1139381 + 6
88419*sqrt(3))/171119622411 + 1924*sqrt(3)*sqrt(1139381 + 688419*sqrt(3))*
sqrt(784371528639*sqrt(3) + 1359975610922)/171119622411) - 867751090756951
0603*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/29281925174083213
452921 - 21752950947364*sqrt(6)*sqrt(784371528639*sqrt(3) + 1359975610922)/
127605100269239577 + 20196165220927340076543947/29281925174083213452921 +
5094503682633631307O*sqrt(3)/127605100269239577) + sqrt(1139381/40310784 +
 2833*sqrt(3)/165888)*log(x**2 + x*(-1924*sqrt(3)*sqrt(1139381 + 688419*sq
rt(3))*sqrt(784371528639*sqrt(3) + 1359975610922)/171119622411 + 769085497
*sqrt(6)*sqrt(1139381 + 688419*sqrt(3))/171119622411 + 3848*sqrt(2)*sqrt(1
139381 + 688419*sqrt(3))/248569) - 8677510907569510603*sqrt(2)*sqrt(784371
528639*sqrt(3) + 1359975610922)/29281925174083213452921 - 21752950947364*s
qrt(6)*sqrt(784371528639*sqrt(3) + 1359975610922)/127605100269239577 + 201
96165220927340076543947/29281925174083213452921 + 50945036826336313070*sqr
t(3)/127605100269239577) + 2*sqrt(-sqrt(2)*sqrt(784371528639*sqrt(3) + 135
9975610922)/20155392 + 1139381/40310784 + 2833*sqrt(3)/55296)*atan(3422392
44822*sqrt(3)*x/(-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*
sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqr
t(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1...
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{(x^4+2\,x^2+3)^2x^6}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-1/3240*(2435*x^8 + 2475*x^6 + 3928*x^4 - 984*x^2 + 864)/(x^9 + 2*x^7 + 3*
x^5) - 1/648*integrate((487*x^2 - 463)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(143) = 286$.

Time = 0.42 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.92

$$\int \frac{4+x^2+3x^4+5x^6}{x^6(3+2x^2+x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
1/1679616*sqrt(2)*(487*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 8766*3^(3/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 8766*3^(3/4)*(sqrt(3) + 3)
*sqrt(-6*sqrt(3) + 18) + 487*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 16668*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 16668*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arc
tan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
1/2)) + 1/1679616*sqrt(2)*(487*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 87
66*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 8766*3^(3/4)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) + 487*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 166
68*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 16668*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) + 1/3359232*sqrt(2)*(8766*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) - 487*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 487*3^(3
/4)*(6*sqrt(3) + 18)^(3/2) + 8766*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) -
3) + 16668*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 16668*3^(1/4)*sqrt(6*sq
rt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1
/3359232*sqrt(2)*(8766*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18)
 - 487*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 487*3^(3/4)*(6*sqrt(3) +
18)^(3/2) + 8766*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 16668*3^(1/4
)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 16668*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(
x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 25/648*(7*x^3 -...
```

Mupad [B] (verification not implemented)

Time = 18.76 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx = -\frac{\frac{487\,x^8}{648}+\frac{55\,x^6}{72}+\frac{491\,x^4}{405}-\frac{41\,x^2}{135}+\frac{4}{15}}{x^9+2\,x^7+3\,x^5} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{3418143-\sqrt{2}\,745707\mathrm{i}}\,248569\mathrm{i}}{306110016\,\left(\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447\mathrm{i}}{204073344}\right)}+\frac{248569\,\sqrt{2}\,x\,\sqrt{3418143-\sqrt{2}\,745707\mathrm{i}}}{612220032\,\left(\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447\mathrm{i}}{204073344}\right)}\right)\,\sqrt{3418143-\sqrt{2}\,745707\mathrm{i}}\,1\mathrm{i}}{3888} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{3418143+\sqrt{2}\,745707\mathrm{i}}\,248569\mathrm{i}}{306110016\,\left(-\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447\mathrm{i}}{204073344}\right)}-\frac{248569\,\sqrt{2}\,x\,\sqrt{3418143+\sqrt{2}\,745707\mathrm{i}}}{612220032\,\left(-\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447\mathrm{i}}{204073344}\right)}\right)\,\sqrt{3418143+\sqrt{2}\,745707\mathrm{i}}\,1\mathrm{i}}{3888}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(2*x^2 + x^4 + 3)^2),x)
```

output
```
(atan((x*(2^(1/2)*745707i + 3418143)^(1/2)*248569i)/(306110016*((2^(1/2)*1
15087447i)/204073344 - 119561689/51018336)) - (248569*2^(1/2)*x*(2^(1/2)*7
45707i + 3418143)^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 - 1195
61689/51018336)))*(2^(1/2)*745707i + 3418143)^(1/2)*1i)/3888 - (atan((x*(3
418143 - 2^(1/2)*745707i)^(1/2)*248569i)/(306110016*((2^(1/2)*115087447i)/
204073344 + 119561689/51018336)) + (248569*2^(1/2)*x*(3418143 - 2^(1/2)*74
5707i)^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 + 119561689/51018
336)))*(3418143 - 2^(1/2)*745707i)^(1/2)*1i)/3888 - ((491*x^4)/405 - (41*x
^2)/135 + (55*x^6)/72 + (487*x^8)/648 + 4/15)/(3*x^5 + 2*x^7 + x^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 825, normalized size of antiderivative = 4.12

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x)
```

output

```
(9620*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sq
rt(sqrt(3) + 1)*sqrt(2)))*x**9 + 19240*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqr
t(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**7 + 28860*sq
rt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(
3) + 1)*sqrt(2)))*x**5 - 14250*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3
) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**9 - 28500*sqrt(sqrt(
3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*
sqrt(2)))*x**7 - 42750*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*s
qrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**5 - 9620*sqrt(sqrt(3) + 1)*s
qrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))
*x**9 - 19240*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) +
2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**7 - 28860*sqrt(sqrt(3) + 1)*sqrt(6)*a
tan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**5 +
14250*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sq
rt(sqrt(3) + 1)*sqrt(2)))*x**9 + 28500*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqr
t(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**7 + 42750*sq
rt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(
3) + 1)*sqrt(2)))*x**5 - 4810*sqrt(sqrt(3) - 1)*sqrt(6)*log( - sqrt(sqrt(3
) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**9 - 9620*sqrt(sqrt(3) - 1)*sqrt(6)*l
og( - sqrt(sqrt(3) - 1)*sqrt(2)*x + sqrt(3) + x**2)*x**7 - 14430*sqrt(s...
```

3.118 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 202

$$\begin{aligned}\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx &= 58x-9x^3+x^5-\frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2}+\frac{x(3305+252x^2)}{64(3+2x^2+x^4)} \\ &+\frac{3}{256}\sqrt{-8595619+7678611\sqrt{3}}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}-2x}{\sqrt{2(1+\sqrt{3})}}\right) \\ &-\frac{3}{256}\sqrt{-8595619+7678611\sqrt{3}}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{2(1+\sqrt{3})}}\right) \\ &-\frac{3}{256}\sqrt{8595619+7678611\sqrt{3}}\operatorname{arctanh}\left(\frac{\sqrt{2(-1+\sqrt{3})}x}{\sqrt{3}+x^2}\right)\end{aligned}$$

output
```
58*x-9*x^3+x^5-25/16*x*(7*x^2+15)/(x^4+2*x^2+3)^2+x*(252*x^2+3305)/(64*x^4
+128*x^2+192)+3/256*(-8595619+7678611*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2)
)^(1/2)-2*x)/(2+2*3^(1/2))^(1/2))-3/256*(-8595619+7678611*3^(1/2))^(1/2)*a
rctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))-3/256*(8595619+76786
11*3^(1/2))^(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.77

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 58x-9x^3+x^5-\frac{25x(15+7x^2)}{16(3+2x^2+x^4)^2}+\frac{x(3305+252x^2)}{64(3+2x^2+x^4)} + \frac{3(4795i+148\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}} + \frac{3(-4795i+148\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(33
05 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*(4795*I + 148*Sqrt[2])*ArcTan[x
/Sqrt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (3*(-4795*I + 148*S
qrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^3}\,dx$$

$$\Big\downarrow \; 2197$$

$$\frac{1}{96}\int \frac{6(80x^{12}-112x^{10}+400x^6-800x^4-475x^2+375)}{(x^4+2x^2+3)^2}dx-\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\int \frac{80x^{12}-112x^{10}+400x^6-800x^4-475x^2+375}{(x^4+2x^2+3)^2}dx-\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{16}\left(\frac{1}{48}\int -\frac{12(-320x^8+1088x^6-1216x^4-2684x^2+2805)}{x^4+2x^2+3}dx+\frac{x(252x^2+3305)}{4(x^4+2x^2+3)}\right)-\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{x(252x^2+3305)}{4(x^4+2x^2+3)}-\frac{1}{4}\int \frac{-320x^8+1088x^6-1216x^4-2684x^2+2805}{x^4+2x^2+3}dx\right)-\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{16}\left(\frac{x(252x^2+3305)}{4(x^4+2x^2+3)}-\frac{1}{4}\int \left(-320x^4+1728x^2+\frac{3(4647-148x^2)}{x^4+2x^2+3}-3712\right)dx\right)-\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(\frac{1}{4}\left(\frac{3}{4}\sqrt{7678611\sqrt{3}-8595619}\arctan\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right)-\frac{3}{4}\sqrt{7678611\sqrt{3}-8595619}\arctan\left(\frac{2x+\sqrt{}}{\sqrt{2(}}\right.\right.\right.$$
$$\frac{25x(7x^2+15)}{16(x^4+2x^2+3)^2}$$

input
```
Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
(-25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + ((x*(3305 + 252*x^2))/(4*(
3 + 2*x^2 + x^4)) + (3712*x - 576*x^3 + 64*x^5 + (3*Sqrt[-8595619 + 767861
1*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4
 - (3*Sqrt[-8595619 + 7678611*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*
x)/Sqrt[2*(1 + Sqrt[3])]])/4 + (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt
[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/8 - (3*Sqrt[8595619 + 7678611*Sqrt[
3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/8)/4)/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.41

method	result
risch	$x^5-9x^3+58x+\frac{\frac{63}{16}x^7+\frac{3809}{64}x^5+\frac{3333}{32}x^3+\frac{8415}{64}x}{(x^4+2x^2+3)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(148\_R^2-4647)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$x^5-9x^3+58x+\frac{\frac{63}{16}x^7+\frac{3809}{64}x^5+\frac{3333}{32}x^3+\frac{8415}{64}x}{(x^4+2x^2+3)^2}+\frac{3\left(-1697\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-4795\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{1024}$

input
```
int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
x^5-9*x^3+58*x+(63/16*x^7+3809/64*x^5+3333/32*x^3+8415/64*x)/(x^4+2*x^2+3)
^2+3/256*sum((148*_R^2-4647)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(161) = 322.

Time = 0.08 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.70

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$
$$= \frac{512\,x^{13}-2560\,x^{11}+16384\,x^9+80864\,x^7+276744\,x^5+368208\,x^3-6\,(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{\cdots}}{\cdots}$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
1/512*(512*x^13 - 2560*x^11 + 16384*x^9 + 80864*x^7 + 276744*x^5 + 368208*
x^3 - 6*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(7678611*sqrt(3) - 8595619
)*arctan(1/14352598*sqrt(7678611*sqrt(3) + 8595619)*sqrt(7678611*sqrt(3) -
 8595619)*(sqrt(3) - 1) + 1/7176299*(1549*sqrt(3)*x + 148*x)*sqrt(7678611*
sqrt(3) - 8595619)) + 6*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(7678611*s
qrt(3) - 8595619)*arctan(1/14352598*sqrt(7678611*sqrt(3) + 8595619)*sqrt(7
678611*sqrt(3) - 8595619)*(sqrt(3) - 1) - 1/7176299*(1549*sqrt(3)*x + 148*
x)*sqrt(7678611*sqrt(3) - 8595619)) + 3*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9
)*sqrt(7678611*sqrt(3) + 8595619)*log(21528897*x^2 + 3*(1697*sqrt(3)*x - 4
795*x)*sqrt(7678611*sqrt(3) + 8595619) + 21528897*sqrt(3)) - 3*(x^8 + 4*x^
6 + 10*x^4 + 12*x^2 + 9)*sqrt(7678611*sqrt(3) + 8595619)*log(21528897*x^2
- 3*(1697*sqrt(3)*x - 4795*x)*sqrt(7678611*sqrt(3) + 8595619) + 21528897*s
qrt(3)) + 334584*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1204 vs. $2(182) = 364$.

Time = 0.95 (sec) , antiderivative size = 1204, normalized size of antiderivative = 5.96

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
x**5 - 9*x**3 + 58*x + (252*x**7 + 3809*x**5 + 6666*x**3 + 8415*x)/(64*x**
8 + 256*x**6 + 640*x**4 + 768*x**2 + 576) - 3*sqrt(8595619/262144 + 767861
1*sqrt(3)/262144)*log(x**2 + x*(-6788*sqrt(3)*sqrt(8595619 + 7678611*sqrt(
3))/7176299 - 2313785528*sqrt(8595619 + 7678611*sqrt(3))/18368002813563 +
1697*sqrt(2)*sqrt(8595619 + 7678611*sqrt(3))*sqrt(66002414605209*sqrt(3) +
 125383933330562)/18368002813563) - 121809524025246887927 9*sqrt(2)*sqrt(66
002414605209*sqrt(3) + 125383933330562)/101215058207717485241026490 7 - 134
353410196228*sqrt(6)*sqrt(66002414605209*sqrt(3) + 125383933330562)/395442
840668908030011 + 1839190299631186746380695988 9/101215058207717485241026 49
07 + 5204579286823805792980*sqrt(3)/395442840668908030011) + 3*sqrt(859561
9/262144 + 7678611*sqrt(3)/262144)*log(x**2 + x*(-1697*sqrt(2)*sqrt(859561
9 + 7678611*sqrt(3))*sqrt(66002414605209*sqrt(3) + 125383933330562)/183680
02813563 + 2313785528*sqrt(8595619 + 7678611*sqrt(3))/18368002813563 + 678
8*sqrt(3)*sqrt(8595619 + 7678611*sqrt(3))/7176299) - 121809524025246887927
9*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)/10121505820771748
52410264907 - 134353410196228*sqrt(6)*sqrt(66002414605209*sqrt(3) + 125383
933330562)/395442840668908030011 + 18391902996311867463806959889/101215058
2077174852410264907 + 5204579286823805792980*sqrt(3)/395442840668908030011
) - 2*sqrt(-9*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)/13107
2 + 77360571/262144 + 207322497*sqrt(3)/262144)*atan(110208016881378*x/...
```

Maxima [F]

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^{10}}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
x^5 - 9*x^3 + 58*x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 8415*x)/(x^8 +
4*x^6 + 10*x^4 + 12*x^2 + 9) + 3/64*integrate((148*x^2 - 4647)/(x^4 + 2*x^
2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(161) = 322$.

Time = 0.62 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.91

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
x^5 - 9*x^3 - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) +
 666*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqr
t(3) + 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 418
23*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2)
 + 666*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(s
qrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 4
1823*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3)
+ 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*
sqrt(3) + 1/2)) - 1/27648*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)
*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) +
 41823*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 41823*3^(1/4)*sqrt(6*sqrt(3
) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/276
48*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3
^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2)
 + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 41823*3^(1/4)*sqrt(2)*
sqrt(-6*sqrt(3) + 18) + 41823*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^
(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 58*x + 1/64*(252*x^7 + 38...
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.91

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 58\,x+\frac{\frac{63\,x^7}{16}+\frac{3809\,x^5}{64}+\frac{3333\,x^3}{32}+\frac{8415\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}-9\,x^3+x^5 - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{17191238-\sqrt{2}\,14352598\mathrm{i}}\,193760073\mathrm{i}}{131072\,\left(-\frac{986432531643}{131072}+\frac{\sqrt{2}\,9004030592 31\mathrm{i}}{131072}\right)}-\frac{193760073\,\sqrt{2}\,x\,\sqrt{17191238-\sqrt{2}\,14352598\mathrm{i}}}{262144\,\left(-\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231\mathrm{i}}{131072}\right)}\right)\,\sqrt{17191238-\sqrt{2}\,143525}}{256} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{17191238+\sqrt{2}\,14352598\mathrm{i}}\,193760073\mathrm{i}}{131072\,\left(\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231\mathrm{i}}{131072}\right)}+\frac{193760073\,\sqrt{2}\,x\,\sqrt{17191238+\sqrt{2}\,14352598\mathrm{i}}}{262144\,\left(\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231\mathrm{i}}{131072}\right)}\right)\,\sqrt{17191238+\sqrt{2}\,1435259}}{256}$$

input
```
int((x^10*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

output
```
58*x - (atan((x*(17191238 - 2^(1/2)*14352598i)^(1/2)*193760073i)/(131072*(
(2^(1/2)*900403059231i)/131072 - 986432531643/131072)) - (193760073*2^(1/2
)*x*(17191238 - 2^(1/2)*14352598i)^(1/2))/(262144*((2^(1/2)*900403059231i)
/131072 - 986432531643/131072)))*(17191238 - 2^(1/2)*14352598i)^(1/2)*3i)/
256 + (atan((x*(2^(1/2)*14352598i + 17191238)^(1/2)*193760073i)/(131072*((
2^(1/2)*900403059231i)/131072 + 986432531643/131072)) + (193760073*2^(1/2)
*x*(2^(1/2)*14352598i + 17191238)^(1/2))/(262144*((2^(1/2)*900403059231i)/
131072 + 986432531643/131072)))*(2^(1/2)*14352598i + 17191238)^(1/2)*3i)/2
56 + ((8415*x)/64 + (3333*x^3)/32 + (3809*x^5)/64 + (63*x^7)/16)/(12*x^2 +
 10*x^4 + 4*x^6 + x^8 + 9) - 9*x^3 + x^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1345, normalized size of antiderivative = 6.66

$$\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

output
```
( - 10182*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)
/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 - 40728*sqrt(sqrt(3) + 1)*sqrt(6)*atan(
(sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 - 1018
20*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(
sqrt(3) + 1)*sqrt(2)))*x**4 - 122184*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(
sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 91638*sqrt
(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3)
 + 1)*sqrt(2))) + 28770*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*
sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 + 115080*sqrt(sqrt(3) + 1
)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2
)))*x**6 + 287700*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2
) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 345240*sqrt(sqrt(3) + 1)*sqrt
(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x*
*2 + 258930*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*
x)/(sqrt(sqrt(3) + 1)*sqrt(2))) + 10182*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sq
rt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 + 40728*s
qrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt
(3) + 1)*sqrt(2)))*x**6 + 101820*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt
(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 122184*sqrt(sq
rt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3)...
```

3.119 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 201

$$\begin{aligned}
&\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx\\
&= -27x+\frac{5x^3}{3}+\frac{25x(3+5x^2)}{16\,(3+2x^2+x^4)^2}-\frac{x(1468+835x^2)}{64\,(3+2x^2+x^4)}\\
&\quad-\frac{21}{256}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{\sqrt{2\,(-1+\sqrt{3})}-2x}{\sqrt{2\,(1+\sqrt{3})}}\right)\\
&\quad+\frac{21}{256}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{2\,(1+\sqrt{3})}}\right)\\
&\quad+\frac{21}{256}\sqrt{-34271+22721\sqrt{3}}\,\mathrm{arctanh}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}x}{\sqrt{3}+x^2}\right)
\end{aligned}$$

output
```
-27*x+5/3*x^3+25/16*x*(5*x^2+3)/(x^4+2*x^2+3)^2-x*(835*x^2+1468)/(64*x^4+1
28*x^2+192)-21/256*(34271+22721*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2
)-2*x)/(2+2*3^(1/2))^(1/2))+21/256*(34271+22721*3^(1/2))^(1/2)*arctan(((-2
+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))+21/256*(-34271+22721*3^(1/2))^
(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.77

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = -27x+\frac{5x^3}{3}+\frac{25x(3+5x^2)}{16\,(3+2x^2+x^4)^2}-\frac{x(1468+835x^2)}{64\,(3+2x^2+x^4)} + \frac{21\left(-175i+137\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}} + \frac{21\left(175i+137\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{128\sqrt{2+2i\sqrt{2}}}$$

input
```
Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
-27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468
 + 835*x^2))/(64*(3 + 2*x^2 + x^4)) + (21*(-175*I + 137*Sqrt[2])*ArcTan[x/
Sqrt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (21*(175*I + 137*Sqr
t[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(5x^6+3x^4+x^2+4\right)}{\left(x^4+2x^2+3\right)^3}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{96}\int -\frac{6\left(-80x^{10}+112x^8-400x^4+175x^2+75\right)}{\left(x^4+2x^2+3\right)^2}dx+\frac{25x\left(5x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{25x(5x^2+3)}{16(x^4+2x^2+3)^2}-\frac{1}{16}\int \frac{-80x^{10}+112x^8-400x^4+175x^2+75}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{16}\left(-\frac{1}{48}\int -\frac{12(320x^6-1088x^4+381x^2+1368)}{x^4+2x^2+3}dx-\frac{x(835x^2+1468)}{4(x^4+2x^2+3)}\right)+\frac{25x(5x^2+3)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{4}\int \frac{320x^6-1088x^4+381x^2+1368}{x^4+2x^2+3}dx-\frac{x(835x^2+1468)}{4(x^4+2x^2+3)}\right)+\frac{25x(5x^2+3)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{16}\left(\frac{1}{4}\int \left(320x^2+\frac{21(137x^2+312)}{x^4+2x^2+3}-1728\right)dx-\frac{x(835x^2+1468)}{4(x^4+2x^2+3)}\right)+\frac{25x(5x^2+3)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(\frac{1}{4}\left(-\frac{21}{4}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right)+\frac{21}{4}\sqrt{34271+22721\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2(\sqrt{}}}{\sqrt{2(1+}}\right.\right.\right.$$
$$\frac{25x(5x^2+3)}{16(x^4+2x^2+3)^2}$$

input
```
Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
(25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (-1/4*(x*(1468 + 835*x^2))/(
3 + 2*x^2 + x^4) + (-1728*x + (320*x^3)/3 - (21*Sqrt[34271 + 22721*Sqrt[3]
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4 + (21*Sq
rt[34271 + 22721*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1
+ Sqrt[3])]])/4 - (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-
1 + Sqrt[3])]*x + x^2])/8 + (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] +
 Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/8)/4)/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.39

method	result
risch	$\frac{5x^3}{3}-27x+\frac{-\frac{835}{64}x^7-\frac{1569}{32}x^5-\frac{4941}{64}x^3-\frac{513}{8}x}{(x^4+2x^2+3)^2}+\frac{21\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(137\_R^2+312)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$\frac{5x^3}{3}-27x+\frac{-\frac{835}{64}x^7-\frac{1569}{32}x^5-\frac{4941}{64}x^3-\frac{513}{8}x}{(x^4+2x^2+3)^2}+\frac{21\left(-33\sqrt{-2+2\sqrt{3}}\sqrt{3}+175\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{1024}+-$

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
5/3*x^3-27*x+(-835/64*x^7-1569/32*x^5-4941/64*x^3-513/8*x)/(x^4+2*x^2+3)^2
+21/256*sum((137*_R^2+312)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(158) = 316.

Time = 0.08 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.58

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$
$$= \frac{2560\,x^{11}-31232\,x^9-160328\,x^7-459312\,x^5-593208\,x^3+126\,(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{22721}}{}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
1/1536*(2560*x^11 - 31232*x^9 - 160328*x^7 - 459312*x^5 - 593208*x^3 + 126
*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(22721*sqrt(3) + 34271)*arctan(1/
27358*(208*sqrt(3)*x + sqrt(22721*sqrt(3) - 34271)*(sqrt(3) - 1) - 274*x)*
sqrt(22721*sqrt(3) + 34271)) - 126*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqr
t(22721*sqrt(3) + 34271)*arctan(-1/27358*(208*sqrt(3)*x - sqrt(22721*sqrt(
3) - 34271)*(sqrt(3) - 1) - 274*x)*sqrt(22721*sqrt(3) + 34271)) + 63*(x^8
+ 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(22721*sqrt(3) - 34271)*log(287259*x^2
+ 21*(33*sqrt(3)*x + 175*x)*sqrt(22721*sqrt(3) - 34271) + 287259*sqrt(3))
- 63*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(22721*sqrt(3) - 34271)*log(2
87259*x^2 - 21*(33*sqrt(3)*x + 175*x)*sqrt(22721*sqrt(3) - 34271) + 287259
*sqrt(3)) - 471744*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.41

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{5x^3}{3} - 27x + \frac{-835x^7-3138x^5-4941x^3-4104x}{64x^8+256x^6+640x^4+768x^2+576} + 21\,\mathrm{RootSum}\left(17179869184t^4+8983937024t^2+1548731523,\ \left(t\mapsto t\log\left(-\frac{1107296256t^3}{310800559}+\frac{4388579}{310800}\right.\right.\right.$$

input
```
integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
5*x**3/3 - 27*x + (-835*x**7 - 3138*x**5 - 4941*x**3 - 4104*x)/(64*x**8 +
256*x**6 + 640*x**4 + 768*x**2 + 576) + 21*RootSum(17179869184*_t**4 + 898
3937024*_t**2 + 1548731523, Lambda(_t, _t*log(-1107296256*_t**3/310800559
+ 438857984*_t/310800559 + x)))
```

Maxima [F]

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^8}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
5/3*x^3 - 27*x - 1/64*(835*x^7 + 3138*x^5 + 4941*x^3 + 4104*x)/(x^8 + 4*x^
6 + 10*x^4 + 12*x^2 + 9) + 21/64*integrate((137*x^2 + 312)/(x^4 + 2*x^2 +
3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(158) = 316$.

Time = 0.56 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.91

$$\int \frac{x^8(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
5/3*x^3 - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 24
66*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 112
32*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2
) + 2466*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)
*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2)
 - 11232*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt
(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(
1/6*sqrt(3) + 1/2)) - 7/110592*sqrt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)
*sqrt(-6*sqrt(3) + 18) - 137*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137
*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 2466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(
3) - 3) - 11232*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt
(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)
) + 7/110592*sqrt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) +
 18) - 137*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137*3^(3/4)*(6*sqrt(3
) + 18)^(3/2) + 2466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11232*3^
(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt(6*sqrt(3) + 18))*
log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 27*x - 1/64...
```

Mupad [B] (verification not implemented)

Time = 18.77 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{5\,x^3}{3} - \frac{\frac{835\,x^7}{64}+\frac{1569\,x^5}{32}+\frac{4941\,x^3}{64}+\frac{513\,x}{8}}{x^8+4\,x^6+10\,x^4+12\,x^2+9} - 27\,x + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-68542-\sqrt{2}\,27358\mathrm{i}}\,126681219\mathrm{i}}{131072\left(\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)} - \frac{126681219\,\sqrt{2}\,x\,\sqrt{-68542-\sqrt{2}\,27358\mathrm{i}}}{262144\left(\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)}\right)\sqrt{-68542-\sqrt{2}\,27358\mathrm{i}}\,21\mathrm{i}}{256} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-68542+\sqrt{2}\,27358\mathrm{i}}\,126681219\mathrm{i}}{131072\left(-\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)} + \frac{126681219\,\sqrt{2}\,x\,\sqrt{-68542+\sqrt{2}\,27358\mathrm{i}}}{262144\left(-\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)}\right)\sqrt{-68542+\sqrt{2}\,27358\mathrm{i}}\,21\mathrm{i}}{256}$$

input
```
int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

output
```
(atan((x*(- 2^(1/2)*27358i - 68542)^(1/2)*126681219i)/(131072*((2^(1/2)*49
40567541i)/16384 + 12541440681/131072)) - (126681219*2^(1/2)*x*(- 2^(1/2)*
27358i - 68542)^(1/2))/(262144*((2^(1/2)*4940567541i)/16384 + 12541440681/
131072)))*(- 2^(1/2)*27358i - 68542)^(1/2)*21i)/256 - ((513*x)/8 + (4941*x
^3)/64 + (1569*x^5)/32 + (835*x^7)/64)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9)
 - 27*x - (atan((x*(2^(1/2)*27358i - 68542)^(1/2)*126681219i)/(131072*((2^
(1/2)*4940567541i)/16384 - 12541440681/131072)) + (126681219*2^(1/2)*x*(2^
(1/2)*27358i - 68542)^(1/2))/(262144*((2^(1/2)*4940567541i)/16384 - 125414
40681/131072)))*(2^(1/2)*27358i - 68542)^(1/2)*21i)/256 + (5*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1340, normalized size of antiderivative = 6.67

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

output
```
( - 4158*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/
(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 - 16632*sqrt(sqrt(3) + 1)*sqrt(6)*atan((
sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 - 41580
*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sq
rt(3) + 1)*sqrt(2)))*x**4 - 49896*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqr
t(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 37422*sqrt(sq
rt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) +
1)*sqrt(2))) - 22050*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqr
t(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 - 88200*sqrt(sqrt(3) + 1)*sq
rt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*
x**6 - 220500*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) -
2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 264600*sqrt(sqrt(3) + 1)*sqrt(2)*
atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 -
 198450*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(
sqrt(sqrt(3) + 1)*sqrt(2))) + 4158*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sq
rt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 + 16632*sqrt(s
qrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) +
 1)*sqrt(2)))*x**6 + 41580*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) -
1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 49896*sqrt(sqrt(3) +
 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*s...
```

3.120 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 194

$$\begin{aligned}
\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx &= 5x+\frac{25x(3-x^2)}{16\,(3+2x^2+x^4)^2}+\frac{7x(11+58x^2)}{64\,(3+2x^2+x^4)} \\
&\quad+\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{\sqrt{2\,(-1+\sqrt{3})}-2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&\quad-\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{\sqrt{2\,(-1+\sqrt{3})}+2x}{\sqrt{2\,(1+\sqrt{3})}}\right) \\
&\quad+\frac{1}{256}\sqrt{-827621+1176531\sqrt{3}}\,\text{arctanh}\left(\frac{\sqrt{2\,(-1+\sqrt{3})}x}{\sqrt{3}+x^2}\right)
\end{aligned}$$

output
```
5*x+25/16*x*(-x^2+3)/(x^4+2*x^2+3)^2+7*x*(58*x^2+11)/(64*x^4+128*x^2+192)+
1/256*(827621+1176531*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+
2*3^(1/2))^(1/2))-1/256*(827621+1176531*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/
2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))+1/256*(-827621+1176531*3^(1/2))^(1/2)*
arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.71

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{1}{256}\left(\frac{4x(3411+5112x^2+4089x^4+1686x^6+320x^8)}{(3+2x^2+x^4)^2} - \frac{i(-2644i+185\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{i(2644i+185\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
((4*x*(3411 + 5112*x^2 + 4089*x^4 + 1686*x^6 + 320*x^8))/(3 + 2*x^2 + x^4)
^2 - (I*(-2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*
Sqrt[2]] + (I*(2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1
 + I*Sqrt[2]])/256
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2197, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^3}dx$$

$$\Big\downarrow \text{ 2197}$$

$$\frac{1}{96}\int -\frac{6(-80x^8+112x^6-275x^2+75)}{(x^4+2x^2+3)^2}dx + \frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}-\frac{1}{16}\int \frac{-80x^8+112x^6-275x^2+75}{(x^4+2x^2+3)^2}dx$$

$$\Big\downarrow \; 2206$$

$$\frac{1}{16}\left(\frac{7x(58x^2+11)}{4(x^4+2x^2+3)}-\frac{1}{48}\int \frac{12(-320x^4+682x^2+177)}{x^4+2x^2+3}dx\right)+\frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\left(\frac{7x(58x^2+11)}{4(x^4+2x^2+3)}-\frac{1}{4}\int \frac{-320x^4+682x^2+177}{x^4+2x^2+3}dx\right)+\frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \; 2205$$

$$\frac{1}{16}\left(\frac{7x(58x^2+11)}{4(x^4+2x^2+3)}-\frac{1}{4}\int \left(\frac{1322x^2+1137}{x^4+2x^2+3}-320\right)dx\right)+\frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{4}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{\sqrt{2(\sqrt{3}-1)}-2x}{\sqrt{2(1+\sqrt{3})}}\right)-\frac{1}{4}\sqrt{827621+1176531\sqrt{3}}\arctan\left(\frac{2x+\sqrt{2}}{\sqrt{2(1}}\right.\right.\right.$$

$$\frac{25x(3-x^2)}{16(x^4+2x^2+3)^2}$$

input
```
Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
(25*x*(3 - x^2))/(16*(3 + 2*x^2 + x^4)^2) + ((7*x*(11 + 58*x^2))/(4*(3 + 2
*x^2 + x^4)) + (320*x + (Sqrt[827621 + 1176531*Sqrt[3]]*ArcTan[(Sqrt[2*(-1
 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4 - (Sqrt[827621 + 1176531*Sqr
t[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4 - (S
qrt[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^
2])/8 + (Sqrt[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/8)/4)/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.38

method	result
risch	$5x+\frac{\frac{203}{32}x^7+\frac{889}{64}x^5+\frac{159}{8}x^3+\frac{531}{64}x}{(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{\left(-1322\_R^2-1137\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{256}$
default	$5x-\frac{-\frac{203}{32}x^7-\frac{889}{64}x^5-\frac{159}{8}x^3-\frac{531}{64}x}{(x^4+2x^2+3)^2}-\frac{\left(-943\sqrt{-2+2\sqrt{3}}\sqrt{3}-185\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{1024}-\frac{\left(1516\sqrt{3}-\frac{(-}{}\right.}{}$

input
```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
5*x+(203/32*x^7+889/64*x^5+159/8*x^3+531/64*x)/(x^4+2*x^2+3)^2+1/256*sum((
-1322*_R^2-1137)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(151) = 302.

Time = 0.08 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.60

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$
$$=\frac{2560\,x^9+13488\,x^7+32712\,x^5+40896\,x^3+2\left(x^8+4\,x^6+10\,x^4+12\,x^2+9\right)\sqrt{1176531\,\sqrt{3}+827621}\,\mathrm{ar}}{}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
1/512*(2560*x^9 + 13488*x^7 + 32712*x^5 + 40896*x^3 + 2*(x^8 + 4*x^6 + 10*
x^4 + 12*x^2 + 9)*sqrt(1176531*sqrt(3) + 827621)*arctan(1/2633522*(758*sqr
t(3)*x + sqrt(1176531*sqrt(3) - 827621)*(sqrt(3) - 1) - 2644*x)*sqrt(11765
31*sqrt(3) + 827621)) - 2*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(1176531
*sqrt(3) + 827621)*arctan(-1/2633522*(758*sqrt(3)*x - sqrt(1176531*sqrt(3)
 - 827621)*(sqrt(3) - 1) - 2644*x)*sqrt(1176531*sqrt(3) + 827621)) + (x^8
+ 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(1176531*sqrt(3) - 827621)*log(1316761*
x^2 + (943*sqrt(3)*x - 185*x)*sqrt(1176531*sqrt(3) - 827621) + 1316761*sqr
t(3)) - (x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(1176531*sqrt(3) - 827621)
*log(1316761*x^2 - (943*sqrt(3)*x - 185*x)*sqrt(1176531*sqrt(3) - 827621)
+ 1316761*sqrt(3)) + 27288*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.37

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 5x + \frac{406x^7+889x^5+1272x^3+531x}{64x^8+256x^6+640x^4+768x^2+576} + \text{RootSum}\left(17179869184t^4+216955879424t^2+4152675581883, \left(t\mapsto t\log\left(-\frac{31641829376t^3}{1549210136091} - \frac{455}{154}\right.\right.\right.$$

input
```
integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
5*x + (406*x**7 + 889*x**5 + 1272*x**3 + 531*x)/(64*x**8 + 256*x**6 + 640*
x**4 + 768*x**2 + 576) + RootSum(17179869184*_t**4 + 216955879424*_t**2 +
4152675581883, Lambda(_t, _t*log(-31641829376*_t**3/1549210136091 - 455309
168896*_t/1549210136091 + x)))
```

Maxima [F]

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^6}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
5*x + 1/64*(406*x^7 + 889*x^5 + 1272*x^3 + 531*x)/(x^8 + 4*x^6 + 10*x^4 +
12*x^2 + 9) - 1/64*integrate((1322*x^2 + 1137)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(151) = 302$.

Time = 0.59 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.99

$$\int \frac{x^6(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt(3) + 3)
*sqrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 20466*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arc
tan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
1/2)) + 1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1189
8*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 204
66*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) +
18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sq
rt(3) + 1/2)) + 1/165888*sqrt(2)*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) - 661*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3
/4)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) -
 3) - 20466*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 20466*3^(1/4)*sqrt(6*s
qrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) -
1/165888*sqrt(2)*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18
) - 661*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3/4)*(6*sqrt(3) +
 18)^(3/2) + 11898*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 20466*3^(1
/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 20466*3^(1/4)*sqrt(6*sqrt(3) + 18))*lo
g(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 5*x + 1/64*(4...
```

Mupad [B] (verification not implemented)

Time = 18.83 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.91

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = 5\,x+\frac{\frac{203\,x^7}{32}+\frac{889\,x^5}{64}+\frac{159\,x^3}{8}+\frac{531\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$
$$+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}\,1316761\mathrm{i}}{131072\,\left(-\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}+\frac{1316761\,\sqrt{2}\,x\,\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}}{262144\,\left(-\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}\right)\,\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}\,1\mathrm{i}}{256}$$
$$-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}\,1316761\mathrm{i}}{131072\,\left(\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}-\frac{1316761\,\sqrt{2}\,x\,\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}}{262144\,\left(\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}\right)\,\sqrt{-1655242+\sqrt{2}\,2633522\mathrm{i}}\,1\mathrm{i}}{256}$$

input
```
int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

output
```
5*x + (atan((x*(- 2^(1/2)*2633522i - 1655242)^(1/2)*1316761i)/(131072*((2^
(1/2)*1497157257i)/131072 - 3725116869/131072)) + (1316761*2^(1/2)*x*(- 2^
(1/2)*2633522i - 1655242)^(1/2))/(262144*((2^(1/2)*1497157257i)/131072 - 3
725116869/131072)))*(- 2^(1/2)*2633522i - 1655242)^(1/2)*1i)/256 - (atan((
x*(2^(1/2)*2633522i - 1655242)^(1/2)*1316761i)/(131072*((2^(1/2)*149715725
7i)/131072 + 3725116869/131072)) - (1316761*2^(1/2)*x*(2^(1/2)*2633522i -
1655242)^(1/2))/(262144*((2^(1/2)*1497157257i)/131072 + 3725116869/131072)
))*(2^(1/2)*2633522i - 1655242)^(1/2)*1i)/256 + ((531*x)/64 + (159*x^3)/8
+ (889*x^5)/64 + (203*x^7)/32)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1335, normalized size of antiderivative = 6.88

$$\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

output
```
(1886*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sq
rt(sqrt(3) + 1)*sqrt(2)))*x**8 + 7544*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt
(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 + 18860*sqr
t(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3
) + 1)*sqrt(2)))*x**4 + 22632*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3)
 - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 16974*sqrt(sqrt(3
) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*s
qrt(2))) - 370*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) -
 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 - 1480*sqrt(sqrt(3) + 1)*sqrt(2)*a
tan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 -
3700*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqr
t(sqrt(3) + 1)*sqrt(2)))*x**4 - 4440*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(
sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 3330*sqrt(
sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3)
+ 1)*sqrt(2))) - 1886*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sq
rt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 - 7544*sqrt(sqrt(3) + 1)*sq
rt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*
x**6 - 18860*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2
*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 22632*sqrt(sqrt(3) + 1)*sqrt(6)*at
an((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 ...
```

3.121 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 195

$$\begin{aligned}
\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} \, dx
&= -\frac{25x(3+x^2)}{16\,(3+2x^2+x^4)^2} + \frac{x(238-59x^2)}{64\,(3+2x^2+x^4)} \\
&\quad - \frac{1}{256}\sqrt{3\left(-48835+32827\sqrt{3}\right)} \arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad + \frac{1}{256}\sqrt{3\left(-48835+32827\sqrt{3}\right)} \arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad - \frac{1}{256}\sqrt{3\left(48835+32827\sqrt{3}\right)} \operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)
\end{aligned}$$

output
```
-25/16*x*(x^2+3)/(x^4+2*x^2+3)^2+x*(-59*x^2+238)/(64*x^4+128*x^2+192)-1/25
6*(-146505+98481*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(
1/2))^(1/2))+1/256*(-146505+98481*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1
/2)+2*x)/(2+2*3^(1/2))^(1/2))-1/256*(146505+98481*3^(1/2))^(1/2)*arctanh((
-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.66

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{1}{256}\left(\frac{4x(414+199x^2+120x^4-59x^6)}{(3+2x^2+x^4)^2} + \frac{3(174+133i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{3(174-133i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
((4*x*(414 + 199*x^2 + 120*x^4 - 59*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(174 +
(133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(
174 - (133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])
/256
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.50, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2197, 27, 2206, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^3}\,dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{96}\int \frac{6(80x^6-112x^4-125x^2+75)}{(x^4+2x^2+3)^2}dx - \frac{25x(x^2+3)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\int \frac{80x^6-112x^4-125x^2+75}{\left(x^4+2x^2+3\right)^2}dx-\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 2206$$

$$\frac{1}{16}\left(\frac{1}{48}\int -\frac{36\left(46-87x^2\right)}{x^4+2x^2+3}dx+\frac{x\left(238-59x^2\right)}{4\left(x^4+2x^2+3\right)}\right)-\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\left(\frac{x\left(238-59x^2\right)}{4\left(x^4+2x^2+3\right)}-\frac{3}{4}\int \frac{46-87x^2}{x^4+2x^2+3}dx\right)-\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 1483$$

$$\frac{1}{16}\left(\frac{x\left(238-59x^2\right)}{4\left(x^4+2x^2+3\right)}-\frac{3}{4}\left(\frac{\int \frac{46\sqrt{2\left(-1+\sqrt{3}\right)}-\left(46+87\sqrt{3}\right)x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\int \frac{\left(46+87\sqrt{3}\right)x+46\sqrt{2\left(-1+\sqrt{3}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right)\right)-$$

$$\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{16}\left(\frac{x\left(238-59x^2\right)}{4\left(x^4+2x^2+3\right)}-\frac{3}{4}\left(\frac{-\frac{1}{2}\sqrt{65654\sqrt{3}-97670}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\left(46+87\sqrt{3}\right)\int -\frac{\sqrt{2\left(-1+\sqrt{3}\right)}}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}}}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{16}\left(\frac{x\left(238-59x^2\right)}{4\left(x^4+2x^2+3\right)}-\frac{3}{4}\left(\frac{\frac{1}{2}\left(46+87\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\sqrt{65654\sqrt{3}-97670}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{\ldots}}}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{16}\left(\frac{x\left(238-59x^2\right)}{4\left(x^4+2x^2+3\right)}-\frac{3}{4}\left(\frac{\frac{1}{2}\left(46+87\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx+\sqrt{65654\sqrt{3}-97670}\int \frac{1}{-\left(2x-\sqrt{2\left(-1+\sqrt{3}\right)}\right)\ldots}}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \ 217$$

$$\frac{1}{16}\left(\frac{x\left(238-59x^2\right)}{4\left(x^4+2x^2+3\right)}-\frac{3}{4}\left(\frac{\frac{1}{2}\left(46+87\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\sqrt{\frac{65654\sqrt{3}-97670}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{16}\left(\frac{x\left(238-59x^2\right)}{4\left(x^4+2x^2+3\right)}-\frac{3}{4}\left(\frac{-\sqrt{\frac{65654\sqrt{3}-97670}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}\left(46+87\sqrt{3}\right)\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\ldots\right.}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}$$

input
```
Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
(-25*x*(3 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + ((x*(238 - 59*x^2))/(4*(3 + 2
*x^2 + x^4)) - (3*((-(Sqrt[(-97670 + 65654*Sqrt[3])/(2*(1 + Sqrt[3]))]*Arc
Tan[(-Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]]) - ((46 + 87*Sq
rt[3])*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + S
qrt[3])]) + (-(Sqrt[(-97670 + 65654*Sqrt[3])/(2*(1 + Sqrt[3]))]*ArcTan[(Sq
rt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]]) + ((46 + 87*Sqrt[3])*L
og[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + Sqrt[3])]
)))/4)/16
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.36

method	result
risch	$\frac{-\frac{59}{64}x^7+\frac{15}{8}x^5+\frac{199}{64}x^3+\frac{207}{32}x}{(x^4+2x^2+3)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(87\_R^2-46)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$\frac{-\frac{59}{64}x^7+\frac{15}{8}x^5+\frac{199}{64}x^3+\frac{207}{32}x}{(x^4+2x^2+3)^2}+\frac{\left(-307\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-399\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{1024}+\frac{\left(-184\sqrt{3}-\frac{(-307\sqrt{-2+}}{}\right.}{}$

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(-59/64*x^7+15/8*x^5+199/64*x^3+207/32*x)/(x^4+2*x^2+3)^2+3/256*sum((87*_R
^2-46)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(148) = 296.

Time = 0.08 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.68

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx =$$
$$-\frac{472\,x^7-960\,x^5-1592\,x^3-2\,(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{98481\,\sqrt{3}-146505}\arctan\left(\frac{1}{123546}\,\sqrt{9\right.}{}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
-1/512*(472*x^7 - 960*x^5 - 1592*x^3 - 2*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
9)*sqrt(98481*sqrt(3) - 146505)*arctan(1/123546*sqrt(98481*sqrt(3) + 14650
5)*sqrt(98481*sqrt(3) - 146505)*(sqrt(3) - 1) + 1/61773*(46*sqrt(3)*x + 26
1*x)*sqrt(98481*sqrt(3) - 146505)) + 2*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
*sqrt(98481*sqrt(3) - 146505)*arctan(1/123546*sqrt(98481*sqrt(3) + 146505)
*sqrt(98481*sqrt(3) - 146505)*(sqrt(3) - 1) - 1/61773*(46*sqrt(3)*x + 261*
x)*sqrt(98481*sqrt(3) - 146505)) + (x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqr
t(98481*sqrt(3) + 146505)*log(61773*x^2 + (307*sqrt(3)*x - 399*x)*sqrt(984
81*sqrt(3) + 146505) + 61773*sqrt(3)) - (x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9
)*sqrt(98481*sqrt(3) + 146505)*log(61773*x^2 - (307*sqrt(3)*x - 399*x)*sqr
t(98481*sqrt(3) + 146505) + 61773*sqrt(3)) - 3312*x)/(x^8 + 4*x^6 + 10*x^4
 + 12*x^2 + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. $2(163) = 326$.

Time = 0.95 (sec) , antiderivative size = 1198, normalized size of antiderivative = 6.14

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
(-59*x**7 + 120*x**5 + 199*x**3 + 414*x)/(64*x**8 + 256*x**6 + 640*x**4 +
768*x**2 + 576) - sqrt(146505/262144 + 98481*sqrt(3)/262144)*log(x**2 + x*
(-307*sqrt(6)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 280884
6506)/675940757 + 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 +
 1228*sqrt(48835 + 32827*sqrt(3))/20591) - 941929306825573*sqrt(2)*sqrt(16
03106545*sqrt(3) + 2808846506)/456895906973733049 - 47771215762*sqrt(6)*sq
rt(1603106545*sqrt(3) + 2808846506)/41754888382161 + 97477949666790882353/
456895906973733049 + 5200450130596150*sqrt(3)/41754888382161) + sqrt(14650
5/262144 + 98481*sqrt(3)/262144)*log(x**2 + x*(-1228*sqrt(48835 + 32827*sq
rt(3))/20591 - 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 + 30
7*sqrt(6)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 2808846506
)/675940757) - 941929306825573*sqrt(2)*sqrt(1603106545*sqrt(3) + 280884650
6)/456895906973733049 - 47771215762*sqrt(6)*sqrt(1603106545*sqrt(3) + 2808
846506)/41754888382161 + 97477949666790882353/456895906973733049 + 5200450
130596150*sqrt(3)/41754888382161) + 2*sqrt(-3*sqrt(2)*sqrt(1603106545*sqrt
(3) + 2808846506)/131072 + 146505/262144 + 295443*sqrt(3)/262144)*atan(135
1881514*sqrt(3)*x/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808
846506) + 48835 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2
808846506)*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 +
 98481*sqrt(3))) - 40311556*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/(-18943...
```

Maxima [F]

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^4}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
-1/64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2
+ 9) + 3/64*integrate((87*x^2 - 46)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(148) = 296$.

Time = 0.61 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.96

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
-1/18432*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt
(2)*sqrt(6*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3
^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1
/18432*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(
3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/3
6864*sqrt(2)*(522*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 29
*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 29*3^(3/4)*(6*sqrt(3) + 18)^(3/
2) + 522*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 552*3^(1/4)*sqrt(2)*
sqrt(-6*sqrt(3) + 18) + 552*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1
/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/36864*sqrt(2)*(522*3^(3/4)*s
qrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 29*3^(3/4)*sqrt(2)*(-6*sqrt(3
) + 18)^(3/2) + 29*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(6*sqr
t(3) + 18)*(sqrt(3) - 3) + 552*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 552
*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1
/2) + sqrt(3)) - 1/64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^4 + 2*x^2...
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.89

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{-\frac{59\,x^7}{64}+\frac{15\,x^5}{8}+\frac{199\,x^3}{64}+\frac{207\,x}{32}}{x^8+4\,x^6+10\,x^4+12\,x^2+9} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}\,61773\mathrm{i}}{131072\,\left(\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}+\frac{61773\,\sqrt{2}\,x\,\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}}{262144\,\left(\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}\right)\,\sqrt{293010-\sqrt{2}\,123546\mathrm{i}}\,1\mathrm{i}}{256} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}\,61773\mathrm{i}}{131072\,\left(-\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}-\frac{61773\,\sqrt{2}\,x\,\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}}{262144\,\left(-\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337\mathrm{i}}{65536}\right)}\right)\,\sqrt{293010+\sqrt{2}\,123546\mathrm{i}}\,1\mathrm{i}}{256}$$

input
```
int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

output
```
((207*x)/32 + (199*x^3)/64 + (15*x^5)/8 - (59*x^7)/64)/(12*x^2 + 10*x^4 +
4*x^6 + x^8 + 9) + (atan((x*(293010 - 2^(1/2)*123546i)^(1/2)*61773i)/(1310
72*((2^(1/2)*4262337i)/65536 + 56892933/131072)) + (61773*2^(1/2)*x*(29301
0 - 2^(1/2)*123546i)^(1/2))/(262144*((2^(1/2)*4262337i)/65536 + 56892933/1
31072)))*(293010 - 2^(1/2)*123546i)^(1/2)*1i)/256 - (atan((x*(2^(1/2)*1235
46i + 293010)^(1/2)*61773i)/(131072*((2^(1/2)*4262337i)/65536 - 56892933/1
31072)) - (61773*2^(1/2)*x*(2^(1/2)*123546i + 293010)^(1/2))/(262144*((2^(
1/2)*4262337i)/65536 - 56892933/131072)))*(2^(1/2)*123546i + 293010)^(1/2)
*1i)/256
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1330, normalized size of antiderivative = 6.82

$$\int \frac{x^4(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

output
```
( - 614*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(
sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 - 2456*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sq
rt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 - 6140*sq
rt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(
3) + 1)*sqrt(2)))*x**4 - 7368*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3)
 - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 5526*sqrt(sqrt(3)
 + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sq
rt(2))) + 798*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) -
2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 + 3192*sqrt(sqrt(3) + 1)*sqrt(2)*at
an((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 + 7
980*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt
(sqrt(3) + 1)*sqrt(2)))*x**4 + 9576*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(s
qrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 7182*sqrt(s
qrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) +
 1)*sqrt(2))) + 614*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt
(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 + 2456*sqrt(sqrt(3) + 1)*sqrt
(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x*
*6 + 6140*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)
/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 + 7368*sqrt(sqrt(3) + 1)*sqrt(6)*atan((
sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 55...
```

3.122 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 201

$$\begin{aligned}
\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx &= \frac{25x(1+x^2)}{16\,(3+2x^2+x^4)^2}-\frac{x(353+88x^2)}{192\,(3+2x^2+x^4)} \\
&\quad -\frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad +\frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad +\frac{11}{768}\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)
\end{aligned}$$

output
```
25/16*x*(x^2+1)/(x^4+2*x^2+3)^2-x*(88*x^2+353)/(192*x^4+384*x^2+576)-11/23
04*(-5475+3267*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/
2))^(1/2))+11/2304*(-5475+3267*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)
+2*x)/(2+2*3^(1/2))^(1/2))+11/2304*(5475+3267*3^(1/2))^(1/2)*arctanh((-2+2
*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \frac{1}{768}\left(-\frac{4x(759+670x^2+529x^4+88x^6)}{(3+2x^2+x^4)^2} - \frac{11i(-16i+31\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{11i(16i+31\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
((-4*x*(759 + 670*x^2 + 529*x^4 + 88*x^6))/(3 + 2*x^2 + x^4)^2 - ((11*I)*(
-16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (
(11*I)*(16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[
2]])/768
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.44, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2197, 27, 2206, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(5x^6+3x^4+x^2+4)}{(x^4+2x^2+3)^3}dx$$

$$\Big\downarrow \text{2197}$$

$$\frac{1}{96}\int -\frac{6(-80x^4-13x^2+25)}{(x^4+2x^2+3)^2}dx + \frac{25x(x^2+1)}{16(x^4+2x^2+3)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}-\frac{1}{16}\int\frac{-80x^4-13x^2+25}{\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \; 2206$$

$$\frac{1}{16}\left(-\frac{1}{48}\int -\frac{44\left(23-8x^2\right)}{x^4+2x^2+3}dx-\frac{x\left(88x^2+353\right)}{12\left(x^4+2x^2+3\right)}\right)+\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\left(\frac{11}{12}\int\frac{23-8x^2}{x^4+2x^2+3}dx-\frac{x\left(88x^2+353\right)}{12\left(x^4+2x^2+3\right)}\right)+\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 1483$$

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\int\frac{23\sqrt{2\left(-1+\sqrt{3}\right)}-\left(23+8\sqrt{3}\right)x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\int\frac{\left(23+8\sqrt{3}\right)x+23\sqrt{2\left(-1+\sqrt{3}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right)-\frac{x\left(88x^2+353\right)}{12\left(x^4+2x^2+3\right)}\right)+$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\frac{1}{2}\sqrt{2178\sqrt{3}-3650}\int\frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\left(23+8\sqrt{3}\right)\int-\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}\sqrt{2178\sqrt{3}}}{}\right.\right.$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\frac{1}{2}\sqrt{2178\sqrt{3}-3650}\int\frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx+\frac{1}{2}\left(23+8\sqrt{3}\right)\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}\sqrt{2178\sqrt{3}-\cdots}}{\cdots}\right.\right.$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 1083

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\frac{1}{2}\left(23+8\sqrt{3}\right)\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\sqrt{2178\sqrt{3}-3650}\int\frac{1}{-\left(2x-\sqrt{2\left(-1+\sqrt{3}\right)}\right)^2-2\left(1+\sqrt{3}\right)}d\left(2x-\sqrt{2\cdots}\right.}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 217

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\frac{1}{2}\left(23+8\sqrt{3}\right)\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx+\sqrt{\frac{2178\sqrt{3}-3650}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\frac{1}{2}\left(23+8\sqrt{3}\right)\int\frac{2\cdots}{x^2+\cdots}}{\cdots}\right.\right.$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

$\downarrow$ 1103

$$\frac{1}{16}\left(\frac{11}{12}\left(\frac{\sqrt{\frac{2178\sqrt{3}-3650}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}\left(23+8\sqrt{3}\right)\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\sqrt{\frac{2178\sqrt{3}-\cdots}{2\left(1+\sqrt{\cdots}\right)}}}{\cdots}\right.\right.$$

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}$$

input
```
Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

output
```
(25*x*(1 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + (-1/12*(x*(353 + 88*x^2))/(3 +
 2*x^2 + x^4) + (11*((Sqrt[(-3650 + 2178*Sqrt[3])/(2*(1 + Sqrt[3]))]*ArcTa
n[(-Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]] - ((23 + 8*Sqrt[3
])*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + Sqrt[
3])]) + (Sqrt[(-3650 + 2178*Sqrt[3])/(2*(1 + Sqrt[3]))]*ArcTan[(Sqrt[2*(-1
 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]] + ((23 + 8*Sqrt[3])*Log[Sqrt[3]
 + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + Sqrt[3])])))/12)/16
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2197

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206

```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.35

method	result
risch	$\frac{-\frac{11}{24}x^7-\frac{529}{192}x^5-\frac{335}{96}x^3-\frac{253}{64}x}{(x^4+2x^2+3)^2}+\frac{11\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^2+3\right)}\frac{\left(-8\_R^2+23\right)\ln\left(x-\_R\right)}{\_R^3+\_R}\right)}{768}$
default	$\frac{-\frac{11}{24}x^7-\frac{529}{192}x^5-\frac{335}{96}x^3-\frac{253}{64}x}{(x^4+2x^2+3)^2}+\frac{11\left(47\sqrt{-2+2\sqrt{3}}\,\sqrt{3}+93\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{9216}+\frac{11\left(92\sqrt{3}-\frac{\left(47\sqrt{-2+2\sqrt{3}}\right.}{}\right.}{}$

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(-11/24*x^7-529/192*x^5-335/96*x^3-253/64*x)/(x^4+2*x^2+3)^2+11/768*sum((-
8*_R^2+23)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(148) = 296$.

Time = 0.08 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.76

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx =$$
$$-\frac{704\,x^7+4232\,x^5+5360\,x^3-22\,\sqrt{\frac{1}{3}}(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{1089\,\sqrt{3}-1825}\arctan\left(\frac{1}{337}\,\sqrt{\frac{1}{3}}\right.}{}$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
-1/1536*(704*x^7 + 4232*x^5 + 5360*x^3 - 22*sqrt(1/3)*(x^8 + 4*x^6 + 10*x^
4 + 12*x^2 + 9)*sqrt(1089*sqrt(3) - 1825)*arctan(1/337*sqrt(1/3)*(23*sqrt(
3)*x + 24*x)*sqrt(1089*sqrt(3) - 1825) + 1/674*sqrt(1089*sqrt(3) + 1825)*s
qrt(1089*sqrt(3) - 1825)*(sqrt(3) - 1)) + 22*sqrt(1/3)*(x^8 + 4*x^6 + 10*x
^4 + 12*x^2 + 9)*sqrt(1089*sqrt(3) - 1825)*arctan(-1/337*sqrt(1/3)*(23*sqr
t(3)*x + 24*x)*sqrt(1089*sqrt(3) - 1825) + 1/674*sqrt(1089*sqrt(3) + 1825)
*sqrt(1089*sqrt(3) - 1825)*(sqrt(3) - 1)) + 11*sqrt(1/3)*(x^8 + 4*x^6 + 10
*x^4 + 12*x^2 + 9)*sqrt(1089*sqrt(3) + 1825)*log(3707*x^2 + 11*sqrt(1/3)*(
47*sqrt(3)*x - 93*x)*sqrt(1089*sqrt(3) + 1825) + 3707*sqrt(3)) - 11*sqrt(1
/3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(1089*sqrt(3) + 1825)*log(3707
*x^2 - 11*sqrt(1/3)*(47*sqrt(3)*x - 93*x)*sqrt(1089*sqrt(3) + 1825) + 3707
*sqrt(3)) + 6072*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(168) = 336$.

Time = 0.93 (sec) , antiderivative size = 1200, normalized size of antiderivative = 5.97

$$\int \frac{x^2(4 + x^2 + 3x^4 + 5x^6)}{(3 + 2x^2 + x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
(-88*x**7 - 529*x**5 - 670*x**3 - 759*x)/(192*x**8 + 768*x**6 + 1920*x**4
+ 2304*x**2 + 1728) - sqrt(220825/7077888 + 14641*sqrt(3)/786432)*log(x**2
 + x*(-47*sqrt(6)*sqrt(1825 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194
)/366993 + 52016*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/366993 + 188*sqrt(1825
+ 1089*sqrt(3))/337) - 24765218375*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)
/134683862049 - 38128468*sqrt(6)*sqrt(1987425*sqrt(3) + 3444194)/371029923
 + 90413874433403/134683862049 + 144251139148*sqrt(3)/371029923) + sqrt(22
0825/7077888 + 14641*sqrt(3)/786432)*log(x**2 + x*(-188*sqrt(1825 + 1089*s
qrt(3))/337 - 52016*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/366993 + 47*sqrt(6)*
sqrt(1825 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/366993) - 247652
18375*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)/134683862049 - 38128468*sqrt
(6)*sqrt(1987425*sqrt(3) + 3444194)/371029923 + 90413874433403/13468386204
9 + 144251139148*sqrt(3)/371029923) + 2*sqrt(-121*sqrt(2)*sqrt(1987425*sqr
t(3) + 3444194)/3538944 + 220825/7077888 + 14641*sqrt(3)/262144)*atan(7339
86*sqrt(3)*x/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825
 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt
(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) - 204732*sqrt(
3)*sqrt(1825 + 1089*sqrt(3))/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) +
 3444194) + 1825 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 34441
94)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(...
```

Maxima [F]

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \int \frac{(5\,x^6+3\,x^4+x^2+4)x^2}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
-1/192*(88*x^7 + 529*x^5 + 670*x^3 + 759*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2
 + 9) - 11/192*integrate((8*x^2 - 23)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(148) = 296$.

Time = 0.57 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.87

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
11/124416*sqrt(2)*(2*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 36*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 2*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 207*3^(1/4)*sqrt(2)
*sqrt(6*sqrt(3) + 18) - 207*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3
/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 11/1
24416*sqrt(2)*(2*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 36*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*s
qrt(3) + 18) + 2*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 207*3^(1/4)*sqrt(2)*sqr
t(6*sqrt(3) + 18) - 207*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*
(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 11/24883
2*sqrt(2)*(36*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 2*3^(3
/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 3
6*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 207*3^(1/4)*sqrt(2)*sqrt(-6
*sqrt(3) + 18) + 207*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 11/248832*sqrt(2)*(36*3^(3/4)*sqrt(2)
*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 2*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)
^(3/2) + 2*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sqrt(6*sqrt(3) + 18
)*(sqrt(3) - 3) + 207*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 207*3^(1/4)*
sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqr
t(3)) - 1/192*(88*x^7 + 529*x^5 + 670*x^3 + 759*x)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = -\frac{\frac{11\,x^7}{24}+\frac{529\,x^5}{192}+\frac{335\,x^3}{96}+\frac{253\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{10950-\sqrt{2}\,2022\mathrm{i}}\,448547\mathrm{i}}{31850496\left(-\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}-\frac{448547\,\sqrt{2}\,x\,\sqrt{10950-\sqrt{2}\,2022\mathrm{i}}}{63700992\left(-\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}\right)\sqrt{10950-\sqrt{2}\,2022\mathrm{i}}\,11\mathrm{i}}{2304} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{10950+\sqrt{2}\,2022\mathrm{i}}\,448547\mathrm{i}}{31850496\left(\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}+\frac{448547\,\sqrt{2}\,x\,\sqrt{10950+\sqrt{2}\,2022\mathrm{i}}}{63700992\left(\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}\right)\sqrt{10950+\sqrt{2}\,2022\mathrm{i}}\,11\mathrm{i}}{2304}$$

input
```
int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

output
```
(atan((x*(10950 - 2^(1/2)*2022i)^(1/2)*448547i)/(31850496*((2^(1/2)*103165
81i)/10616832 - 21081709/10616832)) - (448547*2^(1/2)*x*(10950 - 2^(1/2)*2
022i)^(1/2))/(63700992*((2^(1/2)*10316581i)/10616832 - 21081709/10616832))
)*(10950 - 2^(1/2)*2022i)^(1/2)*11i)/2304 - ((253*x)/64 + (335*x^3)/96 + (
529*x^5)/192 + (11*x^7)/24)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) - (atan((x
*(2^(1/2)*2022i + 10950)^(1/2)*448547i)/(31850496*((2^(1/2)*10316581i)/106
16832 + 21081709/10616832)) + (448547*2^(1/2)*x*(2^(1/2)*2022i + 10950)^(1
/2))/(63700992*((2^(1/2)*10316581i)/10616832 + 21081709/10616832)))*(2^(1/
2)*2022i + 10950)^(1/2)*11i)/2304
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1330, normalized size of antiderivative = 6.62

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

output
```
(1034*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sq
rt(sqrt(3) + 1)*sqrt(2)))*x**8 + 4136*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt
(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 + 10340*sqr
t(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3
) + 1)*sqrt(2)))*x**4 + 12408*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3)
 - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 9306*sqrt(sqrt(3)
 + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sq
rt(2))) - 2046*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) -
 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 - 8184*sqrt(sqrt(3) + 1)*sqrt(2)*a
tan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 -
20460*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sq
rt(sqrt(3) + 1)*sqrt(2)))*x**4 - 24552*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqr
t(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 18414*sq
rt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(
3) + 1)*sqrt(2))) - 1034*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)
*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 - 4136*sqrt(sqrt(3) + 1)
*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)
))*x**6 - 10340*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2)
+ 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 12408*sqrt(sqrt(3) + 1)*sqrt(6)
*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x*...
```

3.123 $\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 203

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^3}\,dx &= \frac{25x(1-x^2)}{48\left(3+2x^2+x^4\right)^2}+\frac{x(64+51x^2)}{192\left(3+2x^2+x^4\right)} \\
&\quad -\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad +\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right) \\
&\quad -\frac{1}{256}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)
\end{aligned}$$

output
```
25/48*x*(-x^2+1)/(x^4+2*x^2+3)^2+x*(51*x^2+64)/(192*x^4+384*x^2+576)-1/768
*(-3873+3057*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2)
)^(1/2))+1/768*(-3873+3057*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x
)/(2+2*3^(1/2))^(1/2))-1/768*(3873+3057*3^(1/2))^(1/2)*arctanh((-2+2*3^(1/
2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.64

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \frac{1}{768}\left(\frac{4x(292+181x^2+166x^4+51x^6)}{(3+2x^2+x^4)^2} + \frac{3(34+21i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{3(34-21i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

output
```
((4*x*(292 + 181*x^2 + 166*x^4 + 51*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(34 + (
21*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(34
 - (21*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/768
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.45, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2206, 27, 1492, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{(x^4+2x^2+3)^3}dx$$

$$\Big\downarrow \text{ 2206}$$

$$\frac{1}{96}\int \frac{2(115x^2+39)}{(x^4+2x^2+3)^2}dx+\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\int \frac{115x^2+39}{(x^4+2x^2+3)^2}dx+\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{48}\left(\frac{1}{48}\int -\frac{36(4-17x^2)}{x^4+2x^2+3}dx+\frac{x(51x^2+64)}{4(x^4+2x^2+3)}\right)+\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{x(51x^2+64)}{4(x^4+2x^2+3)}-\frac{3}{4}\int \frac{4-17x^2}{x^4+2x^2+3}dx\right)+\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{1483}$$

$$\frac{1}{48}\left(\frac{x(51x^2+64)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{\int \frac{4\sqrt{2\left(-1+\sqrt{3}\right)}-\left(4+17\sqrt{3}\right)x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\frac{\int \frac{\left(4+17\sqrt{3}\right)x+4\sqrt{2\left(-1+\sqrt{3}\right)}}{x^2+\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right)\right)+$$

$$\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{48}\left(\frac{x(51x^2+64)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{-\frac{1}{2}\sqrt{2038\sqrt{3}-2582}\int \frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\left(4+17\sqrt{3}\right)\int -\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\cdots}}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{48}\left(\frac{x(51x^2+64)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{\frac{1}{2}(4+17\sqrt{3})\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\frac{1}{2}\sqrt{2038\sqrt{3}-2582}\int\frac{1}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}d}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$\Big\downarrow$ 1083

$$\frac{1}{48}\left(\frac{x(51x^2+64)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{\frac{1}{2}(4+17\sqrt{3})\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx+\sqrt{2038\sqrt{3}-2582}\int\frac{1}{-\left(2x-\sqrt{2\left(-1+\sqrt{3}\right)}\right)^2-2(}}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$\Big\downarrow$ 217

$$\frac{1}{48}\left(\frac{x(51x^2+64)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{\frac{1}{2}(4+17\sqrt{3})\int\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{x^2-\sqrt{2\left(-1+\sqrt{3}\right)}x+\sqrt{3}}dx-\sqrt{\frac{2038\sqrt{3}-2582}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2\sqrt{6\left(\sqrt{3}-1\right)}}+\right.\right.$$

$$\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

$\Big\downarrow$ 1103

$$\frac{1}{48}\left(\frac{x(51x^2+64)}{4(x^4+2x^2+3)}-\frac{3}{4}\left(\frac{-\sqrt{\frac{2038\sqrt{3}-2582}{2\left(1+\sqrt{3}\right)}}\arctan\left(\frac{2x-\sqrt{2\left(\sqrt{3}-1\right)}}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{2}(4+17\sqrt{3})\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x\right.}{2\sqrt{6\left(\sqrt{3}-1\right)}}\right.\right.$$

$$\frac{25x(1-x^2)}{48(x^4+2x^2+3)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

output
```
(25*x*(1 - x^2))/(48*(3 + 2*x^2 + x^4)^2) + ((x*(64 + 51*x^2))/(4*(3 + 2*x
^2 + x^4)) - (3*((-(Sqrt[(-2582 + 2038*Sqrt[3])/(2*(1 + Sqrt[3]))]*ArcTan[
(-Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]]) - ((4 + 17*Sqrt[3]
)*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + Sqrt[3
])]) + (-(Sqrt[(-2582 + 2038*Sqrt[3])/(2*(1 + Sqrt[3]))]*ArcTan[(Sqrt[2*(-
1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]]) + ((4 + 17*Sqrt[3])*Log[Sqrt[
3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/2)/(2*Sqrt[6*(-1 + Sqrt[3])])))/4)/4
8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\frac{17}{64}x^7+\frac{83}{96}x^5+\frac{181}{192}x^3+\frac{73}{48}x}{(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+2\_Z^2+3)}\frac{(17\_R^2-4)\ln(x-\_R)}{\_R^3+\_R}\right)}{256}$
default	$\frac{\frac{17}{64}x^7+\frac{83}{96}x^5+\frac{181}{192}x^3+\frac{73}{48}x}{(x^4+2x^2+3)^2}+\frac{\left(-55\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-63\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{3072}+\frac{\left(-16\sqrt{3}-\frac{\left(-55\sqrt{-2+2\sqrt{3}}\,\sqrt{3}\right.}{}\right.}{}$

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(17/64*x^7+83/96*x^5+181/192*x^3+73/48*x)/(x^4+2*x^2+3)^2+1/256*sum((17*_R
^2-4)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4+2*_Z^2+3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(148) = 296.

Time = 0.08 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.62

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx$$

$$= \frac{408\,x^7+1328\,x^5+1448\,x^3+6\,(x^8+4\,x^6+10\,x^4+12\,x^2+9)\sqrt{\frac{1019}{3}\sqrt{3}-\frac{1291}{3}}\arctan\left(\frac{3}{1702}\sqrt{\frac{1019}{3}\sqrt{3}-\right.}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
1/1536*(408*x^7 + 1328*x^5 + 1448*x^3 + 6*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9)*sqrt(1019/3*sqrt(3) - 1291/3)*arctan(3/1702*sqrt(1019/3*sqrt(3) + 1291
/3)*sqrt(1019/3*sqrt(3) - 1291/3)*(sqrt(3) - 1) + 1/851*(4*sqrt(3)*x + 51*
x)*sqrt(1019/3*sqrt(3) - 1291/3)) - 6*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*
sqrt(1019/3*sqrt(3) - 1291/3)*arctan(3/1702*sqrt(1019/3*sqrt(3) + 1291/3)*
sqrt(1019/3*sqrt(3) - 1291/3)*(sqrt(3) - 1) - 1/851*(4*sqrt(3)*x + 51*x)*s
qrt(1019/3*sqrt(3) - 1291/3)) - 3*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt
(1019/3*sqrt(3) + 1291/3)*log(851*x^2 + (55*sqrt(3)*x - 63*x)*sqrt(1019/3*
sqrt(3) + 1291/3) + 851*sqrt(3)) + 3*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*s
qrt(1019/3*sqrt(3) + 1291/3)*log(851*x^2 - (55*sqrt(3)*x - 63*x)*sqrt(1019
/3*sqrt(3) + 1291/3) + 851*sqrt(3)) + 2336*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x
^2 + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1195 vs. $2(163) = 326$.

Time = 0.93 (sec) , antiderivative size = 1195, normalized size of antiderivative = 5.89

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

output
```
(51*x**7 + 166*x**5 + 181*x**3 + 292*x)/(192*x**8 + 768*x**6 + 1920*x**4 +
 2304*x**2 + 1728) - sqrt(1291/786432 + 1019*sqrt(3)/786432)*log(x**2 + x*
(-55*sqrt(6)*sqrt(1291 + 1019*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/867
169 + 49606*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/867169 + 220*sqrt(1291 + 101
9*sqrt(3))/851) - 26628761029*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)/7519
82074561 - 40176070*sqrt(6)*sqrt(1315529*sqrt(3) + 2390882)/2213882457 + 7
6094994709709/751982074561 + 133967471914*sqrt(3)/2213882457) + sqrt(1291/
786432 + 1019*sqrt(3)/786432)*log(x**2 + x*(-220*sqrt(1291 + 1019*sqrt(3))
/851 - 49606*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/867169 + 55*sqrt(6)*sqrt(12
91 + 1019*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/867169) - 26628761029*s
qrt(2)*sqrt(1315529*sqrt(3) + 2390882)/751982074561 - 40176070*sqrt(6)*sqr
t(1315529*sqrt(3) + 2390882)/2213882457 + 76094994709709/751982074561 + 13
3967471914*sqrt(3)/2213882457) + 2*sqrt(-sqrt(2)*sqrt(1315529*sqrt(3) + 23
90882)/393216 + 1291/786432 + 1019*sqrt(3)/262144)*atan(1734338*sqrt(3)*x/
(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(
3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315
529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) - 224180*sqrt(3)*sqrt(1291
+ 1019*sqrt(3))/(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1
291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*s
qrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) - 148818...
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{(x^4+2\,x^2+3)^3}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2
+ 9) + 1/64*integrate((17*x^2 - 4)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(148) = 296$.

Time = 0.57 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.84

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
-1/165888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 306*3^(3/4)*(sqrt(3) + 3)*sq
rt(-6*sqrt(3) + 18) + 17*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 144*3^(1/4)*sqr
t(2)*sqrt(6*sqrt(3) + 18) - 144*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*
3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
1/165888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 306*3^(3/4)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) + 17*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 144*3^(1/4)*sqrt
(2)*sqrt(6*sqrt(3) + 18) - 144*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3
^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1
/331776*sqrt(2)*(306*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) -
 17*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 17*3^(3/4)*(6*sqrt(3) + 18)^
(3/2) + 306*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 144*3^(1/4)*sqrt(
2)*sqrt(-6*sqrt(3) + 18) + 144*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3
^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/331776*sqrt(2)*(306*3^(3/
4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 17*3^(3/4)*sqrt(2)*(-6*sq
rt(3) + 18)^(3/2) + 17*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*sqrt(6
*sqrt(3) + 18)*(sqrt(3) - 3) + 144*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) +
 144*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3)
 + 1/2) + sqrt(3)) + 1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^4 + ...
```

Mupad [B] (verification not implemented)

Time = 18.86 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \frac{\frac{17\,x^7}{64}+\frac{83\,x^5}{96}+\frac{181\,x^3}{192}+\frac{73\,x}{48}}{x^8+4\,x^6+10\,x^4+12\,x^2+9} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}\,851\mathrm{i}}{1179648\,\left(\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}+\frac{851\,\sqrt{2}\,x\,\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}}{2359296\,\left(\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}\right)\,\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}\,1\mathrm{i}}{768} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{7746+\sqrt{2}\,5106\mathrm{i}}\,851\mathrm{i}}{1179648\,\left(-\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}-\frac{851\,\sqrt{2}\,x\,\sqrt{7746+\sqrt{2}\,5106\mathrm{i}}}{2359296\,\left(-\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}\right)\,\sqrt{7746+\sqrt{2}\,5106\mathrm{i}}\,1\mathrm{i}}{768}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^3,x)
```

output
```
((73*x)/48 + (181*x^3)/192 + (83*x^5)/96 + (17*x^7)/64)/(12*x^2 + 10*x^4 +
 4*x^6 + x^8 + 9) + (atan((x*(7746 - 2^(1/2)*5106i)^(1/2)*851i)/(1179648*(
(2^(1/2)*851i)/98304 + 46805/393216)) + (851*2^(1/2)*x*(7746 - 2^(1/2)*510
6i)^(1/2))/(2359296*((2^(1/2)*851i)/98304 + 46805/393216)))*(7746 - 2^(1/2
)*5106i)^(1/2)*1i)/768 - (atan((x*(2^(1/2)*5106i + 7746)^(1/2)*851i)/(1179
648*((2^(1/2)*851i)/98304 - 46805/393216)) - (851*2^(1/2)*x*(2^(1/2)*5106i
 + 7746)^(1/2))/(2359296*((2^(1/2)*851i)/98304 - 46805/393216)))*(2^(1/2)*
5106i + 7746)^(1/2)*1i)/768
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1330, normalized size of antiderivative = 6.55

$$\int \frac{4+x^2+3x^4+5x^6}{(3+2x^2+x^4)^3}\,dx = \text{Too large to display}$$

input
```
int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

output

```
( - 110*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(
sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 - 440*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqr
t(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 - 1100*sqr
t(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3
) + 1)*sqrt(2)))*x**4 - 1320*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3)
- 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 990*sqrt(sqrt(3) +
 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt
(2))) + 126*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*
x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 + 504*sqrt(sqrt(3) + 1)*sqrt(2)*atan(
(sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 + 1260
*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sq
rt(3) + 1)*sqrt(2)))*x**4 + 1512*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt
(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 1134*sqrt(sqrt
(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)
*sqrt(2))) + 110*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2)
 + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**8 + 440*sqrt(sqrt(3) + 1)*sqrt(6)*
atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**6 +
 1100*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sq
rt(sqrt(3) + 1)*sqrt(2)))*x**4 + 1320*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt
(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 + 990*sq...
```

3.124 $\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 208

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = -\frac{4}{27x}-\frac{25x(5+x^2)}{144\left(3+2x^2+x^4\right)^2}-\frac{x(325+242x^2)}{1728\left(3+2x^2+x^4\right)} + \frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304} - \frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)}{2304} + \frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}x}{\sqrt{3}+x^2}\right)}{2304}$$

output

```
-4/27/x-25/144*x*(x^2+5)/(x^4+2*x^2+3)^2-x*(242*x^2+325)/(1728*x^4+3456*x^
2+5184)+1/6912*(179133+165483*3^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)-
2*x)/(2+2*3^(1/2))^(1/2))-1/6912*(179133+165483*3^(1/2))^(1/2)*arctan(((-2
+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))+1/6912*(-179133+165483*3^(1/2)
)^(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.67

$$\int \frac{4+x^2+3x^4+5x^6}{x^2(3+2x^2+x^4)^3}\,dx = \frac{-\frac{12(768+1849x^2+1412x^4+611x^6+166x^8)}{x(3+2x^2+x^4)^2}+\frac{3i(332i+7\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}-\frac{3i(-332i+7\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{6912}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```

output
```
((-12*(768 + 1849*x^2 + 1412*x^4 + 611*x^6 + 166*x^8))/(x*(3 + 2*x^2 + x^4
)^2) + ((3*I)*(332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 -
I*Sqrt[2]] - ((3*I)*(-332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sq
rt[1 + I*Sqrt[2]])/6912
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2198, 27, 2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^2(x^4+2x^2+3)^3}\,dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{96}\int \frac{2(-125x^4+45x^2+192)}{3x^2(x^4+2x^2+3)^2}dx-\frac{25x(x^2+5)}{144(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{144}\int \frac{-125x^4+45x^2+192}{x^2(x^4+2x^2+3)^2}dx-\frac{25x(x^2+5)}{144(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{144}\left(\frac{1}{48}\int \frac{4(-242x^4-7x^2+768)}{x^2\,(x^4+2x^2+3)}dx-\frac{x(242x^2+325)}{12\,(x^4+2x^2+3)}\right)-\frac{25x\,(x^2+5)}{144\,(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{144}\left(\frac{1}{12}\int \frac{-242x^4-7x^2+768}{x^2\,(x^4+2x^2+3)}dx-\frac{x(242x^2+325)}{12\,(x^4+2x^2+3)}\right)-\frac{25x\,(x^2+5)}{144\,(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{144}\left(\frac{1}{12}\int \left(\frac{256}{x^2}-\frac{3(166x^2+173)}{x^4+2x^2+3}\right)dx-\frac{x(242x^2+325)}{12\,(x^4+2x^2+3)}\right)-\frac{25x\,(x^2+5)}{144\,(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{144}\left(\frac{1}{12}\left(\frac{1}{4}\sqrt{3\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-\frac{1}{4}\sqrt{3\left(59711+55161\sqrt{3}\right)}\arctan\left(\frac{2x+}{\sqrt{2}}\right.\right.\right.$$

$$\frac{25x\,(x^2+5)}{144\,(x^4+2x^2+3)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```

output
```
(-25*x*(5 + x^2))/(144*(3 + 2*x^2 + x^4)^2) + (-1/12*(x*(325 + 242*x^2))/(
3 + 2*x^2 + x^4) + (-256/x + (Sqrt[3*(59711 + 55161*Sqrt[3])]*ArcTan[(Sqrt
[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4 - (Sqrt[3*(59711 + 551
61*Sqrt[3])]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])
/4 - (Sqrt[3*(-59711 + 55161*Sqrt[3])]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])
]*x + x^2])/8 + (Sqrt[3*(-59711 + 55161*Sqrt[3])]*Log[Sqrt[3] + Sqrt[2*(-1
 + Sqrt[3])]*x + x^2])/8)/12)/144
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

method	result
risch	$\frac{-\frac{83}{288}x^8-\frac{611}{576}x^6-\frac{353}{144}x^4-\frac{1849}{576}x^2-\frac{4}{3}}{x(x^4+2x^2+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(12\_Z^4+238844\_Z^2+3042735921\right)}\_R\ln\left(-1950\_R^3-37653769\_R+290\right.\right.}{2304}$
default	$-\frac{\frac{121}{32}x^7+\frac{809}{64}x^5+\frac{419}{16}x^3+\frac{2475}{64}x}{27(x^4+2x^2+3)^2}-\frac{\left(-325\sqrt{-2+2\sqrt{3}}\sqrt{3}+21\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^2+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{27648}-\frac{\left(692\sqrt{3}-\frac{\left(-325\sqrt{-2\right.}{}\right.}{}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(-83/288*x^8-611/576*x^6-353/144*x^4-1849/576*x^2-4/3)/x/(x^4+2*x^2+3)^2+1
/2304*sum(_R*ln(-1950*_R^3-37653769*_R+2909135979*x),_R=RootOf(12*_Z^4+238
844*_Z^2+3042735921))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(153) = 306$.

Time = 0.08 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.54

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = -\frac{1328\,x^8+4888\,x^6+11296\,x^4-2\left(x^9+4\,x^7+10\,x^5+12\,x^3+9\,x\right)\sqrt{18387\,\sqrt{3}+\frac{59711}{3}}\arctan\left(\frac{1}{105478}\right.}{}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
-1/4608*(1328*x^8 + 4888*x^6 + 11296*x^4 - 2*(x^9 + 4*x^7 + 10*x^5 + 12*x^
3 + 9*x)*sqrt(18387*sqrt(3) + 59711/3)*arctan(1/105478*(346*sqrt(3)*x + 3*
sqrt(18387*sqrt(3) - 59711/3)*(sqrt(3) - 1) - 996*x)*sqrt(18387*sqrt(3) +
59711/3)) + 2*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)*sqrt(18387*sqrt(3) + 5
9711/3)*arctan(-1/105478*(346*sqrt(3)*x - 3*sqrt(18387*sqrt(3) - 59711/3)*
(sqrt(3) - 1) - 996*x)*sqrt(18387*sqrt(3) + 59711/3)) - (x^9 + 4*x^7 + 10*
x^5 + 12*x^3 + 9*x)*sqrt(18387*sqrt(3) - 59711/3)*log(52739*x^2 + (325*sqr
t(3)*x + 21*x)*sqrt(18387*sqrt(3) - 59711/3) + 52739*sqrt(3)) + (x^9 + 4*x
^7 + 10*x^5 + 12*x^3 + 9*x)*sqrt(18387*sqrt(3) - 59711/3)*log(52739*x^2 -
(325*sqrt(3)*x + 21*x)*sqrt(18387*sqrt(3) - 59711/3) + 52739*sqrt(3)) + 14
792*x^2 + 6144)/(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.36

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = \frac{-166x^8-611x^6-1412x^4-1849x^2-768}{576x^9+2304x^7+5760x^5+6912x^3+5184x} + \text{RootSum}\left(4174708211712t^4+15652880384t^2+37564641, \left(t\mapsto t\log\left(-\frac{98146713600t^3}{11971753}-\frac{9639364}{323237}\right.\right.\right.$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**3,x)
```

output
```
(-166*x**8 - 611*x**6 - 1412*x**4 - 1849*x**2 - 768)/(576*x**9 + 2304*x**7
 + 5760*x**5 + 6912*x**3 + 5184*x) + RootSum(4174708211712*_t**4 + 1565288
0384*_t**2 + 37564641, Lambda(_t, _t*log(-98146713600*_t**3/11971753 - 963
9364864*_t/323237331 + x)))
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^3x^2}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
-1/576*(166*x^8 + 611*x^6 + 1412*x^4 + 1849*x^2 + 768)/(x^9 + 4*x^7 + 10*x
^5 + 12*x^3 + 9*x) - 1/576*integrate((166*x^2 + 173)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(153) = 306.

Time = 0.58 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.80

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output
```
1/746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1
/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2))
 + 1/746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3
/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3
)*sqrt(-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arcta
n(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/
2)) + 1/1492992*sqrt(2)*(1494*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3
) + 18) - 83*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(
3) + 18)^(3/2) + 1494*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 3114*3^
(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 3114*3^(1/4)*sqrt(6*sqrt(3) + 18))*l
og(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/1492992*sqrt(
2)*(1494*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 83*3^(3/4)*
sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1494
*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 3114*3^(1/4)*sqrt(2)*sqrt(-6
*sqrt(3) + 18) - 3114*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*
sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/1728*(242*x^7 + 809*x^5 + 1676*...
```

Mupad [B] (verification not implemented)

Time = 18.86 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.86

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = -\frac{\frac{83\,x^8}{288}+\frac{611\,x^6}{576}+\frac{353\,x^4}{144}+\frac{1849\,x^2}{576}+\frac{4}{3}}{x^9+4\,x^7+10\,x^5+12\,x^3+9\,x} + \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-358266-\sqrt{2}\,316434{}\mathrm{i}}\,52739{}\mathrm{i}}{859963392\,\left(-\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847{}\mathrm{i}}{286654464}\right)}+\frac{52739\,\sqrt{2}\,x\,\sqrt{-358266-\sqrt{2}\,316434{}\mathrm{i}}}{1719926784\,\left(-\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847{}\mathrm{i}}{286654464}\right)}\right)\,\sqrt{-358266-\sqrt{2}\,316434{}\mathrm{i}}\,1{}\mathrm{i}}{6912} - \frac{\mathrm{atan}\left(\frac{x\,\sqrt{-358266+\sqrt{2}\,316434{}\mathrm{i}}\,52739{}\mathrm{i}}{859963392\,\left(\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847{}\mathrm{i}}{286654464}\right)}-\frac{52739\,\sqrt{2}\,x\,\sqrt{-358266+\sqrt{2}\,316434{}\mathrm{i}}}{1719926784\,\left(\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847{}\mathrm{i}}{286654464}\right)}\right)\,\sqrt{-358266+\sqrt{2}\,316434{}\mathrm{i}}\,1{}\mathrm{i}}{6912}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^3),x)
```

output
```
(atan((x*(- 2^(1/2)*316434i - 358266)^(1/2)*52739i)/(859963392*((2^(1/2)*9
123847i)/286654464 - 17140175/286654464)) + (52739*2^(1/2)*x*(- 2^(1/2)*31
6434i - 358266)^(1/2))/(1719926784*((2^(1/2)*9123847i)/286654464 - 1714017
5/286654464)))*(- 2^(1/2)*316434i - 358266)^(1/2)*1i)/6912 - (atan((x*(2^(
1/2)*316434i - 358266)^(1/2)*52739i)/(859963392*((2^(1/2)*9123847i)/286654
464 + 17140175/286654464)) - (52739*2^(1/2)*x*(2^(1/2)*316434i - 358266)^(
1/2))/(1719926784*((2^(1/2)*9123847i)/286654464 + 17140175/286654464)))*(2
^(1/2)*316434i - 358266)^(1/2)*1i)/6912 - ((1849*x^2)/576 + (353*x^4)/144
+ (611*x^6)/576 + (83*x^8)/288 + 4/3)/(9*x + 12*x^3 + 10*x^5 + 4*x^7 + x^9
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1343, normalized size of antiderivative = 6.46

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x)
```

output
```
(650*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqr
t(sqrt(3) + 1)*sqrt(2)))*x**9 + 2600*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(
sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**7 + 6500*sqrt(
sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3)
+ 1)*sqrt(2)))*x**5 + 7800*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) -
1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**3 + 5850*sqrt(sqrt(3) +
1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(
2)))*x + 42*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*
x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**9 + 168*sqrt(sqrt(3) + 1)*sqrt(2)*atan(
(sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**7 + 420*
sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqr
t(3) + 1)*sqrt(2)))*x**5 + 504*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3
) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**3 + 378*sqrt(sqrt(3)
 + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sq
rt(2)))*x - 650*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2)
+ 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**9 - 2600*sqrt(sqrt(3) + 1)*sqrt(6)*
atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**7 -
 6500*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sq
rt(sqrt(3) + 1)*sqrt(2)))*x**5 - 7800*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt
(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**3 - 5850*s...
```

3.125 $\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 217

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3} dx = -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x(7+5x^2)}{432(3+2x^2+x^4)^2}+\frac{x(1474+1025x^2)}{5184(3+2x^2+x^4)} - \frac{\sqrt{\frac{1}{3}(10004741+11240451\sqrt{3})}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}-2x}{\sqrt{2(1+\sqrt{3})}}\right)}{20736} + \frac{\sqrt{\frac{1}{3}(10004741+11240451\sqrt{3})}\arctan\left(\frac{\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{2(1+\sqrt{3})}}\right)}{20736} - \frac{\sqrt{\frac{1}{3}(-10004741+11240451\sqrt{3})}\operatorname{arctanh}\left(\frac{\sqrt{2(-1+\sqrt{3})}x}{\sqrt{3}+x^2}\right)}{20736}$$

output
```
-4/81/x^3+7/27/x+25/432*x*(5*x^2+7)/(x^4+2*x^2+3)^2+x*(1025*x^2+1474)/(518
4*x^4+10368*x^2+15552)-1/62208*(30014223+33721353*3^(1/2))^(1/2)*arctan(((
-2+2*3^(1/2))^(1/2)-2*x)/(2+2*3^(1/2))^(1/2))+1/62208*(30014223+33721353*3
^(1/2))^(1/2)*arctan(((-2+2*3^(1/2))^(1/2)+2*x)/(2+2*3^(1/2))^(1/2))-1/622
08*(-30014223+33721353*3^(1/2))^(1/2)*arctanh((-2+2*3^(1/2))^(1/2)*x/(3^(1
/2)+x^2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.64

$$\int \frac{4+x^2+3x^4+5x^6}{x^4(3+2x^2+x^4)^3}\,dx = \frac{\frac{4(-2304+9024x^2+20090x^4+19939x^6+8644x^8+2369x^{10})}{x^3(3+2x^2+x^4)^2}+\frac{(4738+127i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{(4738-127i\sqrt{2})\arctan\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{20736}$$

input
```
Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

output
```
((4*(-2304 + 9024*x^2 + 20090*x^4 + 19939*x^6 + 8644*x^8 + 2369*x^10))/(x^
3*(3 + 2*x^2 + x^4)^2) + ((4738 + (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqr
t[2]]])/Sqrt[1 - I*Sqrt[2]] + ((4738 - (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 +
I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/20736
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2198, 27, 2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^6+3x^4+x^2+4}{x^4(x^4+2x^2+3)^3}dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{96}\int \frac{2(625x^6+225x^4-240x^2+576)}{9x^4(x^4+2x^2+3)^2}dx+\frac{25x(5x^2+7)}{432(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{432}\int \frac{625x^6+225x^4-240x^2+576}{x^4(x^4+2x^2+3)^2}dx+\frac{25x(5x^2+7)}{432(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{432}\left(\frac{1}{48}\int \frac{4(1025x^6+322x^4-2496x^2+2304)}{x^4(x^4+2x^2+3)}dx+\frac{x(1025x^2+1474)}{12(x^4+2x^2+3)}\right)+\frac{25x(5x^2+7)}{432(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{432}\left(\frac{1}{12}\int \frac{1025x^6+322x^4-2496x^2+2304}{x^4(x^4+2x^2+3)}dx+\frac{x(1025x^2+1474)}{12(x^4+2x^2+3)}\right)+\frac{25x(5x^2+7)}{432(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{432}\left(\frac{1}{12}\int \left(\frac{2369x^2+2242}{x^4+2x^2+3}-\frac{1344}{x^2}+\frac{768}{x^4}\right)dx+\frac{x(1025x^2+1474)}{12(x^4+2x^2+3)}\right)+\frac{25x(5x^2+7)}{432(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{432}\left(\frac{1}{12}\left(-\frac{1}{4}\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2\left(\sqrt{3}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(10004741+11240451\sqrt{3\right.}}\right.\right.$$

$$\frac{25x(5x^2+7)}{432(x^4+2x^2+3)^2}$$

input
```
Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

output
```
(25*x*(7 + 5*x^2))/(432*(3 + 2*x^2 + x^4)^2) + ((x*(1474 + 1025*x^2))/(12*
(3 + 2*x^2 + x^4)) + (-256/x^3 + 1344/x - (Sqrt[(10004741 + 11240451*Sqrt[
3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/4 + (
Sqrt[(10004741 + 11240451*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x
)/Sqrt[2*(1 + Sqrt[3])]])/4 + (Sqrt[(-10004741 + 11240451*Sqrt[3])/3]*Log[
Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/8 - (Sqrt[(-10004741 + 11240451
*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/8)/12)/432
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\frac{2369}{5184}x^{10}+\frac{2161}{1296}x^{8}+\frac{19939}{5184}x^{6}+\frac{10045}{2592}x^{4}+\frac{47}{27}x^{2}-\frac{4}{9}}{x^{3}(x^{4}+2x^{2}+3)^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(12\_Z^{4}+40018964\_Z^{2}+126347738683401\right)}\_R\ln\left(29190\_R^{3}+1\right.\right.}{20736}$
default	$\frac{\frac{1025}{192}x^{7}+\frac{881}{48}x^{5}+\frac{7523}{192}x^{3}+\frac{1087}{32}x}{27(x^{4}+2x^{2}+3)^{2}}+\frac{\left(-4865\sqrt{-2+2\sqrt{3}}\,\sqrt{3}-381\sqrt{-2+2\sqrt{3}}\right)\ln\left(x^{2}+x\sqrt{-2+2\sqrt{3}}+\sqrt{3}\right)}{248832}+\frac{\left(8968\sqrt{3}-\frac{\left(-4865\right.}{}\right.}{}$

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x,method=_RETURNVERBOSE)
```

output
```
(2369/5184*x^10+2161/1296*x^8+19939/5184*x^6+10045/2592*x^4+47/27*x^2-4/9)
/x^3/(x^4+2*x^2+3)^2+1/20736*sum(_R*ln(29190*_R^3+101628741761*_R+13274881
5833469*x),_R=RootOf(12*_Z^4+40018964*_Z^2+126347738683401))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(160) = 320$.

Time = 0.08 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.54

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx$$

$$= \frac{18952\,x^{10}+69152\,x^8+159512\,x^6+160720\,x^4-2\left(x^{11}+4\,x^9+10\,x^7+12\,x^5+9\,x^3\right)\sqrt{3746817\,\sqrt{3}+ \cdots}}{\cdots}$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

output
```
1/41472*(18952*x^10 + 69152*x^8 + 159512*x^6 + 160720*x^4 - 2*(x^11 + 4*x^
9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(3746817*sqrt(3) + 10004741/3)*arctan(1/2
3619838*(4484*sqrt(3)*x + 3*sqrt(3746817*sqrt(3) - 10004741/3)*(sqrt(3) -
1) - 14214*x)*sqrt(3746817*sqrt(3) + 10004741/3)) + 2*(x^11 + 4*x^9 + 10*x
^7 + 12*x^5 + 9*x^3)*sqrt(3746817*sqrt(3) + 10004741/3)*arctan(-1/23619838
*(4484*sqrt(3)*x - 3*sqrt(3746817*sqrt(3) - 10004741/3)*(sqrt(3) - 1) - 14
214*x)*sqrt(3746817*sqrt(3) + 10004741/3)) - (x^11 + 4*x^9 + 10*x^7 + 12*x
^5 + 9*x^3)*sqrt(3746817*sqrt(3) - 10004741/3)*log(11809919*x^2 + (4865*sq
rt(3)*x - 381*x)*sqrt(3746817*sqrt(3) - 10004741/3) + 11809919*sqrt(3)) +
(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(3746817*sqrt(3) - 10004741/3
)*log(11809919*x^2 - (4865*sqrt(3)*x - 381*x)*sqrt(3746817*sqrt(3) - 10004
741/3) + 11809919*sqrt(3)) + 72192*x^2 - 18432)/(x^11 + 4*x^9 + 10*x^7 + 1
2*x^5 + 9*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.37

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx$$
$$= \text{RootSum}\left(338151365148672t^4+2622682824704t^2+19257390441,\left(t\mapsto t\log\left(\frac{357010935644160t^3}{182097141061}+\right.\right.\right.$$
$$+\frac{2369x^{10}+8644x^8+19939x^6+20090x^4+9024x^2-2304}{5184x^{11}+20736x^9+51840x^7+62208x^5+46656x^3}$$

input
```
integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**3,x)
```

output
```
RootSum(338151365148672*_t**4 + 2622682824704*_t**2 + 19257390441, Lambda(
_t, _t*log(357010935644160*_t**3/182097141061 + 26016957890816*_t/16388742
69549 + x))) + (2369*x**10 + 8644*x**8 + 19939*x**6 + 20090*x**4 + 9024*x*
*2 - 2304)/(5184*x**11 + 20736*x**9 + 51840*x**7 + 62208*x**5 + 46656*x**3
)
```

Maxima [F]

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx = \int \frac{5\,x^6+3\,x^4+x^2+4}{\left(x^4+2\,x^2+3\right)^3x^4}\,dx$$

input
```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

output
```
1/5184*(2369*x^10 + 8644*x^8 + 19939*x^6 + 20090*x^4 + 9024*x^2 - 2304)/(x
^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3) + 1/5184*integrate((2369*x^2 + 2242
)/(x^4 + 2*x^2 + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(160) = 320$.

Time = 0.58 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.71

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx = \text{Too large to display}$$

input

```
integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

output

```
-1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 42642*3
^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(3/4)*(sqrt(3)
 + 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 80712
*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6*sqrt(3) + 18
))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt
(3) + 1/2)) - 1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3
/2) + 42642*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(
3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^
(3/2) - 80712*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6
*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/
sqrt(1/6*sqrt(3) + 1/2)) - 1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3
/2) + 2369*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) +
 18)*(sqrt(3) - 3) - 80712*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3
^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2
) + sqrt(3)) + 1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2369*3
^(3/4)*(6*sqrt(3) + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3
) - 3) - 80712*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3^(1/4)*sqrt(
6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(...
```

Mupad [B] (verification not implemented)

Time = 18.59 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.85

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx = \frac{\frac{2369\,x^{10}}{5184}+\frac{2161\,x^8}{1296}+\frac{19939\,x^6}{5184}+\frac{10045\,x^4}{2592}+\frac{47\,x^2}{27}-\frac{4}{9}}{x^{11}+4\,x^9+10\,x^7+12\,x^5+9\,x^3} - \frac{\operatorname{atan}\left(\frac{x\sqrt{-60028446-\sqrt{2}\,70859514\mathrm{i}}\,11809919\mathrm{i}}{626913312768\left(-\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}+\frac{11809919\sqrt{2}\,x\sqrt{-60028446-\sqrt{2}\,70859514\mathrm{i}}}{1253826625536\left(-\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}\right)\sqrt{-60028446-}}{62208} + \frac{\operatorname{atan}\left(\frac{x\sqrt{-60028446+\sqrt{2}\,70859514\mathrm{i}}\,11809919\mathrm{i}}{626913312768\left(\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}-\frac{11809919\sqrt{2}\,x\sqrt{-60028446+\sqrt{2}\,70859514\mathrm{i}}}{1253826625536\left(\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}\right)\sqrt{-60028446+\sqrt{}}}{62208}$$

input
```
int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^3),x)
```

output
```
((47*x^2)/27 + (10045*x^4)/2592 + (19939*x^6)/5184 + (2161*x^8)/1296 + (23
69*x^10)/5184 - 4/9)/(9*x^3 + 12*x^5 + 10*x^7 + 4*x^9 + x^11) - (atan((x*(
- 2^(1/2)*70859514i - 60028446)^(1/2)*11809919i)/(626913312768*((2^(1/2)*1
3238919199i)/104485552128 - 57455255935/208971104256)) + (11809919*2^(1/2)
*x*(- 2^(1/2)*70859514i - 60028446)^(1/2))/(1253826625536*((2^(1/2)*132389
19199i)/104485552128 - 57455255935/208971104256)))*(- 2^(1/2)*70859514i -
60028446)^(1/2)*1i)/62208 + (atan((x*(2^(1/2)*70859514i - 60028446)^(1/2)*
11809919i)/(626913312768*((2^(1/2)*13238919199i)/104485552128 + 5745525593
5/208971104256)) - (11809919*2^(1/2)*x*(2^(1/2)*70859514i - 60028446)^(1/2
))/(1253826625536*((2^(1/2)*13238919199i)/104485552128 + 57455255935/20897
1104256)))*(2^(1/2)*70859514i - 60028446)^(1/2)*1i)/62208
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1364, normalized size of antiderivative = 6.29

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x)
```

output
```
( - 9730*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/
(sqrt(sqrt(3) + 1)*sqrt(2)))*x**11 - 38920*sqrt(sqrt(3) + 1)*sqrt(6)*atan(
(sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**9 - 9730
0*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(s
qrt(3) + 1)*sqrt(2)))*x**7 - 116760*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(s
qrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**5 - 87570*sqrt(
sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3)
+ 1)*sqrt(2)))*x**3 + 762*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1
)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**11 + 3048*sqrt(sqrt(3) +
1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(
2)))*x**9 + 7620*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2)
 - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**7 + 9144*sqrt(sqrt(3) + 1)*sqrt(2)
*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**5
+ 6858*sqrt(sqrt(3) + 1)*sqrt(2)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(s
qrt(sqrt(3) + 1)*sqrt(2)))*x**3 + 9730*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqr
t(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**11 + 38920*s
qrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt
(3) + 1)*sqrt(2)))*x**9 + 97300*sqrt(sqrt(3) + 1)*sqrt(6)*atan((sqrt(sqrt(
3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**7 + 116760*sqrt(sqr
t(3) + 1)*sqrt(6)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) ...
```

3.126 $\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 33, antiderivative size = 149

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4} \, dx = \frac{(cf-bg)x^2}{2c^2} + \frac{gx^4}{4c} - \frac{(2c^3d - c^2(be+2af) - b^3g + bc(bf+3ag)) \operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(c^2e + b^2g - c(bf+ag)) \log(a+bx^2+cx^4)}{4c^3}$$

output
```
1/2*(-b*g+c*f)*x^2/c^2+1/4*g*x^4/c-1/2*(2*c^3*d-c^2*(2*a*f+b*e)-b^3*g+b*c*
(3*a*g+b*f))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2
)+1/4*(c^2*e+b^2*g-c*(a*g+b*f))*ln(c*x^4+b*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4} \, dx = \frac{2c(cf-bg)x^2 + c^2gx^4 + \frac{2(2c^3d - c^2(be+2af) - b^3g + bc(bf+3ag)) \arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (c^2e + b^2g - c(bf+ag)) \log (a}{4c^3}$$

input
```
Integrate[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*(c*f - b*g)*x^2 + c^2*g*x^4 + (2*(2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g
 + b*c*(b*f + 3*a*g))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2
+ 4*a*c] + (c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2194, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d+ex^2+fx^4+gx^6\right)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int \frac{gx^6+fx^4+ex^2+d}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{2}\int \left(\frac{gx^2}{c}+\frac{cf-bg}{c^2}+\frac{dc^2-afc+\left(gb^2+c^2e-c(bf+ag)\right)x^2+abg}{c^2\left(cx^4+bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)}{c^3\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)-\right.}{2c^3}\right.$$

input
```
Int[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

output
```
(((c*f - b*g)*x^2)/c^2 + (g*x^4)/(2*c) - ((2*c^3*d - c^2*(b*e + 2*a*f) - b
^3*g + b*c*(b*f + 3*a*g))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*S
qrt[b^2 - 4*a*c]) + ((c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x^4
])/(2*c^3))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

method	result
default	$-\frac{-\frac{1}{2} g x^4 c+b g x^2-c f x^2}{2 c^2}+\frac{\frac{\left(-a c g+b^2 g-b c f+e c^2\right)\ln\left(c x^4+b x^2+a\right)}{2 c}+\frac{2\left(a b g-a c f+d c^2-\frac{\left(-a c g+b^2 g-b c f+e c^2\right) b}{2 c}\right)\arctan\left(\frac{2 c x^2+b}{\sqrt{4 a c-b^2}}\right)}{\sqrt{4 a c-b^2}}}{2 c^2}$
risch	Expression too large to display

input
```
int(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2*(-1/2*g*x^4*c+b*g*x^2-c*f*x^2)+1/2/c^2*(1/2*(-a*c*g+b^2*g-b*c*f+c
^2*e)/c*ln(c*x^4+b*x^2+a)+2*(a*b*g-a*c*f+d*c^2-1/2*(-a*c*g+b^2*g-b*c*f+c^2
*e)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.26

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)gx^4+2\,((b^2c^2-4\,ac^3)f-(b^3c-4\,abc^2)g)x^2+(2\,c^3d-bc^2e+(b^2c-2\,ac^2)f-(b^3-3\,a\right.$$

input
```
integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*
b*c^2)*g)*x^2 + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)
*g)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2
+ b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*e - (b
^3*c - 4*a*b*c^2)*f + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 +
 a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4
*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g)*x^2 - 2*(2*c^3*d - b*c^2*e + (b^2*c - 2
*a*c^2)*f - (b^3 - 3*a*b*c)*g)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sq
rt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b*
c^2)*f + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 + a))/(b^2*c^3
 - 4*a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \frac{cgx^4+2\,cfx^2-2\,bgx^2}{4\,c^2}+\frac{(c^2e-bcf+b^2g-acg)\log\left(cx^4+bx^2+a\right)}{4\,c^3} + \frac{(2\,c^3d-bc^2e+b^2cf-2\,ac^2f-b^3g+3\,abcg)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(c*g*x^4 + 2*c*f*x^2 - 2*b*g*x^2)/c^2 + 1/4*(c^2*e - b*c*f + b^2*g - a
*c*g)*log(c*x^4 + b*x^2 + a)/c^3 + 1/2*(2*c^3*d - b*c^2*e + b^2*c*f - 2*a*
c^2*f - b^3*g + 3*a*b*c*g)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(
-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 19.31 (sec) , antiderivative size = 1834, normalized size of antiderivative = 12.31

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x)
```

output
```
x^2*(f/(2*c) - (b*g)/(2*c^2)) + (g*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2
*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f -
 10*a*b^2*c*g))/(2*(16*a*c^4 - 4*b^2*c^3)) + (atan((2*c^4*(4*a*c - b^2)*(x
^2*(((((4*c^6*d + 6*b^2*c^4*f - 6*b^3*c^3*g - 4*a*c^5*f - 6*b*c^5*e + 10*a
*b*c^4*g)/c^4 - (4*b*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e
- 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*c^3*
d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(8*c^3*(4*a*c - b^
2)^(1/2)) - (b*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*
g)*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^
2*f - 10*a*b^2*c*g))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a +
 (b*((((4*c^6*d + 6*b^2*c^4*f - 6*b^3*c^3*g - 4*a*c^5*f - 6*b*c^5*e + 10*a
*b*c^4*g)/c^4 - (4*b*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e
- 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*
g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a
*b^2*c*g))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b^5*g^2 + b*c^4*e^2 + b^3*c^2*f^2
 - c^5*d*e + 2*a^2*b*c^2*g^2 + a*c^4*d*g + a*c^4*e*f + b*c^4*d*f - 2*b^4*c
*f*g - a*b*c^3*f^2 - 3*a*b^3*c*g^2 - b^2*c^3*d*g - 2*b^2*c^3*e*f - a^2*c^3
*f*g + 2*b^3*c^2*e*g + 4*a*b^2*c^2*f*g - 3*a*b*c^3*e*g)/c^4 + (b*(2*c^3*d
- b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g)^2)/(2*c^4*(4*a*c - b^
2))))/(2*a*(4*a*c - b^2)^(1/2)) + ((((8*a^2*c^4*g - 8*a*c^5*e + 8*a*b*...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1196, normalized size of antiderivative = 8.03

$$\int \frac{x(d+ex^2+fx^4+gx^6)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

output
```
( - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*g
+ 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*f +
 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*g - 2*
sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*f + 2*s
qrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c**2*e - 4*sq
rt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*c**3*d - 6*sqrt(
2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*g + 4*sqrt(2*
sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*f + 2*sqrt(2*s
qrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*g - 2*sqrt(2*sqrt
(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*f + 2*sqrt(2*sq...
```

3.127 $\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 35, antiderivative size = 594

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2} dx = \frac{(cf-2bg)x}{c^3} + \frac{gx^3}{3c^2}$$
$$+ \frac{x(a(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))+(b^3cf+bc^2(cd-3af)-b^4g-b^2c(ce-4ag)+2ac^{}}{2c^3(b^2-4ac)(a+bx^2+cx^4)}$$
$$- \frac{\left(3b^3cf-bc^2(cd+13af)-5b^4g-b^2c(ce-24ag)+2ac^2(3ce-7ag)-\frac{3b^4cf-4ac^3(cd-5af)-b^2c^2(cd+19af)-}{\sqrt{b^2-}}\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$- \frac{\left(3b^3cf-bc^2(cd+13af)-5b^4g-b^2c(ce-24ag)+2ac^2(3ce-7ag)+\frac{3b^4cf-4ac^3(cd-5af)-b^2c^2(cd+19af)-}{\sqrt{b^2-}}\right.}{2\sqrt{2}c^{7/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
(-2*b*g+c*f)*x/c^3+1/3*g*x^3/c^2+1/2*x*(a*(2*c^3*d-c^2*(2*a*f+b*e)-b^3*g+b
*c*(3*a*g+b*f))+(b^3*c*f+b*c^2*(-3*a*f+c*d)-b^4*g-b^2*c*(-4*a*g+c*e)+2*a*c
^2*(-a*g+c*e))*x^2)/c^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*(3*b^3*c*f-b*c^2*
(13*a*f+c*d)-5*b^4*g-b^2*c*(-24*a*g+c*e)+2*a*c^2*(-7*a*g+3*c*e)-(3*b^4*c*f
-4*a*c^3*(-5*a*f+c*d)-b^2*c^2*(19*a*f+c*d)-5*b^5*g-b^3*c*(-34*a*g+c*e)+4*a
*b*c^2*(-13*a*g+2*c*e))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(7/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-1/4*(3*b^3*c*f-b*c^2*(13*a*f+c*d)-5*b^4*g-b^2*c*(-24*a*g+c*e)+2*a
*c^2*(-7*a*g+3*c*e)+(3*b^4*c*f-4*a*c^3*(-5*a*f+c*d)-b^2*c^2*(19*a*f+c*d)-5
*b^5*g-b^3*c*(-34*a*g+c*e)+4*a*b*c^2*(-13*a*g+2*c*e))/(-4*a*c+b^2)^(1/2))*
arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(7/2)/(-4
*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.21

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{12\sqrt{c}(cf-2bg)x+4c^{3/2}gx^3+\frac{6\sqrt{c}x\left(b\left(c^3d-bc^2e+b^2cf-b^3g\right)x^2+a^2c\left(3bg-2c\left(f+gx^2\right)\right)+a\left(-b^3g+2c^3\left(d+ex^2\right)-bc^2\left(e+3fx^2\right)+b^2\right.\right.}{(b^2-4ac)(a+bx^2+cx^4)}}{}$$

input
```
Integrate[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(12*Sqrt[c]*(c*f - 2*b*g)*x + 4*c^(3/2)*g*x^3 + (6*Sqrt[c]*x*(b*(c^3*d - b
*c^2*e + b^2*c*f - b^3*g)*x^2 + a^2*c*(3*b*g - 2*c*(f + g*x^2)) + a*(-(b^3
*g) + 2*c^3*(d + e*x^2) - b*c^2*(e + 3*f*x^2) + b^2*c*(f + 4*g*x^2))))/((b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*g - b^3*c*(c*e + 3*S
qrt[b^2 - 4*a*c]*f - 34*a*g) + b^4*(3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + 2*a*c
^2*(-2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e + 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*c]*
g) - b^2*c*(c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f + 24*a*Sqrt[b^2 - 4*a
*c]*g) + b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d + 8*a*e) + 13*a*(Sqrt[b^2 - 4*a*c]*
f - 4*a*g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^
2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b^5*g + b^3*
c*(c*e - 3*Sqrt[b^2 - 4*a*c]*f - 34*a*g) + b^4*(-3*c*f + 5*Sqrt[b^2 - 4*a*
c]*g) + b^2*c*(c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f - 24*a*Sqrt[b^2 -
4*a*c]*g) + 2*a*c^2*(2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e - 10*a*c*f + 7*a*Sq
rt[b^2 - 4*a*c]*g) + b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d - 8*a*e) + 13*a*(Sqrt[b
^2 - 4*a*c]*f + 4*a*g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*c^(7/2))
```

Rubi [A] (verified)

Time = 8.31 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2197, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 2197}$$

$$\frac{x\left(a\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)+x^2\left(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{\int \frac{2a\left(4a-\frac{b^2}{c}\right)gx^6-\frac{2a\left(b^2-4ac\right)(cf-bg)x^4}{c^2}+\frac{a\left(-gb^4+cfb^3-c(ce-6ag)b^2-c^2(cd+5af)b+6ac^2(ce-ag)\right)x^2}{c^3}+\frac{a^2\left(-gb^3+c(bf+3ag)b+2c^3d-c^2(be+2af)\right)}{c^3}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 2205}$$

$$\frac{x\left(a\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)+x^2\left(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{\int\left(-\frac{2a(b^2-4ac)gx^2}{c^2}-\frac{2a(b^2-4ac)(cf-2bg)}{c^3}+\frac{(-5gb^3+c(3bf+19ag)b+2c^3d-c^2(be+10af))a^2+(-5gb^4+3cfb^3-c(ce-24ag)b^2-c^2(cd+13\right.}{c^3(cx^4+bx^2+a)}}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(a\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)+x^2\left(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-b^2c(ce-24ag)-\frac{-b^3c(ce-34ag)-b^2c^2(19af+cd)+4abc^2(2ce-13ag)-4ac^3(cd-5af)-5b^5g+3b^4cf}{\sqrt{b^2-4ac}}-bc^2(13af+cd)+2ac^2(3c\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

input
```
Int[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(a*(2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g)) + (b^3*c*f
 + b*c^2*(c*d - 3*a*f) - b^4*g - b^2*c*(c*e - 4*a*g) + 2*a*c^2*(c*e - a*g)
)*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*a*(b^2 - 4*a*c)*(
c*f - 2*b*g)*x)/c^3 - (2*a*(b^2 - 4*a*c)*g*x^3)/(3*c^2) + (a*(3*b^3*c*f -
b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*c^2*(3*c*e - 7
*a*g) - (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 19*a*f) - 5*b^
5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (a*(3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*
b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*c^2*(3*c*e - 7*a*g) + (3*b^4*c*f - 4*a*
c^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 19*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g
) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]))/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.59

method	result
risch	$\frac{g\,x^{3}}{3c^{2}}-\frac{2bgx}{c^{3}}+\frac{fx}{c^{2}}+\frac{\frac{\left(2a^{2}g\,c^{2}-4a\,b^{2}cg+3ab\,c^{2}f-2a\,c^{3}e+b^{4}g-b^{3}cf+b^{2}c^{2}e-b\,c^{3}d\right)x^{3}}{8ac-2b^{2}}-\frac{a\left(3abcg-2a\,c^{2}f-b^{3}g+b^{2}cf-b\,c^{2}e+2c^{3}d\right)x}{2\left(4ac-b^{2}\right)}}{c^{3}\left(c\,x^{4}+b\,x^{2}+a\right)}+$
default	$-\frac{-\frac{1}{3}x^{3}cg+2bgx-cfx}{c^{3}}+\frac{\frac{\left(2a^{2}g\,c^{2}-4a\,b^{2}cg+3ab\,c^{2}f-2a\,c^{3}e+b^{4}g-b^{3}cf+b^{2}c^{2}e-b\,c^{3}d\right)x^{3}}{8ac-2b^{2}}-\frac{a\left(3abcg-2a\,c^{2}f-b^{3}g+b^{2}cf-b\,c^{2}e+2c^{3}d\right)x}{2\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+$

input
```
int(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*g*x^3/c^2-2/c^3*b*g*x+f*x/c^2+(1/2*(2*a^2*c^2*g-4*a*b^2*c*g+3*a*b*c^2*
f-2*a*c^3*e+b^4*g-b^3*c*f+b^2*c^2*e-b*c^3*d)/(4*a*c-b^2)*x^3-1/2*a*(3*a*b*
c*g-2*a*c^2*f-b^3*g+b^2*c*f-b*c^2*e+2*c^3*d)/(4*a*c-b^2)*x)/c^3/(c*x^4+b*x
^2+a)+1/4/c^3*sum((-(14*a^2*c^2*g-24*a*b^2*c*g+13*a*b*c^2*f-6*a*c^3*e+5*b^
4*g-3*b^3*c*f+b^2*c^2*e+b*c^3*d)/(4*a*c-b^2)*_R^2+a*(19*a*b*c*g-10*a*c^2*f
-5*b^3*g+3*b^2*c*f-b*c^2*e+2*c^3*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),
_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(gx^6+fx^4+ex^2+d)x^4}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
1/2*((b*c^3*d - (b^2*c^2 - 2*a*c^3)*e + (b^3*c - 3*a*b*c^2)*f - (b^4 - 4*a
*b^2*c + 2*a^2*c^2)*g)*x^3 + (2*a*c^3*d - a*b*c^2*e + (a*b^2*c - 2*a^2*c^2
)*f - (a*b^3 - 3*a^2*b*c)*g)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^
5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*integrate(-(2*a*c^3*d - a*b*c^2*
e - (b*c^3*d + (b^2*c^2 - 6*a*c^3)*e - (3*b^3*c - 13*a*b*c^2)*f + (5*b^4 -
 24*a*b^2*c + 14*a^2*c^2)*g)*x^2 + (3*a*b^2*c - 10*a^2*c^2)*f - (5*a*b^3 -
 19*a^2*b*c)*g)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 - 4*a*c^4) + 1/3*(c*g*x^3
 + 3*(c*f - 2*b*g)*x)/c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10752 vs. 2(550) = 1100.

Time = 1.52 (sec) , antiderivative size = 10752, normalized size of antiderivative = 18.10

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c^3*d*x^3 - b^2*c^2*e*x^3 + 2*a*c^3*e*x^3 + b^3*c*f*x^3 - 3*a*b*c^2
*f*x^3 - b^4*g*x^3 + 4*a*b^2*c*g*x^3 - 2*a^2*c^2*g*x^3 + 2*a*c^3*d*x - a*b
*c^2*e*x + a*b^2*c*f*x - 2*a^2*c^2*f*x - a*b^3*g*x + 3*a^2*b*c*g*x)/((b^2*
c^3 - 4*a*c^4)*(c*x^4 + b*x^2 + a)) + 1/16*((2*b^3*c^5 - 8*a*b*c^6 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*(b^2
*c^3 - 4*a*c^4)^2*d + (2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*
c)*a*c^5)*(b^2*c^3 - 4*a*c^4)^2*e - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*
c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c +
25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52...
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 47339, normalized size of antiderivative = 79.70

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x^3*(b^4*g + b^2*c^2*e + 2*a^2*c^2*g - 2*a*c^3*e - b*c^3*d - b^3*c*f + 3
*a*b*c^2*f - 4*a*b^2*c*g))/(2*(4*a*c - b^2)) + (x*(2*a^2*c^2*f - 2*a*c^3*d
 + a*b^3*g + a*b*c^2*e - a*b^2*c*f - 3*a^2*b*c*g))/(2*(4*a*c - b^2)))/(a*c
^3 + c^4*x^4 + b*c^3*x^2) + x*(f/c^2 - (2*b*g)/c^3) + atan(((((2048*a^4*c^
10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*
b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 1
0752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b
^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c
^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a
*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-
(4*a*c - b^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a
*c - b^2)^9)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^
9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*
a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 +
 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a
^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*
b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1
/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767
*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 21504
0*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2...
```

Reduce [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 11215, normalized size of antiderivative = 18.88

$$\int \frac{x^4(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output

```
(168*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**3*g - 174*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**2*g + 96*sqrt(a)*sqrt(2*sqrt(c)*
sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a**2*b*c**3*f + 168*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b
))*a**2*b*c**3*g*x**2 - 72*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c*
*4*e + 168*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a
) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**4*g*x**4 + 30*s
qrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*c*g - 18*sqrt(a)*sqrt(2*sqrt
(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*s
qrt(c)*sqrt(a) + b))*a*b**3*c**2*f - 174*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a*b**3*c**2*g*x**2 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*
*2*c**3*e + 96*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**3*f*x*...
```

3.128 $\int \frac{x^2\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 471

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \frac{gx}{c^2}$$
$$-\frac{x(bc(cd+af)-ab^2g-2ac(ce-ag)+(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag))x^2)}{2c^2(b^2-4ac)(a+bx^2+cx^4)}$$
$$-\frac{\left(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag)+\frac{b^3cf-4bc^2(cd+2af)-3b^4g+4ac^2(ce-5ag)+b^2c(ce+19ag)}{\sqrt{b^2-4ac}}\right)\arctan(\cdots}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(2c^3d-c^2(be-6af)+3b^3g-bc(bf+13ag)-\frac{b^3cf-4bc^2(cd+2af)-3b^4g+4ac^2(ce-5ag)+b^2c(ce+19ag)}{\sqrt{b^2-4ac}}\right)\arctan(\cdots}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
g*x/c^2-1/2*x*(b*c*(a*f+c*d)-a*b^2*g-2*a*c*(-a*g+c*e)+(2*c^3*d-c^2*(2*a*f+
b*e)-b^3*g+b*c*(3*a*g+b*f))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*(2*c
^3*d-c^2*(-6*a*f+b*e)+3*b^3*g-b*c*(13*a*g+b*f)+(b^3*c*f-4*b*c^2*(2*a*f+c*d
)-3*b^4*g+4*a*c^2*(-5*a*g+c*e)+b^2*c*(19*a*g+c*e))/(-4*a*c+b^2)^(1/2))*arc
tan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(-4*a*
c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*(2*c^3*d-c^2*(-6*a*f+b*e)+3*b^3*g-
b*c*(13*a*g+b*f)-(b^3*c*f-4*b*c^2*(2*a*f+c*d)-3*b^4*g+4*a*c^2*(-5*a*g+c*e)
+b^2*c*(19*a*g+c*e))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a
*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.22

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{4\sqrt{c}gx - \frac{2\sqrt{c}x\left(-b^3gx^2+b^2(-ag+cfx^2)+2c(a^2g+c^2dx^2-ac(e+fx^2))+bc(c(d-ex^2)+a(f+3gx^2))\right)}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4g+b^2c\left(ce-\sqrt{b^2-4ac}f\right.\right.}{}}{}$$

input
```
Integrate[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(4*Sqrt[c]*g*x - (2*Sqrt[c]*x*(-(b^3*g*x^2) + b^2*(-(a*g) + c*f*x^2) + 2*c
*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2)) + b*c*(c*(d - e*x^2) + a*(f + 3*g*x
^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*g + b^2*c*(c
*e - Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*
e + 3*a*Sqrt[b^2 - 4*a*c]*f - 10*a^2*g) + b^3*(c*f + 3*Sqrt[b^2 - 4*a*c]*g
) - b*c*(4*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f + 13*a*Sqrt[b^2 - 4*a*c
]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a
*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*g - b^2*c*(c*e +
Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + 3
*a*Sqrt[b^2 - 4*a*c]*f + 10*a^2*g) + b^3*(-(c*f) + 3*Sqrt[b^2 - 4*a*c]*g)
+ b*c*(4*c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f - 13*a*Sqrt[b^2 - 4*a*c]*
g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

Rubi [A] (verified)

Time = 3.99 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2197, 25, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \ 2197$$

$$-\frac{\int -\frac{-2a\left(4a-\frac{b^2}{c}\right)gx^4-\frac{a\left(gb^3-c(bf+5ag)b+2c^3d-c^2(be-6af)\right)x^2}{c^2}+\frac{a\left(-agb^2+c(cd+af)b-2ac(ce-ag)\right)}{c^2}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-2a\left(4a-\frac{b^2}{c}\right)gx^4-\frac{a\left(gb^3-c(bf+5ag)b+2c^3d-c^2(be-6af)\right)x^2}{c^2}+\frac{a\left(-agb^2+c(cd+af)b-2ac(ce-ag)\right)}{c^2}}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 2205$$

$$\frac{\int \left(\frac{2a(b^2-4ac)g}{c^2}+\frac{a(-3agb^2+c(cd+af)b-2ac(ce-5ag))-a(3gb^3-c(bf+13ag)b+2c^3d-c^2(be-6af))x^2}{c^2(cx^4+bx^2+a)}\right)dx}{2a\left(b^2-4ac\right)}-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{a\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(19ag+ce)-4bc^2(2af+cd)+4ac^2(ce-5ag)-3b^4g+b^3cf}{\sqrt{b^2-4ac}}-c^2(be-6af)-bc(13ag+bf)+3b^3g+2c^3d\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{a\arctan\left(\frac{\sqrt{}}{\sqrt{\sqrt{}}}\right.}{}}{2a\left(b^2-4ac\right.}-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(b*c*(c*d + a*f) - a*b^2*g - 2*a*c*(c*e - a*g) + (2*c^3*d - c^2*(b
*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*x^2))/(c^2*(b^2 - 4*a*c)*(a + b*x
^2 + c*x^4)) + ((2*a*(b^2 - 4*a*c)*g*x)/c^2 - (a*(2*c^3*d - c^2*(b*e - 6*a
*f) + 3*b^3*g - b*c*(b*f + 13*a*g) + (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*
b^4*g + 4*a*c^2*(c*e - 5*a*g) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - (a*(2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g -
b*c*(b*f + 13*a*g) - (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*
(c*e - 5*a*g) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.58

method	result
risch	$\frac{gx}{c^2}+\frac{\frac{\left(3abcg-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d\right)x^3}{8ac-2b^2}+\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x}{8ac-2b^2}}{c^2\left(c\,x^4+b\,x^2+a\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\left(-\frac{\left(13abcg-\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{gx}{c^2}-\frac{-\frac{\left(3abcg-2a\,c^2f-b^3g+b^2cf-b\,c^2e+2c^3d\right)x^3}{2\left(4ac-b^2\right)}-\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(13abcg\sqrt{-4ac+b^2}-6a\,c^2f\sqrt{-4ac+b^2}-3b\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
g*x/c^2+(1/2*(3*a*b*c*g-2*a*c^2*f-b^3*g+b^2*c*f-b*c^2*e+2*c^3*d)/(4*a*c-b^
2)*x^3+1/2*(2*a^2*c*g-a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)/(4*a*c-b^2)*x)/c^
2/(c*x^4+b*x^2+a)+1/4/c^2*sum((-(13*a*b*c*g-6*a*c^2*f-3*b^3*g+b^2*c*f+b*c^
2*e-2*c^3*d)/(4*a*c-b^2)*_R^2-(10*a^2*c*g-3*a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^
2*d)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23774 vs. 2(430) = 860.

Time = 171.02 (sec) , antiderivative size = 23774, normalized size of antiderivative = 50.48

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(gx^6+fx^4+ex^2+d)x^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
-1/2*((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*g)*x^3 +
(b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x)/(a*b^2*c^2 - 4*a^
2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + g*x/c^2 + 1
/2*integrate((b*c^2*d - 2*a*c^2*e + a*b*c*f - (2*c^3*d - b*c^2*e - (b^2*c
- 6*a*c^2)*f + (3*b^3 - 13*a*b*c)*g)*x^2 - (3*a*b^2 - 10*a^2*c)*g)/(c*x^4
+ b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9152 vs. $2(430) = 860$.

Time = 1.38 (sec) , antiderivative size = 9152, normalized size of antiderivative = 19.43

$$\int \frac{x^2(d + ex^2 + fx^4 + gx^6)}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
g*x/c^2 - 1/2*(2*c^3*d*x^3 - b*c^2*e*x^3 + b^2*c*f*x^3 - 2*a*c^2*f*x^3 - b
^3*g*x^3 + 3*a*b*c*g*x^3 + b*c^2*d*x - 2*a*c^2*e*x + a*b*c*f*x - a*b^2*g*x
 + 2*a^2*c*g*x)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/16*(2*(2*b^2
*c^5 - 8*a*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c
^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^5 - 2*(b^2 - 4*
a*c)*c^5)*(b^2*c^2 - 4*a*c^3)^2*d - (2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2 - 4
*a*c^3)^2*e - (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^...
```

Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 36589, normalized size of antiderivative = 77.68

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x^3*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(2*(4
*a*c - b^2)) + (x*(b*c^2*d - 2*a*c^2*e - a*b^2*g + 2*a^2*c*g + a*b*c*f))/(
2*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7*g
 - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e +
 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*
c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*
a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^
8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(
(c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b
*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9
)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2
)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2
 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2
*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c
^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 1
0656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*
a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g
 - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d
*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10
*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 1...
```

Reduce [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 9418, normalized size of antiderivative = 20.00

$$\int \frac{x^2(d+ex^2+fx^4+gx^6)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
(32*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**2*g - 24*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**3*f - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a
) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*a**2*b**3*c*g + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*
b**2*c**2*f + 32*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**2*g*
x**2 + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**3*e - 24*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**3*f*x**2 + 32*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**2*b*c**3*g*x**4 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt
(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*s
qrt(a) + b))*a**2*c**4*d - 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2
*c**4*f*x**4 - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*c*g*x**...
```

3.129 $\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 449

$$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2} dx = \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+(bc(cd+af)-ab^2g-2ac(ce-ag))x^2\right)}{2ac(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b(cd+af)+\frac{ab^2g}{c}-2a(ce+3ag)+\frac{b^2c(cd-af)-4ac^2(3cd+af)-ab^3g+4abc(ce+2ag)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b(cd+af)+\frac{ab^2g}{c}-2a(ce+3ag)-\frac{b^2c(cd-af)-4ac^2(3cd+af)-ab^3g+4abc(ce+2ag)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
1/2*x*(c*(b^2*d-2*a*(-a*f+c*d)-a*b*(a*g+c*e)/c)+(b*c*(a*f+c*d)-a*b^2*g-2*a
*c*(-a*g+c*e))*x^2)/a/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*(b*(a*f+c*d)+a*b^
2*g/c-2*a*(3*a*g+c*e)+(b^2*c*(-a*f+c*d)-4*a*c^2*(a*f+3*c*d)-a*b^3*g+4*a*b*
c*(2*a*g+c*e))/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b
^2)^(1/2))^(1/2))*2^(1/2)/a/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2)+1/4*(b*(a*f+c*d)+a*b^2*g/c-2*a*(3*a*g+c*e)-(b^2*c*(-a*f+c*d)-4*a*c^2*(
a*f+3*c*d)-a*b^3*g+4*a*b*c*(2*a*g+c*e))/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/
2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/c^(1/2)/(-4*a*c+b^2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.14

$$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{\frac{2\sqrt{c}x(b(-ace-a^2g+c^2dx^2+acfx^2)+b^2(cd-agx^2)+2ac(-c(d+ex^2)+a(f+gx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-ab^3g+bc\left(c\sqrt{b^2-4ac}d+4ace+a\sqrt{b^2-4ac}f+8a^2g\right.\right.}{}}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*Sqrt[c]*x*(b*(-(a*c*e) - a^2*g + c^2*d*x^2 + a*c*f*x^2) + b^2*(c*d - a
*g*x^2) + 2*a*c*(-(c*(d + e*x^2)) + a*(f + g*x^2))))/((b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*g) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c
*e + a*Sqrt[b^2 - 4*a*c]*f + 8*a^2*g) + b^2*(c^2*d - a*c*f + a*Sqrt[b^2 -
4*a*c]*g) - 2*a*c*(6*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f + 3*a*Sqrt[b^
2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a*b^3*g + b*c*
(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f - 8*a^2*g) + 2*a*
c*(6*c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f - 3*a*Sqrt[b^2 - 4*a*c]*g) +
b^2*(-(c^2*d) + a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/(4*a*c^(3/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2206, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 2206}$$

$$\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{db^2+\frac{a(ce+ag)b}{c}+\left(\frac{agb^2}{c}+(cd+af)b-2a(ce+3ag)\right)x^2-2a(3cd+af)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{db^2+\frac{a(ce+ag)b}{c}+\left(\frac{agb^2}{c}+(cd+af)b-2a(ce+3ag)\right)x^2-2a(3cd+af)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{\frac{1}{2}\left(\frac{ab^2g}{c}+\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{ab^2}{c}\right.}{2a\left(b^2-4ac\right)}$$

$$\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2g}{c}+\frac{-ab^3g+b^2c(cd-af)+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+b(af+cd)-2a(3ag+ce)\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{ab^2g}{c}-\right.}{}}{2a\left(b^2-4ac\right)}$$

$$\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(c*(b^2*d - 2*a*(c*d - a*f) - (a*b*(c*e + a*g))/c) + (b*c*(c*d + a*f) -
 a*b^2*g - 2*a*c*(c*e - a*g))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (((b*(c*d + a*f) + (a*b^2*g)/c - 2*a*(c*e + 3*a*g) + (b^2*c*(c*d - a
*f) - 4*a*c^2*(3*c*d + a*f) - a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2
 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*f) + (a*b^2*g)/c -
 2*a*(c*e + 3*a*g) - (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f) - a*b^3*g
+ 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
]))/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x^3}{2a\left(4ac-b^2\right)c}+\frac{\left(a^2bg-2a^2cf+abce+2a\,c^2d-b^2cd\right)x}{2ac\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{\left(6a^2cg-a\,b^2g-abcf+\right.}{4ac-}\right.}{}}{}$
default	$\frac{-\frac{\left(2a^2cg-a\,b^2g+abcf-2a\,c^2e+b\,c^2d\right)x^3}{2a\left(4ac-b^2\right)c}+\frac{\left(a^2bg-2a^2cf+abce+2a\,c^2d-b^2cd\right)x}{2ac\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(6a^2cg\sqrt{-4ac+b^2}-a\,b^2g\sqrt{-4ac+b^2}-abcf\sqrt{-4ac+b^2}+2a\right.}{}}{}$

input
```
int((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a*(2*a^2*c*g-a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)/(4*a*c-b^2)/c*x^3+1/
2*(a^2*b*g-2*a^2*c*f+a*b*c*e+2*a*c^2*d-b^2*c*d)/a/c/(4*a*c-b^2)*x)/(c*x^4+
b*x^2+a)+1/4/a/c*sum(((6*a^2*c*g-a*b^2*g-a*b*c*f+2*a*c^2*e-b*c^2*d)/(4*a*c
-b^2)*_R^2-(a^2*b*g-2*a^2*c*f+a*b*c*e-6*a*c^2*d+b^2*c*d)/(4*a*c-b^2))/(2*_
R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19375 vs. 2(408) = 816.

Time = 140.97 (sec) , antiderivative size = 19375, normalized size of antiderivative = 43.15

$$\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{gx^6+fx^4+ex^2+d}{\left(cx^4+bx^2+a\right)^2}\,dx$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x^3 - (a*b*c*e
- 2*a^2*c*f + a^2*b*g - (b^2*c - 2*a*c^2)*d)*x)/(a^2*b^2*c - 4*a^3*c^2 + (
a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2) - 1/2*integrate(
-(a*b*c*e - 2*a^2*c*f + a^2*b*g + (b*c^2*d - 2*a*c^2*e + a*b*c*f + (a*b^2
- 6*a^2*c)*g)*x^2 + (b^2*c - 6*a*c^2)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c
- 4*a^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8905 vs. $2(408) = 816$.

Time = 1.20 (sec) , antiderivative size = 8905, normalized size of antiderivative = 19.83

$$\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c^2*d*x^3 - 2*a*c^2*e*x^3 + a*b*c*f*x^3 - a*b^2*g*x^3 + 2*a^2*c*g*x
^3 + b^2*c*d*x - 2*a*c^2*d*x - a*b*c*e*x + 2*a^2*c*f*x - a^2*b*g*x)/((c*x^
4 + b*x^2 + a)*(a*b^2*c - 4*a^2*c^2)) + 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a
*b^2*c - 4*a^2*c^2)^2*d - 2*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2
*c^2)^2*e + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^2*
f + (2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr...
```

Mupad [B] (verification not implemented)

Time = 24.19 (sec) , antiderivative size = 32587, normalized size of antiderivative = 72.58

$$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x*(2*a*c^2*d - b^2*c*d + a^2*b*g - 2*a^2*c*f + a*b*c*e))/(2*a*c*(4*a*c -
 b^2)) - (x^3*(b*c^2*d - 2*a*c^2*e - a*b^2*g + 2*a^2*c*g + a*b*c*f))/(2*a*
c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((6144*a^5*c^7*d + 2048*a^6
*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*
a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f +
 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c
^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*
c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) - (x
*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9
*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^
2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)
^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2
 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^
2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^
9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*
c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2
*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 92
16*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e +
3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^
4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*...
```

Reduce [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 8810, normalized size of antiderivative = 19.62

$$\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
( - 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*c**2*g + 2*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c*g + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt
(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*s
qrt(a) + b))*a**3*b*c**2*f - 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*3*b*c**2*g*x**2 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**3*e -
 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**3*g*x**4 + 2*sqrt(a)*s
qrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x
)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c*g*x**2 - 2*sqrt(a)*sqrt(2*sqrt(
c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sq
rt(c)*sqrt(a) + b))*a**2*b**2*c**2*e + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a**2*b**2*c**2*f*x**2 + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a
**2*b**2*c**2*g*x**4 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*...
```

3.130 $\int \frac{d+ex^2+fx^4+gx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 460

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2(a+bx^2+cx^4)^2}\,dx = -\frac{d}{a^2x} - \frac{x\left(b^3d-ab^2e-ab(3cd-af)+2a^2(ce-ag)+\left(b^2cd-2ac(cd-af)-ab(ce+ag)\right)x^2\right)}{2a^2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^2cd-2ac(5cd-af)-ab(ce+ag)+\frac{3b^3cd-4abc(4cd+af)-ab^2(ce-ag)+4a^2c(3ce+ag)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^2cd-2ac(5cd-af)-ab(ce+ag)-\frac{3b^3cd-4abc(4cd+af)-ab^2(ce-ag)+4a^2c(3ce+ag)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
-d/a^2/x-1/2*x*(b^3*d-a*b^2*e-a*b*(-a*f+3*c*d)+2*a^2*(-a*g+c*e)+(b^2*c*d-2
*a*c*(-a*f+c*d)-a*b*(a*g+c*e))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*(
3*b^2*c*d-2*a*c*(-a*f+5*c*d)-a*b*(a*g+c*e)+(3*b^3*c*d-4*a*b*c*(a*f+4*c*d)-
a*b^2*(-a*g+c*e)+4*a^2*c*(a*g+3*c*e))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c
^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/c^(1/2)/(-4*a*c+b^2)/(b
-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*(3*b^2*c*d-2*a*c*(-a*f+5*c*d)-a*b*(a*g+c*e)
-(3*b^3*c*d-4*a*b*c*(a*f+4*c*d)-a*b^2*(-a*g+c*e)+4*a^2*c*(a*g+3*c*e))/(-4*
a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(
1/2)/a^2/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.15

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx =$$

$$-\frac{\frac{4d}{x}-\frac{2x\left(-b^3d+b^2\left(ae-cdx^2\right)+ab\left(3cd-af+cex^2+agx^2\right)+2a\left(a^2g+c^2dx^2-ac\left(e+fx^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(3b^3cd+b^2\left(3c\sqrt{b^2-4ac}d-ace+a^2g\right)+2a\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/4*((4*d)/x - (2*x*(-(b^3*d) + b^2*(a*e - c*d*x^2) + a*b*(3*c*d - a*f +
c*e*x^2 + a*g*x^2) + 2*a*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2))))/((b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a
*c]*d - a*c*e + a^2*g) + 2*a*c*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e + a*Sqr
t[b^2 - 4*a*c]*f + 2*a^2*g) - a*b*(16*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 4*a*
c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[2]*(-3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]*d + a*c*e - a^2*g) - 2*a*
c*(5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 2*a^2*g) +
a*b*(16*c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f - a*Sqrt[b^2 - 4*a*c]*g))*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a^2
```

Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \text{ 2198}$$

$$-\frac{\int -\frac{-\left(\left(\frac{cdb^2}{a}-c(2cd+be)+a(2cf-bg)\right)x^4\right)-\left(\frac{db^3}{a}-(5cd+be)b+a(6ce-bf)+2a^2g\right)x^2+2\left(b^2-4ac\right)d}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-\frac{\left(cdb^2-a(ce+ag)b-2ac(cd-af)\right)x^4}{a}-\left(\frac{db^3}{a}-eb^2-5cdb-afb+6ace+2a^2g\right)x^2+2\left(b^2-4ac\right)d}{x^2\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 2195}$$

$$\frac{\int \left(\frac{-3db^3+aeb^2+a(13cd+af)b-\left(3cdb^2-a(ce+ag)b-2ac(5cd-af)\right)x^2-2a^2(3ce+ag)}{a\left(cx^4+bx^2+a\right)}-\frac{2\left(4ac-b^2\right)d}{ax^2}\right)dx}{2a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2c(ag+3ce)-ab^2(ce-ag)-4abc(af+4cd)+3b^3cd}{\sqrt{b^2-4ac}}-ab(ag+ce)-2ac(5cd-af)+3b^2cd\right)}{\sqrt{2}a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\Big(-}{}}{2a\left(b^2-4ac\right)}$$

$$\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input

```
Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/2*(x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f) - 2*a^2*g) + (b^
2*c*d - 2*a*c*(c*d - a*f) - a*b*(c*e + a*g))*x^2))/(a^2*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + ((-2*(b^2 - 4*a*c)*d)/(a*x) - ((3*b^2*c*d - 2*a*c*(5*c*
d - a*f) - a*b*(c*e + a*g) + (3*b^3*c*d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c
*e - a*g) + 4*a^2*c*(3*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b - Sqrt[b^2
- 4*a*c]]) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e + a*g) - (3*b^3*
c*d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*a*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.20

method	result
default	$-\frac{d}{a^2x}+\frac{-\frac{\left(a^2bg-2a^2cf+abce+2a\,c^2d-b^2cd\right)x^3}{2\left(4ac-b^2\right)}-\frac{\left(2a^3g-a^2bf-2a^2ce+a\,b^2e+3abcd-b^3d\right)x}{2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{2c\left(\frac{\left(-a^2bg\sqrt{-4ac+b^2}+2a^2cf\sqrt{-4ac+b^2}-\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-d/a^2/x+1/a^2*((-1/2*(a^2*b*g-2*a^2*c*f+a*b*c*e+2*a*c^2*d-b^2*c*d)/(4*a*c
-b^2)*x^3-1/2*(2*a^3*g-a^2*b*f-2*a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(4*a*c-b
^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/8*(-a^2*b*g*(-4*a*c+b^2)^(1/2)+2
*a^2*c*f*(-4*a*c+b^2)^(1/2)-a*b*c*e*(-4*a*c+b^2)^(1/2)-10*a*c^2*d*(-4*a*c+
b^2)^(1/2)+3*b^2*c*d*(-4*a*c+b^2)^(1/2)-4*a^3*g*c-a^2*b^2*g+4*a^2*b*c*f-12
*a^2*c^2*e+a*b^2*c*e+16*a*b*c^2*d-3*b^3*c*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))-1/8*(-a^2*b*g*(-4*a*c+b^2)^(1/2)+2*a^2*c*f*(-4*a*c+b^2)^(1/2)-
a*b*c*e*(-4*a*c+b^2)^(1/2)-10*a*c^2*d*(-4*a*c+b^2)^(1/2)+3*b^2*c*d*(-4*a*c
+b^2)^(1/2)+4*a^3*g*c+a^2*b^2*g-4*a^2*b*c*f+12*a^2*c^2*e-a*b^2*c*e-16*a*b*
c^2*d+3*b^3*c*d)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23991 vs. 2(418) = 836.

Time = 133.38 (sec) , antiderivative size = 23991, normalized size of antiderivative = 52.15

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((g*x**6+f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \int \frac{gx^6 + fx^4 + ex^2 + d}{(cx^4 + bx^2 + a)^2 x^2} \, dx$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
1/2*((a*b*c*e - 2*a^2*c*f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b
*f - 2*a^3*g + (3*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2
- 4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (
a^3*b^2 - 4*a^4*c)*x) + 1/2*integrate((a^2*b*f - 2*a^3*g + (a*b*c*e - 2*a^
2*c*f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^2 - (3*b^3 - 13*a*b*c)*d + (a*
b^2 - 6*a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9167 vs. $2(418) = 836$.

Time = 1.12 (sec) , antiderivative size = 9167, normalized size of antiderivative = 19.93

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^2*c*d*x^4 - 10*a*c^2*d*x^4 - a*b*c*e*x^4 + 2*a^2*c*f*x^4 - a^2*b
*g*x^4 + 3*b^3*d*x^2 - 11*a*b*c*d*x^2 - a*b^2*e*x^2 + 2*a^2*c*e*x^2 + a^2*
b*f*x^2 - 2*a^3*g*x^2 + 2*a*b^2*d - 8*a^2*c*d)/((c*x^5 + b*x^3 + a*x)*(a^2
*b^2 - 4*a^3*c)) - 1/16*((6*b^4*c^3 - 44*a*b^2*c^4 + 80*a^2*c^5 - 3*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 22*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 40*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 20*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 6*(b^2 - 4*a*c)*b^2*c^3 + 20*(b^2
- 4*a*c)*a*c^4)*(a^2*b^2 - 4*a^3*c)^2*d - (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b
*c^3)*(a^2*b^2 - 4*a^3*c)^2*e + 2*(2*a^2*b^2*c^3 - 8*a^3*c^4 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 -...
```

Mupad [B] (verification not implemented)

Time = 25.88 (sec) , antiderivative size = 40860, normalized size of antiderivative = 88.83

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

output
```
atan(((((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/
2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*
e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5
*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b
^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3
*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 -
288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*
c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^
2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^
8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*
a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3
584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*
g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 2
2400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^
4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*
f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 22
40*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*
b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c
^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g
+ 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^...
```

Reduce [F]

$$\int \frac{d+ex^2+fx^4+gx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{g\,x^6+f\,x^4+e\,x^2+d}{x^2\left(c\,x^4+b\,x^2+a\right)^2}dx$$

input
```
int((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x)
```

3.131 $\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 537

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}$$

$$+\frac{x((b^2-2ac)(b^2d-abe-a(cd-af))-ab(bcd-a(ce-ag))+c(b^3d-ab^2e-ab(3cd-af)+2a^2(c}{2a^3(b^2-4ac)(a+bx^2+cx^4)}$$

$$+\frac{\sqrt{c}\left(5b^3d-3ab^2e-ab(19cd-af)+2a^2(5ce-ag)+\frac{5b^4d-3ab^3e+4a^2c(7cd-3af)-ab^2(29cd-af)+4a^2b(4ce+ag)}{\sqrt{b^2-4ac}}\right)}{2\sqrt{2}a^3(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\sqrt{c}\left(5b^3d-3ab^2e-ab(19cd-af)+2a^2(5ce-ag)-\frac{5b^4d-3ab^3e+4a^2c(7cd-3af)-ab^2(29cd-af)+4a^2b(4ce+ag)}{\sqrt{b^2-4ac}}\right)}{2\sqrt{2}a^3(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/3*d/a^2/x^3+(-a*e+2*b*d)/a^3/x+1/2*x*((-2*a*c+b^2)*(b^2*d-a*b*e-a*(-a*f
+c*d))-a*b*(b*c*d-a*(-a*g+c*e))+c*(b^3*d-a*b^2*e-a*b*(-a*f+3*c*d)+2*a^2*(-
a*g+c*e))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/2)*(5*b^3*d-3*a*b
^2*e-a*b*(-a*f+19*c*d)+2*a^2*(-a*g+5*c*e)+(5*b^4*d-3*a*b^3*e+4*a^2*c*(-3*a
*f+7*c*d)-a*b^2*(-a*f+29*c*d)+4*a^2*b*(a*g+4*c*e))/(-4*a*c+b^2)^(1/2))*arc
tan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/(-4*a*c+b^
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*c^(1/2)*(5*b^3*d-3*a*b^2*e-a*b*(-a*f+1
9*c*d)+2*a^2*(-a*g+5*c*e)-(5*b^4*d-3*a*b^3*e+4*a^2*c*(-3*a*f+7*c*d)-a*b^2*
(-a*f+29*c*d)+4*a^2*b*(a*g+4*c*e))/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1
/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/(-4*a*c+b^2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.14

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{4ad}{x^3}+\frac{24bd-12ae}{x}+\frac{6x\left(b^4d+b^3\left(-ae+cdx^2\right)+ab^2\left(af-c\left(4d+ex^2\right)\right)+ab\left(-a^2g-3c^2dx^2+ac\left(3e+fx^2\right)\right)+2a^2c\left(c\left(d+ex^2\right)-a\left(f+gx^2\right)\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+}{}$$

input
```
Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

output

```
((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
 + a*b^2*(a*f - c*(4*d + e*x^2)) + a*b*(-(a^2*g) - 3*c^2*d*x^2 + a*c*(3*e
+ f*x^2)) + 2*a^2*c*(c*(d + e*x^2) - a*(f + g*x^2))))/((b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d
 - 3*a*e) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-19*c*Sqr
t[b^2 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) - 2*a^2*(-1
4*c^2*d - 5*c*Sqrt[b^2 - 4*a*c]*e + 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(5*b^4*d - b^3*(5*Sqrt[b
^2 - 4*a*c]*d + 3*a*e) + a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a
*b*(19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g)
 + 2*a^2*(14*c^2*d - 5*c*Sqrt[b^2 - 4*a*c]*e - 6*a*c*f + a*Sqrt[b^2 - 4*a*
c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Rubi [A] (verified)

Time = 4.02 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2198, 25, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 2198}$$

$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{c\left(\frac{db^3}{a^2}-\frac{(3cd+be)b}{a}+fb+2ce-2ag\right)x^6+\frac{\left(db^4-aeb^3-a(6cd-af)b^2+a^2(5ce+ag)b+6a^2c(cd-af)\right)x^4}{a^2}-\frac{2\left(b^2-4ac\right)(bd-ae)x^2}{a}+2\left(b^2-4ac\right)d}{x^4\left(cx^4+bx^2+a\right)}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{c\left(\frac{db^3}{a^2}-\frac{(3cd+be)b}{a}+fb+2ce-2ag\right)x^6+\frac{\left(db^4-aeb^3-a(6cd-af)b^2+a^2(5ce+ag)b+6a^2c(cd-af)\right)x^4}{a^2}-\frac{2\left(b^2-4ac\right)(bd-ae)x^2}{a}+2(b^2-4ac)d}{x^4(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{2195}$$

$$\frac{\int \left(-\frac{2(4ac-b^2)d}{ax^4}+\frac{5db^4-3aeb^3-a(24cd-af)b^2+a^2(13ce+ag)b+c(5db^3-3aeb^2-a(19cd-af)b+2a^2(5ce-ag))x^2+2a^2c(7cd-3af)}{a^2(cx^4+bx^2+a)}-\frac{2(4ac}{}\right.}{2a\left(b^2-4ac\right)}$$

$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4a^2b(ag+4ce)+4a^2c(7cd-3af)-3ab^3e-ab^2(29cd-af)+5b^4d}{\sqrt{b^2-4ac}}+2a^2(5ce-ag)-3ab^2e-ab(19cd-af)+5b^3d\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan}{}}{}$$

$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f -
 a*(2*c*f + b*g)) + c*(b^3*d - a*b^2*e - a*b*(3*c*d - a*f) + 2*a^2*(c*e -
a*g))*x^2))/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((-2*(b^2 - 4*a*c)
*d)/(3*a*x^3) + (2*(b^2 - 4*a*c)*(2*b*d - a*e))/(a^2*x) + (Sqrt[c]*(5*b^3*
d - 3*a*b^2*e - a*b*(19*c*d - a*f) + 2*a^2*(5*c*e - a*g) + (5*b^4*d - 3*a*
b^3*e + 4*a^2*c*(7*c*d - 3*a*f) - a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e +
a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(5*b^3*d - 3*
a*b^2*e - a*b*(19*c*d - a*f) + 2*a^2*(5*c*e - a*g) - (5*b^4*d - 3*a*b^3*e
+ 4*a^2*c*(7*c*d - 3*a*f) - a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.17

method	result
default	$-\frac{d}{3a^2x^3}-\frac{ae-2bd}{a^3x}+\frac{\frac{c\left(2a^3g-a^2bf-2a^2ce+a\,b^2e+3abcd-b^3d\right)x^3}{8ac-2b^2}+\frac{\left(a^3bg+2a^3cf-a^2b^2f-3a^2bce-2a^2c^2d+a\,b^3e+4a\,b^2cd-b^4d\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+ \cdots 2c$
risch	Expression too large to display

input
```
int((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*d/a^2/x^3-(a*e-2*b*d)/a^3/x+1/a^3*((1/2*c*(2*a^3*g-a^2*b*f-2*a^2*c*e+
a*b^2*e+3*a*b*c*d-b^3*d)/(4*a*c-b^2)*x^3+1/2*(a^3*b*g+2*a^3*c*f-a^2*b^2*f-
3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*b^2*c*d-b^4*d)/(4*a*c-b^2)*x)/(c*x^4+b
*x^2+a)+2/(4*a*c-b^2)*c*(1/8*(2*a^3*g*(-4*a*c+b^2)^(1/2)-a^2*b*f*(-4*a*c+b
^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2)+19*a*
b*c*d*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+b^2)^(1/2)+4*a^3*b*g-12*a^3*c*f+a
^2*b^2*f+16*a^2*b*c*e+28*a^2*c^2*d-3*a*b^3*e-29*a*b^2*c*d+5*b^4*d)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(2*a^3*g*(-4*a*c+b^2)^(1/2)-a^2*b*f*(-
4*a*c+b^2)^(1/2)-10*a^2*c*e*(-4*a*c+b^2)^(1/2)+3*a*b^2*e*(-4*a*c+b^2)^(1/2
)+19*a*b*c*d*(-4*a*c+b^2)^(1/2)-5*b^3*d*(-4*a*c+b^2)^(1/2)-4*a^3*b*g+12*a^
3*c*f-a^2*b^2*f-16*a^2*b*c*e-28*a^2*c^2*d+3*a*b^3*e+29*a*b^2*c*d-5*b^4*d)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((g*x**6+f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2}\,dx=\int \frac{gx^6+fx^4+ex^2+d}{(cx^4+bx^2+a)^2x^4}\,dx$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima
")
```

output
```
1/6*(3*(a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10
*a^2*c^2)*e)*x^6 - (3*a^3*b*g - (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d + 3*(
3*a*b^3 - 11*a^2*b*c)*e - 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 - 4*a
^2*b*c)*d - 3*(a^2*b^2 - 4*a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*
b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x
^3) - 1/2*integrate(-(a^3*b*g + (a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b
*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*b^4 - 24*a*b^2*c + 14*a^2*c
^2)*d - (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6*a^3*c)*f)/(c*x^4 + b*x^2 +
 a), x)/(a^3*b^2 - 4*a^4*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10411 vs. $2(493) = 986$.

Time = 1.30 (sec) , antiderivative size = 10411, normalized size of antiderivative = 19.39

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4(a+bx^2+cx^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 - a*b^2*c*e*x^3 + 2*a^2*c^2*e*x^3 + a^2
*b*c*f*x^3 - 2*a^3*c*g*x^3 + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x - a*b
^3*e*x + 3*a^2*b*c*e*x + a^2*b^2*f*x - 2*a^3*c*f*x - a^3*b*g*x)/((a^3*b^2
- 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a
^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c +
10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*
c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d - (6*a*b^4*
c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(...
```

Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 51386, normalized size of antiderivative = 95.69

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^4 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

output
```
atan((((-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/
2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640
*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f
^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*
b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2
+ 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2
 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)
^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c
^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25
*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5
*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2
 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*
g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f
 + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1
/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*
a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*
g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)
^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) -
 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*
a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161...
```

Reduce [F]

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{g\,x^6+f\,x^4+e\,x^2+d}{x^4\left(c\,x^4+b\,x^2+a\right)^2}dx$$

input
```
int((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x)
```

3.132 $\int x^4\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 761

$$\int x^4\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx =$$

$$-\frac{(176b^4Bc-792ab^2Bc^2+462a^2Bc^3-128b^5C+3abc^2(319Ac-257aC)-24b^3c(11Ac-29aC))\,x\sqrt{a}}{3465c^{9/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$+\frac{x(88b^3Bc+33abBc^2-64b^4C-15ac^2(11Ac-5aC)-12b^2c(11Ac-3aC)+3c(88b^2Bc-77aBc^2-}{3465c^4}$$

$$-\frac{(22bBc-16b^2C-3c(11Ac-5aC))\,x(a+bx^2+cx^4)^{3/2}}{231c^3}$$

$$+\frac{(11Bc-8bC)x^3(a+bx^2+cx^4)^{3/2}}{99c^2}+\frac{Cx^5(a+bx^2+cx^4)^{3/2}}{11c}$$

$$+\frac{\sqrt[4]{a}(176b^4Bc-792ab^2Bc^2+462a^2Bc^3-128b^5C+3abc^2(319Ac-257aC)-24b^3c(11Ac-29aC))\,(}{3465c^{19/4}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(128b^4C-3ac^2\left(77\sqrt{a}B\sqrt{c}+55Ac-25aC\right)+24b^2c\left(11\sqrt{a}B\sqrt{c}+11Ac-13aC\right)+\right.}{}$$

output
```
-1/3465*(176*b^4*B*c-792*a*b^2*B*c^2+462*a^2*B*c^3-128*b^5*C+3*a*b*c^2*(31
9*A*c-257*C*a)-24*b^3*c*(11*A*c-29*C*a))*x*(c*x^4+b*x^2+a)^(1/2)/c^(9/2)/(
a^(1/2)+c^(1/2)*x^2)+1/3465*x*(88*b^3*B*c+33*a*b*B*c^2-64*b^4*C-15*a*c^2*(
11*A*c-5*C*a)-12*b^2*c*(11*A*c-3*C*a)+3*c*(88*b^2*B*c-77*a*B*c^2-64*b^3*C-
4*b*c*(33*A*c-29*C*a))*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^4-1/231*(22*B*b*c-16*b
^2*C-3*c*(11*A*c-5*C*a))*x*(c*x^4+b*x^2+a)^(3/2)/c^3+1/99*(11*B*c-8*C*b)*x
^3*(c*x^4+b*x^2+a)^(3/2)/c^2+1/11*C*x^5*(c*x^4+b*x^2+a)^(3/2)/c+1/3465*a^(
1/4)*(176*b^4*B*c-792*a*b^2*B*c^2+462*a^2*B*c^3-128*b^5*C+3*a*b*c^2*(319*A
*c-257*C*a)-24*b^3*c*(11*A*c-29*C*a))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+
a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(19/4)/(c*x^4+b*x^2+a)^(1/2)+1/6930*
a^(1/4)*(b+2*a^(1/2)*c^(1/2))*(128*b^4*C-3*a*c^2*(77*a^(1/2)*B*c^(1/2)+55*
A*c-25*a*C)+24*b^2*c*(11*a^(1/2)*B*c^(1/2)+11*A*c-13*a*C)+12*a^(1/2)*b*c^(
3/2)*(22*a^(1/2)*B*c^(1/2)-33*A*c+29*a*C)-16*b^3*(11*B*c+12*a^(1/2)*c^(1/2
)*C))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2
)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1
/2))/c^(19/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.21 (sec) , antiderivative size = 845, normalized size of antiderivative = 1.11

$$\int x^4\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)\left(-64b^4C+8b^3c(11B+6Cx^2)-2b^2c(66Ac-138aC+33Bcx^2+20cCx\right.}{}$$

input
```
Integrate[x^4*Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4)*(-64*b^4*C + 8*
b^3*c*(11*B + 6*C*x^2) - 2*b^2*c*(66*A*c - 138*a*C + 33*B*c*x^2 + 20*c*C*x
^4) + b*c^2*(-297*a*B + 99*A*c*x^2 - 157*a*C*x^2 + 55*B*c*x^4 + 35*c*C*x^6
) + c^2*(-150*a^2*C + 2*a*c*(165*A + 77*B*x^2 + 45*C*x^4) + 5*c^2*x^4*(99*
A + 77*B*x^2 + 63*C*x^4))) + I*(-b + Sqrt[b^2 - 4*a*c])*(-176*b^4*B*c + 79
2*a*b^2*B*c^2 - 462*a^2*B*c^3 + 128*b^5*C + 24*b^3*c*(11*A*c - 29*a*C) + 3
*a*b*c^2*(-319*A*c + 257*a*C))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-128*b^6*C + 3*
a*b^2*c^2*(407*A*c + 264*B*Sqrt[b^2 - 4*a*c] - 441*a*C) - 8*b^4*c*(33*A*c
+ 22*B*Sqrt[b^2 - 4*a*c] - 103*a*C) + 6*a^2*c^3*(-110*A*c - 77*B*Sqrt[b^2
- 4*a*c] + 50*a*C) + 16*b^5*(11*B*c + 8*Sqrt[b^2 - 4*a*c]*C) - 8*b^3*c*(12
1*a*B*c - 33*A*c*Sqrt[b^2 - 4*a*c] + 87*a*Sqrt[b^2 - 4*a*c]*C) + 3*a*b*c^2
*(352*a*B*c - 319*A*c*Sqrt[b^2 - 4*a*c] + 257*a*Sqrt[b^2 - 4*a*c]*C))*Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*
Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])])/(13860*c^5*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*
x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 718, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2199, 1596, 27, 1602, 27, 1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \text{2199}$$

$$\int x^4\left(\frac{1}{11}\left(11B-\frac{8bC}{c}\right)x^2+\frac{1}{11}\left(11A-\frac{5aC}{c}\right)\right)\sqrt{cx^4+bx^2+a}dx+\frac{Cx^5\left(a+bx^2+cx^4\right)^{3/2}}{11c}$$

$$\Big\downarrow \text{1596}$$

$$\frac{\int -\frac{x^4\left(\left(-48Cb^3+66Bcb^2-c(99Ac-157aC)b-154aBc^2\right)x^2+ac\left(-\frac{40Cb^2}{c}+55Bb-198Ac+90aC\right)\right)}{11c\sqrt{cx^4+bx^2+a}}dx}{63c}+$$
$$\frac{x^5\sqrt{a+bx^2+cx^4}\left(-45aC+99Ac-\frac{8b^2C}{c}+7x^2(11Bc-8bC)+11bB\right)}{693c}+$$
$$\frac{Cx^5\left(a+bx^2+cx^4\right)^{3/2}}{11c}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{x^4\left(\left(-48Cb^3+66Bcb^2-c(99Ac-157aC)b-154aBc^2\right)x^2+a\left(-40Cb^2+55Bcb-198Ac^2+90acC\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{693c^2}+$$
$$\frac{x^5\sqrt{a+bx^2+cx^4}\left(-45aC+99Ac-\frac{8b^2C}{c}+7x^2(11Bc-8bC)+11bB\right)}{693c}+$$
$$\frac{Cx^5\left(a+bx^2+cx^4\right)^{3/2}}{11c}$$

$\Big\downarrow$ 1602

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-bc(99Ac-157aC)-154aBc^2-48b^3C+66b^2Bc\right)}{5c}-\frac{\int \frac{3x^2\left(\left(-64Cb^4+88Bcb^3-12c(11Ac-23aC)b^2-297aBc^2b+30ac^2(11Ac-5aC)\right.\right.}{\sqrt{cx^4+bx^2+a}}}{5c}}{693c^2}$$
$$\frac{x^5\sqrt{a+bx^2+cx^4}\left(-45aC+99Ac-\frac{8b^2C}{c}+7x^2(11Bc-8bC)+11bB\right)}{693c}+$$
$$\frac{Cx^5\left(a+bx^2+cx^4\right)^{3/2}}{11c}$$

$\Big\downarrow$ 27

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-bc(99Ac-157aC)-154aBc^2-48b^3C+66b^2Bc\right)}{5c}-\frac{3\int \frac{x^2\left(\left(-64Cb^4+88Bcb^3-12c(11Ac-23aC)b^2-297aBc^2b+30ac^2(11Ac-5aC\right.\right.}{\sqrt{cx^4+bx^2+a}}}{5c}}{693c^2}$$
$$\frac{x^5\sqrt{a+bx^2+cx^4}\left(-45aC+99Ac-\frac{8b^2C}{c}+7x^2(11Bc-8bC)+11bB\right)}{693c}+$$
$$\frac{Cx^5\left(a+bx^2+cx^4\right)^{3/2}}{11c}$$

$\Big\downarrow$ 1602

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-bc(99Ac-157aC)-154aBc^2-48b^3C+66b^2Bc\right)}{5c}-\frac{3\left(\frac{x\sqrt{a+bx^2+cx^4}\left(-12b^2c(11Ac-23aC)+30ac^2(11Ac-5aC)-297abBc^2-64b^4\right.}{3c}\right.}{}}{}$$

$$\frac{x^5\sqrt{a+bx^2+cx^4}\left(-45aC+99Ac-\frac{8b^2C}{c}+7x^2(11Bc-8bC)+11bB\right)}{693c}+\frac{Cx^5\left(a+bx^2+cx^4\right)^{3/2}}{11c}$$

$\Big\downarrow$ 1511

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-bc(99Ac-157aC)-154aBc^2-48b^3C+66b^2Bc\right)}{5c}-\frac{3\left(\frac{x\sqrt{a+bx^2+cx^4}\left(-12b^2c(11Ac-23aC)+30ac^2(11Ac-5aC)-297abBc^2-64b^4\right.}{3c}\right.}{}}{}$$

$$\frac{x^5\sqrt{a+bx^2+cx^4}\left(-45aC+99Ac-\frac{8b^2C}{c}+7x^2(11Bc-8bC)+11bB\right)}{693c}+\frac{Cx^5\left(a+bx^2+cx^4\right)^{3/2}}{11c}$$

$\Big\downarrow$ 27

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-bc(99Ac-157aC)-154aBc^2-48b^3C+66b^2Bc\right)}{5c}-\frac{3\left(\frac{x\sqrt{a+bx^2+cx^4}\left(-12b^2c(11Ac-23aC)+30ac^2(11Ac-5aC)-297abBc^2-64b^4\right.}{3c}\right.}{}}{}$$

$$\frac{x^5\sqrt{a+bx^2+cx^4}\left(-45aC+99Ac-\frac{8b^2C}{c}+7x^2(11Bc-8bC)+11bB\right)}{693c}+\frac{Cx^5\left(a+bx^2+cx^4\right)^{3/2}}{11c}$$

$\Big\downarrow$ 1416

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-bc(99Ac-157aC)-154aBc^2-48b^3C+66b^2Bc\right)}{5c}-\frac{3\left(\frac{x\sqrt{a+bx^2+cx^4}\left(-12b^2c(11Ac-23aC)+30ac^2(11Ac-5aC)-297abBc^2-64b^4\right.}{3c}\right.}{}}{}$$

$$\frac{x^5\sqrt{a+bx^2+cx^4}\left(-45aC+99Ac-\frac{8b^2C}{c}+7x^2(11Bc-8bC)+11bB\right)}{693c}+\frac{Cx^5\left(a+bx^2+cx^4\right)^{3/2}}{11c}$$

$\Big\downarrow$ 1509

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-bc(99Ac-157aC)-154aBc^2-48b^3C+66b^2Bc\right)}{5c}-\frac{3\left(\frac{x\sqrt{a+bx^2+cx^4}\left(-12b^2c(11Ac-23aC)+30ac^2(11Ac-5aC)-297abBc^2-64b^4\right.}{3c}\right.}{}}{}$$

$$\frac{x^5\sqrt{a+bx^2+cx^4}\left(-45aC+99Ac-\frac{8b^2C}{c}+7x^2(11Bc-8bC)+11bB\right)}{693c}+\frac{Cx^5\left(a+bx^2+cx^4\right)^{3/2}}{11c}$$

input
```
Int[x^4*Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(x^5*(11*b*B + 99*A*c - 45*a*C - (8*b^2*C)/c + 7*(11*B*c - 8*b*C)*x^2)*Sqr
t[a + b*x^2 + c*x^4])/(693*c) + (C*x^5*(a + b*x^2 + c*x^4)^(3/2))/(11*c) -
 (((66*b^2*B*c - 154*a*B*c^2 - 48*b^3*C - b*c*(99*A*c - 157*a*C))*x^3*Sqrt
[a + b*x^2 + c*x^4])/(5*c) - (3*(((88*b^3*B*c - 297*a*b*B*c^2 - 64*b^4*C -
 12*b^2*c*(11*A*c - 23*a*C) + 30*a*c^2*(11*A*c - 5*a*C))*x*Sqrt[a + b*x^2
+ c*x^4])/(3*c) - (-(((176*b^4*B*c - 792*a*b^2*B*c^2 + 462*a^2*B*c^3 - 128
*b^5*C + 3*a*b*c^2*(319*A*c - 257*a*C) - 24*b^3*c*(11*A*c - 29*a*C))*(-((x
*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt
[a + b*x^2 + c*x^4])))/Sqrt[c]) - (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(128*b^
4*C - 3*a*c^2*(77*Sqrt[a]*B*Sqrt[c] + 55*A*c - 25*a*C) + 24*b^2*c*(11*Sqrt
[a]*B*Sqrt[c] + 11*A*c - 13*a*C) + 12*Sqrt[a]*b*c^(3/2)*(22*Sqrt[a]*B*Sqrt
[c] - 33*A*c + 29*a*C) - 16*b^3*(11*B*c + 12*Sqrt[a]*Sqrt[c]*C))*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)
*Sqrt[a + b*x^2 + c*x^4]))/(3*c)))/(5*c))/(693*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1596
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Simp[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a +
 b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c
*e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &
& NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[
p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 10.49 (sec) , antiderivative size = 899, normalized size of antiderivative = 1.18

method	result
elliptic	$\frac{C\,x^{9}\sqrt{c\,x^{4}+b\,x^{2}+a}}{11}+\frac{\left(Bc+\frac{Cb}{11}\right)x^{7}\sqrt{c\,x^{4}+b\,x^{2}+a}}{9c}+\frac{\left(Ac+Bb+\frac{2aC}{11}-\frac{8b\left(Bc+\frac{Cb}{11}\right)}{9c}\right)x^{5}\sqrt{c\,x^{4}+b\,x^{2}+a}}{7c}+\frac{\left(Ab+Ba-\frac{6b\left(Ac+\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^4*(c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*C*x^9*(c*x^4+b*x^2+a)^(1/2)+1/9*(B*c+1/11*C*b)/c*x^7*(c*x^4+b*x^2+a)^
(1/2)+1/7*(A*c+B*b+2/11*a*C-8/9*b/c*(B*c+1/11*C*b))/c*x^5*(c*x^4+b*x^2+a)^
(1/2)+1/5*(A*b+B*a-6/7*b/c*(A*c+B*b+2/11*a*C-8/9*b/c*(B*c+1/11*C*b))-7/9*a
/c*(B*c+1/11*C*b))/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(A*a-4/5*b/c*(A*b+B*a-6
/7*b/c*(A*c+B*b+2/11*a*C-8/9*b/c*(B*c+1/11*C*b))-7/9*a/c*(B*c+1/11*C*b))-5
/7*a/c*(A*c+B*b+2/11*a*C-8/9*b/c*(B*c+1/11*C*b)))/c*x*(c*x^4+b*x^2+a)^(1/2
)-1/12*a/c*(A*a-4/5*b/c*(A*b+B*a-6/7*b/c*(A*c+B*b+2/11*a*C-8/9*b/c*(B*c+1/
11*C*b))-7/9*a/c*(B*c+1/11*C*b))-5/7*a/c*(A*c+B*b+2/11*a*C-8/9*b/c*(B*c+1/
11*C*b)))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5*a/c*(A*b+B*a-6/7*b/c*(A*
c+B*b+2/11*a*C-8/9*b/c*(B*c+1/11*C*b))-7/9*a/c*(B*c+1/11*C*b))-2/3*b/c*(A*
a-4/5*b/c*(A*b+B*a-6/7*b/c*(A*c+B*b+2/11*a*C-8/9*b/c*(B*c+1/11*C*b))-7/9*a
/c*(B*c+1/11*C*b))-5/7*a/c*(A*c+B*b+2/11*a*C-8/9*b/c*(B*c+1/11*C*b))))*a*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b
+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.15

$$\int x^4\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/6930*(sqrt(1/2)*((128*C*b^5*c - 33*(14*B*a^2 + 29*A*a*b)*c^4 + 3*(257*C*
a^2*b + 264*B*a*b^2 + 88*A*b^3)*c^3 - 8*(87*C*a*b^3 + 22*B*b^4)*c^2)*x*sqr
t((b^2 - 4*a*c)/c^2) - (128*C*b^6 - 33*(14*B*a^2*b + 29*A*a*b^2)*c^3 + 3*(
257*C*a^2*b^2 + 264*B*a*b^3 + 88*A*b^4)*c^2 - 8*(87*C*a*b^4 + 22*B*b^5)*c)
*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt
(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a
*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((128*C*b^5*c + 330*A*a*c^5 - 3
*(2*(77*B + 25*C)*a^2 + 11*(29*A + 9*B)*a*b + 44*A*b^2)*c^4 + (771*C*a^2*b
 + 12*(66*B + 23*C)*a*b^2 + 88*(3*A + B)*b^3)*c^3 - 8*(87*C*a*b^3 + 2*(11*
B + 4*C)*b^4)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (128*C*b^6 - 330*A*a*b*c^4
- 3*(2*(77*B - 25*C)*a^2*b + 11*(29*A - 9*B)*a*b^2 - 44*A*b^3)*c^3 + (771*
C*a^2*b^2 + 12*(66*B - 23*C)*a*b^3 + 88*(3*A - B)*b^4)*c^2 - 8*(87*C*a*b^4
 + 2*(11*B - 4*C)*b^5)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/
c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x),
 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(315*C*c^6*x^1
0 + 35*(C*b*c^5 + 11*B*c^6)*x^8 + 128*C*b^5*c - 5*(8*C*b^2*c^4 - 99*A*c^6
- (18*C*a + 11*B*b)*c^5)*x^6 - 33*(14*B*a^2 + 29*A*a*b)*c^4 + (48*C*b^3*c^
3 + 11*(14*B*a + 9*A*b)*c^5 - (157*C*a*b + 66*B*b^2)*c^4)*x^4 + 3*(257*C*a
^2*b + 264*B*a*b^2 + 88*A*b^3)*c^3 - 8*(87*C*a*b^3 + 22*B*b^4)*c^2 - (64*C
*b^4*c^2 - 330*A*a*c^5 + 3*(50*C*a^2 + 99*B*a*b + 44*A*b^2)*c^4 - 4*(69...
```

Sympy [F]

$$\int x^4\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int x^4\left(A+Bx^2+Cx^4\right)\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**4*(c*x**4+b*x**2+a)**(1/2)*(C*x**4+B*x**2+A),x)
```

output
```
Integral(x**4*(A + B*x**2 + C*x**4)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int x^4\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int (Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}x^4\,dx$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int (Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}x^4\,dx$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int x^4\,\left(C\,x^4+B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(x^4*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^4*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{60\sqrt{c\,x^4+b\,x^2+a}\,a^2c^2x-51\sqrt{c\,x^4+b\,x^2+a}\,a\,b^2cx+32\sqrt{c\,x^4+b\,x^2+a}\,ab\,c^2x^3+195\sqrt{c\,x^4+b\,x^2+}}{}$$

input
```
int(x^4*(c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x)
```

output
```
(60*sqrt(a + b*x**2 + c*x**4)*a**2*c**2*x - 51*sqrt(a + b*x**2 + c*x**4)*a
*b**2*c*x + 32*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*x**3 + 195*sqrt(a + b*x*
*2 + c*x**4)*a*c**3*x**5 + 8*sqrt(a + b*x**2 + c*x**4)*b**4*x - 6*sqrt(a +
 b*x**2 + c*x**4)*b**3*c*x**3 + 5*sqrt(a + b*x**2 + c*x**4)*b**2*c**2*x**5
 + 140*sqrt(a + b*x**2 + c*x**4)*b*c**3*x**7 + 105*sqrt(a + b*x**2 + c*x**
4)*c**4*x**9 - 60*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a
**3*c**2 + 51*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**2*
b**2*c - 8*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b**4 -
 216*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a**2*b*
c**2 + 120*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a
*b**3*c - 16*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)
*b**5)/(1155*c**3)
```

3.133 $\int x^2\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 623

$$\int x^2\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$= \frac{\left(24b^3Bc-87abBc^2-16b^4C-6b^2c(7Ac-12aC)+42ac^2(3Ac-aC)\right)x\sqrt{a+bx^2+cx^4}}{315c^{7/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{x\left(12b^2B+15aBc-\frac{8b^3C}{c}-3b(7Ac+aC)+3\left(12bBc-8b^2C-7c(3Ac-aC)\right)x^2\right)\sqrt{a+bx^2+cx^4}}{315c^2}$$

$$+\frac{(3Bc-2bC)x\left(a+bx^2+cx^4\right)^{3/2}}{21c^2}+\frac{Cx^3\left(a+bx^2+cx^4\right)^{3/2}}{9c}$$

$$-\frac{\sqrt[4]{a}\left(24b^3Bc-87abBc^2-16b^4C-6b^2c(7Ac-12aC)+42ac^2(3Ac-aC)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^{2}}}\ \cdots}{315c^{15/4}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(16b^3C+6bc\left(6\sqrt{a}B\sqrt{c}+7Ac-4aC\right)+3\sqrt{a}c^{3/2}\left(5\sqrt{a}B\sqrt{c}-21Ac+7aC\right)-24b^2\cdots\right.}{630c^{15/4}\sqrt{a+bx^2\cdots}}$$

output
```
1/315*(24*b^3*B*c-87*a*b*B*c^2-16*b^4*C-6*b^2*c*(7*A*c-12*C*a)+42*a*c^2*(3
*A*c-C*a))*x*(c*x^4+b*x^2+a)^(1/2)/c^(7/2)/(a^(1/2)+c^(1/2)*x^2)-1/315*x*(
12*b^2*B+15*a*B*c-8*b^3*C/c-3*b*(7*A*c+C*a)+3*(12*B*b*c-8*b^2*C-7*c*(3*A*c
-C*a))*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/21*(3*B*c-2*C*b)*x*(c*x^4+b*x^2+a)
^(3/2)/c^2+1/9*C*x^3*(c*x^4+b*x^2+a)^(3/2)/c-1/315*a^(1/4)*(24*b^3*B*c-87*
a*b*B*c^2-16*b^4*C-6*b^2*c*(7*A*c-12*C*a)+42*a*c^2*(3*A*c-C*a))*(a^(1/2)+c
^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(15/4)/(c*
x^4+b*x^2+a)^(1/2)-1/630*a^(1/4)*(b+2*a^(1/2)*c^(1/2))*(16*b^3*C+6*b*c*(6*
a^(1/2)*B*c^(1/2)+7*A*c-4*a*C)+3*a^(1/2)*c^(3/2)*(5*a^(1/2)*B*c^(1/2)-21*A
*c+7*a*C)-24*b^2*(B*c+a^(1/2)*c^(1/2)*C))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*
x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a
^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(15/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.09 (sec) , antiderivative size = 737, normalized size of antiderivative = 1.18

$$\int x^2\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)\left(8b^3C-6b^2c(2B+Cx^2)+bc(21Ac-27aC+9Bcx^2+5cCx^4)+c^2(30a\right.}{}$$

input
```
Integrate[x^2*Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4)*(8*b^3*C - 6*b^
2*c*(2*B + C*x^2) + b*c*(21*A*c - 27*a*C + 9*B*c*x^2 + 5*c*C*x^4) + c^2*(3
0*a*B + 63*A*c*x^2 + 14*a*C*x^2 + 45*B*c*x^4 + 35*c*C*x^6)) - I*(-b + Sqrt
[b^2 - 4*a*c])*(-24*b^3*B*c + 87*a*b*B*c^2 + 16*b^4*C + 6*b^2*c*(7*A*c - 1
2*a*C) + 42*a*c^2*(-3*A*c + a*C))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c
])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-16*b^5*C -
2*b^3*c*(21*A*c + 12*B*Sqrt[b^2 - 4*a*c] - 44*a*C) + 3*a*b*c^2*(56*A*c + 2
9*B*Sqrt[b^2 - 4*a*c] - 32*a*C) + 8*b^4*(3*B*c + 2*Sqrt[b^2 - 4*a*c]*C) +
6*a*c^2*(10*a*B*c - 21*A*c*Sqrt[b^2 - 4*a*c] + 7*a*Sqrt[b^2 - 4*a*c]*C) -
3*b^2*c*(37*a*B*c - 14*A*c*Sqrt[b^2 - 4*a*c] + 24*a*Sqrt[b^2 - 4*a*c]*C))*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
- 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
 - Sqrt[b^2 - 4*a*c])])/(1260*c^4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a +
 b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 587, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2199, 1596, 27, 1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \text{2199}$$

$$\int x^2\left(\frac{1}{3}\left(3B-\frac{2bC}{c}\right)x^2+\frac{1}{3}\left(3A-\frac{aC}{c}\right)\right)\sqrt{cx^4+bx^2+a}dx+\frac{Cx^3\left(a+bx^2+cx^4\right)^{3/2}}{9c}$$

$$\Big\downarrow \text{1596}$$

$$\frac{\int -\frac{x^2\left(\left(-8Cb^3+12Bcb^2-3c(7Ac-9aC)b-30aBc^2\right)x^2+ac\left(-\frac{6Cb^2}{c}+9Bb-42Ac+14aC\right)\right)}{3c\sqrt{cx^4+bx^2+a}}dx}{35c}+$$
$$\frac{x^3\sqrt{a+bx^2+cx^4}\left(-7aC+21Ac-\frac{2b^2C}{c}+5x^2(3Bc-2bC)+3bB\right)}{105c}+\frac{Cx^3\left(a+bx^2+cx^4\right)^{3/2}}{9c}$$

$\big\downarrow$ 27

$$-\frac{\int \frac{x^2\left(\left(-8Cb^3+12Bcb^2-3c(7Ac-9aC)b-30aBc^2\right)x^2+a\left(-6Cb^2+9Bcb-14c(3Ac-aC)\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{105c^2}+$$
$$\frac{x^3\sqrt{a+bx^2+cx^4}\left(-7aC+21Ac-\frac{2b^2C}{c}+5x^2(3Bc-2bC)+3bB\right)}{105c}+\frac{Cx^3\left(a+bx^2+cx^4\right)^{3/2}}{9c}$$

$\big\downarrow$ 1602

$$-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(-3bc(7Ac-9aC)-30aBc^2-8b^3C+12b^2Bc\right)}{3c}-\frac{\int \frac{\left(-16Cb^4+24Bcb^3-6c(7Ac-12aC)b^2-87aBc^2b+42ac^2(3Ac-aC)\right)x^2+a\left(-8Cb^3+\cdots\right.}{\sqrt{cx^4+bx^2+a}}\cdots}{3c}}{105c^2}$$
$$\frac{x^3\sqrt{a+bx^2+cx^4}\left(-7aC+21Ac-\frac{2b^2C}{c}+5x^2(3Bc-2bC)+3bB\right)}{105c}+\frac{Cx^3\left(a+bx^2+cx^4\right)^{3/2}}{9c}$$

$\big\downarrow$ 1511

$$-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(-3bc(7Ac-9aC)-30aBc^2-8b^3C+12b^2Bc\right)}{3c}-\frac{-\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(6bc\left(6\sqrt{a}B\sqrt{c}-4aC+7Ac\right)+3\sqrt{a}c^{3/2}\left(5\sqrt{a}B\sqrt{c}+7aC-21Ac\right)-24b^2\left(\cdots\right.\right.}{\sqrt{c}}\cdots}{\cdots}}{105c^2}$$
$$\frac{x^3\sqrt{a+bx^2+cx^4}\left(-7aC+21Ac-\frac{2b^2C}{c}+5x^2(3Bc-2bC)+3bB\right)}{105c}+\frac{Cx^3\left(a+bx^2+cx^4\right)^{3/2}}{9c}$$

$\big\downarrow$ 27

$$-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(-3bc(7Ac-9aC)-30aBc^2-8b^3C+12b^2Bc\right)}{3c}-\frac{-\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(6bc\left(6\sqrt{a}B\sqrt{c}-4aC+7Ac\right)+3\sqrt{a}c^{3/2}\left(5\sqrt{a}B\sqrt{c}+7aC-21Ac\right)-24b^2\left(\cdots\right.\right.}{\sqrt{c}}\cdots}{\cdots}}{105c^2}$$
$$\frac{x^3\sqrt{a+bx^2+cx^4}\left(-7aC+21Ac-\frac{2b^2C}{c}+5x^2(3Bc-2bC)+3bB\right)}{105c}+\frac{Cx^3\left(a+bx^2+cx^4\right)^{3/2}}{9c}$$

$\big\downarrow$ 1416

$$-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(-3bc(7Ac-9aC)-30aBc^2-8b^3C+12b^2Bc\right)}{3c}-\frac{-\frac{\left(-6b^2c(7Ac-12aC)+42ac^2(3Ac-aC)-87abBc^2-16b^4C+24b^3Bc\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}}{\sqrt{c}}}{}}{}$$

$$\frac{x^3\sqrt{a+bx^2+cx^4}\left(-7aC+21Ac-\frac{2b^2C}{c}+5x^2(3Bc-2bC)+3bB\right)}{105c}+\frac{Cx^3\left(a+bx^2+cx^4\right)^{3/2}}{9c}$$

$\downarrow$ 1509

$$-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(-3bc(7Ac-9aC)-30aBc^2-8b^3C+12b^2Bc\right)}{3c}-\frac{-\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6bc\left(6\sqrt{a}B\sqrt{c}-4aC+7Ac\right)+3\sqrt{a}c^{3/2}}{2}}{}}{}$$

$$\frac{x^3\sqrt{a+bx^2+cx^4}\left(-7aC+21Ac-\frac{2b^2C}{c}+5x^2(3Bc-2bC)+3bB\right)}{105c}+\frac{Cx^3\left(a+bx^2+cx^4\right)^{3/2}}{9c}$$

input

```
Int[x^2*Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4),x]
```

output

```
(x^3*(3*b*B + 21*A*c - 7*a*C - (2*b^2*C)/c + 5*(3*B*c - 2*b*C)*x^2)*Sqrt[a
 + b*x^2 + c*x^4])/(105*c) + (C*x^3*(a + b*x^2 + c*x^4)^(3/2))/(9*c) - (((
12*b^2*B*c - 30*a*B*c^2 - 8*b^3*C - 3*b*c*(7*A*c - 9*a*C))*x*Sqrt[a + b*x^
2 + c*x^4])/(3*c) - (-(((24*b^3*B*c - 87*a*b*B*c^2 - 16*b^4*C - 6*b^2*c*(7
*A*c - 12*a*C) + 42*a*c^2*(3*A*c - a*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(S
qrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[
c]) - (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(16*b^3*C + 6*b*c*(6*Sqrt[a]*B*Sqrt
[c] + 7*A*c - 4*a*C) + 3*Sqrt[a]*c^(3/2)*(5*Sqrt[a]*B*Sqrt[c] - 21*A*c + 7
*a*C) - 24*b^2*(B*c + Sqrt[a]*Sqrt[c]*C))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4
]))/(3*c))/(105*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1596
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Simp[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a +
 b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c
*e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &
& NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[
p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 5.69 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{C\,x^{7}\sqrt{c\,x^{4}+b\,x^{2}+a}}{9}+\frac{\left(Bc+\frac{Cb}{9}\right)x^{5}\sqrt{c\,x^{4}+b\,x^{2}+a}}{7c}+\frac{\left(Ac+Bb+\frac{2aC}{9}-\frac{6b\left(Bc+\frac{Cb}{9}\right)}{7c}\right)x^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{5c}+\frac{\left(Ab+Ba-\frac{4b\left(Ac+\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^2*(c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*C*x^7*(c*x^4+b*x^2+a)^(1/2)+1/7*(B*c+1/9*C*b)/c*x^5*(c*x^4+b*x^2+a)^(1
/2)+1/5*(A*c+B*b+2/9*a*C-6/7*b/c*(B*c+1/9*C*b))/c*x^3*(c*x^4+b*x^2+a)^(1/2
)+1/3*(A*b+B*a-4/5*b/c*(A*c+B*b+2/9*a*C-6/7*b/c*(B*c+1/9*C*b))-5/7*a/c*(B*
c+1/9*C*b))/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*a/c*(A*b+B*a-4/5*b/c*(A*c+B*b+2
/9*a*C-6/7*b/c*(B*c+1/9*C*b))-5/7*a/c*(B*c+1/9*C*b))*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2))-1/2*(A*a-3/5*a/c*(A*c+B*b+2/9*a*C-6/7*b/c*(B*c+1/9*C*b))-2/3*b/c*
(A*b+B*a-4/5*b/c*(A*c+B*b+2/9*a*C-6/7*b/c*(B*c+1/9*C*b))-5/7*a/c*(B*c+1/9*
C*b)))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 717, normalized size of antiderivative = 1.15

$$\int x^2\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
-1/630*(sqrt(1/2)*((16*C*b^4*c - 126*A*a*c^4 + 3*(14*C*a^2 + 29*B*a*b + 14
*A*b^2)*c^3 - 24*(3*C*a*b^2 + B*b^3)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (16*
C*b^5 - 126*A*a*b*c^3 + 3*(14*C*a^2*b + 29*B*a*b^2 + 14*A*b^3)*c^2 - 24*(3
*C*a*b^3 + B*b^4)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*el
liptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*
(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((16*C*b^4*
c - 3*(2*(21*A + 5*B)*a + 7*A*b)*c^4 + 3*(14*C*a^2 + (29*B + 9*C)*a*b + 2*
(7*A + 2*B)*b^2)*c^3 - 8*(9*C*a*b^2 + (3*B + C)*b^3)*c^2)*x*sqrt((b^2 - 4*
a*c)/c^2) - (16*C*b^5 - 3*(2*(21*A - 5*B)*a*b - 7*A*b^2)*c^3 + 3*(14*C*a^2
*b + (29*B - 9*C)*a*b^2 + 2*(7*A - 2*B)*b^3)*c^2 - 8*(9*C*a*b^3 + (3*B - C
)*b^4)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(ar
csin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((
b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*(35*C*c^5*x^8 + 5*(C*b*c^4 + 9
*B*c^5)*x^6 - 16*C*b^4*c + 126*A*a*c^4 - (6*C*b^2*c^3 - 63*A*c^5 - (14*C*a
 + 9*B*b)*c^4)*x^4 - 3*(14*C*a^2 + 29*B*a*b + 14*A*b^2)*c^3 + 24*(3*C*a*b^
2 + B*b^3)*c^2 + (8*C*b^3*c^2 + 3*(10*B*a + 7*A*b)*c^4 - 3*(9*C*a*b + 4*B*
b^2)*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^5*x)
```

Sympy [F]

$$\int x^2\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int x^2\left(A+Bx^2+Cx^4\right)\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)**(1/2)*(C*x**4+B*x**2+A),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int x^2\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int\left(Cx^4+Bx^2+A\right)\sqrt{cx^4+bx^2+a}x^2\,dx$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int\left(Cx^4+Bx^2+A\right)\sqrt{cx^4+bx^2+a}x^2\,dx$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int x^2\left(C\,x^4+B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(x^2*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^2*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{24\sqrt{c\,x^4+b\,x^2+a}\,abcx+77\sqrt{c\,x^4+b\,x^2+a}\,a\,c^2x^3-4\sqrt{c\,x^4+b\,x^2+a}\,b^3x+3\sqrt{c\,x^4+b\,x^2+a}\,b^2c\,x^3}{}$$

input
```
int(x^2*(c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x)
```

output
```
(24*sqrt(a + b*x**2 + c*x**4)*a*b*c*x + 77*sqrt(a + b*x**2 + c*x**4)*a*c**
2*x**3 - 4*sqrt(a + b*x**2 + c*x**4)*b**3*x + 3*sqrt(a + b*x**2 + c*x**4)*
b**2*c*x**3 + 50*sqrt(a + b*x**2 + c*x**4)*b*c**2*x**5 + 35*sqrt(a + b*x**
2 + c*x**4)*c**3*x**7 - 24*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x
**4),x)*a**2*b*c + 4*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x
)*a*b**3 + 84*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x
)*a**2*c**2 - 57*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4
),x)*a*b**2*c + 8*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**
4),x)*b**4)/(315*c**2)
```

3.134 $\int \sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 491

$$\begin{aligned}
&\int \sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx \\
&= -\frac{\left(14b^2Bc-42aBc^2-8b^3C-bc(35Ac-29aC)\right)x\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad+\frac{x\left(7bBc+35Ac^2-4b^2C-5acC+3c(7Bc-4bC)x^2\right)\sqrt{a+bx^2+cx^4}}{105c^2} \\
&\quad+\frac{Cx\left(a+bx^2+cx^4\right)^{3/2}}{7c} \\
&\quad+\frac{\sqrt[4]{a}\left(14b^2Bc-42aBc^2-8b^3C-bc(35Ac-29aC)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\right.}{105c^{11/4}\sqrt{a+bx^2+cx^4}} \\
&\quad-\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(14bBc-21\sqrt{a}Bc^{3/2}-35Ac^2-8b^2C+12\sqrt{a}b\sqrt{c}C+5acC\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx}{(\sqrt{a}+}}}{210c^{11/4}\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

output
```
-1/105*(14*b^2*B*c-42*a*B*c^2-8*b^3*C-b*c*(35*A*c-29*C*a))*x*(c*x^4+b*x^2+
a)^(1/2)/c^(5/2)/(a^(1/2)+c^(1/2)*x^2)+1/105*x*(7*B*b*c+35*A*c^2-4*b^2*C-5
*a*c*C+3*c*(7*B*c-4*C*b)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/7*C*x*(c*x^4+b*x
^2+a)^(3/2)/c+1/105*a^(1/4)*(14*b^2*B*c-42*a*B*c^2-8*b^3*C-b*c*(35*A*c-29*
C*a))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2
)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/
2))/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)-1/210*a^(1/4)*(b+2*a^(1/2)*c^(1/2))*(14
*B*b*c-21*a^(1/2)*B*c^(3/2)-35*A*c^2-8*b^2*C+12*a^(1/2)*b*c^(1/2)*C+5*a*c*
C)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*I
nverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2)
)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.81 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.28

$$\int \sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)\left(-4b^2C+bc(7B+3Cx^2)+c(35Ac+10aC+21Bcx^2+15cCx^4)\right)+i\left(\cdot\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4),x]
```

output

```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4)*(-4*b^2*C + b*c
*(7*B + 3*C*x^2) + c*(35*A*c + 10*a*C + 21*B*c*x^2 + 15*c*C*x^4)) + I*(-b
+ Sqrt[b^2 - 4*a*c])*(-14*b^2*B*c + 42*a*B*c^2 + 8*b^3*C + b*c*(35*A*c - 2
9*a*C))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elliptic
E[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^4*C + 2*a*c^2*(70*A*c + 21*B*Sqrt
[b^2 - 4*a*c] - 10*a*C) + b^2*c*(-35*A*c - 14*B*Sqrt[b^2 - 4*a*c] + 37*a*C
) + 2*b^3*(7*B*c + 4*Sqrt[b^2 - 4*a*c]*C) + b*c*(-56*a*B*c + 35*A*c*Sqrt[b
^2 - 4*a*c] - 29*a*Sqrt[b^2 - 4*a*c]*C))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(420*c^3*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2207, 1490, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right) \, dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{\int \left((7Bc-4bC)x^2+7Ac-aC\right)\sqrt{cx^4+bx^2+a}dx}{7c}+\frac{Cx\left(a+bx^2+cx^4\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{1490}$$

$$\frac{\frac{\int -\frac{\left(-8Cb^3+14Bcb^2-c(35Ac-29aC)b-42aBc^2\right)x^2+a\left(-4Cb^2+7Bcb-10c(7Ac-aC)\right)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\frac{x\sqrt{a+bx^2+cx^4}\left(-5acC+35Ac^2-4b^2C+3cx^2(7Bc-4bC)+\right.}{15c}}{7c}$$
$$\frac{Cx\left(a+bx^2+cx^4\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(-5acC+35Ac^2-4b^2C+3cx^2(7Bc-4bC)+7bBc\right)}{15c}-\frac{\int\frac{\left(-8Cb^3+14Bcb^2-c(35Ac-29aC)b-42aBc^2\right)x^2+a\left(-4Cb^2+7Bcb-10c(7Ac-aC\right.}{\sqrt{cx^4+bx^2+a}}}{15c}}{7c}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{3/2}}{7c}$$

$\big\downarrow$ 1511

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(-5acC+35Ac^2-4b^2C+3cx^2(7Bc-4bC)+7bBc\right)}{15c}-\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(12\sqrt{a}b\sqrt{c}C-21\sqrt{a}Bc^{3/2}+5acC-35Ac^2-8b^2C+14bBc\right)\int\frac{1}{\sqrt{cx^4+b}}}{\sqrt{c}}}{15}}{7c}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{3/2}}{7c}$$

$\big\downarrow$ 27

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(-5acC+35Ac^2-4b^2C+3cx^2(7Bc-4bC)+7bBc\right)}{15c}-\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(12\sqrt{a}b\sqrt{c}C-21\sqrt{a}Bc^{3/2}+5acC-35Ac^2-8b^2C+14bBc\right)\int\frac{1}{\sqrt{cx^4+b}}}{\sqrt{c}}}{15c}}{7c}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{3/2}}{7c}$$

$\big\downarrow$ 1416

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(-5acC+35Ac^2-4b^2C+3cx^2(7Bc-4bC)+7bBc\right)}{15c}-\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(12\sqrt{a}b\sqrt{c}C-21\sqrt{a}Bc^{3/2}+5acC-3\right.}{2c^{3/4}\sqrt{a+bx^2+c}}}{7c}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{3/2}}{7c}$$

$\big\downarrow$ 1509

$$\frac{x\sqrt{a+bx^2+cx^4}\left(-5acC+35Ac^2-4b^2C+3cx^2(7Bc-4bC)+7bBc\right)}{15c}-\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(12\sqrt{a}b\sqrt{c}C-21\sqrt{a}Bc^{3/2}+5acC-3\right.}{2c^{3/4}\sqrt{a+bx^2+c}}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{3/2}}{7c}$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(C*x*(a + b*x^2 + c*x^4)^(3/2))/(7*c) + ((x*(7*b*B*c + 35*A*c^2 - 4*b^2*C
- 5*a*c*C + 3*c*(7*B*c - 4*b*C)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) - (-(
((14*b^2*B*c - 42*a*B*c^2 - 8*b^3*C - b*c*(35*A*c - 29*a*C))*(-((x*Sqrt[a
+ b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x
^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(14*b*B*c - 21*
Sqrt[a]*B*c^(3/2) - 35*A*c^2 - 8*b^2*C + 12*Sqrt[a]*b*Sqrt[c]*C + 5*a*c*C)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/(15*c))/(7*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]
```

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{C x^5\sqrt{c x^4+b x^2+a}}{7}+\frac{\left(Bc+\frac{Cb}{7}\right)x^3\sqrt{c x^4+b x^2+a}}{5c}+\frac{\left(Ac+Bb+\frac{2aC}{7}-\frac{4b\left(Bc+\frac{Cb}{7}\right)}{5c}\right)x\sqrt{c x^4+b x^2+a}}{3c}+\frac{\left(Aa-\frac{a\left(Ac+Bb+\frac{2aC}{7}\right.}{}\right.}{}$
risch	$\frac{x\left(15c^2 C x^4+21B c^2x^2+3Ccb x^2+35A c^2+7Bbc+10Cac-4C b^2\right)\sqrt{c x^4+b x^2+a}}{105c^2}+\frac{-\frac{\left(35bA c^2+42Ba c^2-14b^2Bc-29Cacb+8b^3C\right)a\sqrt{}}{}}{}$
default	Expression too large to display

input
```
int((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*C*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(B*c+1/7*C*b)/c*x^3*(c*x^4+b*x^2+a)^(1
/2)+1/3*(A*c+B*b+2/7*a*C-4/5*b/c*(B*c+1/7*C*b))/c*x*(c*x^4+b*x^2+a)^(1/2)+
1/4*(A*a-1/3*a/c*(A*c+B*b+2/7*a*C-4/5*b/c*(B*c+1/7*C*b)))*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2))-1/2*(A*b+B*a-3/5*a/c*(B*c+1/7*C*b)-2/3*b/c*(A*c+B*b+2/7*a*C-
4/5*b/c*(B*c+1/7*C*b)))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(
-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*
b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.17

$$\int \sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$= \frac{\sqrt{\frac{1}{2}}\left(\left(8\,Cb^3c+7\,(6\,Ba+5\,Ab)c^3-(29\,Cab+14\,Bb^2)c^2\right)x\sqrt{\frac{b^2-4\,ac}{c^2}}-\left(8\,Cb^4+7\,(6\,Bab+5\,Ab^2)c^2-(2\right.\right.}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/210*(sqrt(1/2)*((8*C*b^3*c + 7*(6*B*a + 5*A*b)*c^3 - (29*C*a*b + 14*B*b^
2)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (8*C*b^4 + 7*(6*B*a*b + 5*A*b^2)*c^2 -
 (29*C*a*b^2 + 14*B*b^3)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b
)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x
), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((8*
C*b^3*c - 70*A*c^4 + (2*(21*B + 5*C)*a + 7*(5*A + B)*b)*c^3 - (29*C*a*b +
2*(7*B + 2*C)*b^2)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (8*C*b^4 + 70*A*b*c^3
+ (2*(21*B - 5*C)*a*b + 7*(5*A - B)*b^2)*c^2 - (29*C*a*b^2 + 2*(7*B - 2*C)
*b^3)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arc
sin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b
^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(15*C*c^4*x^6 + 8*C*b^3*c + 3*(
C*b*c^3 + 7*B*c^4)*x^4 + 7*(6*B*a + 5*A*b)*c^3 - (29*C*a*b + 14*B*b^2)*c^2
 - (4*C*b^2*c^2 - 35*A*c^4 - (10*C*a + 7*B*b)*c^3)*x^2)*sqrt(c*x^4 + b*x^2
 + a))/(c^4*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(A+Bx^2+Cx^4\right)\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)*(C*x**4+B*x**2+A),x)
```

output
```
Integral((A + B*x**2 + C*x**4)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\sqrt{cx^4+bx^2+a}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\sqrt{cx^4+bx^2+a}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(C\,x^4+B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{15\sqrt{c\,x^4+b\,x^2+a}\,acx+\sqrt{c\,x^4+b\,x^2+a}\,b^2x+8\sqrt{c\,x^4+b\,x^2+a}\,bc\,x^3+5\sqrt{c\,x^4+b\,x^2+a}\,c^2x^5+20\Big(}{35}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A),x)
```

output
```
(15*sqrt(a + b*x**2 + c*x**4)*a*c*x + sqrt(a + b*x**2 + c*x**4)*b**2*x + 8
*sqrt(a + b*x**2 + c*x**4)*b*c*x**3 + 5*sqrt(a + b*x**2 + c*x**4)*c**2*x**
5 + 20*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**2*c - int
(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b**2 + 16*int((sqrt(
a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*b*c - 2*int((sqrt(a
+ b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*b**3)/(35*c)
```

3.135 $\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 444

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \frac{(5bBc-2b^2C+6c(5Ac+aC))\,x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\left(3\left(5A+\frac{aC}{c}\right)-\left(5B-\frac{2bC}{c}\right)x^2\right)\sqrt{a+bx^2+cx^4}}{15x} + \frac{C(a+bx^2+cx^4)^{3/2}}{5cx} - \frac{\sqrt[4]{a}(5bBc-2b^2C+6c(5Ac+aC))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{a+bx^2+cx^4}} + \frac{\left(b+2\sqrt{a}\sqrt{c}\right)\left(15Ac^{3/2}+3a\sqrt{c}C+\sqrt{a}(5Bc-2bC)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\right.}{30\sqrt[4]{a}c^{7/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/15*(5*B*b*c-2*b^2*C+6*c*(5*A*c+C*a))*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^
(1/2)+c^(1/2)*x^2)-1/15*(15*A+3*a*C/c-(5*B-2*b*C/c)*x^2)*(c*x^4+b*x^2+a)^(
1/2)/x+1/5*C*(c*x^4+b*x^2+a)^(3/2)/c/x-1/15*a^(1/4)*(5*B*b*c-2*b^2*C+6*c*(
5*A*c+C*a))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2
))^(1/2))/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)+1/30*(b+2*a^(1/2)*c^(1/2))*(15*A*c
^(3/2)+3*a*c^(1/2)*C+a^(1/2)*(5*B*c-2*C*b))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+
b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x
/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/c^(7/4)/(c*x^4+b*x^2+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.79 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^2}\,dx$$

$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)\left(-15Ac+x^2(5Bc+C(b+3cx^2))\right)-i\left(-b+\sqrt{b^2-4ac}\right)\left(-5bBc+2b^2C\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4))/x^2,x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(-15*A*c + x^2*(5
*B*c + C*(b + 3*c*x^2))) - I*(-b + Sqrt[b^2 - 4*a*c])*(-5*b*B*c + 2*b^2*C
- 6*c*(5*A*c + a*C))*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*
a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-2*b^3*C + b*(-5*B*c*Sq
rt[b^2 - 4*a*c] + 8*a*c*C) + b^2*(5*B*c + 2*Sqrt[b^2 - 4*a*c]*C) - 2*c*(10
*a*B*c + 15*A*c*Sqrt[b^2 - 4*a*c] + 3*a*Sqrt[b^2 - 4*a*c]*C))*x*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b
^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^
2 - 4*a*c])])/(60*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c
*x^4])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2199, 1594, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)}{x^2}dx$$

$$\Big\downarrow \text{ 2199}$$

$$\int \frac{\left(\frac{1}{5}\left(5B-\frac{2bC}{c}\right)x^2+\frac{1}{5}\left(5A+\frac{aC}{c}\right)\right)\sqrt{cx^4+bx^2+a}}{x^2}dx+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{5cx}$$

$$\Big\downarrow \text{ 1594}$$

$$-\frac{1}{3}\int -\frac{\left(-2Cb^2+5Bcb+6c(5Ac+aC)\right)x^2+c\left(15Ab-\frac{aCb}{c}+10aB\right)}{5c\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}\left(3\left(\frac{aC}{c}+5A\right)-x^2\left(5B-\frac{2bC}{c}\right)\right)}{15x}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{5cx}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(-2Cb^2+5Bcb+6c(5Ac+aC)\right)x^2+15Abc+10aBc-abC}{\sqrt{cx^4+bx^2+a}}dx}{15c}-\frac{\sqrt{a+bx^2+cx^4}\left(3\left(\frac{aC}{c}+5A\right)-x^2\left(5B-\frac{2bC}{c}\right)\right)}{15x}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{5cx}$$

$$\Big\downarrow \text{1511}$$

$$\frac{\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}(5Bc-2bC)+3a\sqrt{c}C+15Ac^{3/2}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(6c(aC+5Ac)-2b^2C+5bBc\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{15c}-\frac{\sqrt{a+bx^2+cx^4}\left(3\left(\frac{aC}{c}+5A\right)-x^2\left(5B-\frac{2bC}{c}\right)\right)}{15x}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{5cx}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}(5Bc-2bC)+3a\sqrt{c}C+15Ac^{3/2}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(6c(aC+5Ac)-2b^2C+5bBc\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{15c}-\frac{\sqrt{a+bx^2+cx^4}\left(3\left(\frac{aC}{c}+5A\right)-x^2\left(5B-\frac{2bC}{c}\right)\right)}{15x}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{5cx}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}(5Bc-2bC)+3a\sqrt{c}C+15Ac^{3/2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(6c(aC+5Ac)-2}{}}{15c}-\frac{\sqrt{a+bx^2+cx^4}\left(3\left(\frac{aC}{c}+5A\right)-x^2\left(5B-\frac{2bC}{c}\right)\right)}{15x}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{5cx}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}(5Bc-2bC)+3a\sqrt{c}C+15Ac^{3/2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(6c(aC+5Ac)-2}{}}{15c}-\frac{\sqrt{a+bx^2+cx^4}\left(3\left(\frac{aC}{c}+5A\right)-x^2\left(5B-\frac{2bC}{c}\right)\right)}{15x}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{5cx}$$

input
```
Int[(Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4))/x^2,x]
```

output
```
-1/15*((3*(5*A + (a*C)/c) - (5*B - (2*b*C)/c)*x^2)*Sqrt[a + b*x^2 + c*x^4]
)/x + (C*(a + b*x^2 + c*x^4)^(3/2))/(5*c*x) + (-(((5*b*B*c - 2*b^2*C + 6*c
*(5*A*c + a*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) +
(a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + ((b + 2*Sqrt[a]*Sqr
t[c])*(15*A*c^(3/2) + 3*a*Sqrt[c]*C + Sqrt[a]*(5*B*c - 2*b*C))*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c
^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/(15*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.07

method	result
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{x}+\frac{C\,x^3\sqrt{c\,x^4+b\,x^2+a}}{5}+\frac{\left(Bc+\frac{Cb}{5}\right)x\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\left(Ab+Ba-\frac{a\left(Bc+\frac{Cb}{5}\right)}{3c}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}}{}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(-3Cc\,x^4-5Bc\,x^2-Cb\,x^2+15Ac\right)}{15cx}+\frac{-\frac{\left(30A\,c^2+5Bbc+6Cac-2C\,b^2\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac}\right.}{a}}}{}}{}$
default	Expression too large to display

input
```
int((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-A*(c*x^4+b*x^2+a)^(1/2)/x+1/5*C*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(B*c+1/5*C*
b)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(A*b+B*a-1/3*a/c*(B*c+1/5*C*b))*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2))-1/2*(2*A*c+B*b+2/5*a*C-2/3*b/c*(B*c+1/5*C*b))*a*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4
*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(A+Bx^2+Cx^4)\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)*(C*x**4+B*x**2+A)/x**2,x)
```

output
```
Integral((A + B*x**2 + C*x**4)*sqrt(a + b*x**2 + c*x**4)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(C\,x^4+B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\,dx$$

input
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2))/x^2,x)
```

output
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^2}\,dx$$
$$= \frac{7\sqrt{c\,x^4+b\,x^2+a}\,ac+\sqrt{c\,x^4+b\,x^2+a}\,b^2+2\sqrt{c\,x^4+b\,x^2+a}\,bc\,x^2+\sqrt{c\,x^4+b\,x^2+a}\,c^2x^4+12\left(\int \frac{\sqrt{c}}{c\,x^6}\right.}{5cx}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^2,x)
```

output
```
(7*sqrt(a + b*x**2 + c*x**4)*a*c + sqrt(a + b*x**2 + c*x**4)*b**2 + 2*sqrt
(a + b*x**2 + c*x**4)*b*c*x**2 + sqrt(a + b*x**2 + c*x**4)*c**2*x**4 + 12*
int(sqrt(a + b*x**2 + c*x**4)/(a*x**2 + b*x**4 + c*x**6),x)*a**2*c*x + int
(sqrt(a + b*x**2 + c*x**4)/(a*x**2 + b*x**4 + c*x**6),x)*a*b**2*x + 8*int(
sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b*c*x)/(5*c*x)
```

3.136 $\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 401

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = -\frac{\left(A+\frac{aC}{c}-3Bx^2\right)\sqrt{a+bx^2+cx^4}}{3x^3} - \frac{\left(6aB+b\left(A+\frac{aC}{c}\right)\right)\sqrt{a+bx^2+cx^4}}{3\sqrt{a}x\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{C\left(a+bx^2+cx^4\right)^{3/2}}{3cx^3} - \frac{(Abc+6aBc+abC)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{3/4}c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\left(b+2\sqrt{a}\sqrt{c}\right)\left(3\sqrt{a}B\sqrt{c}+Ac+aC\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{6a^{3/4}c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/3*(A+a*C/c-3*B*x^2)*(c*x^4+b*x^2+a)^(1/2)/x^3-1/3*(6*a*B+b*(A+a*C/c))*(
c*x^4+b*x^2+a)^(1/2)/a^(1/2)/x/(a^(1/2)+c^(1/2)*x^2)+1/3*C*(c*x^4+b*x^2+a)
^(3/2)/c/x^3-1/3*(A*b*c+6*B*a*c+C*a*b)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2
+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4
))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
+1/6*(b+2*a^(1/2)*c^(1/2))*(3*a^(1/2)*B*c^(1/2)+A*c+a*C)*(a^(1/2)+c^(1/2)*
x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arc
tan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/c^(3/4)/(c
*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.62 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^4}\,dx$$
$$= \frac{-4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)\left(Abx^2+a(A+3Bx^2-Cx^4)\right)+i\left(-b+\sqrt{b^2-4ac}\right)\left(Abc+6aBc+abC\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4))/x^4,x]
```

output
```
(-4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(A*b*x^2 + a*(A
+ 3*B*x^2 - C*x^4)) + I*(-b + Sqrt[b^2 - 4*a*c])*(A*b*c + 6*a*B*c + a*b*C)
*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I
*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])] - I*(6*a*B*c*Sqrt[b^2 - 4*a*c] + A*c*(-b^2 + 4
*a*c + b*Sqrt[b^2 - 4*a*c]) + a*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*C)*x^
3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*
b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/
(b - Sqrt[b^2 - 4*a*c])])/(12*a*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^3*Sqrt
[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2199, 1594, 25, 1604, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)}{x^4}dx$$

$$\Big\downarrow \; 2199$$

$$\int \frac{\left(Bx^2+A+\frac{aC}{c}\right)\sqrt{cx^4+bx^2+a}}{x^4}dx+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \; 1594$$

$$-\frac{1}{3}\int -\frac{(3bB+2Ac+2aC)x^2+6aB+b\left(A+\frac{aC}{c}\right)}{x^2\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}\left(aC+Ac-3Bcx^2\right)}{3cx^3}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}\int \frac{(3bB+2Ac+2aC)x^2+6aB+b\left(A+\frac{aC}{c}\right)}{x^2\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}\left(aC+Ac-3Bcx^2\right)}{3cx^3}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \; 1604$$

$$\frac{1}{3}\left(-\frac{\int -\frac{(Abc+6aBc+abC)x^2+a(3bB+2Ac+2aC)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(b\left(\frac{aC}{c}+A\right)+6aB\right)}{ax}\right)-\frac{\sqrt{a+bx^2+cx^4}\left(aC+Ac-3Bcx^2\right)}{3cx^3}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}\left(\frac{\int \frac{(Abc+6aBc+abC)x^2+a(3bB+2Ac+2aC)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(b\left(\frac{aC}{c}+A\right)+6aB\right)}{ax}\right)-\frac{\sqrt{a+bx^2+cx^4}\left(aC+Ac-3Bcx^2\right)}{3cx^3}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \; 1511$$

$$\frac{1}{3}\left(\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(3\sqrt{a}B\sqrt{c}+aC+Ac\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}(abC+6aBc+Abc)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(b\left(\frac{aC}{c}+\right.\right.}{ax}\right.$$

$$-\frac{\sqrt{a+bx^2+cx^4}\left(aC+Ac-3Bcx^2\right)}{3cx^3}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(3\sqrt{a}B\sqrt{c}+aC+Ac\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{(abC+6aBc+Abc)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(b\left(\frac{aC}{c}+A\right)+\right.}{ax}\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}\left(aC+Ac-3Bcx^2\right)}{3cx^3}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{3cx^3}$$

$\Big\downarrow$ 1416

$$\frac{1}{3}\left(\frac{\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}B\sqrt{c}+aC+Ac\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(abC+6aBc+Abc)\int -}{\sqrt{c}}}{a}\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}\left(aC+Ac-3Bcx^2\right)}{3cx^3}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{3cx^3}$$

$\Big\downarrow$ 1509

$$\frac{1}{3}\left(\frac{\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}B\sqrt{c}+aC+Ac\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(abC+6aBc+Abc)\left(\sqrt[4]{} - \right.}{}}{a}\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}\left(aC+Ac-3Bcx^2\right)}{3cx^3}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{3cx^3}$$

input
```
Int[(Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4))/x^4,x]
```

output
```
-1/3*((A*c + a*C - 3*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(c*x^3) + (C*(a + b
*x^2 + c*x^4)^(3/2))/(3*c*x^3) + (-(((6*a*B + b*(A + (a*C)/c))*Sqrt[a + b*
x^2 + c*x^4])/(a*x)) + (-(((A*b*c + 6*a*B*c + a*b*C)*(-((x*Sqrt[a + b*x^2
+ c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x
^4])))/Sqrt[c]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(3*Sqrt[a]*B*Sqrt[c] +
A*c + a*C)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt
[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/a)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.15

method	result
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{3x^3}-\frac{(Ab+3Ba)\sqrt{c\,x^4+b\,x^2+a}}{3ax}+\frac{Cx\sqrt{c\,x^4+b\,x^2+a}}{3}+\frac{\left(\frac{2Ac}{3}+Bb+\frac{2aC}{3}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\cdots}}{4\sqrt{\cdots}}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(-Ca\,x^4+Ab\,x^2+3Ba\,x^2+Aa\right)}{3x^3a}+\frac{-\frac{(Abc+6Bac+Cba)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\mathrm{Elli}\cdots\right.}{2\sqrt{\cdots}}}{\cdots}$
default	Expression too large to display

input
```
int((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*(c*x^4+b*x^2+a)^(1/2)/x^3-1/3*(A*b+3*B*a)/a*(c*x^4+b*x^2+a)^(1/2)/x
+1/3*C*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(2/3*A*c+B*b+2/3*a*C)*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b
+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-1/2*(B*c+1/3*C*b+1/3*c*(A*b+3*B*a)/a)*a*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^4}\,dx=\int \frac{(Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^4}\,dx=\int \frac{(A+Bx^2+Cx^4)\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)*(C*x**4+B*x**2+A)/x**4,x)
```

output
```
Integral((A + B*x**2 + C*x**4)*sqrt(a + b*x**2 + c*x**4)/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^4}\,dx=\int \frac{(Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^4}\,dx=\int \frac{(Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \int \frac{(C\,x^4+B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2+a}}{x^4}\,dx$$

input
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2))/x^4,x)
```

output
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^4}\,dx$$
$$= \frac{-5\sqrt{c\,x^4+b\,x^2+a}\,a+4\sqrt{c\,x^4+b\,x^2+a}\,b\,x^2+\sqrt{c\,x^4+b\,x^2+a}\,c\,x^4-12\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^8+b\,x^6+a\,x^4}dx\right)a^2x^3+3\left(\right.}{3x^3}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^4,x)
```

output
```
( - 5*sqrt(a + b*x**2 + c*x**4)*a + 4*sqrt(a + b*x**2 + c*x**4)*b*x**2 + s
qrt(a + b*x**2 + c*x**4)*c*x**4 - 12*int(sqrt(a + b*x**2 + c*x**4)/(a*x**4
 + b*x**6 + c*x**8),x)*a**2*x**3 + 3*int(sqrt(a + b*x**2 + c*x**4)/(a + b*
x**2 + c*x**4),x)*b**2*x**3)/(3*x**3)
```

3.137 $\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 485

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = -\frac{\left(Ab-\frac{5a(2Bc+3bC)}{c}\right)\sqrt{a+bx^2+cx^4}}{15ax^3}$$
$$+\frac{(2A(b^2-3ac)-5a(bB+6aC))\sqrt{a+bx^2+cx^4}}{15a^{3/2}x\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{\left(A+\frac{5aC}{c}+5\left(B+\frac{2bC}{c}\right)x^2\right)\sqrt{a+bx^2+cx^4}}{5x^5}+\frac{C(a+bx^2+cx^4)^{3/2}}{cx^5}$$
$$+\frac{\sqrt[4]{c}(2A(b^2-3ac)-5a(bB+6aC))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15a^{7/4}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\left(b+2\sqrt{a}\sqrt{c}\right)\left(2Ab\sqrt{c}-5aB\sqrt{c}-3\sqrt{a}Ac-15a^{3/2}C\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\right.}{30a^{7/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/15*(A*b-5*a*(2*B*c+3*C*b)/c)*(c*x^4+b*x^2+a)^(1/2)/a/x^3+1/15*(2*A*(-3*
a*c+b^2)-5*a*(B*b+6*C*a))*(c*x^4+b*x^2+a)^(1/2)/a^(3/2)/x/(a^(1/2)+c^(1/2)
*x^2)-1/5*(A+5*a*C/c+5*(B+2*b*C/c)*x^2)*(c*x^4+b*x^2+a)^(1/2)/x^5+C*(c*x^4
+b*x^2+a)^(3/2)/c/x^5+1/15*c^(1/4)*(2*A*(-3*a*c+b^2)-5*a*(B*b+6*C*a))*(a^(
1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(7/4
)/(c*x^4+b*x^2+a)^(1/2)-1/30*(b+2*a^(1/2)*c^(1/2))*(2*A*b*c^(1/2)-5*a*B*c^
(1/2)-3*a^(1/2)*A*c-15*a^(3/2)*C)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(
a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),
1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(7/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.06 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^6}\,dx$$
$$= \frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)\left(-2Ab^2x^4+a(5bBx^4+Ax^2(b+6cx^2))+a^2(3A+5x^2(B+3Cx^2))\right)-i}{}$$

input
```
Integrate[(Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4))/x^6,x]
```

output
```
(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(-2*A*b^2*x^4 + a*
(5*b*B*x^4 + A*x^2*(b + 6*c*x^2)) + a^2*(3*A + 5*x^2*(B + 3*C*x^2))) - I*(
-b + Sqrt[b^2 - 4*a*c])*(2*A*(b^2 - 3*a*c) - 5*a*(b*B + 6*a*C))*x^5*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sq
rt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sq
rt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c])] + I*(2*A*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*
Sqrt[b^2 - 4*a*c]) - 5*a*(-(b^2*B) + 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] + 6*a
*Sqrt[b^2 - 4*a*c]*C))*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2
- 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
 (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(60*a^2*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x^5*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2199, 1594, 1604, 1604, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}\left(A+Bx^2+Cx^4\right)}{x^6} \, dx$$

$$\Big\downarrow \text{2199}$$

$$\int \frac{\left(\left(B+\frac{2bC}{c}\right)x^2+A+\frac{5aC}{c}\right)\sqrt{cx^4+bx^2+a}}{x^6}dx+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^5}$$

$$\Big\downarrow \text{1594}$$

$$\frac{1}{5}\int \frac{-\left(\left(\frac{10Cb^2}{c}+5Bb-2Ac-10aC\right)x^2\right)+Ab-\frac{5a(2Bc+3bC)}{c}}{x^4\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}\left(\frac{5aC}{c}+A+5x^2\left(\frac{2bC}{c}+B\right)\right)}{5x^5}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^5}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{5}\Bigg(-\frac{\int \frac{(Abc-10aBc-15abC)x^2+2A(b^2-3ac)-5a(bB+6aC)}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2+cx^4}\left(Ab-\frac{5a(3bC+2Bc)}{c}\right)}{3ax^3}\Bigg)-\frac{\sqrt{a+bx^2+cx^4}\left(\frac{5aC}{c}+A+5x^2\left(\frac{2bC}{c}+B\right)\right)}{5x^5}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^5}$$

$\big\downarrow$ 1604

$$\frac{1}{5}\Bigg(-\frac{-\frac{\int -\frac{c\left(2A\left(b^2-3ac\right)-5a(bB+6aC)\right)x^2+a(Abc-10aBc-15abC)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}(2A(b^2-3ac)-5a(6aC+bB))}{ax}}{3a}-\frac{\sqrt{a+bx^2+cx^4}(}{3a}$$
$$\frac{\sqrt{a+bx^2+cx^4}\left(\frac{5aC}{c}+A+5x^2\left(\frac{2bC}{c}+B\right)\right)}{5x^5}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^5}$$

$\big\downarrow$ 25

$$\frac{1}{5}\Bigg(-\frac{\frac{\int \frac{c\left(2A\left(b^2-3ac\right)-5a(bB+6aC)\right)x^2+a(Abc-5a(2Bc+3bC))}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}(2A(b^2-3ac)-5a(6aC+bB))}{ax}}{3a}-\frac{\sqrt{a+bx^2+cx^4}(Ab}{3ax^3}$$
$$\frac{\sqrt{a+bx^2+cx^4}\left(\frac{5aC}{c}+A+5x^2\left(\frac{2bC}{c}+B\right)\right)}{5x^5}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^5}$$

$\big\downarrow$ 1511

$$\frac{1}{5}\Bigg(-\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(-15a^{3/2}C-3\sqrt{a}Ac-5aB\sqrt{c}+2Ab\sqrt{c})\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{a}\sqrt{c}(2A(b^2-3ac)-5a(6aC+bB))\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{a}-}{3a}$$
$$\frac{\sqrt{a+bx^2+cx^4}\left(\frac{5aC}{c}+A+5x^2\left(\frac{2bC}{c}+B\right)\right)}{5x^5}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^5}$$

$\big\downarrow$ 27

$$\frac{1}{5}\Bigg(-\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(-15a^{3/2}C-3\sqrt{a}Ac-5aB\sqrt{c}+2Ab\sqrt{c})\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}(2A(b^2-3ac)-5a(6aC+bB))\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+b}}{}}{3a}$$
$$\frac{\sqrt{a+bx^2+cx^4}\left(\frac{5aC}{c}+A+5x^2\left(\frac{2bC}{c}+B\right)\right)}{5x^5}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^5}$$

$\big\downarrow$ 1416

$$\frac{1}{5}\left(-\frac{\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-15a^{3/2}C-3\sqrt{a}Ac-5aB\sqrt{c}+2Ab\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}(2A(b^2-3ac\cdots}{a}}{3a}\cdots\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}\left(\frac{5aC}{c}+A+5x^2\left(\frac{2bC}{c}+B\right)\right)}{5x^5}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^5}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{5}\left(-\frac{\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-15a^{3/2}C-3\sqrt{a}Ac-5aB\sqrt{c}+2Ab\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}(2A(b^2-3ac\cdots}{a}}{\cdots}\cdots\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}\left(\frac{5aC}{c}+A+5x^2\left(\frac{2bC}{c}+B\right)\right)}{5x^5}+\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^5}$$

input
```
Int[(Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4))/x^6,x]
```

output
```
-1/5*((A + (5*a*C)/c + 5*(B + (2*b*C)/c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/x^5
 + (C*(a + b*x^2 + c*x^4)^(3/2))/(c*x^5) + (-1/3*((A*b - (5*a*(2*B*c + 3*b
*C))/c)*Sqrt[a + b*x^2 + c*x^4])/(a*x^3) - (-((((2*A*(b^2 - 3*a*c) - 5*a*(b
*B + 6*a*C))*Sqrt[a + b*x^2 + c*x^4])/(a*x)) + (-(Sqrt[c]*(2*A*(b^2 - 3*a*
c) - 5*a*(b*B + 6*a*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*
x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a
]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))) + (a^(1/4)*(b + 2*Sqrt
[a]*Sqrt[c])*(2*A*b*Sqrt[c] - 5*a*B*Sqrt[c] - 3*Sqrt[a]*A*c - 15*a^(3/2)*C
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])
/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/a)/(3*a))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 5.39 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.05

method	result
elliptic	$-\frac{A\sqrt{c\,x^{4}+b\,x^{2}+a}}{5x^{5}}-\frac{(Ab+5Ba)\sqrt{c\,x^{4}+b\,x^{2}+a}}{15a\,x^{3}}-\frac{(6Aac-2A\,b^{2}+5Bab+15a^{2}C)\sqrt{c\,x^{4}+b\,x^{2}+a}}{15a^{2}x}+\frac{\left(Bc+Cb-\frac{c(Ab+5Ba)}{15a}\right)\sqrt{2}}{}$
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(6Aac\,x^{4}-2A\,b^{2}x^{4}+5Bab\,x^{4}+15C\,a^{2}x^{4}+Aab\,x^{2}+5B\,a^{2}x^{2}+3A\,a^{2})}{15x^{5}a^{2}}-\frac{c\left(6Aac-2A\,b^{2}+5Bab+30a^{2}C\right)a\sqrt{2}\,\sqrt{4-\frac{2(}{}}}{}$
default	Expression too large to display

input
```
int((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A*(c*x^4+b*x^2+a)^(1/2)/x^5-1/15*(A*b+5*B*a)/a*(c*x^4+b*x^2+a)^(1/2)/
x^3-1/15*(6*A*a*c-2*A*b^2+5*B*a*b+15*C*a^2)/a^2*(c*x^4+b*x^2+a)^(1/2)/x+1/
4*(B*c+C*b-1/15*c*(A*b+5*B*a)/a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(C*c+1/
15*c*(6*A*a*c-2*A*b^2+5*B*a*b+15*C*a^2)/a^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Elli
pticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(A+Bx^2+Cx^4)\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)*(C*x**4+B*x**2+A)/x**6,x)
```

output
```
Integral((A + B*x**2 + C*x**4)*sqrt(a + b*x**2 + c*x**4)/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(C\,x^4+B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2+a}}{x^6}\,dx$$

input
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2))/x^6,x)
```

output
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^6}\,dx$$
$$=\frac{-\sqrt{c\,x^4+b\,x^2+a}\,a^2-10\sqrt{c\,x^4+b\,x^2+a}\,ab\,x^2-7\sqrt{c\,x^4+b\,x^2+a}\,ac\,x^4+15\sqrt{c\,x^4+b\,x^2+a}\,b^2x^4-}{5a\,x^5}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^6,x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4)*a**2 - 10*sqrt(a + b*x**2 + c*x**4)*a*b*x**2
 - 7*sqrt(a + b*x**2 + c*x**4)*a*c*x**4 + 15*sqrt(a + b*x**2 + c*x**4)*b**
2*x**4 - 24*int(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*a*
*2*b*x**5 + 12*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),
x)*a*c**2*x**5 - 15*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x
**4),x)*b**2*c*x**5)/(5*a*x**5)
```

3.138 $\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^8}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 574

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^8}\,dx = -\frac{\left(3Ab-7a\left(2B-\frac{5bC}{c}\right)\right)\sqrt{a+bx^2+cx^4}}{105ax^5}$$
$$+\frac{(4Ab^2-7abB-10aAc+70a^2C)\sqrt{a+bx^2+cx^4}}{105a^2x^3}$$
$$-\frac{(A(8b^3-29abc)-7a(2b^2B-6aBc-5abC))\sqrt{a+bx^2+cx^4}}{105a^{5/2}x\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{\left(3\left(A-\frac{7aC}{c}\right)+7\left(B-\frac{4bC}{c}\right)x^2\right)\sqrt{a+bx^2+cx^4}}{21x^7}-\frac{C(a+bx^2+cx^4)^{3/2}}{cx^7}$$
$$-\frac{\sqrt[4]{c}(A(8b^3-29abc)-7a(2b^2B-6aBc-5abC))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2\right.\right.}{105a^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(b+2\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\left(8Ab^2-14abB-12\sqrt{a}Ab\sqrt{c}+21a^{3/2}B\sqrt{c}-5aAc+35a^2C\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx}{(\sqrt{a}+}}}{210a^{11/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/105*(3*A*b-7*a*(2*B-5*b*C/c))*(c*x^4+b*x^2+a)^(1/2)/a/x^5+1/105*(-10*A*
a*c+4*A*b^2-7*B*a*b+70*C*a^2)*(c*x^4+b*x^2+a)^(1/2)/a^2/x^3-1/105*(A*(-29*
a*b*c+8*b^3)-7*a*(-6*B*a*c+2*B*b^2-5*C*a*b))*(c*x^4+b*x^2+a)^(1/2)/a^(5/2)
/x/(a^(1/2)+c^(1/2)*x^2)-1/21*(3*A-21*a*C/c+7*(B-4*b*C/c)*x^2)*(c*x^4+b*x^
2+a)^(1/2)/x^7-C*(c*x^4+b*x^2+a)^(3/2)/c/x^7-1/105*c^(1/4)*(A*(-29*a*b*c+8
*b^3)-7*a*(-6*B*a*c+2*B*b^2-5*C*a*b))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+
a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(11/4)/(c*x^4+b*x^2+a)^(1/2)+1/210*(
b+2*a^(1/2)*c^(1/2))*c^(1/4)*(8*A*b^2-14*B*a*b-12*a^(1/2)*A*b*c^(1/2)+21*a
^(3/2)*B*c^(1/2)-5*A*a*c+35*C*a^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/
(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4))
,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.37 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^8}\,dx$$

$$= \frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)\left(8Ab^3x^6-abx^4(4Ab+14bBx^2+29Acx^2)+a^3(15A+21Bx^2+35Cx^4)\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4))/x^8,x]
```

output
```
(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(8*A*b^3*x^6 - a*b
*x^4*(4*A*b + 14*b*B*x^2 + 29*A*c*x^2) + a^3*(15*A + 21*B*x^2 + 35*C*x^4)
+ a^2*(3*A*b*x^2 + 7*b*B*x^4 + 10*A*c*x^4 + 42*B*c*x^6 + 35*b*C*x^6)) + I*
(-b + Sqrt[b^2 - 4*a*c])*(A*(8*b^3 - 29*a*b*c) + 7*a*(-2*b^2*B + 6*a*B*c +
 5*a*b*C))*x^7*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*E
llipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(A*(-8*b^4 + 37*a*b^2*c - 20*a^2
*c^2 + 8*b^3*Sqrt[b^2 - 4*a*c] - 29*a*b*c*Sqrt[b^2 - 4*a*c]) + 7*a*(2*b^3*
B - b^2*(2*B*Sqrt[b^2 - 4*a*c] + 5*a*C) + 2*a*c*(3*B*Sqrt[b^2 - 4*a*c] + 1
0*a*C) + a*b*(-8*B*c + 5*Sqrt[b^2 - 4*a*c]*C)))*x^7*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c]
+ 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
)/(420*a^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^7*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2199, 1594, 1604, 27, 1604, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}\,(A+Bx^2+Cx^4)}{x^8}\,dx$$

$$\Big\downarrow \text{2199}$$

$$\int \frac{\left(\left(B-\frac{4bC}{c}\right)x^2+A-\frac{7aC}{c}\right)\sqrt{cx^4+bx^2+a}}{x^8}dx-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

$$\Big\downarrow \text{1594}$$

$$\frac{1}{21}\int \frac{-\left(\left(-\frac{28Cb^2}{c}+7Bb-6Ac+42aC\right)x^2\right)+3Ab-7a\left(2B-\frac{5bC}{c}\right)}{x^6\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}\left(3\left(A-\frac{7aC}{c}\right)+7x^2\left(B-\frac{4bC}{c}\right)\right)}{21x^7}-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{21}\left(-\frac{\int \frac{3\left(70Ca^2-7bBa-10Aca+4Ab^2+(3Abc-14aBc+35abC)x^2\right)}{x^4\sqrt{cx^4+bx^2+a}}dx}{5a}-\frac{\sqrt{a+bx^2+cx^4}\left(3Ab-7a\left(2B-\frac{5bC}{c}\right)\right)}{5ax^5}\right)-\frac{\sqrt{a+bx^2+cx^4}\left(3\left(A-\frac{7aC}{c}\right)+7x^2\left(B-\frac{4bC}{c}\right)\right)}{21x^7}-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

$\downarrow$ 27

$$\frac{1}{21}\left(-\frac{3\int \frac{70Ca^2-7bBa-10Aca+4Ab^2+(3Abc-14aBc+35abC)x^2}{x^4\sqrt{cx^4+bx^2+a}}dx}{5a}-\frac{\sqrt{a+bx^2+cx^4}\left(3Ab-7a\left(2B-\frac{5bC}{c}\right)\right)}{5ax^5}\right)-\frac{\sqrt{a+bx^2+cx^4}\left(3\left(A-\frac{7aC}{c}\right)+7x^2\left(B-\frac{4bC}{c}\right)\right)}{21x^7}-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

$\downarrow$ 1604

$$\frac{1}{21}\left(-\frac{3\left(-\frac{\int \frac{c\left(70Ca^2-7bBa-10Aca+4Ab^2\right)x^2+A\left(8b^3-29abc\right)-7a\left(2Bb^2-5aCb-6aBc\right)}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2+cx^4}\left(70a^2C-10aAc-7abB+4Ab^2\right)}{3ax^3}\right)}{5a}-\right.$$

$$-\frac{\sqrt{a+bx^2+cx^4}\left(3\left(A-\frac{7aC}{c}\right)+7x^2\left(B-\frac{4bC}{c}\right)\right)}{21x^7}-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

$\downarrow$ 1604

$$\frac{1}{21}\left(-\frac{3\left(-\frac{-\frac{\int -\frac{c\left(\left(A\left(8b^3-29abc\right)-7a\left(2Bb^2-5aCb-6aBc\right)\right)x^2+a\left(70Ca^2-7bBa-10Aca+4Ab^2\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(A\left(8b^3-29abc\right)-7a\left(-5abC-6a\right.\right.}{ax}}{3a}\right.}{5a}\right.$$

$$-\frac{\sqrt{a+bx^2+cx^4}\left(3\left(A-\frac{7aC}{c}\right)+7x^2\left(B-\frac{4bC}{c}\right)\right)}{21x^7}-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

$\downarrow$ 25

$$\frac{1}{21}\left(-\frac{3\left(-\frac{\frac{\int \frac{c\left(\left(A\left(8b^3-29abc\right)-7a\left(2Bb^2-5aCb-6aBc\right)\right)x^2+a\left(70Ca^2-7bBa-10Aca+4Ab^2\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(A\left(8b^3-29abc\right)-7a\left(-5abC-6aBc+\cdots\right.\right.}{ax}}{3a}\cdots\right.}{5a}\cdots\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}\left(3\left(A-\frac{7aC}{c}\right)+7x^2\left(B-\frac{4bC}{c}\right)\right)}{21x^7}-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

$\big\downarrow$ 27

$$\frac{1}{21}\left(-\frac{3\left(-\frac{\frac{c\int \frac{\left(A\left(8b^3-29abc\right)-7a\left(2Bb^2-5aCb-6aBc\right)\right)x^2+a\left(70Ca^2-7bBa-10Aca+4Ab^2\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(A\left(8b^3-29abc\right)-7a\left(-5abC-6aBc+2b\cdots\right.\right.}{ax}}{3a}\cdots\right.}{5a}\cdots\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}\left(3\left(A-\frac{7aC}{c}\right)+7x^2\left(B-\frac{4bC}{c}\right)\right)}{21x^7}-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

$\big\downarrow$ 1511

$$\frac{1}{21}\left(-\frac{3\left(-\frac{\frac{c\left(\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(21a^{3/2}B\sqrt{c}+35a^2C-12\sqrt{a}Ab\sqrt{c}-5aAc-14abB+8Ab^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(A\left(8b^3-29abc\right)-7a\left(-5abC-6aBc+2b^2B\cdots\right.\right.}{\sqrt{c}}\cdots\right.}{a}\cdots}{3a}\cdots\right.}{5a\cdots}\cdots\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}\left(3\left(A-\frac{7aC}{c}\right)+7x^2\left(B-\frac{4bC}{c}\right)\right)}{21x^7}-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

$\big\downarrow$ 27

$$\frac{1}{21}\left(-\frac{3\left(-\frac{\frac{c\left(\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(21a^{3/2}B\sqrt{c}+35a^2C-12\sqrt{a}Ab\sqrt{c}-5aAc-14abB+8Ab^2\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(A\left(8b^3-29abc\right)-7a\left(-5abC-6aBc+2b^2B\right)\right)}{\sqrt{c}}\right.}{a}}{3a}\right.}{5a}\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}\left(3\left(A-\frac{7aC}{c}\right)+7x^2\left(B-\frac{4bC}{c}\right)\right)}{21x^7}-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

$\big\downarrow$ 1416

$$\frac{1}{21}\left(-\frac{3\left(-\frac{\frac{c\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(21a^{3/2}B\sqrt{c}+35a^2C-12\sqrt{a}Ab\sqrt{c}-5aAc-14abB+8Ab^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}\right.}{a}}{3a}\right.}{}\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}\left(3\left(A-\frac{7aC}{c}\right)+7x^2\left(B-\frac{4bC}{c}\right)\right)}{21x^7}-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

$\big\downarrow$ 1509

$$\frac{1}{21}\left(-\frac{3\left(-\frac{\sqrt{a+bx^2+cx^4}(70a^2C-10aAc-7abB+4Ab^2)}{3ax^3}-\frac{c\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(21a^{3/2}B\sqrt{c}+35a^2C-12\sqrt{a}Ab\sqrt{c}-5a\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}\right.}{}\right.}{}\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}\left(3\left(A-\frac{7aC}{c}\right)+7x^2\left(B-\frac{4bC}{c}\right)\right)}{21x^7}-\frac{C\left(a+bx^2+cx^4\right)^{3/2}}{cx^7}$$

input
```
Int[(Sqrt[a + b*x^2 + c*x^4]*(A + B*x^2 + C*x^4))/x^8,x]
```

output
```
-1/21*((3*(A - (7*a*C)/c) + 7*(B - (4*b*C)/c)*x^2)*Sqrt[a + b*x^2 + c*x^4]
)/x^7 - (C*(a + b*x^2 + c*x^4)^(3/2))/(c*x^7) + (-1/5*((3*A*b - 7*a*(2*B -
 (5*b*C)/c))*Sqrt[a + b*x^2 + c*x^4])/(a*x^5) - (3*(-1/3*((4*A*b^2 - 7*a*b
*B - 10*a*A*c + 70*a^2*C)*Sqrt[a + b*x^2 + c*x^4])/(a*x^3) - (-(((A*(8*b^3
 - 29*a*b*c) - 7*a*(2*b^2*B - 6*a*B*c - 5*a*b*C))*Sqrt[a + b*x^2 + c*x^4])
/(a*x)) + (c*(-(((A*(8*b^3 - 29*a*b*c) - 7*a*(2*b^2*B - 6*a*B*c - 5*a*b*C)
)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/
4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*
(8*A*b^2 - 14*a*b*B - 12*Sqrt[a]*A*b*Sqrt[c] + 21*a^(3/2)*B*Sqrt[c] - 5*a*
A*c + 35*a^2*C)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])))/a)/(3*a)))/(5*a))/21
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 9.75 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.01

method	result
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{7x^7}-\frac{(Ab+7Ba)\sqrt{c\,x^4+b\,x^2+a}}{35a\,x^5}-\frac{(10Aac-4A\,b^2+7Bab+35a^2C)\sqrt{c\,x^4+b\,x^2+a}}{105a^2x^3}+\frac{(29Aabc-8A\,b^3-42B\,a^2c$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-29Aabc\,x^6+8A\,b^3x^6+42B\,a^2c\,x^6-14Ba\,b^2x^6+35C\,a^2b\,x^6+10A\,a^2c\,x^4-4Aa\,b^2x^4+7B\,a^2b\,x^4+35C\,a^3x^4+3A}{105x^7a^3}$
default	Expression too large to display

input

```
int((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

output

```
-1/7*A*(c*x^4+b*x^2+a)^(1/2)/x^7-1/35*(A*b+7*B*a)/a*(c*x^4+b*x^2+a)^(1/2)/
x^5-1/105/a^2*(10*A*a*c-4*A*b^2+7*B*a*b+35*C*a^2)*(c*x^4+b*x^2+a)^(1/2)/x^
3+1/105*(29*A*a*b*c-8*A*b^3-42*B*a^2*c+14*B*a*b^2-35*C*a^2*b)/a^3*(c*x^4+b
*x^2+a)^(1/2)/x+1/4*(C*c-1/105*c*(10*A*a*c-4*A*b^2+7*B*a*b+35*C*a^2)/a^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*E
llipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/210*c*(29*A*a*b*c-8*A*b^3-42*B*a^2*c+14*B*
a*b^2-35*C*a^2*b)/a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^8}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left((35\,Ca^3b-14\,Ba^2b^2+8\,Aab^3+(42\,Ba^3-29\,Aa^2b)c)x^7\sqrt{\frac{b^2-4\,ac}{a^2}}-(35\,Ca^2b^2-14\,Bab^3+8\,Ab\right.}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^8,x, algorithm="fricas")
```

output
```
-1/210*(sqrt(1/2)*((35*C*a^3*b - 14*B*a^2*b^2 + 8*A*a*b^3 + (42*B*a^3 - 29
*A*a^2*b)*c)*x^7*sqrt((b^2 - 4*a*c)/a^2) - (35*C*a^2*b^2 - 14*B*a*b^3 + 8*
A*b^4 + (42*B*a^2*b - 29*A*a*b^2)*c)*x^7)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*
c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a
^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + sqr
t(1/2)*((70*C*a^4 - 7*(B + 5*C)*a^3*b + 2*(2*A + 7*B)*a^2*b^2 - 8*A*a*b^3
- (2*(5*A + 21*B)*a^3 - 29*A*a^2*b)*c)*x^7*sqrt((b^2 - 4*a*c)/a^2) + (70*C
*a^3*b - 7*(B - 5*C)*a^2*b^2 + 2*(2*A - 7*B)*a*b^3 + 8*A*b^4 - (2*(5*A - 2
1*B)*a^2*b + 29*A*a*b^2)*c)*x^7)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) -
 b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/
a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + 2*((35*C*a^3
*b - 14*B*a^2*b^2 + 8*A*a*b^3 + (42*B*a^3 - 29*A*a^2*b)*c)*x^6 + 15*A*a^4
+ (35*C*a^4 + 7*B*a^3*b - 4*A*a^2*b^2 + 10*A*a^3*c)*x^4 + 3*(7*B*a^4 + A*a
^3*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^7)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^8}\,dx = \int \frac{(A+Bx^2+Cx^4)\sqrt{a+bx^2+cx^4}}{x^8}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)*(C*x**4+B*x**2+A)/x**8,x)
```

output
```
Integral((A + B*x**2 + C*x**4)*sqrt(a + b*x**2 + c*x**4)/x**8, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^8}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}}{x^8}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^8,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^8}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{cx^4+bx^2+a}}{x^8}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^8,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^8}\,dx = \int \frac{(C\,x^4+B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2+a}}{x^8}\,dx$$

input
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2))/x^8,x)
```

output
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(1/2))/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}(A+Bx^2+Cx^4)}{x^8}\,dx$$
$$= \frac{-\sqrt{c\,x^4+b\,x^2+a}\,a-7\sqrt{c\,x^4+b\,x^2+a}\,c\,x^4+8\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^{10}+b\,x^8+a\,x^6}dx\right)ab\,x^7-12\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^8+b\,x^6+a\,x^4}dx\right)ac\,x^7}{7x^7}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)*(C*x^4+B*x^2+A)/x^8,x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4)*a - 7*sqrt(a + b*x**2 + c*x**4)*c*x**4 + 8*i
nt(sqrt(a + b*x**2 + c*x**4)/(a*x**6 + b*x**8 + c*x**10),x)*a*b*x**7 - 12*
int(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*a*c*x**7 + 7*i
nt(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*b**2*x**7)/(7*x
**7)
```

3.139 $\int x^4(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1073

$$\int x^4\left(a+bx^2+cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx = \text{Too large to display}$$

output
```
1/45045*(384*b^6*B*c-3144*a*b^4*B*c^2+7065*a^2*b^2*B*c^3-2772*a^3*B*c^4-25
6*b^7*C+30*a*b^3*c^2*(156*A*c-209*C*a)-24*a^2*b*c^3*(351*A*c-194*C*a)-48*b
^5*c*(13*A*c-48*C*a))*x*(c*x^4+b*x^2+a)^(1/2)/c^(11/2)/(a^(1/2)+c^(1/2)*x^
2)-1/45045*x*(192*b^5*B*c-636*a*b^3*B*c^2-423*a^2*b*B*c^3-128*b^6*C-24*b^4
*c*(13*A*c-22*C*a)+390*a^2*c^3*(3*A*c-C*a)+9*a*b^2*c^2*(91*A*c+C*a)+3*c*(1
92*b^4*B*c-876*a*b^2*B*c^2+462*a^2*B*c^3-128*b^5*C+3*a*b*c^2*(403*A*c-237*
C*a)-8*b^3*c*(39*A*c-86*C*a))*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^5+1/9009*x*(144
*b^3*B*c-27*a*b*B*c^2-96*b^4*C-6*b^2*c*(39*A*c-16*C*a)-39*a*c^2*(3*A*c-C*a
)+7*c*(48*b^2*B*c-33*a*B*c^2-32*b^3*C-6*b*c*(13*A*c-8*C*a))*x^2)*(c*x^4+b*
x^2+a)^(3/2)/c^4-1/429*(24*B*b*c-16*b^2*C-13*c*(3*A*c-C*a))*x*(c*x^4+b*x^2
+a)^(5/2)/c^3+1/39*(3*B*c-2*C*b)*x^3*(c*x^4+b*x^2+a)^(5/2)/c^2+1/15*C*x^5*
(c*x^4+b*x^2+a)^(5/2)/c-1/45045*a^(1/4)*(384*b^6*B*c-3144*a*b^4*B*c^2+7065
*a^2*b^2*B*c^3-2772*a^3*B*c^4-256*b^7*C+30*a*b^3*c^2*(156*A*c-209*C*a)-24*
a^2*b*c^3*(351*A*c-194*C*a)-48*b^5*c*(13*A*c-48*C*a))*(a^(1/2)+c^(1/2)*x^2
)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(23/4)/(c*x^4+b*x^2+
a)^(1/2)-1/90090*a^(1/4)*(b+2*a^(1/2)*c^(1/2))*(256*b^6*C-18*a*b^2*c^2*(14
6*a^(1/2)*B*c^(1/2)+156*A*c-119*a*C)+6*a^2*c^3*(231*a^(1/2)*B*c^(1/2)+195*
A*c-65*a*C)+48*b^4*c*(12*a^(1/2)*B*c^(1/2)+13*A*c-32*a*C)+24*a^(1/2)*b^3*c
^(3/2)*(83*a^(1/2)*B*c^(1/2)-39*A*c+86*a*C)-9*a^(3/2)*b*c^(5/2)*(201*a^...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.47 (sec) , antiderivative size = 5595, normalized size of antiderivative = 5.21

$$\int x^4 \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \text{Result too large to show}$$

input
```
Integrate[x^4*(a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 1006, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2199, 1596, 27, 1596, 25, 1602, 27, 1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+bx^2+cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)dx$$

$$\Big\downarrow \ 2199$$

$$\int x^4\left(\frac{1}{3}\left(3B-\frac{2bC}{c}\right)x^2+\frac{1}{3}\left(3A-\frac{aC}{c}\right)\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{Cx^5\left(a+bx^2+cx^4\right)^{5/2}}{15c}$$

$$\Big\downarrow \ 1596$$

$$\frac{3\int -\frac{x^4\left(\left(-16Cb^3+24Bcb^2-3c(13Ac-19aC)b-66aBc^2\right)x^2+ac\left(-\frac{10Cb^2}{c}+15Bb-78Ac+26aC\right)\right)\sqrt{cx^4+bx^2+a}}{3c}dx}{143c}+\frac{x^5\left(a+bx^2+cx^4\right)^{3/2}\left(-13aC+39Ac-\frac{6b^2C}{c}+11x^2(3Bc-2bC)+9bB\right)}{429c}+\frac{Cx^5\left(a+bx^2+cx^4\right)^{5/2}}{15c}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int x^4\left(\left(-16Cb^3+24Bcb^2-3c(13Ac-19aC)b-66aBc^2\right)x^2+a\left(-10Cb^2+15Bcb-26c(3Ac-aC)\right)\right)\sqrt{cx^4-}}{143c^2}+\frac{x^5\left(a+bx^2+cx^4\right)^{3/2}\left(-13aC+39Ac-\frac{6b^2C}{c}+11x^2(3Bc-2bC)+9bB\right)}{429c}+\frac{Cx^5\left(a+bx^2+cx^4\right)^{5/2}}{15c}$$

$$\Big\downarrow \ 1596$$

$$-\frac{\frac{\int -\frac{x^4\left(\left(-96Cb^5+144Bcb^4-2c(117Ac-328aC)b^3-867aBc^2b^2+24ac^2(52Ac-43aC)b+924a^2Bc^3\right)x^2+a\left(-80Cb^4+120Bcb^3-15c(13Ac-31aC)b^2-600aBc^2b-\right.}{\sqrt{cx^4+bx^2+a}}}{63c}}{}$$

$$\frac{x^5\left(a+bx^2+cx^4\right)^{3/2}\left(-13aC+39Ac-\frac{6b^2C}{c}+11x^2(3Bc-2bC)+9bB\right)}{429c}+\frac{Cx^5\left(a+bx^2+cx^4\right)^{5/2}}{15c}$$

$\downarrow$ 25

$$-\frac{\frac{x^5\sqrt{a+bx^2+cx^4}\left(-3b^2c(11aC+13Ac)+7cx^2\left(-3bc(13Ac-19aC)-66aBc^2-16b^3C+24b^2Bc\right)-234ac^2(3Ac-aC)+69abBc^2-16b^4C+24b^3Bc\right)}{63c}}{}$$

$$\frac{x^5\left(a+bx^2+cx^4\right)^{3/2}\left(-13aC+39Ac-\frac{6b^2C}{c}+11x^2(3Bc-2bC)+9bB\right)}{429c}+\frac{Cx^5\left(a+bx^2+cx^4\right)^{5/2}}{15c}$$

$\downarrow$ 1602

$$-\frac{\frac{x^5\sqrt{a+bx^2+cx^4}\left(-3b^2c(11aC+13Ac)+7cx^2\left(-3bc(13Ac-19aC)-66aBc^2-16b^3C+24b^2Bc\right)-234ac^2(3Ac-aC)+69abBc^2-16b^4C+24b^3Bc\right)}{63c}}{}$$

$$\frac{x^5\left(a+bx^2+cx^4\right)^{3/2}\left(-13aC+39Ac-\frac{6b^2C}{c}+11x^2(3Bc-2bC)+9bB\right)}{429c}+\frac{Cx^5\left(a+bx^2+cx^4\right)^{5/2}}{15c}$$

$\downarrow$ 27

$$-\frac{\frac{x^5\sqrt{a+bx^2+cx^4}\left(-3b^2c(11aC+13Ac)+7cx^2\left(-3bc(13Ac-19aC)-66aBc^2-16b^3C+24b^2Bc\right)-234ac^2(3Ac-aC)+69abBc^2-16b^4C+24b^3Bc\right)}{63c}}{}$$

$$\frac{x^5\left(a+bx^2+cx^4\right)^{3/2}\left(-13aC+39Ac-\frac{6b^2C}{c}+11x^2(3Bc-2bC)+9bB\right)}{429c}+\frac{Cx^5\left(a+bx^2+cx^4\right)^{5/2}}{15c}$$

$\downarrow$ 1602

$$-\frac{\frac{x^5\sqrt{a+bx^2+cx^4}\left(-3b^2c(11aC+13Ac)+7cx^2\left(-3bc(13Ac-19aC)-66aBc^2-16b^3C+24b^2Bc\right)-234ac^2(3Ac-aC)+69abBc^2-16b^4C+24b^3Bc\right)}{63c}}{}$$

$$\frac{x^5\left(a+bx^2+cx^4\right)^{3/2}\left(-13aC+39Ac-\frac{6b^2C}{c}+11x^2(3Bc-2bC)+9bB\right)}{429c}+$$

$$\frac{Cx^5\left(a+bx^2+cx^4\right)^{5/2}}{15c}$$

$$\Big\downarrow \; 1511$$

$$\frac{C\left(cx^4+bx^2+a\right)^{5/2}x^5}{15c}+$$

$$\frac{\left(-\frac{6Cb^2}{c}+9Bb+11(3Bc-2bC)x^2+39Ac-13aC\right)\left(cx^4+bx^2+a\right)^{3/2}x^5}{429c}-$$

$$\frac{\frac{x^5\left(-16Cb^4+24Bcb^3-3c(13Ac+11aC)b^2+69aBc^2b+7c\left(-16Cb^3+24Bcb^2-3c(13Ac-19aC)b-66aBc^2\right)x^2-234ac^2(3Ac-aC)\right)\sqrt{cx^4+bx^2+a}}{63c}-}{}$$

$$\Big\downarrow \; 27$$

$$\frac{C\left(cx^4+bx^2+a\right)^{5/2}x^5}{15c}+$$

$$\frac{\left(-\frac{6Cb^2}{c}+9Bb+11(3Bc-2bC)x^2+39Ac-13aC\right)\left(cx^4+bx^2+a\right)^{3/2}x^5}{429c}-$$

$$\frac{\frac{x^5\left(-16Cb^4+24Bcb^3-3c(13Ac+11aC)b^2+69aBc^2b+7c\left(-16Cb^3+24Bcb^2-3c(13Ac-19aC)b-66aBc^2\right)x^2-234ac^2(3Ac-aC)\right)\sqrt{cx^4+bx^2+a}}{63c}-}{}$$

$$\Big\downarrow \; 1416$$

$$\frac{C\left(cx^4+bx^2+a\right)^{5/2}x^5}{15c}+$$

$$\frac{\left(-\frac{6Cb^2}{c}+9Bb+11(3Bc-2bC)x^2+39Ac-13aC\right)\left(cx^4+bx^2+a\right)^{3/2}x^5}{429c}-$$

$$\frac{x^5\left(-16Cb^4+24Bcb^3-3c(13Ac+11aC)b^2+69aBc^2b+7c\left(-16Cb^3+24Bcb^2-3c(13Ac-19aC)b-66aBc^2\right)x^2-234ac^2(3Ac-aC)\right)\sqrt{cx^4+bx^2+a}}{63c}-$$

$$\Big\downarrow \ 1509$$

$$\frac{C\left(cx^4+bx^2+a\right)^{5/2}x^5}{15c}+$$

$$\frac{\left(-\frac{6Cb^2}{c}+9Bb+11(3Bc-2bC)x^2+39Ac-13aC\right)\left(cx^4+bx^2+a\right)^{3/2}x^5}{429c}-$$

$$\frac{x^5\left(-16Cb^4+24Bcb^3-3c(13Ac+11aC)b^2+69aBc^2b+7c\left(-16Cb^3+24Bcb^2-3c(13Ac-19aC)b-66aBc^2\right)x^2-234ac^2(3Ac-aC)\right)\sqrt{cx^4+bx^2+a}}{63c}-$$

input
```
Int[x^4*(a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
(x^5*(9*b*B + 39*A*c - 13*a*C - (6*b^2*C)/c + 11*(3*B*c - 2*b*C)*x^2)*(a +
 b*x^2 + c*x^4)^(3/2))/(429*c) + (C*x^5*(a + b*x^2 + c*x^4)^(5/2))/(15*c)
- ((x^5*(24*b^3*B*c + 69*a*b*B*c^2 - 16*b^4*C - 234*a*c^2*(3*A*c - a*C) -
3*b^2*c*(13*A*c + 11*a*C) + 7*c*(24*b^2*B*c - 66*a*B*c^2 - 16*b^3*C - 3*b*
c*(13*A*c - 19*a*C))*x^2)*Sqrt[a + b*x^2 + c*x^4])/(63*c) - (((144*b^4*B*c
 - 867*a*b^2*B*c^2 + 924*a^2*B*c^3 - 96*b^5*C - 2*b^3*c*(117*A*c - 328*a*C
) + 24*a*b*c^2*(52*A*c - 43*a*C))*x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) - (3*
(((192*b^5*B*c - 1356*a*b^3*B*c^2 + 2232*a^2*b*B*c^3 - 128*b^6*C + 9*a*b^2
*c^2*(221*A*c - 239*a*C) - 24*b^4*c*(13*A*c - 42*a*C) - 780*a^2*c^3*(3*A*c
 - a*C))*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (-(((384*b^6*B*c - 3144*a*b^4*
B*c^2 + 7065*a^2*b^2*B*c^3 - 2772*a^3*B*c^4 - 256*b^7*C + 30*a*b^3*c^2*(15
6*A*c - 209*a*C) - 24*a^2*b*c^3*(351*A*c - 194*a*C) - 48*b^5*c*(13*A*c - 4
8*a*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) - (a^(1/4)*(b + 2*Sqrt[a]*Sqr
t[c])*(256*b^6*C - 18*a*b^2*c^2*(146*Sqrt[a]*B*Sqrt[c] + 156*A*c - 119*a*C
) + 6*a^2*c^3*(231*Sqrt[a]*B*Sqrt[c] + 195*A*c - 65*a*C) + 48*b^4*c*(12*Sq
rt[a]*B*Sqrt[c] + 13*A*c - 32*a*C) + 24*Sqrt[a]*b^3*c^(3/2)*(83*Sqrt[a]*B*
Sqrt[c] - 39*A*c + 86*a*C) - 9*a^(3/2)*b*c^(5/2)*(201*Sqrt[a]*B*Sqrt[c]...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1596
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Simp[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a +
 b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c
*e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &
& NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[
p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2248 vs. $2(953) = 1906$.

Time = 11.22 (sec) , antiderivative size = 2249, normalized size of antiderivative = 2.10

method	result	size
risch	Expression too large to display	2249
elliptic	Expression too large to display	2615
default	Expression too large to display	2738

input
```
int(x^4*(c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/45045/c^5*x*(3003*C*c^6*x^12+3465*B*c^6*x^10+3696*C*b*c^5*x^10+4095*A*c^
6*x^8+4410*B*b*c^5*x^8+4641*C*a*c^5*x^8+63*C*b^2*c^4*x^8+5460*A*b*c^5*x^6+
5775*B*a*c^5*x^6+105*B*b^2*c^4*x^6+336*C*a*b*c^4*x^6-70*C*b^3*c^3*x^6+7605
*A*a*c^5*x^4+195*A*b^2*c^4*x^4+600*B*a*b*c^4*x^4-120*B*b^3*c^3*x^4+468*C*a
^2*c^4*x^4-465*C*a*b^2*c^3*x^4+80*C*b^4*c^2*x^4+1248*A*a*b*c^4*x^2-234*A*b
^3*c^3*x^2+924*B*a^2*c^4*x^2-867*B*a*b^2*c^3*x^2+144*B*b^4*c^2*x^2-1032*C*
a^2*b*c^3*x^2+656*C*a*b^3*c^2*x^2-96*C*b^5*c*x^2+2340*A*a^2*c^4-1989*A*a*b
^2*c^3+312*A*b^4*c^2-2232*B*a^2*b*c^3+1356*B*a*b^3*c^2-192*B*b^5*c-780*C*a
^3*c^3+2151*C*a^2*b^2*c^2-1008*C*a*b^4*c+128*C*b^6)*(c*x^4+b*x^2+a)^(1/2)-
1/45045/c^5*(-1/2*(8424*A*a^2*b*c^4-4680*A*a*b^3*c^3+624*A*b^5*c^2+2772*B*
a^3*c^4-7065*B*a^2*b^2*c^3+3144*B*a*b^4*c^2-384*B*b^6*c-4656*C*a^3*b*c^3+6
270*C*a^2*b^3*c^2-2304*C*a*b^5*c+256*C*b^7)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ellip
ticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+585*A*a^3*c^4*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1253, normalized size of antiderivative = 1.17

$$\int x^4 (a + bx^2 + cx^4)^{3/2} (A + Bx^2 + Cx^4) \, dx = \text{Too large to display}$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
-1/90090*(sqrt(1/2)*((256*C*b^7*c + 36*(77*B*a^3 + 234*A*a^2*b)*c^5 - 3*(1
552*C*a^3*b + 2355*B*a^2*b^2 + 1560*A*a*b^3)*c^4 + 6*(1045*C*a^2*b^3 + 524
*B*a*b^4 + 104*A*b^5)*c^3 - 384*(6*C*a*b^5 + B*b^6)*c^2)*x*sqrt((b^2 - 4*a
*c)/c^2) - (256*C*b^8 + 36*(77*B*a^3*b + 234*A*a^2*b^2)*c^4 - 3*(1552*C*a^
3*b^2 + 2355*B*a^2*b^3 + 1560*A*a*b^4)*c^3 + 6*(1045*C*a^2*b^4 + 524*B*a*b
^5 + 104*A*b^6)*c^2 - 384*(6*C*a*b^6 + B*b^7)*c)*x)*sqrt(c)*sqrt((c*sqrt((
b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 -
4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a
*c)) - sqrt(1/2)*((256*C*b^7*c - 2340*A*a^2*c^6 + 3*(4*(231*B + 65*C)*a^3
+ 24*(117*A + 31*B)*a^2*b + 663*A*a*b^2)*c^5 - 3*(1552*C*a^3*b + 3*(785*B
+ 239*C)*a^2*b^2 + 4*(390*A + 113*B)*a*b^3 + 104*A*b^4)*c^4 + 6*(1045*C*a^
2*b^3 + 4*(131*B + 42*C)*a*b^4 + 8*(13*A + 4*B)*b^5)*c^3 - 128*(18*C*a*b^5
 + (3*B + C)*b^6)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (256*C*b^8 + 2340*A*a^2
*b*c^5 + 3*(4*(231*B - 65*C)*a^3*b + 24*(117*A - 31*B)*a^2*b^2 - 663*A*a*b
^3)*c^4 - 3*(1552*C*a^3*b^2 + 3*(785*B - 239*C)*a^2*b^3 + 4*(390*A - 113*B
)*a*b^4 - 104*A*b^5)*c^3 + 6*(1045*C*a^2*b^4 + 4*(131*B - 42*C)*a*b^5 + 8*
(13*A - 4*B)*b^6)*c^2 - 128*(18*C*a*b^6 + (3*B - C)*b^7)*c)*x)*sqrt(c)*sqr
t((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*s
qrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2
- 2*a*c)/(a*c)) - 2*(3003*C*c^8*x^14 + 231*(16*C*b*c^7 + 15*B*c^8)*x^12...
```

Sympy [F]

$$\int x^4\left(a+bx^2+cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx=\int x^4\left(A+Bx^2+Cx^4\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**4*(c*x**4+b*x**2+a)**(3/2)*(C*x**4+B*x**2+A),x)
```

output
```
Integral(x**4*(A + B*x**2 + C*x**4)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int x^4 \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \int \left(Cx^4+Bx^2+A\right)\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^4 \, dx$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4 \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \int \left(Cx^4+Bx^2+A\right)\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^4 \, dx$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \int x^4 \left(C\,x^4+B\,x^2+A\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2} \,dx$$

input
```
int(x^4*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^4*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int x^4 (a+bx^2+cx^4)^{3/2} (A+Bx^2+Cx^4)\, dx = \frac{1560\sqrt{c\,x^4+b\,x^2+a}\,a^3c^3x - 2070\sqrt{c\,x^4+b\,x^2+a}\,a^2b^2c^2x + 1140\sqrt{c\,x^4+b\,x^2+a}\,a^2b\,c^3x^3 + \cdots}{\cdots}$$

input
```
int(x^4*(c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x)
```

output
```
(1560*sqrt(a + b*x**2 + c*x**4)*a**3*c**3*x - 2070*sqrt(a + b*x**2 + c*x**
4)*a**2*b**2*c**2*x + 1140*sqrt(a + b*x**2 + c*x**4)*a**2*b*c**3*x**3 + 80
73*sqrt(a + b*x**2 + c*x**4)*a**2*c**4*x**5 + 660*sqrt(a + b*x**2 + c*x**4
)*a*b**4*c*x - 445*sqrt(a + b*x**2 + c*x**4)*a*b**3*c**2*x**3 + 330*sqrt(a
 + b*x**2 + c*x**4)*a*b**2*c**3*x**5 + 11571*sqrt(a + b*x**2 + c*x**4)*a*b
*c**4*x**7 + 8736*sqrt(a + b*x**2 + c*x**4)*a*c**5*x**9 - 64*sqrt(a + b*x*
*2 + c*x**4)*b**6*x + 48*sqrt(a + b*x**2 + c*x**4)*b**5*c*x**3 - 40*sqrt(a
 + b*x**2 + c*x**4)*b**4*c**2*x**5 + 35*sqrt(a + b*x**2 + c*x**4)*b**3*c**
3*x**7 + 4473*sqrt(a + b*x**2 + c*x**4)*b**2*c**4*x**9 + 7161*sqrt(a + b*x
**2 + c*x**4)*b*c**5*x**11 + 3003*sqrt(a + b*x**2 + c*x**4)*c**6*x**13 - 1
560*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**4*c**3 + 207
0*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**3*b**2*c**2 -
660*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**2*b**4*c + 6
4*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b**6 - 6540*int
((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a**3*b*c**3 + 5
475*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a**2*b**
3*c**2 - 1464*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x
)*a*b**5*c + 128*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4
),x)*b**7)/(45045*c**4)
```

3.140 $\int x^2(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 903

$$\int x^2\left(a+bx^2+cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx =$$

$$-\frac{(624b^5Bc-4680ab^3Bc^2+8424a^2bBc^3-384b^6C+3ab^2c^2(2717Ac-2355aC)-8b^4c(143Ac-393aC)}{45045c^{9/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$+\frac{x\left(312b^4B-819ab^2Bc-1170a^2Bc^2-\frac{192b^5C}{c}-b^3(572Ac-636aC)+9abc(143Ac+47aC)+3(312b^3B\right.}{45045c^3}$$

$$-\frac{x\left(3\left(78b^2B-143Abc+39aBc+9abC-\frac{48b^3C}{c}\right)+7(78bBc-48b^2C-11c(13Ac-3aC))x^2\right)(a+bx^2}{9009c^2}$$

$$+\frac{(13Bc-8bC)x(a+bx^2+cx^4)^{5/2}}{143c^2}+\frac{Cx^3(a+bx^2+cx^4)^{5/2}}{13c}$$

$$+\frac{\sqrt[4]{a}(624b^5Bc-4680ab^3Bc^2+8424a^2bBc^3-384b^6C+3ab^2c^2(2717Ac-2355aC)-8b^4c(143Ac-393a}{45045c^{19/4}\sqrt{a+bx}}$$

$$+\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(384b^5C-3abc^2\left(1209\sqrt{a}B\sqrt{c}+1573Ac-603aC\right)+8b^3c\left(117\sqrt{a}B\sqrt{c}+143Ac-249\right.\right.}{}$$

output
```
-1/45045*(624*b^5*B*c-4680*a*b^3*B*c^2+8424*a^2*b*B*c^3-384*b^6*C+3*a*b^2*
c^2*(2717*A*c-2355*C*a)-8*b^4*c*(143*A*c-393*C*a)-924*a^2*c^3*(13*A*c-3*C*
a))*x*(c*x^4+b*x^2+a)^(1/2)/c^(9/2)/(a^(1/2)+c^(1/2)*x^2)+1/45045*x*(312*b
^4*B-819*a*b^2*B*c-1170*a^2*B*c^2-192*b^5*C/c-b^3*(572*A*c-636*C*a)+9*a*b*
c*(143*A*c+47*C*a)+3*(312*b^3*B*c-1209*a*b*B*c^2-192*b^4*C+154*a*c^2*(13*A
*c-3*C*a)-b^2*(572*A*c^2-876*C*a*c))*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^3-1/9009
*x*(234*b^2*B-429*A*b*c+117*a*B*c+27*a*b*C-144*b^3*C/c+7*(78*B*b*c-48*b^2*
C-11*c*(13*A*c-3*C*a))*x^2)*(c*x^4+b*x^2+a)^(3/2)/c^2+1/143*(13*B*c-8*C*b)
*x*(c*x^4+b*x^2+a)^(5/2)/c^2+1/13*C*x^3*(c*x^4+b*x^2+a)^(5/2)/c+1/45045*a^
(1/4)*(624*b^5*B*c-4680*a*b^3*B*c^2+8424*a^2*b*B*c^3-384*b^6*C+3*a*b^2*c^2
*(2717*A*c-2355*C*a)-8*b^4*c*(143*A*c-393*C*a)-924*a^2*c^3*(13*A*c-3*C*a))
*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Ell
ipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c
^(19/4)/(c*x^4+b*x^2+a)^(1/2)+1/90090*a^(1/4)*(b+2*a^(1/2)*c^(1/2))*(384*b
^5*C-3*a*b*c^2*(1209*a^(1/2)*B*c^(1/2)+1573*A*c-603*a*C)+8*b^3*c*(117*a^(1
/2)*B*c^(1/2)+143*A*c-249*a*C)+12*a^(1/2)*b^2*c^(3/2)*(234*a^(1/2)*B*c^(1/
2)-143*A*c+219*a*C)-6*a^(3/2)*c^(5/2)*(195*a^(1/2)*B*c^(1/2)-1001*A*c+231*
a*C)-48*b^4*(13*B*c+12*a^(1/2)*c^(1/2)*C))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b
*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/
a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(19/4)/(c*x^4+b*x^2+a)^(1/2...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.39 (sec) , antiderivative size = 4797, normalized size of antiderivative = 5.31

$$\int x^2\left(a+bx^2+cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx = \text{Result too large to show}$$

input
```
Integrate[x^2*(a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
Sqrt[a + b*x^2 + c*x^4]*(((312*b^4*B*c - 572*A*b^3*c^2 - 1989*a*b^2*B*c^2
+ 3432*a*A*b*c^3 + 2340*a^2*B*c^3 - 192*b^5*C + 1356*a*b^3*c*C - 2232*a^2*
b*c^2*C)*x)/(45045*c^4) + ((-234*b^3*B*c + 429*A*b^2*c^2 + 1248*a*b*B*c^2
+ 11011*a*A*c^3 + 144*b^4*C - 867*a*b^2*c*C + 924*a^2*c^2*C)*x^3)/(45045*c
^3) + ((39*b^2*B*c + 1430*A*b*c^2 + 1521*a*B*c^2 - 24*b^3*C + 120*a*b*c*C)
*x^5)/(9009*c^2) + ((156*b*B*c + 143*A*c^2 + 3*b^2*C + 165*a*c*C)*x^7)/(12
87*c) + ((13*B*c + 14*b*C)*x^9)/143 + (c*C*x^11)/13) + (((-156*I)*Sqrt[2]*
b^5*B*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2
]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + S
qrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 -
 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[
-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) + ((286*I)*Sqrt[2]
*A*b^4*c*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*
c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b
+ Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^
2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]))/(Sq
rt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) + ((1170*I)*Sqr
t[2]*a*b^3*B*c*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b...
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 832, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2199, 1596, 27, 1596, 25, 1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2+cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)dx$$

$$\Big\downarrow \text{ 2199}$$

$$\int x^2\left(\frac{1}{13}\left(13B-\frac{8bC}{c}\right)x^2+\frac{1}{13}\left(13A-\frac{3aC}{c}\right)\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{Cx^3\left(a+bx^2+cx^4\right)^{5/2}}{13c}$$

$$\Big\downarrow \text{ 1596}$$

$$\frac{\int -\frac{x^2\left(\left(-48Cb^3+78Bcb^2-c(143Ac-177aC)b-234aBc^2\right)x^2+ac\left(-\frac{24Cb^2}{c}+39Bb-286Ac+66aC\right)\right)\sqrt{cx^4+bx^2+a}}{13c}dx}{33c}+$$
$$\frac{x^3\left(a+bx^2+cx^4\right)^{3/2}\left(-33aC+143Ac-\frac{24b^2C}{c}+9x^2(13Bc-8bC)+39bB\right)}{1287c}+$$
$$\frac{Cx^3\left(a+bx^2+cx^4\right)^{5/2}}{13c}$$

$\downarrow$ 27

$$-\frac{\int x^2\left(\left(-48Cb^3+78Bcb^2-c(143Ac-177aC)b-234aBc^2\right)x^2+a\left(-24Cb^2+39Bcb-286Ac^2+66acC\right)\right)\sqrt{c}}{429c^2}$$
$$\frac{x^3\left(a+bx^2+cx^4\right)^{3/2}\left(-33aC+143Ac-\frac{24b^2C}{c}+9x^2(13Bc-8bC)+39bB\right)}{1287c}+$$
$$\frac{Cx^3\left(a+bx^2+cx^4\right)^{5/2}}{13c}$$

$\downarrow$ 1596

$$-\frac{\int -\frac{x^2\left(\left(-192Cb^5+312Bcb^4-4c(143Ac-339aC)b^3-1989aBc^2b^2+24ac^2(143Ac-93aC)b+2340a^2Bc^3\right)x^2+a\left(-144Cb^4+234Bcb^3-\left(429Ac^2-867acC\right)b^2-124\right.}{\sqrt{cx^4+bx^2+a}}}{35c}$$
$$\frac{x^3\left(a+bx^2+cx^4\right)^{3/2}\left(-33aC+143Ac-\frac{24b^2C}{c}+9x^2(13Bc-8bC)+39bB\right)}{1287c}+$$
$$\frac{Cx^3\left(a+bx^2+cx^4\right)^{5/2}}{13c}$$

$\downarrow$ 25

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-b^2c(143Ac-9aC)+5cx^2\left(-bc(143Ac-177aC)-234aBc^2-48b^3C+78b^2Bc\right)-154ac^2(13Ac-3aC)+39abBc^2-48b^4C+78b^3\right.}{35c}}{}$$
$$\frac{x^3\left(a+bx^2+cx^4\right)^{3/2}\left(-33aC+143Ac-\frac{24b^2C}{c}+9x^2(13Bc-8bC)+39bB\right)}{1287c}+$$
$$\frac{Cx^3\left(a+bx^2+cx^4\right)^{5/2}}{13c}$$

$\downarrow$ 1602

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-b^2c(143Ac-9aC)+5cx^2\left(-bc(143Ac-177aC)-234aBc^2-48b^3C+78b^2Bc\right)-154ac^2(13Ac-3aC)+39abBc^2-48b^4C+78b^3}{35c}}{}$$

$$\frac{x^3\left(a+bx^2+cx^4\right)^{3/2}\left(-33aC+143Ac-\frac{24b^2C}{c}+9x^2(13Bc-8bC)+39bB\right)}{1287c}+$$

$$\frac{Cx^3\left(a+bx^2+cx^4\right)^{5/2}}{13c}$$

$\downarrow$ 1511

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-b^2c(143Ac-9aC)+5cx^2\left(-bc(143Ac-177aC)-234aBc^2-48b^3C+78b^2Bc\right)-154ac^2(13Ac-3aC)+39abBc^2-48b^4C+78b^3}{35c}}{}$$

$$\frac{x^3\left(a+bx^2+cx^4\right)^{3/2}\left(-33aC+143Ac-\frac{24b^2C}{c}+9x^2(13Bc-8bC)+39bB\right)}{1287c}+$$

$$\frac{Cx^3\left(a+bx^2+cx^4\right)^{5/2}}{13c}$$

$\downarrow$ 27

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-b^2c(143Ac-9aC)+5cx^2\left(-bc(143Ac-177aC)-234aBc^2-48b^3C+78b^2Bc\right)-154ac^2(13Ac-3aC)+39abBc^2-48b^4C+78b^3}{35c}}{}$$

$$\frac{x^3\left(a+bx^2+cx^4\right)^{3/2}\left(-33aC+143Ac-\frac{24b^2C}{c}+9x^2(13Bc-8bC)+39bB\right)}{1287c}+$$

$$\frac{Cx^3\left(a+bx^2+cx^4\right)^{5/2}}{13c}$$

$\downarrow$ 1416

$$-\frac{\frac{x^3\sqrt{a+bx^2+cx^4}\left(-b^2c(143Ac-9aC)+5cx^2\left(-bc(143Ac-177aC)-234aBc^2-48b^3C+78b^2Bc\right)-154ac^2(13Ac-3aC)+39abBc^2-48b^4C+78b^3}{35c}}{}$$

$$\frac{x^3\left(a+bx^2+cx^4\right)^{3/2}\left(-33aC+143Ac-\frac{24b^2C}{c}+9x^2(13Bc-8bC)+39bB\right)}{1287c}+$$

$$\frac{Cx^3\left(a+bx^2+cx^4\right)^{5/2}}{13c}$$

$\downarrow$ 1509

$$\frac{C\left(cx^4+bx^2+a\right)^{5/2}x^3}{13c}+$$
$$\frac{\left(-\frac{24Cb^2}{c}+39Bb+9(13Bc-8bC)x^2+143Ac-33aC\right)\left(cx^4+bx^2+a\right)^{3/2}x^3}{1287c}-$$

$$\frac{x^3\left(-48Cb^4+78Bcb^3-c(143Ac-9aC)b^2+39aBc^2b+5c\left(-48Cb^3+78Bcb^2-c(143Ac-177aC)b-234aBc^2\right)x^2-154ac^2(13Ac-3aC)\right)\sqrt{cx^4+bx^2+}}{35c}$$

input
```
Int[x^2*(a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
(x^3*(39*b*B + 143*A*c - 33*a*C - (24*b^2*C)/c + 9*(13*B*c - 8*b*C)*x^2)*(
a + b*x^2 + c*x^4)^(3/2))/(1287*c) + (C*x^3*(a + b*x^2 + c*x^4)^(5/2))/(13
*c) - ((x^3*(78*b^3*B*c + 39*a*b*B*c^2 - 48*b^4*C - b^2*c*(143*A*c - 9*a*C
) - 154*a*c^2*(13*A*c - 3*a*C) + 5*c*(78*b^2*B*c - 234*a*B*c^2 - 48*b^3*C
- b*c*(143*A*c - 177*a*C))*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c) - (((312*b
^4*B*c - 1989*a*b^2*B*c^2 + 2340*a^2*B*c^3 - 192*b^5*C - 4*b^3*c*(143*A*c
- 339*a*C) + 24*a*b*c^2*(143*A*c - 93*a*C))*x*Sqrt[a + b*x^2 + c*x^4])/(3*
c) - (-(((624*b^5*B*c - 4680*a*b^3*B*c^2 + 8424*a^2*b*B*c^3 - 384*b^6*C +
3*a*b^2*c^2*(2717*A*c - 2355*a*C) - 8*b^4*c*(143*A*c - 393*a*C) - 924*a^2*
c^3*(13*A*c - 3*a*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^
2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*
Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(624
*b^5*B*c - 4680*a*b^3*B*c^2 + 8424*a^2*b*B*c^3 - 384*b^6*C + 3*a*b^2*c^2*(
2717*A*c - 2355*a*C) - 8*b^4*c*(143*A*c - 393*a*C) - 924*a^2*c^3*(13*A*c -
 3*a*C) + Sqrt[a]*Sqrt[c]*(312*b^4*B*c - 1989*a*b^2*B*c^2 + 2340*a^2*B*c^3
 - 192*b^5*C - 4*b^3*c*(143*A*c - 339*a*C) + 24*a*b*c^2*(143*A*c - 93*a*C)
))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/(3*c))/(35*c))/(429*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1596
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Simp[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a +
 b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c
*e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &
& NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[
p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1721 vs. $2(791) = 1582$.

Time = 12.95 (sec) , antiderivative size = 1722, normalized size of antiderivative = 1.91

method	result	size
elliptic	Expression too large to display	1722
risch	Expression too large to display	1810
default	Expression too large to display	2095

input
```
int(x^2*(c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*C*c*x^11*(c*x^4+b*x^2+a)^(1/2)+1/11*(B*c^2+14/13*C*c*b)/c*x^9*(c*x^4+
b*x^2+a)^(1/2)+1/9*(A*c^2+2*B*b*c+15/13*C*a*c+C*b^2-10/11*(B*c^2+14/13*C*c
*b)/c*b)/c*x^7*(c*x^4+b*x^2+a)^(1/2)+1/7*(2*A*b*c+2*B*a*c+B*b^2+2*C*b*a-9/
11*(B*c^2+14/13*C*c*b)/c*a-8/9*(A*c^2+2*B*b*c+15/13*C*a*c+C*b^2-10/11*(B*c
^2+14/13*C*c*b)/c*b)/c*b)/c*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(2*A*a*c+A*b^2+2
*B*a*b+a^2*C-7/9*(A*c^2+2*B*b*c+15/13*C*a*c+C*b^2-10/11*(B*c^2+14/13*C*c*b
)/c*b)/c*a-6/7*(2*A*b*c+2*B*a*c+B*b^2+2*C*b*a-9/11*(B*c^2+14/13*C*c*b)/c*a
-8/9*(A*c^2+2*B*b*c+15/13*C*a*c+C*b^2-10/11*(B*c^2+14/13*C*c*b)/c*b)/c*b)/
c*b)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(2*A*a*b+B*a^2-5/7*(2*A*b*c+2*B*a*c+B
*b^2+2*C*b*a-9/11*(B*c^2+14/13*C*c*b)/c*a-8/9*(A*c^2+2*B*b*c+15/13*C*a*c+C
*b^2-10/11*(B*c^2+14/13*C*c*b)/c*b)/c*b)/c*a-4/5*(2*A*a*c+A*b^2+2*B*a*b+a^
2*C-7/9*(A*c^2+2*B*b*c+15/13*C*a*c+C*b^2-10/11*(B*c^2+14/13*C*c*b)/c*b)/c*
a-6/7*(2*A*b*c+2*B*a*c+B*b^2+2*C*b*a-9/11*(B*c^2+14/13*C*c*b)/c*a-8/9*(A*c
^2+2*B*b*c+15/13*C*a*c+C*b^2-10/11*(B*c^2+14/13*C*c*b)/c*b)/c*b)/c*b)/c*b)
/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(2*A*a*b+B*a^2-5/7*(2*A*b*c+2*B*a*c+B*b^2+
2*C*b*a-9/11*(B*c^2+14/13*C*c*b)/c*a-8/9*(A*c^2+2*B*b*c+15/13*C*a*c+C*b^2-
10/11*(B*c^2+14/13*C*c*b)/c*b)/c*b)/c*a-4/5*(2*A*a*c+A*b^2+2*B*a*b+a^2*C-7
/9*(A*c^2+2*B*b*c+15/13*C*a*c+C*b^2-10/11*(B*c^2+14/13*C*c*b)/c*b)/c*a-6/7
*(2*A*b*c+2*B*a*c+B*b^2+2*C*b*a-9/11*(B*c^2+14/13*C*c*b)/c*a-8/9*(A*c^2+2*
B*b*c+15/13*C*a*c+C*b^2-10/11*(B*c^2+14/13*C*c*b)/c*b)/c*b)/c*b)/c*b)/c...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1063, normalized size of antiderivative = 1.18

$$\int x^2 \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/90090*(sqrt(1/2)*((384*C*b^6*c + 12012*A*a^2*c^5 - 3*(924*C*a^3 + 2808*B
*a^2*b + 2717*A*a*b^2)*c^4 + (7065*C*a^2*b^2 + 4680*B*a*b^3 + 1144*A*b^4)*
c^3 - 24*(131*C*a*b^4 + 26*B*b^5)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (384*C*
b^7 + 12012*A*a^2*b*c^4 - 3*(924*C*a^3*b + 2808*B*a^2*b^2 + 2717*A*a*b^3)*
c^3 + (7065*C*a^2*b^3 + 4680*B*a*b^4 + 1144*A*b^5)*c^2 - 24*(131*C*a*b^5 +
 26*B*b^6)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_
e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sq
rt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((384*C*b^6*c + 15
6*((77*A + 15*B)*a^2 + 22*A*a*b)*c^5 - (2772*C*a^3 + 72*(117*B + 31*C)*a^2
*b + 39*(209*A + 51*B)*a*b^2 + 572*A*b^3)*c^4 + (7065*C*a^2*b^2 + 12*(390*
B + 113*C)*a*b^3 + 104*(11*A + 3*B)*b^4)*c^3 - 24*(131*C*a*b^4 + 2*(13*B +
 4*C)*b^5)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (384*C*b^7 + 156*((77*A - 15*B
)*a^2*b - 22*A*a*b^2)*c^4 - (2772*C*a^3*b + 72*(117*B - 31*C)*a^2*b^2 + 39
*(209*A - 51*B)*a*b^3 - 572*A*b^4)*c^3 + (7065*C*a^2*b^3 + 12*(390*B - 113
*C)*a*b^4 + 104*(11*A - 3*B)*b^5)*c^2 - 24*(131*C*a*b^5 + 2*(13*B - 4*C)*b
^6)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsi
n(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2
 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(3465*C*c^7*x^12 + 315*(14*C*b*c^
6 + 13*B*c^7)*x^10 + 35*(3*C*b^2*c^5 + 143*A*c^7 + 3*(55*C*a + 52*B*b)*c^6
)*x^8 + 384*C*b^6*c + 12012*A*a^2*c^5 - 5*(24*C*b^3*c^4 - 13*(117*B*a +...
```

Sympy [F]

$$\int x^2 \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \int x^2 \left(A+Bx^2+Cx^4\right) \left(a+bx^2+cx^4\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)**(3/2)*(C*x**4+B*x**2+A),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int x^2 \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \int \left(Cx^4 + Bx^2 + A\right) \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2 \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \int \left(Cx^4 + Bx^2 + A\right) \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + bx^2 + cx^4\right)^{3/2} \left(A + Bx^2 + Cx^4\right) \, dx = \int x^2 \left(C\,x^4 + B\,x^2 + A\right) \left(c\,x^4 + b\,x^2 + a\right)^{3/2} \, dx$$

input
```
int(x^2*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^2*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int x^2(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)\,dx = \frac{708\sqrt{c\,x^4+b\,x^2+a}\,a^2b\,c^2x+2387\sqrt{c\,x^4+b\,x^2+a}\,a^2c^3x^3-241\sqrt{c\,x^4+b\,x^2+a}\,a\,b^3cx+16}{}$$

input
```
int(x^2*(c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x)
```

output
```
(708*sqrt(a + b*x**2 + c*x**4)*a**2*b*c**2*x + 2387*sqrt(a + b*x**2 + c*x*
*4)*a**2*c**3*x**3 - 241*sqrt(a + b*x**2 + c*x**4)*a*b**3*c*x + 162*sqrt(a
 + b*x**2 + c*x**4)*a*b**2*c**2*x**3 + 3071*sqrt(a + b*x**2 + c*x**4)*a*b*
c**3*x**5 + 2156*sqrt(a + b*x**2 + c*x**4)*a*c**4*x**7 + 24*sqrt(a + b*x**
2 + c*x**4)*b**5*x - 18*sqrt(a + b*x**2 + c*x**4)*b**4*c*x**3 + 15*sqrt(a
+ b*x**2 + c*x**4)*b**3*c**2*x**5 + 1113*sqrt(a + b*x**2 + c*x**4)*b**2*c*
*3*x**7 + 1701*sqrt(a + b*x**2 + c*x**4)*b*c**4*x**9 + 693*sqrt(a + b*x**2
 + c*x**4)*c**5*x**11 - 708*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*
x**4),x)*a**3*b*c**2 + 241*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x
**4),x)*a**2*b**3*c - 24*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**
4),x)*a*b**5 + 1848*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x
**4),x)*a**3*c**3 - 1902*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2
+ c*x**4),x)*a**2*b**2*c**2 + 536*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a
+ b*x**2 + c*x**4),x)*a*b**4*c - 48*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(
a + b*x**2 + c*x**4),x)*b**6)/(9009*c**3)
```

3.141 $\int (a+bx^2+cx^4)^{3/2} (A+Bx^2+Cx^4)\, dx$

Optimal result

Integrand size = 29, antiderivative size = 741

$$\int \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2\right.$$

$$\left.+Cx^4\right)\, dx = \frac{(88b^4Bc-627ab^2Bc^2+924a^2Bc^3-48b^5C-18b^3c(11Ac-20aC)+72abc^2(22Ac-9aC))\, x}{3465c^{7/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{x(44b^3Bc-99abBc^2-24b^4C-9b^2c(11Ac-7aC)-90ac^2(11Ac-aC)+3c(44b^2Bc-154aBc^2-24b}{3465c^3}$$

$$+\frac{x(3(11bBc+33Ac^2-6b^2C-3acC)+7c(11Bc-6bC)x^2)\left(a+bx^2+cx^4\right)^{3/2}}{693c^2}$$

$$+\frac{Cx(a+bx^2+cx^4)^{5/2}}{11c}$$

$$-\frac{\sqrt[4]{a}(88b^4Bc-627ab^2Bc^2+924a^2Bc^3-48b^5C-18b^3c(11Ac-20aC)+72abc^2(22Ac-9aC))\left(\sqrt{a}+\sqrt{}\right.}{3465c^{15/4}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(48b^4C+6b^2c\left(22\sqrt{a}B\sqrt{c}+33Ac-36aC\right)-6ac^2\left(77\sqrt{a}B\sqrt{c}+165Ac-15aC\right)+3\sqrt{}\right.}{}$$

output
```
1/3465*(88*b^4*B*c-627*a*b^2*B*c^2+924*a^2*B*c^3-48*b^5*C-18*b^3*c*(11*A*c
-20*C*a)+72*a*b*c^2*(22*A*c-9*C*a))*x*(c*x^4+b*x^2+a)^(1/2)/c^(7/2)/(a^(1/
2)+c^(1/2)*x^2)-1/3465*x*(44*b^3*B*c-99*a*b*B*c^2-24*b^4*C-9*b^2*c*(11*A*c
-7*C*a)-90*a*c^2*(11*A*c-C*a)+3*c*(44*b^2*B*c-154*a*B*c^2-24*b^3*C-3*b*c*(
33*A*c-31*C*a))*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^3+1/693*x*(99*A*c^2+33*B*b*c-
9*a*c*C-18*b^2*C+7*c*(11*B*c-6*C*b)*x^2)*(c*x^4+b*x^2+a)^(3/2)/c^2+1/11*C*
x*(c*x^4+b*x^2+a)^(5/2)/c-1/3465*a^(1/4)*(88*b^4*B*c-627*a*b^2*B*c^2+924*a
^2*B*c^3-48*b^5*C-18*b^3*c*(11*A*c-20*C*a)+72*a*b*c^2*(22*A*c-9*C*a))*(a^(
1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(15/
4)/(c*x^4+b*x^2+a)^(1/2)-1/6930*a^(1/4)*(b+2*a^(1/2)*c^(1/2))*(48*b^4*C+6*
b^2*c*(22*a^(1/2)*B*c^(1/2)+33*A*c-36*a*C)-6*a*c^2*(77*a^(1/2)*B*c^(1/2)+1
65*A*c-15*a*C)+3*a^(1/2)*b*c^(3/2)*(121*a^(1/2)*B*c^(1/2)-99*A*c+93*a*C)-8
*b^3*(11*B*c+9*a^(1/2)*c^(1/2)*C))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/
(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4))
,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(15/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.12 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.14

$$\int (a+bx^2+cx^4)^{3/2} (A+Bx^2+Cx^4)\ dx = \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)\left(24b^4C-2b^3c(22B+9Cx^2)+3b^2c(33Ac-51aC+11Bcx^2\right.}{}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4)*(24*b^4*C - 2*b
^3*c*(22*B + 9*C*x^2) + 3*b^2*c*(33*A*c - 51*a*C + 11*B*c*x^2 + 5*c*C*x^4)
 + 2*b*c^2*(132*a*B + 396*A*c*x^2 + 48*a*C*x^2 + 275*B*c*x^4 + 210*c*C*x^6
) + c^2*(180*a^2*C + 5*c^2*x^4*(99*A + 77*B*x^2 + 63*C*x^4) + a*c*(1485*A
+ 847*B*x^2 + 585*C*x^4))) - I*(-b + Sqrt[b^2 - 4*a*c])*(-88*b^4*B*c + 627
*a*b^2*B*c^2 - 924*a^2*B*c^3 + 48*b^5*C + 18*b^3*c*(11*A*c - 20*a*C) + 72*
a*b*c^2*(-22*A*c + 9*a*C))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2
- 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
 (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-48*b^6*C + 3*a*b^2
*c^2*(594*A*c + 209*B*Sqrt[b^2 - 4*a*c] - 318*a*C) - 2*b^4*c*(99*A*c + 44*
B*Sqrt[b^2 - 4*a*c] - 204*a*C) + 12*a^2*c^3*(-330*A*c - 77*B*Sqrt[b^2 - 4*
a*c] + 30*a*C) + 8*b^5*(11*B*c + 6*Sqrt[b^2 - 4*a*c]*C) + 12*a*b*c^2*(121*
a*B*c - 132*A*c*Sqrt[b^2 - 4*a*c] + 54*a*Sqrt[b^2 - 4*a*c]*C) + b^3*c*(198
*A*c*Sqrt[b^2 - 4*a*c] - 5*a*(143*B*c + 72*Sqrt[b^2 - 4*a*c]*C)))*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt
[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt
[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])])/(13860*c^4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2
+ c*x^4])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 699, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2207, 1490, 25, 1490, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{\int \left((11Bc-6bC)x^2+11Ac-aC\right) \left(cx^4+bx^2+a\right)^{3/2} dx}{11c}+\frac{Cx\left(a+bx^2+cx^4\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{ 1490}$$

$$\frac{\frac{\int -\left(\left(\left(-24Cb^3+44Bcb^2-3c(33Ac-31aC)b-154aBc^2\right)x^2+a\left(-6Cb^2+11Bcb-18c(11Ac-aC)\right)\right)\sqrt{cx^4+bx^2+a}\right)dx}{21c}+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(-3\ldots\right.\right.}{\ldots}}{11c}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{5/2}}{11c}$$

$\downarrow$ 25

$$\frac{\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(-3acC+33Ac^2-6b^2C+11bBc\right)+7cx^2(11Bc-6bC)\right)}{63c}-\frac{\int\left(\left(\left(-24Cb^3+44Bcb^2-3c(33Ac-31aC)b-154aBc^2\right)x^2+a\left(-6Cb^2\ldots\right.\right.\right.}{21c}}{11c}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{5/2}}{11c}$$

$\downarrow$ 1490

$$\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(-3acC+33Ac^2-6b^2C+11bBc\right)+7cx^2(11Bc-6bC)\right)}{63c}-\frac{\int -\frac{\left(-48Cb^5+88Bcb^4-18c(11Ac-20aC)b^3-627aBc^2b^2+72ac^2(22Ac-9\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{5/2}}{11c}$$

$\downarrow$ 25

$$\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(-3acC+33Ac^2-6b^2C+11bBc\right)+7cx^2(11Bc-6bC)\right)}{63c}-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(-9b^2c(11Ac-7aC)+3cx^2\left(-3bc(33Ac-31aC)-154aBc^2\ldots\right.\right.}{15c}}{\ldots}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{5/2}}{11c}$$

$\downarrow$ 1511

$$\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(-3acC+33Ac^2-6b^2C+11bBc\right)+7cx^2(11Bc-6bC)\right)}{63c}-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(-9b^2c(11Ac-7aC)+3cx^2\left(-3bc(33Ac-31aC)-154aBc^2\ldots\right.\right.}{15c}}{\ldots}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{5/2}}{11c}$$

$\downarrow$ 27

$$\frac{\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(-3acC+33Ac^2-6b^2C+11bBc\right)+7cx^2(11Bc-6bC)\right)}{63c}-\frac{x\sqrt{a+bx^2+cx^4}\left(-9b^2c(11Ac-7aC)+3cx^2\left(-3bc(33Ac-31aC)-154aBc^2\right.\right.}{15c}}{}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{5/2}}{11c}$$

$\Big\downarrow$ 1416

$$\frac{\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(-3acC+33Ac^2-6b^2C+11bBc\right)+7cx^2(11Bc-6bC)\right)}{63c}-\frac{x\sqrt{a+bx^2+cx^4}\left(-9b^2c(11Ac-7aC)+3cx^2\left(-3bc(33Ac-31aC)-154aBc^2\right.\right.}{15c}}{}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{5/2}}{11c}$$

$\Big\downarrow$ 1509

$$\frac{\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(-3acC+33Ac^2-6b^2C+11bBc\right)+7cx^2(11Bc-6bC)\right)}{63c}-\frac{x\sqrt{a+bx^2+cx^4}\left(-9b^2c(11Ac-7aC)+3cx^2\left(-3bc(33Ac-31aC)-154aBc^2\right.\right.}{15c}}{}$$

$$\frac{Cx\left(a+bx^2+cx^4\right)^{5/2}}{11c}$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
(C*x*(a + b*x^2 + c*x^4)^(5/2))/(11*c) + ((x*(3*(11*b*B*c + 33*A*c^2 - 6*b
^2*C - 3*a*c*C) + 7*c*(11*B*c - 6*b*C)*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(63
*c) - ((x*(44*b^3*B*c - 99*a*b*B*c^2 - 24*b^4*C - 9*b^2*c*(11*A*c - 7*a*C)
 - 90*a*c^2*(11*A*c - a*C) + 3*c*(44*b^2*B*c - 154*a*B*c^2 - 24*b^3*C - 3*
b*c*(33*A*c - 31*a*C))*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) - (-(((88*b^4*
B*c - 627*a*b^2*B*c^2 + 924*a^2*B*c^3 - 48*b^5*C - 18*b^3*c*(11*A*c - 20*a
*C) + 72*a*b*c^2*(22*A*c - 9*a*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a]
 + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2
- b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) -
(a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(48*b^4*C + 6*b^2*c*(22*Sqrt[a]*B*Sqrt[c]
 + 33*A*c - 36*a*C) - 6*a*c^2*(77*Sqrt[a]*B*Sqrt[c] + 165*A*c - 15*a*C) +
3*Sqrt[a]*b*c^(3/2)*(121*Sqrt[a]*B*Sqrt[c] - 99*A*c + 93*a*C) - 8*b^3*(11*
B*c + 9*Sqrt[a]*Sqrt[c]*C))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2
- b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/(15*c))/(2
1*c))/(11*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 6.58 (sec) , antiderivative size = 1174, normalized size of antiderivative = 1.58

method	result	size
elliptic	Expression too large to display	1174
risch	Expression too large to display	1538
default	Expression too large to display	1695

input
```
int((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*C*c*x^9*(c*x^4+b*x^2+a)^(1/2)+1/9*(B*c^2+12/11*C*c*b)/c*x^7*(c*x^4+b*
x^2+a)^(1/2)+1/7*(A*c^2+2*B*b*c+13/11*C*a*c+C*b^2-8/9*b/c*(B*c^2+12/11*C*c
*b))/c*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(2*A*b*c+2*B*a*c+B*b^2+2*C*b*a-6/7*b/
c*(A*c^2+2*B*b*c+13/11*C*a*c+C*b^2-8/9*b/c*(B*c^2+12/11*C*c*b))-7/9*a/c*(B
*c^2+12/11*C*c*b))/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(2*A*a*c+A*b^2+2*B*a*b+
a^2*C-4/5*b/c*(2*A*b*c+2*B*a*c+B*b^2+2*C*b*a-6/7*b/c*(A*c^2+2*B*b*c+13/11*
C*a*c+C*b^2-8/9*b/c*(B*c^2+12/11*C*c*b))-7/9*a/c*(B*c^2+12/11*C*c*b))-5/7*
a/c*(A*c^2+2*B*b*c+13/11*C*a*c+C*b^2-8/9*b/c*(B*c^2+12/11*C*c*b)))/c*x*(c*
x^4+b*x^2+a)^(1/2)+1/4*(A*a^2-1/3*a/c*(2*A*a*c+A*b^2+2*B*a*b+a^2*C-4/5*b/c
*(2*A*b*c+2*B*a*c+B*b^2+2*C*b*a-6/7*b/c*(A*c^2+2*B*b*c+13/11*C*a*c+C*b^2-8
/9*b/c*(B*c^2+12/11*C*c*b))-7/9*a/c*(B*c^2+12/11*C*c*b))-5/7*a/c*(A*c^2+2*
B*b*c+13/11*C*a*c+C*b^2-8/9*b/c*(B*c^2+12/11*C*c*b))))*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2))-1/2*(2*A*a*b+B*a^2-3/5*a/c*(2*A*b*c+2*B*a*c+B*b^2+2*C*b*a-6/7*b
/c*(A*c^2+2*B*b*c+13/11*C*a*c+C*b^2-8/9*b/c*(B*c^2+12/11*C*c*b))-7/9*a/c*(
B*c^2+12/11*C*c*b))-2/3*b/c*(2*A*a*c+A*b^2+2*B*a*b+a^2*C-4/5*b/c*(2*A*b*c+
2*B*a*c+B*b^2+2*C*b*a-6/7*b/c*(A*c^2+2*B*b*c+13/11*C*a*c+C*b^2-8/9*b/c*(B*
c^2+12/11*C*c*b))-7/9*a/c*(B*c^2+12/11*C*c*b))-5/7*a/c*(A*c^2+2*B*b*c+1...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.18

$$\int \left(a+bx^2+cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
-1/6930*(sqrt(1/2)*((48*C*b^5*c - 132*(7*B*a^2 + 12*A*a*b)*c^4 + 3*(216*C*
a^2*b + 209*B*a*b^2 + 66*A*b^3)*c^3 - 8*(45*C*a*b^3 + 11*B*b^4)*c^2)*x*sqr
t((b^2 - 4*a*c)/c^2) - (48*C*b^6 - 132*(7*B*a^2*b + 12*A*a*b^2)*c^3 + 3*(2
16*C*a^2*b^2 + 209*B*a*b^3 + 66*A*b^4)*c^2 - 8*(45*C*a*b^4 + 11*B*b^5)*c)*
x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(
1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*
c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((48*C*b^5*c + 1980*A*a*c^5 - 3*
(4*(77*B + 15*C)*a^2 + 88*(6*A + B)*a*b + 33*A*b^2)*c^4 + (648*C*a^2*b + 3
*(209*B + 51*C)*a*b^2 + 22*(9*A + 2*B)*b^3)*c^3 - 8*(45*C*a*b^3 + (11*B +
3*C)*b^4)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (48*C*b^6 - 1980*A*a*b*c^4 - 3*
(4*(77*B - 15*C)*a^2*b + 88*(6*A - B)*a*b^2 - 33*A*b^3)*c^3 + (648*C*a^2*b
^2 + 3*(209*B - 51*C)*a*b^3 + 22*(9*A - 2*B)*b^4)*c^2 - 8*(45*C*a*b^4 + (1
1*B - 3*C)*b^5)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elli
ptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b
*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*(315*C*c^6*x^10 + 35*
(12*C*b*c^5 + 11*B*c^6)*x^8 - 48*C*b^5*c + 5*(3*C*b^2*c^4 + 99*A*c^6 + (11
7*C*a + 110*B*b)*c^5)*x^6 + 132*(7*B*a^2 + 12*A*a*b)*c^4 - (18*C*b^3*c^3 -
 11*(77*B*a + 72*A*b)*c^5 - 3*(32*C*a*b + 11*B*b^2)*c^4)*x^4 - 3*(216*C*a^
2*b + 209*B*a*b^2 + 66*A*b^3)*c^3 + 8*(45*C*a*b^3 + 11*B*b^4)*c^2 + (24*C*
b^4*c^2 + 1485*A*a*c^5 + 3*(60*C*a^2 + 88*B*a*b + 33*A*b^2)*c^4 - (153*...
```

Sympy [F]

$$\int \left(a+bx^2+cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx = \int \left(A+Bx^2+Cx^4\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)*(C*x**4+B*x**2+A),x)
```

output
```
Integral((A + B*x**2 + C*x**4)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \int \left(Cx^4+Bx^2+A\right)\left(cx^4+bx^2+a\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \int \left(Cx^4+Bx^2+A\right)\left(cx^4+bx^2+a\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2+cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \int \left(C\,x^4+B\,x^2+A\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2} \, dx$$

input
```
int((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int (a + bx^2 + cx^4)^{3/2} (A + Bx^2 + Cx^4)\, dx = \frac{333\sqrt{c\,x^4 + b\,x^2 + a}\,a^2c^2x + 42\sqrt{c\,x^4 + b\,x^2 + a}\,a\,b^2cx + 347\sqrt{c\,x^4 + b\,x^2 + a}\,ab\,c^2x^3 + 216\sqrt{c}}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A),x)
```

output
```
(333*sqrt(a + b*x**2 + c*x**4)*a**2*c**2*x + 42*sqrt(a + b*x**2 + c*x**4)*
a*b**2*c*x + 347*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*x**3 + 216*sqrt(a + b*
x**2 + c*x**4)*a*c**3*x**5 - 4*sqrt(a + b*x**2 + c*x**4)*b**4*x + 3*sqrt(a
 + b*x**2 + c*x**4)*b**3*c*x**3 + 113*sqrt(a + b*x**2 + c*x**4)*b**2*c**2*
x**5 + 161*sqrt(a + b*x**2 + c*x**4)*b*c**3*x**7 + 63*sqrt(a + b*x**2 + c*
x**4)*c**4*x**9 + 360*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),
x)*a**3*c**2 - 42*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a
**2*b**2*c + 4*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b*
*4 + 372*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a**
2*b*c**2 - 93*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x
)*a*b**3*c + 8*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),
x)*b**5)/(693*c**2)
```

3.142 $\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 641

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^2}\,dx =$$

$$-\frac{(18b^3Bc-144abBc^2-8b^4C-3b^2c(21Ac-19aC)-84ac^2(9Ac+aC))\,x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$+\frac{x\left(9b^2B+90aBc-\frac{4b^3C}{c}+9b(49Ac+aC)+3(9bBc-4b^2C+14c(9Ac+aC))\,x^2\right)\sqrt{a+bx^2+cx^4}}{315c}$$

$$-\frac{\left(7\left(9A+\frac{aC}{c}\right)-\left(9B-\frac{4bC}{c}\right)x^2\right)(a+bx^2+cx^4)^{3/2}}{63x}+\frac{C(a+bx^2+cx^4)^{5/2}}{9cx}$$

$$+\frac{\sqrt[4]{a}(18b^3Bc-144abBc^2-8b^4C-3b^2c(21Ac-19aC)-84ac^2(9Ac+aC))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\cdots}{315c^{11/4}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(8b^3C+3bc\left(9\sqrt{a}B\sqrt{c}+21Ac-11aC\right)+6\sqrt{a}c^{3/2}\left(15\sqrt{a}B\sqrt{c}+63Ac+7aC\right)-6b^2\left(\cdots\right.\right.}{630c^{11/4}\sqrt{a+bx^2\cdots}}$$

output
```
-1/315*(18*b^3*B*c-144*a*b*B*c^2-8*b^4*C-3*b^2*c*(21*A*c-19*C*a)-84*a*c^2*
(9*A*c+C*a))*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+c^(1/2)*x^2)+1/315*x
*(9*b^2*B+90*a*B*c-4*b^3*C/c+9*b*(49*A*c+C*a)+3*(9*B*b*c-4*b^2*C+14*c*(9*A
*c+C*a))*x^2)*(c*x^4+b*x^2+a)^(1/2)/c-1/63*(63*A+7*a*C/c-(9*B-4*b*C/c)*x^2
)*(c*x^4+b*x^2+a)^(3/2)/x+1/9*C*(c*x^4+b*x^2+a)^(5/2)/c/x+1/315*a^(1/4)*(1
8*b^3*B*c-144*a*b*B*c^2-8*b^4*C-3*b^2*c*(21*A*c-19*C*a)-84*a*c^2*(9*A*c+C*
a))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*
EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2)
)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)+1/630*a^(1/4)*(b+2*a^(1/2)*c^(1/2))*(8*b^
3*C+3*b*c*(9*a^(1/2)*B*c^(1/2)+21*A*c-11*a*C)+6*a^(1/2)*c^(3/2)*(15*a^(1/2
)*B*c^(1/2)+63*A*c+7*a*C)-6*b^2*(3*B*c+2*a^(1/2)*c^(1/2)*C))*(a^(1/2)+c^(1
/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(11/4)/(c*x^
4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.17 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^2}\,dx = \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)\left(ac(-315Ac+x^2(135Bc+24b\right.}{}$$

input
```
Integrate[((a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/x^2,x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(a*c*(-315*A*c +
x^2*(135*B*c + 24*b*C + 77*c*C*x^2)) + x^2*(-4*b^3*C + 3*b^2*c*(3*B + C*x^
2) + 2*b*c^2*(63*A + 36*B*x^2 + 25*C*x^4) + c^3*x^2*(63*A + 45*B*x^2 + 35*
C*x^4))) + I*(-b + Sqrt[b^2 - 4*a*c])*(-18*b^3*B*c + 144*a*b*B*c^2 + 8*b^4
*C + 3*b^2*c*(21*A*c - 19*a*C) + 84*a*c^2*(9*A*c + a*C))*x*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 -
4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])] - I*(-8*b^5*C + 12*a*b*c^2*(21*A*c + 12*B*Sqrt[b^2 - 4*a*c] - 11*a
*C) + b^3*c*(-63*A*c - 18*B*Sqrt[b^2 - 4*a*c] + 65*a*C) + 2*b^4*(9*B*c + 4
*Sqrt[b^2 - 4*a*c]*C) + 12*a*c^2*(30*a*B*c + 63*A*c*Sqrt[b^2 - 4*a*c] + 7*
a*Sqrt[b^2 - 4*a*c]*C) - 3*b^2*c*(54*a*B*c - 21*A*c*Sqrt[b^2 - 4*a*c] + 19
*a*Sqrt[b^2 - 4*a*c]*C))*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2
- 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
 (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(1260*c^3*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 606, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2199, 1594, 27, 1490, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^2}dx$$

$$\Big\downarrow \text{ 2199}$$

$$\int \frac{\left(\frac{1}{9}\left(9B-\frac{4bC}{c}\right)x^2+\frac{1}{9}\left(9A+\frac{aC}{c}\right)\right)(cx^4+bx^2+a)^{3/2}}{x^2}dx+\frac{C(a+bx^2+cx^4)^{5/2}}{9cx}$$

$$\Big\downarrow \text{ 1594}$$

$$-\frac{3}{7}\int -\frac{\left(\left(-4Cb^2+9Bcb+14c(9Ac+aC)\right)x^2+c\left(63Ab-\frac{aCb}{c}+18aB\right)\right)\sqrt{cx^4+bx^2+a}}{9c}dx-$$
$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(7\left(\frac{aC}{c}+9A\right)-x^2\left(9B-\frac{4bC}{c}\right)\right)}{63x}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{9cx}$$

$\downarrow$ 27

$$\frac{\int \left(\left(-4Cb^2+9Bcb+14c(9Ac+aC)\right)x^2+63Abc+18aBc-abC\right)\sqrt{cx^4+bx^2+a}dx}{21c}-$$
$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(7\left(\frac{aC}{c}+9A\right)-x^2\left(9B-\frac{4bC}{c}\right)\right)}{63x}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{9cx}$$

$\downarrow$ 1490

$$\frac{\frac{\int -\frac{\left(-8Cb^4+18Bcb^3-3c(21Ac-19aC)b^2-144aBc^2b-84ac^2(9Ac+aC)\right)x^2+a\left(-4Cb^3+9Bcb^2-24c(21Ac-aC)b-180aBc^2\right)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14c\cdots\right.\right.}{\cdots}}{21c}$$
$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(7\left(\frac{aC}{c}+9A\right)-x^2\left(9B-\frac{4bC}{c}\right)\right)}{63x}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{9cx}$$

$\downarrow$ 25

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14c(aC+9Ac)-4b^2C+9bBc\right)+9bc(aC+49Ac)+90aBc^2-4b^3C+9b^2Bc\right)}{15c}-\frac{\int \frac{\left(-8Cb^4+18Bcb^3-\left(63Ac^2-57acC\right)b^2-144aBc^2\cdots\right.}{\cdots}}{\cdots}}{21c}$$
$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(7\left(\frac{aC}{c}+9A\right)-x^2\left(9B-\frac{4bC}{c}\right)\right)}{63x}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{9cx}$$

$\downarrow$ 1511

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14c(aC+9Ac)-4b^2C+9bBc\right)+9bc(aC+49Ac)+90aBc^2-4b^3C+9b^2Bc\right)}{15c}-\frac{-\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(3bc\left(9\sqrt{a}B\sqrt{c}-11aC+21Ac\right)+6\sqrt{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}$$
$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(7\left(\frac{aC}{c}+9A\right)-x^2\left(9B-\frac{4bC}{c}\right)\right)}{63x}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{9cx}$$

$\downarrow$ 27

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14c(aC+9Ac)-4b^2C+9bBc\right)+9bc(aC+49Ac)+90aBc^2-4b^3C+9b^2Bc\right)}{15c}-\frac{-\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(3bc\left(9\sqrt{a}B\sqrt{c}-11aC+21Ac\right)+6\sqrt{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}$$
$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(7\left(\frac{aC}{c}+9A\right)-x^2\left(9B-\frac{4bC}{c}\right)\right)}{63x}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{9cx}$$

$\downarrow$ 1416

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14c(aC+9Ac)-4b^2C+9bBc\right)+9bc(aC+49Ac)+90aBc^2-4b^3C+9b^2Bc\right)}{15c}-\frac{-\frac{\left(-3b^2c(21Ac-19aC)-84ac^2(aC+9Ac)-144abB\right.}{\sqrt{c}}}{}}{}$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(7\left(\frac{aC}{c}+9A\right)-x^2\left(9B-\frac{4bC}{c}\right)\right)}{63x}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{9cx}$$

$\downarrow$ 1509

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14c(aC+9Ac)-4b^2C+9bBc\right)+9bc(aC+49Ac)+90aBc^2-4b^3C+9b^2Bc\right)}{15c}-\frac{-\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,(}{}}{}$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(7\left(\frac{aC}{c}+9A\right)-x^2\left(9B-\frac{4bC}{c}\right)\right)}{63x}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{9cx}$$

input

```
Int[((a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/x^2,x]
```

output

```
-1/63*((7*(9*A + (a*C)/c) - (9*B - (4*b*C)/c)*x^2)*(a + b*x^2 + c*x^4)^(3/
2))/x + (C*(a + b*x^2 + c*x^4)^(5/2))/(9*c*x) + ((x*(9*b^2*B*c + 90*a*B*c^
2 - 4*b^3*C + 9*b*c*(49*A*c + a*C) + 3*c*(9*b*B*c - 4*b^2*C + 14*c*(9*A*c
+ a*C))*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) - (-(((18*b^3*B*c - 144*a*b*B
*c^2 - 8*b^4*C - 3*b^2*c*(21*A*c - 19*a*C) - 84*a*c^2*(9*A*c + a*C))*(-((x
*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt
[a + b*x^2 + c*x^4])))/Sqrt[c]) - (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(8*b^3*
C + 3*b*c*(9*Sqrt[a]*B*Sqrt[c] + 21*A*c - 11*a*C) + 6*Sqrt[a]*c^(3/2)*(15*
Sqrt[a]*B*Sqrt[c] + 63*A*c + 7*a*C) - 6*b^2*(3*B*c + 2*Sqrt[a]*Sqrt[c]*C))
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/(15*c))/(21*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 5.89 (sec) , antiderivative size = 849, normalized size of antiderivative = 1.32

method	result
elliptic	$-\frac{Aa\sqrt{c\,x^4+b\,x^2+a}}{x}+\frac{Cc\,x^7\sqrt{c\,x^4+b\,x^2+a}}{9}+\frac{\left(B\,c^2+\frac{10}{9}Ccb\right)x^5\sqrt{c\,x^4+b\,x^2+a}}{7c}+\frac{\left(A\,c^2+2Bbc+\frac{11Cac}{9}+C\,b^2-\frac{6b\left(B\,c^2+\frac{10}{9}C\right.}{7c}\right.}{5c}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-A*a*(c*x^4+b*x^2+a)^(1/2)/x+1/9*C*c*x^7*(c*x^4+b*x^2+a)^(1/2)+1/7*(B*c^2+
10/9*C*c*b)/c*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(A*c^2+2*B*b*c+11/9*C*a*c+C*b^
2-6/7*b/c*(B*c^2+10/9*C*c*b))/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(2*A*b*c+2*B
*a*c+B*b^2+2*C*b*a-4/5*b/c*(A*c^2+2*B*b*c+11/9*C*a*c+C*b^2-6/7*b/c*(B*c^2+
10/9*C*c*b))-5/7*a/c*(B*c^2+10/9*C*c*b))/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(2*
A*a*b+B*a^2-1/3*a/c*(2*A*b*c+2*B*a*c+B*b^2+2*C*b*a-4/5*b/c*(A*c^2+2*B*b*c+
11/9*C*a*c+C*b^2-6/7*b/c*(B*c^2+10/9*C*c*b))-5/7*a/c*(B*c^2+10/9*C*c*b)))*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*E
llipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(3*A*a*c+A*b^2+2*B*a*b+a^2*C-3/5*a/c*(A*
c^2+2*B*b*c+11/9*C*a*c+C*b^2-6/7*b/c*(B*c^2+10/9*C*c*b))-2/3*b/c*(2*A*b*c+
2*B*a*c+B*b^2+2*C*b*a-4/5*b/c*(A*c^2+2*B*b*c+11/9*C*a*c+C*b^2-6/7*b/c*(B*c
^2+10/9*C*c*b))-5/7*a/c*(B*c^2+10/9*C*c*b)))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Elli
pticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^2}\,dx=\int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
integral((C*c*x^8 + (C*b + B*c)*x^6 + (C*a + B*b + A*c)*x^4 + (B*a + A*b)*
x^2 + A*a)*sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(A+Bx^2+Cx^4)(a+bx^2+cx^4)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)*(C*x**4+B*x**2+A)/x**2,x)
```

output
```
Integral((A + B*x**2 + C*x**4)*(a + b*x**2 + c*x**4)**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(C\,x^4+B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{3/2}}{x^2}\,dx$$

input
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2))/x^2,x)
```

output
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^2}\,dx = \frac{105\sqrt{c\,x^4+b\,x^2+a}\,a^2c^2+30\sqrt{c\,x^4+b\,x^2+a}\,a\,b^2c+57\sqrt{c\,x}}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^2,x)
```

output
```
(105*sqrt(a + b*x**2 + c*x**4)*a**2*c**2 + 30*sqrt(a + b*x**2 + c*x**4)*a*
b**2*c + 57*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*x**2 + 28*sqrt(a + b*x**2 +
 c*x**4)*a*c**3*x**4 - 2*sqrt(a + b*x**2 + c*x**4)*b**4 + sqrt(a + b*x**2
+ c*x**4)*b**3*c*x**2 + 15*sqrt(a + b*x**2 + c*x**4)*b**2*c**2*x**4 + 19*s
qrt(a + b*x**2 + c*x**4)*b*c**3*x**6 + 7*sqrt(a + b*x**2 + c*x**4)*c**4*x*
*8 + 168*int(sqrt(a + b*x**2 + c*x**4)/(a*x**2 + b*x**4 + c*x**6),x)*a**3*
c**2*x + 30*int(sqrt(a + b*x**2 + c*x**4)/(a*x**2 + b*x**4 + c*x**6),x)*a*
*2*b**2*c*x - 2*int(sqrt(a + b*x**2 + c*x**4)/(a*x**2 + b*x**4 + c*x**6),x
)*a*b**4*x + 132*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*
*2*b*c**2*x - int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b**
3*c*x)/(63*c**2*x)
```

3.143 $\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 566

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^4}\,dx = \frac{(21b^2Bc+252aBc^2-6b^3C+8bc(35Ac+6aC))\,x\sqrt{a+bx^2}}{105c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{(3(35Abc+42aBc+3abC)-(21bBc+70Ac^2-6b^2C+30acC)\,x^2)\sqrt{a+bx^2+cx^4}}{105cx}$$

$$-\frac{\left(5\left(7A+\frac{3aC}{c}\right)-3\left(7B-\frac{2bC}{c}\right)x^2\right)(a+bx^2+cx^4)^{3/2}}{105x^3}+\frac{C(a+bx^2+cx^4)^{5/2}}{7cx^3}$$

$$-\frac{\sqrt[4]{a}(21b^2Bc+252aBc^2-6b^3C+8bc(35Ac+6aC))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\right.\right.}{105c^{7/4}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\left(b+2\sqrt{a}\sqrt{c}\right)\left(105Abc^{3/2}+30a^{3/2}cC+9a\sqrt{c}(14Bc+bC)+\sqrt{a}(21bBc+70Ac^2-6b^2C)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}{210\sqrt[4]{a}c^{7/4}\sqrt{a+bx^2+cx^4}}$$

output

```
1/105*(21*b^2*B*c+252*a*B*c^2-6*b^3*C+8*b*c*(35*A*c+6*C*a))*x*(c*x^4+b*x^2
+a)^(1/2)/c^(3/2)/(a^(1/2)+c^(1/2)*x^2)-1/105*(105*A*b*c+126*a*B*c+9*a*b*C
-(70*A*c^2+21*B*b*c+30*C*a*c-6*C*b^2)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c/x-1/105
*(35*A+15*a*C/c-3*(7*B-2*b*C/c)*x^2)*(c*x^4+b*x^2+a)^(3/2)/x^3+1/7*C*(c*x^
4+b*x^2+a)^(5/2)/c/x^3-1/105*a^(1/4)*(21*b^2*B*c+252*a*B*c^2-6*b^3*C+8*b*c
*(35*A*c+6*C*a))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x
^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)+1/210*(b+2*a^(1/2)*c^(1/2))*(
105*A*b*c^(3/2)+30*a^(3/2)*c*C+9*a*c^(1/2)*(14*B*c+C*b)+a^(1/2)*(70*A*c^2+
21*B*b*c-6*C*b^2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)
*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2
)/c^(1/2))^(1/2))/a^(1/4)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.17 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^4}\,dx = \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)\left(35Acx^2(-4b+cx^2)-5ac(7A\right.}{}$$

input

```
Integrate[((a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/x^4,x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(35*A*c*x^2*(-4*b
 + c*x^2) - 5*a*c*(7*A + 21*B*x^2 - 9*C*x^4) + 3*x^4*(b^2*C + 2*b*c*(7*B +
 4*C*x^2) + c^2*x^2*(7*B + 5*C*x^2))) - I*(-b + Sqrt[b^2 - 4*a*c])*(-21*b^
2*B*c - 252*a*B*c^2 + 6*b^3*C - 8*b*c*(35*A*c + 6*a*C))*x^3*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 -
 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
4*a*c])] + I*(-6*b^4*C - 4*a*c^2*(70*A*c + 63*B*Sqrt[b^2 - 4*a*c] + 30*a*C
) + b^2*c*(70*A*c - 21*B*Sqrt[b^2 - 4*a*c] + 54*a*C) + 3*b^3*(7*B*c + 2*Sq
rt[b^2 - 4*a*c]*C) - 4*b*c*(70*A*c*Sqrt[b^2 - 4*a*c] + 3*a*(7*B*c + 4*Sqrt
[b^2 - 4*a*c]*C)))*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*
a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(420*c^2*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 541, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2199, 1594, 27, 1594, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)}{x^4}dx$$

$$\Big\downarrow \text{2199}$$

$$\int \frac{\left(\frac{1}{7}\left(7B-\frac{2bC}{c}\right)x^2+\frac{1}{7}\left(7A+\frac{3aC}{c}\right)\right)\left(cx^4+bx^2+a\right)^{3/2}}{x^4}dx+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{7cx^3}$$

$$\Big\downarrow \text{1594}$$

$$-\frac{1}{5}\int -\frac{\left(\left(-6Cb^2+21Bcb+70Ac^2+30acC\right)x^2+c\left(35Ab+\frac{3aCb}{c}+42aB\right)\right)\sqrt{cx^4+bx^2+a}}{7cx^2}dx-\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(5\left(\frac{3aC}{c}+7A\right)-3x^2\left(7B-\frac{2bC}{c}\right)\right)}{105x^3}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{7cx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{((-6Cb^2+21Bcb+10c(7Ac+3aC))x^2+35Abc+42aBc+3abC)\sqrt{cx^4+bx^2+a}}{x^2}dx}{35c} - \frac{(a+bx^2+cx^4)^{3/2}\left(5\left(\frac{3aC}{c}+7A\right)-3x^2\left(7B-\frac{2bC}{c}\right)\right)}{105x^3} + \frac{C(a+bx^2+cx^4)^{5/2}}{7cx^3}$$

$\downarrow$ 1594

$$\frac{-\frac{1}{3}\int -\frac{(-6Cb^3+21Bcb^2+8c(35Ac+6aC)b+252aBc^2)x^2+35Ac(3b^2+4ac)+3a(-Cb^2+56Bcb+20acC)}{\sqrt{cx^4+bx^2+a}}dx - \frac{\sqrt{a+bx^2+cx^4}(3(3abC+42aBc}{}}{35c} - \frac{(a+bx^2+cx^4)^{3/2}\left(5\left(\frac{3aC}{c}+7A\right)-3x^2\left(7B-\frac{2bC}{c}\right)\right)}{105x^3} + \frac{C(a+bx^2+cx^4)^{5/2}}{7cx^3}$$

$\downarrow$ 25

$$\frac{\frac{1}{3}\int \frac{(-6Cb^3+21Bcb^2+8c(35Ac+6aC)b+252aBc^2)x^2+35Ac(3b^2+4ac)+3a(-Cb^2+56Bcb+20acC)}{\sqrt{cx^4+bx^2+a}}dx - \frac{\sqrt{a+bx^2+cx^4}(3(3abC+42aBc+35}{}}{35c} - \frac{(a+bx^2+cx^4)^{3/2}\left(5\left(\frac{3aC}{c}+7A\right)-3x^2\left(7B-\frac{2bC}{c}\right)\right)}{105x^3} + \frac{C(a+bx^2+cx^4)^{5/2}}{7cx^3}$$

$\downarrow$ 1511

$$\frac{\frac{1}{3}\left(\frac{(2\sqrt{a}\sqrt{c}+b)(30a^{3/2}cC+\sqrt{a}(70Ac^2-6b^2C+21bBc)+9a\sqrt{c}(bC+14Bc)+105Abc^{3/2})\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}} - \frac{\sqrt{a}(8bc(6aC+35Ac)+252aBc^2-}{}\right.}{35c} - \frac{(a+bx^2+cx^4)^{3/2}\left(5\left(\frac{3aC}{c}+7A\right)-3x^2\left(7B-\frac{2bC}{c}\right)\right)}{105x^3} + \frac{C(a+bx^2+cx^4)^{5/2}}{7cx^3}$$

$\downarrow$ 27

$$\frac{\frac{1}{3}\left(\frac{(2\sqrt{a}\sqrt{c}+b)(30a^{3/2}cC+\sqrt{a}(70Ac^2-6b^2C+21bBc)+9a\sqrt{c}(bC+14Bc)+105Abc^{3/2})\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}} - \frac{(8bc(6aC+35Ac)+252aBc^2-6b^3}{\sqrt{c}}\right.}{35c} - \frac{(a+bx^2+cx^4)^{3/2}\left(5\left(\frac{3aC}{c}+7A\right)-3x^2\left(7B-\frac{2bC}{c}\right)\right)}{105x^3} + \frac{C(a+bx^2+cx^4)^{5/2}}{7cx^3}$$

$\downarrow$ 1416

$$\frac{1}{3}\left(\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(30a^{3/2}cC+\sqrt{a}\left(70Ac^2-6b^2C+21bBc\right)+9a\sqrt{c}(bC+14Bc)+105Abc^{3/2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(5\left(\frac{3aC}{c}+7A\right)-3x^2\left(7B-\frac{2bC}{c}\right)\right)}{105x^3}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{7cx^3}$$

$\downarrow$ 1509

$$\frac{1}{3}\left(\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(30a^{3/2}cC+\sqrt{a}\left(70Ac^2-6b^2C+21bBc\right)+9a\sqrt{c}(bC+14Bc)+105Abc^{3/2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(5\left(\frac{3aC}{c}+7A\right)-3x^2\left(7B-\frac{2bC}{c}\right)\right)}{105x^3}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{7cx^3}$$

input
```
Int[((a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/x^4,x]
```

output
```
-1/105*((5*(7*A + (3*a*C)/c) - 3*(7*B - (2*b*C)/c)*x^2)*(a + b*x^2 + c*x^4
)^(3/2))/x^3 + (C*(a + b*x^2 + c*x^4)^(5/2))/(7*c*x^3) + (-1/3*((3*(35*A*b
*c + 42*a*B*c + 3*a*b*C) - (21*b*B*c - 6*b^2*C + 10*c*(7*A*c + 3*a*C))*x^2
)*Sqrt[a + b*x^2 + c*x^4])/x + (-(((21*b^2*B*c + 252*a*B*c^2 - 6*b^3*C + 8
*b*c*(35*A*c + 6*a*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x
^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + ((b + 2*Sqrt
[a]*Sqrt[c])*(105*A*b*c^(3/2) + 30*a^(3/2)*c*C + 9*a*Sqrt[c]*(14*B*c + b*C
) + Sqrt[a]*(21*b*B*c + 70*A*c^2 - 6*b^2*C))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(3/4)*Sqrt[a + b*
x^2 + c*x^4]))/3)/(35*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 6.21 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.20

method	result
elliptic	$-\frac{Aa\sqrt{c\,x^{4}+b\,x^{2}+a}}{3x^{3}}-\frac{\left(\frac{4Ab}{3}+Ba\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{x}+\frac{Cc\,x^{5}\sqrt{c\,x^{4}+b\,x^{2}+a}}{7}+\frac{\left(B\,c^{2}+\frac{8}{7}Ccb\right)x^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{5c}+\frac{\left(A\,c^{2}+2Bbc\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*a*(c*x^4+b*x^2+a)^(1/2)/x^3-(4/3*A*b+B*a)*(c*x^4+b*x^2+a)^(1/2)/x+1
/7*C*c*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(B*c^2+8/7*C*c*b)/c*x^3*(c*x^4+b*x^2+
a)^(1/2)+1/3*(A*c^2+2*B*b*c+9/7*C*a*c+C*b^2-4/5*b/c*(B*c^2+8/7*C*c*b))/c*x
*(c*x^4+b*x^2+a)^(1/2)+1/4*(5/3*A*a*c+A*b^2+2*B*a*b+a^2*C-1/3*a/c*(A*c^2+2
*B*b*c+9/7*C*a*c+C*b^2-4/5*b/c*(B*c^2+8/7*C*c*b)))*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
(1/2))-1/2*(2*A*b*c+2*B*a*c+B*b^2+2*C*b*a+1/3*c*(4*A*b+3*B*a)-3/5*a/c*(B*c
^2+8/7*C*c*b)-2/3*b/c*(A*c^2+2*B*b*c+9/7*C*a*c+C*b^2-4/5*b/c*(B*c^2+8/7*C*
c*b)))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^4}\,dx = \int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
integral((C*c*x^8 + (C*b + B*c)*x^6 + (C*a + B*b + A*c)*x^4 + (B*a + A*b)*
x^2 + A*a)*sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^4}\,dx = \int \frac{(A+Bx^2+Cx^4)(a+bx^2+cx^4)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)*(C*x**4+B*x**2+A)/x**4,x)
```

output
```
Integral((A + B*x**2 + C*x**4)*(a + b*x**2 + c*x**4)**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^4}\,dx=\int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^4}\,dx=\int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^4}\,dx=\int \frac{(C\,x^4+B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{3/2}}{x^4}\,dx$$

input
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2))/x^4,x)
```

output
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^4}\,dx = \frac{-130\sqrt{c\,x^4+b\,x^2+a}\,a^2c-3\sqrt{c\,x^4+b\,x^2+a}\,a\,b^2+134\sqrt{c\,x}}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^4,x)
```

output
```
( - 130*sqrt(a + b*x**2 + c*x**4)*a**2*c - 3*sqrt(a + b*x**2 + c*x**4)*a*b
**2 + 134*sqrt(a + b*x**2 + c*x**4)*a*b*c*x**2 + 32*sqrt(a + b*x**2 + c*x*
*4)*a*c**2*x**4 + 6*sqrt(a + b*x**2 + c*x**4)*b**3*x**2 + 18*sqrt(a + b*x*
*2 + c*x**4)*b**2*c*x**4 + 18*sqrt(a + b*x**2 + c*x**4)*b*c**2*x**6 + 6*sq
rt(a + b*x**2 + c*x**4)*c**3*x**8 - 348*int(sqrt(a + b*x**2 + c*x**4)/(a*x
**4 + b*x**6 + c*x**8),x)*a**3*c*x**3 - 9*int(sqrt(a + b*x**2 + c*x**4)/(a
*x**4 + b*x**6 + c*x**8),x)*a**2*b**2*x**3 - 36*int(sqrt(a + b*x**2 + c*x*
*4)/(a + b*x**2 + c*x**4),x)*a**2*c**2*x**3 + 105*int(sqrt(a + b*x**2 + c*
x**4)/(a + b*x**2 + c*x**4),x)*a*b**2*c*x**3)/(42*c*x**3)
```

3.144 $\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 527

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^6}\,dx =$$

$$-\frac{\left(3A(b^2+12ac)+a\left(40bB+36aC+\frac{3b^2C}{c}\right)\right)\sqrt{a+bx^2+cx^4}}{15\sqrt{a}x\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{\left(10aB+3b\left(A+\frac{aC}{c}\right)-3(5bB+6Ac+6aC)x^2\right)\sqrt{a+bx^2+cx^4}}{15x^3}$$

$$-\frac{\left(3\left(A+\frac{aC}{c}\right)-5Bx^2\right)(a+bx^2+cx^4)^{3/2}}{15x^5}+\frac{C(a+bx^2+cx^4)^{5/2}}{5cx^5}$$

$$-\frac{\sqrt[4]{c}\left(3A(b^2+12ac)+a\left(40bB+36aC+\frac{3b^2C}{c}\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{15a^{3/4}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\left(b+2\sqrt{a}\sqrt{c}\right)\left(3Abc+3\sqrt{a}\sqrt{c}(5bB+6Ac)+18a^{3/2}\sqrt{c}C+a(10Bc+3bC)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}}{30a^{3/4}c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/15*(3*A*(12*a*c+b^2)+a*(40*B*b+36*a*C+3*b^2*C/c))*(c*x^4+b*x^2+a)^(1/2)
/a^(1/2)/x/(a^(1/2)+c^(1/2)*x^2)-1/15*(10*a*B+3*b*(A+a*C/c)-3*(6*A*c+5*B*b
+6*C*a)*x^2)*(c*x^4+b*x^2+a)^(1/2)/x^3-1/15*(3*A+3*a*C/c-5*B*x^2)*(c*x^4+b
*x^2+a)^(3/2)/x^5+1/5*C*(c*x^4+b*x^2+a)^(5/2)/c/x^5-1/15*c^(1/4)*(3*A*(12*
a*c+b^2)+a*(40*B*b+36*a*C+3*b^2*C/c))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+
a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/30*(b+
2*a^(1/2)*c^(1/2))*(3*A*b*c+3*a^(1/2)*c^(1/2)*(6*A*c+5*B*b)+18*a^(3/2)*c^(
1/2)*C+a*(10*B*c+3*C*b))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c
^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.89 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^6}\,dx = \frac{-4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)\left(3Ab^2x^4+a(6Abx^2+20bBx\right.}{}$$

input
```
Integrate[((a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/x^6,x]
```

output
```
(-4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(3*A*b^2*x^4 + a
*(6*A*b*x^2 + 20*b*B*x^4 + 21*A*c*x^4 - 5*B*c*x^6 - 6*b*C*x^6 - 3*c*C*x^8)
 + a^2*(3*A + 5*x^2*(B + 3*C*x^2))) + I*(-b + Sqrt[b^2 - 4*a*c])*(3*A*c*(b
^2 + 12*a*c) + a*(40*b*B*c + 3*b^2*C + 36*a*c*C))*x^5*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c
] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] - I*(3*A*c*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 12*a*c*Sqrt[b^2 -
4*a*c]) + a*(-3*b^3*C + 4*b*c*(10*B*Sqrt[b^2 - 4*a*c] + 3*a*C) + b^2*(-10*
B*c + 3*Sqrt[b^2 - 4*a*c]*C) + 4*a*c*(10*B*c + 9*Sqrt[b^2 - 4*a*c]*C)))*x^
5*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*
b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/
(b - Sqrt[b^2 - 4*a*c])])/(60*a*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^5*Sqrt
[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2199, 1594, 25, 1594, 25, 1604, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)}{x^6}dx$$

$$\Big\downarrow \text{2199}$$

$$\int \frac{\left(Bx^2+A+\frac{aC}{c}\right)\left(cx^4+bx^2+a\right)^{3/2}}{x^6}dx+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{1594}$$

$$-\frac{1}{5}\int -\frac{\left((5bB+6Ac+6aC)x^2+10aB+3b\left(A+\frac{aC}{c}\right)\right)\sqrt{cx^4+bx^2+a}}{x^4}dx-\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(\frac{aC}{c}+A\right)-5Bx^2\right)}{15x^5}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\int \frac{\left((5bB+6Ac+6aC)x^2+10aB+3b\left(A+\frac{aC}{c}\right)\right)\sqrt{cx^4+bx^2+a}}{x^4}dx-\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(\frac{aC}{c}+A\right)-5Bx^2\right)}{15x^5}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{1594}$$

$$\frac{1}{5}\left(-\frac{1}{3}\int -\frac{\left(15Bb^2+24(Ac+aC)b+20aBc\right)x^2+6a(5bB+6Ac+6aC)+b\left(10aB+3b\left(A+\frac{aC}{c}\right)\right)}{x^2\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+\cdots}}{\cdots}\right.$$
$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(\frac{aC}{c}+A\right)-5Bx^2\right)}{15x^5}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{\left(15Bb^2+24(Ac+aC)b+20aBc\right)x^2+3A\left(b^2+12ac\right)+a\left(\frac{3Cb^2}{c}+40Bb+36aC\right)}{x^2\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx\cdots}}{\cdots}\right.$$
$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(\frac{aC}{c}+A\right)-5Bx^2\right)}{15x^5}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(-\frac{\int -\frac{c\left(3A\left(b^2+12ac\right)+a\left(\frac{3Cb^2}{c}+40Bb+36aC\right)\right)x^2+a\left(15Bb^2+24(Ac+aC)b+20aBc\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(3A\left(12ac+b^{\cdots}\right.\right.}{a\cdots}\right.\right.$$
$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(\frac{aC}{c}+A\right)-5Bx^2\right)}{15x^5}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int \frac{c\left(3A\left(b^2+12ac\right)+a\left(\frac{3Cb^2}{c}+40Bb+36aC\right)\right)x^2+a\left(15Bb^2+24(Ac+aC)b+20aBc\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(3A\left(12ac+b^2\right)+\cdots\right.}{ax\cdots}\right.\right.$$
$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(\frac{aC}{c}+A\right)-5Bx^2\right)}{15x^5}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(18a^{3/2}\sqrt{c}C+3\sqrt{a}\sqrt{c}(6Ac+5bB)+a(3bC+10Bc)+3Abc\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\sqrt{a}\sqrt{c}\left(3A\left(12ac+b^2\right)+a\left(36\right.\right.}{a}\right.\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(\frac{aC}{c}+A\right)-5Bx^2\right)}{15x^5}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$\big\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(18a^{3/2}\sqrt{c}C+3\sqrt{a}\sqrt{c}(6Ac+5bB)+a(3bC+10Bc)+3Abc\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\sqrt{c}\left(3A\left(12ac+b^2\right)+a\left(36aC\right.\right.}{a}\right.\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(\frac{aC}{c}+A\right)-5Bx^2\right)}{15x^5}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$\big\downarrow$ 1416

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(18a^{3/2}\sqrt{c}C+3\sqrt{a}\sqrt{c}(6Ac+5bB)+a(3bC+10Bc)+3Abc\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}}{a}\right.\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(\frac{aC}{c}+A\right)-5Bx^2\right)}{15x^5}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{5cx^5}$$

$\big\downarrow$ 1509

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(18a^{3/2}\sqrt{c}C+3\sqrt{a}\sqrt{c}(6Ac+5bB)+a(3bC+10Bc)+3Abc\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}}{}\right.\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(\frac{aC}{c}+A\right)-5Bx^2\right)}{15x^5}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{5cx^5}$$

input
```
Int[((a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/x^6,x]
```

output
```
-1/15*((3*(A + (a*C)/c) - 5*B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/x^5 + (C*(a
+ b*x^2 + c*x^4)^(5/2))/(5*c*x^5) + (-1/3*((10*a*B + 3*b*(A + (a*C)/c) - 3
*(5*b*B + 6*A*c + 6*a*C)*x^2)*Sqrt[a + b*x^2 + c*x^4])/x^3 + (-(((3*A*(b^2
 + 12*a*c) + a*(40*b*B + 36*a*C + (3*b^2*C)/c))*Sqrt[a + b*x^2 + c*x^4])/(
a*x)) + (-(Sqrt[c]*(3*A*(b^2 + 12*a*c) + a*(40*b*B + 36*a*C + (3*b^2*C)/c)
)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/
4)*Sqrt[a + b*x^2 + c*x^4]))) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(3*A*b*c
+ 3*Sqrt[a]*Sqrt[c]*(5*b*B + 6*A*c) + 18*a^(3/2)*Sqrt[c]*C + a*(10*B*c + 3
*b*C))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/a)/3)/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 7.98 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.18

method	result
elliptic	$-\frac{Aa\sqrt{c\,x^4+b\,x^2+a}}{5x^5}-\frac{\left(\frac{6Ab}{5}+Ba\right)\sqrt{c\,x^4+b\,x^2+a}}{3x^3}-\frac{\left(21Aac+3A\,b^2+20Bab+15a^2C\right)\sqrt{c\,x^4+b\,x^2+a}}{15ax}+\frac{Cc\,x^3\sqrt{c\,x^4+b\,x^2+a}}{5}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(-3Cac\,x^8-5x^6Bac-6Cab\,x^6+21Aac\,x^4+3A\,b^2x^4+20Bab\,x^4+15C\,a^2x^4+6Aab\,x^2+5B\,a^2x^2+3A\,a^2\right)}{15x^5a}+$
default	Expression too large to display

input

```
int((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output

```
-1/5*A*a*(c*x^4+b*x^2+a)^(1/2)/x^5-1/3*(6/5*A*b+B*a)*(c*x^4+b*x^2+a)^(1/2)
/x^3-1/15*(21*A*a*c+3*A*b^2+20*B*a*b+15*C*a^2)/a*(c*x^4+b*x^2+a)^(1/2)/x+1
/5*C*c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(B*c^2+6/5*C*c*b)/c*x*(c*x^4+b*x^2+a)
^(1/2)+1/4*(2*A*b*c+2*B*a*c+B*b^2+2*C*b*a-1/15*c*(6*A*b+5*B*a)-1/3*a/c*(B*
c^2+6/5*C*c*b))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+
b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(A*c^2+2*B*b*c+7/5*C*a*c
+C*b^2+1/15*c*(21*A*a*c+3*A*b^2+20*B*a*b+15*C*a^2)/a-2/3*b/c*(B*c^2+6/5*C*
c*b))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/
2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
integral((C*c*x^8 + (C*b + B*c)*x^6 + (C*a + B*b + A*c)*x^4 + (B*a + A*b)*
x^2 + A*a)*sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(A+Bx^2+Cx^4)\,(a+bx^2+cx^4)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)*(C*x**4+B*x**2+A)/x**6,x)
```

output
```
Integral((A + B*x**2 + C*x**4)*(a + b*x**2 + c*x**4)**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(C\,x^4+B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{3/2}}{x^6}\,dx$$

input
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2))/x^6,x)
```

output
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^6}\,dx = \frac{-3\sqrt{c\,x^4+b\,x^2+a}\,a^3c-79\sqrt{c\,x^4+b\,x^2+a}\,a^2bc\,x^2-36\sqrt{c}}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^6,x)
```

output
```
( - 3*sqrt(a + b*x**2 + c*x**4)*a**3*c - 79*sqrt(a + b*x**2 + c*x**4)*a**2
*b*c*x**2 - 36*sqrt(a + b*x**2 + c*x**4)*a**2*c**2*x**4 - 15*sqrt(a + b*x*
*2 + c*x**4)*a*b**3*x**2 + 113*sqrt(a + b*x**2 + c*x**4)*a*b**2*c*x**4 + 1
1*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*x**6 + 3*sqrt(a + b*x**2 + c*x**4)*a*
c**3*x**8 + 30*sqrt(a + b*x**2 + c*x**4)*b**4*x**4 - 204*int(sqrt(a + b*x*
*2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*a**3*b*c*x**5 - 45*int(sqrt(a +
 b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*a**2*b**3*x**5 + 72*int((s
qrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a**2*c**3*x**5 - 9
0*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*b**2*c**
2*x**5 - 30*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*
b**4*c*x**5)/(15*a*c*x**5)
```

3.145 $\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^8}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 623

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^8}\,dx =$$

$$-\frac{\left(3A(b^2-20ac)-7a\left(24bB+20aC+\frac{15b^2C}{c}\right)\right)\sqrt{a+bx^2+cx^4}}{105ax^3}$$

$$+\frac{(6A(b^3-8abc)-7a(3b^2B+36aBc+40abC))\sqrt{a+bx^2+cx^4}}{105a^{3/2}x\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{(3Abc+42aBc+35abC+5(21bBc+6Ac^2+14b^2C+14acC)x^2)\sqrt{a+bx^2+cx^4}}{35cx^5}$$

$$-\frac{\left(3A+\frac{7aC}{c}-7\left(3B+\frac{2bC}{c}\right)x^2\right)(a+bx^2+cx^4)^{3/2}}{21x^7}+\frac{C(a+bx^2+cx^4)^{5/2}}{3cx^7}$$

$$+\frac{\sqrt[4]{c}(6A(b^3-8abc)-7a(3b^2B+36aBc+40abC))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{105a^{7/4}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\left(3Ac(b^2-20ac)-7a(24bBc+15b^2C+20acC)+\frac{\sqrt{c}(6A(b^3-8abc)-7a(3b^2B+36aBc+40abC))}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{}}{210a^{5/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/105*(3*A*(-20*a*c+b^2)-7*a*(24*B*b+20*a*C+15*b^2*C/c))*(c*x^4+b*x^2+a)^
(1/2)/a/x^3+1/105*(6*A*(-8*a*b*c+b^3)-7*a*(36*B*a*c+3*B*b^2+40*C*a*b))*(c*
x^4+b*x^2+a)^(1/2)/a^(3/2)/x/(a^(1/2)+c^(1/2)*x^2)-1/35*(3*A*b*c+42*a*B*c+
35*a*b*C+5*(6*A*c^2+21*B*b*c+14*C*a*c+14*C*b^2)*x^2)*(c*x^4+b*x^2+a)^(1/2)
/c/x^5-1/21*(3*A+7*a*C/c-7*(3*B+2*b*C/c)*x^2)*(c*x^4+b*x^2+a)^(3/2)/x^7+1/
3*C*(c*x^4+b*x^2+a)^(5/2)/c/x^7+1/105*c^(1/4)*(6*A*(-8*a*b*c+b^3)-7*a*(36*
B*a*c+3*B*b^2+40*C*a*b))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c
^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a
^(1/2)/c^(1/2))^(1/2))/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/210*(3*A*c*(-20*a*c
+b^2)-7*a*(24*B*b*c+20*C*a*c+15*C*b^2)+c^(1/2)*(6*A*(-8*a*b*c+b^3)-7*a*(36
*B*a*c+3*B*b^2+40*C*a*b))/a^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/
(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4))
,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(5/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.33 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^8}\,dx = \frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)\left(-6Ab^3x^6+a^3(15A+21Bx^2\right.}{}$$

input
```
Integrate[((a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/x^8,x]
```

output
```
(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(-6*A*b^3*x^6 + a^
3*(15*A + 21*B*x^2 + 35*C*x^4) + 3*a*b*x^4*(7*b*B*x^2 + A*(b + 16*c*x^2))
+ a^2*(3*A*(8*b*x^2 + 15*c*x^4) + 7*(6*b*B*x^4 + 21*B*c*x^6 + 20*b*C*x^6 -
 5*c*C*x^8))) - I*(-b + Sqrt[b^2 - 4*a*c])*(6*A*(b^3 - 8*a*b*c) - 7*a*(3*b
^2*B + 36*a*B*c + 40*a*b*C))*x^7*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(6*A*(-b^4 + 9
*a*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])
 - 7*a*(-3*b^3*B + b^2*(3*B*Sqrt[b^2 - 4*a*c] - 10*a*C) + 4*a*c*(9*B*Sqrt[
b^2 - 4*a*c] + 10*a*C) + 4*a*b*(3*B*c + 10*Sqrt[b^2 - 4*a*c]*C)))*x^7*Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*
Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])])/(420*a^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^7*Sqrt[a +
b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2199, 1594, 27, 1594, 1604, 27, 1604, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)}{x^8}dx$$

$$\Big\downarrow \text{2199}$$

$$\int \frac{\left(\frac{1}{3}\left(3B+\frac{2bC}{c}\right)x^2+\frac{1}{3}\left(3A+\frac{7aC}{c}\right)\right)\left(cx^4+bx^2+a\right)^{3/2}}{x^8}dx+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

$$\Big\downarrow \text{1594}$$

$$-\frac{3}{7}\int -\frac{\left(\left(14Cb^2+21Bcb+6Ac^2+14acC\right)x^2+c\left(3Ab+\frac{7a(6Bc+5bC)}{c}\right)\right)\sqrt{cx^4+bx^2+a}}{3cx^6}dx-\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(\frac{7aC}{c}+3A-7x^2\left(\frac{2bC}{c}+3B\right)\right)}{21x^7}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\left((14Cb^2+21Bcb+6Ac^2+14acC)x^2+3Abc+42aBc+35abC\right)\sqrt{cx^4+bx^2+a}}{x^6}dx}{7c} - \frac{\left(a+bx^2+cx^4\right)^{3/2}\left(\frac{7aC}{c}+3A-7x^2\left(\frac{2bC}{c}+3B\right)\right)}{21x^7} + \frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

$\Big\downarrow$ 1594

$$\frac{\frac{1}{5}\int \frac{-\left(\left(70Cb^3+105Bcb^2+24Ac^2b-84aBc^2\right)x^2\right)+3Ac(b^2-20ac)-7a(15Cb^2+24Bcb+20acC)}{x^4\sqrt{cx^4+bx^2+a}}dx - \frac{\sqrt{a+bx^2+cx^4}\left(5x^2\left(14acC+6Ac^2+14b^2C\right.\right.}{5x^5}}{7c} - \frac{\left(a+bx^2+cx^4\right)^{3/2}\left(\frac{7aC}{c}+3A-7x^2\left(\frac{2bC}{c}+3B\right)\right)}{21x^7} + \frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

$\Big\downarrow$ 1604

$$\frac{\frac{1}{5}\left(-\frac{\int \frac{c\left(\left(3Ac\left(b^2-20ac\right)-7a\left(15Cb^2+24Bcb+20acC\right)\right)x^2+6A\left(b^3-8abc\right)-7a\left(3Bb^2+40aCb+36aBc\right)\right)}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a} - \frac{\sqrt{a+bx^2+cx^4}\left(3Ac(b^2-20ac)-7a(20acC\right.}{3ax^3}}{7c} - \frac{\left(a+bx^2+cx^4\right)^{3/2}\left(\frac{7aC}{c}+3A-7x^2\left(\frac{2bC}{c}+3B\right)\right)}{21x^7} + \frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{5}\left(-\frac{c\int \frac{\left(3Ac\left(b^2-20ac\right)-7a\left(15Cb^2+24Bcb+20acC\right)\right)x^2+6A\left(b^3-8abc\right)-7a\left(3Bb^2+40aCb+36aBc\right)}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a} - \frac{\sqrt{a+bx^2+cx^4}\left(3Ac(b^2-20ac)-7a(20acC+\right.}{3ax^3}}{7c} - \frac{\left(a+bx^2+cx^4\right)^{3/2}\left(\frac{7aC}{c}+3A-7x^2\left(\frac{2bC}{c}+3B\right)\right)}{21x^7} + \frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

$\Big\downarrow$ 1604

$$\frac{\frac{1}{5}\left(-\frac{c\left(-\frac{\int -\frac{c\left(6A\left(b^3-8abc\right)-7a\left(3Bb^2+40aCb+36aBc\right)\right)x^2+a\left(3Ac\left(b^2-20ac\right)-7a\left(15Cb^2+24Bcb+20acC\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a} - \frac{\sqrt{a+bx^2+cx^4}\left(6A\left(b^3-8abc\right)-7a\left(40ab\right.\right.}{ax}\right.}{3a}\right.}{} - \frac{\left(a+bx^2+cx^4\right)^{3/2}\left(\frac{7aC}{c}+3A-7x^2\left(\frac{2bC}{c}+3B\right)\right)}{21x^7} + \frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

$\downarrow$ 25

$$\frac{1}{5}\left(-\frac{c\left(\frac{\int \frac{c\left(6A\left(b^3-8abc\right)-7a\left(3Bb^2+40aCb+36aBc\right)\right)x^2+a\left(3Ac\left(b^2-20ac\right)-7a\left(15Cb^2+24Bcb+20acC\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(6A\left(b^3-8abc\right)-7a\left(40abC+3\right.\right.}{ax}\right.}{3a}\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(\frac{7aC}{c}+3A-7x^2\left(\frac{2bC}{c}+3B\right)\right)}{21x^7}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

$\downarrow$ 1511

$$\frac{1}{5}\left(-\frac{c\left(\frac{\sqrt{a}\left(\sqrt{a}\left(3Ac\left(b^2-20ac\right)-7a\left(20acC+15b^2C+24bBc\right)\right)+\sqrt{c}\left(6A\left(b^3-8abc\right)-7a\left(40abC+36aBc+3b^2B\right)\right)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{a}\sqrt{c}\left(6A\left(b^3-8abc\right)-\right.}{a}\right.}{3a}\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(\frac{7aC}{c}+3A-7x^2\left(\frac{2bC}{c}+3B\right)\right)}{21x^7}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

$\downarrow$ 27

$$\frac{1}{5}\left(-\frac{c\left(\frac{\sqrt{a}\left(\sqrt{a}\left(3Ac\left(b^2-20ac\right)-7a\left(20acC+15b^2C+24bBc\right)\right)+\sqrt{c}\left(6A\left(b^3-8abc\right)-7a\left(40abC+36aBc+3b^2B\right)\right)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}\left(6A\left(b^3-8abc\right)-7a\left(\right.\right.}{a}\right.}{3a}\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(\frac{7aC}{c}+3A-7x^2\left(\frac{2bC}{c}+3B\right)\right)}{21x^7}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

$\downarrow$ 1416

$$\frac{1}{5}\left(-\frac{c\left(\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\left(3Ac\left(b^2-20ac\right)-7a\left(20acC+15b^2C+24bBc\right)\right)+\sqrt{c}\left(6A\left(b^3-8abc\right)-7a\left(40abC+36aBc+3b^2B\right)\right)\right)\text{EllipticF}\left(2\right.}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}}{a}\right.}{}\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(\frac{7aC}{c}+3A-7x^2\left(\frac{2bC}{c}+3B\right)\right)}{21x^7}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

$\Big\downarrow$ 1509

$$\frac{1}{5}\left(-\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\left(3Ac\left(b^2-20ac\right)-7a\left(20acC+15b^2C+24bBc\right)\right)+\sqrt{c}\left(6A\left(b^3-8abc\right)-7a\left(40abC+36aBc+3b^2B\right)\right)\right)\text{EllipticF}\left(2\right.}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.}{}\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}\left(\frac{7aC}{c}+3A-7x^2\left(\frac{2bC}{c}+3B\right)\right)}{21x^7}+\frac{C\left(a+bx^2+cx^4\right)^{5/2}}{3cx^7}$$

input
```
Int[((a + b*x^2 + c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/x^8,x]
```

output
```
-1/21*((3*A + (7*a*C)/c - 7*(3*B + (2*b*C)/c)*x^2)*(a + b*x^2 + c*x^4)^(3/
2))/x^7 + (C*(a + b*x^2 + c*x^4)^(5/2))/(3*c*x^7) + (-1/5*((3*A*b*c + 42*a
*B*c + 35*a*b*C + 5*(21*b*B*c + 6*A*c^2 + 14*b^2*C + 14*a*c*C)*x^2)*Sqrt[a
 + b*x^2 + c*x^4])/x^5 + (-1/3*((3*A*c*(b^2 - 20*a*c) - 7*a*(24*b*B*c + 15
*b^2*C + 20*a*c*C))*Sqrt[a + b*x^2 + c*x^4])/(a*x^3) - (c*(-(((6*A*(b^3 -
8*a*b*c) - 7*a*(3*b^2*B + 36*a*B*c + 40*a*b*C))*Sqrt[a + b*x^2 + c*x^4])/(
a*x)) + (-(Sqrt[c]*(6*A*(b^3 - 8*a*b*c) - 7*a*(3*b^2*B + 36*a*B*c + 40*a*b
*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^
(1/4)*Sqrt[a + b*x^2 + c*x^4]))) + (a^(1/4)*(Sqrt[c]*(6*A*(b^3 - 8*a*b*c)
- 7*a*(3*b^2*B + 36*a*B*c + 40*a*b*C)) + Sqrt[a]*(3*A*c*(b^2 - 20*a*c) - 7
*a*(24*b*B*c + 15*b^2*C + 20*a*c*C)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/
a))/(3*a))/5)/(7*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 9.64 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.01

method	result
elliptic	$-\frac{Aa\sqrt{c\,x^4+b\,x^2+a}}{7x^7}-\frac{\left(\frac{8Ab}{7}+Ba\right)\sqrt{c\,x^4+b\,x^2+a}}{5x^5}-\frac{(45Aac+3A\,b^2+42Bab+35a^2C)\sqrt{c\,x^4+b\,x^2+a}}{105a\,x^3}-\frac{(48Aabc-6A\,b^3+147\ldots}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*A*a*(c*x^4+b*x^2+a)^(1/2)/x^7-1/5*(8/7*A*b+B*a)*(c*x^4+b*x^2+a)^(1/2)
/x^5-1/105*(45*A*a*c+3*A*b^2+42*B*a*b+35*C*a^2)/a*(c*x^4+b*x^2+a)^(1/2)/x^
3-1/105/a^2*(48*A*a*b*c-6*A*b^3+147*B*a^2*c+21*B*a*b^2+140*C*a^2*b)*(c*x^4
+b*x^2+a)^(1/2)/x+1/3*C*c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(A*c^2+2*B*b*c+5/3*C
*a*c+C*b^2-1/105*c*(45*A*a*c+3*A*b^2+42*B*a*b+35*C*a^2)/a)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*
(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))-1/2*(B*c^2+4/3*C*c*b+1/105*c*(48*A*a*b*c-6*A*b^3+147*B*a^2*
c+21*B*a*b^2+140*C*a^2*b)/a^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}\,(A+Bx^2+Cx^4)}{x^8}\,dx=\int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^8,x, algorithm="fricas")
```

output
```
integral((C*c*x^8 + (C*b + B*c)*x^6 + (C*a + B*b + A*c)*x^4 + (B*a + A*b)*
x^2 + A*a)*sqrt(c*x^4 + b*x^2 + a)/x^8, x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^8}\,dx = \int \frac{(A+Bx^2+Cx^4)(a+bx^2+cx^4)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)*(C*x**4+B*x**2+A)/x**8,x)
```

output
```
Integral((A + B*x**2 + C*x**4)*(a + b*x**2 + c*x**4)**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^8}\,dx = \int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^8,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^8}\,dx = \int \frac{(Cx^4+Bx^2+A)(cx^4+bx^2+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^8,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^8}\,dx = \int \frac{(C\,x^4+B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{3/2}}{x^8}\,dx$$

input
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2))/x^8,x)
```

output
```
int(((A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^(3/2))/x^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}(A+Bx^2+Cx^4)}{x^8}\,dx = \frac{-3\sqrt{c\,x^4+b\,x^2+a}\,a^2c-21\sqrt{c\,x^4+b\,x^2+a}\,abc\,x^2-56\sqrt{c\,x}}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)*(C*x^4+B*x^2+A)/x^8,x)
```

output
```
( - 3*sqrt(a + b*x**2 + c*x**4)*a**2*c - 21*sqrt(a + b*x**2 + c*x**4)*a*b*
c*x**2 - 56*sqrt(a + b*x**2 + c*x**4)*a*c**2*x**4 + 35*sqrt(a + b*x**2 + c
*x**4)*b**3*x**2 - 63*sqrt(a + b*x**2 + c*x**4)*b**2*c*x**4 + 49*sqrt(a +
b*x**2 + c*x**4)*b*c**2*x**6 + 7*sqrt(a + b*x**2 + c*x**4)*c**3*x**8 - 60*
int(sqrt(a + b*x**2 + c*x**4)/(a*x**6 + b*x**8 + c*x**10),x)*a**2*b*c*x**7
 + 175*int(sqrt(a + b*x**2 + c*x**4)/(a*x**6 + b*x**8 + c*x**10),x)*a*b**3
*x**7 - 120*int(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*a*
*2*c**2*x**7 - 210*int(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8
),x)*a*b**2*c*x**7 + 140*int(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 +
c*x**8),x)*b**4*x**7)/(21*c*x**7)
```

3.146 $\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 517

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(28bB-35Ac+25aC-\frac{24b^2C}{c}\right)x\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{(7Bc-6bC)x^3\sqrt{a+bx^2+cx^4}}{35c^2} + \frac{Cx^5\sqrt{a+bx^2+cx^4}}{7c} + \frac{(56b^2Bc-63aBc^2-48b^3C-2bc(35Ac-52aC))\,x\sqrt{a+bx^2+cx^4}}{105c^{7/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}(56b^2Bc-63aBc^2-48b^3C-2bc(35Ac-52aC))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{105c^{15/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(\sqrt{a}\left(28bB-35Ac+25aC-\frac{24b^2C}{c}\right)+\frac{56b^2Bc-63aBc^2-48b^3C-2bc(35Ac-52aC)}{c^{3/2}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+}{(\sqrt{a}+\sqrt{c}}}}{210c^{9/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/105*(28*B*b-35*A*c+25*a*C-24*b^2*C/c)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/35*
(7*B*c-6*C*b)*x^3*(c*x^4+b*x^2+a)^(1/2)/c^2+1/7*C*x^5*(c*x^4+b*x^2+a)^(1/2
)/c+1/105*(56*b^2*B*c-63*a*B*c^2-48*b^3*C-2*b*c*(35*A*c-52*C*a))*x*(c*x^4+
b*x^2+a)^(1/2)/c^(7/2)/(a^(1/2)+c^(1/2)*x^2)-1/105*a^(1/4)*(56*b^2*B*c-63*
a*B*c^2-48*b^3*C-2*b*c*(35*A*c-52*C*a))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^
2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/
4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(15/4)/(c*x^4+b*x^2+a)^(1/2)+1/210
*a^(1/4)*(a^(1/2)*(28*B*b-35*A*c+25*a*C-24*b^2*C/c)+(56*b^2*B*c-63*a*B*c^2
-48*b^3*C-2*b*c*(35*A*c-52*C*a))/c^(3/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*
x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a
^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(9/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.92 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.22

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)\left(24b^2C-2bc(14B+9Cx^2)+c(35Ac-25aC+21Bcx^2+15cCx^4)\right)-i}{}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4)*(24*b^2*C - 2*b
*c*(14*B + 9*C*x^2) + c*(35*A*c - 25*a*C + 21*B*c*x^2 + 15*c*C*x^4)) - I*(
-b + Sqrt[b^2 - 4*a*c])*(-56*b^2*B*c + 63*a*B*c^2 + 48*b^3*C + 2*b*c*(35*A
*c - 52*a*C))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*El
lipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-48*b^4*C - 2*b^2*c*(35*A*c + 28
*B*Sqrt[b^2 - 4*a*c] - 76*a*C) + a*c^2*(70*A*c + 63*B*Sqrt[b^2 - 4*a*c] -
50*a*C) + 8*b^3*(7*B*c + 6*Sqrt[b^2 - 4*a*c]*C) + b*c*(-119*a*B*c + 70*A*c
*Sqrt[b^2 - 4*a*c] - 104*a*Sqrt[b^2 - 4*a*c]*C))*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(
420*c^4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2199, 1602, 27, 1602, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{2199}$$

$$\int \frac{x^4\left(\frac{1}{7}\left(7B-\frac{6bC}{c}\right)x^2+\frac{1}{7}\left(7A-\frac{5aC}{c}\right)\right)}{\sqrt{cx^4+bx^2+a}}dx+\frac{Cx^5\sqrt{a+bx^2+cx^4}}{7c}$$

$$\Big\downarrow \text{1602}$$

$$-\frac{\int \frac{x^2\left(\left(-\frac{24Cb^2}{c}+28Bb-35Ac+25aC\right)x^2+3a\left(7B-\frac{6bC}{c}\right)\right)}{7\sqrt{cx^4+bx^2+a}}dx}{5c}+\frac{x^3\sqrt{a+bx^2+cx^4}(7Bc-6bC)}{35c^2}+\frac{Cx^5\sqrt{a+bx^2+cx^4}}{7c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{x^2\left(\left(-\frac{24Cb^2}{c}+28Bb-35Ac+25aC\right)x^2+3a\left(7B-\frac{6bC}{c}\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{35c}+\frac{x^3\sqrt{a+bx^2+cx^4}(7Bc-6bC)}{35c^2}+\frac{Cx^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\Big\downarrow$ 1602

$$-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(25aC-35Ac-\frac{24b^2C}{c}+28bB\right)}{3c}-\frac{\int \frac{\left(-48Cb^3+56Bcb^2-2c(35Ac-52aC)b-63aBc^2\right)x^2+ac\left(-\frac{24Cb^2}{c}+28Bb-35Ac+25aC\right)}{c\sqrt{cx^4+bx^2+a}}dx}{3c}}{35c}+\frac{x^3\sqrt{a+bx^2+cx^4}(7Bc-6bC)}{35c^2}+\frac{Cx^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\Big\downarrow$ 27

$$-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(25aC-35Ac-\frac{24b^2C}{c}+28bB\right)}{3c}-\frac{\int \frac{\left(-48Cb^3+56Bcb^2-2c(35Ac-52aC)b-63aBc^2\right)x^2+a\left(-24Cb^2+28Bcb-35Ac^2+25acC\right)}{\sqrt{cx^4+bx^2+a}}dx}{3c^2}}{35c}+\frac{x^3\sqrt{a+bx^2+cx^4}(7Bc-6bC)}{35c^2}+\frac{Cx^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\Big\downarrow$ 1511

$$-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(25aC-35Ac-\frac{24b^2C}{c}+28bB\right)}{3c}-\frac{\frac{\sqrt{a}\left(\sqrt{a}\sqrt{c}\left(-5c(7Ac-5aC)-24b^2C+28bBc\right)-2bc(35Ac-52aC)-63aBc^2-48b^3C+56b^2Bc\right)\int \frac{1}{\sqrt{cx^4+\cdots}}}{\sqrt{c}}\cdots}{3c^2}}{35c}+\frac{x^3\sqrt{a+bx^2+cx^4}(7Bc-6bC)}{35c^2}+\frac{Cx^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\Big\downarrow$ 27

$$-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(25aC-35Ac-\frac{24b^2C}{c}+28bB\right)}{3c}-\frac{\frac{\sqrt{a}\left(\sqrt{a}\sqrt{c}\left(-5c(7Ac-5aC)-24b^2C+28bBc\right)-2bc(35Ac-52aC)-63aBc^2-48b^3C+56b^2Bc\right)\int \frac{1}{\sqrt{cx^4+\cdots}}}{\sqrt{c}}\cdots}{3c^2}}{35c}+\frac{x^3\sqrt{a+bx^2+cx^4}(7Bc-6bC)}{35c^2}+\frac{Cx^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\Big\downarrow$ 1416

$$-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(25aC-35Ac-\frac{24b^2C}{c}+28bB\right)}{3c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\sqrt{c}\left(-5c(7Ac-5aC)-24b^2C+28bBc\right)-2bc(35Ac-52aC)-63aB\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}}{35c}$$

$$\frac{x^3\sqrt{a+bx^2+cx^4}(7Bc-6bC)}{35c^2}+\frac{Cx^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\big\downarrow$ 1509

$$-\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(25aC-35Ac-\frac{24b^2C}{c}+28bB\right)}{3c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\sqrt{c}\left(-5c(7Ac-5aC)-24b^2C+28bBc\right)-2bc(35Ac-52aC)-63aB\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}}{}$$

$$\frac{x^3\sqrt{a+bx^2+cx^4}(7Bc-6bC)}{35c^2}+\frac{Cx^5\sqrt{a+bx^2+cx^4}}{7c}$$

input

```
Int[(x^4*(A + B*x^2 + C*x^4))/Sqrt[a + b*x^2 + c*x^4],x]
```

output

```
((7*B*c - 6*b*C)*x^3*Sqrt[a + b*x^2 + c*x^4])/(35*c^2) + (C*x^5*Sqrt[a + b
*x^2 + c*x^4])/(7*c) - (((28*b*B - 35*A*c + 25*a*C - (24*b^2*C)/c)*x*Sqrt[
a + b*x^2 + c*x^4])/(3*c) - (-(((56*b^2*B*c - 63*a*B*c^2 - 48*b^3*C - 2*b*
c*(35*A*c - 52*a*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2
)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(56*b
^2*B*c - 63*a*B*c^2 - 48*b^3*C - 2*b*c*(35*A*c - 52*a*C) + Sqrt[a]*Sqrt[c]
*(28*b*B*c - 24*b^2*C - 5*c*(7*A*c - 5*a*C)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c
*x^4]))/(3*c^2))/(35*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 9.80 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.03

method	result
elliptic	$\frac{C\,x^{5}\sqrt{c\,x^{4}+b\,x^{2}+a}}{7c}+\frac{\left(B-\frac{6bC}{7c}\right)x^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{5c}+\frac{\left(A-\frac{4b\left(B-\frac{6bC}{7c}\right)}{5c}-\frac{5aC}{7c}\right)x\sqrt{c\,x^{4}+b\,x^{2}+a}}{3c}-\frac{a\left(A-\frac{4b\left(B-\frac{6bC}{7c}\right)}{5c}-\frac{5aC}{7c}\right)}{}$
risch	$\frac{x\left(15c^{2}C\,x^{4}+21B\,c^{2}x^{2}-18Ccb\,x^{2}+35A\,c^{2}-28Bbc-25Cac+24C\,b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{105c^{3}}-\frac{\left(70bA\,c^{2}+63Ba\,c^{2}-56b^{2}Bc-104Cacb+48b^{3}\right.}{}$
default	Expression too large to display

input
```
int(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*C*x^5*(c*x^4+b*x^2+a)^(1/2)/c+1/5*(B-6/7*b*C/c)/c*x^3*(c*x^4+b*x^2+a)^
(1/2)+1/3*(A-4/5*b/c*(B-6/7*b*C/c)-5/7*a*C/c)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/
12*a/c*(A-4/5*b/c*(B-6/7*b*C/c)-5/7*a*C/c)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1
/2*(-3/5*a/c*(B-6/7*b*C/c)-2/3*b/c*(A-4/5*b/c*(B-6/7*b*C/c)-5/7*a*C/c))*a*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.11

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left((48\,Cb^3c+7\,(9\,Ba+10\,Ab)c^3-8\,(13\,Cab+7\,Bb^2)c^2)x\sqrt{\frac{b^2-4\,ac}{c^2}}-(48\,Cb^4+7\,(9\,Bab+10\,Ab^2\right.}{}$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/210*(sqrt(1/2)*((48*C*b^3*c + 7*(9*B*a + 10*A*b)*c^3 - 8*(13*C*a*b + 7*
B*b^2)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (48*C*b^4 + 7*(9*B*a*b + 10*A*b^2)
*c^2 - 8*(13*C*a*b^2 + 7*B*b^3)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c
^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) -
b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/
2)*((48*C*b^3*c - 35*A*c^4 + ((63*B + 25*C)*a + 14*(5*A + 2*B)*b)*c^3 - 8*
(13*C*a*b + (7*B + 3*C)*b^2)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (48*C*b^4 +
35*A*b*c^3 + ((63*B - 25*C)*a*b + 14*(5*A - 2*B)*b^2)*c^2 - 8*(13*C*a*b^2
+ (7*B - 3*C)*b^3)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*e
lliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2
*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*(15*C*c^4*x^6 - 48
*C*b^3*c - 3*(6*C*b*c^3 - 7*B*c^4)*x^4 - 7*(9*B*a + 10*A*b)*c^3 + 8*(13*C*
a*b + 7*B*b^2)*c^2 + (24*C*b^2*c^2 + 35*A*c^4 - (25*C*a + 28*B*b)*c^3)*x^2
)*sqrt(c*x^4 + b*x^2 + a))/(c^5*x)
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**4*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**4*(A + B*x**2 + C*x**4)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^4\,(C\,x^4+B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{10\sqrt{c\,x^4+b\,x^2+a}\,acx - 4\sqrt{c\,x^4+b\,x^2+a}\,b^2x + 3\sqrt{c\,x^4+b\,x^2+a}\,bc\,x^3 + 15\sqrt{c\,x^4+b\,x^2+a}\,c^2x^5 - 1}{1}$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(10*sqrt(a + b*x**2 + c*x**4)*a*c*x - 4*sqrt(a + b*x**2 + c*x**4)*b**2*x +
 3*sqrt(a + b*x**2 + c*x**4)*b*c*x**3 + 15*sqrt(a + b*x**2 + c*x**4)*c**2*
x**5 - 10*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**2*c +
4*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b**2 - 29*int((
sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*b*c + 8*int((sq
rt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*b**3)/(105*c**2)
```

3.147 $\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 429

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{(5Bc-4bC)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Cx^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{(10bBc-8b^2C-3c(5Ac-3aC))\,x\sqrt{a+bx^2+cx^4}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\sqrt[4]{a}(10bBc-15Ac^2-8b^2C+9acC)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\left(10bBc+5\sqrt{a}Bc^{3/2}-15Ac^2-8b^2C-4\sqrt{a}b\sqrt{c}C+9acC\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/15*(5*B*c-4*C*b)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/5*C*x^3*(c*x^4+b*x^2+a)^(
1/2)/c-1/15*(10*B*b*c-8*b^2*C-3*c*(5*A*c-3*C*a))*x*(c*x^4+b*x^2+a)^(1/2)/c
^(5/2)/(a^(1/2)+c^(1/2)*x^2)+1/15*a^(1/4)*(-15*A*c^2+10*B*b*c+9*C*a*c-8*C*
b^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)
*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2
))/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)-1/30*a^(1/4)*(10*B*b*c+5*a^(1/2)*B*c^(3/
2)-15*A*c^2-8*b^2*C-4*a^(1/2)*b*c^(1/2)*C+9*a*c*C)*(a^(1/2)+c^(1/2)*x^2)*(
(c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^
(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(11/4)/(c*x^4+b*x^2+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.01 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.30

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(5Bc-4bC+3cCx^2)\left(a+bx^2+cx^4\right)+i\left(-b+\sqrt{b^2-4ac}\right)\left(-10bBc+8b^2C+3c(5Ac\right.}{}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(5*B*c - 4*b*C + 3*c*C*x^2)*(a + b*
x^2 + c*x^4) + I*(-b + Sqrt[b^2 - 4*a*c])*(-10*b*B*c + 8*b^2*C + 3*c*(5*A*
c - 3*a*C))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elli
pticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^3*C + b*c*(-15*A*c - 10*B*Sqr
t[b^2 - 4*a*c] + 17*a*C) + 2*b^2*(5*B*c + 4*Sqrt[b^2 - 4*a*c]*C) + c*(-10*
a*B*c + 15*A*c*Sqrt[b^2 - 4*a*c] - 9*a*Sqrt[b^2 - 4*a*c]*C))*Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
- 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4]
)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2199, 1602, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}} \, dx$$

$$\Big\downarrow \text{2199}$$

$$\int \frac{x^2 \left(\frac{1}{5}\left(5B-\frac{4bC}{c}\right)x^2+\frac{1}{5}\left(5A-\frac{3aC}{c}\right)\right)}{\sqrt{cx^4+bx^2+a}}dx+\frac{Cx^3\sqrt{a+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \text{1602}$$

$$-\frac{\int \frac{(-8Cb^2+10Bcb-3c(5Ac-3aC))x^2+ac\left(5B-\frac{4bC}{c}\right)}{5c\sqrt{cx^4+bx^2+a}}dx}{3c}+\frac{x\sqrt{a+bx^2+cx^4}(5Bc-4bC)}{15c^2}+\frac{Cx^3\sqrt{a+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(-8Cb^2+10Bcb-3c(5Ac-3aC))x^2+a(5Bc-4bC)}{\sqrt{cx^4+bx^2+a}}dx}{15c^2}+\frac{x\sqrt{a+bx^2+cx^4}(5Bc-4bC)}{15c^2}+\frac{Cx^3\sqrt{a+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \text{1511}$$

$$-\frac{\frac{\sqrt{a}\left(-3c(5Ac-3aC)+\sqrt{a}\sqrt{c}(5Bc-4bC)-8b^2C+10bBc\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(9acC-15Ac^2-8b^2C+10bBc\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{15c^2}+\frac{x\sqrt{a+bx^2+cx^4}(5Bc-4bC)}{15c^2}+\frac{Cx^3\sqrt{a+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\sqrt{a}\left(-3c(5Ac-3aC)+\sqrt{a}\sqrt{c}(5Bc-4bC)-8b^2C+10bBc\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{(9acC-15Ac^2-8b^2C+10bBc)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{15c^2}+\frac{x\sqrt{a+bx^2+cx^4}(5Bc-4bC)}{15c^2}+\frac{Cx^3\sqrt{a+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(-3c(5Ac-3aC)+\sqrt{a}\sqrt{c}(5Bc-4bC)-8b^2C+10bBc\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(9acC}{}}{15c^2}+\frac{x\sqrt{a+bx^2+cx^4}(5Bc-4bC)}{15c^2}+\frac{Cx^3\sqrt{a+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \text{1509}$$

$$-\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(-3c(5Ac-3aC)+\sqrt{a}\sqrt{c}(5Bc-4bC)-8b^2C+10bBc\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(9acC}{}}{15c^2}+\frac{x\sqrt{a+bx^2+cx^4}(5Bc-4bC)}{15c^2}+\frac{Cx^3\sqrt{a+bx^2+cx^4}}{5c}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((5*B*c - 4*b*C)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (C*x^3*Sqrt[a + b*x
^2 + c*x^4])/(5*c) - (-(((10*b*B*c - 15*A*c^2 - 8*b^2*C + 9*a*c*C)*(-((x*S
qrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a
 + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(10*b*B*c - 8*b^2*C - 3*c*(5*A*c
- 3*a*C) + Sqrt[a]*Sqrt[c]*(5*B*c - 4*b*C))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x
^4]))/(15*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 7.29 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.05

method	result
elliptic	$\frac{C x^3 \sqrt{c x^4+b x^2+a}}{5c}+\frac{\left(B-\frac{4bC}{5c}\right)x\sqrt{c x^4+b x^2+a}}{3c}-\frac{a\left(B-\frac{4bC}{5c}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \text{Elliptic}}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4+}}$
risch	$\frac{x(3cC x^2+5Bc-4Cb)\sqrt{c x^4+b x^2+a}}{15c^2}+\frac{-\frac{\left(15A c^2-10Bbc-9Cac+8C b^2\right)a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{Elli}\right.}{2\sqrt{\frac{-b+\sqrt{}}{}}}}{}$
default	Expression too large to display

input
```
int(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*C*x^3*(c*x^4+b*x^2+a)^(1/2)/c+1/3*(B-4/5*b*C/c)/c*x*(c*x^4+b*x^2+a)^(1
/2)-1/12*a/c*(B-4/5*b*C/c)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*
(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(A-3/5*a*C/c-
2/3*b/c*(B-4/5*b*C/c))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A + Bx^2 + Cx^4)}{\sqrt{a + bx^2 + cx^4}}\, dx$$

$$= \frac{\sqrt{\frac{1}{2}}\left((8\,Cb^2c + 15\,Ac^3 - (9\,Ca + 10\,Bb)c^2)x\sqrt{\frac{b^2-4\,ac}{c^2}} - (8\,Cb^3 + 15\,Abc^2 - (9\,Cab + 10\,Bb^2)c)x\right)\sqrt{c}\sqrt{}}{}$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(sqrt(1/2)*((8*C*b^2*c + 15*A*c^3 - (9*C*a + 10*B*b)*c^2)*x*sqrt((b^2
 - 4*a*c)/c^2) - (8*C*b^3 + 15*A*b*c^2 - (9*C*a*b + 10*B*b^2)*c)*x)*sqrt(c
)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt
((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) +
 b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((8*C*b^2*c + 5*(3*A + B)*c^3 - (9*C*a +
2*(5*B + 2*C)*b)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (8*C*b^3 + 5*(3*A - B)*b
*c^2 - (9*C*a*b + 2*(5*B - 2*C)*b^2)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a
*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^
2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*
(3*C*c^3*x^4 + 8*C*b^2*c + 15*A*c^3 - (9*C*a + 10*B*b)*c^2 - (4*C*b*c^2 -
5*B*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^4*x)
```

Sympy [F]

$$\int \frac{x^2(A + Bx^2 + Cx^4)}{\sqrt{a + bx^2 + cx^4}}\, dx = \int \frac{x^2(A + Bx^2 + Cx^4)}{\sqrt{a + bx^2 + cx^4}}\, dx$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2\,\left(C\,x^4+B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{\sqrt{c\,x^4+b\,x^2+a}\,bx+3\sqrt{c\,x^4+b\,x^2+a}\,c\,x^3-\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)ab+6\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)ac-2\left(\int \right.}{15c}$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4)*b*x + 3*sqrt(a + b*x**2 + c*x**4)*c*x**3 - int(
sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b + 6*int((sqrt(a + b
*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*c - 2*int((sqrt(a + b*x**
2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*b**2)/(15*c)
```

3.148 $\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 344

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Cx\sqrt{a+bx^2+cx^4}}{3c}+\frac{(3Bc-2bC)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}(3Bc-2bC)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}} + \frac{\left(3Ac-aC+\frac{\sqrt{a}(3Bc-2bC)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/3*C*x*(c*x^4+b*x^2+a)^(1/2)/c+1/3*(3*B*c-2*C*b)*x*(c*x^4+b*x^2+a)^(1/2)/
c^(3/2)/(a^(1/2)+c^(1/2)*x^2)-1/3*a^(1/4)*(3*B*c-2*C*b)*(a^(1/2)+c^(1/2)*x
^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan
(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(c*x^4+b*x^2
+a)^(1/2)+1/6*(3*A*c-a*C+a^(1/2)*(3*B*c-2*C*b)/c^(1/2))*(a^(1/2)+c^(1/2)*x
^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arct
an(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/c^(5/4)/(c*
x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.81 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.41

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{a + bx^2 + cx^4}} dx = \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}Cx(a + bx^2 + cx^4) - i\left(-b + \sqrt{b^2 - 4ac}\right)(-3Bc + 2bC)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}}{}$$

input

```
Integrate[(A + B*x^2 + C*x^4)/Sqrt[a + b*x^2 + c*x^4],x]
```

output

```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*C*x*(a + b*x^2 + c*x^4) - I*(-b + Sqr
t[b^2 - 4*a*c])*(-3*B*c + 2*b*C)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-2*b^2*C + c*
(-6*A*c - 3*B*Sqrt[b^2 - 4*a*c] + 2*a*C) + b*(3*B*c + 2*Sqrt[b^2 - 4*a*c]*
C))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(
2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])])/(12*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a
 + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2207, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{a + bx^2 + cx^4}} dx$$

$$\Big\downarrow \; 2207$$

$$\frac{\int \frac{(3Bc-2bC)x^2+3Ac-aC}{\sqrt{cx^4+bx^2+a}}dx}{3c}+\frac{Cx\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \; 1511$$

$$\frac{\left(\frac{\sqrt{a}(3Bc-2bC)}{\sqrt{c}}-aC+3Ac\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}(3Bc-2bC)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}+\frac{Cx\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(\frac{\sqrt{a}(3Bc-2bC)}{\sqrt{c}}-aC+3Ac\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{(3Bc-2bC)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}+\frac{Cx\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \; 1416$$

$$\frac{\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}(3Bc-2bC)}{\sqrt{c}}-aC+3Ac\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{(3Bc-2bC)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}+\frac{Cx\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \; 1509$$

$$\frac{\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}(3Bc-2bC)}{\sqrt{c}}-aC+3Ac\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{(3Bc-2bC)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2}{(\sqrt{a}+}}}{}\right.}{}}{3c}+\frac{Cx\sqrt{a+bx^2+cx^4}}{3c}$$

input
```
Int[(A + B*x^2 + C*x^4)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(C*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + (-(((3*B*c - 2*b*C)*(-((x*Sqrt[a + b
*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2
+ c*x^4])))/Sqrt[c]) + ((3*A*c - a*C + (Sqrt[a]*(3*B*c - 2*b*C))/Sqrt[c])*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/(3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 5.76 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.16

method	result
elliptic	$\frac{Cx\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\left(A-\frac{aC}{3c}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b(}{}}}{}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
risch	$\frac{Cx\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{-\frac{(3Bc-2Cb)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b}{}}}{}\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}\,\left(b+\sqrt{}\right.}}{}$
default	$\frac{A\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-\frac{Ba\sqrt{2}\,\sqrt{}}{}$

input
```
int((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*C*x*(c*x^4+b*x^2+a)^(1/2)/c+1/4*(A-1/3*a*C/c)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))-1/2*(B-2/3*b*C/c)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(
-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*
b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.08

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{a + bx^2 + cx^4}}\, dx = -\frac{\sqrt{\frac{1}{2}}\left((2\,Cabc - 3\,Bac^2)x\sqrt{\frac{b^2-4\,ac}{c^2}} - (2\,Cab^2 - 3\,Babc)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)}{}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(1/2)*((2*C*a*b*c - 3*B*a*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (2*C*
a*b^2 - 3*B*a*b*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elli
ptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b
*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*C*a*b*c -
 (3*B + C)*a*c^2 + 3*A*c^3)*x*sqrt((b^2 - 4*a*c)/c^2) - (2*C*a*b^2 - (3*B
- C)*a*b*c - 3*A*b*c^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)
*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1
/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*(C*a*c^2*x^2 - 2
*C*a*b*c + 3*B*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/(a*c^3*x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+bx^2+cx^4}}\,dx=\frac{\sqrt{c\,x^4+b\,x^2+a}\,x}{3}+\frac{2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)a}{3}+\frac{\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)b}{3}$$

input
```
int((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4)*x + 2*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2
 + c*x**4),x)*a + int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**
4),x)*b)/3
```

3.149 $\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 328

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{ax}+\frac{(Ac+aC)x\sqrt{a+bx^2+cx^4}}{a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{(Ac+aC)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\left(\sqrt{a}B\sqrt{c}+Ac+aC\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output

```
-A*(c*x^4+b*x^2+a)^(1/2)/a/x+(A*c+C*a)*x*(c*x^4+b*x^2+a)^(1/2)/a/c^(1/2)/(
a^(1/2)+c^(1/2)*x^2)-(A*c+C*a)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(
1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*
(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/2*(a^
(1/2)*B*c^(1/2)+A*c+a*C)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c
^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.06 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \frac{-4Ac\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)+i\left(-b+\sqrt{b^2-4ac}\right)(Ac+aC)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-4*A*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*(-b + Sqrt
[b^2 - 4*a*c])*(A*c + a*C)*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(2*a*B*c + A*c*(-b
 + Sqrt[b^2 - 4*a*c]) + a*(-b + Sqrt[b^2 - 4*a*c])*C)*x*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a
*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(4*a*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2199, 1604, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \ 2199$$

$$\int \frac{Bx^2+A+\frac{aC}{c}}{x^2\sqrt{cx^4+bx^2+a}}dx+\frac{C\sqrt{a+bx^2+cx^4}}{cx}$$

$\Big\downarrow$ 1604

$$-\frac{\int -\frac{(Ac+aC)x^2+aB}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\left(\frac{aC}{c}+A\right)\sqrt{a+bx^2+cx^4}}{ax}+\frac{C\sqrt{a+bx^2+cx^4}}{cx}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{(Ac+aC)x^2+aB}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\left(\frac{aC}{c}+A\right)\sqrt{a+bx^2+cx^4}}{ax}+\frac{C\sqrt{a+bx^2+cx^4}}{cx}$$

$\Big\downarrow$ 1511

$$\frac{\frac{\sqrt{a}\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}(aC+Ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{a}-\frac{\left(\frac{aC}{c}+A\right)\sqrt{a+bx^2+cx^4}}{ax}+\frac{C\sqrt{a+bx^2+cx^4}}{cx}$$

$\Big\downarrow$ 27

$$\frac{\frac{\sqrt{a}\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{(aC+Ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{a}-\frac{\left(\frac{aC}{c}+A\right)\sqrt{a+bx^2+cx^4}}{ax}+\frac{C\sqrt{a+bx^2+cx^4}}{cx}$$

$\Big\downarrow$ 1416

$$\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(aC+Ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{a}-\frac{\left(\frac{aC}{c}+A\right)\sqrt{a+bx^2+cx^4}}{ax}+\frac{C\sqrt{a+bx^2+cx^4}}{cx}$$

$\Big\downarrow$ 1509

$$\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(aC+Ac)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)}}}{\sqrt[4]{c}}\right.}{}}{a}-\frac{\left(\frac{aC}{c}+A\right)\sqrt{a+bx^2+cx^4}}{ax}+\frac{C\sqrt{a+bx^2+cx^4}}{cx}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(C*Sqrt[a + b*x^2 + c*x^4])/(c*x) - ((A + (a*C)/c)*Sqrt[a + b*x^2 + c*x^4]
)/(a*x) + (-(((A*c + a*C)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c
]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*
(Sqrt[a]*B*Sqrt[c] + A*c + a*C)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199

```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.20

method	result
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{a x}+\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-}\right.}{ac}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
risch	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{a x}+\frac{\frac{B a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac}\right.}{ac}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}}{}$
default	$\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+A\left(-\right.$

input
```
int((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a*A*(c*x^4+b*x^2+a)^(1/2)/x+1/4*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(C+
c/a*A)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2)))
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\,x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)/(c*x^6 + b*x^4 + a*x^
2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^2\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^2+cx^4}}\,dx =\frac{\sqrt{c\,x^4+b\,x^2+a}+2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^6+b\,x^4+a\,x^2}dx\right)ax+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)bx}{x}$$

input
```
int((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4) + 2*int(sqrt(a + b*x**2 + c*x**4)/(a*x**2 + b*x
**4 + c*x**6),x)*a*x + int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4)
,x)*b*x)/x
```

3.150 $\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 347

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}+\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^{3/2}x\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{(2Ab-3aB)\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}} - \frac{\left(\frac{(2Ab-3aB)\sqrt{c}}{\sqrt{a}}+Ac-3aC\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{5/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/3*A*(c*x^4+b*x^2+a)^(1/2)/a/x^3+1/3*(2*A*b-3*B*a)*(c*x^4+b*x^2+a)^(1/2)
/a^(3/2)/x/(a^(1/2)+c^(1/2)*x^2)+1/3*(2*A*b-3*B*a)*c^(1/4)*(a^(1/2)+c^(1/2
)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(7/4)/(c*x^4+b*
x^2+a)^(1/2)-1/6*((2*A*b-3*B*a)*c^(1/2)/a^(1/2)+A*c-3*a*C)*(a^(1/2)+c^(1/2
)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*a
rctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(5/4)/c^(1/4)/
(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.13 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(a+bx^2+cx^4\right)\left(-2Abx^2+a(A+3Bx^2)\right)-i(2Ab-3aB)\left(-b+\sqrt{b^2-4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2-}}{b+\sqrt{}}}}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(-2*A*b*x^2 + a*(A
 + 3*B*x^2)) - I*(2*A*b - 3*a*B)*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])] + I*(2*A*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*c]) - 3*a*(-(b*B) + B*S
qrt[b^2 - 4*a*c] + 2*a*C))*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(12*a^2*Sqrt[c/(b +
 Sqrt[b^2 - 4*a*c])]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2199, 1604, 1604, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{2199}$$

$$\int \frac{\left(B-\frac{2bC}{c}\right)x^2+A-\frac{3aC}{c}}{x^4\sqrt{cx^4+bx^2+a}}dx-\frac{C\sqrt{a+bx^2+cx^4}}{cx^3}$$

$$\Big\downarrow \text{1604}$$

$$-\frac{\int \frac{(Ac-3aC)x^2+2Ab-3aB}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}-\frac{\left(A-\frac{3aC}{c}\right)\sqrt{a+bx^2+cx^4}}{3ax^3}-\frac{C\sqrt{a+bx^2+cx^4}}{cx^3}$$

$$\Big\downarrow \text{1604}$$

$$-\frac{-\frac{\int -\frac{(2Ab-3aB)cx^2+a(Ac-3aC)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{\left(A-\frac{3aC}{c}\right)\sqrt{a+bx^2+cx^4}}{3ax^3}-\frac{C\sqrt{a+bx^2+cx^4}}{cx^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{(2Ab-3aB)cx^2+a(Ac-3aC)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{\left(A-\frac{3aC}{c}\right)\sqrt{a+bx^2+cx^4}}{3ax^3}-\frac{C\sqrt{a+bx^2+cx^4}}{cx^3}$$

$$\Big\downarrow \text{1511}$$

$$-\frac{\frac{\sqrt{a}\left(\sqrt{c}(2Ab-3aB)+\sqrt{a}(Ac-3aC)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{a}\sqrt{c}(2Ab-3aB)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{\left(A-\frac{3aC}{c}\right)\sqrt{a+bx^2+cx^4}}{3ax^3}-\frac{C\sqrt{a+bx^2+cx^4}}{cx^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\sqrt{a}\left(\sqrt{c}(2Ab-3aB)+\sqrt{a}(Ac-3aC)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}(2Ab-3aB)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{\left(A-\frac{3aC}{c}\right)\sqrt{a+bx^2+cx^4}}{3ax^3}-\frac{C\sqrt{a+bx^2+cx^4}}{cx^3}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}(2Ab-3aB)+\sqrt{a}(Ac-3aC)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}(2Ab-3aB)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{a}-}{3a}$$

$$\frac{\left(A-\frac{3aC}{c}\right)\sqrt{a+bx^2+cx^4}}{3ax^3}-\frac{C\sqrt{a+bx^2+cx^4}}{cx^3}$$

$\Big\downarrow$ 1509

$$-\frac{\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}(2Ab-3aB)+\sqrt{a}(Ac-3aC)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}(2Ab-3aB)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\ldots}}{\ldots}\right.}{a}}{3a}$$

$$\frac{\left(A-\frac{3aC}{c}\right)\sqrt{a+bx^2+cx^4}}{3ax^3}-\frac{C\sqrt{a+bx^2+cx^4}}{cx^3}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-((C*Sqrt[a + b*x^2 + c*x^4])/(c*x^3)) - ((A - (3*a*C)/c)*Sqrt[a + b*x^2 +
 c*x^4])/(3*a*x^3) - (-(((2*A*b - 3*a*B)*Sqrt[a + b*x^2 + c*x^4])/(a*x)) +
 (-((2*A*b - 3*a*B)*Sqrt[c]*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt
[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))) + (a^(1/4)*((2*A*b
 - 3*a*B)*Sqrt[c] + Sqrt[a]*(A*c - 3*a*C))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^
4]))/a)/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.25

method	result
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{3a\,x^3}+\frac{(2Ab-3Ba)\sqrt{c\,x^4+b\,x^2+a}}{3a^2x}+\frac{\left(C-\frac{cA}{3a}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\cdots\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2\cdots}}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(-2Ab\,x^2+3Ba\,x^2+Aa\right)}{3a^2x^3}-\frac{-\frac{c(2Ab-3Ba)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\cdots}{\cdots}\right.\right.}{\cdots}}{2\sqrt{\frac{-b+\sqrt{-4\cdots}}{a}}\cdots}$
default	$\frac{C\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+A\left(-\cdots\right.$

input
```
int((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*(c*x^4+b*x^2+a)^(1/2)/a/x^3+1/3*(2*A*b-3*B*a)/a^2*(c*x^4+b*x^2+a)^(
1/2)/x+1/4*(C-1/3*c/a*A)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/6*c*(2*A*b-3*B*a)
/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a+bx^2+cx^4}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left((3\,Ba^2-2\,Aab)cx^3\sqrt{\frac{b^2-4\,ac}{a^2}}-(3\,Bab-2\,Ab^2)cx^3\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}}{a}}\right.\right.}{}$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(1/2)*((3*B*a^2 - 2*A*a*b)*c*x^3*sqrt((b^2 - 4*a*c)/a^2) - (3*B*
a*b - 2*A*b^2)*c*x^3)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elli
ptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a
*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((3*C*a^3 - (
(A + 3*B)*a^2 - 2*A*a*b)*c)*x^3*sqrt((b^2 - 4*a*c)/a^2) + (3*C*a^2*b - ((A
 - 3*B)*a*b + 2*A*b^2)*c)*x^3)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b
)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)
), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + 2*sqrt(c*x^4 +
 b*x^2 + a)*(A*a^2*c + (3*B*a^2 - 2*A*a*b)*c*x^2))/(a^3*c*x^3)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^4\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^4\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{x^4\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{C\,x^4 + B\,x^2 + A}{x^4\,\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\frac{-\sqrt{c\,x^4+b\,x^2+a}-\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^8+b\,x^6+a\,x^4}dx\right)a\,x^3+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)c\,x^3}{2x^3}$$

input
```
int((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4) - int(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*
x**6 + c*x**8),x)*a*x**3 + int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x
**4),x)*c*x**3)/(2*x**3)
```

3.151 $\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 429

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{5ax^5} + \frac{(4Ab-5aB)\sqrt{a+bx^2+cx^4}}{15a^2x^3}$$
$$-\frac{(8Ab^2-10abB-9aAc+15a^2C)\sqrt{a+bx^2+cx^4}}{15a^{5/2}x\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{\sqrt[4]{c}(8Ab^2-10abB-9aAc+15a^2C)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15a^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{c}\left((4Ab-5aB)\sqrt{c}+\frac{A(8b^2-9ac)-5a(2bB-3aC)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{30a^{9/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/5*A*(c*x^4+b*x^2+a)^(1/2)/a/x^5+1/15*(4*A*b-5*B*a)*(c*x^4+b*x^2+a)^(1/2
)/a^2/x^3-1/15*(-9*A*a*c+8*A*b^2-10*B*a*b+15*C*a^2)*(c*x^4+b*x^2+a)^(1/2)/
a^(5/2)/x/(a^(1/2)+c^(1/2)*x^2)-1/15*c^(1/4)*(-9*A*a*c+8*A*b^2-10*B*a*b+15
*C*a^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1
/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))/a^(11/4)/(c*x^4+b*x^2+a)^(1/2)+1/30*c^(1/4)*((4*A*b-5*B*a)*c^(1/2)+(
A*(-9*a*c+8*b^2)-5*a*(2*B*b-3*C*a))/a^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4
+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*
x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(9/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.57 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{-\frac{4(a+bx^2+cx^4)\left(3a^2A+a(-4Ab+5aB)x^2+\left(A\left(8b^2-9ac\right)+5a(-2bB+3aC)\right)x^4\right)}{x^5}+\frac{i\sqrt{2}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\left(-b+\sqrt{b^2-4\right.\right.}{}}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-4*(a + b*x^2 + c*x^4)*(3*a^2*A + a*(-4*A*b + 5*a*B)*x^2 + (A*(8*b^2 - 9
*a*c) + 5*a*(-2*b*B + 3*a*C))*x^4))/x^5 + (I*Sqrt[2]*Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*(A*(8*b^2 - 9*a*c) + 5*a*(-2*b*B +
3*a*C))*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
 + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (A*(8*b^3 - 17*a*b*c - 8*
b^2*Sqrt[b^2 - 4*a*c] + 9*a*c*Sqrt[b^2 - 4*a*c]) + 5*a*(-2*b^2*B + 2*a*B*c
 + 2*b*B*Sqrt[b^2 - 4*a*c] + 3*a*b*C - 3*a*Sqrt[b^2 - 4*a*c]*C))*EllipticF
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(60*a^3*S
qrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2199, 1604, 1604, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a+bx^2+cx^4}}\,dx$$

$\Big\downarrow$ 2199

$$\int \frac{\frac{1}{3}\left(3B-\frac{4bC}{c}\right)x^2+\frac{1}{3}\left(3A-\frac{5aC}{c}\right)}{x^6\sqrt{cx^4+bx^2+a}}dx-\frac{C\sqrt{a+bx^2+cx^4}}{3cx^5}$$

$\Big\downarrow$ 1604

$$-\frac{\int \frac{(3Ac-5aC)x^2+4Ab-5aB}{x^4\sqrt{cx^4+bx^2+a}}dx}{5a}-\frac{\left(3A-\frac{5aC}{c}\right)\sqrt{a+bx^2+cx^4}}{15ax^5}-\frac{C\sqrt{a+bx^2+cx^4}}{3cx^5}$$

$\Big\downarrow$ 1604

$$-\frac{-\frac{\int \frac{15Ca^2-10bBa-9Aca+8Ab^2+(4Ab-5aB)cx^2}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}-\frac{(4Ab-5aB)\sqrt{a+bx^2+cx^4}}{3ax^3}}{5a}-\frac{\left(3A-\frac{5aC}{c}\right)\sqrt{a+bx^2+cx^4}}{15ax^5}-\frac{C\sqrt{a+bx^2+cx^4}}{3cx^5}$$

$\Big\downarrow$ 1604

$$-\frac{-\frac{-\frac{\int -\frac{c\left(\left(15Ca^2-10bBa-9Aca+8Ab^2\right)x^2+a(4Ab-5aB)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(15a^2C-9aAc-10abB+8Ab^2\right)}{ax}}{3a}-\frac{(4Ab-5aB)\sqrt{a+bx^2+cx^4}}{3ax^3}}{5a}-\frac{\left(3A-\frac{5aC}{c}\right)\sqrt{a+bx^2+cx^4}}{15ax^5}-\frac{C\sqrt{a+bx^2+cx^4}}{3cx^5}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\frac{\int \frac{c\left(\left(15Ca^2-10bBa-9Aca+8Ab^2\right)x^2+a(4Ab-5aB)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(15a^2C-9aAc-10abB+8Ab^2\right)}{ax}}{3a}-\frac{(4Ab-5aB)\sqrt{a+bx^2+cx^4}}{3ax^3}}{5a}-\frac{\left(3A-\frac{5aC}{c}\right)\sqrt{a+bx^2+cx^4}}{15ax^5}-\frac{C\sqrt{a+bx^2+cx^4}}{3cx^5}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\frac{c\int \frac{\left(15Ca^2-10bBa-9Aca+8Ab^2\right)x^2+a(4Ab-5aB)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(15a^2C-9aAc-10abB+8Ab^2\right)}{ax}}{3a}-\frac{(4Ab-5aB)\sqrt{a+bx^2+cx^4}}{3ax^3}}{5a}-\frac{\left(3A-\frac{5aC}{c}\right)\sqrt{a+bx^2+cx^4}}{15ax^5}-\frac{C\sqrt{a+bx^2+cx^4}}{3cx^5}$$

$\Big\downarrow$ 1511

$$-\frac{-\frac{c\left(\frac{\sqrt{a}\left(15a^2C+\sqrt{a}\sqrt{c}(4Ab-5aB)-9aAc-10abB+8Ab^2\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(15a^2C-9aAc-10abB+8Ab^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{a+bx^2+cx^4}\cdots}{\cdots}}{3a}\cdots}{5a}$$

$$\frac{\left(3A-\frac{5aC}{c}\right)\sqrt{a+bx^2+cx^4}}{15ax^5}-\frac{C\sqrt{a+bx^2+cx^4}}{3cx^5}$$

$\downarrow$ 27

$$-\frac{-\frac{c\left(\frac{\sqrt{a}\left(15a^2C+\sqrt{a}\sqrt{c}(4Ab-5aB)-9aAc-10abB+8Ab^2\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(15a^2C-9aAc-10abB+8Ab^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(15a^2\cdots\right.}{\cdots}}{3a}\cdots}{5a}$$

$$\frac{\left(3A-\frac{5aC}{c}\right)\sqrt{a+bx^2+cx^4}}{15ax^5}-\frac{C\sqrt{a+bx^2+cx^4}}{3cx^5}$$

$\downarrow$ 1416

$$-\frac{-\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(15a^2C+\sqrt{a}\sqrt{c}(4Ab-5aB)-9aAc-10abB+8Ab^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(15a^2C-9aAc-\cdots\right.}{\cdots}\cdots\right)}{a}\cdots}{3a}\cdots}{5a}$$

$$\frac{\left(3A-\frac{5aC}{c}\right)\sqrt{a+bx^2+cx^4}}{15ax^5}-\frac{C\sqrt{a+bx^2+cx^4}}{3cx^5}$$

$\downarrow$ 1509

$$-\frac{-\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(15a^2C+\sqrt{a}\sqrt{c}(4Ab-5aB)-9aAc-10abB+8Ab^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(15a^2C-9aAc-\cdots\right.}{\cdots}\cdots\right)}{a}\cdots}{\cdots}}{\cdots}$$

$$\frac{\left(3A-\frac{5aC}{c}\right)\sqrt{a+bx^2+cx^4}}{15ax^5}-\frac{C\sqrt{a+bx^2+cx^4}}{3cx^5}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^6*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/3*(C*Sqrt[a + b*x^2 + c*x^4])/(c*x^5) - ((3*A - (5*a*C)/c)*Sqrt[a + b*x
^2 + c*x^4])/(15*a*x^5) - (-1/3*((4*A*b - 5*a*B)*Sqrt[a + b*x^2 + c*x^4])/
(a*x^3) - (-(((8*A*b^2 - 10*a*b*B - 9*a*A*c + 15*a^2*C)*Sqrt[a + b*x^2 + c
*x^4])/(a*x)) + (c*(-(((8*A*b^2 - 10*a*b*B - 9*a*A*c + 15*a^2*C)*(-((x*Sqr
t[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a +
 b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(8*A*b^2 - 10*a*b*B + Sqrt[a]*(4*A*
b - 5*a*B)*Sqrt[c] - 9*a*A*c + 15*a^2*C)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]
)))/a)/(3*a))/(5*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 4.87 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.16

method	result
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{5a\,x^5}+\frac{(4Ab-5Ba)\sqrt{c\,x^4+b\,x^2+a}}{15a^2x^3}+\frac{(9Aac-8A\,b^2+10Bab-15a^2C)\sqrt{c\,x^4+b\,x^2+a}}{15a^3x}+\frac{c(4Ab-5Ba)\sqrt{2}\,\sqrt{4-\frac{2(}{}}}{}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-9Aac\,x^4+8A\,b^2x^4-10Bab\,x^4+15C\,a^2x^4-4Aab\,x^2+5B\,a^2x^2+3A\,a^2)}{15a^3x^5}+\frac{c\left(\frac{(9Aac-8A\,b^2+10Bab-15a^2C)a\sqrt{2}}{}\right.}{}$
default	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/x^6/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*A*(c*x^4+b*x^2+a)^(1/2)/a/x^5+1/15*(4*A*b-5*B*a)*(c*x^4+b*x^2+a)^(1/2
)/a^2/x^3+1/15*(9*A*a*c-8*A*b^2+10*B*a*b-15*C*a^2)/a^3*(c*x^4+b*x^2+a)^(1/
2)/x+1/60*c*(4*A*b-5*B*a)/a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4
-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/30*(9*A*a*c-
8*A*b^2+10*B*a*b-15*C*a^2)*c/a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a+bx^2+cx^4}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left((15\,Ca^3-10\,Ba^2b+8\,Aab^2-9\,Aa^2c)x^5\sqrt{\frac{b^2-4\,ac}{a^2}}-(15\,Ca^2b-10\,Bab^2+8\,Ab^3-9\,Aabc)x^5\right)\sqrt{}}{}$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(sqrt(1/2)*((15*C*a^3 - 10*B*a^2*b + 8*A*a*b^2 - 9*A*a^2*c)*x^5*sqrt
((b^2 - 4*a*c)/a^2) - (15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 - 9*A*a*b*c)*x^5)
*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/
2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)
/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((5*(B + 3*C)*a^3 - 2*(2*A + 5*B)*
a^2*b + 8*A*a*b^2 - 9*A*a^2*c)*x^5*sqrt((b^2 - 4*a*c)/a^2) + (5*(B - 3*C)*
a^2*b - 2*(2*A - 5*B)*a*b^2 - 8*A*b^3 + 9*A*a*b*c)*x^5)*sqrt(a)*sqrt((a*sq
rt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((
b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c
)/(a*c)) + 2*((15*C*a^3 - 10*B*a^2*b + 8*A*a*b^2 - 9*A*a^2*c)*x^4 + 3*A*a^
3 + (5*B*a^3 - 4*A*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^5)
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{a + bx^2 + cx^4}}\, dx = \int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{a + bx^2 + cx^4}}\, dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**6/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**6*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{a + bx^2 + cx^4}}\, dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^6}\, dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{x^6\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^6*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^6*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a+bx^2+cx^4}}\,dx = \frac{-\sqrt{c\,x^4+b\,x^2+a}\,a-2\sqrt{c\,x^4+b\,x^2+a}\,c\,x^4+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^8+b\,x^6+a\,x^4}dx\right)ab\,x^5+2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)c^2x^5}{5a\,x^5}$$

input
```
int((C*x^4+B*x^2+A)/x^6/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4)*a - 2*sqrt(a + b*x**2 + c*x**4)*c*x**4 + int
(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*a*b*x**5 + 2*int(
(sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*c**2*x**5)/(5*a*
x**5)
```

3.152 $\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 538

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{x\left(a(bBc-b^2C-2c(Ac-aC))+(b^2Bc-2aBc^2-b^3C-bc(Ac-3aC))\,x^2\right)}{c^2\,(b^2-4ac)\,\sqrt{a+bx^2+cx^4}}$$

$$+\frac{Cx\sqrt{a+bx^2+cx^4}}{3c^2}+\frac{\left(6b^2B-3Abc-18aBc+29abC-\frac{8b^3C}{c}\right)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\,(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{\sqrt[4]{a}\left(6b^2B-3Abc-18aBc+29abC-\frac{8b^3C}{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\cdots}\right)\right)}{3c^{7/4}\,(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt[4]{a}\left(6bBc-9\sqrt{a}Bc^{3/2}-3Ac^2-8b^2C+12\sqrt{a}b\sqrt{c}C+5acC\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\,\text{ar}\cdots\right)}{6\left(b-2\sqrt{a}\sqrt{c}\right)c^{11/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-x*(a*(B*b*c-b^2*C-2*c*(A*c-C*a))+(b^2*B*c-2*a*B*c^2-b^3*C-b*c*(A*c-3*C*a)
)*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/3*C*x*(c*x^4+b*x^2+a)^(1/2
)/c^2+1/3*(6*b^2*B-3*A*b*c-18*a*B*c+29*a*b*C-8*b^3*C/c)*x*(c*x^4+b*x^2+a)^
(1/2)/c^(3/2)/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)-1/3*a^(1/4)*(6*b^2*B-3*A*
b*c-18*a*B*c+29*a*b*C-8*b^3*C/c)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a
^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+
1/6*a^(1/4)*(6*B*b*c-9*a^(1/2)*B*c^(3/2)-3*A*c^2-8*b^2*C+12*a^(1/2)*b*c^(1
/2)*C+5*a*c*C)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2
)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^
(1/2))^(1/2))/(b-2*a^(1/2)*c^(1/2))/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.01 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.24

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(-10a^2cC+bx^2(-3bBc+3Ac^2+4b^2C+bcCx^2)+a(4b^2C}{}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-10*a^2*c*C + b*x^2*(-3*b*B*c + 3
*A*c^2 + 4*b^2*C + b*c*C*x^2) + a*(4*b^2*C - b*c*(3*B + 13*C*x^2) + 2*c^2*
(3*A + 3*B*x^2 - 2*C*x^4))) + I*(-b + Sqrt[b^2 - 4*a*c])*(-6*b^2*B*c + 18*
a*B*c^2 + 8*b^3*C + b*c*(3*A*c - 29*a*C))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b
^4*C + 2*a*c^2*(6*A*c + 9*B*Sqrt[b^2 - 4*a*c] - 10*a*C) + b^2*c*(-3*A*c -
6*B*Sqrt[b^2 - 4*a*c] + 37*a*C) + b^3*(6*B*c + 8*Sqrt[b^2 - 4*a*c]*C) + b*
c*(-24*a*B*c + 3*A*c*Sqrt[b^2 - 4*a*c] - 29*a*Sqrt[b^2 - 4*a*c]*C))*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sq
rt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sq
rt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c])])/(12*c^3*(-b^2 + 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*S
qrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 516, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2197, 25, 2207, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A + Bx^2 + Cx^4)}{(a + bx^2 + cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{\int -\frac{-a\left(4a-\frac{b^2}{c}\right)Cx^4-a\left(Ab-\frac{-2Cb^3+2Bcb^2+7acCb-6aBc^2}{c^2}\right)x^2+\frac{a^2\left(-Cb^2+Bcb-2c(Ac-aC)\right)}{c^2}}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-a\left(4a-\frac{b^2}{c}\right)Cx^4-a\left(Ab-\frac{-2Cb^3+2Bcb^2+7acCb-6aBc^2}{c^2}\right)x^2+\frac{a^2\left(-Cb^2+Bcb-2c(Ac-aC)\right)}{c^2}}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}}{}$$

$\big\downarrow$ 2207

$$\frac{\frac{\int \frac{a\left(c\left(-\frac{8Cb^3}{c}+6Bb^2-3Acb+29aCb-18aBc\right)x^2+a\left(-4Cb^2+3Bcb-6Ac^2+10acC\right)\right)}{c\sqrt{cx^4+bx^2+a}}dx}{3c}+\frac{aCx\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}{3c^2}}{a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 27

$$\frac{\frac{a\int \frac{c\left(-\frac{8Cb^3}{c}+6Bb^2-3Acb+29aCb-18aBc\right)x^2+a\left(-4Cb^2+3Bcb-6Ac^2+10acC\right)}{\sqrt{cx^4+bx^2+a}}dx}{3c^2}+\frac{aCx\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}{3c^2}}{a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 1511

$$\frac{a\left(\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(12\sqrt{a}b\sqrt{c}C-9\sqrt{a}Bc^{3/2}+5acC-3Ac^2-8b^2C+6bBc\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\sqrt{a}\sqrt{c}\left(29abC-18aBc-3Abc-\frac{8b^3C}{c}+6b^2B\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx}\cdots}}{3c^2}\cdots\right.}{a\left(b^2-4ac\right)}\cdots$$

$$-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 27

$$\frac{a\left(\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(12\sqrt{a}b\sqrt{c}C-9\sqrt{a}Bc^{3/2}+5acC-3Ac^2-8b^2C+6bBc\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\sqrt{c}\left(29abC-18aBc-3Abc-\frac{8b^3C}{c}+6b^2B\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx\cdots}{3c^2}\cdots\right.}{a\left(b^2-4ac\right)}\cdots$$

$$-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 1416

$$\frac{a\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(12\sqrt{a}b\sqrt{c}C-9\sqrt{a}Bc^{3/2}+5acC-3Ac^2-8b^2C+6bBc\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\sqrt{c}\left(29\right.\right.}{3c^2}$$

$$a\left(b^2-4ac\right)$$

$$\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{a\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(12\sqrt{a}b\sqrt{c}C-9\sqrt{a}Bc^{3/2}+5acC-3Ac^2-8b^2C+6bBc\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\sqrt{c}\left(29\right.\right.}{3c^2}$$

$$a$$

$$\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

input

```
Int[(x^4*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output

```
-((x*(a*(b*B*c - b^2*C - 2*c*(A*c - a*C)) + (b^2*B*c - 2*a*B*c^2 - b^3*C -
 b*c*(A*c - 3*a*C))*x^2))/(c^2*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) + (
(a*(b^2 - 4*a*c)*C*x*Sqrt[a + b*x^2 + c*x^4])/(3*c^2) + (a*(-(Sqrt[c]*(6*b
^2*B - 3*A*b*c - 18*a*B*c + 29*a*b*C - (8*b^3*C)/c)*(-((x*Sqrt[a + b*x^2 +
 c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^
4]))) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(6*b*B*c - 9*Sqrt[a]*B*c^(3/2) -
3*A*c^2 - 8*b^2*C + 12*Sqrt[a]*b*Sqrt[c]*C + 5*a*c*C)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b
*x^2 + c*x^4])))/(3*c^2))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 12.03 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.14

method	result
elliptic	$-\frac{2c\left(\frac{\left(bA\,c^{2}+2Ba\,c^{2}-b^{2}Bc-3Cacb+b^{3}C\right)x^{3}}{2c^{3}\left(4ac-b^{2}\right)}+\frac{a\left(2A\,c^{2}-Bbc-2Cac+C\,b^{2}\right)x}{2c^{3}\left(4ac-b^{2}\right)}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{Cx\sqrt{c\,x^{4}+b\,x^{2}+a}}{3c^{2}}+\frac{\left(\frac{a\left(2A\,c^{2}-Bbc-2Cac+C\,b^{2}\right)}{c^{2}\left(4ac-b^{2}\right)}-\cdots\right)}{\cdots}$
default	Expression too large to display
risch	Expression too large to display

input
```
int(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2/c^3*(A*b*c^2+2*B*a*c^2-B*b^2*c-3*C*a*b*c+C*b^3)/(4*a*c-b^2)*x^3+
1/2*a/c^3*(2*A*c^2-B*b*c-2*C*a*c+C*b^2)/(4*a*c-b^2)*x)/((x^4+1/c*b*x^2+1/c
*a)*c)^(1/2)+1/3*C*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/4*(1/c^2*a*(2*A*c^2-B*b*c
-2*C*a*c+C*b^2)/(4*a*c-b^2)-1/3*C/c^2*a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2
*(1/c^2*(B*c-C*b)+1/c^2*(A*b*c^2+2*B*a*c^2-B*b^2*c-3*C*a*b*c+C*b^3)/(4*a*c
-b^2)-2/3*C/c^2*b)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c
*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(459) = 918$.

Time = 0.10 (sec) , antiderivative size = 1175, normalized size of antiderivative = 2.18

$$\int \frac{x^4(A + Bx^2 + Cx^4)}{(a + bx^2 + cx^4)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(1/2)*((8*C*b^4*c + 3*(6*B*a*b + A*b^2)*c^3 - (29*C*a*b^2 + 6*B*b
^3)*c^2)*x^5 + (8*C*b^5 + 3*(6*B*a*b^2 + A*b^3)*c^2 - (29*C*a*b^3 + 6*B*b^
4)*c)*x^3 + (8*C*a*b^4 + 3*(6*B*a^2*b + A*a*b^2)*c^2 - (29*C*a^2*b^2 + 6*B
*a*b^3)*c)*x - ((8*C*b^3*c^2 + 3*(6*B*a + A*b)*c^4 - (29*C*a*b + 6*B*b^2)*
c^3)*x^5 + (8*C*b^4*c + 3*(6*B*a*b + A*b^2)*c^3 - (29*C*a*b^2 + 6*B*b^3)*c
^2)*x^3 + (8*C*a*b^3*c + 3*(6*B*a^2 + A*a*b)*c^3 - (29*C*a^2*b + 6*B*a*b^2
)*c^2)*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2)
 - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/
c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*
((8*C*b^4*c + 6*A*b*c^4 + (2*(9*B - 5*C)*a*b + 3*(A - B)*b^2)*c^3 - (29*C*
a*b^2 + 2*(3*B - 2*C)*b^3)*c^2)*x^5 + (8*C*b^5 + 6*A*b^2*c^3 + (2*(9*B - 5
*C)*a*b^2 + 3*(A - B)*b^3)*c^2 - (29*C*a*b^3 + 2*(3*B - 2*C)*b^4)*c)*x^3 +
 (8*C*a*b^4 + 6*A*a*b*c^3 + (2*(9*B - 5*C)*a^2*b + 3*(A - B)*a*b^2)*c^2 -
(29*C*a^2*b^2 + 2*(3*B - 2*C)*a*b^3)*c)*x - ((8*C*b^3*c^2 - 6*A*c^5 + (2*(
9*B + 5*C)*a + 3*(A + B)*b)*c^4 - (29*C*a*b + 2*(3*B + 2*C)*b^2)*c^3)*x^5
+ (8*C*b^4*c - 6*A*b*c^4 + (2*(9*B + 5*C)*a*b + 3*(A + B)*b^2)*c^3 - (29*C
*a*b^2 + 2*(3*B + 2*C)*b^3)*c^2)*x^3 + (8*C*a*b^3*c - 6*A*a*c^4 + (2*(9*B
+ 5*C)*a^2 + 3*(A + B)*a*b)*c^3 - (29*C*a^2*b + 2*(3*B + 2*C)*a*b^2)*c^2)*
x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c
)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x...
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**4*(A + B*x**2 + C*x**4)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{x^4\,(C\,x^4+B\,x^2+A)}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx=\frac{\sqrt{c\,x^4+b\,x^2+a}\,x-\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)a-2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)b}{3c}$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4)*x - int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 +
 c*x**4),x)*a - 2*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**
4),x)*b)/(3*c)
```

3.153 $\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 452

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{x(Abc-2aBc+abC-(bBc-b^2C-2c(Ac-aC))\,x^2)}{c\,(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(bBc-2Ac^2-2b^2C+6acC)\,x\sqrt{a+bx^2+cx^4}}{c^{3/2}\,(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{\sqrt[4]{a}(bBc-2Ac^2-2b^2C+6acC)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{7/4}\,(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(Ac^{3/2}-3a\sqrt{c}C-\sqrt{a}(Bc-2bC)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right.\right.}{2\sqrt[4]{a}\left(b-2\sqrt{a}\sqrt{c}\right)c^{7/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-x*(A*b*c-2*a*B*c+a*b*C-(B*b*c-b^2*C-2*c*(A*c-C*a))*x^2)/c/(-4*a*c+b^2)/(c
*x^4+b*x^2+a)^(1/2)-(-2*A*c^2+B*b*c+6*C*a*c-2*C*b^2)*x*(c*x^4+b*x^2+a)^(1/
2)/c^(3/2)/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)+a^(1/4)*(-2*A*c^2+B*b*c+6*C*
a*c-2*C*b^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^
2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/
2))^(1/2))/c^(7/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(A*c^(3/2)-3*a*c
^(1/2)*C-a^(1/2)*(B*c-2*C*b))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1
/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*
(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/(b-2*a^(1/2)*c^(1/2))/c^(7/4)/(c*x^4+
b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.95 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.28

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(b(-Bc+bC)x^2+Ac(b+2cx^2)+a(-2Bc+bC-2cCx^2))}{}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b*(-(B*c) + b*C)*x^2 + A*c*(b + 2*
c*x^2) + a*(-2*B*c + b*C - 2*c*C*x^2)) - I*(-b + Sqrt[b^2 - 4*a*c])*(-(b*B
*c) + 2*b^2*C + 2*c*(A*c - 3*a*C))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - S
qrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*
c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-2*b^3*C +
b*(-(B*c*Sqrt[b^2 - 4*a*c]) + 8*a*c*C) + b^2*(B*c + 2*Sqrt[b^2 - 4*a*c]*C)
 + 2*c*(-2*a*B*c + A*c*Sqrt[b^2 - 4*a*c] - 3*a*Sqrt[b^2 - 4*a*c]*C))*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*S
qrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])])/(4*c^2*(-b^2 + 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*S
qrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2197, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{\int -\frac{a\left(-c\left(-\frac{2Cb^2}{c}+Bb-2Ac+6aC\right)x^2+Abc-2aBc+abC\right)}{c\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}-\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a\left(-\left(\left(-2Cb^2+Bcb-2Ac^2+6acC\right)x^2\right)+Abc-2aBc+abC\right)}{c\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}-\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-\left(\left(-2Cb^2+Bcb-2c(Ac-3aC)\right)x^2\right)+Abc-2aBc+abC}{\sqrt{cx^4+bx^2+a}}dx}{c\left(b^2-4ac\right)}-\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{\frac{\sqrt{a}\left(-2c(Ac-3aC)-2b^2C+bBc\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(-\sqrt{a}(Bc-2bC)-3a\sqrt{c}C+Ac^{3/2}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{c\left(b^2-4ac\right)}-\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(-2c(Ac-3aC)-2b^2C+bBc)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{(2\sqrt{a}\sqrt{c}+b)\left(-\sqrt{a}(Bc-2bC)-3a\sqrt{c}C+Ac^{3/2}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{c\left(b^2-4ac\right)}-\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \; 1416$$

$$\frac{\frac{(-2c(Ac-3aC)-2b^2C+bBc)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-\sqrt{a}(Bc-2bC)-3a\sqrt{c}C+Ac^{3/2}\right)\text{EllipticF}\left(2\text{arc}\right.}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}}{c\left(b^2-4ac\right)}$$

$$\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \; 1509$$

$$\frac{\frac{(-2c(Ac-3aC)-2b^2C+bBc)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}+\frac{(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}\right.}{}}{c\left(b^2-4ac\right)}$$

$$\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-((x*(A*b*c - 2*a*B*c + a*b*C - (b*B*c - b^2*C - 2*c*(A*c - a*C))*x^2))/(c
*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) + (((b*B*c - 2*b^2*C - 2*c*(A*c -
 3*a*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])
/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c] + ((b + 2*Sqrt[a]*Sqrt[c])*(A
*c^(3/2) - 3*a*Sqrt[c]*C - Sqrt[a]*(B*c - 2*b*C))*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(3/4)*Sqrt[a
 + b*x^2 + c*x^4]))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.18

method	result
elliptic	$-\frac{2c\left(-\frac{\left(2A\,c^2-Bbc-2Cac+C\,b^2\right)x^3}{2c^2\left(4ac-b^2\right)}-\frac{(Abc-2Bac+Cba)x}{2c^2\left(4ac-b^2\right)}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}-\frac{(Abc-2Bac+Cba)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b\cdots}\right.}{a}}}{4c\left(4ac-b^2\right)\sqrt{\frac{-b+\sqrt{-4\cdots}}{a}}}$
default	Expression too large to display

input
```
int(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/2/c^2*(2*A*c^2-B*b*c-2*C*a*c+C*b^2)/(4*a*c-b^2)*x^3-1/2*(A*b*c-2*
B*a*c+C*a*b)/c^2/(4*a*c-b^2)*x)/((x^4+1/c*b*x^2+1/c*a)*c)^(1/2)-1/4*(A*b*c
-2*B*a*c+C*a*b)/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4
-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(C/c-(2*A*
c^2-B*b*c-2*C*a*c+C*b^2)/c/(4*a*c-b^2))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF
(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^
2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(386) = 772.

Time = 0.10 (sec) , antiderivative size = 992, normalized size of antiderivative = 2.19

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output

```
-1/2*(sqrt(1/2)*((2*C*a*b^3*c + 2*A*a*b*c^3 - (6*C*a^2*b + B*a*b^2)*c^2)*x
^5 + (2*C*a*b^4 + 2*A*a*b^2*c^2 - (6*C*a^2*b^2 + B*a*b^3)*c)*x^3 + (2*C*a^
2*b^3 + 2*A*a^2*b*c^2 - (6*C*a^3*b + B*a^2*b^2)*c)*x - ((2*C*a*b^2*c^2 + 2
*A*a*c^4 - (6*C*a^2 + B*a*b)*c^3)*x^5 + (2*C*a*b^3*c + 2*A*a*b*c^3 - (6*C*
a^2*b + B*a*b^2)*c^2)*x^3 + (2*C*a^2*b^2*c + 2*A*a^2*c^3 - (6*C*a^3 + B*a^
2*b)*c^2)*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c
^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) -
b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/
2)*((2*C*a*b^3*c + (2*(A - B)*a*b + A*b^2)*c^3 - (6*C*a^2*b + (B - C)*a*b^
2)*c^2)*x^5 + (2*C*a*b^4 + (2*(A - B)*a*b^2 + A*b^3)*c^2 - (6*C*a^2*b^2 +
(B - C)*a*b^3)*c)*x^3 + (2*C*a^2*b^3 + (2*(A - B)*a^2*b + A*a*b^2)*c^2 - (
6*C*a^3*b + (B - C)*a^2*b^2)*c)*x - ((2*C*a*b^2*c^2 + (2*(A + B)*a - A*b)*
c^4 - (6*C*a^2 + (B + C)*a*b)*c^3)*x^5 + (2*C*a*b^3*c + (2*(A + B)*a*b - A
*b^2)*c^3 - (6*C*a^2*b + (B + C)*a*b^2)*c^2)*x^3 + (2*C*a^2*b^2*c + (2*(A
+ B)*a^2 - A*a*b)*c^3 - (6*C*a^3 + (B + C)*a^2*b)*c^2)*x)*sqrt((b^2 - 4*a*
c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin
(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2
- 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*(2*C*a^2*b^2*c + 2*A*a^2*c^3 + (C*
a*b^2*c^2 - 4*C*a^2*c^3)*x^4 - (6*C*a^3 + B*a^2*b)*c^2 + (2*C*a*b^3*c + (2
*B*a^2 + A*a*b)*c^3 - (7*C*a^2*b + B*a*b^2)*c^2)*x^2)*sqrt(c*x^4 + b*x^...
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x**2*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**(3/2),x)
```

output

```
Integral(x**2*(A + B*x**2 + C*x**4)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2\,(C\,x^4+B\,x^2+A)}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)
```

3.154 $\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 426

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x(Ab^2-abB-2aAc+2a^2C+(Abc-2aBc+abC)x^2)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(Abc-2aBc+abC)x\sqrt{a+bx^2+cx^4}}{a\sqrt{c}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{(Abc-2aBc+abC)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(\sqrt{a}B\sqrt{c}-Ac-aC\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
x*(A*b^2-B*a*b-2*A*a*c+2*C*a^2+(A*b*c-2*B*a*c+C*a*b)*x^2)/a/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)^(1/2)-(A*b*c-2*B*a*c+C*a*b)*x*(c*x^4+b*x^2+a)^(1/2)/a/c^(1/
2)/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)+(A*b*c-2*B*a*c+C*a*b)*(a^(1/2)+c^(1/
2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*ar
ctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/c^(3/4)/
(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(a^(1/2)*B*c^(1/2)-A*c-a*C)*(a^(1/2
)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/
(b-2*a^(1/2)*c^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.47 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(A(b^2-2ac+bcx^2)+a(-bB+2aC-2Bcx^2+bCx^2))+i(\cdot}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(A*(b^2 - 2*a*c + b*c*x^2) + a*(-(
b*B) + 2*a*C - 2*B*c*x^2 + b*C*x^2)) + I*(-b + Sqrt[b^2 - 4*a*c])*(A*b*c -
 2*a*B*c + a*b*C)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqr
t[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-2*a*B*c*Sqrt[b^2 - 4*a*c] +
 A*c*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c]) + a*(-b^2 + 4*a*c + b*Sqrt[b^2 -
 4*a*c])*C)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elli
pticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(4*a*c*(-b^2 + 4*a*c)*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2206, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2206$$

$$\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\int -\frac{a(bB-2Ac-2aC)-(Abc-2aBc+abC)x^2}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{a(bB-2(Ac+aC))-(Abc-2aBc+abC)x^2}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}+\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}B\sqrt{c}-aC-Ac\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\sqrt{a}(abC-2aBc+Abc)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{a\left(b^2-4ac\right)}+\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}B\sqrt{c}-aC-Ac\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{(abC-2aBc+Abc)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{a\left(b^2-4ac\right)}+\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\frac{(abC-2aBc+Abc)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B\sqrt{c}-aC-Ac\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\cdots\right.\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}}{a\left(b^2-4ac\right)}+\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+$$

$$\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B\sqrt{c}-aC-Ac\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{(abC-2aBc+Abc)\left(\frac{\sqrt[4]{a}(\sqrt{a}\ldots}{\ldots}\right.}{\ldots}}{a\left(b^2-4ac\right)}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(A*b^2 - a*b*B - 2*a*A*c + 2*a^2*C + (A*b*c - 2*a*B*c + a*b*C)*x^2))/(a
*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (((A*b*c - 2*a*B*c + a*b*C)*(-((
x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqr
t[a + b*x^2 + c*x^4])))/Sqrt[c] + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[a
]*B*Sqrt[c] - A*c - a*C)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/(a*(b^2 - 4*a
*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.29

method	result
elliptic	$-\frac{2c\left(\frac{(Abc-2Bac+Cba)x^{3}}{2ca\left(4ac-b^{2}\right)}-\frac{\left(2Aac-A\,b^{2}+Bab-2a^{2}C\right)x}{2ca\left(4ac-b^{2}\right)}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{C}{c}+\frac{Ac-aC}{ca}-\frac{2Aac-A\,b^{2}+Bab-2a^{2}C}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}}{4\sqrt{\frac{-b+\sqrt{}}{}}}$
default	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2/c*(A*b*c-2*B*a*c+C*a*b)/a/(4*a*c-b^2)*x^3-1/2/c*(2*A*a*c-A*b^2+B
*a*b-2*C*a^2)/a/(4*a*c-b^2)*x)/((x^4+1/c*b*x^2+1/c*a)*c)^(1/2)+1/4*(C/c+1/
c*(A*c-C*a)/a-(2*A*a*c-A*b^2+B*a*b-2*C*a^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*
(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))-1/2*(A*b*c-2*B*a*c+C*a*b)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(358) = 716$.

Time = 0.09 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.86

$$\int \frac{A + Bx^2 + Cx^4}{(a + bx^2 + cx^4)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(C*a^2*b^2 + (C*a*b^2*c - (2*B*a*b - A*b^2)*c^2)*x^4 + (C*
a*b^3 - (2*B*a*b^2 - A*b^3)*c)*x^2 - (2*B*a^2*b - A*a*b^2)*c - (C*a^3*b +
(C*a^2*b*c - (2*B*a^2 - A*a*b)*c^2)*x^4 + (C*a^2*b^2 - (2*B*a^2*b - A*a*b^
2)*c)*x^2 - (2*B*a^3 - A*a^2*b)*c)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((
a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sq
rt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2
*a*c)/(a*c)) - sqrt(1/2)*(2*C*a^3*b - (B - C)*a^2*b^2 + ((2*(A - B)*a*b +
A*b^2)*c^2 + (2*C*a^2*b - (B - C)*a*b^2)*c)*x^4 + (2*C*a^2*b^2 - (B - C)*a
*b^3 + (2*(A - B)*a*b^2 + A*b^3)*c)*x^2 + (2*(A - B)*a^2*b + A*a*b^2)*c +
(2*C*a^4 - (B + C)*a^3*b + ((2*(A + B)*a^2 - A*a*b)*c^2 + (2*C*a^3 - (B +
C)*a^2*b)*c)*x^4 + (2*C*a^3*b - (B + C)*a^2*b^2 + (2*(A + B)*a^2*b - A*a*b
^2)*c)*x^2 + (2*(A + B)*a^3 - A*a^2*b)*c)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)
*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqr
t((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) +
b^2 - 2*a*c)/(a*c)) - 2*sqrt(c*x^4 + b*x^2 + a)*((C*a^2*b*c - (2*B*a^2 - A
*a*b)*c^2)*x^3 - (2*A*a^2*c^2 - (2*C*a^3 - B*a^2*b + A*a*b^2)*c)*x))/(a^3*
b^2*c - 4*a^4*c^2 + (a^2*b^2*c^2 - 4*a^3*c^3)*x^4 + (a^2*b^3*c - 4*a^3*b*c
^2)*x^2)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx$$

input
```
int((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)
```

3.155 $\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 504

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x\left(a\left(\frac{Ab^3}{a}-b(bB+3Ac)+a(2Bc+bC)\right)+c\left(Ab^2-abB-2aAc+2a^2C\right)x^2\right)}{a^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{A\sqrt{a+bx^2+cx^4}}{a^2x} + \frac{\sqrt{c}(2A(b^2-3ac)-a(bB-2aC))\,x\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{c}(2A(b^2-3ac)-a(bB-2aC))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(2Ab\sqrt{c}-aB\sqrt{c}-3\sqrt{a}Ac+a^{3/2}C\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
-x*(a*(A*b^3/a-b*(3*A*c+B*b)+a*(2*B*c+C*b))+c*(-2*A*a*c+A*b^2-B*a*b+2*C*a^
2)*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-A*(c*x^4+b*x^2+a)^(1/2)/a^2
/x+c^(1/2)*(2*A*(-3*a*c+b^2)-a*(B*b-2*C*a))*x*(c*x^4+b*x^2+a)^(1/2)/a^2/(-
4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)-c^(1/4)*(2*A*(-3*a*c+b^2)-a*(B*b-2*C*a))*
(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Elli
pticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^
(7/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(2*A*b*c^(1/2)-a*B*c^(1/2)-3*
a^(1/2)*A*c+a^(3/2)*C)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(
1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^
(1/2)/c^(1/2))^(1/2))/a^(7/4)/(b-2*a^(1/2)*c^(1/2))/c^(1/4)/(c*x^4+b*x^2+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.88 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(2Ab^2x^2(b+cx^2)+a^2(-4Ac+x^2(2Bc+bC+2cCx^2))+a(-bBx^2(b+cx^2)+A(b^2-7bc}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/4*(4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(2*A*b^2*x^2*(b + c*x^2) + a^2*(-4
*A*c + x^2*(2*B*c + b*C + 2*c*C*x^2)) + a*(-(b*B*x^2*(b + c*x^2)) + A*(b^2
 - 7*b*c*x^2 - 6*c^2*x^4))) - I*(-b + Sqrt[b^2 - 4*a*c])*(2*A*(b^2 - 3*a*c
) + a*(-(b*B) + 2*a*C))*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
(b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(2*A*(-b^3 + 4*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c]) + a*(b^2*B - 4*a*B*c -
b*B*Sqrt[b^2 - 4*a*c] + 2*a*Sqrt[b^2 - 4*a*c]*C))*x*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c]
+ 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
)/(a^2*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*
x^4])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2198, 25, 2199, 1604, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 2198$$

$$-\frac{\int -\frac{\frac{c\left(A\left(b^2-2ac\right)-a(bB-2aC)\right)x^4}{a}+(Abc-2aBc+abC)x^2+A\left(b^2-4ac\right)}{x^2\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}-\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{a^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{c\left(A\left(b^2-2ac\right)-a(bB-2aC)\right)x^4}{a}+(Abc-2aBc+abC)x^2+A\left(b^2-4ac\right)}{x^2\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}-\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{a^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 2199

$$\frac{\int \frac{(Abc-2aBc+abC)x^2+2A\left(b^2-3ac\right)-a(bB-2aC)}{x^2\sqrt{cx^4+bx^2+a}}dx+\frac{\sqrt{a+bx^2+cx^4}\left(A\left(b^2-2ac\right)-a(bB-2aC)\right)}{ax}}{a\left(b^2-4ac\right)}-\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{a^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 1604

$$\frac{-\frac{\int -\frac{c\left(2A\left(b^2-3ac\right)-a(bB-2aC)\right)x^2+a(Abc-2aBc+abC)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(2A\left(b^2-3ac\right)-a(bB-2aC)\right)}{ax}+\frac{\sqrt{a+bx^2+cx^4}\left(A\left(b^2-2ac\right)-a(bB-\right.}{ax}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{a^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 25

$$\frac{\frac{\int \frac{c\left(2Ca^2-bBa-6Aca+2Ab^2\right)x^2+a(Abc-2aBc+abC)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(2A\left(b^2-3ac\right)-a(bB-2aC)\right)}{ax}+\frac{\sqrt{a+bx^2+cx^4}\left(A\left(b^2-2ac\right)-a(bB-2aC)\right.}{ax}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{a^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 1511

$$\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(a^{3/2}C-3\sqrt{a}Ac-aB\sqrt{c}+2Ab\sqrt{c}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{a}\sqrt{c}\left(2A\left(b^2-3ac\right)-a(bB-2aC)\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{a}-\sqrt{a+bx^2+cx^4}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{a^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 27

$$\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(a^{3/2}C-3\sqrt{a}Ac-aB\sqrt{c}+2Ab\sqrt{c}\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}\left(2A\left(b^2-3ac\right)-a(bB-2aC)\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(2A(b\right.}{}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{a^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\frac{\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(a^{3/2}C-3\sqrt{a}Ac-aB\sqrt{c}+2Ab\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}\left(2A\left(b^2-3ac\right)-a(bB-2a\right.}{a}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{a^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{\frac{\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(a^{3/2}C-3\sqrt{a}Ac-aB\sqrt{c}+2Ab\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}\left(2A\left(b^2-3ac\right)-a(bB-2a\right.}{a}}{}$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{a^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-((x*(a*((A*b^3)/a - b*(b*B + 3*A*c) + a*(2*B*c + b*C)) + c*(A*b^2 - a*b*B
 - 2*a*A*c + 2*a^2*C)*x^2))/(a^2*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) +
 (-(((2*A*(b^2 - 3*a*c) - a*(b*B - 2*a*C))*Sqrt[a + b*x^2 + c*x^4])/(a*x))
 + ((A*(b^2 - 2*a*c) - a*(b*B - 2*a*C))*Sqrt[a + b*x^2 + c*x^4])/(a*x) + (
-(Sqrt[c]*(2*A*(b^2 - 3*a*c) - a*(b*B - 2*a*C))*(-((x*Sqrt[a + b*x^2 + c*x
^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))
) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(2*A*b*Sqrt[c] - a*B*Sqrt[c] - 3*Sqrt
[a]*A*c + a^(3/2)*C)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/a)/(a*(b^2 - 4*a*
c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 5.88 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.22

method	result
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{a^2x}-\frac{2c\left(\frac{\left(2Aac-A\,b^2+Bab-2a^2C\right)x^3}{2a^2\left(4ac-b^2\right)}+\frac{\left(3Aabc-A\,b^3-2B\,a^2c+Ba\,b^2-C\,a^2b\right)x}{2\left(4ac-b^2\right)c\,a^2}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(-\frac{Ab-Ba}{a^2}+\frac{3Aabc-A\,b^3-2B\,a}{(4ac-b}\right.}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(c*x^4+b*x^2+a)^(1/2)/a^2/x-2*c*(1/2*(2*A*a*c-A*b^2+B*a*b-2*C*a^2)/a^2/
(4*a*c-b^2)*x^3+1/2*(3*A*a*b*c-A*b^3-2*B*a^2*c+B*a*b^2-C*a^2*b)/(4*a*c-b^2
)/c/a^2*x)/((x^4+1/c*b*x^2+1/c*a)*c)^(1/2)+1/4*(-(A*b-B*a)/a^2+(3*A*a*b*c-
A*b^3-2*B*a^2*c+B*a*b^2-C*a^2*b)/(4*a*c-b^2)/a^2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))-1/2*(1/a^2*A*c+(2*A*a*c-A*b^2+B*a*b-2*C*a^2)/a^2/(4*a*c-b^2)*c)*a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+
(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. 2(431) = 862.

Time = 0.10 (sec) , antiderivative size = 989, normalized size of antiderivative = 1.96

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*((6*A*a*b*c^3 - (2*C*a^2*b - B*a*b^2 + 2*A*b^3)*c^2)*x^5 +
 (6*A*a*b^2*c^2 - (2*C*a^2*b^2 - B*a*b^3 + 2*A*b^4)*c)*x^3 + (6*A*a^2*b*c^
2 - (2*C*a^3*b - B*a^2*b^2 + 2*A*a*b^3)*c)*x - ((6*A*a^2*c^3 - (2*C*a^3 -
B*a^2*b + 2*A*a*b^2)*c^2)*x^5 + (6*A*a^2*b*c^2 - (2*C*a^3*b - B*a^2*b^2 +
2*A*a*b^3)*c)*x^3 + (6*A*a^3*c^2 - (2*C*a^4 - B*a^3*b + 2*A*a^2*b^2)*c)*x)
*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*
elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/
2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((C*a^2*b
^2*c - 6*A*a*b*c^3 - (2*(B - C)*a^2*b - (A - B)*a*b^2 - 2*A*b^3)*c^2)*x^5
+ (C*a^2*b^3 - 6*A*a*b^2*c^2 - (2*(B - C)*a^2*b^2 - (A - B)*a*b^3 - 2*A*b^
4)*c)*x^3 + (C*a^3*b^2 - 6*A*a^2*b*c^2 - (2*(B - C)*a^3*b - (A - B)*a^2*b^
2 - 2*A*a*b^3)*c)*x + ((C*a^3*b*c + 6*A*a^2*c^3 - (2*(B + C)*a^3 - (A + B)
*a^2*b + 2*A*a*b^2)*c^2)*x^5 + (C*a^3*b^2 + 6*A*a^2*b*c^2 - (2*(B + C)*a^3
*b - (A + B)*a^2*b^2 + 2*A*a*b^3)*c)*x^3 + (C*a^4*b + 6*A*a^3*c^2 - (2*(B
+ C)*a^4 - (A + B)*a^3*b + 2*A*a^2*b^2)*c)*x)*sqrt((b^2 - 4*a*c)/a^2))*sqr
t(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x
*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2
) + b^2 - 2*a*c)/(a*c)) + 2*(A*a^2*b^2*c - 4*A*a^3*c^2 - (6*A*a^2*c^3 - (2
*C*a^3 - B*a^2*b + 2*A*a*b^2)*c^2)*x^4 + ((2*B*a^3 - 7*A*a^2*b)*c^2 + (C*a
^3*b - B*a^2*b^2 + 2*A*a*b^3)*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^...
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^2 \left(a + bx^2 + cx^4\right)^{3/2}}\, dx = \int \frac{A + Bx^2 + Cx^4}{x^2 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**2*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^2\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^6+b\,x^4+a\,x^2}dx$$

input
```
int((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a*x**2 + b*x**4 + c*x**6),x)
```

3.156 $\int \frac{A+Bx^2+Cx^4}{x^4(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 610

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x\left(a^2\left(\frac{Ab^4}{a^2}+3bBc+2Ac^2-\frac{b^2(bB+4Ac)}{a}+b^2C-2acC\right)+c(A(b^3-3abc)-c\right.}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{A\sqrt{a+bx^2+cx^4}}{3a^2x^3}+\frac{(5Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^3x}$$
$$-\frac{\sqrt{c}(A(8b^3-29abc)-3a(2b^2B-6aBc-abC))x\sqrt{a+bx^2+cx^4}}{3a^3\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{\sqrt[4]{c}(A(8b^3-29abc)-3a(2b^2B-6aBc-abC))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{3a^{11/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{c}\left(A\left(8b^2-12\sqrt{a}b\sqrt{c}-5ac\right)-3a\left(2bB-3\sqrt{a}B\sqrt{c}-aC\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\operatorname{ar}\right.}{6a^{11/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}$$

output
```
x*(a^2*(A*b^4/a^2+3*B*b*c+2*A*c^2-b^2*(4*A*c+B*b)/a+b^2*C-2*a*c*C)+c*(A*(-
3*a*b*c+b^3)-a*(-2*B*a*c+B*b^2-C*a*b))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)^(1/2)-1/3*A*(c*x^4+b*x^2+a)^(1/2)/a^2/x^3+1/3*(5*A*b-3*B*a)*(c*x^4+b*x^
2+a)^(1/2)/a^3/x-1/3*c^(1/2)*(A*(-29*a*b*c+8*b^3)-3*a*(-6*B*a*c+2*B*b^2-C*
a*b))*x*(c*x^4+b*x^2+a)^(1/2)/a^3/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)+1/3*c
^(1/4)*(A*(-29*a*b*c+8*b^3)-3*a*(-6*B*a*c+2*B*b^2-C*a*b))*(a^(1/2)+c^(1/2)
*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arct
an(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(11/4)/(-4*a*c+b
^2)/(c*x^4+b*x^2+a)^(1/2)-1/6*c^(1/4)*(A*(8*b^2-12*a^(1/2)*b*c^(1/2)-5*a*c
)-3*a*(2*B*b-3*a^(1/2)*B*c^(1/2)-a*C))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2
+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1
/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(11/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^
4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.04 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^2+cx^4\right)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(-8Ab^3x^4(b+cx^2)-2a^3c(2A+6Bx^2-3Cx^4)+abx^2(6bBx^2(b+cx^2)+A(-4b^2+33bcx\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/12*(4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-8*A*b^3*x^4*(b + c*x^2) - 2*a^3
*c*(2*A + 6*B*x^2 - 3*C*x^4) + a*b*x^2*(6*b*B*x^2*(b + c*x^2) + A*(-4*b^2
+ 33*b*c*x^2 + 29*c^2*x^4)) + a^2*(A*(b^2 + 16*b*c*x^2 - 10*c^2*x^4) - 3*(
6*B*c^2*x^6 + b*c*x^4*(7*B + C*x^2) + b^2*(-(B*x^2) + C*x^4)))) + I*(-b +
Sqrt[b^2 - 4*a*c])*(A*(8*b^3 - 29*a*b*c) + 3*a*(-2*b^2*B + 6*a*B*c + a*b*C
))*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(A*(-8*b^4 + 37*a*b^2*c - 20*a^2*c^2 + 8
*b^3*Sqrt[b^2 - 4*a*c] - 29*a*b*c*Sqrt[b^2 - 4*a*c]) + 3*a*(2*b^3*B - b^2*
(2*B*Sqrt[b^2 - 4*a*c] + a*C) + 2*a*c*(3*B*Sqrt[b^2 - 4*a*c] + 2*a*C) + a*
b*(-8*B*c + Sqrt[b^2 - 4*a*c]*C)))*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a^3*(b^2 -
 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2198, 25, 2199, 2199, 1604, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 2198$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\int -\frac{-\frac{c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^6}{a^2}-c\left(\frac{Ab^2}{a}-Bb-2Ac+2aC\right)x^4-\frac{(Ab-aB)\left(b^2-4ac\right)x^2}{a}+A\left(b^2-4ac\right)}{x^4\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^6}{a^2}-c\left(\frac{Ab^2}{a}-Bb-2Ac+2aC\right)x^4-\frac{(Ab-aB)\left(b^2-4ac\right)x^2}{a}+A\left(b^2-4ac\right)}{x^4\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}+$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 2199

$$\frac{\int \frac{-c\left(\frac{Ab^2}{a}-Bb-2Ac+2aC\right)x^4-\frac{\left(A\left(2b^3-7abc\right)-a\left(2Bb^2-aCb-6aBc\right)\right)x^2}{a}+A\left(b^2-4ac\right)}{x^4\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right.\right.}{a^2x}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 2199

$$\frac{\int \frac{6Ca^2-3bBa-10Aca+4Ab^2+3(Abc-2aBc+abC)x^2}{x^4\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)}{a^2x}+\frac{\sqrt{a+bx^2+cx^4}\left(\frac{Ab^2}{a}+2aC\right.}{x^3}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1604

$$\frac{-\frac{\int \frac{c\left(6Ca^2-3bBa-10Aca+4Ab^2\right)x^2+A\left(8b^3-29abc\right)-3a\left(2Bb^2-aCb-6aBc\right)}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2+cx^4}\left(6a^2C-10aAc-3abB+4Ab^2\right)}{3ax^3}-\frac{\sqrt{a+bx^2+cx^4}\left(A\right.}{}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1604

$$-\frac{-\frac{\int -\frac{c\left(\left(A\left(8b^3-29abc\right)-3a\left(2Bb^2-aCb-6aBc\right)\right)x^2+a\left(6Ca^2-3bBa-10Aca+4Ab^2\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(A\left(8b^3-29abc\right)-3a\left(-abC-6aBc+2b^2B\right)\right)}{ax}}{3a}-\ldots$$

$$a\left(b^2-\right.$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int \frac{c\left(\left(A\left(8b^3-29abc\right)-3a\left(2Bb^2-aCb-6aBc\right)\right)x^2+a\left(6Ca^2-3bBa-10Aca+4Ab^2\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(A\left(8b^3-29abc\right)-3a\left(-abC-6aBc+2b^2B\right)\right)}{ax}}{3a}-\frac{\sqrt{a+}}{a\left(b^2-4\right.}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 27

$$-\frac{\frac{c\int \frac{\left(A\left(8b^3-29abc\right)-3a\left(2Bb^2-aCb-6aBc\right)\right)x^2+a\left(6Ca^2-3bBa-10Aca+4Ab^2\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(A\left(8b^3-29abc\right)-3a\left(-abC-6aBc+2b^2B\right)\right)}{ax}}{3a}-\frac{\sqrt{a+b}}{a\left(b^2-4\right.}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 1511

$$-\frac{\frac{c\left(\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(A\left(-12\sqrt{a}b\sqrt{c}-5ac+8b^2\right)-3a(-3\sqrt{a}B\sqrt{c}-aC+2bB)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(A\left(8b^3-29abc\right)-3a\left(-abC-6aBc+2b^2B\right)\right)\int \frac{\sqrt{a}-\sqrt{c}}{\sqrt{a}\sqrt{cx^4+b}}}{\sqrt{c}}\right.}{a}}{3a}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 27

$$-\frac{\frac{c\left(\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(A\left(-12\sqrt{a}b\sqrt{c}-5ac+8b^2\right)-3a(-3\sqrt{a}B\sqrt{c}-aC+2bB)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(A\left(8b^3-29abc\right)-3a\left(-abC-6aBc+2b^2B\right)\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}}{\sqrt{c}}\right.}{a}}{3a}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 1416

$$-\frac{\frac{c\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(A\left(-12\sqrt{a}b\sqrt{c}-5ac+8b^2\right)-3a\left(-3\sqrt{a}B\sqrt{c}-aC+2bB\right)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(A}{}\right.}{a}}{3a}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1509

$$-\frac{\sqrt{a+bx^2+cx^4}\left(6a^2C-10aAc-3abB+4Ab^2\right)}{3ax^3}-\frac{\sqrt{a+bx^2+cx^4}\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)}{a^2x}-\frac{c\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+b}{\left(\sqrt{a}\right.}}}{}\right.}{}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^4*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(x*(a^2*((A*b^4)/a^2 + 3*b*B*c + 2*A*c^2 - (b^2*(b*B + 4*A*c))/a + b^2*C -
 2*a*c*C) + c*(A*(b^3 - 3*a*b*c) - a*(b^2*B - 2*a*B*c - a*b*C))*x^2))/(a^3
*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((((A*b^2)/a - b*B - 2*A*c + 2*a
*C)*Sqrt[a + b*x^2 + c*x^4])/x^3 - ((4*A*b^2 - 3*a*b*B - 10*a*A*c + 6*a^2*
C)*Sqrt[a + b*x^2 + c*x^4])/(3*a*x^3) - ((A*(b^3 - 3*a*b*c) - a*(b^2*B - 2
*a*B*c - a*b*C))*Sqrt[a + b*x^2 + c*x^4])/(a^2*x) - (-(((A*(8*b^3 - 29*a*b
*c) - 3*a*(2*b^2*B - 6*a*B*c - a*b*C))*Sqrt[a + b*x^2 + c*x^4])/(a*x)) + (
c*(-(((A*(8*b^3 - 29*a*b*c) - 3*a*(2*b^2*B - 6*a*B*c - a*b*C))*(-((x*Sqrt[
a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b
*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(A*(8*b^2 - 1
2*Sqrt[a]*b*Sqrt[c] - 5*a*c) - 3*a*(2*b*B - 3*Sqrt[a]*B*Sqrt[c] - a*C))*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*
c^(3/4)*Sqrt[a + b*x^2 + c*x^4])))/a)/(3*a))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2198

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199

```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 9.90 (sec) , antiderivative size = 736, normalized size of antiderivative = 1.21

method	result
elliptic	$-\frac{A\sqrt{c x^4+b x^2+a}}{3a^2x^3}+\frac{(5Ab-3Ba)\sqrt{c x^4+b x^2+a}}{3a^3x}-\frac{2c\left(-\frac{\left(3Aabc-A b^3-2B a^2c+B a b^2-C a^2b\right)x^3}{2\left(4ac-b^2\right)a^3}+\frac{\left(2A a^2c^2-4Aa b^2c+A b^4+3B a^2 \cdots\right.}{2a^3\left(4ac \cdots\right.}\right.}{\sqrt{\left(x^4+\frac{b x^2}{c}+\frac{a}{c}\right)c}}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*(c*x^4+b*x^2+a)^(1/2)/a^2/x^3+1/3*(5*A*b-3*B*a)*(c*x^4+b*x^2+a)^(1/
2)/a^3/x-2*c*(-1/2*(3*A*a*b*c-A*b^3-2*B*a^2*c+B*a*b^2-C*a^2*b)/(4*a*c-b^2)
/a^3*x^3+1/2*(2*A*a^2*c^2-4*A*a*b^2*c+A*b^4+3*B*a^2*b*c-B*a*b^3-2*C*a^3*c+
C*a^2*b^2)/a^3/(4*a*c-b^2)/c*x)/((x^4+1/c*b*x^2+1/c*a)*c)^(1/2)+1/4*(-1/3/
a^2*A*c-(A*a*c-A*b^2+B*a*b-C*a^2)/a^3+(2*A*a^2*c^2-4*A*a*b^2*c+A*b^4+3*B*a
^2*b*c-B*a*b^3-2*C*a^3*c+C*a^2*b^2)/a^3/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2))-1/2*(-1/3*c*(5*A*b-3*B*a)/a^3-(3*A*a*b*c-A*b^3-2*B*a^2*c+B*a*b^2-
C*a^2*b)/(4*a*c-b^2)/a^3*c)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4
-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. $2(523) = 1046$.

Time = 0.10 (sec) , antiderivative size = 1203, normalized size of antiderivative = 1.97

$$\int \frac{A + Bx^2 + Cx^4}{x^4 (a + bx^2 + cx^4)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(1/2)*(((18*B*a^2*b - 29*A*a*b^2)*c^2 + (3*C*a^2*b^2 - 6*B*a*b^3
 + 8*A*b^4)*c)*x^7 + (3*C*a^2*b^3 - 6*B*a*b^4 + 8*A*b^5 + (18*B*a^2*b^2 -
29*A*a*b^3)*c)*x^5 + (3*C*a^3*b^2 - 6*B*a^2*b^3 + 8*A*a*b^4 + (18*B*a^3*b
- 29*A*a^2*b^2)*c)*x^3 - (((18*B*a^3 - 29*A*a^2*b)*c^2 + (3*C*a^3*b - 6*B*
a^2*b^2 + 8*A*a*b^3)*c)*x^7 + (3*C*a^3*b^2 - 6*B*a^2*b^3 + 8*A*a*b^4 + (18
*B*a^3*b - 29*A*a^2*b^2)*c)*x^5 + (3*C*a^4*b - 6*B*a^3*b^2 + 8*A*a^2*b^3 +
 (18*B*a^4 - 29*A*a^3*b)*c)*x^3)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*
sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt
((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a
*c)/(a*c)) + sqrt(1/2)*(((2*(5*A - 9*B)*a^2*b + 29*A*a*b^2)*c^2 - (6*C*a^3
*b - 3*(B - C)*a^2*b^2 + 2*(2*A - 3*B)*a*b^3 + 8*A*b^4)*c)*x^7 - (6*C*a^3*
b^2 - 3*(B - C)*a^2*b^3 + 2*(2*A - 3*B)*a*b^4 + 8*A*b^5 - (2*(5*A - 9*B)*a
^2*b^2 + 29*A*a*b^3)*c)*x^5 - (6*C*a^4*b - 3*(B - C)*a^3*b^2 + 2*(2*A - 3*
B)*a^2*b^3 + 8*A*a*b^4 - (2*(5*A - 9*B)*a^3*b + 29*A*a^2*b^2)*c)*x^3 + (((
2*(5*A + 9*B)*a^3 - 29*A*a^2*b)*c^2 - (6*C*a^4 - 3*(B + C)*a^3*b + 2*(2*A
+ 3*B)*a^2*b^2 - 8*A*a*b^3)*c)*x^7 - (6*C*a^4*b - 3*(B + C)*a^3*b^2 + 2*(2
*A + 3*B)*a^2*b^3 - 8*A*a*b^4 - (2*(5*A + 9*B)*a^3*b - 29*A*a^2*b^2)*c)*x^
5 - (6*C*a^5 - 3*(B + C)*a^4*b + 2*(2*A + 3*B)*a^3*b^2 - 8*A*a^2*b^3 - (2*
(5*A + 9*B)*a^4 - 29*A*a^3*b)*c)*x^3)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqr
t((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt...
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^4 (a + bx^2 + cx^4)^{3/2}}\, dx = \int \frac{A + Bx^2 + Cx^4}{x^4 (a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**4*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^4 (a + bx^2 + cx^4)^{3/2}}\, dx = \int \frac{Cx^4 + Bx^2 + A}{(cx^4 + bx^2 + a)^{\frac{3}{2}} x^4}\, dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^4 (a + bx^2 + cx^4)^{3/2}}\, dx = \int \frac{Cx^4 + Bx^2 + A}{(cx^4 + bx^2 + a)^{\frac{3}{2}} x^4}\, dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^4\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^8+b\,x^6+a\,x^4}dx$$

input
```
int((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)
```

3.157 $\int \frac{A+Bx^2+Cx^4}{x^6(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 773

$$\int \frac{A+Bx^2+Cx^4}{x^6(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{x(A(b^5-5ab^3c+5a^2bc^2)-a(b^4B-4ab^2Bc+2a^2Bc^2-ab^3C+3a^2bcC)+c(A(b^4-4ab^2c+2a^2c^2)-}{a^4(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$-\frac{A\sqrt{a+bx^2+cx^4}}{5a^2x^5}+\frac{(9Ab-5aB)\sqrt{a+bx^2+cx^4}}{15a^3x^3}$$

$$-\frac{(33Ab^2-25abB-24aAc+15a^2C)\sqrt{a+bx^2+cx^4}}{15a^4x}$$

$$+\frac{\sqrt{c}(6A(8b^4-36ab^2c+21a^2c^2)-5a(8b^3B-29abBc-6ab^2C+18a^2cC))x\sqrt{a+bx^2+cx^4}}{15a^4(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)}$$

$$-\frac{\sqrt[4]{c}(6A(8b^4-36ab^2c+21a^2c^2)-5a(8b^3B-29abBc-6ab^2C+18a^2cC))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E}{15a^{15/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt[4]{c}\left(A\left(48b^3-72\sqrt{a}b^2\sqrt{c}-72abc+63a^{3/2}c^{3/2}\right)-5a\left(8b^2B-5aBc+9a^{3/2}\sqrt{c}C-6b\left(2\sqrt{a}B\sqrt{c}+aC\right)\right)\right)}{30a^{15/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+}}$$

output
```
-x*(A*(5*a^2*b*c^2-5*a*b^3*c+b^5)-a*(2*B*a^2*c^2-4*B*a*b^2*c+B*b^4+3*C*a^2
*b*c-C*a*b^3)+c*(A*(2*a^2*c^2-4*a*b^2*c+b^4)-a*(-3*B*a*b*c+B*b^3+2*C*a^2*c
-C*a*b^2))*x^2)/a^4/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/5*A*(c*x^4+b*x^2+
a)^(1/2)/a^2/x^5+1/15*(9*A*b-5*B*a)*(c*x^4+b*x^2+a)^(1/2)/a^3/x^3-1/15*(-2
4*A*a*c+33*A*b^2-25*B*a*b+15*C*a^2)*(c*x^4+b*x^2+a)^(1/2)/a^4/x+1/15*c^(1/
2)*(6*A*(21*a^2*c^2-36*a*b^2*c+8*b^4)-5*a*(-29*B*a*b*c+8*B*b^3+18*C*a^2*c-
6*C*a*b^2))*x*(c*x^4+b*x^2+a)^(1/2)/a^4/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)
-1/15*c^(1/4)*(6*A*(21*a^2*c^2-36*a*b^2*c+8*b^4)-5*a*(-29*B*a*b*c+8*B*b^3+
18*C*a^2*c-6*C*a*b^2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(
1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(
1/2)/c^(1/2))^(1/2))/a^(15/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/30*c^(1
/4)*(A*(48*b^3-72*a^(1/2)*b^2*c^(1/2)-72*a*b*c+63*a^(3/2)*c^(3/2))-5*a*(8*
b^2*B-5*a*B*c+9*a^(3/2)*c^(1/2)*C-6*b*(2*a^(1/2)*B*c^(1/2)+a*C)))*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacob
iAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(15/4)/
(b-2*a^(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.81 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(a+bx^2+cx^4\right)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(48Ab^4x^6(b+cx^2)-4a^4c(3A+5x^2(B+3Cx^2))+8ab^2x^4(-5bBx^2(b+cx^2)+3A(b^2-10a}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/60*(4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(48*A*b^4*x^6*(b + c*x^2) - 4*a^4
*c*(3*A + 5*x^2*(B + 3*C*x^2)) + 8*a*b^2*x^4*(-5*b*B*x^2*(b + c*x^2) + 3*A
*(b^2 - 10*b*c*x^2 - 9*c^2*x^4)) + a^2*(5*b*x^4*(-4*b^2*B + 33*b*B*c*x^2 +
 6*b^2*C*x^2 + 29*B*c^2*x^4 + 6*b*c*C*x^4) + 3*A*(-2*b^3*x^2 - 39*b^2*c*x^
4 + 69*b*c^2*x^6 + 42*c^3*x^8)) + a^3*(3*A*(b^2 + 8*b*c*x^2 + 28*c^2*x^4)
+ 5*(b*c*x^4*(16*B - 21*C*x^2) + b^2*x^2*(B + 3*C*x^2) - 2*c^2*x^6*(5*B +
9*C*x^2)))) - I*(-b + Sqrt[b^2 - 4*a*c])*(6*A*(8*b^4 - 36*a*b^2*c + 21*a^2
*c^2) + 5*a*(-8*b^3*B + 29*a*b*B*c + 6*a*b^2*C - 18*a^2*c*C))*x^5*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt
[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt
[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])] + I*(6*A*(-8*b^5 + 44*a*b^3*c - 48*a^2*b*c^2 + 8*b^4*Sqrt[b
^2 - 4*a*c] - 36*a*b^2*c*Sqrt[b^2 - 4*a*c] + 21*a^2*c^2*Sqrt[b^2 - 4*a*c])
 - 5*a*(-8*b^4*B + b^3*(8*B*Sqrt[b^2 - 4*a*c] + 6*a*C) - a*b*c*(29*B*Sqrt[
b^2 - 4*a*c] + 24*a*C) + a*b^2*(37*B*c - 6*Sqrt[b^2 - 4*a*c]*C) + 2*a^2*c*
(-10*B*c + 9*Sqrt[b^2 - 4*a*c]*C)))*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a^4*(b^2
- 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^5*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 2.76 (sec) , antiderivative size = 975, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2198, 25, 2199, 2199, 2199, 1604, 1604, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{2198}$$

$$-\frac{\int -\frac{\frac{c\left(A\left(b^4-4acb^2+2a^2c^2\right)-a\left(Bb^3-aCb^2-3aBcb+2a^2cC\right)\right)x^8}{a^3}+\frac{c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^6}{a^2}+\frac{\left(b^2-4ac\right)\left(A\left(b^2-ac\right)-a(bB-aC)\right)x^4}{a^2}-\frac{(A\cdots}{\cdots}}{x^6\sqrt{cx^4+bx^2+a}}\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\cdots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 25

$$\frac{\int \frac{\frac{c\left(A\left(b^4-4acb^2+2a^2c^2\right)-a\left(Bb^3-aCb^2-3aBcb+2a^2cC\right)\right)x^8}{a^3}+\frac{c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^6}{a^2}+\frac{\left(b^2-4ac\right)\left(A\left(b^2-ac\right)-a(bB-aC)\right)x^4}{a^2}-\frac{(Ab-aB\cdots}{\cdots}}{x^6\sqrt{cx^4+bx^2+a}}\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\cdots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 2199

$$\frac{\int \frac{\frac{c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^6}{a^2}+\frac{\left(A\left(2b^4-9acb^2+6a^2c^2\right)-a\left(2Bb^3-2aCb^2-7aBcb+6a^2cC\right)\right)x^4}{a^2}-\frac{(Ab-aB)\left(b^2-4ac\right)x^2}{a}+A\left(b^2-4ac\right)}{x^6\sqrt{cx^4+bx^2+a}}dx+\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\cdots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 2199

$$\frac{\int \frac{-3c\left(\frac{Ab^2}{a}-Bb-2Ac+2aC\right)x^4-\left(\frac{4Ab^3}{a}-4Bb^2-13Acb+3aCb+10aBc\right)x^2+A\left(b^2-4ac\right)}{x^6\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b\cdots\right.\right.}{a^2x^3}\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\cdots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 2199

$$\frac{\int \frac{10Ca^2-5bBa-14Aca+6Ab^2+5(Abc-2aBc+abC)x^2}{x^6\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)}{a^2x^3}+\frac{\sqrt{a+bx^2+cx^4}\left(A\left(2a^2c^2\cdots\right.\right.}{\cdots}\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\cdots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 1604

$$-\frac{\int \frac{24Ab^3-20aBb^2-81aAcb+15a^2Cb+3c\left(10Ca^2-5bBa-14Aca+6Ab^2\right)x^2+50a^2Bc}{x^4\sqrt{cx^4+bx^2+a}}dx}{5a}-\frac{\sqrt{a+bx^2+cx^4}\left(10a^2C-14aAc-5abB+6Ab^2\right)}{5ax^5}-\frac{\sqrt{a+bx^2+}\cdots}{\cdots}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\cdots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 1604

$$-\frac{-\frac{\int \frac{c\left(3A\left(8b^3-27abc\right)-5a\left(4Bb^2-3aCb-10aBc\right)\right)x^2+6A\left(8b^4-36acb^2+21a^2c^2\right)-5a\left(8Bb^3-6aCb^2-29aBcb+18a^2cC\right)}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2+cx^4}\left(15a^2bC+50a^2B\cdots\right.}{3ax\cdots}}{5a}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\cdots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 1604

$$-\frac{-\frac{-\frac{\int -\frac{c\left(\left(6A\left(8b^4-36acb^2+21a^2c^2\right)-5a\left(8Bb^3-6aCb^2-29aBcb+18a^2cC\right)\right)x^2+a\left(24Ab^3-20aBb^2-81aAcb+15a^2Cb+50a^2Bc\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(6A\left(2\cdots\right.\right.}{\cdots}}{3a}}{5a}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\cdots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 25

$$-\frac{-\frac{\frac{\int \frac{c\left(\left(6A\left(8b^4-36acb^2+21a^2c^2\right)-5a\left(8Bb^3-6aCb^2-29aBcb+18a^2cC\right)\right)x^2+a\left(24Ab^3-20aBb^2-81aAcb+15a^2Cb+50a^2Bc\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(6A\left(21a^2\cdots\right.\right.}{\cdots}}{3a}}{5a}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\cdots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\frac{c\int \frac{\left(6A\left(8b^4-36acb^2+21a^2c^2\right)-5a\left(8Bb^3-6aCb^2-29aBcb+18a^2cC\right)\right)x^2+a\left(24Ab^3-20aBb^2-81aAcb+15a^2Cb+50a^2Bc\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}\left(6A\left(21a^2c^2\cdots\right.\right.}{\cdots}}{3a}}{5a}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\cdots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\big\downarrow$ 1511

$$-\frac{-\frac{c\left(\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(A\left(63a^{3/2}c^{3/2}-72\sqrt{a}b^2\sqrt{c}-72abc+48b^3\right)-5a\left(9a^{3/2}\sqrt{c}C-6b(2\sqrt{a}B\sqrt{c}+aC)-5aBc+8b^2B\right)\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(6A\left(21a^2c^2-36\ldots\right.\right.}{\ldots}}{a}}{\ldots}}{\ldots}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\ldots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 27

$$-\frac{-\frac{c\left(\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(A\left(63a^{3/2}c^{3/2}-72\sqrt{a}b^2\sqrt{c}-72abc+48b^3\right)-5a\left(9a^{3/2}\sqrt{c}C-6b(2\sqrt{a}B\sqrt{c}+aC)-5aBc+8b^2B\right)\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(6A\left(21a^2c^2-36ab\ldots\right.\right.}{\ldots}}{a}}{3a\ldots}}{\ldots}$$

$$\frac{x\left(A\left(5a^2bc^2-5ab^3c+b^5\right)+cx^2\left(A\left(2a^2c^2-4ab^2c+b^4\right)-a\left(2a^2cC-ab^2C-3abBc+b^3B\right)\right)-a\left(3a^2bcC+2a\ldots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 1416

$$\frac{\frac{\sqrt{cx^4+bx^2+a}\left(\frac{Ab^2}{a}-Bb-2Ac+2aC\right)}{x^5}-\frac{-\frac{\sqrt{cx^4+bx^2+a}\left(24Ab^3-20aBb^2-81aAcb+15a^2Cb+50a^2Bc\right)}{3ax^3}-\frac{c\left(\frac{\sqrt[4]{a}(b+2\sqrt{a}\sqrt{c})\left(A\left(48b^3-72\sqrt{a}\sqrt{c}b^2-72ac\ldots\right.\right.}{\ldots}\right.}{\ldots}}{\ldots}}{\ldots}$$

$$\frac{x\left(c\left(A\left(b^4-4acb^2+2a^2c^2\right)-a\left(Bb^3-aCb^2-3aBcb+2a^2cC\right)\right)x^2+A\left(b^5-5acb^3+5a^2c^2b\right)-a\left(Bb^4-aCb^3\ldots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{cx^4+bx^2+a}}$$

$\downarrow$ 1509

$$\frac{\frac{\sqrt{cx^4+bx^2+a}\left(\frac{Ab^2}{a}-Bb-2Ac+2aC\right)}{x^5}-\frac{-\frac{\sqrt{cx^4+bx^2+a}\left(24Ab^3-20aBb^2-81aAcb+15a^2Cb+50a^2Bc\right)}{3ax^3}-\frac{c\left(\frac{\sqrt[4]{a}(b+2\sqrt{a}\sqrt{c})\left(A\left(48b^3-72\sqrt{a}\sqrt{c}b^2-72ac\ldots\right.\right.}{\ldots}\right.}{\ldots}}{\ldots}}{\ldots}$$

$$\frac{x\left(c\left(A\left(b^4-4acb^2+2a^2c^2\right)-a\left(Bb^3-aCb^2-3aBcb+2a^2cC\right)\right)x^2+A\left(b^5-5acb^3+5a^2c^2b\right)-a\left(Bb^4-aCb^3\ldots\right.\right.}{a^4\left(b^2-4ac\right)\sqrt{cx^4+bx^2+a}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^6*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-((x*(A*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2) - a*(b^4*B - 4*a*b^2*B*c + 2*a^2*B
*c^2 - a*b^3*C + 3*a^2*b*c*C) + c*(A*(b^4 - 4*a*b^2*c + 2*a^2*c^2) - a*(b^
3*B - 3*a*b*B*c - a*b^2*C + 2*a^2*c*C))*x^2))/(a^4*(b^2 - 4*a*c)*Sqrt[a +
b*x^2 + c*x^4])) + ((((A*b^2)/a - b*B - 2*A*c + 2*a*C)*Sqrt[a + b*x^2 + c*
x^4])/x^5 - ((6*A*b^2 - 5*a*b*B - 14*a*A*c + 10*a^2*C)*Sqrt[a + b*x^2 + c*
x^4])/(5*a*x^5) - ((A*(b^3 - 3*a*b*c) - a*(b^2*B - 2*a*B*c - a*b*C))*Sqrt[
a + b*x^2 + c*x^4])/(a^2*x^3) + ((A*(b^4 - 4*a*b^2*c + 2*a^2*c^2) - a*(b^3
*B - 3*a*b*B*c - a*b^2*C + 2*a^2*c*C))*Sqrt[a + b*x^2 + c*x^4])/(a^3*x) -
(-1/3*((24*A*b^3 - 20*a*b^2*B - 81*a*A*b*c + 50*a^2*B*c + 15*a^2*b*C)*Sqrt
[a + b*x^2 + c*x^4])/(a*x^3) - (-(((6*A*(8*b^4 - 36*a*b^2*c + 21*a^2*c^2)
- 5*a*(8*b^3*B - 29*a*b*B*c - 6*a*b^2*C + 18*a^2*c*C))*Sqrt[a + b*x^2 + c*
x^4])/(a*x)) + (c*(-(((6*A*(8*b^4 - 36*a*b^2*c + 21*a^2*c^2) - 5*a*(8*b^3*
B - 29*a*b*B*c - 6*a*b^2*C + 18*a^2*c*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(
Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt
[c]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(A*(48*b^3 - 72*Sqrt[a]*b^2*Sqrt[c
] - 72*a*b*c + 63*a^(3/2)*c^(3/2)) - 5*a*(8*b^2*B - 5*a*B*c + 9*a^(3/2)*Sq
rt[c]*C - 6*b*(2*Sqrt[a]*B*Sqrt[c] + a*C)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 11.28 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.14

method	result
elliptic	$-\frac{A\sqrt{c\,x^{4}+b\,x^{2}+a}}{5a^{2}x^{5}}+\frac{(9Ab-5Ba)\sqrt{c\,x^{4}+b\,x^{2}+a}}{15a^{3}x^{3}}+\frac{\left(24Aac-33A\,b^{2}+25Bab-15a^{2}C\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{15a^{4}x}-\frac{2c\left(-\frac{\left(2A\,a^{2}c^{2}-4Aa\,b^{2}\right.}{}\right.}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/x^6/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*A*(c*x^4+b*x^2+a)^(1/2)/a^2/x^5+1/15*(9*A*b-5*B*a)*(c*x^4+b*x^2+a)^(1
/2)/a^3/x^3+1/15/a^4*(24*A*a*c-33*A*b^2+25*B*a*b-15*C*a^2)*(c*x^4+b*x^2+a)
^(1/2)/x-2*c*(-1/2*(2*A*a^2*c^2-4*A*a*b^2*c+A*b^4+3*B*a^2*b*c-B*a*b^3-2*C*
a^3*c+C*a^2*b^2)/a^4/(4*a*c-b^2)*x^3-1/2*(5*A*a^2*b*c^2-5*A*a*b^3*c+A*b^5-
2*B*a^3*c^2+4*B*a^2*b^2*c-B*a*b^4-3*C*a^3*b*c+C*a^2*b^3)/a^4/(4*a*c-b^2)/c
*x)/((x^4+1/c*b*x^2+1/c*a)*c)^(1/2)+1/4*(1/15*c*(9*A*b-5*B*a)/a^3+(2*A*a*b
*c-A*b^3-B*a^2*c+B*a*b^2-C*a^2*b)/a^4-(5*A*a^2*b*c^2-5*A*a*b^3*c+A*b^5-2*B
*a^3*c^2+4*B*a^2*b^2*c-B*a*b^4-3*C*a^3*b*c+C*a^2*b^3)/a^4/(4*a*c-b^2))*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Elli
pticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-1/15*c*(24*A*a*c-33*A*b^2+25*B*a*b-15*C*a
^2)/a^4-c*(2*A*a^2*c^2-4*A*a*b^2*c+A*b^4+3*B*a^2*b*c-B*a*b^3-2*C*a^3*c+C*a
^2*b^2)/(4*a*c-b^2)/a^4)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*
(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1606 vs. $2(677) = 1354$.

Time = 0.10 (sec) , antiderivative size = 1606, normalized size of antiderivative = 2.08

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/30*(sqrt(1/2)*((126*A*a^2*b*c^3 - (90*C*a^3*b - 145*B*a^2*b^2 + 216*A*a*
b^3)*c^2 + 2*(15*C*a^2*b^3 - 20*B*a*b^4 + 24*A*b^5)*c)*x^9 + (30*C*a^2*b^4
 - 40*B*a*b^5 + 48*A*b^6 + 126*A*a^2*b^2*c^2 - (90*C*a^3*b^2 - 145*B*a^2*b
^3 + 216*A*a*b^4)*c)*x^7 + (30*C*a^3*b^3 - 40*B*a^2*b^4 + 48*A*a*b^5 + 126
*A*a^3*b*c^2 - (90*C*a^4*b - 145*B*a^3*b^2 + 216*A*a^2*b^3)*c)*x^5 - ((126
*A*a^3*c^3 - (90*C*a^4 - 145*B*a^3*b + 216*A*a^2*b^2)*c^2 + 2*(15*C*a^3*b^
2 - 20*B*a^2*b^3 + 24*A*a*b^4)*c)*x^9 + (30*C*a^3*b^3 - 40*B*a^2*b^4 + 48*
A*a*b^5 + 126*A*a^3*b*c^2 - (90*C*a^4*b - 145*B*a^3*b^2 + 216*A*a^2*b^3)*c
)*x^7 + (30*C*a^4*b^2 - 40*B*a^3*b^3 + 48*A*a^2*b^4 + 126*A*a^4*c^2 - (90*
C*a^5 - 145*B*a^4*b + 216*A*a^3*b^2)*c)*x^5)*sqrt((b^2 - 4*a*c)/a^2))*sqrt
(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*
sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2)
 + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((126*A*a^2*b*c^3 + (10*(5*B - 9*C)*a^3
*b - (81*A - 145*B)*a^2*b^2 - 216*A*a*b^3)*c^2 + (15*C*a^3*b^2 - 10*(2*B -
 3*C)*a^2*b^3 + 8*(3*A - 5*B)*a*b^4 + 48*A*b^5)*c)*x^9 + (15*C*a^3*b^3 - 1
0*(2*B - 3*C)*a^2*b^4 + 8*(3*A - 5*B)*a*b^5 + 48*A*b^6 + 126*A*a^2*b^2*c^2
 + (10*(5*B - 9*C)*a^3*b^2 - (81*A - 145*B)*a^2*b^3 - 216*A*a*b^4)*c)*x^7
+ (15*C*a^4*b^2 - 10*(2*B - 3*C)*a^3*b^3 + 8*(3*A - 5*B)*a^2*b^4 + 48*A*a*
b^5 + 126*A*a^3*b*c^2 + (10*(5*B - 9*C)*a^4*b - (81*A - 145*B)*a^3*b^2 - 2
16*A*a^2*b^3)*c)*x^5 - ((126*A*a^3*c^3 - (10*(5*B + 9*C)*a^4 - (81*A + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{x^6 (a + bx^2 + cx^4)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/x**6/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^6\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^6*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^6*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^6 (a + bx^2 + cx^4)^{3/2}} \, dx = \frac{-\sqrt{c\,x^4 + b\,x^2 + a}\,a + 3\sqrt{c\,x^4 + b\,x^2 + a}\,c\,x^4 - 4\left(\int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{c\,x^8 + b\,x^6 + a\,x^4}dx\right)ab\,x^5 -}{5a^2x^5}$$

input
```
int((C*x^4+B*x^2+A)/x^6/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4)*a + 3*sqrt(a + b*x**2 + c*x**4)*c*x**4 - 4*i
nt(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*a*b*x**5 - 3*in
t((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*c**2*x**5)/(5*
a**2*x**5)
```

3.158 $\int \frac{x^6(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 776

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{x(a(b^2Bc-2aBc^2-b^3C-bc(Ac-3aC))+(b^3Bc-3abBc^2-b^4C-b^2c(A}{3c^3(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$
$$-\frac{x(b^4Bc-7ab^2Bc^2+28a^2Bc^3-b^5C-b^3c(Ac-11aC)-4abc^2(Ac+9aC)-c(2b^3Bc-16abBc^2-5b^4}{3c^3(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(2b^3Bc-16abBc^2-8b^4C+12ac^2(Ac-7aC)+b^2c(Ac+57aC))x\sqrt{a+bx^2+cx^4}}{3c^{5/2}(b^2-4ac)^2(\sqrt{a}+\sqrt{c}x^2)}$$
$$+\frac{\sqrt[4]{a}(2b^3Bc-16abBc^2-8b^4C+12ac^2(Ac-7aC)+b^2c(Ac+57aC))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\Big(2\,\mathrm{ar}}{3c^{11/4}(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(8b^3C+bc\left(3\sqrt{a}B\sqrt{c}-Ac-33aC\right)+2\sqrt{a}c^{3/2}\left(5\sqrt{a}B\sqrt{c}-3Ac+21aC\right)-2b^2\left(Bc+6\sqrt{a}\sqrt{c}C\right)\right)(}{6\left(b-2\sqrt{a}\sqrt{c}\right)c^{11/4}(b^2-4ac)\sqrt{a+bx^2}}$$

output
```
1/3*x*(a*(b^2*B*c-2*a*B*c^2-b^3*C-b*c*(A*c-3*C*a))+(b^3*B*c-3*a*b*B*c^2-b^
4*C-b^2*c*(A*c-4*C*a)+2*a*c^2*(A*c-C*a))*x^2)/c^3/(-4*a*c+b^2)/(c*x^4+b*x^
2+a)^(3/2)-1/3*x*(b^4*B*c-7*a*b^2*B*c^2+28*a^2*B*c^3-b^5*C-b^3*c*(A*c-11*C
*a)-4*a*b*c^2*(A*c+9*C*a)-c*(2*b^3*B*c-16*a*b*B*c^2-5*b^4*C+12*a*c^2*(A*c-
3*C*a)+b^2*c*(A*c+33*C*a))*x^2)/c^3/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1
/3*(2*b^3*B*c-16*a*b*B*c^2-8*b^4*C+12*a*c^2*(A*c-7*C*a)+b^2*c*(A*c+57*C*a)
)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(-4*a*c+b^2)^2/(a^(1/2)+c^(1/2)*x^2)+1/3
*a^(1/4)*(2*b^3*B*c-16*a*b*B*c^2-8*b^4*C+12*a*c^2*(A*c-7*C*a)+b^2*c*(A*c+5
7*C*a))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1
/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))/c^(11/4)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)+1/6*a^(1/4)*(8*b^3*C+b
*c*(3*a^(1/2)*B*c^(1/2)-A*c-33*a*C)+2*a^(1/2)*c^(3/2)*(5*a^(1/2)*B*c^(1/2)
-3*A*c+21*a*C)-2*b^2*(B*c+6*a^(1/2)*c^(1/2)*C))*(a^(1/2)+c^(1/2)*x^2)*((c*
x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/
4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/(b-2*a^(1/2)*c^(1/2))/c^(11
/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.35 (sec) , antiderivative size = 740, normalized size of antiderivative = 0.95

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{4cx(-4a^3c(5Bc-6bC+7cCx^2)+b^2x^4(-4b^3C+Ac^3x^2+2bc^2(A+Bx^2)+\cdots}{\cdots}$$

input
```
Integrate[(x^6*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output

```
(4*c*x*(-4*a^3*c*(5*B*c - 6*b*C + 7*c*C*x^2) + b^2*x^4*(-4*b^3*C + A*c^3*x
^2 + 2*b*c^2*(A + B*x^2) + b^2*c*(B - 5*C*x^2)) + a^2*(-4*b^3*C + 8*b*c^2*
(A - 4*B*x^2) + b^2*c*(B + 51*C*x^2) + 4*c^3*x^2*(A - 7*B*x^2 - 9*C*x^4))
+ a*x^2*(-8*b^4*C + 12*A*c^4*x^4 + 16*b*c^3*x^2*(A - B*x^2) + 2*b^3*c*(B +
 11*C*x^2) + b^2*c^2*(11*A - 9*B*x^2 + 33*C*x^4))) + (I*Sqrt[2]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*((-b + Sqrt[b^2 - 4*a*c])*(-2*
b^3*B*c + 16*a*b*B*c^2 + 8*b^4*C + 12*a*c^2*(-(A*c) + 7*a*C) - b^2*c*(A*c
+ 57*a*C))*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
 (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (8*b^5*C + 4*a*b*c^2*(
A*c - 4*B*Sqrt[b^2 - 4*a*c] + 33*a*C) - b^3*c*(A*c - 2*B*Sqrt[b^2 - 4*a*c]
 + 65*a*C) - 2*b^4*(B*c + 4*Sqrt[b^2 - 4*a*c]*C) - 4*a*c^2*(10*a*B*c - 3*A
*c*Sqrt[b^2 - 4*a*c] + 21*a*Sqrt[b^2 - 4*a*c]*C) + b^2*c*(18*a*B*c + A*c*S
qrt[b^2 - 4*a*c] + 57*a*Sqrt[b^2 - 4*a*c]*C))*EllipticF[I*ArcSinh[Sqrt[2]*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(12*c^3*(b^2 - 4*a*c)^2*(a +
 b*x^2 + c*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 715, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2197, 2206, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (A + Bx^2 + Cx^4)}{(a + bx^2 + cx^4)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 2197}$$

$$\frac{x\left(a\left(-bc(Ac - 3aC) - 2aBc^2 + b^3(-C) + b^2Bc\right) + x^2\left(-b^2c(Ac - 4aC) + 2ac^2(Ac - aC) - 3abBc^2 + b^4(-C) - \right.\right.}{3c^3 (b^2 - 4ac) (a + bx^2 + cx^4)^{3/2}}$$

$$\frac{\int \frac{3a\left(4a - \frac{b^2}{c}\right)Cx^6 - \frac{3a\left(b^2 - 4ac\right)(Bc - bC)x^4}{c^2} - \frac{3a^2\left(-Cb^2 + Bcb - 2c(Ac - aC)\right)x^2}{c^2} + \frac{a^2\left(-Cb^3 + Bcb^2 - c(Ac - 3aC)b - 2aBc^2\right)}{c^3}}{(cx^4 + bx^2 + a)^{3/2}} dx}{3a(b^2 - 4ac)}$$

$$\Big\downarrow \text{ 2206}$$

$$\frac{x\left(a\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)+x^2\left(-b^2c(Ac-4aC)+2ac^2(Ac-aC)-3abBc^2+b^4(-C)\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\frac{\frac{ax\left(28a^2Bc^3-b^3c(Ac-11aC)-cx^2\left(b^2c(33aC+Ac)+12ac^2(Ac-3aC)-16abBc^2-5b^4C+2b^3Bc\right)-4abc^2(9aC+Ac)-7ab^2Bc^2+b^5(-C)+b^4Bc\right)}{c^3(b^2-4ac)\sqrt{a+bx^2+cx^4}}}{3a\left(b^2-4ac\right)}$$

$\big\downarrow$ 25

$$\frac{x\left(a\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)+x^2\left(-b^2c(Ac-4aC)+2ac^2(Ac-aC)-3abBc^2+b^4(-C)\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\frac{\frac{\int \frac{a^2\left(\left(-8Cb^4+2Bcb^3+c(Ac+57aC)b^2-16aBc^2b+12ac^2(Ac-7aC)\right)x^2+a\left(-4Cb^3+Bcb^2+8c(Ac+3aC)b-20aBc^2\right)\right)}{c^2\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}+\frac{ax\left(28a^2Bc^3-b^3c(Ac-11aC)-\right.}{}}{3a\left(b^2-4ac\right)}$$

$\big\downarrow$ 27

$$\frac{x\left(a\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)+x^2\left(-b^2c(Ac-4aC)+2ac^2(Ac-aC)-3abBc^2+b^4(-C)\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\frac{\frac{a\int \frac{\left(-8Cb^4+2Bcb^3+c(Ac+57aC)b^2-16aBc^2b+12ac^2(Ac-7aC)\right)x^2+a\left(-4Cb^3+Bcb^2+8c(Ac+3aC)b-20aBc^2\right)}{\sqrt{cx^4+bx^2+a}}dx}{c^2(b^2-4ac)}+\frac{ax\left(28a^2Bc^3-b^3c(Ac-11aC)-cx^2\right.}{}}{3a\left(b^2-4ac\right)}$$

$\big\downarrow$ 1511

$$\frac{x\left(a\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)+x^2\left(-b^2c(Ac-4aC)+2ac^2(Ac-aC)-3abBc^2+b^4(-C)\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\frac{a\left(\frac{\sqrt{a}\left(b^2c(57aC+Ac)+\sqrt{a}\sqrt{c}\left(8bc(3aC+Ac)-20aBc^2-4b^3C+b^2Bc\right)+12ac^2(Ac-7aC)-16abBc^2-8b^4C+2b^3Bc\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(b^2c(57aC+Ac)+12\right.}{}\right.}{c^2(b^2-4ac)}$$

$\big\downarrow$ 27

$$\frac{x\left(a\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)+x^2\left(-b^2c(Ac-4aC)+2ac^2(Ac-aC)-3abBc^2+b^4(-C)\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\frac{a\left(\frac{\sqrt{a}\left(b^2c(57aC+Ac)+\sqrt{a}\sqrt{c}\left(8bc(3aC+Ac)-20aBc^2-4b^3C+b^2Bc\right)+12ac^2(Ac-7aC)-16abBc^2-8b^4C+2b^3Bc\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(b^2c(57aC+Ac)+12ac^2\right.}{}\right.}{c^2(b^2-4ac)}$$

$\big\downarrow$ 1416

$$\frac{x(a(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc)+x^2(-b^2c(Ac-4aC)+2ac^2(Ac-aC)-3abBc^2+b^4(-C)-}{3c^3(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$$\frac{a\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(b^2c(57aC+Ac)+\sqrt{a}\sqrt{c}\left(8bc(3aC+Ac)-20aBc^2-4b^3C+b^2Bc\right)+12ac^2(Ac-7aC)-16abBc^2-8b^4C+2b^3Bc\right)\text{EllipticF}\left(2a\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}\right.}{c^2(b^2-4ac)}$$

$\downarrow$ 1509

$$\frac{x(a(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc)+x^2(-b^2c(Ac-4aC)+2ac^2(Ac-aC)-3abBc^2+b^4(-C)-}{3c^3(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$$\frac{\frac{ax(28a^2Bc^3-b^3c(Ac-11aC)-cx^2(b^2c(33aC+Ac)+12ac^2(Ac-3aC)-16abBc^2-5b^4C+2b^3Bc)-4abc^2(9aC+Ac)-7ab^2Bc^2+b^5(-C)+b^4Bc)}{c^3(b^2-4ac)\sqrt{a+bx^2+cx^4}}-}{}$$

input
```
Int[(x^6*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
(x*(a*(b^2*B*c - 2*a*B*c^2 - b^3*C - b*c*(A*c - 3*a*C)) + (b^3*B*c - 3*a*b
*B*c^2 - b^4*C - b^2*c*(A*c - 4*a*C) + 2*a*c^2*(A*c - a*C))*x^2))/(3*c^3*(
b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^(3/2)) - ((a*x*(b^4*B*c - 7*a*b^2*B*c^2 +
 28*a^2*B*c^3 - b^5*C - b^3*c*(A*c - 11*a*C) - 4*a*b*c^2*(A*c + 9*a*C) - c
*(2*b^3*B*c - 16*a*b*B*c^2 - 5*b^4*C + 12*a*c^2*(A*c - 3*a*C) + b^2*c*(A*c
 + 33*a*C))*x^2))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (a*(-(((2*
b^3*B*c - 16*a*b*B*c^2 - 8*b^4*C + 12*a*c^2*(A*c - 7*a*C) + b^2*c*(A*c + 5
7*a*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(2*b^3*B*c - 16*a*
b*B*c^2 - 8*b^4*C + 12*a*c^2*(A*c - 7*a*C) + b^2*c*(A*c + 57*a*C) + Sqrt[a
]*Sqrt[c]*(b^2*B*c - 20*a*B*c^2 - 4*b^3*C + 8*b*c*(A*c + 3*a*C)))*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4
)*Sqrt[a + b*x^2 + c*x^4])))/(c^2*(b^2 - 4*a*c)))/(3*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 9.46 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\left(-\frac{\left(2Aa\,c^{3}-A\,b^{2}c^{2}-3Bab\,c^{2}+b^{3}Bc-2C\,a^{2}c^{2}+4Ca\,b^{2}c-b^{4}C\right)x^{3}}{3c^{5}\left(4ac-b^{2}\right)}+\frac{a\left(bA\,c^{2}+2Ba\,c^{2}-b^{2}Bc-3Cacb+b^{3}C\right)x}{3c^{5}\left(4ac-b^{2}\right)}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)^{2}}-2c\left(-\frac{(12\cdots\right.$
default	Expression too large to display

input
```
int(x^6*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3/c^5*(2*A*a*c^3-A*b^2*c^2-3*B*a*b*c^2+B*b^3*c-2*C*a^2*c^2+4*C*a*b^2*c
-C*b^4)/(4*a*c-b^2)*x^3+1/3*a/c^5*(A*b*c^2+2*B*a*c^2-B*b^2*c-3*C*a*b*c+C*b
^3)/(4*a*c-b^2)*x)*(c*x^4+b*x^2+a)^(1/2)/(x^4+1/c*b*x^2+1/c*a)^2-2*c*(-1/6
/c^3*(12*A*a*c^3+A*b^2*c^2-16*B*a*b*c^2+2*B*b^3*c-36*C*a^2*c^2+33*C*a*b^2*
c-5*C*b^4)/(4*a*c-b^2)^2*x^3-1/6*(4*A*a*b*c^3+A*b^3*c^2-28*B*a^2*c^3+7*B*a
*b^2*c^2-B*b^4*c+36*C*a^2*b*c^2-11*C*a*b^3*c+C*b^5)/(4*a*c-b^2)^2/c^4*x)/(
(x^4+1/c*b*x^2+1/c*a)*c)^(1/2)+1/4*((B*c-2*C*b)/c^3-1/3*(A*b*c^2+14*B*a*c^
2-4*B*b^2*c-27*C*a*b*c+7*C*b^3)/c^3/(4*a*c-b^2)-1/3/c^3*(4*A*a*b*c^3+A*b^3
*c^2-28*B*a^2*c^3+7*B*a*b^2*c^2-B*b^4*c+36*C*a^2*b*c^2-11*C*a*b^3*c+C*b^5)
/(4*a*c-b^2)^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+
b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(C/c^2-1/3*(12*A*a*c^3+A
*b^2*c^2-16*B*a*b*c^2+2*B*b^3*c-36*C*a^2*c^2+33*C*a*b^2*c-5*C*b^4)/c^2/(4*
a*c-b^2)^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*
x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ellipt
icE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2560 vs. $2(696) = 1392$.

Time = 0.13 (sec) , antiderivative size = 2560, normalized size of antiderivative = 3.30

$$\int \frac{x^6(A + Bx^2 + Cx^4)}{(a + bx^2 + cx^4)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(1/2)*((8*C*b^5*c^2 - 12*A*a*b*c^5 + (84*C*a^2*b + 16*B*a*b^2 -
A*b^3)*c^4 - (57*C*a*b^3 + 2*B*b^4)*c^3)*x^9 + 2*(8*C*b^6*c - 12*A*a*b^2*c
^4 + (84*C*a^2*b^2 + 16*B*a*b^3 - A*b^4)*c^3 - (57*C*a*b^4 + 2*B*b^5)*c^2)
*x^7 + (8*C*b^7 - 24*A*a^2*b*c^4 + 2*(84*C*a^3*b + 16*B*a^2*b^2 - 7*A*a*b^
3)*c^3 - (30*C*a^2*b^3 - 12*B*a*b^4 + A*b^5)*c^2 - (41*C*a*b^5 + 2*B*b^6)*
c)*x^5 + 2*(8*C*a*b^6 - 12*A*a^2*b^2*c^3 + (84*C*a^3*b^2 + 16*B*a^2*b^3 -
A*a*b^4)*c^2 - (57*C*a^2*b^4 + 2*B*a*b^5)*c)*x^3 + (8*C*a^2*b^5 - 12*A*a^3
*b*c^3 + (84*C*a^4*b + 16*B*a^3*b^2 - A*a^2*b^3)*c^2 - (57*C*a^3*b^3 + 2*B
*a^2*b^4)*c)*x - ((8*C*b^4*c^3 - 12*A*a*c^6 + (84*C*a^2 + 16*B*a*b - A*b^2
)*c^5 - (57*C*a*b^2 + 2*B*b^3)*c^4)*x^9 + 2*(8*C*b^5*c^2 - 12*A*a*b*c^5 +
(84*C*a^2*b + 16*B*a*b^2 - A*b^3)*c^4 - (57*C*a*b^3 + 2*B*b^4)*c^3)*x^7 +
(8*C*b^6*c - 24*A*a^2*c^5 + 2*(84*C*a^3 + 16*B*a^2*b - 7*A*a*b^2)*c^4 - (3
0*C*a^2*b^2 - 12*B*a*b^3 + A*b^4)*c^3 - (41*C*a*b^4 + 2*B*b^5)*c^2)*x^5 +
2*(8*C*a*b^5*c - 12*A*a^2*b*c^4 + (84*C*a^3*b + 16*B*a^2*b^2 - A*a*b^3)*c^
3 - (57*C*a^2*b^3 + 2*B*a*b^4)*c^2)*x^3 + (8*C*a^2*b^4*c - 12*A*a^3*c^4 +
(84*C*a^4 + 16*B*a^3*b - A*a^2*b^2)*c^3 - (57*C*a^3*b^2 + 2*B*a^2*b^3)*c^2
)*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)
/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x)
, 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((8*C
*b^5*c^2 - 4*((3*A - 5*B)*a*b + 2*A*b^2)*c^5 + (84*C*a^2*b + 8*(2*B - 3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(A + Bx^2 + Cx^4)}{(a + bx^2 + cx^4)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate(x**6*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^6}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^6/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^6}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^6/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{x^6\,(C\,x^4+B\,x^2+A)}{(c\,x^4+b\,x^2+a)^{5/2}}\,dx$$

input
```
int((x^6*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2),x)
```

output
```
int((x^6*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{2\sqrt{c\,x^4+b\,x^2+a}\,bx+\sqrt{c\,x^4+b\,x^2+a}\,c\,x^3-2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2}\right.}{}$$

input
```
int(x^6*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
(2*sqrt(a + b*x**2 + c*x**4)*b*x + sqrt(a + b*x**2 + c*x**4)*c*x**3 - 2*in
t(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 +
2*b*c*x**6 + c**2*x**8),x)*a**2*b - 2*int(sqrt(a + b*x**2 + c*x**4)/(a**2
+ 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b**2*
x**2 - 2*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b
**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b*c*x**4 - 3*int((sqrt(a + b*x**2
+ c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 +
 c**2*x**8),x)*a**2*c - 3*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a
*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b*c*x**2 -
 3*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 +
b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*c**2*x**4)/(c**2*(a + b*x**2 + c*
x**4))
```

3.159 $\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 658

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx =$$

$$-\frac{x(a(bBc-b^2C-2c(Ac-aC))+(b^2Bc-2aBc^2-b^3C-bc(Ac-3aC))\,x^2)}{3c^2\,(b^2-4ac)\,(a+bx^2+cx^4)^{3/2}}$$

$$+\frac{x(b^3Bc+4abBc^2-b^4C+4ac^2(Ac-7aC)-b^2c(5Ac-7aC)+c(b^2Bc+12aBc^2+2b^3C-8bc(Ac+2}{3c^2\,(b^2-4ac)^2\,\sqrt{a+bx^2+cx^4}}$$

$$-\frac{(b^2Bc+12aBc^2+2b^3C-8bc(Ac+2aC))\,x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\,(b^2-4ac)^2\,(\sqrt{a}+\sqrt{c}x^2)}$$

$$+\frac{\sqrt[4]{a}(b^2Bc+12aBc^2+2b^3C-8bc(Ac+2aC))\,(\sqrt{a}+\sqrt{c}x^2)\,\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{3c^{7/4}\,(b^2-4ac)^2\,\sqrt{a+bx^2+cx^4}}$$

$$+\frac{(3Abc^{3/2}+10a^{3/2}cC-3a\sqrt{c}(2Bc-bC)-\sqrt{a}(bBc-2Ac^2+2b^2C))\,(\sqrt{a}+\sqrt{c}x^2)\,\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{Ellipti}}{6\sqrt[4]{a}\,(b-2\sqrt{a}\sqrt{c})\,c^{7/4}\,(b^2-4ac)\,\sqrt{a+bx^2+cx^4}}$$

output
```
-1/3*x*(a*(B*b*c-b^2*C-2*c*(A*c-C*a))+(b^2*B*c-2*a*B*c^2-b^3*C-b*c*(A*c-3*
C*a))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(3/2)+1/3*x*(b^3*B*c+4*a*b*B*c
^2-b^4*C+4*a*c^2*(A*c-7*C*a)-b^2*c*(5*A*c-7*C*a)+c*(b^2*B*c+12*a*B*c^2+2*b
^3*C-8*b*c*(A*c+2*C*a))*x^2)/c^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/3*
(b^2*B*c+12*a*B*c^2+2*b^3*C-8*b*c*(A*c+2*C*a))*x*(c*x^4+b*x^2+a)^(1/2)/c^(
3/2)/(-4*a*c+b^2)^2/(a^(1/2)+c^(1/2)*x^2)+1/3*a^(1/4)*(b^2*B*c+12*a*B*c^2+
2*b^3*C-8*b*c*(A*c+2*C*a))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)
+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b
/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)+1/6*
(3*A*b*c^(3/2)+10*a^(3/2)*c*C-3*a*c^(1/2)*(2*B*c-C*b)-a^(1/2)*(-2*A*c^2+B*
b*c+2*C*b^2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)
^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(
1/2))^(1/2))/a^(1/4)/(b-2*a^(1/2)*c^(1/2))/c^(7/4)/(-4*a*c+b^2)/(c*x^4+b*x
^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.72 (sec) , antiderivative size = 638, normalized size of antiderivative = 0.97

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{4x((a+bx^2+cx^4)(-b^4C+b^2c(-5Ac+7aC+Bcx^2)+4ac^2(Ac-7aC+3}{}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
(4*x*((a + b*x^2 + c*x^4)*(-(b^4*C) + b^2*c*(-5*A*c + 7*a*C + B*c*x^2) + 4
*a*c^2*(A*c - 7*a*C + 3*B*c*x^2) + b^3*c*(B + 2*C*x^2) + 4*b*c^2*(-2*A*c*x
^2 + a*(B - 4*C*x^2))) + (b^2 - 4*a*c)*(-2*a^2*c*C + b*(-(b*B*c) + A*c^2 +
 b^2*C)*x^2 + a*(b^2*C + 2*c^2*(A + B*x^2) - b*c*(B + 3*C*x^2)))) - (I*Sqr
t[2]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*((-b + Sqrt[b^2
 - 4*a*c])*(b^2*B*c + 12*a*B*c^2 + 2*b^3*C - 8*b*c*(A*c + 2*a*C))*Elliptic
E[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] + (2*b^4*C + 4*a*c^2*(2*A*c - 3*B*Sqrt[b^2
- 4*a*c] + 10*a*C) - b^2*c*(2*A*c + B*Sqrt[b^2 - 4*a*c] + 18*a*C) + b^3*(B
*c - 2*Sqrt[b^2 - 4*a*c]*C) + 4*b*c*(-(a*B*c) + 2*A*c*Sqrt[b^2 - 4*a*c] +
4*a*Sqrt[b^2 - 4*a*c]*C))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])])/(12*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^(3/2
))
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2197, 25, 2206, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{\int -\frac{-3a\left(4a-\frac{b^2}{c}\right)Cx^4+3a\left(Ab+\frac{aCb}{c}-2aB\right)x^2+\frac{a^2\left(-Cb^2+Bcb-2c(Ac-aC)\right)}{c^2}}{(cx^4+bx^2+a)^{3/2}}dx}{3a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-3a\left(4a-\frac{b^2}{c}\right)Cx^4+3a\left(Ab+\frac{aCb}{c}-2aB\right)x^2+\frac{a^2\left(-Cb^2+Bcb-2c(Ac-aC)\right)}{c^2}}{\left(cx^4+bx^2+a\right)^{3/2}}dx}{3a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 2206

$$\frac{\frac{ax\left(-b^2c(5Ac-7aC)+cx^2\left(-8bc(2aC+Ac)+12aBc^2+2b^3C+b^2Bc\right)+4ac^2(Ac-7aC)+4abBc^2+b^4(-C)+b^3Bc\right)}{c^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\int -\frac{a^2\left(c\left(A\left(3b^2+4ac\right)-a\left(\frac{Cb}{c}\cdots\right.\right.\right.}{\cdots}}{\cdots}}{3a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{a^2\left(-\left(\left(2Cb^3+Bcb^2-8c(Ac+2aC)b+12aBc^2\right)x^2\right)+Ac\left(3b^2+4ac\right)-a\left(Cb^2+8Bcb-20acC\right)\right)}{c\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}+\frac{ax\left(-b^2c(5Ac-7aC)+cx^2\left(-8bc(2aC+Ac)+12aBc^2\cdots\right.\right.}{c^2\left(b^2-4a\cdots\right.}}{3a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{a\int \frac{-\left(\left(2Cb^3+Bcb^2-8c(Ac+2aC)b+12aBc^2\right)x^2\right)+Ac\left(3b^2+4ac\right)-a\left(Cb^2+8Bcb-20acC\right)}{\sqrt{cx^4+bx^2+a}}dx}{c\left(b^2-4ac\right)}+\frac{ax\left(-b^2c(5Ac-7aC)+cx^2\left(-8bc(2aC+Ac)+12aBc^2+2\cdots\right.\right.}{c^2\left(b^2-4ac\right)\cdots}}{3a\left(b^2-4ac\right)}-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 1511

$$\frac{\frac{a\left(\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(10a^{3/2}cC-\sqrt{a}\left(-2Ac^2+2b^2C+bBc\right)-3a\sqrt{c}(2Bc-bC)+3Abc^{3/2}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\sqrt{a}\left(-8bc(2aC+Ac)+12aBc^2+2b^3C+b^2Bc\right)\int \frac{\sqrt{a}-\cdots}{\sqrt{a}\sqrt{cx^4\cdots}}}{\sqrt{c}}\cdots\right.}{c\left(b^2-4ac\right)}\cdots}{3a\left(b^2-4ac\cdots\right.}-\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 27

$$a\left(\frac{\frac{(2\sqrt{a}\sqrt{c}+b)\left(10a^{3/2}cC-\sqrt{a}\left(-2Ac^2+2b^2C+bBc\right)-3a\sqrt{c}(2Bc-bC)+3Abc^{3/2}\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\left(-8bc(2aC+Ac)+12aBc^2+2b^3C+b^2Bc\right)\int\frac{\sqrt{a}-\sqrt{cx^2}}{\sqrt{cx^4+bx^2}}}{\sqrt{c}}}{c(b^2-4ac)}\right.$$

$$3a\left(b^2-4ac\right)$$

$$\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \ 1416$$

$$a\left(\frac{\frac{\left(-8bc(2aC+Ac)+12aBc^2+2b^3C+b^2Bc\right)\int\frac{\sqrt{a}-\sqrt{cx^2}}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{cx^2}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{cx^2}\right)^2}}\left(10a^{3/2}cC-\sqrt{a}\left(-2Ac^2+2b^2C+bBc\right)-3a\sqrt{c}(2Bc-\right.}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}}{c(b^2-4ac)}\right.$$

$$\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \ 1509$$

$$a\left(\frac{\frac{(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{cx^2}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{cx^2}\right)^2}}\left(10a^{3/2}cC-\sqrt{a}\left(-2Ac^2+2b^2C+bBc\right)-3a\sqrt{c}(2Bc-bC)+3Abc^{3/2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}}{c(b^2-4ac)}\right.$$

$$\frac{x\left(a\left(-2c(Ac-aC)+b^2(-C)+bBc\right)+x^2\left(-bc(Ac-3aC)-2aBc^2+b^3(-C)+b^2Bc\right)\right)}{3c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
-1/3*(x*(a*(b*B*c - b^2*C - 2*c*(A*c - a*C)) + (b^2*B*c - 2*a*B*c^2 - b^3*
C - b*c*(A*c - 3*a*C))*x^2))/(c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^(3/2))
 + ((a*x*(b^3*B*c + 4*a*b*B*c^2 - b^4*C + 4*a*c^2*(A*c - 7*a*C) - b^2*c*(5
*A*c - 7*a*C) + c*(b^2*B*c + 12*a*B*c^2 + 2*b^3*C - 8*b*c*(A*c + 2*a*C))*x
^2))/(c^2*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (a*(((b^2*B*c + 12*a*B*
c^2 + 2*b^3*C - 8*b*c*(A*c + 2*a*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[
a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c] +
 ((b + 2*Sqrt[a]*Sqrt[c])*(3*A*b*c^(3/2) + 10*a^(3/2)*c*C - 3*a*Sqrt[c]*(2
*B*c - b*C) - Sqrt[a]*(b*B*c - 2*A*c^2 + 2*b^2*C))*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(3/4)*Sqrt[
a + b*x^2 + c*x^4])))/(c*(b^2 - 4*a*c)))/(3*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 805, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\left(-\frac{\left(bA\,c^2+2Ba\,c^2-b^2Bc-3Cacb+b^3C\right)x^3}{3c^4\left(4ac-b^2\right)}-\frac{a\left(2A\,c^2-Bbc-2Cac+C\,b^2\right)x}{3c^4\left(4ac-b^2\right)}\right)\sqrt{c\,x^4+b\,x^2+a}}{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)^2}-\frac{2c\left(\frac{\left(8bA\,c^2-12Ba\,c^2-b^2Bc+16Cacb-2b^3\right.}{6c^2\left(4ac-b^2\right)^2}\right.}{}$
default	Expression too large to display

input
```
int(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3/c^4*(A*b*c^2+2*B*a*c^2-B*b^2*c-3*C*a*b*c+C*b^3)/(4*a*c-b^2)*x^3-1/3*
a/c^4*(2*A*c^2-B*b*c-2*C*a*c+C*b^2)/(4*a*c-b^2)*x)*(c*x^4+b*x^2+a)^(1/2)/(
x^4+1/c*b*x^2+1/c*a)^2-2*c*(1/6/c^2*(8*A*b*c^2-12*B*a*c^2-B*b^2*c+16*C*a*b
*c-2*C*b^3)/(4*a*c-b^2)^2*x^3-1/6*(4*A*a*c^3-5*A*b^2*c^2+4*B*a*b*c^2+B*b^3
*c-28*C*a^2*c^2+7*C*a*b^2*c-C*b^4)/(4*a*c-b^2)^2/c^3*x)/((x^4+1/c*b*x^2+1/
c*a)*c)^(1/2)+1/4*(C/c^2+1/3*(2*A*c^2-B*b*c-14*C*a*c+4*C*b^2)/c^2/(4*a*c-b
^2)-1/3/c^2*(4*A*a*c^3-5*A*b^2*c^2+4*B*a*b*c^2+B*b^3*c-28*C*a^2*c^2+7*C*a*
b^2*c-C*b^4)/(4*a*c-b^2)^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2
*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/6/c*(8*A*b*c^2
-12*B*a*c^2-B*b^2*c+16*C*a*b*c-2*C*b^3)/(4*a*c-b^2)^2*a*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/
2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. $2(579) = 1158$.

Time = 0.12 (sec) , antiderivative size = 2116, normalized size of antiderivative = 3.22

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(1/2)*((2*C*b^4*c^2 + 4*(3*B*a*b - 2*A*b^2)*c^4 - (16*C*a*b^2 -
B*b^3)*c^3)*x^8 + 2*C*a^2*b^4 + 2*(2*C*b^5*c + 4*(3*B*a*b^2 - 2*A*b^3)*c^3
 - (16*C*a*b^3 - B*b^4)*c^2)*x^6 + (2*C*b^6 + 8*(3*B*a^2*b - 2*A*a*b^2)*c^
3 - 2*(16*C*a^2*b^2 - 7*B*a*b^3 + 4*A*b^4)*c^2 - (12*C*a*b^4 - B*b^5)*c)*x
^4 + 4*(3*B*a^3*b - 2*A*a^2*b^2)*c^2 + 2*(2*C*a*b^5 + 4*(3*B*a^2*b^2 - 2*A
*a*b^3)*c^2 - (16*C*a^2*b^3 - B*a*b^4)*c)*x^2 - (16*C*a^3*b^2 - B*a^2*b^3)
*c - ((2*C*a*b^3*c^2 + 4*(3*B*a^2 - 2*A*a*b)*c^4 - (16*C*a^2*b - B*a*b^2)*
c^3)*x^8 + 2*C*a^3*b^3 + 2*(2*C*a*b^4*c + 4*(3*B*a^2*b - 2*A*a*b^2)*c^3 -
(16*C*a^2*b^2 - B*a*b^3)*c^2)*x^6 + (2*C*a*b^5 + 8*(3*B*a^3 - 2*A*a^2*b)*c
^3 - 2*(16*C*a^3*b - 7*B*a^2*b^2 + 4*A*a*b^3)*c^2 - (12*C*a^2*b^3 - B*a*b^
4)*c)*x^4 + 4*(3*B*a^4 - 2*A*a^3*b)*c^2 + 2*(2*C*a^2*b^4 + 4*(3*B*a^3*b -
2*A*a^2*b^2)*c^2 - (16*C*a^3*b^2 - B*a^2*b^3)*c)*x^2 - (16*C*a^4*b - B*a^3
*b^2)*c)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2)
- b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)
/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*(
(4*((A - 3*B)*a*b + 2*A*b^2)*c^4 + (20*C*a^2*b - 8*(B - 2*C)*a*b^2 + (3*A
- B)*b^3)*c^3 - (C*a*b^3 + 2*C*b^4)*c^2)*x^8 - C*a^3*b^3 - 2*C*a^2*b^4 + 2
*(4*((A - 3*B)*a*b^2 + 2*A*b^3)*c^3 + (20*C*a^2*b^2 - 8*(B - 2*C)*a*b^3 +
(3*A - B)*b^4)*c^2 - (C*a*b^4 + 2*C*b^5)*c)*x^6 - (C*a*b^5 + 2*C*b^6 - 8*(
(A - 3*B)*a^2*b + 2*A*a*b^2)*c^3 - 2*(20*C*a^3*b - 8*(B - 2*C)*a^2*b^2 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A + Bx^2 + Cx^4)}{(a + bx^2 + cx^4)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(x**4*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{x^4\,(C\,x^4+B\,x^2+A)}{(c\,x^4+b\,x^2+a)^{5/2}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{-\sqrt{c\,x^4+b\,x^2+a}\,x+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx\right)a^2+\left(\int \frac{}{c^2x^8+2bc\,x}\right.}{c\,(c\,x^4+b}$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4)*x + int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*
a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2 + int(
sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*
b*c*x**6 + c**2*x**8),x)*a*b*x**2 + int(sqrt(a + b*x**2 + c*x**4)/(a**2 +
2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*c*x**4)
/(c*(a + b*x**2 + c*x**4))
```

3.160 $\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 627

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = -\frac{x(Abc-2aBc+abC-(bBc-b^2C-2c(Ac-aC))x^2)}{3c(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$
$$+\frac{x(Abc(b^2+4ac)-a(5b^2Bc-4aBc^2-b^3C-4abcC)+c(Ac(b^2+12ac)-a(8bBc-b^2C-12acC))x^2)}{3ac(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(Ac(b^2+12ac)-a(8bBc-b^2C-12acC))x\sqrt{a+bx^2+cx^4}}{3a\sqrt{c}(b^2-4ac)^2(\sqrt{a}+\sqrt{c}x^2)}$$
$$+\frac{(Ac(b^2+12ac)-a(8bBc-b^2C-12acC))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right.\right.}{3a^{3/4}c^{3/4}(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\left(Abc-3\sqrt{a}\sqrt{c}(bB-2Ac)+6a^{3/2}\sqrt{c}C-a(2Bc-bC)\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\right.}{6a^{3/4}(b-2\sqrt{a}\sqrt{c})c^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

output
```
-1/3*x*(A*b*c-2*a*B*c+a*b*C-(B*b*c-b^2*C-2*c*(A*c-C*a))*x^2)/c/(-4*a*c+b^2
)/(c*x^4+b*x^2+a)^(3/2)+1/3*x*(A*b*c*(4*a*c+b^2)-a*(-4*B*a*c^2+5*B*b^2*c-4
*C*a*b*c-C*b^3)+c*(A*c*(12*a*c+b^2)-a*(8*B*b*c-12*C*a*c-C*b^2))*x^2)/a/c/(
-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/3*(A*c*(12*a*c+b^2)-a*(8*B*b*c-12*C*
a*c-C*b^2))*x*(c*x^4+b*x^2+a)^(1/2)/a/c^(1/2)/(-4*a*c+b^2)^2/(a^(1/2)+c^(1
/2)*x^2)+1/3*(A*c*(12*a*c+b^2)-a*(8*B*b*c-12*C*a*c-C*b^2))*(a^(1/2)+c^(1/2
)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/c^(3/4)/(
-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/6*(A*b*c-3*a^(1/2)*c^(1/2)*(-2*A*c+B
*b)+6*a^(3/2)*c^(1/2)*C-a*(2*B*c-C*b))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2
+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1
/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/c^(3/4
)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.87 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.02

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{i\left(4i\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a(b^2-4ac)\,x(b(-Bc+bC)x^2+Ac(b+2cx^2)+a(-2Bc+\right.}{}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
((I/12)*((4*I)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a*(b^2 - 4*a*c)*x*(b*(-(B*
c) + b*C)*x^2 + A*c*(b + 2*c*x^2) + a*(-2*B*c + b*C - 2*c*C*x^2)) - x*(a +
 b*x^2 + c*x^4)*(A*c*(b^3 + 4*a*b*c + b^2*c*x^2 + 12*a*c^2*x^2) + a*(b^3*C
 + 4*b*c*(a*C - 2*B*c*x^2) + 4*a*c^2*(B + 3*C*x^2) + b^2*(-5*B*c + c*C*x^2
)))) + Sqrt[2]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(-((-
b + Sqrt[b^2 - 4*a*c])*(A*c*(b^2 + 12*a*c) + a*(-8*b*B*c + b^2*C + 12*a*c*
C))*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (A*c*(-b^3 + 4*a*b*c + b^2*S
qrt[b^2 - 4*a*c] + 12*a*c*Sqrt[b^2 - 4*a*c]) + a*(-(b^3*C) + b*(-8*B*c*Sqr
t[b^2 - 4*a*c] + 4*a*c*C) + b^2*(2*B*c + Sqrt[b^2 - 4*a*c]*C) + 4*a*c*(-2*
B*c + 3*Sqrt[b^2 - 4*a*c]*C)))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])))/(
a*c*(b^2 - 4*a*c)^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)^(3
/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 585, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2197, 25, 27, 1492, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}} dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{\int -\frac{a(3c(bB-2(Ac+aC))x^2+Abc-2aBc+abC)}{c(cx^4+bx^2+a)^{3/2}}dx}{3a(b^2-4ac)}-\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{3c(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(3c(bB-2(Ac+aC))x^2+Abc-2aBc+abC)}{c(cx^4+bx^2+a)^{3/2}}dx}{3a(b^2-4ac)}-\frac{x(-(x^2(-2c(Ac-aC)+b^2(-C)+bBc))+abC-2aBc+Abc)}{3c(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{3c(bB-2(Ac+aC))x^2+Abc-2aBc+abC}{(cx^4+bx^2+a)^{3/2}}dx}{3c(b^2-4ac)}-\frac{x(-(x^2(-2c(Ac-aC)+b^2(-C)+bBc))+abC-2aBc+Abc)}{3c(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$\Big\downarrow$ 1492

$$\frac{\frac{x(cx^2(Ac(12ac+b^2)-a(-12acC+b^2(-C)+8bBc))+Abc(4ac+b^2)-a(-4abcC-4aBc^2+b^3(-C)+5b^2Bc))}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\int -\frac{c\left(a\left(3Bb^2-8(Ac+aC)b+4aBc\right)\right.}{}}{3c(b^2-4ac)}-\frac{x(-(x^2(-2c(Ac-aC)+b^2(-C)+bBc))+abC-2aBc+Abc)}{3c(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{c\left(a\left(3Bb^2-8(Ac+aC)b+4aBc\right)-\left(Ac\left(b^2+12ac\right)-a\left(-Cb^2+8Bcb-12acC\right)\right)x^2\right)}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}+\frac{x(cx^2(Ac(12ac+b^2)-a(-12acC+b^2(-C)+8bBc))+Abc(4ac+}{a(b^2-4ac)\sqrt{a+bx^2+}}}{3c(b^2-4ac)}-\frac{x(-(x^2(-2c(Ac-aC)+b^2(-C)+bBc))+abC-2aBc+Abc)}{3c(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{c\int \frac{a\left(3Bb^2-8(Ac+aC)b+4aBc\right)-\left(Ac\left(b^2+12ac\right)-a\left(-Cb^2+8Bcb-12acC\right)\right)x^2}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}+\frac{x(cx^2(Ac(12ac+b^2)-a(-12acC+b^2(-C)+8bBc))+Abc(4ac+b}{a(b^2-4ac)\sqrt{a+bx^2+cx}}}{3c(b^2-4ac)}-\frac{x(-(x^2(-2c(Ac-aC)+b^2(-C)+bBc))+abC-2aBc+Abc)}{3c(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$\Big\downarrow$ 1511

$$\frac{c\left(\frac{\sqrt{a}\left(Ac\left(12ac+b^2\right)-a\left(-12acC+b^2(-C)+8bBc\right)\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(6a^{3/2}\sqrt{c}C-3\sqrt{a}\sqrt{c}(bB-2Ac)-a(2Bc-bC)+Abc\right)\int \frac{1}{\sqrt{cx^4+bx^2+}}}{\sqrt{c}}}{a(b^2-4ac)}$$

$$3c\left(b^2-4ac\right)$$

$$\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{3c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{c\left(\frac{\left(Ac\left(12ac+b^2\right)-a\left(-12acC+b^2(-C)+8bBc\right)\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(6a^{3/2}\sqrt{c}C-3\sqrt{a}\sqrt{c}(bB-2Ac)-a(2Bc-bC)+Abc\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a(b^2-4ac)}$$

$$3c\left(b^2-4ac\right)$$

$$\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{3c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 1416$$

$$\frac{c\left(\frac{\left(Ac\left(12ac+b^2\right)-a\left(-12acC+b^2(-C)+8bBc\right)\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6a^{3/2}\sqrt{c}C-3\sqrt{a}\sqrt{c}(bB-2Ac)-a(2Bc-\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}}{a(b^2-4ac)}$$

$$\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{3c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 1509$$

$$\frac{c\left(\frac{\left(Ac\left(12ac+b^2\right)-a\left(-12acC+b^2(-C)+8bBc\right)\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}-\frac{\sqrt[4]{a}(2\sqrt{a}\cdots}{\cdots}\right.}{a(b^2-4ac)}$$

$$\frac{x\left(-\left(x^2\left(-2c(Ac-aC)+b^2(-C)+bBc\right)\right)+abC-2aBc+Abc\right)}{3c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
-1/3*(x*(A*b*c - 2*a*B*c + a*b*C - (b*B*c - b^2*C - 2*c*(A*c - a*C))*x^2))
/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^(3/2)) + ((x*(A*b*c*(b^2 + 4*a*c) -
a*(5*b^2*B*c - 4*a*B*c^2 - b^3*C - 4*a*b*c*C) + c*(A*c*(b^2 + 12*a*c) - a*
(8*b*B*c - b^2*C - 12*a*c*C))*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^
4]) + (c*(((A*c*(b^2 + 12*a*c) - a*(8*b*B*c - b^2*C - 12*a*c*C))*(-((x*Sqr
t[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a +
 b*x^2 + c*x^4])))/Sqrt[c] - (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(A*b*c - 3*S
qrt[a]*Sqrt[c]*(b*B - 2*A*c) + 6*a^(3/2)*Sqrt[c]*C - a*(2*B*c - b*C))*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^
(3/4)*Sqrt[a + b*x^2 + c*x^4])))/(a*(b^2 - 4*a*c)))/(3*c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\left(\frac{\left(2A\,c^2-Bbc-2Cac+C\,b^2\right)x^3}{3c^3\left(4ac-b^2\right)}+\frac{(Abc-2Bac+Cba)x}{3c^3\left(4ac-b^2\right)}\right)\sqrt{c\,x^4+b\,x^2+a}}{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)^2}-\frac{2c\left(-\frac{\left(12A\,c^2a+A\,b^2c-8Babc+12C\,a^2c+Ca\,b^2\right)x^3}{6ca\left(4ac-b^2\right)^2}-\frac{\left(4Aab\,c^2+\right.}{}\right.}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}$
default	Expression too large to display

input
```
int(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3/c^3*(2*A*c^2-B*b*c-2*C*a*c+C*b^2)/(4*a*c-b^2)*x^3+1/3/c^3*(A*b*c-2*B*
a*c+C*a*b)/(4*a*c-b^2)*x)*(c*x^4+b*x^2+a)^(1/2)/(x^4+1/c*b*x^2+1/c*a)^2-2*
c*(-1/6/c*(12*A*a*c^2+A*b^2*c-8*B*a*b*c+12*C*a^2*c+C*a*b^2)/a/(4*a*c-b^2)^
2*x^3-1/6*(4*A*a*b*c^2+A*b^3*c+4*B*a^2*c^2-5*B*a*b^2*c+4*C*a^2*b*c+C*a*b^3
)/a/(4*a*c-b^2)^2/c^2*x)/((x^4+1/c*b*x^2+1/c*a)*c)^(1/2)+1/4*(-1/3/c/(4*a*
c-b^2)*(A*b*c-2*B*a*c+C*a*b)/a-1/3*(4*A*a*b*c^2+A*b^3*c+4*B*a^2*c^2-5*B*a*
b^2*c+4*C*a^2*b*c+C*a*b^3)/(4*a*c-b^2)^2/c/a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)+1/6*(12*A*a*c^2+A*b^2*c-8*B*a*b*c+12*C*a^2*c+C*a*b^2)/(4*a*c-b^2)^2*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1985 vs. $2(535) = 1070$.

Time = 0.12 (sec) , antiderivative size = 1985, normalized size of antiderivative = 3.17

$$\int \frac{x^2(A + Bx^2 + Cx^4)}{(a + bx^2 + cx^4)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(1/2)*((C*a*b^3*c^2 + 12*A*a*b*c^4 + (12*C*a^2*b - 8*B*a*b^2 + A
*b^3)*c^3)*x^8 + C*a^3*b^3 + 12*A*a^3*b*c^2 + 2*(C*a*b^4*c + 12*A*a*b^2*c^
3 + (12*C*a^2*b^2 - 8*B*a*b^3 + A*b^4)*c^2)*x^6 + (C*a*b^5 + 24*A*a^2*b*c^
3 + 2*(12*C*a^3*b - 8*B*a^2*b^2 + 7*A*a*b^3)*c^2 + (14*C*a^2*b^3 - 8*B*a*b
^4 + A*b^5)*c)*x^4 + 2*(C*a^2*b^4 + 12*A*a^2*b^2*c^2 + (12*C*a^3*b^2 - 8*B
*a^2*b^3 + A*a*b^4)*c)*x^2 + (12*C*a^4*b - 8*B*a^3*b^2 + A*a^2*b^3)*c - ((
C*a^2*b^2*c^2 + 12*A*a^2*c^4 + (12*C*a^3 - 8*B*a^2*b + A*a*b^2)*c^3)*x^8 +
 C*a^4*b^2 + 12*A*a^4*c^2 + 2*(C*a^2*b^3*c + 12*A*a^2*b*c^3 + (12*C*a^3*b
- 8*B*a^2*b^2 + A*a*b^3)*c^2)*x^6 + (C*a^2*b^4 + 24*A*a^3*c^3 + 2*(12*C*a^
4 - 8*B*a^3*b + 7*A*a^2*b^2)*c^2 + (14*C*a^3*b^2 - 8*B*a^2*b^3 + A*a*b^4)*
c)*x^4 + 2*(C*a^3*b^3 + 12*A*a^3*b*c^2 + (12*C*a^4*b - 8*B*a^3*b^2 + A*a^2
*b^3)*c)*x^2 + (12*C*a^5 - 8*B*a^4*b + A*a^3*b^2)*c)*sqrt((b^2 - 4*a*c)/a^
2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt
(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a
*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((12*A*a*b*c^4 - (4*(B - 3*C)*a
^2*b - 8*(A - B)*a*b^2 - A*b^3)*c^3 + (8*C*a^2*b^2 - (3*B - C)*a*b^3)*c^2)
*x^8 + 8*C*a^4*b^2 - (3*B - C)*a^3*b^3 + 12*A*a^3*b*c^2 + 2*(12*A*a*b^2*c^
3 - (4*(B - 3*C)*a^2*b^2 - 8*(A - B)*a*b^3 - A*b^4)*c^2 + (8*C*a^2*b^3 - (
3*B - C)*a*b^4)*c)*x^6 + (8*C*a^2*b^4 - (3*B - C)*a*b^5 + 24*A*a^2*b*c^3 -
 2*(4*(B - 3*C)*a^3*b - 8*(A - B)*a^2*b^2 - 7*A*a*b^3)*c^2 + (16*C*a^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{x^2\,(C\,x^4+B\,x^2+A)}{(c\,x^4+b\,x^2+a)^{5/2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**
2*x**4 + 2*b*c*x**6 + c**2*x**8),x)
```

3.161 $\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 648

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{x(Ab^2-abB-2aAc+2a^2C+(Abc-2aBc+abC)x^2)}{3a(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$
$$+\frac{x(A(2b^4-17ab^2c+20a^2c^2)+a(b^3B+4abBc-5ab^2C+4a^2cC)+c(2A(b^3-8abc)+a(b^2B+12aBc-}{3a^2(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt{c}(2A(b^3-8abc)+a(b^2B+12aBc-8abC))\,x\sqrt{a+bx^2+cx^4}}{3a^2(b^2-4ac)^2(\sqrt{a}+\sqrt{c}x^2)}$$
$$+\frac{\sqrt[4]{c}(2A(b^3-8abc)+a(b^2B+12aBc-8abC))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{}{\sqrt{}}\right.\right.}{3a^{7/4}(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\left(A\left(2b^2\sqrt{c}-3\sqrt{a}bc-10ac^{3/2}\right)+a\left(bB\sqrt{c}+6\sqrt{a}Bc-3\sqrt{a}bC-2a\sqrt{c}C\right)\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\mathrm{E}}{6a^{7/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\,(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

output
```
1/3*x*(A*b^2-B*a*b-2*A*a*c+2*C*a^2+(A*b*c-2*B*a*c+C*a*b)*x^2)/a/(-4*a*c+b^
2)/(c*x^4+b*x^2+a)^(3/2)+1/3*x*(A*(20*a^2*c^2-17*a*b^2*c+2*b^4)+a*(4*B*a*b
*c+B*b^3+4*C*a^2*c-5*C*a*b^2)+c*(2*A*(-8*a*b*c+b^3)+a*(12*B*a*c+B*b^2-8*C*
a*b))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/3*c^(1/2)*(2*A*(-8*a
*b*c+b^3)+a*(12*B*a*c+B*b^2-8*C*a*b))*x*(c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+
b^2)^2/(a^(1/2)+c^(1/2)*x^2)+1/3*c^(1/4)*(2*A*(-8*a*b*c+b^3)+a*(12*B*a*c+B
*b^2-8*C*a*b))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2
)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(
1/2))^(1/2))/a^(7/4)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/6*(A*(2*b^2*c^
(1/2)-3*a^(1/2)*b*c-10*a*c^(3/2))+a*(b*B*c^(1/2)+6*a^(1/2)*B*c-3*a^(1/2)*b
*C-2*a*c^(1/2)*C))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)
*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2
)/c^(1/2))^(1/2))/a^(7/4)/(b-2*a^(1/2)*c^(1/2))/c^(1/4)/(-4*a*c+b^2)/(c*x^
4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.41 (sec) , antiderivative size = 634, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{4(a(b^2-4ac)\,x(A(b^2-2ac+bcx^2)+a(-bB+2aC-2Bcx^2+bCx^2))+x(a}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
(4*(a*(b^2 - 4*a*c)*x*(A*(b^2 - 2*a*c + b*c*x^2) + a*(-(b*B) + 2*a*C - 2*B
*c*x^2 + b*C*x^2)) + x*(a + b*x^2 + c*x^4)*(A*(2*b^4 - 17*a*b^2*c + 20*a^2
*c^2 + 2*b^3*c*x^2 - 16*a*b*c^2*x^2) + a*(b^3*B + b^2*(-5*a*C + B*c*x^2) +
 4*a*c*(a*C + 3*B*c*x^2) + 4*a*b*c*(B - 2*C*x^2)))) - (I*Sqrt[2]*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*((-b + Sqrt[b^2 - 4*a*c])*(2*
A*(b^3 - 8*a*b*c) + a*(b^2*B + 12*a*B*c - 8*a*b*C))*EllipticE[I*ArcSinh[Sq
rt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c])] + (-2*A*(-b^4 + 9*a*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4
*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(b^3*B + 4*a*c*(-3*B*Sqrt[b^2 - 4*a
*c] + 2*a*C) - b^2*(B*Sqrt[b^2 - 4*a*c] + 2*a*C) + a*b*(-4*B*c + 8*Sqrt[b^
2 - 4*a*c]*C)))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])])/(12*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 604, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2206, 25, 1492, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{5/2}}\,dx$$

$$\Big\downarrow \; 2206$$

$$\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}-\frac{\int -\frac{3(Abc-2aBc+abC)x^2+2A\left(b^2-5ac\right)+a(bB-2aC)}{\left(cx^4+bx^2+a\right)^{3/2}}dx}{3a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3(Abc-2aBc+abC)x^2+2A\left(b^2-5ac\right)+a(bB-2aC)}{\left(cx^4+bx^2+a\right)^{3/2}}dx}{3a\left(b^2-4ac\right)}+\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\downarrow$ 1492

$$-\frac{-\frac{\int \frac{c\left(2A\left(b^3-8abc\right)+a\left(Bb^2-8aCb+12aBc\right)\right)x^2+a\left(Ac\left(b^2-20ac\right)+a\left(-3Cb^2+8Bcb-4acC\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}-\frac{x(-(b^2-2ac)(2A(b^2-5ac)+a(bB-2aC))-cx^2(2A}{a(b^2-}}{3a\left(b^2-4ac\right)}$$

$$\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\downarrow$ 1511

$$-\frac{\sqrt{a}(\sqrt{a}(Ac(b^2-20ac)+a(-4acC-3b^2C+8bBc))+\sqrt{c}(2A(b^3-8abc)+a(-8abC+12aBc+b^2B)))\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{a}\sqrt{c}(2A(b^3-8abc)+a}{a(b^2-4ac)}$$

$$\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\downarrow$ 27

$$-\frac{\sqrt{a}(\sqrt{a}(Ac(b^2-20ac)+a(-4acC-3b^2C+8bBc))+\sqrt{c}(2A(b^3-8abc)+a(-8abC+12aBc+b^2B)))\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}(2A(b^3-8abc)+a(-}{a(b^2-4ac)}$$

3

$$\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\downarrow$ 1416

$$-\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\left(Ac\left(b^2-20ac\right)+a\left(-4acC-3b^2C+8bBc\right)\right)+\sqrt{c}\left(2A\left(b^3-8abc\right)+a\left(-8abC+12aBc+b^2B\right)\right)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}}{a(b^2-4ac)}$$

$$\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\downarrow$ 1509

$$\frac{x\left(2a^2C+x^2(abC-2aBc+Abc)-2aAc-abB+Ab^2\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}+$$

$$-\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\left(Ac\left(b^2-20ac\right)+a\left(-4acC-3b^2C+8bBc\right)\right)+\sqrt{c}\left(2A\left(b^3-8abc\right)+a\left(-8abC+12aBc+b^2B\right)\right)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}}{}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
(x*(A*b^2 - a*b*B - 2*a*A*c + 2*a^2*C + (A*b*c - 2*a*B*c + a*b*C)*x^2))/(3
*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^(3/2)) + (-((x*(3*a*b*(A*b*c - 2*a*B*
c + a*b*C) - (b^2 - 2*a*c)*(2*A*(b^2 - 5*a*c) + a*(b*B - 2*a*C)) - c*(2*A*
(b^3 - 8*a*b*c) + a*(b^2*B + 12*a*B*c - 8*a*b*C))*x^2))/(a*(b^2 - 4*a*c)*S
qrt[a + b*x^2 + c*x^4])) - (-(Sqrt[c]*(2*A*(b^3 - 8*a*b*c) + a*(b^2*B + 12
*a*B*c - 8*a*b*C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2))
 + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqr
t[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))) + (a^(1/4)*(Sqrt[c]*(2*A*(b
^3 - 8*a*b*c) + a*(b^2*B + 12*a*B*c - 8*a*b*C)) + Sqrt[a]*(A*c*(b^2 - 20*a
*c) + a*(8*b*B*c - 3*b^2*C - 4*a*c*C)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
)/(a*(b^2 - 4*a*c)))/(3*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 790, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\left(-\frac{(Abc-2Bac+Cba)x^3}{3c^2a\left(4ac-b^2\right)}+\frac{\left(2Aac-A\,b^2+Bab-2a^2C\right)x}{3c^2a\left(4ac-b^2\right)}\right)\sqrt{c\,x^4+b\,x^2+a}}{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)^2}-\frac{2c\left(\frac{\left(16Aabc-2A\,b^3-12B\,a^2c-Ba\,b^2+8C\,a^2b\right)x^3}{6a^2\left(4ac-b^2\right)^2}-\frac{\left(20A\,a^2c^2\right.}{}\right.}{\sqrt{\left(x^4+\frac{b\,x^2}{c}\right.}}$
default	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3/c^2*(A*b*c-2*B*a*c+C*a*b)/a/(4*a*c-b^2)*x^3+1/3*(2*A*a*c-A*b^2+B*a*b
-2*C*a^2)/c^2/a/(4*a*c-b^2)*x)*(c*x^4+b*x^2+a)^(1/2)/(x^4+1/c*b*x^2+1/c*a)
^2-2*c*(1/6*(16*A*a*b*c-2*A*b^3-12*B*a^2*c-B*a*b^2+8*C*a^2*b)/a^2/(4*a*c-b
^2)^2*x^3-1/6*(20*A*a^2*c^2-17*A*a*b^2*c+2*A*b^4+4*B*a^2*b*c+B*a*b^3+4*C*a
^3*c-5*C*a^2*b^2)/a^2/(4*a*c-b^2)^2/c*x)/((x^4+1/c*b*x^2+1/c*a)*c)^(1/2)+1
/4*(1/3/(4*a*c-b^2)*(10*A*a*c-2*A*b^2-B*a*b+2*C*a^2)/a^2-1/3*(20*A*a^2*c^2
-17*A*a*b^2*c+2*A*b^4+4*B*a^2*b*c+B*a*b^3+4*C*a^3*c-5*C*a^2*b^2)/a^2/(4*a*
c-b^2)^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/6*c*(16*A*a*b*c-2*A*b^3-12*B*a^2
*c-B*a*b^2+8*C*a^2*b)/a/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(
1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2198 vs. $2(571) = 1142$.

Time = 0.12 (sec) , antiderivative size = 2198, normalized size of antiderivative = 3.39

$$\int \frac{A + Bx^2 + Cx^4}{(a + bx^2 + cx^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(1/2)*((4*(3*B*a^2*b - 4*A*a*b^2)*c^4 - (8*C*a^2*b^2 - B*a*b^3 -
 2*A*b^4)*c^3)*x^8 + 2*(4*(3*B*a^2*b^2 - 4*A*a*b^3)*c^3 - (8*C*a^2*b^3 - B
*a*b^4 - 2*A*b^5)*c^2)*x^6 + (8*(3*B*a^3*b - 4*A*a^2*b^2)*c^3 - 2*(8*C*a^3
*b^2 - 7*B*a^2*b^3 + 6*A*a*b^4)*c^2 - (8*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c)
*x^4 + 4*(3*B*a^4*b - 4*A*a^3*b^2)*c^2 + 2*(4*(3*B*a^3*b^2 - 4*A*a^2*b^3)*
c^2 - (8*C*a^3*b^3 - B*a^2*b^4 - 2*A*a*b^5)*c)*x^2 - (8*C*a^4*b^2 - B*a^3*
b^3 - 2*A*a^2*b^4)*c - ((4*(3*B*a^3 - 4*A*a^2*b)*c^4 - (8*C*a^3*b - B*a^2*
b^2 - 2*A*a*b^3)*c^3)*x^8 + 2*(4*(3*B*a^3*b - 4*A*a^2*b^2)*c^3 - (8*C*a^3*
b^2 - B*a^2*b^3 - 2*A*a*b^4)*c^2)*x^6 + (8*(3*B*a^4 - 4*A*a^3*b)*c^3 - 2*(
8*C*a^4*b - 7*B*a^3*b^2 + 6*A*a^2*b^3)*c^2 - (8*C*a^3*b^3 - B*a^2*b^4 - 2*
A*a*b^5)*c)*x^4 + 4*(3*B*a^5 - 4*A*a^4*b)*c^2 + 2*(4*(3*B*a^4*b - 4*A*a^3*
b^2)*c^2 - (8*C*a^4*b^2 - B*a^3*b^3 - 2*A*a^2*b^4)*c)*x^2 - (8*C*a^5*b - B
*a^4*b^2 - 2*A*a^3*b^3)*c)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((
b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2
- 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a
*c)) + sqrt(1/2)*((3*C*a^2*b^3*c^2 + 4*((5*A - 3*B)*a^2*b + 4*A*a*b^2)*c^4
 + (4*C*a^3*b - 8*(B - C)*a^2*b^2 - (A + B)*a*b^3 - 2*A*b^4)*c^3)*x^8 + 3*
C*a^4*b^3 + 2*(3*C*a^2*b^4*c + 4*((5*A - 3*B)*a^2*b^2 + 4*A*a*b^3)*c^3 + (
4*C*a^3*b^2 - 8*(B - C)*a^2*b^3 - (A + B)*a*b^4 - 2*A*b^5)*c^2)*x^6 + (3*C
*a^2*b^5 + 8*((5*A - 3*B)*a^3*b + 4*A*a^2*b^2)*c^3 + 2*(4*C*a^4*b - 8*(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{(c\,x^4+b\,x^2+a)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^2 + c*x^4)^(5/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a + b*x^2 + c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int((C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4
+ 2*b*c*x**6 + c**2*x**8),x)
```

3.162 $\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 787

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{5/2}}\,dx =$$

$$-\frac{x\left(a\left(\frac{Ab^3}{a}-b(bB+3Ac)+a(2Bc+bC)\right)+c\left(Ab^2-abB-2aAc+2a^2C\right)x^2\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$-\frac{x\left(a\left(\frac{5Ab^5}{a}-b^3(2bB+37Ac)-4a^2c(5Bc+bC)+ab\left(17bBc+60Ac^2-b^2C\right)\right)+c\left(A\left(5b^4-33ab^2c+36\right.\right.}{3a^3\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}}$$

$$-\frac{A\sqrt{a+bx^2+cx^4}}{a^3x}$$

$$+\frac{\sqrt{c}\left(A\left(8b^4-57ab^2c+84a^2c^2\right)-a\left(2b^3B-16abBc+ab^2C+12a^2cC\right)\right)x\sqrt{a+bx^2+cx^4}}{3a^3\left(b^2-4ac\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{\sqrt[4]{c}\left(A\left(8b^4-57ab^2c+84a^2c^2\right)-a\left(2b^3B-16abBc+ab^2C+12a^2cC\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\,a\right.}{3a^{11/4}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt[4]{c}\left(A\left(8b^3-12\sqrt{a}b^2\sqrt{c}-33abc+42a^{3/2}c^{3/2}\right)-a\left(2b^2B-3\sqrt{a}bB\sqrt{c}-10aBc+abC+6a^{3/2}\sqrt{c}C\right)\right)\left(\sqrt{\right.}{6a^{11/4}\left(b-2\sqrt{a}\sqrt{c}\right)\left(b^2-4ac\right)\sqrt{a+bx^2}}$$

output
```
-1/3*x*(a*(A*b^3/a-b*(3*A*c+B*b)+a*(2*B*c+C*b))+c*(-2*A*a*c+A*b^2-B*a*b+2*
C*a^2)*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(3/2)-1/3*x*(a*(5*A*b^5/a-b^3
*(37*A*c+2*B*b)-4*a^2*c*(5*B*c+C*b)+a*b*(60*A*c^2+17*B*b*c-C*b^2))+c*(A*(3
6*a^2*c^2-33*a*b^2*c+5*b^4)-a*(-16*B*a*b*c+2*B*b^3+12*C*a^2*c+C*a*b^2))*x^
2)/a^3/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-A*(c*x^4+b*x^2+a)^(1/2)/a^3/x+
1/3*c^(1/2)*(A*(84*a^2*c^2-57*a*b^2*c+8*b^4)-a*(-16*B*a*b*c+2*B*b^3+12*C*a
^2*c+C*a*b^2))*x*(c*x^4+b*x^2+a)^(1/2)/a^3/(-4*a*c+b^2)^2/(a^(1/2)+c^(1/2)
*x^2)-1/3*c^(1/4)*(A*(84*a^2*c^2-57*a*b^2*c+8*b^4)-a*(-16*B*a*b*c+2*B*b^3+
12*C*a^2*c+C*a*b^2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/
2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/
2)/c^(1/2))^(1/2))/a^(11/4)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)+1/6*c^(1/
4)*(A*(8*b^3-12*a^(1/2)*b^2*c^(1/2)-33*a*b*c+42*a^(3/2)*c^(3/2))-a*(2*b^2*
B-3*a^(1/2)*b*B*c^(1/2)-10*a*B*c+a*b*C+6*a^(3/2)*c^(1/2)*C))*(a^(1/2)+c^(1
/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(11/4)/(b-2*
a^(1/2)*c^(1/2))/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.42 (sec) , antiderivative size = 827, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{5/2}}\,dx = \frac{4\left(-8Ab^4x^4\left(b+cx^2\right)^2+4a^4c\left(-12Ac+x^2\left(7Bc+2bC+5cCx^2\right)\right)+ab^2x^2\left(b+cx^2\right)\left(2bBx^2\left(b+cx^2\right)+A\left(-12b\right.\right.\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*(a + b*x^2 + c*x^4)^(5/2)),x]
```

output
```
((4*(-8*A*b^4*x^4*(b + c*x^2)^2 + 4*a^4*c*(-12*A*c + x^2*(7*B*c + 2*b*C +
5*c*C*x^2)) + a*b^2*x^2*(b + c*x^2)*(2*b*B*x^2*(b + c*x^2) + A*(-12*b^2 +
61*b*c*x^2 + 57*c^2*x^4)) + a^3*c*(16*b*c*C*x^6 + 4*c^2*x^6*(5*B + 3*C*x^2
) + 4*A*(6*b^2 - 42*b*c*x^2 - 35*c^2*x^4) + b^2*(-23*B*x^2 + 3*C*x^4)) + a
^2*(-(A*(3*b^4 - 92*b^3*c*x^2 + 21*b^2*c^2*x^4 + 192*b*c^3*x^6 + 84*c^4*x^
8)) + b*x^2*(b + c*x^2)*(-16*B*c^2*x^4 + b^2*(3*B + C*x^2) + b*(-17*B*c*x^
2 + c*C*x^4)))))/x + (I*Sqrt[2]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(a + b*x
^2 + c*x^4)*((-b + Sqrt[b^2 - 4*a*c])*(A*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2)
 - a*(2*b^3*B - 16*a*b*B*c + a*b^2*C + 12*a^2*c*C))*EllipticE[I*ArcSinh[Sq
rt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c])] + (A*(8*b^5 - 65*a*b^3*c + 132*a^2*b*c^2 - 8*b^4*Sqrt[b^2
 - 4*a*c] + 57*a*b^2*c*Sqrt[b^2 - 4*a*c] - 84*a^2*c^2*Sqrt[b^2 - 4*a*c]) +
 a*(-2*b^4*B + b^3*(2*B*Sqrt[b^2 - 4*a*c] - a*C) + 4*a*b*c*(-4*B*Sqrt[b^2
- 4*a*c] + a*C) + a*b^2*(18*B*c + Sqrt[b^2 - 4*a*c]*C) + 4*a^2*c*(-10*B*c
+ 3*Sqrt[b^2 - 4*a*c]*C)))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]])/(12*a^3*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^(3/
2))
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 857, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2198, 25, 2198, 25, 2199, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{2198}$$

$$-\frac{\int -\frac{-\frac{3c\left(A\left(b^2-2ac\right)-a(bB-2aC)\right)x^4}{a}-\frac{\left(A\left(2b^3-9abc\right)-a\left(2Bb^2+aCb-10aBc\right)\right)x^2}{a}+3A\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)^{3/2}}dx}{3a\left(b^2-4ac\right)}-\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{-\frac{3c\left(A\left(b^2-2ac\right)-a(bB-2aC)\right)x^4}{a}-\frac{\left(A\left(2b^3-9abc\right)-a\left(2Bb^2+aCb-10aBc\right)\right)x^2}{a}+3A\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)^{3/2}}dx}{3a\left(b^2-4ac\right)}-\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 2198$$

$$\frac{-\frac{\int -\frac{\frac{c\left(A\left(5b^4-33acb^2+36a^2c^2\right)-a\left(2Bb^3+aCb^2-16aBcb+12a^2cC\right)\right)x^4}{a}+c\left(4A\left(b^3-6abc\right)-a\left(Bb^2+8aCb-20aBc\right)\right)x^2+3A\left(b^2-4ac\right)^2}{x^2\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}-\frac{x\left(a\left(-4a^2c(\ldots\right.\right.}{\ldots}}{3a\left(b^2-4\ldots\right.}-\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\frac{c\left(A\left(5b^4-33acb^2+36a^2c^2\right)-a\left(2Bb^3+aCb^2-16aBcb+12a^2cC\right)\right)x^4}{a}+c\left(4A\left(b^3-6abc\right)-a\left(Bb^2+8aCb-20aBc\right)\right)x^2+3A\left(b^2-4ac\right)^2}{x^2\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}-\frac{x\left(a\left(-4a^2c(bC+\ldots\right.\right.}{\ldots}}{3a\left(b^2-4ac\ldots\right.}-\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 2199$$

$$\frac{\frac{\int \frac{c\left(4A\left(b^3-6abc\right)-a\left(Bb^2+8aCb-20aBc\right)\right)x^2+A\left(8b^4-57acb^2+84a^2c^2\right)-a\left(2Bb^3+aCb^2-16aBcb+12a^2cC\right)}{x^2\sqrt{cx^4+bx^2+a}}dx+\frac{\sqrt{a+bx^2+cx^4}\left(A\left(36a^2c^2-33ab^2c+5b^4\right)-a\ldots\right.}{ax}}{a\left(b^2-4ac\right)}}{\ldots}-\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 1604$$

$$-\frac{\int -\frac{c\left(\left(A\left(8b^4-57acb^2+84a^2c^2\right)-a\left(2Bb^3+aCb^2-16aBcb+12a^2cC\right)\right)x^2+a\left(4A\left(b^3-6abc\right)-a\left(Bb^2+8aCb-20aBc\right)\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}+\frac{\sqrt{a+bx^2+cx^4}\left(A\left(36a^2c^2-33ab^2c\cdots\right.\right.}{\cdots}$$

$$a(b^2-4ac)$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{c\left(\left(A\left(8b^4-57acb^2+84a^2c^2\right)-a\left(2Bb^3+aCb^2-16aBcb+12a^2cC\right)\right)x^2+a\left(4A\left(b^3-6abc\right)-a\left(Bb^2+8aCb-20aBc\right)\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}+\frac{\sqrt{a+bx^2+cx^4}\left(A\left(36a^2c^2-33ab^2c+5b\cdots\right.\right.}{\cdots}$$

$$a(b^2-4ac)$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{\left(A\left(8b^4-57acb^2+84a^2c^2\right)-a\left(2Bb^3+aCb^2-16aBcb+12a^2cC\right)\right)x^2+a\left(4A\left(b^3-6abc\right)-a\left(Bb^2+8aCb-20aBc\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}+\frac{\sqrt{a+bx^2+cx^4}\left(A\left(36a^2c^2-33ab^2c+5b^4\cdots\right.\right.}{ax\cdots}$$

$$a(b^2-4ac)$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 1511$$

$$\frac{c\left(\frac{\left(\sqrt{a}\left(A\left(84a^2c^2-57ab^2c+8b^4\right)-a\left(12a^2cC+ab^2C-16abBc+2b^3B\right)\right)+\sqrt{a}\sqrt{c}\left(4A\left(b^3-6abc\right)-a\left(8abC-20aBc+b^2B\right)\right)\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(A\left(84a^2c^2-57\cdots\right.\right.}{\cdots}\right.}{a}$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{c\left(\frac{\sqrt{a}\left(A\left(84a^2c^2-57ab^2c+8b^4\right)-a\left(12a^2cC+ab^2C-16abBc+2b^3B\right)+\sqrt{a}\sqrt{c}\left(4A\left(b^3-6abc\right)-a\left(8abC-20aBc+b^2B\right)\right)\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\left(A\left(84a^2c^2-57ab^2\right.\right.}{a}$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 1416

$$\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(A\left(84a^2c^2-57ab^2c+8b^4\right)-a\left(12a^2cC+ab^2C-16abBc+2b^3B\right)+\sqrt{a}\sqrt{c}\left(4A\left(b^3-6abc\right)-a\left(8abC-20aBc+b^2B\right)\right)\right)\text{EllipticF}}{2c^{3/4}\sqrt{a+bx^2+cx^4}}\right.}{a}$$

$$\frac{x\left(cx^2\left(2a^2C-2aAc-abB+Ab^2\right)+a\left(\frac{Ab^3}{a}+a(bC+2Bc)-b(3Ac+bB)\right)\right)}{3a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 1509

$$\frac{\sqrt{cx^4+bx^2+a}\left(A\left(5b^4-33acb^2+36a^2c^2\right)-a\left(2Bb^3+aCb^2-16aBcb+12a^2cC\right)\right)}{ax}+\frac{c\left(\frac{\sqrt[4]{a}\left(A\left(8b^4-57acb^2+84a^2c^2\right)-a\left(2Bb^3+aCb^2-16aBcb+12a^2cC\right)+\sqrt{a}\sqrt{c}\right.}{}\right.}{}$$

$$\frac{x\left(c\left(2Ca^2-bBa-2Aca+Ab^2\right)x^2+a\left(\frac{Ab^3}{a}-(bB+3Ac)b+a(2Bc+bC)\right)\right)}{3a^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^{3/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*(a + b*x^2 + c*x^4)^(5/2)),x]
```

output
```
-1/3*(x*(a*((A*b^3)/a - b*(b*B + 3*A*c) + a*(2*B*c + b*C)) + c*(A*b^2 - a*
b*B - 2*a*A*c + 2*a^2*C)*x^2))/(a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^(3/2
)) + (-((x*(a*((5*A*b^5)/a - b^3*(2*b*B + 37*A*c) - 4*a^2*c*(5*B*c + b*C)
+ a*b*(17*b*B*c + 60*A*c^2 - b^2*C)) + c*(A*(5*b^4 - 33*a*b^2*c + 36*a^2*c
^2) - a*(2*b^3*B - 16*a*b*B*c + a*b^2*C + 12*a^2*c*C))*x^2))/(a^2*(b^2 - 4
*a*c)*Sqrt[a + b*x^2 + c*x^4])) + (((A*(5*b^4 - 33*a*b^2*c + 36*a^2*c^2) -
 a*(2*b^3*B - 16*a*b*B*c + a*b^2*C + 12*a^2*c*C))*Sqrt[a + b*x^2 + c*x^4])
/(a*x) - ((A*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2) - a*(2*b^3*B - 16*a*b*B*c +
 a*b^2*C + 12*a^2*c*C))*Sqrt[a + b*x^2 + c*x^4])/(a*x) + (c*(-(((A*(8*b^4
- 57*a*b^2*c + 84*a^2*c^2) - a*(2*b^3*B - 16*a*b*B*c + a*b^2*C + 12*a^2*c*
C))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(
1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(A*(8*b^4 - 57*a*b^2*c
 + 84*a^2*c^2) - a*(2*b^3*B - 16*a*b*B*c + a*b^2*C + 12*a^2*c*C) + Sqrt[a]
*Sqrt[c]*(4*A*(b^3 - 6*a*b*c) - a*(b^2*B - 20*a*B*c + 8*a*b*C)))*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)
*Sqrt[a + b*x^2 + c*x^4])))/a)/(a*(b^2 - 4*a*c)))/(3*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 10.57 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.17

method	result
elliptic	$-\frac{A\sqrt{c\,x^{4}+b\,x^{2}+a}}{a^{3}x}+\frac{\left(-\frac{\left(2Aac-A\,b^{2}+Bab-2a^{2}C\right)x^{3}}{3c\,a^{2}\left(4ac-b^{2}\right)}-\frac{\left(3Aabc-A\,b^{3}-2B\,a^{2}c+Ba\,b^{2}-C\,a^{2}b\right)x}{3a^{2}\left(4ac-b^{2}\right)c^{2}}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)^{2}}-\frac{2c\left(\frac{\left(36A\,a^{2}c^{2}-3\right.}{}\right.}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(c*x^4+b*x^2+a)^(1/2)/a^3/x+(-1/3/c*(2*A*a*c-A*b^2+B*a*b-2*C*a^2)/a^2/(
4*a*c-b^2)*x^3-1/3*(3*A*a*b*c-A*b^3-2*B*a^2*c+B*a*b^2-C*a^2*b)/a^2/(4*a*c-
b^2)/c^2*x)*(c*x^4+b*x^2+a)^(1/2)/(x^4+1/c*b*x^2+1/c*a)^2-2*c*(1/6*(36*A*a
^2*c^2-33*A*a*b^2*c+5*A*b^4+16*B*a^2*b*c-2*B*a*b^3-12*C*a^3*c-C*a^2*b^2)/a
^3/(4*a*c-b^2)^2*x^3+1/6*(60*A*a^2*b*c^2-37*A*a*b^3*c+5*A*b^5-20*B*a^3*c^2
+17*B*a^2*b^2*c-2*B*a*b^4-4*C*a^3*b*c-C*a^2*b^3)/a^3/(4*a*c-b^2)^2/c*x)/((
x^4+1/c*b*x^2+1/c*a)*c)^(1/2)+1/4*(-1/3/(4*a*c-b^2)*(21*A*a*b*c-5*A*b^3-10
*B*a^2*c+2*B*a*b^2+C*a^2*b)/a^3+1/3*(60*A*a^2*b*c^2-37*A*a*b^3*c+5*A*b^5-2
0*B*a^3*c^2+17*B*a^2*b^2*c-2*B*a*b^4-4*C*a^3*b*c-C*a^2*b^3)/a^3/(4*a*c-b^2
)^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(A*c/a^3+1/3*c*(36*A*a^2*c^2-33*A*a
*b^2*c+5*A*b^4+16*B*a^2*b*c-2*B*a*b^3-12*C*a^3*c-C*a^2*b^2)/(4*a*c-b^2)^2/
a^3)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(
1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2597 vs. $2(696) = 1392$.

Time = 0.13 (sec) , antiderivative size = 2597, normalized size of antiderivative = 3.30

$$\int \frac{A + Bx^2 + Cx^4}{x^2 \left(a + bx^2 + cx^4\right)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(1/2)*((84*A*a^2*b*c^4 - (12*C*a^3*b - 16*B*a^2*b^2 + 57*A*a*b^3)
*c^3 - (C*a^2*b^3 + 2*B*a*b^4 - 8*A*b^5)*c^2)*x^9 + 2*(84*A*a^2*b^2*c^3 -
(12*C*a^3*b^2 - 16*B*a^2*b^3 + 57*A*a*b^4)*c^2 - (C*a^2*b^4 + 2*B*a*b^5 -
8*A*b^6)*c)*x^7 - (C*a^2*b^5 + 2*B*a*b^6 - 8*A*b^7 - 168*A*a^3*b*c^3 + 2*(
12*C*a^4*b - 16*B*a^3*b^2 + 15*A*a^2*b^3)*c^2 + (14*C*a^3*b^3 - 12*B*a^2*b
^4 + 41*A*a*b^5)*c)*x^5 - 2*(C*a^3*b^4 + 2*B*a^2*b^5 - 8*A*a*b^6 - 84*A*a^
3*b^2*c^2 + (12*C*a^4*b^2 - 16*B*a^3*b^3 + 57*A*a^2*b^4)*c)*x^3 - (C*a^4*b
^3 + 2*B*a^3*b^4 - 8*A*a^2*b^5 - 84*A*a^4*b*c^2 + (12*C*a^5*b - 16*B*a^4*b
^2 + 57*A*a^3*b^3)*c)*x - ((84*A*a^3*c^4 - (12*C*a^4 - 16*B*a^3*b + 57*A*a
^2*b^2)*c^3 - (C*a^3*b^2 + 2*B*a^2*b^3 - 8*A*a*b^4)*c^2)*x^9 + 2*(84*A*a^3
*b*c^3 - (12*C*a^4*b - 16*B*a^3*b^2 + 57*A*a^2*b^3)*c^2 - (C*a^3*b^3 + 2*B
*a^2*b^4 - 8*A*a*b^5)*c)*x^7 - (C*a^3*b^4 + 2*B*a^2*b^5 - 8*A*a*b^6 - 168*
A*a^4*c^3 + 2*(12*C*a^5 - 16*B*a^4*b + 15*A*a^3*b^2)*c^2 + (14*C*a^4*b^2 -
 12*B*a^3*b^3 + 41*A*a^2*b^4)*c)*x^5 - 2*(C*a^4*b^3 + 2*B*a^3*b^4 - 8*A*a^
2*b^5 - 84*A*a^4*b*c^2 + (12*C*a^5*b - 16*B*a^4*b^2 + 57*A*a^3*b^3)*c)*x^3
 - (C*a^5*b^2 + 2*B*a^4*b^3 - 8*A*a^3*b^4 - 84*A*a^5*c^2 + (12*C*a^6 - 16*
B*a^5*b + 57*A*a^4*b^2)*c)*x)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqr
t((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b
^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)
/(a*c)) - sqrt(1/2)*((84*A*a^2*b*c^4 + (4*(5*B - 3*C)*a^3*b - 8*(3*A - ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{x^2 (a + bx^2 + cx^4)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(cx^4+bx^2+a)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(cx^4+bx^2+a)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^2\,\left(c\,x^4+b\,x^2+a\right)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^2 + c*x^4)^(5/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^2 + c*x^4)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^2+cx^4\right)^{5/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^{10}+2bc\,x^8+2ac\,x^6+b^2x^6+2ab\,x^4+a^2x^2}dx$$

input
```
int((C*x^4+B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a**2*x**2 + 2*a*b*x**4 + 2*a*c*x**6 + b**2*
x**6 + 2*b*c*x**8 + c**2*x**10),x)
```

3.163 $\int \frac{A+Bx^2+Cx^4}{x^4(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 956

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^2+cx^4\right)^{5/2}}\,dx = \text{Too large to display}$$

output
```
1/3*x*(a^2*(A*b^4/a^2+3*B*b*c+2*A*c^2-b^2*(4*A*c+B*b)/a+b^2*C-2*a*c*C)+c*(
A*(-3*a*b*c+b^3)-a*(-2*B*a*c+B*b^2-C*a*b))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*
x^2+a)^(3/2)+1/3*x*(a^2*(8*A*b^6/a^2-b^4*(63*A*c+5*B*b)/a+20*a^2*c^2*C+b^2
*(131*A*c^2+37*B*b*c+2*C*b^2)-a*c*(44*A*c^2+60*B*b*c+17*C*b^2))+c*(8*A*(11
*a^2*b*c^2-7*a*b^3*c+b^5)-a*(36*B*a^2*c^2-33*B*a*b^2*c+5*B*b^4+16*C*a^2*b*
c-2*C*a*b^3))*x^2)/a^4/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/3*A*(c*x^4+b
*x^2+a)^(1/2)/a^3/x^3+1/3*(8*A*b-3*B*a)*(c*x^4+b*x^2+a)^(1/2)/a^4/x-1/3*c^
(1/2)*(8*A*(27*a^2*b*c^2-15*a*b^3*c+2*b^5)-a*(84*B*a^2*c^2-57*B*a*b^2*c+8*
B*b^4+16*C*a^2*b*c-2*C*a*b^3))*x*(c*x^4+b*x^2+a)^(1/2)/a^4/(-4*a*c+b^2)^2/
(a^(1/2)+c^(1/2)*x^2)+1/3*c^(1/4)*(8*A*(27*a^2*b*c^2-15*a*b^3*c+2*b^5)-a*(
84*B*a^2*c^2-57*B*a*b^2*c+8*B*b^4+16*C*a^2*b*c-2*C*a*b^3))*(a^(1/2)+c^(1/2
)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(15/4)/(-4*a*c+
b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/6*c^(1/4)*(A*(16*b^4-24*a^(1/2)*b^3*c^(1/2)
-72*a*b^2*c+93*a^(3/2)*b*c^(3/2)+30*c^2*a^2)-a*(8*B*b^3+42*a^(3/2)*B*c^(3/
2)+10*a^2*c*C-2*b^2*(6*a^(1/2)*B*c^(1/2)+a*C)-3*b*(11*a*B*c-a^(3/2)*c^(1/2
)*C)))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/
2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))/a^(15/4)/(b-2*a^(1/2)*c^(1/2))/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.83 (sec) , antiderivative size = 957, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2+Cx^4}{x^4(a+bx^2+cx^4)^{5/2}}\,dx = \frac{4\left(-\frac{aA(b^2-4ac)^2(a+bx^2+cx^4)^2}{x^3}+\frac{(8Ab-3aB)(b^2-4ac)^2(a+bx^2+cx^4)^2}{x}+a(b^2-4ac)\,x(A\cdots\right.}{\cdots}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*(a + b*x^2 + c*x^4)^(5/2)),x]
```

output
```
(4*(-((a*A*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)/x^3) + ((8*A*b - 3*a*B)*
(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)/x + a*(b^2 - 4*a*c)*x*(A*(b^4 - 4*a
*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*x^2) + a*(-(b^3*B) + b^2*(a*C -
 B*c*x^2) + 2*a*c*(-(a*C) + B*c*x^2) + a*b*c*(3*B + C*x^2))) + x*(a + b*x^
2 + c*x^4)*(A*(8*b^6 - 63*a*b^4*c + 131*a^2*b^2*c^2 - 44*a^3*c^3 + 8*b^5*c
*x^2 - 56*a*b^3*c^2*x^2 + 88*a^2*b*c^3*x^2) + a*(-5*b^5*B + 4*a^2*c^2*(5*a
*C - 9*B*c*x^2) + b^4*(2*a*C - 5*B*c*x^2) + a*b^2*c*(-17*a*C + 33*B*c*x^2)
 + a*b^3*c*(37*B + 2*C*x^2) - 4*a^2*b*c^2*(15*B + 4*C*x^2)))) - (I*Sqrt[2]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
(2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*((-b + Sqrt[b^2 - 4
*a*c])*(8*A*(2*b^5 - 15*a*b^3*c + 27*a^2*b*c^2) + a*(-8*b^4*B + 57*a*b^2*B
*c - 84*a^2*B*c^2 + 2*a*b^3*C - 16*a^2*b*c*C))*EllipticE[I*ArcSinh[Sqrt[2]
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])] + (-2*A*(-8*b^6 + 68*a*b^4*c - 159*a^2*b^2*c^2 + 60*a^3*c^3 +
8*b^5*Sqrt[b^2 - 4*a*c] - 60*a*b^3*c*Sqrt[b^2 - 4*a*c] + 108*a^2*b*c^2*Sqr
t[b^2 - 4*a*c]) + a*(-8*b^5*B + 2*b^4*(4*B*Sqrt[b^2 - 4*a*c] + a*C) - 3*a*
b^2*c*(19*B*Sqrt[b^2 - 4*a*c] + 6*a*C) + 4*a^2*c^2*(21*B*Sqrt[b^2 - 4*a*c]
 + 10*a*C) - 4*a^2*b*c*(33*B*c - 4*Sqrt[b^2 - 4*a*c]*C) + a*b^3*(65*B*c -
2*Sqrt[b^2 - 4*a*c]*C)))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[...
```

Rubi [A] (verified)

Time = 3.29 (sec) , antiderivative size = 1161, normalized size of antiderivative = 1.21, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {2198, 25, 2198, 25, 2199, 2199, 1604, 27, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4 \left(a+bx^2+cx^4\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{2198}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{3a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}-\frac{\int -\frac{\frac{3c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^6}{a^2}+\left(\frac{2Ab^4}{a^2}-\frac{(2bB+11Ac)b^2}{a}+2Cb^2+9Bcb+10Ac^2-10acC\right)x^4-\frac{3(Ab-aB)\left(b^2-4ac\right)x^2}{a}+3A\left(b^2-4ac\right)}{x^4\left(cx^4+bx^2+a\right)^{3/2}}dx}{3a\left(b^2-4ac\right)}$$

$\downarrow$ 25

$$\frac{\int \frac{\frac{3c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^6}{a^2}+\left(\frac{2Ab^4}{a^2}-\frac{(2bB+11Ac)b^2}{a}+2Cb^2+9Bcb+10Ac^2-10acC\right)x^4-\frac{3(Ab-aB)\left(b^2-4ac\right)x^2}{a}+3A\left(b^2-4ac\right)}{x^4\left(cx^4+bx^2+a\right)^{3/2}}dx}{3a\left(b^2-4ac\right)}+$$
$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{3a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\downarrow$ 2198

$$\frac{\frac{x\left(a^2\left(\frac{8Ab^6}{a^2}+20a^2c^2C-\frac{b^4(63Ac+5bB)}{a}-ac\left(44Ac^2+17b^2C+60bBc\right)+b^2\left(131Ac^2+2b^2C+37bBc\right)\right)+cx^2\left(8A\left(11a^2bc^2-7ab^3c+b^5\right)-a\left(16a^2bcC+\cdots\right)\right)\right)}{a^3\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}\cdots}{\cdots}$$
$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{3a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\downarrow$ 25

$$\frac{\int \frac{-\frac{c\left(8A\left(b^5-7acb^3+11a^2c^2b\right)-a\left(5Bb^4-2aCb^3-33aBcb^2+16a^2cCb+36a^2Bc^2\right)\right)x^6}{a^2}-\frac{c\left(A\left(7b^4-43acb^2+44a^2c^2\right)-a\left(4Bb^3-aCb^2-24aBcb+20a^2cC\right)\right)x^4}{a}-\cdots}{x^4\sqrt{cx^4+bx^2+a}}\cdots}{a\left(b^2-4ac\right)}\cdots$$
$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{3a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\downarrow$ 2199

$$\frac{\int -\frac{\frac{c\left(A\left(7b^4-43acb^2+44a^2c^2\right)-a\left(4Bb^3-aCb^2-24aBcb+20a^2cC\right)\right)x^4}{a}-\frac{\left(2A\left(7b^5-52acb^3+92a^2c^2b\right)-a\left(8Bb^4-2aCb^3-57aBcb^2+16a^2cCb+84a^2Bc^2\right)\right)x^2}{a}+}{x^4\sqrt{cx^4+bx^2+a}}}{a(b^2-4ac)}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{3a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 2199

$$\frac{\int \frac{3c\left(A\left(6b^3-32abc\right)-a\left(3Bb^2+8aCb-28aBc\right)\right)x^2+3\left(A\left(8b^4-51acb^2+60a^2c^2\right)-a\left(4Bb^3-aCb^2-24aBcb+20a^2cC\right)\right)}{x^4\sqrt{cx^4+bx^2+a}}dx+\frac{\sqrt{a+bx^2+cx^4}\left(A\left(44a^2c^2-43ab^2c+7\right.\right.}{}}{a(b^2-4ac)}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{3a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 1604

$$-\frac{\int \frac{3\left(c\left(A\left(8b^4-51acb^2+60a^2c^2\right)-a\left(4Bb^3-aCb^2-24aBcb+20a^2cC\right)\right)x^2+8A\left(2b^5-15acb^3+27a^2c^2b\right)-a\left(8Bb^4-2aCb^3-57aBcb^2+16a^2cCb+84a^2Bc^2\right)\right)}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}+$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{3a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{c\left(A\left(8b^4-51acb^2+60a^2c^2\right)-a\left(4Bb^3-aCb^2-24aBcb+20a^2cC\right)\right)x^2+8A\left(2b^5-15acb^3+27a^2c^2b\right)-a\left(8Bb^4-2aCb^3-57aBcb^2+16a^2cCb+84a^2Bc^2\right)}{x^2\sqrt{cx^4+bx^2+a}}dx}{a}+\frac{\sqrt{c}}{}$$

$$\frac{x\left(a^2\left(\frac{Ab^4}{a^2}-\frac{b^2(4Ac+bB)}{a}-2acC+2Ac^2+b^2C+3bBc\right)+cx^2\left(A\left(b^3-3abc\right)-a\left(-abC-2aBc+b^2B\right)\right)\right)}{3a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

$\Big\downarrow$ 1604

$$\frac{x\left(\left(\frac{Ab^4}{a^2}-\frac{(bB+4Ac)b^2}{a}+Cb^2+3Bcb+2Ac^2-2acC\right)a^2+c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^2\right)}{3a^3\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^{3/2}}+$$

$$\frac{\frac{x\left(\left(\frac{8Ab^6}{a^2}-\frac{(5bB+63Ac)b^4}{a}+\left(2Cb^2+37Bcb+131Ac^2\right)b^2+20a^2c^2C-ac\left(17Cb^2+60Bcb+44Ac^2\right)\right)a^2+c\left(8A\left(b^5-7acb^3+11a^2c^2b\right)-a\left(5Bb^4-2aCb^3 \cdots\right.\right.\right.}{a^3\left(b^2-4ac\right)\sqrt{cx^4+bx^2+a}}}{\cdots}$$

$\downarrow$ 25

$$\frac{x\left(\left(\frac{Ab^4}{a^2}-\frac{(bB+4Ac)b^2}{a}+Cb^2+3Bcb+2Ac^2-2acC\right)a^2+c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^2\right)}{3a^3\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^{3/2}}+$$

$$\frac{\frac{x\left(\left(\frac{8Ab^6}{a^2}-\frac{(5bB+63Ac)b^4}{a}+\left(2Cb^2+37Bcb+131Ac^2\right)b^2+20a^2c^2C-ac\left(17Cb^2+60Bcb+44Ac^2\right)\right)a^2+c\left(8A\left(b^5-7acb^3+11a^2c^2b\right)-a\left(5Bb^4-2aCb^3 \cdots\right.\right.\right.}{a^3\left(b^2-4ac\right)\sqrt{cx^4+bx^2+a}}}{\cdots}$$

$\downarrow$ 27

$$\frac{x\left(\left(\frac{Ab^4}{a^2}-\frac{(bB+4Ac)b^2}{a}+Cb^2+3Bcb+2Ac^2-2acC\right)a^2+c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^2\right)}{3a^3\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^{3/2}}+$$

$$\frac{\frac{x\left(\left(\frac{8Ab^6}{a^2}-\frac{(5bB+63Ac)b^4}{a}+\left(2Cb^2+37Bcb+131Ac^2\right)b^2+20a^2c^2C-ac\left(17Cb^2+60Bcb+44Ac^2\right)\right)a^2+c\left(8A\left(b^5-7acb^3+11a^2c^2b\right)-a\left(5Bb^4-2aCb^3 \cdots\right.\right.\right.}{a^3\left(b^2-4ac\right)\sqrt{cx^4+bx^2+a}}}{\cdots}$$

$\downarrow$ 1511

$$\frac{x\left(\left(\frac{Ab^4}{a^2}-\frac{(bB+4Ac)b^2}{a}+Cb^2+3Bcb+2Ac^2-2acC\right)a^2+c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^2\right)}{3a^3\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^{3/2}}+$$

$$\frac{\frac{x\left(\left(\frac{8Ab^6}{a^2}-\frac{(5bB+63Ac)b^4}{a}+\left(2Cb^2+37Bcb+131Ac^2\right)b^2+20a^2c^2C-ac\left(17Cb^2+60Bcb+44Ac^2\right)\right)a^2+c\left(8A\left(b^5-7acb^3+11a^2c^2b\right)-a\left(5Bb^4-2aCb^3 \cdots\right.\right.\right.}{a^3\left(b^2-4ac\right)\sqrt{cx^4+bx^2+a}}}{\cdots}$$

$\downarrow$ 27

$$\frac{x\left(\left(\frac{Ab^4}{a^2}-\frac{(bB+4Ac)b^2}{a}+Cb^2+3Bcb+2Ac^2-2acC\right)a^2+c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^2\right)}{3a^3\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^{3/2}}+$$

$$\frac{\frac{x\left(\left(\frac{8Ab^6}{a^2}-\frac{(5bB+63Ac)b^4}{a}+(2Cb^2+37Bcb+131Ac^2)b^2+20a^2c^2C-ac(17Cb^2+60Bcb+44Ac^2)\right)a^2+c\left(8A\left(b^5-7acb^3+11a^2c^2b\right)-a\left(5Bb^4-2aCb^3\right.\right.\right.}{a^3(b^2-4ac)\sqrt{cx^4+bx^2+a}}}{}$$

$\downarrow$ 1416

$$\frac{x\left(\left(\frac{Ab^4}{a^2}-\frac{(bB+4Ac)b^2}{a}+Cb^2+3Bcb+2Ac^2-2acC\right)a^2+c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^2\right)}{3a^3\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^{3/2}}+$$

$$\frac{\frac{x\left(\left(\frac{8Ab^6}{a^2}-\frac{(5bB+63Ac)b^4}{a}+(2Cb^2+37Bcb+131Ac^2)b^2+20a^2c^2C-ac(17Cb^2+60Bcb+44Ac^2)\right)a^2+c\left(8A\left(b^5-7acb^3+11a^2c^2b\right)-a\left(5Bb^4-2aCb^3\right.\right.\right.}{a^3(b^2-4ac)\sqrt{cx^4+bx^2+a}}}{}$$

$\downarrow$ 1509

$$\frac{x\left(\left(\frac{Ab^4}{a^2}-\frac{(bB+4Ac)b^2}{a}+Cb^2+3Bcb+2Ac^2-2acC\right)a^2+c\left(A\left(b^3-3abc\right)-a\left(Bb^2-aCb-2aBc\right)\right)x^2\right)}{3a^3\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^{3/2}}+$$

$$\frac{\frac{x\left(\left(\frac{8Ab^6}{a^2}-\frac{(5bB+63Ac)b^4}{a}+(2Cb^2+37Bcb+131Ac^2)b^2+20a^2c^2C-ac(17Cb^2+60Bcb+44Ac^2)\right)a^2+c\left(8A\left(b^5-7acb^3+11a^2c^2b\right)-a\left(5Bb^4-2aCb^3\right.\right.\right.}{a^3(b^2-4ac)\sqrt{cx^4+bx^2+a}}}{}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^4*(a + b*x^2 + c*x^4)^(5/2)),x]
```

output
```
(x*(a^2*((A*b^4)/a^2 + 3*b*B*c + 2*A*c^2 - (b^2*(b*B + 4*A*c))/a + b^2*C -
 2*a*c*C) + c*(A*(b^3 - 3*a*b*c) - a*(b^2*B - 2*a*B*c - a*b*C))*x^2))/(3*a
^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^(3/2)) + ((x*(a^2*((8*A*b^6)/a^2 - (b
^4*(5*b*B + 63*A*c))/a + 20*a^2*c^2*C + b^2*(37*b*B*c + 131*A*c^2 + 2*b^2*
C) - a*c*(60*b*B*c + 44*A*c^2 + 17*b^2*C)) + c*(8*A*(b^5 - 7*a*b^3*c + 11*
a^2*b*c^2) - a*(5*b^4*B - 33*a*b^2*B*c + 36*a^2*B*c^2 - 2*a*b^3*C + 16*a^2
*b*c*C))*x^2))/(a^3*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (((A*(7*b^4 -
 43*a*b^2*c + 44*a^2*c^2) - a*(4*b^3*B - 24*a*b*B*c - a*b^2*C + 20*a^2*c*C
))*Sqrt[a + b*x^2 + c*x^4])/(a*x^3) - ((A*(8*b^4 - 51*a*b^2*c + 60*a^2*c^2
) - a*(4*b^3*B - 24*a*b*B*c - a*b^2*C + 20*a^2*c*C))*Sqrt[a + b*x^2 + c*x^
4])/(a*x^3) - ((8*A*(b^5 - 7*a*b^3*c + 11*a^2*b*c^2) - a*(5*b^4*B - 33*a*b
^2*B*c + 36*a^2*B*c^2 - 2*a*b^3*C + 16*a^2*b*c*C))*Sqrt[a + b*x^2 + c*x^4]
)/(a^2*x) - (-(((8*A*(2*b^5 - 15*a*b^3*c + 27*a^2*b*c^2) - a*(8*b^4*B - 57
*a*b^2*B*c + 84*a^2*B*c^2 - 2*a*b^3*C + 16*a^2*b*c*C))*Sqrt[a + b*x^2 + c*
x^4])/(a*x)) + (c*(-(((8*A*(2*b^5 - 15*a*b^3*c + 27*a^2*b*c^2) - a*(8*b^4*
B - 57*a*b^2*B*c + 84*a^2*B*c^2 - 2*a*b^3*C + 16*a^2*b*c*C))*(-((x*Sqrt[a
+ b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x
^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(8*A*(2*b^5 - 15*a*b^3*c + 27*a^2*b*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 13.43 (sec) , antiderivative size = 1094, normalized size of antiderivative = 1.14

method	result	size
elliptic	Expression too large to display	1094
default	Expression too large to display	2121
risch	Expression too large to display	5720

input
```
int((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*(c*x^4+b*x^2+a)^(1/2)/a^3/x^3+1/3*(8*A*b-3*B*a)*(c*x^4+b*x^2+a)^(1/
2)/a^4/x+(1/3/c*(3*A*a*b*c-A*b^3-2*B*a^2*c+B*a*b^2-C*a^2*b)/a^3/(4*a*c-b^2
)*x^3-1/3*(2*A*a^2*c^2-4*A*a*b^2*c+A*b^4+3*B*a^2*b*c-B*a*b^3-2*C*a^3*c+C*a
^2*b^2)/a^3/(4*a*c-b^2)/c^2*x)*(c*x^4+b*x^2+a)^(1/2)/(x^4+1/c*b*x^2+1/c*a)
^2-2*c*(-1/6*(88*A*a^2*b*c^2-56*A*a*b^3*c+8*A*b^5-36*B*a^3*c^2+33*B*a^2*b^
2*c-5*B*a*b^4-16*C*a^3*b*c+2*C*a^2*b^3)/a^4/(4*a*c-b^2)^2*x^3+1/6*(44*A*a^
3*c^3-131*A*a^2*b^2*c^2+63*A*a*b^4*c-8*A*b^6+60*B*a^3*b*c^2-37*B*a^2*b^3*c
+5*B*a*b^5-20*C*a^4*c^2+17*C*a^3*b^2*c-2*C*a^2*b^4)/a^4/(4*a*c-b^2)^2/c*x)
/((x^4+1/c*b*x^2+1/c*a)*c)^(1/2)+1/4*(-1/3*A*c/a^3-1/3/(4*a*c-b^2)*(22*A*a
^2*c^2-38*A*a*b^2*c+8*A*b^4+21*B*a^2*b*c-5*B*a*b^3-10*C*a^3*c+2*C*a^2*b^2)
/a^4+1/3*(44*A*a^3*c^3-131*A*a^2*b^2*c^2+63*A*a*b^4*c-8*A*b^6+60*B*a^3*b*c
^2-37*B*a^2*b^3*c+5*B*a*b^5-20*C*a^4*c^2+17*C*a^3*b^2*c-2*C*a^2*b^4)/a^4/(
4*a*c-b^2)^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*
x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-1/3*(8*A*b-3*B*a)*c/a^4-
1/3*c*(88*A*a^2*b*c^2-56*A*a*b^3*c+8*A*b^5-36*B*a^3*c^2+33*B*a^2*b^2*c-5*B
*a*b^4-16*C*a^3*b*c+2*C*a^2*b^3)/(4*a*c-b^2)^2/a^4)*a*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3140 vs. $2(850) = 1700$.

Time = 0.15 (sec) , antiderivative size = 3140, normalized size of antiderivative = 3.28

$$\int \frac{A + Bx^2 + Cx^4}{x^4 (a + bx^2 + cx^4)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^2+cx^4\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^2+cx^4\right)^{5/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{\left(cx^4+bx^2+a\right)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^(5/2)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^2+cx^4\right)^{5/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{\left(cx^4+bx^2+a\right)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^(5/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^4\,\left(c\,x^4+b\,x^2+a\right)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(a + b*x^2 + c*x^4)^(5/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(a + b*x^2 + c*x^4)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\frac{-2\sqrt{c\,x^4+b\,x^2+a}\,ab-10\sqrt{c\,x^4+b\,x^2+a}\,bc\,x^4-5\sqrt{c\,x^4+b\,x^2+a}\,c^2x^6-}{}$$

input
```
int((C*x^4+B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x**2 + c*x**4)*a*b - 10*sqrt(a + b*x**2 + c*x**4)*b*c*x**
4 - 5*sqrt(a + b*x**2 + c*x**4)*c**2*x**6 - 8*int(sqrt(a + b*x**2 + c*x**4
)/(a**2*x**2 + 2*a*b*x**4 + 2*a*c*x**6 + b**2*x**6 + 2*b*c*x**8 + c**2*x**
10),x)*a**2*b**2*x**3 - 8*int(sqrt(a + b*x**2 + c*x**4)/(a**2*x**2 + 2*a*b
*x**4 + 2*a*c*x**6 + b**2*x**6 + 2*b*c*x**8 + c**2*x**10),x)*a*b**3*x**5 -
 8*int(sqrt(a + b*x**2 + c*x**4)/(a**2*x**2 + 2*a*b*x**4 + 2*a*c*x**6 + b*
*2*x**6 + 2*b*c*x**8 + c**2*x**10),x)*a*b**2*c*x**7 + 5*int((sqrt(a + b*x*
*2 + c*x**4)*x**6)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**
6 + c**2*x**8),x)*a*c**3*x**3 + 5*int((sqrt(a + b*x**2 + c*x**4)*x**6)/(a*
*2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*b*c*
*3*x**5 + 5*int((sqrt(a + b*x**2 + c*x**4)*x**6)/(a**2 + 2*a*b*x**2 + 2*a*
c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*c**4*x**7 + 15*int((sqrt(a
 + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*
b*c*x**6 + c**2*x**8),x)*a**2*c**2*x**3 + 15*int((sqrt(a + b*x**2 + c*x**4
)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x*
*8),x)*a*b*c**2*x**5 + 15*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a
*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*c**3*x**7)
/(6*a**2*b*x**3*(a + b*x**2 + c*x**4))
```

3.164 $\int \frac{21+28x^2+10(3+\sqrt{2})x^4}{x^4\sqrt{1+2x^2+2x^4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 118

$$\int \frac{21+28x^2+10(3+\sqrt{2})\,x^4}{x^4\sqrt{1+2x^2+2x^4}}\,dx = -\frac{7\sqrt{1+2x^2+2x^4}}{x^3} + \frac{\sqrt[4]{2}(5+4\sqrt{2})\,(1+\sqrt{2}x^2)\,\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{\sqrt{1+2x^2+2x^4}}$$

output
```
-7*(2*x^4+2*x^2+1)^(1/2)/x^3+2^(1/4)*(5+4*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4
+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(2^(1/4)*x),1/2
*(2-2^(1/2))^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int \frac{21+28x^2+10\left(3+\sqrt{2}\right)x^4}{x^4\sqrt{1+2x^2+2x^4}}\,dx$$
$$= \frac{-7(1+2x^2+2x^4)+(1-i)^{3/2}\left(8+5\sqrt{2}\right)x^3\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-i}\right.\right.}{x^3\sqrt{1+2x^2+2x^4}}$$

input
```
Integrate[(21 + 28*x^2 + 10*(3 + Sqrt[2])*x^4)/(x^4*Sqrt[1 + 2*x^2 + 2*x^4
]),x]
```

output
```
(-7*(1 + 2*x^2 + 2*x^4) + (1 - I)^(3/2)*(8 + 5*Sqrt[2])*x^3*Sqrt[1 + (1 -
I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I])/(x^3
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2199, 1604, 27, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{10\left(3+\sqrt{2}\right)x^4+28x^2+21}{x^4\sqrt{2x^4+2x^2+1}}\,dx$$
$$\Big\downarrow \text{2199}$$
$$\int \frac{-4\left(8+5\sqrt{2}\right)x^2-3\left(8+5\sqrt{2}\right)}{x^4\sqrt{2x^4+2x^2+1}}dx-\frac{5\left(3+\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}$$
$$\Big\downarrow \text{1604}$$
$$-\frac{1}{3}\int -\frac{6\left(8+5\sqrt{2}\right)}{\sqrt{2x^4+2x^2+1}}dx+\frac{\left(8+5\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}-\frac{5\left(3+\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}$$

$$\Big\downarrow \text{27}$$

$$2\left(8+5\sqrt{2}\right)\int \frac{1}{\sqrt{2x^4+2x^2+1}}dx+\frac{\left(8+5\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}-\frac{5\left(3+\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(8+5\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\left(8+5\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}-\frac{5\left(3+\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}{x^3}$$

input
```
Int[(21 + 28*x^2 + 10*(3 + Sqrt[2])*x^4)/(x^4*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

output
```
(-5*(3 + Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])/x^3 + ((8 + 5*Sqrt[2])*Sqrt[1 +
 2*x^2 + 2*x^4])/x^3 + ((8 + 5*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{7\sqrt{2x^4+2x^2+1}}{x^3}+\frac{\left(10\sqrt{2}+16\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{7\sqrt{2x^4+2x^2+1}}{x^3}+\frac{\left(10\sqrt{2}+16\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$\frac{16\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{10\sqrt{2}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

input
```
int((21+28*x^2+10*(3+2^(1/2))*x^4)/x^4/(2*x^4+2*x^2+1)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-7*(2*x^4+2*x^2+1)^(1/2)/x^3+(10*2^(1/2)+16)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1
/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{21+28x^2+10\left(3+\sqrt{2}\right)x^4}{x^4\sqrt{1+2x^2+2x^4}}\,dx = \frac{\sqrt{i-1}\left(-(5i+5)\sqrt{2}x^3-(8i+8)\,x^3\right)F\left(\arcsin\left(\sqrt{i-1}x\right)\mid i\right)-7\sqrt{2\,x^4+2\,x^2+1}}{x^3}$$

input
```
integrate((21+28*x^2+10*(3+2^(1/2))*x^4)/x^4/(2*x^4+2*x^2+1)^(1/2),x, algo
rithm="fricas")
```

output
```
(sqrt(I - 1)*(-(5*I + 5)*sqrt(2)*x^3 - (8*I + 8)*x^3)*elliptic_f(arcsin(sq
rt(I - 1)*x), I) - 7*sqrt(2*x^4 + 2*x^2 + 1))/x^3
```

Sympy [F]

$$\int \frac{21+28x^2+10\left(3+\sqrt{2}\right)x^4}{x^4\sqrt{1+2x^2+2x^4}}\,dx = \int \frac{10\sqrt{2}x^4+30x^4+28x^2+21}{x^4\sqrt{2x^4+2x^2+1}}\,dx$$

input
```
integrate((21+28*x**2+10*(3+2**(1/2))*x**4)/x**4/(2*x**4+2*x**2+1)**(1/2),
x)
```

output
```
Integral((10*sqrt(2)*x**4 + 30*x**4 + 28*x**2 + 21)/(x**4*sqrt(2*x**4 + 2*
x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{21+28x^2+10\left(3+\sqrt{2}\right)x^4}{x^4\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{10\,x^4\left(\sqrt{2}+3\right)+28\,x^2+21}{\sqrt{2\,x^4+2\,x^2+1}x^4}\,dx$$

input
```
integrate((21+28*x^2+10*(3+2^(1/2))*x^4)/x^4/(2*x^4+2*x^2+1)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((10*x^4*(sqrt(2) + 3) + 28*x^2 + 21)/(sqrt(2*x^4 + 2*x^2 + 1)*x^
4), x)
```

Giac [F]

$$\int \frac{21+28x^2+10\left(3+\sqrt{2}\right)x^4}{x^4\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{10\,x^4\left(\sqrt{2}+3\right)+28\,x^2+21}{\sqrt{2\,x^4+2\,x^2+1}x^4}\,dx$$

input
```
integrate((21+28*x^2+10*(3+2^(1/2))*x^4)/x^4/(2*x^4+2*x^2+1)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((10*x^4*(sqrt(2) + 3) + 28*x^2 + 21)/(sqrt(2*x^4 + 2*x^2 + 1)*x^
4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{21+28x^2+10\left(3+\sqrt{2}\right)x^4}{x^4\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{28\,x^2+10\,x^4\,\left(\sqrt{2}+3\right)+21}{x^4\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

input
```
int((28*x^2 + 10*x^4*(2^(1/2) + 3) + 21)/(x^4*(2*x^2 + 2*x^4 + 1)^(1/2)),x
)
```

output
```
int((28*x^2 + 10*x^4*(2^(1/2) + 3) + 21)/(x^4*(2*x^2 + 2*x^4 + 1)^(1/2)),
x)
```

Reduce [F]

$$\int \frac{21+28x^2+10\left(3+\sqrt{2}\right)x^4}{x^4\sqrt{1+2x^2+2x^4}}\,dx$$
$$= \frac{-7\sqrt{2x^4+2x^2+1}\,\sqrt{2}+16\sqrt{2}\left(\int \frac{\sqrt{2x^4+2x^2+1}}{2x^4+2x^2+1}dx\right)x^3+20\left(\int \frac{\sqrt{2x^4+2x^2+1}}{2x^4+2x^2+1}dx\right)x^3}{\sqrt{2}\,x^3}$$

input
```
int((21+28*x^2+10*(3+2^(1/2))*x^4)/x^4/(2*x^4+2*x^2+1)^(1/2),x)
```

output
```
( - 7*sqrt(2*x**4 + 2*x**2 + 1)*sqrt(2) + 16*sqrt(2)*int(sqrt(2*x**4 + 2*x
**2 + 1)/(2*x**4 + 2*x**2 + 1),x)*x**3 + 20*int(sqrt(2*x**4 + 2*x**2 + 1)/
(2*x**4 + 2*x**2 + 1),x)*x**3)/(sqrt(2)*x**3)
```

3.165 $\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 254

$$\begin{aligned}\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx &= -\frac{2(525A-511B+480C)x(2+3x^2)}{2835\sqrt{2+5x^2+3x^4}}+\frac{1}{189}(21A-28B+30C)x\sqrt{2+5x^2+3x^4}\\ &+\frac{1}{105}(7B-10C)x^3\sqrt{2+5x^2+3x^4}+\frac{1}{21}Cx^5\sqrt{2+5x^2+3x^4}\\ &+\frac{2\sqrt{2}(525A-511B+480C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{2835\sqrt{2+5x^2+3x^4}}\\ &-\frac{\sqrt{2}(21A-28B+30C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{189\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
-2/2835*(525*A-511*B+480*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/189*(21*A-
28*B+30*C)*x*(3*x^4+5*x^2+2)^(1/2)+1/105*(7*B-10*C)*x^3*(3*x^4+5*x^2+2)^(1
/2)+1/21*C*x^5*(3*x^4+5*x^2+2)^(1/2)+2/2835*2^(1/2)*(525*A-511*B+480*C)*(x
^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(
3*x^4+5*x^2+2)^(1/2)-1/189*2^(1/2)*(21*A-28*B+30*C)*(x^2+1)*((3*x^2+2)/(x^
2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.70

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx$$
$$= \frac{3x(2+5x^2+3x^4)\left(105A+7B(-20+9x^2)+15C(10-6x^2+3x^4)\right)+2i\sqrt{3}(525A-511B+480C)\sqrt{1}}{28}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(3*x*(2 + 5*x^2 + 3*x^4)*(105*A + 7*B*(-20 + 9*x^2) + 15*C*(10 - 6*x^2 + 3
*x^4)) + (2*I)*Sqrt[3]*(525*A - 511*B + 480*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^
2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (2*I)*Sqrt[3]*(420*A - 371*B +
 330*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/
3])/(2835*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2199, 1602, 27, 1602, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{3x^4+5x^2+2}}\,dx$$

$$\Big\downarrow \text{ 2199}$$

$$\int \frac{x^4\left(\frac{1}{7}(7B-10C)x^2+\frac{1}{21}(21A-10C)\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{21}C\sqrt{3x^4+5x^2+2}x^5$$

$$\Big\downarrow \text{ 1602}$$

$$-\frac{1}{15}\int \frac{x^2\left(6(7B-10C)-5(21A-28B+30C)x^2\right)}{7\sqrt{3x^4+5x^2+2}}dx+\frac{1}{105}\sqrt{3x^4+5x^2+2}x^3(7B-10C)+\frac{1}{21}C\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 27

$$-\frac{1}{105}\int \frac{x^2\left(6(7B-10C)-5(21A-28B+30C)x^2\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{105}\sqrt{3x^4+5x^2+2}x^3(7B-10C)+\frac{1}{21}C\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 1602

$$\frac{1}{105}\left(\frac{1}{9}\int -\frac{2\left((525A-511B+480C)x^2+5(21A-28B+30C)\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{5}{9}\sqrt{3x^4+5x^2+2}x(21A-28B+30C)\right.$$
$$\frac{1}{105}\sqrt{3x^4+5x^2+2}x^3(7B-10C)+\frac{1}{21}C\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 27

$$\frac{1}{105}\left(\frac{5}{9}x\sqrt{3x^4+5x^2+2}(21A-28B+30C)-\frac{2}{9}\int \frac{(525A-511B+480C)x^2+5(21A-28B+30C)}{\sqrt{3x^4+5x^2+2}}dx\right)+$$
$$\frac{1}{105}\sqrt{3x^4+5x^2+2}x^3(7B-10C)+\frac{1}{21}C\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 1503

$$\frac{1}{105}\left(\frac{5}{9}x\sqrt{3x^4+5x^2+2}(21A-28B+30C)-\frac{2}{9}\left(5(21A-28B+30C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+(525A-511B\right.\right.$$
$$\frac{1}{105}\sqrt{3x^4+5x^2+2}x^3(7B-10C)+\frac{1}{21}C\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 1413

$$\frac{1}{105}\left(\frac{5}{9}x\sqrt{3x^4+5x^2+2}(21A-28B+30C)-\frac{2}{9}\left((525A-511B+480C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{5\left(x^2+1\right)}{}\right.\right.$$
$$\frac{1}{105}\sqrt{3x^4+5x^2+2}x^3(7B-10C)+\frac{1}{21}C\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 1456

$$\frac{1}{105}\left(\frac{5}{9}x\sqrt{3x^4+5x^2+2}(21A-28B+30C)-\frac{2}{9}\left(\frac{5(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(21A-28B+30C)\operatorname{EllipticF}\left(\arctan(x),\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.\right.$$

$$\frac{1}{105}\sqrt{3x^4+5x^2+2}x^3(7B-10C)+\frac{1}{21}C\sqrt{3x^4+5x^2+2}x^5$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4))/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
((7*B - 10*C)*x^3*Sqrt[2 + 5*x^2 + 3*x^4])/105 + (C*x^5*Sqrt[2 + 5*x^2 + 3
*x^4])/21 + ((5*(21*A - 28*B + 30*C)*x*Sqrt[2 + 5*x^2 + 3*x^4])/9 - (2*((5
25*A - 511*B + 480*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt
[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*S
qrt[2 + 5*x^2 + 3*x^4])) + (5*(21*A - 28*B + 30*C)*(1 + x^2)*Sqrt[(2 + 3*x
^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4
])))/9)/105
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 16.89 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.76

method	result
elliptic	$\frac{C\,x^{5}\sqrt{3x^{4}+5x^{2}+2}}{21}+\left(\frac{B}{15}-\frac{2C}{21}\right)x^{3}\sqrt{3x^{4}+5x^{2}+2}+\left(\frac{A}{9}-\frac{4B}{27}+\frac{10C}{63}\right)x\sqrt{3x^{4}+5x^{2}+2}-\frac{i\left(-\frac{2A}{9}-\frac{20C}{63}\right.}{}$
risch	$\frac{x\left(45C\,x^{4}+63B\,x^{2}-90C\,x^{2}+105A-140B+150C\right)\sqrt{3x^{4}+5x^{2}+2}}{945}-\frac{2i\left(525A-511B+480C\right)\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\right.}{2835\sqrt{3x^{4}+5x^{2}+2}}$
default	$A\left(\frac{x\sqrt{3x^{4}+5x^{2}+2}}{9}+\frac{i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{9\sqrt{3x^{4}+5x^{2}+2}}-\frac{10i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{27\sqrt{3x^{4}+5x^{2}+2}}\right)$

input
```
int(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*C*x^5*(3*x^4+5*x^2+2)^(1/2)+(1/15*B-2/21*C)*x^3*(3*x^4+5*x^2+2)^(1/2)
+(1/9*A-4/27*B+10/63*C)*x*(3*x^4+5*x^2+2)^(1/2)-1/2*I*(-2/9*A-20/63*C+8/27
*B)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*
6^(1/2))+1/3*I*(146/135*B-64/63*C-10/9*A)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3
*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.48

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx$$
$$= \frac{4\sqrt{3}\sqrt{-\frac{2}{3}}(525\,A-511\,B+480\,C)xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2})-\sqrt{3}\sqrt{-\frac{2}{3}}(3045\,A-3304\,B+3270\,C)xF(a}{}$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/8505*(4*sqrt(3)*sqrt(-2/3)*(525*A - 511*B + 480*C)*x*elliptic_e(arcsin(s
qrt(-2/3)/x), 3/2) - sqrt(3)*sqrt(-2/3)*(3045*A - 3304*B + 3270*C)*x*ellip
tic_f(arcsin(sqrt(-2/3)/x), 3/2) + 3*(135*C*x^6 + 27*(7*B - 10*C)*x^4 + 15
*(21*A - 28*B + 30*C)*x^2 - 1050*A + 1022*B - 960*C)*sqrt(3*x^4 + 5*x^2 +
2))/x
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{(x^2+1)\,(3x^2+2)}}\,dx$$

input
```
integrate(x**4*(C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(x**4*(A + B*x**2 + C*x**4)/sqrt((x**2 + 1)*(3*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4\,(C\,x^4+B\,x^2+A)}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{\sqrt{3x^4+5x^2+2}\,ax}{9} + \frac{\sqrt{3x^4+5x^2+2}\,b\,x^3}{15} - \frac{4\sqrt{3x^4+5x^2+2}\,bx}{27} + \frac{\sqrt{3x^4+5x^2+2}\,c\,x^5}{21} - \frac{2\sqrt{3x^4+5x^2+2}\,c\,x^3}{21} + \frac{10\sqrt{3x^4+5x^2+2}\,cx}{63} - \frac{2\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)a}{9} + \frac{8\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)b}{27} - \frac{20\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)c}{63} - \frac{10\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)a}{9} + \frac{146\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)b}{135} - \frac{64\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)c}{63}$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(105*sqrt(3*x**4 + 5*x**2 + 2)*a*x + 63*sqrt(3*x**4 + 5*x**2 + 2)*b*x**3 -
 140*sqrt(3*x**4 + 5*x**2 + 2)*b*x + 45*sqrt(3*x**4 + 5*x**2 + 2)*c*x**5 -
 90*sqrt(3*x**4 + 5*x**2 + 2)*c*x**3 + 150*sqrt(3*x**4 + 5*x**2 + 2)*c*x -
 210*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x)*a + 280*int(sq
rt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x)*b - 300*int(sqrt(3*x**4 +
 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x)*c - 1050*int((sqrt(3*x**4 + 5*x**2 +
 2)*x**2)/(3*x**4 + 5*x**2 + 2),x)*a + 1022*int((sqrt(3*x**4 + 5*x**2 + 2)
*x**2)/(3*x**4 + 5*x**2 + 2),x)*b - 960*int((sqrt(3*x**4 + 5*x**2 + 2)*x**
2)/(3*x**4 + 5*x**2 + 2),x)*c)/945
```

3.166 $\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 218

$$
\begin{aligned}
\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx
&= \frac{(135A-150B+146C)x(2+3x^2)}{405\sqrt{2+5x^2+3x^4}} \\
&\quad + \frac{1}{27}(3B-4C)x\sqrt{2+5x^2+3x^4} + \frac{1}{15}Cx^3\sqrt{2+5x^2+3x^4} \\
&\quad - \frac{\sqrt{2}(135A-150B+146C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{405\sqrt{2+5x^2+3x^4}} \\
&\quad - \frac{\sqrt{2}(3B-4C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}\,\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{27\sqrt{2+5x^2+3x^4}}
\end{aligned}
$$

output
```
1/405*(135*A-150*B+146*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/27*(3*B-4*C)
*x*(3*x^4+5*x^2+2)^(1/2)+1/15*C*x^3*(3*x^4+5*x^2+2)^(1/2)-1/405*2^(1/2)*(1
35*A-150*B+146*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1
/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)-1/27*2^(1/2)*(3*B-4*C)*(x^2+1)*((
3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*
x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx$$
$$= \frac{3x(2+5x^2+3x^4)(15B+C(-20+9x^2)) - i\sqrt{3}(135A-150B+146C)\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\right.\right.}{405\sqrt{2+5x^2+}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(3*x*(2 + 5*x^2 + 3*x^4)*(15*B + C*(-20 + 9*x^2)) - I*Sqrt[3]*(135*A - 150
*B + 146*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x]
, 2/3] + I*Sqrt[3]*(135*A - 120*B + 106*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*E
llipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(405*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2199, 1602, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^2+Cx^4\right)}{\sqrt{3x^4+5x^2+2}}\,dx$$
$$\Big\downarrow \text{ 2199}$$
$$\int \frac{x^2\left(\frac{1}{3}(3B-4C)x^2+\frac{1}{5}(5A-2C)\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{15}C\sqrt{3x^4+5x^2+2}x^3$$
$$\Big\downarrow \text{ 1602}$$

$$-\frac{1}{9}\int \frac{10(3B-4C)-(135A-150B+146C)x^2}{15\sqrt{3x^4+5x^2+2}}dx+\frac{1}{27}\sqrt{3x^4+5x^2+2}x(3B-4C)+\frac{1}{15}C\sqrt{3x^4+5x^2+2}x^3$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{135}\int \frac{10(3B-4C)-(135A-150B+146C)x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{27}\sqrt{3x^4+5x^2+2}x(3B-4C)+\frac{1}{15}C\sqrt{3x^4+5x^2+2}x^3$$

$$\Big\downarrow \; 1503$$

$$\frac{1}{135}\left((135A-150B+146C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx-10(3B-4C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{27}\sqrt{3x^4+5x^2+2}x(3B-4C)+\frac{1}{15}C\sqrt{3x^4+5x^2+2}x^3$$

$$\Big\downarrow \; 1413$$

$$\frac{1}{135}\left((135A-150B+146C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx-\frac{5\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(3B-4C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right.}{\sqrt{3x^4+5x^2+2}}\right.$$
$$\frac{1}{27}\sqrt{3x^4+5x^2+2}x(3B-4C)+\frac{1}{15}C\sqrt{3x^4+5x^2+2}x^3$$

$$\Big\downarrow \; 1456$$

$$\frac{1}{135}\left((135A-150B+146C)\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{3x^4+5x^2+2}}\right)-\frac{5\sqrt{2}(x^2+1)\sqrt{}}{}\right.$$
$$\frac{1}{27}\sqrt{3x^4+5x^2+2}x(3B-4C)+\frac{1}{15}C\sqrt{3x^4+5x^2+2}x^3$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
((3*B - 4*C)*x*Sqrt[2 + 5*x^2 + 3*x^4])/27 + (C*x^3*Sqrt[2 + 5*x^2 + 3*x^4
])/15 + ((135*A - 150*B + 146*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^
4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
-1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) - (5*Sqrt[2]*(3*B - 4*C)*(1 + x^2)*Sqr
t[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^
4])/135
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 10.59 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.73

method	result
elliptic	$\frac{C x^3\sqrt{3x^4+5x^2+2}}{15}+\left(\frac{B}{9}-\frac{4C}{27}\right)x\sqrt{3x^4+5x^2+2}-\frac{i\left(-\frac{2B}{9}+\frac{8C}{27}\right)\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{i\left(A+\frac{146C}{135}\right.}{}$
risch	$\frac{x(9Cx^2+15B-20C)\sqrt{3x^4+5x^2+2}}{135}+\frac{i(135A-150B+146C)\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{405\sqrt{3x^4+5x^2+2}}+\frac{iB\sqrt{}}{}$
default	$\frac{iA\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^4+5x^2+2}}+B\left(\frac{x\sqrt{3x^4+5x^2+2}}{9}+\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{9\sqrt{3x^4+5x^2+2}}-\right.$

input
```
int(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*C*x^3*(3*x^4+5*x^2+2)^(1/2)+(1/9*B-4/27*C)*x*(3*x^4+5*x^2+2)^(1/2)-1/
2*I*(-2/9*B+8/27*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*El
lipticF(I*x,1/2*6^(1/2))+1/3*I*(A+146/135*C-10/9*B)*(x^2+1)^(1/2)*(6*x^2+4
)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/
2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.50

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx =$$
$$-\frac{2\sqrt{3}\sqrt{-\frac{2}{3}}(135\,A-150\,B+146\,C)xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2})-\sqrt{3}\sqrt{-\frac{2}{3}}(270\,A-435\,B+472\,C)xF(\mathrm{ar}}{1215\,x}$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/1215*(2*sqrt(3)*sqrt(-2/3)*(135*A - 150*B + 146*C)*x*elliptic_e(arcsin(
sqrt(-2/3)/x), 3/2) - sqrt(3)*sqrt(-2/3)*(270*A - 435*B + 472*C)*x*ellipti
c_f(arcsin(sqrt(-2/3)/x), 3/2) - 3*(27*C*x^4 + 15*(3*B - 4*C)*x^2 + 135*A
- 150*B + 146*C)*sqrt(3*x^4 + 5*x^2 + 2))/x
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{(x^2+1)\,(3x^2+2)}}\,dx$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4)/sqrt((x**2 + 1)*(3*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^2}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{x^2\,\left(C\,x^4+B\,x^2+A\right)}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{2+5x^2+3x^4}}\,dx&=\frac{\sqrt{3x^4+5x^2+2}\,bx}{9}+\frac{\sqrt{3x^4+5x^2+2}\,c\,x^3}{15}\\&\quad-\frac{4\sqrt{3x^4+5x^2+2}\,cx}{27}-\frac{2\left(\int\frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)b}{9}\\&\quad+\frac{8\left(\int\frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)c}{27}+\left(\int\frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)a\\&\quad-\frac{10\left(\int\frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)b}{9}+\frac{146\left(\int\frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)c}{135}\end{aligned}$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(15*sqrt(3*x**4 + 5*x**2 + 2)*b*x + 9*sqrt(3*x**4 + 5*x**2 + 2)*c*x**3 - 2
0*sqrt(3*x**4 + 5*x**2 + 2)*c*x - 30*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4
 + 5*x**2 + 2),x)*b + 40*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 +
2),x)*c + 135*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2 + 2),x
)*a - 150*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2 + 2),x)*b
+ 146*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2 + 2),x)*c)/135
```

3.167 $\int \frac{A+Bx^2+Cx^4}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 182

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{(9B-10C)x(2+3x^2)}{27\sqrt{2+5x^2+3x^4}}+\frac{1}{9}Cx\sqrt{2+5x^2+3x^4} - \frac{\sqrt{2}(9B-10C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{27\sqrt{2+5x^2+3x^4}} + \frac{(9A-2C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{9\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
1/27*(9*B-10*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/9*C*x*(3*x^4+5*x^2+2)^
(1/2)-1/27*2^(1/2)*(9*B-10*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(
x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+1/18*(9*A-2*C)*(x^2+1
)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/
2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{2+5x^2+3x^4}}\,dx$$
$$= \frac{3Cx(2+5x^2+3x^4)-i\sqrt{3}(9B-10C)\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-i\sqrt{3}(9A-9B+8C)}{27\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(3*C*x*(2 + 5*x^2 + 3*x^4) - I*Sqrt[3]*(9*B - 10*C)*Sqrt[1 + x^2]*Sqrt[2 +
 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - I*Sqrt[3]*(9*A - 9*B + 8*
C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(
27*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2207, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{3x^4+5x^2+2}}\,dx$$
$$\Big\downarrow \text{2207}$$
$$\frac{1}{9}\int \frac{(9B-10C)x^2+9A-2C}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{9}C\sqrt{3x^4+5x^2+2}x$$
$$\Big\downarrow \text{1503}$$

$$\frac{1}{9}\left((9A-2C)\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx+(9B-10C)\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{9}C\sqrt{3x^4+5x^2+2}x$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{9}\left((9B-10C)\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(9A-2C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+\frac{1}{9}C\sqrt{3x^4+5x^2+2}x$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{9}\left(\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(9A-2C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+(9B-10C)\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x}{x}}}{3\sqrt{3x}}\right.\right.$$
$$\frac{1}{9}C\sqrt{3x^4+5x^2+2}x$$

input
```
Int[(A + B*x^2 + C*x^4)/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(C*x*Sqrt[2 + 5*x^2 + 3*x^4])/9 + ((9*B - 10*C)*((x*(2 + 3*x^2))/(3*Sqrt[2
 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Ellipt
icE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + ((9*A - 2*C)*(1 + x^2
)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2
+ 5*x^2 + 3*x^4]))/9
```

Defintions of rubi rules used

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.71

method	result
elliptic	$\frac{Cx\sqrt{3x^4+5x^2+2}}{9}-\frac{i\left(A-\frac{2C}{9}\right)\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{i\left(B-\frac{10C}{9}\right)\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{Elliptic}\right.}{3\sqrt{3x^4+5x^2+2}}$
risch	$\frac{Cx\sqrt{3x^4+5x^2+2}}{9}+\frac{i(9B-10C)\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{27\sqrt{3x^4+5x^2+2}}-\frac{iA\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\right.}{2\sqrt{3x^4+5x^2+2}}$
default	$-\frac{iA\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{iB\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^4+5x^2+2}}+C\left(\frac{x\sqrt{3x^4+5x^2+2}}{9}\right.$

input
```
int((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*C*x*(3*x^4+5*x^2+2)^(1/2)-1/2*I*(A-2/9*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2
)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3*I*(B-10/9*C)*(x^2+1
)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-
EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{2+5x^2+3x^4}}\,dx =$$
$$-\frac{4\sqrt{3}\sqrt{-\frac{2}{3}}(9B-10C)xE\left(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}\right) - \sqrt{3}\sqrt{-\frac{2}{3}}(81A+36B-58C)xF\left(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}\right.}{162\,x}$$

input
```
integrate((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/162*(4*sqrt(3)*sqrt(-2/3)*(9*B - 10*C)*x*elliptic_e(arcsin(sqrt(-2/3)/x
), 3/2) - sqrt(3)*sqrt(-2/3)*(81*A + 36*B - 58*C)*x*elliptic_f(arcsin(sqrt
(-2/3)/x), 3/2) - 6*sqrt(3*x^4 + 5*x^2 + 2)*(3*C*x^2 + 9*B - 10*C))/x
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{\sqrt{(x^2+1)(3x^2+2)}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/sqrt((x**2 + 1)*(3*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{\sqrt{3x^4+5x^2+2}\,cx}{9} + \left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)a - \frac{2\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)c}{9} + \left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)b - \frac{10\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)c}{9}$$

input
```
int((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(sqrt(3*x**4 + 5*x**2 + 2)*c*x + 9*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 +
 5*x**2 + 2),x)*a - 2*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),
x)*c + 9*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2 + 2),x)*b -
 10*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2 + 2),x)*c)/9
```

3.168 $\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 175

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{2+5x^2+3x^4}}\,dx = \frac{(3A+2C)x(2+3x^2)}{6\sqrt{2+5x^2+3x^4}} - \frac{A\sqrt{2+5x^2+3x^4}}{2x} - \frac{(3A+2C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{2}\sqrt{2+5x^2+3x^4}} + \frac{B(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
1/6*(3*A+2*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-1/2*A*(3*x^4+5*x^2+2)^(1/2
)/x-1/6*(3*A+2*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1
/2),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)+1/2*B*(x^2+1)*((3*x^2+2)/
(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x
^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{2+5x^2+3x^4}}\,dx = \frac{-3A(2+5x^2+3x^4)-i\sqrt{3}(3A+2C)x\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)+i\sqrt{3}(3A-2B+2C}{6x\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
(-3*A*(2 + 5*x^2 + 3*x^4) - I*Sqrt[3]*(3*A + 2*C)*x*Sqrt[1 + x^2]*Sqrt[2 +
 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + I*Sqrt[3]*(3*A - 2*B + 2*
C)*x*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])
/(6*x*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2199, 1604, 25, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{3x^4+5x^2+2}}\,dx$$

$$\Big\downarrow \text{2199}$$

$$\int \frac{Bx^2+\frac{1}{3}(3A+2C)}{x^2\sqrt{3x^4+5x^2+2}}dx+\frac{C\sqrt{3x^4+5x^2+2}}{3x}$$

$$\Big\downarrow \text{1604}$$

$$-\frac{1}{2}\int -\frac{(3A+2C)x^2+2B}{\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(3A+2C)}{6x}+\frac{C\sqrt{3x^4+5x^2+2}}{3x}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{(3A+2C)x^2+2B}{\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(3A+2C)}{6x}+\frac{C\sqrt{3x^4+5x^2+2}}{3x}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{2}\left((3A+2C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+2B\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{\sqrt{3x^4+5x^2+2}(3A+2C)}{6x}+\frac{C\sqrt{3x^4+5x^2+2}}{3x}$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{2}\left((3A+2C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{2}B(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)-\frac{\sqrt{3x^4+5x^2+2}(3A+2C)}{6x}+\frac{C\sqrt{3x^4+5x^2+2}}{3x}$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{2}\left((3A+2C)\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{3x^4+5x^2+2}}\right)+\frac{\sqrt{2}B(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{Ellipticl}}{\sqrt{3x^4+5x^2+}}\right.$$
$$\frac{\sqrt{3x^4+5x^2+2}(3A+2C)}{6x}+\frac{C\sqrt{3x^4+5x^2+2}}{3x}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
(C*Sqrt[2 + 5*x^2 + 3*x^4])/(3*x) - ((3*A + 2*C)*Sqrt[2 + 5*x^2 + 3*x^4])/
(6*x) + ((3*A + 2*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[
2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sq
rt[2 + 5*x^2 + 3*x^4])) + (Sqrt[2]*B*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]
*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.73

method	result
elliptic	$-\frac{A\sqrt{3x^4+5x^2+2}}{2x}-\frac{iB\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{i\left(C+\frac{3A}{2}\right)\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^4+5x^2+2}}$
risch	$-\frac{A\sqrt{3x^4+5x^2+2}}{2x}+\frac{i(3A+2C)\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{6\sqrt{3x^4+5x^2+2}}-\frac{iB\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$
default	$-\frac{iB\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+A\left(-\frac{\sqrt{3x^4+5x^2+2}}{2x}+\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^4+5x^2+2}}\right.$

input
```
int((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A*(3*x^4+5*x^2+2)^(1/2)/x-1/2*I*B*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^
4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3*I*(C+3/2*A)*(x^2+1)^(1/2)*
(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-Elliptic
E(I*x,1/2*6^(1/2)))
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{3\,x^4+5\,x^2+2}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(3*x^4 + 5*x^2 + 2)/(3*x^6 + 5*x^4 + 2*x^
2), x)
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^2\sqrt{2 + 5x^2 + 3x^4}}\,dx = \int \frac{A + Bx^2 + Cx^4}{x^2\sqrt{(x^2 + 1)\,(3x^2 + 2)}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**2*sqrt((x**2 + 1)*(3*x**2 + 2))), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^2\sqrt{2 + 5x^2 + 3x^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{3\,x^4 + 5\,x^2 + 2}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(3*x^4 + 5*x^2 + 2)*x^2), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^2\sqrt{2 + 5x^2 + 3x^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{3\,x^4 + 5\,x^2 + 2}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(3*x^4 + 5*x^2 + 2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^2\,\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(5*x^2 + 3*x^4 + 2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(5*x^2 + 3*x^4 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{2+5x^2+3x^4}}\,dx = \frac{\sqrt{3x^4+5x^2+2}\,c+3\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^6+5x^4+2x^2}dx\right)ax+2\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^6+5x^4+2x^2}dx\right)cx+3\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)bx}{3x}$$

input
```
int((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(sqrt(3*x**4 + 5*x**2 + 2)*c + 3*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**6 + 5
*x**4 + 2*x**2),x)*a*x + 2*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**6 + 5*x**4
+ 2*x**2),x)*c*x + 3*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x
)*b*x)/(3*x)
```

3.169 $\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 184

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{2+5x^2+3x^4}}\,dx = -\frac{A\sqrt{2+5x^2+3x^4}}{6x^3}+\frac{(5A-3B)\sqrt{2+5x^2+3x^4}}{6x\,(1+x^2)}+\frac{(5A-3B)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+5x^2+3x^4}}-\frac{(A-2C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
-1/6*A*(3*x^4+5*x^2+2)^(1/2)/x^3+1/6*(5*A-3*B)*(3*x^4+5*x^2+2)^(1/2)/x/(x^
2+1)+1/6*(5*A-3*B)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(
1/2),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)-1/4*(A-2*C)*(x^2+1)*((3*
x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*
x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{2+5x^2+3x^4}}\,dx = \frac{-((A-5Ax^2+3Bx^2)(2+5x^2+3x^4))+i\sqrt{3}(5A-3B)x^3\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-}{6x^3\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
(-((A - 5*A*x^2 + 3*B*x^2)*(2 + 5*x^2 + 3*x^4)) + I*Sqrt[3]*(5*A - 3*B)*x^
3*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - I
*Sqrt[3]*(4*A - 3*B + 2*C)*x^3*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*A
rcSinh[Sqrt[3/2]*x], 2/3])/(6*x^3*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2199, 1604, 1604, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{3x^4+5x^2+2}}\,dx$$

$$\Big\downarrow \ 2199$$

$$\int \frac{\frac{1}{3}(3B-10C)x^2+A-2C}{x^4\sqrt{3x^4+5x^2+2}}dx-\frac{C\sqrt{3x^4+5x^2+2}}{3x^3}$$

$$\Big\downarrow \ 1604$$

$$-\frac{1}{6}\int \frac{3(A-2C)x^2+2(5A-3B)}{x^2\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(A-2C)}{6x^3}-\frac{C\sqrt{3x^4+5x^2+2}}{3x^3}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{6}\left(\frac{1}{2}\int -\frac{6\left((5A-3B)x^2+A-2C\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(5A-3B)}{x}\right)-\frac{\sqrt{3x^4+5x^2+2}(A-2C)}{6x^3}-\frac{C\sqrt{3x^4+5x^2+2}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{\sqrt{3x^4+5x^2+2}(5A-3B)}{x}-3\int \frac{(5A-3B)x^2+A-2C}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{\sqrt{3x^4+5x^2+2}(A-2C)}{6x^3}-\frac{C\sqrt{3x^4+5x^2+2}}{3x^3}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{6}\left(\frac{\sqrt{3x^4+5x^2+2}(5A-3B)}{x}-3\left((5A-3B)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+(A-2C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx\right)\right)-\frac{\sqrt{3x^4+5x^2+2}(A-2C)}{6x^3}-\frac{C\sqrt{3x^4+5x^2+2}}{3x^3}$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{6}\left(\frac{\sqrt{3x^4+5x^2+2}(5A-3B)}{x}-3\left((5A-3B)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(A-2C)\,\text{EllipticF}\,(\text{ar}}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.\right.$$
$$-\frac{\sqrt{3x^4+5x^2+2}(A-2C)}{6x^3}-\frac{C\sqrt{3x^4+5x^2+2}}{3x^3}$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{6}\left(\frac{\sqrt{3x^4+5x^2+2}(5A-3B)}{x}-3\left((5A-3B)\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{3x^4+5x^2+2}}\right)\right.\right.$$
$$-\frac{\sqrt{3x^4+5x^2+2}(A-2C)}{6x^3}-\frac{C\sqrt{3x^4+5x^2+2}}{3x^3}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^4*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
-1/6*((A - 2*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x^3 - (C*Sqrt[2 + 5*x^2 + 3*x^4])
/(3*x^3) + (((5*A - 3*B)*Sqrt[2 + 5*x^2 + 3*x^4])/x - 3*((5*A - 3*B)*((x*(
2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x
^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) +
((A - 2*C)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2
])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 5.31 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{A\sqrt{3x^4+5x^2+2}}{6x^3}-\frac{\left(\frac{B}{2}-\frac{5A}{6}\right)\sqrt{3x^4+5x^2+2}}{x}-\frac{i\left(C-\frac{A}{2}\right)\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{i\left(\frac{3B}{2}-\frac{5A}{2}\right)\sqrt{x^2+1}\,\sqrt{6x^2+4}}{3}$
risch	$\frac{\sqrt{3x^4+5x^2+2}\,(5A\,x^2-3B\,x^2-A)}{6x^3}+\frac{iA\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{4\sqrt{3x^4+5x^2+2}}-\frac{i(5A-3B)\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\right.}{6\sqrt{3x^4+5x^2+2}}$
default	$-\frac{iC\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+A\left(-\frac{\sqrt{3x^4+5x^2+2}}{6x^3}+\frac{5\sqrt{3x^4+5x^2+2}}{6x}+\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{4\sqrt{3x^4+5x^2+2}}-\right.$

input
```
int((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*A*(3*x^4+5*x^2+2)^(1/2)/x^3-(1/2*B-5/6*A)*(3*x^4+5*x^2+2)^(1/2)/x-1/2
*I*(C-1/2*A)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF
(I*x,1/2*6^(1/2))+1/3*I*(3/2*B-5/2*A)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4
+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.45

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{2+5x^2+3x^4}}\,dx = \frac{-2i\sqrt{2}(5A-3B)x^3E(\arcsin(ix)\mid\frac{3}{2})+i\sqrt{2}(13A-6B-6C)x^3F(\arcsin(ix)\mid\frac{3}{2})+2\sqrt{3x^4+5x^2}}{12x^3}$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(-2*I*sqrt(2)*(5*A - 3*B)*x^3*elliptic_e(arcsin(I*x), 3/2) + I*sqrt(2
)*(13*A - 6*B - 6*C)*x^3*elliptic_f(arcsin(I*x), 3/2) + 2*sqrt(3*x^4 + 5*x
^2 + 2)*((5*A - 3*B)*x^2 - A))/x^3
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^4\sqrt{(x^2+1)(3x^2+2)}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**4*sqrt((x**2 + 1)*(3*x**2 + 2))), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{3x^4+5x^2+2}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(3*x^4 + 5*x^2 + 2)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{3\,x^4+5\,x^2+2}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(3*x^4 + 5*x^2 + 2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^4\,\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(5*x^2 + 3*x^4 + 2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(5*x^2 + 3*x^4 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{2+5x^2+3x^4}}\,dx$$
$$=\frac{-\sqrt{3x^4+5x^2+2}\,b+10\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^8+5x^6+2x^4}dx\right)a\,x^3-6\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^8+5x^6+2x^4}dx\right)b\,x^3-3\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)b\,x^3+}{10x^3}$$

input
```
int((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
( - sqrt(3*x**4 + 5*x**2 + 2)*b + 10*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**8
 + 5*x**6 + 2*x**4),x)*a*x**3 - 6*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**8 +
5*x**6 + 2*x**4),x)*b*x**3 - 3*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x
**2 + 2),x)*b*x**3 + 10*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2
),x)*c*x**3)/(10*x**3)
```

3.170 $\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 222

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{2+5x^2+3x^4}}\,dx = -\frac{A\sqrt{2+5x^2+3x^4}}{10x^5}+\frac{(2A-B)\sqrt{2+5x^2+3x^4}}{6x^3} - \frac{(73A-50B+30C)\sqrt{2+5x^2+3x^4}}{60x(1+x^2)} - \frac{(73A-50B+30C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{30\sqrt{2}\sqrt{2+5x^2+3x^4}} + \frac{(2A-B)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
-1/10*A*(3*x^4+5*x^2+2)^(1/2)/x^5+1/6*(2*A-B)*(3*x^4+5*x^2+2)^(1/2)/x^3-1/
60*(73*A-50*B+30*C)*(3*x^4+5*x^2+2)^(1/2)/x/(x^2+1)-1/60*(73*A-50*B+30*C)*
(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))
*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)+1/4*(2*A-B)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/
2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{2+5x^2+3x^4}}\,dx$$
$$= \frac{-((2+5x^2+3x^4)(10x^2(B-5Bx^2+3Cx^2)+A(6-20x^2+73x^4)))-i\sqrt{3}(73A-50B+30C)x^5\sqrt{1-}}{60x^5}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
(-((2 + 5*x^2 + 3*x^4)*(10*x^2*(B - 5*B*x^2 + 3*C*x^2) + A*(6 - 20*x^2 + 7
3*x^4))) - I*Sqrt[3]*(73*A - 50*B + 30*C)*x^5*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2
]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + I*Sqrt[3]*(53*A - 40*B + 30*C)*
x^5*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/
(60*x^5*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2199, 1604, 1604, 27, 1604, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{3x^4+5x^2+2}}\,dx$$
$$\Big\downarrow \text{ 2199}$$
$$\int \frac{\frac{1}{9}(9B-20C)x^2+\frac{1}{9}(9A-10C)}{x^6\sqrt{3x^4+5x^2+2}}dx-\frac{C\sqrt{3x^4+5x^2+2}}{9x^5}$$
$$\Big\downarrow \text{ 1604}$$
$$-\frac{1}{10}\int \frac{(9A-10C)x^2+10(2A-B)}{x^4\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(9A-10C)}{90x^5}-\frac{C\sqrt{3x^4+5x^2+2}}{9x^5}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{10}\left(\frac{1}{6}\int \frac{2\left(15(2A-B)x^2+73A-50B+30C\right)}{x^2\sqrt{3x^4+5x^2+2}}dx+\frac{5\sqrt{3x^4+5x^2+2}(2A-B)}{3x^3}\right)-\frac{\sqrt{3x^4+5x^2+2}(9A-10C)}{90x^5}-\frac{C\sqrt{3x^4+5x^2+2}}{9x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{1}{3}\int \frac{15(2A-B)x^2+73A-50B+30C}{x^2\sqrt{3x^4+5x^2+2}}dx+\frac{5\sqrt{3x^4+5x^2+2}(2A-B)}{3x^3}\right)-\frac{\sqrt{3x^4+5x^2+2}(9A-10C)}{90x^5}-\frac{C\sqrt{3x^4+5x^2+2}}{9x^5}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(-\frac{1}{2}\int -\frac{3\left((73A-50B+30C)x^2+10(2A-B)\right)}{\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(73A-50B+30C)}{2x}\right)+\frac{5\sqrt{3x^4}}{}\right.$$
$$-\frac{\sqrt{3x^4+5x^2+2}(9A-10C)}{90x^5}-\frac{C\sqrt{3x^4+5x^2+2}}{9x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{(73A-50B+30C)x^2+10(2A-B)}{\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(73A-50B+30C)}{2x}\right)+\frac{5\sqrt{3x^4+5x^2}}{3x}\right.$$
$$-\frac{\sqrt{3x^4+5x^2+2}(9A-10C)}{90x^5}-\frac{C\sqrt{3x^4+5x^2+2}}{9x^5}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{3}{2}\left((73A-50B+30C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+10(2A-B)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{\sqrt{3x^4+5x^2+2}}{}\right.\right.$$
$$-\frac{\sqrt{3x^4+5x^2+2}(9A-10C)}{90x^5}-\frac{C\sqrt{3x^4+5x^2+2}}{9x^5}$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{3}{2}\left((73A-50B+30C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{5\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}(2A-B)\,\text{EllipticF}\left(\arctan(x),-\right.}{\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$
$$-\frac{\sqrt{3x^4+5x^2+2}(9A-10C)}{90x^5}-\frac{C\sqrt{3x^4+5x^2+2}}{9x^5}$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{10}\left(\frac{1}{3}\left(\frac{3}{2}\left((73A-50B+30C)\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{3x^4+5x^2+2}}\right)+\frac{5\sqrt{2}(x^2+1)\right.\right.\right.$$

$$\frac{\sqrt{3x^4+5x^2+2}(9A-10C)}{90x^5}-\frac{C\sqrt{3x^4+5x^2+2}}{9x^5}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^6*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
-1/90*((9*A - 10*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x^5 - (C*Sqrt[2 + 5*x^2 + 3*x
^4])/(9*x^5) + ((5*(2*A - B)*Sqrt[2 + 5*x^2 + 3*x^4])/(3*x^3) + (-1/2*((73
*A - 50*B + 30*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x + (3*((73*A - 50*B + 30*C)*((
x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 +
3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4]))
 + (5*Sqrt[2]*(2*A - B)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[Ar
cTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4]))/2)/3)/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 11.03 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{A\sqrt{3x^4+5x^2+2}}{10x^5}-\frac{\left(\frac{B}{2}-A\right)\sqrt{3x^4+5x^2+2}}{3x^3}-\frac{\left(\frac{C}{2}+\frac{73A}{60}-\frac{5B}{6}\right)\sqrt{3x^4+5x^2+2}}{x}-\frac{i\left(A-\frac{B}{2}\right)\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}$
risch	$-\frac{\sqrt{3x^4+5x^2+2}\,(73A\,x^4-50B\,x^4+30C\,x^4-20A\,x^2+10B\,x^2+6A)}{60x^5}+\frac{i(73A-50B+30C)\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{E}\right.}{60\sqrt{3x^4+5x^2+2}}$
default	$A\left(-\frac{\sqrt{3x^4+5x^2+2}}{10x^5}+\frac{\sqrt{3x^4+5x^2+2}}{3x^3}-\frac{73\sqrt{3x^4+5x^2+2}}{60x}-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{73i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{E}\right.}{6}\right.$

input
```
int((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*A*(3*x^4+5*x^2+2)^(1/2)/x^5-1/3*(1/2*B-A)*(3*x^4+5*x^2+2)^(1/2)/x^3-
(1/2*C+73/60*A-5/6*B)*(3*x^4+5*x^2+2)^(1/2)/x-1/2*I*(A-1/2*B)*(x^2+1)^(1/2
)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3*I*(
73/20*A+3/2*C-5/2*B)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(
EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.45

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{2+5x^2+3x^4}}\,dx = \frac{i\sqrt{2}(73\,A-50\,B+30\,C)x^5E(\arcsin{(i\,x)}\mid\frac{3}{2})-i\sqrt{2}(103\,A-65\,B+30\,C)x^5F(\arcsin{(i\,x)}\mid\frac{3}{2})-((73}{60\,x^5}$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/60*(I*sqrt(2)*(73*A - 50*B + 30*C)*x^5*elliptic_e(arcsin(I*x), 3/2) - I*
sqrt(2)*(103*A - 65*B + 30*C)*x^5*elliptic_f(arcsin(I*x), 3/2) - ((73*A -
50*B + 30*C)*x^4 - 10*(2*A - B)*x^2 + 6*A)*sqrt(3*x^4 + 5*x^2 + 2))/x^5
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^6\sqrt{(x^2+1)\,(3x^2+2)}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**6/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**6*sqrt((x**2 + 1)*(3*x**2 + 2))), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{3\,x^4+5\,x^2+2}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(3*x^4 + 5*x^2 + 2)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{3\,x^4+5\,x^2+2}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(3*x^4 + 5*x^2 + 2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^6\,\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^6*(5*x^2 + 3*x^4 + 2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^6*(5*x^2 + 3*x^4 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{2+5x^2+3x^4}}\,dx$$
$$= \frac{9\sqrt{3x^4+5x^2+2}\,a\,x^4-2\sqrt{3x^4+5x^2+2}\,a-10\sqrt{3x^4+5x^2+2}\,c\,x^4-40\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^8+5x^6+2x^4}dx\right)a\,x^5+20\left(\right.}{20x^5}$$

input
```
int((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(9*sqrt(3*x**4 + 5*x**2 + 2)*a*x**4 - 2*sqrt(3*x**4 + 5*x**2 + 2)*a - 10*s
qrt(3*x**4 + 5*x**2 + 2)*c*x**4 - 40*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**8
 + 5*x**6 + 2*x**4),x)*a*x**5 + 20*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**8 +
 5*x**6 + 2*x**4),x)*b*x**5 - 27*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x
**4 + 5*x**2 + 2),x)*a*x**5 + 30*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x
**4 + 5*x**2 + 2),x)*c*x**5)/(20*x**5)
```

3.171 $\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 258

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{2+5x^2+3x^4}}\,dx &= -\frac{A\sqrt{2+5x^2+3x^4}}{14x^7}+\frac{(15A-7B)\sqrt{2+5x^2+3x^4}}{70x^5}\\ &\quad-\frac{(45A-28B+14C)\sqrt{2+5x^2+3x^4}}{84x^3}+\frac{(720A-511B+350C)\sqrt{2+5x^2+3x^4}}{420x\,(1+x^2)}\\ &\quad+\frac{(720A-511B+350C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{210\sqrt{2}\sqrt{2+5x^2+3x^4}}\\ &\quad-\frac{(45A-28B+14C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{28\sqrt{2}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output

```
-1/14*A*(3*x^4+5*x^2+2)^(1/2)/x^7+1/70*(15*A-7*B)*(3*x^4+5*x^2+2)^(1/2)/x^
5-1/84*(45*A-28*B+14*C)*(3*x^4+5*x^2+2)^(1/2)/x^3+1/420*(720*A-511*B+350*C
)*(3*x^4+5*x^2+2)^(1/2)/x/(x^2+1)+1/420*(720*A-511*B+350*C)*(x^2+1)*((3*x^
2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)/(3*x^
4+5*x^2+2)^(1/2)-1/56*(45*A-28*B+14*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*I
nverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{2+5x^2+3x^4}}\,dx = \frac{(2+5x^2+3x^4)\left(70Cx^4(-1+5x^2)-7Bx^2(6-20x^2+73x^4)+15A(-2+6x^2-15x^4+48x^6)\right)+i\sqrt{3}(}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^8*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
((2 + 5*x^2 + 3*x^4)*(70*C*x^4*(-1 + 5*x^2) - 7*B*x^2*(6 - 20*x^2 + 73*x^4
) + 15*A*(-2 + 6*x^2 - 15*x^4 + 48*x^6)) + I*Sqrt[3]*(720*A - 511*B + 350*
C)*x^7*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3
] - I*Sqrt[3]*(495*A - 371*B + 280*C)*x^7*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*El
lipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(420*x^7*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2199, 1604, 1604, 27, 1604, 1604, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{3x^4+5x^2+2}}\,dx$$

$$\Big\downarrow \ 2199$$

$$\int \frac{(B-2C)x^2+\frac{1}{15}(15A-14C)}{x^8\sqrt{3x^4+5x^2+2}}dx-\frac{C\sqrt{3x^4+5x^2+2}}{15x^7}$$

$$\Big\downarrow \ 1604$$

$$-\frac{1}{14}\int \frac{(15A-14C)x^2+2(15A-7B)}{x^6\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(15A-14C)}{210x^7}-\frac{C\sqrt{3x^4+5x^2+2}}{15x^7}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{14}\left(\frac{1}{10}\int \frac{2\big(9(15A-7B)x^2+5(45A-14(2B-C))\big)}{x^4\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(15A-7B)}{5x^5}\right)-\frac{\sqrt{3x^4+5x^2+2}(15A-14C)}{210x^7}-\frac{C\sqrt{3x^4+5x^2+2}}{15x^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{14}\left(\frac{1}{5}\int \frac{9(15A-7B)x^2+5(45A-14(2B-C))}{x^4\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(15A-7B)}{5x^5}\right)-\frac{\sqrt{3x^4+5x^2+2}(15A-14C)}{210x^7}-\frac{C\sqrt{3x^4+5x^2+2}}{15x^7}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{14}\Bigg(\frac{1}{5}\Bigg(-\frac{1}{6}\int \frac{15(45A-28B+14C)x^2+2(720A-511B+350C)}{x^2\sqrt{3x^4+5x^2+2}}dx-\frac{5\sqrt{3x^4+5x^2+2}(45A-28B+14C)}{6x^3}$$
$$-\frac{\sqrt{3x^4+5x^2+2}(15A-14C)}{210x^7}-\frac{C\sqrt{3x^4+5x^2+2}}{15x^7}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{14}\Bigg(\frac{1}{5}\Bigg(\frac{1}{6}\Bigg(\frac{1}{2}\int -\frac{6\big((720A-511B+350C)x^2+5(45A-28B+14C)\big)}{\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(720A-511B+}{x}$$
$$-\frac{\sqrt{3x^4+5x^2+2}(15A-14C)}{210x^7}-\frac{C\sqrt{3x^4+5x^2+2}}{15x^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{14}\Bigg(\frac{1}{5}\Bigg(\frac{1}{6}\Bigg(\frac{\sqrt{3x^4+5x^2+2}(720A-511B+350C)}{x}-3\int \frac{(720A-511B+350C)x^2+5(45A-14(2B-C))}{\sqrt{3x^4+5x^2+2}}d$$
$$-\frac{\sqrt{3x^4+5x^2+2}(15A-14C)}{210x^7}-\frac{C\sqrt{3x^4+5x^2+2}}{15x^7}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{14}\Bigg(\frac{1}{5}\Bigg(\frac{1}{6}\Bigg(\frac{\sqrt{3x^4+5x^2+2}(720A-511B+350C)}{x}-3\Bigg(5(45A-28B+14C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+(720A-$$
$$-\frac{\sqrt{3x^4+5x^2+2}(15A-14C)}{210x^7}-\frac{C\sqrt{3x^4+5x^2+2}}{15x^7}$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{1}{6}\left(\frac{\sqrt{3x^4+5x^2+2}(720A-511B+350C)}{x}-3\left((720A-511B+350C)\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{5(x^2\right.\right.\right.\right.$$

$$\frac{\sqrt{3x^4+5x^2+2}(15A-14C)}{210x^7}-\frac{C\sqrt{3x^4+5x^2+2}}{15x^7}$$

$$\Big\downarrow \text{ 1456}$$

$$\frac{1}{14}\left(\frac{1}{5}\left(\frac{1}{6}\left(\frac{\sqrt{3x^4+5x^2+2}(720A-511B+350C)}{x}-3\left(\frac{5(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(45A-28B+14C)\operatorname{EllipticF}(\operatorname{arct}}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.\right.\right.\right.$$

$$\frac{\sqrt{3x^4+5x^2+2}(15A-14C)}{210x^7}-\frac{C\sqrt{3x^4+5x^2+2}}{15x^7}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^8*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
-1/210*((15*A - 14*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x^7 - (C*Sqrt[2 + 5*x^2 + 3
*x^4])/(15*x^7) + (((15*A - 7*B)*Sqrt[2 + 5*x^2 + 3*x^4])/(5*x^5) + ((-5*(
45*A - 28*B + 14*C)*Sqrt[2 + 5*x^2 + 3*x^4])/(6*x^3) + (((720*A - 511*B +
350*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x - 3*((720*A - 511*B + 350*C)*((x*(2 + 3*
x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1
 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (5*(45
*A - 28*B + 14*C)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x
], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])))/6)/5)/14
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 14.07 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{A\sqrt{3x^4+5x^2+2}}{14x^7}-\frac{\left(\frac{B}{2}-\frac{15A}{14}\right)\sqrt{3x^4+5x^2+2}}{5x^5}-\frac{\left(\frac{C}{2}+\frac{45A}{28}-B\right)\sqrt{3x^4+5x^2+2}}{3x^3}-\frac{\left(-\frac{12A}{7}+\frac{73B}{60}-\frac{5C}{6}\right)\sqrt{3x^4+5x^2+2}}{x}-\frac{i\left(B\right.}{}$
risch	$\frac{\sqrt{3x^4+5x^2+2}\,(720x^6A-511B\,x^6+350C\,x^6-225A\,x^4+140B\,x^4-70C\,x^4+90A\,x^2-42B\,x^2-30A)}{420x^7}-\frac{i(720A-511B+350C)\sqrt{x^2+}}{}$
default	$A\left(-\frac{\sqrt{3x^4+5x^2+2}}{14x^7}+\frac{3\sqrt{3x^4+5x^2+2}}{14x^5}-\frac{15\sqrt{3x^4+5x^2+2}}{28x^3}+\frac{12\sqrt{3x^4+5x^2+2}}{7x}+\frac{45i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{56\sqrt{3x^4+5x^2+2}}-\right.$

input
```
int((C*x^4+B*x^2+A)/x^8/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/14*A*(3*x^4+5*x^2+2)^(1/2)/x^7-1/5*(1/2*B-15/14*A)*(3*x^4+5*x^2+2)^(1/2
)/x^5-1/3*(1/2*C+45/28*A-B)*(3*x^4+5*x^2+2)^(1/2)/x^3-(-12/7*A+73/60*B-5/6
*C)*(3*x^4+5*x^2+2)^(1/2)/x-1/2*I*(B-1/2*C-45/28*A)*(x^2+1)^(1/2)*(6*x^2+4
)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3*I*(73/20*B-5/
2*C-36/7*A)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF
(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.45

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{2+5x^2+3x^4}}\,dx$$
$$=\frac{-2i\sqrt{2}(720\,A-511\,B+350\,C)x^7E(\arcsin{(i\,x)}\mid\frac{3}{2})+i\sqrt{2}(2115\,A-1442\,B+910\,C)x^7F(\arcsin{(i\,x)}}{}$$

input
```
integrate((C*x^4+B*x^2+A)/x^8/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/840*(-2*I*sqrt(2)*(720*A - 511*B + 350*C)*x^7*elliptic_e(arcsin(I*x), 3/
2) + I*sqrt(2)*(2115*A - 1442*B + 910*C)*x^7*elliptic_f(arcsin(I*x), 3/2)
+ 2*((720*A - 511*B + 350*C)*x^6 - 5*(45*A - 28*B + 14*C)*x^4 + 6*(15*A -
7*B)*x^2 - 30*A)*sqrt(3*x^4 + 5*x^2 + 2))/x^7
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{(x^2+1)\,(3x^2+2)}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**8/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**8*sqrt((x**2 + 1)*(3*x**2 + 2))), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{3\,x^4+5\,x^2+2}x^8}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^8/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(3*x^4 + 5*x^2 + 2)*x^8), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{3\,x^4+5\,x^2+2}x^8}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^8/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(3*x^4 + 5*x^2 + 2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^8\,\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^8*(5*x^2 + 3*x^4 + 2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^8*(5*x^2 + 3*x^4 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{2+5x^2+3x^4}}\,dx$$
$$=\frac{-\sqrt{3x^4+5x^2+2}\,a-30\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^{10}+5x^8+2x^6}dx\right)a\,x^7+14\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^{10}+5x^8+2x^6}dx\right)b\,x^7-15\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^8+5x^6+2x^4}dx\right)a\,x}{14x^7}$$

input
```
int((C*x^4+B*x^2+A)/x^8/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
( - sqrt(3*x**4 + 5*x**2 + 2)*a - 30*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**1
0 + 5*x**8 + 2*x**6),x)*a*x**7 + 14*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**10
 + 5*x**8 + 2*x**6),x)*b*x**7 - 15*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**8 +
 5*x**6 + 2*x**4),x)*a*x**7 + 14*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**8 + 5
*x**6 + 2*x**4),x)*c*x**7)/(14*x**7)
```

3.172 $\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 267

$$\begin{aligned}\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx &= \frac{2(315A-325B+356C)x(2+3x^2)}{405\sqrt{2+5x^2+3x^4}} \\ &- \frac{x(2(45A-39B+35C)+(117A-105B+97C)x^2)}{27\sqrt{2+5x^2+3x^4}} \\ &+ \frac{1}{27}(B-3C)x\sqrt{2+5x^2+3x^4}+\frac{1}{45}Cx^3\sqrt{2+5x^2+3x^4} \\ &- \frac{2\sqrt{2}(315A-325B+356C)\,(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{405\sqrt{2+5x^2+3x^4}} \\ &+ \frac{\sqrt{2}(45A-40B+38C)\,(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{27\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output

```
2/405*(315*A-325*B+356*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-1/27*x*(90*A-7
8*B+70*C+(117*A-105*B+97*C)*x^2)/(3*x^4+5*x^2+2)^(1/2)+1/27*(B-3*C)*x*(3*x
^4+5*x^2+2)^(1/2)+1/45*C*x^3*(3*x^4+5*x^2+2)^(1/2)-2/405*2^(1/2)*(315*A-32
5*B+356*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2
*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+1/27*2^(1/2)*(45*A-40*B+38*C)*(x^2+1)*((
3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*
x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.69

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{3x(-45A(10+13x^2)+5B(80+110x^2+3x^4)+C(-380-554x^2-30x^4+\cdots}{}$$

input
```
Integrate[(x^6*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(3*x*(-45*A*(10 + 13*x^2) + 5*B*(80 + 110*x^2 + 3*x^4) + C*(-380 - 554*x^2
 - 30*x^4 + 9*x^6)) - (2*I)*Sqrt[3]*(315*A - 325*B + 356*C)*Sqrt[1 + x^2]*
Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (2*I)*Sqrt[3]*(90
*A - 125*B + 166*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt
[3/2]*x], 2/3])/(405*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2197, 27, 2207, 27, 2207, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(3x^4+5x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{1}{2}\int -\frac{2(9Cx^6+3(3B-5C)x^4+2(63A-60B+58C)x^2+2(45A-39B+35C))}{27\sqrt{3x^4+5x^2+2}}dx - \frac{x(x^2(117A-105B+97C)+2(45A-39B+35C))}{27\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27}\int \frac{9Cx^6+3(3B-5C)x^4+2(63A-60B+58C)x^2+2(45A-39B+35C)}{\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{27\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{1}{27}\left(\frac{1}{15}\int \frac{3\left(45(B-3C)x^4+2(315A-300B+281C)x^2+10(45A-39B+35C)\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{3}{5}C\sqrt{3x^4+5x^2+2}x^3\right.$$
$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{27\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{27}\left(\frac{1}{5}\int \frac{45(B-3C)x^4+2(315A-300B+281C)x^2+10(45A-39B+35C)}{\sqrt{3x^4+5x^2+2}}dx+\frac{3}{5}C\sqrt{3x^4+5x^2+2}x^3\right)-$$
$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{27\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{1}{27}\left(\frac{1}{5}\left(\frac{1}{9}\int \frac{18\left((315A-325B+356C)x^2+5(45A-40B+38C)\right)}{\sqrt{3x^4+5x^2+2}}dx+5\sqrt{3x^4+5x^2+2}x(B-3C)\right)+\frac{3}{5}C\sqrt{}\right.$$
$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{27\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{27}\left(\frac{1}{5}\left(2\int \frac{(315A-325B+356C)x^2+5(45A-40B+38C)}{\sqrt{3x^4+5x^2+2}}dx+5\sqrt{3x^4+5x^2+2}x(B-3C)\right)+\frac{3}{5}C\sqrt{3x^4+}\right.$$
$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{27\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{27}\left(\frac{1}{5}\left(2\left(5(45A-40B+38C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+(315A-325B+356C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+5\sqrt{3}\right.\right.$$
$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{27\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{ 1413}$$

$$\frac{1}{27}\left(\frac{1}{5}\left(2\left((315A-325B+356C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{5(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(45A-40B+38C)\,\text{EllipticF}\left(\text{ar}\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$

$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{27\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{27}\left(\frac{1}{5}\left(2\left(\frac{5(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(45A-40B+38C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+(315A-325B+356C)\left(\frac{x(3}{3\sqrt{3x^4}}\right.\right.\right.\right.$$

$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{27\sqrt{3x^4+5x^2+2}}$$

input
```
Int[(x^6*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
-1/27*(x*(2*(45*A - 39*B + 35*C) + (117*A - 105*B + 97*C)*x^2))/Sqrt[2 + 5
*x^2 + 3*x^4] + ((3*C*x^3*Sqrt[2 + 5*x^2 + 3*x^4])/5 + (5*(B - 3*C)*x*Sqrt
[2 + 5*x^2 + 3*x^4] + 2*((315*A - 325*B + 356*C)*((x*(2 + 3*x^2))/(3*Sqrt[
2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Ellip
ticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (5*(45*A - 40*B + 38
*C)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqr
t[2]*Sqrt[2 + 5*x^2 + 3*x^4])))/5)/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 18.20 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.78

method	result
elliptic	$-\frac{6\left(\left(\frac{13A}{18}-\frac{35B}{54}+\frac{97C}{162}\right)x^{3}+\left(\frac{5A}{9}-\frac{13B}{27}+\frac{35C}{81}\right)x\right)}{\sqrt{3x^{4}+5x^{2}+2}}+\frac{C\,x^{3}\sqrt{3x^{4}+5x^{2}+2}}{45}+\left(\frac{B}{27}-\frac{C}{9}\right)x\sqrt{3x^{4}+5x^{2}+2}-\frac{i\left(\frac{10A}{3}-\frac{80}{27}\right.}{}$
risch	$-\frac{x\left(-9C\,x^{6}-15B\,x^{4}+30C\,x^{4}+585A\,x^{2}-550B\,x^{2}+554C\,x^{2}+450A-400B+380C\right)}{135\sqrt{3x^{4}+5x^{2}+2}}+\frac{2i(315A-325B+356C)\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\right.}{405\sqrt{3x^{4}+}}$
default	$B\left(-\frac{6\left(-\frac{35}{54}x^{3}-\frac{13}{27}x\right)}{\sqrt{3x^{4}+5x^{2}+2}}+\frac{x\sqrt{3x^{4}+5x^{2}+2}}{27}+\frac{40i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{27\sqrt{3x^{4}+5x^{2}+2}}-\frac{130i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right.\right.}{81\sqrt{3x^{4}+5x^{2}+}}\right.$

input
```
int(x^6*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6*((13/18*A-35/54*B+97/162*C)*x^3+(5/9*A-13/27*B+35/81*C)*x)/(3*x^4+5*x^2
+2)^(1/2)+1/45*C*x^3*(3*x^4+5*x^2+2)^(1/2)+(1/27*B-1/9*C)*x*(3*x^4+5*x^2+2
)^(1/2)-1/2*I*(10/3*A-80/27*B+76/27*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^
4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3*I*(14/3*A-130/27*B+712/135
*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2
*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.81

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{3}\sqrt{-\frac{2}{3}}(3\,(315\,A-325\,B+356\,C)x^5+5\,(315\,A-325\,B+356\,C)x^3+2\,(315\,A-325\,B+356\,C)x)}{}$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/1215*(4*sqrt(3)*sqrt(-2/3)*(3*(315*A - 325*B + 356*C)*x^5 + 5*(315*A -
325*B + 356*C)*x^3 + 2*(315*A - 325*B + 356*C)*x)*elliptic_e(arcsin(sqrt(-
2/3)/x), 3/2) - sqrt(3)*sqrt(-2/3)*(3*(3285*A - 3100*B + 3134*C)*x^5 + 5*(
3285*A - 3100*B + 3134*C)*x^3 + 2*(3285*A - 3100*B + 3134*C)*x)*elliptic_f
(arcsin(sqrt(-2/3)/x), 3/2) - 3*(27*C*x^8 + 45*(B - 2*C)*x^6 + 3*(45*A - 1
00*B + 158*C)*x^4 + 10*(180*A - 205*B + 242*C)*x^2 + 1260*A - 1300*B + 142
4*C)*sqrt(3*x^4 + 5*x^2 + 2))/(3*x^5 + 5*x^3 + 2*x)
```

Sympy [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^6(A+Bx^2+Cx^4)}{((x^2+1)(3x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6*(C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral(x**6*(A + B*x**2 + C*x**4)/((x**2 + 1)*(3*x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^6}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^6/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^6}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^6/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{x^6\,(C\,x^4+B\,x^2+A)}{(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int((x^6*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int((x^6*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(x^6*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x)
```

output

```
(135*sqrt(3*x**4 + 5*x**2 + 2)*a*x**3 + 450*sqrt(3*x**4 + 5*x**2 + 2)*a*x
+ 45*sqrt(3*x**4 + 5*x**2 + 2)*b*x**5 - 300*sqrt(3*x**4 + 5*x**2 + 2)*b*x*
*3 - 850*sqrt(3*x**4 + 5*x**2 + 2)*b*x + 27*sqrt(3*x**4 + 5*x**2 + 2)*c*x*
*7 - 90*sqrt(3*x**4 + 5*x**2 + 2)*c*x**5 + 474*sqrt(3*x**4 + 5*x**2 + 2)*c
*x**3 + 1280*sqrt(3*x**4 + 5*x**2 + 2)*c*x - 2700*int(sqrt(3*x**4 + 5*x**2
 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a*x**4 - 4500*int(sqrt
(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a*x**2
 - 1800*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**
2 + 4),x)*a + 5100*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x*
*4 + 20*x**2 + 4),x)*b*x**4 + 8500*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 +
 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b*x**2 + 3400*int(sqrt(3*x**4 + 5*x**
2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b - 7680*int(sqrt(3*x
**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*c*x**4 - 1
2800*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 +
 4),x)*c*x**2 - 5120*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*
x**4 + 20*x**2 + 4),x)*c - 2430*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x*
*8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a*x**4 - 4050*int((sqrt(3*x**4 +
5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a*x**2 - 1
620*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*
x**2 + 4),x)*a + 5400*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30...
```

3.173 $\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 239

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = -\frac{(135A-126B+130C)x(2+3x^2)}{81\sqrt{2+5x^2+3x^4}} + \frac{x(2(18A-15B+13C)+(45A-39B+35C)x^2)}{9\sqrt{2+5x^2+3x^4}} + \frac{1}{27}Cx\sqrt{2+5x^2+3x^4} + \frac{\sqrt{2}(135A-126B+130C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{81\sqrt{2+5x^2+3x^4}} - \frac{\sqrt{2}(54A-45B+40C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{27\sqrt{2+5x^2+3x^4}}$$

output
```
-1/81*(135*A-126*B+130*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/9*x*(36*A-30
*B+26*C+(45*A-39*B+35*C)*x^2)/(3*x^4+5*x^2+2)^(1/2)+1/27*C*x*(3*x^4+5*x^2+
2)^(1/2)+1/81*2^(1/2)*(135*A-126*B+130*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2
)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)-1/27*2^(1
/2)*(54*A-45*B+40*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arc
tan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.72

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{3x(27A(4+5x^2)-9B(10+13x^2)+C(80+110x^2+3x^4))+i\sqrt{3}(135A-1}{}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(3*x*(27*A*(4 + 5*x^2) - 9*B*(10 + 13*x^2) + C*(80 + 110*x^2 + 3*x^4)) + I
*Sqrt[3]*(135*A - 126*B + 130*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I
*ArcSinh[Sqrt[3/2]*x], 2/3] - I*Sqrt[3]*(27*A - 36*B + 50*C)*Sqrt[1 + x^2]
*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(81*Sqrt[2 + 5*x^
2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2197, 27, 2207, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(3x^4+5x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2197}$$

$$\frac{x(x^2(45A-39B+35C)+2(18A-15B+13C))}{9\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int \frac{2(-3Cx^4+(45A-42B+40C)x^2+2(18A-15B+13C))}{9\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{9\sqrt{3x^4+5x^2+2}}-\frac{1}{9}\int\frac{-3Cx^4+(45A-42B+40C)x^2+2(18A-15B+13C)}{\sqrt{3x^4+5x^2+2}}dx$$

$\downarrow$ 2207

$$\frac{1}{9}\left(\frac{1}{3}Cx\sqrt{3x^4+5x^2+2}-\frac{1}{9}\int\frac{3\left((135A-126B+130C)x^2+2(54A-45B+40C)\right)}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{9\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 27

$$\frac{1}{9}\left(\frac{1}{3}Cx\sqrt{3x^4+5x^2+2}-\frac{1}{3}\int\frac{(135A-126B+130C)x^2+2(54A-45B+40C)}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{9\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1503

$$\frac{1}{9}\left(\frac{1}{3}\left(-2(54A-45B+40C)\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx-(135A-126B+130C)\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{3}C\sqrt{3\right.$$
$$\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{9\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1413

$$\frac{1}{9}\left(\frac{1}{3}\left(-(135A-126B+130C)\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(54A-45B+40C)\,\text{EllipticF}\left(\text{arc}\right.}{\sqrt{3x^4+5x^2+2}}\right.\right.$$
$$\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{9\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1456

$$\frac{1}{9}\left(\frac{1}{3}\left(-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(54A-45B+40C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}-\left((135A-126B+130C)\left(\frac{x}{3\sqrt{3}}\right.\right.\right.\right.$$
$$\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{9\sqrt{3x^4+5x^2+2}}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(x*(2*(18*A - 15*B + 13*C) + (45*A - 39*B + 35*C)*x^2))/(9*Sqrt[2 + 5*x^2
+ 3*x^4]) + ((C*x*Sqrt[2 + 5*x^2 + 3*x^4])/3 + (-((135*A - 126*B + 130*C)*
((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2
+ 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4]
))) - (Sqrt[2]*(54*A - 45*B + 40*C)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*
EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4])/3)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 13.58 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.76

method	result
elliptic	$-\frac{6\left(\left(-\frac{5A}{6}+\frac{13B}{18}-\frac{35C}{54}\right)x^3+\left(-\frac{2A}{3}+\frac{5B}{9}-\frac{13C}{27}\right)x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{Cx\sqrt{3x^4+5x^2+2}}{27}-\frac{i\left(-4A+\frac{10B}{3}-\frac{80C}{27}\right)\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\right.}{2\sqrt{3x^4+5x^2+2}}$
risch	$\frac{x\left(3C\,x^4+135A\,x^2-117B\,x^2+110C\,x^2+108A-90B+80C\right)}{27\sqrt{3x^4+5x^2+2}}-\frac{i(135A-126B+130C)\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{Ellipt}\right.}{81\sqrt{3x^4+5x^2+2}}$
default	$A\left(-\frac{6\left(-\frac{5}{6}x^3-\frac{2}{3}x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{2i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4+5x^2+2}}-\frac{5i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^4+5x^2+2}}\right)$

input
```
int(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6*((-5/6*A+13/18*B-35/54*C)*x^3+(-2/3*A+5/9*B-13/27*C)*x)/(3*x^4+5*x^2+2)
^(1/2)+1/27*C*x*(3*x^4+5*x^2+2)^(1/2)-1/2*I*(-4*A+10/3*B-80/27*C)*(x^2+1)^
(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3
*I*(14/3*B-130/27*C-5*A)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/
2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.85

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{2\sqrt{3}\sqrt{-\frac{2}{3}}(3\,(135\,A-126\,B+130\,C)x^5+5\,(135\,A-126\,B+130\,C)x^3+2$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/243*(2*sqrt(3)*sqrt(-2/3)*(3*(135*A - 126*B + 130*C)*x^5 + 5*(135*A - 12
6*B + 130*C)*x^3 + 2*(135*A - 126*B + 130*C)*x)*elliptic_e(arcsin(sqrt(-2/
3)/x), 3/2) - sqrt(3)*sqrt(-2/3)*(3*(756*A - 657*B + 620*C)*x^5 + 5*(756*A
 - 657*B + 620*C)*x^3 + 2*(756*A - 657*B + 620*C)*x)*elliptic_f(arcsin(sqr
t(-2/3)/x), 3/2) + 3*(9*C*x^6 + 3*(9*B - 20*C)*x^4 - (351*A - 360*B + 410*
C)*x^2 - 270*A + 252*B - 260*C)*sqrt(3*x^4 + 5*x^2 + 2))/(3*x^5 + 5*x^3 +
2*x)
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4(A+Bx^2+Cx^4)}{((x^2+1)\,(3x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral(x**4*(A + B*x**2 + C*x**4)/((x**2 + 1)*(3*x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4\,(C\,x^4+B\,x^2+A)}{(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{-27\sqrt{3x^4+5x^2+2}\,ax+27\sqrt{3x^4+5x^2+2}\,b\,x^3+90\sqrt{3x^4+5x^2+2}\,bx+9}{}$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
( - 27*sqrt(3*x**4 + 5*x**2 + 2)*a*x + 27*sqrt(3*x**4 + 5*x**2 + 2)*b*x**3
 + 90*sqrt(3*x**4 + 5*x**2 + 2)*b*x + 9*sqrt(3*x**4 + 5*x**2 + 2)*c*x**5 -
 60*sqrt(3*x**4 + 5*x**2 + 2)*c*x**3 - 170*sqrt(3*x**4 + 5*x**2 + 2)*c*x +
 162*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 +
 4),x)*a*x**4 + 270*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x
**4 + 20*x**2 + 4),x)*a*x**2 + 108*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 +
 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a - 540*int(sqrt(3*x**4 + 5*x**2 + 2)
/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b*x**4 - 900*int(sqrt(3*x**
4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b*x**2 - 360
*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),
x)*b + 1020*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20
*x**2 + 4),x)*c*x**4 + 1700*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**
6 + 37*x**4 + 20*x**2 + 4),x)*c*x**2 + 680*int(sqrt(3*x**4 + 5*x**2 + 2)/(
9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*c - 486*int((sqrt(3*x**4 + 5*
x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b*x**4 - 810
*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**
2 + 4),x)*b*x**2 - 324*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x
**6 + 37*x**4 + 20*x**2 + 4),x)*b + 1080*int((sqrt(3*x**4 + 5*x**2 + 2)*x*
*2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*c*x**4 + 1800*int((sqrt(
3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x...
```

3.174 $\int \frac{x^2\left(A+Bx^2+Cx^4\right)}{\left(2+5x^2+3x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 214

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{(18A-15B+14C)x(2+3x^2)}{9\sqrt{2+5x^2+3x^4}} - \frac{x(15A-12B+10C+(18A-15B+13C)x^2)}{3\sqrt{2+5x^2+3x^4}} - \frac{\sqrt{2}(18A-15B+14C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{9\sqrt{2+5x^2+3x^4}} + \frac{(15A-12B+10C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output

```
1/9*(18*A-15*B+14*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-1/3*x*(15*A-12*B+10
*C+(18*A-15*B+13*C)*x^2)/(3*x^4+5*x^2+2)^(1/2)-1/9*2^(1/2)*(18*A-15*B+14*C
)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2
))/(3*x^4+5*x^2+2)^(1/2)+1/6*(15*A-12*B+10*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^
(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.78

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{36Bx-30Cx+45Bx^3-39Cx^3-9Ax(5+6x^2)-i\sqrt{3}(18A-15B+14C)}{}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(36*B*x - 30*C*x + 45*B*x^3 - 39*C*x^3 - 9*A*x*(5 + 6*x^2) - I*Sqrt[3]*(18
*A - 15*B + 14*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3
/2]*x], 2/3] + I*Sqrt[3]*(3*A - 3*B + 4*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*E
llipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(9*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2197, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^2+Cx^4\right)}{\left(3x^4+5x^2+2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{2197}$$

$$-\frac{1}{2}\int -\frac{2\left((18A-15B+14C)x^2+15A-12B+10C\right)}{3\sqrt{3x^4+5x^2+2}}dx - \frac{x\left(x^2(18A-15B+13C)+15A-12B+10C\right)}{3\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(18A-15B+14C)x^2+15A-12B+10C}{\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(x^2(18A-15B+13C)+15A-12B+10C\right)}{3\sqrt{3x^4+5x^2+2}}$$

$\Big\downarrow$ 1503

$$\frac{1}{3}\left((15A-12B+10C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+(18A-15B+14C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{x\left(x^2(18A-15B+13C)+15A-12B+10C\right)}{3\sqrt{3x^4+5x^2+2}}$$

$\Big\downarrow$ 1413

$$\frac{1}{3}\left((18A-15B+14C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(15A-12B+10C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.$$
$$-\frac{x\left(x^2(18A-15B+13C)+15A-12B+10C\right)}{3\sqrt{3x^4+5x^2+2}}$$

$\Big\downarrow$ 1456

$$\frac{1}{3}\left(\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(15A-12B+10C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+(18A-15B+14C)\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\right.\right.$$
$$-\frac{x\left(x^2(18A-15B+13C)+15A-12B+10C\right)}{3\sqrt{3x^4+5x^2+2}}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
-1/3*(x*(15*A - 12*B + 10*C + (18*A - 15*B + 13*C)*x^2))/Sqrt[2 + 5*x^2 +
3*x^4] + ((18*A - 15*B + 14*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]
) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1
/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + ((15*A - 12*B + 10*C)*(1 + x^2)*Sqrt[(
2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2
+ 3*x^4]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

Maple [A] (verified)

Time = 10.77 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

method	result
elliptic	$-\frac{6\left(\left(A-\frac{5B}{6}+\frac{13C}{18}\right)x^3+\left(\frac{5A}{6}-\frac{2B}{3}+\frac{5C}{9}\right)x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{i\left(-4B+\frac{10C}{3}+5A\right)\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{i\left(\frac{14C}{3}+6A-5B\right)\sqrt{x^2}}{}$
risch	$-\frac{x(18A\,x^2-15B\,x^2+13C\,x^2+15A-12B+10C)}{3\sqrt{3x^4+5x^2+2}}+\frac{i(18A-15B+14C)\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{9\sqrt{3x^4+5x^2+2}}$
default	$A\left(-\frac{6\left(x^3+\frac{5}{6}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{5i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{2i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{\sqrt{3x^4+5x^2+2}}\right)$

input
```
int(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6*((A-5/6*B+13/18*C)*x^3+(5/6*A-2/3*B+5/9*C)*x)/(3*x^4+5*x^2+2)^(1/2)-1/2
*I*(-4*B+10/3*C+5*A)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*E
llipticF(I*x,1/2*6^(1/2))+1/3*I*(14/3*C+6*A-5*B)*(x^2+1)^(1/2)*(6*x^2+4)^(
1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6
^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{3}\sqrt{-\frac{2}{3}}(3\,(18\,A-15\,B+14\,C)x^5+5\,(18\,A-15\,B+14\,C)x^3+2\,(18\,A-15\,B+14\,C)x)E(\arcsin\Big(}{}$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/54*(4*sqrt(3)*sqrt(-2/3)*(3*(18*A - 15*B + 14*C)*x^5 + 5*(18*A - 15*B +
 14*C)*x^3 + 2*(18*A - 15*B + 14*C)*x)*elliptic_e(arcsin(sqrt(-2/3)/x), 3/
2) - sqrt(3)*sqrt(-2/3)*(3*(207*A - 168*B + 146*C)*x^5 + 5*(207*A - 168*B
+ 146*C)*x^3 + 2*(207*A - 168*B + 146*C)*x)*elliptic_f(arcsin(sqrt(-2/3)/x
), 3/2) - 6*(3*C*x^4 + (45*A - 39*B + 40*C)*x^2 + 36*A - 30*B + 28*C)*sqrt
(3*x^4 + 5*x^2 + 2))/(3*x^5 + 5*x^3 + 2*x)
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4)}{((x^2+1)(3x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4)/((x**2 + 1)*(3*x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(3x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^2\,(C\,x^4+B\,x^2+A)}{(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{-3\sqrt{3x^4+5x^2+2}\,bx+3\sqrt{3x^4+5x^2+2}\,c\,x^3+10\sqrt{3x^4+5x^2+2}\,cx+18\big(}{}$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
( - 3*sqrt(3*x**4 + 5*x**2 + 2)*b*x + 3*sqrt(3*x**4 + 5*x**2 + 2)*c*x**3 +
 10*sqrt(3*x**4 + 5*x**2 + 2)*c*x + 18*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x*
*8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b*x**4 + 30*int(sqrt(3*x**4 + 5*x
**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b*x**2 + 12*int(sqr
t(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b - 6
0*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4)
,x)*c*x**4 - 100*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4
 + 20*x**2 + 4),x)*c*x**2 - 40*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*
x**6 + 37*x**4 + 20*x**2 + 4),x)*c + 27*int((sqrt(3*x**4 + 5*x**2 + 2)*x**
2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a*x**4 + 45*int((sqrt(3*x
**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a*x*
*2 + 18*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 +
 20*x**2 + 4),x)*a - 54*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*
x**6 + 37*x**4 + 20*x**2 + 4),x)*c*x**4 - 90*int((sqrt(3*x**4 + 5*x**2 + 2
)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*c*x**2 - 36*int((sqr
t(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)
*c)/(9*(3*x**4 + 5*x**2 + 2))
```

3.175 $\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 160

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{(9A-6B+4C)x}{6\sqrt{2+5x^2+3x^4}} + \frac{(15A-12B+10C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{2}\sqrt{2+5x^2+3x^4}} - \frac{(6A-5B+4C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
1/6*(9*A-6*B+4*C)*x/(3*x^4+5*x^2+2)^(1/2)+1/6*(15*A-12*B+10*C)*(x^2+1)*((3
*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)/(3
*x^4+5*x^2+2)^(1/2)-1/2*(6*A-5*B+4*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*In
verseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{3x(2C(4+5x^2)-2B(5+6x^2)+A(13+15x^2))+i\sqrt{3}(15A-12B+10C)\sqrt{1}}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(3*x*(2*C*(4 + 5*x^2) - 2*B*(5 + 6*x^2) + A*(13 + 15*x^2)) + I*Sqrt[3]*(15
*A - 12*B + 10*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3
/2]*x], 2/3] - I*Sqrt[3]*(3*A - 2*B + 2*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*E
llipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(6*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2206, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 2206}$$

$$\frac{x\left(x^2(15A-12B+10C)+13A-10B+8C\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int \frac{(15A-12B+10C)x^2+2(6A-5B+4C)}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{2}\left(-2(6A-5B+4C)\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx-(15A-12B+10C)\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{x\left(x^2(15A-12B+10C)+13A-10B+8C\right)}{2\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 1413

$$\frac{1}{2}\left(-(15A-12B+10C)\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}(6A-5B+4C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right.$$
$$\frac{x\left(x^2(15A-12B+10C)+13A-10B+8C\right)}{2\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 1456

$$\frac{1}{2}\left(-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}(6A-5B+4C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}-\left((15A-12B+10C)\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+\cdots}}\right.\right.\right.$$
$$\frac{x\left(x^2(15A-12B+10C)+13A-10B+8C\right)}{2\sqrt{3x^4+5x^2+2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(x*(13*A - 10*B + 8*C + (15*A - 12*B + 10*C)*x^2))/(2*Sqrt[2 + 5*x^2 + 3*x
^4]) + (-((15*A - 12*B + 10*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]
) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1
/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4]))) - (Sqrt[2]*(6*A - 5*B + 4*C)*(1 + x^2)*
Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3
*x^4])/2
```

Defintions of rubi rules used

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 9.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01

method	result
elliptic	$-\frac{6\left(\left(B-\frac{5C}{6}-\frac{5A}{4}\right)x^{3}+\left(\frac{5B}{6}-\frac{2C}{3}-\frac{13A}{12}\right)x\right)}{\sqrt{3x^{4}+5x^{2}+2}}-\frac{i(-4C-6A+5B)\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^{4}+5x^{2}+2}}+\frac{i\left(-\frac{15A}{2}+6B-5C\right)\sqrt{x^{2}}}{}$
risch	$\frac{x\left(15A\,x^{2}-12B\,x^{2}+10C\,x^{2}+13A-10B+8C\right)}{2\sqrt{3x^{4}+5x^{2}+2}}-\frac{i(15A-12B+10C)\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{6\sqrt{3x^{4}+5x^{2}+2}}+$
default	$A\left(-\frac{6\left(-\frac{5}{4}x^{3}-\frac{13}{12}x\right)}{\sqrt{3x^{4}+5x^{2}+2}}+\frac{3i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^{4}+5x^{2}+2}}-\frac{5i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^{4}+5x^{2}+2}}\right.$

input
```
int((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6*((B-5/6*C-5/4*A)*x^3+(5/6*B-2/3*C-13/12*A)*x)/(3*x^4+5*x^2+2)^(1/2)-1/2
*I*(-4*C-6*A+5*B)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*Elli
pticF(I*x,1/2*6^(1/2))+1/3*I*(-15/2*A+6*B-5*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/
2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{\sqrt{2}(-3i\,(15\,A-12\,B+10\,C)x^4-5i\,(15\,A-12\,B+10\,C)x^2-30i\,A+24i\,B}{}$$

input
```
integrate((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(2)*(-3*I*(15*A - 12*B + 10*C)*x^4 - 5*I*(15*A - 12*B + 10*C)*x^2
 - 30*I*A + 24*I*B - 20*I*C)*elliptic_e(arcsin(I*x), 3/2) + sqrt(2)*(3*I*(
33*A - 27*B + 22*C)*x^4 + 5*I*(33*A - 27*B + 22*C)*x^2 + 66*I*A - 54*I*B +
 44*I*C)*elliptic_f(arcsin(I*x), 3/2) + 3*((15*A - 12*B + 10*C)*x^3 + (13*
A - 10*B + 8*C)*x)*sqrt(3*x^4 + 5*x^2 + 2))/(3*x^4 + 5*x^2 + 2)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{A+Bx^2+Cx^4}{\left((x^2+1)\,(3x^2+2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/((x**2 + 1)*(3*x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{-\sqrt{3x^4+5x^2+2}\,cx+9\left(\int \frac{\sqrt{3x^4+5x^2+2}}{9x^8+30x^6+37x^4+20x^2+4}dx\right)a\,x^4+15\left(\int \frac{\sqrt{3x^4+5x^2}}{9x^8+30x^6+37x^4}\right.}{}$$

input
```
int((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
( - sqrt(3*x**4 + 5*x**2 + 2)*c*x + 9*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**
8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a*x**4 + 15*int(sqrt(3*x**4 + 5*x*
*2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a*x**2 + 6*int(sqrt(
3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a + 6*i
nt(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)
*c*x**4 + 10*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 2
0*x**2 + 4),x)*c*x**2 + 4*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6
+ 37*x**4 + 20*x**2 + 4),x)*c + 9*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*
x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b*x**4 + 15*int((sqrt(3*x**4 +
5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b*x**2 + 6
*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**
2 + 4),x)*b)/(3*(3*x**4 + 5*x**2 + 2))
```

3.176 $\int \frac{A+Bx^2+Cx^4}{x^2\left(2+5x^2+3x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 190

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(2+5x^2+3x^4\right)^{3/2}}\,dx=-\frac{A\sqrt{2+5x^2+3x^4}}{4x}+\frac{x(6B-4C+A(-7+3x^2))}{4\sqrt{2+5x^2+3x^4}}-\frac{(7A-5B+4C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}+\frac{(15A-12B+10C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
-1/4*A*(3*x^4+5*x^2+2)^(1/2)/x+1/4*x*(6*B-4*C+A*(3*x^2-7))/(3*x^4+5*x^2+2)
^(1/2)-1/2*(7*A-5*B+4*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^
2+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)+1/4*(15*A-12*B+10*
C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/
2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4}{x^2(2+5x^2+3x^4)^{3/2}}\,dx = \frac{-3(A(1+20x^2+21x^4)+x^2(2C(5+6x^2)-B(13+15x^2)))-3i\sqrt{3}(7A-}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output
```
(-3*(A*(1 + 20*x^2 + 21*x^4) + x^2*(2*C*(5 + 6*x^2) - B*(13 + 15*x^2))) -
(3*I)*Sqrt[3]*(7*A - 5*B + 4*C)*x*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[
I*ArcSinh[Sqrt[3/2]*x], 2/3] + I*Sqrt[3]*(6*A - 3*B + 2*C)*x*Sqrt[1 + x^2]
*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(6*x*Sqrt[2 + 5*x
^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.44, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2198, 27, 2199, 1604, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2(3x^4+5x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2198}$$

$$-\frac{1}{2}\int -\frac{3(13A-10B+8C)x^4+2(15A-12B+10C)x^2+2A}{2x^2\sqrt{3x^4+5x^2+2}}dx - \frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{4\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{3(13A-10B+8C)x^4+2(15A-12B+10C)x^2+2A}{x^2\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{4\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 2199

$$\frac{1}{4}\left(\int \frac{2(15A-12B+10C)x^2+4(7A-5B+4C)}{x^2\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(13A-10B+8C)}{x}\right)-\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{4\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 1604

$$\frac{1}{4}\left(-\frac{1}{2}\int -\frac{4\left(3(7A-5B+4C)x^2+15A-12B+10C\right)}{\sqrt{3x^4+5x^2+2}}dx-\frac{2\sqrt{3x^4+5x^2+2}(7A-5B+4C)}{x}+\frac{\sqrt{3x^4+5x^2}}{}\right.$$
$$-\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{4\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 27

$$\frac{1}{4}\left(2\int \frac{3(7A-5B+4C)x^2+15A-12B+10C}{\sqrt{3x^4+5x^2+2}}dx-\frac{2\sqrt{3x^4+5x^2+2}(7A-5B+4C)}{x}+\frac{\sqrt{3x^4+5x^2+2}(13}{x}\right.$$
$$-\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{4\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 1503

$$\frac{1}{4}\left(2\left((15A-12B+10C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+3(7A-5B+4C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{2\sqrt{3x^4+5x^2+}}{}\right.$$
$$-\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{4\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 1413

$$\frac{1}{4}\left(2\left(3(7A-5B+4C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(15A-12B+10C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.\right.$$
$$-\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{4\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 1456

$$\frac{1}{4}\left(2\left(\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(15A-12B+10C)\,\mathrm{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+3(7A-5B+4C)\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$

$$\frac{x(3x^2(13A-10B+8C)+35A-26B+20C)}{4\sqrt{3x^4+5x^2+2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output
```
-1/4*(x*(35*A - 26*B + 20*C + 3*(13*A - 10*B + 8*C)*x^2))/Sqrt[2 + 5*x^2 +
 3*x^4] + ((-2*(7*A - 5*B + 4*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x + ((13*A - 10*
B + 8*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x + 2*(3*(7*A - 5*B + 4*C)*((x*(2 + 3*x^
2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 +
 x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + ((15*A -
 12*B + 10*C)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -
1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.96

method	result
elliptic	$-\frac{6\left(\left(\frac{13A}{8}+C-\frac{5B}{4}\right)x^3+\left(\frac{35A}{24}+\frac{5C}{6}-\frac{13B}{12}\right)x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{A\sqrt{3x^4+5x^2+2}}{4x}-\frac{i\left(\frac{15A}{2}-6B+5C\right)\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+$
risch	$-\frac{21A\,x^4-15B\,x^4+12C\,x^4+20A\,x^2-13B\,x^2+10C\,x^2+A}{2x\sqrt{3x^4+5x^2+2}}+\frac{i(21A-15B+12C)\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(\right.\right.}{6\sqrt{3x^4+5x^2+2}}$
default	$B\left(-\frac{6\left(-\frac{5}{4}x^3-\frac{13}{12}x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{3i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4+5x^2+2}}-\frac{5i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^4+5x^2+2}}\right.$

input
```
int((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6*((13/8*A+C-5/4*B)*x^3+(35/24*A+5/6*C-13/12*B)*x)/(3*x^4+5*x^2+2)^(1/2)-
1/4*A*(3*x^4+5*x^2+2)^(1/2)/x-1/2*I*(15/2*A-6*B+5*C)*(x^2+1)^(1/2)*(6*x^2+
4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3*I*(21/2*A+6*
C-15/2*B)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I
*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(2+5x^2+3x^4\right)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{2}(-3i\,(7\,A-5\,B+4\,C)x^5-5i\,(7\,A-5\,B+4\,C)x^3-2i\,(7\,A-5\,B+4\,C)x)E(\arcsin{(i\,x)}\mid\frac{3}{2})-\sqrt{}}{}$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*(-3*I*(7*A - 5*B + 4*C)*x^5 - 5*I*(7*A - 5*B + 4*C)*x^3 -
2*I*(7*A - 5*B + 4*C)*x)*elliptic_e(arcsin(I*x), 3/2) - sqrt(2)*(-3*I*(29*
A - 22*B + 18*C)*x^5 - 5*I*(29*A - 22*B + 18*C)*x^3 - 2*I*(29*A - 22*B + 1
8*C)*x)*elliptic_f(arcsin(I*x), 3/2) + 2*(3*(7*A - 5*B + 4*C)*x^4 + (20*A
- 13*B + 10*C)*x^2 + A)*sqrt(3*x^4 + 5*x^2 + 2))/(3*x^5 + 5*x^3 + 2*x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2 (2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^2\left((x^2+1)\,(3x^2+2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**2*((x**2 + 1)*(3*x**2 + 2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2 (2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2 (2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(2+5x^2+3x^4\right)^{3/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^2\,\left(3\,x^4+5\,x^2+2\right)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(5*x^2 + 3*x^4 + 2)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(5*x^2 + 3*x^4 + 2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(2+5x^2+3x^4\right)^{3/2}}\,dx=\frac{-\sqrt{3x^4+5x^2+2}\,b+30\left(\int \frac{\sqrt{3x^4+5x^2+2}}{9x^{10}+30x^8+37x^6+20x^4+4x^2}dx\right)a\,x^5+50\left(\int \frac{\sqrt{3}}{9x^{10}+30x^8}\right.}{}$$

input
```
int((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
( - sqrt(3*x**4 + 5*x**2 + 2)*b + 30*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**1
0 + 30*x**8 + 37*x**6 + 20*x**4 + 4*x**2),x)*a*x**5 + 50*int(sqrt(3*x**4 +
 5*x**2 + 2)/(9*x**10 + 30*x**8 + 37*x**6 + 20*x**4 + 4*x**2),x)*a*x**3 +
20*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**10 + 30*x**8 + 37*x**6 + 20*x**4 +
4*x**2),x)*a*x - 6*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**10 + 30*x**8 + 37*x
**6 + 20*x**4 + 4*x**2),x)*b*x**5 - 10*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x*
*10 + 30*x**8 + 37*x**6 + 20*x**4 + 4*x**2),x)*b*x**3 - 4*int(sqrt(3*x**4
+ 5*x**2 + 2)/(9*x**10 + 30*x**8 + 37*x**6 + 20*x**4 + 4*x**2),x)*b*x - 27
*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**
2 + 4),x)*b*x**5 - 45*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x*
*6 + 37*x**4 + 20*x**2 + 4),x)*b*x**3 - 18*int((sqrt(3*x**4 + 5*x**2 + 2)*
x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b*x + 30*int((sqrt(3*x
**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*c*x*
*5 + 50*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 +
 20*x**2 + 4),x)*c*x**3 + 20*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8
+ 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*c*x)/(10*x*(3*x**4 + 5*x**2 + 2))
```

3.177 $\int \frac{A+Bx^2+Cx^4}{x^4(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 228

$$\int \frac{A+Bx^2+Cx^4}{x^4(2+5x^2+3x^4)^{3/2}}\,dx = \frac{x(31A-42B+36C-3(25A-6B)x^2)}{24\sqrt{2+5x^2+3x^4}} - \frac{A\sqrt{2+5x^2+3x^4}}{12x^3} + \frac{(25A-6B)\sqrt{2+5x^2+3x^4}}{24x} + \frac{(65A-42B+30C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{6\sqrt{2}\sqrt{2+5x^2+3x^4}} - \frac{(20A-15B+12C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
1/24*x*(31*A-42*B+36*C-3*(25*A-6*B)*x^2)/(3*x^4+5*x^2+2)^(1/2)-1/12*A*(3*x
^4+5*x^2+2)^(1/2)/x^3+1/24*(25*A-6*B)*(3*x^4+5*x^2+2)^(1/2)/x+1/12*(65*A-4
2*B+30*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*
I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)-1/4*(20*A-15*B+12*C)*(x^2+1)*((3*
x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*
x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.26 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(2+5x^2+3x^4\right)^{3/2}}\,dx = \frac{-2A+20Ax^2-6Bx^2+205Ax^4-120Bx^4+78Cx^4+195Ax^6-126Bx^6 -$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output
```
(-2*A + 20*A*x^2 - 6*B*x^2 + 205*A*x^4 - 120*B*x^4 + 78*C*x^4 + 195*A*x^6
- 126*B*x^6 + 90*C*x^6 + I*Sqrt[3]*(65*A - 42*B + 30*C)*x^3*Sqrt[1 + x^2]*
Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - I*Sqrt[3]*(25*A -
 12*B + 6*C)*x^3*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/
2]*x], 2/3])/(12*x^3*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.52, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2198, 27, 2199, 2199, 1604, 27, 1604, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(3x^4+5x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2198}$$

$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{8\sqrt{3x^4+5x^2+2}} - \frac{1}{2}\int -\frac{-3(35A-26B+20C)x^6-6(13A-10B+8C)x^4-2(5A-2B)x^2+4A}{4x^4\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\int \frac{-3(35A-26B+20C)x^6-6(13A-10B+8C)x^4-2(5A-2B)x^2+4A}{x^4\sqrt{3x^4+5x^2+2}}dx+\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{8\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 2199

$$\frac{1}{8}\left(\int \frac{-6(13A-10B+8C)x^4-8(10A-7B+5C)x^2+4A}{x^4\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(35A-26B+20C)}{x}\right)+\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{8\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 2199

$$\frac{1}{8}\left(\int \frac{12(15A-12B+10C)x^2+8(20A-15B+12C)}{x^4\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(35A-26B+20C)}{x}+\frac{2\sqrt{3x^4+5}}{}\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{8\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 1604

$$\frac{1}{8}\left(-\frac{1}{6}\int \frac{8\left(3(20A-15B+12C)x^2+65A-42B+30C\right)}{x^2\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(35A-26B+20C)}{x}+\frac{2\sqrt{3x^4}}{}\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{8\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 27

$$\frac{1}{8}\left(-\frac{4}{3}\int \frac{3(20A-15B+12C)x^2+65A-42B+30C}{x^2\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(35A-26B+20C)}{x}+\frac{2\sqrt{3x^4+5}}{}\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{8\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 1604

$$\frac{1}{8}\left(-\frac{4}{3}\left(-\frac{1}{2}\int -\frac{3\left((65A-42B+30C)x^2+2(20A-15B+12C)\right)}{\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(65A-42B+30C)}{2x}\right)\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{8\sqrt{3x^4+5x^2+2}}$$

$\big\downarrow$ 27

$$\frac{1}{8}\left(-\frac{4}{3}\left(\frac{3}{2}\int \frac{(65A-42B+30C)x^2+2(20A-15B+12C)}{\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(65A-42B+30C)}{2x}\right)-\frac{\sqrt{3x}}{}\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{8\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1503

$$\frac{1}{8}\left(-\frac{4}{3}\left(\frac{3}{2}\left(2(20A-15B+12C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+(65A-42B+30C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{\sqrt{3x^4}}{}\right.\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{8\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1413

$$\frac{1}{8}\left(-\frac{4}{3}\left(\frac{3}{2}\left((65A-42B+30C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(20A-15B+12C)\,\text{EllipticF}\left(\text{arc}\right.}{\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{8\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1456

$$\frac{1}{8}\left(-\frac{4}{3}\left(\frac{3}{2}\left(\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(20A-15B+12C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+(65A-42B+30C)\left(\frac{x(3}{3\sqrt{3x^4}}\right.\right.\right.\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{8\sqrt{3x^4+5x^2+2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^4*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output
```
(x*(97*A - 70*B + 52*C + 3*(35*A - 26*B + 20*C)*x^2))/(8*Sqrt[2 + 5*x^2 +
3*x^4]) + ((2*(13*A - 10*B + 8*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x^3 - (4*(20*A
- 15*B + 12*C)*Sqrt[2 + 5*x^2 + 3*x^4])/(3*x^3) - ((35*A - 26*B + 20*C)*Sq
rt[2 + 5*x^2 + 3*x^4])/x - (4*(-1/2*((65*A - 42*B + 30*C)*Sqrt[2 + 5*x^2 +
 3*x^4])/x + (3*((65*A - 42*B + 30*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 +
 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan
[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (Sqrt[2]*(20*A - 15*B + 12*C)*(
1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 +
5*x^2 + 3*x^4]))/2))/3)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 8.79 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.92

method	result
elliptic	$-\frac{6\left(\left(-\frac{35A}{16}+\frac{13B}{8}-\frac{5C}{4}\right)x^3+\left(\frac{35B}{24}-\frac{97A}{48}-\frac{13C}{12}\right)x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{A\sqrt{3x^4+5x^2+2}}{12x^3}-\frac{\left(\frac{B}{4}-\frac{25A}{24}\right)\sqrt{3x^4+5x^2+2}}{x}-\frac{i\left(-10A+\frac{15B}{2}-6C\right)\sqrt{3}\cdots}{2\sqrt{\cdots}}$
risch	$\frac{195x^6A-126Bx^6+90Cx^6+205Ax^4-120Bx^4+78Cx^4+20Ax^2-6Bx^2-2A}{12x^3\sqrt{3x^4+5x^2+2}}-\frac{i(65A-42B+30C)\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(i\cdots\right.\right.}{12\sqrt{3x^4+5x^2+2}}$
default	$C\left(-\frac{6\left(-\frac{5}{4}x^3-\frac{13}{12}x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{3i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4+5x^2+2}}-\frac{5i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{2\sqrt{3x^4+5x^2+2}}\cdots\right.$

input
```
int((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6*((-35/16*A+13/8*B-5/4*C)*x^3+(35/24*B-97/48*A-13/12*C)*x)/(3*x^4+5*x^2+
2)^(1/2)-1/12*A*(3*x^4+5*x^2+2)^(1/2)/x^3-(1/4*B-25/24*A)*(3*x^4+5*x^2+2)^
(1/2)/x-1/2*I*(-10*A+15/2*B-6*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^
2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3*I*(-65/4*A+21/2*B-15/2*C)*(x^2+1
)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-
EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(2+5x^2+3x^4\right)^{3/2}}\,dx=\frac{\sqrt{2}(-3i\,(65\,A-42\,B+30\,C)x^7-5i\,(65\,A-42\,B+30\,C)x^5-2i\,(65\,A-4\cdots}{\cdots}$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(2)*(-3*I*(65*A - 42*B + 30*C)*x^7 - 5*I*(65*A - 42*B + 30*C)*x^
5 - 2*I*(65*A - 42*B + 30*C)*x^3)*elliptic_e(arcsin(I*x), 3/2) + sqrt(2)*(
3*I*(125*A - 87*B + 66*C)*x^7 + 5*I*(125*A - 87*B + 66*C)*x^5 + 2*I*(125*A
 - 87*B + 66*C)*x^3)*elliptic_f(arcsin(I*x), 3/2) + (3*(65*A - 42*B + 30*C
)*x^6 + (205*A - 120*B + 78*C)*x^4 + 2*(10*A - 3*B)*x^2 - 2*A)*sqrt(3*x^4
+ 5*x^2 + 2))/(3*x^7 + 5*x^5 + 2*x^3)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^4\left((x^2+1)(3x^2+2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**4*((x**2 + 1)*(3*x**2 + 2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(2+5x^2+3x^4\right)^{3/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^4\,{\left(3\,x^4+5\,x^2+2\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(5*x^2 + 3*x^4 + 2)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(5*x^2 + 3*x^4 + 2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(2+5x^2+3x^4\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(3/2),x)
```

output

```
( - 300*sqrt(3*x**4 + 5*x**2 + 2)*a*x**4 - 40*sqrt(3*x**4 + 5*x**2 + 2)*a
- 27*sqrt(3*x**4 + 5*x**2 + 2)*b*x**6 + 36*sqrt(3*x**4 + 5*x**2 + 2)*c*x**
6 + 120*sqrt(3*x**4 + 5*x**2 + 2)*c*x**4 - 2400*int(sqrt(3*x**4 + 5*x**2 +
 2)/(9*x**10 + 30*x**8 + 37*x**6 + 20*x**4 + 4*x**2),x)*a*x**7 - 4000*int(
sqrt(3*x**4 + 5*x**2 + 2)/(9*x**10 + 30*x**8 + 37*x**6 + 20*x**4 + 4*x**2)
,x)*a*x**5 - 1600*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**10 + 30*x**8 + 37*x*
*6 + 20*x**4 + 4*x**2),x)*a*x**3 + 720*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x*
*10 + 30*x**8 + 37*x**6 + 20*x**4 + 4*x**2),x)*b*x**7 + 1200*int(sqrt(3*x*
*4 + 5*x**2 + 2)/(9*x**10 + 30*x**8 + 37*x**6 + 20*x**4 + 4*x**2),x)*b*x**
5 + 480*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**10 + 30*x**8 + 37*x**6 + 20*x*
*4 + 4*x**2),x)*b*x**3 + 243*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(9*x**8
+ 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b*x**7 + 405*int((sqrt(3*x**4 + 5*x*
*2 + 2)*x**6)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b*x**5 + 162*i
nt((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2
+ 4),x)*b*x**3 - 324*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(9*x**8 + 30*x**
6 + 37*x**4 + 20*x**2 + 4),x)*c*x**7 - 540*int((sqrt(3*x**4 + 5*x**2 + 2)*
x**6)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*c*x**5 - 216*int((sqrt
(3*x**4 + 5*x**2 + 2)*x**6)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*
c*x**3 - 2700*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(9*x**8 + 30*x**6 + 37*
x**4 + 20*x**2 + 4),x)*a*x**7 - 4500*int((sqrt(3*x**4 + 5*x**2 + 2)*x**...
```

3.178 $\int \frac{A+Bx^2+Cx^4}{x^6\left(2+5x^2+3x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 302

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(2+5x^2+3x^4\right)^{3/2}}\,dx = \frac{(534A-325B+210C)x(2+3x^2)}{60\sqrt{2+5x^2+3x^4}} - \frac{x(275A-194B+140C+3(97A-70B+52C)x^2)}{16\sqrt{2+5x^2+3x^4}} - \frac{A\sqrt{2+5x^2+3x^4}}{20x^5} + \frac{(9A-2B)\sqrt{2+5x^2+3x^4}}{24x^3} - \frac{(681A-250B+60C)\sqrt{2+5x^2+3x^4}}{240x} - \frac{(534A-325B+210C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\,E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{30\sqrt{2}\sqrt{2+5x^2+3x^4}} + \frac{(57A-40B+30C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{4\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
1/60*(534*A-325*B+210*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-1/16*x*(275*A-1
94*B+140*C+3*(97*A-70*B+52*C)*x^2)/(3*x^4+5*x^2+2)^(1/2)-1/20*A*(3*x^4+5*x
^2+2)^(1/2)/x^5+1/24*(9*A-2*B)*(3*x^4+5*x^2+2)^(1/2)/x^3-1/240*(681*A-250*
B+60*C)*(3*x^4+5*x^2+2)^(1/2)/x-1/60*(534*A-325*B+210*C)*(x^2+1)*((3*x^2+2
)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5
*x^2+2)^(1/2)+1/8*(57*A-40*B+30*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*Inver
seJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(2+5x^2+3x^4\right)^{3/2}}\,dx = \frac{-6A+30Ax^2-10Bx^2-237Ax^4+100Bx^4-30Cx^4-1815Ax^6+1025B\ldots}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output
```
(-6*A + 30*A*x^2 - 10*B*x^2 - 237*A*x^4 + 100*B*x^4 - 30*C*x^4 - 1815*A*x^
6 + 1025*B*x^6 - 600*C*x^6 - 1602*A*x^8 + 975*B*x^8 - 630*C*x^8 - I*Sqrt[3
]*(534*A - 325*B + 210*C)*x^5*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*Ar
cSinh[Sqrt[3/2]*x], 2/3] + I*Sqrt[3]*(249*A - 125*B + 60*C)*x^5*Sqrt[1 + x
^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(60*x^5*Sqrt[2
 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2198, 27, 2199, 2199, 2199, 1604, 27, 1604, 1604, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(3x^4+5x^2+2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2198}$$

$$-\frac{1}{2}\int -\frac{3(97A-70B+52C)x^8+6(35A-26B+20C)x^6+2(19A-10B+4C)x^4-4(5A-2B)x^2+8A}{8x^6\sqrt{3x^4+5x^2+2}}dx - \frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{16}\int \frac{3(97A-70B+52C)x^8+6(35A-26B+20C)x^6+2(19A-10B+4C)x^4-4(5A-2B)x^2+8A}{x^6\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 2199

$$\frac{1}{16}\left(\int \frac{6(35A-26B+20C)x^6+8(29A-20B+14C)x^4-4(5A-2B)x^2+8A}{x^6\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(97A-7\cdots}{x}\cdots\right)-\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 2199

$$\frac{1}{16}\left(\int \frac{-36(13A-10B+8C)x^4-40(11A-8B+6C)x^2+8A}{x^6\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(97A-70B+52C)}{x}-\frac{2\sqrt{\cdots}}{\cdots}\right)-\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 2199

$$\frac{1}{16}\left(\int \frac{40(15A-12B+10C)x^2+16(33A-25B+20C)}{x^6\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(97A-70B+52C)}{x}+\frac{4\sqrt{3x^4+\cdots}}{\cdots}\right)-\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1604

$$\frac{1}{16}\left(-\frac{1}{10}\int \frac{16\left(9(33A-25B+20C)x^2+5(57A-40B+30C)\right)}{x^4\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(97A-70B+52C)}{x}+\frac{4\cdots}{\cdots}\right)-\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 27

$$\frac{1}{16}\left(-\frac{8}{5}\int \frac{9(33A-25B+20C)x^2+5(57A-40B+30C)}{x^4\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(97A-70B+52C)}{x}+\frac{4\sqrt{3x^4\cdots}}{\cdots}\right)-\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1604

$$\frac{1}{16}\left(-\frac{8}{5}\left(-\frac{1}{6}\int\frac{15(57A-40B+30C)x^2+2(534A-325B+210C)}{x^2\sqrt{3x^4+5x^2+2}}dx-\frac{5\sqrt{3x^4+5x^2+2}(57A-40B+30C}{6x^3}\right.\right.$$
$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1604

$$\frac{1}{16}\left(-\frac{8}{5}\left(\frac{1}{6}\left(\frac{1}{2}\int-\frac{6\left((534A-325B+210C)x^2+5(57A-40B+30C)\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(534A-325B+}{x}\right.\right.\right.$$
$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 27

$$\frac{1}{16}\left(-\frac{8}{5}\left(\frac{1}{6}\left(\frac{\sqrt{3x^4+5x^2+2}(534A-325B+210C)}{x}-3\int\frac{(534A-325B+210C)x^2+5(57A-40B+30C)}{\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$
$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1503

$$\frac{1}{16}\left(-\frac{8}{5}\left(\frac{1}{6}\left(\frac{\sqrt{3x^4+5x^2+2}(534A-325B+210C)}{x}-3\left(5(57A-40B+30C)\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx+(534A\right.\right.\right.\right.$$
$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1413

$$\frac{1}{16}\left(-\frac{8}{5}\left(\frac{1}{6}\left(\frac{\sqrt{3x^4+5x^2+2}(534A-325B+210C)}{x}-3\left((534A-325B+210C)\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{5(}{}\right.\right.\right.\right.$$
$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1456

$$\frac{1}{16}\left(-\frac{8}{5}\left(\frac{1}{6}\left(\frac{\sqrt{3x^4+5x^2+2}(534A-325B+210C)}{x}-3\left(\frac{5\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}(57A-40B+30C)\,\text{EllipticF}\left(\text{ar}\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.\right.\right.\right.$$
$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{16\sqrt{3x^4+5x^2+2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^6*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output
```
-1/16*(x*(275*A - 194*B + 140*C + 3*(97*A - 70*B + 52*C)*x^2))/Sqrt[2 + 5*
x^2 + 3*x^4] + ((4*(13*A - 10*B + 8*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x^5 - (8*(
33*A - 25*B + 20*C)*Sqrt[2 + 5*x^2 + 3*x^4])/(5*x^5) - (2*(35*A - 26*B + 2
0*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x^3 + ((97*A - 70*B + 52*C)*Sqrt[2 + 5*x^2 +
 3*x^4])/x - (8*((-5*(57*A - 40*B + 30*C)*Sqrt[2 + 5*x^2 + 3*x^4])/(6*x^3)
 + (((534*A - 325*B + 210*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x - 3*((534*A - 325*
B + 210*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^
2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x
^2 + 3*x^4])) + (5*(57*A - 40*B + 30*C)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^
2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])))/6))/5)
/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 10.77 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.79

method	result
elliptic	$-\frac{6\left(\left(\frac{97A}{32}-\frac{35B}{16}+\frac{13C}{8}\right)x^3+\left(\frac{275A}{96}-\frac{97B}{48}+\frac{35C}{24}\right)x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{A\sqrt{3x^4+5x^2+2}}{20x^5}-\frac{\left(\frac{B}{4}-\frac{9A}{8}\right)\sqrt{3x^4+5x^2+2}}{3x^3}-\frac{\left(\frac{C}{4}+\frac{227A}{80}-\frac{25B}{24}\right)\sqrt{3x^4}}{x}$
risch	$-\frac{1602A\,x^8-975B\,x^8+630C\,x^8+1815x^6A-1025B\,x^6+600C\,x^6+237A\,x^4-100B\,x^4+30C\,x^4-30A\,x^2+10B\,x^2+6A}{60x^5\sqrt{3x^4+5x^2+2}}+\frac{i(534A-3}{}$
default	$A\left(-\frac{6\left(\frac{97}{32}x^3+\frac{275}{96}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{3x^4+5x^2+2}}{20x^5}+\frac{3\sqrt{3x^4+5x^2+2}}{8x^3}-\frac{227\sqrt{3x^4+5x^2+2}}{80x}-\frac{57i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{8\sqrt{3x^4+5x^2+2}}+\right.$

input
```
int((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6*((97/32*A-35/16*B+13/8*C)*x^3+(275/96*A-97/48*B+35/24*C)*x)/(3*x^4+5*x^
2+2)^(1/2)-1/20*A*(3*x^4+5*x^2+2)^(1/2)/x^5-1/3*(1/4*B-9/8*A)*(3*x^4+5*x^2
+2)^(1/2)/x^3-(1/4*C+227/80*A-25/24*B)*(3*x^4+5*x^2+2)^(1/2)/x-1/2*I*(57/4
*A-10*B+15/2*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*Ellipt
icF(I*x,1/2*6^(1/2))+1/3*I*(267/10*A-65/4*B+21/2*C)*(x^2+1)^(1/2)*(6*x^2+4
)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/
2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2+Cx^4}{x^6\,(2+5x^2+3x^4)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{2}(-3i\,(534\,A-325\,B+210\,C)x^9-5i\,(534\,A-325\,B+210\,C)x^7-2i\,(534\,A-325\,B+210\,C)x^5)}{}$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/120*(2*sqrt(2)*(-3*I*(534*A - 325*B + 210*C)*x^9 - 5*I*(534*A - 325*B +
 210*C)*x^7 - 2*I*(534*A - 325*B + 210*C)*x^5)*elliptic_e(arcsin(I*x), 3/2
) - sqrt(2)*(-3*I*(1923*A - 1250*B + 870*C)*x^9 - 5*I*(1923*A - 1250*B + 8
70*C)*x^7 - 2*I*(1923*A - 1250*B + 870*C)*x^5)*elliptic_f(arcsin(I*x), 3/2
) + 2*(3*(534*A - 325*B + 210*C)*x^8 + 5*(363*A - 205*B + 120*C)*x^6 + (23
7*A - 100*B + 30*C)*x^4 - 10*(3*A - B)*x^2 + 6*A)*sqrt(3*x^4 + 5*x^2 + 2))
/(3*x^9 + 5*x^7 + 2*x^5)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6 (2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^6 \left((x^2+1)(3x^2+2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**6/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**6*((x**2 + 1)*(3*x**2 + 2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6 (2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6 \left(2+5x^2+3x^4\right)^{3/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{\left(3\,x^4+5\,x^2+2\right)^{\frac{3}{2}} x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^6 \left(2+5x^2+3x^4\right)^{3/2}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{x^6\,\left(3\,x^4+5\,x^2+2\right)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^6*(5*x^2 + 3*x^4 + 2)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^6*(5*x^2 + 3*x^4 + 2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6 \left(2+5x^2+3x^4\right)^{3/2}}\,dx = \frac{-\sqrt{3x^4+5x^2+2}\,a-90\left(\int \frac{\sqrt{3x^4+5x^2+2}}{9x^{12}+30x^{10}+37x^8+20x^6+4x^4}dx\right)a\,x^9-150\left(\int \frac{}{9x^{12}+30}\right.}{}$$

input
```
int((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(3/2),x)
```

output

```
( - sqrt(3*x**4 + 5*x**2 + 2)*a - 90*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**1
2 + 30*x**10 + 37*x**8 + 20*x**6 + 4*x**4),x)*a*x**9 - 150*int(sqrt(3*x**4
 + 5*x**2 + 2)/(9*x**12 + 30*x**10 + 37*x**8 + 20*x**6 + 4*x**4),x)*a*x**7
 - 60*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**12 + 30*x**10 + 37*x**8 + 20*x**
6 + 4*x**4),x)*a*x**5 + 30*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**12 + 30*x**
10 + 37*x**8 + 20*x**6 + 4*x**4),x)*b*x**9 + 50*int(sqrt(3*x**4 + 5*x**2 +
 2)/(9*x**12 + 30*x**10 + 37*x**8 + 20*x**6 + 4*x**4),x)*b*x**7 + 20*int(s
qrt(3*x**4 + 5*x**2 + 2)/(9*x**12 + 30*x**10 + 37*x**8 + 20*x**6 + 4*x**4)
,x)*b*x**5 - 63*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**10 + 30*x**8 + 37*x**6
 + 20*x**4 + 4*x**2),x)*a*x**9 - 105*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**1
0 + 30*x**8 + 37*x**6 + 20*x**4 + 4*x**2),x)*a*x**7 - 42*int(sqrt(3*x**4 +
 5*x**2 + 2)/(9*x**10 + 30*x**8 + 37*x**6 + 20*x**4 + 4*x**2),x)*a*x**5 +
30*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**10 + 30*x**8 + 37*x**6 + 20*x**4 +
4*x**2),x)*c*x**9 + 50*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**10 + 30*x**8 +
37*x**6 + 20*x**4 + 4*x**2),x)*c*x**7 + 20*int(sqrt(3*x**4 + 5*x**2 + 2)/(
9*x**10 + 30*x**8 + 37*x**6 + 20*x**4 + 4*x**2),x)*c*x**5)/(10*x**5*(3*x**
4 + 5*x**2 + 2))
```

3.179 $\int \frac{x^8(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 285

$$\begin{aligned}\int \frac{x^8(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx &= \frac{x(2(117A-105B+97C)+(315A-291B+275C)x^2)}{243\,(2+5x^2+3x^4)^{3/2}}\\
&+\frac{2(1035A-789B+560C)x(2+3x^2)}{243\sqrt{2+5x^2+3x^4}}\\
&-\frac{x(5247A-4065B+3043C+3(2070A-1587B+1160C)x^2)}{243\sqrt{2+5x^2+3x^4}}\\
&+\frac{1}{81}Cx\sqrt{2+5x^2+3x^4}\\
&-\frac{2\sqrt{2}(1035A-789B+560C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{243\sqrt{2+5x^2+3x^4}}\\
&+\frac{5\sqrt{2}(171A-132B+98C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{81\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
1/243*x*(234*A-210*B+194*C+(315*A-291*B+275*C)*x^2)/(3*x^4+5*x^2+2)^(3/2)+
2/243*(1035*A-789*B+560*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-1/243*x*(5247
*A-4065*B+3043*C+3*(2070*A-1587*B+1160*C)*x^2)/(3*x^4+5*x^2+2)^(1/2)+1/81*
C*x*(3*x^4+5*x^2+2)^(1/2)-2/243*2^(1/2)*(1035*A-789*B+560*C)*(x^2+1)*((3*x
^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2
+2)^(1/2)+5/81*2^(1/2)*(171*A-132*B+98*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2
)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.80

$$\int \frac{x^8(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{-3x(9A(380+1420x^2+1733x^4+690x^6)-3B(880+3284x^2+4000x^4+1\cdots}{}$$

input
```
Integrate[(x^8*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(-3*x*(9*A*(380 + 1420*x^2 + 1733*x^4 + 690*x^6) - 3*B*(880 + 3284*x^2 + 4
000*x^4 + 1587*x^6) + C*(1960 + 7280*x^2 + 8806*x^4 + 3450*x^6 - 9*x^8)) -
 (2*I)*Sqrt[3]*(1035*A - 789*B + 560*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 +
 5*x^2 + 3*x^4)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (2*I)*Sqrt[3]*(18
0*A - 129*B + 70*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*Elli
pticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(243*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2197, 27, 2206, 27, 2207, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(A+Bx^2+Cx^4\right)}{\left(3x^4+5x^2+2\right)^{5/2}}dx$$

$\Big\downarrow$ 2197

$$\frac{x\left(x^2(315A-291B+275C)+2(117A-105B+97C)\right)}{243\left(3x^4+5x^2+2\right)^{3/2}}-$$
$$\frac{1}{6}\int \frac{2\left(-81Cx^8-27(3B-5C)x^6-9(9A-15B+19C)x^4-18(45A-39B+35C)x^2+2(117A-105B+97C\right.}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{x\left(x^2(315A-291B+275C)+2(117A-105B+97C)\right)}{243\left(3x^4+5x^2+2\right)^{3/2}}-$$
$$\frac{1}{243}\int \frac{-81Cx^8-27(3B-5C)x^6-9(9A-15B+19C)x^4-18(45A-39B+35C)x^2+2(117A-105B+97C}{\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 2206

$$\frac{1}{243}\left(\frac{1}{2}\int \frac{6\left(9Cx^4+2(1035A-789B+565C)x^2+2(855A-660B+491C)\right)}{\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(3x^2(2070A-1587B+1\right.}{\sqrt{3x}}\right.$$
$$\frac{x\left(x^2(315A-291B+275C)+2(117A-105B+97C)\right)}{243\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{1}{243}\left(3\int \frac{9Cx^4+2(1035A-789B+565C)x^2+2(855A-660B+491C)}{\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(3x^2(2070A-1587B+1160C\right.}{\sqrt{3x^4+}}\right.$$
$$\frac{x\left(x^2(315A-291B+275C)+2(117A-105B+97C)\right)}{243\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 2207

$$\frac{1}{243}\left(3\left(\frac{1}{9}\int \frac{18\left((1035A-789B+560C)x^2+5(171A-132B+98C)\right)}{\sqrt{3x^4+5x^2+2}}dx+C\sqrt{3x^4+5x^2+2}x\right)-\frac{x\left(3x^2(20\right.}{}\right.$$
$$\frac{x\left(x^2(315A-291B+275C)+2(117A-105B+97C)\right)}{243\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{1}{243}\left(3\left(2\int \frac{(1035A-789B+560C)x^2+5(171A-132B+98C)}{\sqrt{3x^4+5x^2+2}}dx+C\sqrt{3x^4+5x^2+2}x\right)-\frac{x\left(3x^2(2070A-\right.}{}\right.$$
$$\frac{x\left(x^2(315A-291B+275C)+2(117A-105B+97C)\right)}{243\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 1503

$$\frac{1}{243}\left(3\left(2\left(5(171A-132B+98C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+(1035A-789B+560C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\right.\right.$$
$$\frac{x\left(x^2(315A-291B+275C)+2(117A-105B+97C)\right)}{243\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 1413

$$\frac{1}{243}\left(3\left(2\left((1035A-789B+560C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{5(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(171A-132B+98C)\text{ EllipticF}}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$
$$\frac{x\left(x^2(315A-291B+275C)+2(117A-105B+97C)\right)}{243\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 1456

$$\frac{1}{243}\left(3\left(2\left(\frac{5(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(171A-132B+98C)\text{ EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+(1035A-789B+560C)\left(\frac{}{3}\right.\right.\right.\right.$$
$$\frac{x\left(x^2(315A-291B+275C)+2(117A-105B+97C)\right)}{243\left(3x^4+5x^2+2\right)^{3/2}}$$

input
```
Int[(x^8*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(x*(2*(117*A - 105*B + 97*C) + (315*A - 291*B + 275*C)*x^2))/(243*(2 + 5*x
^2 + 3*x^4)^(3/2)) + (-((x*(5247*A - 4065*B + 3043*C + 3*(2070*A - 1587*B
+ 1160*C)*x^2))/Sqrt[2 + 5*x^2 + 3*x^4]) + 3*(C*x*Sqrt[2 + 5*x^2 + 3*x^4]
+ 2*((1035*A - 789*B + 560*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4])
 - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/
2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (5*(171*A - 132*B + 98*C)*(1 + x^2)*Sqr
t[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x
^2 + 3*x^4]))))/243
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 20.38 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.83

method	result
elliptic	$\frac{\left(\left(\frac{35A}{243}-\frac{97B}{729}+\frac{275C}{2187}\right)x^3+\left(\frac{26A}{243}-\frac{70B}{729}+\frac{194C}{2187}\right)x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\left(\frac{115A}{27}-\frac{529B}{162}+\frac{580C}{243}\right)x^3+\left(\frac{583A}{162}-\frac{1355B}{486}+\frac{3043C}{1458}\right)x\right)}{\sqrt{3x^4+5x^2+2}}+$
risch	$-\frac{x\left(-9C\,x^8+6210x^6A-4761B\,x^6+3450C\,x^6+15597A\,x^4-12000B\,x^4+8806C\,x^4+12780A\,x^2-9852B\,x^2+7280C\,x^2+3420A-26\right.}{81\left(3x^4+5x^2+2\right)^{\frac{3}{2}}}$
default	$A\left(\frac{\left(\frac{35}{243}x^3+\frac{26}{243}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\frac{115}{27}x^3+\frac{583}{162}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{95i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{9\sqrt{3x^4+5x^2+2}}+\frac{230i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,(\mathrm{Ellip}}{27\sqrt{}}\right.$

input
```
int(x^8*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((35/243*A-97/729*B+275/2187*C)*x^3+(26/243*A-70/729*B+194/2187*C)*x)*(3*x
^4+5*x^2+2)^(1/2)/(x^4+5/3*x^2+2/3)^2-6*((115/27*A-529/162*B+580/243*C)*x^
3+(583/162*A-1355/486*B+3043/1458*C)*x)/(3*x^4+5*x^2+2)^(1/2)+1/81*C*x*(3*
x^4+5*x^2+2)^(1/2)-1/2*I*(190/9*A-440/27*B+980/81*C)*(x^2+1)^(1/2)*(6*x^2+
4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3*I*(-526/27*B
+1120/81*C+230/9*A)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(E
llipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.06

$$\int \frac{x^8(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx =$$

$$-\frac{4\sqrt{3}\sqrt{-\frac{2}{3}}(9\,(1035\,A-789\,B+560\,C)x^9+30\,(1035\,A-789\,B+560\,C)x^7+37\,(1035\,A-789\,B+560}{}$$

input
```
integrate(x^8*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/729*(4*sqrt(3)*sqrt(-2/3)*(9*(1035*A - 789*B + 560*C)*x^9 + 30*(1035*A
- 789*B + 560*C)*x^7 + 37*(1035*A - 789*B + 560*C)*x^5 + 20*(1035*A - 789*
B + 560*C)*x^3 + 4*(1035*A - 789*B + 560*C)*x)*elliptic_e(arcsin(sqrt(-2/3
)/x), 3/2) - sqrt(3)*sqrt(-2/3)*(9*(11835*A - 9096*B + 6650*C)*x^9 + 30*(1
1835*A - 9096*B + 6650*C)*x^7 + 37*(11835*A - 9096*B + 6650*C)*x^5 + 20*(1
1835*A - 9096*B + 6650*C)*x^3 + 4*(11835*A - 9096*B + 6650*C)*x)*elliptic_
f(arcsin(sqrt(-2/3)/x), 3/2) - 3*(27*C*x^10 + 27*(3*B - 10*C)*x^8 + 189*(8
1*A - 60*B + 38*C)*x^6 + 10*(3825*A - 2883*B + 1960*C)*x^4 + 20*(1557*A -
1182*B + 826*C)*x^2 + 8280*A - 6312*B + 4480*C)*sqrt(3*x^4 + 5*x^2 + 2))/(
9*x^9 + 30*x^7 + 37*x^5 + 20*x^3 + 4*x)
```

Sympy [F]

$$\int \frac{x^8(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^8(A+Bx^2+Cx^4)}{((x^2+1)\,(3x^2+2))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**8*(C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral(x**8*(A + B*x**2 + C*x**4)/((x**2 + 1)*(3*x**2 + 2))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^8(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^8}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^8*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^8/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{x^8(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^8}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^8*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^8/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^8\,(C\,x^4+B\,x^2+A)}{(3\,x^4+5\,x^2+2)^{5/2}}\,dx$$

input
```
int((x^8*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(5/2),x)
```

output
```
int((x^8*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^8(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \text{too large to display}$$

input
```
int(x^8*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x)
```

output

```
( - 405*sqrt(3*x**4 + 5*x**2 + 2)*a*x**5 - 450*sqrt(3*x**4 + 5*x**2 + 2)*a
*x**3 - 270*sqrt(3*x**4 + 5*x**2 + 2)*a*x + 405*sqrt(3*x**4 + 5*x**2 + 2)*
b*x**7 + 2700*sqrt(3*x**4 + 5*x**2 + 2)*b*x**5 + 3630*sqrt(3*x**4 + 5*x**2
 + 2)*b*x**3 + 2178*sqrt(3*x**4 + 5*x**2 + 2)*b*x + 135*sqrt(3*x**4 + 5*x*
*2 + 2)*c*x**9 - 1350*sqrt(3*x**4 + 5*x**2 + 2)*c*x**7 - 8190*sqrt(3*x**4
+ 5*x**2 + 2)*c*x**5 - 11200*sqrt(3*x**4 + 5*x**2 + 2)*c*x**3 - 6720*sqrt(
3*x**4 + 5*x**2 + 2)*c*x + 4860*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 +
135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**8 + 1620
0*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**
6 + 186*x**4 + 60*x**2 + 8),x)*a*x**6 + 19980*int(sqrt(3*x**4 + 5*x**2 + 2
)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)
*a*x**4 + 10800*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*
x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**2 + 2160*int(sqrt(3*x**4
 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60
*x**2 + 8),x)*a - 39204*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**1
0 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**8 - 130680*int(s
qrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186
*x**4 + 60*x**2 + 8),x)*b*x**6 - 161172*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*
x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**
4 - 87120*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**...
```

3.180 $\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 263

$$
\begin{aligned}
\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx &= \\
&-\frac{x(2(45A-39B+35C)+(117A-105B+97C)x^2)}{81(2+5x^2+3x^4)^{3/2}} \\
&-\frac{(873A-690B+526C)x(2+3x^2)}{81\sqrt{2+5x^2+3x^4}} \\
&+\frac{x(2205A-1749B+1355C+3(873A-690B+529C)x^2)}{81\sqrt{2+5x^2+3x^4}} \\
&+\frac{\sqrt{2}(873A-690B+526C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{81\sqrt{2+5x^2+3x^4}} \\
&-\frac{5\sqrt{2}(72A-57B+44C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{27\sqrt{2+5x^2+3x^4}}
\end{aligned}
$$

output
```
-1/81*x*(90*A-78*B+70*C+(117*A-105*B+97*C)*x^2)/(3*x^4+5*x^2+2)^(3/2)-1/81
*(873*A-690*B+526*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/81*x*(2205*A-1749
*B+1355*C+3*(873*A-690*B+529*C)*x^2)/(3*x^4+5*x^2+2)^(1/2)+1/81*2^(1/2)*(8
73*A-690*B+526*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1
/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)-5/27*2^(1/2)*(72*A-57*B+44*C)*(x^
2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3
*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.40 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.84

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{3x(9A(160+598x^2+730x^4+291x^6)-3B(380+1420x^2+1733x^4+690x^6}{}$$

input
```
Integrate[(x^6*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(3*x*(9*A*(160 + 598*x^2 + 730*x^4 + 291*x^6) - 3*B*(380 + 1420*x^2 + 1733
*x^4 + 690*x^6) + C*(880 + 3284*x^2 + 4000*x^4 + 1587*x^6)) + I*Sqrt[3]*(8
73*A - 690*B + 526*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*El
lipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - I*Sqrt[3]*(153*A - 120*B + 86*C)*Sq
rt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticF[I*ArcSinh[Sqrt[3
/2]*x], 2/3])/(81*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2197, 27, 2206, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(A+Bx^2+Cx^4\right)}{\left(3x^4+5x^2+2\right)^{5/2}}dx$$

$\downarrow$ 2197

$$-\frac{1}{6}\int -\frac{2\left(27Cx^6+9(3B-5C)x^4-18(18A-15B+13C)x^2+2(45A-39B+35C)\right)}{27\left(3x^4+5x^2+2\right)^{3/2}}dx-\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{81}\int \frac{27Cx^6+9(3B-5C)x^4-18(18A-15B+13C)x^2+2(45A-39B+35C)}{\left(3x^4+5x^2+2\right)^{3/2}}dx-\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 2206

$$\frac{1}{81}\left(\frac{x\left(3x^2(873A-690B+529C)+2205A-1749B+1355C\right)}{\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int \frac{6\left((873A-690B+526C)x^2+10(72A-\right.}{\sqrt{3x^4+5x^2+2}}\right.$$
$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{81}\left(\frac{x\left(3x^2(873A-690B+529C)+2205A-1749B+1355C\right)}{\sqrt{3x^4+5x^2+2}}-3\int \frac{(873A-690B+526C)x^2+10(72A-5}{\sqrt{3x^4+5x^2+2}}\right.$$
$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1503

$$\frac{1}{81}\left(\frac{x\left(3x^2(873A-690B+529C)+2205A-1749B+1355C\right)}{\sqrt{3x^4+5x^2+2}}-3\left(10(72A-57B+44C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}\right.\right.$$
$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1413

$$\frac{1}{81}\left(\frac{x\left(3x^2(873A-690B+529C)+2205A-1749B+1355C\right)}{\sqrt{3x^4+5x^2+2}}-3\left((873A-690B+526C)\int\frac{x^2}{\sqrt{3x^4+5x^2+}}\right.\right.$$
$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{81}\left(\frac{x\left(3x^2(873A-690B+529C)+2205A-1749B+1355C\right)}{\sqrt{3x^4+5x^2+2}}-3\left(\frac{5\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(72A-57B+44C)}{\sqrt{3x^4+5x^2+2}}\right.\right.$$
$$\frac{x\left(x^2(117A-105B+97C)+2(45A-39B+35C)\right)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

input
```
Int[(x^6*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
-1/81*(x*(2*(45*A - 39*B + 35*C) + (117*A - 105*B + 97*C)*x^2))/(2 + 5*x^2
 + 3*x^4)^(3/2) + ((x*(2205*A - 1749*B + 1355*C + 3*(873*A - 690*B + 529*C
)*x^2))/Sqrt[2 + 5*x^2 + 3*x^4] - 3*((873*A - 690*B + 526*C)*((x*(2 + 3*x^
2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 +
 x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (5*Sqrt[
2]*(72*A - 57*B + 44*C)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[Ar
cTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4]))/81
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 18.82 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.83

method	result
elliptic	$\frac{\left(\left(-\frac{13A}{81}+\frac{35B}{243}-\frac{97C}{729}\right)x^3+\left(-\frac{10A}{81}+\frac{26B}{243}-\frac{70C}{729}\right)x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\left(-\frac{97A}{18}+\frac{115B}{27}-\frac{529C}{162}\right)x^3+\left(-\frac{245A}{54}+\frac{583B}{162}-\frac{1355C}{486}\right)x\right)}{\sqrt{3x^4+5x^2+2}}$
risch	$\frac{x\left(2619x^6A-2070Bx^6+1587Cx^6+6570Ax^4-5199Bx^4+4000Cx^4+5382Ax^2-4260Bx^2+3284Cx^2+1440A-1140B+880C\right)}{27\left(3x^4+5x^2+2\right)^{\frac{3}{2}}}$
default	$B\left(\frac{\left(\frac{35}{243}x^3+\frac{26}{243}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\frac{115}{27}x^3+\frac{583}{162}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{95i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{9\sqrt{3x^4+5x^2+2}}+\frac{230i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,(\mathrm{Elli}}{27\sqrt{}}\right.$

input
```
int(x^6*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((-13/81*A+35/243*B-97/729*C)*x^3+(-10/81*A+26/243*B-70/729*C)*x)*(3*x^4+5
*x^2+2)^(1/2)/(x^4+5/3*x^2+2/3)^2-6*((-97/18*A+115/27*B-529/162*C)*x^3+(-2
45/54*A+583/162*B-1355/486*C)*x)/(3*x^4+5*x^2+2)^(1/2)-1/2*I*(190/9*B-440/
27*C-80/3*A)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF
(I*x,1/2*6^(1/2))+1/3*I*(-526/27*C-97/3*A+230/9*B)*(x^2+1)^(1/2)*(6*x^2+4)
^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2
*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.10

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{2\sqrt{3}\sqrt{-\frac{2}{3}}(9\,(873\,A-690\,B+526\,C)x^9+30\,(873\,A-690\,B+526\,C)x^7+3}{}$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/243*(2*sqrt(3)*sqrt(-2/3)*(9*(873*A - 690*B + 526*C)*x^9 + 30*(873*A - 6
90*B + 526*C)*x^7 + 37*(873*A - 690*B + 526*C)*x^5 + 20*(873*A - 690*B + 5
26*C)*x^3 + 4*(873*A - 690*B + 526*C)*x)*elliptic_e(arcsin(sqrt(-2/3)/x),
3/2) - sqrt(3)*sqrt(-2/3)*(9*(4986*A - 3945*B + 3032*C)*x^9 + 30*(4986*A -
 3945*B + 3032*C)*x^7 + 37*(4986*A - 3945*B + 3032*C)*x^5 + 20*(4986*A - 3
945*B + 3032*C)*x^3 + 4*(4986*A - 3945*B + 3032*C)*x)*elliptic_f(arcsin(sq
rt(-2/3)/x), 3/2) + 3*(27*C*x^8 - 27*(240*A - 189*B + 140*C)*x^6 - 5*(3231
*A - 2550*B + 1922*C)*x^4 - 20*(657*A - 519*B + 394*C)*x^2 - 3492*A + 2760
*B - 2104*C)*sqrt(3*x^4 + 5*x^2 + 2))/(9*x^9 + 30*x^7 + 37*x^5 + 20*x^3 +
4*x)
```

Sympy [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^6(A+Bx^2+Cx^4)}{((x^2+1)\,(3x^2+2))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**6*(C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral(x**6*(A + B*x**2 + C*x**4)/((x**2 + 1)*(3*x**2 + 2))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^6}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^6/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^6}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^6/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx=\int \frac{x^6\,(C\,x^4+B\,x^2+A)}{(3\,x^4+5\,x^2+2)^{5/2}}\,dx$$

input
```
int((x^6*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(5/2),x)
```

output
```
int((x^6*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx=\text{too large to display}$$

input
```
int(x^6*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x)
```

output

```
( - 45*sqrt(3*x**4 + 5*x**2 + 2)*a*x**3 - 27*sqrt(3*x**4 + 5*x**2 + 2)*a*x
 - 135*sqrt(3*x**4 + 5*x**2 + 2)*b*x**5 - 150*sqrt(3*x**4 + 5*x**2 + 2)*b*
x**3 - 90*sqrt(3*x**4 + 5*x**2 + 2)*b*x + 135*sqrt(3*x**4 + 5*x**2 + 2)*c*
x**7 + 900*sqrt(3*x**4 + 5*x**2 + 2)*c*x**5 + 1210*sqrt(3*x**4 + 5*x**2 +
2)*c*x**3 + 726*sqrt(3*x**4 + 5*x**2 + 2)*c*x + 486*int(sqrt(3*x**4 + 5*x*
*2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 +
 8),x)*a*x**8 + 1620*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 +
 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**6 + 1998*int(sqrt(3
*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4
 + 60*x**2 + 8),x)*a*x**4 + 1080*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 +
 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**2 + 216
*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6
 + 186*x**4 + 60*x**2 + 8),x)*a + 1620*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x
**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**8
 + 5400*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 3
05*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**6 + 6660*int(sqrt(3*x**4 + 5*x**
2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 +
8),x)*b*x**4 + 3600*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 +
279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**2 + 720*int(sqrt(3*x
**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**...
```

3.181 $\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 210

$$\begin{aligned}\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx &= \frac{x(2(18A-15B+13C)+(45A-39B+35C)x^2)}{27\,(2+5x^2+3x^4)^{3/2}} \\ &- \frac{(63A-51B+41C)x}{9\sqrt{2+5x^2+3x^4}} \\ &- \frac{\sqrt{2}(360A-291B+230C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|\,-\frac{1}{2}\right)}{27\sqrt{2+5x^2+3x^4}} \\ &+ \frac{(297A-240B+190C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}\,\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{9\sqrt{2}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
1/27*x*(36*A-30*B+26*C+(45*A-39*B+35*C)*x^2)/(3*x^4+5*x^2+2)^(3/2)-1/9*(63
*A-51*B+41*C)*x/(3*x^4+5*x^2+2)^(1/2)-1/27*2^(1/2)*(360*A-291*B+230*C)*(x^
2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3
*x^4+5*x^2+2)^(1/2)+1/18*(297*A-240*B+190*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(
1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{-3x(-3B(160+598x^2+730x^4+291x^6)+3A(198+740x^2+903x^4+360x}{}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(-3*x*(-3*B*(160 + 598*x^2 + 730*x^4 + 291*x^6) + 3*A*(198 + 740*x^2 + 903
*x^4 + 360*x^6) + C*(380 + 1420*x^2 + 1733*x^4 + 690*x^6)) - I*Sqrt[3]*(36
0*A - 291*B + 230*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*Ell
ipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + I*Sqrt[3]*(63*A - 51*B + 40*C)*Sqrt[
1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticF[I*ArcSinh[Sqrt[3/2]
*x], 2/3])/(27*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2197, 27, 2206, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx^2+Cx^4\right)}{\left(3x^4+5x^2+2\right)^{5/2}}dx$$

$$\Big\downarrow \; 2197$$

$$\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{27\left(3x^4+5x^2+2\right)^{3/2}}-\frac{1}{6}\int \frac{2\left(-9Cx^4-9(15A-12B+10C)x^2+2(18A-15B+13C)\right)}{9\left(3x^4+5x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{27\left(3x^4+5x^2+2\right)^{3/2}}-\frac{1}{27}\int\frac{-9Cx^4-9(15A-12B+10C)x^2+2(18A-15B+13C)}{\left(3x^4+5x^2+2\right)^{3/2}}dx$$

$\downarrow$ 2206

$$\frac{1}{27}\left(\frac{1}{2}\int\frac{6\left((360A-291B+230C)x^2+297A-240B+190C\right)}{\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(3x^2(360A-291B+230C)+909A-73\right.}{\sqrt{3x^4+5x^2+2}}\right.$$
$$\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{27\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{27}\left(3\int\frac{(360A-291B+230C)x^2+297A-240B+190C}{\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(3x^2(360A-291B+230C)+909A-735B\right.}{\sqrt{3x^4+5x^2+2}}\right.$$
$$\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{27\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1503

$$\frac{1}{27}\left(3\left((297A-240B+190C)\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx+(360A-291B+230C)\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{x(3x}{}\right.$$
$$\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{27\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1413

$$\frac{1}{27}\left(3\left((360A-291B+230C)\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}(297A-240B+190C)\,\text{EllipticF}\left(\text{arc}\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.\right.$$
$$\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{27\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1456

$$\frac{1}{27}\left(3\left(\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}(297A-240B+190C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+(360A-291B+230C)\left(\frac{x(3x}{3\sqrt{3x^4}}\right.\right.\right.$$
$$\frac{x\left(x^2(45A-39B+35C)+2(18A-15B+13C)\right)}{27\left(3x^4+5x^2+2\right)^{3/2}}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(x*(2*(18*A - 15*B + 13*C) + (45*A - 39*B + 35*C)*x^2))/(27*(2 + 5*x^2 + 3
*x^4)^(3/2)) + (-((x*(909*A - 735*B + 583*C + 3*(360*A - 291*B + 230*C)*x^
2))/Sqrt[2 + 5*x^2 + 3*x^4]) + 3*((360*A - 291*B + 230*C)*((x*(2 + 3*x^2))
/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^
2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + ((297*A - 2
40*B + 190*C)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -
1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])))/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 10.64 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.04

method	result
elliptic	$\frac{\left(\left(\frac{5A}{27}-\frac{13B}{81}+\frac{35C}{243}\right)x^3+\left(\frac{4A}{27}-\frac{10B}{81}+\frac{26C}{243}\right)x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\left(\frac{20A}{3}-\frac{97B}{18}+\frac{115C}{27}\right)x^3+\left(\frac{101A}{18}-\frac{245B}{54}+\frac{583C}{162}\right)x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{i\left(\frac{190C}{9}+\cdots\right.}{\cdots}$
risch	$-\frac{x\left(1080A\,x^6-873B\,x^6+690C\,x^6+2709A\,x^4-2190B\,x^4+1733C\,x^4+2220A\,x^2-1794B\,x^2+1420C\,x^2+594A-480B+380C\right)}{9\left(3x^4+5x^2+2\right)^{\frac{3}{2}}}+\cdots$
default	$A\left(\frac{\left(\frac{5}{27}x^3+\frac{4}{27}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\frac{20}{3}x^3+\frac{101}{18}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{33i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{40i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\operatorname{EllipticE}\cdots\right.}{3\sqrt{3x^4\cdots}}\right.$

input
```
int(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((5/27*A-13/81*B+35/243*C)*x^3+(4/27*A-10/81*B+26/243*C)*x)*(3*x^4+5*x^2+2
)^(1/2)/(x^4+5/3*x^2+2/3)^2-6*((20/3*A-97/18*B+115/27*C)*x^3+(101/18*A-245
/54*B+583/162*C)*x)/(3*x^4+5*x^2+2)^(1/2)-1/2*I*(190/9*C+33*A-80/3*B)*(x^2
+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))
+1/3*I*(40*A-97/3*B+230/9*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)
^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.25

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = -\frac{2\sqrt{2}(-9i\,(360\,A-291\,B+230\,C)x^8-30i\,(360\,A-291\,B+230\,C)x^6-37i\,(360\,A-291\,B+230\,C)x$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/54*(2*sqrt(2)*(-9*I*(360*A - 291*B + 230*C)*x^8 - 30*I*(360*A - 291*B +
 230*C)*x^6 - 37*I*(360*A - 291*B + 230*C)*x^4 - 20*I*(360*A - 291*B + 230
*C)*x^2 - 1440*I*A + 1164*I*B - 920*I*C)*elliptic_e(arcsin(I*x), 3/2) - sq
rt(2)*(-9*I*(1611*A - 1302*B + 1030*C)*x^8 - 30*I*(1611*A - 1302*B + 1030*
C)*x^6 - 37*I*(1611*A - 1302*B + 1030*C)*x^4 - 20*I*(1611*A - 1302*B + 103
0*C)*x^2 - 6444*I*A + 5208*I*B - 4120*I*C)*elliptic_f(arcsin(I*x), 3/2) +
6*(3*(360*A - 291*B + 230*C)*x^7 + (2709*A - 2190*B + 1733*C)*x^5 + 2*(111
0*A - 897*B + 710*C)*x^3 + 2*(297*A - 240*B + 190*C)*x)*sqrt(3*x^4 + 5*x^2
 + 2))/(9*x^8 + 30*x^6 + 37*x^4 + 20*x^2 + 4)
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4(A+Bx^2+Cx^4)}{((x^2+1)\,(3x^2+2))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral(x**4*(A + B*x**2 + C*x**4)/((x**2 + 1)*(3*x**2 + 2))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4\,(C\,x^4+B\,x^2+A)}{(3\,x^4+5\,x^2+2)^{5/2}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(5/2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x)
```

output
```
( - 15*sqrt(3*x**4 + 5*x**2 + 2)*b*x**3 - 9*sqrt(3*x**4 + 5*x**2 + 2)*b*x
- 45*sqrt(3*x**4 + 5*x**2 + 2)*c*x**5 - 50*sqrt(3*x**4 + 5*x**2 + 2)*c*x**
3 - 30*sqrt(3*x**4 + 5*x**2 + 2)*c*x + 162*int(sqrt(3*x**4 + 5*x**2 + 2)/(
27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*
x**8 + 540*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8
+ 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**6 + 666*int(sqrt(3*x**4 + 5*x
**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2
+ 8),x)*b*x**4 + 360*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 +
 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**2 + 72*int(sqrt(3*x
**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 +
 60*x**2 + 8),x)*b + 540*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**
10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*c*x**8 + 1800*int(sq
rt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*
x**4 + 60*x**2 + 8),x)*c*x**6 + 2220*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**
12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*c*x**4 +
 1200*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305
*x**6 + 186*x**4 + 60*x**2 + 8),x)*c*x**2 + 240*int(sqrt(3*x**4 + 5*x**2 +
 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),
x)*c + 1215*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 2
79*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**8 + 4050*int((sqrt...
```

3.182 $\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 207

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = -\frac{x(15A-12B+10C+(18A-15B+13C)x^2)}{9(2+5x^2+3x^4)^{3/2}}+\frac{(51A-42B+34C)x}{6\sqrt{2+5x^2+3x^4}}+\frac{(291A-240B+194C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,-\frac{1}{2}\right)}{9\sqrt{2}\sqrt{2+5x^2+3x^4}}-\frac{(120A-99B+80C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
-1/9*x*(15*A-12*B+10*C+(18*A-15*B+13*C)*x^2)/(3*x^4+5*x^2+2)^(3/2)+1/6*(51
*A-42*B+34*C)*x/(3*x^4+5*x^2+2)^(1/2)+1/18*(291*A-240*B+194*C)*(x^2+1)*((3
*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)/(3
*x^4+5*x^2+2)^(1/2)-1/6*(120*A-99*B+80*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2
)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.40 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{3x(2C(160+598x^2+730x^4+291x^6)-2B(198+740x^2+903x^4+360x^6)\cdots}{\cdots}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(3*x*(2*C*(160 + 598*x^2 + 730*x^4 + 291*x^6) - 2*B*(198 + 740*x^2 + 903*x
^4 + 360*x^6) + A*(480 + 1795*x^2 + 2190*x^4 + 873*x^6)) + I*Sqrt[3]*(291*
A - 240*B + 194*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*Ellip
ticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - I*Sqrt[3]*(51*A - 42*B + 34*C)*Sqrt[1
+ x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticF[I*ArcSinh[Sqrt[3/2]*x
], 2/3])/(18*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2197, 27, 1492, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(3x^4+5x^2+2)^{5/2}}\,dx$$

$$\Big\downarrow \; 2197$$

$$-\frac{1}{6}\int -\frac{2(-9(6A-5B+4C)x^2+15A-12B+10C)}{3(3x^4+5x^2+2)^{3/2}}dx - \frac{x(x^2(18A-15B+13C)+15A-12B+10C)}{9(3x^4+5x^2+2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\int \frac{-9(6A-5B+4C)x^2+15A-12B+10C}{(3x^4+5x^2+2)^{3/2}}dx-\frac{x\left(x^2(18A-15B+13C)+15A-12B+10C\right)}{9\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \ 1492$$

$$\frac{1}{9}\left(\frac{x\left(3x^2(291A-240B+194C)+735A-606B+490C\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int \frac{3\left((291A-240B+194C)x^2+2(120A-99B\right.}{\sqrt{3x^4+5x^2+2}}\right.$$
$$\frac{x\left(x^2(18A-15B+13C)+15A-12B+10C\right)}{9\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{9}\left(\frac{x\left(3x^2(291A-240B+194C)+735A-606B+490C\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\int \frac{(291A-240B+194C)x^2+2(120A-99B}{\sqrt{3x^4+5x^2+2}}\right.$$
$$\frac{x\left(x^2(18A-15B+13C)+15A-12B+10C\right)}{9\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \ 1503$$

$$\frac{1}{9}\left(\frac{x\left(3x^2(291A-240B+194C)+735A-606B+490C\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(2(120A-99B+80C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+\right.\right.$$
$$\frac{x\left(x^2(18A-15B+13C)+15A-12B+10C\right)}{9\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \ 1413$$

$$\frac{1}{9}\left(\frac{x\left(3x^2(291A-240B+194C)+735A-606B+490C\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left((291A-240B+194C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right.\right.$$
$$\frac{x\left(x^2(18A-15B+13C)+15A-12B+10C\right)}{9\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \ 1456$$

$$\frac{1}{9}\left(\frac{x\left(3x^2(291A-240B+194C)+735A-606B+490C\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}(120A-99B+80C)\,\text{Elli}}{\sqrt{3x^4+5x^2+2}}\right.\right.$$
$$\frac{x\left(x^2(18A-15B+13C)+15A-12B+10C\right)}{9\left(3x^4+5x^2+2\right)^{3/2}}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
-1/9*(x*(15*A - 12*B + 10*C + (18*A - 15*B + 13*C)*x^2))/(2 + 5*x^2 + 3*x^
4)^(3/2) + ((x*(735*A - 606*B + 490*C + 3*(291*A - 240*B + 194*C)*x^2))/(2
*Sqrt[2 + 5*x^2 + 3*x^4]) - (3*((291*A - 240*B + 194*C)*((x*(2 + 3*x^2))/(
3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (Sqrt[2]*(120
*A - 99*B + 80*C)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x
], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4]))/2)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2197
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x
^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Qx + b^2*d*(2*p + 3) - 2*
a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; Fre
eQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && IGtQ[m/2, 0]
```

Maple [A] (verified)

Time = 6.76 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.05

method	result
elliptic	$\frac{\left(\left(-\frac{2A}{9}+\frac{5B}{27}-\frac{13C}{81}\right)x^3+\left(-\frac{5A}{27}+\frac{4B}{27}-\frac{10C}{81}\right)x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\left(-\frac{97A}{12}+\frac{20B}{3}-\frac{97C}{18}\right)x^3+\left(-\frac{245A}{36}+\frac{101B}{18}-\frac{245C}{54}\right)x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{i\left(-4\right.}{}$
risch	$\frac{x\left(873A\,x^6-720B\,x^6+582C\,x^6+2190A\,x^4-1806B\,x^4+1460C\,x^4+1795A\,x^2-1480B\,x^2+1196C\,x^2+480A-396B+320C\right)}{6\left(3x^4+5x^2+2\right)^{\frac{3}{2}}}-\frac{i(29}{}$
default	$A\left(\frac{\left(-\frac{2}{9}x^3-\frac{5}{27}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(-\frac{97}{12}x^3-\frac{245}{36}x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{20i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4+5x^2+2}}-\frac{97i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{Ellipt}\right.}{6\sqrt{3}}\right.$

input
```
int(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((-2/9*A+5/27*B-13/81*C)*x^3+(-5/27*A+4/27*B-10/81*C)*x)*(3*x^4+5*x^2+2)^(
1/2)/(x^4+5/3*x^2+2/3)^2-6*((-97/12*A+20/3*B-97/18*C)*x^3+(-245/36*A+101/1
8*B-245/54*C)*x)/(3*x^4+5*x^2+2)^(1/2)-1/2*I*(-40*A+33*B-80/3*C)*(x^2+1)^(
1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3*
I*(-97/2*A+40*B-97/3*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2
)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.26

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{\sqrt{2}(-9i\,(291\,A-240\,B+194\,C)x^8-30i\,(291\,A-240\,B+194\,C)x^6-37i\,(}{}$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/18*(sqrt(2)*(-9*I*(291*A - 240*B + 194*C)*x^8 - 30*I*(291*A - 240*B + 19
4*C)*x^6 - 37*I*(291*A - 240*B + 194*C)*x^4 - 20*I*(291*A - 240*B + 194*C)
*x^2 - 1164*I*A + 960*I*B - 776*I*C)*elliptic_e(arcsin(I*x), 3/2) + sqrt(2
)*(9*I*(651*A - 537*B + 434*C)*x^8 + 30*I*(651*A - 537*B + 434*C)*x^6 + 37
*I*(651*A - 537*B + 434*C)*x^4 + 20*I*(651*A - 537*B + 434*C)*x^2 + 2604*I
*A - 2148*I*B + 1736*I*C)*elliptic_f(arcsin(I*x), 3/2) + 3*(3*(291*A - 240
*B + 194*C)*x^7 + 2*(1095*A - 903*B + 730*C)*x^5 + (1795*A - 1480*B + 1196
*C)*x^3 + 4*(120*A - 99*B + 80*C)*x)*sqrt(3*x^4 + 5*x^2 + 2))/(9*x^8 + 30*
x^6 + 37*x^4 + 20*x^2 + 4)
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4)}{((x^2+1)(3x^2+2))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4)/((x**2 + 1)*(3*x**2 + 2))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx=\int \frac{x^2\,(C\,x^4+B\,x^2+A)}{(3\,x^4+5\,x^2+2)^{5/2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(5/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/(5*x^2 + 3*x^4 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(2+5x^2+3x^4)^{5/2}}\,dx=\text{too large to display}$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x)
```

output
```
( - 9*sqrt(3*x**4 + 5*x**2 + 2)*a*x - 10*sqrt(3*x**4 + 5*x**2 + 2)*c*x**3
- 6*sqrt(3*x**4 + 5*x**2 + 2)*c*x + 162*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*
x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**
8 + 540*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 3
05*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**6 + 666*int(sqrt(3*x**4 + 5*x**2
 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8
),x)*a*x**4 + 360*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 27
9*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**2 + 72*int(sqrt(3*x**4
 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60
*x**2 + 8),x)*a + 108*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10
+ 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*c*x**8 + 360*int(sqrt(3
*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4
 + 60*x**2 + 8),x)*c*x**6 + 444*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 +
135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*c*x**4 + 240*
int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6
+ 186*x**4 + 60*x**2 + 8),x)*c*x**2 + 48*int(sqrt(3*x**4 + 5*x**2 + 2)/(27
*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*c -
1215*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x**8
 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**8 - 4050*int((sqrt(3*x**4 +
5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**...
```

3.183 $\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 207

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{x(13A-10B+8C+(15A-12B+10C)x^2)}{6\,(2+5x^2+3x^4)^{3/2}} - \frac{(20A-17B+14C)x}{2\sqrt{2+5x^2+3x^4}} - \frac{(115A-97B+80C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+5x^2+3x^4}} + \frac{(95A-80B+66C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output

```
1/6*x*(13*A-10*B+8*C+(15*A-12*B+10*C)*x^2)/(3*x^4+5*x^2+2)^(3/2)-1/2*(20*A
-17*B+14*C)*x/(3*x^4+5*x^2+2)^(1/2)-1/6*(115*A-97*B+80*C)*(x^2+1)*((3*x^2+
2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+
5*x^2+2)^(1/2)+1/4*(95*A-80*B+66*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*Inve
rseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.40 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{-2Cx(198+740x^2+903x^4+360x^6)+Bx(480+1795x^2+2190x^4+873x^6)}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(-2*C*x*(198 + 740*x^2 + 903*x^4 + 360*x^6) + B*x*(480 + 1795*x^2 + 2190*x
^4 + 873*x^6) - A*x*(567 + 2125*x^2 + 2595*x^4 + 1035*x^6) - I*Sqrt[3]*(11
5*A - 97*B + 80*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*Ellip
ticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + I*Sqrt[3]*(20*A - 17*B + 14*C)*Sqrt[1
+ x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticF[I*ArcSinh[Sqrt[3/2]*x
], 2/3])/(6*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2206, 1492, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(3x^4+5x^2+2)^{5/2}}dx$$

$$\Big\downarrow \text{ 2206}$$

$$\frac{x\left(x^2(15A-12B+10C)+13A-10B+8C\right)}{6\left(3x^4+5x^2+2\right)^{3/2}}-\frac{1}{6}\int \frac{2(5A-5B+4C)-3(15A-12B+10C)x^2}{(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1492}$$

$$\frac{1}{6}\left(\frac{1}{2}\int \frac{6\left((115A-97B+80C)x^2+95A-80B+66C\right)}{\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(3x^2(115A-97B+80C)+290A-245B+20\right.}{\sqrt{3x^4+5x^2+2}}\right.$$
$$\frac{x\left(x^2(15A-12B+10C)+13A-10B+8C\right)}{6\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{6}\left(3\int \frac{(115A-97B+80C)x^2+95A-80B+66C}{\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(3x^2(115A-97B+80C)+290A-245B+202C\right)}{\sqrt{3x^4+5x^2+2}}\right.$$
$$\frac{x\left(x^2(15A-12B+10C)+13A-10B+8C\right)}{6\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1503

$$\frac{1}{6}\left(3\left((95A-80B+66C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+(115A-97B+80C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{x\left(3x^2(115A\right.}{}\right.$$
$$\frac{x\left(x^2(15A-12B+10C)+13A-10B+8C\right)}{6\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1413

$$\frac{1}{6}\left(3\left((115A-97B+80C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}(95A-80B+66C)\,\text{EllipticF}\left(\arctan(x),\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.\right.$$
$$\frac{x\left(x^2(15A-12B+10C)+13A-10B+8C\right)}{6\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1456

$$\frac{1}{6}\left(3\left(\frac{\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}(95A-80B+66C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+(115A-97B+80C)\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+}}\right.\right.\right.$$
$$\frac{x\left(x^2(15A-12B+10C)+13A-10B+8C\right)}{6\left(3x^4+5x^2+2\right)^{3/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(x*(13*A - 10*B + 8*C + (15*A - 12*B + 10*C)*x^2))/(6*(2 + 5*x^2 + 3*x^4)^
(3/2)) + (-((x*(290*A - 245*B + 202*C + 3*(115*A - 97*B + 80*C)*x^2))/Sqrt
[2 + 5*x^2 + 3*x^4]) + 3*((115*A - 97*B + 80*C)*((x*(2 + 3*x^2))/(3*Sqrt[2
 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Ellipt
icE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + ((95*A - 80*B + 66*C)
*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2
]*Sqrt[2 + 5*x^2 + 3*x^4])))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.05

method	result
elliptic	$\frac{\left(\left(-\frac{2B}{9}+\frac{5C}{27}+\frac{5A}{18}\right)x^3+\left(-\frac{5B}{27}+\frac{4C}{27}+\frac{13A}{54}\right)x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\left(\frac{115A}{12}-\frac{97B}{12}+\frac{20C}{3}\right)x^3+\left(\frac{145A}{18}-\frac{245B}{36}+\frac{101C}{18}\right)x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{i\left(\frac{95A}{2}-\right.}{}$
risch	$-\frac{x\left(1035Ax^6-873Bx^6+720Cx^6+2595Ax^4-2190Bx^4+1806Cx^4+2125Ax^2-1795Bx^2+1480Cx^2+567A-480B+396C\right)}{6\left(3x^4+5x^2+2\right)^{\frac{3}{2}}}+\frac{i}{}$
default	$A\left(\frac{\left(\frac{5}{18}x^3+\frac{13}{54}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\frac{115}{12}x^3+\frac{145}{18}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{95i\sqrt{x^2+1}\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{4\sqrt{3x^4+5x^2+2}}+\frac{115i\sqrt{x^2+1}\sqrt{6x^2+4}\left(\mathrm{Ellipti}\right.}{6\sqrt{3x}}\right.$

input
```
int((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((-2/9*B+5/27*C+5/18*A)*x^3+(-5/27*B+4/27*C+13/54*A)*x)*(3*x^4+5*x^2+2)^(1
/2)/(x^4+5/3*x^2+2/3)^2-6*((115/12*A-97/12*B+20/3*C)*x^3+(145/18*A-245/36*
B+101/18*C)*x)/(3*x^4+5*x^2+2)^(1/2)-1/2*I*(95/2*A-40*B+33*C)*(x^2+1)^(1/2
)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3*I*(
115/2*A-97/2*B+40*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(
EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{5/2}}\,dx = -\frac{2\sqrt{2}(-9i\,(115\,A-97\,B+80\,C)x^8-30i\,(115\,A-97\,B+80\,C)x^6-37i\,(115\,A-97\,B+80\,C)x^4-20i}{}$$

input
```
integrate((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(2)*(-9*I*(115*A - 97*B + 80*C)*x^8 - 30*I*(115*A - 97*B + 80
*C)*x^6 - 37*I*(115*A - 97*B + 80*C)*x^4 - 20*I*(115*A - 97*B + 80*C)*x^2
- 460*I*A + 388*I*B - 320*I*C)*elliptic_e(arcsin(I*x), 3/2) - sqrt(2)*(-9*
I*(515*A - 434*B + 358*C)*x^8 - 30*I*(515*A - 434*B + 358*C)*x^6 - 37*I*(5
15*A - 434*B + 358*C)*x^4 - 20*I*(515*A - 434*B + 358*C)*x^2 - 2060*I*A +
1736*I*B - 1432*I*C)*elliptic_f(arcsin(I*x), 3/2) + 2*(9*(115*A - 97*B + 8
0*C)*x^7 + 3*(865*A - 730*B + 602*C)*x^5 + 5*(425*A - 359*B + 296*C)*x^3 +
 3*(189*A - 160*B + 132*C)*x)*sqrt(3*x^4 + 5*x^2 + 2))/(9*x^8 + 30*x^6 + 3
7*x^4 + 20*x^2 + 4)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{A+Bx^2+Cx^4}{((x^2+1)\,(3x^2+2))^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/((x**2 + 1)*(3*x**2 + 2))**(5/2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(3\,x^4+5\,x^2+2)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(5*x^2 + 3*x^4 + 2)^(5/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(5*x^2 + 3*x^4 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{(2 + 5x^2 + 3x^4)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/(3*x^4+5*x^2+2)^(5/2),x)
```

output
```
( - sqrt(3*x**4 + 5*x**2 + 2)*b*x + 90*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x
**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**8
 + 300*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 30
5*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**6 + 370*int(sqrt(3*x**4 + 5*x**2
+ 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8)
,x)*a*x**4 + 200*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279
*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**2 + 40*int(sqrt(3*x**4
+ 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*
x**2 + 8),x)*a + 18*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 +
279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**8 + 60*int(sqrt(3*x*
*4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 +
60*x**2 + 8),x)*b*x**6 + 74*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*
x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**4 + 40*int(s
qrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186
*x**4 + 60*x**2 + 8),x)*b*x**2 + 8*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12
 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b - 135*in
t((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x**8 + 305*
x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**8 - 450*int((sqrt(3*x**4 + 5*x**2 +
 2)*x**4)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2
 + 8),x)*b*x**6 - 555*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*x**12 + ...
```

3.184 $\int \frac{A+Bx^2+Cx^4}{x^2\left(2+5x^2+3x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 288

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{x^2\left(2+5x^2+3x^4\right)^{5/2}}\,dx &= -\frac{x(35A-26B+20C+3(13A-10B+8C)x^2)}{12\left(2+5x^2+3x^4\right)^{3/2}}\\ &-\frac{(263A-230B+194C)x(2+3x^2)}{12\sqrt{2+5x^2+3x^4}}\\ &+\frac{x(5(265A-232B+196C)+3(529A-460B+388C)x^2)}{24\sqrt{2+5x^2+3x^4}}\\ &-\frac{A\sqrt{2+5x^2+3x^4}}{8x}\\ &+\frac{(263A-230B+194C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{6\sqrt{2}\sqrt{2+5x^2+3x^4}}\\ &-\frac{5(22A-19B+16C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
-1/12*x*(35*A-26*B+20*C+3*(13*A-10*B+8*C)*x^2)/(3*x^4+5*x^2+2)^(3/2)-1/12*
(263*A-230*B+194*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/24*x*(1325*A-1160*
B+980*C+3*(529*A-460*B+388*C)*x^2)/(3*x^4+5*x^2+2)^(1/2)-1/8*A*(3*x^4+5*x^
2+2)^(1/2)/x+1/12*(263*A-230*B+194*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*El
lipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)-5/4*(
22*A-19*B+16*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x
),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.40 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2+Cx^4}{x^2(2+5x^2+3x^4)^{5/2}}\,dx = \frac{-6A+1260Ax^2-1134Bx^2+960Cx^2+4805Ax^4-4250Bx^4+3590Cx^4}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*(2 + 5*x^2 + 3*x^4)^(5/2)),x]
```

output
```
(-6*A + 1260*A*x^2 - 1134*B*x^2 + 960*C*x^2 + 4805*A*x^4 - 4250*B*x^4 + 35
90*C*x^4 + 5910*A*x^6 - 5190*B*x^6 + 4380*C*x^6 + 2367*A*x^8 - 2070*B*x^8
+ 1746*C*x^8 + I*Sqrt[3]*(263*A - 230*B + 194*C)*x*Sqrt[1 + x^2]*Sqrt[2 +
3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - I*Sqrt
[3]*(43*A - 40*B + 34*C)*x*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^
4)*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(12*x*(2 + 5*x^2 + 3*x^4)^(3/2)
)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2198, 27, 2198, 27, 2199, 1604, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(3x^4+5x^2+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2198}$$

$$-\frac{1}{6}\int -\frac{-9(13A-10B+8C)x^4+20(A-B+C)x^2+6A}{2x^2\left(3x^4+5x^2+2\right)^{3/2}}dx-\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{12\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\int \frac{-9(13A-10B+8C)x^4+20(A-B+C)x^2+6A}{x^2\left(3x^4+5x^2+2\right)^{3/2}}dx-\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{12\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{12}\left(\frac{x\left(3x^2(529A-460B+388C)+5(265A-232B+196C)\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int -\frac{3\left(-\left((529A-460B+388C)x^4\right)-20(\right.}{x^2\sqrt{3x^4+5x^2}}\right.$$
$$\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{12\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\left(\frac{3}{2}\int \frac{-\left((529A-460B+388C)x^4\right)-20(22A-19B+16C)x^2+2A}{x^2\sqrt{3x^4+5x^2+2}}dx+\frac{x\left(3x^2(529A-460B+388C)+\right.}{2\sqrt{3x^4+5x}}\right.$$
$$\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{12\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{2199}$$

$$\frac{1}{12}\left(\frac{3}{2}\left(\int \frac{-20(22A-19B+16C)x^2-\frac{4}{3}(263A-230B+194C)}{x^2\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(529A-460B+388C)}{3x}\right.\right.$$
$$\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{12\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{12}\left(\frac{3}{2}\left(-\frac{1}{2}\int \frac{4\left((263A-230B+194C)x^2+10(22A-19B+16C)\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{2\sqrt{3x^4+5x^2+2}(263A-230B+19}{3x}\right.\right.$$
$$\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{12\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\left(\frac{3}{2}\left(-2\int \frac{(263A-230B+194C)x^2+10(22A-19B+16C)}{\sqrt{3x^4+5x^2+2}}dx+\frac{2\sqrt{3x^4+5x^2+2}(263A-230B+194C}{3x}\right.\right.$$
$$\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{12\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{12}\left(\frac{3}{2}\left(-2\left(10(22A-19B+16C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+(263A-230B+194C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{2}{}\right.\right.$$
$$\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{12\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{12}\left(\frac{3}{2}\left(-2\left((263A-230B+194C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{5\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(22A-19B+16C)\,\text{Elliptic}}{\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$
$$\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{12\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{12}\left(\frac{3}{2}\left(-2\left(\frac{5\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(22A-19B+16C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+(263A-230B+194C)\left(\frac{}{3}\right.\right.\right.\right.$$
$$\frac{x\left(3x^2(13A-10B+8C)+35A-26B+20C\right)}{12\left(3x^4+5x^2+2\right)^{3/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*(2 + 5*x^2 + 3*x^4)^(5/2)),x]
```

output
```
-1/12*(x*(35*A - 26*B + 20*C + 3*(13*A - 10*B + 8*C)*x^2))/(2 + 5*x^2 + 3*
x^4)^(3/2) + ((x*(5*(265*A - 232*B + 196*C) + 3*(529*A - 460*B + 388*C)*x^
2))/(2*Sqrt[2 + 5*x^2 + 3*x^4]) + (3*((2*(263*A - 230*B + 194*C)*Sqrt[2 +
5*x^2 + 3*x^4])/(3*x) - ((529*A - 460*B + 388*C)*Sqrt[2 + 5*x^2 + 3*x^4])/
(3*x) - 2*((263*A - 230*B + 194*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*
x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x]
, -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (5*Sqrt[2]*(22*A - 19*B + 16*C)*(1
 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5
*x^2 + 3*x^4])))/2)/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.83

method	result
elliptic	$\frac{\left(\left(-\frac{13A}{36}-\frac{2C}{9}+\frac{5B}{18}\right)x^3+\left(-\frac{35A}{108}-\frac{5C}{27}+\frac{13B}{54}\right)x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\left(-\frac{529A}{48}+\frac{115B}{12}-\frac{97C}{12}\right)x^3+\left(-\frac{1325A}{144}+\frac{145B}{18}-\frac{245C}{36}\right)x\right)}{\sqrt{3x^4+5x^2+2}}-$
risch	$\frac{2367A\,x^8-2070B\,x^8+1746C\,x^8+5910A\,x^6-5190B\,x^6+4380C\,x^6+4805A\,x^4-4250B\,x^4+3590C\,x^4+1260A\,x^2-1134B\,x^2+960C}{12x(3x^4+5x^2+2)^{\frac{3}{2}}}$
default	$B\left(\frac{\left(\frac{5}{18}x^3+\frac{13}{54}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\frac{115}{12}x^3+\frac{145}{18}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{95i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{4\sqrt{3x^4+5x^2+2}}+\frac{115i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{Ellipt}\right.}{6\sqrt{3x}}\right.$

input
```
int((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((-13/36*A-2/9*C+5/18*B)*x^3+(-35/108*A-5/27*C+13/54*B)*x)*(3*x^4+5*x^2+2)
^(1/2)/(x^4+5/3*x^2+2/3)^2-6*((-529/48*A+115/12*B-97/12*C)*x^3+(-1325/144*
A+145/18*B-245/36*C)*x)/(3*x^4+5*x^2+2)^(1/2)-1/8*A*(3*x^4+5*x^2+2)^(1/2)/
x-1/2*I*(-55*A+95/2*B-40*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^
(1/2)*EllipticF(I*x,1/2*6^(1/2))+1/3*I*(-263/4*A+115/2*B-97/2*C)*(x^2+1)^(
1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-Ell
ipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(2+5x^2+3x^4\right)^{5/2}}\,dx=\frac{\sqrt{2}(-9i\,(263\,A-230\,B+194\,C)x^9-30i\,(263\,A-230\,B+194\,C)x^7-37i}{}$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(2)*(-9*I*(263*A - 230*B + 194*C)*x^9 - 30*I*(263*A - 230*B + 19
4*C)*x^7 - 37*I*(263*A - 230*B + 194*C)*x^5 - 20*I*(263*A - 230*B + 194*C)
*x^3 - 4*I*(263*A - 230*B + 194*C)*x)*elliptic_e(arcsin(I*x), 3/2) + sqrt(
2)*(9*I*(593*A - 515*B + 434*C)*x^9 + 30*I*(593*A - 515*B + 434*C)*x^7 + 3
7*I*(593*A - 515*B + 434*C)*x^5 + 20*I*(593*A - 515*B + 434*C)*x^3 + 4*I*(
593*A - 515*B + 434*C)*x)*elliptic_f(arcsin(I*x), 3/2) + (9*(263*A - 230*B
 + 194*C)*x^8 + 30*(197*A - 173*B + 146*C)*x^6 + 5*(961*A - 850*B + 718*C)
*x^4 + 6*(210*A - 189*B + 160*C)*x^2 - 6*A)*sqrt(3*x^4 + 5*x^2 + 2))/(9*x^
9 + 30*x^7 + 37*x^5 + 20*x^3 + 4*x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2(2+5x^2+3x^4)^{5/2}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^2\left((x^2+1)\,(3x^2+2)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**2*((x**2 + 1)*(3*x**2 + 2))**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2(2+5x^2+3x^4)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(2+5x^2+3x^4\right)^{5/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{\left(3\,x^4+5\,x^2+2\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(2+5x^2+3x^4\right)^{5/2}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{x^2\,\left(3\,x^4+5\,x^2+2\right)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(5*x^2 + 3*x^4 + 2)^(5/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(5*x^2 + 3*x^4 + 2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(2+5x^2+3x^4\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/x^2/(3*x^4+5*x^2+2)^(5/2),x)
```

output
```
( - 21*sqrt(3*x**4 + 5*x**2 + 2)*a*x**4 - 6*sqrt(3*x**4 + 5*x**2 + 2)*a +
2*sqrt(3*x**4 + 5*x**2 + 2)*c*x**4 - 1080*int(sqrt(3*x**4 + 5*x**2 + 2)/(2
7*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x
**9 - 3600*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8
+ 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**7 - 4440*int(sqrt(3*x**4 + 5*
x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2
 + 8),x)*a*x**5 - 2400*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10
 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**3 - 480*int(sqrt(
3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**
4 + 60*x**2 + 8),x)*a*x + 108*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 13
5*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**9 + 360*in
t(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 +
186*x**4 + 60*x**2 + 8),x)*b*x**7 + 444*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*
x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**
5 + 240*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 3
05*x**6 + 186*x**4 + 60*x**2 + 8),x)*b*x**3 + 48*int(sqrt(3*x**4 + 5*x**2
+ 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8)
,x)*b*x - 1701*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(27*x**12 + 135*x**10
+ 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*a*x**9 - 5670*int((sqrt
(3*x**4 + 5*x**2 + 2)*x**6)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6...
```

3.185 $\int \frac{A+Bx^2+Cx^4}{x^4(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 315

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(2+5x^2+3x^4\right)^{5/2}}\,dx=\frac{x(97A-70B+52C+3(35A-26B+20C)x^2)}{24\left(2+5x^2+3x^4\right)^{3/2}}+\frac{(280A-263B+230C)x(2+3x^2)}{12\sqrt{2+5x^2+3x^4}}-\frac{x(1427A-1325B+1160C+3(580A-529B+460C)x^2)}{24\sqrt{2+5x^2+3x^4}}-\frac{A\sqrt{2+5x^2+3x^4}}{24x^3}+\frac{(20A-3B)\sqrt{2+5x^2+3x^4}}{24x}-\frac{(280A-263B+230C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{6\sqrt{2}\sqrt{2+5x^2+3x^4}}+\frac{5(49A-44B+38C)\left(1+x^2\right)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{4\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
1/24*x*(97*A-70*B+52*C+3*(35*A-26*B+20*C)*x^2)/(3*x^4+5*x^2+2)^(3/2)+1/12*
(280*A-263*B+230*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-1/24*x*(1427*A-1325*
B+1160*C+3*(580*A-529*B+460*C)*x^2)/(3*x^4+5*x^2+2)^(1/2)-1/24*A*(3*x^4+5*
x^2+2)^(1/2)/x^3+1/24*(20*A-3*B)*(3*x^4+5*x^2+2)^(1/2)/x-1/12*(280*A-263*B
+230*C)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*
2^(1/2))*2^(1/2)/(3*x^4+5*x^2+2)^(1/2)+5/8*(49*A-44*B+38*C)*(x^2+1)*((3*x^
2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^
4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.41 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(2+5x^2+3x^4\right)^{5/2}}\,dx = \frac{-2A+30Ax^2-6Bx^2-1197Ax^4+1260Bx^4-1134Cx^4-4900Ax^6+480}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*(2 + 5*x^2 + 3*x^4)^(5/2)),x]
```

output
```
(-2*A + 30*A*x^2 - 6*B*x^2 - 1197*A*x^4 + 1260*B*x^4 - 1134*C*x^4 - 4900*A
*x^6 + 4805*B*x^6 - 4250*C*x^6 - 6195*A*x^8 + 5910*B*x^8 - 5190*C*x^8 - 25
20*A*x^10 + 2367*B*x^10 - 2070*C*x^10 - I*Sqrt[3]*(280*A - 263*B + 230*C)*
x^3*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticE[I*ArcSinh[
Sqrt[3/2]*x], 2/3] + I*Sqrt[3]*(35*A - 43*B + 40*C)*x^3*Sqrt[1 + x^2]*Sqrt
[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(1
2*x^3*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2198, 27, 2198, 27, 2199, 2199, 1604, 27, 1604, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(3x^4+5x^2+2\right)^{5/2}}dx$$

$\Big\downarrow$ 2198

$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}-\frac{1}{6}\int -\frac{9(35A-26B+20C)x^6-40(A-B+C)x^4-6(5A-2B)x^2+12A}{4x^4\left(3x^4+5x^2+2\right)^{3/2}}dx$$

$\Big\downarrow$ 27

$$\frac{1}{24}\int \frac{9(35A-26B+20C)x^6-40(A-B+C)x^4-6(5A-2B)x^2+12A}{x^4\left(3x^4+5x^2+2\right)^{3/2}}dx+\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 2198

$$\frac{1}{24}\left(-\frac{1}{2}\int -\frac{6\left((580A-529B+460C)x^6+(491A-440B+380C)x^4-2(5A-B)x^2+2A\right)}{x^4\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(3x^2(580A\right.}{}\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{1}{24}\left(3\int \frac{(580A-529B+460C)x^6+(491A-440B+380C)x^4-2(5A-B)x^2+2A}{x^4\sqrt{3x^4+5x^2+2}}dx-\frac{x\left(3x^2(580A-529B\right.}{}\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 2199

$$\frac{1}{24}\left(3\left(\int \frac{(491A-440B+380C)x^4+\frac{2}{3}(565A-526B+460C)x^2+2A}{x^4\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(580A-529B+}{3x}\right.\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 2199

$$\frac{1}{24}\left(3\left(\int \frac{-12(105A-93B+80C)x^2-20(49A-44B+38C)}{x^4\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(580A-529B+460C)}{3x}-\right.\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1604

$$\frac{1}{24}\left(3\left(-\frac{1}{6}\int -\frac{4\left(15(49A-44B+38C)x^2+2(280A-263B+230C)\right)}{x^2\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(580A-529B+}{3x}\right.\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{24}\left(3\left(\frac{2}{3}\int \frac{15(49A-44B+38C)x^2+2(280A-263B+230C)}{x^2\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(580A-529B+460C)}{3x}\right.\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1604

$$\frac{1}{24}\left(3\left(\frac{2}{3}\left(-\frac{1}{2}\int -\frac{6\left((280A-263B+230C)x^2+5(49A-44B+38C)\right)}{\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(280A-263B+}{x}\right.\right.\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{24}\left(3\left(\frac{2}{3}\left(3\int \frac{(280A-263B+230C)x^2+5(49A-44B+38C)}{\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(280A-263B+230C)}{x}\right.\right.\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1503

$$\frac{1}{24}\left(3\left(\frac{2}{3}\left(3\left(5(49A-44B+38C)\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+(280A-263B+230C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\right.\right.\right.$$
$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{24}\left(3\left(\frac{2}{3}\left(3\left((280A-263B+230C)\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{5(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(49A-44B+38C)\,\text{EllipticF}}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.\right.\right.\right.$$

$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{24}\left(3\left(\frac{2}{3}\left(3\left(\frac{5(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(49A-44B+38C)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+(280A-263B+230C)\left(\frac{}{3\sqrt{}}\right.\right.\right.\right.\right.$$

$$\frac{x\left(3x^2(35A-26B+20C)+97A-70B+52C\right)}{24\left(3x^4+5x^2+2\right)^{3/2}}$$

input

```
Int[(A + B*x^2 + C*x^4)/(x^4*(2 + 5*x^2 + 3*x^4)^(5/2)),x]
```

output

```
(x*(97*A - 70*B + 52*C + 3*(35*A - 26*B + 20*C)*x^2))/(24*(2 + 5*x^2 + 3*x
^4)^(3/2)) + (-((x*(1427*A - 1325*B + 1160*C + 3*(580*A - 529*B + 460*C)*x
^2))/Sqrt[2 + 5*x^2 + 3*x^4]) + 3*((10*(49*A - 44*B + 38*C)*Sqrt[2 + 5*x^2
 + 3*x^4])/(3*x^3) - ((491*A - 440*B + 380*C)*Sqrt[2 + 5*x^2 + 3*x^4])/(3*
x^3) + ((580*A - 529*B + 460*C)*Sqrt[2 + 5*x^2 + 3*x^4])/(3*x) + (2*(-(((2
80*A - 263*B + 230*C)*Sqrt[2 + 5*x^2 + 3*x^4])/x) + 3*((280*A - 263*B + 23
0*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqr
t[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3
*x^4])) + (5*(49*A - 44*B + 38*C)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*El
lipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]))))/3))/24
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.84

method	result
elliptic	$\frac{\left(\left(\frac{35A}{72}-\frac{13B}{36}+\frac{5C}{18}\right)x^3+\left(\frac{97A}{216}-\frac{35B}{108}+\frac{13C}{54}\right)x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\left(\frac{145A}{12}-\frac{529B}{48}+\frac{115C}{12}\right)x^3+\left(\frac{1427A}{144}-\frac{1325B}{144}+\frac{145C}{18}\right)x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{A\sqrt{}}{}$
risch	$-\frac{2520A\,x^{10}-2367B\,x^{10}+2070C\,x^{10}+6195A\,x^8-5910B\,x^8+5190C\,x^8+4900A\,x^6-4805B\,x^6+4250C\,x^6+1197A\,x^4-1260B\,x^4+1}{12x^3\left(3x^4+5x^2+2\right)^{\frac{3}{2}}}$
default	$C\left(\frac{\left(\frac{5}{18}x^3+\frac{13}{54}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\frac{115}{12}x^3+\frac{145}{18}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{95i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{4\sqrt{3x^4+5x^2+2}}+\frac{115i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\text{Ellipti}\right.}{6\sqrt{3x}}\right.$

input
```
int((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((35/72*A-13/36*B+5/18*C)*x^3+(97/216*A-35/108*B+13/54*C)*x)*(3*x^4+5*x^2+
2)^(1/2)/(x^4+5/3*x^2+2/3)^2-6*((145/12*A-529/48*B+115/12*C)*x^3+(1427/144
*A-1325/144*B+145/18*C)*x)/(3*x^4+5*x^2+2)^(1/2)-1/24*A*(3*x^4+5*x^2+2)^(1
/2)/x^3-(1/8*B-5/6*A)*(3*x^4+5*x^2+2)^(1/2)/x-1/2*I*(245/4*A-55*B+95/2*C)*
(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1
/2))+1/3*I*(70*A-263/4*B+115/2*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x
^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(2+5x^2+3x^4\right)^{5/2}}\,dx =$$
$$-\frac{2\sqrt{2}(-9i\,(280\,A-263\,B+230\,C)x^{11}-30i\,(280\,A-263\,B+230\,C)x^9-37i\,(280\,A-263\,B+230\,C)}{}$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/24*(2*sqrt(2)*(-9*I*(280*A - 263*B + 230*C)*x^11 - 30*I*(280*A - 263*B
+ 230*C)*x^9 - 37*I*(280*A - 263*B + 230*C)*x^7 - 20*I*(280*A - 263*B + 23
0*C)*x^5 - 4*I*(280*A - 263*B + 230*C)*x^3)*elliptic_e(arcsin(I*x), 3/2) -
 sqrt(2)*(-9*I*(1295*A - 1186*B + 1030*C)*x^11 - 30*I*(1295*A - 1186*B + 1
030*C)*x^9 - 37*I*(1295*A - 1186*B + 1030*C)*x^7 - 20*I*(1295*A - 1186*B +
 1030*C)*x^5 - 4*I*(1295*A - 1186*B + 1030*C)*x^3)*elliptic_f(arcsin(I*x),
 3/2) + 2*(9*(280*A - 263*B + 230*C)*x^10 + 15*(413*A - 394*B + 346*C)*x^8
 + 5*(980*A - 961*B + 850*C)*x^6 + 63*(19*A - 20*B + 18*C)*x^4 - 6*(5*A -
B)*x^2 + 2*A)*sqrt(3*x^4 + 5*x^2 + 2))/(9*x^11 + 30*x^9 + 37*x^7 + 20*x^5
+ 4*x^3)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4(2+5x^2+3x^4)^{5/2}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^4\left((x^2+1)\,(3x^2+2)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**4*((x**2 + 1)*(3*x**2 + 2))**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4(2+5x^2+3x^4)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(5/2)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4(2+5x^2+3x^4)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(5/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(2+5x^2+3x^4\right)^{5/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^4\,{\left(3\,x^4+5\,x^2+2\right)}^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(5*x^2 + 3*x^4 + 2)^(5/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(5*x^2 + 3*x^4 + 2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(2+5x^2+3x^4\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/x^4/(3*x^4+5*x^2+2)^(5/2),x)
```

output
```
( - sqrt(3*x**4 + 5*x**2 + 2)*a - 270*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x*
*14 + 135*x**12 + 279*x**10 + 305*x**8 + 186*x**6 + 60*x**4 + 8*x**2),x)*a
*x**11 - 900*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**14 + 135*x**12 + 279*x**
10 + 305*x**8 + 186*x**6 + 60*x**4 + 8*x**2),x)*a*x**9 - 1110*int(sqrt(3*x
**4 + 5*x**2 + 2)/(27*x**14 + 135*x**12 + 279*x**10 + 305*x**8 + 186*x**6
+ 60*x**4 + 8*x**2),x)*a*x**7 - 600*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**1
4 + 135*x**12 + 279*x**10 + 305*x**8 + 186*x**6 + 60*x**4 + 8*x**2),x)*a*x
**5 - 120*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**14 + 135*x**12 + 279*x**10
+ 305*x**8 + 186*x**6 + 60*x**4 + 8*x**2),x)*a*x**3 + 54*int(sqrt(3*x**4 +
 5*x**2 + 2)/(27*x**14 + 135*x**12 + 279*x**10 + 305*x**8 + 186*x**6 + 60*
x**4 + 8*x**2),x)*b*x**11 + 180*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**14 +
135*x**12 + 279*x**10 + 305*x**8 + 186*x**6 + 60*x**4 + 8*x**2),x)*b*x**9
+ 222*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**14 + 135*x**12 + 279*x**10 + 30
5*x**8 + 186*x**6 + 60*x**4 + 8*x**2),x)*b*x**7 + 120*int(sqrt(3*x**4 + 5*
x**2 + 2)/(27*x**14 + 135*x**12 + 279*x**10 + 305*x**8 + 186*x**6 + 60*x**
4 + 8*x**2),x)*b*x**5 + 24*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**14 + 135*x
**12 + 279*x**10 + 305*x**8 + 186*x**6 + 60*x**4 + 8*x**2),x)*b*x**3 - 243
*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6
 + 186*x**4 + 60*x**2 + 8),x)*a*x**11 - 810*int(sqrt(3*x**4 + 5*x**2 + 2)/
(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x...
```

3.186 $\int \frac{A+Bx^2+Cx^4}{x^6(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 348

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{x^6(2+5x^2+3x^4)^{5/2}}\,dx = &-\frac{x(275A-194B+140C+3(97A-70B+52C)x^2)}{48(2+5x^2+3x^4)^{3/2}} \\ &-\frac{(2521A-2800B+2630C)x(2+3x^2)}{120\sqrt{2+5x^2+3x^4}} \\ &+\frac{x(5495A-5708B+5300C+3(2335A-2320B+2116C)x^2)}{96\sqrt{2+5x^2+3x^4}} \\ &-\frac{A\sqrt{2+5x^2+3x^4}}{40x^5}+\frac{(7A-B)\sqrt{2+5x^2+3x^4}}{24x^3} \\ &-\frac{(1591A-400B+60C)\sqrt{2+5x^2+3x^4}}{480x} \\ &+\frac{(2521A-2800B+2630C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{60\sqrt{2}\sqrt{2+5x^2+3x^4}} \\ &-\frac{(254A-245B+220C)(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{4\sqrt{2}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
-1/48*x*(275*A-194*B+140*C+3*(97*A-70*B+52*C)*x^2)/(3*x^4+5*x^2+2)^(3/2)-1
/120*(2521*A-2800*B+2630*C)*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/96*x*(5495
*A-5708*B+5300*C+3*(2335*A-2320*B+2116*C)*x^2)/(3*x^4+5*x^2+2)^(1/2)-1/40*
A*(3*x^4+5*x^2+2)^(1/2)/x^5+1/24*(7*A-B)*(3*x^4+5*x^2+2)^(1/2)/x^3-1/480*(
1591*A-400*B+60*C)*(3*x^4+5*x^2+2)^(1/2)/x+1/120*(2521*A-2800*B+2630*C)*(x
^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*2
^(1/2)/(3*x^4+5*x^2+2)^(1/2)-1/8*(254*A-245*B+220*C)*(x^2+1)*((3*x^2+2)/(x
^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))*2^(1/2)/(3*x^4+5*x^2
+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^2+Cx^4}{x^6(2+5x^2+3x^4)^{5/2}}\,dx = \frac{-12A+80Ax^2-20Bx^2-1002Ax^4+300Bx^4-60Cx^4+6300Ax^6-1197}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*(2 + 5*x^2 + 3*x^4)^(5/2)),x]
```

output
```
(-12*A + 80*A*x^2 - 20*B*x^2 - 1002*A*x^4 + 300*B*x^4 - 60*C*x^4 + 6300*A*
x^6 - 11970*B*x^6 + 12600*C*x^6 + 37435*A*x^8 - 49000*B*x^8 + 48050*C*x^8
+ 52770*A*x^10 - 61950*B*x^10 + 59100*C*x^10 + 22689*A*x^12 - 25200*B*x^12
 + 23670*C*x^12 + I*Sqrt[3]*(2521*A - 2800*B + 2630*C)*x^5*Sqrt[1 + x^2]*S
qrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3]
+ I*Sqrt[3]*(19*A + 350*B - 430*C)*x^5*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 +
5*x^2 + 3*x^4)*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(120*x^5*(2 + 5*x^2
 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.36, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2198, 27, 2198, 27, 2199, 2199, 2199, 1604, 27, 1604, 27, 1604, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(3x^4+5x^2+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2198}$$

$$-\frac{1}{6}\int -\frac{-9(97A-70B+52C)x^8+80(A-B+C)x^6+6(19A-10B+4C)x^4-12(5A-2B)x^2+24A}{8x^6\left(3x^4+5x^2+2\right)^{3/2}}dx-\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{48}\int \frac{-9(97A-70B+52C)x^8+80(A-B+C)x^6+6(19A-10B+4C)x^4-12(5A-2B)x^2+24A}{x^6\left(3x^4+5x^2+2\right)^{3/2}}dx-\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 2198}$$

$$\frac{1}{48}\left(\frac{x\left(3x^2(2335A-2320B+2116C)+5495A-5708B+5300C\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int -\frac{3(-((2335A-2320B+2116C)x^8}{}\right.-\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{48}\left(\frac{3}{2}\int \frac{-((2335A-2320B+2116C)x^8)-4(515A-491B+440C)x^6+2(63A-20B+4C)x^4-8(5A-B)}{x^6\sqrt{3x^4+5x^2+2}}\right.-\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{2199}$$

$$\frac{1}{48}\left(\frac{3}{2}\left(\int \frac{-4(515A-491B+440C)x^6-\frac{4}{3}(1073A-1130B+1052C)x^4-8(5A-B)x^2+8A}{x^6\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x}}{}\right.\right.$$
$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{2199}$$

$$\frac{1}{48}\left(\frac{3}{2}\left(\int \frac{36(151A-140B+124C)x^4+80(51A-49B+44C)x^2+8A}{x^6\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(2335A-2320B}{3x}\right.\right.$$
$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{2199}$$

$$\frac{1}{48}\left(\frac{3}{2}\left(\int \frac{-80(100A-91B+80C)x^2-16(377A-350B+310C)}{x^6\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(2335A-2320B+21}{3x}\right.\right.$$
$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{48}\left(\frac{3}{2}\left(-\frac{1}{10}\int -\frac{16\left(9(377A-350B+310C)x^2+10(254A-245B+220C)\right)}{x^4\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(2335A-2}{3x}\right.\right.$$
$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{3}{2}\left(\frac{8}{5}\int \frac{9(377A-350B+310C)x^2+10(254A-245B+220C)}{x^4\sqrt{3x^4+5x^2+2}}dx-\frac{\sqrt{3x^4+5x^2+2}(2335A-2320B+2}{3x}\right.\right.$$
$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{48}\left(\frac{3}{2}\left(\frac{8}{5}\left(-\frac{1}{6}\int\frac{2\left(15(254A-245B+220C)x^2+2521A-2800B+2630C\right)}{x^2\sqrt{3x^4+5x^2+2}}dx-\frac{5\sqrt{3x^4+5x^2+2}(254A-2}{3x^3}\right.\right.\right.$$

$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{48}\left(\frac{3}{2}\left(\frac{8}{5}\left(-\frac{1}{3}\int\frac{15(254A-245B+220C)x^2+2521A-2800B+2630C}{x^2\sqrt{3x^4+5x^2+2}}dx-\frac{5\sqrt{3x^4+5x^2+2}(254A-245B}{3x^3}\right.\right.\right.$$

$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1604

$$\frac{1}{48}\left(\frac{3}{2}\left(\frac{8}{5}\left(\frac{1}{3}\left(\frac{1}{2}\int-\frac{3\left((2521A-2800B+2630C)x^2+10(254A-245B+220C)\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{\sqrt{3x^4+5x^2+2}(2521A}{2x}\right.\right.\right.\right.$$

$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{1}{48}\left(\frac{3}{2}\left(\frac{8}{5}\left(\frac{1}{3}\left(\frac{\sqrt{3x^4+5x^2+2}(2521A-2800B+2630C)}{2x}-\frac{3}{2}\int\frac{(2521A-2800B+2630C)x^2+10(254A-2}{\sqrt{3x^4+5x^2+2}}\right.\right.\right.\right.$$

$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1503

$$\frac{1}{48}\left(\frac{3}{2}\left(\frac{8}{5}\left(\frac{1}{3}\left(\frac{\sqrt{3x^4+5x^2+2}(2521A-2800B+2630C)}{2x}-\frac{3}{2}\left(10(254A-245B+220C)\int\frac{1}{\sqrt{3x^4+5x^2+2}}d\right.\right.\right.\right.\right.$$

$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1413

$$\frac{1}{48}\left(\frac{3}{2}\left(\frac{8}{5}\left(\frac{1}{3}\left(\frac{\sqrt{3x^4+5x^2+2}(2521A-2800B+2630C)}{2x}-\frac{3}{2}\left((2521A-2800B+2630C)\int\frac{x^2}{\sqrt{3x^4+5x^2+}}\right.\right.\right.\right.\right.$$

$$\frac{x\left(3x^2(97A-70B+52C)+275A-194B+140C\right)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

$\downarrow$ 1456

$$\frac{1}{48}\left(\frac{3}{2}\left(\frac{8}{5}\left(\frac{1}{3}\left(\frac{\sqrt{3x^4+5x^2+2}(2521A-2800B+2630C)}{2x}-\frac{3}{2}\left(\frac{5\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}(254A-245B+220C)}{\sqrt{3x^4+5x^2+2}}\right.\right.\right.\right.\right.$$
$$\frac{x(3x^2(97A-70B+52C)+275A-194B+140C)}{48\left(3x^4+5x^2+2\right)^{3/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^6*(2 + 5*x^2 + 3*x^4)^(5/2)),x]
```

output
```
-1/48*(x*(275*A - 194*B + 140*C + 3*(97*A - 70*B + 52*C)*x^2))/(2 + 5*x^2
+ 3*x^4)^(3/2) + ((x*(5495*A - 5708*B + 5300*C + 3*(2335*A - 2320*B + 2116
*C)*x^2))/(2*Sqrt[2 + 5*x^2 + 3*x^4]) + (3*((-4*(151*A - 140*B + 124*C)*Sq
rt[2 + 5*x^2 + 3*x^4])/x^5 + (8*(377*A - 350*B + 310*C)*Sqrt[2 + 5*x^2 + 3
*x^4])/(5*x^5) + (4*(515*A - 491*B + 440*C)*Sqrt[2 + 5*x^2 + 3*x^4])/(3*x^
3) - ((2335*A - 2320*B + 2116*C)*Sqrt[2 + 5*x^2 + 3*x^4])/(3*x) + (8*((-5*
(254*A - 245*B + 220*C)*Sqrt[2 + 5*x^2 + 3*x^4])/(3*x^3) + (((2521*A - 280
0*B + 2630*C)*Sqrt[2 + 5*x^2 + 3*x^4])/(2*x) - (3*((2521*A - 2800*B + 2630
*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt
[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*
x^4])) + (5*Sqrt[2]*(254*A - 245*B + 220*C)*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1
+ x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4]))/2)/3))/5))/2
)/48
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 7.08 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.84

method	result
elliptic	$\frac{\left(\left(-\frac{97A}{144}+\frac{35B}{72}-\frac{13C}{36}\right)x^3+\left(-\frac{275A}{432}+\frac{97B}{216}-\frac{35C}{108}\right)x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2} - \frac{6\left(\left(-\frac{2335A}{192}+\frac{145B}{12}-\frac{529C}{48}\right)x^3+\left(-\frac{5495A}{576}+\frac{1427B}{144}-\frac{1325C}{144}\right)\cdots\right.}{\sqrt{3x^4+5x^2+2}}$
risch	$\frac{22689A\,x^{12}-25200B\,x^{12}+23670C\,x^{12}+52770A\,x^{10}-61950B\,x^{10}+59100C\,x^{10}+37435A\,x^8-49000B\,x^8+48050C\,x^8+6300A\,x^6-1\cdots}{120x^5\left(3x^4+5x^2+2\right)^{\frac{3}{2}}}$
default	$A\left(\frac{\left(-\frac{97}{144}x^3-\frac{275}{432}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(-\frac{2335}{192}x^3-\frac{5495}{576}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{3x^4+5x^2+2}}{40x^5}+\frac{7\sqrt{3x^4+5x^2+2}}{24x^3}-\frac{1591\sqrt{3x^4+5x^2+2}}{480x}+\cdots\right.$

input
```
int((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((-97/144*A+35/72*B-13/36*C)*x^3+(-275/432*A+97/216*B-35/108*C)*x)*(3*x^4+
5*x^2+2)^(1/2)/(x^4+5/3*x^2+2/3)^2-6*((-2335/192*A+145/12*B-529/48*C)*x^3+
(-5495/576*A+1427/144*B-1325/144*C)*x)/(3*x^4+5*x^2+2)^(1/2)-1/40*A*(3*x^4
+5*x^2+2)^(1/2)/x^5-1/3*(1/8*B-7/8*A)*(3*x^4+5*x^2+2)^(1/2)/x^3-(1/8*C-5/6
*B+1591/480*A)*(3*x^4+5*x^2+2)^(1/2)/x-1/2*I*(-127/2*A+245/4*B-55*C)*(x^2+
1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+
1/3*I*(-2521/40*A+70*B-263/4*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2
+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2+Cx^4}{x^6 \left(2+5x^2+3x^4\right)^{5/2}} \, dx = \frac{\sqrt{2}(-9i\,(2521\,A-2800\,B+2630\,C)x^{13}-30i\,(2521\,A-2800\,B+2630\,C)x}{}$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/120*(sqrt(2)*(-9*I*(2521*A - 2800*B + 2630*C)*x^13 - 30*I*(2521*A - 2800
*B + 2630*C)*x^11 - 37*I*(2521*A - 2800*B + 2630*C)*x^9 - 20*I*(2521*A - 2
800*B + 2630*C)*x^7 - 4*I*(2521*A - 2800*B + 2630*C)*x^5)*elliptic_e(arcsi
n(I*x), 3/2) + sqrt(2)*(9*I*(6331*A - 6475*B + 5930*C)*x^13 + 30*I*(6331*A
 - 6475*B + 5930*C)*x^11 + 37*I*(6331*A - 6475*B + 5930*C)*x^9 + 20*I*(633
1*A - 6475*B + 5930*C)*x^7 + 4*I*(6331*A - 6475*B + 5930*C)*x^5)*elliptic_
f(arcsin(I*x), 3/2) + (9*(2521*A - 2800*B + 2630*C)*x^12 + 30*(1759*A - 20
65*B + 1970*C)*x^10 + 5*(7487*A - 9800*B + 9610*C)*x^8 + 630*(10*A - 19*B
+ 20*C)*x^6 - 6*(167*A - 50*B + 10*C)*x^4 + 20*(4*A - B)*x^2 - 12*A)*sqrt(
3*x^4 + 5*x^2 + 2))/(9*x^13 + 30*x^11 + 37*x^9 + 20*x^7 + 4*x^5)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6 \left(2+5x^2+3x^4\right)^{5/2}} \, dx = \int \frac{A+Bx^2+Cx^4}{x^6 \left((x^2+1)\left(3x^2+2\right)\right)^{\frac{5}{2}}} \, dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**6/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**6*((x**2 + 1)*(3*x**2 + 2))**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(2+5x^2+3x^4\right)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(3\,x^4+5\,x^2+2\right)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(5/2)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(2+5x^2+3x^4\right)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(3\,x^4+5\,x^2+2\right)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((3*x^4 + 5*x^2 + 2)^(5/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(2+5x^2+3x^4\right)^{5/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^6\,\left(3\,x^4+5\,x^2+2\right)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^6*(5*x^2 + 3*x^4 + 2)^(5/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^6*(5*x^2 + 3*x^4 + 2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^6 (2 + 5x^2 + 3x^4)^{5/2}} \, dx = \text{too large to display}$$

input
```
int((C*x^4+B*x^2+A)/x^6/(3*x^4+5*x^2+2)^(5/2),x)
```

output
```
( - 16*sqrt(3*x**4 + 5*x**2 + 2)*a + 231*sqrt(3*x**4 + 5*x**2 + 2)*b*x**8
+ 66*sqrt(3*x**4 + 5*x**2 + 2)*b*x**4 - 280*sqrt(3*x**4 + 5*x**2 + 2)*c*x*
*8 - 80*sqrt(3*x**4 + 5*x**2 + 2)*c*x**4 - 5760*int(sqrt(3*x**4 + 5*x**2 +
 2)/(27*x**16 + 135*x**14 + 279*x**12 + 305*x**10 + 186*x**8 + 60*x**6 + 8
*x**4),x)*a*x**13 - 19200*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**16 + 135*x*
*14 + 279*x**12 + 305*x**10 + 186*x**8 + 60*x**6 + 8*x**4),x)*a*x**11 - 23
680*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**16 + 135*x**14 + 279*x**12 + 305*
x**10 + 186*x**8 + 60*x**6 + 8*x**4),x)*a*x**9 - 12800*int(sqrt(3*x**4 + 5
*x**2 + 2)/(27*x**16 + 135*x**14 + 279*x**12 + 305*x**10 + 186*x**8 + 60*x
**6 + 8*x**4),x)*a*x**7 - 2560*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**16 + 1
35*x**14 + 279*x**12 + 305*x**10 + 186*x**8 + 60*x**6 + 8*x**4),x)*a*x**5
+ 1440*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**16 + 135*x**14 + 279*x**12 + 3
05*x**10 + 186*x**8 + 60*x**6 + 8*x**4),x)*b*x**13 + 4800*int(sqrt(3*x**4
+ 5*x**2 + 2)/(27*x**16 + 135*x**14 + 279*x**12 + 305*x**10 + 186*x**8 + 6
0*x**6 + 8*x**4),x)*b*x**11 + 5920*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**16
 + 135*x**14 + 279*x**12 + 305*x**10 + 186*x**8 + 60*x**6 + 8*x**4),x)*b*x
**9 + 3200*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**16 + 135*x**14 + 279*x**12
 + 305*x**10 + 186*x**8 + 60*x**6 + 8*x**4),x)*b*x**7 + 640*int(sqrt(3*x**
4 + 5*x**2 + 2)/(27*x**16 + 135*x**14 + 279*x**12 + 305*x**10 + 186*x**8 +
 60*x**6 + 8*x**4),x)*b*x**5 - 4752*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x...
```

3.187 $\int \frac{13A-10B+8C+2(15A-12B+10C)x^2+(18A-15B+13C)x^4}{(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 56, antiderivative size = 97

$$\int \frac{13A-10B+8C+2(15A-12B+10C)x^2+(18A-15B+13C)x^4}{(2+5x^2+3x^4)^{3/2}}\,dx=\frac{(9A-6B+4C)x}{6\sqrt{2+5x^2+3x^4}}+\frac{(15A-12B+10C)\,(1+x^2)\,\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
1/6*(9*A-6*B+4*C)*x/(3*x^4+5*x^2+2)^(1/2)+1/6*(15*A-12*B+10*C)*(x^2+1)*((3
*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)/(3
*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.36

$$\int \frac{13A-10B+8C+2(15A-12B+10C)x^2+(18A-15B+13C)x^4}{(2+5x^2+3x^4)^{3/2}}\,dx=\frac{6x(2C(4+5x^2)-2B(5+6

input
```
Integrate[(13*A - 10*B + 8*C + 2*(15*A - 12*B + 10*C)*x^2 + (18*A - 15*B +
 13*C)*x^4)/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(6*x*(2*C*(4 + 5*x^2) - 2*B*(5 + 6*x^2) + A*(13 + 15*x^2)) + (2*I)*Sqrt[3]
*(15*A - 12*B + 10*C)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(EllipticE[I*ArcSinh[S
qrt[3/2]*x], 2/3] - EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3]))/(12*Sqrt[2 +
5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.48, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2206, 27, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(18A-15B+13C)+2x^2(15A-12B+10C)+13A-10B+8C}{(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{x(x^2(15A-12B+10C)+13A-10B+8C)}{2\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int \frac{(15A-12B+10C)x^2}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(x^2(15A-12B+10C)+13A-10B+8C)}{2\sqrt{3x^4+5x^2+2}}-\frac{1}{2}(15A-12B+10C)\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1456}$$

$$\frac{x(x^2(15A-12B+10C)+13A-10B+8C)}{2\sqrt{3x^4+5x^2+2}}-\frac{1}{2}(15A-12B+10C)\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{3x^4+5x^2+2}}\right)$$

input
```
Int[(13*A - 10*B + 8*C + 2*(15*A - 12*B + 10*C)*x^2 + (18*A - 15*B + 13*C)
*x^4)/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(x*(13*A - 10*B + 8*C + (15*A - 12*B + 10*C)*x^2))/(2*Sqrt[2 + 5*x^2 + 3*x
^4]) - ((15*A - 12*B + 10*C)*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4])
- (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2
])/(3*Sqrt[2 + 5*x^2 + 3*x^4])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12

method	result
elliptic	$-\frac{6\left(\left(B-\frac{5C}{6}-\frac{5A}{4}\right)x^{3}+\left(\frac{5B}{6}-\frac{2C}{3}-\frac{13A}{12}\right)x\right)}{\sqrt{3x^{4}+5x^{2}+2}}+\frac{i\left(-\frac{15A}{2}+6B-5C\right)\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^{4}+5x^{2}+2}}$
risch	$\frac{x\left(15A\,x^{2}-12B\,x^{2}+10C\,x^{2}+13A-10B+8C\right)}{2\sqrt{3x^{4}+5x^{2}+2}}+\frac{i\left(-\frac{15A}{2}+6B-5C\right)\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^{4}+5x^{2}+2}}$
default	$(18A-15B+13C)\left(-\frac{6\left(-\frac{5}{6}x^{3}-\frac{2}{3}x\right)}{\sqrt{3x^{4}+5x^{2}+2}}+\frac{2i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^{4}+5x^{2}+2}}-\frac{5i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\right.\right.}{3\sqrt{3x^{4}+5x^{2}+}}\right.$

input
```
int((13*A-10*B+8*C+2*(15*A-12*B+10*C)*x^2+(18*A-15*B+13*C)*x^4)/(3*x^4+5*x
^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6*((B-5/6*C-5/4*A)*x^3+(5/6*B-2/3*C-13/12*A)*x)/(3*x^4+5*x^2+2)^(1/2)+1/3
*I*(-15/2*A+6*B-5*C)*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(
EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.65

$$\int \frac{13A-10B+8C+2(15A-12B+10C)x^2+(18A-15B+13C)x^4}{\left(2+5x^2+3x^4\right)^{3/2}}\,dx = \frac{\sqrt{2}(-3i\,(15\,A-12\,B+10}{}$$

input
```
integrate((13*A-10*B+8*C+2*(15*A-12*B+10*C)*x^2+(18*A-15*B+13*C)*x^4)/(3*x
^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(2)*(-3*I*(15*A - 12*B + 10*C)*x^4 - 5*I*(15*A - 12*B + 10*C)*x^2
 - 30*I*A + 24*I*B - 20*I*C)*elliptic_e(arcsin(I*x), 3/2) + sqrt(2)*(3*I*(
15*A - 12*B + 10*C)*x^4 + 5*I*(15*A - 12*B + 10*C)*x^2 + 30*I*A - 24*I*B +
 20*I*C)*elliptic_f(arcsin(I*x), 3/2) + 3*((15*A - 12*B + 10*C)*x^3 + (13*
A - 10*B + 8*C)*x)*sqrt(3*x^4 + 5*x^2 + 2))/(3*x^4 + 5*x^2 + 2)
```

Sympy [F]

$$\int \frac{13A - 10B + 8C + 2(15A - 12B + 10C)x^2 + (18A - 15B + 13C)x^4}{(2 + 5x^2 + 3x^4)^{3/2}}\, dx = \int \frac{18Ax^4 + 30Ax^2 + 13A -}{}$$

input
```
integrate((13*A-10*B+8*C+2*(15*A-12*B+10*C)*x**2+(18*A-15*B+13*C)*x**4)/(3
*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((18*A*x**4 + 30*A*x**2 + 13*A - 15*B*x**4 - 24*B*x**2 - 10*B + 13
*C*x**4 + 20*C*x**2 + 8*C)/((x**2 + 1)*(3*x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{13A - 10B + 8C + 2(15A - 12B + 10C)x^2 + (18A - 15B + 13C)x^4}{(2 + 5x^2 + 3x^4)^{3/2}}\, dx = \int \frac{(18\,A - 15\,B + 13\,C)x^4}{}$$

input
```
integrate((13*A-10*B+8*C+2*(15*A-12*B+10*C)*x^2+(18*A-15*B+13*C)*x^4)/(3*x
^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((18*A - 15*B + 13*C)*x^4 + 2*(15*A - 12*B + 10*C)*x^2 + 13*A -
10*B + 8*C)/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{13A - 10B + 8C + 2(15A - 12B + 10C)x^2 + (18A - 15B + 13C)x^4}{(2 + 5x^2 + 3x^4)^{3/2}}\, dx = \int \frac{(18\,A - 15\,B + 13\,C)x^4}{}$$

input
```
integrate((13*A-10*B+8*C+2*(15*A-12*B+10*C)*x^2+(18*A-15*B+13*C)*x^4)/(3*x
^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate(((18*A - 15*B + 13*C)*x^4 + 2*(15*A - 12*B + 10*C)*x^2 + 13*A -
10*B + 8*C)/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{13A-10B+8C+2(15A-12B+10C)x^2+(18A-15B+13C)x^4}{(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{13\,A-10\,B+8\,C+2\,x^2}{}$$

input
```
int((13*A - 10*B + 8*C + 2*x^2*(15*A - 12*B + 10*C) + x^4*(18*A - 15*B + 1
3*C))/(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int((13*A - 10*B + 8*C + 2*x^2*(15*A - 12*B + 10*C) + x^4*(18*A - 15*B + 1
3*C))/(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{13A-10B+8C+2(15A-12B+10C)x^2+(18A-15B+13C)x^4}{(2+5x^2+3x^4)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((13*A-10*B+8*C+2*(15*A-12*B+10*C)*x^2+(18*A-15*B+13*C)*x^4)/(3*x^4+5*x
^2+2)^(3/2),x)
```

output
```
( - 18*sqrt(3*x**4 + 5*x**2 + 2)*a*x + 15*sqrt(3*x**4 + 5*x**2 + 2)*b*x -
13*sqrt(3*x**4 + 5*x**2 + 2)*c*x + 225*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x*
*8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a*x**4 + 375*int(sqrt(3*x**4 + 5*
x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a*x**2 + 150*int(s
qrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a -
 180*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 +
 4),x)*b*x**4 - 300*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x
**4 + 20*x**2 + 4),x)*b*x**2 - 120*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 +
 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b + 150*int(sqrt(3*x**4 + 5*x**2 + 2)
/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*c*x**4 + 250*int(sqrt(3*x**
4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*c*x**2 + 100
*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),
x)*c + 270*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**
4 + 20*x**2 + 4),x)*a*x**4 + 450*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x
**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a*x**2 + 180*int((sqrt(3*x**4 +
5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*a - 216*in
t((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 +
 4),x)*b*x**4 - 360*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6
 + 37*x**4 + 20*x**2 + 4),x)*b*x**2 - 144*int((sqrt(3*x**4 + 5*x**2 + 2)*x
**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*b + 180*int((sqrt(3*...
```

3.188 $\int (dx)^m (a+bx^2+cx^4)^p (A+Bx^2+Cx^4)\, dx$

Optimal result

Integrand size = 32, antiderivative size = 411

$$\int (dx)^m (a+bx^2+cx^4)^p (A+Bx^2+Cx^4)\, dx = \frac{C(dx)^{1+m}(a+bx^2+cx^4)^{1+p}}{cd(5+m+4p)} - \frac{(aC(1+m)-Ac(5+m+4p))(dx)^{1+m}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p \text{Appel}}{cd(1+m)(5+m+4p)} - \frac{(bC(3+m+2p)-Bc(5+m+4p))(dx)^{3+m}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\, A}{cd^3(3+m)(5+m+4p)}$$

output
```
C*(d*x)^(1+m)*(c*x^4+b*x^2+a)^(p+1)/c/d/(5+m+4*p)-(a*C*(1+m)-A*c*(5+m+4*p)
)*(d*x)^(1+m)*(c*x^4+b*x^2+a)^p*AppellF1(1/2+1/2*m,-p,-p,3/2+1/2*m,-2*c*x^
2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/c/d/(1+m)/(5+m+4
*p)/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^p)-(b*C*(3+m+2*p)-B*c*(5+m+4*p))*(d*x)^(3+m)*(c*x^4+b*x^2+a)^p*Appell
F1(3/2+1/2*m,-p,-p,5/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(
-4*a*c+b^2)^(1/2)))/c/d^3/(3+m)/(5+m+4*p)/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2
)))^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.86

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(A+Bx^2+Cx^4\right) \, dx$$
$$= \frac{x(dx)^m \left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p} \left(a+bx^2+cx^4\right)^p \left(A(15+8m+m^2)\,\text{AppellF1}\left(\frac{1+m}{2},-p\right.\right.}{}$$

input
```
Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^p*(A + B*x^2 + C*x^4),x]
```

output
```
(x*(d*x)^m*(a + b*x^2 + c*x^4)^p*(A*(15 + 8*m + m^2)*AppellF1[(1 + m)/2, -
p, -p, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])] + (1 + m)*x^2*(B*(5 + m)*AppellF1[(3 + m)/2, -p, -p, (5 +
m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])] + C*(3 + m)*x^2*AppellF1[(5 + m)/2, -p, -p, (7 + m)/2, (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 + m)*(3 +
m)*(5 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2199, 1674, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(A+Bx^2+Cx^4\right) \left(a+bx^2+cx^4\right)^p \, dx$$

$$\Big\downarrow \; 2199$$

$$\int (dx)^m \left(\left(B-\frac{bC(m+2p+3)}{c(m+4p+5)}\right)x^2+A-\frac{aC(m+1)}{c(m+4p+5)}\right) \left(cx^4+bx^2+a\right)^p dx+ \frac{C(dx)^{m+1} \left(a+bx^2+cx^4\right)^{p+1}}{cd(m+4p+5)}$$

$$\Big\downarrow \text{1674}$$

$$\int \left(A\left(1-\frac{aC(m+1)}{Ac(m+4p+5)}\right)\left(cx^4+bx^2+a\right)^p(dx)^m+\frac{(Bc(m+4p+5)-bC(m+2p+3))\left(cx^4+bx^2+a\right)^p}{cd^2(m+4p+5)}\right.$$

$$\frac{C(dx)^{m+1}\left(a+bx^2+cx^4\right)^{p+1}}{cd(m+4p+5)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(dx)^{m+1}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p\left(\frac{A}{m+1}-\frac{aC}{c(m+4p+5)}\right)\text{AppellF1}\left(\frac{m+1}{2},-p,-p,\right.}{d}$$

$$\frac{(dx)^{m+3}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p(bC(m+2p+3)-Bc(m+4p+5))\text{AppellF1}}{cd^3(m+3)(m+4p+5)}$$

$$\frac{C(dx)^{m+1}\left(a+bx^2+cx^4\right)^{p+1}}{cd(m+4p+5)}$$

input

```
Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*(A + B*x^2 + C*x^4),x]
```

output

```
(C*(d*x)^(1 + m)*(a + b*x^2 + c*x^4)^(1 + p))/(c*d*(5 + m + 4*p)) + ((A/(1
 + m) - (a*C)/(c*(5 + m + 4*p)))*(d*x)^(1 + m)*(a + b*x^2 + c*x^4)^p*Appel
lF1[(1 + m)/2, -p, -p, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
)^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p) - ((b*C*(3 + m + 2*p) - B*c
*(5 + m + 4*p))*(d*x)^(3 + m)*(a + b*x^2 + c*x^4)^p*AppellF1[(3 + m)/2, -p
, -p, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(c*d^3*(3 + m)*(5 + m + 4*p)*(1 + (2*c*x^2)/(b - Sqrt[b^2
- 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 1674

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2199
```
Int[(Px_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> With[{q = Expon[Px, x^2]}, Simp[Coeff[Px, x^2, q]*(d*x)^(m + 2*q
- 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*d^(2*q - 3)*(m + 4*p + 2*q + 1))), x]
+ Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*ExpandToSum[Px - Coeff[Px, x^2, q]*x^(2
*q) - Coeff[Px, x^2, q]*((a*(m + 2*q - 3)*x^(2*(q - 2)) + b*(m + 2*p + 2*q
- 1)*x^(2*(q - 1)))/(c*(m + 4*p + 2*q + 1))), x], x] /; GtQ[q, 1] && NeQ[m
+ 4*p + 2*q + 1, 0]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Px, x^2] && N
eQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int (dx)^m \left(c\,x^4+b\,x^2+a\right)^p \left(C\,x^4+B\,x^2+A\right)dx$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^p*(C*x^4+B*x^2+A),x)
```

output
```
int((d*x)^m*(c*x^4+b*x^2+a)^p*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx = \int \left(Cx^4+Bx^2+A\right)\left(cx^4+bx^2+a\right)^p (dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^p*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(A+Bx^2+Cx^4\right) \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(c*x**4+b*x**2+a)**p*(C*x**4+B*x**2+A),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(A+Bx^2+Cx^4\right) \, dx = \int \left(Cx^4+Bx^2+A\right)\left(cx^4+bx^2+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^p*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(A+Bx^2+Cx^4\right) \, dx = \int \left(Cx^4+Bx^2+A\right)\left(cx^4+bx^2+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^p*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(A+Bx^2+Cx^4\right)\, dx = \int (d\,x)^m \left(C\,x^4+B\,x^2+A\right)\, \left(c\,x^4+b\,x^2+a\right)^p\, dx$$

input
```
int((d*x)^m*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^p,x)
```

output
```
int((d*x)^m*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(A+Bx^2+Cx^4\right)\, dx = \text{too large to display}$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^p*(C*x^4+B*x^2+A),x)
```

output
```
(d**m*(x**m*(a + b*x**2 + c*x**4)**p*a*c*m**2*x + 12*x**m*(a + b*x**2 + c*
x**4)**p*a*c*m*p*x + 8*x**m*(a + b*x**2 + c*x**4)**p*a*c*m*x + 32*x**m*(a
+ b*x**2 + c*x**4)**p*a*c*p**2*x + 44*x**m*(a + b*x**2 + c*x**4)**p*a*c*p*
x + 15*x**m*(a + b*x**2 + c*x**4)**p*a*c*x + 4*x**m*(a + b*x**2 + c*x**4)*
*p*b**2*p**2*x + 4*x**m*(a + b*x**2 + c*x**4)**p*b**2*p*x + x**m*(a + b*x*
*2 + c*x**4)**p*b*c*m**2*x**3 + 10*x**m*(a + b*x**2 + c*x**4)**p*b*c*m*p*x
**3 + 6*x**m*(a + b*x**2 + c*x**4)**p*b*c*m*x**3 + 24*x**m*(a + b*x**2 + c
*x**4)**p*b*c*p**2*x**3 + 26*x**m*(a + b*x**2 + c*x**4)**p*b*c*p*x**3 + 5*
x**m*(a + b*x**2 + c*x**4)**p*b*c*x**3 + x**m*(a + b*x**2 + c*x**4)**p*c**
2*m**2*x**5 + 8*x**m*(a + b*x**2 + c*x**4)**p*c**2*m*p*x**5 + 4*x**m*(a +
b*x**2 + c*x**4)**p*c**2*m*x**5 + 16*x**m*(a + b*x**2 + c*x**4)**p*c**2*p*
*2*x**5 + 16*x**m*(a + b*x**2 + c*x**4)**p*c**2*p*x**5 + 3*x**m*(a + b*x**
2 + c*x**4)**p*c**2*x**5 + 16*int((x**m*(a + b*x**2 + c*x**4)**p*x**2)/(a*
m**3 + 12*a*m**2*p + 9*a*m**2 + 48*a*m*p**2 + 72*a*m*p + 23*a*m + 64*a*p**
3 + 144*a*p**2 + 92*a*p + 15*a + b*m**3*x**2 + 12*b*m**2*p*x**2 + 9*b*m**2
*x**2 + 48*b*m*p**2*x**2 + 72*b*m*p*x**2 + 23*b*m*x**2 + 64*b*p**3*x**2 +
144*b*p**2*x**2 + 92*b*p*x**2 + 15*b*x**2 + c*m**3*x**4 + 12*c*m**2*p*x**4
 + 9*c*m**2*x**4 + 48*c*m*p**2*x**4 + 72*c*m*p*x**4 + 23*c*m*x**4 + 64*c*p
**3*x**4 + 144*c*p**2*x**4 + 92*c*p*x**4 + 15*c*x**4),x)*a*b*c*m**4*p**2 +
 16*int((x**m*(a + b*x**2 + c*x**4)**p*x**2)/(a*m**3 + 12*a*m**2*p + 9*...
```

3.189 $\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(a(1+m)+b(3+m+2p\right.$

Optimal result

Integrand size = 48, antiderivative size = 27

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(a(1+m)+b(3+m+2p)x^2+c(5+m+4p)x^4\right) dx = \frac{(dx)^{1+m} \left(a+bx^2+cx^4\right)^{1+p}}{d}$$

output
```
(d*x)^(1+m)*(c*x^4+b*x^2+a)^(p+1)/d
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(a(1+m)+b(3+m+2p)x^2+c(5+m+4p)x^4\right) dx = x(dx)^m \left(a+bx^2+cx^4\right)^{1+p}$$

input
```
Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^p*(a*(1 + m) + b*(3 + m + 2*p)*x^2 +
 c*(5 + m + 4*p)*x^4),x]
```

output
```
x*(d*x)^m*(a + b*x^2 + c*x^4)^(1 + p)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(a(m+1)+bx^2(m+2p+3)+cx^4(m+4p+5)\right) \, dx$$

$$\Big\downarrow \text{ 2023}$$

$$\frac{(dx)^{m+1} \left(a+bx^2+cx^4\right)^{p+1}}{d}$$

input
```
Int[(d*x)^m*(a + b*x^2 + c*x^4)^p*(a*(1 + m) + b*(3 + m + 2*p)*x^2 + c*(5
+ m + 4*p)*x^4),x]
```

output
```
((d*x)^(1 + m)*(a + b*x^2 + c*x^4)^(1 + p))/d
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
gosper	$x(dx)^{m} \left(c\,x^{4}+b\,x^{2}+a\right)^{p+1}$	24
risch	$\left(c\,x^{4}+b\,x^{2}+a\right) x\left(c\,x^{4}+b\,x^{2}+a\right)^{p} x^{m} d^{m} \mathrm{e}^{\frac{i\,\mathrm{csgn}(idx)\pi m(\mathrm{csgn}(idx)-\mathrm{csgn}(ix))(-\mathrm{csgn}(idx)+\mathrm{csgn}(id))}{2}}$	75
parallelrisch	$\frac{x^{5}(dx)^{m} \left(c\,x^{4}+b\,x^{2}+a\right)^{p} ac+x^{3}(dx)^{m} \left(c\,x^{4}+b\,x^{2}+a\right)^{p} ab+x(dx)^{m} \left(c\,x^{4}+b\,x^{2}+a\right)^{p} a^{2}}{a}$	80
orering	$\frac{x\left(c\,x^{4}+b\,x^{2}+a\right)(dx)^{m} \left(c\,x^{4}+b\,x^{2}+a\right)^{p} \left(a(1+m)+b(3+m+2p)x^{2}+c(5+m+4p)x^{4}\right)}{c\,x^{4}m+4c\,x^{4}p+5c\,x^{4}+b\,x^{2}m+2b\,x^{2}p+3b\,x^{2}+am+a}$	107

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^p*(a*(1+m)+b*(3+m+2*p)*x^2+c*(5+m+4*p)*x^4),x,
method=_RETURNVERBOSE)
```

output
```
x*(d*x)^m*(c*x^4+b*x^2+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int (dx)^{m} \left(a+bx^{2}+cx^{4}\right)^{p} \left(a(1+m)+b(3+m+2p)x^{2}+c(5+m+4p)x^{4}\right) \, dx \\ = \left(cx^{5}+bx^{3}+ax\right) \left(cx^{4}+bx^{2}+a\right)^{p} (dx)^{m}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^p*(a*(1+m)+b*(3+m+2*p)*x^2+c*(5+m+4*p)*x
^4),x, algorithm="fricas")
```

output
```
(c*x^5 + b*x^3 + a*x)*(c*x^4 + b*x^2 + a)^p*(d*x)^m
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(a(1+m)+b(3+m+2p)x^2+c(5+m+4p)x^4\right)\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(c*x**4+b*x**2+a)**p*(a*(1+m)+b*(3+m+2*p)*x**2+c*(5+m+4
*p)*x**4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(a(1+m)+b(3+m+2p)x^2+c(5+m+4p)x^4\right)\,dx = \left(cd^mx^5+bd^mx^3+ad^mx\right)e^{\left(p\log\left(cx^4+bx^2+a\right)+m\log(x)\right)}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^p*(a*(1+m)+b*(3+m+2*p)*x^2+c*(5+m+4*p)*x
^4),x, algorithm="maxima")
```

output
```
(c*d^m*x^5 + b*d^m*x^3 + a*d^m*x)*e^(p*log(c*x^4 + b*x^2 + a) + m*log(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(27) = 54.

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(a(1+m)+b(3+m+2p)x^2+c(5+m+4p)x^4\right)\,dx = \left(cx^4+bx^2+a\right)^p(dx)^m cx^5+\left(cx^4+bx^2+a\right)^p(dx)^m bx^3+\left(cx^4+bx^2+a\right)^p(dx)^m ax$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^p*(a*(1+m)+b*(3+m+2*p)*x^2+c*(5+m+4*p)*x
^4),x, algorithm="giac")
```

output
```
(c*x^4 + b*x^2 + a)^p*(d*x)^m*c*x^5 + (c*x^4 + b*x^2 + a)^p*(d*x)^m*b*x^3
+ (c*x^4 + b*x^2 + a)^p*(d*x)^m*a*x
```

Mupad [B] (verification not implemented)

Time = 19.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(a(1+m)+b(3+m+2p)x^2+c(5+m+4p)x^4\right) dx$$
$$= \left(a\,x\,(d\,x)^m+b\,x^3\,(d\,x)^m+c\,x^5\,(d\,x)^m\right)\,\left(c\,x^4+b\,x^2+a\right)^p$$

input
```
int((d*x)^m*(a*(m + 1) + b*x^2*(m + 2*p + 3) + c*x^4*(m + 4*p + 5))*(a + b
*x^2 + c*x^4)^p,x)
```

output
```
(a*x*(d*x)^m + b*x^3*(d*x)^m + c*x^5*(d*x)^m)*(a + b*x^2 + c*x^4)^p
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \left(a(1+m)+b(3+m+2p)x^2+c(5+m+4p)x^4\right) dx$$
$$= x^m d^m \left(c\,x^4+b\,x^2+a\right)^p x\left(c\,x^4+b\,x^2+a\right)$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^p*(a*(1+m)+b*(3+m+2*p)*x^2+c*(5+m+4*p)*x^4),x)
```

output
```
x**m*d**m*(a + b*x**2 + c*x**4)**p*x*(a + b*x**2 + c*x**4)
```

3.190 $\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)\right.$

Optimal result

Integrand size = 42, antiderivative size = 20

$$\int x^2\left(a+bx^2+cx^4\right)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx=x^3\left(a+bx^2+cx^4\right)^{1+p}$$

output
```
x^3*(c*x^4+b*x^2+a)^(p+1)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2+cx^4\right)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx=x^3\left(a+bx^2+cx^4\right)^{1+p}$$

input
```
Integrate[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x
^4),x]
```

output
```
x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + bx^2 + cx^4)^p (3a + b(2p + 5)x^2 + c(4p + 7)x^4) \, dx$$

$$\Big\downarrow \text{ 2021}$$

$$x^3 (a + bx^2 + cx^4)^{p+1}$$

input
```
Int[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4),x]
```

output
```
x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$x^3(c\,x^4+b\,x^2+a)^{p+1}$	21
risch	$(c\,x^4+b\,x^2+a)^p\,x^3(c\,x^4+b\,x^2+a)$	31
norman	$a\,x^3\mathrm{e}^{p\ln(c\,x^4+b\,x^2+a)}+b\,x^5\mathrm{e}^{p\ln(c\,x^4+b\,x^2+a)}+c\,x^7\mathrm{e}^{p\ln(c\,x^4+b\,x^2+a)}$	65
parallelrisch	$\frac{x^7(c\,x^4+b\,x^2+a)^p ac+ab(c\,x^4+b\,x^2+a)^p x^5+a^2(c\,x^4+b\,x^2+a)^p x^3}{a}$	67
orering	$\frac{(c\,x^4+b\,x^2+a)x^3(c\,x^4+b\,x^2+a)^p(3a+b(5+2p)x^2+c(7+4p)x^4)}{4c\,x^4p+7c\,x^4+2b\,x^2p+5b\,x^2+3a}$	87

input
```
int(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x,method=_RETU
RNVERBOSE)
```

output
```
x^3*(c*x^4+b*x^2+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx$$
$$=\left(cx^7+bx^5+ax^3\right)\left(cx^4+bx^2+a\right)^p$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algor
ithm="fricas")
```

output
```
(c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.

Time = 177.98 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx = ax^3(a+bx^2+cx^4)^p+bx^5(a+bx^2+cx^4)^p+cx^7(a+bx^2+cx^4)^p$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)**p*(3*a+b*(5+2*p)*x**2+c*(7+4*p)*x**4),x)
```

output
```
a*x**3*(a + b*x**2 + c*x**4)**p + b*x**5*(a + b*x**2 + c*x**4)**p + c*x**7
*(a + b*x**2 + c*x**4)**p
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx = (cx^7+bx^5+ax^3)(cx^4+bx^2+a)^p$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algor
ithm="maxima")
```

output
```
(c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx = \left(cx^4+bx^2+a\right)^p cx^7 + \left(cx^4+bx^2+a\right)^p bx^5 + \left(cx^4+bx^2+a\right)^p ax^3$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algor
ithm="giac")
```

output
```
(c*x^4 + b*x^2 + a)^p*c*x^7 + (c*x^4 + b*x^2 + a)^p*b*x^5 + (c*x^4 + b*x^2
 + a)^p*a*x^3
```

Mupad [B] (verification not implemented)

Time = 19.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx = \left(c\,x^7+b\,x^5+a\,x^3\right)\,\left(c\,x^4+b\,x^2+a\right)^p$$

input
```
int(x^2*(3*a + b*x^2*(2*p + 5) + c*x^4*(4*p + 7))*(a + b*x^2 + c*x^4)^p,x)
```

output
```
(a*x^3 + b*x^5 + c*x^7)*(a + b*x^2 + c*x^4)^p
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int x^2(a+bx^2+cx^4)^p\left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx = \left(c\,x^4+b\,x^2+a\right)^p x^3\left(c\,x^4+b\,x^2+a\right)$$

input
```
int(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x)
```

output
```
(a + b*x**2 + c*x**4)**p*x**3*(a + b*x**2 + c*x**4)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
    if m:
        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")


  return m

def is_special_function(func):
  #debug=False
  if debug:
    print ("type(func)=", type(func))

  m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral',
        'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
        'weierstrassPPrime','weierstrassSigma']

  if debug:
    print ("m=",m)
    if m:
        print ("func ", func ," is special_function")
    else:
        print ("func ", func ," is NOT special_function")


  return m


def is_hypergeometric_function(func):
  return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
  return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

  #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file