Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.2-Quartic-trinomial/121-1.2.2.7

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May 18, 2024 Compiled on May 18, 2024 at 8:15pm

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**262**]. This is test number [121].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	72.14 (189)	27.86 (73)
Rubi	69.85 (183)	30.15 (79)
Maple	68.70 (180)	31.30 (82)
Fricas	35.88 (94)	64.12 (168)
Sympy	11.83 (31)	88.17 (231)
Reduce	11.45 (30)	88.55 (232)
Giac	5.73 (15)	94.27 (247)
Mupad	0.00 (0)	100.00 (262)
Maxima	0.00 (0)	100.00 (262)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

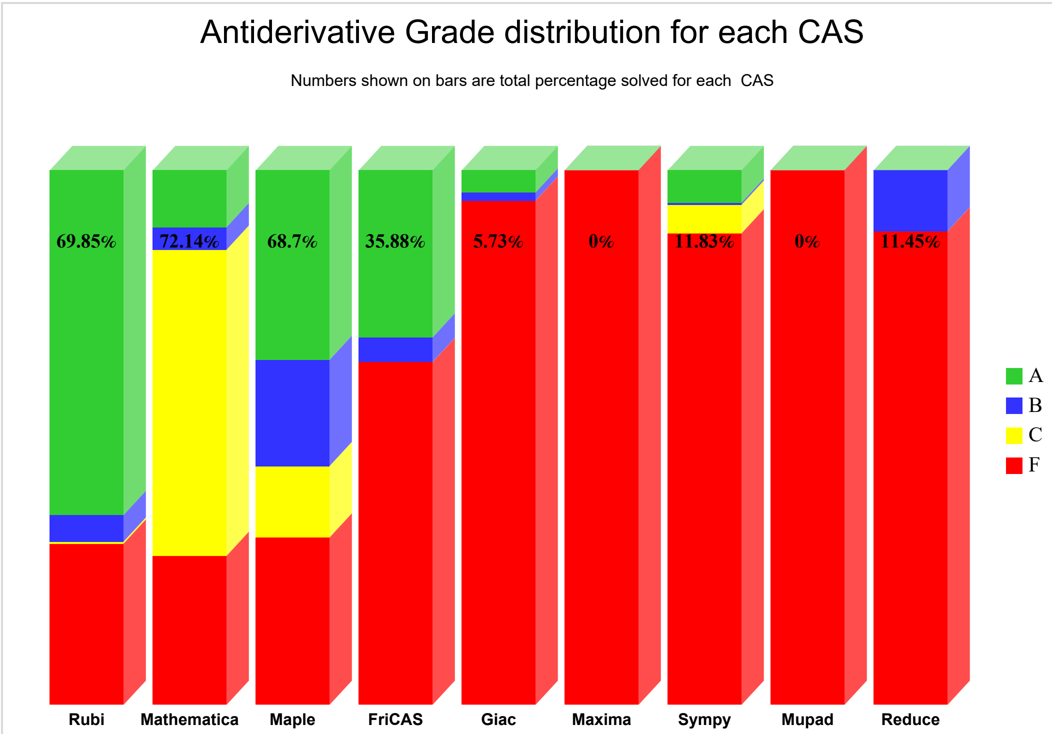
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

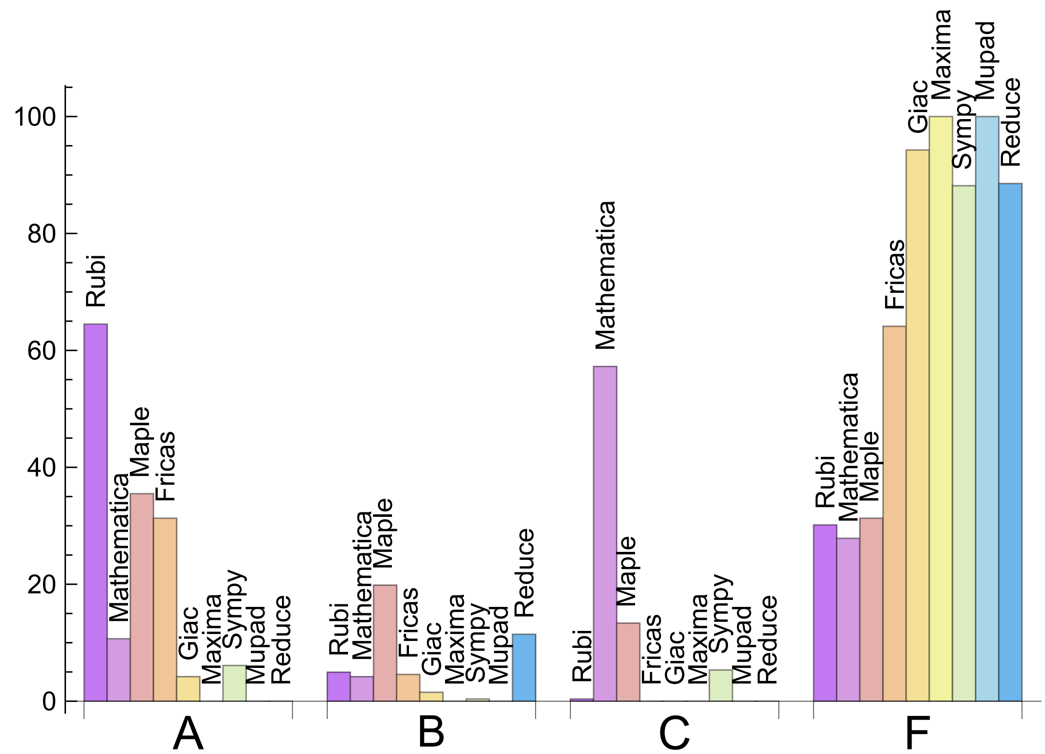
System	% A grade	% B grade	% C grade	% F grade
Rubi	64.504	4.962	0.382	30.153
Maple	35.496	19.847	13.359	31.298
Fricas	31.298	4.580	0.000	64.122
Mathematica	10.687	4.198	57.252	27.863
Sympy	6.107	0.382	5.344	88.168
Giac	4.198	1.527	0.000	94.275
Mupad	0.000	0.000	0.000	100.000
Maxima	0.000	0.000	0.000	100.000
Reduce	0.000	11.450	0.000	88.550

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	73	100.00	0.00	0.00
Rubi	79	100.00	0.00	0.00
Maple	82	98.78	1.22	0.00
Fricas	168	54.17	45.24	0.60
Sympy	231	78.35	21.65	0.00
Reduce	232	100.00	0.00	0.00
Giac	247	85.02	8.50	6.48
Mupad	262	0.00	100.00	0.00
Maxima	262	96.18	0.00	3.82

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.12
Reduce	0.22
Giac	0.30
Rubi	0.85
Maple	4.58
Mathematica	9.55
Sympy	12.30
Maxima	-nan(ind)
Mupad	-nan(ind)

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	281.39	0.89	178.00	0.74
Fricas	422.48	1.75	278.50	1.16
Rubi	438.79	1.28	288.00	1.08
Reduce	595.17	2.61	289.00	2.42
Maple	702.89	1.97	378.00	1.26
Giac	970.20	2.06	139.00	1.17
Mathematica	2946.01	3.56	193.00	0.92
Maxima	-nan(ind)	-nan(ind)	nan	nan
Mupad	-nan(ind)	-nan(ind)	nan	nan

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

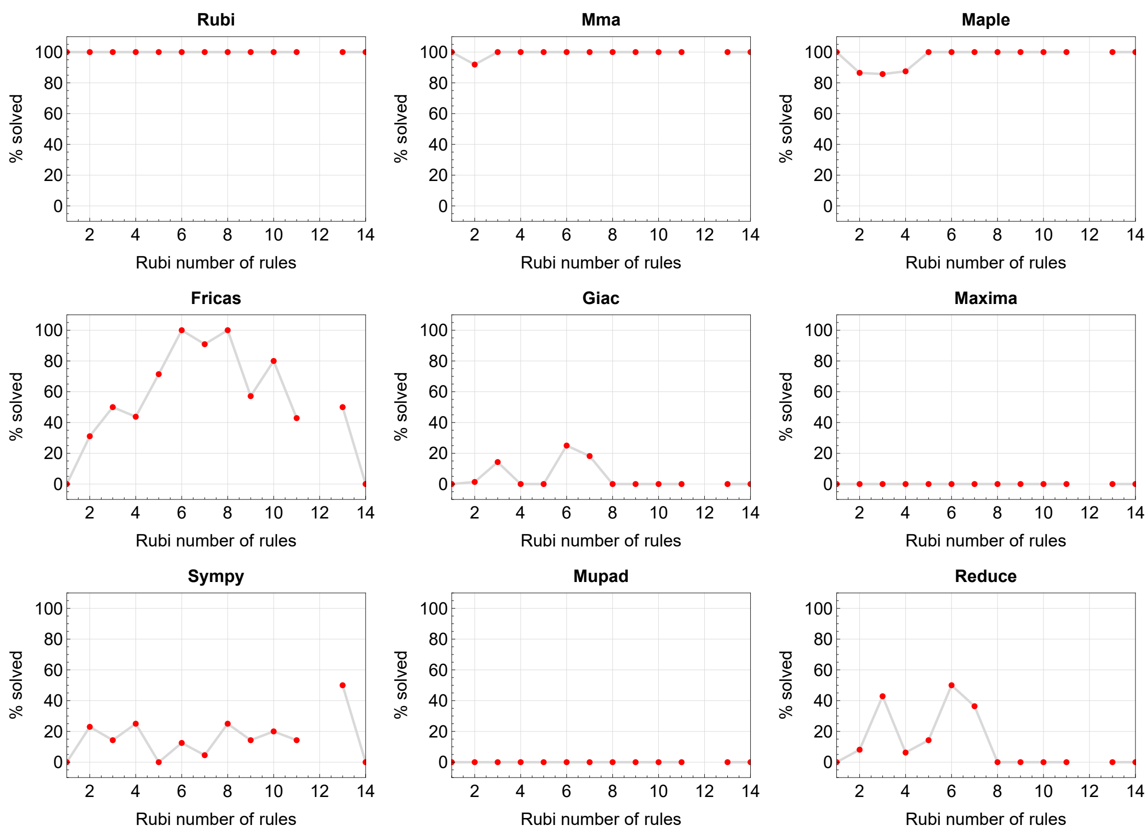


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

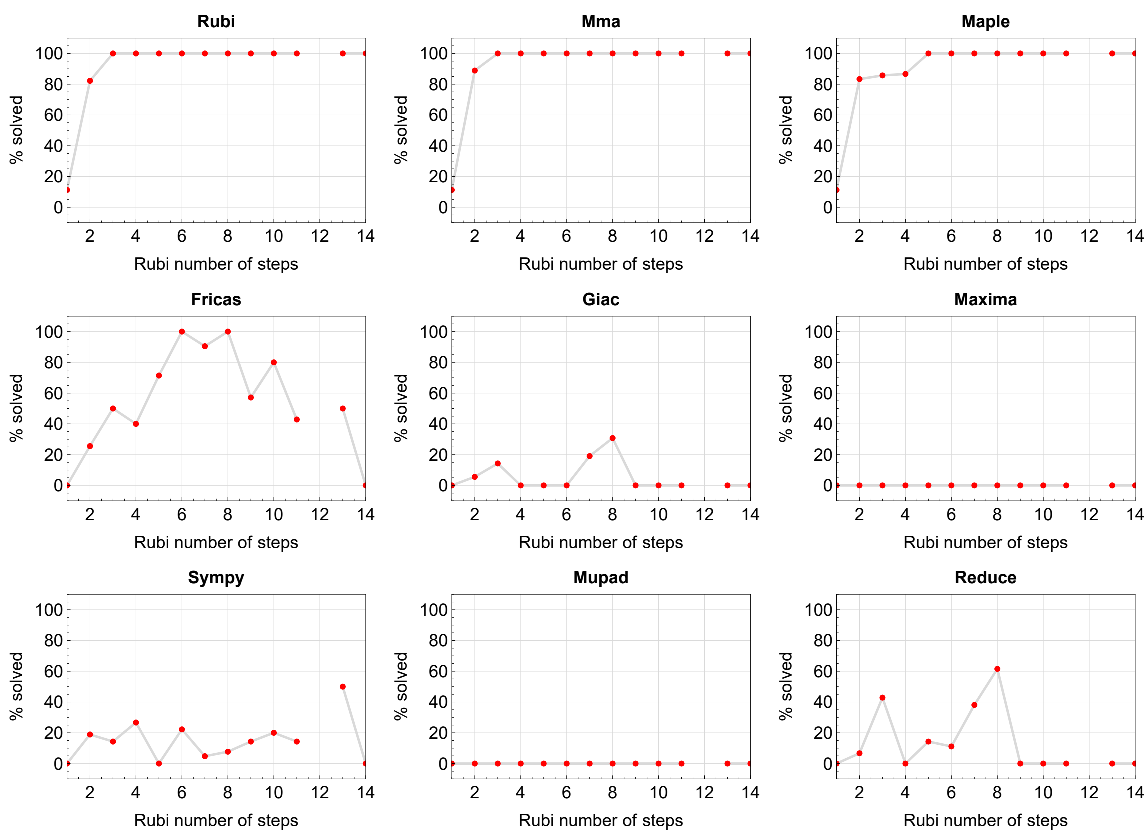


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

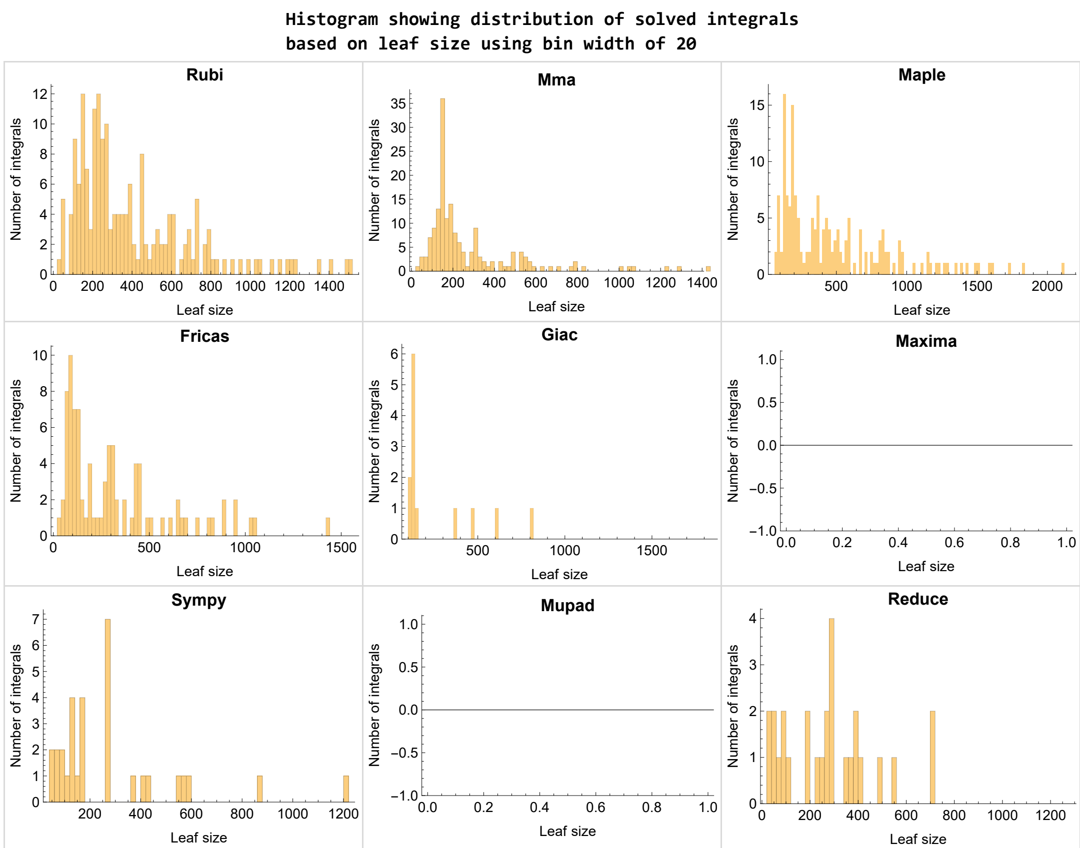


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

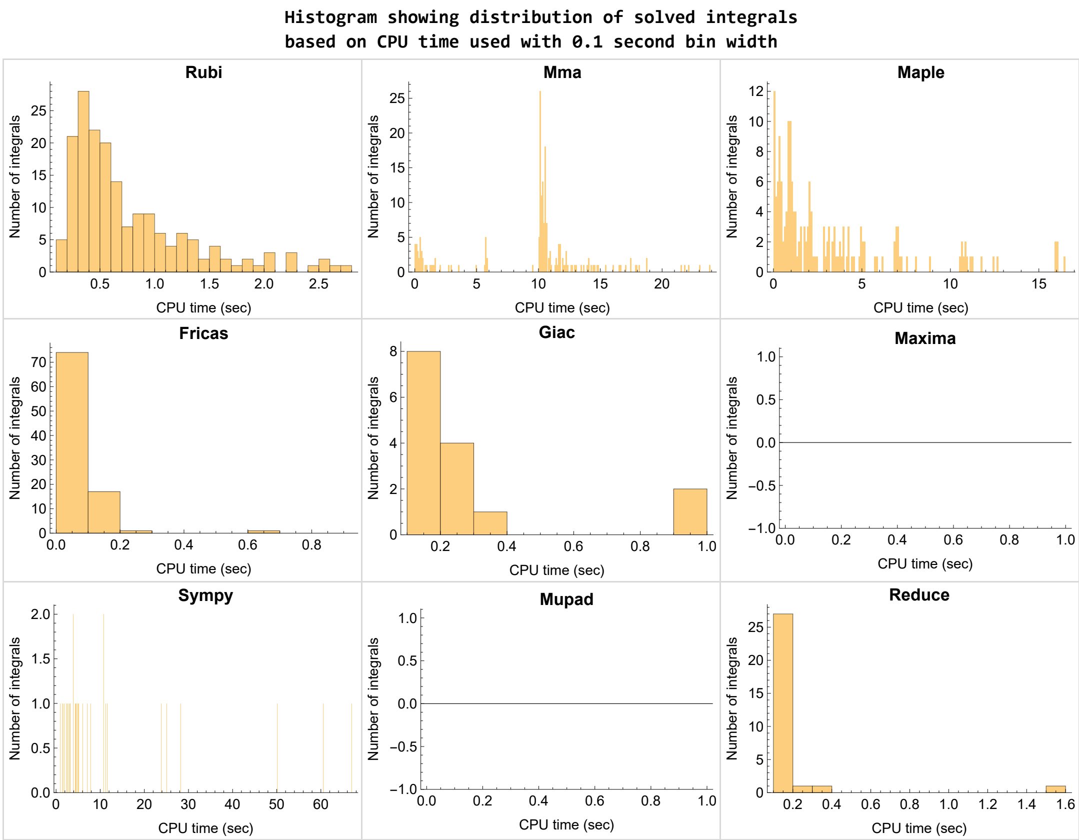


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

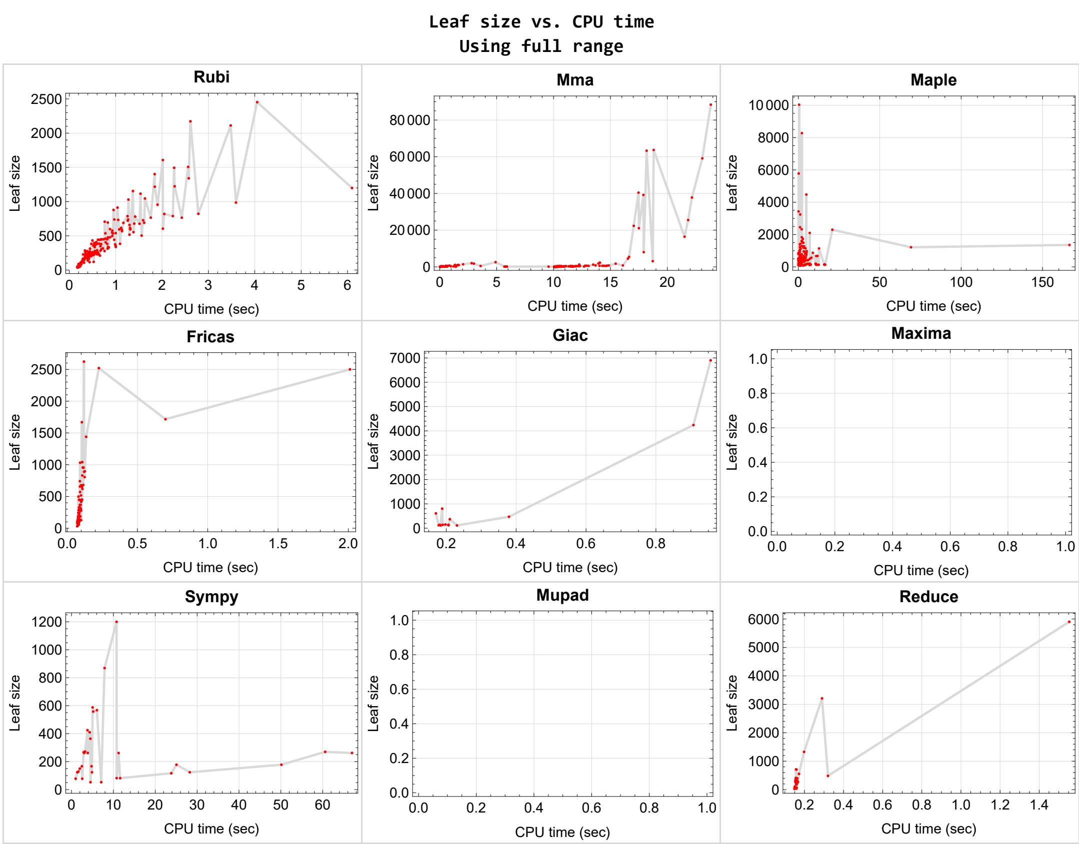


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{86, 88, 90, 132, 179, 189, 261\}$

Mathematica $\{37, 38, 58, 73, 79, 80, 160, 164, 165\}$

Maple $\{152\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

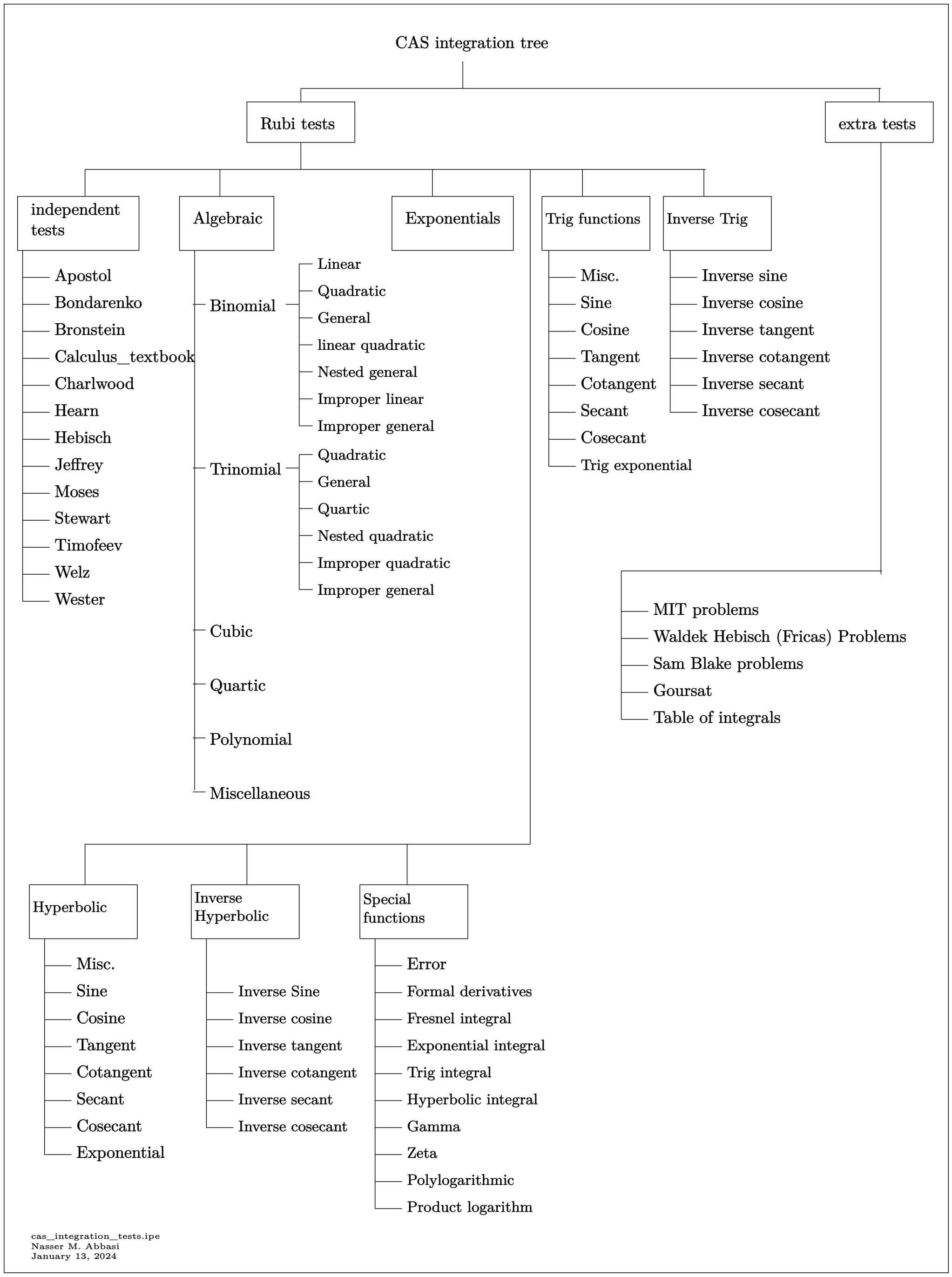
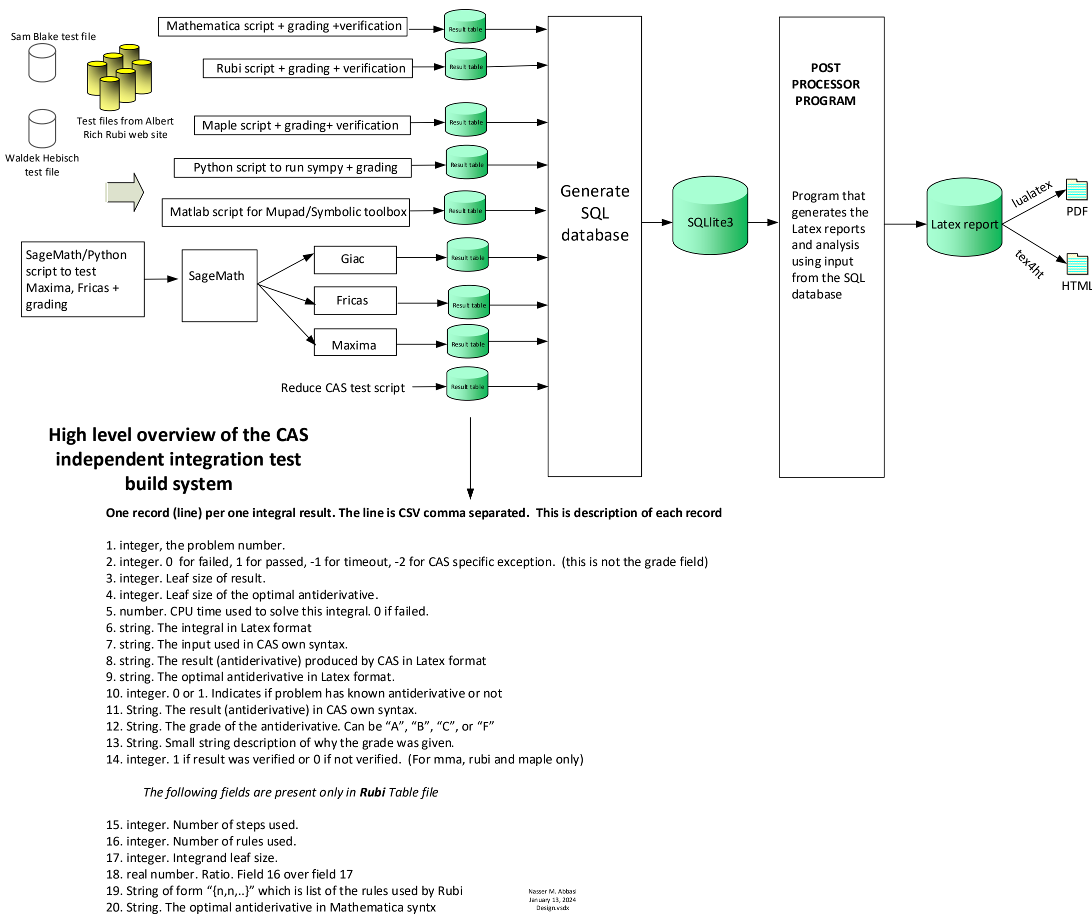


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 25, 29, 30, 31, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 87, 89, 90, 123, 124, 125, 127, 128, 129, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 255, 256, 257, 259, 260, 261 }

B grade { 26, 27, 28, 32, 33, 34, 35, 39, 66, 80, 86, 88, 132 }

C grade { 153 }

F normal fail { 19, 20, 21, 22, 23, 24, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 126, 130, 160, 161, 162, 163, 164, 165, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 258, 262 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 6, 8, 9, 11, 63, 64, 65, 66, 123, 124, 127, 128, 137, 138, 139, 142, 143, 146, 147, 148, 150, 151, 152, 157, 255, 256, 259, 260 }

B grade { 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165 }

C grade { 1, 2, 3, 4, 5, 7, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 131, 132, 133, 134, 135, 136, 140, 141, 144, 145, 149, 153, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223 }

F normal fail { 25, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 125, 126, 129, 130, 166, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 261, 262 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 3, 4, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 47, 48, 49, 50, 53, 54, 55, 56, 61, 62, 132, 137, 146, 147, 148, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 167, 168, 169, 170, 171, 174, 175, 176, 177, 180, 181, 182, 183, 184, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223 }

B grade { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 31, 32, 38, 39, 45, 46, 51, 52, 57, 58, 63, 64, 65, 66, 131, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 172, 173, 178, 179 }

C grade { 1, 59, 60, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 133, 134, 135, 136, 153, 185, 188, 189 }

F normal fail { 25, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 166, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262 }

F(-1) timedout fail { 165 }

F(-2) exception fail { }

Fricas

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 26, 27, 28, 29, 33, 34, 35, 36, 40, 41, 42, 43, 47, 48, 49, 50, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 167, 168, 169, 170, 177, 190, 191, 192, 193, 197, 198, 199, 200, 204, 205, 206, 207, 211, 212, 213, 214, 218, 219, 220, 221 }

B grade { 1, 2, 53, 131, 136, 149, 150, 151, 152, 174, 175, 176 }

C grade { }

F normal fail { 30, 32, 60, 62, 71, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 172, 185, 188, 189, 194, 195, 196, 201, 202, 203, 208, 209, 210, 215, 216, 217, 222, 223, 225, 227, 229, 230, 231, 232, 233, 235, 237, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262 }

F(-1) timedout fail { 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 31, 37, 38, 39, 44, 45, 46, 51, 52, 57, 58, 59, 61, 72, 73, 78, 79, 80, 82, 83, 84, 85, 86, 87, 101, 112, 113, 117, 132, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 171, 173, 178, 179, 180, 181, 182, 183, 184, 186, 187, 224, 226, 228, 234, 236, 238, 239, 243, 244, 254 }

F(-2) exception fail { 81 }

Maxima

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262 }

F(-1) timedout fail { }

F(-2) exception fail { 3, 4, 85, 86, 89, 131, 132, 180, 182, 186 }

Giac

A grade { 5, 6, 7, 8, 9, 10, 11, 12, 19, 20, 149 }

B grade { 150, 153, 160, 161 }

C grade { }

F normal fail { 1, 2, 3, 4, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 84, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 146, 147, 148, 151, 152, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262 }

F(-1) timedout fail { 17, 18, 21, 22, 23, 24, 138, 139, 140, 141, 142, 143, 144, 145, 158, 159, 162, 163, 164, 165, 186 }

F(-2) exception fail { 13, 14, 15, 16, 61, 81, 83, 85, 86, 89, 154, 155, 156, 157, 180, 182 }

Mupad

A grade { }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262 }

F(-2) exception fail { }

Sympy

A grade { 26, 27, 28, 29, 33, 34, 35, 36, 40, 41, 42, 43, 49, 50, 55, 56 }

B grade { 1 }

C grade { 67, 68, 69, 70, 76, 77, 123, 124, 127, 128, 255, 256, 259, 260 }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 25, 30, 31, 32, 37, 38, 39, 44, 45, 46, 47, 48, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 71, 72, 73, 74, 75, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 231, 232, 233, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 254 }

F(-1) timedout fail { 17, 18, 19, 20, 21, 22, 23, 24, 51, 52, 57, 58, 80, 97, 98, 104, 105, 106, 125, 126, 129, 130, 146, 152, 158, 159, 160, 161, 162, 163, 164, 165, 178, 179, 229, 230, 234, 235, 236, 237, 248, 249, 250, 251, 252, 253, 257, 258, 261, 262 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 5, 6, 7, 8, 9, 10, 11, 12, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159 }

C grade { }

F normal fail { 1, 2, 3, 4, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 153, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	81	77	77	0	109	150	0	56	0
N.S.	1	1.25	1.18	1.18	0.00	1.68	2.31	0.00	0.86	0.00
time (sec)	N/A	0.258	10.068	2.869	0.000	0.081	1.967	0.000	0.156	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	81	82	143	0	116	0	0	68	0
N.S.	1	1.25	1.26	2.20	0.00	1.78	0.00	0.00	1.05	0.00
time (sec)	N/A	0.237	1.418	1.796	0.000	0.082	0.000	0.000	0.197	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	255	158	238	0	190	0	0	210	0
N.S.	1	1.36	0.84	1.27	0.00	1.02	0.00	0.00	1.12	0.00
time (sec)	N/A	0.460	11.603	1.229	0.000	0.081	0.000	0.000	0.197	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	252	153	239	0	192	0	0	213	0
N.S.	1	1.35	0.82	1.28	0.00	1.03	0.00	0.00	1.15	0.00
time (sec)	N/A	0.456	11.603	0.970	0.000	0.089	0.000	0.000	0.175	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	116	138	434	0	301	0	134	184	0
N.S.	1	1.09	1.30	4.09	0.00	2.84	0.00	1.26	1.74	0.00
time (sec)	N/A	0.263	5.751	0.337	0.000	0.094	0.000	0.182	0.158	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	113	153	449	0	304	0	113	298	0
N.S.	1	1.06	1.43	4.20	0.00	2.84	0.00	1.06	2.79	0.00
time (sec)	N/A	0.269	5.782	0.197	0.000	0.096	0.000	0.231	0.160	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	124	150	480	0	308	0	139	289	0
N.S.	1	1.14	1.38	4.40	0.00	2.83	0.00	1.28	2.65	0.00
time (sec)	N/A	0.286	5.900	0.143	0.000	0.095	0.000	0.190	0.154	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	123	159	513	0	313	0	120	183	0
N.S.	1	1.10	1.42	4.58	0.00	2.79	0.00	1.07	1.63	0.00
time (sec)	N/A	0.286	5.793	0.102	0.000	0.095	0.000	0.207	0.160	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	152	511	0	275	0	121	267	0
N.S.	1	1.11	1.41	4.73	0.00	2.55	0.00	1.12	2.47	0.00
time (sec)	N/A	0.273	5.679	0.073	0.000	0.098	0.000	0.178	0.156	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	123	140	480	0	278	0	138	289	0
N.S.	1	1.13	1.28	4.40	0.00	2.55	0.00	1.27	2.65	0.00
time (sec)	N/A	0.271	5.737	0.069	0.000	0.095	0.000	0.206	0.153	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	116	163	451	0	282	0	114	298	0
N.S.	1	1.05	1.47	4.06	0.00	2.54	0.00	1.03	2.68	0.00
time (sec)	N/A	0.269	5.748	0.062	0.000	0.099	0.000	0.184	0.165	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	121	141	438	0	287	0	133	268	0
N.S.	1	1.06	1.24	3.84	0.00	2.52	0.00	1.17	2.35	0.00
time (sec)	N/A	0.269	5.892	0.066	0.000	0.096	0.000	0.181	0.159	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	717	819	1063	1275	0	0	0	0	250	0
N.S.	1	1.14	1.48	1.78	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	2.048	1.327	3.503	0.000	0.000	0.000	0.000	0.659	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	593	705	1059	0	0	0	0	154	0
N.S.	1	0.96	1.15	1.72	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.318	0.851	0.953	0.000	0.000	0.000	0.000	0.419	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	497	382	557	832	0	0	0	0	73	0
N.S.	1	0.77	1.12	1.67	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.818	0.524	0.390	0.000	0.000	0.000	0.000	0.270	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	230	325	913	0	0	0	0	99	0
N.S.	1	0.54	0.76	2.13	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.635	0.308	0.688	0.000	0.000	0.000	0.000	0.238	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	356	549	946	0	0	0	0	189	0
N.S.	1	0.74	1.14	1.97	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.999	0.675	0.871	0.000	0.000	0.000	0.000	0.350	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	637	553	839	1287	0	0	0	0	33	0
N.S.	1	0.87	1.32	2.02	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.384	1.510	2.225	0.000	0.000	0.000	0.000	200.022	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	A	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	752	0	2013	1733	0	0	0	803	349	0
N.S.	1	0.00	2.68	2.30	0.00	0.00	0.00	1.07	0.46	0.00
time (sec)	N/A	0.000	2.798	2.802	0.000	0.000	0.000	0.188	8.372	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	A	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	586	0	1422	1111	0	0	0	610	220	0
N.S.	1	0.00	2.43	1.90	0.00	0.00	0.00	1.04	0.38	0.00
time (sec)	N/A	0.000	2.054	1.371	0.000	0.000	0.000	0.170	3.147	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	0	1059	978	0	0	0	0	106	0
N.S.	1	0.00	2.31	2.13	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	1.607	0.636	0.000	0.000	0.000	0.000	1.676	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	0	1010	1179	0	0	0	0	156	0
N.S.	1	0.00	1.81	2.12	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.000	1.667	3.163	0.000	0.000	0.000	0.000	1.025	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	719	0	1758	1501	0	0	0	0	309	0
N.S.	1	0.00	2.45	2.09	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.000	2.999	2.459	0.000	0.000	0.000	0.000	46.919	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	928	0	2576	2297	0	0	0	0	33	0
N.S.	1	0.00	2.78	2.48	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	4.905	20.850	0.000	0.000	0.000	0.000	200.029	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	466	173	0	0	0	0	0	0	50	0
N.S.	1	0.37	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.414	0.000	0.000	0.000	0.000	0.000	0.000	0.432	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	1401	461	742	0	570	588	0	649	0
N.S.	1	2.43	0.80	1.29	0.00	0.99	1.02	0.00	1.12	0.00
time (sec)	N/A	1.842	10.559	4.918	0.000	0.092	5.031	0.000	0.262	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	1029	222	527	0	369	425	0	419	0
N.S.	1	2.47	0.53	1.26	0.00	0.88	1.02	0.00	1.00	0.00
time (sec)	N/A	1.276	10.469	3.105	0.000	0.091	3.819	0.000	0.241	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	738	149	352	0	241	272	0	244	0
N.S.	1	2.64	0.53	1.26	0.00	0.86	0.97	0.00	0.87	0.00
time (sec)	N/A	0.971	10.253	1.467	0.000	0.080	3.231	0.000	0.205	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	202	115	229	0	128	129	0	100	0
N.S.	1	1.04	0.59	1.17	0.00	0.66	0.66	0.00	0.51	0.00
time (sec)	N/A	0.468	10.099	0.733	0.000	0.086	1.638	0.000	0.181	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	598	388	412	0	0	0	0	424	0
N.S.	1	1.57	1.02	1.08	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.878	11.905	2.861	0.000	0.000	0.000	0.000	0.405	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	737	1282	970	0	0	0	0	1082	0
N.S.	1	1.83	3.18	2.41	0.00	0.00	0.00	0.00	2.68	0.00
time (sec)	N/A	1.269	12.256	5.134	0.000	0.000	0.000	0.000	1.016	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	597	1223	530	1832	0	0	0	0	0	0
N.S.	1	2.05	0.89	3.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.272	14.043	2.089	0.000	0.000	0.000	0.000	1.691	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	716	2173	483	860	0	741	1200	0	841	0
N.S.	1	3.03	0.67	1.20	0.00	1.03	1.68	0.00	1.17	0.00
time (sec)	N/A	2.612	10.647	4.939	0.000	0.091	10.748	0.000	0.283	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	1607	352	673	0	498	869	0	558	0
N.S.	1	3.14	0.69	1.31	0.00	0.97	1.70	0.00	1.09	0.00
time (sec)	N/A	2.017	10.408	3.159	0.000	0.081	7.886	0.000	0.261	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	1154	154	467	0	326	558	0	330	0
N.S.	1	3.37	0.45	1.37	0.00	0.95	1.63	0.00	0.96	0.00
time (sec)	N/A	1.373	10.246	1.886	0.000	0.084	5.169	0.000	0.223	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	243	119	302	0	166	267	0	139	0
N.S.	1	1.04	0.51	1.29	0.00	0.71	1.14	0.00	0.59	0.00
time (sec)	N/A	0.566	10.161	0.713	0.000	0.077	2.884	0.000	0.197	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	586	1115	568	636	0	0	0	0	892	0
N.S.	1	1.90	0.97	1.09	0.00	0.00	0.00	0.00	1.52	0.00
time (sec)	N/A	1.536	14.291	3.357	0.000	0.000	0.000	0.000	0.643	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	1217	2295	1146	0	0	0	0	0	0
N.S.	1	1.98	3.74	1.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.843	14.070	4.402	0.000	0.000	0.000	0.000	1.800	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	1506	579	2100	0	0	0	0	0	0
N.S.	1	2.32	0.89	3.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.562	14.433	6.948	0.000	0.000	0.000	0.000	3.571	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	676	304	408	0	445	568	0	459	0
N.S.	1	1.51	0.68	0.91	0.00	0.99	1.27	0.00	1.02	0.00
time (sec)	N/A	1.515	10.515	4.852	0.000	0.095	6.063	0.000	0.306	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	535	224	311	0	279	410	0	288	0
N.S.	1	1.65	0.69	0.96	0.00	0.86	1.26	0.00	0.89	0.00
time (sec)	N/A	0.940	10.340	3.039	0.000	0.092	4.337	0.000	0.259	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	407	150	227	0	186	262	0	164	0
N.S.	1	1.81	0.67	1.01	0.00	0.83	1.16	0.00	0.73	0.00
time (sec)	N/A	0.584	10.175	1.734	0.000	0.094	3.036	0.000	0.224	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	113	179	0	106	124	0	65	0
N.S.	1	1.05	0.73	1.16	0.00	0.69	0.81	0.00	0.42	0.00
time (sec)	N/A	0.422	10.102	0.714	0.000	0.086	1.373	0.000	0.182	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	229	207	342	0	0	0	0	127	0
N.S.	1	1.01	0.92	1.51	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.599	10.573	0.937	0.000	0.000	0.000	0.000	0.237	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	382	1223	718	0	0	0	0	199	0
N.S.	1	0.95	3.05	1.79	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	1.094	13.480	1.059	0.000	0.000	0.000	0.000	0.601	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	604	521	1616	0	0	0	0	271	0
N.S.	1	0.98	0.85	2.63	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	2.018	14.592	2.164	0.000	0.000	0.000	0.000	0.984	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	787	313	567	0	676	0	0	846	0
N.S.	1	1.74	0.69	1.25	0.00	1.50	0.00	0.00	1.87	0.00
time (sec)	N/A	1.256	10.515	8.008	0.000	0.100	0.000	0.000	0.361	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	549	227	409	0	436	0	0	551	0
N.S.	1	1.66	0.69	1.24	0.00	1.32	0.00	0.00	1.66	0.00
time (sec)	N/A	0.924	10.352	5.734	0.000	0.103	0.000	0.000	0.301	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	385	148	280	0	290	262	0	308	0
N.S.	1	1.73	0.66	1.26	0.00	1.30	1.17	0.00	1.38	0.00
time (sec)	N/A	0.643	10.211	0.872	0.000	0.092	11.237	0.000	0.259	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	176	117	222	0	144	124	0	103	0
N.S.	1	1.05	0.70	1.33	0.00	0.86	0.74	0.00	0.62	0.00
time (sec)	N/A	0.419	10.106	0.464	0.000	0.088	4.883	0.000	0.207	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	348	340	756	0	0	0	0	193	0
N.S.	1	1.00	0.97	2.17	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.671	11.955	1.005	0.000	0.000	0.000	0.000	0.831	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	582	780	480	1422	0	0	0	0	316	0
N.S.	1	1.34	0.82	2.44	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	1.420	12.172	1.089	0.000	0.000	0.000	0.000	1.240	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	954	363	596	0	954	0	0	1438	0
N.S.	1	1.84	0.70	1.15	0.00	1.84	0.00	0.00	2.77	0.00
time (sec)	N/A	1.903	10.696	4.576	0.000	0.117	0.000	0.000	0.448	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	678	280	447	0	651	0	0	975	0
N.S.	1	1.69	0.70	1.11	0.00	1.62	0.00	0.00	2.43	0.00
time (sec)	N/A	1.401	10.496	1.991	0.000	0.106	0.000	0.000	0.365	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	449	191	328	0	422	262	0	562	0
N.S.	1	1.59	0.67	1.16	0.00	1.49	0.93	0.00	1.99	0.00
time (sec)	N/A	0.821	10.291	0.827	0.000	0.101	66.994	0.000	0.290	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	222	151	270	0	239	124	0	355	0
N.S.	1	1.08	0.73	1.31	0.00	1.16	0.60	0.00	1.72	0.00
time (sec)	N/A	0.489	10.165	0.469	0.000	0.087	28.235	0.000	0.215	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	592	689	545	1221	0	0	0	0	271	0
N.S.	1	1.16	0.92	2.06	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	1.166	12.323	0.954	0.000	0.000	0.000	0.000	1.381	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	889	1340	782	2456	0	0	0	0	445	0
N.S.	1	1.51	0.88	2.76	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	2.576	13.691	1.004	0.000	0.000	0.000	0.000	2.260	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	114	149	187	0	0	0	0	91	0
N.S.	1	1.01	1.32	1.65	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.267	10.544	0.315	0.000	0.000	0.000	0.000	0.206	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	146	188	0	0	0	0	91	0
N.S.	1	1.00	1.28	1.65	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.281	10.367	0.930	0.000	0.000	0.000	0.000	0.207	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	243	149	187	0	0	0	0	92	0
N.S.	1	1.40	0.86	1.07	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.484	10.570	0.302	0.000	0.000	0.000	0.000	0.211	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	245	146	186	0	0	0	0	92	0
N.S.	1	1.37	0.82	1.04	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.486	10.371	0.918	0.000	0.000	0.000	0.000	0.208	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	38	88	0	31	0	0	28	0
N.S.	1	1.06	1.19	2.75	0.00	0.97	0.00	0.00	0.88	0.00
time (sec)	N/A	0.178	0.022	0.361	0.000	0.071	0.000	0.000	0.169	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	43	46	88	0	42	0	0	62	0
N.S.	1	1.19	1.28	2.44	0.00	1.17	0.00	0.00	1.72	0.00
time (sec)	N/A	0.211	0.038	0.370	0.000	0.077	0.000	0.000	0.157	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	53	46	116	0	58	0	0	127	0
N.S.	1	1.10	0.96	2.42	0.00	1.21	0.00	0.00	2.65	0.00
time (sec)	N/A	0.234	10.232	0.821	0.000	0.083	0.000	0.000	0.172	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	119	54	125	0	90	0	0	277	0
N.S.	1	2.12	0.96	2.23	0.00	1.61	0.00	0.00	4.95	0.00
time (sec)	N/A	0.525	10.246	1.137	0.000	0.079	0.000	0.000	0.178	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	878	217	327	0	283	364	0	357	0
N.S.	1	1.94	0.48	0.72	0.00	0.62	0.80	0.00	0.79	0.00
time (sec)	N/A	0.954	10.305	3.732	0.000	0.081	4.494	0.000	0.242	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	706	159	258	0	206	262	0	222	0
N.S.	1	1.92	0.43	0.70	0.00	0.56	0.71	0.00	0.60	0.00
time (sec)	N/A	0.763	10.189	2.118	0.000	0.084	3.876	0.000	0.223	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	432	121	203	0	127	167	0	124	0
N.S.	1	1.57	0.44	0.74	0.00	0.46	0.61	0.00	0.45	0.00
time (sec)	N/A	0.500	10.119	0.800	0.000	0.083	2.465	0.000	0.182	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	228	77	169	0	90	78	0	48	0
N.S.	1	1.00	0.34	0.74	0.00	0.40	0.34	0.00	0.21	0.00
time (sec)	N/A	0.299	10.042	0.275	0.000	0.073	0.984	0.000	0.177	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	382	138	192	0	0	0	0	78	0
N.S.	1	1.03	0.37	0.52	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.598	10.393	0.490	0.000	0.000	0.000	0.000	0.207	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	641	600	297	679	0	0	0	0	126	0
N.S.	1	0.94	0.46	1.06	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.311	11.315	0.869	0.000	0.000	0.000	0.000	0.676	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	875	788	453	1591	0	0	0	0	174	0
N.S.	1	0.90	0.52	1.82	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.233	12.266	2.015	0.000	0.000	0.000	0.000	1.605	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	912	222	426	0	448	0	0	829	0
N.S.	1	1.94	0.47	0.90	0.00	0.95	0.00	0.00	1.76	0.00
time (sec)	N/A	1.040	10.304	6.129	0.000	0.084	0.000	0.000	0.353	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	694	166	323	0	332	0	0	561	0
N.S.	1	1.83	0.44	0.85	0.00	0.88	0.00	0.00	1.48	0.00
time (sec)	N/A	0.830	10.215	0.872	0.000	0.087	0.000	0.000	0.291	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	395	126	253	0	188	167	0	345	0
N.S.	1	1.30	0.41	0.83	0.00	0.62	0.55	0.00	1.13	0.00
time (sec)	N/A	0.540	10.136	0.443	0.000	0.078	4.766	0.000	0.226	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	99	212	0	115	78	0	70	0
N.S.	1	1.00	0.38	0.81	0.00	0.44	0.30	0.00	0.27	0.00
time (sec)	N/A	0.347	10.059	0.290	0.000	0.082	2.558	0.000	0.194	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	732	432	564	0	0	0	0	126	0
N.S.	1	1.20	0.71	0.93	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.055	11.086	0.515	0.000	0.000	0.000	0.000	1.044	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	830	1494	427	1384	0	0	0	0	208	0
N.S.	1	1.80	0.51	1.67	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	2.261	11.675	0.904	0.000	0.000	0.000	0.000	1.640	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1142	2452	630	2326	0	0	0	0	290	0
N.S.	1	2.15	0.55	2.04	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	4.051	12.906	2.030	0.000	0.000	0.000	0.000	3.225	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	231	155	203	0	0	0	0	85	0
N.S.	1	1.00	0.67	0.88	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.351	10.541	0.344	0.000	0.000	0.000	0.000	0.217	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	228	152	204	0	0	0	0	85	0
N.S.	1	0.93	0.62	0.83	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.378	10.392	0.898	0.000	0.000	0.000	0.000	0.225	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	376	155	203	0	0	0	0	86	0
N.S.	1	1.05	0.43	0.57	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.544	10.536	0.330	0.000	0.000	0.000	0.000	0.210	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	372	152	202	0	0	0	0	86	0
N.S.	1	1.00	0.41	0.54	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.570	10.394	0.931	0.000	0.000	0.000	0.000	0.220	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	205	192	433	0	0	0	0	252	0
N.S.	1	1.04	0.97	2.19	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.410	10.966	3.427	0.000	0.000	0.000	0.000	3.099	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	615	207	588	0	0	0	0	252	0
N.S.	1	3.11	1.05	2.97	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	1.311	12.678	3.963	0.000	0.000	0.000	0.000	3.059	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	210	192	435	0	0	0	0	252	0
N.S.	1	1.04	0.95	2.15	0.00	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.353	10.639	2.950	0.000	0.000	0.000	0.000	3.007	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	616	190	582	0	0	0	0	342	0
N.S.	1	3.21	0.99	3.03	0.00	0.00	0.00	0.00	1.78	0.00
time (sec)	N/A	1.119	10.804	3.446	0.000	0.000	0.000	0.000	6.408	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	379	207	456	0	0	0	0	342	0
N.S.	1	1.26	0.69	1.51	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.687	10.593	4.265	0.000	0.000	0.000	0.000	6.497	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	587	190	580	0	0	0	0	342	0
N.S.	1	1.95	0.63	1.93	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	1.134	10.923	3.462	0.000	0.000	0.000	0.000	6.526	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	732	0	0	0	0	0	0	0	737	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.341	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	623	0	0	0	0	0	0	0	458	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.925	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	538	0	0	0	0	0	0	0	291	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.691	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	546	0	0	0	0	0	0	0	679	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.150	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	584	0	0	0	0	0	0	0	1630	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.79	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.743	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	577	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.770	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	781	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	11.271	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1002	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	63.213	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1007	0	0	0	0	0	0	0	1395	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.39	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.452	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	868	0	0	0	0	0	0	0	1026	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.664	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	732	0	0	0	0	0	0	0	788	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.403	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	725	0	0	0	0	0	0	0	1659	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.334	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	740	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.310	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	885	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	12.421	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	933	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	13.436	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	922	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	58.114	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	545	0	0	0	0	0	0	0	410	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.856	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	469	0	0	0	0	0	0	0	68	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.317	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	0	0	0	0	0	0	0	100	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.299	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	354	0	0	0	0	0	0	0	148	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.385	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	510	0	0	0	0	0	0	0	1005	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.97	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.185	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	669	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	7.789	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	568	0	0	0	0	0	0	0	1350	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.38	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.090	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	374	0	0	0	0	0	0	0	852	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.28	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.292	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	413	0	0	0	0	0	0	0	144	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.522	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	576	0	0	0	0	0	0	0	226	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.655	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	476	0	0	0	0	0	0	0	151	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.395	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	322	0	0	0	0	0	0	0	156	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.372	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1110	0	0	0	0	0	0	0	275	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.869	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	660	0	0	0	0	0	0	0	94	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.325	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1059	0	0	0	0	0	0	0	142	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.459	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	569	0	0	0	0	0	0	0	147	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.429	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	276	200	132	0	0	0	117	0	0	0
N.S.	1	0.72	0.48	0.00	0.00	0.00	0.42	0.00	0.00	0.00
time (sec)	N/A	0.340	0.229	0.000	0.000	0.000	23.822	0.000	0.182	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	53	0	498	0
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.57	0.00	5.35	0.00
time (sec)	N/A	0.198	0.106	0.000	0.000	0.000	7.091	0.000	0.158	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	0	0	0	0	0	0	50	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.390	0.000	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	449	0	0	0	0	0	0	0	72	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.369	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	425	256	163	0	0	0	178	0	0	0
N.S.	1	0.60	0.38	0.00	0.00	0.00	0.42	0.00	0.00	0.00
time (sec)	N/A	0.427	0.421	0.000	0.000	0.000	50.123	0.000	0.190	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	159	101	0	0	0	82	0	1394	0
N.S.	1	0.99	0.63	0.00	0.00	0.00	0.51	0.00	8.66	0.00
time (sec)	N/A	0.296	0.140	0.000	0.000	0.000	11.607	0.000	0.172	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	0	0	0	0	0	0	748	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	3.20	0.00
time (sec)	N/A	0.539	0.000	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	512	0	0	0	0	0	0	0	1493	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.92	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.065	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	312	247	559	0	516	0	0	192	0
N.S.	1	1.20	0.95	2.14	0.00	1.98	0.00	0.00	0.74	0.00
time (sec)	N/A	0.446	11.602	2.366	0.000	0.099	0.000	0.000	0.787	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F(-2)	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	721	152	285	0	0	0	0	174	0
N.S.	1	2.47	0.52	0.98	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	1.260	10.688	1.882	0.000	0.000	0.000	0.000	0.372	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	50	94	80	0	66	0	0	26	0
N.S.	1	1.04	1.96	1.67	0.00	1.38	0.00	0.00	0.54	0.00
time (sec)	N/A	0.175	10.156	0.934	0.000	0.077	0.000	0.000	0.151	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	94	80	0	66	0	0	26	0
N.S.	1	1.00	1.96	1.67	0.00	1.38	0.00	0.00	0.54	0.00
time (sec)	N/A	0.172	10.066	1.257	0.000	0.081	0.000	0.000	0.166	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	94	80	0	66	0	0	26	0
N.S.	1	1.00	1.96	1.67	0.00	1.38	0.00	0.00	0.54	0.00
time (sec)	N/A	0.177	10.148	0.590	0.000	0.080	0.000	0.000	0.163	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	115	226	329	0	127	0	0	113	0
N.S.	1	1.72	3.37	4.91	0.00	1.90	0.00	0.00	1.69	0.00
time (sec)	N/A	0.437	10.599	1.506	0.000	0.100	0.000	0.000	0.187	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	106	106	97	0	283	0	0	40	0
N.S.	1	0.98	0.98	0.90	0.00	2.62	0.00	0.00	0.37	0.00
time (sec)	N/A	0.239	0.251	0.068	0.000	0.093	0.000	0.000	0.158	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	141	176	842	0	804	0	0	709	0
N.S.	1	0.99	1.23	5.89	0.00	5.62	0.00	0.00	4.96	0.00
time (sec)	N/A	0.295	0.550	0.405	0.000	0.124	0.000	0.000	0.160	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	149	183	829	0	888	0	0	412	0
N.S.	1	1.01	1.24	5.60	0.00	6.00	0.00	0.00	2.78	0.00
time (sec)	N/A	0.315	0.555	0.386	0.000	0.120	0.000	0.000	0.156	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	148	178	805	0	896	0	0	230	0
N.S.	1	0.99	1.19	5.37	0.00	5.97	0.00	0.00	1.53	0.00
time (sec)	N/A	0.307	11.778	0.089	0.000	0.126	0.000	0.000	0.157	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	154	182	837	0	830	0	0	710	0
N.S.	1	1.03	1.22	5.62	0.00	5.57	0.00	0.00	4.77	0.00
time (sec)	N/A	0.305	11.708	0.094	0.000	0.108	0.000	0.000	0.157	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	146	170	858	0	416	0	0	393	0
N.S.	1	1.01	1.17	5.92	0.00	2.87	0.00	0.00	2.71	0.00
time (sec)	N/A	0.303	0.462	0.207	0.000	0.100	0.000	0.000	0.159	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	140	188	807	0	446	0	0	370	0
N.S.	1	0.96	1.29	5.53	0.00	3.05	0.00	0.00	2.53	0.00
time (sec)	N/A	0.296	11.713	0.189	0.000	0.105	0.000	0.000	0.157	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	151	190	824	0	450	0	0	351	0
N.S.	1	1.02	1.28	5.57	0.00	3.04	0.00	0.00	2.37	0.00
time (sec)	N/A	0.308	11.705	0.081	0.000	0.106	0.000	0.000	0.155	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	147	176	807	0	428	0	0	394	0
N.S.	1	0.97	1.17	5.34	0.00	2.83	0.00	0.00	2.61	0.00
time (sec)	N/A	0.300	11.478	0.086	0.000	0.100	0.000	0.000	0.163	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	536	320	385	0	955	0	0	484	0
N.S.	1	1.20	0.72	0.87	0.00	2.15	0.00	0.00	1.09	0.00
time (sec)	N/A	0.778	0.994	0.079	0.000	0.111	0.000	0.000	0.320	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	335	195	217	0	619	0	0	258	0
N.S.	1	1.18	0.69	0.76	0.00	2.18	0.00	0.00	0.91	0.00
time (sec)	N/A	0.548	0.457	0.063	0.000	0.109	0.000	0.000	0.167	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	154	130	131	0	367	0	0	105	0
N.S.	1	0.89	0.75	0.76	0.00	2.12	0.00	0.00	0.61	0.00
time (sec)	N/A	0.307	0.467	0.055	0.000	0.097	0.000	0.000	0.151	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	210	486	465	0	1716	0	150	554	0
N.S.	1	0.99	2.28	2.18	0.00	8.06	0.00	0.70	2.60	0.00
time (sec)	N/A	0.661	3.590	0.291	0.000	0.699	0.000	0.198	0.172	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	277	246	3431	0	2501	0	469	1328	0
N.S.	1	1.13	1.00	13.95	0.00	10.17	0.00	1.91	5.40	0.00
time (sec)	N/A	0.761	1.228	0.112	0.000	2.011	0.000	0.380	0.198	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	442	392	5779	0	1439	0	0	3209	0
N.S.	1	1.49	1.32	19.46	0.00	4.85	0.00	0.00	10.80	0.00
time (sec)	N/A	0.969	12.030	0.210	0.000	0.135	0.000	0.000	0.290	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	464	658	537	10036	0	2520	0	0	5903	0
N.S.	1	1.42	1.16	21.63	0.00	5.43	0.00	0.00	12.72	0.00
time (sec)	N/A	1.299	13.077	0.456	0.000	0.226	0.000	0.000	1.554	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	139	143	71	0	61	0	368	107	0
N.S.	1	2.57	2.65	1.31	0.00	1.13	0.00	6.81	1.98	0.00
time (sec)	N/A	0.398	0.319	1.257	0.000	0.078	0.000	0.210	0.161	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1233	1198	63315	954	0	0	0	0	80	0
N.S.	1	0.97	51.35	0.77	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	6.097	18.177	3.322	0.000	0.000	0.000	0.000	0.154	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	825	821	40458	702	0	0	0	0	61	0
N.S.	1	1.00	49.04	0.85	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	2.783	17.466	1.507	0.000	0.000	0.000	0.000	0.161	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	516	22359	490	0	0	0	0	40	0
N.S.	1	1.02	44.36	0.97	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.298	17.046	0.758	0.000	0.000	0.000	0.000	0.152	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	329	327	0	0	0	0	25	0
N.S.	1	1.00	0.98	0.97	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.002	11.599	0.297	0.000	0.000	0.000	0.000	0.149	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	503	16421	522	0	0	0	0	39	0
N.S.	1	1.12	36.57	1.16	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.559	21.508	0.681	0.000	0.000	0.000	0.000	0.158	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	687	764	25519	779	0	0	0	0	84	0
N.S.	1	1.11	37.15	1.13	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	2.426	21.825	2.053	0.000	0.000	0.000	0.000	0.150	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1335	0	63689	1214	0	0	0	6900	94	0
N.S.	1	0.00	47.71	0.91	0.00	0.00	0.00	5.17	0.07	0.00
time (sec)	N/A	0.000	18.797	69.178	0.000	0.000	0.000	0.957	0.546	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	A	F	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	867	0	39198	948	0	0	0	4241	58	0
N.S.	1	0.00	45.21	1.09	0.00	0.00	0.00	4.89	0.07	0.00
time (sec)	N/A	0.000	17.895	3.682	0.000	0.000	0.000	0.907	0.251	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	A	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	0	21061	579	0	0	0	0	25	0
N.S.	1	0.00	35.82	0.98	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	17.498	1.753	0.000	0.000	0.000	0.000	0.179	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	A	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	833	0	37781	872	0	0	0	0	41	0
N.S.	1	0.00	45.36	1.05	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	22.159	8.884	0.000	0.000	0.000	0.000	0.164	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	A	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1348	0	59143	1358	0	0	0	0	86	0
N.S.	1	0.00	43.87	1.01	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	23.068	166.522	0.000	0.000	0.000	0.000	0.354	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	F(-1)	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	2548	0	88367	0	0	0	0	0	146	0
N.S.	1	0.00	34.68	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	23.819	0.000	0.000	0.000	0.000	0.000	139.406	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	547	222	0	0	0	0	0	0	60	0
N.S.	1	0.41	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.511	0.000	0.000	0.000	0.000	0.000	0.000	0.570	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	755	690	4473	673	0	1041	0	0	810	0
N.S.	1	0.91	5.92	0.89	0.00	1.38	0.00	0.00	1.07	0.00
time (sec)	N/A	1.615	16.530	10.950	0.000	0.108	0.000	0.000	0.341	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	498	674	508	0	681	0	0	452	0
N.S.	1	0.94	1.28	0.96	0.00	1.29	0.00	0.00	0.86	0.00
time (sec)	N/A	0.928	14.704	7.101	0.000	0.114	0.000	0.000	0.270	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	358	521	409	0	435	0	0	208	0
N.S.	1	0.97	1.42	1.11	0.00	1.18	0.00	0.00	0.57	0.00
time (sec)	N/A	0.519	12.248	3.724	0.000	0.097	0.000	0.000	0.224	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	285	302	362	0	300	0	0	68	0
N.S.	1	1.00	1.06	1.27	0.00	1.06	0.00	0.00	0.24	0.00
time (sec)	N/A	0.337	0.187	0.804	0.000	0.088	0.000	0.000	0.183	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	449	298	359	0	0	0	0	112	0
N.S.	1	1.02	0.68	0.82	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.699	10.628	1.170	0.000	0.000	0.000	0.000	0.248	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	782	727	1853	1495	0	0	0	0	184	0
N.S.	1	0.93	2.37	1.91	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.592	13.946	3.070	0.000	0.000	0.000	0.000	1.410	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1125	985	781	4476	0	0	0	0	256	0
N.S.	1	0.88	0.69	3.98	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	3.594	16.092	4.904	0.000	0.000	0.000	0.000	8.745	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	859	765	5432	1141	0	2623	0	0	0	0
N.S.	1	0.89	6.32	1.33	0.00	3.05	0.00	0.00	0.00	0.00
time (sec)	N/A	1.758	16.635	12.678	0.000	0.120	0.000	0.000	0.947	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	576	766	799	0	1669	0	0	1565	0
N.S.	1	0.91	1.21	1.26	0.00	2.64	0.00	0.00	2.47	0.00
time (sec)	N/A	0.928	14.854	4.227	0.000	0.105	0.000	0.000	0.625	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	446	597	593	0	1031	0	0	831	0
N.S.	1	0.93	1.24	1.23	0.00	2.14	0.00	0.00	1.73	0.00
time (sec)	N/A	0.682	12.767	1.441	0.000	0.092	0.000	0.000	0.419	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	382	497	509	0	656	0	0	122	0
N.S.	1	0.96	1.25	1.28	0.00	1.65	0.00	0.00	0.31	0.00
time (sec)	N/A	0.476	11.492	0.827	0.000	0.091	0.000	0.000	0.256	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	882	1045	1736	3241	0	0	0	0	232	0
N.S.	1	1.18	1.97	3.67	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.631	15.456	1.157	0.000	0.000	0.000	0.000	2.881	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1247	2112	8031	8276	0	0	0	0	394	0
N.S.	1	1.69	6.44	6.64	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	3.481	17.921	2.172	0.000	0.000	0.000	0.000	18.875	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	272	315	370	0	0	0	0	119	0
N.S.	1	1.00	1.15	1.36	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.388	10.903	1.008	0.000	0.000	0.000	0.000	0.260	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	271	312	371	0	0	0	0	119	0
N.S.	1	0.94	1.08	1.29	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.444	10.574	2.007	0.000	0.000	0.000	0.000	0.286	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	443	315	370	0	0	0	0	120	0
N.S.	1	1.04	0.74	0.87	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.620	10.695	1.037	0.000	0.000	0.000	0.000	0.278	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	441	312	368	0	0	0	0	120	0
N.S.	1	1.00	0.71	0.83	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.654	10.542	2.102	0.000	0.000	0.000	0.000	0.279	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	449	298	359	0	0	0	0	112	0
N.S.	1	1.02	0.68	0.82	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.743	0.213	1.135	0.000	0.000	0.000	0.000	0.242	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	74	93	0	0	0	0	74	0
N.S.	1	1.00	0.70	0.88	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.318	10.324	1.234	0.000	0.000	0.000	0.000	0.192	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	310	366	0	0	0	0	114	0
N.S.	1	1.00	1.14	1.34	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.362	10.534	0.988	0.000	0.000	0.000	0.000	0.257	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	271	312	371	0	0	0	0	119	0
N.S.	1	0.94	1.08	1.29	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.387	0.116	1.964	0.000	0.000	0.000	0.000	0.261	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	108	199	0	0	0	0	159	0
N.S.	1	1.00	0.73	1.34	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	0.325	10.511	1.576	0.000	0.000	0.000	0.000	0.205	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	593	3113	674	0	0	0	0	56	0
N.S.	1	1.49	7.84	1.70	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.077	18.715	11.763	0.000	0.000	0.000	0.000	200.038	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	283	155	145	0	92	0	0	174	0
N.S.	1	1.10	0.60	0.56	0.00	0.36	0.00	0.00	0.68	0.00
time (sec)	N/A	0.674	10.187	16.055	0.000	0.078	0.000	0.000	0.201	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	255	150	140	0	87	0	0	156	0
N.S.	1	1.09	0.64	0.60	0.00	0.37	0.00	0.00	0.67	0.00
time (sec)	N/A	0.547	10.159	10.841	0.000	0.072	0.000	0.000	0.179	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	227	145	135	0	82	0	0	138	0
N.S.	1	1.08	0.69	0.64	0.00	0.39	0.00	0.00	0.65	0.00
time (sec)	N/A	0.445	10.120	7.081	0.000	0.075	0.000	0.000	0.191	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	199	140	130	0	77	0	0	120	0
N.S.	1	1.06	0.74	0.69	0.00	0.41	0.00	0.00	0.64	0.00
time (sec)	N/A	0.347	9.561	3.922	0.000	0.073	0.000	0.000	0.182	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	359	183	172	0	0	0	0	152	0
N.S.	1	1.44	0.73	0.69	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.549	10.363	5.141	0.000	0.000	0.000	0.000	0.193	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	215	179	0	0	0	0	318	0
N.S.	1	1.00	0.84	0.70	0.00	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	0.617	10.460	7.241	0.000	0.000	0.000	0.000	0.209	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	329	213	182	0	0	0	0	767	0
N.S.	1	1.24	0.80	0.69	0.00	0.00	0.00	0.00	2.89	0.00
time (sec)	N/A	0.853	10.528	10.582	0.000	0.000	0.000	0.000	0.224	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	316	165	155	0	102	0	0	210	0
N.S.	1	1.11	0.58	0.54	0.00	0.36	0.00	0.00	0.74	0.00
time (sec)	N/A	0.725	10.186	16.057	0.000	0.086	0.000	0.000	0.194	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	288	160	150	0	97	0	0	192	0
N.S.	1	1.10	0.61	0.57	0.00	0.37	0.00	0.00	0.73	0.00
time (sec)	N/A	0.600	10.170	10.861	0.000	0.084	0.000	0.000	0.192	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	260	155	145	0	92	0	0	174	0
N.S.	1	1.09	0.65	0.61	0.00	0.38	0.00	0.00	0.73	0.00
time (sec)	N/A	0.503	10.131	7.027	0.000	0.073	0.000	0.000	0.195	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	232	150	140	0	87	0	0	156	0
N.S.	1	1.07	0.69	0.65	0.00	0.40	0.00	0.00	0.72	0.00
time (sec)	N/A	0.399	10.118	3.950	0.000	0.078	0.000	0.000	0.180	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	405	193	183	0	0	0	0	188	0
N.S.	1	1.37	0.65	0.62	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.785	10.386	5.083	0.000	0.000	0.000	0.000	0.201	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	478	225	192	0	0	0	0	354	0
N.S.	1	1.58	0.75	0.64	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	0.881	10.517	6.889	0.000	0.000	0.000	0.000	0.208	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	375	204	192	0	0	0	0	803	0
N.S.	1	1.21	0.66	0.62	0.00	0.00	0.00	0.00	2.60	0.00
time (sec)	N/A	0.989	10.651	10.690	0.000	0.000	0.000	0.000	0.256	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	250	145	135	0	82	0	0	138	0
N.S.	1	1.09	0.63	0.59	0.00	0.36	0.00	0.00	0.60	0.00
time (sec)	N/A	0.623	10.174	15.972	0.000	0.072	0.000	0.000	0.207	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	222	138	130	0	77	0	0	120	0
N.S.	1	1.08	0.67	0.63	0.00	0.37	0.00	0.00	0.58	0.00
time (sec)	N/A	0.531	10.156	11.268	0.000	0.078	0.000	0.000	0.219	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	194	135	125	0	72	0	0	102	0
N.S.	1	1.06	0.74	0.68	0.00	0.39	0.00	0.00	0.56	0.00
time (sec)	N/A	0.411	10.120	6.826	0.000	0.083	0.000	0.000	0.190	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	166	128	118	0	67	0	0	84	0
N.S.	1	1.04	0.80	0.74	0.00	0.42	0.00	0.00	0.52	0.00
time (sec)	N/A	0.305	10.115	4.108	0.000	0.077	0.000	0.000	0.175	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	267	106	136	0	0	0	0	116	0
N.S.	1	1.33	0.53	0.68	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.568	10.277	2.096	0.000	0.000	0.000	0.000	0.187	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	302	189	162	0	0	0	0	131	0
N.S.	1	1.29	0.80	0.69	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.690	10.482	6.924	0.000	0.000	0.000	0.000	0.204	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	337	213	182	0	0	0	0	767	0
N.S.	1	1.27	0.80	0.69	0.00	0.00	0.00	0.00	2.89	0.00
time (sec)	N/A	0.834	10.598	10.767	0.000	0.000	0.000	0.000	0.222	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	223	133	135	0	121	0	0	359	0
N.S.	1	1.06	0.63	0.64	0.00	0.57	0.00	0.00	1.70	0.00
time (sec)	N/A	0.519	10.180	15.969	0.000	0.077	0.000	0.000	0.194	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	199	128	130	0	116	0	0	341	0
N.S.	1	1.06	0.68	0.69	0.00	0.62	0.00	0.00	1.81	0.00
time (sec)	N/A	0.440	10.150	12.485	0.000	0.075	0.000	0.000	0.194	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	173	123	125	0	111	0	0	323	0
N.S.	1	1.04	0.74	0.75	0.00	0.66	0.00	0.00	1.93	0.00
time (sec)	N/A	0.346	10.130	6.913	0.000	0.074	0.000	0.000	0.189	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	168	125	124	0	88	0	0	305	0
N.S.	1	1.29	0.96	0.95	0.00	0.68	0.00	0.00	2.35	0.00
time (sec)	N/A	0.317	10.109	4.269	0.000	0.079	0.000	0.000	0.180	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	278	173	161	0	0	0	0	146	0
N.S.	1	1.25	0.78	0.72	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.551	10.325	5.030	0.000	0.000	0.000	0.000	0.224	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	476	254	186	0	0	0	0	161	0
N.S.	1	1.87	1.00	0.73	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.833	10.537	7.055	0.000	0.000	0.000	0.000	0.221	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	567	194	192	0	0	0	0	176	0
N.S.	1	1.94	0.66	0.66	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	1.137	10.875	10.617	0.000	0.000	0.000	0.000	0.249	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	206	159	135	0	151	0	0	655	0
N.S.	1	1.06	0.82	0.69	0.00	0.77	0.00	0.00	3.36	0.00
time (sec)	N/A	0.444	10.251	16.479	0.000	0.082	0.000	0.000	0.206	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	206	159	135	0	128	0	0	637	0
N.S.	1	1.29	0.99	0.84	0.00	0.80	0.00	0.00	3.98	0.00
time (sec)	N/A	0.444	10.211	11.136	0.000	0.089	0.000	0.000	0.215	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	206	159	135	0	128	0	0	619	0
N.S.	1	1.29	0.99	0.84	0.00	0.80	0.00	0.00	3.87	0.00
time (sec)	N/A	0.390	10.202	7.589	0.000	0.080	0.000	0.000	0.212	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	206	159	135	0	128	0	0	601	0
N.S.	1	1.30	1.01	0.85	0.00	0.81	0.00	0.00	3.80	0.00
time (sec)	N/A	0.354	10.179	3.809	0.000	0.074	0.000	0.000	0.233	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	458	241	197	0	0	0	0	176	0
N.S.	1	1.81	0.95	0.78	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.779	10.421	1.980	0.000	0.000	0.000	0.000	0.231	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	539	267	197	0	0	0	0	191	0
N.S.	1	1.86	0.92	0.68	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	1.001	10.498	5.853	0.000	0.000	0.000	0.000	0.252	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1241	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4.756	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	964	0	0	0	0	0	0	0	1034	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.557	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	805	0	0	0	0	0	0	0	502	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.441	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	754	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	7.792	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	837	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	38.180	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	810	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.033	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1240	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.037	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1564	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1243	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.531	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1097	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1035	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1311	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1429	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1593	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	814	0	0	0	0	0	0	0	653	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.742	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	690	0	0	0	0	0	0	0	84	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.433	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	430	0	0	0	0	0	0	0	128	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.399	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	470	0	0	0	0	0	0	0	200	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.281	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	660	0	0	0	0	0	0	0	272	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	10.160	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1091	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	82.952	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	895	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	47.276	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	533	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	13.990	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	611	0	0	0	0	0	0	0	248	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	7.581	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	905	0	0	0	0	0	0	0	410	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	33.340	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1562	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	956	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	845	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	965	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1344	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.032	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	2023	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.050	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	697	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	318	210	140	0	0	0	178	0	0	0
N.S.	1	0.66	0.44	0.00	0.00	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.380	0.419	0.000	0.000	0.000	25.058	0.000	0.197	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	53	0	498	0
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.57	0.00	5.35	0.00
time (sec)	N/A	0.205	0.038	0.000	0.000	0.000	4.526	0.000	0.176	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	0	0	0	0	0	0	60	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.506	0.000	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	857	0	0	0	0	0	0	0	114	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.122	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	529	269	174	0	0	0	270	0	0	0
N.S.	1	0.51	0.33	0.00	0.00	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.441	0.666	0.000	0.000	0.000	60.586	0.000	0.213	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	159	101	0	0	0	82	0	1394	0
N.S.	1	0.99	0.63	0.00	0.00	0.00	0.51	0.00	8.66	0.00
time (sec)	N/A	0.291	0.021	0.000	0.000	0.000	10.778	0.000	0.185	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	0	0	0	0	0	0	94	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.826	0.000	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1114	0	0	0	0	0	0	0	26	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.182	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [36] had the largest ratio of [.52000000000000018]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	7	1.25	29	0.241
2	A	6	6	1.25	31	0.194
3	A	10	10	1.36	36	0.278
4	A	10	10	1.35	37	0.270
5	A	7	6	1.09	35	0.171
6	A	7	6	1.06	36	0.167
7	A	8	7	1.14	37	0.189
8	A	8	7	1.10	38	0.184
9	A	7	6	1.11	36	0.167
10	A	7	6	1.13	37	0.162
11	A	8	7	1.05	38	0.184
12	A	8	7	1.06	39	0.179
13	A	2	2	1.14	33	0.061
14	A	2	2	0.96	33	0.061
15	A	2	2	0.77	33	0.061
16	A	2	2	0.54	33	0.061
17	A	2	2	0.74	33	0.061
18	A	2	2	0.87	33	0.061
19	F	0	0	N/A	0.000	N/A
20	F	0	0	N/A	0.000	N/A
21	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	F	0	0	N/A	0.000	N/A
23	F	0	0	N/A	0.000	N/A
24	F	0	0	N/A	0.000	N/A
25	A	2	2	0.37	28	0.071
26	B	2	2	2.43	34	0.059
27	B	2	2	2.47	34	0.059
28	B	2	2	2.64	32	0.062
29	A	11	11	1.04	25	0.440
30	A	2	2	1.57	34	0.059
31	A	2	2	1.83	34	0.059
32	B	2	2	2.05	34	0.059
33	B	2	2	3.03	34	0.059
34	B	2	2	3.14	34	0.059
35	B	2	2	3.37	32	0.062
36	A	13	13	1.04	25	0.520
37	A	2	2	1.90	34	0.059
38	A	2	2	1.98	34	0.059
39	B	2	2	2.32	34	0.059
40	A	6	6	1.51	34	0.176
41	A	4	4	1.65	34	0.118
42	A	2	2	1.81	32	0.062
43	A	9	9	1.05	25	0.360
44	A	10	10	1.01	34	0.294
45	A	11	11	0.95	34	0.324
46	A	14	14	0.98	34	0.412
47	A	2	2	1.74	34	0.059
48	A	2	2	1.66	34	0.059
49	A	2	2	1.73	32	0.062
50	A	8	8	1.05	25	0.320
51	A	2	2	1.00	34	0.059
52	A	2	2	1.34	34	0.059
53	A	2	2	1.84	34	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.69	34	0.059
55	A	2	2	1.59	32	0.062
56	A	10	10	1.08	25	0.400
57	A	2	2	1.16	34	0.059
58	A	2	2	1.51	34	0.059
59	A	2	2	1.01	38	0.053
60	A	2	2	1.00	37	0.054
61	A	7	7	1.40	39	0.179
62	A	7	7	1.37	38	0.184
63	A	4	4	1.06	19	0.211
64	A	6	6	1.19	22	0.273
65	A	7	7	1.10	26	0.269
66	B	3	3	2.12	31	0.097
67	A	2	2	1.94	28	0.071
68	A	2	2	1.92	28	0.071
69	A	2	2	1.57	26	0.077
70	A	4	4	1.00	19	0.211
71	A	4	4	1.03	28	0.143
72	A	9	9	0.94	28	0.321
73	A	11	11	0.90	28	0.393
74	A	2	2	1.94	28	0.071
75	A	2	2	1.83	28	0.071
76	A	2	2	1.30	26	0.077
77	A	6	6	1.00	19	0.316
78	A	2	2	1.20	28	0.071
79	A	2	2	1.80	28	0.071
80	B	2	2	2.15	28	0.071
81	A	1	1	1.00	37	0.027
82	A	1	1	0.93	36	0.028
83	A	4	4	1.05	38	0.105
84	A	3	3	1.00	37	0.081
85	A	1	1	1.04	40	0.025

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	B	3	3	3.11	72	0.042
87	A	2	2	1.04	47	0.043
88	B	4	4	3.21	68	0.059
89	A	4	4	1.26	69	0.058
90	A	3	3	1.95	76	0.039
91	F	0	0	N/A	0.000	N/A
92	F	0	0	N/A	0.000	N/A
93	F	0	0	N/A	0.000	N/A
94	F	0	0	N/A	0.000	N/A
95	F	0	0	N/A	0.000	N/A
96	F	0	0	N/A	0.000	N/A
97	F	0	0	N/A	0.000	N/A
98	F	0	0	N/A	0.000	N/A
99	F	0	0	N/A	0.000	N/A
100	F	0	0	N/A	0.000	N/A
101	F	0	0	N/A	0.000	N/A
102	F	0	0	N/A	0.000	N/A
103	F	0	0	N/A	0.000	N/A
104	F	0	0	N/A	0.000	N/A
105	F	0	0	N/A	0.000	N/A
106	F	0	0	N/A	0.000	N/A
107	F	0	0	N/A	0.000	N/A
108	F	0	0	N/A	0.000	N/A
109	F	0	0	N/A	0.000	N/A
110	F	0	0	N/A	0.000	N/A
111	F	0	0	N/A	0.000	N/A
112	F	0	0	N/A	0.000	N/A
113	F	0	0	N/A	0.000	N/A
114	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
115	F	0	0	N/A	0.000	N/A
116	F	0	0	N/A	0.000	N/A
117	F	0	0	N/A	0.000	N/A
118	F	0	0	N/A	0.000	N/A
119	F	0	0	N/A	0.000	N/A
120	F	0	0	N/A	0.000	N/A
121	F	0	0	N/A	0.000	N/A
122	F	0	0	N/A	0.000	N/A
123	A	2	2	0.72	24	0.083
124	A	3	3	0.91	17	0.176
125	A	2	2	1.00	26	0.077
126	F	0	0	N/A	0.000	N/A
127	A	2	2	0.60	29	0.069
128	A	4	4	0.99	22	0.182
129	A	2	2	1.00	31	0.065
130	F	0	0	N/A	0.000	N/A
131	A	4	4	1.20	50	0.080
132	B	4	4	2.47	51	0.078
133	A	2	2	1.04	22	0.091
134	A	2	2	1.00	22	0.091
135	A	2	2	1.00	27	0.074
136	A	8	7	1.72	30	0.233
137	A	5	4	0.98	44	0.091
138	A	7	6	0.99	44	0.136
139	A	7	6	1.01	47	0.128
140	A	8	7	0.99	48	0.146
141	A	8	7	1.03	47	0.149
142	A	7	6	1.01	47	0.128
143	A	7	6	0.96	48	0.125
144	A	8	7	1.02	49	0.143
145	A	8	7	0.97	50	0.140

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	3	3	1.20	49	0.061
147	A	3	3	1.18	49	0.061
148	A	6	5	0.89	49	0.102
149	A	3	3	0.99	49	0.061
150	A	3	3	1.13	49	0.061
151	A	3	3	1.49	49	0.061
152	A	3	3	1.42	49	0.061
153	C	2	2	2.57	36	0.056
154	A	2	2	0.97	38	0.053
155	A	2	2	1.00	38	0.053
156	A	2	2	1.02	38	0.053
157	A	2	2	1.00	38	0.053
158	A	2	2	1.12	38	0.053
159	A	2	2	1.11	38	0.053
160	F	0	0	N/A	0.000	N/A
161	F	0	0	N/A	0.000	N/A
162	F	0	0	N/A	0.000	N/A
163	F	0	0	N/A	0.000	N/A
164	F	0	0	N/A	0.000	N/A
165	F	0	0	N/A	0.000	N/A
166	A	2	2	0.41	33	0.061
167	A	7	7	0.91	33	0.212
168	A	7	7	0.94	33	0.212
169	A	5	5	0.97	31	0.161
170	A	4	4	1.00	24	0.167
171	A	4	4	1.02	33	0.121
172	A	9	9	0.93	33	0.273
173	A	11	11	0.88	33	0.333
174	A	7	7	0.89	33	0.212
175	A	6	6	0.91	33	0.182
176	A	6	6	0.93	31	0.194
177	A	6	6	0.96	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
178	A	2	2	1.18	33	0.061
179	A	2	2	1.69	33	0.061
180	A	1	1	1.00	42	0.024
181	A	1	1	0.94	41	0.024
182	A	4	4	1.04	43	0.093
183	A	3	3	1.00	42	0.071
184	A	5	5	1.02	38	0.132
185	A	5	5	1.00	31	0.161
186	A	1	1	1.00	41	0.024
187	A	1	1	0.94	41	0.024
188	A	1	1	1.00	47	0.021
189	A	3	3	1.49	64	0.047
190	A	10	10	1.10	36	0.278
191	A	9	9	1.09	36	0.250
192	A	9	9	1.08	34	0.265
193	A	6	6	1.06	27	0.222
194	A	2	2	1.44	36	0.056
195	A	2	2	1.00	36	0.056
196	A	2	2	1.24	36	0.056
197	A	11	11	1.11	36	0.306
198	A	11	11	1.10	36	0.306
199	A	8	8	1.09	34	0.235
200	A	7	7	1.07	27	0.259
201	A	2	2	1.37	36	0.056
202	A	2	2	1.58	36	0.056
203	A	2	2	1.21	36	0.056
204	A	8	8	1.09	36	0.222
205	A	8	8	1.08	36	0.222
206	A	5	5	1.06	34	0.147
207	A	4	4	1.04	27	0.148
208	A	9	9	1.33	36	0.250
209	A	11	11	1.29	36	0.306

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
210	A	13	13	1.27	36	0.361
211	A	9	9	1.06	36	0.250
212	A	7	7	1.06	36	0.194
213	A	5	5	1.04	34	0.147
214	A	5	5	1.29	27	0.185
215	A	2	2	1.25	36	0.056
216	A	2	2	1.87	36	0.056
217	A	2	2	1.94	36	0.056
218	A	7	7	1.06	36	0.194
219	A	7	7	1.29	36	0.194
220	A	7	7	1.29	34	0.206
221	A	7	7	1.30	27	0.259
222	A	2	2	1.81	36	0.056
223	A	2	2	1.86	36	0.056
224	F	0	0	N/A	0.000	N/A
225	F	0	0	N/A	0.000	N/A
226	F	0	0	N/A	0.000	N/A
227	F	0	0	N/A	0.000	N/A
228	F	0	0	N/A	0.000	N/A
229	F	0	0	N/A	0.000	N/A
230	F	0	0	N/A	0.000	N/A
231	F	0	0	N/A	0.000	N/A
232	F	0	0	N/A	0.000	N/A
233	F	0	0	N/A	0.000	N/A
234	F	0	0	N/A	0.000	N/A
235	F	0	0	N/A	0.000	N/A
236	F	0	0	N/A	0.000	N/A
237	F	0	0	N/A	0.000	N/A
238	F	0	0	N/A	0.000	N/A
239	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
240	F	0	0	N/A	0.000	N/A
241	F	0	0	N/A	0.000	N/A
242	F	0	0	N/A	0.000	N/A
243	F	0	0	N/A	0.000	N/A
244	F	0	0	N/A	0.000	N/A
245	F	0	0	N/A	0.000	N/A
246	F	0	0	N/A	0.000	N/A
247	F	0	0	N/A	0.000	N/A
248	F	0	0	N/A	0.000	N/A
249	F	0	0	N/A	0.000	N/A
250	F	0	0	N/A	0.000	N/A
251	F	0	0	N/A	0.000	N/A
252	F	0	0	N/A	0.000	N/A
253	F	0	0	N/A	0.000	N/A
254	F	0	0	N/A	0.000	N/A
255	A	2	2	0.66	29	0.069
256	A	3	3	0.91	17	0.176
257	A	2	2	1.00	31	0.065
258	F	0	0	N/A	0.000	N/A
259	A	2	2	0.51	34	0.059
260	A	4	4	0.99	22	0.182
261	A	2	2	1.00	36	0.056
262	F	0	0	N/A	0.000	N/A

CHAPTER 3

Listing of integrals

3.1 $\int \frac{(1+bx^2)(A+Bx^2)}{\sqrt{1-b^2x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int \frac{(1+bx^2)(A+Bx^2)}{\sqrt{1-b^2x^4}}\,dx = -\frac{Bx\sqrt{1-b^2x^4}}{3b} + \frac{(Ab+B)E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{b^{3/2}} - \frac{2B\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{3b^{3/2}}$$

output
```
-1/3*B*x*(-b^2*x^4+1)^(1/2)/b+(A*b+B)*EllipticE(b^(1/2)*x,I)/b^(3/2)-2/3*B
*EllipticF(b^(1/2)*x,I)/b^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \frac{(1+bx^2)(A+Bx^2)}{\sqrt{1-b^2x^4}}\,dx = \frac{x\left(-B\sqrt{1-b^2x^4} + (3Ab+B)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},b^2x^4\right) + b(Ab+B)x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{3b}$$

input
```
Integrate[((1 + b*x^2)*(A + B*x^2))/Sqrt[1 - b^2*x^4],x]
```

output
```
(x*(-(B*Sqrt[1 - b^2*x^4]) + (3*A*b + B)*Hypergeometric2F1[1/4, 1/2, 5/4,
b^2*x^4] + b*(A*b + B)*x^2*Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/(3*
b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1388, 403, 25, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+1)(A+Bx^2)}{\sqrt{1-b^2x^4}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\sqrt{bx^2+1}(A+Bx^2)}{\sqrt{1-bx^2}}dx$$

$$\Big\downarrow \text{403}$$

$$-\frac{\int -\frac{3b(Ab+B)x^2+3Ab+B}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3b(Ab+B)x^2+3Ab+B}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

$$\Big\downarrow \text{399}$$

$$\frac{3(Ab+B)\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx-2B\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

$$\Big\downarrow \text{284}$$

$$\frac{3(Ab+B)\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx-2B\int \frac{1}{\sqrt{1-b^2x^4}}dx}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{3(Ab+B)E\left(\arcsin\left(\sqrt{b}x\right)|-1\right)}{\sqrt{b}}-2B\int\frac{1}{\sqrt{1-b^2x^4}}dx}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{3(Ab+B)E\left(\arcsin\left(\sqrt{b}x\right)|-1\right)}{\sqrt{b}}-\frac{2B\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

input
```
Int[((1 + b*x^2)*(A + B*x^2))/Sqrt[1 - b^2*x^4],x]
```

output
```
-1/3*(B*x*Sqrt[1 - b*x^2]*Sqrt[1 + b*x^2])/b + ((3*(A*b + B)*EllipticE[Arc
Sin[Sqrt[b]*x], -1])/Sqrt[b] - (2*B*EllipticF[ArcSin[Sqrt[b]*x], -1])/Sqrt
[b])/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] && !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.87 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

method	result
meijerg	$\frac{bB\,x^{5} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{9}{4}\right],b^{2}x^{4}\right)}{5}+\frac{bA\,x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^{2}x^{4}\right)}{3}+\frac{B\,x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^{2}x^{4}\right)}{3}+Ax\,\text{hyper}$
elliptic	$-\frac{Bx\sqrt{-b^{2}x^{4}+1}}{3b}+\frac{\left(A+\frac{B}{3b}\right)\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}-\frac{(bA+B)\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)-\text{Ellipt}\right.}{b^{\frac{3}{2}}\sqrt{-b^{2}x^{4}+1}}$
default	$\frac{A\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}-\frac{(bA+B)\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)-\operatorname{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{b^{\frac{3}{2}}\sqrt{-b^{2}x^{4}+1}}+Bb\left(-\frac{x}{}\right.$
risch	$\frac{Bx\left(b^{2}x^{4}-1\right)}{3b\sqrt{-b^{2}x^{4}+1}}+\frac{\frac{B\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}-\frac{3(bA+B)\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)-\operatorname{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}+\frac{3\sqrt{b}\,A\sqrt{-}}{}}{3b}$

input
```
int((b*x^2+1)*(B*x^2+A)/(-b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*b*B*x^5*hypergeom([1/2,5/4],[9/4],b^2*x^4)+1/3*b*A*x^3*hypergeom([1/2,
3/4],[7/4],b^2*x^4)+1/3*B*x^3*hypergeom([1/2,3/4],[7/4],b^2*x^4)+A*x*hyper
geom([1/4,1/2],[5/4],b^2*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(49) = 98$.

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{(1+bx^2)(A+Bx^2)}{\sqrt{1-b^2x^4}}\,dx = -\frac{\frac{3\,(Ab+B)\sqrt{-b^2}xE(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1)}{\sqrt{b}} - \frac{(3\,Ab^2+(3\,A+B)b+3\,B)\sqrt{-b^2}xF(\arcsin\left(\frac{1}{\sqrt{b}x}\right)|-1)}{\sqrt{b}} + \sqrt{-b^2x^4+1}(Bb^2x^2+3\,Ab^2}{3\,b^3x}$$

input
```
integrate((b*x^2+1)*(B*x^2+A)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(3*(A*b + B)*sqrt(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(
b) - (3*A*b^2 + (3*A + B)*b + 3*B)*sqrt(-b^2)*x*elliptic_f(arcsin(1/(sqrt(
b)*x)), -1)/sqrt(b) + sqrt(-b^2*x^4 + 1)*(B*b^2*x^2 + 3*A*b^2 + 3*B*b))/(b
^3*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(54) = 108$.

Time = 1.97 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.31

$$\int \frac{(1+bx^2)(A+Bx^2)}{\sqrt{1-b^2x^4}}\,dx = \frac{Abx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{Bbx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((b*x**2+1)*(B*x**2+A)/(-b**2*x**4+1)**(1/2),x)
```

output
```
A*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))
/(4*gamma(7/4)) + A*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_p
olar(2*I*pi))/(4*gamma(5/4)) + B*b*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,
), b**2*x**4*exp_polar(2*I*pi))/(4*gamma(9/4)) + B*x**3*gamma(3/4)*hyper((
1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(4*gamma(7/4))
```

Maxima [F]

$$\int \frac{(1+bx^2)(A+Bx^2)}{\sqrt{1-b^2x^4}}\,dx = \int \frac{(Bx^2+A)(bx^2+1)}{\sqrt{-b^2x^4+1}}\,dx$$

input
```
integrate((b*x^2+1)*(B*x^2+A)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

Giac [F]

$$\int \frac{(1+bx^2)(A+Bx^2)}{\sqrt{1-b^2x^4}}\,dx = \int \frac{(Bx^2+A)(bx^2+1)}{\sqrt{-b^2x^4+1}}\,dx$$

input
```
integrate((b*x^2+1)*(B*x^2+A)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+bx^2)(A+Bx^2)}{\sqrt{1-b^2x^4}}\,dx = \int \frac{(B\,x^2+A)\,(b\,x^2+1)}{\sqrt{1-b^2\,x^4}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + 1))/(1 - b^2*x^4)^(1/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + 1))/(1 - b^2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1+bx^2)(A+Bx^2)}{\sqrt{1-b^2x^4}}\,dx = -\left(\int \frac{\sqrt{-b^2x^4+1}}{b\,x^2-1}dx\right)a - \left(\int \frac{\sqrt{-b^2x^4+1}\,x^2}{b\,x^2-1}dx\right)b$$

input
```
int((b*x^2+1)*(B*x^2+A)/(-b^2*x^4+1)^(1/2),x)
```

output
```
 - (int(sqrt( - b**2*x**4 + 1)/(b*x**2 - 1),x)*a + int((sqrt( - b**2*x**4
+ 1)*x**2)/(b*x**2 - 1),x)*b)
```

3.2 $\int \frac{\sqrt{1+bx^2}(A+Bx^2)}{\sqrt{1-bx^2}} dx$

Optimal result

Integrand size = 31, antiderivative size = 65

$$\int \frac{\sqrt{1+bx^2}(A+Bx^2)}{\sqrt{1-bx^2}} dx = -\frac{Bx\sqrt{1-b^2x^4}}{3b} + \frac{(Ab+B)E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{b^{3/2}} - \frac{2B\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{3b^{3/2}}$$

output
```
-1/3*B*x*(-b^2*x^4+1)^(1/2)/b+(A*b+B)*EllipticE(b^(1/2)*x,I)/b^(3/2)-2/3*B
*EllipticF(b^(1/2)*x,I)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{1+bx^2}(A+Bx^2)}{\sqrt{1-bx^2}} dx = \frac{\sqrt{-b}Bx\sqrt{1-b^2x^4} + 3i(Ab+B)E\left(i\text{arcsinh}\left(\sqrt{-b}x\right)\middle|-1\right) - 2iB\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-b}x\right),-1\right)}{3(-b)^{3/2}}$$

input
```
Integrate[(Sqrt[1 + b*x^2]*(A + B*x^2))/Sqrt[1 - b*x^2],x]
```

output
```
(Sqrt[-b]*B*x*Sqrt[1 - b^2*x^4] + (3*I)*(A*b + B)*EllipticE[I*ArcSinh[Sqrt
[-b]*x], -1] - (2*I)*B*EllipticF[I*ArcSinh[Sqrt[-b]*x], -1])/(3*(-b)^(3/2)
)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {403, 25, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+1}(A+Bx^2)}{\sqrt{1-bx^2}}dx$$

$$\Big\downarrow \text{403}$$

$$-\frac{\int -\frac{3b(Ab+B)x^2+3Ab+B}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3b(Ab+B)x^2+3Ab+B}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

$$\Big\downarrow \text{399}$$

$$\frac{3(Ab+B)\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx-2B\int \frac{1}{\sqrt{1-bx^2}\sqrt{bx^2+1}}dx}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

$$\Big\downarrow \text{284}$$

$$\frac{3(Ab+B)\int \frac{\sqrt{bx^2+1}}{\sqrt{1-bx^2}}dx-2B\int \frac{1}{\sqrt{1-b^2x^4}}dx}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{3(Ab+B)E\left(\arcsin\left(\sqrt{b}x\right)|-1\right)}{\sqrt{b}}-2B\int \frac{1}{\sqrt{1-b^2x^4}}dx}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{3(Ab+B)E\left(\arcsin\left(\sqrt{b}x\right)\middle|-1\right)}{\sqrt{b}}-\frac{2B\,\text{EllipticF}\left(\arcsin\left(\sqrt{b}x\right),-1\right)}{\sqrt{b}}}{3b}-\frac{Bx\sqrt{1-bx^2}\sqrt{bx^2+1}}{3b}$$

input
```
Int[(Sqrt[1 + b*x^2]*(A + B*x^2))/Sqrt[1 - b*x^2],x]
```

output
```
-1/3*(B*x*Sqrt[1 - b*x^2]*Sqrt[1 + b*x^2])/b + ((3*(A*b + B)*EllipticE[Arc
Sin[Sqrt[b]*x], -1])/Sqrt[b] - (2*B*EllipticF[ArcSin[Sqrt[b]*x], -1])/Sqrt
[b])/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(51) = 102$.

Time = 1.80 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.20

method	result
default	$-\frac{\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\,\left(B\,b^{\frac{5}{2}}x^{5}+3A\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\,\operatorname{EllipticE}\left(\sqrt{b}\,x,i\right)b-2B\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)+3B\sqrt{b\,x^{2}}\cdots\right.}{3(b^{2}x^{4}-1)b^{\frac{3}{2}}}$
elliptic	$\frac{\sqrt{-b^{2}x^{4}+1}\,\left(-\frac{Bx\sqrt{-b^{2}x^{4}+1}}{3b}+\frac{\left(A+\frac{B}{3b}\right)\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}-\frac{(bA+B)\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)-\operatorname{EllipticE}\left(\cdots\right)\right.}{b^{\frac{3}{2}}\sqrt{-b^{2}x^{4}+1}}\cdots\right.}{\sqrt{b\,x^{2}+1}\,\sqrt{-b\,x^{2}+1}}$
risch	$\frac{Bx\sqrt{b\,x^{2}+1}\,\left(b\,x^{2}-1\right)\sqrt{\left(b\,x^{2}+1\right)\left(-b\,x^{2}+1\right)}}{3b\sqrt{-\left(b\,x^{2}+1\right)\left(b\,x^{2}-1\right)}\,\sqrt{-b\,x^{2}+1}}+\frac{\left(\frac{B\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\operatorname{EllipticF}\left(\sqrt{b}\,x,i\right)}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}-\frac{3(bA+B)\sqrt{-b\,x^{2}+1}\,\sqrt{b\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(\sqrt{b}\,x,\cdots\right)\right.}{\sqrt{b}\,\sqrt{-b^{2}x^{4}+1}}\cdots\right.}{3b\sqrt{b\,x^{2}+\cdots}}$

input
```
int((b*x^2+1)^(1/2)*(B*x^2+A)/(-b*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)*(B*b^(5/2)*x^5+3*A*(b*x^2+1)^(1/2)*(
-b*x^2+1)^(1/2)*EllipticE(b^(1/2)*x,I)*b-2*B*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1
/2)*EllipticF(b^(1/2)*x,I)+3*B*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)*EllipticE(
b^(1/2)*x,I)-B*b^(1/2)*x)/(b^2*x^4-1)/b^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(49) = 98$.

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{1+bx^2}(A+Bx^2)}{\sqrt{1-bx^2}}\,dx = -\frac{\frac{3\,(Ab+B)\sqrt{-b^2}xE\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\mid -1\right)}{\sqrt{b}} - \frac{(3\,Ab^2+(3\,A+B)b+3\,B)\sqrt{-b^2}xF\left(\arcsin\left(\frac{1}{\sqrt{b}x}\right)\mid -1\right)}{\sqrt{b}} + (Bb^2x^2 + 3\,Ab^2 + 3\,Bb)\sqrt{bx}}{3\,b^3x}$$

input
```
integrate((b*x^2+1)^(1/2)*(B*x^2+A)/(-b*x^2+1)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/3*(3*(A*b + B)*sqrt(-b^2)*x*elliptic_e(arcsin(1/(sqrt(b)*x)), -1)/sqrt(
b) - (3*A*b^2 + (3*A + B)*b + 3*B)*sqrt(-b^2)*x*elliptic_f(arcsin(1/(sqrt(
b)*x)), -1)/sqrt(b) + (B*b^2*x^2 + 3*A*b^2 + 3*B*b)*sqrt(b*x^2 + 1)*sqrt(-
b*x^2 + 1))/(b^3*x)
```

Sympy [F]

$$\int \frac{\sqrt{1+bx^2}(A+Bx^2)}{\sqrt{1-bx^2}}\,dx = \int \frac{(A+Bx^2)\sqrt{bx^2+1}}{\sqrt{-bx^2+1}}\,dx$$

input
```
integrate((b*x**2+1)**(1/2)*(B*x**2+A)/(-b*x**2+1)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(b*x**2 + 1)/sqrt(-b*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+bx^2}(A+Bx^2)}{\sqrt{1-bx^2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+1}}{\sqrt{-bx^2+1}}\,dx$$

input
```
integrate((b*x^2+1)^(1/2)*(B*x^2+A)/(-b*x^2+1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + 1)/sqrt(-b*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+bx^2}(A+Bx^2)}{\sqrt{1-bx^2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+1}}{\sqrt{-bx^2+1}}\,dx$$

input
```
integrate((b*x^2+1)^(1/2)*(B*x^2+A)/(-b*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + 1)/sqrt(-b*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+bx^2}(A+Bx^2)}{\sqrt{1-bx^2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+1}}{\sqrt{1-b\,x^2}}\,dx$$

input
```
int(((A + B*x^2)*(b*x^2 + 1)^(1/2))/(1 - b*x^2)^(1/2),x)
```

output
```
int(((A + B*x^2)*(b*x^2 + 1)^(1/2))/(1 - b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+bx^2}(A+Bx^2)}{\sqrt{1-bx^2}}\,dx = -\left(\int \frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,x^2}{b\,x^2-1}dx\right)b - \left(\int \frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}}{b\,x^2-1}dx\right)a$$

input
```
int((b*x^2+1)^(1/2)*(B*x^2+A)/(-b*x^2+1)^(1/2),x)
```

output
```
 - (int((sqrt(b*x**2 + 1)*sqrt( - b*x**2 + 1)*x**2)/(b*x**2 - 1),x)*b + in
t((sqrt(b*x**2 + 1)*sqrt( - b*x**2 + 1))/(b*x**2 - 1),x)*a)
```

3.3 $\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2-ae^2x^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 187

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = -\frac{(Bd-Ae)x\sqrt{ad^2-ae^2x^4}}{2ad^2e\,(d+ex^2)} - \frac{(Bd-Ae)\sqrt{1-\frac{e^2x^4}{d^2}}E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}e^{3/2}\sqrt{ad^2-ae^2x^4}} + \frac{B\sqrt{d}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{e^{3/2}\sqrt{ad^2-ae^2x^4}}$$

output
```
-1/2*(-A*e+B*d)*x*(-a*e^2*x^4+a*d^2)^(1/2)/a/d^2/e/(e*x^2+d)-1/2*(-A*e+B*d
)*(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(3/2)/(-a
*e^2*x^4+a*d^2)^(1/2)+B*d^(1/2)*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/
d^(1/2),I)/e^(3/2)/(-a*e^2*x^4+a*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.60 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2-ae^2x^4}}\,dx =$$
$$-\frac{-\sqrt{-\frac{e}{d}}(Bd-Ae)x(d-ex^2)+id(Bd-Ae)\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-2iBd^2\sqrt{1-\frac{e^2x^4}{d^2}}}{2d^3\left(-\frac{e}{d}\right)^{3/2}\sqrt{a(d^2-e^2x^4)}}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)*Sqrt[a*d^2 - a*e^2*x^4]),x]
```

output
```
-1/2*(-(Sqrt[-(e/d)]*(B*d - A*e)*x*(d - e*x^2)) + I*d*(B*d - A*e)*Sqrt[1 -
 (e^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - (2*I)*B*d^2*Sqr
t[1 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/(d^3*(-(e/d
))^(3/2)*Sqrt[a*(d^2 - e^2*x^4)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1396, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2-ae^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d+ex^2}\sqrt{ad-aex^2}\int \frac{Bx^2+A}{(ex^2+d)^{3/2}\sqrt{ad-aex^2}}dx}{\sqrt{ad^2-ae^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d+ex^2}\sqrt{ad-aex^2}\left(-\frac{\int -\frac{a\left(d(Bd+Ae)-e(Bd-Ae)x^2\right)}{\sqrt{ex^2+d}\sqrt{ad-aex^2}}dx}{2ad^2e}-\frac{x\sqrt{ad-aex^2}(Bd-Ae)}{2ad^2e\sqrt{d+ex^2}}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\downarrow$ 25

$$\frac{\sqrt{d+ex^2}\sqrt{ad-aex^2}\left(\frac{\int \frac{a\left(d(Bd+Ae)-e(Bd-Ae)x^2\right)}{\sqrt{ex^2+d}\sqrt{ad-aex^2}}dx}{2ad^2e}-\frac{x\sqrt{ad-aex^2}(Bd-Ae)}{2ad^2e\sqrt{d+ex^2}}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\downarrow$ 27

$$\frac{\sqrt{d+ex^2}\sqrt{ad-aex^2}\left(\frac{\int \frac{d(Bd+Ae)-e(Bd-Ae)x^2}{\sqrt{ex^2+d}\sqrt{ad-aex^2}}dx}{2d^2e}-\frac{x\sqrt{ad-aex^2}(Bd-Ae)}{2ad^2e\sqrt{d+ex^2}}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\downarrow$ 399

$$\frac{\sqrt{d+ex^2}\sqrt{ad-aex^2}\left(\frac{2Bd^2\int \frac{1}{\sqrt{ex^2+d}\sqrt{ad-aex^2}}dx-(Bd-Ae)\int \frac{\sqrt{ex^2+d}}{\sqrt{ad-aex^2}}dx}{2d^2e}-\frac{x\sqrt{ad-aex^2}(Bd-Ae)}{2ad^2e\sqrt{d+ex^2}}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\downarrow$ 289

$$\frac{\sqrt{d+ex^2}\sqrt{ad-aex^2}\left(\frac{\frac{2Bd^2\sqrt{ad^2-ae^2x^4}\int \frac{1}{\sqrt{ad^2-ae^2x^4}}dx}{\sqrt{d+ex^2}\sqrt{ad-aex^2}}-(Bd-Ae)\int \frac{\sqrt{ex^2+d}}{\sqrt{ad-aex^2}}dx}{2d^2e}-\frac{x\sqrt{ad-aex^2}(Bd-Ae)}{2ad^2e\sqrt{d+ex^2}}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\downarrow$ 329

$$\frac{\sqrt{d+ex^2}\sqrt{ad-aex^2}\left(\frac{\frac{2Bd^2\sqrt{ad^2-ae^2x^4}\int \frac{1}{\sqrt{ad^2-ae^2x^4}}dx}{\sqrt{d+ex^2}\sqrt{ad-aex^2}}-\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}(Bd-Ae)\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d+ex^2}\sqrt{ad-aex^2}}}{2d^2e}-\frac{x\sqrt{ad-aex^2}(Bd-Ae)}{2ad^2e\sqrt{d+ex^2}}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\downarrow$ 327

$$\frac{\sqrt{d+ex^2}\sqrt{ad-aex^2}\left(\frac{\frac{2Bd^2\sqrt{ad^2-ae^2x^4}\int \frac{1}{\sqrt{ad^2-ae^2x^4}}dx}{\sqrt{d+ex^2}\sqrt{ad-aex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}(Bd-Ae)E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d+ex^2}\sqrt{ad-aex^2}}}{2d^2e}-\frac{x\sqrt{ad-aex^2}(Bd-Ae)}{2ad^2e\sqrt{d+ex^2}}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\sqrt{d+ex^2}\sqrt{ad-aex^2}\left(\frac{\frac{2Bd^2\sqrt{1-\frac{e^2x^4}{d^2}}\int \frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d+ex^2}\sqrt{ad-aex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}(Bd-Ae)E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d+ex^2}\sqrt{ad-aex^2}}}{2d^2e}-\frac{x\sqrt{ad-aex^2}(Bd-Ae)}{2ad^2e\sqrt{d+ex^2}}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{\sqrt{d+ex^2}\sqrt{ad-aex^2}\left(\frac{\frac{2Bd^{5/2}\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{e}\sqrt{d+ex^2}\sqrt{ad-aex^2}}-\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}(Bd-Ae)E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d+ex^2}\sqrt{ad-aex^2}}}{2d^2e}-\frac{x\sqrt{ad-aex^2}(Bd-A}{2ad^2e\sqrt{d+ex^2}}\right.}{\sqrt{ad^2-ae^2x^4}}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)*Sqrt[a*d^2 - a*e^2*x^4]),x]
```

output
```
(Sqrt[d + e*x^2]*Sqrt[a*d - a*e*x^2]*(-1/2*((B*d - A*e)*x*Sqrt[a*d - a*e*x
^2])/(a*d^2*e*Sqrt[d + e*x^2]) + (-((d^(3/2)*(B*d - A*e)*Sqrt[1 - (e^2*x^4
)/d^2]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d + e*x^2
]*Sqrt[a*d - a*e*x^2])) + (2*B*d^(5/2)*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF[A
rcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d + e*x^2]*Sqrt[a*d - a*e*x
^2]))/(2*d^2*e)))/Sqrt[a*d^2 - a*e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.27

method	result
elliptic	$\frac{(-a e^2 x^2+ade)x(Ae-Bd)}{2d^2 a e^2\sqrt{\left(x^2+\frac{d}{e}\right)(-a e^2 x^2+ade)}}+\frac{\left(\frac{B}{e}+\frac{Ae-Bd}{2de}\right)\sqrt{1-\frac{e x^2}{d}}\sqrt{1+\frac{e x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\sqrt{-a e^2 x^4+a d^2}}-\frac{(Ae-Bd)\sqrt{1-\frac{e x^2}{d}}\sqrt{1+\frac{e x^2}{d}}\,(\mathrm{E}\cdots}{2d\sqrt{\frac{e}{d}}\sqrt{\cdots}}$
default	$\frac{B\sqrt{1-\frac{e x^2}{d}}\sqrt{1+\frac{e x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{e\sqrt{\frac{e}{d}}\sqrt{-a e^2 x^4+a d^2}}+\frac{(Ae-Bd)\left(\frac{(-a e^2 x^2+ade)x}{2d^2 ae\sqrt{\left(x^2+\frac{d}{e}\right)(-a e^2 x^2+ade)}}+\frac{\sqrt{1-\frac{e x^2}{d}}\sqrt{1+\frac{e x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{2d\sqrt{\frac{e}{d}}\sqrt{-a e^2 x^4+a d^2}}-\sqrt{1\cdots}\right.}{e}$

input
```
int((B*x^2+A)/(e*x^2+d)/(-a*e^2*x^4+a*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a*e^2*x^2+a*d*e)/d^2*x/a*(A*e-B*d)/e^2/((x^2+d/e)*(-a*e^2*x^2+a*d*e)
)^(1/2)+(B/e+1/2/d/e*(A*e-B*d))/(1/d*e)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d
)^(1/2)/(-a*e^2*x^4+a*d^2)^(1/2)*EllipticF(x*(1/d*e)^(1/2),I)-1/2*(A*e-B*d
)/d/(1/d*e)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-a*e^2*x^4+a*d^2)^(
1/2)/e*(EllipticF(x*(1/d*e)^(1/2),I)-EllipticE(x*(1/d*e)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = -\frac{(Bd^2e-Ade^2+(Bde^2-Ae^3)x^2)\sqrt{ad^2}\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1)-(Bd^3+(A+B)d^2e-Ade^2+(B}{2\left(ad^3e^3x^2+\right.}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(-a*e^2*x^4+a*d^2)^(1/2),x, algorithm="frica
s")
```

output
```
-1/2*((B*d^2*e - A*d*e^2 + (B*d*e^2 - A*e^3)*x^2)*sqrt(a*d^2)*sqrt(e/d)*el
liptic_e(arcsin(x*sqrt(e/d)), -1) - (B*d^3 + (A + B)*d^2*e - A*d*e^2 + (B*
d^2*e + (A + B)*d*e^2 - A*e^3)*x^2)*sqrt(a*d^2)*sqrt(e/d)*elliptic_f(arcsi
n(x*sqrt(e/d)), -1) + sqrt(-a*e^2*x^4 + a*d^2)*(B*d^2*e - A*d*e^2)*x)/(a*d
^3*e^3*x^2 + a*d^4*e^2)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{-a\left(-d+ex^2\right)\left(d+ex^2\right)}\left(d+ex^2\right)}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)/(-a*e**2*x**4+a*d**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(-a*(-d + e*x**2)*(d + e*x**2))*(d + e*x**2)),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(-a*e^2*x^4+a*d^2)^(1/2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-ae^2x^4+ad^2}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(-a*e^2*x^4+a*d^2)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((B*x^2 + A)/(sqrt(-a*e^2*x^4 + a*d^2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{a\,d^2-a\,e^2\,x^4}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((A + B*x^2)/((a*d^2 - a*e^2*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int((A + B*x^2)/((a*d^2 - a*e^2*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2-ae^2x^4}}\,dx$$
$$=\frac{\sqrt{a}\left(-\sqrt{-e^2x^4+d^2}\,bx+2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^3x^6-d\,e^2x^4+d^2e\,x^2+d^3}dx\right)a\,d^2e+2\left(\int \frac{\sqrt{-e^2x^4+d^2}}{-e^3x^6-d\,e^2x^4+d^2e\,x^2+d^3}dx\right)ad\,e^2x^2+\left(\right.\right.}{2ade\,(e\,x^2+d)}$$

input
```
int((B*x^2+A)/(e*x^2+d)/(-a*e^2*x^4+a*d^2)^(1/2),x)
```

output
```
(sqrt(a)*( - sqrt(d**2 - e**2*x**4)*b*x + 2*int(sqrt(d**2 - e**2*x**4)/(d*
*3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),x)*a*d**2*e + 2*int(sqrt(d**2
- e**2*x**4)/(d**3 + d**2*e*x**2 - d*e**2*x**4 - e**3*x**6),x)*a*d*e**2*x*
*2 + int(sqrt(d**2 - e**2*x**4)/(d + e*x**2),x)*b*d + int(sqrt(d**2 - e**2
*x**4)/(d + e*x**2),x)*b*e*x**2))/(2*a*d*e*(d + e*x**2))
```

3.4 $\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2-ae^2x^4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 186

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = \frac{(Bd+Ae)x\sqrt{ad^2-ae^2x^4}}{2ad^2e\,(d-ex^2)} - \frac{(Bd+Ae)\sqrt{1-\frac{e^2x^4}{d^2}}\,E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{2\sqrt{d}e^{3/2}\sqrt{ad^2-ae^2x^4}} + \frac{A\sqrt{1-\frac{e^2x^4}{d^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),-1\right)}{\sqrt{d}\sqrt{e}\sqrt{ad^2-ae^2x^4}}$$

output
```
1/2*(A*e+B*d)*x*(-a*e^2*x^4+a*d^2)^(1/2)/a/d^2/e/(-e*x^2+d)-1/2*(A*e+B*d)*
(1-e^2*x^4/d^2)^(1/2)*EllipticE(e^(1/2)*x/d^(1/2),I)/d^(1/2)/e^(3/2)/(-a*e
^2*x^4+a*d^2)^(1/2)+A*(1-e^2*x^4/d^2)^(1/2)*EllipticF(e^(1/2)*x/d^(1/2),I)
/d^(1/2)/e^(1/2)/(-a*e^2*x^4+a*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.60 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = -\frac{\sqrt{-\frac{e}{d}}(Bd+Ae)x(d+ex^2)+id(Bd+Ae)\sqrt{1-\frac{e^2x^4}{d^2}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{e}{d}}x\right)\middle|-1\right)-2iAde\sqrt{1-\frac{e^2x^4}{d^2}}\,\mathrm{E}}{2d^3\left(-\frac{e}{d}\right)^{3/2}\sqrt{a(d^2-e^2x^4)}}$$

input
```
Integrate[(A + B*x^2)/((d - e*x^2)*Sqrt[a*d^2 - a*e^2*x^4]),x]
```

output
```
-1/2*(Sqrt[-(e/d)]*(B*d + A*e)*x*(d + e*x^2) + I*d*(B*d + A*e)*Sqrt[1 - (e
^2*x^4)/d^2]*EllipticE[I*ArcSinh[Sqrt[-(e/d)]*x], -1] - (2*I)*A*d*e*Sqrt[1
 - (e^2*x^4)/d^2]*EllipticF[I*ArcSinh[Sqrt[-(e/d)]*x], -1])/(d^3*(-(e/d))^
(3/2)*Sqrt[a*(d^2 - e^2*x^4)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.35, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {1396, 402, 25, 27, 399, 289, 329, 327, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2-ae^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{ad+aex^2}\int \frac{Bx^2+A}{(d-ex^2)^{3/2}\sqrt{aex^2+ad}}dx}{\sqrt{ad^2-ae^2x^4}}$$

$$\Big\downarrow \text{402}$$

$$\frac{\sqrt{d-ex^2}\sqrt{ad+aex^2}\left(\frac{\int -\frac{a\left(e(Bd+Ae)x^2+d(Bd-Ae)\right)}{\sqrt{d-ex^2}\sqrt{aex^2+ad}}dx}{2ad^2e}+\frac{x\sqrt{ad+aex^2}(Ae+Bd)}{2ad^2e\sqrt{d-ex^2}}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{d-ex^2}\sqrt{ad+aex^2}\left(\frac{x\sqrt{ad+aex^2}(Ae+Bd)}{2ad^2e\sqrt{d-ex^2}}-\frac{\int \frac{a\left(e(Bd+Ae)x^2+d(Bd-Ae)\right)}{\sqrt{d-ex^2}\sqrt{aex^2+ad}}dx}{2ad^2e}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{d-ex^2}\sqrt{ad+aex^2}\left(\frac{x\sqrt{ad+aex^2}(Ae+Bd)}{2ad^2e\sqrt{d-ex^2}}-\frac{\int \frac{e(Bd+Ae)x^2+d(Bd-Ae)}{\sqrt{d-ex^2}\sqrt{aex^2+ad}}dx}{2d^2e}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\Big\downarrow$ 399

$$\frac{\sqrt{d-ex^2}\sqrt{ad+aex^2}\left(\frac{x\sqrt{ad+aex^2}(Ae+Bd)}{2ad^2e\sqrt{d-ex^2}}-\frac{\frac{(Ae+Bd)\int \frac{\sqrt{aex^2+ad}}{\sqrt{d-ex^2}}dx}{a}-2Ade\int \frac{1}{\sqrt{d-ex^2}\sqrt{aex^2+ad}}dx}{2d^2e}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\Big\downarrow$ 289

$$\frac{\sqrt{d-ex^2}\sqrt{ad+aex^2}\left(\frac{x\sqrt{ad+aex^2}(Ae+Bd)}{2ad^2e\sqrt{d-ex^2}}-\frac{\frac{(Ae+Bd)\int \frac{\sqrt{aex^2+ad}}{\sqrt{d-ex^2}}dx}{a}-\frac{2Ade\sqrt{ad^2-ae^2x^4}\int \frac{1}{\sqrt{ad^2-ae^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{ad+aex^2}}}{2d^2e}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\Big\downarrow$ 329

$$\frac{\sqrt{d-ex^2}\sqrt{ad+aex^2}\left(\frac{x\sqrt{ad+aex^2}(Ae+Bd)}{2ad^2e\sqrt{d-ex^2}}-\frac{\frac{d\sqrt{1-\frac{e^2x^4}{d^2}}(Ae+Bd)\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{ex^2}{d}}}dx}{\sqrt{d-ex^2}\sqrt{ad+aex^2}}-\frac{2Ade\sqrt{ad^2-ae^2x^4}\int \frac{1}{\sqrt{ad^2-ae^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{ad+aex^2}}}{2d^2e}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{d-ex^2}\sqrt{ad+aex^2}\left(\frac{x\sqrt{ad+aex^2}(Ae+Bd)}{2ad^2e\sqrt{d-ex^2}}-\frac{\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}(Ae+Bd)E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{ad+aex^2}}-\frac{2Ade\sqrt{ad^2-ae^2x^4}\int\frac{1}{\sqrt{ad^2-ae^2x^4}}dx}{\sqrt{d-ex^2}\sqrt{ad+aex^2}}}{2d^2e}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{d-ex^2}\sqrt{ad+aex^2}\left(\frac{x\sqrt{ad+aex^2}(Ae+Bd)}{2ad^2e\sqrt{d-ex^2}}-\frac{\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}(Ae+Bd)E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{ad+aex^2}}-\frac{2Ade\sqrt{1-\frac{e^2x^4}{d^2}}\int\frac{1}{\sqrt{1-\frac{e^2x^4}{d^2}}}dx}{\sqrt{d-ex^2}\sqrt{ad+aex^2}}}{2d^2e}\right)}{\sqrt{ad^2-ae^2x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{d-ex^2}\sqrt{ad+aex^2}\left(\frac{x\sqrt{ad+aex^2}(Ae+Bd)}{2ad^2e\sqrt{d-ex^2}}-\frac{\frac{d^{3/2}\sqrt{1-\frac{e^2x^4}{d^2}}(Ae+Bd)E\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{e}\sqrt{d-ex^2}\sqrt{ad+aex^2}}-\frac{2Ad^{3/2}\sqrt{e}\sqrt{1-\frac{e^2x^4}{d^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right.}{\sqrt{d-ex^2}\sqrt{ad+aex^2}}}{2d^2e}\right.}{\sqrt{ad^2-ae^2x^4}}$$

input
```
Int[(A + B*x^2)/((d - e*x^2)*Sqrt[a*d^2 - a*e^2*x^4]),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[a*d + a*e*x^2]*(((B*d + A*e)*x*Sqrt[a*d + a*e*x^2])/
(2*a*d^2*e*Sqrt[d - e*x^2]) - ((d^(3/2)*(B*d + A*e)*Sqrt[1 - (e^2*x^4)/d^2
]*EllipticE[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[e]*Sqrt[d - e*x^2]*Sqr
t[a*d + a*e*x^2]) - (2*A*d^(3/2)*Sqrt[e]*Sqrt[1 - (e^2*x^4)/d^2]*EllipticF
[ArcSin[(Sqrt[e]*x)/Sqrt[d]], -1])/(Sqrt[d - e*x^2]*Sqrt[a*d + a*e*x^2]))/
(2*d^2*e))/Sqrt[a*d^2 - a*e^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 329
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a*(Sqrt[1 - b^2*(x^4/a^2)]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]))   Int[Sqrt[1
+ b*(x^2/a)]/Sqrt[1 - b*(x^2/a)], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] &&  !(LtQ[a*c, 0] && GtQ[a*b, 0])
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.28

method	result
elliptic	$-\frac{(-a\,e^2x^2-ade)x(Ae+Bd)}{2d^2a\,e^2\sqrt{\left(x^2-\frac{d}{e}\right)(-a\,e^2x^2-ade)}}+\frac{\left(-\frac{B}{e}+\frac{Ae+Bd}{2de}\right)\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{\sqrt{\frac{e}{d}}\,\sqrt{-a\,e^2x^4+a\,d^2}}+\frac{(Ae+Bd)\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}}{2d\sqrt{}}$
default	$\frac{(Ae+Bd)\left(-\frac{\left(-a\,e^2x^2-ade\right)x}{2d^2ae\sqrt{\left(x^2-\frac{d}{e}\right)\left(-a\,e^2x^2-ade\right)}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)}{2d\sqrt{\frac{e}{d}}\,\sqrt{-a\,e^2x^4+a\,d^2}}+\frac{\sqrt{1-\frac{e\,x^2}{d}}\,\sqrt{1+\frac{e\,x^2}{d}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{e}{d}},i\right)-\mathrm{EllipticE}\left(\right.\right.}{2d\sqrt{\frac{e}{d}}\,\sqrt{-a\,e^2x^4+a\,d^2}}\right.}{e}$

input
```
int((B*x^2+A)/(-e*x^2+d)/(-a*e^2*x^4+a*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a*e^2*x^2-a*d*e)/d^2*x/a/e^2*(A*e+B*d)/((x^2-d/e)*(-a*e^2*x^2-a*d*e
))^(1/2)+(-B/e+1/2/d/e*(A*e+B*d))/(1/d*e)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2
/d)^(1/2)/(-a*e^2*x^4+a*d^2)^(1/2)*EllipticF(x*(1/d*e)^(1/2),I)+1/2*(A*e+B
*d)/d/(1/d*e)^(1/2)*(1-e*x^2/d)^(1/2)*(1+e*x^2/d)^(1/2)/(-a*e^2*x^4+a*d^2)
^(1/2)/e*(EllipticF(x*(1/d*e)^(1/2),I)-EllipticE(x*(1/d*e)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2-ae^2x^4}}\,dx$$
$$= \frac{(Bd^2e+Ade^2-(Bde^2+Ae^3)x^2)\sqrt{ad^2}\sqrt{\frac{e}{d}}E(\arcsin\left(x\sqrt{\frac{e}{d}}\right)|-1)+(Bd^3-(A+B)d^2e-Ade^2-(Bd^2}{2\left(ad^3e^3x^2-a\right.}$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)/(-a*e^2*x^4+a*d^2)^(1/2),x, algorithm="fric
as")
```

output
```
1/2*((B*d^2*e + A*d*e^2 - (B*d*e^2 + A*e^3)*x^2)*sqrt(a*d^2)*sqrt(e/d)*ell
iptic_e(arcsin(x*sqrt(e/d)), -1) + (B*d^3 - (A + B)*d^2*e - A*d*e^2 - (B*d
^2*e - (A + B)*d*e^2 - A*e^3)*x^2)*sqrt(a*d^2)*sqrt(e/d)*elliptic_f(arcsin
(x*sqrt(e/d)), -1) - sqrt(-a*e^2*x^4 + a*d^2)*(B*d^2*e + A*d*e^2)*x)/(a*d^
3*e^3*x^2 - a*d^4*e^2)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = -\int \frac{A}{-d\sqrt{ad^2-ae^2x^4}+ex^2\sqrt{ad^2-ae^2x^4}}\,dx - \int \frac{Bx^2}{-d\sqrt{ad^2-ae^2x^4}+ex^2\sqrt{ad^2-ae^2x^4}}\,dx$$

input
```
integrate((B*x**2+A)/(-e*x**2+d)/(-a*e**2*x**4+a*d**2)**(1/2),x)
```

output
```
-Integral(A/(-d*sqrt(a*d**2 - a*e**2*x**4) + e*x**2*sqrt(a*d**2 - a*e**2*x
**4)), x) - Integral(B*x**2/(-d*sqrt(a*d**2 - a*e**2*x**4) + e*x**2*sqrt(a
*d**2 - a*e**2*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)/(-a*e^2*x^4+a*d^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = \int -\frac{Bx^2+A}{\sqrt{-ae^2x^4+ad^2}(ex^2-d)}\,dx$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)/(-a*e^2*x^4+a*d^2)^(1/2),x, algorithm="giac
")
```

output
```
integrate(-(B*x^2 + A)/(sqrt(-a*e^2*x^4 + a*d^2)*(e*x^2 - d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2-ae^2x^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{a\,d^2-a\,e^2\,x^4}\,\left(d-e\,x^2\right)}\,dx$$

input
```
int((A + B*x^2)/((a*d^2 - a*e^2*x^4)^(1/2)*(d - e*x^2)),x)
```

output
```
int((A + B*x^2)/((a*d^2 - a*e^2*x^4)^(1/2)*(d - e*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2-ae^2x^4}}\,dx$$
$$=\frac{\sqrt{a}\left(\sqrt{-e^2x^4+d^2}\,bx+2\left(\int\frac{\sqrt{-e^2x^4+d^2}}{e^3x^6-d\,e^2x^4-d^2e\,x^2+d^3}dx\right)a\,d^2e-2\left(\int\frac{\sqrt{-e^2x^4+d^2}}{e^3x^6-d\,e^2x^4-d^2e\,x^2+d^3}dx\right)ad\,e^2x^2-\left(\int\frac{\sqrt{-}}{-}\right.\right.}{2ade\left(-e\,x^2+d\right)}$$

input
```
int((B*x^2+A)/(-e*x^2+d)/(-a*e^2*x^4+a*d^2)^(1/2),x)
```

output
```
(sqrt(a)*(sqrt(d**2 - e**2*x**4)*b*x + 2*int(sqrt(d**2 - e**2*x**4)/(d**3
- d**2*e*x**2 - d*e**2*x**4 + e**3*x**6),x)*a*d**2*e - 2*int(sqrt(d**2 - e
**2*x**4)/(d**3 - d**2*e*x**2 - d*e**2*x**4 + e**3*x**6),x)*a*d*e**2*x**2
- int(sqrt(d**2 - e**2*x**4)/(d - e*x**2),x)*b*d + int(sqrt(d**2 - e**2*x*
*4)/(d - e*x**2),x)*b*e*x**2))/(2*a*d*e*(d - e*x**2))
```

3.5 $\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}} dx$

Optimal result

Integrand size = 35, antiderivative size = 106

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}} dx = \frac{B\arctan\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{e^{3/2}} - \frac{(Bd-Ae)\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{2}de^{3/2}}$$

output
```
B*arctan(e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(3/2)-1/2*(-A*e
+B*d)*arctan(2^(1/2)*e^(1/2)*x*(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(1/
2)/d/e^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}} dx = \frac{(-Bd+Ae)\sqrt{d^2-e^2x^4}\arctan\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d-ex^2}}\right)}{\sqrt{2}de^{3/2}\sqrt{d-ex^2}\sqrt{d+ex^2}} + \frac{iB\log\left(-2i\sqrt{e}x+\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{d+ex^2}}\right)}{e^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
((-(B*d) + A*e)*Sqrt[d^2 - e^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*
x^2]])/(Sqrt[2]*d*e^(3/2)*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + (I*B*Log[(-2*
I)*Sqrt[e]*x + (2*Sqrt[d^2 - e^2*x^4])/Sqrt[d + e*x^2]])/e^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1396, 398, 224, 216, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{Bx^2+A}{\sqrt{d-ex^2}(ex^2+d)}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{B\int \frac{1}{\sqrt{d-ex^2}}dx}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{B\int \frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{d-ex^2}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\int\frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{2}de^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*((B*ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^2]])/
e^(3/2) - ((B*d - A*e)*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]])/(Sqrt[
2]*d*e^(3/2))))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] && !IntegerQ[p] && !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(86) = 172$.

Time $= 0.34$ (sec) , antiderivative size $= 434$, normalized size of antiderivative $= 4.09$

method	result
default	$-\frac{\sqrt{-e^2x^4+d^2}\left(-A\sqrt{2}\,\sqrt{d}\,\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^2+d}-\sqrt{-de}\,x+d\right)}{ex-\sqrt{-de}}\right)e^{\frac{3}{2}}+A\sqrt{2}\,\sqrt{d}\,\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^2+d}+\sqrt{-de}\,x+d\right)}{ex+\sqrt{-de}}\right)e^{\frac{3}{2}}+B\sqrt{2}\,d^{\frac{3}{2}}\right.}{}$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/2*(-e^2*x^4+d^2)^(1/2)*(-A*2^(1/2)*d^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*
x^2+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*e^(3/2)+A*2^(1/2)*d^(1/
2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^
(1/2)))*e^(3/2)+B*2^(1/2)*d^(3/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)
-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*e^(1/2)-B*2^(1/2)*d^(3/2)*ln(2*e*(2
^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*e^(1
/2)-2*A*(-d*e)^(1/2)*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*e+2*A*(-d*e)^(1/2)
*arctan(e^(1/2)*x/(1/e*(-e*x+(d*e)^(1/2))*(e*x+(d*e)^(1/2)))^(1/2))*e+2*B*
(-d*e)^(1/2)*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*d+2*B*(-d*e)^(1/2)*arctan(
e^(1/2)*x/(1/e*(-e*x+(d*e)^(1/2))*(e*x+(d*e)^(1/2)))^(1/2))*d)/(e*x^2+d)^(
1/2)/(-e*x^2+d)^(1/2)/((-d*e)^(1/2)-(d*e)^(1/2))/((-d*e)^(1/2)+(d*e)^(1/2)
)/(-d*e)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.84

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \left[-\frac{2\,Bd\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{-e}x-d^2}{ex^2+d}\right)-\sqrt{2}(Bd-Ae)\sqrt{-e}\log\left(-\frac{3\,e^2x^4+2\,dex^2-2\,\sqrt{2}\sqrt{-\cdots}}{e^2x^4+\cdots}\right.}{4\,de^2}\right.$$
$$\left.-\frac{2\,Bd\sqrt{e}\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)-\sqrt{2}(Bd-Ae)\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}\sqrt{e}x}{e^2x^4-d^2}\right)}{2\,de^2}\right]$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fri
cas")
```

output
```
[-1/4*(2*B*d*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(-e^2*x^4 + d^2)*s
qrt(e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) - sqrt(2)*(B*d - A*e)*sqrt(-
e)*log(-(3*e^2*x^4 + 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2
 + d)*sqrt(-e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)))/(d*e^2), -1/2*(2*B*d
*sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 -
d^2)) - sqrt(2)*(B*d - A*e)*sqrt(e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sq
rt(e*x^2 + d)*sqrt(e)*x/(e^2*x^4 - d^2)))/(d*e^2)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{-(-d+ex^2)(d+ex^2)}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(1/2)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(-(-d + e*x**2)*(d + e*x**2))*sqrt(d + e*x**2))
, x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-e^2x^4+d^2}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = -\frac{B\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2+d}\right|\right)}{\sqrt{-e}e} + \frac{\sqrt{2}(Bd-Ae)\sqrt{-e}\log\left(\frac{\left|2\left(\sqrt{-e}x-\sqrt{-ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{-e}x-\sqrt{-ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{4e^2|d|}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="gia
c")
```

output
```
-B*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 + d)))/(sqrt(-e)*e) + 1/4*sqrt(2)*(B*
d - A*e)*sqrt(-e)*log(abs(2*(sqrt(-e)*x - sqrt(-e*x^2 + d))^2 - 4*sqrt(2)*
abs(d) - 6*d)/abs(2*(sqrt(-e)*x - sqrt(-e*x^2 + d))^2 + 4*sqrt(2)*abs(d) -
 6*d))/(e^2*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{d^2-e^2\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2)/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((d^2 - e^2*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.74

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \frac{\sqrt{e}\left(4asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)bd+2\sqrt{2}\,atan\left(\frac{\tan\left(\frac{asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2}\right)}{\sqrt{2}+1}\right)ae-2\sqrt{2}\,atan\left(\frac{\tan\left(\frac{asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2}\right)}{\sqrt{2}+1}\right)bd-\sqrt{2}\log\left(-\sqrt{\cdots}\right.\right.}{\cdots}$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(sqrt(e)*(4*asin((sqrt(e)*x)/sqrt(d))*b*d + 2*sqrt(2)*atan(tan(asin((sqrt(
e)*x)/sqrt(d))/2)/(sqrt(2) + 1))*a*e - 2*sqrt(2)*atan(tan(asin((sqrt(e)*x)
/sqrt(d))/2)/(sqrt(2) + 1))*b*d - sqrt(2)*log( - sqrt(2)*i + tan(asin((sqr
t(e)*x)/sqrt(d))/2) + i)*a*e*i + sqrt(2)*log( - sqrt(2)*i + tan(asin((sqrt
(e)*x)/sqrt(d))/2) + i)*b*d*i + sqrt(2)*log(sqrt(2)*i + tan(asin((sqrt(e)*
x)/sqrt(d))/2) - i)*a*e*i - sqrt(2)*log(sqrt(2)*i + tan(asin((sqrt(e)*x)/s
qrt(d))/2) - i)*b*d*i))/(4*d*e**2)
```

3.6 $\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 107

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}}\,dx = -\frac{B\text{arctanh}\left(\frac{\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{e^{3/2}}+\frac{(Bd+Ae)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{2}de^{3/2}}$$

output
```
-B*arctanh(e^(1/2)*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(3/2)+1/2*(A
*e+B*d)*arctanh(2^(1/2)*e^(1/2)*x*(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2
^(1/2)/d/e^(3/2)
```

Mathematica [A] (verified)

Time = 5.78 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}}\,dx = \frac{\frac{\sqrt{2}(Bd+Ae)\sqrt{d^2-e^2x^4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{d\sqrt{d-ex^2}\sqrt{d+ex^2}}+2B\left(\log\left(-d+ex^2\right)-\log\left(dex-e^2x^3+\sqrt{e}\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}\right)\right)}{2e^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d - e*x^2]*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
((Sqrt[2]*(B*d + A*e)*Sqrt[d^2 - e^2*x^4]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt
[d + e*x^2]])/(d*Sqrt[d - e*x^2]*Sqrt[d + e*x^2]) + 2*B*(Log[-d + e*x^2] -
 Log[d*e*x - e^2*x^3 + Sqrt[e]*Sqrt[d - e*x^2]*Sqrt[d^2 - e^2*x^4]]))/(2*e
^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1396, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\int \frac{Bx^2+A}{(d-ex^2)\sqrt{ex^2+d}}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{398}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{(d-ex^2)\sqrt{ex^2+d}}dx}{e}-\frac{B\int \frac{1}{\sqrt{ex^2+d}}dx}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{(d-ex^2)\sqrt{ex^2+d}}dx}{e}-\frac{B\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{(d-ex^2)\sqrt{ex^2+d}}dx}{e}-\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}-\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d-ex^2}\sqrt{d+ex^2}\left(\frac{(Ae+Bd)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}de^{3/2}}-\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(A + B*x^2)/(Sqrt[d - e*x^2]*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(Sqrt[d - e*x^2]*Sqrt[d + e*x^2]*(-((B*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/e^(3/2)) + ((B*d + A*e)*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]])/(
Sqrt[2]*d*e^(3/2))))/Sqrt[d^2 - e^2*x^4]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(88) = 176$.

Time = 0.20 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.20

method	result
default	$\frac{\sqrt{-e^2x^4+d^2}\left(A\ln\left(\frac{2e\left(\sqrt{2}\sqrt{d}\sqrt{e\,x^2+d}+\sqrt{de}\,x+d\right)}{ex-\sqrt{de}}\right)\sqrt{2}\sqrt{d}\,e^{\frac{3}{2}}-A\ln\left(\frac{2e\left(\sqrt{2}\sqrt{d}\sqrt{e\,x^2+d}-\sqrt{de}\,x+d\right)}{ex+\sqrt{de}}\right)\sqrt{2}\sqrt{d}\,e^{\frac{3}{2}}+B\ln\left(\frac{2e\left(\sqrt{2}\sqrt{d}\sqrt{e\right.}}{\ldots}\right.\right.}{\ldots}$

input
```
int((B*x^2+A)/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/2/(-e*x^2+d)^(1/2)*(-e^2*x^4+d^2)^(1/2)*(A*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^
2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*e)^(1/2)))*2^(1/2)*d^(1/2)*e^(3/2)-A*l
n(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))
*2^(1/2)*d^(1/2)*e^(3/2)+B*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e)^(
1/2)*x+d)/(e*x-(d*e)^(1/2)))*2^(1/2)*d^(3/2)*e^(1/2)-B*ln(2*e*(2^(1/2)*d^(
1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*2^(1/2)*d^(3/2)*e
^(1/2)+2*A*(d*e)^(1/2)*ln((e^(1/2)*(-1/e*(-e*x+(-d*e)^(1/2))*(e*x+(-d*e)^(
1/2)))^(1/2)+e*x)/e^(1/2))*e-2*A*(d*e)^(1/2)*ln(((e*x^2+d)^(1/2)*e^(1/2)+e
*x)/e^(1/2))*e-2*B*(d*e)^(1/2)*ln((e^(1/2)*(-1/e*(-e*x+(-d*e)^(1/2))*(e*x+
(-d*e)^(1/2)))^(1/2)+e*x)/e^(1/2))*d-2*B*(d*e)^(1/2)*ln(((e*x^2+d)^(1/2)*e
^(1/2)+e*x)/e^(1/2))*d)/(e*x^2+d)^(1/2)/(-(-d*e)^(1/2)+(d*e)^(1/2))/((-d*e
)^(1/2)+(d*e)^(1/2))/e^(1/2)/(d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.84

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}}\,dx$$
$$= \left[\frac{2\,Bd\sqrt{e}\log\left(\frac{2\,e^2x^4-dex^2+2\,\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}\sqrt{e}x-d^2}{ex^2-d}\right)+\sqrt{2}(Bd+Ae)\sqrt{e}\log\left(-\frac{3\,e^2x^4-2\,dex^2-2\,\sqrt{2}\sqrt{-e^2x^4+d^2}\cdots}{e^2x^4-2\,dex^2+d^2}\cdots\right)}{4\,de^2}\right.$$
$$\left.-\frac{2\,Bd\sqrt{-e}\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}\sqrt{-e}x}{e^2x^4-d^2}\right)-\sqrt{2}(Bd+Ae)\sqrt{-e}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}\sqrt{-e}x}{e^2x^4-d^2}\right)}{2\,de^2}\right]$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fr
icas")
```

output
```
[1/4*(2*B*d*sqrt(e)*log((2*e^2*x^4 - d*e*x^2 + 2*sqrt(-e^2*x^4 + d^2)*sqrt
(-e*x^2 + d)*sqrt(e)*x - d^2)/(e*x^2 - d)) + sqrt(2)*(B*d + A*e)*sqrt(e)*l
og(-(3*e^2*x^4 - 2*d*e*x^2 - 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 +
d)*sqrt(e)*x - d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)))/(d*e^2), -1/2*(2*B*d*sqr
t(-e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d
^2)) - sqrt(2)*(B*d + A*e)*sqrt(-e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sq
rt(-e*x^2 + d)*sqrt(-e)*x/(e^2*x^4 - d^2)))/(d*e^2)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}\sqrt{d-ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(-e*x**2+d)**(1/2)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(-(-d + e*x**2)*(d + e*x**2))*sqrt(d - e*x**2))
, x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-e^2x^4+d^2}\sqrt{-ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}}\,dx = \frac{B\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{e^{\frac{3}{2}}} + \frac{\sqrt{2}(Bd+Ae)\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6\,d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6\,d\right|}\right)}{4\,e^{\frac{3}{2}}|d|}$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="gi
ac")
```

output
```
B*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(3/2) + 1/4*sqrt(2)*(B*d + A*e)
*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d) - 6*d)/abs(2
*(sqrt(e)*x - sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(e^(3/2)*abs(d
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{d^2-e^2\,x^4}\,\sqrt{d-e\,x^2}}\,dx$$

input
```
int((A + B*x^2)/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((d^2 - e^2*x^4)^(1/2)*(d - e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.79

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{d^2-e^2x^4}}\,dx$$
$$= \frac{\sqrt{e}\left(-\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae-\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)bd+\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}+\sqrt{}}{\sqrt{d}}\right.}{}$$

input
```
int((B*x^2+A)/(-e*x^2+d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(sqrt(e)*( - sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + s
qrt(e)*x)/sqrt(d))*a*e - sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) -
 sqrt(d) + sqrt(e)*x)/sqrt(d))*b*d + sqrt(2)*log((sqrt(d + e*x**2) - sqrt(
d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e + sqrt(2)*log((sqrt(d + e*x
**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*b*d + sqrt(2)*log((
sqrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e + s
qrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt
(d))*b*d - sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqr
t(e)*x)/sqrt(d))*a*e - sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + s
qrt(d) + sqrt(e)*x)/sqrt(d))*b*d - 4*log((sqrt(d + e*x**2) + sqrt(e)*x)/sq
rt(d))*b*d))/(4*d*e**2)
```

3.7 $\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 109

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{d^2-e^2x^4}}\,dx=\frac{B\arctan\left(\frac{\sqrt{e}x\sqrt{-d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{e^{3/2}}-\frac{(Bd+Ae)\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{-d+ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{2}de^{3/2}}$$

output
```
B*arctan(e^(1/2)*x*(e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(3/2)-1/2*(A*e+
B*d)*arctan(2^(1/2)*e^(1/2)*x*(e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*2^(1/2
)/d/e^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.90 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{d^2-e^2x^4}}\,dx=\frac{\frac{(Bd+Ae)\sqrt{-2d+2ex^2}\sqrt{d^2-e^2x^4}\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{-d-ex^2}}\right)}{d\sqrt{-d-ex^2}(d-ex^2)}-2iB\log\left(-2i\sqrt{e}x-\frac{2\sqrt{d^2-e^2x^4}}{\sqrt{-d+ex^2}}\right)}{2e^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[-d + e*x^2]*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
(((B*d + A*e)*Sqrt[-2*d + 2*e*x^2]*Sqrt[d^2 - e^2*x^4]*ArcTan[(Sqrt[2]*Sqr
t[e]*x)/Sqrt[-d - e*x^2]])/(d*Sqrt[-d - e*x^2]*(d - e*x^2)) - (2*I)*B*Log[
(-2*I)*Sqrt[e]*x - (2*Sqrt[d^2 - e^2*x^4])/Sqrt[-d + e*x^2]])/(2*e^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {1396, 25, 398, 224, 216, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{ex^2-d}\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\int -\frac{Bx^2+A}{\sqrt{-ex^2-d}(d-ex^2)}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\int \frac{Bx^2+A}{\sqrt{-ex^2-d}(d-ex^2)}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{-ex^2-d}\left(d-ex^2\right)}dx}{e}-\frac{B\int \frac{1}{\sqrt{-ex^2-d}}dx}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{-ex^2-d}\left(d-ex^2\right)}dx}{e}-\frac{B\int \frac{1}{\frac{ex^2}{-ex^2-d}+1}d\frac{x}{\sqrt{-ex^2-d}}}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{-ex^2-d}\left(d-ex^2\right)}dx}{e}-\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{-d-ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 291}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{\frac{2dex^2}{-ex^2-d}+d}d\frac{x}{\sqrt{-ex^2-d}}}{e}-\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{-d-ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{-d-ex^2}}\right)}{\sqrt{2}de^{3/2}}-\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{-d-ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(A + B*x^2)/(Sqrt[-d + e*x^2]*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
-((Sqrt[-d - e*x^2]*Sqrt[-d + e*x^2]*(-((B*ArcTan[(Sqrt[e]*x)/Sqrt[-d - e*
x^2]])/e^(3/2)) + ((B*d + A*e)*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[-d - e*x^2]
])/(Sqrt[2]*d*e^(3/2))))/Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(89) = 178$.

Time = 0.14 (sec) , antiderivative size = 480, normalized size of antiderivative = 4.40

method	result
default	$\frac{\sqrt{-e^{2} x^{4}+d^{2}}\left(A \sqrt{d e}\, \arctan\left(\frac{\sqrt{e}\, x}{\sqrt{\frac{\left(-e x+\sqrt{-d e}\right)\left(e x+\sqrt{-d e}\right)}{e}}}\right)\sqrt{2}\, \sqrt{-d}\, e-A \sqrt{d e}\, \arctan\left(\frac{\sqrt{e}\, x}{\sqrt{-e\, x^{2}-d}}\right)\sqrt{2}\, \sqrt{-d}\, e+A \ln\left(\frac{2 e\left(\sqrt{2}\, \sqrt{-d}\, \sqrt{-}\right.}{e x}\right.\right.}{}$

input
```
int((B*x^2+A)/(e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*(-e^2*x^4+d^2)^(1/2)*(A*(d*e)^(1/2)*arctan(e^(1/2)*x/(1/e*(-e*x+(-d*e)
^(1/2))*(e*x+(-d*e)^(1/2)))^(1/2))*2^(1/2)*(-d)^(1/2)*e-A*(d*e)^(1/2)*arct
an(e^(1/2)*x/(-e*x^2-d)^(1/2))*2^(1/2)*(-d)^(1/2)*e+A*ln(2*e*(2^(1/2)*(-d)
^(1/2)*(-e*x^2-d)^(1/2)-(d*e)^(1/2)*x-d)/(e*x-(d*e)^(1/2)))*e^(3/2)*d-A*ln
(2*e*((d*e)^(1/2)*x+2^(1/2)*(-d)^(1/2)*(-e*x^2-d)^(1/2)-d)/(e*x+(d*e)^(1/2
)))*e^(3/2)*d-B*(d*e)^(1/2)*arctan(e^(1/2)*x/(1/e*(-e*x+(-d*e)^(1/2))*(e*x
+(-d*e)^(1/2)))^(1/2))*2^(1/2)*(-d)^(1/2)*d-B*(d*e)^(1/2)*arctan(e^(1/2)*x
/(-e*x^2-d)^(1/2))*2^(1/2)*(-d)^(1/2)*d+B*ln(2*e*(2^(1/2)*(-d)^(1/2)*(-e*x
^2-d)^(1/2)-(d*e)^(1/2)*x-d)/(e*x-(d*e)^(1/2)))*e^(1/2)*d^2-B*ln(2*e*((d*e
)^(1/2)*x+2^(1/2)*(-d)^(1/2)*(-e*x^2-d)^(1/2)-d)/(e*x+(d*e)^(1/2)))*e^(1/2
)*d^2)/(e*x^2-d)^(1/2)/(-e*x^2-d)^(1/2)/((-d*e)^(1/2)-(d*e)^(1/2))/((-d*e)
^(1/2)+(d*e)^(1/2))*2^(1/2)/e^(1/2)/(d*e)^(1/2)/(-d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.83

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{d^2-e^2x^4}}\,dx$$
$$= \left[-\frac{2\,Bd\sqrt{-e}\log\left(\frac{2\,e^2x^4-dex^2-2\,\sqrt{-e^2x^4+d^2}\sqrt{ex^2-d}\sqrt{-e}x-d^2}{ex^2-d}\right)+\sqrt{2}(Bd+Ae)\sqrt{-e}\log\left(-\frac{3\,e^2x^4-2\,dex^2+2\,\sqrt{2}\sqrt{-e^2}\cdots}{e^2x^4-2\,d\cdots}\right)}{4\,de^2}\right.$$
$$\left.-\frac{2\,Bd\sqrt{e}\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\sqrt{ex^2-d}\sqrt{e}x}{e^2x^4-d^2}\right)-\sqrt{2}(Bd+Ae)\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{ex^2-d}\sqrt{e}x}{e^2x^4-d^2}\right)}{2\,de^2}\right]$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fri
cas")
```

output
```
[-1/4*(2*B*d*sqrt(-e)*log((2*e^2*x^4 - d*e*x^2 - 2*sqrt(-e^2*x^4 + d^2)*sq
rt(e*x^2 - d)*sqrt(-e)*x - d^2)/(e*x^2 - d)) + sqrt(2)*(B*d + A*e)*sqrt(-e
)*log(-(3*e^2*x^4 - 2*d*e*x^2 + 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2
- d)*sqrt(-e)*x - d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)))/(d*e^2), -1/2*(2*B*d*
sqrt(e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 - d)*sqrt(e)*x/(e^2*x^4 - d
^2)) - sqrt(2)*(B*d + A*e)*sqrt(e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqr
t(e*x^2 - d)*sqrt(e)*x/(e^2*x^4 - d^2)))/(d*e^2)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{-\left(-d+ex^2\right)\left(d+ex^2\right)}\sqrt{-d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2-d)**(1/2)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(-(-d + e*x**2)*(d + e*x**2))*sqrt(-d + e*x**2)
), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-e^2x^4+d^2}\sqrt{ex^2-d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-e^2*x^4 + d^2)*sqrt(e*x^2 - d)), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{d^2-e^2x^4}}\,dx = -\frac{B\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2-d}\right|\right)}{\sqrt{-e}e} - \frac{\sqrt{2}(Bd+Ae)\sqrt{-e}\log\left(\frac{\left|2\left(\sqrt{-e}x-\sqrt{-ex^2-d}\right)^2-4\sqrt{2}|d|+6d\right|}{\left|2\left(\sqrt{-e}x-\sqrt{-ex^2-d}\right)^2+4\sqrt{2}|d|+6d\right|}\right)}{4e^2|d|}$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="gia
c")
```

output
```
-B*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 - d)))/(sqrt(-e)*e) - 1/4*sqrt(2)*(B*
d + A*e)*sqrt(-e)*log(abs(2*(sqrt(-e)*x - sqrt(-e*x^2 - d))^2 - 4*sqrt(2)*
abs(d) + 6*d)/abs(2*(sqrt(-e)*x - sqrt(-e*x^2 - d))^2 + 4*sqrt(2)*abs(d) +
 6*d))/(e^2*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{d^2-e^2x^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{d^2-e^2\,x^4}\,\sqrt{e\,x^2-d}}\,dx$$

input
```
int((A + B*x^2)/((d^2 - e^2*x^4)^(1/2)*(e*x^2 - d)^(1/2)),x)
```

output
```
int((A + B*x^2)/((d^2 - e^2*x^4)^(1/2)*(e*x^2 - d)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.65

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{d^2-e^2x^4}}\,dx$$
$$=\frac{\sqrt{e}\,i\left(-4asinh\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)bd-\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae-\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)bd+\sqrt{2}}{}$$

input
```
int((B*x^2+A)/(e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(sqrt(e)*i*( - 4*asinh((sqrt(e)*x)/sqrt(d))*b*d - sqrt(2)*log((sqrt(d + e*
x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e - sqrt(2)*log(
(sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*b*d +
sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqr
t(d))*a*e + sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sq
rt(e)*x)/sqrt(d))*b*d + sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) -
sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e + sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d
)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*b*d - sqrt(2)*log((sqrt(d + e*x*
*2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e - sqrt(2)*log((s
qrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*b*d))/(4
*d*e**2)
```

3.8 $\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{d^2-e^2x^4}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 112

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{d^2-e^2x^4}}\,dx=-\frac{B\operatorname{arctanh}\left(\frac{\sqrt{e}x\sqrt{-d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{e^{3/2}}+\frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{-d-ex^2}}{\sqrt{d^2-e^2x^4}}\right)}{\sqrt{2}de^{3/2}}$$

output
```
-B*arctanh(e^(1/2)*x*(-e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2))/e^(3/2)+1/2*(-
A*e+B*d)*arctanh(2^(1/2)*e^(1/2)*x*(-e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2))*
2^(1/2)/d/e^(3/2)
```

Mathematica [A] (verified)

Time = 5.79 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{d^2-e^2x^4}}\,dx$$

$$=\frac{\frac{\sqrt{2}(Bd-Ae)\sqrt{d^2-e^2x^4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{-d+ex^2}}\right)}{d\sqrt{-d-ex^2}\sqrt{-d+ex^2}}+2B\left(-\log\left(d+ex^2\right)+\log\left(dex+e^2x^3+\sqrt{e}\sqrt{-d-ex^2}\sqrt{d^2-e^2x\right.\right.}{2e^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[-d - e*x^2]*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
((Sqrt[2]*(B*d - A*e)*Sqrt[d^2 - e^2*x^4]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt
[-d + e*x^2]])/(d*Sqrt[-d - e*x^2]*Sqrt[-d + e*x^2]) + 2*B*(-Log[d + e*x^2
] + Log[d*e*x + e^2*x^3 + Sqrt[e]*Sqrt[-d - e*x^2]*Sqrt[d^2 - e^2*x^4]]))/
(2*e^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1396, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{d^2-e^2x^4}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\int -\frac{Bx^2+A}{\sqrt{ex^2-d}(ex^2+d)}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\int \frac{Bx^2+A}{\sqrt{ex^2-d}(ex^2+d)}dx}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\left(\frac{B\int \frac{1}{\sqrt{ex^2-d}}dx}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{ex^2-d}(ex^2+d)}dx}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\left(\frac{B\int \frac{1}{1-\frac{ex^2}{ex^2-d}}d\frac{x}{\sqrt{ex^2-d}}}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{ex^2-d}(ex^2+d)}dx}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2-d}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\int\frac{1}{\sqrt{ex^2-d}(ex^2+d)}dx}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2-d}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\int\frac{1}{d-\frac{2dex^2}{ex^2-d}}d\frac{x}{\sqrt{ex^2-d}}}{e}\right)}{\sqrt{d^2-e^2x^4}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{ex^2-d}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2-d}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{ex^2-d}}\right)}{\sqrt{2}de^{3/2}}\right)}{\sqrt{d^2-e^2x^4}}$$

input
```
Int[(A + B*x^2)/(Sqrt[-d - e*x^2]*Sqrt[d^2 - e^2*x^4]),x]
```

output
```
-((Sqrt[-d - e*x^2]*Sqrt[-d + e*x^2]*((B*ArcTanh[(Sqrt[e]*x)/Sqrt[-d + e*x
^2]])/e^(3/2) - ((B*d - A*e)*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[-d + e*x^2]]
)/(Sqrt[2]*d*e^(3/2))))/Sqrt[d^2 - e^2*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] && !IntegerQ[p] && !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(93) = 186$.

Time = 0.10 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.58

method	result
default	$\frac{\sqrt{-e^2x^4+d^2}\left(A\sqrt{-de}\,\ln\left(\frac{\sqrt{e\,x^2-d}\,\sqrt{e}+ex}{\sqrt{e}}\right)\sqrt{2}\,\sqrt{-d}\,e-A\sqrt{-de}\,\ln\left(\frac{\sqrt{e}\,\sqrt{-\frac{\left(-ex+\sqrt{de}\right)\left(ex+\sqrt{de}\right)}{e}}+ex}{\sqrt{e}}\right)\sqrt{2}\,\sqrt{-d}\,e+A\ln\left(\frac{2e\left(\sqrt{-d}\right.}{}\right.\right.}{}$

input
```
int((B*x^2+A)/(-e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/2*(-e^2*x^4+d^2)^(1/2)*(A*(-d*e)^(1/2)*ln(((e*x^2-d)^(1/2)*e^(1/2)+e*x)/
e^(1/2))*2^(1/2)*(-d)^(1/2)*e-A*(-d*e)^(1/2)*ln((e^(1/2)*(-1/e*(-e*x+(d*e)
^(1/2))*(e*x+(d*e)^(1/2)))^(1/2)+e*x)/e^(1/2))*2^(1/2)*(-d)^(1/2)*e+A*ln(2
*e*((-d*e)^(1/2)*x+2^(1/2)*(-d)^(1/2)*(e*x^2-d)^(1/2)-d)/(e*x-(-d*e)^(1/2)
))*e^(3/2)*d-A*ln(2*e*(2^(1/2)*(-d)^(1/2)*(e*x^2-d)^(1/2)-(-d*e)^(1/2)*x-d
)/(e*x+(-d*e)^(1/2)))*e^(3/2)*d-B*(-d*e)^(1/2)*ln(((e*x^2-d)^(1/2)*e^(1/2)
+e*x)/e^(1/2))*2^(1/2)*(-d)^(1/2)*d-B*(-d*e)^(1/2)*ln((e^(1/2)*(-1/e*(-e*x
+(d*e)^(1/2))*(e*x+(d*e)^(1/2)))^(1/2)+e*x)/e^(1/2))*2^(1/2)*(-d)^(1/2)*d-
B*ln(2*e*((-d*e)^(1/2)*x+2^(1/2)*(-d)^(1/2)*(e*x^2-d)^(1/2)-d)/(e*x-(-d*e)
^(1/2)))*e^(1/2)*d^2+B*ln(2*e*(2^(1/2)*(-d)^(1/2)*(e*x^2-d)^(1/2)-(-d*e)^(
1/2)*x-d)/(e*x+(-d*e)^(1/2)))*e^(1/2)*d^2)/(-e*x^2-d)^(1/2)/(e*x^2-d)^(1/2
)/(-(-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)+(d*e)^(1/2))/(-d*e)^(1/2)/e^(1
/2)*2^(1/2)/(-d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.79

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{d^2-e^2x^4}}\,dx = \left[\frac{2\,Bd\sqrt{e}\log\left(-\frac{2\,e^2x^4+dex^2+2\,\sqrt{-e^2x^4+d^2}\sqrt{-ex^2-d}\sqrt{e}x-d^2}{ex^2+d}\right)-\sqrt{2}(Bd-Ae)\sqrt{e}\log\left(-\frac{3\,e^2x^4+2\,dex^2+2\,\sqrt{2}\sqrt{-e^2x^4+\cdots}}{e^2x^4+2\,dex^2\cdots}\right)\cdots}{4\,de^2}\right.$$
$$\left. -\frac{2\,Bd\sqrt{-e}\arctan\left(\frac{\sqrt{-e^2x^4+d^2}\sqrt{-ex^2-d}\sqrt{-e}x}{e^2x^4-d^2}\right)-\sqrt{2}(Bd-Ae)\sqrt{-e}\arctan\left(\frac{\sqrt{2}\sqrt{-e^2x^4+d^2}\sqrt{-ex^2-d}\sqrt{-e}x}{e^2x^4-d^2}\right)}{2\,de^2}\right]$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="fr
icas")
```

output
```
[1/4*(2*B*d*sqrt(e)*log(-(2*e^2*x^4 + d*e*x^2 + 2*sqrt(-e^2*x^4 + d^2)*sqr
t(-e*x^2 - d)*sqrt(e)*x - d^2)/(e*x^2 + d)) - sqrt(2)*(B*d - A*e)*sqrt(e)*
log(-(3*e^2*x^4 + 2*d*e*x^2 + 2*sqrt(2)*sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 -
 d)*sqrt(e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)))/(d*e^2), -1/2*(2*B*d*sq
rt(-e)*arctan(sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 - d)*sqrt(-e)*x/(e^2*x^4 -
d^2)) - sqrt(2)*(B*d - A*e)*sqrt(-e)*arctan(sqrt(2)*sqrt(-e^2*x^4 + d^2)*s
qrt(-e*x^2 - d)*sqrt(-e)*x/(e^2*x^4 - d^2)))/(d*e^2)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{-(-d+ex^2)(d+ex^2)}\sqrt{-d-ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(-e*x**2-d)**(1/2)/(-e**2*x**4+d**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(-(-d + e*x**2)*(d + e*x**2))*sqrt(-d - e*x**2)
), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{d^2-e^2x^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-e^2x^4+d^2}\sqrt{-ex^2-d}}\,dx$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-e^2*x^4 + d^2)*sqrt(-e*x^2 - d)), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{d^2-e^2x^4}}\,dx = \frac{B\log\left(\left|-\sqrt{e}x+\sqrt{ex^2-d}\right|\right)}{e^{\frac{3}{2}}} - \frac{\sqrt{2}(Bd-Ae)\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2-d}\right)^2-4\sqrt{2}|d|+6d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2-d}\right)^2+4\sqrt{2}|d|+6d\right|}\right)}{4e^{\frac{3}{2}}|d|}$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x, algorithm="gi
ac")
```

output
```
B*log(abs(-sqrt(e)*x + sqrt(e*x^2 - d)))/e^(3/2) - 1/4*sqrt(2)*(B*d - A*e)
*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 - d))^2 - 4*sqrt(2)*abs(d) + 6*d)/abs(2
*(sqrt(e)*x - sqrt(e*x^2 - d))^2 + 4*sqrt(2)*abs(d) + 6*d))/(e^(3/2)*abs(d
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{d^2-e^2x^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{d^2-e^2\,x^4}\,\sqrt{-e\,x^2-d}}\,dx$$

input
```
int((A + B*x^2)/((d^2 - e^2*x^4)^(1/2)*(- d - e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((d^2 - e^2*x^4)^(1/2)*(- d - e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.63

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{d^2-e^2x^4}}\,dx = \frac{\sqrt{e}\left(-4 asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right) bdi-2\sqrt{2}\,atan\left(\frac{\tan\left(\frac{asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2}\right)}{\sqrt{2}+1}\right) aei+2\sqrt{2}\,atan\left(\frac{\tan\left(\frac{asin\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2}\right)}{\sqrt{2}+1}\right) bdi-\sqrt{2}\log\left(\right.\right.}{}$$

input
```
int((B*x^2+A)/(-e*x^2-d)^(1/2)/(-e^2*x^4+d^2)^(1/2),x)
```

output
```
(sqrt(e)*( - 4*asin((sqrt(e)*x)/sqrt(d))*b*d*i - 2*sqrt(2)*atan(tan(asin((
sqrt(e)*x)/sqrt(d))/2)/(sqrt(2) + 1))*a*e*i + 2*sqrt(2)*atan(tan(asin((sqr
t(e)*x)/sqrt(d))/2)/(sqrt(2) + 1))*b*d*i - sqrt(2)*log( - sqrt(2)*i + tan(
asin((sqrt(e)*x)/sqrt(d))/2) + i)*a*e + sqrt(2)*log( - sqrt(2)*i + tan(asi
n((sqrt(e)*x)/sqrt(d))/2) + i)*b*d + sqrt(2)*log(sqrt(2)*i + tan(asin((sqr
t(e)*x)/sqrt(d))/2) - i)*a*e - sqrt(2)*log(sqrt(2)*i + tan(asin((sqrt(e)*x
)/sqrt(d))/2) - i)*b*d))/(4*d*e**2)
```

3.9 $\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 108

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \frac{B\operatorname{arctanh}\left(\frac{\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{-d^2+e^2x^4}}\right)}{e^{3/2}} - \frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d+ex^2}}{\sqrt{-d^2+e^2x^4}}\right)}{\sqrt{2}de^{3/2}}$$

output
```
B*arctanh(e^(1/2)*x*(e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2))/e^(3/2)-1/2*(-A*e
+B*d)*arctanh(2^(1/2)*e^(1/2)*x*(e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2))*2^(1/
2)/d/e^(3/2)
```

Mathematica [A] (verified)

Time = 5.68 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \frac{-\frac{(Bd-Ae)\sqrt{-2d^2+2e^2x^4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{-d+ex^2}}\right)}{d\sqrt{-d+ex^2}\sqrt{d+ex^2}} + 2B\left(-\log\left(d+ex^2\right)+\log\left(dex+e^2x^3+\sqrt{e}\sqrt{d+ex^2}\sqrt{-d^2+e\right.\right.}{2e^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[-d^2 + e^2*x^4]),x]
```

output
```
(-(((B*d - A*e)*Sqrt[-2*d^2 + 2*e^2*x^4]*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[
-d + e*x^2]])/(d*Sqrt[-d + e*x^2]*Sqrt[d + e*x^2])) + 2*B*(-Log[d + e*x^2]
 + Log[d*e*x + e^2*x^3 + Sqrt[e]*Sqrt[d + e*x^2]*Sqrt[-d^2 + e^2*x^4]]))/(
2*e^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1396, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{e^2x^4-d^2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\int \frac{Bx^2+A}{\sqrt{ex^2-d}(ex^2+d)}dx}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{398}$$

$$\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\left(\frac{B\int \frac{1}{\sqrt{ex^2-d}}dx}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{ex^2-d}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\left(\frac{B\int \frac{1}{1-\frac{ex^2}{ex^2-d}}d\frac{x}{\sqrt{ex^2-d}}}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{ex^2-d}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2-d}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\int\frac{1}{\sqrt{ex^2-d}(ex^2+d)}dx}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2-d}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\int\frac{1}{d-\frac{2dex^2}{ex^2-d}}d\frac{x}{\sqrt{ex^2-d}}}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2-d}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{ex^2-d}}\right)}{\sqrt{2}de^{3/2}}\right)}{\sqrt{e^2x^4-d^2}}$$

input
```
Int[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[-d^2 + e^2*x^4]),x]
```

output
```
(Sqrt[-d + e*x^2]*Sqrt[d + e*x^2]*((B*ArcTanh[(Sqrt[e]*x)/Sqrt[-d + e*x^2]
])/e^(3/2) - ((B*d - A*e)*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[-d + e*x^2]])/(
Sqrt[2]*d*e^(3/2))))/Sqrt[-d^2 + e^2*x^4]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(88) = 176$.

Time = 0.07 (sec) , antiderivative size = 511, normalized size of antiderivative = 4.73

method	result
default	$-\frac{\sqrt{e^{2} x^{4}-d^{2}}\left(A\sqrt{-de}\,\ln\left(\frac{\sqrt{e\,x^{2}-d}\,\sqrt{e}+ex}{\sqrt{e}}\right)\sqrt{2}\,\sqrt{-d}\,e-A\sqrt{-de}\,\ln\left(\frac{\sqrt{e}\,\sqrt{-\frac{\left(-ex+\sqrt{de}\right)\left(ex+\sqrt{de}\right)}{e}}+ex}{\sqrt{e}}\right)\sqrt{2}\,\sqrt{-d}\,e+A\ln\left(\frac{2e\left(\sqrt{-d}\right.}{}\right.\right.}{}$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e^2*x^4-d^2)^(1/2)*(A*(-d*e)^(1/2)*ln(((e*x^2-d)^(1/2)*e^(1/2)+e*x)/
e^(1/2))*2^(1/2)*(-d)^(1/2)*e-A*(-d*e)^(1/2)*ln((e^(1/2)*(-1/e*(-e*x+(d*e)
^(1/2))*(e*x+(d*e)^(1/2)))^(1/2)+e*x)/e^(1/2))*2^(1/2)*(-d)^(1/2)*e+A*ln(2
*e*((-d*e)^(1/2)*x+2^(1/2)*(-d)^(1/2)*(e*x^2-d)^(1/2)-d)/(e*x-(-d*e)^(1/2)
))*e^(3/2)*d-A*ln(2*e*(2^(1/2)*(-d)^(1/2)*(e*x^2-d)^(1/2)-(-d*e)^(1/2)*x-d
)/(e*x+(-d*e)^(1/2)))*e^(3/2)*d-B*(-d*e)^(1/2)*ln(((e*x^2-d)^(1/2)*e^(1/2)
+e*x)/e^(1/2))*2^(1/2)*(-d)^(1/2)*d-B*(-d*e)^(1/2)*ln((e^(1/2)*(-1/e*(-e*x
+(d*e)^(1/2))*(e*x+(d*e)^(1/2)))^(1/2)+e*x)/e^(1/2))*2^(1/2)*(-d)^(1/2)*d-
B*ln(2*e*((-d*e)^(1/2)*x+2^(1/2)*(-d)^(1/2)*(e*x^2-d)^(1/2)-d)/(e*x-(-d*e)
^(1/2)))*e^(1/2)*d^2+B*ln(2*e*(2^(1/2)*(-d)^(1/2)*(e*x^2-d)^(1/2)-(-d*e)^(
1/2)*x-d)/(e*x+(-d*e)^(1/2)))*e^(1/2)*d^2)*2^(1/2)/(e*x^2+d)^(1/2)/(e*x^2-
d)^(1/2)/(-(-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)+(d*e)^(1/2))/(-d*e)^(1/
2)/e^(1/2)/(-d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.55

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx$$

$$= \left[\frac{2\,Bd\sqrt{e}\log\left(-\frac{2\,e^2x^4+dex^2+2\,\sqrt{e^2x^4-d^2}\sqrt{ex^2+d}\sqrt{e}x-d^2}{ex^2+d}\right)-\sqrt{2}(Bd-Ae)\sqrt{e}\log\left(-\frac{3\,e^2x^4+2\,dex^2+2\,\sqrt{2}\sqrt{e^2x^4-d^2}\sqrt{e}\cdots}{e^2x^4+2\,dex^2+d^2}\cdots\right.}{4\,de^2}\right.$$

$$\left.-\frac{2\,Bd\sqrt{-e}\arctan\left(\frac{\sqrt{ex^2+d}\sqrt{-e}x}{\sqrt{e^2x^4-d^2}}\right)-\sqrt{2}(Bd-Ae)\sqrt{-e}\arctan\left(\frac{\sqrt{2}\sqrt{ex^2+d}\sqrt{-e}x}{\sqrt{e^2x^4-d^2}}\right)}{2\,de^2}\right]$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="fric
as")
```

output
```
[1/4*(2*B*d*sqrt(e)*log(-(2*e^2*x^4 + d*e*x^2 + 2*sqrt(e^2*x^4 - d^2)*sqrt
(e*x^2 + d)*sqrt(e)*x - d^2)/(e*x^2 + d)) - sqrt(2)*(B*d - A*e)*sqrt(e)*lo
g(-(3*e^2*x^4 + 2*d*e*x^2 + 2*sqrt(2)*sqrt(e^2*x^4 - d^2)*sqrt(e*x^2 + d)*
sqrt(e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)))/(d*e^2), -1/2*(2*B*d*sqrt(-
e)*arctan(sqrt(e*x^2 + d)*sqrt(-e)*x/sqrt(e^2*x^4 - d^2)) - sqrt(2)*(B*d -
 A*e)*sqrt(-e)*arctan(sqrt(2)*sqrt(e*x^2 + d)*sqrt(-e)*x/sqrt(e^2*x^4 - d^
2)))/(d*e^2)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{(-d+ex^2)\,(d+ex^2)}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(1/2)/(e**2*x**4-d**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt((-d + e*x**2)*(d + e*x**2))*sqrt(d + e*x**2)),
 x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{e^2x^4-d^2}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((B*x^2 + A)/(sqrt(e^2*x^4 - d^2)*sqrt(e*x^2 + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = -\frac{B\log\left(\left|-\sqrt{e}x+\sqrt{ex^2-d}\right|\right)}{e^{\frac{3}{2}}} + \frac{\sqrt{2}(Bd-Ae)\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2-d}\right)^2-4\sqrt{2}|d|+6d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2-d}\right)^2+4\sqrt{2}|d|+6d\right|}\right)}{4e^{\frac{3}{2}}|d|}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="giac
")
```

output
```
-B*log(abs(-sqrt(e)*x + sqrt(e*x^2 - d)))/e^(3/2) + 1/4*sqrt(2)*(B*d - A*e
)*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 - d))^2 - 4*sqrt(2)*abs(d) + 6*d)/abs(
2*(sqrt(e)*x - sqrt(e*x^2 - d))^2 + 4*sqrt(2)*abs(d) + 6*d))/(e^(3/2)*abs(
d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{e^2\,x^4-d^2}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2)/((e^2*x^4 - d^2)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((e^2*x^4 - d^2)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.47

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \frac{\sqrt{e}\left(\sqrt{2}\log\left(\frac{\sqrt{e\,x^2-d}-\sqrt{d}\,\sqrt{2}\,i+\sqrt{d}\,i+\sqrt{e}\,x}{\sqrt{d}}\right)ae-\sqrt{2}\log\left(\frac{\sqrt{e\,x^2-d}-\sqrt{d}\,\sqrt{2}\,i+\sqrt{d}\,i+\sqrt{e}\,x}{\sqrt{d}}\right)bd+\sqrt{2}\log\left(\frac{\sqrt{e\,x^2-d}+\sqrt{d}\,\sqrt{2}\,i}{\sqrt{d}}\right.}{}$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2),x)
```

output
```
(sqrt(e)*(sqrt(2)*log((sqrt( - d + e*x**2) - sqrt(d)*sqrt(2)*i + sqrt(d)*i
 + sqrt(e)*x)/sqrt(d))*a*e - sqrt(2)*log((sqrt( - d + e*x**2) - sqrt(d)*sq
rt(2)*i + sqrt(d)*i + sqrt(e)*x)/sqrt(d))*b*d + sqrt(2)*log((sqrt( - d + e
*x**2) + sqrt(d)*sqrt(2)*i - sqrt(d)*i + sqrt(e)*x)/sqrt(d))*a*e - sqrt(2)
*log((sqrt( - d + e*x**2) + sqrt(d)*sqrt(2)*i - sqrt(d)*i + sqrt(e)*x)/sqr
t(d))*b*d - sqrt(2)*log((2*sqrt(e)*sqrt( - d + e*x**2)*x + 2*sqrt(2)*d + 2
*d + 2*e*x**2)/d)*a*e + sqrt(2)*log((2*sqrt(e)*sqrt( - d + e*x**2)*x + 2*s
qrt(2)*d + 2*d + 2*e*x**2)/d)*b*d + 4*log((sqrt( - d + e*x**2) + sqrt(e)*x
)/sqrt(d))*b*d))/(4*d*e**2)
```

3.10 $\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 109

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx = -\frac{B\arctan\left(\frac{\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{-d^2+e^2x^4}}\right)}{e^{3/2}} + \frac{(Bd+Ae)\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{d-ex^2}}{\sqrt{-d^2+e^2x^4}}\right)}{\sqrt{2}de^{3/2}}$$

output

```
-B*arctan(e^(1/2)*x*(-e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2))/e^(3/2)+1/2*(A*e
+B*d)*arctan(2^(1/2)*e^(1/2)*x*(-e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2))*2^(1/
2)/d/e^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.74 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \frac{\frac{(Bd+Ae)\sqrt{-2d^2+2e^2x^4}\arctan\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{-d-ex^2}}\right)}{d\sqrt{-d-ex^2}\sqrt{d-ex^2}} - 2iB\log\left(-2i\sqrt{ex}+\frac{2\sqrt{-d^2+e^2x^4}}{\sqrt{d-ex^2}}\right)}{2e^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d - e*x^2]*Sqrt[-d^2 + e^2*x^4]),x]
```

output
```
(((B*d + A*e)*Sqrt[-2*d^2 + 2*e^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[-d
- e*x^2]])/(d*Sqrt[-d - e*x^2]*Sqrt[d - e*x^2]) - (2*I)*B*Log[(-2*I)*Sqrt[
e]*x + (2*Sqrt[-d^2 + e^2*x^4])/Sqrt[d - e*x^2]])/(2*e^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1396, 398, 224, 216, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{e^2x^4-d^2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\int \frac{Bx^2+A}{\sqrt{-ex^2-d}(d-ex^2)}dx}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{398}$$

$$\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{-ex^2-d}\left(d-ex^2\right)}dx}{e}-\frac{B\int \frac{1}{\sqrt{-ex^2-d}}dx}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{-ex^2-d}\left(d-ex^2\right)}dx}{e}-\frac{B\int \frac{1}{\frac{ex^2}{-ex^2-d}+1}d\frac{x}{\sqrt{-ex^2-d}}}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{-ex^2-d}\left(d-ex^2\right)}dx}{e}-\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{-d-ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{\frac{2dex^2}{-ex^2-d}+d}d\frac{x}{\sqrt{-ex^2-d}}}{e}-\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{-d-ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{-d-ex^2}}\right)}{\sqrt{2}de^{3/2}}-\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{-d-ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{e^2x^4-d^2}}$$

input
```
Int[(A + B*x^2)/(Sqrt[d - e*x^2]*Sqrt[-d^2 + e^2*x^4]),x]
```

output
```
(Sqrt[-d - e*x^2]*Sqrt[d - e*x^2]*(-((B*ArcTan[(Sqrt[e]*x)/Sqrt[-d - e*x^2
]])/e^(3/2)) + ((B*d + A*e)*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[-d - e*x^2]])/
(Sqrt[2]*d*e^(3/2))))/Sqrt[-d^2 + e^2*x^4]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] && !IntegerQ[p] && !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(90) = 180$.

Time = 0.07 (sec) , antiderivative size = 480, normalized size of antiderivative = 4.40

method	result
default	$-\frac{\sqrt{e^2x^4-d^2}\left(A\sqrt{de}\,\arctan\left(\frac{\sqrt{e}\,x}{\sqrt{\frac{(-ex+\sqrt{-de})(ex+\sqrt{-de})}{e}}}\right)\sqrt{2}\,\sqrt{-d}\,e-A\sqrt{de}\,\arctan\left(\frac{\sqrt{e}\,x}{\sqrt{-e\,x^2-d}}\right)\sqrt{2}\,\sqrt{-d}\,e+A\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{-d}\,\sqrt{\cdots}\right.}{e\cdots}\right.\cdots\right.}{\cdots}$

input
```
int((B*x^2+A)/(-e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/2*(e^2*x^4-d^2)^(1/2)*(A*(d*e)^(1/2)*arctan(e^(1/2)*x/(1/e*(-e*x+(-d*e)
^(1/2))*(e*x+(-d*e)^(1/2)))^(1/2))*2^(1/2)*(-d)^(1/2)*e-A*(d*e)^(1/2)*arct
an(e^(1/2)*x/(-e*x^2-d)^(1/2))*2^(1/2)*(-d)^(1/2)*e+A*ln(2*e*(2^(1/2)*(-d)
^(1/2)*(-e*x^2-d)^(1/2)-(d*e)^(1/2)*x-d)/(e*x-(d*e)^(1/2)))*e^(3/2)*d-A*ln
(2*e*((d*e)^(1/2)*x+2^(1/2)*(-d)^(1/2)*(-e*x^2-d)^(1/2)-d)/(e*x+(d*e)^(1/2
)))*e^(3/2)*d-B*(d*e)^(1/2)*arctan(e^(1/2)*x/(1/e*(-e*x+(-d*e)^(1/2))*(e*x
+(-d*e)^(1/2)))^(1/2))*2^(1/2)*(-d)^(1/2)*d-B*(d*e)^(1/2)*arctan(e^(1/2)*x
/(-e*x^2-d)^(1/2))*2^(1/2)*(-d)^(1/2)*d+B*ln(2*e*(2^(1/2)*(-d)^(1/2)*(-e*x
^2-d)^(1/2)-(d*e)^(1/2)*x-d)/(e*x-(d*e)^(1/2)))*e^(1/2)*d^2-B*ln(2*e*((d*e
)^(1/2)*x+2^(1/2)*(-d)^(1/2)*(-e*x^2-d)^(1/2)-d)/(e*x+(d*e)^(1/2)))*e^(1/2
)*d^2)*2^(1/2)/(-e*x^2+d)^(1/2)/(-e*x^2-d)^(1/2)/((-d*e)^(1/2)-(d*e)^(1/2)
)/((-d*e)^(1/2)+(d*e)^(1/2))/e^(1/2)/(d*e)^(1/2)/(-d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.55

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx$$
$$= \left[-\frac{2\,Bd\sqrt{-e}\log\left(\frac{2\,e^2x^4-dex^2-2\,\sqrt{e^2x^4-d^2}\sqrt{-ex^2+d}\sqrt{-e}x-d^2}{ex^2-d}\right)+\sqrt{2}(Bd+Ae)\sqrt{-e}\log\left(-\frac{3\,e^2x^4-2\,dex^2+2\,\sqrt{2}\sqrt{e^2x}}{e^2x^4-2}\right.}{4\,de^2}\right.$$
$$\left.-\frac{2\,Bd\sqrt{e}\arctan\left(\frac{\sqrt{-ex^2+d}\sqrt{e}x}{\sqrt{e^2x^4-d^2}}\right)-\sqrt{2}(Bd+Ae)\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{-ex^2+d}\sqrt{e}x}{\sqrt{e^2x^4-d^2}}\right)}{2\,de^2}\right]$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="fri
cas")
```

output
```
[-1/4*(2*B*d*sqrt(-e)*log((2*e^2*x^4 - d*e*x^2 - 2*sqrt(e^2*x^4 - d^2)*sqr
t(-e*x^2 + d)*sqrt(-e)*x - d^2)/(e*x^2 - d)) + sqrt(2)*(B*d + A*e)*sqrt(-e
)*log(-(3*e^2*x^4 - 2*d*e*x^2 + 2*sqrt(2)*sqrt(e^2*x^4 - d^2)*sqrt(-e*x^2
+ d)*sqrt(-e)*x - d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)))/(d*e^2), -1/2*(2*B*d*
sqrt(e)*arctan(sqrt(-e*x^2 + d)*sqrt(e)*x/sqrt(e^2*x^4 - d^2)) - sqrt(2)*(
B*d + A*e)*sqrt(e)*arctan(sqrt(2)*sqrt(-e*x^2 + d)*sqrt(e)*x/sqrt(e^2*x^4
- d^2)))/(d*e^2)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{(-d+ex^2)\,(d+ex^2)}\sqrt{d-ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(-e*x**2+d)**(1/2)/(e**2*x**4-d**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt((-d + e*x**2)*(d + e*x**2))*sqrt(d - e*x**2)),
 x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{e^2x^4-d^2}\sqrt{-ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((B*x^2 + A)/(sqrt(e^2*x^4 - d^2)*sqrt(-e*x^2 + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

$$\begin{aligned}&\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx\\&=\frac{B\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2-d}\right|\right)}{\sqrt{-e}e}\\&\quad+\frac{\sqrt{2}(Bd+Ae)\sqrt{-e}\log\left(\frac{\left|2\left(\sqrt{-e}x-\sqrt{-ex^2-d}\right)^2-4\sqrt{2}|d|+6\,d\right|}{\left|2\left(\sqrt{-e}x-\sqrt{-ex^2-d}\right)^2+4\sqrt{2}|d|+6\,d\right|}\right)}{4\,e^2|d|}\end{aligned}$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="gia
c")
```

output
```
B*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 - d)))/(sqrt(-e)*e) + 1/4*sqrt(2)*(B*d
 + A*e)*sqrt(-e)*log(abs(2*(sqrt(-e)*x - sqrt(-e*x^2 - d))^2 - 4*sqrt(2)*a
bs(d) + 6*d)/abs(2*(sqrt(-e)*x - sqrt(-e*x^2 - d))^2 + 4*sqrt(2)*abs(d) +
6*d))/(e^2*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{e^2\,x^4-d^2}\,\sqrt{d-e\,x^2}}\,dx$$

input
```
int((A + B*x^2)/((e^2*x^4 - d^2)^(1/2)*(d - e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((e^2*x^4 - d^2)^(1/2)*(d - e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.65

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \frac{\sqrt{e}\,i\left(4asinh\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)bd+\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae+\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)bd-\sqrt{2}\,\mathrm{lo}\cdots}{\cdots}$$

input
```
int((B*x^2+A)/(-e*x^2+d)^(1/2)/(e^2*x^4-d^2)^(1/2),x)
```

output
```
(sqrt(e)*i*(4*asinh((sqrt(e)*x)/sqrt(d))*b*d + sqrt(2)*log((sqrt(d + e*x**
2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e + sqrt(2)*log((sq
rt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*b*d - sqr
t(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d
))*a*e - sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(
e)*x)/sqrt(d))*b*d - sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqr
t(d) + sqrt(e)*x)/sqrt(d))*a*e - sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*s
qrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*b*d + sqrt(2)*log((sqrt(d + e*x**2)
 + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e + sqrt(2)*log((sqrt
(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*b*d))/(4*d*
e**2)
```

3.11 $\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 111

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx=\frac{B\,\text{arctanh}\left(\frac{\sqrt{e}x\sqrt{-d+ex^2}}{\sqrt{-d^2+e^2x^4}}\right)}{e^{3/2}}-\frac{(Bd+Ae)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{-d+ex^2}}{\sqrt{-d^2+e^2x^4}}\right)}{\sqrt{2}de^{3/2}}$$

output

```
B*arctanh(e^(1/2)*x*(e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2))/e^(3/2)-1/2*(A*e+
B*d)*arctanh(2^(1/2)*e^(1/2)*x*(e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2))*2^(1/2
)/d/e^(3/2)
```

Mathematica [A] (verified)

Time = 5.75 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx=\frac{\frac{(Bd+Ae)\sqrt{-2d+2ex^2}\sqrt{-d^2+e^2x^4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{d(d-ex^2)\sqrt{d+ex^2}}+2B\left(\log\left(-d+ex^2\right)-\log\left(dex-e^2x^3+\sqrt{e}\sqrt{-d+ex^2}\sqrt{-\right.\right.}{2e^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[-d + e*x^2]*Sqrt[-d^2 + e^2*x^4]),x]
```

output
```
(((B*d + A*e)*Sqrt[-2*d + 2*e*x^2]*Sqrt[-d^2 + e^2*x^4]*ArcTanh[(Sqrt[2]*S
qrt[e]*x)/Sqrt[d + e*x^2]])/(d*(d - e*x^2)*Sqrt[d + e*x^2]) + 2*B*(Log[-d
+ e*x^2] - Log[d*e*x - e^2*x^3 + Sqrt[e]*Sqrt[-d + e*x^2]*Sqrt[-d^2 + e^2*
x^4]]))/(2*e^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1396, 25, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{ex^2-d}\sqrt{e^2x^4-d^2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\int -\frac{Bx^2+A}{(d-ex^2)\sqrt{ex^2+d}}dx}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\int \frac{Bx^2+A}{(d-ex^2)\sqrt{ex^2+d}}dx}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{(d-ex^2)\sqrt{ex^2+d}}dx}{e}-\frac{B\int \frac{1}{\sqrt{ex^2+d}}dx}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{(d-ex^2)\sqrt{ex^2+d}}dx}{e}-\frac{B\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{\left(d-ex^2\right)\sqrt{ex^2+d}}dx}{e}-\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{d-\frac{2dex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}-\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{ex^2-d}\sqrt{d+ex^2}\left(\frac{(Ae+Bd)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}de^{3/2}}-\frac{B\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}\right)}{\sqrt{e^2x^4-d^2}}$$

input
```
Int[(A + B*x^2)/(Sqrt[-d + e*x^2]*Sqrt[-d^2 + e^2*x^4]),x]
```

output
```
-((Sqrt[-d + e*x^2]*Sqrt[d + e*x^2]*(-((B*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x
^2]])/e^(3/2)) + ((B*d + A*e)*ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/(Sqrt[2]*d*e^(3/2))))/Sqrt[-d^2 + e^2*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(91) = 182$.

Time = 0.06 (sec) , antiderivative size = 451, normalized size of antiderivative = 4.06

method	result
default	$\frac{\sqrt{e^{2}x^{4}-d^{2}}\left(A\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e\,x^{2}+d}+\sqrt{de}\,x+d\right)}{ex-\sqrt{de}}\right)\sqrt{2}\,\sqrt{d}\,e^{\frac{3}{2}}-A\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e\,x^{2}+d}-\sqrt{de}\,x+d\right)}{ex+\sqrt{de}}\right)\sqrt{2}\,\sqrt{d}\,e^{\frac{3}{2}}+B\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e\,x}\right.}{ex-}\right.\right.}{}$

input
```
int((B*x^2+A)/(e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(e^2*x^4-d^2)^(1/2)*(A*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e)^(
1/2)*x+d)/(e*x-(d*e)^(1/2)))*2^(1/2)*d^(1/2)*e^(3/2)-A*ln(2*e*(2^(1/2)*d^(
1/2)*(e*x^2+d)^(1/2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*2^(1/2)*d^(1/2)*e
^(3/2)+B*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/2)+(d*e)^(1/2)*x+d)/(e*x-(d*
e)^(1/2)))*2^(1/2)*d^(3/2)*e^(1/2)-B*ln(2*e*(2^(1/2)*d^(1/2)*(e*x^2+d)^(1/
2)-(d*e)^(1/2)*x+d)/(e*x+(d*e)^(1/2)))*2^(1/2)*d^(3/2)*e^(1/2)+2*A*(d*e)^(
1/2)*ln((e^(1/2)*(-1/e*(-e*x+(-d*e)^(1/2))*(e*x+(-d*e)^(1/2)))^(1/2)+e*x)/
e^(1/2))*e-2*A*(d*e)^(1/2)*ln(((e*x^2+d)^(1/2)*e^(1/2)+e*x)/e^(1/2))*e-2*B
*(d*e)^(1/2)*ln((e^(1/2)*(-1/e*(-e*x+(-d*e)^(1/2))*(e*x+(-d*e)^(1/2)))^(1/
2)+e*x)/e^(1/2))*d-2*B*(d*e)^(1/2)*ln(((e*x^2+d)^(1/2)*e^(1/2)+e*x)/e^(1/2
))*d)/(e*x^2-d)^(1/2)/(e*x^2+d)^(1/2)/((-d*e)^(1/2)-(d*e)^(1/2))/((-d*e)^(
1/2)+(d*e)^(1/2))/e^(1/2)/(d*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.54

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx$$

$$= \left[\frac{2\,Bd\sqrt{e}\log\left(\frac{2\,e^2x^4-dex^2+2\,\sqrt{e^2x^4-d^2}\sqrt{ex^2-d}\sqrt{e}x-d^2}{ex^2-d}\right)+\sqrt{2}(Bd+Ae)\sqrt{e}\log\left(-\frac{3\,e^2x^4-2\,dex^2-2\,\sqrt{2}\sqrt{e^2x^4-d^2}\sqrt{ex^2}}{e^2x^4-2\,dex^2+d^2}\right.}{4\,de^2}\right.$$

$$\left.-\frac{2\,Bd\sqrt{-e}\arctan\left(\frac{\sqrt{ex^2-d}\sqrt{-e}x}{\sqrt{e^2x^4-d^2}}\right)-\sqrt{2}(Bd+Ae)\sqrt{-e}\arctan\left(\frac{\sqrt{2}\sqrt{ex^2-d}\sqrt{-e}x}{\sqrt{e^2x^4-d^2}}\right)}{2\,de^2}\right]$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="fric
as")
```

output
```
[1/4*(2*B*d*sqrt(e)*log((2*e^2*x^4 - d*e*x^2 + 2*sqrt(e^2*x^4 - d^2)*sqrt(
e*x^2 - d)*sqrt(e)*x - d^2)/(e*x^2 - d)) + sqrt(2)*(B*d + A*e)*sqrt(e)*log
(-(3*e^2*x^4 - 2*d*e*x^2 - 2*sqrt(2)*sqrt(e^2*x^4 - d^2)*sqrt(e*x^2 - d)*s
qrt(e)*x - d^2)/(e^2*x^4 - 2*d*e*x^2 + d^2)))/(d*e^2), -1/2*(2*B*d*sqrt(-e
)*arctan(sqrt(e*x^2 - d)*sqrt(-e)*x/sqrt(e^2*x^4 - d^2)) - sqrt(2)*(B*d +
A*e)*sqrt(-e)*arctan(sqrt(2)*sqrt(e*x^2 - d)*sqrt(-e)*x/sqrt(e^2*x^4 - d^2
)))/(d*e^2)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{(-d+ex^2)\,(d+ex^2)}\sqrt{-d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2-d)**(1/2)/(e**2*x**4-d**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt((-d + e*x**2)*(d + e*x**2))*sqrt(-d + e*x**2))
, x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{e^2x^4-d^2}\sqrt{ex^2-d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((B*x^2 + A)/(sqrt(e^2*x^4 - d^2)*sqrt(e*x^2 - d)), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = -\frac{B\log\left(\left|-\sqrt{e}x+\sqrt{ex^2+d}\right|\right)}{e^{\frac{3}{2}}} - \frac{\sqrt{2}(Bd+Ae)\log\left(\frac{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{4e^{\frac{3}{2}}|d|}$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="giac
")
```

output
```
-B*log(abs(-sqrt(e)*x + sqrt(e*x^2 + d)))/e^(3/2) - 1/4*sqrt(2)*(B*d + A*e
)*log(abs(2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 - 4*sqrt(2)*abs(d) - 6*d)/abs(
2*(sqrt(e)*x - sqrt(e*x^2 + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(e^(3/2)*abs(
d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{e^2\,x^4-d^2}\,\sqrt{e\,x^2-d}}\,dx$$

input
```
int((A + B*x^2)/((e^2*x^4 - d^2)^(1/2)*(e*x^2 - d)^(1/2)),x)
```

output
```
int((A + B*x^2)/((e^2*x^4 - d^2)^(1/2)*(e*x^2 - d)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.68

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \frac{\sqrt{e}\left(\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae+\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)bd-\sqrt{2}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}\,\sqrt{2}+\sqrt{d}-}{\sqrt{d}}\right.}{}$$

input
```
int((B*x^2+A)/(e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2),x)
```

output
```
(sqrt(e)*(sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sqrt(d) + sqrt
(e)*x)/sqrt(d))*a*e + sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*sqrt(2) - sq
rt(d) + sqrt(e)*x)/sqrt(d))*b*d - sqrt(2)*log((sqrt(d + e*x**2) - sqrt(d)*
sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e - sqrt(2)*log((sqrt(d + e*x**2
) - sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*b*d - sqrt(2)*log((sqr
t(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e - sqrt
(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) - sqrt(d) + sqrt(e)*x)/sqrt(d)
)*b*d + sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt(d) + sqrt(e
)*x)/sqrt(d))*a*e + sqrt(2)*log((sqrt(d + e*x**2) + sqrt(d)*sqrt(2) + sqrt
(d) + sqrt(e)*x)/sqrt(d))*b*d + 4*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*b*d))/(4*d*e**2)
```

3.12 $\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 114

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx=-\frac{B\arctan\left(\frac{\sqrt{e}x\sqrt{-d-ex^2}}{\sqrt{-d^2+e^2x^4}}\right)}{e^{3/2}}+\frac{(Bd-Ae)\arctan\left(\frac{\sqrt{2}\sqrt{e}x\sqrt{-d-ex^2}}{\sqrt{-d^2+e^2x^4}}\right)}{\sqrt{2}de^{3/2}}$$

output
```
-B*arctan(e^(1/2)*x*(-e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2))/e^(3/2)+1/2*(-A*
e+B*d)*arctan(2^(1/2)*e^(1/2)*x*(-e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2))*2^(1
/2)/d/e^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.89 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx=\frac{\frac{(Bd-Ae)\sqrt{-2d^2+2e^2x^4}\arctan\left(\frac{\sqrt{2}\sqrt{ex}}{\sqrt{d-ex^2}}\right)}{d\sqrt{-d-ex^2}\sqrt{d-ex^2}}+2iB\log\left(-2i\sqrt{e}x-\frac{2\sqrt{-d^2+e^2x^4}}{\sqrt{-d-ex^2}}\right)}{2e^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[-d - e*x^2]*Sqrt[-d^2 + e^2*x^4]),x]
```

output
```
(((B*d - A*e)*Sqrt[-2*d^2 + 2*e^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d -
 e*x^2]])/(d*Sqrt[-d - e*x^2]*Sqrt[d - e*x^2]) + (2*I)*B*Log[(-2*I)*Sqrt[e
]*x - (2*Sqrt[-d^2 + e^2*x^4])/Sqrt[-d - e*x^2]])/(2*e^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1396, 25, 398, 224, 216, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{e^2x^4-d^2}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\int -\frac{Bx^2+A}{\sqrt{d-ex^2}(ex^2+d)}dx}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\int \frac{Bx^2+A}{\sqrt{d-ex^2}(ex^2+d)}dx}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\left(\frac{B\int \frac{1}{\sqrt{d-ex^2}}dx}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\left(\frac{B\int \frac{1}{\frac{ex^2}{d-ex^2}+1}d\frac{x}{\sqrt{d-ex^2}}}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\int\frac{1}{\sqrt{d-ex^2}(ex^2+d)}dx}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$\downarrow$ 291

$$-\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\int\frac{1}{\frac{2dex^2}{d-ex^2}+d}d\frac{x}{\sqrt{d-ex^2}}}{e}\right)}{\sqrt{e^2x^4-d^2}}$$

$\downarrow$ 218

$$-\frac{\sqrt{-d-ex^2}\sqrt{d-ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{e^{3/2}}-\frac{(Bd-Ae)\arctan\left(\frac{\sqrt{2}\sqrt{e}x}{\sqrt{d-ex^2}}\right)}{\sqrt{2}de^{3/2}}\right)}{\sqrt{e^2x^4-d^2}}$$

input
```
Int[(A + B*x^2)/(Sqrt[-d - e*x^2]*Sqrt[-d^2 + e^2*x^4]),x]
```

output
```
-((Sqrt[-d - e*x^2]*Sqrt[d - e*x^2]*((B*ArcTan[(Sqrt[e]*x)/Sqrt[d - e*x^2]
])/e^(3/2) - ((B*d - A*e)*ArcTan[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d - e*x^2]])/(Sq
rt[2]*d*e^(3/2))))/Sqrt[-d^2 + e^2*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(95) = 190$.

Time = 0.07 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.84

method	result
default	$\frac{\sqrt{e^{2} x^{4}-d^{2}}\left(A\sqrt{2}\,\sqrt{d}\,\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^{2}+d}-\sqrt{-de}\,x+d\right)}{ex-\sqrt{-de}}\right)e^{\frac{3}{2}}-A\sqrt{2}\,\sqrt{d}\,\ln\left(\frac{2e\left(\sqrt{2}\,\sqrt{d}\,\sqrt{-e\,x^{2}+d}+\sqrt{-de}\,x+d\right)}{ex+\sqrt{-de}}\right)e^{\frac{3}{2}}-B\sqrt{2}\,d^{\frac{3}{2}}\ln\left(\cdots\right.\right.}{\cdots}$

input
```
int((B*x^2+A)/(-e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*(e^2*x^4-d^2)^(1/2)*(A*2^(1/2)*d^(1/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2
+d)^(1/2)-(-d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*e^(3/2)-A*2^(1/2)*d^(1/2)*
ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/
2)))*e^(3/2)-B*2^(1/2)*d^(3/2)*ln(2*e*(2^(1/2)*d^(1/2)*(-e*x^2+d)^(1/2)-(-
d*e)^(1/2)*x+d)/(e*x-(-d*e)^(1/2)))*e^(1/2)+B*2^(1/2)*d^(3/2)*ln(2*e*(2^(1
/2)*d^(1/2)*(-e*x^2+d)^(1/2)+(-d*e)^(1/2)*x+d)/(e*x+(-d*e)^(1/2)))*e^(1/2)
+2*A*(-d*e)^(1/2)*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*e-2*A*(-d*e)^(1/2)*ar
ctan(e^(1/2)*x/(1/e*(-e*x+(d*e)^(1/2))*(e*x+(d*e)^(1/2)))^(1/2))*e-2*B*(-d
*e)^(1/2)*arctan(e^(1/2)*x/(-e*x^2+d)^(1/2))*d-2*B*(-d*e)^(1/2)*arctan(e^(
1/2)*x/(1/e*(-e*x+(d*e)^(1/2))*(e*x+(d*e)^(1/2)))^(1/2))*d)/(-e*x^2-d)^(1/
2)/(-e*x^2+d)^(1/2)/(-(-d*e)^(1/2)+(d*e)^(1/2))/((-d*e)^(1/2)+(d*e)^(1/2))
/(-d*e)^(1/2)/e^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.52

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \left[-\frac{2\,Bd\sqrt{-e}\log\left(-\frac{2\,e^2x^4+dex^2-2\,\sqrt{e^2x^4-d^2}\sqrt{-ex^2-d}\sqrt{-e}x-d^2}{ex^2+d}\right)-\sqrt{2}(Bd-Ae)\sqrt{-e}\log\left(-\frac{3\,e^2x^4+2\,dex^2-2\,\sqrt{2}\sqrt{e}\cdots}{e^2x^4+\cdots}\right.}{4\,de^2}\right.$$

$$\left.-\frac{2\,Bd\sqrt{e}\arctan\left(\frac{\sqrt{-ex^2-d}\sqrt{e}x}{\sqrt{e^2x^4-d^2}}\right)-\sqrt{2}(Bd-Ae)\sqrt{e}\arctan\left(\frac{\sqrt{2}\sqrt{-ex^2-d}\sqrt{e}x}{\sqrt{e^2x^4-d^2}}\right)}{2\,de^2}\right]$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="fri
cas")
```

output
```
[-1/4*(2*B*d*sqrt(-e)*log(-(2*e^2*x^4 + d*e*x^2 - 2*sqrt(e^2*x^4 - d^2)*sq
rt(-e*x^2 - d)*sqrt(-e)*x - d^2)/(e*x^2 + d)) - sqrt(2)*(B*d - A*e)*sqrt(-
e)*log(-(3*e^2*x^4 + 2*d*e*x^2 - 2*sqrt(2)*sqrt(e^2*x^4 - d^2)*sqrt(-e*x^2
 - d)*sqrt(-e)*x - d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)))/(d*e^2), -1/2*(2*B*d
*sqrt(e)*arctan(sqrt(-e*x^2 - d)*sqrt(e)*x/sqrt(e^2*x^4 - d^2)) - sqrt(2)*
(B*d - A*e)*sqrt(e)*arctan(sqrt(2)*sqrt(-e*x^2 - d)*sqrt(e)*x/sqrt(e^2*x^4
 - d^2)))/(d*e^2)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{(-d+ex^2)\,(d+ex^2)}\sqrt{-d-ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(-e*x**2-d)**(1/2)/(e**2*x**4-d**2)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt((-d + e*x**2)*(d + e*x**2))*sqrt(-d - e*x**2))
, x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{e^2x^4-d^2}\sqrt{-ex^2-d}}\,dx$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((B*x^2 + A)/(sqrt(e^2*x^4 - d^2)*sqrt(-e*x^2 - d)), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx = \frac{B\log\left(\left|-\sqrt{-e}x+\sqrt{-ex^2+d}\right|\right)}{\sqrt{-e}e} - \frac{\sqrt{2}(Bd-Ae)\sqrt{-e}\log\left(\frac{\left|2\left(\sqrt{-e}x-\sqrt{-ex^2+d}\right)^2-4\sqrt{2}|d|-6d\right|}{\left|2\left(\sqrt{-e}x-\sqrt{-ex^2+d}\right)^2+4\sqrt{2}|d|-6d\right|}\right)}{4e^2|d|}$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2),x, algorithm="gia
c")
```

output
```
B*log(abs(-sqrt(-e)*x + sqrt(-e*x^2 + d)))/(sqrt(-e)*e) - 1/4*sqrt(2)*(B*d
 - A*e)*sqrt(-e)*log(abs(2*(sqrt(-e)*x - sqrt(-e*x^2 + d))^2 - 4*sqrt(2)*a
bs(d) - 6*d)/abs(2*(sqrt(-e)*x - sqrt(-e*x^2 + d))^2 + 4*sqrt(2)*abs(d) -
6*d))/(e^2*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{e^2\,x^4-d^2}\,\sqrt{-e\,x^2-d}}\,dx$$

input
```
int((A + B*x^2)/((e^2*x^4 - d^2)^(1/2)*(- d - e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((e^2*x^4 - d^2)^(1/2)*(- d - e*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.35

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-d^2+e^2x^4}}\,dx$$
$$=\frac{\sqrt{e}\,i\left(-\sqrt{2}\log\left(\frac{\sqrt{e\,x^2-d}-\sqrt{d}\,\sqrt{2}\,i+\sqrt{d}\,i+\sqrt{e}\,x}{\sqrt{d}}\right)ae+\sqrt{2}\log\left(\frac{\sqrt{e\,x^2-d}-\sqrt{d}\,\sqrt{2}\,i+\sqrt{d}\,i+\sqrt{e}\,x}{\sqrt{d}}\right)bd-\sqrt{2}\log\left(\frac{\sqrt{e\,x^2-d}+\sqrt{d}\cdots}{\cdots}\right.}{\cdots}$$

input
```
int((B*x^2+A)/(-e*x^2-d)^(1/2)/(e^2*x^4-d^2)^(1/2),x)
```

output
```
(sqrt(e)*i*( - sqrt(2)*log((sqrt( - d + e*x**2) - sqrt(d)*sqrt(2)*i + sqrt
(d)*i + sqrt(e)*x)/sqrt(d))*a*e + sqrt(2)*log((sqrt( - d + e*x**2) - sqrt(
d)*sqrt(2)*i + sqrt(d)*i + sqrt(e)*x)/sqrt(d))*b*d - sqrt(2)*log((sqrt( -
d + e*x**2) + sqrt(d)*sqrt(2)*i - sqrt(d)*i + sqrt(e)*x)/sqrt(d))*a*e + sq
rt(2)*log((sqrt( - d + e*x**2) + sqrt(d)*sqrt(2)*i - sqrt(d)*i + sqrt(e)*x
)/sqrt(d))*b*d + sqrt(2)*log((2*sqrt(e)*sqrt( - d + e*x**2)*x + 2*sqrt(2)*
d + 2*d + 2*e*x**2)/d)*a*e - sqrt(2)*log((2*sqrt(e)*sqrt( - d + e*x**2)*x
+ 2*sqrt(2)*d + 2*d + 2*e*x**2)/d)*b*d - 4*log((sqrt( - d + e*x**2) + sqrt
(e)*x)/sqrt(d))*b*d))/(4*d*e**2)
```

3.13 $\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 717

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \frac{(5cCd^2+14Bcde+8Ace^2-8aCe^2)\,x\sqrt{d+ex^2}}{16c^2}$$

$$+\frac{(5Cd+6Be)x(d+ex^2)^{3/2}}{24c}+\frac{Cx(d+ex^2)^{5/2}}{6c}$$

$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(d(Bcd(cd^2-3ae^2)+(Ac-aC)e(3cd^2-ae^2))-\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(Acd(cd^2-3ae^2}{2\sqrt{2}\sqrt[4]{a}c^{5/2}d\sqrt{cd^2+ae}$$

$$-\frac{(8ae^2(5Cd+2Be)-5cd(Cd^2+6Bde+8Ae^2))\,\mathrm{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16c^2\sqrt{e}}$$

$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(d(Bcd(cd^2-3ae^2)+(Ac-aC)e(3cd^2-ae^2))-\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(Acd(cd^2-3a}{2\sqrt{2}\sqrt[4]{a}c^{5/2}d\sqrt{cd^2+}$$

output
```
1/16*(8*A*c*e^2+14*B*c*d*e-8*C*a*e^2+5*C*c*d^2)*x*(e*x^2+d)^(1/2)/c^2+1/24
*(6*B*e+5*C*d)*x*(e*x^2+d)^(3/2)/c+1/6*C*x*(e*x^2+d)^(5/2)/c+1/4*(a^(1/2)*
e+(a*e^2+c*d^2)^(1/2))^(1/2)*(d*(B*c*d*(-3*a*e^2+c*d^2)+(A*c-C*a)*e*(-a*e^
2+3*c*d^2))-(e-(a*e^2+c*d^2)^(1/2)/a^(1/2))*(A*c*d*(-3*a*e^2+c*d^2)+a*(a*e
^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d))))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2
)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^
2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(1/4)/c^(5/2)/d/(a*e^2+c*d^2)^(1/2)-1/
16*(8*a*e^2*(2*B*e+5*C*d)-5*c*d*(8*A*e^2+6*B*d*e+C*d^2))*arctanh(e^(1/2)*x
/(e*x^2+d)^(1/2))/c^2/e^(1/2)+1/4*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(
d*(B*c*d*(-3*a*e^2+c*d^2)+(A*c-C*a)*e*(-a*e^2+3*c*d^2))-(e+(a*e^2+c*d^2)^(
1/2)/a^(1/2))*(A*c*d*(-3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*
d))))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/
2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^
(1/2)/a^(1/4)/c^(5/2)/d/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.33 (sec) , antiderivative size = 1063, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + c*x^4),x]
```

output
```
(Sqrt[e]*x*Sqrt[d + e*x^2]*(-24*a*C*e^2 + 6*c*e*(9*B*d + 4*A*e + 2*B*e*x^2
) + c*C*(33*d^2 + 26*d*e*x^2 + 8*e^2*x^4)) - 3*(-8*a*e^2*(5*C*d + 2*B*e) +
 5*c*d*(C*d^2 + 6*B*d*e + 8*A*e^2))*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] +
24*e*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^
3 + c*#1^4 & , (B*c^2*d^5*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] -
#1] + 3*A*c^2*d^4*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] -
3*a*c*C*d^4*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 3*a*B*
c*d^3*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - a*A*c*d^2*
e^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + a^2*C*d^2*e^3*Lo
g[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2*B*c^2*d^4*Log[d + 2*
e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 2*A*c^2*d^3*e*Log[d + 2*e*x
^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 2*a*c*C*d^3*e*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 6*a*B*c*d^2*e^2*Log[d + 2*e*x^2 -
 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 10*a*A*c*d*e^3*Log[d + 2*e*x^2 - 2
*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 10*a^2*C*d*e^3*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 4*a^2*B*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e
]*x*Sqrt[d + e*x^2] - #1]*#1 + B*c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqr
t[d + e*x^2] - #1]*#1^2 + 3*A*c^2*d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt
[d + e*x^2] - #1]*#1^2 - 3*a*c*C*d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[
d + e*x^2] - #1]*#1^2 - 3*a*B*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqr...
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 819, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx$$

$$\Big\downarrow \text{2257}$$

$$\int \left(\frac{(d+ex^2)^{5/2}(-aC+Ac+Bcx^2)}{c(a+cx^4)}+\frac{C(d+ex^2)^{5/2}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
\frac{5C\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)d^3}{16c\sqrt{e}}+\frac{5Cx\sqrt{ex^2+d}d^2}{16c}+\frac{5Cx(ex^2+d)^{3/2}d}{24c}+\frac{Cx(ex^2+d)^{5/2}}{6c}+\frac{(\sqrt{-a}\sqrt{c}B+Ac-aC)\,ex(ex^2+d)^{3/2}}{8\sqrt{-a}c^{3/2}}+\frac{(\sqrt{-a}\sqrt{c}B-Ac+aC)\,ex(ex^2+d)^{3/2}}{8\sqrt{-a}c^{3/2}}+\frac{(\sqrt{-a}\sqrt{c}B-Ac+aC)\,(c^{3/2}d^3-3\sqrt{-a}ced^2-3a\sqrt{c}e^2d+\sqrt{-a}ae^3)\arctan\left(\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}x}{\sqrt[4]{-a}\sqrt{ex^2+d}}\right)}{2(-a)^{3/4}c^{5/2}\sqrt{\sqrt{c}d-\sqrt{-a}e}}+\frac{(\sqrt{-a}\sqrt{c}B-Ac+aC)\sqrt{e}(15cd^2-20\sqrt{-a}\sqrt{c}ed-8ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{16\sqrt{-a}c^{5/2}}+\frac{(\sqrt{-a}\sqrt{c}B+Ac-aC)\sqrt{e}(15cd^2+20\sqrt{-a}\sqrt{c}ed-8ae^2)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{16\sqrt{-a}c^{5/2}}-\frac{(\sqrt{-a}\sqrt{c}B+Ac-aC)\,(c^{3/2}d^3+3\sqrt{-a}ced^2-3a\sqrt{c}e^2d+(-a)^{3/2}e^3)\,\text{arctanh}\left(\frac{\sqrt{\sqrt{c}d+\sqrt{-a}e}x}{\sqrt[4]{-a}\sqrt{ex^2+d}}\right)}{2(-a)^{3/4}c^{5/2}\sqrt{\sqrt{c}d+\sqrt{-a}e}}+\frac{(\sqrt{-a}\sqrt{c}B-Ac+aC)\,e(7\sqrt{c}d-4\sqrt{-a}e)\,x\sqrt{ex^2+d}}{16\sqrt{-a}c^2}+\frac{(\sqrt{-a}\sqrt{c}B+Ac-aC)\,e(7\sqrt{c}d+4\sqrt{-a}e)\,x\sqrt{ex^2+d}}{16\sqrt{-a}c^2}
$$

input
```
Int[((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + c*x^4),x]
```

output
```
(5*C*d^2*x*Sqrt[d + e*x^2])/(16*c) + ((Sqrt[-a]*B*Sqrt[c] - A*c + a*C)*e*(
7*Sqrt[c]*d - 4*Sqrt[-a]*e)*x*Sqrt[d + e*x^2])/(16*Sqrt[-a]*c^2) + ((Sqrt[
-a]*B*Sqrt[c] + A*c - a*C)*e*(7*Sqrt[c]*d + 4*Sqrt[-a]*e)*x*Sqrt[d + e*x^2
])/(16*Sqrt[-a]*c^2) + (5*C*d*x*(d + e*x^2)^(3/2))/(24*c) + ((Sqrt[-a]*B*S
qrt[c] + A*c - a*C)*e*x*(d + e*x^2)^(3/2))/(8*Sqrt[-a]*c^(3/2)) + ((Sqrt[-
a]*B*Sqrt[c] - A*c + a*C)*e*x*(d + e*x^2)^(3/2))/(8*Sqrt[-a]*c^(3/2)) + (C
*x*(d + e*x^2)^(5/2))/(6*c) + ((Sqrt[-a]*B*Sqrt[c] - A*c + a*C)*(c^(3/2)*d
^3 - 3*Sqrt[-a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + Sqrt[-a]*a*e^3)*ArcTan[(Sqrt
[Sqrt[c]*d - Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/(2*(-a)^(3/4)*c
^(5/2)*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]) + (5*C*d^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/(16*c*Sqrt[e]) + ((Sqrt[-a]*B*Sqrt[c] - A*c + a*C)*Sqrt[e]*(15
*c*d^2 - 20*Sqrt[-a]*Sqrt[c]*d*e - 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e
*x^2]])/(16*Sqrt[-a]*c^(5/2)) + ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*Sqrt[e]*
(15*c*d^2 + 20*Sqrt[-a]*Sqrt[c]*d*e - 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d
+ e*x^2]])/(16*Sqrt[-a]*c^(5/2)) - ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*(c^(3
/2)*d^3 + 3*Sqrt[-a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + (-a)^(3/2)*e^3)*ArcTanh
[(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/(2*(-a)^(
3/4)*c^(5/2)*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1274 vs. $2(617) = 1234$.

Time = 3.50 (sec) , antiderivative size = 1275, normalized size of antiderivative = 1.78

method	result	size
pseudoelliptic	Expression too large to display	1275
risch	Expression too large to display	1374
default	Expression too large to display	5484

input
```
int((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-(-1/4*(((d*(A*c*a^(1/2)-C*a^(3/2))*e^(3/2)+1/2*B*(c*d^2*a^(1/2)*e^(1/2)-a
^(3/2)*e^(5/2)))*(a*e^2+c*d^2)^(1/2)+3/2*a*d*(A*c-C*a)*e^(5/2)+3/2*B*a*c*d
^2*e^(3/2)-1/2*B*a^2*e^(7/2)-1/2*e^(1/2)*c*d^3*(A*c-C*a))*(a*(a*e^2+c*d^2)
)^(1/2)+(-1/2*B*a^(3/2)*c*d^2*e^(3/2)-A*a^(3/2)*c*d*e^(5/2)+a^(5/2)*(C*d*e
^(5/2)+1/2*B*e^(7/2)))*(a*e^2+c*d^2)^(1/2)+1/2*(c*d^3*(A*c-C*a)*e^(3/2)-3*
(e^(5/2)*B*c*d^2+d*(A*c-C*a)*e^(7/2)-1/3*e^(9/2)*B*a)*a)*a)*(2*(a*(a*e^2+c
*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2
))^(1/2)-2*a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c
*d^2))^(1/2)+2*a*e)^(1/2)*x+x^2*(a*e^2+c*d^2)^(1/2))/x^2)+1/4*(((d*(A*c*a^
(1/2)-C*a^(3/2))*e^(3/2)+1/2*B*(c*d^2*a^(1/2)*e^(1/2)-a^(3/2)*e^(5/2)))*(a
*e^2+c*d^2)^(1/2)+3/2*a*d*(A*c-C*a)*e^(5/2)+3/2*B*a*c*d^2*e^(3/2)-1/2*B*a^
2*e^(7/2)-1/2*e^(1/2)*c*d^3*(A*c-C*a))*(a*(a*e^2+c*d^2))^(1/2)+(-1/2*B*a^(
3/2)*c*d^2*e^(3/2)-A*a^(3/2)*c*d*e^(5/2)+a^(5/2)*(C*d*e^(5/2)+1/2*B*e^(7/2
)))*(a*e^2+c*d^2)^(1/2)+1/2*(c*d^3*(A*c-C*a)*e^(3/2)-3*(e^(5/2)*B*c*d^2+d*
(A*c-C*a)*e^(7/2)-1/3*e^(9/2)*B*a)*a)*a)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)
^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/
2)*ln((a^(1/2)*(e*x^2+d)+x^2*(a*e^2+c*d^2)^(1/2)+(e*x^2+d)^(1/2)*(2*(a*(a*
e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x^2)+c*d^2*(((-5/2*c*(A*e^2+3/4*B*d*e+1/
8*C*d^2)*d*a^(3/2)+a^(5/2)*e^2*(B*e+5/2*C*d))*arctanh((e*x^2+d)^(1/2)/x/e^
(1/2))-1/2*(e*x^2+d)^(1/2)*x*(9/4*c*d*a^(3/2)*(13/27*C*x^2+B)*e^(3/2)+c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \int \frac{(d+ex^2)^{\frac{5}{2}}(A+Bx^2+Cx^4)}{a+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(C*x**4+B*x**2+A)/(c*x**4+a),x)
```

output
```
Integral((d + e*x**2)**(5/2)*(A + B*x**2 + C*x**4)/(a + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{cx^4+a}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(5/2)/(c*x^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \int \frac{(e\,x^2+d)^{5/2}\,(C\,x^4+B\,x^2+A)}{c\,x^4+a}\,dx$$

input
```
int(((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + c*x^4),x)
```

output
```
int(((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + c*x^4), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx &= \left(\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+a}dx\right) a\,d^2 \\ &+ \left(\int \frac{\sqrt{e\,x^2+d}\,x^8}{c\,x^4+a}dx\right) c\,e^2 + \left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{c\,x^4+a}dx\right) b\,e^2 \\ &+ 2\left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{c\,x^4+a}dx\right) cde + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c\,x^4+a}dx\right) a\,e^2 \\ &+ 2\left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c\,x^4+a}dx\right) bde + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c\,x^4+a}dx\right) c\,d^2 \\ &+ 2\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c\,x^4+a}dx\right) ade + \left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c\,x^4+a}dx\right) b\,d^2\end{aligned}$$

input
```
int((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x)
```

output
```
int(sqrt(d + e*x**2)/(a + c*x**4),x)*a*d**2 + int((sqrt(d + e*x**2)*x**8)/
(a + c*x**4),x)*c*e**2 + int((sqrt(d + e*x**2)*x**6)/(a + c*x**4),x)*b*e**
2 + 2*int((sqrt(d + e*x**2)*x**6)/(a + c*x**4),x)*c*d*e + int((sqrt(d + e*
x**2)*x**4)/(a + c*x**4),x)*a*e**2 + 2*int((sqrt(d + e*x**2)*x**4)/(a + c*
x**4),x)*b*d*e + int((sqrt(d + e*x**2)*x**4)/(a + c*x**4),x)*c*d**2 + 2*in
t((sqrt(d + e*x**2)*x**2)/(a + c*x**4),x)*a*d*e + int((sqrt(d + e*x**2)*x*
*2)/(a + c*x**4),x)*b*d**2
```

3.14 $\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 615

$$\int \frac{(d+ex^2)^{3/2}\left(A+Bx^2+Cx^4\right)}{a+cx^4}\,dx = \frac{(3Cd+4Be)x\sqrt{d+ex^2}}{8c} + \frac{Cx(d+ex^2)^{3/2}}{4c}$$

$$+ \frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd(Bcd^2+2Acde-2aCde-aBe^2)-\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(Ac(cd^2-ae^2)+a(aCe^2-\cdots}{2\sqrt{2}\sqrt[4]{a}c^{5/2}d\sqrt{cd^2+ae^2}}$$

$$+ \frac{(3cCd^2+12Bcde+8Ace^2-8aCe^2)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}}$$

$$+ \frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd(Bcd^2+2Acde-2aCde-aBe^2)-\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(Ac(cd^2-ae^2)+a(aCe^2\cdots}{2\sqrt{2}\sqrt[4]{a}c^{5/2}d\sqrt{cd^2+ae^2}}$$

output
```
1/8*(4*B*e+3*C*d)*x*(e*x^2+d)^(1/2)/c+1/4*C*x*(e*x^2+d)^(3/2)/c+1/4*(a^(1/
2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(c*d*(2*A*c*d*e-B*a*e^2+B*c*d^2-2*C*a*d*e)
-(e-(a*e^2+c*d^2)^(1/2)/a^(1/2))*(A*c*(-a*e^2+c*d^2)+a*(C*a*e^2-c*d*(2*B*e
+C*d))))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1
/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2
^(1/2)/a^(1/4)/c^(5/2)/d/(a*e^2+c*d^2)^(1/2)+1/8*(8*A*c*e^2+12*B*c*d*e-8*C
*a*e^2+3*C*c*d^2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^2/e^(1/2)+1/4*(-a^(
1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(c*d*(2*A*c*d*e-B*a*e^2+B*c*d^2-2*C*a*d*
e)-(e+(a*e^2+c*d^2)^(1/2)/a^(1/2))*(A*c*(-a*e^2+c*d^2)+a*(C*a*e^2-c*d*(2*B
*e+C*d))))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2)
)^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2
))*2^(1/2)/a^(1/4)/c^(5/2)/d/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.85 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \frac{c\sqrt{e}x\sqrt{d+ex^2}(5Cd+4Be+2Cex^2)+(-3cCd^2+8aCe^2-4ce(3$$

input
```
Integrate[((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + c*x^4),x]
```

output
```
(c*Sqrt[e]*x*Sqrt[d + e*x^2]*(5*C*d + 4*B*e + 2*C*e*x^2) + (-3*c*C*d^2 + 8
*a*C*e^2 - 4*c*e*(3*B*d + 2*A*e))*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] + 4*
e*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 +
 c*#1^4 & , (B*c^2*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]
 + 2*A*c^2*d^3*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2*a
*c*C*d^3*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - a*B*c*d^2
*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2*B*c^2*d^3*Log
[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 6*a*B*c*d*e^2*Log[d
+ 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 4*a*A*c*e^3*Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 4*a^2*C*e^3*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + B*c^2*d^2*Log[d + 2*e*x^2 - 2*Sqr
t[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + 2*A*c^2*d*e*Log[d + 2*e*x^2 - 2*Sqrt[e
]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 2*a*c*C*d*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1]*#1^2 - a*B*c*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt
[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*
#1^3) & ])/(8*c^2*Sqrt[e])
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 593, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx$$

$$\Big\downarrow \text{ 2257}$$

$$\int \left(\frac{(d+ex^2)^{3/2}(-aC+Ac+Bcx^2)}{c(a+cx^4)}+\frac{C(d+ex^2)^{3/2}}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\left(\sqrt{-a}B\sqrt{c}+aC-Ac\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c^2\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{\left(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2\right)\left(\sqrt{-a}B\sqrt{c}-aC+Ac\right)\operatorname{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c^2\sqrt{\sqrt{-a}e+\sqrt{c}d}}+\frac{\sqrt{e}\left(3\sqrt{c}d-2\sqrt{-a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\sqrt{-a}B\sqrt{c}+aC-Ac\right)}{4\sqrt{-a}c^2}+\frac{\sqrt{e}\left(2\sqrt{-a}e+3\sqrt{c}d\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\sqrt{-a}B\sqrt{c}-aC+Ac\right)}{4\sqrt{-a}c^2}+\frac{ex\sqrt{d+ex^2}\left(\sqrt{-a}B\sqrt{c}-aC+Ac\right)}{4\sqrt{-a}c^{3/2}}+\frac{ex\sqrt{d+ex^2}\left(\sqrt{-a}B\sqrt{c}+aC-Ac\right)}{4\sqrt{-a}c^{3/2}}+\frac{3Cd^2\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c\sqrt{e}}+\frac{3Cdx\sqrt{d+ex^2}}{8c}+\frac{Cx\left(d+ex^2\right)^{3/2}}{4c}$$

input
```
Int[((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + c*x^4),x]
```

output
```
(3*C*d*x*Sqrt[d + e*x^2])/(8*c) + ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*e*x*Sq
rt[d + e*x^2])/(4*Sqrt[-a]*c^(3/2)) + ((Sqrt[-a]*B*Sqrt[c] - A*c + a*C)*e*
x*Sqrt[d + e*x^2])/(4*Sqrt[-a]*c^(3/2)) + (C*x*(d + e*x^2)^(3/2))/(4*c) +
((Sqrt[-a]*B*Sqrt[c] - A*c + a*C)*(c*d^2 - 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)
*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/(2
*(-a)^(3/4)*c^2*Sqrt[Sqrt[c]*d - Sqrt[-a]*e]) + (3*C*d^2*ArcTanh[(Sqrt[e]*
x)/Sqrt[d + e*x^2]])/(8*c*Sqrt[e]) + ((Sqrt[-a]*B*Sqrt[c] - A*c + a*C)*Sqr
t[e]*(3*Sqrt[c]*d - 2*Sqrt[-a]*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(4
*Sqrt[-a]*c^2) + ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*Sqrt[e]*(3*Sqrt[c]*d +
2*Sqrt[-a]*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(4*Sqrt[-a]*c^2) - ((S
qrt[-a]*B*Sqrt[c] + A*c - a*C)*(c*d^2 + 2*Sqrt[-a]*Sqrt[c]*d*e - a*e^2)*Ar
cTanh[(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/(2*(
-a)^(3/4)*c^2*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. 2(519) = 1038.

Time = 0.95 (sec) , antiderivative size = 1059, normalized size of antiderivative = 1.72

method	result	size
pseudoelliptic	Expression too large to display	1059
risch	Expression too large to display	1156
default	Expression too large to display	3422

input
```
int((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((((-C*a^(3/2)*e^(3/2)+a^(1/2)*c*(B*d*e^(1/2)+A*e^(3/2)))*(a*e^2+c*d^
2)^(1/2)+2*B*a*c*d*e^(3/2)-(e^(1/2)*c*d^2-a*e^(5/2))*(A*c-C*a))*(a*(a*e^2+
c*d^2))^(1/2)+(-c*(B*d*e^(3/2)+e^(5/2)*A)*a^(3/2)+C*a^(5/2)*e^(5/2))*(a*e^
2+c*d^2)^(1/2)+(c*d^2*(A*c-C*a)*e^(3/2)-(2*B*c*d*e^(5/2)+e^(7/2)*(A*c-C*a)
)*a)*a)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1
/2)*(-ln(((e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-x^2*(a
*e^2+c*d^2)^(1/2)-a^(1/2)*(e*x^2+d))/x^2)+ln((a^(1/2)*(e*x^2+d)+x^2*(a*e^2
+c*d^2)^(1/2)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x
^2))*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)-4*c*(2*(c*(A*e^2+3/2*B*d*e+3/
8*C*d^2)*a^(3/2)-a^(5/2)*C*e^2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2
+c*d^2))^(1/2)-2*a*e)^(1/2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+(arctan(((2
*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a
*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan((
2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*
(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))*(c*(B
*d*e^(1/2)+A*e^(3/2))*a^(3/2)-C*a^(5/2)*e^(3/2))*(a*e^2+c*d^2)^(1/2)+5/4*c
*a^(3/2)*x*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)
^(1/2)*(2/5*(C*x^2+2*B)*e^(3/2)+C*d*e^(1/2))*(e*x^2+d)^(1/2)+(-2*B*a*c*d*e
^(3/2)+(e^(1/2)*c*d^2-a*e^(5/2))*(A*c-C*a))*(arctan(((2*(a*(a*e^2+c*d^2))^
(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \int \frac{(d+ex^2)^{\frac{3}{2}}(A+Bx^2+Cx^4)}{a+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(C*x**4+B*x**2+A)/(c*x**4+a),x)
```

output
```
Integral((d + e*x**2)**(3/2)*(A + B*x**2 + C*x**4)/(a + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{cx^4+a}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(3/2)/(c*x^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,(C\,x^4+B\,x^2+A)}{c\,x^4+a}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + c*x^4),x)
```

output
```
int(((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + c*x^4), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+a}dx\right)ad + \left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{c\,x^4+a}dx\right)ce + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c\,x^4+a}dx\right)be + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c\,x^4+a}dx\right)cd + \left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c\,x^4+a}dx\right)ae + \left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c\,x^4+a}dx\right)bd$$

input
```
int((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x)
```

output
```
int(sqrt(d + e*x**2)/(a + c*x**4),x)*a*d + int((sqrt(d + e*x**2)*x**6)/(a
+ c*x**4),x)*c*e + int((sqrt(d + e*x**2)*x**4)/(a + c*x**4),x)*b*e + int((
sqrt(d + e*x**2)*x**4)/(a + c*x**4),x)*c*d + int((sqrt(d + e*x**2)*x**2)/(
a + c*x**4),x)*a*e + int((sqrt(d + e*x**2)*x**2)/(a + c*x**4),x)*b*d
```

3.15 $\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 497

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \frac{Cx\sqrt{d+ex^2}}{2c}$$
$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(d(Bcd+Ace-aCe)-(Acd-a(Cd+Be))\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{\cdot}}{\sqrt{a}(\cdot}\right.}{2\sqrt{2}\sqrt[4]{a}c^{3/2}d\sqrt{cd^2+ae^2}}$$
$$+\frac{(Cd+2Be)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{e}}$$
$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(d(Bcd+Ace-aCe)-(Acd-a(Cd+Be))\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cdot}}{\cdot}\right.}{2\sqrt{2}\sqrt[4]{a}c^{3/2}d\sqrt{cd^2+ae^2}}$$

output
```
1/2*C*x*(e*x^2+d)^(1/2)/c+1/4*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(d*(A*
c*e+B*c*d-C*a*e)-(A*c*d-a*(B*e+C*d))*(e-(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arct
an(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+
d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(1/4
)/c^(3/2)/d/(a*e^2+c*d^2)^(1/2)+1/2*(2*B*e+C*d)*arctanh(e^(1/2)*x/(e*x^2+d
)^(1/2))/c/e^(1/2)+1/4*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(d*(A*c*e+B*
c*d-C*a*e)-(A*c*d-a*(B*e+C*d))*(e+(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctanh(2^
(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(
1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(1/4)/c^
(3/2)/d/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.52 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx$$

$$= \frac{C\sqrt{e}x\sqrt{d+ex^2}-(Cd+2Be)\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)+e\text{RootSum}\Big[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2}{}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(A + B*x^2 + C*x^4))/(a + c*x^4),x]
```

output
```
(C*Sqrt[e]*x*Sqrt[d + e*x^2] - (C*d + 2*B*e)*Log[-(Sqrt[e]*x) + Sqrt[d + e
*x^2]] + e*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c
*d*#1^3 + c*#1^4 & , (B*c*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2
] - #1] + A*c*d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] -
a*C*d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2*B*c*d^2*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 2*A*c*d*e*Log[d +
 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 2*a*C*d*e*Log[d + 2*e*x^
2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 4*a*B*e^2*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + B*c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sq
rt[d + e*x^2] - #1]*#1^2 + A*c*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*
x^2] - #1]*#1^2 - a*C*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1
]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(2*c*
Sqrt[e])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+cx^4}dx$$

$$\Big\downarrow \text{2257}$$

$$\int \left(\frac{\sqrt{d+ex^2}(-aC+Ac+Bcx^2)}{c(a+cx^4)}+\frac{C\sqrt{d+ex^2}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\sqrt{c}d-\sqrt{-a}e}\left(\sqrt{-a}B\sqrt{c}+aC-Ac\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c^{3/2}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\sqrt{-a}B\sqrt{c}-aC+Ac\right)}{2\sqrt{-a}c^{3/2}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\sqrt{-a}B\sqrt{c}+aC-Ac\right)}{2\sqrt{-a}c^{3/2}}-\frac{\sqrt{\sqrt{-a}e+\sqrt{c}d}\left(\sqrt{-a}B\sqrt{c}-aC+Ac\right)\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c^{3/2}}+\frac{Cd\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{e}}+\frac{Cx\sqrt{d+ex^2}}{2c}$$

input
```
Int[(Sqrt[d + e*x^2]*(A + B*x^2 + C*x^4))/(a + c*x^4),x]
```

output
```
(C*x*Sqrt[d + e*x^2])/(2*c) + ((Sqrt[-a]*B*Sqrt[c] - A*c + a*C)*Sqrt[Sqrt[
c]*d - Sqrt[-a]*e]*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqr
t[d + e*x^2])])/(2*(-a)^(3/4)*c^(3/2)) + (C*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]])/(2*c*Sqrt[e]) + ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*Sqrt[e]*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[-a]*c^(3/2)) + ((Sqrt[-a]*B*Sqrt[c
] - A*c + a*C)*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*Sqrt[-a]*c
^(3/2)) - ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*A
rcTanh[(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/(2*
(-a)^(3/4)*c^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(405) = 810$.

Time = 0.39 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.67

method	result
pseudoelliptic	$-\frac{4c\,d^2\left(a^{\frac{3}{2}}\sqrt{e}\,B\sqrt{a\,e^2+c\,d^2}+\left(-Ba\,e^{\frac{3}{2}}+\sqrt{e}\,d(Ac-Ca)\right)a\right)\arctan\left(\frac{2\sqrt{a}\,\sqrt{e\,x^2+d}+\sqrt{2\sqrt{a\left(a\,e^2+c\,d^2\right)}+2ae}\,x}{x\sqrt{4\sqrt{a\,e^2+c\,d^2}\,\sqrt{a}-2\sqrt{a\left(a\,e^2+c\,d^2\right)}-2ae}}\right)-4c\,d^2\left(a^{\frac{3}{2}}\right.\cdots}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8/e^(1/2)/a^(3/2)/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1
/2)-2*a*e)^(1/2)*(4*c*d^2*(a^(3/2)*e^(1/2)*B*(a*e^2+c*d^2)^(1/2)+(-B*a*e^(
3/2)+e^(1/2)*d*(A*c-C*a))*a)*arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^
2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e
^2+c*d^2))^(1/2)-2*a*e)^(1/2))-4*c*d^2*(a^(3/2)*e^(1/2)*B*(a*e^2+c*d^2)^(1
/2)+(-B*a*e^(3/2)+e^(1/2)*d*(A*c-C*a))*a)*arctan(((2*(a*(a*e^2+c*d^2))^(1/
2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1
/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))+(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)
-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(((-a^(1/2)*e^(1/2)*B*(a*e^2+c*d^2
)^(1/2)-B*a*e^(3/2)+e^(1/2)*d*(A*c-C*a))*(a*(a*e^2+c*d^2))^(1/2)+a^(3/2)*e
^(3/2)*(a*e^2+c*d^2)^(1/2)*B-a*(d*(A*c-C*a)*e^(3/2)-e^(5/2)*B*a))*(2*(a*(a
*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*ln(((e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(
1/2)+2*a*e)^(1/2)*x-x^2*(a*e^2+c*d^2)^(1/2)-a^(1/2)*(e*x^2+d))/x^2)-((-a^(
1/2)*e^(1/2)*B*(a*e^2+c*d^2)^(1/2)-B*a*e^(3/2)+e^(1/2)*d*(A*c-C*a))*(a*(a*
e^2+c*d^2))^(1/2)+a^(3/2)*e^(3/2)*(a*e^2+c*d^2)^(1/2)*B-a*(d*(A*c-C*a)*e^(
3/2)-e^(5/2)*B*a))*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*ln((a^(1/2)*(e*
x^2+d)+x^2*(a*e^2+c*d^2)^(1/2)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+
2*a*e)^(1/2)*x)/x^2)-4*c*((2*B*e+C*d)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+e
^(1/2)*(e*x^2+d)^(1/2)*C*x)*d^2*a^(3/2)))/d^2/c^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(C*x**4+B*x**2+A)/(c*x**4+a),x)
```

output
```
Integral(sqrt(d + e*x**2)*(A + B*x**2 + C*x**4)/(a + c*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{ex^2+d}}{cx^4+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(e*x^2 + d)/(c*x^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx=\int \frac{\sqrt{e\,x^2+d}\,(C\,x^4+B\,x^2+A)}{c\,x^4+a}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(A + B*x^2 + C*x^4))/(a + c*x^4),x)
```

output
```
int(((d + e*x^2)^(1/2)*(A + B*x^2 + C*x^4))/(a + c*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+cx^4}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+a}dx\right)a+\left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c\,x^4+a}dx\right)c+\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c\,x^4+a}dx\right)b$$

input
```
int((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+a),x)
```

output
```
int(sqrt(d + e*x**2)/(a + c*x**4),x)*a + int((sqrt(d + e*x**2)*x**4)/(a +
c*x**4),x)*c + int((sqrt(d + e*x**2)*x**2)/(a + c*x**4),x)*b
```

3.16 $\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}(a+cx^4)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 429

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx = \frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(Bcd-(Ac-aC)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\,x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{2\sqrt{2}\sqrt[4]{a}c^{3/2}d\sqrt{cd^2+ae^2}} + \frac{C\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} + \frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(Bcd-(Ac-aC)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\,x\sqrt{d+ex^2}}{\sqrt{a}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)-cdx^2}\right)}{2\sqrt{2}\sqrt[4]{a}c^{3/2}d\sqrt{cd^2+ae^2}}$$

output
```
1/4*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(B*c*d-(A*c-C*a)*(e-(a*e^2+c*d^2
)^(1/2)/a^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^
(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*
d*x^2))*2^(1/2)/a^(1/4)/c^(3/2)/d/(a*e^2+c*d^2)^(1/2)+C*arctanh(e^(1/2)*x/
(e*x^2+d)^(1/2))/c/e^(1/2)+1/4*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(B*c
*d-(A*c-C*a)*(e+(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctanh(2^(1/2)*a^(1/4)*c^(1
/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(
1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(1/4)/c^(3/2)/d/(a*e^2+c*d
^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx$$

$$= \frac{-2C\log\left(-\sqrt{e}x+\sqrt{d+ex^2}\right)+e\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-4cd\#1^3+c\#1^4\&,\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*(a + c*x^4)),x]
```

output
```
(-2*C*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] + e*RootSum[c*d^4 - 4*c*d^3*#1 +
 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (B*c*d^2*Log[d + 2
*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2*B*c*d*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 4*A*c*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*
Sqrt[d + e*x^2] - #1]*#1 - 4*a*C*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d +
e*x^2] - #1]*#1 + B*c*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*
#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(2*c*Sq
rt[e])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.54, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{2257}$$

$$\int \left(\frac{-aC+Ac+Bcx^2}{c(a+cx^4)\sqrt{d+ex^2}}+\frac{C}{c\sqrt{d+ex^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\sqrt{-a}B\sqrt{c}+aC-Ac\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{\left(\sqrt{-a}B\sqrt{c}-aC+Ac\right)\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c\sqrt{\sqrt{-a}e+\sqrt{c}d}}+\frac{C\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*(a + c*x^4)),x]
```

output
```
((Sqrt[-a]*B*Sqrt[c] - A*c + a*C)*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*x)/
((-a)^(1/4)*Sqrt[d + e*x^2])])/(2*(-a)^(3/4)*c*Sqrt[Sqrt[c]*d - Sqrt[-a]*e
]) + (C*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c*Sqrt[e]) - ((Sqrt[-a]*B*S
qrt[c] + A*c - a*C)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[-a]*e]*x)/((-a)^(1/4)*S
qrt[d + e*x^2])])/(2*(-a)^(3/4)*c*Sqrt[Sqrt[c]*d + Sqrt[-a]*e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(343) = 686$.

Time = 0.69 (sec) , antiderivative size = 913, normalized size of antiderivative = 2.13

method	result
pseudoelliptic	$-\frac{\left(\left(\sqrt{e}\,(Ac-Ca)\sqrt{a\,e^{2}+c\,d^{2}}-C\,a^{\frac{3}{2}}\,e^{\frac{3}{2}}+\sqrt{a}\,c\left(-Bd\sqrt{e}+A\,e^{\frac{3}{2}}\right)\right)\sqrt{a\left(a\,e^{2}+c\,d^{2}\right)}-e^{\frac{3}{2}}\,a(Ac-Ca)\sqrt{a\,e^{2}+c\,d^{2}}-c\left(-Bd\,e^{\frac{3}{2}}+e^{\frac{5}{2}}\,A\right)a^{\frac{3}{2}}+ \cdots}{\cdots}$
default	$\frac{C\ln\left(\sqrt{e\,x^{2}+d}+x\sqrt{e}\right)}{c\sqrt{e}}-\frac{\left(-\sqrt{-a}\,B\sqrt{c}+Ac-Ca\right)\left(-\frac{\ln\left(\frac{\frac{-2\sqrt{-a}\,\sqrt{c}\,e+2cd}{c}+\frac{2e\sqrt{-\sqrt{-a}\,\sqrt{c}}\left(x-\frac{\sqrt{-\sqrt{-a}\,\sqrt{c}}}{\sqrt{c}}\right)}{\sqrt{c}}+2\sqrt{\frac{-\sqrt{-a}\,\sqrt{c}}{c}\cdots}}{x\cdots}\right)}{2\sqrt{-\sqrt{-a}\cdots}}\cdots\right)}{\cdots}$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-1/4*((e^(1/2)*(A*c-C*a)*(a*e^2+c*d^2)^(1/2)-C*a^(3/2)*e^(3/2)+a^(1/2
)*c*(-B*d*e^(1/2)+A*e^(3/2)))*(a*(a*e^2+c*d^2))^(1/2)-e^(3/2)*a*(A*c-C*a)*
(a*e^2+c*d^2)^(1/2)-c*(-B*d*e^(3/2)+e^(5/2)*A)*a^(3/2)+C*a^(5/2)*e^(5/2))*
(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(
a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*
(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+x^2*(a*e^2+c*d^2)^(1/2))/x^2)+1/
4*((e^(1/2)*(A*c-C*a)*(a*e^2+c*d^2)^(1/2)-C*a^(3/2)*e^(3/2)+a^(1/2)*c*(-B*
d*e^(1/2)+A*e^(3/2)))*(a*(a*e^2+c*d^2))^(1/2)-e^(3/2)*a*(A*c-C*a)*(a*e^2+c
*d^2)^(1/2)-c*(-B*d*e^(3/2)+e^(5/2)*A)*a^(3/2)+C*a^(5/2)*e^(5/2))*(2*(a*(a
*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2
+c*d^2))^(1/2)-2*a*e)^(1/2)*ln((a^(1/2)*(e*x^2+d)+x^2*(a*e^2+c*d^2)^(1/2)+
(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x^2)+c*d^2*(2*a
^(3/2)*(a*e^2+c*d^2)^(1/2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))*(4*(a*e^2+c*
d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*C+(arctan((2*a^(
1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^
2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan(((2*
(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*
e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))*(-e^(1/2
)*a*(A*c-C*a)*(a*e^2+c*d^2)^(1/2)+c*(-B*d*e^(1/2)+A*e^(3/2))*a^(3/2)-C*a^(
5/2)*e^(3/2))))/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\,(a + cx^4)}\,dx = \int \frac{A + Bx^2 + Cx^4}{(a + cx^4)\,\sqrt{d + ex^2}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+a),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/((a + c*x**4)*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\,(a + cx^4)}\,dx = \int \frac{Cx^4 + Bx^2 + A}{(cx^4 + a)\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\,(a + cx^4)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(c\,x^4+a)\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)}\,dx = \left(\int \frac{x^4}{\sqrt{e\,x^2+d}\,a+\sqrt{e\,x^2+d}\,c\,x^4}dx\right)c + \left(\int \frac{x^2}{\sqrt{e\,x^2+d}\,a+\sqrt{e\,x^2+d}\,c\,x^4}dx\right)b + \left(\int \frac{1}{\sqrt{e\,x^2+d}\,a+\sqrt{e\,x^2+d}\,c\,x^4}dx\right)a$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a),x)
```

output
```
int(x**4/(sqrt(d + e*x**2)*a + sqrt(d + e*x**2)*c*x**4),x)*c + int(x**2/(s
qrt(d + e*x**2)*a + sqrt(d + e*x**2)*c*x**4),x)*b + int(1/(sqrt(d + e*x**2
)*a + sqrt(d + e*x**2)*c*x**4),x)*a
```

3.17 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 481

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)}\,dx = \frac{(Cd^2-Bde+Ae^2)\,x}{d\,(cd^2+ae^2)\sqrt{d+ex^2}}$$

$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(d(Bcd-Ace+aCe)-(Acd-aCd+aBe)\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{\cdots}}}{\sqrt{a}\left(\sqrt{ae+\cdots}\right.}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}d\,(cd^2+ae^2)^{3/2}}$$

$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(d(Bcd-Ace+aCe)-(Acd-aCd+aBe)\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\cdots}{\sqrt{a}(\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}d\,(cd^2+ae^2)^{3/2}}$$

output
```
(A*e^2-B*d*e+C*d^2)*x/d/(a*e^2+c*d^2)/(e*x^2+d)^(1/2)+1/4*(a^(1/2)*e+(a*e^
2+c*d^2)^(1/2))^(1/2)*(d*(-A*c*e+B*c*d+C*a*e)-(A*c*d+B*a*e-C*a*d)*(e-(a*e^
2+c*d^2)^(1/2)/a^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+
c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1
/2))-c*d*x^2))*2^(1/2)/a^(1/4)/c^(1/2)/d/(a*e^2+c*d^2)^(3/2)+1/4*(-a^(1/2)
*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(d*(-A*c*e+B*c*d+C*a*e)-(A*c*d+B*a*e-C*a*d)*
(e+(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)
*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2
+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(1/4)/c^(1/2)/d/(a*e^2+c*d^2)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.67 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}\,(a+cx^4)}\,dx = \frac{\frac{2(Cd^2+e(-Bd+Ae))x}{d\sqrt{d+ex^2}} + \sqrt{e}\text{RootSum}\left[cd^4-4cd^3\#1+6cd^2\#1^2+16ae^2\#1^2-\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + c*x^4)),x]
```

output
```
((2*(C*d^2 + e*(-(B*d) + A*e))*x)/(d*Sqrt[d + e*x^2]) + Sqrt[e]*RootSum[c*
d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & ,
(B*c*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - A*c*d^2*e*L
og[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + a*C*d^2*e*Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2*B*c*d^2*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 6*A*c*d*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*
x*Sqrt[d + e*x^2] - #1]*#1 - 6*a*C*d*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[
d + e*x^2] - #1]*#1 + 4*a*B*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x
^2] - #1]*#1 + B*c*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#
1^2 - A*c*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + a*C
*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*
d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(2*(c*d^2 + a*e^2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)\,(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2257$$

$$\int \left(\frac{-aC + Ac + Bcx^2}{c(a + cx^4)(d + ex^2)^{3/2}} + \frac{C}{c(d + ex^2)^{3/2}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\sqrt{-a}B\sqrt{c} + aC - Ac\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d - \sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\sqrt{c}\left(\sqrt{c}d - \sqrt{-a}e\right)^{3/2}} - \frac{\left(\sqrt{-a}B\sqrt{c} - aC + Ac\right)\operatorname{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e + \sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\sqrt{c}\left(\sqrt{-a}e + \sqrt{c}d\right)^{3/2}} - \frac{ex\left(\sqrt{-a}B\sqrt{c} - aC + Ac\right)}{2cd\sqrt{d+ex^2}\left(\sqrt{-a}\sqrt{c}d - ae\right)} - \frac{ex\left(\sqrt{-a}B\sqrt{c} + aC - Ac\right)}{2cd\sqrt{d+ex^2}\left(\sqrt{-a}\sqrt{c}d + ae\right)} + \frac{Cx}{cd\sqrt{d+ex^2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + c*x^4)),x]
```

output
```
(C*x)/(c*d*Sqrt[d + e*x^2]) - ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*e*x)/(2*c*
d*(Sqrt[-a]*Sqrt[c]*d - a*e)*Sqrt[d + e*x^2]) - ((Sqrt[-a]*B*Sqrt[c] - A*c
 + a*C)*e*x)/(2*c*d*(Sqrt[-a]*Sqrt[c]*d + a*e)*Sqrt[d + e*x^2]) + ((Sqrt[-
a]*B*Sqrt[c] - A*c + a*C)*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]*x)/((-a)^(1
/4)*Sqrt[d + e*x^2])])/(2*(-a)^(3/4)*Sqrt[c]*(Sqrt[c]*d - Sqrt[-a]*e)^(3/2
)) - ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[-a]*
e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/(2*(-a)^(3/4)*Sqrt[c]*(Sqrt[c]*d + Sq
rt[-a]*e)^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(399) = 798$.

Time = 0.87 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.97

method	result
pseudoelliptic	$-\frac{\sqrt{4\sqrt{a\,e^{2}+c\,d^{2}}\,\sqrt{a}-2\sqrt{a(a\,e^{2}+c\,d^{2})}-2ae}\,\sqrt{e\,x^{2}+d}\left(\ln\left(\frac{\sqrt{e\,x^{2}+d}\,\sqrt{2\sqrt{a\left(a\,e^{2}+c\,d^{2}\right)}+2ae}\,x-x^{2}\sqrt{a\,e^{2}+c\,d^{2}}-\sqrt{a}\left(e\,x^{2}+d\right)}{x^{2}}\right)\right.}{}$
default	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)
/a^(3/2)/(e*x^2+d)^(1/2)/(a*e^2+c*d^2)^(3/2)*((4*(a*e^2+c*d^2)^(1/2)*a^(1/
2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(e*x^2+d)^(1/2)*(ln(((e*x^2+d)^(
1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-x^2*(a*e^2+c*d^2)^(1/2)-a^(
1/2)*(e*x^2+d))/x^2)-ln((a^(1/2)*(e*x^2+d)+x^2*(a*e^2+c*d^2)^(1/2)+(e*x^2+
d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x^2))*((1/2*((A*c-C*a)
*d+B*a*e)*(a*e^2+c*d^2)^(1/2)+(1/2*B*e^2-C*d*e)*a^(3/2)+c*d*a^(1/2)*(A*e-1
/2*B*d))*(a*(a*e^2+c*d^2))^(1/2)-(1/2*((A*c-C*a)*d+B*a*e)*a*(a*e^2+c*d^2)^
(1/2)+c*d*(A*e-1/2*B*d)*a^(3/2)+1/2*a^(5/2)*e*(B*e-2*C*d))*e)*(2*(a*(a*e^2
+c*d^2))^(1/2)+2*a*e)^(1/2)-4*c*((-1/2*d*(arctan((2*a^(1/2)*(e*x^2+d)^(1/2
)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1
/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan(((2*(a*(a*e^2+c*d^2))^(
1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^
(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))*((A*c-C*a)*d+B*a*e)*a*(e*x^
2+d)^(1/2)+(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)
^(1/2)*a^(3/2)*x*(A*e^2-B*d*e+C*d^2))*(a*e^2+c*d^2)^(1/2)+(c*d*(A*e-1/2*B*
d)*a^(3/2)+1/2*a^(5/2)*e*(B*e-2*C*d))*d*(arctan((2*a^(1/2)*(e*x^2+d)^(1/2)
+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/
2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan(((2*(a*(a*e^2+c*d^2))^(1
/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(3/2)/(c*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)}\,dx = \int \frac{Cx^4+Bx^2+A}{(cx^4+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + a)*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(c\,x^4+a)\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)*(d + e*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)}\,dx = \left(\int \frac{x^4}{\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2+\sqrt{e\,x^2+d}\,cd\,x^4+\sqrt{e\,x^2+d}\,ce\,x^6}dx\right)$$
$$+\left(\int \frac{x^2}{\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2+\sqrt{e\,x^2+d}\,cd\,x^4+\sqrt{e\,x^2+d}\,ce\,x^6}dx\right)b$$
$$+\left(\int \frac{1}{\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2+\sqrt{e\,x^2+d}\,cd\,x^4+\sqrt{e\,x^2+d}\,ce\,x^6}dx\right)a$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+a),x)
```

output
```
int(x**4/(sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 + sqrt(d + e*x*
*2)*c*d*x**4 + sqrt(d + e*x**2)*c*e*x**6),x)*c + int(x**2/(sqrt(d + e*x**2
)*a*d + sqrt(d + e*x**2)*a*e*x**2 + sqrt(d + e*x**2)*c*d*x**4 + sqrt(d + e
*x**2)*c*e*x**6),x)*b + int(1/(sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e
*x**2 + sqrt(d + e*x**2)*c*d*x**4 + sqrt(d + e*x**2)*c*e*x**6),x)*a
```

3.18 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 637

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)}\,dx = \frac{(Cd^2-Bde+Ae^2)\,x}{3d\,(cd^2+ae^2)\,(d+ex^2)^{3/2}}$$
$$+\frac{(cd^2(2Cd^2-e(5Bd-8Ae))-ae^2(4Cd^2-e(Bd+2Ae)))\,x}{3d^2\,(cd^2+ae^2)^2\,\sqrt{d+ex^2}}$$
$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd(Bcd^2-2Acde+2aCde-aBe^2)-\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(Ac(cd^2-ae^2)+a(aCe^2-\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}d\,(cd^2+ae^2)^{5/2}}$$
$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd(Bcd^2-2Acde+2aCde-aBe^2)-\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(Ac(cd^2-ae^2)+a(aCe^2\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}d\,(cd^2+ae^2)^{5/2}}$$

output
```
1/3*(A*e^2-B*d*e+C*d^2)*x/d/(a*e^2+c*d^2)/(e*x^2+d)^(3/2)+1/3*(c*d^2*(2*C*
d^2-e*(-8*A*e+5*B*d))-a*e^2*(4*C*d^2-e*(2*A*e+B*d)))*x/d^2/(a*e^2+c*d^2)^2
/(e*x^2+d)^(1/2)+1/4*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(c*d*(-2*A*c*d*
e-B*a*e^2+B*c*d^2+2*C*a*d*e)-(e-(a*e^2+c*d^2)^(1/2)/a^(1/2))*(A*c*(-a*e^2+
c*d^2)+a*(C*a*e^2-c*d*(-2*B*e+C*d))))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1
/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*
e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(1/4)/c^(1/2)/d/(a*e^2+c*d^2)^(5/2)+
1/4*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(c*d*(-2*A*c*d*e-B*a*e^2+B*c*d^
2+2*C*a*d*e)-(e+(a*e^2+c*d^2)^(1/2)/a^(1/2))*(A*c*(-a*e^2+c*d^2)+a*(C*a*e^
2-c*d*(-2*B*e+C*d))))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c
*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/
2))-c*d*x^2))*2^(1/2)/a^(1/4)/c^(1/2)/d/(a*e^2+c*d^2)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.51 (sec) , antiderivative size = 839, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)}\,dx = \frac{2x(cCd^4(3d+2ex^2)-aCd^2e^2(3d+4ex^2)+ae^4(3Ad+Bdx^2+2Aex^2)+c}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + c*x^4)),x]
```

output
```
(2*x*(c*C*d^4*(3*d + 2*e*x^2) - a*C*d^2*e^2*(3*d + 4*e*x^2) + a*e^4*(3*A*d
 + B*d*x^2 + 2*A*e*x^2) + c*d^2*e*(-(B*d*(6*d + 5*e*x^2)) + A*e*(9*d + 8*e
*x^2))) + 3*d^2*Sqrt[e]*(d + e*x^2)^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c
*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (B*c^2*d^4*Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2*A*c^2*d^3*e*Log[d + 2*e*x^2 -
 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 2*a*c*C*d^3*e*Log[d + 2*e*x^2 - 2*Sqr
t[e]*x*Sqrt[d + e*x^2] - #1] - a*B*c*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x
*Sqrt[d + e*x^2] - #1] - 2*B*c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d
+ e*x^2] - #1]*#1 + 8*A*c^2*d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e
*x^2] - #1]*#1 - 8*a*c*C*d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1 + 10*a*B*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2]
 - #1]*#1 - 4*a*A*c*e^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1
]*#1 + 4*a^2*C*e^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1
+ B*c^2*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 2*A
*c^2*d*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + 2*a*c*
C*d*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - a*B*c*e^2
*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^
2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(6*(c*d^3 + a*d*e^2)^2*(d +
e*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 553, normalized size of antiderivative = 0.87, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{2257}$$

$$\int \left(\frac{-aC+Ac+Bcx^2}{c(a+cx^4)(d+ex^2)^{5/2}}+\frac{C}{c(d+ex^2)^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\sqrt{-a}B\sqrt{c}+aC-Ac\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\left(\sqrt{c}d-\sqrt{-a}e\right)^{5/2}}-\frac{\left(\sqrt{-a}B\sqrt{c}-aC+Ac\right)\operatorname{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\left(\sqrt{-a}e+\sqrt{c}d\right)^{5/2}}-\frac{ex\left(2\sqrt{-a}e+5\sqrt{c}d\right)\left(\sqrt{-a}B\sqrt{c}-aC+Ac\right)}{6\sqrt{-a}cd^2\sqrt{d+ex^2}\left(\sqrt{-a}e+\sqrt{c}d\right)^2}-\frac{ex\left(5\sqrt{c}d-2\sqrt{-a}e\right)\left(\sqrt{-a}B\sqrt{c}+aC-Ac\right)}{6\sqrt{-a}cd^2\sqrt{d+ex^2}\left(\sqrt{c}d-\sqrt{-a}e\right)^2}-\frac{ex\left(\sqrt{-a}B\sqrt{c}-aC+Ac\right)}{6cd\left(d+ex^2\right)^{3/2}\left(\sqrt{-a}\sqrt{c}d-ae\right)}-\frac{ex\left(\sqrt{-a}B\sqrt{c}+aC-Ac\right)}{6cd\left(d+ex^2\right)^{3/2}\left(\sqrt{-a}\sqrt{c}d+ae\right)}+\frac{2Cx}{3cd^2\sqrt{d+ex^2}}+\frac{Cx}{3cd\left(d+ex^2\right)^{3/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + c*x^4)),x]
```

output
```
(C*x)/(3*c*d*(d + e*x^2)^(3/2)) - ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*e*x)/(
6*c*d*(Sqrt[-a]*Sqrt[c]*d - a*e)*(d + e*x^2)^(3/2)) - ((Sqrt[-a]*B*Sqrt[c]
 - A*c + a*C)*e*x)/(6*c*d*(Sqrt[-a]*Sqrt[c]*d + a*e)*(d + e*x^2)^(3/2)) +
(2*C*x)/(3*c*d^2*Sqrt[d + e*x^2]) - ((Sqrt[-a]*B*Sqrt[c] - A*c + a*C)*e*(5
*Sqrt[c]*d - 2*Sqrt[-a]*e)*x)/(6*Sqrt[-a]*c*d^2*(Sqrt[c]*d - Sqrt[-a]*e)^2
*Sqrt[d + e*x^2]) - ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*e*(5*Sqrt[c]*d + 2*S
qrt[-a]*e)*x)/(6*Sqrt[-a]*c*d^2*(Sqrt[c]*d + Sqrt[-a]*e)^2*Sqrt[d + e*x^2]
) + ((Sqrt[-a]*B*Sqrt[c] - A*c + a*C)*ArcTan[(Sqrt[Sqrt[c]*d - Sqrt[-a]*e]
*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/(2*(-a)^(3/4)*(Sqrt[c]*d - Sqrt[-a]*e)^
(5/2)) - ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*ArcTanh[(Sqrt[Sqrt[c]*d + Sqrt[
-a]*e]*x)/((-a)^(1/4)*Sqrt[d + e*x^2])])/(2*(-a)^(3/4)*(Sqrt[c]*d + Sqrt[-
a]*e)^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1286 vs. $2(549) = 1098$.

Time = 2.22 (sec) , antiderivative size = 1287, normalized size of antiderivative = 2.02

method	result	size
pseudoelliptic	Expression too large to display	1287
default	Expression too large to display	3916

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
3/2/(a*e^2+c*d^2)^(5/2)*(-1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*((1/
3*(-a*(A*c-C*a)*e^2+2*B*a*c*d*e+c*d^2*(A*c-C*a))*(a*e^2+c*d^2)^(1/2)-1/3*c
*e*(A*e^2-3*B*d*e+3*C*d^2)*a^(3/2)+1/3*a^(5/2)*C*e^3+a^(1/2)*c^2*d^2*(A*e-
1/3*B*d))*(a*(a*e^2+c*d^2))^(1/2)-(-1/3*((A*a*c-C*a^2)*e^2-2*B*a*c*d*e-c*d
^2*(A*c-C*a))*a*(a*e^2+c*d^2)^(1/2)+c^2*d^2*(A*e-1/3*B*d)*a^(3/2)-1/3*e*(c
*(A*e^2-3*B*d*e+3*C*d^2)*a^(5/2)-a^(7/2)*C*e^2))*e)*(e*x^2+d)^(3/2)*(4*(a*
e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*ln((a^(1/2
)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+x^2*
(a*e^2+c*d^2)^(1/2))/x^2)+1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*((1/
3*(-a*(A*c-C*a)*e^2+2*B*a*c*d*e+c*d^2*(A*c-C*a))*(a*e^2+c*d^2)^(1/2)-1/3*c
*e*(A*e^2-3*B*d*e+3*C*d^2)*a^(3/2)+1/3*a^(5/2)*C*e^3+a^(1/2)*c^2*d^2*(A*e-
1/3*B*d))*(a*(a*e^2+c*d^2))^(1/2)-(-1/3*((A*a*c-C*a^2)*e^2-2*B*a*c*d*e-c*d
^2*(A*c-C*a))*a*(a*e^2+c*d^2)^(1/2)+c^2*d^2*(A*e-1/3*B*d)*a^(3/2)-1/3*e*(c
*(A*e^2-3*B*d*e+3*C*d^2)*a^(5/2)-a^(7/2)*C*e^2))*e)*(e*x^2+d)^(3/2)*(4*(a*
e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*ln((a^(1/2
)*(e*x^2+d)+x^2*(a*e^2+c*d^2)^(1/2)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(
1/2)+2*a*e)^(1/2)*x)/x^2)+c*(2*x*(a*e^2+c*d^2)^(1/2)*(c*d^2*(8/9*A*e^3*x^2
+d*(-5/9*B*x^2+A)*e^2-2/3*d^2*(-1/3*C*x^2+B)*e+1/3*C*d^3)*a^(3/2)+1/3*a^(5
/2)*(2/3*A*e^3*x^2+d*(1/3*B*x^2+A)*e^2-4/3*C*d^2*e*x^2-C*d^3)*e^2)*(4*(a*e
^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)+(1/3*((A...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(5/2)/(c*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)}\,dx = \int \frac{Cx^4+Bx^2+A}{(cx^4+a)(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + a)*(e*x^2 + d)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\,(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\,(a+cx^4)}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(c\,x^4+a)\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)*(d + e*x^2)^(5/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)*(d + e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\,(a+cx^4)}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(e\,x^2+d)^{\frac{5}{2}}\,(c\,x^4+a)}dx$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a),x)
```

output
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a),x)
```

3.19 $\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2} dx$

Optimal result

Integrand size = 33, antiderivative size = 752

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = -\frac{e(Bcd+Ace-3aCe)x\sqrt{d+ex^2}}{4ac^2}$$

$$-\frac{Bex(d+ex^2)^{3/2}}{4ac}+\frac{x(Ac-aC+Bcx^2)(d+ex^2)^{5/2}}{4ac(a+cx^4)}$$

$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(\sqrt{a}c^2d^3(Bd+Ae)+a^{5/2}e^3(7Cd+4Be)+3Ac^2d^3\sqrt{cd^2+ae^2}-a^2e^2(13Cd+4Be\right.}{8\ldots}$$

$$+\frac{e^{3/2}(5Cd+2Be)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2}$$

$$-\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(Acd(3cd^2+ae^2)+a(cd^2(Cd+2Be)-ae^2(13Cd+4Be)))-a\ldots\right.}{8\sqrt{2}a^{5/4}c^{5/2}\ldots}$$

output
```
-1/4*e*(A*c*e+B*c*d-3*C*a*e)*x*(e*x^2+d)^(1/2)/a/c^2-1/4*B*e*x*(e*x^2+d)^(
3/2)/a/c+1/4*x*(B*c*x^2+A*c-C*a)*(e*x^2+d)^(5/2)/a/c/(c*x^4+a)+1/16*(a^(1/
2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(a^(1/2)*c^2*d^3*(A*e+B*d)+a^(5/2)*e^3*(4*
B*e+7*C*d)+3*A*c^2*d^3*(a*e^2+c*d^2)^(1/2)-a^2*e^2*(4*B*e+13*C*d)*(a*e^2+c
*d^2)^(1/2)+a*c*d*(a*e^2+c*d^2)^(1/2)*(C*d^2+e*(A*e+2*B*d))+a^(3/2)*c*d*e*
(7*C*d^2+e*(A*e+5*B*d)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+
c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1
/2))-c*d*x^2))*2^(1/2)/a^(7/4)/c^(5/2)/d/(a*e^2+c*d^2)^(1/2)+1/2*e^(3/2)*(
2*B*e+5*C*d)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^2-1/16*(-a^(1/2)*e+(a*e^
2+c*d^2)^(1/2))^(1/2)*((e+(a*e^2+c*d^2)^(1/2)/a^(1/2))*(A*c*d*(a*e^2+3*c*d
^2)+a*(c*d^2*(2*B*e+C*d)-a*e^2*(4*B*e+13*C*d)))-d*(B*c*d*(7*a*e^2+c*d^2)+2
*e*(a*C*(-3*a*e^2+4*c*d^2)+A*c*(a*e^2+2*c*d^2))))*arctanh(2^(1/2)*a^(1/4)*
c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*
(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(5/4)/c^(5/2)/d/(a*e^2
+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.80 (sec) , antiderivative size = 2013, normalized size of antiderivative = 2.68

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x]
```

output
```
((2*c*x*Sqrt[d + e*x^2]*(3*a^2*C*e^2 + B*c^2*d^2*x^2 + A*c*(-(a*e^2) + c*d
*(d + 2*e*x^2)) - a*c*(B*e*(2*d + e*x^2) + C*(d^2 + 2*d*e*x^2 - 2*e^2*x^4)
)))/(a*(a + c*x^4)) - 4*c*e^(3/2)*(5*C*d + 2*B*e)*Log[-(Sqrt[e]*x) + Sqrt[
d + e*x^2]] + 4*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e
^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (3*c^2*C*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]
*x*Sqrt[d + e*x^2] - #1] + 19*B*c^2*d^3*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sq
rt[d + e*x^2] - #1] + 49*A*c^2*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[
d + e*x^2] - #1] - 50*a*c*C*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d +
 e*x^2] - #1] - 48*a*B*c*d*e^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1] - 16*a*A*c*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1
] + 16*a^2*C*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2*c
^2*C*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 6*B*c^2*
d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 10*A*c^2*d*
e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 20*a*c*C*d*e^
2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 8*a*B*c*e^3*Log
[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 3*c^2*C*d^2*Log[d +
2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + 3*B*c^2*d*e*Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + A*c^2*e^2*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - 2*a*c*C*e^2*Log[d + 2*e*x^2 - 2
*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 2257}$$

$$\int \left(\frac{(d+ex^2)^{5/2}(-aC+Ac+Bcx^2)}{c(a+cx^4)^2}+\frac{C(d+ex^2)^{5/2}}{c(a+cx^4)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Ac-aC)\int \frac{(ex^2+d)^{5/2}}{(cx^4+a)^2}dx}{c}+B\int \frac{x^2(ex^2+d)^{5/2}}{(cx^4+a)^2}dx-\frac{C\left(-3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+\sqrt{-a}ae^3+c^{3/2}d^3\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c^{5/2}\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{C\sqrt{e}\left(-20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16\sqrt{-a}c^{5/2}}+\frac{C\sqrt{e}\left(20\sqrt{-a}\sqrt{c}de-8ae^2+15cd^2\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16\sqrt{-a}c^{5/2}}-\frac{C\left(3\sqrt{-a}cd^2e-3a\sqrt{c}de^2+(-a)^{3/2}e^3+c^{3/2}d^3\right)\operatorname{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c^{5/2}\sqrt{\sqrt{-a}e+\sqrt{c}d}}+\frac{Cex\sqrt{d+ex^2}\left(4\sqrt{-a}e+7\sqrt{c}d\right)}{16\sqrt{-a}c^2}+\frac{Cex\sqrt{d+ex^2}\left(7\sqrt{-a}\sqrt{c}d+4ae\right)}{16ac^2}$$

input
```
Int[((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1732 vs. 2(644) = 1288.

Time = 2.80 (sec) , antiderivative size = 1733, normalized size of antiderivative = 2.30

method	result	size
pseudoelliptic	Expression too large to display	1733
default	Expression too large to display	13770
risch	Expression too large to display	37570

input

```
int((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/2*(1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*e^(1/2)*(4*(a*e^2+c*d^2)
^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(((-9/20*c^3*d^3*x^4
*(A*e+B*d)*a^(5/2)-(a*B*(c*x^4+a)*e^4+1/4*d*(c*x^4+a)*(A*c+7*C*a)*e^3+41/2
0*B*c*d^2*(c*x^4+a)*e^2+9/20*c*d^3*(7*C*a+c*(7*C*x^4+A))*e+9/20*B*c^2*d^4)
*a^(7/2))*(a*e^2+c*d^2)^(1/2)+1/4*(-4*a^2*B*e^3+a*d*(A*c-13*C*a)*e^2+2*B*a
*c*d^2*e+(3*A*c^2+C*a*c)*d^3)*(a*e^2+9/5*c*d^2)*(c*x^4+a)*a^2)*(a*(a*e^2+c
*d^2))^(1/2)+1/4*(a*e^2+9/5*c*d^2)*(c*x^4+a)*((A*c*d*e+4*B*a*e^2+B*c*d^2+7
*C*a*d*e)*a^(7/2)*(a*e^2+c*d^2)^(1/2)-(-4*a^2*B*e^3+a*d*(A*c-13*C*a)*e^2+2
*B*a*c*d^2*e+(3*A*c^2+C*a*c)*d^3)*a^3)*e)*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^
(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+x^2*(a*e^2+c*d^2)^(1/2))/x
^2)-1/4*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*e^(1/2)*(4*(a*e^2+c*d^2)^(
1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(((-9/20*c^3*d^3*x^4*(
A*e+B*d)*a^(5/2)-(a*B*(c*x^4+a)*e^4+1/4*d*(c*x^4+a)*(A*c+7*C*a)*e^3+41/20*
B*c*d^2*(c*x^4+a)*e^2+9/20*c*d^3*(7*C*a+c*(7*C*x^4+A))*e+9/20*B*c^2*d^4)*a
^(7/2))*(a*e^2+c*d^2)^(1/2)+1/4*(-4*a^2*B*e^3+a*d*(A*c-13*C*a)*e^2+2*B*a*c
*d^2*e+(3*A*c^2+C*a*c)*d^3)*(a*e^2+9/5*c*d^2)*(c*x^4+a)*a^2)*(a*(a*e^2+c*d
^2))^(1/2)+1/4*(a*e^2+9/5*c*d^2)*(c*x^4+a)*((A*c*d*e+4*B*a*e^2+B*c*d^2+7*C
*a*d*e)*a^(7/2)*(a*e^2+c*d^2)^(1/2)-(-4*a^2*B*e^3+a*d*(A*c-13*C*a)*e^2+2*B
*a*c*d^2*e+(3*A*c^2+C*a*c)*d^3)*a^3)*e)*ln((a^(1/2)*(e*x^2+d)+x^2*(a*e^2+c
*d^2)^(1/2)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(5/2)*(C*x**4+B*x**2+A)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{(cx^4+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(5/2)/(c*x^4 + a)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(e*x^2 + d)*C*e^2*x/c^2 - 1/4*(5*C*d*e^(3/2) + 2*B*e^(5/2))*log((s
qrt(e)*x - sqrt(e*x^2 + d))^2)/c^2 - 1/2*((sqrt(e)*x - sqrt(e*x^2 + d))^6*
B*c^2*d^3*sqrt(e) - 4*(sqrt(e)*x - sqrt(e*x^2 + d))^6*C*a*c*d^2*e^(3/2) +
4*(sqrt(e)*x - sqrt(e*x^2 + d))^6*A*c^2*d^2*e^(3/2) - 5*(sqrt(e)*x - sqrt(
e*x^2 + d))^6*B*a*c*d*e^(5/2) + 2*(sqrt(e)*x - sqrt(e*x^2 + d))^6*C*a^2*e^
(7/2) - 2*(sqrt(e)*x - sqrt(e*x^2 + d))^6*A*a*c*e^(7/2) - 3*(sqrt(e)*x - s
qrt(e*x^2 + d))^4*B*c^2*d^4*sqrt(e) + 6*(sqrt(e)*x - sqrt(e*x^2 + d))^4*C*
a*c*d^3*e^(3/2) - 6*(sqrt(e)*x - sqrt(e*x^2 + d))^4*A*c^2*d^3*e^(3/2) - 5*
(sqrt(e)*x - sqrt(e*x^2 + d))^4*B*a*c*d^2*e^(5/2) + 16*(sqrt(e)*x - sqrt(e
*x^2 + d))^4*C*a^2*d*e^(7/2) - 16*(sqrt(e)*x - sqrt(e*x^2 + d))^4*A*a*c*d*
e^(7/2) + 8*(sqrt(e)*x - sqrt(e*x^2 + d))^4*B*a^2*e^(9/2) + 3*(sqrt(e)*x -
 sqrt(e*x^2 + d))^2*B*c^2*d^5*sqrt(e) - 4*(sqrt(e)*x - sqrt(e*x^2 + d))^2*
C*a*c*d^4*e^(3/2) + 4*(sqrt(e)*x - sqrt(e*x^2 + d))^2*A*c^2*d^4*e^(3/2) +
(sqrt(e)*x - sqrt(e*x^2 + d))^2*B*a*c*d^3*e^(5/2) - 2*(sqrt(e)*x - sqrt(e*
x^2 + d))^2*C*a^2*d^2*e^(7/2) + 2*(sqrt(e)*x - sqrt(e*x^2 + d))^2*A*a*c*d^
2*e^(7/2) - B*c^2*d^6*sqrt(e) + 2*C*a*c*d^5*e^(3/2) - 2*A*c^2*d^5*e^(3/2)
+ B*a*c*d^4*e^(5/2))/(((sqrt(e)*x - sqrt(e*x^2 + d))^8*c - 4*(sqrt(e)*x -
sqrt(e*x^2 + d))^6*c*d + 6*(sqrt(e)*x - sqrt(e*x^2 + d))^4*c*d^2 + 16*(sqr
t(e)*x - sqrt(e*x^2 + d))^4*a*e^2 - 4*(sqrt(e)*x - sqrt(e*x^2 + d))^2*c*d^
3 + c*d^4)*a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \int \frac{(e\,x^2+d)^{5/2}\,(C\,x^4+B\,x^2+A)}{(c\,x^4+a)^2}\,dx$$

input
```
int(((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x)
```

output
```
int(((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c^2x^8+2ac\,x^4+a^2}dx\right)a\,d^2 + \left(\int \frac{\sqrt{e\,x^2+d}\,x^8}{c^2x^8+2ac\,x^4+a^2}dx\right)c\,e^2 + \left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{c^2x^8+2ac\,x^4+a^2}dx\right)b\,e^2 + 2\left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{c^2x^8+2ac\,x^4+a^2}dx\right)cde + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2x^8+2ac\,x^4+a^2}dx\right)a\,e^2 + 2\left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2x^8+2ac\,x^4+a^2}dx\right)bde + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2x^8+2ac\,x^4+a^2}dx\right)c\,d^2 + 2\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8+2ac\,x^4+a^2}dx\right)ade + \left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8+2ac\,x^4+a^2}dx\right)b\,d^2$$

input
```
int((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*d**2 + int((sqrt
(d + e*x**2)*x**8)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*c*e**2 + int((sqrt(d
 + e*x**2)*x**6)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*b*e**2 + 2*int((sqrt(d
 + e*x**2)*x**6)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*c*d*e + int((sqrt(d +
e*x**2)*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*e**2 + 2*int((sqrt(d +
e*x**2)*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*b*d*e + int((sqrt(d + e*x
**2)*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*c*d**2 + 2*int((sqrt(d + e*x
**2)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*d*e + int((sqrt(d + e*x**2
)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*b*d**2
```

3.20 $\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 586

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx =$$
$$-\frac{Bex\sqrt{d+ex^2}}{4ac}+\frac{x(Ac-aC+Bcx^2)(d+ex^2)^{3/2}}{4ac(a+cx^4)}$$
$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd(Bcd^2+3Acde+5aCde+2aBe^2)+(4a^2Ce^2-cd(3Acd+aCd+aBe))\left(e-\right.\right.}{8\sqrt{2}a^{5/4}c^{5/2}d\sqrt{cd^2+ae^2}}$$
$$+\frac{Ce^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}$$
$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(cd(Bcd^2+3Acde+5aCde+2aBe^2)+(4a^2Ce^2-cd(3Acd+aCd+aBe))\left(e-\right.\right.}{8\sqrt{2}a^{5/4}c^{5/2}d\sqrt{cd^2+ae^2}}$$

output
```
-1/4*B*e*x*(e*x^2+d)^(1/2)/a/c+1/4*x*(B*c*x^2+A*c-C*a)*(e*x^2+d)^(3/2)/a/c
/(c*x^4+a)+1/16*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(c*d*(3*A*c*d*e+2*B*
a*e^2+B*c*d^2+5*C*a*d*e)+(4*a^2*C*e^2-c*d*(3*A*c*d+B*a*e+C*a*d))*(e-(a*e^2
+c*d^2)^(1/2)/a^(1/2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c
*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/
2))-c*d*x^2))*2^(1/2)/a^(5/4)/c^(5/2)/d/(a*e^2+c*d^2)^(1/2)+C*e^(3/2)*arct
anh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^2+1/16*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(
1/2)*(c*d*(3*A*c*d*e+2*B*a*e^2+B*c*d^2+5*C*a*d*e)+(4*a^2*C*e^2-c*d*(3*A*c*
d+B*a*e+C*a*d))*(e+(a*e^2+c*d^2)^(1/2)/a^(1/2)))*arctanh(2^(1/2)*a^(1/4)*c
^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(
a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(5/4)/c^(5/2)/d/(a*e^2+
c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.05 (sec) , antiderivative size = 1422, normalized size of antiderivative = 2.43

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x]
```

output
```
((2*c*x*Sqrt[d + e*x^2]*(B*c*d*x^2 + A*c*(d + e*x^2) - a*(C*d + B*e + C*e*
x^2)))/(a*(a + c*x^4)) - 8*C*e^(3/2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]] +
 4*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c
*d*#1^3 + c*#1^4 & , (2*c*C*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x
^2] - #1] + 17*B*c*d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #
1] + 32*A*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 32
*a*C*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 16*a*B*e^
3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 6*B*c*d*e*Log[d +
2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 4*A*c*e^2*Log[d + 2*e*x^2
 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 8*a*C*e^2*Log[d + 2*e*x^2 - 2*Sq
rt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 2*c*C*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*S
qrt[d + e*x^2] - #1]*#1^2 + B*c*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e
*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3)
& ] + (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 -
 4*c*d*#1^3 + c*#1^4 & , (B*c^2*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d +
 e*x^2] - #1] + 3*A*c^2*d^3*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2
] - #1] - 3*a*c*C*d^3*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1
] - 66*a*B*c*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] -
 128*a*A*c*d*e^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 128
*a^2*C*d*e^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 64*a...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}\left(A+Bx^2+Cx^4\right)}{\left(a+cx^4\right)^2}\,dx$$

$$\Big\downarrow \text{ 2257}$$

$$\int \left(\frac{\left(d+ex^2\right)^{3/2}\left(-aC+Ac+Bcx^2\right)}{c\left(a+cx^4\right)^2}+\frac{C\left(d+ex^2\right)^{3/2}}{c\left(a+cx^4\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Ac-aC)\int \frac{(ex^2+d)^{3/2}}{(cx^4+a)^2}dx}{c}+B\int \frac{x^2(ex^2+d)^{3/2}}{(cx^4+a)^2}dx-\frac{C(-2\sqrt{-a}\sqrt{c}de-ae^2+cd^2)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c^2\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{C(2\sqrt{-a}\sqrt{c}de-ae^2+cd^2)\operatorname{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c^2\sqrt{\sqrt{-a}e+\sqrt{c}d}}-\frac{C\sqrt{e}(3\sqrt{c}d-2\sqrt{-a}e)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{-a}c^2}+\frac{C\sqrt{e}(2\sqrt{-a}e+3\sqrt{c}d)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4\sqrt{-a}c^2}$$

input
```
Int[((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1110 vs. 2(492) = 984.

Time = 1.37 (sec) , antiderivative size = 1111, normalized size of antiderivative = 1.90

method	result	size
pseudoelliptic	Expression too large to display	1111
default	Expression too large to display	9082

input
```
int((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*((((-C*a^(9/2)*e-1/4*c*((4*C*e*x^4+B*d)*a^(7/2)+B*a^(5/2)*c*d*x^4))*(
a*e^2+c*d^2)^(1/2)+3/4*(-4/3*a^2*C*e^2+1/3*c*d*(B*e+C*d)*a+A*c^2*d^2)*(c*x
^4+a)*a^2)*(a*(a*e^2+c*d^2))^(1/2)+((1/4*c*(4*C*e*x^4+B*d)*a^(9/2)+C*a^(11
/2)*e+1/4*B*a^(7/2)*c^2*d*x^4)*(a*e^2+c*d^2)^(1/2)-3/4*(-4/3*a^2*C*e^2+1/3
*c*d*(B*e+C*d)*a+A*c^2*d^2)*(c*x^4+a)*a^3)*e)*(4*(a*e^2+c*d^2)^(1/2)*a^(1/
2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)
^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+x^2*(a*e^2+c*d^2)^(1/2))/
x^2)-ln((a^(1/2)*(e*x^2+d)+x^2*(a*e^2+c*d^2)^(1/2)+(e*x^2+d)^(1/2)*(2*(a*(
a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x^2))*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e
)^(1/2)-8*c*d^2*((4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-
2*a*e)^(1/2)*e^(3/2)*(a^(11/2)+a^(9/2)*c*x^4)*C*arctanh((e*x^2+d)^(1/2)/x/
e^(1/2))+1/2*(arctan(((2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*
(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2
)-2*a*e)^(1/2))-arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/
2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1
/2)-2*a*e)^(1/2)))*(1/4*c*(4*C*e*x^4+B*d)*a^(9/2)+C*a^(11/2)*e+1/4*B*a^(7/
2)*c^2*d*x^4)*(a*e^2+c*d^2)^(1/2)+1/4*c*x*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2
*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*((-C*d-(C*x^2+B)*e)*a^(9/2)+c*a^(7/2
)*(A*e*x^2+(B*x^2+A)*d))*(e*x^2+d)^(1/2)+3/8*(arctan((((2*(a*(a*e^2+c*d^2))
^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(C*x**4+B*x**2+A)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{(cx^4+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(3/2)/(c*x^4 + a)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = -\frac{Ce^{\frac{3}{2}}\log\left(\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^2\right)}{2\,c^2} - \frac{\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6Bc^2d^2\sqrt{e}-3\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6Cacde^{\frac{3}{2}}+3\left(\sqrt{e}x-\sqrt{ex^2+d}\right)^6Ac^2de^{\frac{3}{2}}-2\left(\sqrt{e}x\right.}{}$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*C*e^(3/2)*log((sqrt(e)*x - sqrt(e*x^2 + d))^2)/c^2 - 1/2*((sqrt(e)*x
- sqrt(e*x^2 + d))^6*B*c^2*d^2*sqrt(e) - 3*(sqrt(e)*x - sqrt(e*x^2 + d))^6
*C*a*c*d*e^(3/2) + 3*(sqrt(e)*x - sqrt(e*x^2 + d))^6*A*c^2*d*e^(3/2) - 2*(
sqrt(e)*x - sqrt(e*x^2 + d))^6*B*a*c*e^(5/2) - 3*(sqrt(e)*x - sqrt(e*x^2 +
 d))^4*B*c^2*d^3*sqrt(e) + 3*(sqrt(e)*x - sqrt(e*x^2 + d))^4*C*a*c*d^2*e^(
3/2) - 3*(sqrt(e)*x - sqrt(e*x^2 + d))^4*A*c^2*d^2*e^(3/2) - 8*(sqrt(e)*x
- sqrt(e*x^2 + d))^4*B*a*c*d*e^(5/2) + 8*(sqrt(e)*x - sqrt(e*x^2 + d))^4*C
*a^2*e^(7/2) - 8*(sqrt(e)*x - sqrt(e*x^2 + d))^4*A*a*c*e^(7/2) + 3*(sqrt(e
)*x - sqrt(e*x^2 + d))^2*B*c^2*d^4*sqrt(e) - (sqrt(e)*x - sqrt(e*x^2 + d))
^2*C*a*c*d^3*e^(3/2) + (sqrt(e)*x - sqrt(e*x^2 + d))^2*A*c^2*d^3*e^(3/2) +
 2*(sqrt(e)*x - sqrt(e*x^2 + d))^2*B*a*c*d^2*e^(5/2) - B*c^2*d^5*sqrt(e) +
 C*a*c*d^4*e^(3/2) - A*c^2*d^4*e^(3/2))/(((sqrt(e)*x - sqrt(e*x^2 + d))^8*
c - 4*(sqrt(e)*x - sqrt(e*x^2 + d))^6*c*d + 6*(sqrt(e)*x - sqrt(e*x^2 + d)
)^4*c*d^2 + 16*(sqrt(e)*x - sqrt(e*x^2 + d))^4*a*e^2 - 4*(sqrt(e)*x - sqrt
(e*x^2 + d))^2*c*d^3 + c*d^4)*a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx=\int \frac{(e\,x^2+d)^{3/2}\,(C\,x^4+B\,x^2+A)}{(c\,x^4+a)^2}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x)
```

output
```
int(((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c^2x^8+2ac\,x^4+a^2}dx\right)ad + \left(\int \frac{\sqrt{e\,x^2+d}\,x^6}{c^2x^8+2ac\,x^4+a^2}dx\right)ce + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2x^8+2ac\,x^4+a^2}dx\right)be + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2x^8+2ac\,x^4+a^2}dx\right)cd + \left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8+2ac\,x^4+a^2}dx\right)ae + \left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8+2ac\,x^4+a^2}dx\right)bd$$

input
```
int((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*d + int((sqrt(d
+ e*x**2)*x**6)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*c*e + int((sqrt(d + e*x
**2)*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*b*e + int((sqrt(d + e*x**2)*
x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*c*d + int((sqrt(d + e*x**2)*x**2)
/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*e + int((sqrt(d + e*x**2)*x**2)/(a**
2 + 2*a*c*x**4 + c**2*x**8),x)*b*d
```

3.21 $\int \frac{\sqrt{d+ex^2}\left(A+Bx^2+Cx^4\right)}{\left(a+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 459

$$\int \frac{\sqrt{d+ex^2}\left(A+Bx^2+Cx^4\right)}{\left(a+cx^4\right)^2}\,dx=\frac{x\left(Ac-aC+Bcx^2\right)\sqrt{d+ex^2}}{4ac\left(a+cx^4\right)}$$
$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(a^{3/2}Ce+\sqrt{a}c(Bd-Ae)+(3Ac+aC)\sqrt{cd^2+ae^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{\sqrt{a}e+\sqrt{cd^2}}\cdots}{\sqrt{a}\left(\sqrt{a}e+\sqrt{cd^2+a}\cdots\right.}\right.}{8\sqrt{2}a^{7/4}c^{3/2}\sqrt{cd^2+ae^2}}$$
$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(a^{3/2}Ce+\sqrt{a}c(Bd-Ae)-(3Ac+aC)\sqrt{cd^2+ae^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{c}\sqrt{-\sqrt{a}e+}\cdots}{\sqrt{a}\left(\sqrt{a}e-\sqrt{c}\cdots\right.}\right.}{8\sqrt{2}a^{7/4}c^{3/2}\sqrt{cd^2+ae^2}}$$

output
```
1/4*x*(B*c*x^2+A*c-C*a)*(e*x^2+d)^(1/2)/a/c/(c*x^4+a)+1/16*(a^(1/2)*e+(a*e
^2+c*d^2)^(1/2))^(1/2)*(a^(3/2)*C*e+a^(1/2)*c*(-A*e+B*d)+(3*A*c+C*a)*(a*e^
2+c*d^2)^(1/2))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1
/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*
x^2))*2^(1/2)/a^(7/4)/c^(3/2)/(a*e^2+c*d^2)^(1/2)+1/16*(-a^(1/2)*e+(a*e^2+
c*d^2)^(1/2))^(1/2)*(a^(3/2)*C*e+a^(1/2)*c*(-A*e+B*d)-(3*A*c+C*a)*(a*e^2+c
*d^2)^(1/2))*arctanh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/
2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x
^2))*2^(1/2)/a^(7/4)/c^(3/2)/(a*e^2+c*d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.61 (sec) , antiderivative size = 1059, normalized size of antiderivative = 2.31

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x]
```

output
```
(4*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c
*d*#1^3 + c*#1^4 & , (c*C*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2
] - #1] + 16*B*c*d*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] +
 16*A*c*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 16*a*C*e
^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 2*c*C*d*Log[d + 2
*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 4*B*c*e*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c*C*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*S
qrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 -
 c*#1^3) & ] + ((2*c*x*(A*c - a*C + B*c*x^2)*Sqrt[d + e*x^2])/(a + c*x^4)
+ Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*
d*#1^3 + c*#1^4 & , (B*c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1] + 2*A*c^2*d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #
1] - 2*a*c*C*d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 6
4*a*B*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 64*a*A
*c*e^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 64*a^2*C*e^3*
Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 2*B*c^2*d^2*Log[d +
2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 8*A*c^2*d*e*Log[d + 2*e*x
^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 8*a*c*C*d*e*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 16*a*B*c*e^2*Log[d + 2*e*x^2 - 2*Sq
rt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + B*c^2*d*Log[d + 2*e*x^2 - 2*Sqrt[e]*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\left(A+Bx^2+Cx^4\right)}{\left(a+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 2257}$$

$$\int \left(\frac{\sqrt{d+ex^2}\left(-aC+Ac+Bcx^2\right)}{c\left(a+cx^4\right)^2}+\frac{C\sqrt{d+ex^2}}{c\left(a+cx^4\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Ac-aC)\int \frac{\sqrt{ex^2+d}}{\left(cx^4+a\right)^2}dx}{c}+B\int \frac{x^2\sqrt{ex^2+d}}{\left(cx^4+a\right)^2}dx-\frac{C\sqrt{\sqrt{c}d-\sqrt{-a}e}\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c^{3/2}}-\frac{C\sqrt{\sqrt{-a}e+\sqrt{c}d}\,\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c^{3/2}}$$

input
```
Int[(Sqrt[d + e*x^2]*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(371) = 742$.

Time = 0.64 (sec) , antiderivative size = 978, normalized size of antiderivative = 2.13

method	result
pseudoelliptic	$-\frac{\left(\ln\left(\frac{\sqrt{e\,x^2+d}\,\sqrt{2\sqrt{a\left(a\,e^2+c\,d^2\right)}+2ae}\,x-x^2\sqrt{a\,e^2+c\,d^2}-\sqrt{a}\left(e\,x^2+d\right)}{x^2}\right)-\ln\left(\frac{\sqrt{a}\left(e\,x^2+d\right)+x^2\sqrt{a\,e^2+c\,d^2}+\sqrt{e\,x^2+d}\,\sqrt{2\sqrt{a\left(a\,e^2\right.}}}{x^2}\right.\right.}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)
/a^(5/2)*(1/4*(ln(((e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)
*x-x^2*(a*e^2+c*d^2)^(1/2)-a^(1/2)*(e*x^2+d))/x^2)-ln((a^(1/2)*(e*x^2+d)+x
^2*(a*e^2+c*d^2)^(1/2)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(
1/2)*x)/x^2))*((3*(A*c+1/3*C*a)*(c*x^4+a)*(a*e^2+c*d^2)^(1/2)+c*(-C*e*x^4+
A*e-B*d)*a^(3/2)-a^(5/2)*C*e+a^(1/2)*c^2*x^4*(A*e-B*d))*(a*(a*e^2+c*d^2))^
(1/2)-(3*(A*c+1/3*C*a)*(c*x^4+a)*a*(a*e^2+c*d^2)^(1/2)+c^2*x^4*(A*e-B*d)*a
^(3/2)+c*(-C*e*x^4+A*e-B*d)*a^(5/2)-C*a^(7/2)*e)*e)*(4*(a*e^2+c*d^2)^(1/2)
*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)
+2*a*e)^(1/2)+c*d*((-2*x*(c*(B*x^2+A)*a^(3/2)-a^(5/2)*C)*(4*(a*e^2+c*d^2)^
(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(e*x^2+d)^(1/2)-3*(ar
ctan(((2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))
/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-
arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*
x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)
))*d*(A*c+1/3*C*a)*(c*x^4+a)*a)*(a*e^2+c*d^2)^(1/2)+(arctan(((2*(a*(a*e^2+
c*d^2))^(1/2)+2*a*e)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)
^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan((2*a^(1/2)*(
e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^
2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))*d*(c^2*x^4*(A...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(1/2)*(C*x**4+B*x**2+A)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{ex^2+d}}{(cx^4+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(e*x^2 + d)/(c*x^4 + a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \int \frac{\sqrt{e\,x^2+d}\,(C\,x^4+B\,x^2+A)}{(c\,x^4+a)^2}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x)
```

output
```
int(((d + e*x^2)^(1/2)*(A + B*x^2 + C*x^4))/(a + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c^2x^8+2ac\,x^4+a^2}dx\right)a + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c^2x^8+2ac\,x^4+a^2}dx\right)c + \left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c^2x^8+2ac\,x^4+a^2}dx\right)b$$

input
```
int((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a + int((sqrt(d +
e*x**2)*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*c + int((sqrt(d + e*x**2)
*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*b
```

3.22 $\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}(a+cx^4)^2} dx$

Optimal result

Integrand size = 33, antiderivative size = 557

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \frac{x\sqrt{d+ex^2}(Acd-aCd+aBe+(Bcd-Ace+aCe)x^2)}{4a\,(cd^2+ae^2)\,(a+cx^4)}$$
$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(d(Bcd^2+Acde-aCde+2aBe^2)-\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(ad(Cd-Be)+A(3cd^2+\ldots}{8\sqrt{2}a^{5/4}\sqrt{c}d\,(cd^2+ae^2)^{3/2}}$$
$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(d(Bcd^2+Acde-aCde+2aBe^2)-\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(ad(Cd-Be)+A(3cd^2\ldots}{8\sqrt{2}a^{5/4}\sqrt{c}d\,(cd^2+ae^2)^{3/2}}$$

output
```
1/4*x*(e*x^2+d)^(1/2)*(A*c*d-C*a*d+B*a*e+(-A*c*e+B*c*d+C*a*e)*x^2)/a/(a*e^
2+c*d^2)/(c*x^4+a)+1/16*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(d*(A*c*d*e+
2*B*a*e^2+B*c*d^2-C*a*d*e)-(e-(a*e^2+c*d^2)^(1/2)/a^(1/2))*(a*d*(-B*e+C*d)
+A*(4*a*e^2+3*c*d^2)))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^(1/2)*e+(a*e^2+c*
d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2
))-c*d*x^2))*2^(1/2)/a^(5/4)/c^(1/2)/d/(a*e^2+c*d^2)^(3/2)+1/16*(-a^(1/2)*
e+(a*e^2+c*d^2)^(1/2))^(1/2)*(d*(A*c*d*e+2*B*a*e^2+B*c*d^2-C*a*d*e)-(e+(a*
e^2+c*d^2)^(1/2)/a^(1/2))*(a*d*(-B*e+C*d)+A*(4*a*e^2+3*c*d^2)))*arctanh(2^
(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(
1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(5/4)/c^
(1/2)/d/(a*e^2+c*d^2)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.67 (sec) , antiderivative size = 1010, normalized size of antiderivative = 1.81

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx=\text{Too large to display}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*(a + c*x^4)^2),x]
```

output
```
(x*Sqrt[d + e*x^2]*(A*c*d - a*C*d + a*B*e + B*c*d*x^2 - A*c*e*x^2 + a*C*e*
x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3
*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + c*#1^4 & , (B*c^2*d^4*Lo
g[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + A*c^2*d^3*e*Log[d + 2*
e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - a*c*C*d^3*e*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 62*a*B*c*d^2*e^2*Log[d + 2*e*x^2 - 2*S
qrt[e]*x*Sqrt[d + e*x^2] - #1] - 64*a^2*B*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*
x*Sqrt[d + e*x^2] - #1] - 2*B*c^2*d^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d
 + e*x^2] - #1]*#1 + 10*A*c^2*d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d +
 e*x^2] - #1]*#1 - 10*a*c*C*d^2*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e
*x^2] - #1]*#1 - 8*a*B*c*d*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^
2] - #1]*#1 + 16*a*A*c*e^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] -
 #1]*#1 - 16*a^2*C*e^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]
*#1 + B*c^2*d^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 +
 A*c^2*d*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - a*c*
C*d*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + 2*a*B*c*e
^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*
d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1^2 - c*#1^3) & ])/(8*a*c*(c*d^2 + a*e^2)) -
(2*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c
*d*#1^3 + c*#1^4 & , (4*B*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{ 2257}$$

$$\int \left(\frac{-aC+Ac+Bcx^2}{c(a+cx^4)^2\sqrt{d+ex^2}}+\frac{C}{c(a+cx^4)\sqrt{d+ex^2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Ac-aC)\int \frac{1}{\sqrt{ex^2+d}(cx^4+a)^2}dx}{c}+B\int \frac{x^2}{\sqrt{ex^2+d}\,(cx^4+a)^2}dx-\frac{C\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c\sqrt{\sqrt{c}d-\sqrt{-a}e}}-\frac{C\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}c\sqrt{\sqrt{-a}e+\sqrt{c}d}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*(a + c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. $2(473) = 946$.

Time = 3.16 (sec) , antiderivative size = 1179, normalized size of antiderivative = 2.12

method	result	size
pseudoelliptic	Expression too large to display	1179
default	Expression too large to display	2750

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2
)/a^(5/2)/(a*e^2+c*d^2)^(3/2)*(((2*(1/4*(3*A*c+C*a)*d^2-1/4*a*B*d*e+A*a*e^
2)*(c*x^4+a)*(a*e^2+c*d^2)^(1/2)+c*(-1/2*B*d^3+e*(C*x^4+A)*d^2-3/2*B*e^2*d
*x^4+2*A*e^3*x^4)*a^(3/2)+(2*A*e^3-3/2*B*d*e^2+C*d^2*e)*a^(5/2)+c^2*d^2*x^
4*a^(1/2)*(A*e-1/2*B*d))*(a*(a*e^2+c*d^2))^(1/2)-e*(2*(1/4*(3*A*c+C*a)*d^2
-1/4*a*B*d*e+A*a*e^2)*(c*x^4+a)*a*(a*e^2+c*d^2)^(1/2)+c*(-1/2*B*d^3+e*(C*x
^4+A)*d^2-3/2*B*e^2*d*x^4+2*A*e^3*x^4)*a^(5/2)+c^2*d^2*x^4*(A*e-1/2*B*d)*a
^(3/2)+2*(A*e^2-3/4*B*d*e+1/2*C*d^2)*e*a^(7/2)))*(ln((a^(1/2)*(e*x^2+d)-(e
*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x+x^2*(a*e^2+c*d^2)^
(1/2))/x^2)-ln((a^(1/2)*(e*x^2+d)+x^2*(a*e^2+c*d^2)^(1/2)+(e*x^2+d)^(1/2)*
(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x^2))*(4*(a*e^2+c*d^2)^(1/2)*a^
(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*
a*e)^(1/2)-4*c*d^2*((x*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^
(1/2)-2*a*e)^(1/2)*((-C*d+(C*x^2+B)*e)*a^(5/2)+c*((B*x^2+A)*d-A*e*x^2)*a^(
3/2))*(e*x^2+d)^(1/2)-2*(arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a*e^2+c*
d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2+c
*d^2))^(1/2)-2*a*e)^(1/2))-arctan(((2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)
*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e^2
+c*d^2))^(1/2)-2*a*e)^(1/2)))*(1/4*(3*A*c+C*a)*d^2-1/4*a*B*d*e+A*a*e^2)*(c
*x^4+a)*a)*(a*e^2+c*d^2)^(1/2)+(arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \int \frac{Cx^4+Bx^2+A}{(cx^4+a)^2\sqrt{ex^2+d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + a)^2*sqrt(e*x^2 + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx = \int \frac{C\,x^4+B\,x^2+A}{\left(c\,x^4+a\right)^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)^2*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)^2*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+cx^4)^2}\,dx\\&=\left(\int \frac{x^4}{\sqrt{e\,x^2+d}\,a^2+2\sqrt{e\,x^2+d}\,ac\,x^4+\sqrt{e\,x^2+d}\,c^2x^8}dx\right)c\\&\quad+\left(\int \frac{x^2}{\sqrt{e\,x^2+d}\,a^2+2\sqrt{e\,x^2+d}\,ac\,x^4+\sqrt{e\,x^2+d}\,c^2x^8}dx\right)b\\&\quad+\left(\int \frac{1}{\sqrt{e\,x^2+d}\,a^2+2\sqrt{e\,x^2+d}\,ac\,x^4+\sqrt{e\,x^2+d}\,c^2x^8}dx\right)a\end{aligned}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^2,x)
```

output

```
int(x**4/(sqrt(d + e*x**2)*a**2 + 2*sqrt(d + e*x**2)*a*c*x**4 + sqrt(d + e
*x**2)*c**2*x**8),x)*c + int(x**2/(sqrt(d + e*x**2)*a**2 + 2*sqrt(d + e*x*
*2)*a*c*x**4 + sqrt(d + e*x**2)*c**2*x**8),x)*b + int(1/(sqrt(d + e*x**2)*
a**2 + 2*sqrt(d + e*x**2)*a*c*x**4 + sqrt(d + e*x**2)*c**2*x**8),x)*a
```

3.23 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 719

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)^2}\,dx = \frac{e(Bcd^3-2Acd^2e+6aCd^2e-5aBde^2+4aAe^3)\,x}{4ad\,(cd^2+ae^2)^2\sqrt{d+ex^2}}$$
$$+\frac{x(Acd-aCd+aBe+(Bcd-Ace+aCe)x^2)}{4a\,(cd^2+ae^2)\sqrt{d+ex^2}\,(a+cx^4)}$$
$$+\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\Big(d(Bcd(cd^2+7ae^2)-2ae(2cCd^2+3Ace^2-aCe^2))-\frac{\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)(3Acd(cd^2+3ae^2)}{\sqrt{}}}{8\sqrt{2}a^{5/4}\sqrt{c}d\,(cd^2+ae^2)^{5/2}}$$
$$+\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\Big(d(Bcd(cd^2+7ae^2)-2ae(2cCd^2+3Ace^2-aCe^2))-\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(3Acd(cd^2}{8\sqrt{2}a^{5/4}\sqrt{c}d\,(cd^2+}$$

output
```
1/4*e*(4*A*a*e^3-2*A*c*d^2*e-5*B*a*d*e^2+B*c*d^3+6*C*a*d^2*e)*x/a/d/(a*e^2
+c*d^2)^2/(e*x^2+d)^(1/2)+1/4*x*(A*c*d-C*a*d+B*a*e+(-A*c*e+B*c*d+C*a*e)*x^
2)/a/(a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(c*x^4+a)+1/16*(a^(1/2)*e+(a*e^2+c*d^2)
^(1/2))^(1/2)*(d*(B*c*d*(7*a*e^2+c*d^2)-2*a*e*(3*A*c*e^2-C*a*e^2+2*C*c*d^2
))-(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))*(3*A*c*d*(3*a*e^2+c*d^2)-a*(a*e^2*(-4*B
*e+5*C*d)-c*d^2*(-2*B*e+C*d)))/a^(1/2))*arctan(2^(1/2)*a^(1/4)*c^(1/2)*(a^
(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(1/2)*e+(
a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(5/4)/c^(1/2)/d/(a*e^2+c*d^2)^(5/2
)+1/16*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*(d*(B*c*d*(7*a*e^2+c*d^2)-2*
a*e*(3*A*c*e^2-C*a*e^2+2*C*c*d^2))-(e+(a*e^2+c*d^2)^(1/2)/a^(1/2))*(3*A*c*
d*(3*a*e^2+c*d^2)-a*(a*e^2*(-4*B*e+5*C*d)-c*d^2*(-2*B*e+C*d))))*arctanh(2^
(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(
1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(5/4)/c^
(1/2)/d/(a*e^2+c*d^2)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.00 (sec) , antiderivative size = 1758, normalized size of antiderivative = 2.45

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + c*x^4)^2),x]
```

output
```
((-4*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4
*c*d*#1^3 + c*#1^4 & , (c^2*C*d^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e
*x^2] - #1] - 17*B*c^2*d^3*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2]
 - #1] + 17*A*c^2*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] -
#1] - 16*a*c*C*d^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]
 - 16*a*B*c*d*e^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 16
*a*A*c*e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 16*a^2*C*
e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 6*c^2*C*d^3*Log[
d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 6*B*c^2*d^2*e*Log[d +
 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 6*A*c^2*d*e^2*Log[d + 2*
e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + c^2*C*d^2*Log[d + 2*e*x^2 -
 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 - B*c^2*d*e*Log[d + 2*e*x^2 - 2*Sq
rt[e]*x*Sqrt[d + e*x^2] - #1]*#1^2 + A*c^2*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]
*x*Sqrt[d + e*x^2] - #1]*#1^2)/(c*d^3 - 3*c*d^2*#1 - 8*a*e^2*#1 + 3*c*d*#1
^2 - c*#1^3) & ])/c + ((2*x*(4*a^2*A*e^4 + B*c^2*d^3*x^2*(d + e*x^2) + a^2
*d*e^2*(5*C*d - 4*B*e + C*e*x^2) - a*A*c*e^2*(d^2 + d*e*x^2 - 4*e^2*x^4) +
 A*c^2*d^2*(d^2 - d*e*x^2 - 2*e^2*x^4) + a*c*d*(B*e*(2*d^2 + d*e*x^2 - 5*e
^2*x^4) + C*d*(-d^2 + d*e*x^2 + 6*e^2*x^4))))/(d*Sqrt[d + e*x^2]*(a + c*x^
4)) + (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 -
 4*c*d*#1^3 + c*#1^4 & , (B*c^3*d^5*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2 (d+ex^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{2257}$$

$$\int \left(\frac{-aC+Ac+Bcx^2}{c(a+cx^4)^2 (d+ex^2)^{3/2}}+\frac{C}{c(a+cx^4)(d+ex^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(Ac-aC)\int \frac{1}{(ex^2+d)^{3/2}(cx^4+a)^2}dx}{c}+B\int \frac{x^2}{(ex^2+d)^{3/2}(cx^4+a)^2}dx-\frac{C(\sqrt{-a}e+\sqrt{c}d)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\sqrt{c}\sqrt{\sqrt{c}d-\sqrt{-a}e}\,(ae^2+cd^2)}-\frac{C(\sqrt{c}d-\sqrt{-a}e)\,\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\sqrt{c}\sqrt{\sqrt{-a}e+\sqrt{c}d}\,(ae^2+cd^2)}+\frac{Ce^2x}{cd\sqrt{d+ex^2}\,(ae^2+cd^2)}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1500 vs. 2(629) = 1258.

Time = 2.46 (sec) , antiderivative size = 1501, normalized size of antiderivative = 2.09

method	result	size
pseudoelliptic	Expression too large to display	1501
default	Expression too large to display	5642

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/32/a^(5/2)*((e*x^2+d)^(1/2)*(ln(((e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2))^(
1/2)+2*a*e)^(1/2)*x-x^2*(a*e^2+c*d^2)^(1/2)-a^(1/2)*(e*x^2+d))/x^2)-ln((a^
(1/2)*(e*x^2+d)+x^2*(a*e^2+c*d^2)^(1/2)+(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2
))^(1/2)+2*a*e)^(1/2)*x)/x^2))*((3*(1/3*(A*c^2+1/3*C*c*a)*d^3-2/9*B*a*c*d^
2*e+a*e^2*(A*c-5/9*C*a)*d+4/9*a^2*B*e^3)*(c*x^4+a)*(a*e^2+c*d^2)^(1/2)+c^2
*d*(-1/3*B*d^3+e*(A+5/3*C*x^4)*d^2-3*B*e^2*d*x^4+5*A*e^3*x^4)*a^(3/2)+5*c*
e*(1/3*C*d^3-3/5*B*e*d^2+e^2*(-7/15*C*x^4+A)*d+4/15*B*e^3*x^4)*a^(5/2)+1/3
*(4*B*e^4-7*C*d*e^3)*a^(7/2)+a^(1/2)*c^3*d^3*x^4*(A*e-1/3*B*d))*(a*(a*e^2+
c*d^2))^(1/2)-e*(3*(1/3*(A*c^2+1/3*C*c*a)*d^3-2/9*B*a*c*d^2*e+a*e^2*(A*c-5
/9*C*a)*d+4/9*a^2*B*e^3)*(c*x^4+a)*a*(a*e^2+c*d^2)^(1/2)+c^3*d^3*x^4*(A*e-
1/3*B*d)*a^(3/2)+c^2*d*(-1/3*B*d^3+e*(A+5/3*C*x^4)*d^2-3*B*e^2*d*x^4+5*A*e
^3*x^4)*a^(5/2)+5*e*(c*(1/3*C*d^3-3/5*B*e*d^2+e^2*(-7/15*C*x^4+A)*d+4/15*B
*e^3*x^4)*a^(7/2)+4/15*(B*e-7/4*C*d)*a^(9/2)*e^2)))*(4*(a*e^2+c*d^2)^(1/2)
*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)
+2*a*e)^(1/2)-8/3*c*d*((-9/2*d*(arctan((2*a^(1/2)*(e*x^2+d)^(1/2)+(2*(a*(a
*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*x)/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(
a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2))-arctan(((2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e
)^(1/2)*x-2*a^(1/2)*(e*x^2+d)^(1/2))/x/(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a
*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)))*(1/3*(A*c^2+1/3*C*c*a)*d^3-2/9*B*a*c*
d^2*e+a*e^2*(A*c-5/9*C*a)*d+4/9*a^2*B*e^3)*(c*x^4+a)*a*(e*x^2+d)^(1/2)+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)^{3/2} (a + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(3/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)^2}\,dx = \int \frac{Cx^4+Bx^2+A}{(cx^4+a)^2(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+a)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + a)^2*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)^2}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\left(c\,x^4+a\right)^2\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)^2*(d + e*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)^2*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^{3/2}\left(a+cx^4\right)^2}\,dx=\left(\int \frac{x^4}{\sqrt{e\,x^2+d}\,a^2d+\sqrt{e\,x^2+d}\,a^2e\,x^2+2\sqrt{e\,x^2+d}\,acd\,x^4+2\sqrt{e\,x^2+d}\,ac}\right.$$
$$+\left(\int \frac{x^2}{\sqrt{e\,x^2+d}\,a^2d+\sqrt{e\,x^2+d}\,a^2e\,x^2+2\sqrt{e\,x^2+d}\,acd\,x^4+2\sqrt{e\,x^2+d}\,ace\,x^6+\sqrt{e\,x^2+d}\,c^2d\,x^8+\sqrt{e}}\right.$$
$$+\left(\int \frac{1}{\sqrt{e\,x^2+d}\,a^2d+\sqrt{e\,x^2+d}\,a^2e\,x^2+2\sqrt{e\,x^2+d}\,acd\,x^4+2\sqrt{e\,x^2+d}\,ace\,x^6+\sqrt{e\,x^2+d}\,c^2d\,x^8+\sqrt{e}}\right.$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+a)^2,x)
```

output
```
int(x**4/(sqrt(d + e*x**2)*a**2*d + sqrt(d + e*x**2)*a**2*e*x**2 + 2*sqrt(
d + e*x**2)*a*c*d*x**4 + 2*sqrt(d + e*x**2)*a*c*e*x**6 + sqrt(d + e*x**2)*
c**2*d*x**8 + sqrt(d + e*x**2)*c**2*e*x**10),x)*c + int(x**2/(sqrt(d + e*x
**2)*a**2*d + sqrt(d + e*x**2)*a**2*e*x**2 + 2*sqrt(d + e*x**2)*a*c*d*x**4
 + 2*sqrt(d + e*x**2)*a*c*e*x**6 + sqrt(d + e*x**2)*c**2*d*x**8 + sqrt(d +
 e*x**2)*c**2*e*x**10),x)*b + int(1/(sqrt(d + e*x**2)*a**2*d + sqrt(d + e*
x**2)*a**2*e*x**2 + 2*sqrt(d + e*x**2)*a*c*d*x**4 + 2*sqrt(d + e*x**2)*a*c
*e*x**6 + sqrt(d + e*x**2)*c**2*d*x**8 + sqrt(d + e*x**2)*c**2*e*x**10),x)
*a
```

3.24 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 928

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)^2}\,dx = \frac{e(3Bcd^3-6Acd^2e+10aCd^2e-7aBde^2+4aAe^3)\,x}{12ad\,(cd^2+ae^2)^2\,(d+ex^2)^{3/2}}$$
$$+\frac{e(aCd^2e(41cd^2-19ae^2)+B(3c^2d^5-53acd^3e^2+4a^2de^4)-A(9c^2d^4e-59acd^2e^3-8a^2e^5))\,x}{12ad^2\,(cd^2+ae^2)^3\,\sqrt{d+ex^2}}$$
$$+\frac{x(Acd-aCd+aBe+(Bcd-Ace+aCe)x^2)}{4a\,(cd^2+ae^2)\,(d+ex^2)^{3/2}\,(a+cx^4)}$$
$$-\frac{\sqrt{\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(\left(e-\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(Ac(3c^2d^4+15acd^2e^2-8a^2e^4)+a(4a^2Ce^4-acde^2(15Cd-17B\cdots}{\cdots}$$
$$-\frac{\sqrt{-\sqrt{a}e+\sqrt{cd^2+ae^2}}\left(\left(e+\frac{\sqrt{cd^2+ae^2}}{\sqrt{a}}\right)(Ac(3c^2d^4+15acd^2e^2-8a^2e^4)+a(4a^2Ce^4-acde^2(15Cd-17\cdots}{\cdots}$$

output
```
1/12*e*(4*A*a*e^3-6*A*c*d^2*e-7*B*a*d*e^2+3*B*c*d^3+10*C*a*d^2*e)*x/a/d/(a
*e^2+c*d^2)^2/(e*x^2+d)^(3/2)+1/12*e*(a*C*d^2*e*(-19*a*e^2+41*c*d^2)+B*(4*
a^2*d*e^4-53*a*c*d^3*e^2+3*c^2*d^5)-A*(-8*a^2*e^5-59*a*c*d^2*e^3+9*c^2*d^4
*e))*x/a/d^2/(a*e^2+c*d^2)^3/(e*x^2+d)^(1/2)+1/4*x*(A*c*d-C*a*d+B*a*e+(-A*
c*e+B*c*d+C*a*e)*x^2)/a/(a*e^2+c*d^2)/(e*x^2+d)^(3/2)/(c*x^4+a)-1/16*(a^(1
/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*((e-(a*e^2+c*d^2)^(1/2)/a^(1/2))*(A*c*(-8
*a^2*e^4+15*a*c*d^2*e^2+3*c^2*d^4)+a*(4*a^2*C*e^4-a*c*d*e^2*(-17*B*e+15*C*
d)+c^2*d^3*(-3*B*e+C*d)))-c*d*(B*(-6*a^2*e^4+15*a*c*d^2*e^2+c^2*d^4)-d*e*(
a*C*(-13*a*e^2+7*c*d^2)+A*c*(21*a*e^2+c*d^2))))*arctan(2^(1/2)*a^(1/4)*c^(
1/2)*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2+d)^(1/2)/(a^(1/2)*(a^(
1/2)*e+(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(5/4)/c^(1/2)/d/(a*e^2+c*d
^2)^(7/2)-1/16*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*((e+(a*e^2+c*d^2)^(1
/2)/a^(1/2))*(A*c*(-8*a^2*e^4+15*a*c*d^2*e^2+3*c^2*d^4)+a*(4*a^2*C*e^4-a*c
*d*e^2*(-17*B*e+15*C*d)+c^2*d^3*(-3*B*e+C*d)))-c*d*(B*(-6*a^2*e^4+15*a*c*d
^2*e^2+c^2*d^4)-d*e*(a*C*(-13*a*e^2+7*c*d^2)+A*c*(21*a*e^2+c*d^2))))*arcta
nh(2^(1/2)*a^(1/4)*c^(1/2)*(-a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^(1/2)*x*(e*x^2
+d)^(1/2)/(a^(1/2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))-c*d*x^2))*2^(1/2)/a^(5/
4)/c^(1/2)/d/(a*e^2+c*d^2)^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 4.90 (sec) , antiderivative size = 2576, normalized size of antiderivative = 2.78

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + c*x^4)^2),x]
```

output
```
((2*x*(A*(3*c^3*d^4*(d - 3*e*x^2)*(d + e*x^2)^2 + 4*a^3*e^6*(3*d + 2*e*x^2
) + 4*a^2*c*e^4*(15*d^3 + 14*d^2*e*x^2 + 3*d*e^2*x^4 + 2*e^3*x^6) + a*c^2*
d^2*e^2*(-9*d^3 - 15*d^2*e*x^2 + 57*d*e^2*x^4 + 59*e^3*x^6)) + d*(3*B*c^3*
d^4*x^2*(d + e*x^2)^2 + 4*a^3*e^4*(B*e^2*x^2 - C*d*(3*d + 4*e*x^2)) + a^2*
c*e^2*(C*d*(45*d^3 + 47*d^2*e*x^2 - 9*d*e^2*x^4 - 19*e^3*x^6) + B*e*(-51*d
^3 - 50*d^2*e*x^2 - 3*d*e^2*x^4 + 4*e^3*x^6)) + a*c^2*d^2*(B*e*(9*d^3 + 9*
d^2*e*x^2 - 57*d*e^2*x^4 - 53*e^3*x^6) + C*d*(-3*d^3 + 3*d^2*e*x^2 + 51*d*
e^2*x^4 + 41*e^3*x^6)))))/(a*d^2*(d + e*x^2)^(3/2)*(a + c*x^4)) - 12*e^(3/
2)*RootSum[c*d^4 - 4*c*d^3*#1 + 6*c*d^2*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3
+ c*#1^4 & , (2*c^2*C*d^5*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] -
#1] - 19*B*c^2*d^4*e*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] +
 36*A*c^2*d^3*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 34
*a*c*C*d^3*e^2*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + a*B*c
*d^2*e^3*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 32*a*A*c*d*
e^4*Log[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 32*a^2*C*d*e^4*L
og[d + 2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] + 16*a^2*B*e^5*Log[d +
2*e*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1] - 8*c^2*C*d^4*Log[d + 2*e*x^2
- 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 14*B*c^2*d^3*e*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 - 20*A*c^2*d^2*e^2*Log[d + 2*e*x^2 -
2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1 + 16*a*c*C*d^2*e^2*Log[d + 2*e*x^2...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2 (d+ex^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 2257}$$

$$\int \left(\frac{-aC+Ac+Bcx^2}{c(a+cx^4)^2 (d+ex^2)^{5/2}}+\frac{C}{c(a+cx^4)(d+ex^2)^{5/2}}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Ac-aC)\int \frac{1}{(ex^2+d)^{5/2}(cx^4+a)^2}dx}{c}+B\int \frac{x^2}{(ex^2+d)^{5/2}\left(cx^4+a\right)^2}dx-\frac{C\left(\sqrt{-a}e+\sqrt{c}d\right)\arctan\left(\frac{x\sqrt{\sqrt{c}d-\sqrt{-a}e}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\left(\sqrt{c}d-\sqrt{-a}e\right)^{3/2}(ae^2+cd^2)}-\frac{C\left(\sqrt{c}d-\sqrt{-a}e\right)\text{arctanh}\left(\frac{x\sqrt{\sqrt{-a}e+\sqrt{c}d}}{\sqrt[4]{-a}\sqrt{d+ex^2}}\right)}{2(-a)^{3/4}\left(\sqrt{-a}e+\sqrt{c}d\right)^{3/2}(ae^2+cd^2)}+\frac{2Ce^2x}{3cd^2\sqrt{d+ex^2}\,(ae^2+cd^2)}+\frac{Ce^2x}{3cd\,(d+ex^2)^{3/2}\,(ae^2+cd^2)}-\frac{Cex\left(\sqrt{c}d-\sqrt{-a}e\right)}{2\sqrt{c}d\sqrt{d+ex^2}\left(\sqrt{-a}\sqrt{c}d-ae\right)(ae^2+cd^2)}+\frac{Cex\left(\sqrt{-a}e+\sqrt{c}d\right)}{2\sqrt{c}d\sqrt{d+ex^2}\left(\sqrt{-a}\sqrt{c}d+ae\right)(ae^2+cd^2)}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2296 vs. 2(836) = 1672.

Time = 20.85 (sec) , antiderivative size = 2297, normalized size of antiderivative = 2.48

method	result	size
pseudoelliptic	Expression too large to display	2297
default	Expression too large to display	9630

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(e*x^2+d)^(3/2)/a^(5/2)*(-1/4*(4*(a*e^2+c*d^2)^(1/2)*a^(1/2)-2*(a*(a*e
^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(2*(a*(a*e^2+c*d^2))^(1/2)+2*a*e)^(1/2)*(e*x
^2+d)^(3/2)*((-2*(c*x^4+a)*(a^2*(A*c-1/2*C*a)*e^4-17/8*B*a^2*c*d*e^3-15/8*
a*c*d^2*(A*c-C*a)*e^2+3/8*B*a*c^2*d^3*e-3/8*c^2*d^4*(A*c+1/3*C*a))*(a*e^2+
c*d^2)^(1/2)+c^3*(9*A*e^3*x^4-9/2*B*e^2*d*x^4+d^2*(2*C*x^4+A)*e-1/4*B*d^3)
*d^2*a^(3/2)+9*c^2*(-2/9*A*e^4*x^4+23/36*B*d*e^3*x^4+d^2*(-7/9*C*x^4+A)*e^
2-1/2*B*d^3*e+2/9*C*d^4)*e*a^(5/2)-2*c*e^3*((-1/2*C*x^4+A)*e^2-23/8*B*d*e+
7/2*C*d^2)*a^(7/2)+a^(9/2)*C*e^5+a^(1/2)*c^4*d^4*x^4*(-1/4*B*d+A*e))*(a*(a
*e^2+c*d^2))^(1/2)-e*(-2*(c*x^4+a)*(a^2*(A*c-1/2*C*a)*e^4-17/8*B*a^2*c*d*e
^3-15/8*a*c*d^2*(A*c-C*a)*e^2+3/8*B*a*c^2*d^3*e-3/8*c^2*d^4*(A*c+1/3*C*a))
*a*(a*e^2+c*d^2)^(1/2)+c^4*d^4*x^4*(-1/4*B*d+A*e)*a^(3/2)+c^3*(9*A*e^3*x^4
-9/2*B*e^2*d*x^4+d^2*(2*C*x^4+A)*e-1/4*B*d^3)*d^2*a^(5/2)+9*e*(c^2*(-2/9*A
*e^4*x^4+23/36*B*d*e^3*x^4+d^2*(-7/9*C*x^4+A)*e^2-1/2*B*d^3*e+2/9*C*d^4)*a
^(7/2)-2/9*(c*((-1/2*C*x^4+A)*e^2-23/8*B*d*e+7/2*C*d^2)*a^(9/2)-1/2*a^(11/
2)*C*e^2)*e^2)))*ln((a^(1/2)*(e*x^2+d)-(e*x^2+d)^(1/2)*(2*(a*(a*e^2+c*d^2)
)^(1/2)+2*a*e)^(1/2)*x+x^2*(a*e^2+c*d^2)^(1/2))/x^2)+1/4*(4*(a*e^2+c*d^2)^
(1/2)*a^(1/2)-2*(a*(a*e^2+c*d^2))^(1/2)-2*a*e)^(1/2)*(2*(a*(a*e^2+c*d^2))^
(1/2)+2*a*e)^(1/2)*(e*x^2+d)^(3/2)*((-2*(c*x^4+a)*(a^2*(A*c-1/2*C*a)*e^4-1
7/8*B*a^2*c*d*e^3-15/8*a*c*d^2*(A*c-C*a)*e^2+3/8*B*a*c^2*d^3*e-3/8*c^2*d^4
*(A*c+1/3*C*a))*(a*e^2+c*d^2)^(1/2)+c^3*(9*A*e^3*x^4-9/2*B*e^2*d*x^4+d^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^{5/2}\left(a+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(5/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)^2}\,dx = \int \frac{Cx^4+Bx^2+A}{(cx^4+a)^2(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + a)^2*(e*x^2 + d)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)^2}\,dx=\int \frac{C\,x^4+B\,x^2+A}{(c\,x^4+a)^2\,(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)^2*(d + e*x^2)^(5/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)^2*(d + e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+cx^4)^2}\,dx=\int \frac{C\,x^4+B\,x^2+A}{(e\,x^2+d)^{\frac{5}{2}}\,(c\,x^4+a)^2}dx$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a)^2,x)
```

output
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+a)^2,x)
```

3.25 $\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 466

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx = \frac{\left(\sqrt{c}(Bd+Ae)-\frac{Acd-aBe}{\sqrt{-a}}\right)x\left(1+\frac{ex^2}{d}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{2}{3},1,\frac{3}{2},-\frac{ex^2}{d},-\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)}{2\sqrt{-a}c\,(d+ex^2)^{2/3}}$$
$$-\frac{\left(\sqrt{c}(Bd+Ae)+\frac{Acd-aBe}{\sqrt{-a}}\right)x\left(1+\frac{ex^2}{d}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{2}{3},1,\frac{3}{2},-\frac{ex^2}{d},\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)}{2\sqrt{-a}c\,(d+ex^2)^{2/3}}$$
$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}B\left(\sqrt[3]{d}-\sqrt[3]{d+ex^2}\right)\sqrt{\frac{d^{2/3}+\sqrt[3]{d}\sqrt[3]{d+ex^2}+(d+ex^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{d}-\sqrt[3]{d+ex^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{d}-\sqrt[3]{d}}{\left(1-\sqrt{3}\right)\sqrt[3]{d}-\sqrt[3]{d}}\right.\right.}{cx\sqrt{-\frac{\sqrt[3]{d}\left(\sqrt[3]{d}-\sqrt[3]{d+ex^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{d}-\sqrt[3]{d+ex^2}\right)^2}}}$$

output
```
1/2*(c^(1/2)*(A*e+B*d)-(A*c*d-B*a*e)/(-a)^(1/2))*x*(1+e*x^2/d)^(2/3)*Appel
lF1(1/2,1,2/3,3/2,-c^(1/2)*x^2/(-a)^(1/2),-e*x^2/d)/(-a)^(1/2)/c/(e*x^2+d)
^(2/3)-1/2*(c^(1/2)*(A*e+B*d)+(A*c*d-B*a*e)/(-a)^(1/2))*x*(1+e*x^2/d)^(2/3
)*AppellF1(1/2,1,2/3,3/2,c^(1/2)*x^2/(-a)^(1/2),-e*x^2/d)/(-a)^(1/2)/c/(e*
x^2+d)^(2/3)-3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*B*(d^(1/3)-(e*x^2+d)^(1/3))
*((d^(2/3)+d^(1/3)*(e*x^2+d)^(1/3)+(e*x^2+d)^(2/3))/((1-3^(1/2))*d^(1/3)-(
e*x^2+d)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*d^(1/3)-(e*x^2+d)^(1/3))/(
(1-3^(1/2))*d^(1/3)-(e*x^2+d)^(1/3)),2*I-I*3^(1/2))/c/x/(-d^(1/3)*(d^(1/3)
-(e*x^2+d)^(1/3))/((1-3^(1/2))*d^(1/3)-(e*x^2+d)^(1/3))^2)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx=\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(1/3))/(a + c*x^4),x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(1/3))/(a + c*x^4), x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx$$

$$\Big\downarrow \text{2257}$$

$$\int \left(\frac{\sqrt[3]{d+ex^2}\left(\sqrt{-a}B-A\sqrt{c}\right)}{2\sqrt{-a}\sqrt{c}\left(\sqrt{-a}+\sqrt{c}x^2\right)}-\frac{\sqrt[3]{d+ex^2}\left(\sqrt{-a}B+A\sqrt{c}\right)}{2\sqrt{-a}\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\sqrt[3]{d+ex^2}\left(A-\frac{\sqrt{-a}B}{\sqrt{c}}\right)\text{AppellF1}\left(\frac{1}{2},1,-\frac{1}{3},\frac{3}{2},-\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a\sqrt[3]{\frac{ex^2}{d}+1}}+\frac{x\sqrt[3]{d+ex^2}\left(\frac{\sqrt{-a}B}{\sqrt{c}}+A\right)\text{AppellF1}\left(\frac{1}{2},1,-\frac{1}{3},\frac{3}{2},\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a\sqrt[3]{\frac{ex^2}{d}+1}}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(1/3))/(a + c*x^4),x]
```

output
```
((A - (Sqrt[-a]*B)/Sqrt[c])*x*(d + e*x^2)^(1/3)*AppellF1[1/2, 1, -1/3, 3/2
, -((Sqrt[c]*x^2)/Sqrt[-a]), -((e*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^(1/3)) +
((A + (Sqrt[-a]*B)/Sqrt[c])*x*(d + e*x^2)^(1/3)*AppellF1[1/2, 1, -1/3, 3/2
, (Sqrt[c]*x^2)/Sqrt[-a], -((e*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{1}{3}}}{c\,x^4+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+a),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx=\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/3)/(c*x**4+a),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(1/3)/(a + c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^{\frac{1}{3}}}{cx^4+a}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(1/3)/(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{1}{3}}}{cx^4+a}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+a),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(1/3)/(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{1/3}}{c\,x^4+a}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/3))/(a + c*x^4),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/3))/(a + c*x^4), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+cx^4}\,dx = \left(\int \frac{(e\,x^2+d)^{\frac{1}{3}}}{c\,x^4+a}dx\right)a+\left(\int \frac{(e\,x^2+d)^{\frac{1}{3}}\,x^2}{c\,x^4+a}dx\right)b$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+a),x)
```

output
```
int((d + e*x**2)**(1/3)/(a + c*x**4),x)*a + int(((d + e*x**2)**(1/3)*x**2)
/(a + c*x**4),x)*b
```

3.26 $\int (d+ex^2)^3 \sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 577

$$\int (d+ex^2)^3 \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$= \frac{x(65(11Acd(7cd^2+3ae^2)+a(5ae^2(3Cd+Be)+11cd^2(Cd+3Be)))+77(39Bcd(cd^2+ae^2)+e(13Ac}{15015c^2}$$

$$- \frac{(5ae^2(3Cd+Be)+11cd(Cd^2+3e(Bd+Ae)))\,x(a-cx^4)^{3/2}}{77c^2}$$

$$- \frac{e(7aCe^2+13c(3Cd^2+e(3Bd+Ae)))\,x^3(a-cx^4)^{3/2}}{117c^2}$$

$$- \frac{e^2(3Cd+Be)x^5(a-cx^4)^{3/2}}{11c} - \frac{Ce^3x^7(a-cx^4)^{3/2}}{13c}$$

$$+ \frac{2a^{7/4}(39Bcd(cd^2+ae^2)+e(13Ac(9cd^2+ae^2)+aC(39cd^2+7ae^2)))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-\right.}{195c^{11/4}\sqrt{a-cx^4}}$$

$$+ \frac{2a^{5/4}\left(65\sqrt{c}(11Acd(7cd^2+3ae^2)+a(5ae^2(3Cd+Be)+11cd^2(Cd+3Be)))-77\sqrt{a}(39Bcd(cd^2+ae\right.}{15015c^{11/4}\sqrt{a-}}$$

output
```
1/15015*x*(715*A*c*d*(3*a*e^2+7*c*d^2)+65*a*(5*a*e^2*(B*e+3*C*d)+11*c*d^2*
(3*B*e+C*d))+77*(39*B*c*d*(a*e^2+c*d^2)+e*(13*A*c*(a*e^2+9*c*d^2)+a*C*(7*a
*e^2+39*c*d^2)))*x^2)*(-c*x^4+a)^(1/2)/c^2-1/77*(5*a*e^2*(B*e+3*C*d)+11*c*
d*(C*d^2+3*e*(A*e+B*d)))*x*(-c*x^4+a)^(3/2)/c^2-1/117*e*(7*C*a*e^2+13*c*(3
*C*d^2+e*(A*e+3*B*d)))*x^3*(-c*x^4+a)^(3/2)/c^2-1/11*e^2*(B*e+3*C*d)*x^5*(
-c*x^4+a)^(3/2)/c-1/13*C*e^3*x^7*(-c*x^4+a)^(3/2)/c+2/195*a^(7/4)*(39*B*c*
d*(a*e^2+c*d^2)+e*(13*A*c*(a*e^2+9*c*d^2)+a*C*(7*a*e^2+39*c*d^2)))*(1-c*x^
4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(11/4)/(-c*x^4+a)^(1/2)+2/1501
5*a^(5/4)*(65*c^(1/2)*(11*A*c*d*(3*a*e^2+7*c*d^2)+a*(5*a*e^2*(B*e+3*C*d)+1
1*c*d^2*(3*B*e+C*d)))-77*a^(1/2)*(39*B*c*d*(a*e^2+c*d^2)+e*(13*A*c*(a*e^2+
9*c*d^2)+a*C*(7*a*e^2+39*c*d^2))))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a
^(1/4),I)/c^(11/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.56 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.80

$$\int (d+ex^2)^3\sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx$$

$$= \frac{x\sqrt{a-cx^4}\Big(-1287cd(Cd^2+3e(Bd+Ae))\,(a-cx^4)\sqrt{1-\frac{cx^4}{a}}-1001ce(3Cd^2+e(3Bd+Ae))\,x^2(a-}{}$$

input
```
Integrate[(d + e*x^2)^3*Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(x*Sqrt[a - c*x^4]*(-1287*c*d*(C*d^2 + 3*e*(B*d + A*e))*(a - c*x^4)*Sqrt[1
 - (c*x^4)/a] - 1001*c*e*(3*C*d^2 + e*(3*B*d + A*e))*x^2*(a - c*x^4)*Sqrt[
1 - (c*x^4)/a] - 819*c*e^2*(3*C*d + B*e)*x^4*(a - c*x^4)*Sqrt[1 - (c*x^4)/
a] - 693*c*C*e^3*x^6*(a - c*x^4)*Sqrt[1 - (c*x^4)/a] + 9009*A*c^2*d^3*Hype
rgeometric2F1[-1/2, 1/4, 5/4, (c*x^4)/a] + 1287*a*c*d*(C*d^2 + 3*e*(B*d +
A*e))*Hypergeometric2F1[-1/2, 1/4, 5/4, (c*x^4)/a] - 585*a*e^2*(3*C*d + B*
e)*((a - c*x^4)*Sqrt[1 - (c*x^4)/a] - a*Hypergeometric2F1[-1/2, 1/4, 5/4,
(c*x^4)/a]) + 3003*c^2*d^2*(B*d + 3*A*e)*x^2*Hypergeometric2F1[-1/2, 3/4,
7/4, (c*x^4)/a] + 1001*a*c*e*(3*C*d^2 + e*(3*B*d + A*e))*x^2*Hypergeometri
c2F1[-1/2, 3/4, 7/4, (c*x^4)/a] - 539*a*C*e^3*x^2*((a - c*x^4)*Sqrt[1 - (c
*x^4)/a] - a*Hypergeometric2F1[-1/2, 3/4, 7/4, (c*x^4)/a])))/(9009*c^2*Sqr
t[1 - (c*x^4)/a])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1401 vs. $2(577) = 1154$.

Time = 1.84 (sec) , antiderivative size = 1401, normalized size of antiderivative = 2.43, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^4}\left(d+ex^2\right)^3\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \text{ 2259}$$

$$\int \left(\frac{x^8\left(ae^2(Be+3Cd)-c\left(3de(Ae+Bd)+Cd^3\right)\right)}{\sqrt{a-cx^4}}+\frac{dx^4\left(ad(3Be+Cd)-A\left(cd^2-3ae^2\right)\right)}{\sqrt{a-cx^4}}+\frac{ex^{10}\left(aCe^2-ce\right.}{\sqrt{}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{13}Ce^3\sqrt{a-cx^4}x^{11}+\frac{1}{11}e^2(3Cd+Be)\sqrt{a-cx^4}x^9+\frac{11aCe^3\sqrt{a-cx^4}x^7}{117c}-\frac{e\left(aCe^2-c\left(3Cd^2+e(3Bd+Ae)\right)\right)\sqrt{a-cx^4}x^7}{9c}+\frac{9ae^2(3Cd+Be)\sqrt{a-cx^4}x^5}{77c}-\frac{\left(ae^2(3Cd+Be)-c\left(Cd^3+3e(Bd+Ae)d\right)\right)\sqrt{a-cx^4}x^5}{7c}+\frac{77a^2Ce^3\sqrt{a-cx^4}x^3}{585c^2}+\frac{\left(Bcd^3+3Aced^2-3aCed^2-3aBe^2d-aAe^3\right)\sqrt{a-cx^4}x^3}{5c}-\frac{7ae\left(aCe^2-c\left(3Cd^2+e(3Bd+Ae)\right)\right)\sqrt{a-cx^4}x^3}{45c^2}+\frac{15a^2e^2(3Cd+Be)\sqrt{a-cx^4}x}{77c^2}-\frac{d\left(ad(Cd+3Be)-A\left(cd^2-3ae^2\right)\right)\sqrt{a-cx^4}x}{3c}-\frac{5a\left(ae^2(3Cd+Be)-c\left(Cd^3+3e(Bd+Ae)d\right)\right)\sqrt{a-cx^4}x}{21c^2}-\frac{77a^{15/4}Ce^3\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{195c^{11/4}\sqrt{a-cx^4}}+\frac{a^{7/4}d^2(Bd+3Ae)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{3a^{7/4}\left(Bcd^3+3Aced^2-3aCed^2-3aBe^2d-aAe^3\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{7a^{11/4}e\left(aCe^2-c\left(3Cd^2+e(3Bd+Ae)\right)\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15c^{11/4}\sqrt{a-cx^4}}+\frac{a^{5/4}Ad^3\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}+\frac{77a^{15/4}Ce^3\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{195c^{11/4}\sqrt{a-cx^4}}-\frac{a^{7/4}d^2(Bd+3Ae)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{15a^{13/4}e^2(3Cd+Be)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{77c^{9/4}\sqrt{a-cx^4}}+\frac{3a^{7/4}\left(Bcd^3+3Aced^2-3aCed^2-3aBe^2d-aAe^3\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{a^{5/4}d\left(ad(Cd+3Be)-A\left(cd^2-3ae^2\right)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}}+\frac{5a^{9/4}\left(ae^2(3Cd+Be)-c\left(Cd^3+3e(Bd+Ae)d\right)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{21c^{9/4}\sqrt{a-cx^4}}-\frac{7a^{11/4}e\left(aCe^2-c\left(3Cd^2+e(3Bd+Ae)\right)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{15c^{11/4}\sqrt{a-cx^4}}$$

input
```
Int[(d + e*x^2)^3*Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(15*a^2*e^2*(3*C*d + B*e)*x*Sqrt[a - c*x^4])/(77*c^2) - (d*(a*d*(C*d + 3*B
*e) - A*(c*d^2 - 3*a*e^2))*x*Sqrt[a - c*x^4])/(3*c) - (5*a*(a*e^2*(3*C*d +
 B*e) - c*(C*d^3 + 3*d*e*(B*d + A*e)))*x*Sqrt[a - c*x^4])/(21*c^2) + (77*a
^2*C*e^3*x^3*Sqrt[a - c*x^4])/(585*c^2) + ((B*c*d^3 + 3*A*c*d^2*e - 3*a*C*
d^2*e - 3*a*B*d*e^2 - a*A*e^3)*x^3*Sqrt[a - c*x^4])/(5*c) - (7*a*e*(a*C*e^
2 - c*(3*C*d^2 + e*(3*B*d + A*e)))*x^3*Sqrt[a - c*x^4])/(45*c^2) + (9*a*e^
2*(3*C*d + B*e)*x^5*Sqrt[a - c*x^4])/(77*c) - ((a*e^2*(3*C*d + B*e) - c*(C
*d^3 + 3*d*e*(B*d + A*e)))*x^5*Sqrt[a - c*x^4])/(7*c) + (11*a*C*e^3*x^7*Sq
rt[a - c*x^4])/(117*c) - (e*(a*C*e^2 - c*(3*C*d^2 + e*(3*B*d + A*e)))*x^7*
Sqrt[a - c*x^4])/(9*c) + (e^2*(3*C*d + B*e)*x^9*Sqrt[a - c*x^4])/11 + (C*e
^3*x^11*Sqrt[a - c*x^4])/13 - (77*a^(15/4)*C*e^3*Sqrt[1 - (c*x^4)/a]*Ellip
ticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(195*c^(11/4)*Sqrt[a - c*x^4]) + (a
^(7/4)*d^2*(B*d + 3*A*e)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/
a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) - (3*a^(7/4)*(B*c*d^3 + 3*A*c*d^2
*e - 3*a*C*d^2*e - 3*a*B*d*e^2 - a*A*e^3)*Sqrt[1 - (c*x^4)/a]*EllipticE[Ar
cSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (7*a^(11/4)*
e*(a*C*e^2 - c*(3*C*d^2 + e*(3*B*d + A*e)))*Sqrt[1 - (c*x^4)/a]*EllipticE[
ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(15*c^(11/4)*Sqrt[a - c*x^4]) + (a^(5/4)
*A*d^3*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^
(1/4)*Sqrt[a - c*x^4]) + (77*a^(15/4)*C*e^3*Sqrt[1 - (c*x^4)/a]*Ellipti...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.29

method	result
elliptic	$\frac{C e^3 x^{11}\sqrt{-c x^4+a}}{13}-\frac{(-B e^3 c-3d e^2 c C) x^9\sqrt{-c x^4+a}}{11c}-\frac{(-A c e^3-3B c d e^2+\frac{2}{13}C e^3 a-3e C c d^2) x^7\sqrt{-c x^4+a}}{9c}-\frac{\left(-3A c d e^2-\right.}{}$
default	$A d^3\left(\frac{x\sqrt{-c x^4+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\ \text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-c x^4+a}}\right)+d^2(3Ae+Bd)\left(\frac{x^3\sqrt{-c x^4+a}}{5}-\frac{2a^{\frac{3}{2}}\sqrt{1-}}{}\right.$
risch	$-\frac{x(-3465C e^3 x^{10} c^2-4095B c^2 e^3 x^8-12285C c^2 d e^2 x^8-5005A c^2 e^3 x^6-15015B c^2 d e^2 x^6+770C e^3 a x^6 c-15015C c^2 d^2 e x^6-1930}{}$

input
```
int((e*x^2+d)^3*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*C*e^3*x^11*(-c*x^4+a)^(1/2)-1/11*(-B*c*e^3-3*C*c*d*e^2)/c*x^9*(-c*x^4
+a)^(1/2)-1/9*(-A*c*e^3-3*B*c*d*e^2+2/13*C*e^3*a-3*e*C*c*d^2)/c*x^7*(-c*x^
4+a)^(1/2)-1/7*(-3*A*c*d*e^2+B*a*e^3-3*B*c*d^2*e+3*a*C*d*e^2-C*c*d^3+9/11*
(-B*c*e^3-3*C*c*d*e^2)/c*a)/c*x^5*(-c*x^4+a)^(1/2)-1/5*(A*a*e^3-3*A*c*d^2*
e+3*B*a*d*e^2-B*c*d^3+3*C*a*d^2*e+7/9*(-A*c*e^3-3*B*c*d*e^2+2/13*C*e^3*a-3
*e*C*c*d^2)/c*a)/c*x^3*(-c*x^4+a)^(1/2)-1/3*(3*a*d*A*e^2-A*d^3*c+3*B*a*d^2
*e+C*a*d^3+5/7*(-3*A*c*d*e^2+B*a*e^3-3*B*c*d^2*e+3*a*C*d*e^2-C*c*d^3+9/11*
(-B*c*e^3-3*C*c*d*e^2)/c*a)/c*a)/c*x*(-c*x^4+a)^(1/2)+(A*a*d^3+1/3*(3*a*d*
A*e^2-A*d^3*c+3*B*a*d^2*e+C*a*d^3+5/7*(-3*A*c*d*e^2+B*a*e^3-3*B*c*d^2*e+3*
a*C*d*e^2-C*c*d^3+9/11*(-B*c*e^3-3*C*c*d*e^2)/c*a)/c*a)/c*a)/(c^(1/2)/a^(1
/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c
*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-(3*A*a*d^2*e+B*a*d^3+
3/5*(A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2-B*c*d^3+3*C*a*d^2*e+7/9*(-A*c*e^3-3*B
*c*d*e^2+2/13*C*e^3*a-3*e*C*c*d^2)/c*a)/c*a)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/
2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^
(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)
/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.99

$$\int \left(d+ex^2\right)^3 \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx =$$

$$-\frac{462\,(39\,Bac^2d^3+39\,Ba^2cde^2+39\,(Ca^2c+3\,Aac^2)d^2e+(7\,Ca^3+13\,Aa^2c)e^3)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\right.}{}$$

input
```
integrate((e*x^2+d)^3*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="frica
s")
```

output
```
-1/45045*(462*(39*B*a*c^2*d^3 + 39*B*a^2*c*d*e^2 + 39*(C*a^2*c + 3*A*a*c^2
)*d^2*e + (7*C*a^3 + 13*A*a^2*c)*e^3)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_e(ar
csin((a/c)^(1/4)/x), -1) - 6*(143*((21*B + 5*C)*a*c^2 + 35*A*c^3)*d^3 + 42
9*(7*C*a^2*c + (21*A + 5*B)*a*c^2)*d^2*e + 39*((77*B + 25*C)*a^2*c + 55*A*
a*c^2)*d*e^2 + (539*C*a^3 + 13*(77*A + 25*B)*a^2*c)*e^3)*sqrt(-c)*x*(a/c)^
(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) - (3465*C*c^3*e^3*x^12 + 4095*
(3*C*c^3*d*e^2 + B*c^3*e^3)*x^10 + 385*(39*C*c^3*d^2*e + 39*B*c^3*d*e^2 -
(2*C*a*c^2 - 13*A*c^3)*e^3)*x^8 - 18018*B*a*c^2*d^3 - 18018*B*a^2*c*d*e^2
+ 585*(11*C*c^3*d^3 + 33*B*c^3*d^2*e - 2*B*a*c^2*e^3 - 3*(2*C*a*c^2 - 11*A
*c^3)*d*e^2)*x^6 + 77*(117*B*c^3*d^3 - 78*B*a*c^2*d*e^2 - 39*(2*C*a*c^2 -
9*A*c^3)*d^2*e - 2*(7*C*a^2*c + 13*A*a*c^2)*e^3)*x^4 - 18018*(C*a^2*c + 3*
A*a*c^2)*d^2*e - 462*(7*C*a^3 + 13*A*a^2*c)*e^3 - 195*(66*B*a*c^2*d^2*e +
10*B*a^2*c*e^3 + 11*(2*C*a*c^2 - 7*A*c^3)*d^3 + 6*(5*C*a^2*c + 11*A*a*c^2)
*d*e^2)*x^2)*sqrt(-c*x^4 + a))/(c^3*x)
```

Sympy [A] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.02

$$\int \left(d+ex^2\right)^3 \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)**3*(-c*x**4+a)**(1/2)*(C*x**4+B*x**2+A),x)
```

output
```
A*sqrt(a)*d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(2*
I*pi)/a)/(4*gamma(5/4)) + 3*A*sqrt(a)*d**2*e*x**3*gamma(3/4)*hyper((-1/2,
3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) + 3*A*sqrt(a)*d*e
**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)
/(4*gamma(9/4)) + A*sqrt(a)*e**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,
), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11/4)) + B*sqrt(a)*d**3*x**3*gamma
(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)
) + 3*B*sqrt(a)*d**2*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*e
xp_polar(2*I*pi)/a)/(4*gamma(9/4)) + 3*B*sqrt(a)*d*e**2*x**7*gamma(7/4)*hy
per((-1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11/4)) + B*
sqrt(a)*e**3*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), c*x**4*exp_polar(
2*I*pi)/a)/(4*gamma(13/4)) + C*sqrt(a)*d**3*x**5*gamma(5/4)*hyper((-1/2, 5
/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(9/4)) + 3*C*sqrt(a)*d**2
*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)
/(4*gamma(11/4)) + 3*C*sqrt(a)*d*e**2*x**9*gamma(9/4)*hyper((-1/2, 9/4), (
13/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(13/4)) + C*sqrt(a)*e**3*x**11
*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*g
amma(15/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)^3 \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\int \left(Cx^4+Bx^2+A\right)\sqrt{-cx^4+a}\left(ex^2+d\right)^3\,dx$$

input
```
integrate((e*x^2+d)^3*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*(e*x^2 + d)^3, x)
```

Giac [F]

$$\int \left(d+ex^2\right)^3 \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx = \int \left(Cx^4+Bx^2+A\right)\sqrt{-cx^4+a}\left(ex^2+d\right)^3\,dx$$

input
```
integrate((e*x^2+d)^3*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^3 \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx = \int \sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^3\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((a - c*x^4)^(1/2)*(d + e*x^2)^3*(A + B*x^2 + C*x^4),x)
```

output
```
int((a - c*x^4)^(1/2)*(d + e*x^2)^3*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^3 \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx = \frac{1950\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right) a^3 b\,e^3 + 34320\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right) a^2 c^2 d^3 + 9240\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right) a^3 c\,e^3 - 18720\sqrt{-c}}{}$$

input
```
int((e*x^2+d)^3*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x)
```

output

```
( - 1950*sqrt(a - c*x**4)*a**2*b*e**3*x - 18720*sqrt(a - c*x**4)*a**2*c*d*
e**2*x - 3080*sqrt(a - c*x**4)*a**2*c*e**3*x**3 - 12870*sqrt(a - c*x**4)*a
*b*c*d**2*e*x - 6006*sqrt(a - c*x**4)*a*b*c*d*e**2*x**3 - 1170*sqrt(a - c*
x**4)*a*b*c*e**3*x**5 + 10725*sqrt(a - c*x**4)*a*c**2*d**3*x + 21021*sqrt(
a - c*x**4)*a*c**2*d**2*e*x**3 + 15795*sqrt(a - c*x**4)*a*c**2*d*e**2*x**5
 + 4235*sqrt(a - c*x**4)*a*c**2*e**3*x**7 + 9009*sqrt(a - c*x**4)*b*c**2*d
**3*x**3 + 19305*sqrt(a - c*x**4)*b*c**2*d**2*e*x**5 + 15015*sqrt(a - c*x*
*4)*b*c**2*d*e**2*x**7 + 4095*sqrt(a - c*x**4)*b*c**2*e**3*x**9 + 6435*sqr
t(a - c*x**4)*c**3*d**3*x**5 + 15015*sqrt(a - c*x**4)*c**3*d**2*e*x**7 + 1
2285*sqrt(a - c*x**4)*c**3*d*e**2*x**9 + 3465*sqrt(a - c*x**4)*c**3*e**3*x
**11 + 1950*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a**3*b*e**3 + 18720*int(s
qrt(a - c*x**4)/(a - c*x**4),x)*a**3*c*d*e**2 + 12870*int(sqrt(a - c*x**4)
/(a - c*x**4),x)*a**2*b*c*d**2*e + 34320*int(sqrt(a - c*x**4)/(a - c*x**4)
,x)*a**2*c**2*d**3 + 9240*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a**3
*c*e**3 + 18018*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a**2*b*c*d*e**
2 + 72072*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a**2*c**2*d**2*e + 1
8018*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a*b*c**2*d**3)/(45045*c**
2)
```

3.27 $\int (d+ex^2)^2 \sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 417

$$\int (d+ex^2)^2 \sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx$$

$$= \frac{x(5(11Ac(7cd^2+ae^2)+a(5aCe^2+11cd(Cd+2Be)))+77c(3Bcd^2+6Acde+2aCde+aBe^2)\,x^2)\sqrt{a}}{1155c^2}$$

$$-\frac{(5aCe^2+11c(Cd^2+e(2Bd+Ae)))\,x(a-cx^4)^{3/2}}{77c^2}$$

$$-\frac{e(2Cd+Be)x^3(a-cx^4)^{3/2}}{9c}-\frac{Ce^2x^5(a-cx^4)^{3/2}}{11c}$$

$$+\frac{2a^{7/4}(3Bcd^2+6Acde+2aCde+aBe^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15c^{7/4}\sqrt{a-cx^4}}$$

$$-\frac{2a^{5/4}(77\sqrt{a}\sqrt{c}(3Bcd^2+6Acde+2aCde+aBe^2)-5(11Ac(7cd^2+ae^2)+a(5aCe^2+11cd(Cd+2B}{1155c^{9/4}\sqrt{a-cx^4}}$$

output
```
1/1155*x*(55*A*c*(a*e^2+7*c*d^2)+5*a*(5*C*a*e^2+11*c*d*(2*B*e+C*d))+77*c*(
6*A*c*d*e+B*a*e^2+3*B*c*d^2+2*C*a*d*e)*x^2)*(-c*x^4+a)^(1/2)/c^2-1/77*(5*C
*a*e^2+11*c*(C*d^2+e*(A*e+2*B*d)))*x*(-c*x^4+a)^(3/2)/c^2-1/9*e*(B*e+2*C*d
)*x^3*(-c*x^4+a)^(3/2)/c-1/11*C*e^2*x^5*(-c*x^4+a)^(3/2)/c+2/15*a^(7/4)*(6
*A*c*d*e+B*a*e^2+3*B*c*d^2+2*C*a*d*e)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*
x/a^(1/4),I)/c^(7/4)/(-c*x^4+a)^(1/2)-2/1155*a^(5/4)*(77*a^(1/2)*c^(1/2)*(
6*A*c*d*e+B*a*e^2+3*B*c*d^2+2*C*a*d*e)-55*A*c*(a*e^2+7*c*d^2)-5*a*(5*C*a*e
^2+11*c*d*(2*B*e+C*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c
^(9/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.47 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.53

$$\int (d+ex^2)^2\sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx$$
$$=\frac{x\sqrt{a-cx^4}\left(-\left(\left(a-cx^4\right)\sqrt{1-\frac{cx^4}{a}}(45aCe^2+11ce(18Bd+9Ae+7Bex^2)+cC(99d^2+154dex^2+63e\right.\right.\right.}{}$$

input
```
Integrate[(d + e*x^2)^2*Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(x*Sqrt[a - c*x^4]*(-((a - c*x^4)*Sqrt[1 - (c*x^4)/a]*(45*a*C*e^2 + 11*c*e
*(18*B*d + 9*A*e + 7*B*e*x^2) + c*C*(99*d^2 + 154*d*e*x^2 + 63*e^2*x^4)))
+ 9*(11*A*c*(7*c*d^2 + a*e^2) + a*(5*a*C*e^2 + 11*c*d*(C*d + 2*B*e)))*Hype
rgeometric2F1[-1/2, 1/4, 5/4, (c*x^4)/a] + 77*c*(3*B*c*d^2 + 6*A*c*d*e + 2
*a*C*d*e + a*B*e^2)*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, (c*x^4)/a]))/(69
3*c^2*Sqrt[1 - (c*x^4)/a])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1029 vs. 2(417) = 834.

Time = 1.28 (sec) , antiderivative size = 1029, normalized size of antiderivative = 2.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^4}\left(d+ex^2\right)^2\left(A+Bx^2+Cx^4\right)dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{x^4\left(ad(2Be+Cd)-A\left(cd^2-ae^2\right)\right)}{\sqrt{a-cx^4}}+\frac{x^8\left(aCe^2-c\left(e(Ae+2Bd)+Cd^2\right)\right)}{\sqrt{a-cx^4}}-\frac{x^6\left(-aBe^2-2aCde+2Acde\right.}{\sqrt{a-cx^4}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{11}Ce^2\sqrt{a-cx^4}x^9+\frac{1}{9}e(2Cd+Be)\sqrt{a-cx^4}x^7+\frac{9aCe^2\sqrt{a-cx^4}x^5}{77c}-\frac{(aCe^2-c(Cd^2+e(2Bd+Ae)))\sqrt{a-cx^4}x^5}{7c}+\frac{7ae(2Cd+Be)\sqrt{a-cx^4}x^3}{45c}+\frac{(Bcd^2+2Aced-2aCed-aBe^2)\sqrt{a-cx^4}x^3}{5c}+\frac{15a^2Ce^2\sqrt{a-cx^4}x}{77c^2}-\frac{(ad(Cd+2Be)-A(cd^2-ae^2))\sqrt{a-cx^4}x}{3c}-\frac{5a(aCe^2-c(Cd^2+e(2Bd+Ae)))\sqrt{a-cx^4}x}{21c^2}+\frac{a^{7/4}d(Bd+2Ae)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{7a^{11/4}e(2Cd+Be)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15c^{7/4}\sqrt{a-cx^4}}-\frac{3a^{7/4}(Bcd^2+2Aced-2aCed-aBe^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{a^{5/4}Ad^2\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{15a^{13/4}Ce^2\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{77c^{9/4}\sqrt{a-cx^4}}-\frac{a^{7/4}d(Bd+2Ae)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{7a^{11/4}e(2Cd+Be)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{15c^{7/4}\sqrt{a-cx^4}}+\frac{3a^{7/4}(Bcd^2+2Aced-2aCed-aBe^2)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{a^{5/4}(ad(Cd+2Be)-A(cd^2-ae^2))\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}}+\frac{5a^{9/4}(aCe^2-c(Cd^2+e(2Bd+Ae)))\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{21c^{9/4}\sqrt{a-cx^4}}$$

input
```
Int[(d + e*x^2)^2*Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(15*a^2*C*e^2*x*Sqrt[a - c*x^4])/(77*c^2) - ((a*d*(C*d + 2*B*e) - A*(c*d^2
 - a*e^2))*x*Sqrt[a - c*x^4])/(3*c) - (5*a*(a*C*e^2 - c*(C*d^2 + e*(2*B*d
+ A*e)))*x*Sqrt[a - c*x^4])/(21*c^2) + (7*a*e*(2*C*d + B*e)*x^3*Sqrt[a - c
*x^4])/(45*c) + ((B*c*d^2 + 2*A*c*d*e - 2*a*C*d*e - a*B*e^2)*x^3*Sqrt[a -
c*x^4])/(5*c) + (9*a*C*e^2*x^5*Sqrt[a - c*x^4])/(77*c) - ((a*C*e^2 - c*(C*
d^2 + e*(2*B*d + A*e)))*x^5*Sqrt[a - c*x^4])/(7*c) + (e*(2*C*d + B*e)*x^7*
Sqrt[a - c*x^4])/9 + (C*e^2*x^9*Sqrt[a - c*x^4])/11 + (a^(7/4)*d*(B*d + 2*
A*e)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3
/4)*Sqrt[a - c*x^4]) - (7*a^(11/4)*e*(2*C*d + B*e)*Sqrt[1 - (c*x^4)/a]*Ell
ipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(15*c^(7/4)*Sqrt[a - c*x^4]) - (3
*a^(7/4)*(B*c*d^2 + 2*A*c*d*e - 2*a*C*d*e - a*B*e^2)*Sqrt[1 - (c*x^4)/a]*E
llipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (
a^(5/4)*A*d^2*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(c^(1/4)*Sqrt[a - c*x^4]) - (15*a^(13/4)*C*e^2*Sqrt[1 - (c*x^4)/a]*Ell
ipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(77*c^(9/4)*Sqrt[a - c*x^4]) - (a
^(7/4)*d*(B*d + 2*A*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^
(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (7*a^(11/4)*e*(2*C*d + B*e)*Sqrt[
1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(15*c^(7/4)*Sqr
t[a - c*x^4]) + (3*a^(7/4)*(B*c*d^2 + 2*A*c*d*e - 2*a*C*d*e - a*B*e^2)*Sqr
t[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{C e^2 x^9 \sqrt{-c x^4+a}}{11} - \frac{(-B c e^2-2C c d e) x^7 \sqrt{-c x^4+a}}{9c} - \frac{(-A c e^2-2B c d e+\frac{2}{11} C a e^2-C c d^2) x^5 \sqrt{-c x^4+a}}{7c} - \frac{\left(-2A c d e+B a e^2- \cdots\right.}{\cdots}$
default	$A d^2 \left(\frac{x\sqrt{-c x^4+a}}{3} + \frac{2a\sqrt{1-\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-c x^4+a}}\right) + d(2Ae+Bd)\left(\frac{x^3\sqrt{-c x^4+a}}{5} - \frac{2a^{\frac{3}{2}}\sqrt{1- \cdots}}{\cdots}\right.$
risch	$-\frac{x(-315C c^2 e^2 x^8-385B c^2 e^2 x^6-770C c^2 d e x^6-495A c^2 e^2 x^4-990B c^2 d e x^4+90C a c e^2 x^4-495C c^2 d^2 x^4-1386A c^2 d e x^2+154B \cdots}{3465c^2}$

input
```
int((e*x^2+d)^2*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*C*e^2*x^9*(-c*x^4+a)^(1/2)-1/9*(-B*c*e^2-2*C*c*d*e)/c*x^7*(-c*x^4+a)^
(1/2)-1/7*(-A*c*e^2-2*B*c*d*e+2/11*C*a*e^2-C*c*d^2)/c*x^5*(-c*x^4+a)^(1/2)
-1/5*(-2*A*c*d*e+B*a*e^2-B*c*d^2+2*C*a*d*e+7/9*a/c*(-B*c*e^2-2*C*c*d*e))/c
*x^3*(-c*x^4+a)^(1/2)-1/3*(A*a*e^2-A*c*d^2+2*a*B*d*e+C*a*d^2+5/7*a/c*(-A*c
*e^2-2*B*c*d*e+2/11*C*a*e^2-C*c*d^2))/c*x*(-c*x^4+a)^(1/2)+(A*a*d^2+1/3*a/
c*(A*a*e^2-A*c*d^2+2*a*B*d*e+C*a*d^2+5/7*a/c*(-A*c*e^2-2*B*c*d*e+2/11*C*a*
e^2-C*c*d^2)))/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^
(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1
/2),I)-(2*A*a*d*e+B*a*d^2+3/5*a/c*(-2*A*c*d*e+B*a*e^2-B*c*d^2+2*C*a*d*e+7/
9*a/c*(-B*c*e^2-2*C*c*d*e)))*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^
2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(E
llipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),
I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.88

$$\int \left(d+ex^2\right)^2\sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx =$$

$$-\frac{462\,(3\,Bacd^2+Ba^2e^2+2\,(Ca^2+3\,Aac)de)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)-6\,(11\,((21\,B+5\,C)}{}$$

input
```
integrate((e*x^2+d)^2*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="frica
s")
```

output
```
-1/3465*(462*(3*B*a*c*d^2 + B*a^2*e^2 + 2*(C*a^2 + 3*A*a*c)*d*e)*sqrt(-c)*
x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - 6*(11*((21*B + 5*C)*
a*c + 35*A*c^2)*d^2 + 22*(7*C*a^2 + (21*A + 5*B)*a*c)*d*e + ((77*B + 25*C)
*a^2 + 55*A*a*c)*e^2)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)
/x), -1) - (315*C*c^2*e^2*x^10 + 385*(2*C*c^2*d*e + B*c^2*e^2)*x^8 + 45*(1
1*C*c^2*d^2 + 22*B*c^2*d*e - (2*C*a*c - 11*A*c^2)*e^2)*x^6 - 1386*B*a*c*d^
2 - 462*B*a^2*e^2 + 77*(9*B*c^2*d^2 - 2*B*a*c*e^2 - 2*(2*C*a*c - 9*A*c^2)*
d*e)*x^4 - 924*(C*a^2 + 3*A*a*c)*d*e - 15*(44*B*a*c*d*e + 11*(2*C*a*c - 7*
A*c^2)*d^2 + 2*(5*C*a^2 + 11*A*a*c)*e^2)*x^2)*sqrt(-c*x^4 + a))/(c^2*x)
```

Sympy [A] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.02

$$\int \left(d+ex^2\right)^2\sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)**2*(-c*x**4+a)**(1/2)*(C*x**4+B*x**2+A),x)
```

output
```
A*sqrt(a)*d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(2*
I*pi)/a)/(4*gamma(5/4)) + A*sqrt(a)*d*e*x**3*gamma(3/4)*hyper((-1/2, 3/4),
 (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(2*gamma(7/4)) + A*sqrt(a)*e**2*x**5*
gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma
(9/4)) + B*sqrt(a)*d**2*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*
exp_polar(2*I*pi)/a)/(4*gamma(7/4)) + B*sqrt(a)*d*e*x**5*gamma(5/4)*hyper(
(-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(2*gamma(9/4)) + B*sqrt(a
)*e**2*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi
)/a)/(4*gamma(11/4)) + C*sqrt(a)*d**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (
9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(9/4)) + C*sqrt(a)*d*e*x**7*gam
ma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(2*gamma(1
1/4)) + C*sqrt(a)*e**2*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), c*x**4*
exp_polar(2*I*pi)/a)/(4*gamma(13/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)^2 \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx = \int \left(Cx^4+Bx^2+A\right)\sqrt{-cx^4+a}\left(ex^2+d\right)^2\,dx$$

input
```
integrate((e*x^2+d)^2*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*(e*x^2 + d)^2, x)
```

Giac [F]

$$\int \left(d+ex^2\right)^2 \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx = \int \left(Cx^4+Bx^2+A\right)\sqrt{-cx^4+a}\left(ex^2+d\right)^2\,dx$$

input
```
integrate((e*x^2+d)^2*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\int \sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^2\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((a - c*x^4)^(1/2)*(d + e*x^2)^2*(A + B*x^2 + C*x^4),x)
```

output
```
int((a - c*x^4)^(1/2)*(d + e*x^2)^2*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^2 \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{-480\sqrt{-c\,x^4+a}\,a^2e^2x-660\sqrt{-c\,x^4+a}\,abdex-154\sqrt{-c\,x^4+a}\,ab\,e^2x^3+825\sqrt{-c\,x^4+a}\,ac\,d^2x+10}{}$$

input
```
int((e*x^2+d)^2*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x)
```

output
```
( - 480*sqrt(a - c*x**4)*a**2*e**2*x - 660*sqrt(a - c*x**4)*a*b*d*e*x - 15
4*sqrt(a - c*x**4)*a*b*e**2*x**3 + 825*sqrt(a - c*x**4)*a*c*d**2*x + 1078*
sqrt(a - c*x**4)*a*c*d*e*x**3 + 405*sqrt(a - c*x**4)*a*c*e**2*x**5 + 693*s
qrt(a - c*x**4)*b*c*d**2*x**3 + 990*sqrt(a - c*x**4)*b*c*d*e*x**5 + 385*sq
rt(a - c*x**4)*b*c*e**2*x**7 + 495*sqrt(a - c*x**4)*c**2*d**2*x**5 + 770*s
qrt(a - c*x**4)*c**2*d*e*x**7 + 315*sqrt(a - c*x**4)*c**2*e**2*x**9 + 480*
int(sqrt(a - c*x**4)/(a - c*x**4),x)*a**3*e**2 + 660*int(sqrt(a - c*x**4)/
(a - c*x**4),x)*a**2*b*d*e + 2640*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a**
2*c*d**2 + 462*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a**2*b*e**2 + 3
696*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a**2*c*d*e + 1386*int((sqr
t(a - c*x**4)*x**2)/(a - c*x**4),x)*a*b*c*d**2)/(3465*c)
```

3.28 $\int (d+ex^2)\sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 280

$$\int (d+ex^2)\sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx$$
$$= \frac{x(5(7Acd+aCd+aBe)+7(3Bcd+3Ace+aCe)x^2)\sqrt{a-cx^4}}{105c}$$
$$- \frac{(Cd+Be)x(a-cx^4)^{3/2}}{7c} - \frac{Cex^3(a-cx^4)^{3/2}}{9c}$$
$$+ \frac{2a^{7/4}(3Bcd+3Ace+aCe)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15c^{7/4}\sqrt{a-cx^4}}$$
$$+ \frac{2a^{5/4}\left(5\sqrt{c}(7Acd+aCd+aBe)-7\sqrt{a}(3Bcd+3Ace+aCe)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-\right.}{105c^{7/4}\sqrt{a-cx^4}}$$

output

```
1/105*x*(35*A*c*d+5*B*a*e+5*C*a*d+7*(3*A*c*e+3*B*c*d+C*a*e)*x^2)*(-c*x^4+a
)^(1/2)/c-1/7*(B*e+C*d)*x*(-c*x^4+a)^(3/2)/c-1/9*C*e*x^3*(-c*x^4+a)^(3/2)/
c+2/15*a^(7/4)*(3*A*c*e+3*B*c*d+C*a*e)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)
*x/a^(1/4),I)/c^(7/4)/(-c*x^4+a)^(1/2)+2/105*a^(5/4)*(5*c^(1/2)*(7*A*c*d+B
*a*e+C*a*d)-7*a^(1/2)*(3*A*c*e+3*B*c*d+C*a*e))*(1-c*x^4/a)^(1/2)*EllipticF
(c^(1/4)*x/a^(1/4),I)/c^(7/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.53

$$\int (d+ex^2)\sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx = \frac{x\sqrt{a-cx^4}\left((9Cd+9Be+7Cex^2)(-a+cx^4)\sqrt{1-\frac{cx^4}{a}}+9(7Acd+aCd+aBe)\,\text{Hypergeometric2F1}\right.}{63c\sqrt{1-\frac{cx^4}{a}}}$$

input
```
Integrate[(d + e*x^2)*Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(x*Sqrt[a - c*x^4]*((9*C*d + 9*B*e + 7*C*e*x^2)*(-a + c*x^4)*Sqrt[1 - (c*x
^4)/a] + 9*(7*A*c*d + a*C*d + a*B*e)*Hypergeometric2F1[-1/2, 1/4, 5/4, (c*
x^4)/a] + 7*(3*B*c*d + 3*A*c*e + a*C*e)*x^2*Hypergeometric2F1[-1/2, 3/4, 7
/4, (c*x^4)/a]))/(63*c*Sqrt[1 - (c*x^4)/a])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 738 vs. 2(280) = 560.

Time = 0.97 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.64, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^4}(d+ex^2)\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(-\frac{x^4(-aBe-aCd+Acd)}{\sqrt{a-cx^4}}-\frac{x^6(-aCe+Ace+Bcd)}{\sqrt{a-cx^4}}+\frac{ax^2(Ae+Bd)}{\sqrt{a-cx^4}}+\frac{aAd}{\sqrt{a-cx^4}}-\frac{cx^8(Be+Cd)}{\sqrt{a-cx^4}}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{5/4}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)(-aBe-aCd+Acd)}{3c^{5/4}\sqrt{a-cx^4}}+\frac{3a^{7/4}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)(-aCe+Ace+Bcd)}{5c^{7/4}\sqrt{a-cx^4}}-\frac{3a^{7/4}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)(-aCe+Ace+Bcd)}{5c^{7/4}\sqrt{a-cx^4}}-\frac{a^{7/4}\sqrt{1-\frac{cx^4}{a}}(Ae+Bd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{7/4}\sqrt{1-\frac{cx^4}{a}}(Ae+Bd)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{5/4}Ad\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{5a^{9/4}\sqrt{1-\frac{cx^4}{a}}(Be+Cd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{21c^{5/4}\sqrt{a-cx^4}}+\frac{7a^{11/4}Ce\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{15c^{7/4}\sqrt{a-cx^4}}-\frac{7a^{11/4}Ce\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15c^{7/4}\sqrt{a-cx^4}}+\frac{x\sqrt{a-cx^4}(-aBe-aCd+Acd)}{3c}+\frac{x^3\sqrt{a-cx^4}(-aCe+Ace+Bcd)}{5c}+\frac{5ax\sqrt{a-cx^4}(Be+Cd)}{21c}+\frac{1}{7}x^5\sqrt{a-cx^4}(Be+Cd)+\frac{1}{9}Cex^7\sqrt{a-cx^4}+\frac{7aCex^3\sqrt{a-cx^4}}{45c}$$

input
```
Int[(d + e*x^2)*Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(5*a*(C*d + B*e)*x*Sqrt[a - c*x^4])/(21*c) + ((A*c*d - a*C*d - a*B*e)*x*Sq
rt[a - c*x^4])/(3*c) + (7*a*C*e*x^3*Sqrt[a - c*x^4])/(45*c) + ((B*c*d + A*
c*e - a*C*e)*x^3*Sqrt[a - c*x^4])/(5*c) + ((C*d + B*e)*x^5*Sqrt[a - c*x^4]
)/7 + (C*e*x^7*Sqrt[a - c*x^4])/9 - (7*a^(11/4)*C*e*Sqrt[1 - (c*x^4)/a]*El
lipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(15*c^(7/4)*Sqrt[a - c*x^4]) + (
a^(7/4)*(B*d + A*e)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/
4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) - (3*a^(7/4)*(B*c*d + A*c*e - a*C*e)*S
qrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*
Sqrt[a - c*x^4]) + (a^(5/4)*A*d*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1
/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]) + (7*a^(11/4)*C*e*Sqrt[1 -
 (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(15*c^(7/4)*Sqrt[a
 - c*x^4]) - (a^(7/4)*(B*d + A*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^
(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) - (5*a^(9/4)*(C*d + B*e)
*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(21*c^(5/
4)*Sqrt[a - c*x^4]) - (a^(5/4)*(A*c*d - a*C*d - a*B*e)*Sqrt[1 - (c*x^4)/a]
*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(5/4)*Sqrt[a - c*x^4]) +
 (3*a^(7/4)*(B*c*d + A*c*e - a*C*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(
c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{eC\,x^{7}\sqrt{-c\,x^{4}+a}}{9}-\frac{(-Bce-Ccd)x^{5}\sqrt{-c\,x^{4}+a}}{7c}-\frac{\left(-Ace-Bcd+\frac{2}{9}Cae\right)x^{3}\sqrt{-c\,x^{4}+a}}{5c}-\frac{\left(-Acd+Bae+Cad+\frac{5a(-Bce-Ccd)}{7c}\right)}{3c}$
risch	$\frac{x(35eC\,x^{6}c+45Bce\,x^{4}+45Ccd\,x^{4}+63Ace\,x^{2}+63Bcd\,x^{2}-14Cae\,x^{2}+105Acd-30Bae-30Cad)\sqrt{-c\,x^{4}+a}}{315c}+\frac{2a\left(-\frac{(21Ace+21Bcd}{}\right.}{}$
default	$Ad\left(\frac{x\sqrt{-c\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right)+(Ae+Bd)\left(\frac{x^{3}\sqrt{-c\,x^{4}+a}}{5}-\frac{2a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}}{}\right.$

input
```
int((e*x^2+d)*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*e*C*x^7*(-c*x^4+a)^(1/2)-1/7*(-B*c*e-C*c*d)/c*x^5*(-c*x^4+a)^(1/2)-1/5
*(-A*c*e-B*c*d+2/9*C*a*e)/c*x^3*(-c*x^4+a)^(1/2)-1/3*(-A*c*d+B*a*e+C*a*d+5
/7*a/c*(-B*c*e-C*c*d))/c*x*(-c*x^4+a)^(1/2)+(a*d*A+1/3*a/c*(-A*c*d+B*a*e+C
*a*d+5/7*a/c*(-B*c*e-C*c*d)))/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/
2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1
/2)/a^(1/2))^(1/2),I)-(A*a*e+B*a*d+3/5*a/c*(-A*c*e-B*c*d+2/9*C*a*e))*a^(1/
2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^
(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2)
,I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.86

$$\int \left(d+ex^{2}\right)\sqrt{a-cx^{4}}\left(A+Bx^{2}+Cx^{4}\right)\,dx =$$

$$-\frac{42\,(3\,Bacd+(Ca^{2}+3\,Aac)e)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)-6\left(((21\,B+5\,C)ac+35\,Ac^{2})d+\right.}{}$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="fricas"
)
```

output
```
-1/315*(42*(3*B*a*c*d + (C*a^2 + 3*A*a*c)*e)*sqrt(-c)*x*(a/c)^(3/4)*ellipt
ic_e(arcsin((a/c)^(1/4)/x), -1) - 6*(((21*B + 5*C)*a*c + 35*A*c^2)*d + (7*
C*a^2 + (21*A + 5*B)*a*c)*e)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c
)^(1/4)/x), -1) - (35*C*c^2*e*x^8 + 45*(C*c^2*d + B*c^2*e)*x^6 + 7*(9*B*c^
2*d - (2*C*a*c - 9*A*c^2)*e)*x^4 - 126*B*a*c*d - 15*(2*B*a*c*e + (2*C*a*c
- 7*A*c^2)*d)*x^2 - 42*(C*a^2 + 3*A*a*c)*e)*sqrt(-c*x^4 + a))/(c^2*x)
```

Sympy [A] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.97

$$\int (d+ex^2)\sqrt{a-cx^4}(A+Bx^2+Cx^4)\ dx = \frac{A\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{A\sqrt{a}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{a}dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{a}ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{C\sqrt{a}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{C\sqrt{a}ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+d)*(-c*x**4+a)**(1/2)*(C*x**4+B*x**2+A),x)
```

output
```
A*sqrt(a)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(2*I*p
i)/a)/(4*gamma(5/4)) + A*sqrt(a)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4
,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) + B*sqrt(a)*d*x**3*gamma(3/
4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) +
 B*sqrt(a)*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2
*I*pi)/a)/(4*gamma(9/4)) + C*sqrt(a)*d*x**5*gamma(5/4)*hyper((-1/2, 5/4),
(9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(9/4)) + C*sqrt(a)*e*x**7*gamm
a(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11
/4))
```

Maxima [F]

$$\int \left(d+ex^2\right)\sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\sqrt{-cx^4+a}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="maxima"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*(e*x^2 + d), x)
```

Giac [F]

$$\int \left(d+ex^2\right)\sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\sqrt{-cx^4+a}\left(ex^2+d\right)dx$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx$$
$$=\int \sqrt{a-c\,x^4}\,(e\,x^2+d)\,(C\,x^4+B\,x^2+A)\,dx$$

input
```
int((a - c*x^4)^(1/2)*(d + e*x^2)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a - c*x^4)^(1/2)*(d + e*x^2)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (d+ex^2)\sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx$$
$$=\frac{-30\sqrt{-c\,x^4+a}\,abex+75\sqrt{-c\,x^4+a}\,acdx+49\sqrt{-c\,x^4+a}\,ace\,x^3+63\sqrt{-c\,x^4+a}\,bcd\,x^3+45\sqrt{-c\,x^4}}{}$$

input
```
int((e*x^2+d)*(-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x)
```

output
```
( - 30*sqrt(a - c*x**4)*a*b*e*x + 75*sqrt(a - c*x**4)*a*c*d*x + 49*sqrt(a
- c*x**4)*a*c*e*x**3 + 63*sqrt(a - c*x**4)*b*c*d*x**3 + 45*sqrt(a - c*x**4
)*b*c*e*x**5 + 45*sqrt(a - c*x**4)*c**2*d*x**5 + 35*sqrt(a - c*x**4)*c**2*
e*x**7 + 30*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a**2*b*e + 240*int(sqrt(a
 - c*x**4)/(a - c*x**4),x)*a**2*c*d + 168*int((sqrt(a - c*x**4)*x**2)/(a -
 c*x**4),x)*a**2*c*e + 126*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a*b
*c*d)/(315*c)
```

3.29 $\int \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 195

$$\begin{aligned}\int \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx &= \frac{x(5(7Ac+aC)+21Bcx^2)\sqrt{a-cx^4}}{105c}-\frac{Cx(a-cx^4)^{3/2}}{7c}\\&\quad+\frac{2a^{7/4}B\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{3/4}\sqrt{a-cx^4}}\\&\quad-\frac{2a^{5/4}\left(21\sqrt{a}B\sqrt{c}-35Ac-5aC\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{105c^{5/4}\sqrt{a-cx^4}}\end{aligned}$$

output

```
1/105*x*(21*B*c*x^2+35*A*c+5*C*a)*(-c*x^4+a)^(1/2)/c-1/7*C*x*(-c*x^4+a)^(3
/2)/c+2/5*a^(7/4)*B*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(3/
4)/(-c*x^4+a)^(1/2)-2/105*a^(5/4)*(21*a^(1/2)*B*c^(1/2)-35*A*c-5*a*C)*(1-c
*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(5/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.59

$$\int \sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx = \frac{x\sqrt{a-cx^4}\left(-3C(a-cx^4)\sqrt{1-\frac{cx^4}{a}}+3(7Ac+aC)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{cx^4}{a}\right)+7Bcx^2\,\text{Hype}\right.}{21c\sqrt{1-\frac{cx^4}{a}}}$$

input
```
Integrate[Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(x*Sqrt[a - c*x^4]*(-3*C*(a - c*x^4)*Sqrt[1 - (c*x^4)/a] + 3*(7*A*c + a*C)
*Hypergeometric2F1[-1/2, 1/4, 5/4, (c*x^4)/a] + 7*B*c*x^2*Hypergeometric2F
1[-1/2, 3/4, 7/4, (c*x^4)/a]))/(21*c*Sqrt[1 - (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2427, 25, 1491, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^4}(A+Bx^2+Cx^4)\,dx$$

$$\Big\downarrow \text{2427}$$

$$-\frac{\int -\left((7Bcx^2+7Ac+aC)\sqrt{a-cx^4}\right)dx}{7c}-\frac{Cx(a-cx^4)^{3/2}}{7c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (7Bcx^2+7Ac+aC)\sqrt{a-cx^4}dx}{7c}-\frac{Cx(a-cx^4)^{3/2}}{7c}$$

$$\Big\downarrow \text{1491}$$

$$\frac{\frac{1}{15}\int \frac{2a\left(21Bcx^2+5(7Ac+aC)\right)}{\sqrt{a-cx^4}}dx+\frac{1}{15}x\sqrt{a-cx^4}\left(5(aC+7Ac)+21Bcx^2\right)}{7c}-\frac{Cx\left(a-cx^4\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2}{15}a\int \frac{21Bcx^2+5(7Ac+aC)}{\sqrt{a-cx^4}}dx+\frac{1}{15}x\sqrt{a-cx^4}\left(5(aC+7Ac)+21Bcx^2\right)}{7c}-\frac{Cx\left(a-cx^4\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{1513}$$

$$\frac{\frac{2}{15}a\left(21\sqrt{a}B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx-\left(21\sqrt{a}B\sqrt{c}-5aC-35Ac\right)\int \frac{1}{\sqrt{a-cx^4}}dx\right)+\frac{1}{15}x\sqrt{a-cx^4}\left(5(aC+7Ac)+21Bc\right.}{7c}$$
$$\frac{Cx\left(a-cx^4\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2}{15}a\left(21B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\left(21\sqrt{a}B\sqrt{c}-5aC-35Ac\right)\int \frac{1}{\sqrt{a-cx^4}}dx\right)+\frac{1}{15}x\sqrt{a-cx^4}\left(5(aC+7Ac)+21Bcx^2\right)}{7c}$$
$$\frac{Cx\left(a-cx^4\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{2}{15}a\left(21B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\frac{\sqrt{1-\frac{cx^4}{a}}\left(21\sqrt{a}B\sqrt{c}-5aC-35Ac\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}\right)+\frac{1}{15}x\sqrt{a-cx^4}\left(5(aC+7Ac)+21Bcx^2\right)}{7c}$$
$$\frac{Cx\left(a-cx^4\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{2}{15}a\left(21B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(21\sqrt{a}B\sqrt{c}-5aC-35Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+\frac{1}{15}x\sqrt{a-cx^4}\left(5(aC+\right.}{7c}$$
$$\frac{Cx\left(a-cx^4\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\frac{2}{15}a\left(\frac{21B\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(21\sqrt{a}B\sqrt{c}-5aC-35Ac)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+\frac{1}{15}x\sqrt{a-cx^4}(5(a}{7c}$$

$$\frac{Cx(a-cx^4)^{3/2}}{7c}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\frac{2}{15}a\left(\frac{21\sqrt{a}B\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(21\sqrt{a}B\sqrt{c}-5aC-35Ac)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+\frac{1}{15}x\sqrt{a-cx^4}(}{7c}$$

$$\frac{Cx(a-cx^4)^{3/2}}{7c}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{2}{15}a\left(\frac{21a^{3/4}B\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(21\sqrt{a}B\sqrt{c}-5aC-35Ac)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+\frac{1}{15}x\sqrt{}}{7c}$$

$$\frac{Cx(a-cx^4)^{3/2}}{7c}$$

input

```
Int[Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4),x]
```

output

```
-1/7*(C*x*(a - c*x^4)^(3/2))/c + ((x*(5*(7*A*c + a*C) + 21*B*c*x^2)*Sqrt[a
 - c*x^4])/15 + (2*a*((21*a^(3/4)*B*c^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[
ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^4] - (a^(1/4)*(21*Sqrt[a]*B
*Sqrt[c] - 35*A*c - 5*a*C)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x
)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4])))/15)/(7*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.17

method	result
elliptic	$\frac{C x^5\sqrt{-c x^4+a}}{7}+\frac{B x^3\sqrt{-c x^4+a}}{5}-\frac{\left(-Ac+\frac{2Ca}{7}\right)x\sqrt{-c x^4+a}}{3c}+\frac{\left(Aa+\frac{a\left(-Ac+\frac{2Ca}{7}\right)}{3c}\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}$
risch	$\frac{x\left(15Ccx^4+21Bx^2c+35Ac-10Ca\right)\sqrt{-cx^4+a}}{105c}+\frac{2a\left(\frac{35Ac\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}+\frac{5Ca\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{E}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4}}\right.}{}$
default	$A\left(\frac{x\sqrt{-cx^4+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}\right)+B\left(\frac{x^3\sqrt{-cx^4+a}}{5}-\frac{2a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\right.}{5\sqrt{}}\right.$

input
```
int((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*C*x^5*(-c*x^4+a)^(1/2)+1/5*B*x^3*(-c*x^4+a)^(1/2)-1/3*(-A*c+2/7*C*a)/c
*x*(-c*x^4+a)^(1/2)+(A*a+1/3*a/c*(-A*c+2/7*C*a))/(c^(1/2)/a^(1/2))^(1/2)*(
1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2
)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-2/5*B*a^(3/2)/(c^(1/2)/a^(1/2))^(
1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a
)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/
2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.66

$$\int \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx = -\frac{42\,Ba\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)-2\left((21\,B+5\,C)a+35\,Ac\right)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-}{105\,cx}$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
-1/105*(42*B*a*sqrt(-c)*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1
) - 2*((21*B + 5*C)*a + 35*A*c)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((
a/c)^(1/4)/x), -1) - (15*C*c*x^6 + 21*B*c*x^4 - 5*(2*C*a - 7*A*c)*x^2 - 42
*B*a)*sqrt(-c*x^4 + a))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.66

$$\int \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx = \frac{A\sqrt{a}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{B\sqrt{a}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{C\sqrt{a}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((-c*x**4+a)**(1/2)*(C*x**4+B*x**2+A),x)
```

output
```
A*sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(2*I*pi)
/a)/(4*gamma(5/4)) + B*sqrt(a)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) + C*sqrt(a)*x**5*gamma(5/4)*hyp
er((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx = \int \left(Cx^4+Bx^2+A\right)\sqrt{-cx^4+a}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\sqrt{-cx^4+a}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \sqrt{a-c\,x^4}\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((a - c*x^4)^(1/2)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a - c*x^4)^(1/2)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int \sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)\,dx=\frac{5\sqrt{-c\,x^4+a}\,ax}{21}+\frac{\sqrt{-c\,x^4+a}\,b\,x^3}{5}+\frac{\sqrt{-c\,x^4+a}\,c\,x^5}{7}+\frac{16\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)a^2}{21}+\frac{2\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right)ab}{5}$$

input
```
int((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x)
```

output
```
(25*sqrt(a - c*x**4)*a*x + 21*sqrt(a - c*x**4)*b*x**3 + 15*sqrt(a - c*x**4
)*c*x**5 + 80*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a**2 + 42*int((sqrt(a -
 c*x**4)*x**2)/(a - c*x**4),x)*a*b)/105
```

3.30 $\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{d+ex^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 382

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = -\frac{(Cd-Be)x\sqrt{a-cx^4}}{3e^2}+\frac{Cx^3\sqrt{a-cx^4}}{5e}$$
$$+\frac{a^{3/4}(2aCe^2-5c(Cd^2-e(Bd-Ae)))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{3/4}e^3\sqrt{a-cx^4}}$$
$$-\frac{\sqrt[4]{a}\left(3\sqrt{a}e(2aCe^2-5c(Cd^2-e(Bd-Ae)))+5\sqrt{c}(2ae^2(Cd-Be)-3cd(Cd^2-e(Bd-Ae)))\right)\sqrt{1}}{15c^{3/4}e^4\sqrt{a-cx^4}}$$
$$-\frac{\sqrt[4]{a}(cd^2-ae^2)\left(Cd^2-Bde+Ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de^4\sqrt{a-cx^4}}$$

output
```
-1/3*(-B*e+C*d)*x*(-c*x^4+a)^(1/2)/e^2+1/5*C*x^3*(-c*x^4+a)^(1/2)/e+1/5*a^
(3/4)*(2*C*a*e^2-5*c*(C*d^2-e*(-A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticE(c^(
1/4)*x/a^(1/4),I)/c^(3/4)/e^3/(-c*x^4+a)^(1/2)-1/15*a^(1/4)*(3*a^(1/2)*e*(
2*C*a*e^2-5*c*(C*d^2-e*(-A*e+B*d)))+5*c^(1/2)*(2*a*e^2*(-B*e+C*d)-3*c*d*(C
*d^2-e*(-A*e+B*d))))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(3
/4)/e^4/(-c*x^4+a)^(1/2)-a^(1/4)*(-a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*(1-c*x
^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4)/d
/e^4/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.91 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{d+ex^2}\,dx$$
$$= \frac{-3i\sqrt{a}de(2aCe^2-5c(Cd^2+e(-Bd+Ae)))\sqrt{1-\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+id\left(6a^{3/2}Ce^3+1\right.}{}$$

input
```
Integrate[(Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4))/(d + e*x^2),x]
```

output
```
((-3*I)*Sqrt[a]*d*e*(2*a*C*e^2 - 5*c*(C*d^2 + e*(-(B*d) + A*e)))*Sqrt[1 -
(c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*d*(6*a
^(3/2)*C*e^3 + 10*a*Sqrt[c]*e^2*(C*d - B*e) - 15*Sqrt[a]*c*e*(C*d^2 + e*(-
(B*d) + A*e)) - 15*c^(3/2)*(C*d^3 + d*e*(-(B*d) + A*e)))*Sqrt[1 - (c*x^4)/
a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*(Sqrt[-(
Sqrt[c]/Sqrt[a])]*d*e^2*x*(-5*C*d + 5*B*e + 3*C*e*x^2)*(a - c*x^4) - (15*I
)*(-(c*d^2) + a*e^2)*(C*d^2 + e*(-(B*d) + A*e))*Sqrt[1 - (c*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1
]))/(15*Sqrt[-(Sqrt[c]/Sqrt[a])]*Sqrt[c]*d*e^4*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.57, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{d+ex^2}\,dx$$
$$\Big\downarrow \text{ 2259}$$

$$\int \left(\frac{-ae^2(Cd-Be)-cde(Bd-Ae)+cCd^3}{e^4\sqrt{a-cx^4}}-\frac{x^2\left(-aCe^2-ce(Bd-Ae)+cCd^2\right)}{e^3\sqrt{a-cx^4}}+\frac{aAe^4-aBde^3+aCd^2e^2}{e^4\sqrt{a-c}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(-aCe^2-ce(Bd-Ae)+cCd^2\right)}{c^{3/4}e^3\sqrt{a-cx^4}}-\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\left(-aCe^2-ce(Bd-Ae)+cCd^2\right)}{c^{3/4}e^3\sqrt{a-cx^4}}+\frac{a^{5/4}\sqrt{1-\frac{cx^4}{a}}(Cd-Be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{c}e^2\sqrt{a-cx^4}}+\frac{3a^{7/4}C\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{3/4}e\sqrt{a-cx^4}}-\frac{3a^{7/4}C\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{3/4}e\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(-ae^2(Cd-Be)-cde(Bd-Ae)+cCd^3\right)}{\sqrt[4]{c}e^4\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(cd^2-ae^2\right)\left(Ae^2-Bde+Cd^2\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de^4\sqrt{a-cx^4}}-\frac{x\sqrt{a-cx^4}(Cd-Be)}{3e^2}+\frac{Cx^3\sqrt{a-cx^4}}{5e}$$

input
```
Int[(Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4))/(d + e*x^2),x]
```

output
```
-1/3*((C*d - B*e)*x*Sqrt[a - c*x^4])/e^2 + (C*x^3*Sqrt[a - c*x^4])/(5*e) -
 (3*a^(7/4)*C*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(5*c^(3/4)*e*Sqrt[a - c*x^4]) - (a^(3/4)*(c*C*d^2 - a*C*e^2 - c*e*(B*d
 - A*e))*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(
c^(3/4)*e^3*Sqrt[a - c*x^4]) + (3*a^(7/4)*C*Sqrt[1 - (c*x^4)/a]*EllipticF[
ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(3/4)*e*Sqrt[a - c*x^4]) + (a^(5/4)
*(C*d - B*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1
])/(3*c^(1/4)*e^2*Sqrt[a - c*x^4]) + (a^(3/4)*(c*C*d^2 - a*C*e^2 - c*e*(B*
d - A*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(c^(3/4)*e^3*Sqrt[a - c*x^4]) + (a^(1/4)*(c*C*d^3 - c*d*e*(B*d - A*e) - a*
e^2*(C*d - B*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)]
, -1])/(c^(1/4)*e^4*Sqrt[a - c*x^4]) - (a^(1/4)*(c*d^2 - a*e^2)*(C*d^2 - B
*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), A
rcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e^4*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.08

method	result
risch	$\frac{x(3Cx^2e+5Be-5Cd)\sqrt{-cx^4+a}}{15e^2}-\frac{-\frac{15\left(Aae^4-Acd^2e^2-Bade^3+Bcd^3e+Cad^2e^2-Ccd^4\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\right)}{e^2d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}}{}$
default	$\frac{Be\left(\frac{x\sqrt{-cx^4+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}\right)+Ce\left(\frac{x^3\sqrt{-cx^4+a}}{5}-\frac{2a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}\sqrt{c}}\right)}{e^2}$
elliptic	Expression too large to display

input
```
int((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*C*e*x^2+5*B*e-5*C*d)*(-c*x^4+a)^(1/2)/e^2-1/15/e^2*(-15*(A*a*e^4
-A*c*d^2*e^2-B*a*d*e^3+B*c*d^3*e+C*a*d^2*e^2-C*c*d^4)/e^2/d/(c^(1/2)/a^(1/
2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*
x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c
^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))-5*(3*A*c*d*e^2+2*B*a*e^3-3*
B*c*d^2*e-2*C*a*d*e^2+3*C*c*d^3)/e^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^
2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(
x*(c^(1/2)/a^(1/2))^(1/2),I)-3/e*(5*A*c*e^2-5*B*c*d*e-2*C*a*e^2+5*C*c*d^2)
*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*
x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))
^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))
```

Fricas [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{d+ex^2}\,dx=\int \frac{(Cx^4+Bx^2+A)\sqrt{-cx^4+a}}{ex^2+d}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d),x, algorithm="fricas"
)
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{d+ex^2}\,dx$$

input
```
integrate((-c*x**4+a)**(1/2)*(C*x**4+B*x**2+A)/(e*x**2+d),x)
```

output
```
Integral(sqrt(a - c*x**4)*(A + B*x**2 + C*x**4)/(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-cx^4+a}}{ex^2+d}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d),x, algorithm="maxima"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)/(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-cx^4+a}}{ex^2+d}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \int \frac{\sqrt{a-c\,x^4}\,(C\,x^4+B\,x^2+A)}{e\,x^2+d}\,dx$$

input
```
int(((a - c*x^4)^(1/2)*(A + B*x^2 + C*x^4))/(d + e*x^2),x)
```

output
```
int(((a - c*x^4)^(1/2)*(A + B*x^2 + C*x^4))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \frac{5\sqrt{-c\,x^4+a}\,bex-5\sqrt{-c\,x^4+a}\,cdx+3\sqrt{-c\,x^4+a}\,ce\,x^3+15\left(\int\frac{\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)a^2e^2-5\left(\int \frac{}{-c}\right.}{}$$

input
```
int((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d),x)
```

output
```
(5*sqrt(a - c*x**4)*b*e*x - 5*sqrt(a - c*x**4)*c*d*x + 3*sqrt(a - c*x**4)*
c*e*x**3 + 15*int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),
x)*a**2*e**2 - 5*int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**
6),x)*a*b*d*e + 5*int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x*
*6),x)*a*c*d**2 - 9*int((sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4
 - c*e*x**6),x)*a*c*e**2 + 15*int((sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2
- c*d*x**4 - c*e*x**6),x)*b*c*d*e - 15*int((sqrt(a - c*x**4)*x**4)/(a*d +
a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c**2*d**2 + 10*int((sqrt(a - c*x**4)*x*
*2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*e**2 - 4*int((sqrt(a - c
*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*d*e)/(15*e**2)
```

3.31 $\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 403

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx = \frac{Cx\sqrt{a-cx^4}}{3e^2}+\frac{(Cd^2-Bde+Ae^2)\,x\sqrt{a-cx^4}}{2de^2\,(d+ex^2)}$$
$$+\frac{a^{3/4}\sqrt[4]{c}(5Cd^2-e(3Bd-Ae))\,\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2de^3\sqrt{a-cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(4aCde^2-3cd(5Cd^2-e(3Bd-Ae))-3\sqrt{a}\sqrt{c}e(5Cd^2-e(3Bd-Ae))\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\text{arc}\right.}{6\sqrt[4]{c}de^4\sqrt{a-cx^4}}$$
$$+\frac{\sqrt[4]{a}(cd^2(5Cd^2-e(3Bd-Ae))-ae^2(3Cd^2-e(Bd+Ae)))\,\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{2\sqrt[4]{c}d^2e^4\sqrt{a-cx^4}}$$

output
```
1/3*C*x*(-c*x^4+a)^(1/2)/e^2+1/2*(A*e^2-B*d*e+C*d^2)*x*(-c*x^4+a)^(1/2)/d/
e^2/(e*x^2+d)+1/2*a^(3/4)*c^(1/4)*(5*C*d^2-e*(-A*e+3*B*d))*(1-c*x^4/a)^(1/
2)*EllipticE(c^(1/4)*x/a^(1/4),I)/d/e^3/(-c*x^4+a)^(1/2)+1/6*a^(1/4)*(4*a*
C*d*e^2-3*c*d*(5*C*d^2-e*(-A*e+3*B*d))-3*a^(1/2)*c^(1/2)*e*(5*C*d^2-e*(-A*
e+3*B*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(1/4)/d/e^4/
(-c*x^4+a)^(1/2)+1/2*a^(1/4)*(c*d^2*(5*C*d^2-e*(-A*e+3*B*d))-a*e^2*(3*C*d^
2-e*(A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/
c^(1/2)/d,I)/c^(1/4)/d^2/e^4/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.26 (sec) , antiderivative size = 1282, normalized size of antiderivative = 3.18

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4))/(d + e*x^2)^2,x]
```

output
```
(5*a*Sqrt[-(Sqrt[c]/Sqrt[a])]*C*d^3*e^2*x - 3*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]
*d^2*e^3*x + 3*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e^4*x + 2*a*Sqrt[-(Sqrt[c]/S
qrt[a])]*C*d^2*e^3*x^3 - 5*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*C*d^3*e^2*x^5 + 3*B*
Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^2*e^3*x^5 - 3*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d*
e^4*x^5 - 2*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*C*d^2*e^3*x^7 - (3*I)*Sqrt[a]*Sqrt[
c]*d*e*(5*C*d^2 + e*(-3*B*d + A*e))*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*Ellipt
icE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*d*(4*a*C*d*e^2 - 3*c*d*
(5*C*d^2 - 3*B*d*e + A*e^2) - 3*Sqrt[a]*Sqrt[c]*e*(5*C*d^2 - 3*B*d*e + A*e
^2))*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sq
rt[a])]*x], -1] - (15*I)*c*C*d^5*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]
*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (9*I)*B*c*d
^4*e*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (3*I)*A*c*d^3*e^2*Sqrt[1 - (c*x^4)/a]*E
llipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x
], -1] + (9*I)*a*C*d^3*e^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(S
qrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (3*I)*a*B*d^2*e^3
*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt
[-(Sqrt[c]/Sqrt[a])]*x], -1] - (3*I)*a*A*d*e^4*Sqrt[1 - (c*x^4)/a]*Ellipti
cPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]
 - (15*I)*c*C*d^4*e*x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(S...
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 737, normalized size of antiderivative = 1.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 2259}$$

$$\int \left(\frac{-ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3}{e^4\sqrt{a-cx^4}\left(d+ex^2\right)}+\frac{aCe^2-c\left(3Cd^2-e(2Bd-Ae)\right)}{e^4\sqrt{a-cx^4}}+\frac{\left(ae^2-cd^2\right)\left(Ae^2-Bc\right.}{e^4\sqrt{a-cx^4}\left(d+e\right.}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^{3/4}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\left(Ae^2-Bde+Cd^2\right)}{2de^3\sqrt{a-cx^4}}-$$
$$\frac{a^{3/4}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(2Cd-Be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{e^3\sqrt{a-cx^4}}+$$
$$\frac{a^{3/4}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(2Cd-Be)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{e^3\sqrt{a-cx^4}}-$$
$$\frac{a^{5/4}C\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{c}e^2\sqrt{a-cx^4}}+$$
$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de^4\sqrt{a-cx^4}}+$$
$$\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(Ae^2-Bde+Cd^2\right)}{2de^4\sqrt{a-cx^4}}+$$
$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(aCe^2-c\left(3Cd^2-e(2Bd-Ae)\right)\right)}{\sqrt[4]{c}e^4\sqrt{a-cx^4}}-$$
$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)\left(Ae^2-Bde+Cd^2\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}d^2e^4\sqrt{a-cx^4}}+$$
$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{2de^2\left(d+ex^2\right)}+\frac{Cx\sqrt{a-cx^4}}{3e^2}$$

input
```
Int[(Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4))/(d + e*x^2)^2,x]
```

output
```
(C*x*Sqrt[a - c*x^4])/(3*e^2) + ((C*d^2 - B*d*e + A*e^2)*x*Sqrt[a - c*x^4]
)/(2*d*e^2*(d + e*x^2)) + (a^(3/4)*c^(1/4)*(2*C*d - B*e)*Sqrt[1 - (c*x^4)/
a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(e^3*Sqrt[a - c*x^4]) + (a^
(3/4)*c^(1/4)*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin
[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*e^3*Sqrt[a - c*x^4]) - (a^(5/4)*C*Sqrt[1
- (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(1/4)*e^2*Sq
rt[a - c*x^4]) - (a^(3/4)*c^(1/4)*(2*C*d - B*e)*Sqrt[1 - (c*x^4)/a]*Ellipt
icF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(e^3*Sqrt[a - c*x^4]) + (a^(1/4)*c^(
1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*E
llipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*e^4*Sqrt[a - c*x^4]) + (a^
(1/4)*(a*C*e^2 - c*(3*C*d^2 - e*(2*B*d - A*e)))*Sqrt[1 - (c*x^4)/a]*Ellipt
icF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e^4*Sqrt[a - c*x^4]) - (a^(
1/4)*(3*c*d^2 - a*e^2)*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*Ellipti
cPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(1/
4)*d^2*e^4*Sqrt[a - c*x^4]) + (a^(1/4)*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e)
- a*e^2*(2*C*d - B*e))*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[
c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e^4*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(345) = 690$.

Time = 5.13 (sec) , antiderivative size = 970, normalized size of antiderivative = 2.41

method	result	size
default	Expression too large to display	970
risch	Expression too large to display	1114
elliptic	Expression too large to display	1396

input
```
int((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
C/e^2*(1/3*x*(-c*x^4+a)^(1/2)+2/3*a/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2
/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x
*(c^(1/2)/a^(1/2))^(1/2),I))+1/e^2*(B*e-2*C*d)*(c*d/e^2/(c^(1/2)/a^(1/2))^
(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+
a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/e*c^(1/2)*a^(1/2)/(c^(1/
2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1
/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-1/e*c^(1/2)*a^
(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2
/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/
d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(
1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)
*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))*a-1/e^2*d/(
c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2
))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/
c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))*c)+1/e^2*(A*e^
2-B*d*e+C*d^2)*(1/2/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)-1/2*c/e^2/(c^(1/2)/a^(1
/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c
*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-1/2*c^(1/2)/d/e*a^(1/
2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^
(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx=\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx$$

input
```
integrate((-c*x**4+a)**(1/2)*(C*x**4+B*x**2+A)/(e*x**2+d)**2,x)
```

output
```
Integral(sqrt(a - c*x**4)*(A + B*x**2 + C*x**4)/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx=\int \frac{(Cx^4+Bx^2+A)\sqrt{-cx^4+a}}{(ex^2+d)^2}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)/(e*x^2 + d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-cx^4+a}}{(ex^2+d)^2}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{a-c\,x^4}\,(C\,x^4+B\,x^2+A)}{(e\,x^2+d)^2}\,dx$$

input
```
int(((a - c*x^4)^(1/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^2,x)
```

output
```
int(((a - c*x^4)^(1/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
int((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^2,x)
```

output
```
(sqrt(a - c*x**4)*a*e*x + 3*sqrt(a - c*x**4)*c*d*x**3 + 8*int(sqrt(a - c*x
**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c
*e**2*x**8),x)*a**2*d**2*e + 8*int(sqrt(a - c*x**4)/(a*d**2 + 2*a*d*e*x**2
 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*d*e**2*
x**2 + int((sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 -
c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*c*d*e**2 + int((sqrt(a - c*
x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x
**6 - c*e**2*x**8),x)*a*c*e**3*x**2 - 9*int((sqrt(a - c*x**4)*x**6)/(a*d**
2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8)
,x)*b*c*d**2*e - 9*int((sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*
e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*b*c*d*e**2*x**2 +
 15*int((sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d
**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*c**2*d**3 + 15*int((sqrt(a - c*x
**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x*
*6 - c*e**2*x**8),x)*c**2*d**2*e*x**2 + int((sqrt(a - c*x**4)*x**2)/(a*d**
2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8)
,x)*a**2*d*e**2 + int((sqrt(a - c*x**4)*x**2)/(a*d**2 + 2*a*d*e*x**2 + a*e
**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*e**3*x**2 + 9
*int((sqrt(a - c*x**4)*x**2)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2
*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*b*d**2*e + 9*int((sqrt(a - c*x...
```

3.32 $\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 597

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \frac{(Cd^2-Bde+Ae^2)\,x\sqrt{a-cx^4}}{4de^2\,(d+ex^2)^2}$$

$$-\frac{(cd^2(7Cd^2-e(3Bd+Ae))-ae^2(5Cd^2-e(Bd+3Ae)))\,x\sqrt{a-cx^4}}{8d^2e^2\,(cd^2-ae^2)\,(d+ex^2)}$$

$$-\frac{a^{3/4}\sqrt[4]{c}(cd^2(15Cd^2-e(3Bd+Ae))-ae^2(13Cd^2-e(Bd+3Ae)))\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2e^3\,(cd^2-ae^2)\sqrt{a-cx^4}}$$

$$+\frac{\sqrt[4]{a}\sqrt[4]{c}\big(cd^2(15Cd^2-e(3Bd+Ae))+2\sqrt{a}\sqrt{c}de(15Cd^2-e(3Bd+Ae))+ae^2(13Cd^2-e(Bd+3Ae)}{8d^2e^4\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{a-cx^4}}$$

$$+\frac{\sqrt[4]{a}(2acd^2e^2(11Cd^2-3e(Bd-Ae))-c^2d^4(15Cd^2-e(3Bd+Ae))-a^2e^4(3Cd^2+e(Bd+3Ae)))\sqrt{\;}}{8\sqrt[4]{c}d^3e^4\,(cd^2-ae^2)\sqrt{a-cx^4}}$$

output
```
1/4*(A*e^2-B*d*e+C*d^2)*x*(-c*x^4+a)^(1/2)/d/e^2/(e*x^2+d)^2-1/8*(c*d^2*(7
*C*d^2-e*(A*e+3*B*d))-a*e^2*(5*C*d^2-e*(3*A*e+B*d)))*x*(-c*x^4+a)^(1/2)/d^
2/e^2/(-a*e^2+c*d^2)/(e*x^2+d)-1/8*a^(3/4)*c^(1/4)*(c*d^2*(15*C*d^2-e*(A*e
+3*B*d))-a*e^2*(13*C*d^2-e*(3*A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/
4)*x/a^(1/4),I)/d^2/e^3/(-a*e^2+c*d^2)/(-c*x^4+a)^(1/2)+1/8*a^(1/4)*c^(1/4
)*(c*d^2*(15*C*d^2-e*(A*e+3*B*d))+2*a^(1/2)*c^(1/2)*d*e*(15*C*d^2-e*(A*e+3
*B*d))+a*e^2*(13*C*d^2-e*(3*A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)
*x/a^(1/4),I)/d^2/e^4/(c^(1/2)*d+a^(1/2)*e)/(-c*x^4+a)^(1/2)+1/8*a^(1/4)*(
2*a*c*d^2*e^2*(11*C*d^2-3*e*(-A*e+B*d))-c^2*d^4*(15*C*d^2-e*(A*e+3*B*d))-a
^2*e^4*(3*C*d^2+e*(3*A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(
1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4)/d^3/e^4/(-a*e^2+c*d^2)/(-c*x^4+a)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.04 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx$$

$$= \frac{\frac{de^2x(a-cx^4)\left(2d(cd^2-ae^2)\left(Cd^2+e(-Bd+Ae)\right)-\left(c\left(7Cd^4-d^2e(3Bd+Ae)\right)+ae^2\left(-5Cd^2+e(Bd+3Ae)\right)\right)(d+ex^2)\right)}{(cd^2-ae^2)(d+ex^2)^2} - \frac{i\sqrt{1-\frac{cx^4}{a}}\Big(-\sqrt{a}\sqrt{c}de}{}}{}$$

input
```
Integrate[(Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4))/(d + e*x^2)^3,x]
```

output
```
((d*e^2*x*(a - c*x^4)*(2*d*(c*d^2 - a*e^2)*(C*d^2 + e*(-(B*d) + A*e)) - (c
*(7*C*d^4 - d^2*e*(3*B*d + A*e)) + a*e^2*(-5*C*d^2 + e*(B*d + 3*A*e)))*(d
+ e*x^2)))/((c*d^2 - a*e^2)*(d + e*x^2)^2) - (I*Sqrt[1 - (c*x^4)/a]*(-(Sqr
t[a]*Sqrt[c]*d*e*(-15*c*C*d^4 + 13*a*C*d^2*e^2 + c*d^2*e*(3*B*d + A*e) - a
*e^3*(B*d + 3*A*e))*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])
- Sqrt[c]*d*(Sqrt[c]*d - Sqrt[a]*e)*(2*Sqrt[a]*Sqrt[c]*d*e*(15*C*d^2 - e*(
3*B*d + A*e)) + c*(15*C*d^4 - d^2*e*(3*B*d + A*e)) - a*e^2*(-13*C*d^2 + e*
(B*d + 3*A*e)))*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (-2
*a*c*d^2*e^2*(11*C*d^2 + 3*e*(-(B*d) + A*e)) + c^2*(15*C*d^6 - d^4*e*(3*B*
d + A*e)) + a^2*e^4*(3*C*d^2 + e*(B*d + 3*A*e)))*EllipticPi[-((Sqrt[a]*e)/
(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(Sqrt[-(Sqrt[c]
/Sqrt[a])]*(-(c*d^2) + a*e^2)))/(8*d^3*e^4*Sqrt[a - c*x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1223 vs. 2(597) = 1194.

Time = 2.27 (sec) , antiderivative size = 1223, normalized size of antiderivative = 2.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}\left(A+Bx^2+Cx^4\right)}{\left(d+ex^2\right)^3} \, dx$$

$$\Big\downarrow \text{ 2259}$$

$$\int \left(\frac{-ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3}{e^4\sqrt{a-cx^4}\left(d+ex^2\right)^2}+\frac{aCe^2-c\left(6Cd^2-e(3Bd-Ae)\right)}{e^4\sqrt{a-cx^4}\left(d+ex^2\right)}+\frac{\left(ae^2-cd^2\right)\left(Ae^2-Bc\right.}{e^4\sqrt{a-cx^4}\left(d+e\right.}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3(3cd^2-ae^2)(Cd^2-Bed+Ae^2)\sqrt{a-cx^4}x}{8d^2e^2(cd^2-ae^2)(ex^2+d)}-$$
$$\frac{(4cCd^3-ce(3Bd-2Ae)d-ae^2(2Cd-Be))\sqrt{a-cx^4}x}{2de^2(cd^2-ae^2)(ex^2+d)}+\frac{(Cd^2-Bed+Ae^2)\sqrt{a-cx^4}x}{4de^2(ex^2+d)^2}+$$
$$\frac{3a^{3/4}\sqrt[4]{c}(3cd^2-ae^2)(Cd^2-Bed+Ae^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2e^3(cd^2-ae^2)\sqrt{a-cx^4}}-$$
$$\frac{a^{3/4}\sqrt[4]{c}(4cCd^3-ce(3Bd-2Ae)d-ae^2(2Cd-Be))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2de^3(cd^2-ae^2)\sqrt{a-cx^4}}-$$
$$\frac{a^{3/4}\sqrt[4]{c}C\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{e^3\sqrt{a-cx^4}}+$$
$$\frac{\sqrt[4]{a}c^{3/4}(3Cd-Be)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{e^4\sqrt{a-cx^4}}+$$
$$\frac{\sqrt[4]{a}\sqrt[4]{c}(7cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2)(Cd^2-Bed+Ae^2)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8d^2e^4(\sqrt{c}d+\sqrt{a}e)\sqrt{a-cx^4}}-$$
$$\frac{\sqrt[4]{a}\sqrt[4]{c}(4cCd^3-ce(3Bd-2Ae)d-ae^2(2Cd-Be))\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2de^4(\sqrt{c}d+\sqrt{a}e)\sqrt{a-cx^4}}+$$
$$\frac{a^{3/4}\sqrt[4]{c}C\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{e^3\sqrt{a-cx^4}}-$$
$$\frac{3\sqrt[4]{a}(Cd^2-Bed+Ae^2)(5c^2d^4-2ace^2d^2+a^2e^4)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{c}d^3e^4(cd^2-ae^2)\sqrt{a-cx^4}}+$$
$$\frac{\sqrt[4]{a}(3cd^2-ae^2)(4cCd^3-ce(3Bd-2Ae)d-ae^2(2Cd-Be))\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}d^2e^4(cd^2-ae^2)\sqrt{a-cx^4}}$$
$$\frac{\sqrt[4]{a}(aCe^2-c(6Cd^2-e(3Bd-Ae)))\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de^4\sqrt{a-cx^4}}$$

input
```
Int[(Sqrt[a - c*x^4]*(A + B*x^2 + C*x^4))/(d + e*x^2)^3,x]
```

output
```
((C*d^2 - B*d*e + A*e^2)*x*Sqrt[a - c*x^4])/(4*d*e^2*(d + e*x^2)^2) + (3*(
3*c*d^2 - a*e^2)*(C*d^2 - B*d*e + A*e^2)*x*Sqrt[a - c*x^4])/(8*d^2*e^2*(c*
d^2 - a*e^2)*(d + e*x^2)) - ((4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) - a*e^2*(2
*C*d - B*e))*x*Sqrt[a - c*x^4])/(2*d*e^2*(c*d^2 - a*e^2)*(d + e*x^2)) - (a
^(3/4)*c^(1/4)*C*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)]
, -1])/(e^3*Sqrt[a - c*x^4]) + (3*a^(3/4)*c^(1/4)*(3*c*d^2 - a*e^2)*(C*d^2
 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)
], -1])/(8*d^2*e^3*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) - (a^(3/4)*c^(1/4)*(4*
c*C*d^3 - c*d*e*(3*B*d - 2*A*e) - a*e^2*(2*C*d - B*e))*Sqrt[1 - (c*x^4)/a]
*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*e^3*(c*d^2 - a*e^2)*Sqrt
[a - c*x^4]) + (a^(3/4)*c^(1/4)*C*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^
(1/4)*x)/a^(1/4)], -1])/(e^3*Sqrt[a - c*x^4]) + (a^(1/4)*c^(3/4)*(3*C*d -
B*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(e^4*
Sqrt[a - c*x^4]) + (a^(1/4)*c^(1/4)*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a
*e^2)*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4
)*x)/a^(1/4)], -1])/(8*d^2*e^4*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt[a - c*x^4]) -
(a^(1/4)*c^(1/4)*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) - a*e^2*(2*C*d - B*e))
*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*e^4*
(Sqrt[c]*d + Sqrt[a]*e)*Sqrt[a - c*x^4]) - (3*a^(1/4)*(C*d^2 - B*d*e + A*e
^2)*(5*c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticP...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1831 vs. $2(535) = 1070$.

Time = 2.09 (sec) , antiderivative size = 1832, normalized size of antiderivative = 3.07

method	result	size
default	Expression too large to display	1832
elliptic	Expression too large to display	3049

input
```
int((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
C/e^2*(c*d/e^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^
(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1
/2),I)+1/e*c^(1/2)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))
^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)
/a^(1/2))^(1/2),I)-1/e*c^(1/2)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*
x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
E(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/
a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x
*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^
(1/2)/a^(1/2))^(1/2))*a-1/e^2*d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(
1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c
^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/
2)/a^(1/2))^(1/2))*c)+(B*e-2*C*d)/e^2*(1/2/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)-
1/2*c/e^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)
*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I
)-1/2*c^(1/2)/d/e*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^
(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/
a^(1/2))^(1/2),I)+1/2*c^(1/2)/d/e*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/
2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticE(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/2/d^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^...
```

Fricas [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-cx^4+a}}{(ex^2+d)^3}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^3,x, algorithm="frica
s")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d
^2*e*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx$$

input
```
integrate((-c*x**4+a)**(1/2)*(C*x**4+B*x**2+A)/(e*x**2+d)**3,x)
```

output
```
Integral(sqrt(a - c*x**4)*(A + B*x**2 + C*x**4)/(d + e*x**2)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-cx^4+a}}{(ex^2+d)^3}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^3,x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)/(e*x^2 + d)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-cx^4+a}}{(ex^2+d)^3}\,dx$$

input
```
integrate((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^3,x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \int \frac{\sqrt{a-c\,x^4}\,(C\,x^4+B\,x^2+A)}{(e\,x^2+d)^3}\,dx$$

input
```
int(((a - c*x^4)^(1/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^3,x)
```

output
```
int(((a - c*x^4)^(1/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \text{too large to display}$$

input
```
int((-c*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^3,x)
```

output
```
( - sqrt(a - c*x**4)*a*e*x + 3*sqrt(a - c*x**4)*c*d*x**3 + 4*int(sqrt(a -
c*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3
*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*a**2*d**3*e +
 8*int(sqrt(a - c*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e*
*3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),
x)*a**2*d**2*e**2*x**2 + 4*int(sqrt(a - c*x**4)/(a*d**3 + 3*a*d**2*e*x**2
+ 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e*
*2*x**8 - c*e**3*x**10),x)*a**2*d*e**3*x**4 + int((sqrt(a - c*x**4)*x**6)/
(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 -
3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*a*c*d**2*e**2 + 2*int
((sqrt(a - c*x**4)*x**6)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e
**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10)
,x)*a*c*d*e**3*x**2 + int((sqrt(a - c*x**4)*x**6)/(a*d**3 + 3*a*d**2*e*x**
2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*
e**2*x**8 - c*e**3*x**10),x)*a*c*e**4*x**4 - 3*int((sqrt(a - c*x**4)*x**6)
/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 -
 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*b*c*d**3*e - 6*int((
sqrt(a - c*x**4)*x**6)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**
3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x
)*b*c*d**2*e**2*x**2 - 3*int((sqrt(a - c*x**4)*x**6)/(a*d**3 + 3*a*d**2...
```

3.33 $\int (d+ex^2)^3 (a-cx^4)^{3/2} (A+Bx^2+Cx^4)\, dx$

Optimal result

Integrand size = 34, antiderivative size = 716

$$\int (d+ex^2)^3 (a-cx^4)^{3/2} (A+Bx^2+Cx^4)\, dx = \frac{2ax(1105(3Acd(11cd^2+3ae^2)+a(ae^2(3Cd+Be)+3cd^2(Cd+3Be)))+77(17Bcd(13cd^2+\cdots}{255255c^2}$$
$$+\frac{x(663(3Acd(11cd^2+3ae^2)+a(ae^2(3Cd+Be)+3cd^2(Cd+3Be)))+77(17Bcd(13cd^2+9ae^2)+3e(17\cdots}{153153c^2}$$
$$-\frac{(ae^2(3Cd+Be)+3cd(Cd^2+3e(Bd+Ae)))x(a-cx^4)^{5/2}}{33c^2}$$
$$-\frac{e(7aCe^2+17c(3Cd^2+e(3Bd+Ae)))x^3(a-cx^4)^{5/2}}{221c^2}$$
$$-\frac{e^2(3Cd+Be)x^5(a-cx^4)^{5/2}}{15c}-\frac{Ce^3x^7(a-cx^4)^{5/2}}{17c}$$
$$+\frac{4a^{11/4}(17Bcd(13cd^2+9ae^2)+3e(17Ac(13cd^2+ae^2)+aC(51cd^2+7ae^2)))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\cdots\right.}{3315c^{11/4}\sqrt{a-cx^4}}$$
$$+\frac{4a^{9/4}(1105\sqrt{c}(3Acd(11cd^2+3ae^2)+a(ae^2(3Cd+Be)+3cd^2(Cd+3Be)))-77\sqrt{a}(17Bcd(13cd^2+9a\cdots}{255255c^{11/4}\sqrt{a-\cdots}}$$

output
```
2/255255*a*x*(3315*A*c*d*(3*a*e^2+11*c*d^2)+1105*a*(a*e^2*(B*e+3*C*d)+3*c*
d^2*(3*B*e+C*d))+77*(17*B*c*d*(9*a*e^2+13*c*d^2)+3*e*(17*A*c*(a*e^2+13*c*d
^2)+a*C*(7*a*e^2+51*c*d^2)))*x^2)*(-c*x^4+a)^(1/2)/c^2+1/153153*x*(1989*A*
c*d*(3*a*e^2+11*c*d^2)+663*a*(a*e^2*(B*e+3*C*d)+3*c*d^2*(3*B*e+C*d))+77*(1
7*B*c*d*(9*a*e^2+13*c*d^2)+3*e*(17*A*c*(a*e^2+13*c*d^2)+a*C*(7*a*e^2+51*c*
d^2)))*x^2)*(-c*x^4+a)^(3/2)/c^2-1/33*(a*e^2*(B*e+3*C*d)+3*c*d*(C*d^2+3*e*
(A*e+B*d)))*x*(-c*x^4+a)^(5/2)/c^2-1/221*e*(7*C*a*e^2+17*c*(3*C*d^2+e*(A*e
+3*B*d)))*x^3*(-c*x^4+a)^(5/2)/c^2-1/15*e^2*(B*e+3*C*d)*x^5*(-c*x^4+a)^(5/
2)/c-1/17*C*e^3*x^7*(-c*x^4+a)^(5/2)/c+4/3315*a^(11/4)*(17*B*c*d*(9*a*e^2+
13*c*d^2)+3*e*(17*A*c*(a*e^2+13*c*d^2)+a*C*(7*a*e^2+51*c*d^2)))*(1-c*x^4/a
)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(11/4)/(-c*x^4+a)^(1/2)+4/255255*
a^(9/4)*(1105*c^(1/2)*(3*A*c*d*(3*a*e^2+11*c*d^2)+a*(a*e^2*(B*e+3*C*d)+3*c
*d^2*(3*B*e+C*d)))-77*a^(1/2)*(17*B*c*d*(9*a*e^2+13*c*d^2)+3*e*(17*A*c*(a*
e^2+13*c*d^2)+a*C*(7*a*e^2+51*c*d^2))))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4
)*x/a^(1/4),I)/c^(11/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.65 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.67

$$\int (d+ex^2)^3 (a-cx^4)^{3/2} (A+Bx^2+Cx^4) \, dx = \frac{x\sqrt{a-cx^4}\left(-3315cd(Cd^2+3e(Bd+Ae))(a-cx^4)^2\sqrt{1-\frac{cx^4}{a}}-2805ce(3Cd^2+e(3Bd+A\right.}{}$$

input
```
Integrate[(d + e*x^2)^3*(a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
(x*Sqrt[a - c*x^4]*(-3315*c*d*(C*d^2 + 3*e*(B*d + A*e))*(a - c*x^4)^2*Sqrt
[1 - (c*x^4)/a] - 2805*c*e*(3*C*d^2 + e*(3*B*d + A*e))*x^2*(a - c*x^4)^2*S
qrt[1 - (c*x^4)/a] - 2431*c*e^2*(3*C*d + B*e)*x^4*(a - c*x^4)^2*Sqrt[1 - (
c*x^4)/a] - 2145*c*C*e^3*x^6*(a - c*x^4)^2*Sqrt[1 - (c*x^4)/a] + 36465*a*A
*c^2*d^3*Hypergeometric2F1[-3/2, 1/4, 5/4, (c*x^4)/a] + 3315*a^2*c*d*(C*d^
2 + 3*e*(B*d + A*e))*Hypergeometric2F1[-3/2, 1/4, 5/4, (c*x^4)/a] - 1105*a
*e^2*(3*C*d + B*e)*((a - c*x^4)^2*Sqrt[1 - (c*x^4)/a] - a^2*Hypergeometric
2F1[-3/2, 1/4, 5/4, (c*x^4)/a]) + 12155*a*c^2*d^2*(B*d + 3*A*e)*x^2*Hyperg
eometric2F1[-3/2, 3/4, 7/4, (c*x^4)/a] + 2805*a^2*c*e*(3*C*d^2 + e*(3*B*d
+ A*e))*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, (c*x^4)/a] - 1155*a*C*e^3*x^
2*((a - c*x^4)^2*Sqrt[1 - (c*x^4)/a] - a^2*Hypergeometric2F1[-3/2, 3/4, 7/
4, (c*x^4)/a])))/(36465*c^2*Sqrt[1 - (c*x^4)/a])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2173 vs. 2(716) = 1432.

Time = 2.61 (sec) , antiderivative size = 2173, normalized size of antiderivative = 3.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^{3/2} \left(d+ex^2\right)^3 \left(A+Bx^2+Cx^4\right) \, dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{a^2d^2x^2(3Ae+Bd)}{\sqrt{a-cx^4}}+\frac{a^2Ad^3}{\sqrt{a-cx^4}}+\frac{cx^{12}\left(-2ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)}{\sqrt{a-cx^4}}+\frac{adx^4\left(ad(3Be+C\right.}{\sqrt{a}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{17}cCe^3\sqrt{a-cx^4}x^{15}-\frac{1}{15}ce^2(3Cd+Be)\sqrt{a-cx^4}x^{13}-\frac{15}{221}aCe^3\sqrt{a-cx^4}x^{11}-\frac{1}{13}e\left(3cCd^2-2aCe^2+ce(3Bd+Ae)\right)\sqrt{a-cx^4}x^{11}-\frac{13}{165}ae^2(3Cd+Be)\sqrt{a-cx^4}x^9+\frac{1}{11}\left(2ae^2(3Cd+Be)-c\left(Cd^3+3e(Bd+Ae)d\right)\right)\sqrt{a-cx^4}x^9-\frac{55a^2Ce^3\sqrt{a-cx^4}x^7}{663c}-\frac{11ae\left(3cCd^2-2aCe^2+ce(3Bd+Ae)\right)\sqrt{a-cx^4}x^7}{117c}-\frac{\left(Bcd\left(cd^2-6ae^2\right)+Ace\left(3cd^2-2ae^2\right)-aCe\left(6cd^2-ae^2\right)\right)\sqrt{a-cx^4}x^7}{9c}-\frac{39a^2e^2(3Cd+Be)\sqrt{a-cx^4}x^5}{385c}-\frac{9a\left(cCd^3+3ce(Bd+Ae)d-2ae^2(3Cd+Be)\right)\sqrt{a-cx^4}x^5}{77c}-\frac{\left(Acd\left(cd^2-6ae^2\right)+a\left(ae^2(3Cd+Be)-2cd^2(Cd+3Be)\right)\right)\sqrt{a-cx^4}x^5}{7c}-\frac{77a^3Ce^3\sqrt{a-cx^4}x^3}{663c^2}+\frac{a\left(2Bcd^3+6Aced^2-3aCed^2-3aBe^2d-aAe^3\right)\sqrt{a-cx^4}x^3}{5c}-\frac{77a^2e\left(3cCd^2-2aCe^2+ce(3Bd+Ae)\right)\sqrt{a-cx^4}x^3}{585c^2}-\frac{7a\left(Bcd\left(cd^2-6ae^2\right)+Ace\left(3cd^2-2ae^2\right)-aCe\left(6cd^2-ae^2\right)\right)\sqrt{a-cx^4}x^3}{45c^2}-\frac{13a^3e^2(3Cd+Be)\sqrt{a-cx^4}x}{77c^2}-\frac{15a^2\left(cCd^3+3ce(Bd+Ae)d-2ae^2(3Cd+Be)\right)\sqrt{a-cx^4}x}{77c^2}-\frac{ad\left(ad(Cd+3Be)-A\left(2cd^2-3ae^2\right)\right)\sqrt{a-cx^4}x}{3c}-\frac{5a\left(Acd\left(cd^2-6ae^2\right)+a\left(ae^2(3Cd+Be)-2cd^2(Cd+3Be)\right)\right)\sqrt{a-cx^4}x}{21c^2}+\frac{77a^{19/4}Ce^3\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{221c^{11/4}\sqrt{a-cx^4}}+\frac{a^{11/4}d^2(Bd+3Ae)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{3a^{11/4}\left(2Bcd^3+6Aced^2-3aCed^2-3aBe^2d-aAe^3\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{77a^{15/4}e\left(3cCd^2-2aCe^2+ce(3Bd+Ae)\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{195c^{11/4}\sqrt{a-cx^4}}+\frac{7a^{11/4}\left(Bcd\left(cd^2-6ae^2\right)+Ace\left(3cd^2-2ae^2\right)-aCe\left(6cd^2-ae^2\right)\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15c^{11/4}\sqrt{a-cx^4}}+\frac{a^{9/4}Ad^3\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{77a^{19/4}Ce^3\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{221c^{11/4}\sqrt{a-cx^4}}-\frac{a^{11/4}d^2(Bd+3Ae)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}+$$

input
```
Int[(d + e*x^2)^3*(a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
(-13*a^3*e^2*(3*C*d + B*e)*x*Sqrt[a - c*x^4])/(77*c^2) - (15*a^2*(c*C*d^3
+ 3*c*d*e*(B*d + A*e) - 2*a*e^2*(3*C*d + B*e))*x*Sqrt[a - c*x^4])/(77*c^2)
 - (a*d*(a*d*(C*d + 3*B*e) - A*(2*c*d^2 - 3*a*e^2))*x*Sqrt[a - c*x^4])/(3*
c) - (5*a*(A*c*d*(c*d^2 - 6*a*e^2) + a*(a*e^2*(3*C*d + B*e) - 2*c*d^2*(C*d
 + 3*B*e)))*x*Sqrt[a - c*x^4])/(21*c^2) - (77*a^3*C*e^3*x^3*Sqrt[a - c*x^4
])/(663*c^2) + (a*(2*B*c*d^3 + 6*A*c*d^2*e - 3*a*C*d^2*e - 3*a*B*d*e^2 - a
*A*e^3)*x^3*Sqrt[a - c*x^4])/(5*c) - (77*a^2*e*(3*c*C*d^2 - 2*a*C*e^2 + c*
e*(3*B*d + A*e))*x^3*Sqrt[a - c*x^4])/(585*c^2) - (7*a*(B*c*d*(c*d^2 - 6*a
*e^2) + A*c*e*(3*c*d^2 - 2*a*e^2) - a*C*e*(6*c*d^2 - a*e^2))*x^3*Sqrt[a -
c*x^4])/(45*c^2) - (39*a^2*e^2*(3*C*d + B*e)*x^5*Sqrt[a - c*x^4])/(385*c)
- (9*a*(c*C*d^3 + 3*c*d*e*(B*d + A*e) - 2*a*e^2*(3*C*d + B*e))*x^5*Sqrt[a
- c*x^4])/(77*c) - ((A*c*d*(c*d^2 - 6*a*e^2) + a*(a*e^2*(3*C*d + B*e) - 2*
c*d^2*(C*d + 3*B*e)))*x^5*Sqrt[a - c*x^4])/(7*c) - (55*a^2*C*e^3*x^7*Sqrt[
a - c*x^4])/(663*c) - (11*a*e*(3*c*C*d^2 - 2*a*C*e^2 + c*e*(3*B*d + A*e))*
x^7*Sqrt[a - c*x^4])/(117*c) - ((B*c*d*(c*d^2 - 6*a*e^2) + A*c*e*(3*c*d^2
- 2*a*e^2) - a*C*e*(6*c*d^2 - a*e^2))*x^7*Sqrt[a - c*x^4])/(9*c) - (13*a*e
^2*(3*C*d + B*e)*x^9*Sqrt[a - c*x^4])/165 + ((2*a*e^2*(3*C*d + B*e) - c*(C
*d^3 + 3*d*e*(B*d + A*e)))*x^9*Sqrt[a - c*x^4])/11 - (15*a*C*e^3*x^11*Sqrt
[a - c*x^4])/221 - (e*(3*c*C*d^2 - 2*a*C*e^2 + c*e*(3*B*d + A*e))*x^11*Sqr
t[a - c*x^4])/13 - (c*e^2*(3*C*d + B*e)*x^13*Sqrt[a - c*x^4])/15 - (c*C...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 860, normalized size of antiderivative = 1.20

method	result
default	$A\,d^3\left(-\frac{c\,x^5\sqrt{-c\,x^4+a}}{7}+\frac{3ax\sqrt{-c\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)+d^2(3Ae+Bd)\left(-\right.$
risch	Expression too large to display
elliptic	Expression too large to display

input
```
int((e*x^2+d)^3*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
A*d^3*(-1/7*c*x^5*(-c*x^4+a)^(1/2)+3/7*a*x*(-c*x^4+a)^(1/2)+4/7*a^2/(c^(1/
2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1
/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))+d^2*(3*A*e+B*
d)*(-1/9*c*x^7*(-c*x^4+a)^(1/2)+11/45*a*x^3*(-c*x^4+a)^(1/2)-4/15*a^(5/2)/
(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/
2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)
-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))+e^2*(B*e+3*C*d)*(-1/15*c*x^13*(-
c*x^4+a)^(1/2)+17/165*a*x^9*(-c*x^4+a)^(1/2)-4/385*a^2/c*x^5*(-c*x^4+a)^(1
/2)-4/231*a^3/c^2*x*(-c*x^4+a)^(1/2)+4/231*a^4/c^2/(c^(1/2)/a^(1/2))^(1/2)
*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1
/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))+e*(A*e^2+3*B*d*e+3*C*d^2)*(-1/
13*c*x^11*(-c*x^4+a)^(1/2)+5/39*a*x^7*(-c*x^4+a)^(1/2)-4/195*a^2/c*x^3*(-c
*x^4+a)^(1/2)-4/65*a^(7/2)/c^(3/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/
a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x
*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))+d*(3*
A*e^2+3*B*d*e+C*d^2)*(-1/11*c*x^9*(-c*x^4+a)^(1/2)+13/77*a*x^5*(-c*x^4+a)^
(1/2)-4/77*a^2/c*x*(-c*x^4+a)^(1/2)+4/77*a^3/c/(c^(1/2)/a^(1/2))^(1/2)*(1-
c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*
EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))+e^3*C*(-1/17*c*x^15*(-c*x^4+a)^(1/
2)+19/221*a*x^11*(-c*x^4+a)^(1/2)-4/663*a^2/c*x^7*(-c*x^4+a)^(1/2)-28/3...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.03

$$\int \left(d+ex^2\right)^3 \left(a-cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="frica
s")
```

output
```
-1/765765*(924*(221*B*a^2*c^2*d^3 + 153*B*a^3*c*d*e^2 + 51*(3*C*a^3*c + 13
*A*a^2*c^2)*d^2*e + 3*(7*C*a^4 + 17*A*a^3*c)*e^3)*sqrt(-c)*x*(a/c)^(3/4)*e
lliptic_e(arcsin((a/c)^(1/4)/x), -1) - 12*(221*((77*B + 15*C)*a^2*c^2 + 16
5*A*a*c^3)*d^3 + 51*(231*C*a^3*c + 13*(77*A + 15*B)*a^2*c^2)*d^2*e + 51*((
231*B + 65*C)*a^3*c + 195*A*a^2*c^2)*d*e^2 + (1617*C*a^4 + 17*(231*A + 65*
B)*a^3*c)*e^3)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1
) + (45045*C*c^4*e^3*x^16 + 51051*(3*C*c^4*d*e^2 + B*c^4*e^3)*x^14 + 3465*
(51*C*c^4*d^2*e + 51*B*c^4*d*e^2 - (19*C*a*c^3 - 17*A*c^4)*e^3)*x^12 + 464
1*(15*C*c^4*d^3 + 45*B*c^4*d^2*e - 17*B*a*c^3*e^3 - 3*(17*C*a*c^3 - 15*A*c
^4)*d*e^2)*x^10 + 385*(221*B*c^4*d^3 - 765*B*a*c^3*d*e^2 - 51*(15*C*a*c^3
- 13*A*c^4)*d^2*e + 3*(4*C*a^2*c^2 - 85*A*a*c^3)*e^3)*x^8 + 204204*B*a^2*c
^2*d^3 + 141372*B*a^3*c*d*e^2 - 1989*(195*B*a*c^3*d^2*e - 4*B*a^2*c^2*e^3
+ 5*(13*C*a*c^3 - 11*A*c^4)*d^3 - 3*(4*C*a^2*c^2 - 65*A*a*c^3)*d*e^2)*x^6
- 77*(2431*B*a*c^3*d^3 - 612*B*a^2*c^2*d*e^2 - 51*(12*C*a^2*c^2 - 143*A*a*
c^3)*d^2*e - 12*(7*C*a^3*c + 17*A*a^2*c^2)*e^3)*x^4 + 47124*(3*C*a^3*c + 1
3*A*a^2*c^2)*d^2*e + 2772*(7*C*a^4 + 17*A*a^3*c)*e^3 + 3315*(36*B*a^2*c^2*
d^2*e + 4*B*a^3*c*e^3 + 3*(4*C*a^2*c^2 - 33*A*a*c^3)*d^3 + 12*(C*a^3*c + 3
*A*a^2*c^2)*d*e^2)*x^2)*sqrt(-c*x^4 + a))/(c^3*x)
```

Sympy [A] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 1200, normalized size of antiderivative = 1.68

$$\int \left(d+ex^2\right)^3 \left(a-cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)**3*(-c*x**4+a)**(3/2)*(C*x**4+B*x**2+A),x)
```

output
```
A*a**(3/2)*d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(2
*I*pi)/a)/(4*gamma(5/4)) + 3*A*a**(3/2)*d**2*e*x**3*gamma(3/4)*hyper((-1/2
, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) + 3*A*a**(3/2)*
d*e**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)
/a)/(4*gamma(9/4)) + A*a**(3/2)*e**3*x**7*gamma(7/4)*hyper((-1/2, 7/4), (1
1/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11/4)) - A*sqrt(a)*c*d**3*x**5
*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamm
a(9/4)) - 3*A*sqrt(a)*c*d**2*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,),
 c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11/4)) - 3*A*sqrt(a)*c*d*e**2*x**9*g
amma(9/4)*hyper((-1/2, 9/4), (13/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma
(13/4)) - A*sqrt(a)*c*e**3*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,),
c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(15/4)) + B*a**(3/2)*d**3*x**3*gamma(3
/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4))
+ 3*B*a**(3/2)*d**2*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*ex
p_polar(2*I*pi)/a)/(4*gamma(9/4)) + 3*B*a**(3/2)*d*e**2*x**7*gamma(7/4)*hy
per((-1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11/4)) + B*
a**(3/2)*e**3*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), c*x**4*exp_polar
(2*I*pi)/a)/(4*gamma(13/4)) - B*sqrt(a)*c*d**3*x**7*gamma(7/4)*hyper((-1/2
, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11/4)) - 3*B*sqrt(a)
*c*d**2*e*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), c*x**4*exp_polar(...
```

Maxima [F]

$$\int (d+ex^2)^3 (a-cx^4)^{3/2} (A+Bx^2+Cx^4)\,dx = \int (Cx^4+Bx^2+A)(-cx^4+a)^{\frac{3}{2}}(ex^2+d)^3\,dx$$

input
```
integrate((e*x^2+d)^3*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)*(e*x^2 + d)^3, x)
```

Giac [F]

$$\int \left(d+ex^2\right)^3 \left(a-cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right)\, dx = \int \left(Cx^4+Bx^2+A\right)\left(-cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)^3 dx$$

input
```
integrate((e*x^2+d)^3*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)*(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^3 \left(a-cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right)\, dx = \int \left(a-c\,x^4\right)^{3/2} \left(e\,x^2+d\right)^3 \left(C\,x^4+B\,x^2+A\right)\, dx$$

input
```
int((a - c*x^4)^(3/2)*(d + e*x^2)^3*(A + B*x^2 + C*x^4),x)
```

output
```
int((a - c*x^4)^(3/2)*(d + e*x^2)^3*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^3 \left(a-cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right)\, dx = \text{Too large to display}$$

input
```
int((e*x^2+d)^3*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x)
```

output
```
( - 13260*sqrt(a - c*x**4)*a**3*b*e**3*x - 159120*sqrt(a - c*x**4)*a**3*c*
d*e**2*x - 22176*sqrt(a - c*x**4)*a**3*c*e**3*x**3 - 119340*sqrt(a - c*x**
4)*a**2*b*c*d**2*e*x - 47124*sqrt(a - c*x**4)*a**2*b*c*d*e**2*x**3 - 7956*
sqrt(a - c*x**4)*a**2*b*c*e**3*x**5 + 288405*sqrt(a - c*x**4)*a**2*c**2*d*
*3*x + 514437*sqrt(a - c*x**4)*a**2*c**2*d**2*e*x**3 + 363987*sqrt(a - c*x
**4)*a**2*c**2*d*e**2*x**5 + 93555*sqrt(a - c*x**4)*a**2*c**2*e**3*x**7 +
187187*sqrt(a - c*x**4)*a*b*c**2*d**3*x**3 + 387855*sqrt(a - c*x**4)*a*b*c
**2*d**2*e*x**5 + 294525*sqrt(a - c*x**4)*a*b*c**2*d*e**2*x**7 + 78897*sqr
t(a - c*x**4)*a*b*c**2*e**3*x**9 + 19890*sqrt(a - c*x**4)*a*c**3*d**3*x**5
 + 39270*sqrt(a - c*x**4)*a*c**3*d**2*e*x**7 + 27846*sqrt(a - c*x**4)*a*c*
*3*d*e**2*x**9 + 6930*sqrt(a - c*x**4)*a*c**3*e**3*x**11 - 85085*sqrt(a -
c*x**4)*b*c**3*d**3*x**7 - 208845*sqrt(a - c*x**4)*b*c**3*d**2*e*x**9 - 17
6715*sqrt(a - c*x**4)*b*c**3*d*e**2*x**11 - 51051*sqrt(a - c*x**4)*b*c**3*
e**3*x**13 - 69615*sqrt(a - c*x**4)*c**4*d**3*x**9 - 176715*sqrt(a - c*x**
4)*c**4*d**2*e*x**11 - 153153*sqrt(a - c*x**4)*c**4*d*e**2*x**13 - 45045*s
qrt(a - c*x**4)*c**4*e**3*x**15 + 13260*int(sqrt(a - c*x**4)/(a - c*x**4),
x)*a**4*b*e**3 + 159120*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a**4*c*d*e**2
 + 119340*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a**3*b*c*d**2*e + 477360*in
t(sqrt(a - c*x**4)/(a - c*x**4),x)*a**3*c**2*d**3 + 66528*int((sqrt(a - c*
x**4)*x**2)/(a - c*x**4),x)*a**4*c*e**3 + 141372*int((sqrt(a - c*x**4)*...
```

3.34 $\int (d+ex^2)^2 (a-cx^4)^{3/2} (A+Bx^2+Cx^4) \, dx$

Optimal result

Integrand size = 34, antiderivative size = 512

$$\int (d+ex^2)^2 (a-cx^4)^{3/2} (A+Bx^2+Cx^4) \, dx = \frac{2ax(65(3Ac(11cd^2+ae^2)+a(aCe^2+3cd(Cd+2Be)))+77c(13Bcd^2+26Acde+6aCde+ \ldots}{15015c^2}$$
$$+\frac{x(39(3Ac(11cd^2+ae^2)+a(aCe^2+3cd(Cd+2Be)))+77c(13Bcd^2+26Acde+6aCde+3aBe^2)x^2)(a \ldots}{9009c^2}$$
$$-\frac{(aCe^2+3c(Cd^2+e(2Bd+Ae)))x(a-cx^4)^{5/2}}{33c^2}$$
$$-\frac{e(2Cd+Be)x^3(a-cx^4)^{5/2}}{13c}-\frac{Ce^2x^5(a-cx^4)^{5/2}}{15c}$$
$$+\frac{4a^{11/4}(13Bcd^2+26Acde+6aCde+3aBe^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{195c^{7/4}\sqrt{a-cx^4}}$$
$$-\frac{4a^{9/4}(77\sqrt{a}\sqrt{c}(13Bcd^2+26Acde+6aCde+3aBe^2)-65(3Ac(11cd^2+ae^2)+a(aCe^2+3cd(Cd+2B \ldots}{15015c^{9/4}\sqrt{a-cx^4}}$$

output
```
2/15015*a*x*(195*A*c*(a*e^2+11*c*d^2)+65*a*(C*a*e^2+3*c*d*(2*B*e+C*d))+77*
c*(26*A*c*d*e+3*B*a*e^2+13*B*c*d^2+6*C*a*d*e)*x^2)*(-c*x^4+a)^(1/2)/c^2+1/
9009*x*(117*A*c*(a*e^2+11*c*d^2)+39*a*(C*a*e^2+3*c*d*(2*B*e+C*d))+77*c*(26
*A*c*d*e+3*B*a*e^2+13*B*c*d^2+6*C*a*d*e)*x^2)*(-c*x^4+a)^(3/2)/c^2-1/33*(C
*a*e^2+3*c*(C*d^2+e*(A*e+2*B*d)))*x*(-c*x^4+a)^(5/2)/c^2-1/13*e*(B*e+2*C*d
)*x^3*(-c*x^4+a)^(5/2)/c-1/15*C*e^2*x^5*(-c*x^4+a)^(5/2)/c+4/195*a^(11/4)*
(26*A*c*d*e+3*B*a*e^2+13*B*c*d^2+6*C*a*d*e)*(1-c*x^4/a)^(1/2)*EllipticE(c^
(1/4)*x/a^(1/4),I)/c^(7/4)/(-c*x^4+a)^(1/2)-4/15015*a^(9/4)*(77*a^(1/2)*c^
(1/2)*(26*A*c*d*e+3*B*a*e^2+13*B*c*d^2+6*C*a*d*e)-195*A*c*(a*e^2+11*c*d^2)
-65*a*(C*a*e^2+3*c*d*(2*B*e+C*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a
^(1/4),I)/c^(9/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.41 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.69

$$\int (d+ex^2)^2 (a-cx^4)^{3/2} (A+Bx^2+Cx^4)\, dx = \frac{x\sqrt{a-cx^4}\Big(-195c(Cd^2+e(2Bd+Ae))(a-cx^4)^2\sqrt{1-\frac{cx^4}{a}}-165ce(2Cd+Be)x^2(a-cx^4)^2 \cdots}{\cdots}$$

input
```
Integrate[(d + e*x^2)^2*(a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
(x*Sqrt[a - c*x^4]*(-195*c*(C*d^2 + e*(2*B*d + A*e))*(a - c*x^4)^2*Sqrt[1
- (c*x^4)/a] - 165*c*e*(2*C*d + B*e)*x^2*(a - c*x^4)^2*Sqrt[1 - (c*x^4)/a]
 - 143*c*C*e^2*x^4*(a - c*x^4)^2*Sqrt[1 - (c*x^4)/a] + 2145*a*A*c^2*d^2*Hy
pergeometric2F1[-3/2, 1/4, 5/4, (c*x^4)/a] + 195*a^2*c*(C*d^2 + e*(2*B*d +
 A*e))*Hypergeometric2F1[-3/2, 1/4, 5/4, (c*x^4)/a] - 65*a*C*e^2*((a - c*x
^4)^2*Sqrt[1 - (c*x^4)/a] - a^2*Hypergeometric2F1[-3/2, 1/4, 5/4, (c*x^4)/
a]) + 715*a*c^2*d*(B*d + 2*A*e)*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, (c*x
^4)/a] + 165*a^2*c*e*(2*C*d + B*e)*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, (
c*x^4)/a]))/(2145*c^2*Sqrt[1 - (c*x^4)/a])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1607 vs. 2(512) = 1024.

Time = 2.02 (sec) , antiderivative size = 1607, normalized size of antiderivative = 3.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^{3/2} \left(d+ex^2\right)^2 \left(A+Bx^2+Cx^4\right) dx$$

$$\Big\downarrow \; 2259$$

$$\int \left(\frac{a^2dx^2(2Ae+Bd)}{\sqrt{a-cx^4}}+\frac{a^2Ad^2}{\sqrt{a-cx^4}}+\frac{ax^4\left(ad(2Be+Cd)-A\left(2cd^2-ae^2\right)\right)}{\sqrt{a-cx^4}}+\frac{cx^{12}\left(-2aCe^2+ce(Ae+2Bd)+\right.}{\sqrt{a-cx^4}}\right.$$

$$\Big\downarrow \; 2009$$

$$
-\frac{1}{15}cCe^2\sqrt{a-cx^4}x^{13}-\frac{1}{13}ce(2Cd+Be)\sqrt{a-cx^4}x^{11}-\frac{13}{165}aCe^2\sqrt{a-cx^4}x^9-
\frac{1}{11}\left(cCd^2-2aCe^2+ce(2Bd+Ae)\right)\sqrt{a-cx^4}x^9-\frac{11}{117}ae(2Cd+Be)\sqrt{a-cx^4}x^7-
\frac{1}{9}\left(Bcd^2+2Aced-4aCed-2aBe^2\right)\sqrt{a-cx^4}x^7-\frac{39a^2Ce^2\sqrt{a-cx^4}x^5}{385c}-
\frac{9a\left(cCd^2-2aCe^2+ce(2Bd+Ae)\right)\sqrt{a-cx^4}x^5}{77c}-
\frac{\left(Ac\left(cd^2-2ae^2\right)+a\left(aCe^2-2cd(Cd+2Be)\right)\right)\sqrt{a-cx^4}x^5}{7c}-
\frac{77a^2e(2Cd+Be)\sqrt{a-cx^4}x^3}{585c}-\frac{7a\left(Bcd^2+2Aced-4aCed-2aBe^2\right)\sqrt{a-cx^4}x^3}{45c}+
\frac{a\left(2Bcd^2+4Aced-2aCed-aBe^2\right)\sqrt{a-cx^4}x^3}{5c}-\frac{13a^3Ce^2\sqrt{a-cx^4}x}{77c^2}-
\frac{15a^2\left(cCd^2-2aCe^2+ce(2Bd+Ae)\right)\sqrt{a-cx^4}x}{77c^2}-
\frac{a\left(ad(Cd+2Be)-A\left(2cd^2-ae^2\right)\right)\sqrt{a-cx^4}x}{3c}-
\frac{5a\left(Ac\left(cd^2-2ae^2\right)+a\left(aCe^2-2cd(Cd+2Be)\right)\right)\sqrt{a-cx^4}x}{21c^2}+
\frac{a^{11/4}d(Bd+2Ae)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+
\frac{77a^{15/4}e(2Cd+Be)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{195c^{7/4}\sqrt{a-cx^4}}+
\frac{7a^{11/4}\left(Bcd^2+2Aced-4aCed-2aBe^2\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15c^{7/4}\sqrt{a-cx^4}}-
\frac{3a^{11/4}\left(2Bcd^2+4Aced-2aCed-aBe^2\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+
\frac{a^{9/4}Ad^2\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}+
\frac{13a^{17/4}Ce^2\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{77c^{9/4}\sqrt{a-cx^4}}-
\frac{a^{11/4}d(Bd+2Ae)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}-
\frac{77a^{15/4}e(2Cd+Be)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{195c^{7/4}\sqrt{a-cx^4}}-
\frac{7a^{11/4}\left(Bcd^2+2Aced-4aCed-2aBe^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{15c^{7/4}\sqrt{a-cx^4}}+
\frac{3a^{11/4}\left(2Bcd^2+4Aced-2aCed-aBe^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+
15a^{13/4}\left(cCd^2-2aCe^2+ce(2Bd+Ae)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)
$$

input
```
Int[(d + e*x^2)^2*(a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
(-13*a^3*C*e^2*x*Sqrt[a - c*x^4])/(77*c^2) - (15*a^2*(c*C*d^2 - 2*a*C*e^2
+ c*e*(2*B*d + A*e))*x*Sqrt[a - c*x^4])/(77*c^2) - (a*(a*d*(C*d + 2*B*e) -
 A*(2*c*d^2 - a*e^2))*x*Sqrt[a - c*x^4])/(3*c) - (5*a*(A*c*(c*d^2 - 2*a*e^
2) + a*(a*C*e^2 - 2*c*d*(C*d + 2*B*e)))*x*Sqrt[a - c*x^4])/(21*c^2) - (77*
a^2*e*(2*C*d + B*e)*x^3*Sqrt[a - c*x^4])/(585*c) - (7*a*(B*c*d^2 + 2*A*c*d
*e - 4*a*C*d*e - 2*a*B*e^2)*x^3*Sqrt[a - c*x^4])/(45*c) + (a*(2*B*c*d^2 +
4*A*c*d*e - 2*a*C*d*e - a*B*e^2)*x^3*Sqrt[a - c*x^4])/(5*c) - (39*a^2*C*e^
2*x^5*Sqrt[a - c*x^4])/(385*c) - (9*a*(c*C*d^2 - 2*a*C*e^2 + c*e*(2*B*d +
A*e))*x^5*Sqrt[a - c*x^4])/(77*c) - ((A*c*(c*d^2 - 2*a*e^2) + a*(a*C*e^2 -
 2*c*d*(C*d + 2*B*e)))*x^5*Sqrt[a - c*x^4])/(7*c) - (11*a*e*(2*C*d + B*e)*
x^7*Sqrt[a - c*x^4])/117 - ((B*c*d^2 + 2*A*c*d*e - 4*a*C*d*e - 2*a*B*e^2)*
x^7*Sqrt[a - c*x^4])/9 - (13*a*C*e^2*x^9*Sqrt[a - c*x^4])/165 - ((c*C*d^2
- 2*a*C*e^2 + c*e*(2*B*d + A*e))*x^9*Sqrt[a - c*x^4])/11 - (c*e*(2*C*d + B
*e)*x^11*Sqrt[a - c*x^4])/13 - (c*C*e^2*x^13*Sqrt[a - c*x^4])/15 + (a^(11/
4)*d*(B*d + 2*A*e)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4
)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (77*a^(15/4)*e*(2*C*d + B*e)*Sqrt[1 -
 (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(195*c^(7/4)*Sqrt[
a - c*x^4]) + (7*a^(11/4)*(B*c*d^2 + 2*A*c*d*e - 4*a*C*d*e - 2*a*B*e^2)*Sq
rt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(15*c^(7/4)*
Sqrt[a - c*x^4]) - (3*a^(11/4)*(2*B*c*d^2 + 4*A*c*d*e - 2*a*C*d*e - a*B...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.31

method	result
default	$A\,d^{2}\left(-\frac{c\,x^{5}\sqrt{-c\,x^{4}+a}}{7}+\frac{3ax\sqrt{-c\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right)+d(2Ae+Bd)\left(-\right.$
risch	$-\frac{x\left(3003C\,e^{2}x^{12}c^{3}+3465B\,c^{3}e^{2}x^{10}+6930C\,c^{3}de\,x^{10}+4095A\,c^{3}e^{2}x^{8}+8190B\,c^{3}de\,x^{8}-4641Ca\,c^{2}e^{2}x^{8}+4095C\,c^{3}d^{2}x^{8}+10010A\,c^{3}\right.}{}$
elliptic	$-\frac{Cc\,e^{2}x^{13}\sqrt{-c\,x^{4}+a}}{15}-\frac{\left(B\,c^{2}e^{2}+2c^{2}deC\right)x^{11}\sqrt{-c\,x^{4}+a}}{13c}-\frac{\left(A\,c^{2}e^{2}+2B\,c^{2}de-\frac{17}{15}Cac\,e^{2}+C\,c^{2}d^{2}\right)x^{9}\sqrt{-c\,x^{4}+a}}{11c}-\frac{\left(2A\,c^{2}de\right.}{}$

input
```
int((e*x^2+d)^2*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
A*d^2*(-1/7*c*x^5*(-c*x^4+a)^(1/2)+3/7*a*x*(-c*x^4+a)^(1/2)+4/7*a^2/(c^(1/
2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1
/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))+d*(2*A*e+B*d)
*(-1/9*c*x^7*(-c*x^4+a)^(1/2)+11/45*a*x^3*(-c*x^4+a)^(1/2)-4/15*a^(5/2)/(c
^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2)
)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-E
llipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))+e*(B*e+2*C*d)*(-1/13*c*x^11*(-c*x^
4+a)^(1/2)+5/39*a*x^7*(-c*x^4+a)^(1/2)-4/195*a^2/c*x^3*(-c*x^4+a)^(1/2)-4/
65*a^(7/2)/c^(3/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(
1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2
))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))+(A*e^2+2*B*d*e+C*d^2)
*(-1/11*c*x^9*(-c*x^4+a)^(1/2)+13/77*a*x^5*(-c*x^4+a)^(1/2)-4/77*a^2/c*x*(
-c*x^4+a)^(1/2)+4/77*a^3/c/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))
^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)
/a^(1/2))^(1/2),I))+C*e^2*(-1/15*c*x^13*(-c*x^4+a)^(1/2)+17/165*a*x^9*(-c*
x^4+a)^(1/2)-4/385*a^2/c*x^5*(-c*x^4+a)^(1/2)-4/231*a^3/c^2*x*(-c*x^4+a)^(
1/2)+4/231*a^4/c^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(
1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2)
)^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.97

$$\int \left(d+ex^2\right)^2 \left(a-cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right)\,dx =$$

$$-\frac{924\,(13\,Ba^2cd^2+3\,Ba^3e^2+2\,(3\,Ca^3+13\,Aa^2c)de)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-12\,(13\,((77\,B+\cdots}{\cdots}$$

input
```
integrate((e*x^2+d)^2*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="frica
s")
```

output
```
-1/45045*(924*(13*B*a^2*c*d^2 + 3*B*a^3*e^2 + 2*(3*C*a^3 + 13*A*a^2*c)*d*e
)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - 12*(13*((
77*B + 15*C)*a^2*c + 165*A*a*c^2)*d^2 + 2*(231*C*a^3 + 13*(77*A + 15*B)*a^
2*c)*d*e + ((231*B + 65*C)*a^3 + 195*A*a^2*c)*e^2)*sqrt(-c)*x*(a/c)^(3/4)*
elliptic_f(arcsin((a/c)^(1/4)/x), -1) + (3003*C*c^3*e^2*x^14 + 3465*(2*C*c
^3*d*e + B*c^3*e^2)*x^12 + 273*(15*C*c^3*d^2 + 30*B*c^3*d*e - (17*C*a*c^2
- 15*A*c^3)*e^2)*x^10 + 385*(13*B*c^3*d^2 - 15*B*a*c^2*e^2 - 2*(15*C*a*c^2
 - 13*A*c^3)*d*e)*x^8 - 117*(130*B*a*c^2*d*e + 5*(13*C*a*c^2 - 11*A*c^3)*d
^2 - (4*C*a^2*c - 65*A*a*c^2)*e^2)*x^6 + 12012*B*a^2*c*d^2 + 2772*B*a^3*e^
2 - 77*(143*B*a*c^2*d^2 - 12*B*a^2*c*e^2 - 2*(12*C*a^2*c - 143*A*a*c^2)*d*
e)*x^4 + 1848*(3*C*a^3 + 13*A*a^2*c)*d*e + 195*(24*B*a^2*c*d*e + 3*(4*C*a^
2*c - 33*A*a*c^2)*d^2 + 4*(C*a^3 + 3*A*a^2*c)*e^2)*x^2)*sqrt(-c*x^4 + a))/
(c^2*x)
```

Sympy [A] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.70

$$\int \left(d+ex^2\right)^2 \left(a-cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)**2*(-c*x**4+a)**(3/2)*(C*x**4+B*x**2+A),x)
```

output
```
A*a**(3/2)*d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(2
*I*pi)/a)/(4*gamma(5/4)) + A*a**(3/2)*d*e*x**3*gamma(3/4)*hyper((-1/2, 3/4
), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(2*gamma(7/4)) + A*a**(3/2)*e**2*x*
*5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*ga
mma(9/4)) - A*sqrt(a)*c*d**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*
x**4*exp_polar(2*I*pi)/a)/(4*gamma(9/4)) - A*sqrt(a)*c*d*e*x**7*gamma(7/4)
*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(2*gamma(11/4)) -
 A*sqrt(a)*c*e**2*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), c*x**4*exp_p
olar(2*I*pi)/a)/(4*gamma(13/4)) + B*a**(3/2)*d**2*x**3*gamma(3/4)*hyper((-
1/2, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) + B*a**(3/2)
*d*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a
)/(2*gamma(9/4)) + B*a**(3/2)*e**2*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/
4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11/4)) - B*sqrt(a)*c*d**2*x**7*g
amma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma
(11/4)) - B*sqrt(a)*c*d*e*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), c*x*
*4*exp_polar(2*I*pi)/a)/(2*gamma(13/4)) - B*sqrt(a)*c*e**2*x**11*gamma(11/
4)*hyper((-1/2, 11/4), (15/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(15/4)
) + C*a**(3/2)*d**2*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_
polar(2*I*pi)/a)/(4*gamma(9/4)) + C*a**(3/2)*d*e*x**7*gamma(7/4)*hyper((-1
/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(2*gamma(11/4)) + C*a**(...
```

Maxima [F]

$$\int (d+ex^2)^2 (a - cx^4)^{3/2} (A+Bx^2+Cx^4)\,dx = \int (Cx^4+Bx^2+A)(-cx^4+a)^{\frac{3}{2}}(ex^2+d)^2\,dx$$

input
```
integrate((e*x^2+d)^2*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)*(e*x^2 + d)^2, x)
```

Giac [F]

$$\int (d+ex^2)^2 (a-cx^4)^{3/2} (A+Bx^2+Cx^4)\,dx = \int (Cx^4+Bx^2+A)(-cx^4+a)^{\frac{3}{2}}(ex^2+d)^2\,dx$$

input
```
integrate((e*x^2+d)^2*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)*(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^2 (a-cx^4)^{3/2} (A+Bx^2+Cx^4)\,dx = \int (a-c\,x^4)^{3/2}\,(e\,x^2+d)^2\,(C\,x^4+B\,x^2+A)\,dx$$

input
```
int((a - c*x^4)^(3/2)*(d + e*x^2)^2*(A + B*x^2 + C*x^4),x)
```

output
```
int((a - c*x^4)^(3/2)*(d + e*x^2)^2*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (d+ex^2)^2 (a-cx^4)^{3/2} (A+Bx^2+Cx^4)\,dx = \frac{-3120\sqrt{-c\,x^4+a}\,a^3e^2x-4680\sqrt{-c\,x^4+a}\,a^2bdex-924\sqrt{-c\,x^4+a}\,a^2b\,e^2x^3+16965\sqrt{-c\,x}}{}$$

input
```
int((e*x^2+d)^2*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x)
```

output

```
( - 3120*sqrt(a - c*x**4)*a**3*e**2*x - 4680*sqrt(a - c*x**4)*a**2*b*d*e*x
 - 924*sqrt(a - c*x**4)*a**2*b*e**2*x**3 + 16965*sqrt(a - c*x**4)*a**2*c*d
**2*x + 20174*sqrt(a - c*x**4)*a**2*c*d*e*x**3 + 7137*sqrt(a - c*x**4)*a**
2*c*e**2*x**5 + 11011*sqrt(a - c*x**4)*a*b*c*d**2*x**3 + 15210*sqrt(a - c*
x**4)*a*b*c*d*e*x**5 + 5775*sqrt(a - c*x**4)*a*b*c*e**2*x**7 + 1170*sqrt(a
 - c*x**4)*a*c**2*d**2*x**5 + 1540*sqrt(a - c*x**4)*a*c**2*d*e*x**7 + 546*
sqrt(a - c*x**4)*a*c**2*e**2*x**9 - 5005*sqrt(a - c*x**4)*b*c**2*d**2*x**7
 - 8190*sqrt(a - c*x**4)*b*c**2*d*e*x**9 - 3465*sqrt(a - c*x**4)*b*c**2*e*
*2*x**11 - 4095*sqrt(a - c*x**4)*c**3*d**2*x**9 - 6930*sqrt(a - c*x**4)*c*
*3*d*e*x**11 - 3003*sqrt(a - c*x**4)*c**3*e**2*x**13 + 3120*int(sqrt(a - c
*x**4)/(a - c*x**4),x)*a**4*e**2 + 4680*int(sqrt(a - c*x**4)/(a - c*x**4),
x)*a**3*b*d*e + 28080*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a**3*c*d**2 + 2
772*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a**3*b*e**2 + 29568*int((s
qrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a**3*c*d*e + 12012*int((sqrt(a - c*x
**4)*x**2)/(a - c*x**4),x)*a**2*b*c*d**2)/(45045*c)
```

3.35 $\int (d+ex^2)(a-cx^4)^{3/2}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 342

$$\int (d+ex^2)(a-cx^4)^{3/2}(A+Bx^2+Cx^4)\,dx = \frac{2ax(195(11Acd+aCd+aBe)+77(13Bcd+13Ace+3aCe)x^2)\sqrt{a-cx^4}}{15015c} + \frac{x(117(11Acd+aCd+aBe)+77(13Bcd+13Ace+3aCe)x^2)(a-cx^4)^{3/2}}{9009c} - \frac{(Cd+Be)x(a-cx^4)^{5/2}}{11c} - \frac{Cex^3(a-cx^4)^{5/2}}{13c} + \frac{4a^{11/4}(13Bcd+13Ace+3aCe)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{195c^{7/4}\sqrt{a-cx^4}} + \frac{4a^{9/4}\left(195\sqrt{c}(11Acd+aCd+aBe)-77\sqrt{a}(13Bcd+13Ace+3aCe)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{15015c^{7/4}\sqrt{a-cx^4}}$$

output
```
2/15015*a*x*(2145*A*c*d+195*B*a*e+195*C*a*d+77*(13*A*c*e+13*B*c*d+3*C*a*e)
*x^2)*(-c*x^4+a)^(1/2)/c+1/9009*x*(1287*A*c*d+117*B*a*e+117*C*a*d+77*(13*A
*c*e+13*B*c*d+3*C*a*e)*x^2)*(-c*x^4+a)^(3/2)/c-1/11*(B*e+C*d)*x*(-c*x^4+a)
^(5/2)/c-1/13*C*e*x^3*(-c*x^4+a)^(5/2)/c+4/195*a^(11/4)*(13*A*c*e+13*B*c*d
+3*C*a*e)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(7/4)/(-c*x^4
+a)^(1/2)+4/15015*a^(9/4)*(195*c^(1/2)*(11*A*c*d+B*a*e+C*a*d)-77*a^(1/2)*(
13*A*c*e+13*B*c*d+3*C*a*e))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),
I)/c^(7/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.45

$$\int (d+ex^2)(a-cx^4)^{3/2}(A+Bx^2+Cx^4)\,dx =$$

$$-\frac{x\sqrt{a-cx^4}\left(3(13Cd+13Be+11Cex^2)(a-cx^4)^2\sqrt{1-\frac{cx^4}{a}}-39a(11Acd+aCd+aBe)\,\text{Hypergeomet}\right.}{429c\sqrt{1-\text{}}}$$

input
```
Integrate[(d + e*x^2)*(a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
-1/429*(x*Sqrt[a - c*x^4]*(3*(13*C*d + 13*B*e + 11*C*e*x^2)*(a - c*x^4)^2*
Sqrt[1 - (c*x^4)/a] - 39*a*(11*A*c*d + a*C*d + a*B*e)*Hypergeometric2F1[-3
/2, 1/4, 5/4, (c*x^4)/a] - 11*a*(13*B*c*d + 13*A*c*e + 3*a*C*e)*x^2*Hyperg
eometric2F1[-3/2, 3/4, 7/4, (c*x^4)/a]))/(c*Sqrt[1 - (c*x^4)/a])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1154 vs. 2(342) = 684.

Time = 1.37 (sec) , antiderivative size = 1154, normalized size of antiderivative = 3.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^{3/2} \left(d+ex^2\right) \left(A+Bx^2+Cx^4\right) \, dx$$

$$\Big\downarrow \; 2259$$

$$\int \left(\frac{a^2x^2(Ae+Bd)}{\sqrt{a-cx^4}}+\frac{a^2Ad}{\sqrt{a-cx^4}}+\frac{ax^4(aBe+aCd-2Acd)}{\sqrt{a-cx^4}}+\frac{cx^{10}(-2aCe+Ace+Bcd)}{\sqrt{a-cx^4}}+\frac{cx^8(Acd-2a(Be}{\sqrt{a-cx^4}}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{13}cCe\sqrt{a-cx^4}x^{11}-\frac{1}{11}c(Cd+Be)\sqrt{a-cx^4}x^9-\frac{11}{117}aCe\sqrt{a-cx^4}x^7-\frac{1}{9}(Bcd+Ace-2aCe)\sqrt{a-cx^4}x^7-\frac{9}{77}a(Cd+Be)\sqrt{a-cx^4}x^5-\frac{1}{7}(Acd-2a(Cd+Be))\sqrt{a-cx^4}x^5-\frac{77a^2Ce\sqrt{a-cx^4}x^3}{585c}-\frac{7a(Bcd+Ace-2aCe)\sqrt{a-cx^4}x^3}{45c}+\frac{a(2Bcd+2Ace-aCe)\sqrt{a-cx^4}x^3}{5c}-\frac{15a^2(Cd+Be)\sqrt{a-cx^4}x}{77c}+\frac{a(2Acd-aCd-aBe)\sqrt{a-cx^4}x}{3c}-\frac{5a(Acd-2a(Cd+Be))\sqrt{a-cx^4}x}{21c}+\frac{77a^{15/4}Ce\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{195c^{7/4}\sqrt{a-cx^4}}+\frac{a^{11/4}(Bd+Ae)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{7a^{11/4}(Bcd+Ace-2aCe)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15c^{7/4}\sqrt{a-cx^4}}-\frac{3a^{11/4}(2Bcd+2Ace-aCe)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{a^{9/4}Ad\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{77a^{15/4}Ce\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{195c^{7/4}\sqrt{a-cx^4}}-\frac{a^{11/4}(Bd+Ae)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{15a^{13/4}(Cd+Be)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{77c^{5/4}\sqrt{a-cx^4}}-\frac{a^{9/4}(2Acd-aCd-aBe)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}}-\frac{7a^{11/4}(Bcd+Ace-2aCe)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{15c^{7/4}\sqrt{a-cx^4}}+\frac{3a^{11/4}(2Bcd+2Ace-aCe)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{5a^{9/4}(Acd-2a(Cd+Be))\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{21c^{5/4}\sqrt{a-cx^4}}$$

input
```
Int[(d + e*x^2)*(a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
(-15*a^2*(C*d + B*e)*x*Sqrt[a - c*x^4])/(77*c) + (a*(2*A*c*d - a*C*d - a*B
*e)*x*Sqrt[a - c*x^4])/(3*c) - (5*a*(A*c*d - 2*a*(C*d + B*e))*x*Sqrt[a - c
*x^4])/(21*c) - (77*a^2*C*e*x^3*Sqrt[a - c*x^4])/(585*c) - (7*a*(B*c*d + A
*c*e - 2*a*C*e)*x^3*Sqrt[a - c*x^4])/(45*c) + (a*(2*B*c*d + 2*A*c*e - a*C*
e)*x^3*Sqrt[a - c*x^4])/(5*c) - (9*a*(C*d + B*e)*x^5*Sqrt[a - c*x^4])/77 -
 ((A*c*d - 2*a*(C*d + B*e))*x^5*Sqrt[a - c*x^4])/7 - (11*a*C*e*x^7*Sqrt[a
- c*x^4])/117 - ((B*c*d + A*c*e - 2*a*C*e)*x^7*Sqrt[a - c*x^4])/9 - (c*(C*
d + B*e)*x^9*Sqrt[a - c*x^4])/11 - (c*C*e*x^11*Sqrt[a - c*x^4])/13 + (77*a
^(15/4)*C*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1]
)/(195*c^(7/4)*Sqrt[a - c*x^4]) + (a^(11/4)*(B*d + A*e)*Sqrt[1 - (c*x^4)/a
]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) +
(7*a^(11/4)*(B*c*d + A*c*e - 2*a*C*e)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin
[(c^(1/4)*x)/a^(1/4)], -1])/(15*c^(7/4)*Sqrt[a - c*x^4]) - (3*a^(11/4)*(2*
B*c*d + 2*A*c*e - a*C*e)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/
a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (a^(9/4)*A*d*Sqrt[1 - (c*x^4)
/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4])
- (77*a^(15/4)*C*e*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4
)], -1])/(195*c^(7/4)*Sqrt[a - c*x^4]) - (a^(11/4)*(B*d + A*e)*Sqrt[1 - (c
*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x
^4]) + (15*a^(13/4)*(C*d + B*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.37

method	result
risch	$\frac{x(-3465eCx^{10}c^2-4095Bc^2ex^8-4095Cc^2dx^8-5005Ac^2ex^6-5005Bc^2dx^6+5775Cacex^6-6435Ac^2dx^4+7605Bacex^4+7605C}{45045c}$
default	$Ad\left(-\frac{cx^5\sqrt{-cx^4+a}}{7}+\frac{3ax\sqrt{-cx^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}\right)+(Ae+Bd)\left(-\frac{cx^7}{}\right.$
elliptic	$-\frac{Cecx^{11}\sqrt{-cx^4+a}}{13}-\frac{(Bc^2e+dc^2C)x^9\sqrt{-cx^4+a}}{11c}-\frac{(Ac^2e+dc^2B-\frac{15}{13}Caec)x^7\sqrt{-cx^4+a}}{9c}-\frac{\left(Ac^2d-2Bace-2acCd+\right.}{}$

input
```
int((e*x^2+d)*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/45045/c*x*(-3465*C*c^2*e*x^10-4095*B*c^2*e*x^8-4095*C*c^2*d*x^8-5005*A*c
^2*e*x^6-5005*B*c^2*d*x^6+5775*C*a*c*e*x^6-6435*A*c^2*d*x^4+7605*B*a*c*e*x
^4+7605*C*a*c*d*x^4+11011*A*a*c*e*x^2+11011*B*a*c*d*x^2-924*C*a^2*e*x^2+19
305*A*a*c*d-2340*B*a^2*e-2340*C*a^2*d)*(-c*x^4+a)^(1/2)+4/15015*a^2/c*(-(1
001*A*c*e+1001*B*c*d+231*C*a*e)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)
*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)
*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/
2),I))+2145*A*c*d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1
+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))
^(1/2),I)+195*B*a*e/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*
(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2
))^(1/2),I)+195*C*a*d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2
)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1
/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.95

$$\int (d+ex^2)\left(a-cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx =$$

$$-\frac{924\,(13\,Ba^2cd+(3\,Ca^3+13\,Aa^2c)e)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-12\,(13\,((77\,B+15\,C)a^2c+1\ldots}{}$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="fricas"
)
```

output
```
-1/45045*(924*(13*B*a^2*c*d + (3*C*a^3 + 13*A*a^2*c)*e)*sqrt(-c)*x*(a/c)^(
3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - 12*(13*((77*B + 15*C)*a^2*c +
 165*A*a*c^2)*d + (231*C*a^3 + 13*(77*A + 15*B)*a^2*c)*e)*sqrt(-c)*x*(a/c)
^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + (3465*C*c^3*e*x^12 + 4095*(
C*c^3*d + B*c^3*e)*x^10 + 385*(13*B*c^3*d - (15*C*a*c^2 - 13*A*c^3)*e)*x^8
 - 585*(13*B*a*c^2*e + (13*C*a*c^2 - 11*A*c^3)*d)*x^6 + 12012*B*a^2*c*d -
77*(143*B*a*c^2*d - (12*C*a^2*c - 143*A*a*c^2)*e)*x^4 + 585*(4*B*a^2*c*e +
 (4*C*a^2*c - 33*A*a*c^2)*d)*x^2 + 924*(3*C*a^3 + 13*A*a^2*c)*e)*sqrt(-c*x
^4 + a))/(c^2*x)
```

Sympy [A] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.63

$$\int (d+ex^2)\left(a-cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)*(-c*x**4+a)**(3/2)*(C*x**4+B*x**2+A),x)
```

output
```
A*a**(3/2)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(2*I*
pi)/a)/(4*gamma(5/4)) + A*a**(3/2)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7
/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) - A*sqrt(a)*c*d*x**5*gamm
a(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(9/4
)) - A*sqrt(a)*c*e*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_
polar(2*I*pi)/a)/(4*gamma(11/4)) + B*a**(3/2)*d*x**3*gamma(3/4)*hyper((-1/
2, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) + B*a**(3/2)*e
*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4
*gamma(9/4)) - B*sqrt(a)*c*d*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), c
*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11/4)) - B*sqrt(a)*c*e*x**9*gamma(9/4)
*hyper((-1/2, 9/4), (13/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(13/4)) +
 C*a**(3/2)*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(
2*I*pi)/a)/(4*gamma(9/4)) + C*a**(3/2)*e*x**7*gamma(7/4)*hyper((-1/2, 7/4)
, (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11/4)) - C*sqrt(a)*c*d*x**
9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*ga
mma(13/4)) - C*sqrt(a)*c*e*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,),
c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(15/4))
```

Maxima [F]

$$\int (d+ex^2)\,(a-cx^4)^{3/2}\,(A+Bx^2+Cx^4)\;dx = \int (Cx^4+Bx^2+A)\,(-cx^4+a)^{\frac{3}{2}}\,(ex^2+d)\,dx$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="maxima"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)*(e*x^2 + d), x)
```

Giac [F]

$$\int (d+ex^2) (a - cx^4)^{3/2} (A+Bx^2+Cx^4) \, dx = \int (Cx^4+Bx^2+A) (-cx^4+a)^{\frac{3}{2}} (ex^2+d) \, dx$$

input
```
integrate((e*x^2+d)*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)*(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) (a - cx^4)^{3/2} (A+Bx^2+Cx^4) \, dx = \int (a-c\,x^4)^{3/2} (e\,x^2+d) (C\,x^4+B\,x^2+A) \, dx$$

input
```
int((a - c*x^4)^(3/2)*(d + e*x^2)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a - c*x^4)^(3/2)*(d + e*x^2)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (d+ex^2) (a-cx^4)^{3/2} (A+Bx^2 +Cx^4) \, dx = \frac{-2340\sqrt{-c\,x^4+a}\,a^2bex + 16965\sqrt{-c\,x^4+a}\,a^2cdx + 10087\sqrt{-c\,x^4+a}\,a^2ce\,x^3 + 11011\sqrt{-c}}{}$$

input
```
int((e*x^2+d)*(-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x)
```

output
```
( - 2340*sqrt(a - c*x**4)*a**2*b*e*x + 16965*sqrt(a - c*x**4)*a**2*c*d*x +
 10087*sqrt(a - c*x**4)*a**2*c*e*x**3 + 11011*sqrt(a - c*x**4)*a*b*c*d*x**
3 + 7605*sqrt(a - c*x**4)*a*b*c*e*x**5 + 1170*sqrt(a - c*x**4)*a*c**2*d*x*
*5 + 770*sqrt(a - c*x**4)*a*c**2*e*x**7 - 5005*sqrt(a - c*x**4)*b*c**2*d*x
**7 - 4095*sqrt(a - c*x**4)*b*c**2*e*x**9 - 4095*sqrt(a - c*x**4)*c**3*d*x
**9 - 3465*sqrt(a - c*x**4)*c**3*e*x**11 + 2340*int(sqrt(a - c*x**4)/(a -
c*x**4),x)*a**3*b*e + 28080*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a**3*c*d
+ 14784*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a**3*c*e + 12012*int((
sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a**2*b*c*d)/(45045*c)
```

3.36 $\int (a - cx^4)^{3/2} (A + Bx^2 + Cx^4)\, dx$

Optimal result

Integrand size = 25, antiderivative size = 234

$$\int (a-cx^4)^{3/2} (A+Bx^2+Cx^4)\, dx = \frac{2ax(15(11Ac+aC)+77Bcx^2)\sqrt{a-cx^4}}{1155c} + \frac{x(9(11Ac+aC)+77Bcx^2)(a-cx^4)^{3/2}}{693c} - \frac{Cx(a-cx^4)^{5/2}}{11c} + \frac{4a^{11/4}B\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15c^{3/4}\sqrt{a-cx^4}} - \frac{4a^{9/4}(77\sqrt{a}B\sqrt{c}-165Ac-15aC)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{1155c^{5/4}\sqrt{a-cx^4}}$$

output

```
2/1155*a*x*(77*B*c*x^2+165*A*c+15*C*a)*(-c*x^4+a)^(1/2)/c+1/693*x*(77*B*c*
x^2+99*A*c+9*C*a)*(-c*x^4+a)^(3/2)/c-1/11*C*x*(-c*x^4+a)^(5/2)/c+4/15*a^(1
1/4)*B*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(3/4)/(-c*x^4+a)
^(1/2)-4/1155*a^(9/4)*(77*a^(1/2)*B*c^(1/2)-165*A*c-15*a*C)*(1-c*x^4/a)^(1
/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(5/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.51

$$\int \left(a-cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx = \frac{x\sqrt{a-cx^4}\left(-3C(a-cx^4)^2\sqrt{1-\frac{cx^4}{a}}+3a(11Ac+aC)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{cx^4}{a}\right)\right.}{33c\sqrt{1-\frac{cx^4}{a}}}$$

input
```
Integrate[(a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
(x*Sqrt[a - c*x^4]*(-3*C*(a - c*x^4)^2*Sqrt[1 - (c*x^4)/a] + 3*a*(11*A*c +
 a*C)*Hypergeometric2F1[-3/2, 1/4, 5/4, (c*x^4)/a] + 11*a*B*c*x^2*Hypergeo
metric2F1[-3/2, 3/4, 7/4, (c*x^4)/a]))/(33*c*Sqrt[1 - (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {2427, 25, 1491, 27, 1491, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^{3/2} \left(A+Bx^2+Cx^4\right) \, dx$$

$$\Big\downarrow \text{2427}$$

$$-\frac{\int -\left(\left(11Bcx^2+11Ac+aC\right)\left(a-cx^4\right)^{3/2}\right)dx}{11c}-\frac{Cx\left(a-cx^4\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \left(11Bcx^2+11Ac+aC\right)\left(a-cx^4\right)^{3/2}dx}{11c}-\frac{Cx\left(a-cx^4\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{1491}$$

$$\frac{\frac{1}{21}\int 2a\left(77Bcx^2+9(11Ac+aC)\right)\sqrt{a-cx^4}dx+\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9(aC+11Ac)+77Bcx^2\right)}{11c}-\frac{Cx\left(a-cx^4\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2}{21}a\int \left(77Bcx^2+9(11Ac+aC)\right)\sqrt{a-cx^4}dx+\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9(aC+11Ac)+77Bcx^2\right)}{11c}-\frac{Cx\left(a-cx^4\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{1491}$$

$$\frac{\frac{2}{21}a\left(\frac{1}{15}\int \frac{6a\left(77Bcx^2+15(11Ac+aC)\right)}{\sqrt{a-cx^4}}dx+\frac{1}{5}x\sqrt{a-cx^4}\left(15(aC+11Ac)+77Bcx^2\right)\right)+\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9(aC+11A\right.}{11c}$$

$$\frac{Cx\left(a-cx^4\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2}{21}a\left(\frac{2}{5}a\int \frac{77Bcx^2+15(11Ac+aC)}{\sqrt{a-cx^4}}dx+\frac{1}{5}x\sqrt{a-cx^4}\left(15(aC+11Ac)+77Bcx^2\right)\right)+\frac{1}{63}x\left(a-cx^4\right)^{3/2}\left(9(aC+11Ac)\right.}{11c}$$

$$\frac{Cx\left(a-cx^4\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{1513}$$

$$\frac{\frac{2}{21}a\left(\frac{2}{5}a\left(77\sqrt{a}B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx-\left(77\sqrt{a}B\sqrt{c}-15aC-165Ac\right)\int \frac{1}{\sqrt{a-cx^4}}dx\right)+\frac{1}{5}x\sqrt{a-cx^4}\left(15(aC+11Ac)\right.\right.}{11c}$$

$$\frac{Cx\left(a-cx^4\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2}{21}a\left(\frac{2}{5}a\left(77B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\left(77\sqrt{a}B\sqrt{c}-15aC-165Ac\right)\int \frac{1}{\sqrt{a-cx^4}}dx\right)+\frac{1}{5}x\sqrt{a-cx^4}\left(15(aC+11Ac)+7\right.\right.}{11c}$$

$$\frac{Cx\left(a-cx^4\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{2}{21}a\left(\frac{2}{5}a\left(77B\sqrt{c}\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\frac{\sqrt{1-\frac{cx^4}{a}}(77\sqrt{a}B\sqrt{c}-15aC-165Ac)\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}\right)+\frac{1}{5}x\sqrt{a-cx^4}(15(aC+11Ac)+7}{11c}$$

$$\frac{Cx(a-cx^4)^{5/2}}{11c}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{2}{21}a\left(\frac{2}{5}a\left(77B\sqrt{c}\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(77\sqrt{a}B\sqrt{c}-15aC-165Ac)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+\frac{1}{5}x\sqrt{a-cx^4}(1}{11c}$$

$$\frac{Cx(a-cx^4)^{5/2}}{11c}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\frac{2}{21}a\left(\frac{2}{5}a\left(\frac{77B\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(77\sqrt{a}B\sqrt{c}-15aC-165Ac)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+\frac{1}{5}x\sqrt{a-cx}}{11c}$$

$$\frac{Cx(a-cx^4)^{5/2}}{11c}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\frac{2}{21}a\left(\frac{2}{5}a\left(\frac{77\sqrt{a}B\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(77\sqrt{a}B\sqrt{c}-15aC-165Ac)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)+\frac{1}{5}x\sqrt{a-}}{11c}$$

$$\frac{Cx(a-cx^4)^{5/2}}{11c}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{2}{21}a\left(\frac{2}{5}a\left(\frac{77a^{3/4}B\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(77\sqrt{a}B\sqrt{c}-15aC-165Ac)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}\right)\right.}{11c}$$

$$\frac{Cx\left(a-cx^4\right)^{5/2}}{11c}$$

input
```
Int[(a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x]
```

output
```
-1/11*(C*x*(a - c*x^4)^(5/2))/c + ((x*(9*(11*A*c + a*C) + 77*B*c*x^2)*(a -
 c*x^4)^(3/2))/63 + (2*a*((x*(15*(11*A*c + a*C) + 77*B*c*x^2)*Sqrt[a - c*x
^4])/5 + (2*a*((77*a^(3/4)*B*c^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^4] - (a^(1/4)*(77*Sqrt[a]*B*Sqrt[c
] - 165*A*c - 15*a*C)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(
1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4])))/5))/21)/(11*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.29

method	result
risch	$\frac{x\left(-315 C\, x^{8} c^{2}-385 B\, x^{6} c^{2}-495 A\, c^{2} x^{4}+585 x^{4} a C c+847 B a c\, x^{2}+1485 a A c-180 C\, a^{2}\right) \sqrt{-c\, x^{4}+a}}{3465 c}+\frac{4 a^{2}\left(\frac{165 A c \sqrt{1-\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-}}\right.}{}$
elliptic	$-\frac{C c\, x^{9} \sqrt{-c\, x^{4}+a}}{11}-\frac{B c\, x^{7} \sqrt{-c\, x^{4}+a}}{9}-\frac{\left(A\, c^{2}-\frac{13}{11} C c a\right) x^{5} \sqrt{-c\, x^{4}+a}}{7 c}+\frac{11 B a\, x^{3} \sqrt{-c\, x^{4}+a}}{45}-\frac{\left(-2 a A c+C\, a^{2}+\frac{5 a\left(A\, c^{2}-\frac{1}{1}\right.}{7 c}\right.}{3 c}$
default	$A\left(-\frac{c\, x^{5} \sqrt{-c\, x^{4}+a}}{7}+\frac{3 a x \sqrt{-c\, x^{4}+a}}{7}+\frac{4 a^{2} \sqrt{1-\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{7 \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\, x^{4}+a}}\right)+B\left(-\frac{c\, x^{7} \sqrt{-c\, x^{4}+a}}{9}+\right.$

input
```
int((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/3465/c*x*(-315*C*c^2*x^8-385*B*c^2*x^6-495*A*c^2*x^4+585*C*a*c*x^4+847*B
*a*c*x^2+1485*A*a*c-180*C*a^2)*(-c*x^4+a)^(1/2)+4/1155*a^2/c*(165*A*c/(c^(
1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^
(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+15*C*a/(c^(1
/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(
1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-77*B*c^(1/2)*
a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x
^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)
-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.71

$$\int \left(a-cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx =$$

$$-\frac{924\,Ba^2\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)-12\left((77\,B+15\,C)a^2+165\,Aac\right)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right.}{}$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
-1/3465*(924*B*a^2*sqrt(-c)*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x)
, -1) - 12*((77*B + 15*C)*a^2 + 165*A*a*c)*sqrt(-c)*x*(a/c)^(3/4)*elliptic
_f(arcsin((a/c)^(1/4)/x), -1) + (315*C*c^2*x^10 + 385*B*c^2*x^8 - 847*B*a*
c*x^4 - 45*(13*C*a*c - 11*A*c^2)*x^6 + 924*B*a^2 + 45*(4*C*a^2 - 33*A*a*c)
*x^2)*sqrt(-c*x^4 + a))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.14

$$\int \left(a-cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx = \frac{Aa^{\frac{3}{2}}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{A\sqrt{a}cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{Ba^{\frac{3}{2}}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{B\sqrt{a}cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{Ca^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{C\sqrt{a}cx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((-c*x**4+a)**(3/2)*(C*x**4+B*x**2+A),x)
```

output
```
A*a**(3/2)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(2*I*pi
)/a)/(4*gamma(5/4)) - A*sqrt(a)*c*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,
), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(9/4)) + B*a**(3/2)*x**3*gamma(3/4)
*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) - B
*sqrt(a)*c*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(2*
I*pi)/a)/(4*gamma(11/4)) + C*a**(3/2)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (
9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(9/4)) - C*sqrt(a)*c*x**9*gamma
(9/4)*hyper((-1/2, 9/4), (13/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*gamma(13/
4))
```

Maxima [F]

$$\int (a-cx^4)^{3/2} (A+Bx^2+Cx^4) \, dx = \int (Cx^4+Bx^2+A)(-cx^4+a)^{\frac{3}{2}} \, dx$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int (a-cx^4)^{3/2} (A+Bx^2+Cx^4) \, dx = \int (Cx^4+Bx^2+A)(-cx^4+a)^{\frac{3}{2}} \, dx$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(a-c\,x^4\right)^{3/2}\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int \left(a-cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)\,dx=\frac{29\sqrt{-c\,x^4+a}\,a^2x}{77}+\frac{11\sqrt{-c\,x^4+a}\,ab\,x^3}{45}+\frac{2\sqrt{-c\,x^4+a}\,ac\,x^5}{77}-\frac{\sqrt{-c\,x^4+a}\,bc\,x^7}{9}-\frac{\sqrt{-c\,x^4+a}\,c^2x^9}{11}+\frac{48\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)a^3}{77}+\frac{4\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right)a^2b}{15}$$

input
```
int((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A),x)
```

output
```
(1305*sqrt(a - c*x**4)*a**2*x + 847*sqrt(a - c*x**4)*a*b*x**3 + 90*sqrt(a
- c*x**4)*a*c*x**5 - 385*sqrt(a - c*x**4)*b*c*x**7 - 315*sqrt(a - c*x**4)*
c**2*x**9 + 2160*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a**3 + 924*int((sqrt
(a - c*x**4)*x**2)/(a - c*x**4),x)*a**2*b)/3465
```

3.37 $\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{d+ex^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 586

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{d+ex^2}\,dx =$$

$$-\frac{(9ae^2(Cd-Be)-7cd(Cd^2-e(Bd-Ae)))\,x\sqrt{a-cx^4}}{21e^4}$$

$$+\frac{(11aCe^2-9c(Cd^2-e(Bd-Ae)))\,x^3\sqrt{a-cx^4}}{45e^3}$$

$$+\frac{c(Cd-Be)x^5\sqrt{a-cx^4}}{7e^2}-\frac{cCx^7\sqrt{a-cx^4}}{9e}$$

$$+\frac{a^{3/4}(4a^2Ce^4+15c^2d^2(Cd^2-e(Bd-Ae))-21ace^2(Cd^2-e(Bd-Ae)))\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle| -\right.}{15c^{3/4}e^5\sqrt{a-cx^4}}$$

$$-\frac{\sqrt[4]{a}\left(7\sqrt{a}e(4a^2Ce^4+15c^2d^2(Cd^2-e(Bd-Ae))-21ace^2(Cd^2-e(Bd-Ae)))+5\sqrt{c}(12a^2e^4(Cd-Be\right.}{105c^{3/4}}$$

$$+\frac{\sqrt[4]{a}(cd^2-ae^2)^2\,(Cd^2-Bde+Ae^2)\sqrt{1-\frac{cx^4}{a}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de^6\sqrt{a-cx^4}}$$

output
```
-1/21*(9*a*e^2*(-B*e+C*d)-7*c*d*(C*d^2-e*(-A*e+B*d)))*x*(-c*x^4+a)^(1/2)/e
^4+1/45*(11*C*a*e^2-9*c*(C*d^2-e*(-A*e+B*d)))*x^3*(-c*x^4+a)^(1/2)/e^3+1/7
*c*(-B*e+C*d)*x^5*(-c*x^4+a)^(1/2)/e^2-1/9*c*C*x^7*(-c*x^4+a)^(1/2)/e+1/15
*a^(3/4)*(4*a^2*C*e^4+15*c^2*d^2*(C*d^2-e*(-A*e+B*d))-21*a*c*e^2*(C*d^2-e*
(-A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(3/4)/e^5/
(-c*x^4+a)^(1/2)-1/105*a^(1/4)*(7*a^(1/2)*e*(4*a^2*C*e^4+15*c^2*d^2*(C*d^2
-e*(-A*e+B*d))-21*a*c*e^2*(C*d^2-e*(-A*e+B*d)))+5*c^(1/2)*(12*a^2*e^4*(-B*
e+C*d)+21*c^2*d^3*(C*d^2-e*(-A*e+B*d))-35*a*c*d*e^2*(C*d^2-e*(-A*e+B*d))))
*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(3/4)/e^6/(-c*x^4+a)^(
1/2)+a^(1/4)*(-a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)*(1-c*x^4/a)^(1/2)*Ellipt
icPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4)/d/e^6/(-c*x^4+a)^(1
/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.29 (sec) , antiderivative size = 568, normalized size of antiderivative = 0.97

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \frac{-21i\sqrt{a}de(4a^2Ce^4-21ace^2(Cd^2+e(-Bd+Ae))+15c^2(Cd^4+c}{}$$

input
```
Integrate[((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2),x]
```

output
```
((-21*I)*Sqrt[a]*d*e*(4*a^2*C*e^4 - 21*a*c*e^2*(C*d^2 + e*(-(B*d) + A*e))
+ 15*c^2*(C*d^4 + d^2*e*(-(B*d) + A*e)))*Sqrt[1 - (c*x^4)/a]*EllipticE[I*A
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (3*I)*d*(28*a^(5/2)*C*e^5 + 60*a
^2*Sqrt[c]*e^4*(C*d - B*e) + 105*Sqrt[a]*c^2*d^2*e*(C*d^2 + e*(-(B*d) + A*
e)) - 175*a*c^(3/2)*d*e^2*(C*d^2 + e*(-(B*d) + A*e)) - 147*a^(3/2)*c*e^3*(
C*d^2 + e*(-(B*d) + A*e)) + 105*c^(5/2)*(C*d^5 + d^3*e*(-(B*d) + A*e)))*Sq
rt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - S
qrt[c]*(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e^2*x*(a - c*x^4)*(a*e^2*(135*C*d - 135
*B*e - 77*C*e*x^2) + c*C*(-105*d^3 + 63*d^2*e*x^2 - 45*d*e^2*x^4 + 35*e^3*
x^6) + 3*c*e*(7*A*e*(-5*d + 3*e*x^2) + B*(35*d^2 - 21*d*e*x^2 + 15*e^2*x^4
))) + (315*I)*(c*d^2 - a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e))*Sqrt[1 - (c*x^4
)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[
a])]*x], -1]))/(315*Sqrt[-(Sqrt[c]/Sqrt[a])]*Sqrt[c]*d*e^6*Sqrt[a - c*x^4]
)
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 1115, normalized size of antiderivative = 1.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)}{d+ex^2}\,dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{x^2\left(a^2Ce^4-2ace^2\left(Cd^2-e(Bd-Ae)\right)+c^2\left(Cd^4-d^2e(Bd-Ae)\right)\right)}{e^5\sqrt{a-cx^4}}-\frac{a^2e^4(Cd-Be)-2acde^2\left(Cd^2-e(\cdots\right.}{e^6\sqrt{\cdots}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{cC\sqrt{a-cx^4}x^7}{9e}+\frac{c(Cd-Be)\sqrt{a-cx^4}x^5}{7e^2}-\frac{\left(cCd^2-2aCe^2-ce(Bd-Ae)\right)\sqrt{a-cx^4}x^3}{5e^3}-\frac{7aC\sqrt{a-cx^4}x^3}{45e}+\frac{5a(Cd-Be)\sqrt{a-cx^4}x}{21e^2}+\frac{\left(cCd^3-ce(Bd-Ae)d-2ae^2(Cd-Be)\right)\sqrt{a-cx^4}x}{3e^4}+\frac{3a^{7/4}\left(cCd^2-2aCe^2-ce(Bd-Ae)\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{3/4}e^3\sqrt{a-cx^4}}+\frac{a^{3/4}\left(a^2Ce^4-2ac\left(Cd^2-e(Bd-Ae)\right)e^2+c^2\left(Cd^4-d^2e(Bd-Ae)\right)\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}e^5\sqrt{a-cx^4}}+\frac{7a^{11/4}C\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15c^{3/4}e\sqrt{a-cx^4}}-\frac{5a^{9/4}(Cd-Be)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{21\sqrt[4]{c}e^2\sqrt{a-cx^4}}-\frac{3a^{7/4}\left(cCd^2-2aCe^2-ce(Bd-Ae)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{3/4}e^3\sqrt{a-cx^4}}-\frac{a^{5/4}\left(cCd^3-ce(Bd-Ae)d-2ae^2(Cd-Be)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{c}e^4\sqrt{a-cx^4}}-\frac{a^{3/4}\left(a^2Ce^4-2ac\left(Cd^2-e(Bd-Ae)\right)e^2+c^2\left(Cd^4-d^2e(Bd-Ae)\right)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right.}{c^{3/4}e^5\sqrt{a-cx^4}}$$

$$\frac{\sqrt[4]{a}\left(a^2(Cd-Be)e^4-2acd\left(Cd^2-e(Bd-Ae)\right)e^2+c^2\left(Cd^5-d^3e(Bd-Ae)\right)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{\sqrt[4]{c}e^6\sqrt{a-cx^4}}$$

$$\frac{7a^{11/4}C\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{15c^{3/4}e\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\left(cd^2-ae^2\right)^2\left(Cd^2-Bed+Ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de^6\sqrt{a-cx^4}}$$

input
```
Int[((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2),x]
```

output
```
(5*a*(C*d - B*e)*x*Sqrt[a - c*x^4])/(21*e^2) + ((c*C*d^3 - c*d*e*(B*d - A*
e) - 2*a*e^2*(C*d - B*e))*x*Sqrt[a - c*x^4])/(3*e^4) - (7*a*C*x^3*Sqrt[a -
 c*x^4])/(45*e) - ((c*C*d^2 - 2*a*C*e^2 - c*e*(B*d - A*e))*x^3*Sqrt[a - c*
x^4])/(5*e^3) + (c*(C*d - B*e)*x^5*Sqrt[a - c*x^4])/(7*e^2) - (c*C*x^7*Sqr
t[a - c*x^4])/(9*e) + (7*a^(11/4)*C*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(
c^(1/4)*x)/a^(1/4)], -1])/(15*c^(3/4)*e*Sqrt[a - c*x^4]) + (3*a^(7/4)*(c*C
*d^2 - 2*a*C*e^2 - c*e*(B*d - A*e))*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(
c^(1/4)*x)/a^(1/4)], -1])/(5*c^(3/4)*e^3*Sqrt[a - c*x^4]) + (a^(3/4)*(a^2*
C*e^4 - 2*a*c*e^2*(C*d^2 - e*(B*d - A*e)) + c^2*(C*d^4 - d^2*e*(B*d - A*e)
))*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4
)*e^5*Sqrt[a - c*x^4]) - (7*a^(11/4)*C*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(15*c^(3/4)*e*Sqrt[a - c*x^4]) - (5*a^(9/4)*(
C*d - B*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(21*c^(1/4)*e^2*Sqrt[a - c*x^4]) - (3*a^(7/4)*(c*C*d^2 - 2*a*C*e^2 - c*e*
(B*d - A*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1
])/(5*c^(3/4)*e^3*Sqrt[a - c*x^4]) - (a^(5/4)*(c*C*d^3 - c*d*e*(B*d - A*e)
 - 2*a*e^2*(C*d - B*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a
^(1/4)], -1])/(3*c^(1/4)*e^4*Sqrt[a - c*x^4]) - (a^(3/4)*(a^2*C*e^4 - 2*a*
c*e^2*(C*d^2 - e*(B*d - A*e)) + c^2*(C*d^4 - d^2*e*(B*d - A*e)))*Sqrt[1 -
(c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*e^5*Sqr...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.09

method	result
risch	$\frac{x(-35C\,x^6 c\,e^3-45Bc\,e^3x^4+45Ccd\,e^2x^4-63Ac\,e^3x^2+63Bcd\,e^2x^2+77Ca\,e^3x^2-63Cc\,d^2e\,x^2+105Acd\,e^2+135Ba\,e^3-105Bc\,d^2e-1}{315e^4}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/315*x*(-35*C*c*e^3*x^6-45*B*c*e^3*x^4+45*C*c*d*e^2*x^4-63*A*c*e^3*x^2+63
*B*c*d*e^2*x^2+77*C*a*e^3*x^2-63*C*c*d^2*e*x^2+105*A*c*d*e^2+135*B*a*e^3-1
05*B*c*d^2*e-135*C*a*d*e^2+105*C*c*d^3)*(-c*x^4+a)^(1/2)/e^4-1/105/e^4*(-1
05*(A*a^2*e^6-2*A*a*c*d^2*e^4+A*c^2*d^4*e^2-B*a^2*d*e^5+2*B*a*c*d^3*e^3-B*
c^2*d^5*e+C*a^2*d^2*e^4-2*C*a*c*d^4*e^2+C*c^2*d^6)/e^2/d/(c^(1/2)/a^(1/2))
^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4
+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1
/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))-5*(35*A*a*c*d*e^4-21*A*c^2*d^3
*e^2+12*B*a^2*e^5-35*B*a*c*d^2*e^3+21*B*c^2*d^4*e-12*C*a^2*d*e^4+35*C*a*c*
d^3*e^2-21*C*c^2*d^5)/e^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^
(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/
a^(1/2))^(1/2),I)-7/e*(21*A*a*c*e^4-15*A*c^2*d^2*e^2-21*B*a*c*d*e^3+15*B*c
^2*d^3*e-4*C*a^2*e^4+21*C*a*c*d^2*e^2-15*C*c^2*d^4)*a^(1/2)/(c^(1/2)/a^(1/
2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*
x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(
c^(1/2)/a^(1/2))^(1/2),I)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \text{Timed out}$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \int \frac{(a-cx^4)^{\frac{3}{2}}(A+Bx^2+Cx^4)}{d+ex^2}\,dx$$

input
```
integrate((-c*x**4+a)**(3/2)*(C*x**4+B*x**2+A)/(e*x**2+d),x)
```

output
```
Integral((a - c*x**4)**(3/2)*(A + B*x**2 + C*x**4)/(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(-cx^4+a)^{\frac{3}{2}}}{ex^2+d}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d),x, algorithm="maxima"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)/(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(-cx^4+a)^{\frac{3}{2}}}{ex^2+d}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)/(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \int \frac{(a-c\,x^4)^{3/2}\,(C\,x^4+B\,x^2+A)}{e\,x^2+d}\,dx$$

input
```
int(((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2),x)
```

output
```
int(((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
int((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d),x)
```

output

```
(135*sqrt(a - c*x**4)*a*b*e**3*x - 30*sqrt(a - c*x**4)*a*c*d*e**2*x + 14*s
qrt(a - c*x**4)*a*c*e**3*x**3 - 105*sqrt(a - c*x**4)*b*c*d**2*e*x + 63*sqr
t(a - c*x**4)*b*c*d*e**2*x**3 - 45*sqrt(a - c*x**4)*b*c*e**3*x**5 + 105*sq
rt(a - c*x**4)*c**2*d**3*x - 63*sqrt(a - c*x**4)*c**2*d**2*e*x**3 + 45*sqr
t(a - c*x**4)*c**2*d*e**2*x**5 - 35*sqrt(a - c*x**4)*c**2*e**3*x**7 + 315*
int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**3*e**4 -
 135*int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*b
*d*e**3 + 30*int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x
)*a**2*c*d**2*e**2 + 105*int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 -
 c*e*x**6),x)*a*b*c*d**3*e - 105*int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*
d*x**4 - c*e*x**6),x)*a*c**2*d**4 - 357*int((sqrt(a - c*x**4)*x**4)/(a*d +
 a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*e**4 + 441*int((sqrt(a - c*x**4
)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d*e**3 - 126*int((
sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c**2*d*
*2*e**2 - 315*int((sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e
*x**6),x)*b*c**2*d**3*e + 315*int((sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2
- c*d*x**4 - c*e*x**6),x)*c**3*d**4 + 180*int((sqrt(a - c*x**4)*x**2)/(a*d
 + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*b*e**4 - 12*int((sqrt(a - c*x**
4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d*e**3 - 84*int(
(sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c...
```

3.38 $\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 614

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx = \frac{(9aCe^2-7c(3Cd^2-e(2Bd-Ae)))x\sqrt{a-cx^4}}{21e^4} + \frac{c(2Cd-Be)x^3\sqrt{a-cx^4}}{5e^3} - \frac{cCx^5\sqrt{a-cx^4}}{7e^2} - \frac{(cd^2-ae^2)(Cd^2-Bde+Ae^2)x\sqrt{a-cx^4}}{2de^4(d+ex^2)}$$
$$- \frac{a^{3/4}\sqrt[4]{c}(5cd^2(9Cd^2-e(7Bd-5Ae))-ae^2(33Cd^2-e(19Bd-5Ae)))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right.}{10de^5\sqrt{a-cx^4}}$$
$$+ \frac{\sqrt[4]{a}\left(120a^2Cde^4-35acde^2(27Cd^2-e(17Bd-7Ae))+105c^2d^3(9Cd^2-e(7Bd-5Ae))+105\sqrt{a}c^{3/2}d^2e\right.}{210\sqrt[4]{c}d}$$
$$- \frac{\sqrt[4]{a}(cd^2-ae^2)(cd^2(9Cd^2-e(7Bd-5Ae))-ae^2(3Cd^2-e(Bd+Ae)))\sqrt{1-\frac{cx^4}{a}}\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\text{arc}\right.}{2\sqrt[4]{c}d^2e^6\sqrt{a-cx^4}}$$

output
```
1/21*(9*C*a*e^2-7*c*(3*C*d^2-e*(-A*e+2*B*d)))*x*(-c*x^4+a)^(1/2)/e^4+1/5*c
*(-B*e+2*C*d)*x^3*(-c*x^4+a)^(1/2)/e^3-1/7*c*C*x^5*(-c*x^4+a)^(1/2)/e^2-1/
2*(-a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*x*(-c*x^4+a)^(1/2)/d/e^4/(e*x^2+d)-1/
10*a^(3/4)*c^(1/4)*(5*c*d^2*(9*C*d^2-e*(-5*A*e+7*B*d))-a*e^2*(33*C*d^2-e*(
-5*A*e+19*B*d)))*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/d/e^5/(-
c*x^4+a)^(1/2)+1/210*a^(1/4)*(120*a^2*C*d*e^4-35*a*c*d*e^2*(27*C*d^2-e*(-7
*A*e+17*B*d))+105*c^2*d^3*(9*C*d^2-e*(-5*A*e+7*B*d))+105*a^(1/2)*c^(3/2)*d
^2*e*(9*C*d^2-e*(-5*A*e+7*B*d))-21*a^(3/2)*c^(1/2)*e^3*(33*C*d^2-e*(-5*A*e
+19*B*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(1/4)/d/e^6/
(-c*x^4+a)^(1/2)-1/2*a^(1/4)*(-a*e^2+c*d^2)*(c*d^2*(9*C*d^2-e*(-5*A*e+7*B*
d))-a*e^2*(3*C*d^2-e*(A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^
(1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4)/d^2/e^6/(-c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.07 (sec) , antiderivative size = 2295, normalized size of antiderivative = 3.74

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^2,x]
```

output

```
(-315*a*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*C*d^5*e^2*x + 245*a*B*Sqrt[-(Sqrt[c]/Sq
rt[a])]*c*d^4*e^3*x - 175*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^3*e^4*x + 195*a
^2*Sqrt[-(Sqrt[c]/Sqrt[a])]*C*d^3*e^4*x - 105*a^2*B*Sqrt[-(Sqrt[c]/Sqrt[a]
)]*d^2*e^5*x + 105*a^2*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e^6*x - 126*a*Sqrt[-(S
qrt[c]/Sqrt[a])]*c*C*d^4*e^3*x^3 + 98*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^3*e
^4*x^3 - 70*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^2*e^5*x^3 + 90*a^2*Sqrt[-(Sqr
t[c]/Sqrt[a])]*C*d^2*e^5*x^3 + 315*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*C*d^5*e^2*
x^5 - 245*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d^4*e^3*x^5 + 175*A*Sqrt[-(Sqrt[c
]/Sqrt[a])]*c^2*d^3*e^4*x^5 - 141*a*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*C*d^3*e^4*x
^5 + 63*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^2*e^5*x^5 - 105*a*A*Sqrt[-(Sqrt[c
]/Sqrt[a])]*c*d*e^6*x^5 + 126*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*C*d^4*e^3*x^7 -
 98*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d^3*e^4*x^7 + 70*A*Sqrt[-(Sqrt[c]/Sqrt[
a])]*c^2*d^2*e^5*x^7 - 120*a*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*C*d^2*e^5*x^7 - 54
*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*C*d^3*e^4*x^9 + 42*B*Sqrt[-(Sqrt[c]/Sqrt[a])
]*c^2*d^2*e^5*x^9 + 30*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*C*d^2*e^5*x^11 - (21*I
)*Sqrt[a]*Sqrt[c]*d*e*(a*e^2*(33*C*d^2 - 19*B*d*e + 5*A*e^2) - 5*c*d^2*(9*
C*d^2 - 7*B*d*e + 5*A*e^2))*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*Ar
cSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*d*(120*a^2*C*d*e^4 - 21*a^(3/2)
*Sqrt[c]*e^3*(33*C*d^2 - 19*B*d*e + 5*A*e^2) + 105*c^2*d^3*(9*C*d^2 - 7*B*
d*e + 5*A*e^2) + 105*Sqrt[a]*c^(3/2)*d^2*e*(9*C*d^2 - 7*B*d*e + 5*A*e^2...
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 1217, normalized size of antiderivative = 1.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{a^2Ce^4-2ace^2(3Cd^2-e(2Bd-Ae))+c^2(5Cd^4-d^2e(4Bd-3Ae))}{e^6\sqrt{a-cx^4}}+\frac{cx^2(2ae^2(2Cd-Be)+cde(3Bd-}{e^5\sqrt{a-cx^4}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{cC\sqrt{a-cx^4}x^5}{7e^2}+\frac{c(2Cd-Be)\sqrt{a-cx^4}x^3}{5e^3}-\frac{(3cCd^2-2aCe^2-ce(2Bd-Ae))\sqrt{a-cx^4}x}{3e^4}-\frac{(cd^2-ae^2)(Cd^2-Bed+Ae^2)\sqrt{a-cx^4}x}{2de^4(ex^2+d)}-\frac{5aC\sqrt{a-cx^4}x}{21e^2}-\frac{3a^{7/4}\sqrt[4]{c}(2Cd-Be)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5e^3\sqrt{a-cx^4}}-\frac{a^{3/4}\sqrt[4]{c}(cd^2-ae^2)(Cd^2-Bed+Ae^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2de^5\sqrt{a-cx^4}}-\frac{a^{3/4}\sqrt[4]{c}(4cCd^3-ce(3Bd-2Ae)d-2ae^2(2Cd-Be))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{e^5\sqrt{a-cx^4}}+\frac{3a^{7/4}\sqrt[4]{c}(2Cd-Be)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5e^3\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt[4]{c}(cd^2-ae^2)^2(Cd^2-Bed+Ae^2)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2de^6(\sqrt{c}d+\sqrt{a}e)\sqrt{a-cx^4}}+\frac{a^{5/4}(3cCd^2-2aCe^2-ce(2Bd-Ae))\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{c}e^4\sqrt{a-cx^4}}+\frac{a^{3/4}\sqrt[4]{c}(4cCd^3-ce(3Bd-2Ae)d-2ae^2(2Cd-Be))\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{e^5\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}(a^2Ce^4-2ac(3Cd^2-e(2Bd-Ae))e^2+c^2(5Cd^4-d^2e(4Bd-3Ae)))\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{\sqrt[4]{c}e^6\sqrt{a-cx^4}}\frac{5a^{9/4}C\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{21\sqrt[4]{c}e^2\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}(cd^2-ae^2)(3cd^2-ae^2)(Cd^2-Bed+Ae^2)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}d^2e^6\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}(cd^2-ae^2)(6cCd^3-ce(5Bd-4Ae)d-ae^2(2Cd-Be))\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de^6\sqrt{a-cx^4}}$$

input
```
Int[((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^2,x]
```

output
```
(-5*a*C*x*Sqrt[a - c*x^4])/(21*e^2) - ((3*c*C*d^2 - 2*a*C*e^2 - c*e*(2*B*d
 - A*e))*x*Sqrt[a - c*x^4])/(3*e^4) + (c*(2*C*d - B*e)*x^3*Sqrt[a - c*x^4]
)/(5*e^3) - (c*C*x^5*Sqrt[a - c*x^4])/(7*e^2) - ((c*d^2 - a*e^2)*(C*d^2 -
B*d*e + A*e^2)*x*Sqrt[a - c*x^4])/(2*d*e^4*(d + e*x^2)) - (3*a^(7/4)*c^(1/
4)*(2*C*d - B*e)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)]
, -1])/(5*e^3*Sqrt[a - c*x^4]) - (a^(3/4)*c^(1/4)*(c*d^2 - a*e^2)*(C*d^2 -
 B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)],
 -1])/(2*d*e^5*Sqrt[a - c*x^4]) - (a^(3/4)*c^(1/4)*(4*c*C*d^3 - c*d*e*(3*B
*d - 2*A*e) - 2*a*e^2*(2*C*d - B*e))*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(e^5*Sqrt[a - c*x^4]) + (5*a^(9/4)*C*Sqrt[1 - (
c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(21*c^(1/4)*e^2*Sqrt
[a - c*x^4]) + (3*a^(7/4)*c^(1/4)*(2*C*d - B*e)*Sqrt[1 - (c*x^4)/a]*Ellipt
icF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*e^3*Sqrt[a - c*x^4]) - (a^(1/4)*c
^(1/4)*(c*d^2 - a*e^2)^2*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*Ellip
ticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*e^6*(Sqrt[c]*d + Sqrt[a]*e)*Sq
rt[a - c*x^4]) + (a^(5/4)*(3*c*C*d^2 - 2*a*C*e^2 - c*e*(2*B*d - A*e))*Sqrt
[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(1/4)*e^4
*Sqrt[a - c*x^4]) + (a^(3/4)*c^(1/4)*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) -
2*a*e^2*(2*C*d - B*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^
(1/4)], -1])/(e^5*Sqrt[a - c*x^4]) + (a^(1/4)*(a^2*C*e^4 + c^2*(5*C*d^4...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1145 vs. 2(544) = 1088.

Time = 4.40 (sec) , antiderivative size = 1146, normalized size of antiderivative = 1.87

method	result	size
risch	Expression too large to display	1146
default	Expression too large to display	1718
elliptic	Expression too large to display	2521

input
```
int((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/105*x*(15*C*c*e^2*x^4+21*B*c*e^2*x^2-42*C*c*d*e*x^2+35*A*c*e^2-70*B*c*d
*e-45*C*a*e^2+105*C*c*d^2)*(-c*x^4+a)^(1/2)/e^4-1/105/e^4*(-105/e^2*(4*A*a
*c*d*e^4-4*A*c^2*d^3*e^2+B*a^2*e^5-6*B*a*c*d^2*e^3+5*B*c^2*d^4*e-2*C*a^2*d
*e^4+8*C*a*c*d^3*e^2-6*C*c^2*d^5)/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2
/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(
x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c
^(1/2)/a^(1/2))^(1/2))-105*(A*a^2*e^6-2*A*a*c*d^2*e^4+A*c^2*d^4*e^2-B*a^2*
d*e^5+2*B*a*c*d^3*e^3-B*c^2*d^5*e+C*a^2*d^2*e^4-2*C*a*c*d^4*e^2+C*c^2*d^6)
/e^2*(1/2*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/2*c/(a*e^2-c*
d^2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/
a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-1/2
*e*c^(1/2)/(a*e^2-c*d^2)/d*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/
a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*
(c^(1/2)/a^(1/2))^(1/2),I)+1/2*e*c^(1/2)/(a*e^2-c*d^2)/d*a^(1/2)/(c^(1/2)/
a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)
/(-c*x^4+a)^(1/2)*EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/2/(a*e^2-c*d^2)
/d^2*e^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*
x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-
a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))*a-3/
2/(a*e^2-c*d^2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a-cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)}{\left(d+ex^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\left(a-cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{\left(a-cx^4\right)^{\frac{3}{2}}\left(A+Bx^2+Cx^4\right)}{\left(d+ex^2\right)^2}\,dx$$

input
```
integrate((-c*x**4+a)**(3/2)*(C*x**4+B*x**2+A)/(e*x**2+d)**2,x)
```

output
```
Integral((a - c*x**4)**(3/2)*(A + B*x**2 + C*x**4)/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\left(a-cx^4\right)^{3/2}\left(A+Bx^2+Cx^4\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{\left(Cx^4+Bx^2+A\right)\left(-cx^4+a\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)/(e*x^2 + d)^2, x)
```

Giac [F]

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^2}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^2,x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx = \int \frac{(a-c\,x^4)^{3/2}\,(C\,x^4+B\,x^2+A)}{(e\,x^2+d)^2}\,dx$$

input
```
int(((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^2,x)
```

output
```
int(((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^2}\,dx = \text{too large to display}$$

input
```
int((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^2,x)
```

output

```
(115*sqrt(a - c*x**4)*a**2*e**3*x - 56*sqrt(a - c*x**4)*a*b*d*e**2*x + 72*
sqrt(a - c*x**4)*a*c*d**2*e*x + 30*sqrt(a - c*x**4)*a*c*d*e**2*x**3 + 147*
sqrt(a - c*x**4)*b*c*d**2*e*x**3 - 63*sqrt(a - c*x**4)*b*c*d*e**2*x**5 - 1
89*sqrt(a - c*x**4)*c**2*d**3*x**3 + 81*sqrt(a - c*x**4)*c**2*d**2*e*x**5
- 45*sqrt(a - c*x**4)*c**2*d*e**2*x**7 + 200*int(sqrt(a - c*x**4)/(a*d**2
+ 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x
)*a**3*d**2*e**3 + 200*int(sqrt(a - c*x**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**
2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**3*d*e**4*x**2 + 5
6*int(sqrt(a - c*x**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4
- 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*b*d**3*e**2 + 56*int(sqrt(a - c*x**4
)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e*
*2*x**8),x)*a**2*b*d**2*e**3*x**2 - 72*int(sqrt(a - c*x**4)/(a*d**2 + 2*a*
d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2
*c*d**4*e - 72*int(sqrt(a - c*x**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 -
 c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*c*d**3*e**2*x**2 + 115*
int((sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*
x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*c*d*e**4 + 115*int((sqrt(a - c*
x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x
**6 - c*e**2*x**8),x)*a**2*c*e**5*x**2 - 497*int((sqrt(a - c*x**4)*x**6)/(
a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e*...
```

3.39 $\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^3} dx$

Optimal result

Integrand size = 34, antiderivative size = 648

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^3} dx = \frac{c(3Cd-Be)x\sqrt{a-cx^4}}{3e^4}$$
$$-\frac{cCx^3\sqrt{a-cx^4}}{5e^3}-\frac{(cd^2-ae^2)(Cd^2-Bde+Ae^2)x\sqrt{a-cx^4}}{4de^4(d+ex^2)^2}$$
$$+\frac{(cd^2(15Cd^2-e(11Bd-7Ae))-ae^2(5Cd^2-e(Bd+3Ae)))x\sqrt{a-cx^4}}{8d^2e^4(d+ex^2)}$$
$$+\frac{a^{3/4}\sqrt[4]{c}(5cd^2(63Cd^2-5e(7Bd-3Ae))-ae^2(81Cd^2-5e(Bd+3Ae)))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{40d^2e^5\sqrt{a-cx^4}}$$
$$-\frac{\sqrt[4]{a}\sqrt[4]{c}(15c^{3/2}d^3(63Cd^2-5e(7Bd-3Ae))+15\sqrt{a}cd^2e(63Cd^2-5e(7Bd-3Ae))-5a\sqrt{c}de^2(99Cd^2-}{120d^2e^6\sqrt{a-}}$$
$$+\frac{\sqrt[4]{a}(c^2d^4(63Cd^2-5e(7Bd-3Ae))-2acd^2e^2(27Cd^2-e(11Bd-3Ae))+a^2e^4(3Cd^2+e(Bd+3Ae)))}{8\sqrt[4]{c}d^3e^6\sqrt{a-cx^4}}$$

output
```
1/3*c*(-B*e+3*C*d)*x*(-c*x^4+a)^(1/2)/e^4-1/5*c*C*x^3*(-c*x^4+a)^(1/2)/e^3
-1/4*(-a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*x*(-c*x^4+a)^(1/2)/d/e^4/(e*x^2+d)
^2+1/8*(c*d^2*(15*C*d^2-e*(-7*A*e+11*B*d))-a*e^2*(5*C*d^2-e*(3*A*e+B*d)))*
x*(-c*x^4+a)^(1/2)/d^2/e^4/(e*x^2+d)+1/40*a^(3/4)*c^(1/4)*(5*c*d^2*(63*C*d
^2-5*e*(-3*A*e+7*B*d))-a*e^2*(81*C*d^2-5*e*(3*A*e+B*d)))*(1-c*x^4/a)^(1/2)
*EllipticE(c^(1/4)*x/a^(1/4),I)/d^2/e^5/(-c*x^4+a)^(1/2)-1/120*a^(1/4)*c^(
1/4)*(15*c^(3/2)*d^3*(63*C*d^2-5*e*(-3*A*e+7*B*d))+15*a^(1/2)*c*d^2*e*(63*
C*d^2-5*e*(-3*A*e+7*B*d))-5*a*c^(1/2)*d*e^2*(99*C*d^2-e*(-3*A*e+31*B*d))-3
*a^(3/2)*e^3*(81*C*d^2-5*e*(3*A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/
4)*x/a^(1/4),I)/d^2/e^6/(-c*x^4+a)^(1/2)+1/8*a^(1/4)*(c^2*d^4*(63*C*d^2-5*
e*(-3*A*e+7*B*d))-2*a*c*d^2*e^2*(27*C*d^2-e*(-3*A*e+11*B*d))+a^2*e^4*(3*C*
d^2+e*(3*A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2
)*e/c^(1/2)/d,I)/c^(1/4)/d^3/e^6/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.43 (sec) , antiderivative size = 579, normalized size of antiderivative = 0.89

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \frac{-\frac{de^2x(a-cx^4)(-15ae^2(-Cd^2(3d+5ex^2)+e(Bd(-d+ex^2)+Ae(5d+3ex^2)))+cd^2(-3C(1\cdots}{(d+e\cdots}}{\cdots}$$

input
```
Integrate[((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^3,x]
```

output
```
(-((d*e^2*x*(a - c*x^4)*(-15*a*e^2*(-(C*d^2*(3*d + 5*e*x^2)) + e*(B*d*(-d
+ e*x^2) + A*e*(5*d + 3*e*x^2))) + c*d^2*(-3*C*(105*d^3 + 147*d^2*e*x^2 +
24*d*e^2*x^4 - 8*e^3*x^6) + 5*e*(-3*A*e*(5*d + 7*e*x^2) + B*(35*d^2 + 49*d
*e*x^2 + 8*e^2*x^4)))))/(d + e*x^2)^2) - (I*Sqrt[1 - (c*x^4)/a]*(-3*Sqrt[a
]*Sqrt[c]*d*e*(-5*c*(63*C*d^4 + 5*d^2*e*(-7*B*d + 3*A*e)) + a*e^2*(81*C*d^
2 - 5*e*(B*d + 3*A*e)))*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -
1] - Sqrt[c]*d*(-5*a*Sqrt[c]*d*e^2*(99*C*d^2 + e*(-31*B*d + 3*A*e)) + 15*S
qrt[a]*c*d^2*e*(63*C*d^2 + 5*e*(-7*B*d + 3*A*e)) + 15*c^(3/2)*(63*C*d^5 +
5*d^3*e*(-7*B*d + 3*A*e)) + 3*a^(3/2)*e^3*(-81*C*d^2 + 5*e*(B*d + 3*A*e)))
*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + 15*(-2*a*c*d^2*e^2
*(27*C*d^2 + e*(-11*B*d + 3*A*e)) + c^2*(63*C*d^6 + 5*d^4*e*(-7*B*d + 3*A*
e)) + a^2*e^4*(3*C*d^2 + e*(B*d + 3*A*e)))*EllipticPi[-((Sqrt[a]*e)/(Sqrt[
c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/Sqrt[-(Sqrt[c]/Sqrt[a
])])/(120*d^3*e^6*Sqrt[a - c*x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1506 vs. $2(648) = 1296$.

Time = 2.56 (sec) , antiderivative size = 1506, normalized size of antiderivative = 2.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{a^2Ce^4-2ace^2(6Cd^2-e(3Bd-Ae))+c^2(15Cd^4-2d^2e(5Bd-3Ae))}{e^6\sqrt{a-cx^4}(d+ex^2)}+\frac{c(2ae^2(3Cd-Be)+3cde(2Bd}{e^6\sqrt{a-cx^4}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{cC\sqrt{a-cx^4}x^3}{5e^3}+\frac{c(3Cd-Be)\sqrt{a-cx^4}x}{3e^4}-\frac{3(3cd^2-ae^2)\left(Cd^2-Bed+Ae^2\right)\sqrt{a-cx^4}x}{8d^2e^4\left(ex^2+d\right)}+$$

$$\frac{\left(6cCd^3-ce(5Bd-4Ae)d-ae^2(2Cd-Be)\right)\sqrt{a-cx^4}x}{2de^4\left(ex^2+d\right)}-$$

$$\frac{\left(cd^2-ae^2\right)\left(Cd^2-Bed+Ae^2\right)\sqrt{a-cx^4}x}{4de^4\left(ex^2+d\right)^2}-$$

$$\frac{3a^{3/4}\sqrt[4]{c}(3cd^2-ae^2)\left(Cd^2-Bed+Ae^2\right)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2e^5\sqrt{a-cx^4}}+$$

$$\frac{a^{3/4}\sqrt[4]{c}(6cCd^2-2aCe^2-ce(3Bd-Ae))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{e^5\sqrt{a-cx^4}}+$$

$$\frac{a^{3/4}\sqrt[4]{c}(6cCd^3-ce(5Bd-4Ae)d-ae^2(2Cd-Be))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2de^5\sqrt{a-cx^4}}+$$

$$\frac{3a^{7/4}\sqrt[4]{c}C\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5e^3\sqrt{a-cx^4}}-$$

$$\frac{a^{5/4}c^{3/4}(3Cd-Be)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3e^4\sqrt{a-cx^4}}-$$

$$\frac{\sqrt[4]{a}\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\left(7cd^2-2\sqrt{a}\sqrt{c}ed-3ae^2\right)\left(Cd^2-Bed+Ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8d^2e^6\sqrt{a-cx^4}}-$$

$$\frac{a^{3/4}\sqrt[4]{c}(6cCd^2-2aCe^2-ce(3Bd-Ae))\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{e^5\sqrt{a-cx^4}}+$$

$$\frac{\sqrt[4]{a}\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\left(6cCd^3-ce(5Bd-4Ae)d-ae^2(2Cd-Be)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2de^6\sqrt{a-cx^4}}+$$

$$\frac{\sqrt[4]{a}c^{3/4}\left(2ae^2(3Cd-Be)-cd\left(10Cd^2-6Bed+3Ae^2\right)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{e^6\sqrt{a-cx^4}}-$$

$$\frac{3a^{7/4}\sqrt[4]{c}C\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5e^3\sqrt{a-cx^4}}+$$

$$\frac{3\sqrt[4]{a}\left(Cd^2-Bed+Ae^2\right)\left(5c^2d^4-2ace^2d^2+a^2e^4\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{c}d^3e^6\sqrt{a-cx^4}}-$$

$$\frac{\sqrt[4]{a}(3cd^2-ae^2)\left(6cCd^3-ce(5Bd-4Ae)d-ae^2(2Cd-Be)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}d^2e^6\sqrt{a-cx^4}}$$

$$\frac{\sqrt[4]{a}\left(a^2Ce^4-2ac\left(6Cd^2-e(3Bd-Ae)\right)e^2+c^2\left(15Cd^4-2d^2e(5Bd-3Ae)\right)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\text{arcsi}\right.}{\sqrt[4]{c}de^6\sqrt{a-cx^4}}$$

input
```
Int[((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^3,x]
```

output
```
(c*(3*C*d - B*e)*x*Sqrt[a - c*x^4])/(3*e^4) - (c*C*x^3*Sqrt[a - c*x^4])/(5
*e^3) - ((c*d^2 - a*e^2)*(C*d^2 - B*d*e + A*e^2)*x*Sqrt[a - c*x^4])/(4*d*e
^4*(d + e*x^2)^2) - (3*(3*c*d^2 - a*e^2)*(C*d^2 - B*d*e + A*e^2)*x*Sqrt[a
- c*x^4])/(8*d^2*e^4*(d + e*x^2)) + ((6*c*C*d^3 - c*d*e*(5*B*d - 4*A*e) -
a*e^2*(2*C*d - B*e))*x*Sqrt[a - c*x^4])/(2*d*e^4*(d + e*x^2)) + (3*a^(7/4)
*c^(1/4)*C*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(5*e^3*Sqrt[a - c*x^4]) - (3*a^(3/4)*c^(1/4)*(3*c*d^2 - a*e^2)*(C*d^2 - B
*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(8*d^2*e^5*Sqrt[a - c*x^4]) + (a^(3/4)*c^(1/4)*(6*c*C*d^2 - 2*a*C*e^2
- c*e*(3*B*d - A*e))*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1
/4)], -1])/(e^5*Sqrt[a - c*x^4]) + (a^(3/4)*c^(1/4)*(6*c*C*d^3 - c*d*e*(5*
B*d - 4*A*e) - a*e^2*(2*C*d - B*e))*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(
c^(1/4)*x)/a^(1/4)], -1])/(2*d*e^5*Sqrt[a - c*x^4]) - (3*a^(7/4)*c^(1/4)*C
*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*e^3*Sq
rt[a - c*x^4]) - (a^(5/4)*c^(3/4)*(3*C*d - B*e)*Sqrt[1 - (c*x^4)/a]*Ellipt
icF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*e^4*Sqrt[a - c*x^4]) - (a^(1/4)*c
^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)
*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/
a^(1/4)], -1])/(8*d^2*e^6*Sqrt[a - c*x^4]) - (a^(3/4)*c^(1/4)*(6*c*C*d^2 -
 2*a*C*e^2 - c*e*(3*B*d - A*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2099 vs. $2(578) = 1156$.

Time = 6.95 (sec) , antiderivative size = 2100, normalized size of antiderivative = 3.24

method	result	size
risch	Expression too large to display	2100
default	Expression too large to display	2446
elliptic	Expression too large to display	2466

input
```
int((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*c*x*(3*C*e*x^2+5*B*e-15*C*d)*(-c*x^4+a)^(1/2)/e^4+1/15/e^4*(-15/e^2*
(2*A*a*c*e^4-6*A*c^2*d^2*e^2-6*B*a*c*d*e^3+10*B*c^2*d^3*e-C*a^2*e^4+12*C*a
*c*d^2*e^2-15*C*c^2*d^4)/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))
^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2
)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^
(1/2))^(1/2))+15/e^2*(4*A*a*c*d*e^4-4*A*c^2*d^3*e^2+B*a^2*e^5-6*B*a*c*d^2*
e^3+5*B*c^2*d^4*e-2*C*a^2*d*e^4+8*C*a*c*d^3*e^2-6*C*c^2*d^5)*(1/2*e^2/(a*e
^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/2*c/(a*e^2-c*d^2)/(c^(1/2)/a^(1
/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c
*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-1/2*e*c^(1/2)/(a*e^2-
c*d^2)/d*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+
c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^
(1/2),I)+1/2*e*c^(1/2)/(a*e^2-c*d^2)/d*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-
c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*
EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/2/(a*e^2-c*d^2)/d^2*e^2/(c^(1/2)/
a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)
/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/
d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))*a-3/2/(a*e^2-c*d^2)/(c
^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2)
)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^3,x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \int \frac{(a-cx^4)^{\frac{3}{2}}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx$$

input
```
integrate((-c*x**4+a)**(3/2)*(C*x**4+B*x**2+A)/(e*x**2+d)**3,x)
```

output
```
Integral((a - c*x**4)**(3/2)*(A + B*x**2 + C*x**4)/(d + e*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \int \frac{(Cx^4+Bx^2+A)(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^3}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^3,x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)/(e*x^2 + d)^3, x)
```

Giac [F]

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \int \frac{(Cx^4+Bx^2+A)(-cx^4+a)^{\frac{3}{2}}}{(ex^2+d)^3}\,dx$$

input
```
integrate((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^3,x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(-c*x^4 + a)^(3/2)/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \int \frac{(a-c\,x^4)^{3/2}\,(C\,x^4+B\,x^2+A)}{(e\,x^2+d)^3}\,dx$$

input
```
int(((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^3,x)
```

output
```
int(((a - c*x^4)^(3/2)*(A + B*x^2 + C*x^4))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2+Cx^4)}{(d+ex^2)^3}\,dx = \text{too large to display}$$

input
```
int((-c*x^4+a)^(3/2)*(C*x^4+B*x^2+A)/(e*x^2+d)^3,x)
```

output
```
(3*sqrt(a - c*x**4)*a**2*e**3*x - 20*sqrt(a - c*x**4)*a*b*d*e**2*x + 36*sq
rt(a - c*x**4)*a*c*d**2*e*x + 6*sqrt(a - c*x**4)*a*c*d*e**2*x**3 + 35*sqrt
(a - c*x**4)*b*c*d**2*e*x**3 - 5*sqrt(a - c*x**4)*b*c*d*e**2*x**5 - 63*sqr
t(a - c*x**4)*c**2*d**3*x**3 + 9*sqrt(a - c*x**4)*c**2*d**2*e*x**5 - 3*sqr
t(a - c*x**4)*c**2*d*e**2*x**7 + 12*int(sqrt(a - c*x**4)/(a*d**3 + 3*a*d**
2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 -
 3*c*d*e**2*x**8 - c*e**3*x**10),x)*a**3*d**3*e**3 + 24*int(sqrt(a - c*x**
4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4
 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*a**3*d**2*e**4*x**
2 + 12*int(sqrt(a - c*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 +
a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**
10),x)*a**3*d*e**5*x**4 + 20*int(sqrt(a - c*x**4)/(a*d**3 + 3*a*d**2*e*x**
2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*
e**2*x**8 - c*e**3*x**10),x)*a**2*b*d**4*e**2 + 40*int(sqrt(a - c*x**4)/(a
*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*
c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*a**2*b*d**3*e**3*x**2 +
 20*int(sqrt(a - c*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e
**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10)
,x)*a**2*b*d**2*e**4*x**4 - 36*int(sqrt(a - c*x**4)/(a*d**3 + 3*a*d**2*e*x
**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3...
```

3.40 $\int \frac{(d+ex^2)^3(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 449

$$\begin{aligned}
&\int \frac{(d+ex^2)^3(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx \\
&= -\frac{(5ae^2(3Cd+Be)+7cd(Cd^2+3e(Bd+Ae)))\,x\sqrt{a-cx^4}}{21c^2} \\
&\quad -\frac{e(7aCe^2+9c(3Cd^2+e(3Bd+Ae)))\,x^3\sqrt{a-cx^4}}{45c^2} \\
&\quad -\frac{e^2(3Cd+Be)x^5\sqrt{a-cx^4}}{7c}-\frac{Ce^3x^7\sqrt{a-cx^4}}{9c} \\
&\quad +\frac{a^{3/4}(3Bcd(5cd^2+9ae^2)+e(9Ac(5cd^2+ae^2)+aC(27cd^2+7ae^2)))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right.}{15c^{11/4}\sqrt{a-cx^4}} \\
&\quad +\frac{\sqrt[4]{a}\left(5\sqrt{c}(21Acd(cd^2+ae^2)+a(5ae^2(3Cd+Be)+7cd^2(Cd+3Be)))-7\sqrt{a}(3Bcd(5cd^2+9ae^2)+e\right.}{105c^{11/4}\sqrt{a-cx^4}}
\end{aligned}$$

output
```
-1/21*(5*a*e^2*(B*e+3*C*d)+7*c*d*(C*d^2+3*e*(A*e+B*d)))*x*(-c*x^4+a)^(1/2)
/c^2-1/45*e*(7*C*a*e^2+9*c*(3*C*d^2+e*(A*e+3*B*d)))*x^3*(-c*x^4+a)^(1/2)/c
^2-1/7*e^2*(B*e+3*C*d)*x^5*(-c*x^4+a)^(1/2)/c-1/9*C*e^3*x^7*(-c*x^4+a)^(1/
2)/c+1/15*a^(3/4)*(3*B*c*d*(9*a*e^2+5*c*d^2)+e*(9*A*c*(a*e^2+5*c*d^2)+a*C*
(7*a*e^2+27*c*d^2)))*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(1
1/4)/(-c*x^4+a)^(1/2)+1/105*a^(1/4)*(5*c^(1/2)*(21*A*c*d*(a*e^2+c*d^2)+a*(
5*a*e^2*(B*e+3*C*d)+7*c*d^2*(3*B*e+C*d)))-7*a^(1/2)*(3*B*c*d*(9*a*e^2+5*c*
d^2)+e*(9*A*c*(a*e^2+5*c*d^2)+a*C*(7*a*e^2+27*c*d^2))))*(1-c*x^4/a)^(1/2)*
EllipticF(c^(1/4)*x/a^(1/4),I)/c^(11/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.51 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx$$

$$= \frac{x(-a+cx^4)\,(ae^2(225Cd+75Be+49Cex^2)+cC(105d^3+189d^2ex^2+135de^2x^4+35e^3x^6)+9ce(7Ae(}{}$$

input
```
Integrate[((d + e*x^2)^3*(A + B*x^2 + C*x^4))/Sqrt[a - c*x^4],x]
```

output
```
(x*(-a + c*x^4)*(a*e^2*(225*C*d + 75*B*e + 49*C*e*x^2) + c*C*(105*d^3 + 18
9*d^2*e*x^2 + 135*d*e^2*x^4 + 35*e^3*x^6) + 9*c*e*(7*A*e*(5*d + e*x^2) + B
*(35*d^2 + 21*d*e*x^2 + 5*e^2*x^4))) + 15*(21*A*c*d*(c*d^2 + a*e^2) + a*(5
*a*e^2*(3*C*d + B*e) + 7*c*d^2*(C*d + 3*B*e)))*x*Sqrt[1 - (c*x^4)/a]*Hyper
geometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + 7*(9*A*c*e*(5*c*d^2 + a*e^2) + a*
C*e*(27*c*d^2 + 7*a*e^2) + 3*B*c*d*(5*c*d^2 + 9*a*e^2))*x^3*Sqrt[1 - (c*x^
4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a])/(315*c^2*Sqrt[a - c*x^4
])
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.51, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2209, 25, 2209, 25, 2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}dx$$

$$\Big\downarrow \text{2209}$$

$$-\frac{\int -\frac{(ex^2+d)^2(3c(2Cd+3Be)x^4+(9Bcd+9Ace+7aCe)x^2+(9Ac+aC)d)}{\sqrt{a-cx^4}}dx}{9c}-\frac{Cx\sqrt{a-cx^4}(d+ex^2)^3}{9c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex^2+d)^2(3c(2Cd+3Be)x^4+(9Bcd+9Ace+7aCe)x^2+(9Ac+aC)d)}{\sqrt{a-cx^4}}dx}{9c}-\frac{Cx\sqrt{a-cx^4}(d+ex^2)^3}{9c}$$

$$\Big\downarrow \text{2209}$$

$$\frac{-\frac{\int -\frac{(ex^2+d)\left(c\left(24cCd^2+49aCe^2+9ce(11Bd+7Ae)\right)x^4+c\left(63Bcd^2+126Aced+86aCed+45aBe^2\right)x^2+cd(63Acd+13aCd+9aBe)\right)}{\sqrt{a-cx^4}}dx}{7c}-\frac{3}{7}x\sqrt{a-cx^4}(d}{9c}$$

$$\frac{Cx\sqrt{a-cx^4}(d+ex^2)^3}{9c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{(ex^2+d)\left(c\left(24cCd^2+49aCe^2+9ce(11Bd+7Ae)\right)x^4+c\left(63Bcd^2+126Aced+86aCed+45aBe^2\right)x^2+cd(63Acd+13aCd+9aBe)\right)}{\sqrt{a-cx^4}}dx}{7c}-\frac{3}{7}x\sqrt{a-cx^4}(d+e}{9c}$$

$$\frac{Cx\sqrt{a-cx^4}(d+ex^2)^3}{9c}$$

$$\Big\downarrow \text{2259}$$

$$\frac{\frac{\int \left(\frac{ce\left(24cCd^2+49aCe^2+9ce(11Bd+7Ae)\right)x^6}{\sqrt{a-cx^4}}+\frac{3c\left(8cCd^3+9ce(6Bd+7Ae)d+15ae^2(3Cd+Be)\right)x^4}{\sqrt{a-cx^4}}+\frac{9cd\left(7Bcd^2+21Aced+11aCed+6aBe^2\right)x^2}{\sqrt{a-cx^4}}+\frac{cd^2(63Acd+13\cdots}{\sqrt{a-c\cdots}}}{7c}}{9c}$$

$$\frac{Cx\sqrt{a-cx^4}\left(d+ex^2\right)^3}{9c}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{3a^{7/4}e\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(49aCe^2+9ce(7Ae+11Bd)+24cCd^2\right)}{5c^{3/4}\sqrt{a-cx^4}}+\frac{3a^{7/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\left(49aCe^2+9ce(7Ae+11Bd)\cdots\right.}{5c^{3/4}\sqrt{a-cx^4}}}{\cdots}$$

$$\frac{Cx\sqrt{a-cx^4}\left(d+ex^2\right)^3}{9c}$$

input
```
Int[((d + e*x^2)^3*(A + B*x^2 + C*x^4))/Sqrt[a - c*x^4],x]
```

output
```
-1/9*(C*x*(d + e*x^2)^3*Sqrt[a - c*x^4])/c + ((-3*(2*C*d + 3*B*e)*x*(d + e
*x^2)^2*Sqrt[a - c*x^4])/7 + (-((8*c*C*d^3 + 9*c*d*e*(6*B*d + 7*A*e) + 15*
a*e^2*(3*C*d + B*e))*x*Sqrt[a - c*x^4]) - (e*(24*c*C*d^2 + 49*a*C*e^2 + 9*
c*e*(11*B*d + 7*A*e))*x^3*Sqrt[a - c*x^4])/5 + (9*a^(3/4)*c^(1/4)*d*(7*B*c
*d^2 + 21*A*c*d*e + 11*a*C*d*e + 6*a*B*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[
ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^4] + (3*a^(7/4)*e*(24*c*C*d
^2 + 49*a*C*e^2 + 9*c*e*(11*B*d + 7*A*e))*Sqrt[1 - (c*x^4)/a]*EllipticE[Ar
cSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*c^(
3/4)*d^2*(63*A*c*d + 13*a*C*d + 9*a*B*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[Arc
Sin[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^4] - (9*a^(3/4)*c^(1/4)*d*(7*B
*c*d^2 + 21*A*c*d*e + 11*a*C*d*e + 6*a*B*e^2)*Sqrt[1 - (c*x^4)/a]*Elliptic
F[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^4] - (3*a^(7/4)*e*(24*c*C
*d^2 + 49*a*C*e^2 + 9*c*e*(11*B*d + 7*A*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[
ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(3/4)*Sqrt[a - c*x^4]) + (a^(5/4)*(
8*c*C*d^3 + 9*c*d*e*(6*B*d + 7*A*e) + 15*a*e^2*(3*C*d + B*e))*Sqrt[1 - (c*
x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^
4]))/(7*c))/(9*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2209
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[C*x*(d + e*x^2)^q*(Sqrt[a + c*x^4]/(c*(2*q + 3))), x] + Simp[1/(c*(
2*q + 3))   Int[((d + e*x^2)^(q - 1)/Sqrt[a + c*x^4])*Simp[A*c*d*(2*q + 3)
- a*C*d + (c*(B*d + A*e)*(2*q + 3) - a*C*e*(2*q + 1))*x^2 + (B*c*e*(2*q + 3
) + 2*c*C*d*q)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2
] && EqQ[Expon[P4x, x], 4] && IGtQ[q, 0]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.91

method	result
elliptic	$-\frac{C e^3 x^7 \sqrt{-c x^4+a}}{9c}-\frac{\left(e^3 B+3 d e^2 C\right) x^5 \sqrt{-c x^4+a}}{7c}-\frac{\left(A e^3+3 B d e^2+3 C d^2 e+\frac{7 a C e^3}{9c}\right) x^3 \sqrt{-c x^4+a}}{5c}-\frac{\left(3 A d e^2+3 B e d^2+C\right.}{}$
default	$\frac{A d^3 \sqrt{1-\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c x^4+a}}-\frac{d^2 (3 A e+B d) \sqrt{a}\, \sqrt{1-\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\, x^2}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(\right.\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c x^4+a}\, \sqrt{c}}$
risch	$-\frac{x \left(35 C x^6 c e^3+45 B c e^3 x^4+135 C c d e^2 x^4+63 A c e^3 x^2+189 B c d e^2 x^2+49 C a e^3 x^2+189 C c d^2 e x^2+315 A c d e^2+75 B a e^3+315 B c d^2 \right.}{315 c^2}$

input
```
int((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*C*e^3*x^7*(-c*x^4+a)^(1/2)/c-1/7*(B*e^3+3*C*d*e^2)/c*x^5*(-c*x^4+a)^(
1/2)-1/5*(A*e^3+3*B*d*e^2+3*C*d^2*e+7/9*a/c*C*e^3)/c*x^3*(-c*x^4+a)^(1/2)-
1/3*(3*A*d*e^2+3*B*e*d^2+C*d^3+5/7*a/c*(B*e^3+3*C*d*e^2))/c*x*(-c*x^4+a)^(
1/2)+(A*d^3+1/3*a/c*(3*A*d*e^2+3*B*e*d^2+C*d^3+5/7*a/c*(B*e^3+3*C*d*e^2)))
/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1
/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-(3*A*d^
2*e+B*d^3+3/5*a/c*(A*e^3+3*B*d*e^2+3*C*d^2*e+7/9*a/c*C*e^3))*a^(1/2)/(c^(1
/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(
1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-Elli
pticE(x*(c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx =$$

$$-\frac{21\,(15\,Bac^2d^3+27\,Ba^2cde^2+9\,(3\,Ca^2c+5\,Aac^2)d^2e+(7\,Ca^3+9\,Aa^2c)e^3)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right.}{}\right.}{}$$

input
```
integrate((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="frica
s")
```

output
```
-1/315*(21*(15*B*a*c^2*d^3 + 27*B*a^2*c*d*e^2 + 9*(3*C*a^2*c + 5*A*a*c^2)*
d^2*e + (7*C*a^3 + 9*A*a^2*c)*e^3)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_e(arcsi
n((a/c)^(1/4)/x), -1) - 3*(35*((3*B + C)*a*c^2 + 3*A*c^3)*d^3 + 21*(9*C*a^
2*c + 5*(3*A + B)*a*c^2)*d^2*e + 3*((63*B + 25*C)*a^2*c + 35*A*a*c^2)*d*e^
2 + (49*C*a^3 + (63*A + 25*B)*a^2*c)*e^3)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_
f(arcsin((a/c)^(1/4)/x), -1) + (35*C*a*c^2*e^3*x^8 + 315*B*a*c^2*d^3 + 567
*B*a^2*c*d*e^2 + 45*(3*C*a*c^2*d*e^2 + B*a*c^2*e^3)*x^6 + 7*(27*C*a*c^2*d^
2*e + 27*B*a*c^2*d*e^2 + (7*C*a^2*c + 9*A*a*c^2)*e^3)*x^4 + 189*(3*C*a^2*c
 + 5*A*a*c^2)*d^2*e + 21*(7*C*a^3 + 9*A*a^2*c)*e^3 + 15*(7*C*a*c^2*d^3 + 2
1*B*a*c^2*d^2*e + 5*B*a^2*c*e^3 + 3*(5*C*a^2*c + 7*A*a*c^2)*d*e^2)*x^2)*sq
rt(-c*x^4 + a))/(a*c^3*x)
```

Sympy [A] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)**3*(C*x**4+B*x**2+A)/(-c*x**4+a)**(1/2),x)
```

output
```
A*d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/
(4*sqrt(a)*gamma(5/4)) + 3*A*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4
,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*A*d*e**2*x**5*g
amma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)
*gamma(9/4)) + A*e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*ex
p_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(11/4)) + B*d**3*x**3*gamma(3/4)*hyper(
(1/2, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3
*B*d**2*e*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*p
i)/a)/(4*sqrt(a)*gamma(9/4)) + 3*B*d*e**2*x**7*gamma(7/4)*hyper((1/2, 7/4)
, (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(11/4)) + B*e**3*x*
*9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sq
rt(a)*gamma(13/4)) + C*d**3*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x*
*4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + 3*C*d**2*e*x**7*gamma(7/4
)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(
11/4)) + 3*C*d*e**2*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), c*x**4*exp_
polar(2*I*pi)/a)/(4*sqrt(a)*gamma(13/4)) + C*e**3*x**11*gamma(11/4)*hyper(
(1/2, 11/4), (15/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(15/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^3}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\, dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^3}{\sqrt{-cx^4+a}}\, dx$$

input
```
integrate((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^3/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\, dx = \int \frac{(e\,x^2+d)^3\,(C\,x^4+B\,x^2+A)}{\sqrt{a-c\,x^4}}\, dx$$

input
```
int(((d + e*x^2)^3*(A + B*x^2 + C*x^4))/(a - c*x^4)^(1/2),x)
```

output
```
int(((d + e*x^2)^3*(A + B*x^2 + C*x^4))/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\, dx = \frac{-75\sqrt{-c\,x^4+a}\,ab\,e^3x-540\sqrt{-c\,x^4+a}\,acd\,e^2x-112\sqrt{-c\,x^4+a}\,ac\,e^3x^3-315\sqrt{-c\,x^4+a}\,bc\,d^2ex-1}{}$$

input
```
int((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 75*sqrt(a - c*x**4)*a*b*e**3*x - 540*sqrt(a - c*x**4)*a*c*d*e**2*x - 1
12*sqrt(a - c*x**4)*a*c*e**3*x**3 - 315*sqrt(a - c*x**4)*b*c*d**2*e*x - 18
9*sqrt(a - c*x**4)*b*c*d*e**2*x**3 - 45*sqrt(a - c*x**4)*b*c*e**3*x**5 - 1
05*sqrt(a - c*x**4)*c**2*d**3*x - 189*sqrt(a - c*x**4)*c**2*d**2*e*x**3 -
135*sqrt(a - c*x**4)*c**2*d*e**2*x**5 - 35*sqrt(a - c*x**4)*c**2*e**3*x**7
 + 75*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a**2*b*e**3 + 540*int(sqrt(a -
c*x**4)/(a - c*x**4),x)*a**2*c*d*e**2 + 315*int(sqrt(a - c*x**4)/(a - c*x*
*4),x)*a*b*c*d**2*e + 420*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a*c**2*d**3
 + 336*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a**2*c*e**3 + 567*int((
sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*a*b*c*d*e**2 + 1512*int((sqrt(a - c
*x**4)*x**2)/(a - c*x**4),x)*a*c**2*d**2*e + 315*int((sqrt(a - c*x**4)*x**
2)/(a - c*x**4),x)*b*c**2*d**3)/(315*c**2)
```

3.41 $\int \frac{(d+ex^2)^2(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 325

$$\int \frac{(d+ex^2)^2(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = -\frac{(5aCe^2+7c(Cd^2+e(2Bd+Ae)))x\sqrt{a-cx^4}}{21c^2} - \frac{e(2Cd+Be)x^3\sqrt{a-cx^4}}{5c} - \frac{Ce^2x^5\sqrt{a-cx^4}}{7c} + \frac{a^{3/4}(5Bcd^2+10Acde+6aCde+3aBe^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\left(21\sqrt{a}\sqrt{c}(5Bcd^2+10Acde+6aCde+3aBe^2)-5(7Ac(3cd^2+ae^2)+a(5aCe^2+7cd(Cd+2Be)\right.}{105c^{9/4}\sqrt{a-cx^4}}$$

output
```
-1/21*(5*C*a*e^2+7*c*(C*d^2+e*(A*e+2*B*d)))*x*(-c*x^4+a)^(1/2)/c^2-1/5*e*(
B*e+2*C*d)*x^3*(-c*x^4+a)^(1/2)/c-1/7*C*e^2*x^5*(-c*x^4+a)^(1/2)/c+1/5*a^(
3/4)*(10*A*c*d*e+3*B*a*e^2+5*B*c*d^2+6*C*a*d*e)*(1-c*x^4/a)^(1/2)*Elliptic
E(c^(1/4)*x/a^(1/4),I)/c^(7/4)/(-c*x^4+a)^(1/2)-1/105*a^(1/4)*(21*a^(1/2)*
c^(1/2)*(10*A*c*d*e+3*B*a*e^2+5*B*c*d^2+6*C*a*d*e)-35*A*c*(a*e^2+3*c*d^2)-
5*a*(5*C*a*e^2+7*c*d*(2*B*e+C*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a
^(1/4),I)/c^(9/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx$$
$$= \frac{x(-a+cx^4)\left(25aCe^2+7ce(10Bd+5Ae+3Bex^2)+cC(35d^2+42dex^2+15e^2x^4)\right)+5(7Ac(3cd^2+ae^2}{}$$

input
```
Integrate[((d + e*x^2)^2*(A + B*x^2 + C*x^4))/Sqrt[a - c*x^4],x]
```

output
```
(x*(-a + c*x^4)*(25*a*C*e^2 + 7*c*e*(10*B*d + 5*A*e + 3*B*e*x^2) + c*C*(35
*d^2 + 42*d*e*x^2 + 15*e^2*x^4)) + 5*(7*A*c*(3*c*d^2 + a*e^2) + a*(5*a*C*e
^2 + 7*c*d*(C*d + 2*B*e)))*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/
2, 5/4, (c*x^4)/a] + 7*c*(5*B*c*d^2 + 10*A*c*d*e + 6*a*C*d*e + 3*a*B*e^2)*
x^3*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a])/(105*
c^2*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.65, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2209, 25, 2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx$$

$$\Big\downarrow \ 2209$$

$$-\frac{\int -\frac{(ex^2+d)\left(c(4Cd+7Be)x^4+(7Bcd+7Ace+5aCe)x^2+(7Ac+aC)d\right)}{\sqrt{a-cx^4}}dx}{7c}-\frac{Cx\sqrt{a-cx^4}(d+ex^2)^2}{7c}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(ex^2+d)\left(c(4Cd+7Be)x^4+(7Bcd+7Ace+5aCe)x^2+(7Ac+aC)d\right)}{\sqrt{a-cx^4}}dx}{7c}-\frac{Cx\sqrt{a-cx^4}\left(d+ex^2\right)^2}{7c}$$

$\Big\downarrow$ 2259

$$\frac{\int \left(\frac{ce(4Cd+7Be)x^6}{\sqrt{a-cx^4}}+\frac{(4cCd^2+5aCe^2+7ce(2Bd+Ae))x^4}{\sqrt{a-cx^4}}+\frac{d(7Bcd+14Ace+6aCe)x^2}{\sqrt{a-cx^4}}+\frac{(7Ac+aC)d^2}{\sqrt{a-cx^4}}\right)dx}{7c}-\frac{Cx\sqrt{a-cx^4}\left(d+ex^2\right)^2}{7c}$$

$\Big\downarrow$ 2009

$$\frac{\frac{a^{5/4}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(5aCe^2+7ce(Ae+2Bd)+4cCd^2\right)}{3c^{5/4}\sqrt{a-cx^4}}-\frac{a^{3/4}d\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)(6aCe+14A}{c^{3/4}\sqrt{a-cx^4}}}{}$$

$$\frac{Cx\sqrt{a-cx^4}\left(d+ex^2\right)^2}{7c}$$

input
```
Int[((d + e*x^2)^2*(A + B*x^2 + C*x^4))/Sqrt[a - c*x^4],x]
```

output
```
-1/7*(C*x*(d + e*x^2)^2*Sqrt[a - c*x^4])/c + (-1/3*((4*c*C*d^2 + 5*a*C*e^2
 + 7*c*e*(2*B*d + A*e))*x*Sqrt[a - c*x^4])/c - (e*(4*C*d + 7*B*e)*x^3*Sqrt
[a - c*x^4])/5 + (3*a^(7/4)*e*(4*C*d + 7*B*e)*Sqrt[1 - (c*x^4)/a]*Elliptic
E[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(3/4)*Sqrt[a - c*x^4]) + (a^(3/4)
*d*(7*B*c*d + 14*A*c*e + 6*a*C*e)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^
(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(7*A*c + a*C)
*d^2*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1
/4)*Sqrt[a - c*x^4]) - (3*a^(7/4)*e*(4*C*d + 7*B*e)*Sqrt[1 - (c*x^4)/a]*El
lipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(3/4)*Sqrt[a - c*x^4]) - (a
^(3/4)*d*(7*B*c*d + 14*A*c*e + 6*a*C*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcS
in[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(5/4)*(4*c*C*
d^2 + 5*a*C*e^2 + 7*c*e*(2*B*d + A*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(5/4)*Sqrt[a - c*x^4]))/(7*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2209
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[C*x*(d + e*x^2)^q*(Sqrt[a + c*x^4]/(c*(2*q + 3))), x] + Simp[1/(c*(
2*q + 3))   Int[((d + e*x^2)^(q - 1)/Sqrt[a + c*x^4])*Simp[A*c*d*(2*q + 3)
- a*C*d + (c*(B*d + A*e)*(2*q + 3) - a*C*e*(2*q + 1))*x^2 + (B*c*e*(2*q + 3
) + 2*c*C*d*q)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2
] && EqQ[Expon[P4x, x], 4] && IGtQ[q, 0]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.96

method	result
elliptic	$-\frac{C e^{2} x^{5} \sqrt{-c x^{4}+a}}{7 c}-\frac{\left(B e^{2}+2 C d e\right) x^{3} \sqrt{-c x^{4}+a}}{5 c}-\frac{\left(A e^{2}+2 B d e+C d^{2}+\frac{5 a C e^{2}}{7 c}\right) x \sqrt{-c x^{4}+a}}{3 c}+\frac{\left(A d^{2}+\frac{a\left(A e^{2}+2 B d e+C d^{2}\right.}{3 c}\right.}{}$
default	$\frac{A d^{2} \sqrt{1-\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c x^{4}+a}}-\frac{d(2 A e+B d) \sqrt{a}\, \sqrt{1-\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x\right.\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c x^{4}+a}\, \sqrt{c}}$
risch	$-\frac{x\left(15 C x^{4} c e^{2}+21 B c e^{2} x^{2}+42 C c d e x^{2}+35 A c e^{2}+70 B c d e+25 C a e^{2}+35 C c d^{2}\right) \sqrt{-c x^{4}+a}}{105 c^{2}}+\frac{-\frac{21 \sqrt{c}\left(10 A c d e+3 B a e^{2}+5 B c d^{2}+6 C\right.}{}}{}$

input
```
int((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*C*e^2*x^5*(-c*x^4+a)^(1/2)/c-1/5*(B*e^2+2*C*d*e)/c*x^3*(-c*x^4+a)^(1/
2)-1/3*(A*e^2+2*B*d*e+C*d^2+5/7*a/c*C*e^2)/c*x*(-c*x^4+a)^(1/2)+(A*d^2+1/3
*a/c*(A*e^2+2*B*d*e+C*d^2+5/7*a/c*C*e^2))/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/
2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-(2*d*e*A+B*d^2+3/5*a/c*(B*e^2+2*C*d*e))*
a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x
^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^
(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx =$$
$$-\frac{21\,(5\,Bacd^2+3\,Ba^2e^2+2\,(3\,Ca^2+5\,Aac)de)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)-(35\,((3\,B+C)ac}{}$$

input
```
integrate((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="frica
s")
```

output
```
-1/105*(21*(5*B*a*c*d^2 + 3*B*a^2*e^2 + 2*(3*C*a^2 + 5*A*a*c)*d*e)*sqrt(-c
)*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - (35*((3*B + C)*a*c
 + 3*A*c^2)*d^2 + 14*(9*C*a^2 + 5*(3*A + B)*a*c)*d*e + ((63*B + 25*C)*a^2
+ 35*A*a*c)*e^2)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x),
-1) + (15*C*a*c*e^2*x^6 + 105*B*a*c*d^2 + 63*B*a^2*e^2 + 21*(2*C*a*c*d*e +
 B*a*c*e^2)*x^4 + 42*(3*C*a^2 + 5*A*a*c)*d*e + 5*(7*C*a*c*d^2 + 14*B*a*c*d
*e + (5*C*a^2 + 7*A*a*c)*e^2)*x^2)*sqrt(-c*x^4 + a))/(a*c^2*x)
```

Sympy [A] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)**2*(C*x**4+B*x**2+A)/(-c*x**4+a)**(1/2),x)
```

output
```
A*d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/
(4*sqrt(a)*gamma(5/4)) + A*d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c
*x**4*exp_polar(2*I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + A*e**2*x**5*gamma(5/4)
*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/
4)) + B*d**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(2*
I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + B*d*e*x**5*gamma(5/4)*hyper((1/2, 5/4),
(9/4,), c*x**4*exp_polar(2*I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + B*e**2*x**7*g
amma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a
)*gamma(11/4)) + C*d**2*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*e
xp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + C*d*e*x**7*gamma(7/4)*hyper((
1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(2*sqrt(a)*gamma(11/4)) +
C*e**2*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), c*x**4*exp_polar(2*I*pi)
/a)/(4*sqrt(a)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^2}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^2}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^2/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \int \frac{(e\,x^2+d)^2\,(C\,x^4+B\,x^2+A)}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int(((d + e*x^2)^2*(A + B*x^2 + C*x^4))/(a - c*x^4)^(1/2),x)
```

output
```
int(((d + e*x^2)^2*(A + B*x^2 + C*x^4))/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx$$
$$= \frac{-60\sqrt{-c\,x^4+a}\,a\,e^2x - 70\sqrt{-c\,x^4+a}\,b\,d\,e\,x - 21\sqrt{-c\,x^4+a}\,b\,e^2x^3 - 35\sqrt{-c\,x^4+a}\,c\,d^2x - 42\sqrt{-c\,x^4+}}{}$$

input
```
int((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 60*sqrt(a - c*x**4)*a*e**2*x - 70*sqrt(a - c*x**4)*b*d*e*x - 21*sqrt(a
 - c*x**4)*b*e**2*x**3 - 35*sqrt(a - c*x**4)*c*d**2*x - 42*sqrt(a - c*x**4
)*c*d*e*x**3 - 15*sqrt(a - c*x**4)*c*e**2*x**5 + 60*int(sqrt(a - c*x**4)/(
a - c*x**4),x)*a**2*e**2 + 70*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a*b*d*e
 + 140*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a*c*d**2 + 63*int((sqrt(a - c*
x**4)*x**2)/(a - c*x**4),x)*a*b*e**2 + 336*int((sqrt(a - c*x**4)*x**2)/(a
- c*x**4),x)*a*c*d*e + 105*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*b*c
*d**2)/(105*c)
```

3.42 $\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 225

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = -\frac{(Cd+Be)x\sqrt{a-cx^4}}{3c} - \frac{Cex^3\sqrt{a-cx^4}}{5c} + \frac{a^{3/4}(5Bcd+5Ace+3aCe)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(5\sqrt{c}(3Acd+aCd+aBe)-3\sqrt{a}(5Bcd+5Ace+3aCe)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-\right.}{15c^{7/4}\sqrt{a-cx^4}}$$

output
```
-1/3*(B*e+C*d)*x*(-c*x^4+a)^(1/2)/c-1/5*C*e*x^3*(-c*x^4+a)^(1/2)/c+1/5*a^(
3/4)*(5*A*c*e+5*B*c*d+3*C*a*e)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/
4),I)/c^(7/4)/(-c*x^4+a)^(1/2)+1/15*a^(1/4)*(5*c^(1/2)*(3*A*c*d+B*a*e+C*a*
d)-3*a^(1/2)*(5*A*c*e+5*B*c*d+3*C*a*e))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4
)*x/a^(1/4),I)/c^(7/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \frac{x(5Cd+5Be+3Cex^2)(-a+cx^4)+5(3Acd+aCd+aBe)x\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)}{15c\sqrt{a-cx^4}}$$

input
```
Integrate[((d + e*x^2)*(A + B*x^2 + C*x^4))/Sqrt[a - c*x^4],x]
```

output
```
(x*(5*C*d + 5*B*e + 3*C*e*x^2)*(-a + c*x^4) + 5*(3*A*c*d + a*C*d + a*B*e)*
x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + (5*B*c
*d + 5*A*c*e + 3*a*C*e)*x^3*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4
, 7/4, (c*x^4)/a])/(15*c*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.81, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx$$

$$\Big\downarrow \ 2259$$

$$\int \left(\frac{x^2(Ae+Bd)}{\sqrt{a-cx^4}}+\frac{Ad}{\sqrt{a-cx^4}}+\frac{x^4(Be+Cd)}{\sqrt{a-cx^4}}+\frac{Cex^6}{\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}(Ae+Bd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}(Ae+Bd)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{5/4}\sqrt{1-\frac{cx^4}{a}}(Be+Cd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}}-\frac{3a^{7/4}Ce\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{3a^{7/4}Ce\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}Ad\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{x\sqrt{a-cx^4}(Be+Cd)}{3c}-\frac{Cex^3\sqrt{a-cx^4}}{5c}$$

input
```
Int[((d + e*x^2)*(A + B*x^2 + C*x^4))/Sqrt[a - c*x^4],x]
```

output
```
-1/3*((C*d + B*e)*x*Sqrt[a - c*x^4])/c - (C*e*x^3*Sqrt[a - c*x^4])/(5*c) +
 (3*a^(7/4)*C*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)],
 -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (a^(3/4)*(B*d + A*e)*Sqrt[1 - (c*x^4)/
a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) +
 (a^(1/4)*A*d*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(c^(1/4)*Sqrt[a - c*x^4]) - (3*a^(7/4)*C*e*Sqrt[1 - (c*x^4)/a]*Ellipti
cF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) - (a^(3/4
)*(B*d + A*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(5/4)*(C*d + B*e)*Sqrt[1 - (c*x^4)/a]*E
llipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(5/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.01

method	result
elliptic	$-\frac{Ce\,x^3\sqrt{-c\,x^4+a}}{5c}-\frac{(Be+Cd)x\sqrt{-c\,x^4+a}}{3c}+\frac{\left(Ad+\frac{a(Be+Cd)}{3c}\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{\left(Ae+Bd+3\right.}{}$
risch	$-\frac{x\left(3C\,x^2e+5Be+5Cd\right)\sqrt{-c\,x^4+a}}{15c}+\frac{-\frac{(15Ace+15Bcd+9Cae)\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}}{}$
default	$\frac{Ad\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{(Ae+Bd)\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\right.\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}$

input
```
int((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*C*e*x^3*(-c*x^4+a)^(1/2)/c-1/3*(B*e+C*d)*x*(-c*x^4+a)^(1/2)/c+(A*d+1/
3*a/c*(B*e+C*d))/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+
c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^
(1/2),I)-(A*e+B*d+3/5*a/c*e*C)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*
x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*
(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2
),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx =$$

$$-\frac{3\left(5\,Bacd+(3\,Ca^2+5\,Aac)e\right)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-\left(5\left((3\,B+C)ac+3\,Ac^2\right)d+(9\,C\right.}{}$$

input
```
integrate((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/15*(3*(5*B*a*c*d + (3*C*a^2 + 5*A*a*c)*e)*sqrt(-c)*x*(a/c)^(3/4)*ellipt
ic_e(arcsin((a/c)^(1/4)/x), -1) - (5*((3*B + C)*a*c + 3*A*c^2)*d + (9*C*a^
2 + 5*(3*A + B)*a*c)*e)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/
4)/x), -1) + (3*C*a*c*e*x^4 + 15*B*a*c*d + 5*(C*a*c*d + B*a*c*e)*x^2 + 3*(
3*C*a^2 + 5*A*a*c)*e)*sqrt(-c*x^4 + a))/(a*c^2*x)
```

Sympy [A] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \frac{Adx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Aex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Bdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Bex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Cex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+d)*(C*x**4+B*x**2+A)/(-c*x**4+a)**(1/2),x)
```

output
```
A*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + A*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4
*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + B*d*x**3*gamma(3/4)*hyper((
1/2, 3/4), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + B*
e*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4
*sqrt(a)*gamma(9/4)) + C*d*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**
4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + C*e*x**7*gamma(7/4)*hyper(
(1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \int \frac{(e\,x^2+d)\,(C\,x^4+B\,x^2+A)}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int(((d + e*x^2)*(A + B*x^2 + C*x^4))/(a - c*x^4)^(1/2),x)
```

output
```
int(((d + e*x^2)*(A + B*x^2 + C*x^4))/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}}\,dx = \frac{-5\sqrt{-c\,x^4+a}\,bex-5\sqrt{-c\,x^4+a}\,cdx-3\sqrt{-c\,x^4+a}\,ce\,x^3+5\left(\int\frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)abe+20\left(\int\frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)}{15c}$$

input
```
int((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 5*sqrt(a - c*x**4)*b*e*x - 5*sqrt(a - c*x**4)*c*d*x - 3*sqrt(a - c*x**
4)*c*e*x**3 + 5*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a*b*e + 20*int(sqrt(a
 - c*x**4)/(a - c*x**4),x)*a*c*d + 24*int((sqrt(a - c*x**4)*x**2)/(a - c*x
**4),x)*a*c*e + 15*int((sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*b*c*d)/(15*
c)
```

3.43 $\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 154

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}}\,dx = -\frac{Cx\sqrt{a-cx^4}}{3c}+\frac{a^{3/4}B\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\left(3\sqrt{a}B\sqrt{c}-3Ac-aC\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}}$$

output
```
-1/3*C*x*(-c*x^4+a)^(1/2)/c+a^(3/4)*B*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*
x/a^(1/4),I)/c^(3/4)/(-c*x^4+a)^(1/2)-1/3*a^(1/4)*(3*a^(1/2)*B*c^(1/2)-3*A
*c-a*C)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(5/4)/(-c*x^4+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}}\,dx$$
$$= \frac{-aCx+cCx^5+(3Ac+aC)x\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+Bcx^3\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeom}}{3c\sqrt{a-cx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/Sqrt[a - c*x^4],x]
```

output
```
(-(a*C*x) + c*C*x^5 + (3*A*c + a*C)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F
1[1/4, 1/2, 5/4, (c*x^4)/a] + B*c*x^3*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F
1[1/2, 3/4, 7/4, (c*x^4)/a])/(3*c*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2427, 25, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}}\,dx$$

$$\Big\downarrow \ 2427$$

$$-\frac{\int -\frac{3Bcx^2+3Ac+aC}{\sqrt{a-cx^4}}dx}{3c}-\frac{Cx\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{3Bcx^2+3Ac+aC}{\sqrt{a-cx^4}}dx}{3c}-\frac{Cx\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \ 1513$$

$$\frac{3\sqrt{a}B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx-\left(3\sqrt{a}B\sqrt{c}-aC-3Ac\right)\int \frac{1}{\sqrt{a-cx^4}}dx}{3c}-\frac{Cx\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \; 27$$

$$\frac{3B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\left(3\sqrt{a}B\sqrt{c}-aC-3Ac\right)\int \frac{1}{\sqrt{a-cx^4}}dx}{3c}-\frac{Cx\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \; 765$$

$$\frac{3B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\frac{\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}B\sqrt{c}-aC-3Ac\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}}{3c}-\frac{Cx\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \; 762$$

$$\frac{3B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}B\sqrt{c}-aC-3Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{Cx\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \; 1390$$

$$\frac{\frac{3B\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}B\sqrt{c}-aC-3Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{Cx\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \; 1389$$

$$\frac{\frac{3\sqrt{a}B\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}B\sqrt{c}-aC-3Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{Cx\sqrt{a-cx^4}}{3c}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{3a^{3/4}B\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}B\sqrt{c}-aC-3Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{3c}-\frac{Cx\sqrt{a-cx^4}}{3c}$$

input
```
Int[(A + B*x^2 + C*x^4)/Sqrt[a - c*x^4],x]
```

output
```
-1/3*(C*x*Sqrt[a - c*x^4])/c + ((3*a^(3/4)*B*c^(1/4)*Sqrt[1 - (c*x^4)/a]*E
llipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^4] - (a^(1/4)*(3*S
qrt[a]*B*Sqrt[c] - 3*A*c - a*C)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1
/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]))/(3*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.16

method	result
elliptic	$-\frac{Cx\sqrt{-cx^4+a}}{3c}+\frac{\left(A+\frac{aC}{3c}\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}-\frac{B\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},\right.\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}\,\sqrt{c}}$
default	$\frac{A\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}-\frac{B\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}\,\sqrt{c}}-$
risch	$-\frac{Cx\sqrt{-cx^4+a}}{3c}+\frac{\frac{Ca\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}+\frac{3Ac\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-cx^4+a}}-\frac{3B\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}}{}}}{}}{3c}$

input
```
int((C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*C*x*(-c*x^4+a)^(1/2)/c+(A+1/3*a*C/c)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/
2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-B*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(
1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(
1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))
^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}}\,dx = -\frac{3\,Ba\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)-((3\,B+C)a+3\,Ac)\sqrt{-c}x\left(\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+}{3\,acx}$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(3*B*a*sqrt(-c)*x*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) -
 ((3*B + C)*a + 3*A*c)*sqrt(-c)*x*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4
)/x), -1) + sqrt(-c*x^4 + a)*(C*a*x^2 + 3*B*a))/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}}\,dx = \frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}+\frac{Cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/(-c*x**4+a)**(1/2),x)
```

output
```
A*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp
_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + C*x**5*gamma(5/4)*hyper((1/2, 5
/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a - c*x^4)^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}}\,dx=-\frac{\sqrt{-c\,x^4+a}\,x}{3}+\frac{4\left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)a}{3}+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right)b$$

input
```
int((C*x^4+B*x^2+A)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a - c*x**4)*x + 4*int(sqrt(a - c*x**4)/(a - c*x**4),x)*a + 3*int(
(sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*b)/3
```

3.44 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)\sqrt{a-cx^4}} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 226

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)\sqrt{a-cx^4}} \, dx = \frac{a^{3/4}C\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}e\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(Be-C\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e^2\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}(Cd^2-Bde+Ae^2)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de^2\sqrt{a-cx^4}}$$

output
```
a^(3/4)*C*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(3/4)/e/(-c*x
^4+a)^(1/2)+a^(1/4)*(B*e-C*(d+a^(1/2)*e/c^(1/2)))*(1-c*x^4/a)^(1/2)*Ellipt
icF(c^(1/4)*x/a^(1/4),I)/c^(1/4)/e^2/(-c*x^4+a)^(1/2)+a^(1/4)*(A*e^2-B*d*e
+C*d^2)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/
d,I)/c^(1/4)/d/e^2/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.57 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \frac{i\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}CdeE\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-d\left(\sqrt{a}Ce+\sqrt{c}(Cd-Be)\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\right.\right.\right.}{\sqrt{a}\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2}de^2\sqrt{a-}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(I*Sqrt[1 - (c*x^4)/a]*(Sqrt[a]*C*d*e*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/S
qrt[a])]*x], -1] - d*(Sqrt[a]*C*e + Sqrt[c]*(C*d - B*e))*EllipticF[I*ArcSi
nh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*(C*d^2 + e*(-(B*d) + A*e))*E
llipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x
], -1]))/(Sqrt[a]*(-(Sqrt[c]/Sqrt[a]))^(3/2)*d*e^2*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2235, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}\,(d+ex^2)}\,dx$$

$$\Big\downarrow \ 2235$$

$$\frac{(Ae^2-Bde+Cd^2)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{e^2}-\frac{\int \frac{-Cex^2+Cd-Be}{\sqrt{a-cx^4}}dx}{e^2}$$

$$\Big\downarrow \ 1513$$

$$\frac{\left(Ae^2-Bde+Cd^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{e^2}-\frac{-\left(\left(Be-C\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\right)\int \frac{1}{\sqrt{a-cx^4}}dx\right)-\frac{\sqrt{a}Ce\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}}{e^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(Ae^2-Bde+Cd^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{e^2}-\frac{-\left(\left(Be-C\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\right)\int \frac{1}{\sqrt{a-cx^4}}dx\right)-\frac{Ce\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{e^2}$$

$$\Big\downarrow \ 765$$

$$\frac{\left(Ae^2-Bde+Cd^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{e^2}-\frac{-\frac{\sqrt{1-\frac{cx^4}{a}}\left(Be-C\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-\frac{Ce\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{e^2}$$

$$\Big\downarrow \ 762$$

$$\frac{\left(Ae^2-Bde+Cd^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{e^2}-\frac{-\frac{Ce\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(Be-C\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{e^2}$$

$$\Big\downarrow \ 1390$$

$$\frac{\left(Ae^2-Bde+Cd^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{e^2}-\frac{-\frac{Ce\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(Be-C\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{e^2}$$

$$\Big\downarrow \ 1389$$

$$\frac{\left(Ae^2-Bde+Cd^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{e^2}-\frac{-\frac{\sqrt{a}Ce\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(Be-C\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{e^2}$$

$$\Big\downarrow \ 327$$

$$\frac{\frac{\left(Ae^2-Bde+Cd^2\right)\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{e^2}-\frac{-\frac{a^{3/4}Ce\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(Be-C\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{e^2}}{}$$

$$\Big\downarrow \text{ 1543}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(Ae^2-Bde+Cd^2\right)\int \frac{1}{(ex^2+d)\sqrt{1-\frac{cx^4}{a}}}dx}{e^2\sqrt{a-cx^4}}-\frac{-\frac{a^{3/4}Ce\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(Be-C\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{e^2}$$

$$\Big\downarrow \text{ 1542}$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(Ae^2-Bde+Cd^2\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de^2\sqrt{a-cx^4}}-\frac{-\frac{a^{3/4}Ce\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(Be-C\left(\frac{\sqrt{a}e}{\sqrt{c}}+d\right)\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{e^2}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
-((-((a^(3/4)*C*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)
], -1])/(c^(3/4)*Sqrt[a - c*x^4])) - (a^(1/4)*(B*e - C*(d + (Sqrt[a]*e)/Sq
rt[c]))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c
^(1/4)*Sqrt[a - c*x^4]))/e^2) + (a^(1/4)*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 -
(c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/
4)], -1])/(c^(1/4)*d*e^2*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.51

method	result
default	$\frac{\frac{B e\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}-\frac{C e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}\,\sqrt{c}}-\frac{C d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{}}{e^2}$
elliptic	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)B}{e\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)C d}{e^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}-\frac{C\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}}{}}}{e\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-}}$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(B*e/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2
)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),
I)-C*e*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^
(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^
(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))-C*d/(c^(1/2)/a^(1/
2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*
x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))+(A*e^2-B*d*e+C*d^2)/e
^2/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/
a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1
/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)\sqrt{a - cx^4}} \, dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)\sqrt{a - cx^4}} \, dx = \int \frac{A + Bx^2 + Cx^4}{\sqrt{a - cx^4}\,(d + ex^2)} \, dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt(a - c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \left(\int \frac{\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)a + \left(\int \frac{\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)c + \left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)b$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a + int((sq
rt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c + int((sq
rt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b
```

3.45 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 401

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = -\frac{(Cd^2-Bde+Ae^2)\,x\sqrt{a-cx^4}}{2d\,(cd^2-ae^2)\,(d+ex^2)}$$
$$-\frac{a^{3/4}\sqrt[4]{c}(Cd^2-Bde+Ae^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2de\,(cd^2-ae^2)\sqrt{a-cx^4}}$$
$$+\frac{\sqrt[4]{a}(2\sqrt{a}Cde+\sqrt{c}(Cd^2+e(Bd-Ae)))\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}de^2\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{a-cx^4}}$$
$$-\frac{\sqrt[4]{a}(cd^2(Cd^2+e(Bd-3Ae))-ae^2(3Cd^2-e(Bd+Ae)))\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{2\sqrt[4]{c}d^2e^2\,(cd^2-ae^2)\sqrt{a-cx^4}}$$

output
```
-1/2*(A*e^2-B*d*e+C*d^2)*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)-1/2
*a^(3/4)*c^(1/4)*(A*e^2-B*d*e+C*d^2)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x
/a^(1/4),I)/d/e/(-a*e^2+c*d^2)/(-c*x^4+a)^(1/2)+1/2*a^(1/4)*(2*a^(1/2)*C*d
*e+c^(1/2)*(C*d^2+e*(-A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(
1/4),I)/c^(1/4)/d/e^2/(c^(1/2)*d+a^(1/2)*e)/(-c*x^4+a)^(1/2)-1/2*a^(1/4)*(
c*d^2*(C*d^2+e*(-3*A*e+B*d))-a*e^2*(3*C*d^2-e*(A*e+B*d)))*(1-c*x^4/a)^(1/2
)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4)/d^2/e^2/(-a
*e^2+c*d^2)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.48 (sec) , antiderivative size = 1223, normalized size of antiderivative = 3.05

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

output
```
(-(a*Sqrt[-(Sqrt[c]/Sqrt[a])]*C*d^3*e^2*x) + a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*
d^2*e^3*x - a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e^4*x + Sqrt[-(Sqrt[c]/Sqrt[a])
]*c*C*d^3*e^2*x^5 - B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^2*e^3*x^5 + A*Sqrt[-(Sq
rt[c]/Sqrt[a])]*c*d*e^4*x^5 + I*Sqrt[a]*Sqrt[c]*d*e*(C*d^2 + e*(-(B*d) + A
*e))*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sq
rt[a])]*x], -1] + I*d*(-(Sqrt[c]*d) + Sqrt[a]*e)*(2*Sqrt[a]*C*d*e + Sqrt[c
]*(C*d^2 + B*d*e - A*e^2))*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*Arc
Sinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*c*C*d^5*Sqrt[1 - (c*x^4)/a]*Elli
pticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x],
-1] + I*B*c*d^4*e*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)
), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (3*I)*A*c*d^3*e^2*Sqrt[1 -
 (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c
]/Sqrt[a])]*x], -1] - (3*I)*a*C*d^3*e^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((
Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*
B*d^2*e^3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*Arc
Sinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*A*d*e^4*Sqrt[1 - (c*x^4)/a]*El
lipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x]
, -1] + I*c*C*d^4*e*x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt
[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*B*c*d^3*e^2*x^2*Sq
rt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt...
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2211, 2235, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \ 2211$$

$$\frac{\int \frac{-c\left(Cd^2-Bed+Ae^2\right)x^4+2d(Bcd-(Ac+aC)e)x^2+ad(Cd-Be)+A\left(2cd^2-ae^2\right)}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{2d\left(cd^2-ae^2\right)}-\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 2235$$

$$\frac{-\frac{\int \frac{ce\left(Cd^2-Bed+Ae^2\right)x^2+d\left(2aCe^2-c\left(Cd^2+e(Bd-Ae)\right)\right)}{\sqrt{a-cx^4}}dx}{e^2}-\frac{\left(c\left(d^2e(Bd-3Ae)+Cd^4\right)-ae^2\left(3Cd^2-e(Ae+Bd)\right)\right)\int \frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{e^2}}{2d\left(cd^2-ae^2\right)}-\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 1513$$

$$\frac{-\frac{\sqrt{a}\sqrt{c}e\left(Ae^2-Bde+Cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx-\left(\sqrt{c}d-\sqrt{a}e\right)\left(2\sqrt{a}Cde+\sqrt{c}\left(e(Bd-Ae)+Cd^2\right)\right)\int \frac{1}{\sqrt{a-cx^4}}dx}{e^2}-\frac{\left(c\left(d^2e(Bd-3Ae)+Cd^4\right)-ae^2\left(3C\cdots\right.\right.}{\cdots}}{2d\left(cd^2-ae^2\right)}-\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\sqrt{c}e\left(Ae^2-Bde+Cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\left(\sqrt{c}d-\sqrt{a}e\right)\left(2\sqrt{a}Cde+\sqrt{c}\left(e(Bd-Ae)+Cd^2\right)\right)\int \frac{1}{\sqrt{a-cx^4}}dx}{e^2}-\frac{\left(c\left(d^2e(Bd-3Ae)+Cd^4\right)-ae^2\left(3Cd^2-\cdots\right.\right.}{e^2}}{2d\left(cd^2-ae^2\right)}-\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 765$$

$$-\frac{\frac{\sqrt{c}e\left(Ae^2-Bde+Cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\frac{\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\left(2\sqrt{a}Cde+\sqrt{c}\left(e(Bd-Ae)+Cd^2\right)\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}}{e^2}-\frac{\left(c\left(d^2e(Bd-3Ae)+Cd^4\right)-ae^2\left(3C\cdots\right.\right.}{\cdots}}{2d\left(cd^2-ae^2\right)}$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 762$$

$$-\frac{\frac{\sqrt{c}e\left(Ae^2-Bde+Cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(2\sqrt{a}Cde+\sqrt{c}\left(e(Bd-Ae)+Cd^2\right)\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{e^2}-\frac{\left(c\left(d^2e(Bd-3\cdots\right.\right.}{\cdots}}{2d\left(cd^2-ae^2\right)}$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1390$$

$$-\frac{\frac{\frac{\sqrt{c}e\sqrt{1-\frac{cx^4}{a}}\left(Ae^2-Bde+Cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(2\sqrt{a}Cde+\sqrt{c}\left(e(Bd-Ae)+Cd^2\right)\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{e^2}-\frac{\left(c\left(d^2e\left(\cdots\right.\right.\right.}{\cdots}}{2d\left(cd^2-ae^2\right)}$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1389$$

$$-\frac{\frac{\frac{\sqrt{a}\sqrt{c}e\sqrt{1-\frac{cx^4}{a}}\left(Ae^2-Bde+Cd^2\right)\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(2\sqrt{a}Cde+\sqrt{c}\left(e(Bd-Ae)+Cd^2\right)\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{e^2}-\frac{\left(c\left(\cdots\right.\right.}{\cdots}}{2d\left(cd^2-ae^2\right)}$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 327$$

$$-\frac{\frac{(c(d^2e(Bd-3Ae)+Cd^4)-ae^2(3Cd^2-e(Ae+Bd)))\int \frac{1}{(ex^2+d)\sqrt{a-cx^4}}dx}{e^2}-\frac{\frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)(Ae^2-Bde+Cd^2)}{\sqrt{a-cx^4}}-\sqrt[4]{c}}{2d(cd^2-ae^2)}$$

$$\frac{x\sqrt{a-cx^4}(Ae^2-Bde+Cd^2)}{2d(d+ex^2)(cd^2-ae^2)}$$

$\Big\downarrow$ 1543

$$-\frac{\frac{\sqrt{1-\frac{cx^4}{a}}(c(d^2e(Bd-3Ae)+Cd^4)-ae^2(3Cd^2-e(Ae+Bd)))\int \frac{1}{(ex^2+d)\sqrt{1-\frac{cx^4}{a}}}dx}{e^2\sqrt{a-cx^4}}-\frac{\frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)(Ae^2-Bde+C}{\sqrt{a-cx^4}}}{2d(cd^2-ae^2)}$$

$$\frac{x\sqrt{a-cx^4}(Ae^2-Bde+Cd^2)}{2d(d+ex^2)(cd^2-ae^2)}$$

$\Big\downarrow$ 1542

$$-\frac{\frac{\frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)(Ae^2-Bde+Cd^2)}{\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(\sqrt{c}d-\sqrt{a}e)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(2\sqrt{a}Cde+\sqrt{c}\left(e(Bd-Ae)+Cd\right.\right.}{\sqrt[4]{c}\sqrt{a-cx^4}}}{e^2}}{2d(cd^2-ae^2)}$$

$$\frac{x\sqrt{a-cx^4}(Ae^2-Bde+Cd^2)}{2d(d+ex^2)(cd^2-ae^2)}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

output
```
-1/2*((C*d^2 - B*d*e + A*e^2)*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e
*x^2)) + (-(((a^(3/4)*c^(1/4)*e*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a
]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^4] - (a^(1/4)*(
Sqrt[c]*d - Sqrt[a]*e)*(2*Sqrt[a]*C*d*e + Sqrt[c]*(C*d^2 + e*(B*d - A*e)))
*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*
Sqrt[a - c*x^4]))/e^2) - (a^(1/4)*(c*(C*d^4 + d^2*e*(B*d - 3*A*e)) - a*e^2
*(3*C*d^2 - e*(B*d + A*e)))*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(
Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e^2*Sqrt[a - c*x
^4]))/(2*d*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 2211
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))
Int[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(
2*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(
C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(343) = 686$.

Time = 1.06 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.79

method	result
default	$\frac{C\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e^{2}\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{(Be-2Cd)\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{e^{2}d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{(A}{}$
elliptic	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
C/e^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2
/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/
e^2*(B*e-2*C*d)/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1
+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2)
)^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1
/2))+1/e^2*(A*e^2-B*d*e+C*d^2)*(1/2*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)
/(e*x^2+d)+1/2*c/(a*e^2-c*d^2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1
/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(
1/2)/a^(1/2))^(1/2),I)-1/2*e*c^(1/2)/(a*e^2-c*d^2)/d*a^(1/2)/(c^(1/2)/a^(1
/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c
*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/2*e*c^(1/2)/(a*e^2-
c*d^2)/d*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+
c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(c^(1/2)/a^(1/2))^
(1/2),I)+1/2/(a*e^2-c*d^2)/d^2*e^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/
a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x
*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^
(1/2)/a^(1/2))^(1/2))*a-3/2/(a*e^2-c*d^2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/
2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1
/2)/(c^(1/2)/a^(1/2))^(1/2))*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}\,(d+ex^2)^2}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt(a - c*x**4)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^2}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)^2),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx &= \left(\int \frac{\sqrt{-c\,x^4+a}}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx\right)a\\ &\quad+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^4}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx\right)c\\ &\quad+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx\right)b\end{aligned}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 -
2*c*d*e*x**6 - c*e**2*x**8),x)*a + int((sqrt(a - c*x**4)*x**4)/(a*d**2 + 2
*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*c
 + int((sqrt(a - c*x**4)*x**2)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d*
*2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*b
```

3.46 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 615

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = -\frac{(Cd^2-Bde+Ae^2)\,x\sqrt{a-cx^4}}{4d\,(cd^2-ae^2)\,(d+ex^2)^2}$$
$$-\frac{(cd^2(Cd^2-e(5Bd-9Ae))+ae^2(5Cd^2-e(Bd+3Ae)))\,x\sqrt{a-cx^4}}{8d^2\,(cd^2-ae^2)^2\,(d+ex^2)}$$
$$-\frac{a^{3/4}\sqrt[4]{c}(cd^2(Cd^2-e(5Bd-9Ae))+ae^2(5Cd^2-e(Bd+3Ae)))\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2e\,(cd^2-ae^2)^2\sqrt{a-cx^4}}$$
$$+\frac{\sqrt[4]{a}\sqrt[4]{c}\left(cd^2(Cd^2+e(3Bd-7Ae))+2\sqrt{a}\sqrt{c}de(Cd^2-e(Bd-Ae))-ae^2(5Cd^2-e(Bd+3Ae))\right)\sqrt{1}}{8d^2e^2\left(\sqrt{c}d+\sqrt{a}e\right)(cd^2-ae^2)\sqrt{a-cx^4}}$$
$$-\frac{\sqrt[4]{a}(c^2d^4(Cd^2+3e(Bd-5Ae))-a^2e^4(3Cd^2+e(Bd+3Ae))-2acd^2e^2(5Cd^2-e(5Bd+3Ae)))\sqrt{1}}{8\sqrt[4]{c}d^3e^2\,(cd^2-ae^2)^2\sqrt{a-cx^4}}$$

output
```
-1/4*(A*e^2-B*d*e+C*d^2)*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^2-1
/8*(c*d^2*(C*d^2-e*(-9*A*e+5*B*d))+a*e^2*(5*C*d^2-e*(3*A*e+B*d)))*x*(-c*x^
4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)-1/8*a^(3/4)*c^(1/4)*(c*d^2*(C*d^
2-e*(-9*A*e+5*B*d))+a*e^2*(5*C*d^2-e*(3*A*e+B*d)))*(1-c*x^4/a)^(1/2)*Ellip
ticE(c^(1/4)*x/a^(1/4),I)/d^2/e/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)+1/8*a^(1
/4)*c^(1/4)*(c*d^2*(C*d^2+e*(-7*A*e+3*B*d))+2*a^(1/2)*c^(1/2)*d*e*(C*d^2-e
*(-A*e+B*d))-a*e^2*(5*C*d^2-e*(3*A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^
(1/4)*x/a^(1/4),I)/d^2/e^2/(c^(1/2)*d+a^(1/2)*e)/(-a*e^2+c*d^2)/(-c*x^4+a)
^(1/2)-1/8*a^(1/4)*(c^2*d^4*(C*d^2+3*e*(-5*A*e+B*d))-a^2*e^4*(3*C*d^2+e*(3
*A*e+B*d))-2*a*c*d^2*e^2*(5*C*d^2-e*(3*A*e+5*B*d)))*(1-c*x^4/a)^(1/2)*Elli
pticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4)/d^3/e^2/(-a*e^2+c
*d^2)^2/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.59 (sec) , antiderivative size = 521, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx$$

$$= \frac{-\frac{dx(a-cx^4)\left(2d(cd^2-ae^2)\left(Cd^2+e(-Bd+Ae)\right)+\left(cCd^4+5aCd^2e^2-ae^3(Bd+3Ae)+cd^2e(-5Bd+9Ae)\right)(d+ex^2)\right)}{(cd^2-ae^2)^2(d+ex^2)^2} - \frac{i\sqrt{1-\frac{cx^4}{a}}\left(-\sqrt{a}\sqrt{c}de\right.}{}}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

output
```
(-((d*x*(a - c*x^4)*(2*d*(c*d^2 - a*e^2)*(C*d^2 + e*(-(B*d) + A*e)) + (c*C
*d^4 + 5*a*C*d^2*e^2 - a*e^3*(B*d + 3*A*e) + c*d^2*e*(-5*B*d + 9*A*e))*(d
+ e*x^2)))/((c*d^2 - a*e^2)^2*(d + e*x^2)^2)) - (I*Sqrt[1 - (c*x^4)/a]*(-(
Sqrt[a]*Sqrt[c]*d*e*(c*C*d^4 + 5*a*C*d^2*e^2 - a*e^3*(B*d + 3*A*e) + c*d^2
*e*(-5*B*d + 9*A*e))*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])
 + Sqrt[c]*d*(Sqrt[c]*d - Sqrt[a]*e)*(c*(C*d^4 + d^2*e*(3*B*d - 7*A*e)) +
2*Sqrt[a]*Sqrt[c]*d*e*(C*d^2 + e*(-(B*d) + A*e)) + a*e^2*(-5*C*d^2 + e*(B*
d + 3*A*e)))*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (-(c^2
*(C*d^6 + 3*d^4*e*(B*d - 5*A*e))) + a^2*e^4*(3*C*d^2 + e*(B*d + 3*A*e)) +
2*a*c*d^2*e^2*(5*C*d^2 - e*(5*B*d + 3*A*e)))*EllipticPi[-((Sqrt[a]*e)/(Sqr
t[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(Sqrt[-(Sqrt[c]/Sqr
t[a])]*(c*d^2*e - a*e^3)^2))/(8*d^3*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 604, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2211, 2211, 2235, 25, 27, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{2211}$$

$$\frac{\int \frac{c\left(Cd^2-Bed+Ae^2\right)x^4+4d(Bcd-(Ac+aC)e)x^2+ad(Cd-Be)+A\left(4cd^2-3ae^2\right)}{\left(ex^2+d\right)^2\sqrt{a-cx^4}}dx}{4d\left(cd^2-ae^2\right)}-\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \text{2211}$$

$$\frac{\frac{\int \frac{-c\left(cCd^4+5aCe^2d^2-ce(5Bd-9Ae)d^2-ae^3(Bd+3Ae)\right)x^4+4cd\left(2Bcd^3-4Aced^2-3aCed^2+aBe^2d+aAe^3\right)x^2+A\left(8c^2d^4-5ace^2d^2+3a^2e^4\right)+ad\left(c(3Cd-7Be)d\right.}{\left(ex^2+d\right)\sqrt{a-cx^4}}}{2d(cd^2-ae^2)}}{4d\left(cd^2-ae^2\right)}-\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\downarrow 2235$$

$$-\frac{\left(-a^2e^4\left(e(3Ae+Bd)+3Cd^2\right)-2acd^2e^2\left(5Cd^2-e(3Ae+5Bd)\right)+c^2\left(3d^4e(Bd-5Ae)+Cd^6\right)\right)\int\frac{1}{\left(ex^2+d\right)\sqrt{a-cx^4}}dx}{e^2}-\int-\frac{c\left(d\left(c\left(Cd^4+e(3Bd-7Ae)d^2\right)-ae^2\left(7\right.\right.\right.}{}$$

$$2d(cd^2-ae^2)$$

$$4d\left(cd^2\right.$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\downarrow 25$$

$$\frac{\int\frac{c\left(d\left(c\left(Cd^4+e(3Bd-7Ae)d^2\right)-ae^2\left(7Cd^2-e(3Bd+Ae)\right)\right)-e\left(cCd^4+5aCe^2d^2-ce(5Bd-9Ae)d^2-ae^3(Bd+3Ae)\right)x^2\right)}{\sqrt{a-cx^4}}dx}{e^2}-\frac{\left(-a^2e^4\left(e(3Ae+Bd)+3Cd^2\right)-2acd\right.}{}$$

$$2d(cd^2-ae^2)$$

$$4d\left(cd^2-\right.$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\downarrow 27$$

$$\frac{c\int\frac{d\left(c\left(Cd^4+e(3Bd-7Ae)d^2\right)-ae^2\left(7Cd^2-e(3Bd+Ae)\right)\right)-e\left(cCd^4+5aCe^2d^2-ce(5Bd-9Ae)d^2-ae^3(Bd+3Ae)\right)x^2}{\sqrt{a-cx^4}}dx}{e^2}-\frac{\left(-a^2e^4\left(e(3Ae+Bd)+3Cd^2\right)-2acd^2e\right.}{}$$

$$2d(cd^2-ae^2)$$

$$4d\left(cd^2-\right.$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\downarrow 1513$$

$$c\left(\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\left(2\sqrt{a}\sqrt{c}de\left(Cd^2-e(Bd-Ae)\right)-ae^2\left(5Cd^2-e(3Ae+Bd)\right)+c\left(d^2e(3Bd-7Ae)+Cd^4\right)\right)\int\frac{1}{\sqrt{a-cx^4}}dx}{\sqrt{c}}-\frac{\sqrt{a}e\left(-ae^3(3Ae+Bd)+5aCd^2e^2-cd^2e(5Bd\right.}{\sqrt{c}}\right.$$

$$e^2$$

$$2d(cd^2-ae$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\downarrow 27$$

$$c\left(\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\left(2\sqrt{a}\sqrt{c}de\left(Cd^2-e(Bd-Ae)\right)-ae^2\left(5Cd^2-e(3Ae+Bd)\right)+c\left(d^2e(3Bd-7Ae)+Cd^4\right)\right)\int\frac{1}{\sqrt{a-cx^4}}dx}{\sqrt{c}}-\frac{e\left(-ae^3(3Ae+Bd)+5aCd^2e^2-cd^2e(5Bd-9\right.}{\sqrt{c}}\right.$$
$$\overline{\qquad e^2 \qquad}$$
$$\overline{\qquad 2d(cd^2-ae^2) \qquad}$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 765

$$c\left(\frac{\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\left(2\sqrt{a}\sqrt{c}de\left(Cd^2-e(Bd-Ae)\right)-ae^2\left(5Cd^2-e(3Ae+Bd)\right)+c\left(d^2e(3Bd-7Ae)+Cd^4\right)\right)\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}-\frac{e\left(-ae^3(3Ae+Bd)+5aCd^2e^2-cd^2\right.}{}\right.$$
$$\overline{\qquad e^2 \qquad}$$
$$\overline{\qquad 2d(cd^2- \qquad}$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 762

$$c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(2\sqrt{a}\sqrt{c}de\left(Cd^2-e(Bd-Ae)\right)-ae^2\left(5Cd^2-e(3Ae+Bd)\right)+c\left(d^2e(3Bd-7Ae)+Cd^4\right)\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{e\left(-ae^3(3A\right.}{}\right.$$
$$\overline{\qquad e^2 \qquad}$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 1390

$$c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(2\sqrt{a}\sqrt{c}de\left(Cd^2-e(Bd-Ae)\right)-ae^2\left(5Cd^2-e(3Ae+Bd)\right)+c\left(d^2e(3Bd-7Ae)+Cd^4\right)\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{e\sqrt{1-\frac{cx^4}{a}}}{}\right.$$
$$\overline{\qquad e^2 \qquad}$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$\Big\downarrow$ 1389

$$\frac{c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(2\sqrt{a}\sqrt{c}de\left(Cd^2-e(Bd-Ae)\right)-ae^2\left(5Cd^2-e(3Ae+Bd)\right)+c\left(d^2e(3Bd-7Ae)+Cd^4\right)\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{\sqrt{a}e\sqrt{1-\frac{c}{\cdots}}\cdots}{\cdots}\right.}{e^2}\cdots$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 327$$

$$\frac{c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(2\sqrt{a}\sqrt{c}de\left(Cd^2-e(Bd-Ae)\right)-ae^2\left(5Cd^2-e(3Ae+Bd)\right)+c\left(d^2e(3Bd-7Ae)+Cd^4\right)\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{a^{3/4}e\sqrt{1-\cdots}\cdots}{\cdots}\right.}{e^2}\cdots$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1543$$

$$\frac{c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(2\sqrt{a}\sqrt{c}de\left(Cd^2-e(Bd-Ae)\right)-ae^2\left(5Cd^2-e(3Ae+Bd)\right)+c\left(d^2e(3Bd-7Ae)+Cd^4\right)\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{a^{3/4}e\sqrt{1-\cdots}\cdots}{\cdots}\right.}{e^2}\cdots$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

$$\Big\downarrow \; 1542$$

$$\frac{c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(2\sqrt{a}\sqrt{c}de\left(Cd^2-e(Bd-Ae)\right)-ae^2\left(5Cd^2-e(3Ae+Bd)\right)+c\left(d^2e(3Bd-7Ae)+Cd^4\right)\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{a^{3/4}e\sqrt{1-\cdots}\cdots}{\cdots}\right.}{e^2}\cdots$$

$$\frac{x\sqrt{a-cx^4}\left(Ae^2-Bde+Cd^2\right)}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

output
```
-1/4*((C*d^2 - B*d*e + A*e^2)*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e
*x^2)^2) + (-1/2*((c*C*d^4 + 5*a*C*d^2*e^2 - c*d^2*e*(5*B*d - 9*A*e) - a*e
^3*(B*d + 3*A*e))*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)) + ((c
*(-((a^(3/4)*e*(c*C*d^4 + 5*a*C*d^2*e^2 - c*d^2*e*(5*B*d - 9*A*e) - a*e^3*
(B*d + 3*A*e))*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)],
-1])/(c^(3/4)*Sqrt[a - c*x^4])) + (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*(C*d
^4 + d^2*e*(3*B*d - 7*A*e)) + 2*Sqrt[a]*Sqrt[c]*d*e*(C*d^2 - e*(B*d - A*e)
) - a*e^2*(5*C*d^2 - e*(B*d + 3*A*e)))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4])))/e^2 - (a^(1/4)*(c
^2*(C*d^6 + 3*d^4*e*(B*d - 5*A*e)) - a^2*e^4*(3*C*d^2 + e*(B*d + 3*A*e)) -
 2*a*c*d^2*e^2*(5*C*d^2 - e*(5*B*d + 3*A*e)))*Sqrt[1 - (c*x^4)/a]*Elliptic
Pi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*
d*e^2*Sqrt[a - c*x^4]))/(2*d*(c*d^2 - a*e^2)))/(4*d*(c*d^2 - a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 2211
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))
Int[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(
2*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(
C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1615 vs. $2(553) = 1106$.

Time = 2.16 (sec) , antiderivative size = 1616, normalized size of antiderivative = 2.63

method	result	size
default	Expression too large to display	1616
elliptic	Expression too large to display	2762

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
C/e^2/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x
^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a
^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))+(B*e-
2*C*d)/e^2*(1/2*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/2*c/(a*
e^2-c*d^2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2
)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),
I)-1/2*e*c^(1/2)/(a*e^2-c*d^2)/d*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2
)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ellipt
icF(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/2*e*c^(1/2)/(a*e^2-c*d^2)/d*a^(1/2)/(c^
(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))
^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/2/(a*e^2-
c*d^2)/d^2*e^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^
(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(
1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2)
)*a-3/2/(a*e^2-c*d^2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2
)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(
1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2)
)^(1/2))*c)+(A*e^2-B*d*e+C*d^2)/e^2*(1/4*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^
(1/2)/(e*x^2+d)^2+3/8*e^2*(a*e^2-3*c*d^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)
^(1/2)/(e*x^2+d)+1/8*c/d/(a*e^2-c*d^2)^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}\left(d+ex^2\right)^3}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt(a - c*x**4)*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^3\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)^3 \sqrt{a - cx^4}} \, dx = \int \frac{C\,x^4 + B\,x^2 + A}{\sqrt{a - c\,x^4}\,(e\,x^2 + d)^3} \, dx$$

input
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)^3),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)^3 \sqrt{a - cx^4}} \, dx$$
$$= \left(\int \frac{\sqrt{-c\,x^4 + a}}{-c\,e^3 x^{10} - 3cd\,e^2 x^8 + a\,e^3 x^6 - 3c\,d^2 e\,x^6 + 3ad\,e^2 x^4 - c\,d^3 x^4 + 3a\,d^2 e\,x^2 + a\,d^3} dx \right) a$$
$$+ \left(\int \frac{\sqrt{-c\,x^4 + a}\,x^4}{-c\,e^3 x^{10} - 3cd\,e^2 x^8 + a\,e^3 x^6 - 3c\,d^2 e\,x^6 + 3ad\,e^2 x^4 - c\,d^3 x^4 + 3a\,d^2 e\,x^2 + a\,d^3} dx \right) c$$
$$+ \left(\int \frac{\sqrt{-c\,x^4 + a}\,x^2}{-c\,e^3 x^{10} - 3cd\,e^2 x^8 + a\,e^3 x^6 - 3c\,d^2 e\,x^6 + 3ad\,e^2 x^4 - c\,d^3 x^4 + 3a\,d^2 e\,x^2 + a\,d^3} dx \right) b$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*
x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*
a + int((sqrt(a - c*x**4)*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**
4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3
*x**10),x)*c + int((sqrt(a - c*x**4)*x**2)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a
*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x*
*8 - c*e**3*x**10),x)*b
```

3.47 $\int \frac{(d+ex^2)^3(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 452

$$\int \frac{(d+ex^2)^3(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \frac{x(Acd(cd^2+3ae^2)+a(ae^2(3Cd+Be)+cd^2(Cd+3Be))+((Ac+}{2ac^2\sqrt{a-cx^4}}$$

$$+\frac{e^2(3Cd+Be)x\sqrt{a-cx^4}}{3c^2}+\frac{Ce^3x^3\sqrt{a-cx^4}}{5c^2}$$

$$-\frac{(5Bcd(cd^2+9ae^2)+3e(5Ac(cd^2+ae^2)+aC(15cd^2+7ae^2)))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{10\sqrt[4]{a}c^{11/4}\sqrt{a-cx^4}}$$

$$+\frac{\left(5(3Acd(cd^2-3ae^2)-a(5ae^2(3Cd+Be)+3cd^2(Cd+3Be)))+\frac{3\sqrt{a}(5Bcd(cd^2+9ae^2)+3e(5Ac(cd^2+ae^2)+aC(15}{\sqrt{c}}\right.}{30a^{3/4}c^{9/4}\sqrt{a-cx^4}}$$

output
```
1/2*x*(A*c*d*(3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*d))+((A*c
+C*a)*e*(a*e^2+3*c*d^2)+B*c*d*(3*a*e^2+c*d^2))*x^2)/a/c^2/(-c*x^4+a)^(1/2)
+1/3*e^2*(B*e+3*C*d)*x*(-c*x^4+a)^(1/2)/c^2+1/5*C*e^3*x^3*(-c*x^4+a)^(1/2)
/c^2-1/10*(5*B*c*d*(9*a*e^2+c*d^2)+3*e*(5*A*c*(a*e^2+c*d^2)+a*C*(7*a*e^2+1
5*c*d^2)))*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4)/c^(11/
4)/(-c*x^4+a)^(1/2)+1/30*(15*A*c*d*(-3*a*e^2+c*d^2)-5*a*(5*a*e^2*(B*e+3*C*
d)+3*c*d^2*(3*B*e+C*d))+3*a^(1/2)*(5*B*c*d*(9*a*e^2+c*d^2)+3*e*(5*A*c*(a*e
^2+c*d^2)+a*C*(7*a*e^2+15*c*d^2)))/c^(1/2))*(1-c*x^4/a)^(1/2)*EllipticF(c^
(1/4)*x/a^(1/4),I)/a^(3/4)/c^(9/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.52 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}} \, dx = \frac{x(15Ac(cd^3+ae^2(3d-2ex^2))+a(ae^2(75Cd+25Be-42Cex^2)-5}{}$$

input
```
Integrate[((d + e*x^2)^3*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2),x]
```

output
```
(x*(15*A*c*(c*d^3 + a*e^2*(3*d - 2*e*x^2)) + a*(a*e^2*(75*C*d + 25*B*e - 4
2*C*e*x^2) - 5*B*c*e*(-9*d^2 + 18*d*e*x^2 + 2*e^2*x^4) + 3*c*C*(5*d^3 - 30
*d^2*e*x^2 - 10*d*e^2*x^4 - 2*e^3*x^6))) - 5*(-3*A*c*d*(c*d^2 - 3*a*e^2) +
 a*(5*a*e^2*(3*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x*Sqrt[1 - (c*x^4)/a]*
Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + 2*(5*B*c*d*(c*d^2 + 9*a*e^2)
 + 3*e*(5*A*c*(c*d^2 + a*e^2) + a*C*(15*c*d^2 + 7*a*e^2)))*x^3*Sqrt[1 - (c
*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, (c*x^4)/a])/(30*a*c^2*Sqrt[a - c
*x^4])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(-\frac{ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3}{c^2\sqrt{a-cx^4}} - \frac{ex^2(aCe^2+ce(Ae+3Bd)+3cCd^2)}{c^2\sqrt{a-cx^4}} + \frac{x^2(e(aC+Ac)(ae^2}{}\right.$$

$\downarrow$ 2009

$$\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(aCe^2+ce(Ae+3Bd)+3cCd^2\right)}{c^{11/4}\sqrt{a-cx^4}}-\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\left(aCe^2+ce(Ae+3Bd)+3cCd^2\right)}{c^{11/4}\sqrt{a-cx^4}}+\frac{\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}e+\sqrt{c}d\right)^3\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}c^{11/4}\sqrt{a-cx^4}}-\frac{a^{5/4}e^2\sqrt{1-\frac{cx^4}{a}}(Be+3Cd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{9/4}\sqrt{a-cx^4}}+\frac{3a^{7/4}Ce^3\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{11/4}\sqrt{a-cx^4}}-\frac{3a^{7/4}Ce^3\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{11/4}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)}{c^{9/4}\sqrt{a-cx^4}}-\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\left(e(aC+Ac)\left(ae^2+3cd^2\right)+Bcd\left(3ae^2+cd^2\right)\right)}{2\sqrt[4]{a}c^{11/4}\sqrt{a-cx^4}}+\frac{x\left(x^2\left(e(aC+Ac)\left(ae^2+3cd^2\right)+Bcd\left(3ae^2+cd^2\right)\right)+Acd\left(3ae^2+cd^2\right)+a\left(ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right)}{2ac^2\sqrt{a-cx^4}}$$

$$\frac{e^2x\sqrt{a-cx^4}(Be+3Cd)}{3c^2}+\frac{Ce^3x^3\sqrt{a-cx^4}}{5c^2}$$

input
```
Int[((d + e*x^2)^3*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2),x]
```

output
```
(x*(A*c*d*(c*d^2 + 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)
) + ((A*c + a*C)*e*(3*c*d^2 + a*e^2) + B*c*d*(c*d^2 + 3*a*e^2))*x^2))/(2*a
*c^2*Sqrt[a - c*x^4]) + (e^2*(3*C*d + B*e)*x*Sqrt[a - c*x^4])/(3*c^2) + (C
*e^3*x^3*Sqrt[a - c*x^4])/(5*c^2) - (3*a^(7/4)*C*e^3*Sqrt[1 - (c*x^4)/a]*E
llipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(11/4)*Sqrt[a - c*x^4]) -
(a^(3/4)*e*(3*c*C*d^2 + a*C*e^2 + c*e*(3*B*d + A*e))*Sqrt[1 - (c*x^4)/a]*E
llipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(11/4)*Sqrt[a - c*x^4]) - ((
(A*c + a*C)*e*(3*c*d^2 + a*e^2) + B*c*d*(c*d^2 + 3*a*e^2))*Sqrt[1 - (c*x^4
)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(1/4)*c^(11/4)*Sqrt[
a - c*x^4]) + (3*a^(7/4)*C*e^3*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/
4)*x)/a^(1/4)], -1])/(5*c^(11/4)*Sqrt[a - c*x^4]) + ((Sqrt[a]*B*Sqrt[c] +
A*c + a*C)*(Sqrt[c]*d + Sqrt[a]*e)^3*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*c^(11/4)*Sqrt[a - c*x^4]) - (a^(5/4)
*e^2*(3*C*d + B*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4
)], -1])/(3*c^(9/4)*Sqrt[a - c*x^4]) + (a^(3/4)*e*(3*c*C*d^2 + a*C*e^2 + c
*e*(3*B*d + A*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)
], -1])/(c^(11/4)*Sqrt[a - c*x^4]) - (a^(1/4)*(c*C*d^3 + 3*c*d*e*(B*d + A*
e) + a*e^2*(3*C*d + B*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)
/a^(1/4)], -1])/(c^(9/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
 + 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 8.01 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{2c\left(\frac{\left(Aac\,e^3+3A\,d^2e\,c^2+3Bacd\,e^2+B\,d^3c^2+C\,a^2e^3+3Cac\,d^2e\right)x^3}{4a\,c^3}+\frac{\left(3Aacd\,e^2+A\,c^2d^3+a^2B\,e^3+3Bac\,d^2e+3C\,a^2d\,e^2+Cac\,d^3\right)x}{4a\,c^3}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{C\,e^3}{}$
default	$A\,d^3\left(\frac{x}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-c\,x^4+a}}\right)+d^2(3Ae+Bd)\left(\frac{x^3}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-}}{}\right.$
risch	$\frac{e^2x\left(3C\,x^2e+5Be+15Cd\right)\sqrt{-c\,x^4+a}}{15c^2}-\frac{-\frac{3e\left(5Ac\,e^2+15Bcde+8Ca\,e^2+15Cc\,d^2\right)\sqrt{a}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{Ellipti}\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-c\,x^4+a}\,\sqrt{c}}}{}$

input
```
int((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(1/4/a/c^3*(A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^2+B*c^2*d^3+C*a^2*e^3+
3*C*a*c*d^2*e)*x^3+1/4/a/c^3*(3*A*a*c*d*e^2+A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^
2*e+3*C*a^2*d*e^2+C*a*c*d^3)*x)/(-(x^4-a/c)*c)^(1/2)+1/5*C*e^3*x^3*(-c*x^4
+a)^(1/2)/c^2+1/3*e^2*(B*e+3*C*d)*x*(-c*x^4+a)^(1/2)/c^2+(-(3*A*c*d*e^2+B*
a*e^3+3*B*c*d^2*e+3*C*a*d*e^2+C*c*d^3)/c^2+1/2/c^2/a*(3*A*a*c*d*e^2+A*c^2*
d^3+B*a^2*e^3+3*B*a*c*d^2*e+3*C*a^2*d*e^2+C*a*c*d^3)-1/3/c^2*e^2*(B*e+3*C*
d)*a)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2
/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-(-
e*(A*c*e^2+3*B*c*d*e+C*a*e^2+3*C*c*d^2)/c^2-1/2/c^2/a*(A*a*c*e^3+3*A*c^2*d
^2*e+3*B*a*c*d*e^2+B*c^2*d^3+C*a^2*e^3+3*C*a*c*d^2*e)-3/5*C/c^2*e^3*a)*a^(
1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/
a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/
2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="frica
s")
```

output
```
1/30*(3*((5*B*a*c^3*d^3 + 45*B*a^2*c^2*d*e^2 + 15*(3*C*a^2*c^2 + A*a*c^3)*
d^2*e + 3*(7*C*a^3*c + 5*A*a^2*c^2)*e^3)*x^5 - (5*B*a^2*c^2*d^3 + 45*B*a^3
*c*d*e^2 + 15*(3*C*a^3*c + A*a^2*c^2)*d^2*e + 3*(7*C*a^4 + 5*A*a^3*c)*e^3)
*x)*sqrt(-c)*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - ((15*((B
+ C)*a*c^3 - A*c^4)*d^3 + 45*(3*C*a^2*c^2 + (A + B)*a*c^3)*d^2*e + 15*((9*
B + 5*C)*a^2*c^2 + 3*A*a*c^3)*d*e^2 + (63*C*a^3*c + 5*(9*A + 5*B)*a^2*c^2)
*e^3)*x^5 - (15*((B + C)*a^2*c^2 - A*a*c^3)*d^3 + 45*(3*C*a^3*c + (A + B)*
a^2*c^2)*d^2*e + 15*((9*B + 5*C)*a^3*c + 3*A*a^2*c^2)*d*e^2 + (63*C*a^4 +
5*(9*A + 5*B)*a^3*c)*e^3)*x)*sqrt(-c)*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^
(1/4)/x), -1) + (6*C*a^2*c^2*e^3*x^8 - 15*B*a^2*c^2*d^3 - 135*B*a^3*c*d*e^
2 + 10*(3*C*a^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^6 + 6*(15*C*a^2*c^2*d^2*e + 1
5*B*a^2*c^2*d*e^2 + (7*C*a^3*c + 5*A*a^2*c^2)*e^3)*x^4 - 45*(3*C*a^3*c + A
*a^2*c^2)*d^2*e - 9*(7*C*a^4 + 5*A*a^3*c)*e^3 - 5*(9*B*a^2*c^2*d^2*e + 5*B
*a^3*c*e^3 + 3*(C*a^2*c^2 + A*a*c^3)*d^3 + 3*(5*C*a^3*c + 3*A*a^2*c^2)*d*e
^2)*x^2)*sqrt(-c*x^4 + a))/(a^2*c^4*x^5 - a^3*c^3*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**3*(C*x**4+B*x**2+A)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((d + e*x**2)**3*(A + B*x**2 + C*x**4)/(a - c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^3}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^3/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^3}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^3/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \int \frac{(e\,x^2+d)^3\,(C\,x^4+B\,x^2+A)}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int(((d + e*x^2)^3*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2),x)
```

output
```
int(((d + e*x^2)^3*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x)
```

output
```
(25*sqrt(a - c*x**4)*a*b*e**3*x + 120*sqrt(a - c*x**4)*a*c*d*e**2*x - 36*s
qrt(a - c*x**4)*a*c*e**3*x**3 + 45*sqrt(a - c*x**4)*b*c*d**2*e*x - 45*sqrt
(a - c*x**4)*b*c*d*e**2*x**3 - 5*sqrt(a - c*x**4)*b*c*e**3*x**5 + 15*sqrt(
a - c*x**4)*c**2*d**3*x - 45*sqrt(a - c*x**4)*c**2*d**2*e*x**3 - 15*sqrt(a
 - c*x**4)*c**2*d*e**2*x**5 - 3*sqrt(a - c*x**4)*c**2*e**3*x**7 - 25*int(s
qrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a**3*b*e**3 - 120*int(s
qrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a**3*c*d*e**2 - 45*int(
sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a**2*b*c*d**2*e + 25*i
nt(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a**2*b*c*e**3*x**4
 + 120*int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a**2*c**2*d*
e**2*x**4 + 45*int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*b
*c**2*d**2*e*x**4 + 108*int((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c
**2*x**8),x)*a**3*c*e**3 + 135*int((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x
**4 + c**2*x**8),x)*a**2*b*c*d*e**2 + 180*int((sqrt(a - c*x**4)*x**2)/(a**
2 - 2*a*c*x**4 + c**2*x**8),x)*a**2*c**2*d**2*e - 108*int((sqrt(a - c*x**4
)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a**2*c**2*e**3*x**4 + 15*int((s
qrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*b*c**2*d**3 - 1
35*int((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*b*c**2
*d*e**2*x**4 - 180*int((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x
**8),x)*a*c**3*d**2*e*x**4 - 15*int((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a...
```

3.48 $\int \frac{(d+ex^2)^2(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 331

$$\int \frac{(d+ex^2)^2(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \frac{x(Ac(cd^2+ae^2)+a(aCe^2+cd(Cd+2Be))+c(2(Ac+aC)de+B}{2ac^2\sqrt{a-cx^4}}$$

$$+\frac{Ce^2x\sqrt{a-cx^4}}{3c^2}$$

$$-\frac{(Bcd^2+2Acde+6aCde+3aBe^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}c^{7/4}\sqrt{a-cx^4}}$$

$$+\frac{\left(3A(cd^2-ae^2)+\frac{3\sqrt{a}(Bcd^2+2Acde+6aCde+3aBe^2)}{\sqrt{c}}-\frac{a(5aCe^2+3cd(Cd+2Be))}{c}\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{6a^{3/4}c^{5/4}\sqrt{a-cx^4}}$$

output
```
1/2*x*(A*c*(a*e^2+c*d^2)+a*(C*a*e^2+c*d*(2*B*e+C*d))+c*(2*(A*c+C*a)*d*e+B*
(a*e^2+c*d^2))*x^2)/a/c^2/(-c*x^4+a)^(1/2)+1/3*C*e^2*x*(-c*x^4+a)^(1/2)/c^
2-1/2*(2*A*c*d*e+3*B*a*e^2+B*c*d^2+6*C*a*d*e)*(1-c*x^4/a)^(1/2)*EllipticE(
c^(1/4)*x/a^(1/4),I)/a^(1/4)/c^(7/4)/(-c*x^4+a)^(1/2)+1/6*(3*A*(-a*e^2+c*d
^2)+3*a^(1/2)*(2*A*c*d*e+3*B*a*e^2+B*c*d^2+6*C*a*d*e)/c^(1/2)-a*(5*C*a*e^2
+3*c*d*(2*B*e+C*d))/c)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^
(3/4)/c^(5/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}} dx = \frac{3Ac(cd^2+ae^2)x+ax(5aCe^2+6Bce(d-ex^2)+cC(3d^2-12dex^2}{}$$

input
```
Integrate[((d + e*x^2)^2*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2),x]
```

output
```
(3*A*c*(c*d^2 + a*e^2)*x + a*x*(5*a*C*e^2 + 6*B*c*e*(d - e*x^2) + c*C*(3*d
^2 - 12*d*e*x^2 - 2*e^2*x^4)) - (3*A*c*(-(c*d^2) + a*e^2) + a*(5*a*C*e^2 +
 3*c*d*(C*d + 2*B*e)))*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5
/4, (c*x^4)/a] + 2*c*(B*c*d^2 + 2*A*c*d*e + 6*a*C*d*e + 3*a*B*e^2)*x^3*Sqr
t[1 - (c*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, (c*x^4)/a])/(6*a*c^2*Sqr
t[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.66, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}} dx$$

$$\Big\downarrow \text{ 2259}$$

$$\int \left(-\frac{aCe^2+ce(Ae+2Bd)+cCd^2}{c^2\sqrt{a-cx^4}}+\frac{cx^2\left(2de(aC+Ac)+B\left(ae^2+cd^2\right)\right)+Ac\left(ae^2+cd^2\right)+a\left(aCe^2+cd(2B\right.}{c^2\left(a-cx^4\right)^{3/2}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}e+\sqrt{c}d\right)^2\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}c^{9/4}\sqrt{a-cx^4}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}(Be+2Cd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{7/4}\sqrt{a-cx^4}}-\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}(Be+2Cd)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{7/4}\sqrt{a-cx^4}}-\frac{a^{5/4}Ce^2\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{9/4}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(aCe^2+ce(Ae+2Bd)+cCd^2\right)}{c^{9/4}\sqrt{a-cx^4}}-\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\left(2de(aC+Ac)+B\left(ae^2+cd^2\right)\right)}{2\sqrt[4]{a}c^{7/4}\sqrt{a-cx^4}}+\frac{x\left(cx^2\left(2de(aC+Ac)+B\left(ae^2+cd^2\right)\right)+Ac\left(ae^2+cd^2\right)+a\left(aCe^2+cd(2Be+Cd)\right)\right)}{2ac^2\sqrt{a-cx^4}}+\frac{Ce^2x\sqrt{a-cx^4}}{3c^2}$$

input

```
Int[((d + e*x^2)^2*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2),x]
```

output

```
(x*(A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d + 2*B*e)) + c*(2*(A*c + a*
C)*d*e + B*(c*d^2 + a*e^2))*x^2))/(2*a*c^2*Sqrt[a - c*x^4]) + (C*e^2*x*Sqr
t[a - c*x^4])/(3*c^2) - (a^(3/4)*e*(2*C*d + B*e)*Sqrt[1 - (c*x^4)/a]*Ellip
ticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(7/4)*Sqrt[a - c*x^4]) - ((2*(A*
c + a*C)*d*e + B*(c*d^2 + a*e^2))*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^
(1/4)*x)/a^(1/4)], -1])/(2*a^(1/4)*c^(7/4)*Sqrt[a - c*x^4]) - (a^(5/4)*C*e
^2*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(9
/4)*Sqrt[a - c*x^4]) + ((Sqrt[a]*B*Sqrt[c] + A*c + a*C)*(Sqrt[c]*d + Sqrt[
a]*e)^2*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2
*a^(3/4)*c^(9/4)*Sqrt[a - c*x^4]) + (a^(3/4)*e*(2*C*d + B*e)*Sqrt[1 - (c*x
^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(7/4)*Sqrt[a - c*x^4
]) - (a^(1/4)*(c*C*d^2 + a*C*e^2 + c*e*(2*B*d + A*e))*Sqrt[1 - (c*x^4)/a]*
EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(9/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 5.73 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{2c\left(\frac{\left(2Acde+Bae^2+Bcd^2+2Cade\right)x^3}{4c^2a}+\frac{\left(Aace^2+Ac^2d^2+2Bacde+a^2Ce^2+Cacd^2\right)x}{4ac^3}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{Ce^2x\sqrt{-cx^4+a}}{3c^2}+\frac{\left(-\frac{Ace^2+2Bcde+Cae^2}{c^2}\right.}{}$
default	$Ad^2\left(\frac{x}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}\right)+d(2Ae+Bd)\left(\frac{x^3}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-}}{}\right.$
risch	$\frac{Ce^2x\sqrt{-cx^4+a}}{3c^2}-\frac{\frac{6c\left(-\frac{\left(2Acde+Bae^2+Bcd^2+2Cade\right)x^3}{4a}-\frac{\left(Aace^2+Ac^2d^2+2Bacde+a^2Ce^2+Cacd^2\right)x}{4ac}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}-\frac{3\left(Aace^2+Ac^2d^2+2Bacde\right.}{}}{}$

input
```
int((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(1/4/c^2*(2*A*c*d*e+B*a*e^2+B*c*d^2+2*C*a*d*e)/a*x^3+1/4/a/c^3*(A*a*c*
e^2+A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)*x)/(-(x^4-a/c)*c)^(1/2)+1/3
*C*e^2*x*(-c*x^4+a)^(1/2)/c^2+(-(A*c*e^2+2*B*c*d*e+C*a*e^2+C*c*d^2)/c^2+1/
2/c^2/a*(A*a*c*e^2+A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)-1/3*C/c^2*e^
2*a)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/
a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-(-1
/c*e*(B*e+2*C*d)-1/2/c*(2*A*c*d*e+B*a*e^2+B*c*d^2+2*C*a*d*e)/a)*a^(1/2)/(c
^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2)
)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-E
llipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex^2)^2(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \frac{3\left((Bac^2d^2+3\,Ba^2ce^2+2\,(3\,Ca^2c+Aac^2)de)x^5-(Ba^2cd^2+3\,Ba^3\right.}{}$$

input
```
integrate((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="frica
s")
```

output
```
1/6*(3*((B*a*c^2*d^2 + 3*B*a^2*c*e^2 + 2*(3*C*a^2*c + A*a*c^2)*d*e)*x^5 -
(B*a^2*c*d^2 + 3*B*a^3*e^2 + 2*(3*C*a^3 + A*a^2*c)*d*e)*x)*sqrt(-c)*(a/c)^
(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1) - ((3*((B + C)*a*c^2 - A*c^3)*
d^2 + 6*(3*C*a^2*c + (A + B)*a*c^2)*d*e + ((9*B + 5*C)*a^2*c + 3*A*a*c^2)*
e^2)*x^5 - (3*((B + C)*a^2*c - A*a*c^2)*d^2 + 6*(3*C*a^3 + (A + B)*a^2*c)*
d*e + ((9*B + 5*C)*a^3 + 3*A*a^2*c)*e^2)*x)*sqrt(-c)*(a/c)^(3/4)*elliptic_
f(arcsin((a/c)^(1/4)/x), -1) + (2*C*a^2*c*e^2*x^6 - 3*B*a^2*c*d^2 - 9*B*a^
3*e^2 + 6*(2*C*a^2*c*d*e + B*a^2*c*e^2)*x^4 - 6*(3*C*a^3 + A*a^2*c)*d*e -
(6*B*a^2*c*d*e + 3*(C*a^2*c + A*a*c^2)*d^2 + (5*C*a^3 + 3*A*a^2*c)*e^2)*x^
2)*sqrt(-c*x^4 + a))/(a^2*c^3*x^5 - a^3*c^2*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}} \, dx = \int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{\frac{3}{2}}} \, dx$$

input
```
integrate((e*x**2+d)**2*(C*x**4+B*x**2+A)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((d + e*x**2)**2*(A + B*x**2 + C*x**4)/(a - c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}} \, dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^2}{(-cx^4+a)^{\frac{3}{2}}} \, dx$$

input
```
integrate((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^2/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}} \, dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^2}{(-cx^4+a)^{\frac{3}{2}}} \, dx$$

input
```
integrate((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^2/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \int \frac{(e\,x^2+d)^2\,(C\,x^4+B\,x^2+A)}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int(((d + e*x^2)^2*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2),x)
```

output
```
int(((d + e*x^2)^2*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \frac{8\sqrt{-c\,x^4+a}\,a\,e^2x+6\sqrt{-c\,x^4+a}\,bdex-3\sqrt{-c\,x^4+a}\,b\,e^2x^3+3\sqrt{-}}{}$$

input
```
int((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x)
```

output
```
(8*sqrt(a - c*x**4)*a*e**2*x + 6*sqrt(a - c*x**4)*b*d*e*x - 3*sqrt(a - c*x
**4)*b*e**2*x**3 + 3*sqrt(a - c*x**4)*c*d**2*x - 6*sqrt(a - c*x**4)*c*d*e*
x**3 - sqrt(a - c*x**4)*c*e**2*x**5 - 8*int(sqrt(a - c*x**4)/(a**2 - 2*a*c
*x**4 + c**2*x**8),x)*a**3*e**2 - 6*int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**
4 + c**2*x**8),x)*a**2*b*d*e + 8*int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 +
 c**2*x**8),x)*a**2*c*e**2*x**4 + 6*int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**
4 + c**2*x**8),x)*a*b*c*d*e*x**4 + 9*int((sqrt(a - c*x**4)*x**2)/(a**2 - 2
*a*c*x**4 + c**2*x**8),x)*a**2*b*e**2 + 24*int((sqrt(a - c*x**4)*x**2)/(a*
*2 - 2*a*c*x**4 + c**2*x**8),x)*a**2*c*d*e + 3*int((sqrt(a - c*x**4)*x**2)
/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*b*c*d**2 - 9*int((sqrt(a - c*x**4)*x
**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*b*c*e**2*x**4 - 24*int((sqrt(a -
 c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*c**2*d*e*x**4 - 3*int(
(sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*b*c**2*d**2*x**
4)/(3*c*(a - c*x**4))
```

3.49 $\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 223

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \frac{x(Acd+aCd+aBe+(Bcd+Ace+aCe)x^2)}{2ac\sqrt{a-cx^4}} - \frac{(Bcd+Ace+3aCe)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}c^{7/4}\sqrt{a-cx^4}} + \frac{\left(\sqrt{a}\left(Bd+\left(A+\frac{3aC}{c}\right)e\right)+\frac{Acd-a(Cd+Be)}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}c^{3/4}\sqrt{a-cx^4}}$$

output
```
1/2*x*(A*c*d+C*a*d+B*a*e+(A*c*e+B*c*d+C*a*e)*x^2)/a/c/(-c*x^4+a)^(1/2)-1/2
*(A*c*e+B*c*d+3*C*a*e)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^
(1/4)/c^(7/4)/(-c*x^4+a)^(1/2)+1/2*(a^(1/2)*(B*d+(A+3*a*C/c)*e)+(A*c*d-a*(
B*e+C*d))/c^(1/2))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(3/4
)/c^(3/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \frac{3x(Acd+aBe+aC(d-2ex^2))-3(-Acd+aCd+aBe)x\sqrt{1-\frac{cx^4}{a}}}{}$$

input
```
Integrate[((d + e*x^2)*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2),x]
```

output
```
(3*x*(A*c*d + a*B*e + a*C*(d - 2*e*x^2)) - 3*(-(A*c*d) + a*C*d + a*B*e)*x*
Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + 2*(B*c*d
 + A*c*e + 3*a*C*e)*x^3*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/
4, (c*x^4)/a])/(6*a*c*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.73, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2259}$$

$$\int \left(\frac{x^2(aCe+Ace+Bcd)+aBe+aCd+Acd}{c(a-cx^4)^{3/2}}-\frac{Be+Cd}{c\sqrt{a-cx^4}}-\frac{Cex^2}{c\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}e+\sqrt{c}d\right)\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}c^{7/4}\sqrt{a-cx^4}}+\frac{a^{3/4}Ce\sqrt{1-\frac{cx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{7/4}\sqrt{a-cx^4}}-\frac{a^{3/4}Ce\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{7/4}\sqrt{a-cx^4}}-\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)(aCe+Ace+Bcd)}{2\sqrt[4]{a}c^{7/4}\sqrt{a-cx^4}}+\frac{x\left(x^2(aCe+Ace+Bcd)+aBe+aCd+Acd\right)}{2ac\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(Be+Cd)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{5/4}\sqrt{a-cx^4}}$$

input
```
Int[((d + e*x^2)*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2),x]
```

output
```
(x*(A*c*d + a*C*d + a*B*e + (B*c*d + A*c*e + a*C*e)*x^2))/(2*a*c*Sqrt[a -
c*x^4]) - (a^(3/4)*C*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^
(1/4)], -1])/(c^(7/4)*Sqrt[a - c*x^4]) - ((B*c*d + A*c*e + a*C*e)*Sqrt[1 -
 (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(1/4)*c^(7/4)
*Sqrt[a - c*x^4]) + (a^(3/4)*C*e*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(
1/4)*x)/a^(1/4)], -1])/(c^(7/4)*Sqrt[a - c*x^4]) + ((Sqrt[a]*B*Sqrt[c] + A
*c + a*C)*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^
(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*c^(7/4)*Sqrt[a - c*x^4]) - (a^(1/4)*(C*
d + B*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(
c^(5/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{2c\left(\frac{(Ace+Bcd+Cae)x^3}{4a\,c^2}+\frac{(Acd+Bae+Cad)x}{4c^2a}\right)}{\sqrt{-(x^4-\frac{a}{c})c}}+\frac{\left(-\frac{Be+Cd}{c}+\frac{Acd+Bae+Cad}{2ac}\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\Big(-$
default	$Ad\left(\frac{x}{2a\sqrt{-(x^4-\frac{a}{c})c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)+(Ae+Bd)\left(\frac{x^3}{2a\sqrt{-(x^4-\frac{a}{c})c}}+\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\right.$

input
```
int((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(1/4*(A*c*e+B*c*d+C*a*e)/a/c^2*x^3+1/4*(A*c*d+B*a*e+C*a*d)/c^2/a*x)/(-
(x^4-a/c)*c)^(1/2)+(-(B*e+C*d)/c+1/2*(A*c*d+B*a*e+C*a*d)/a/c)/(c^(1/2)/a^(
1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-
c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-(-1/c*e*C-1/2*(A*c*e
+B*c*d+C*a*e)/a/c)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))
^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x
*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \frac{((Bac^2d+(3\,Ca^2c+Aac^2)e)x^5-(Ba^2cd+(3\,Ca^3+Aa^2c)e)x)\sqrt{-c}}{}$$

input
```
integrate((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="fricas"
)
```

output
```
1/2*(((B*a*c^2*d + (3*C*a^2*c + A*a*c^2)*e)*x^5 - (B*a^2*c*d + (3*C*a^3 +
A*a^2*c)*e)*x)*sqrt(-c)*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x), -1)
- ((((B + C)*a*c^2 - A*c^3)*d + (3*C*a^2*c + (A + B)*a*c^2)*e)*x^5 - (((B
+ C)*a^2*c - A*a*c^2)*d + (3*C*a^3 + (A + B)*a^2*c)*e)*x)*sqrt(-c)*(a/c)^(
3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) + (2*C*a^2*c*e*x^4 - B*a^2*c*d
- (B*a^2*c*e + (C*a^2*c + A*a*c^2)*d)*x^2 - (3*C*a^3 + A*a^2*c)*e)*sqrt(-c
*x^4 + a))/(a^2*c^3*x^5 - a^3*c^2*x)
```

Sympy [A] (verification not implemented)

Time = 11.24 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \frac{Adx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Aex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{Cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{Cex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+d)*(C*x**4+B*x**2+A)/(-c*x**4+a)**(3/2),x)
```

output
```
A*d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*
a**(3/2)*gamma(5/4)) + A*e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), c*x**
4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + B*d*x**3*gamma(3/4)*hyper
((3/4, 3/2), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(7/4)) +
 B*e*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), c*x**4*exp_polar(2*I*pi)/a)
/(4*a**(3/2)*gamma(9/4)) + C*d*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), c
*x**4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + C*e*x**7*gamma(7/4)*h
yper((3/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(11
/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{3/2}}\,dx = \int \frac{(e\,x^2+d)\,(C\,x^4+B\,x^2+A)}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int(((d + e*x^2)*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2),x)
```

output
```
int(((d + e*x^2)*(A + B*x^2 + C*x^4))/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)\left(A+Bx^2+Cx^4\right)}{\left(a-cx^4\right)^{3/2}}\,dx = \frac{\sqrt{-c\,x^4+a}\,bex+\sqrt{-c\,x^4+a}\,cdx-\sqrt{-c\,x^4+a}\,ce\,x^3-\left(\int \frac{\sqrt{-c\,x^4+}}{c^2x^8-2ac\,x}\right.}{}$$

input
```
int((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x)
```

output
```
(sqrt(a - c*x**4)*b*e*x + sqrt(a - c*x**4)*c*d*x - sqrt(a - c*x**4)*c*e*x*
*3 - int(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a**2*b*e + in
t(sqrt(a - c*x**4)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*b*c*e*x**4 + 4*int
((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a**2*c*e + int
((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*b*c*d - 4*in
t((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*a*c**2*e*x**4
 - int((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**8),x)*b*c**2*d
*x**4)/(c*(a - c*x**4))
```

3.50 $\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{3/2}}\,dx = \frac{x\left(A+\frac{aC}{c}+Bx^2\right)}{2a\sqrt{a-cx^4}} - \frac{B\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a-cx^4}} + \frac{\left(\sqrt{a}B\sqrt{c}+Ac-aC\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}c^{5/4}\sqrt{a-cx^4}}$$

output
```
1/2*x*(A+a*C/c+B*x^2)/a/(-c*x^4+a)^(1/2)-1/2*B*(1-c*x^4/a)^(1/2)*EllipticE
(c^(1/4)*x/a^(1/4),I)/a^(1/4)/c^(3/4)/(-c*x^4+a)^(1/2)+1/2*(a^(1/2)*B*c^(1
/2)+A*c-a*C)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(3/4)/c^(5
/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{3/2}}\,dx = \frac{3(Ac+aC)x+3(Ac-aC)x\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{cx^4}{a}\right)+2Bcx}{6ac\sqrt{a-cx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a - c*x^4)^(3/2),x]
```

output
```
(3*(A*c + a*C)*x + 3*(A*c - a*C)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1
/4, 1/2, 5/4, (c*x^4)/a] + 2*B*c*x^3*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1
[3/4, 3/2, 7/4, (c*x^4)/a])/(6*a*c*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2397, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{\int \frac{-Bcx^2+Ac-aC}{\sqrt{a-cx^4}}dx}{2ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{2ac\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1513}$$

$$\frac{\left(\sqrt{a}B\sqrt{c}-aC+Ac\right)\int \frac{1}{\sqrt{a-cx^4}}dx-\sqrt{a}B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{2ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{2ac\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\sqrt{a}B\sqrt{c}-aC+Ac\right)\int\frac{1}{\sqrt{a-cx^4}}dx-B\sqrt{c}\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{2ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{2ac\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}B\sqrt{c}-aC+Ac\right)\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-B\sqrt{c}\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{2ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{2ac\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}B\sqrt{c}-aC+Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-B\sqrt{c}\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{2ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{2ac\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}B\sqrt{c}-aC+Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{B\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}}{2ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{2ac\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}B\sqrt{c}-aC+Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{\sqrt{a}B\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{a-cx^4}}}{2ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{2ac\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}B\sqrt{c}-aC+Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{a^{3/4}B\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-cx^4}}}{2ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{2ac\sqrt{a-cx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a - c*x^4)^(3/2),x]
```

output
```
(x*(A*c + a*C + B*c*x^2))/(2*a*c*Sqrt[a - c*x^4]) + (-((a^(3/4)*B*c^(1/4)*
Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c
*x^4]) + (a^(1/4)*(Sqrt[a]*B*Sqrt[c] + A*c - a*C)*Sqrt[1 - (c*x^4)/a]*Elli
pticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]))/(2*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{2c\left(\frac{B x^3}{4ac}+\frac{(Ac+Ca)x}{4a c^2}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\left(-\frac{C}{c}+\frac{Ac+Ca}{2ac}\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}+\frac{B\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(\right.\right.}{2\sqrt{a}\,\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-}}$
default	$A\left(\frac{x}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)+B\left(\frac{x^3}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\right.}{2\sqrt{}}\right.$

input
```
int((C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*c*(1/4/a*B/c*x^3+1/4*(A*c+C*a)/a/c^2*x)/(-(x^4-a/c)*c)^(1/2)+(-C/c+1/2*(
A*c+C*a)/a/c)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(
1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/
2),I)+1/2/a^(1/2)*B/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*
(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/
2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{3/2}}\,dx = -\frac{(Bc^2x^4-Bac)\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1)+((Cac-(A+B)c^2)x^4-Ca^2+(A+B)ac)\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}}}{2\,(ac^3x^4-a^2c^2)}$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((B*c^2*x^4 - B*a*c)*sqrt(a)*(c/a)^(3/4)*elliptic_e(arcsin(x*(c/a)^(1
/4)), -1) + ((C*a*c - (A + B)*c^2)*x^4 - C*a^2 + (A + B)*a*c)*sqrt(a)*(c/a
)^(3/4)*elliptic_f(arcsin(x*(c/a)^(1/4)), -1) + (B*c^2*x^3 + (C*a*c + A*c^
2)*x)*sqrt(-c*x^4 + a))/(a*c^3*x^4 - a^2*c^2)
```

Sympy [A] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{3/2}}\,dx = \frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/(-c*x**4+a)**(3/2),x)
```

output
```
A*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a*
*(3/2)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), c*x**4*ex
p_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + C*x**5*gamma(5/4)*hyper((5/4,
 3/2), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{3/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{3/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{3/2}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a - c*x^4)^(3/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{(a - cx^4)^{3/2}}\,dx = \frac{\sqrt{-c\,x^4 + a}\,x + \left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{c^2x^8-2ac\,x^4+a^2}dx\right)ab - \left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{c^2x^8-2ac\,x^4+a^2}dx\right)bc\,x^4}{-c\,x^4 + a}$$

input

```
int((C*x^4+B*x^2+A)/(-c*x^4+a)^(3/2),x)
```

output

```
(sqrt(a - c*x**4)*x + int((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**
2*x**8),x)*a*b - int((sqrt(a - c*x**4)*x**2)/(a**2 - 2*a*c*x**4 + c**2*x**
8),x)*b*c*x**4)/(a - c*x**4)
```

3.51 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 349

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)(a-cx^4)^{3/2}}\,dx = \frac{x((Ac+aC)d-aBe+(Bcd-(Ac+aC)e)x^2)}{2a(cd^2-ae^2)\sqrt{a-cx^4}} - \frac{(Bcd-(Ac+aC)e)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}c^{3/4}(cd^2-ae^2)\sqrt{a-cx^4}} + \frac{(\sqrt{a}B\sqrt{c}+Ac+aC)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}c^{3/4}(\sqrt{c}d+\sqrt{a}e)\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}(Cd^2-Bde+Ae^2)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d(cd^2-ae^2)\sqrt{a-cx^4}}$$

output
```
1/2*x*((A*c+C*a)*d-B*a*e+(B*c*d-(A*c+C*a)*e)*x^2)/a/(-a*e^2+c*d^2)/(-c*x^4
+a)^(1/2)-1/2*(B*c*d-(A*c+C*a)*e)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^
(1/4),I)/a^(1/4)/c^(3/4)/(-a*e^2+c*d^2)/(-c*x^4+a)^(1/2)+1/2*(a^(1/2)*B*c^
(1/2)+A*c+a*C)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(3/4)/c^
(3/4)/(c^(1/2)*d+a^(1/2)*e)/(-c*x^4+a)^(1/2)-a^(1/4)*(A*e^2-B*d*e+C*d^2)*(
1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1
/4)/d/(-a*e^2+c*d^2)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.96 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)\,(a-cx^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{c}dx(-Bcdx^2+Ac(-d+ex^2)+a(-Cd+Be+Cex^2))+i\sqrt{a}d(-Bcd+Ace+aCe)\sqrt{1-\frac{cx^4}{a}}}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
-1/2*(Sqrt[-(Sqrt[c]/Sqrt[a])]*Sqrt[c]*d*x*(-(B*c*d*x^2) + A*c*(-d + e*x^2
) + a*(-(C*d) + B*e + C*e*x^2)) + I*Sqrt[a]*d*(-(B*c*d) + A*c*e + a*C*e)*S
qrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] -
I*(Sqrt[a]*B*Sqrt[c] + A*c + a*C)*d*(-(Sqrt[c]*d) + Sqrt[a]*e)*Sqrt[1 - (c
*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (2*I)*a*Sq
rt[c]*(C*d^2 + e*(-(B*d) + A*e))*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]
*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(a^(3/2)*(-(
Sqrt[c]/Sqrt[a]))^(3/2)*(-(c*d^3) + a*d*e^2)*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{3/2}\,(d+ex^2)}\,dx$$

$$\Big\downarrow \; 2259$$

$$\int \left(\frac{-Ae^2+Bde-Cd^2}{\sqrt{a-cx^4}\,(d+ex^2)\,(cd^2-ae^2)}+\frac{x^2(-aCe-Ace+Bcd)-aBe+aCd+Acd}{(a-cx^4)^{3/2}\,(cd^2-ae^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}c^{3/4}\sqrt{a-cx^4}\left(\sqrt{a}e+\sqrt{c}d\right)}-\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)(-aCe-Ace+Bcd)}{2\sqrt[4]{a}c^{3/4}\sqrt{a-cx^4}\,(cd^2-ae^2)}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(Ae^2-Bde+Cd^2\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}\,(cd^2-ae^2)}+\frac{x\left(x^2(Bcd-e(aC+Ac))-aBe+aCd+Acd\right)}{2a\sqrt{a-cx^4}\,(cd^2-ae^2)}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(x*(A*c*d + a*C*d - a*B*e + (B*c*d - (A*c + a*C)*e)*x^2))/(2*a*(c*d^2 - a*
e^2)*Sqrt[a - c*x^4]) - ((B*c*d - A*c*e - a*C*e)*Sqrt[1 - (c*x^4)/a]*Ellip
ticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(1/4)*c^(3/4)*(c*d^2 - a*e^2)*
Sqrt[a - c*x^4]) + ((Sqrt[a]*B*Sqrt[c] + A*c + a*C)*Sqrt[1 - (c*x^4)/a]*El
lipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*c^(3/4)*(Sqrt[c]*d +
Sqrt[a]*e)*Sqrt[a - c*x^4]) - (a^(1/4)*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c
*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)
], -1])/(c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(293) = 586.

Time = 1.00 (sec) , antiderivative size = 756, normalized size of antiderivative = 2.17

method	result
default	$\frac{Be\left(\frac{x}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)+Ce\left(\frac{x^3}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right.\right.}{2\sqrt{a}\,\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}}{e^2}$
elliptic	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(B*e*(1/2/a*x/(-(x^4-a/c)*c)^(1/2)+1/2/a/(c^(1/2)/a^(1/2))^(1/2)*(1-
c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*
EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))+C*e*(1/2/a*x^3/(-(x^4-a/c)*c)^(1/2
)+1/2/a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(
1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(
1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))-C*d*(1/2/a*x/(-(x^
4-a/c)*c)^(1/2)+1/2/a/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2
)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1
/2))^(1/2),I)))+(A*e^2-B*d*e+C*d^2)/e^2*(2*c*(1/4/a*e/(a*e^2-c*d^2)*x^3-1/
4/a*d/(a*e^2-c*d^2)*x)/(-(x^4-a/c)*c)^(1/2)-1/2*c/a*d/(a*e^2-c*d^2)/(c^(1/
2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1
/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/2*c^(1/2)/a^
(1/2)*e/(a*e^2-c*d^2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2
)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1
/2))^(1/2),I)-1/2*c^(1/2)/a^(1/2)*e/(a*e^2-c*d^2)/(c^(1/2)/a^(1/2))^(1/2)*
(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/
2)*EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/(a*e^2-c*d^2)*e^2/d/(c^(1/2)/a
^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/
(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d
,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)(a-cx^4)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)(a-cx^4)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)/(-c*x**4+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)(a-cx^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-c*x^4 + a)^(3/2)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)(a - cx^4)^{3/2}}\, dx = \int \frac{Cx^4 + Bx^2 + A}{(-cx^4 + a)^{\frac{3}{2}}(ex^2 + d)}\, dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-c*x^4 + a)^(3/2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)(a - cx^4)^{3/2}}\, dx = \int \frac{C\,x^4 + B\,x^2 + A}{(a - c\,x^4)^{3/2}\,(e\,x^2 + d)}\, dx$$

input
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(3/2)*(d + e*x^2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(3/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)(a - cx^4)^{3/2}}\, dx = \left(\int \frac{\sqrt{-c\,x^4 + a}}{c^2e\,x^{10} + c^2d\,x^8 - 2ace\,x^6 - 2acd\,x^4 + a^2e\,x^2 + a^2d}dx\right) a + \left(\int \frac{\sqrt{-c\,x^4 + a}\,x^4}{c^2e\,x^{10} + c^2d\,x^8 - 2ace\,x^6 - 2acd\,x^4 + a^2e\,x^2 + a^2d}dx\right) c + \left(\int \frac{\sqrt{-c\,x^4 + a}\,x^2}{c^2e\,x^{10} + c^2d\,x^8 - 2ace\,x^6 - 2acd\,x^4 + a^2e\,x^2 + a^2d}dx\right) b$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 +
 c**2*d*x**8 + c**2*e*x**10),x)*a + int((sqrt(a - c*x**4)*x**4)/(a**2*d +
a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)
*c + int((sqrt(a - c*x**4)*x**2)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*
a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*b
```

3.52 $\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^2\left(a-cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 582

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^2\left(a-cx^4\right)^{3/2}}\,dx=-\frac{\left(Cd^2-Bde+Ae^2\right)x}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)\sqrt{a-cx^4}}$$
$$+\frac{x\left(d\left(Ac\left(cd^2+2ae^2\right)+a\left(aCe^2+cd(2Cd-3Be)\right)\right)+c\left(Bcd^3-2Acd^2e-3aCd^2e+2aBde^2-aAe^3\right)x^2\right)}{2ad\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}}$$
$$+\frac{\sqrt[4]{c}\left(e\left(2Acd^2+3aCd^2+aAe^2\right)-B\left(cd^3+2ade^2\right)\right)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}d\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}}$$
$$+\frac{\left(Ac^{3/2}d^2-a^{3/2}Cde+\sqrt{a}cd(Bd-Ae)+a\sqrt{c}\left(2Cd^2-e(2Bd-Ae)\right)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{2a^{3/4}\sqrt[4]{c}d\left(\sqrt{c}d+\sqrt{a}e\right)\left(cd^2-ae^2\right)\sqrt{a-cx^4}}$$
$$-\frac{\sqrt[4]{a}\left(cd^2\left(3Cd^2-e(5Bd-7Ae)\right)+ae^2\left(3Cd^2-e(Bd+Ae)\right)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{2\sqrt[4]{c}d^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}}$$

output
```
-1/2*(A*e^2-B*d*e+C*d^2)*x/d/(-a*e^2+c*d^2)/(e*x^2+d)/(-c*x^4+a)^(1/2)+1/2
*x*(d*(A*c*(2*a*e^2+c*d^2)+a*(C*a*e^2+c*d*(-3*B*e+2*C*d)))+c*(-A*a*e^3-2*A
*c*d^2*e+2*B*a*d*e^2+B*c*d^3-3*C*a*d^2*e)*x^2)/a/d/(-a*e^2+c*d^2)^2/(-c*x^
4+a)^(1/2)+1/2*c^(1/4)*(e*(A*a*e^2+2*A*c*d^2+3*C*a*d^2)-B*(2*a*d*e^2+c*d^3
))*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4)/d/(-a*e^2+c*d^
2)^2/(-c*x^4+a)^(1/2)+1/2*(A*c^(3/2)*d^2-a^(3/2)*C*d*e+a^(1/2)*c*d*(-A*e+B
*d)+a*c^(1/2)*(2*C*d^2-e*(-A*e+2*B*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4
)*x/a^(1/4),I)/a^(3/4)/c^(1/4)/d/(c^(1/2)*d+a^(1/2)*e)/(-a*e^2+c*d^2)/(-c*
x^4+a)^(1/2)-1/2*a^(1/4)*(c*d^2*(3*C*d^2-e*(-7*A*e+5*B*d))+a*e^2*(3*C*d^2-
e*(A*e+B*d)))*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^
(1/2)/d,I)/c^(1/4)/d^2/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.17 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2(a-cx^4)^{3/2}}\,dx = \frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d(ae^2(Cd^2+e(-Bd+Ae))\,x(a-cx^4)+dx(d+ex^2)\,(a^2Ce^2+Bc$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^2*(a - c*x^4)^(3/2)),x]
```

output
```
(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*(a*e^2*(C*d^2 + e*(-(B*d) + A*e))*x*(a - c*x^4
) + d*x*(d + e*x^2)*(a^2*C*e^2 + B*c^2*d^2*x^2 + A*c*(a*e^2 + c*d*(d - 2*e
*x^2)) + a*c*(C*d*(d - 2*e*x^2) + B*e*(-2*d + e*x^2)))) - I*(d + e*x^2)*Sq
rt[1 - (c*x^4)/a]*(Sqrt[a]*Sqrt[c]*d*(-(B*c*d^3) + 2*A*c*d^2*e + 3*a*C*d^2
*e - 2*a*B*d*e^2 + a*A*e^3)*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x
], -1] + d*(Sqrt[c]*d - Sqrt[a]*e)*(A*c^(3/2)*d^2 - a^(3/2)*C*d*e + Sqrt[a
]*c*d*(B*d - A*e) + a*Sqrt[c]*(2*C*d^2 + e*(-2*B*d + A*e)))*EllipticF[I*Ar
cSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + a*(c*(-3*C*d^4 + d^2*e*(5*B*d - 7
*A*e)) + a*e^2*(-3*C*d^2 + e*(B*d + A*e)))*EllipticPi[-((Sqrt[a]*e)/(Sqrt[
c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(2*a*Sqrt[-(Sqrt[c]/S
qrt[a])]*(c*d^3 - a*d*e^2)^2*(d + e*x^2)*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{3/2}(d+ex^2)^2}dx$$

$$\Big\downarrow \ 2259$$

$$\int \left(\frac{-Ae^2+Bde-Cd^2}{\sqrt{a-cx^4}(d+ex^2)^2(cd^2-ae^2)}+\frac{e\left(B\left(ae^2+cd^2\right)-2de(aC+Ac)\right)}{\sqrt{a-cx^4}(d+ex^2)(cd^2-ae^2)^2}+\frac{-cx^2\left(2de(aC+Ac)-B\left(ae^2+cd\right.\right.}{(a-}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{a^{3/4}\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)(Ae^2-Bde+Cd^2)}{2d\sqrt{a-cx^4}(cd^2-ae^2)^2}+\frac{\sqrt{1-\frac{cx^4}{a}}(\sqrt{a}B\sqrt{c}+aC+Ac)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a-cx^4}(\sqrt{a}e+\sqrt{c}d)^2}+\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)(Ae^2-Bde+Cd^2)}{2d\sqrt{a-cx^4}(\sqrt{a}e+\sqrt{c}d)(cd^2-ae^2)}+\frac{\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\left(2de(aC+Ac)-B\left(ae^2+cd^2\right)\right)}{2\sqrt[4]{a}\sqrt{a-cx^4}(cd^2-ae^2)^2}-\frac{\sqrt[4]{a}e\sqrt{1-\frac{cx^4}{a}}\left(2de(aC+Ac)-B\left(ae^2+cd^2\right)\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}(cd^2-ae^2)^2}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(3cd^2-ae^2)(Ae^2-Bde+Cd^2)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{c}d^2\sqrt{a-cx^4}(cd^2-ae^2)^2}+\frac{e^2x\sqrt{a-cx^4}(Ae^2-Bde+Cd^2)}{2d(d+ex^2)(cd^2-ae^2)^2}+\frac{x\left(-cx^2\left(2de(aC+Ac)-B\left(ae^2+cd^2\right)\right)+Ac\left(ae^2+cd^2\right)+a\left(aCe^2+cd(Cd-2Be)\right)\right)}{2a\sqrt{a-cx^4}(cd^2-ae^2)^2}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^2*(a - c*x^4)^(3/2)),x]
```

output
```
(x*(A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d - 2*B*e)) - c*(2*(A*c + a*
C)*d*e - B*(c*d^2 + a*e^2))*x^2))/(2*a*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4])
+ (e^2*(C*d^2 - B*d*e + A*e^2)*x*Sqrt[a - c*x^4])/(2*d*(c*d^2 - a*e^2)^2*(
d + e*x^2)) + (a^(3/4)*c^(1/4)*e*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/
a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*(c*d^2 - a*e^2)^2*Sqrt
[a - c*x^4]) + (c^(1/4)*(2*(A*c + a*C)*d*e - B*(c*d^2 + a*e^2))*Sqrt[1 - (
c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(1/4)*(c*d^2 -
a*e^2)^2*Sqrt[a - c*x^4]) + ((Sqrt[a]*B*Sqrt[c] + A*c + a*C)*Sqrt[1 - (c*x
^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*c^(1/4)*(Sqr
t[c]*d + Sqrt[a]*e)^2*Sqrt[a - c*x^4]) + (a^(1/4)*c^(1/4)*(C*d^2 - B*d*e +
 A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2
*d*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) - (a^(1/4)*(3*
c*d^2 - a*e^2)*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((S
qrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(1/4)*d^2*(
c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) - (a^(1/4)*e*(2*(A*c + a*C)*d*e - B*(c*d
^2 + a*e^2))*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), Ar
cSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^
4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1421 vs. 2(516) = 1032.

Time = 1.09 (sec) , antiderivative size = 1422, normalized size of antiderivative = 2.44

method	result	size
default	Expression too large to display	1422
elliptic	Expression too large to display	2169

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
C/e^2*(1/2/a*x/(-(x^4-a/c)*c)^(1/2)+1/2/a/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/
2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticF(x*(c^(1/2)/a^(1/2))^(1/2),I))+1/e^2*(B*e-2*C*d)*(2*c*(1/4/a*e/(a*e^2-
c*d^2)*x^3-1/4/a*d/(a*e^2-c*d^2)*x)/(-(x^4-a/c)*c)^(1/2)-1/2*c/a*d/(a*e^2-
c*d^2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^
2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+1
/2*c^(1/2)/a^(1/2)*e/(a*e^2-c*d^2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/
a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*
(c^(1/2)/a^(1/2))^(1/2),I)-1/2*c^(1/2)/a^(1/2)*e/(a*e^2-c*d^2)/(c^(1/2)/a^
(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(
-c*x^4+a)^(1/2)*EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/(a*e^2-c*d^2)*e^2
/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^
(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2
)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2)))+1/e^2*(A*
e^2-B*d*e+C*d^2)*(1/2*e^4/(a*e^2-c*d^2)^2/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+2
*c*(-1/2/a*c*d*e/(a*e^2-c*d^2)^2*x^3+1/4/a*(a*e^2+c*d^2)/(a*e^2-c*d^2)^2*x
)/(-(x^4-a/c)*c)^(1/2)+1/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(
1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a
^(1/2))^(1/2),I)*e^2*c/(a*e^2-c*d^2)^2+1/2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1
/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*E...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2(a-cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(3/2),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2(a-cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**2/(-c*x**4+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2(a-cx^4)^{3/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}(ex^2+d)^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2 (a-cx^4)^{3/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}(ex^2+d)^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2 (a-cx^4)^{3/2}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(a-c\,x^4)^{3/2}\,(e\,x^2+d)^2}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(3/2)*(d + e*x^2)^2),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(3/2)*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2 (a-cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{-c\,x^4+a}}{c^2e^2x^{12}+2c^2de\,x^{10}-2ac\,e^2x^8+c^2d^2x^8-4acde\,x^6+a^2e^2x^4-2ac\,d^2}\right.$$
$$+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^4}{c^2e^2x^{12}+2c^2de\,x^{10}-2ac\,e^2x^8+c^2d^2x^8-4acde\,x^6+a^2e^2x^4-2ac\,d^2x^4+2a^2de\,x^2+a^2d^2}dx\right)c$$
$$+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{c^2e^2x^{12}+2c^2de\,x^{10}-2ac\,e^2x^8+c^2d^2x^8-4acde\,x^6+a^2e^2x^4-2ac\,d^2x^4+2a^2de\,x^2+a^2d^2}dx\right)b$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a**2*d**2 + 2*a**2*d*e*x**2 + a**2*e**2*x**4 - 2*a*c
*d**2*x**4 - 4*a*c*d*e*x**6 - 2*a*c*e**2*x**8 + c**2*d**2*x**8 + 2*c**2*d*
e*x**10 + c**2*e**2*x**12),x)*a + int((sqrt(a - c*x**4)*x**4)/(a**2*d**2 +
 2*a**2*d*e*x**2 + a**2*e**2*x**4 - 2*a*c*d**2*x**4 - 4*a*c*d*e*x**6 - 2*a
*c*e**2*x**8 + c**2*d**2*x**8 + 2*c**2*d*e*x**10 + c**2*e**2*x**12),x)*c +
 int((sqrt(a - c*x**4)*x**2)/(a**2*d**2 + 2*a**2*d*e*x**2 + a**2*e**2*x**4
 - 2*a*c*d**2*x**4 - 4*a*c*d*e*x**6 - 2*a*c*e**2*x**8 + c**2*d**2*x**8 + 2
*c**2*d*e*x**10 + c**2*e**2*x**12),x)*b
```

3.53 $\int \frac{(d+ex^2)^3(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 519

$$\int \frac{(d+ex^2)^3(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \frac{x(Acd(cd^2+3ae^2)+a(ae^2(3Cd+Be)+cd^2(Cd+3Be))+((Ac+}{6ac^2(a-cx^4)^{3/2}}$$

$$+\frac{x(Acd(5cd^2-3ae^2)-a(7ae^2(3Cd+Be)+cd^2(Cd+3Be))+3(Bcd(cd^2-3ae^2)+Ace(3cd^2-ae^2)-}{12a^2c^2\sqrt{a-cx^4}}$$

$$-\frac{(Bcd(cd^2-3ae^2)+Ace(3cd^2-ae^2)-aCe(3cd^2+7ae^2))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{5/4}c^{11/4}\sqrt{a-cx^4}}$$

$$+\frac{(5Ac^{5/2}d^3-21a^{5/2}Ce^3+3\sqrt{a}c^2d^2(Bd+3Ae)+5a^2\sqrt{c}e^2(3Cd+Be)-ac^{3/2}d(Cd^2+3e(Bd+Ae))-3}{12a^{7/4}c^{11/4}\sqrt{a-cx^4}}$$

output
```
1/6*x*(A*c*d*(3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*d))+((A*c
+C*a)*e*(a*e^2+3*c*d^2)+B*c*d*(3*a*e^2+c*d^2))*x^2)/a/c^2/(-c*x^4+a)^(3/2)
+1/12*x*(A*c*d*(-3*a*e^2+5*c*d^2)-a*(7*a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*d)
)+3*(B*c*d*(-3*a*e^2+c*d^2)+A*c*e*(-a*e^2+3*c*d^2)-3*a*C*e*(a*e^2+c*d^2))*
x^2)/a^2/c^2/(-c*x^4+a)^(1/2)-1/4*(B*c*d*(-3*a*e^2+c*d^2)+A*c*e*(-a*e^2+3*
c*d^2)-a*C*e*(7*a*e^2+3*c*d^2))*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1
/4),I)/a^(5/4)/c^(11/4)/(-c*x^4+a)^(1/2)+1/12*(5*A*c^(5/2)*d^3-21*a^(5/2)*
C*e^3+3*a^(1/2)*c^2*d^2*(3*A*e+B*d)+5*a^2*c^(1/2)*e^2*(B*e+3*C*d)-a*c^(3/2
)*d*(C*d^2+3*e*(A*e+B*d))-3*a^(3/2)*c*e*(3*C*d^2+e*(A*e+3*B*d)))*(1-c*x^4/
a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(7/4)/c^(11/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.70 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex^2)^3\,(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \frac{x(-5Ac^3d^3x^4+a^3e^2(-15Cd-5Be+28Cex^2)+ac^2d(d(Cd+3Be)$$

input
```
Integrate[((d + e*x^2)^3*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2),x]
```

output
```
(x*(-5*A*c^3*d^3*x^4 + a^3*e^2*(-15*C*d - 5*B*e + 28*C*e*x^2) + a*c^2*d*(d
*(C*d + 3*B*e)*x^4 + A*(7*d^2 + 3*e^2*x^4)) + a^2*c*(C*(d^3 + 12*d^2*e*x^2
 + 21*d*e^2*x^4 - 12*e^3*x^6) + e*(A*e*(3*d + 4*e*x^2) + B*(3*d^2 + 12*d*e
*x^2 + 7*e^2*x^4)))) + (A*c*d*(5*c*d^2 - 3*a*e^2) + a*(5*a*e^2*(3*C*d + B*
e) - c*d^2*(C*d + 3*B*e)))*x*(a - c*x^4)*Sqrt[1 - (c*x^4)/a]*Hypergeometri
c2F1[1/4, 1/2, 5/4, (c*x^4)/a] - 4*(A*c*e*(-3*c*d^2 + a*e^2) + B*c*d*(-(c*
d^2) + 3*a*e^2) + a*C*e*(3*c*d^2 + 7*a*e^2))*x^3*(a - c*x^4)*Sqrt[1 - (c*x
^4)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, (c*x^4)/a])/(12*a^2*c^2*(a - c*x^4
)^(3/2))
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 954, normalized size of antiderivative = 1.84, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}} dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{x^2 (e(aC+Ac) (ae^2+3cd^2)+Bcd(3ae^2+cd^2))+Acd(3ae^2+cd^2)+a(ae^2(Be+3Cd)+cd^2(3Be+Cd)}{c^2 (a-cx^4)^{5/2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/4}C\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)e^3}{c^{11/4}\sqrt{a-cx^4}} - \frac{a^{3/4}C\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^3}{c^{11/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}(3Cd+Be)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{c^{9/4}\sqrt{a-cx^4}} + \frac{(3cCd^2+2aCe^2+ce(3Bd+Ae))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)e}{2\sqrt[4]{a}c^{11/4}\sqrt{a-cx^4}} - \frac{((Ac+aC)e(3cd^2+ae^2)+Bcd(cd^2+3ae^2))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{5/4}c^{11/4}\sqrt{a-cx^4}} - \frac{\left(e(3cCd^2+2aCe^2+ce(3Bd+Ae))+\frac{\sqrt{c}(cCd^3+3ce(Bd+Ae)d+2ae^2(3Cd+Be))}{\sqrt{a}}\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{2\sqrt[4]{a}c^{11/4}\sqrt{a-cx^4}}$$

$$\frac{\left(3((Ac+aC)e(3cd^2+ae^2)+Bcd(cd^2+3ae^2))+\frac{5\sqrt{c}(Acd(cd^2+3ae^2)+a(c(Cd+3Be)d^2+ae^2(3Cd+Be)))}{\sqrt{a}}\right)\sqrt{1-\frac{cx^4}{a}}\text{El}}{12a^{5/4}c^{11/4}\sqrt{a-cx^4}}$$

$$\frac{x\left(cCd^3+3ce(Bd+Ae)d+e(3cCd^2+2aCe^2+ce(3Bd+Ae))x^2+2ae^2(3Cd+Be)\right)}{2ac^2\sqrt{a-cx^4}} + \frac{x\left(3((Ac+aC)e(3cd^2+ae^2)+Bcd(cd^2+3ae^2))x^2+5(Acd(cd^2+3ae^2)+a(c(Cd+3Be)d^2+ae^2(3Cd+Be\right.}{12a^2c^2\sqrt{a-cx^4}}$$

$$\frac{x\left(((Ac+aC)e(3cd^2+ae^2)+Bcd(cd^2+3ae^2))x^2+Acd(cd^2+3ae^2)+a(c(Cd+3Be)d^2+ae^2(3Cd+Be))\right)}{6ac^2(a-cx^4)^{3/2}}$$

input
```
Int[((d + e*x^2)^3*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2),x]
```

output
```
(x*(A*c*d*(c*d^2 + 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)
) + ((A*c + a*C)*e*(3*c*d^2 + a*e^2) + B*c*d*(c*d^2 + 3*a*e^2))*x^2))/(6*a
*c^2*(a - c*x^4)^(3/2)) - (x*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 2*a*e^2*(3*C
*d + B*e) + e*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(3*B*d + A*e))*x^2))/(2*a*c^2*S
qrt[a - c*x^4]) + (x*(5*(A*c*d*(c*d^2 + 3*a*e^2) + a*(a*e^2*(3*C*d + B*e)
+ c*d^2*(C*d + 3*B*e))) + 3*((A*c + a*C)*e*(3*c*d^2 + a*e^2) + B*c*d*(c*d^
2 + 3*a*e^2))*x^2))/(12*a^2*c^2*Sqrt[a - c*x^4]) + (a^(3/4)*C*e^3*Sqrt[1 -
 (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(11/4)*Sqrt[a -
 c*x^4]) + (e*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(3*B*d + A*e))*Sqrt[1 - (c*x^4)
/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(1/4)*c^(11/4)*Sqrt[a
 - c*x^4]) - (((A*c + a*C)*e*(3*c*d^2 + a*e^2) + B*c*d*(c*d^2 + 3*a*e^2))*
Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(4*a^(5/4)
*c^(11/4)*Sqrt[a - c*x^4]) - (a^(3/4)*C*e^3*Sqrt[1 - (c*x^4)/a]*EllipticF[
ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(11/4)*Sqrt[a - c*x^4]) + (a^(1/4)*e^
2*(3*C*d + B*e)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)],
 -1])/(c^(9/4)*Sqrt[a - c*x^4]) - ((e*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(3*B*d
+ A*e)) + (Sqrt[c]*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 2*a*e^2*(3*C*d + B*e))
)/Sqrt[a])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(2*a^(1/4)*c^(11/4)*Sqrt[a - c*x^4]) + ((3*((A*c + a*C)*e*(3*c*d^2 + a*e^
2) + B*c*d*(c*d^2 + 3*a*e^2)) + (5*Sqrt[c]*(A*c*d*(c*d^2 + 3*a*e^2) + a...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{\left(\frac{\left(Aac\,e^{3}+3A\,d^{2}e\,c^{2}+3Bacd\,e^{2}+B\,d^{3}c^{2}+C\,a^{2}e^{3}+3Cac\,d^{2}e\right)x^{3}}{6a\,c^{4}}+\frac{\left(3Aacd\,e^{2}+A\,c^{2}d^{3}+a^{2}B\,e^{3}+3Bac\,d^{2}e+3C\,a^{2}d\,e^{2}+Cac\,d^{3}\right)x}{6a\,c^{4}}\right)\sqrt{-c\,x^{4}+a}}{\left(x^{4}-\frac{a}{c}\right)^{2}}$
default	$A\,d^{3}\left(\frac{x\sqrt{-c\,x^{4}+a}}{6a\,c^{2}\left(x^{4}-\frac{a}{c}\right)^{2}}+\frac{5x}{12a^{2}\sqrt{-\left(x^{4}-\frac{a}{c}\right)c}}+\frac{5\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{12a^{2}\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right)+d^{2}(3Ae+Bd)\left(\frac{x^{3}}{6a\cdots}\right.$

input
```
int((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/a/c^4*(A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^2+B*c^2*d^3+C*a^2*e^3+3*C*
a*c*d^2*e)*x^3+1/6/a/c^4*(3*A*a*c*d*e^2+A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e+
3*C*a^2*d*e^2+C*a*c*d^3)*x)*(-c*x^4+a)^(1/2)/(x^4-a/c)^2+2*c*(-1/8*(A*a*c*
e^3-3*A*c^2*d^2*e+3*B*a*c*d*e^2-B*c^2*d^3+3*C*a^2*e^3+3*C*a*c*d^2*e)/a^2/c
^3*x^3-1/24/a^2/c^3*(3*A*a*c*d*e^2-5*A*c^2*d^3+7*B*a^2*e^3+3*B*a*c*d^2*e+2
1*C*a^2*d*e^2+C*a*c*d^3)*x)/(-(x^4-a/c)*c)^(1/2)+(e^2*(B*e+3*C*d)/c^2-1/12
/a^2/c^2*(3*A*a*c*d*e^2-5*A*c^2*d^3+7*B*a^2*e^3+3*B*a*c*d^2*e+21*C*a^2*d*e
^2+C*a*c*d^3))/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^
(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1
/2),I)-(1/c^2*e^3*C+1/4*(A*a*c*e^3-3*A*c^2*d^2*e+3*B*a*c*d*e^2-B*c^2*d^3+3
*C*a^2*e^3+3*C*a*c*d^2*e)/a^2/c^2)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1
/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1
/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^
(1/2),I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 954 vs. 2(469) = 938.

Time = 0.12 (sec) , antiderivative size = 954, normalized size of antiderivative = 1.84

$$\int \frac{\left(d+ex^2\right)^3\left(A+Bx^2+Cx^4\right)}{\left(a-cx^4\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="frica
s")
```

output
```
1/12*(3*((B*a*c^4*d^3 - 3*B*a^2*c^3*d*e^2 - 3*(C*a^2*c^3 - A*a*c^4)*d^2*e
- (7*C*a^3*c^2 + A*a^2*c^3)*e^3)*x^9 - 2*(B*a^2*c^3*d^3 - 3*B*a^3*c^2*d*e^
2 - 3*(C*a^3*c^2 - A*a^2*c^3)*d^2*e - (7*C*a^4*c + A*a^3*c^2)*e^3)*x^5 + (
B*a^3*c^2*d^3 - 3*B*a^4*c*d*e^2 - 3*(C*a^4*c - A*a^3*c^2)*d^2*e - (7*C*a^5
 + A*a^4*c)*e^3)*x)*sqrt(-c)*(a/c)^(3/4)*elliptic_e(arcsin((a/c)^(1/4)/x),
 -1) - ((((3*B + C)*a*c^4 - 5*A*c^5)*d^3 - 3*(3*C*a^2*c^3 - (3*A + B)*a*c^
4)*d^2*e - 3*((3*B + 5*C)*a^2*c^3 - A*a*c^4)*d*e^2 - (21*C*a^3*c^2 + (3*A
+ 5*B)*a^2*c^3)*e^3)*x^9 - 2*(((3*B + C)*a^2*c^3 - 5*A*a*c^4)*d^3 - 3*(3*C
*a^3*c^2 - (3*A + B)*a^2*c^3)*d^2*e - 3*((3*B + 5*C)*a^3*c^2 - A*a^2*c^3)*
d*e^2 - (21*C*a^4*c + (3*A + 5*B)*a^3*c^2)*e^3)*x^5 + (((3*B + C)*a^3*c^2
- 5*A*a^2*c^3)*d^3 - 3*(3*C*a^4*c - (3*A + B)*a^3*c^2)*d^2*e - 3*((3*B + 5
*C)*a^4*c - A*a^3*c^2)*d*e^2 - (21*C*a^5 + (3*A + 5*B)*a^4*c)*e^3)*x)*sqrt
(-c)*(a/c)^(3/4)*elliptic_f(arcsin((a/c)^(1/4)/x), -1) - (12*C*a^3*c^2*e^3
*x^8 - 3*B*a^3*c^2*d^3 + 9*B*a^4*c*d*e^2 - (3*B*a^2*c^3*d^2*e + 7*B*a^3*c^
2*e^3 + (C*a^2*c^3 - 5*A*a*c^4)*d^3 + 3*(7*C*a^3*c^2 + A*a^2*c^3)*d*e^2)*x
^6 + (B*a^2*c^3*d^3 - 15*B*a^3*c^2*d*e^2 - 3*(5*C*a^3*c^2 - A*a^2*c^3)*d^2
*e - 5*(7*C*a^4*c + A*a^3*c^2)*e^3)*x^4 + 9*(C*a^4*c - A*a^3*c^2)*d^2*e +
3*(7*C*a^5 + A*a^4*c)*e^3 - (3*B*a^3*c^2*d^2*e - 5*B*a^4*c*e^3 + (C*a^3*c^
2 + 7*A*a^2*c^3)*d^3 - 3*(5*C*a^4*c - A*a^3*c^2)*d*e^2)*x^2)*sqrt(-c*x^4 +
 a))/(a^3*c^5*x^9 - 2*a^4*c^4*x^5 + a^5*c^3*x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**3*(C*x**4+B*x**2+A)/(-c*x**4+a)**(5/2),x)
```

output
```
Integral((d + e*x**2)**3*(A + B*x**2 + C*x**4)/(a - c*x**4)**(5/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3\,(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^3}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^3/(-c*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^3\,(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^3}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^3/(-c*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3\,(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \int \frac{(e\,x^2+d)^3\,(C\,x^4+B\,x^2+A)}{(a-c\,x^4)^{5/2}}\,dx$$

input
```
int(((d + e*x^2)^3*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2),x)
```

output
```
int(((d + e*x^2)^3*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^3 (A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \text{Too large to display}$$

input

```
int((e*x^2+d)^3*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x)
```

output

```
( - 15*sqrt(a - c*x**4)*a*b*e**3*x - 36*sqrt(a - c*x**4)*a*c*d*e**2*x + 40
*sqrt(a - c*x**4)*a*c*e**3*x**3 + 9*sqrt(a - c*x**4)*b*c*d**2*e*x + 15*sqr
t(a - c*x**4)*b*c*d*e**2*x**3 + 15*sqrt(a - c*x**4)*b*c*e**3*x**5 + 3*sqrt
(a - c*x**4)*c**2*d**3*x + 15*sqrt(a - c*x**4)*c**2*d**2*e*x**3 + 45*sqrt(
a - c*x**4)*c**2*d*e**2*x**5 - 15*sqrt(a - c*x**4)*c**2*e**3*x**7 + 15*int
(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a
**4*b*e**3 + 36*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8
 - c**3*x**12),x)*a**4*c*d*e**2 - 9*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*
x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**3*b*c*d**2*e - 30*int(sqrt(a - c*
x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**3*b*c*e**3
*x**4 + 12*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c*
*3*x**12),x)*a**3*c**2*d**3 - 72*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**
4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**3*c**2*d*e**2*x**4 + 18*int(sqrt(a -
 c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**2*b*c**
2*d**2*e*x**4 + 15*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x
**8 - c**3*x**12),x)*a**2*b*c**2*e**3*x**8 - 24*int(sqrt(a - c*x**4)/(a**3
 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**2*c**3*d**3*x**4 + 36
*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),
x)*a**2*c**3*d*e**2*x**8 - 9*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 +
3*a*c**2*x**8 - c**3*x**12),x)*a*b*c**3*d**2*e*x**8 + 12*int(sqrt(a - c...
```

3.54 $\int \frac{(d+ex^2)^2(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 402

$$\int \frac{(d+ex^2)^2(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \frac{x(Ac(cd^2+ae^2)+a(aCe^2+cd(Cd+2Be))+c(2(Ac+aC)de+B}{6ac^2(a-cx^4)^{3/2}}$$

$$+\frac{x(Ac(5cd^2-ae^2)-a(7aCe^2+cd(Cd+2Be))+3c(Bcd^2+2Acde-2aCde-aBe^2)x^2)}{12a^2c^2\sqrt{a-cx^4}}$$

$$-\frac{(Bcd^2+2Acde-2aCde-aBe^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{5/4}c^{7/4}\sqrt{a-cx^4}}$$

$$+\frac{(5Ac^2d^2+5a^2Ce^2+3\sqrt{a}c^{3/2}d(Bd+2Ae)-3a^{3/2}\sqrt{c}e(2Cd+Be)-ac(Cd^2+e(2Bd+Ae)))\sqrt{1-\frac{cx^4}{a}}}{12a^{7/4}c^{9/4}\sqrt{a-cx^4}}$$

output

```
1/6*x*(A*c*(a*e^2+c*d^2)+a*(C*a*e^2+c*d*(2*B*e+C*d))+c*(2*(A*c+C*a)*d*e+B*
(a*e^2+c*d^2))*x^2)/a/c^2/(-c*x^4+a)^(3/2)+1/12*x*(A*c*(-a*e^2+5*c*d^2)-a*
(7*C*a*e^2+c*d*(2*B*e+C*d))+3*c*(2*A*c*d*e-B*a*e^2+B*c*d^2-2*C*a*d*e)*x^2)
/a^2/c^2/(-c*x^4+a)^(1/2)-1/4*(2*A*c*d*e-B*a*e^2+B*c*d^2-2*C*a*d*e)*(1-c*x
^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(5/4)/c^(7/4)/(-c*x^4+a)^(1/2
)+1/12*(5*A*c^2*d^2+5*a^2*C*e^2+3*a^(1/2)*c^(3/2)*d*(2*A*e+B*d)-3*a^(3/2)*
c^(1/2)*e*(B*e+2*C*d)-a*c*(C*d^2+e*(A*e+2*B*d)))*(1-c*x^4/a)^(1/2)*Ellipti
cF(c^(1/4)*x/a^(1/4),I)/a^(7/4)/c^(9/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.50 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \frac{x(-5a^3Ce^2-5Ac^3d^2x^4+ac^2(d(Cd+2Be)x^4+A(7d^2+e^2x^4))+a}{}$$

input
```
Integrate[((d + e*x^2)^2*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2),x]
```

output
```
(x*(-5*a^3*C*e^2 - 5*A*c^3*d^2*x^4 + a*c^2*(d*(C*d + 2*B*e)*x^4 + A*(7*d^2
 + e^2*x^4)) + a^2*c*(e*(2*B*d + A*e + 4*B*e*x^2) + C*(d^2 + 8*d*e*x^2 + 7
*e^2*x^4))) + (A*c*(5*c*d^2 - a*e^2) + a*(5*a*C*e^2 - c*d*(C*d + 2*B*e)))*
x*(a - c*x^4)*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)
/a] - 4*c*(-(B*c*d^2) - 2*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^3*(a - c*x^4)*S
qrt[1 - (c*x^4)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, (c*x^4)/a])/(12*a^2*c^
2*(a - c*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.69, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{-2aCe^2-ce(Ae+2Bd)-cex^2(Be+2Cd)-cCd^2}{c^2(a-cx^4)^{3/2}}+\frac{cx^2(2de(aC+Ac)+B(ae^2+cd^2))+Ac(ae^2+cd}{c^2(a-cx^4)^{5/2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(\sqrt{a}\sqrt{c}e(Be+2Cd)+2aCe^2+ce(Ae+2Bd)+cCd^2\right)}{2a^{3/4}c^{9/4}\sqrt{a-cx^4}}+$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(3\sqrt{c}\left(2de(aC+Ac)+B\left(ae^2+cd^2\right)\right)+\frac{5\left(Ac\left(ae^2+cd^2\right)+a\left(aCe^2+cd(2Be+\right.\right.}{\sqrt{a}}\right.}{12a^{5/4}c^{9/4}\sqrt{a-cx^4}}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)\left(2de(aC+Ac)+B\left(ae^2+cd^2\right)\right)}{4a^{5/4}c^{7/4}\sqrt{a-cx^4}}+$$

$$\frac{x\left(3cx^2\left(2de(aC+Ac)+B\left(ae^2+cd^2\right)\right)+5\left(Ac\left(ae^2+cd^2\right)+a\left(aCe^2+cd(2Be+Cd)\right)\right)\right)}{12a^2c^2\sqrt{a-cx^4}}-$$

$$\frac{x\left(2aCe^2+ce(Ae+2Bd)+cex^2(Be+2Cd)+cCd^2\right)}{2ac^2\sqrt{a-cx^4}}+$$

$$\frac{x\left(cx^2\left(2de(aC+Ac)+B\left(ae^2+cd^2\right)\right)+Ac\left(ae^2+cd^2\right)+a\left(aCe^2+cd(2Be+Cd)\right)\right)}{6ac^2\left(a-cx^4\right)^{3/2}}+$$

$$\frac{e\sqrt{1-\frac{cx^4}{a}}(Be+2Cd)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}c^{7/4}\sqrt{a-cx^4}}+$$

$$\frac{\sqrt[4]{a}Ce^2\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{9/4}\sqrt{a-cx^4}}$$

input
```
Int[((d + e*x^2)^2*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2),x]
```

output
```
(x*(A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d + 2*B*e)) + c*(2*(A*c + a*
C)*d*e + B*(c*d^2 + a*e^2))*x^2))/(6*a*c^2*(a - c*x^4)^(3/2)) - (x*(c*C*d^
2 + 2*a*C*e^2 + c*e*(2*B*d + A*e) + c*e*(2*C*d + B*e)*x^2))/(2*a*c^2*Sqrt[
a - c*x^4]) + (x*(5*(A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d + 2*B*e))
) + 3*c*(2*(A*c + a*C)*d*e + B*(c*d^2 + a*e^2))*x^2))/(12*a^2*c^2*Sqrt[a -
 c*x^4]) + (e*(2*C*d + B*e)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*
x)/a^(1/4)], -1])/(2*a^(1/4)*c^(7/4)*Sqrt[a - c*x^4]) - ((2*(A*c + a*C)*d*
e + B*(c*d^2 + a*e^2))*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^
(1/4)], -1])/(4*a^(5/4)*c^(7/4)*Sqrt[a - c*x^4]) + (a^(1/4)*C*e^2*Sqrt[1 -
 (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(9/4)*Sqrt[a -
c*x^4]) - ((c*C*d^2 + 2*a*C*e^2 + c*e*(2*B*d + A*e) + Sqrt[a]*Sqrt[c]*e*(2
*C*d + B*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1
])/(2*a^(3/4)*c^(9/4)*Sqrt[a - c*x^4]) + ((3*Sqrt[c]*(2*(A*c + a*C)*d*e +
B*(c*d^2 + a*e^2)) + (5*(A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d + 2*B
*e))))/Sqrt[a])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)],
 -1])/(12*a^(5/4)*c^(9/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\left(\frac{\left(2Acde+Ba\,e^2+Bc\,d^2+2Cade\right)x^3}{6c^3a}+\frac{\left(Aac\,e^2+A\,c^2d^2+2Bacde+a^2C\,e^2+Cac\,d^2\right)x}{6a\,c^4}\right)\sqrt{-c\,x^4+a}}{\left(x^4-\frac{a}{c}\right)^2}+\frac{2c\left(\frac{\left(2Acde-Ba\,e^2+Bc\,d^2-2Cade\right)x^3}{8c^2a^2}\cdots\right.}{\cdots}$
default	$A\,d^2\left(\frac{x\sqrt{-c\,x^4+a}}{6a\,c^2\left(x^4-\frac{a}{c}\right)^2}+\frac{5x}{12a^2\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{5\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{12a^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)+d(2Ae+Bd)\left(\frac{x^3\sqrt{\cdots}}{6a\,c\cdots}\right.$

input
```
int((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/c^3*(2*A*c*d*e+B*a*e^2+B*c*d^2+2*C*a*d*e)/a*x^3+1/6/a/c^4*(A*a*c*e^2+
A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)*x)*(-c*x^4+a)^(1/2)/(x^4-a/c)^2
+2*c*(1/8/c^2*(2*A*c*d*e-B*a*e^2+B*c*d^2-2*C*a*d*e)/a^2*x^3-1/24/a^2/c^3*(
A*a*c*e^2-5*A*c^2*d^2+2*B*a*c*d*e+7*C*a^2*e^2+C*a*c*d^2)*x)/(-(x^4-a/c)*c)
^(1/2)+(C*e^2/c^2-1/12/c^2/a^2*(A*a*c*e^2-5*A*c^2*d^2+2*B*a*c*d*e+7*C*a^2*
e^2+C*a*c*d^2))/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c
^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(
1/2),I)+1/4/c^(3/2)*(2*A*c*d*e-B*a*e^2+B*c*d^2-2*C*a*d*e)/a^(3/2)/(c^(1/2)
/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2
)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^
(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex^2)^2(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx =$$

$$-\frac{3\left((Bc^4d^2-Bac^3e^2-2(Cac^3-Ac^4)de)x^8+Ba^2c^2d^2-Ba^3ce^2-2(Bac^3d^2-Ba^2c^2e^2-2(Ca^2c^2-A\cdots\right.}{\cdots}$$

input
```
integrate((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="frica
s")
```

output
```
-1/12*(3*((B*c^4*d^2 - B*a*c^3*e^2 - 2*(C*a*c^3 - A*c^4)*d*e)*x^8 + B*a^2*
c^2*d^2 - B*a^3*c*e^2 - 2*(B*a*c^3*d^2 - B*a^2*c^2*e^2 - 2*(C*a^2*c^2 - A*
a*c^3)*d*e)*x^4 - 2*(C*a^3*c - A*a^2*c^2)*d*e)*sqrt(a)*(c/a)^(3/4)*ellipti
c_e(arcsin(x*(c/a)^(1/4)), -1) + (((C*a*c^3 - (5*A + 3*B)*c^4)*d^2 + 2*((B
 + 3*C)*a*c^3 - 3*A*c^4)*d*e - (5*C*a^2*c^2 - (A + 3*B)*a*c^3)*e^2)*x^8 -
2*((C*a^2*c^2 - (5*A + 3*B)*a*c^3)*d^2 + 2*((B + 3*C)*a^2*c^2 - 3*A*a*c^3)
*d*e - (5*C*a^3*c - (A + 3*B)*a^2*c^2)*e^2)*x^4 + (C*a^3*c - (5*A + 3*B)*a
^2*c^2)*d^2 + 2*((B + 3*C)*a^3*c - 3*A*a^2*c^2)*d*e - (5*C*a^4 - (A + 3*B)
*a^3*c)*e^2)*sqrt(a)*(c/a)^(3/4)*elliptic_f(arcsin(x*(c/a)^(1/4)), -1) + (
3*(B*c^4*d^2 - B*a*c^3*e^2 - 2*(C*a*c^3 - A*c^4)*d*e)*x^7 - (2*B*a*c^3*d*e
 + (C*a*c^3 - 5*A*c^4)*d^2 + (7*C*a^2*c^2 + A*a*c^3)*e^2)*x^5 - (5*B*a*c^3
*d^2 - B*a^2*c^2*e^2 - 2*(C*a^2*c^2 - 5*A*a*c^3)*d*e)*x^3 - (2*B*a^2*c^2*d
*e + (C*a^2*c^2 + 7*A*a*c^3)*d^2 - (5*C*a^3*c - A*a^2*c^2)*e^2)*x)*sqrt(-c
*x^4 + a))/(a^2*c^5*x^8 - 2*a^3*c^4*x^4 + a^4*c^3)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x**2+d)**2*(C*x**4+B*x**2+A)/(-c*x**4+a)**(5/2),x)
```

output
```
Integral((d + e*x**2)**2*(A + B*x**2 + C*x**4)/(a - c*x**4)**(5/2), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^2}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^2/(-c*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^2}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^2/(-c*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \int \frac{(e\,x^2+d)^2\,(C\,x^4+B\,x^2+A)}{(a-c\,x^4)^{5/2}}\,dx$$

input
```
int(((d + e*x^2)^2*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2),x)
```

output
```
int(((d + e*x^2)^2*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2 (A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x^2+d)^2*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x)
```

output
```
( - 12*sqrt(a - c*x**4)*a*e**2*x + 6*sqrt(a - c*x**4)*b*d*e*x + 5*sqrt(a -
 c*x**4)*b*e**2*x**3 + 3*sqrt(a - c*x**4)*c*d**2*x + 10*sqrt(a - c*x**4)*c
*d*e*x**3 + 15*sqrt(a - c*x**4)*c*e**2*x**5 + 12*int(sqrt(a - c*x**4)/(a**
3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**4*e**2 - 6*int(sqrt(
a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**3*b*
d*e + 12*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3
*x**12),x)*a**3*c*d**2 - 24*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3
*a*c**2*x**8 - c**3*x**12),x)*a**3*c*e**2*x**4 + 12*int(sqrt(a - c*x**4)/(
a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**2*b*c*d*e*x**4 -
24*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12
),x)*a**2*c**2*d**2*x**4 + 12*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 +
 3*a*c**2*x**8 - c**3*x**12),x)*a**2*c**2*e**2*x**8 - 6*int(sqrt(a - c*x**
4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a*b*c**2*d*e*x**
8 + 12*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x
**12),x)*a*c**3*d**2*x**8 - 15*int((sqrt(a - c*x**4)*x**2)/(a**3 - 3*a**2*
c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**3*b*e**2 + 15*int((sqrt(a - c*x
**4)*x**2)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**2*b*c
*d**2 + 30*int((sqrt(a - c*x**4)*x**2)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x*
*8 - c**3*x**12),x)*a**2*b*c*e**2*x**4 - 30*int((sqrt(a - c*x**4)*x**2)/(a
**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a*b*c**2*d**2*x**4...
```

3.55 $\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 283

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \frac{x(Acd+aCd+aBe+(Bcd+Ace+aCe)x^2)}{6ac(a-cx^4)^{3/2}} + \frac{x(5Acd-a(Cd+Be)+3(Bcd+Ace-aCe)x^2)}{12a^2c\sqrt{a-cx^4}} - \frac{(Bcd+Ace-aCe)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{5/4}c^{7/4}\sqrt{a-cx^4}} + \frac{\left(3\sqrt{a}\left(Bd+\left(A-\frac{aC}{c}\right)e\right)+\frac{5Acd-a(Cd+Be)}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}c^{3/4}\sqrt{a-cx^4}}$$

output
```
1/6*x*(A*c*d+C*a*d+B*a*e+(A*c*e+B*c*d+C*a*e)*x^2)/a/c/(-c*x^4+a)^(3/2)+1/1
2*x*(5*A*c*d-a*(B*e+C*d)+3*(A*c*e+B*c*d-C*a*e)*x^2)/a^2/c/(-c*x^4+a)^(1/2)
-1/4*(A*c*e+B*c*d-C*a*e)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/
a^(5/4)/c^(7/4)/(-c*x^4+a)^(1/2)+1/12*(3*a^(1/2)*(B*d+(A-a*C/c)*e)+(5*A*c*
d-a*(B*e+C*d))/c^(1/2))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a
^(7/4)/c^(3/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \frac{-5Ac^2dx^5+acx(7Ad+(Cd+Be)x^4)+a^2x(Be+C(d+4ex^2))-(}{}$$

input
```
Integrate[((d + e*x^2)*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2),x]
```

output
```
(-5*A*c^2*d*x^5 + a*c*x*(7*A*d + (C*d + B*e)*x^4) + a^2*x*(B*e + C*(d + 4*
e*x^2)) - (-5*A*c*d + a*C*d + a*B*e)*x*(a - c*x^4)*Sqrt[1 - (c*x^4)/a]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] - 4*(B*c*d + A*c*e - a*C*e)*x^3*(
-a + c*x^4)*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, (c*x^4)/a
])/(12*a^2*c*(a - c*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.59, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx$$

$$\Big\downarrow \; 2259$$

$$\int \left(\frac{x^2(aCe+Ace+Bcd)+aBe+aCd+Acd}{c\,(a-cx^4)^{5/2}}+\frac{Be+Cd+Cex^2}{c\sqrt{a-cx^4}\,(cx^4-a)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(\frac{5\sqrt{c}(aBe+aCd+Acd)}{\sqrt{a}}+3(aCe+Ace+Bcd)\right)}{12a^{5/4}c^{7/4}\sqrt{a-cx^4}}-\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)(aCe+Ace+Bcd)}{4a^{5/4}c^{7/4}\sqrt{a-cx^4}}-\frac{\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(\sqrt{a}Ce+\sqrt{c}(Be+Cd)\right)}{2a^{3/4}c^{7/4}\sqrt{a-cx^4}}+\frac{x\left(3x^2(aCe+Ace+Bcd)+5(aBe+aCd+Acd)\right)}{12a^2c\sqrt{a-cx^4}}+\frac{x\left(x^2(aCe+Ace+Bcd)+aBe+aCd+Acd\right)}{6ac\left(a-cx^4\right)^{3/2}}+\frac{Ce\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}c^{7/4}\sqrt{a-cx^4}}-\frac{x\left(Be+Cd+Cex^2\right)}{2ac\sqrt{a-cx^4}}$$

input
```
Int[((d + e*x^2)*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2),x]
```

output
```
(x*(A*c*d + a*C*d + a*B*e + (B*c*d + A*c*e + a*C*e)*x^2))/(6*a*c*(a - c*x^
4)^(3/2)) - (x*(C*d + B*e + C*e*x^2))/(2*a*c*Sqrt[a - c*x^4]) + (x*(5*(A*c
*d + a*C*d + a*B*e) + 3*(B*c*d + A*c*e + a*C*e)*x^2))/(12*a^2*c*Sqrt[a - c
*x^4]) + (C*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(2*a^(1/4)*c^(7/4)*Sqrt[a - c*x^4]) - ((B*c*d + A*c*e + a*C*e)*Sqrt[1
- (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(4*a^(5/4)*c^(7/4
)*Sqrt[a - c*x^4]) - ((Sqrt[a]*C*e + Sqrt[c]*(C*d + B*e))*Sqrt[1 - (c*x^4)
/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*c^(7/4)*Sqrt[a
- c*x^4]) + (((5*Sqrt[c]*(A*c*d + a*C*d + a*B*e))/Sqrt[a] + 3*(B*c*d + A*c
*e + a*C*e))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1
])/(12*a^(5/4)*c^(7/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.16

method	result
elliptic	$\frac{\left(\frac{(Ace+Bcd+Cae)x^3}{6a\,c^3}+\frac{(Acd+Bae+Cad)x}{6a\,c^3}\right)\sqrt{-c\,x^4+a}}{\left(x^4-\frac{a}{c}\right)^2}+\frac{2c\left(\frac{(Ace+Bcd-Cae)x^3}{8a^2c^2}+\frac{(5Acd-Bae-Cad)x}{24a^2c^2}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{(5Acd-Bae-Cad)\sqrt{1-}}{12a}$
default	$Ad\left(\frac{x\sqrt{-c\,x^4+a}}{6a\,c^2\left(x^4-\frac{a}{c}\right)^2}+\frac{5x}{12a^2\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{5\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{12a^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)+(Ae+Bd)\left(\frac{x^3\sqrt{-c\,x}}{6a\,c^2\left(x^4\right.}\right.$

input
```
int((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/a/c^3*(A*c*e+B*c*d+C*a*e)*x^3+1/6/a/c^3*(A*c*d+B*a*e+C*a*d)*x)*(-c*x^
4+a)^(1/2)/(x^4-a/c)^2+2*c*(1/8*(A*c*e+B*c*d-C*a*e)/a^2/c^2*x^3+1/24/a^2/c
^2*(5*A*c*d-B*a*e-C*a*d)*x)/(-(x^4-a/c)*c)^(1/2)+1/12/a^2/c*(5*A*c*d-B*a*e
-C*a*d)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x
^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+
1/4*(A*c*e+B*c*d-C*a*e)/a^(3/2)/c^(3/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)
*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(Ellipt
icF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx =$$

$$-\frac{3\left((Bc^3d-(Cac^2-Ac^3)e)x^8+Ba^2cd-2\left(Bac^2d-(Ca^2c-Aac^2)e\right)x^4-(Ca^3-Aa^2c)e\right)\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}}E(\mathrm{a}$$

input
```
integrate((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="fricas"
)
```

output
```
-1/12*(3*((B*c^3*d - (C*a*c^2 - A*c^3)*e)*x^8 + B*a^2*c*d - 2*(B*a*c^2*d -
 (C*a^2*c - A*a*c^2)*e)*x^4 - (C*a^3 - A*a^2*c)*e)*sqrt(a)*(c/a)^(3/4)*ell
iptic_e(arcsin(x*(c/a)^(1/4)), -1) + (((C*a*c^2 - (5*A + 3*B)*c^3)*d + ((B
 + 3*C)*a*c^2 - 3*A*c^3)*e)*x^8 - 2*((C*a^2*c - (5*A + 3*B)*a*c^2)*d + ((B
 + 3*C)*a^2*c - 3*A*a*c^2)*e)*x^4 + (C*a^3 - (5*A + 3*B)*a^2*c)*d + ((B +
3*C)*a^3 - 3*A*a^2*c)*e)*sqrt(a)*(c/a)^(3/4)*elliptic_f(arcsin(x*(c/a)^(1/
4)), -1) + (3*(B*c^3*d - (C*a*c^2 - A*c^3)*e)*x^7 - (B*a*c^2*e + (C*a*c^2
- 5*A*c^3)*d)*x^5 - (5*B*a*c^2*d - (C*a^2*c - 5*A*a*c^2)*e)*x^3 - (B*a^2*c
*e + (C*a^2*c + 7*A*a*c^2)*d)*x)*sqrt(-c*x^4 + a))/(a^2*c^4*x^8 - 2*a^3*c^
3*x^4 + a^4*c^2)
```

Sympy [A] (verification not implemented)

Time = 66.99 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \frac{Adx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{5}{2}\\ \frac{5}{4}\end{matrix}\right| \frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Aex^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{5}{2}\\ \frac{7}{4}\end{matrix}\right| \frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bdx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{4}, \frac{5}{2}\\ \frac{7}{4}\end{matrix}\right| \frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bex^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, \frac{5}{2}\\ \frac{9}{4}\end{matrix}\right| \frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{Cdx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{5}{4}, \frac{5}{2}\\ \frac{9}{4}\end{matrix}\right| \frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{Cex^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{7}{4}, \frac{5}{2}\\ \frac{11}{4}\end{matrix}\right| \frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x**2+d)*(C*x**4+B*x**2+A)/(-c*x**4+a)**(5/2),x)
```

output
```
A*d*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*
a**(5/2)*gamma(5/4)) + A*e*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), c*x**
4*exp_polar(2*I*pi)/a)/(4*a**(5/2)*gamma(7/4)) + B*d*x**3*gamma(3/4)*hyper
((3/4, 5/2), (7/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a**(5/2)*gamma(7/4)) +
 B*e*x**5*gamma(5/4)*hyper((5/4, 5/2), (9/4,), c*x**4*exp_polar(2*I*pi)/a)
/(4*a**(5/2)*gamma(9/4)) + C*d*x**5*gamma(5/4)*hyper((5/4, 5/2), (9/4,), c
*x**4*exp_polar(2*I*pi)/a)/(4*a**(5/2)*gamma(9/4)) + C*e*x**7*gamma(7/4)*h
yper((7/4, 5/2), (11/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a**(5/2)*gamma(11
/4))
```

Maxima [F]

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)/(-c*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)}{(-cx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)/(-c*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \int \frac{(e\,x^2+d)\,(C\,x^4+B\,x^2+A)}{(a-c\,x^4)^{5/2}}\,dx$$

input
```
int(((d + e*x^2)*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2),x)
```

output
```
int(((d + e*x^2)*(A + B*x^2 + C*x^4))/(a - c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)(A+Bx^2+Cx^4)}{(a-cx^4)^{5/2}}\,dx = \frac{3\sqrt{-c\,x^4+a}\,bex+3\sqrt{-c\,x^4+a}\,cdx+5\sqrt{-c\,x^4+a}\,ce\,x^3-3\left(\int \frac{}{-c^3x}\right.}{}$$

input
```
int((e*x^2+d)*(C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x)
```

output
```
(3*sqrt(a - c*x**4)*b*e*x + 3*sqrt(a - c*x**4)*c*d*x + 5*sqrt(a - c*x**4)*
c*e*x**3 - 3*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 -
c**3*x**12),x)*a**3*b*e + 12*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 +
3*a*c**2*x**8 - c**3*x**12),x)*a**3*c*d + 6*int(sqrt(a - c*x**4)/(a**3 - 3
*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**2*b*c*e*x**4 - 24*int(sqr
t(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**2*
c**2*d*x**4 - 3*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8
 - c**3*x**12),x)*a*b*c**2*e*x**8 + 12*int(sqrt(a - c*x**4)/(a**3 - 3*a**2
*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a*c**3*d*x**8 + 15*int((sqrt(a -
c*x**4)*x**2)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**2*
b*c*d - 30*int((sqrt(a - c*x**4)*x**2)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x*
*8 - c**3*x**12),x)*a*b*c**2*d*x**4 + 15*int((sqrt(a - c*x**4)*x**2)/(a**3
 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*b*c**3*d*x**8)/(15*c*(a*
*2 - 2*a*c*x**4 + c**2*x**8))
```

3.56 $\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{5/2}}\,dx=\frac{x\left(A+\frac{aC}{c}+Bx^2\right)}{6a\,(a-cx^4)^{3/2}}+\frac{x\left(5A-\frac{aC}{c}+3Bx^2\right)}{12a^2\sqrt{a-cx^4}}-\frac{B\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{5/4}c^{3/4}\sqrt{a-cx^4}}+\frac{\left(3\sqrt{a}B\sqrt{c}+5Ac-aC\right)\sqrt{1-\frac{cx^4}{a}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}c^{5/4}\sqrt{a-cx^4}}$$

output
```
1/6*x*(A+a*C/c+B*x^2)/a/(-c*x^4+a)^(3/2)+1/12*x*(5*A-a*C/c+3*B*x^2)/a^2/(-
c*x^4+a)^(1/2)-1/4*B*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(5
/4)/c^(3/4)/(-c*x^4+a)^(1/2)+1/12*(3*a^(1/2)*B*c^(1/2)+5*A*c-a*C)*(1-c*x^4
/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(7/4)/c^(5/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{5/2}}\,dx=\frac{a^2Cx-5Ac^2x^5+acx(7A+Cx^4)-(-5Ac+aC)x(a-cx^4)\sqrt{1-\frac{cx^4}{a}}\,\text{Hypergeo}}{12a^2c}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a - c*x^4)^(5/2),x]
```

output
```
(a^2*C*x - 5*A*c^2*x^5 + a*c*x*(7*A + C*x^4) - (-5*A*c + a*C)*x*(a - c*x^4
)*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + 4*B*c*
x^3*(a - c*x^4)*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, (c*x^
4)/a])/(12*a^2*c*(a - c*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2397, 1493, 25, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{\int \frac{3Bcx^2+5Ac-aC}{(a-cx^4)^{3/2}}dx}{6ac}+\frac{x(aC+Ac+Bcx^2)}{6ac\,(a-cx^4)^{3/2}}$$

$$\Big\downarrow \text{1493}$$

$$\frac{\frac{x(-aC+5Ac+3Bcx^2)}{2a\sqrt{a-cx^4}}-\frac{\int -\frac{-3Bcx^2+5Ac-aC}{\sqrt{a-cx^4}}dx}{2a}}{6ac}+\frac{x(aC+Ac+Bcx^2)}{6ac\,(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{-3Bcx^2+5Ac-aC}{\sqrt{a-cx^4}}dx}{2a}+\frac{x(-aC+5Ac+3Bcx^2)}{2a\sqrt{a-cx^4}}}{6ac}+\frac{x(aC+Ac+Bcx^2)}{6ac(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 1513$$

$$\frac{\frac{(3\sqrt{a}B\sqrt{c}-aC+5Ac)\int \frac{1}{\sqrt{a-cx^4}}dx-3\sqrt{a}B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{2a}+\frac{x(-aC+5Ac+3Bcx^2)}{2a\sqrt{a-cx^4}}}{6ac}+\frac{x(aC+Ac+Bcx^2)}{6ac(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{(3\sqrt{a}B\sqrt{c}-aC+5Ac)\int \frac{1}{\sqrt{a-cx^4}}dx-3B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{2a}+\frac{x(-aC+5Ac+3Bcx^2)}{2a\sqrt{a-cx^4}}}{6ac}+\frac{x(aC+Ac+Bcx^2)}{6ac(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 765$$

$$\frac{\frac{\frac{\sqrt{1-\frac{cx^4}{a}}(3\sqrt{a}B\sqrt{c}-aC+5Ac)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-3B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{2a}+\frac{x(-aC+5Ac+3Bcx^2)}{2a\sqrt{a-cx^4}}}{6ac}+\frac{x(aC+Ac+Bcx^2)}{6ac(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 762$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(3\sqrt{a}B\sqrt{c}-aC+5Ac)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-3B\sqrt{c}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{2a}+\frac{x(-aC+5Ac+3Bcx^2)}{2a\sqrt{a-cx^4}}}{6ac}+\frac{x(aC+Ac+Bcx^2)}{6ac(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 1390$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(3\sqrt{a}B\sqrt{c}-aC+5Ac)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3B\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}}{2a}+\frac{x(-aC+5Ac+3Bcx^2)}{2a\sqrt{a-cx^4}}}{6ac}+\frac{x(aC+Ac+Bcx^2)}{6ac(a-cx^4)^{3/2}}$$

$$\Big\downarrow \ 1389$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(3\sqrt{a}B\sqrt{c}-aC+5Ac)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3\sqrt{a}B\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{a-cx^4}}}{2a}+\frac{x(-aC+5Ac+3Bcx^2)}{2a\sqrt{a-cx^4}}}{6ac}+\frac{x(aC+Ac+Bcx^2)}{6ac(a-cx^4)^{3/2}}$$

$\downarrow$ 327

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(3\sqrt{a}B\sqrt{c}-aC+5Ac)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3a^{3/4}B\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-cx^4}}}{2a}+\frac{x(-aC+5Ac+3Bcx^2)}{2a\sqrt{a-cx^4}}}{6ac}+\frac{x(aC+Ac+Bcx^2)}{6ac(a-cx^4)^{3/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a - c*x^4)^(5/2),x]
```

output
```
(x*(A*c + a*C + B*c*x^2))/(6*a*c*(a - c*x^4)^(3/2)) + ((x*(5*A*c - a*C + 3
*B*c*x^2))/(2*a*Sqrt[a - c*x^4]) + ((-3*a^(3/4)*B*c^(1/4)*Sqrt[1 - (c*x^4)
/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^4] + (a^(1/4)
*(3*Sqrt[a]*B*Sqrt[c] + 5*A*c - a*C)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4]))/(2*a))/(6*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.31

method	result
elliptic	$\frac{\left(\frac{B x^3}{6 a c^2}+\frac{(A c+C a) x}{6 a c^3}\right)\sqrt{-c x^4+a}}{\left(x^4-\frac{a}{c}\right)^2}+\frac{2c\left(\frac{B x^3}{8a^2 c}+\frac{(5Ac-Ca)x}{24a^2c^2}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{(5Ac-Ca)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{12a^2c\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}+\frac{B\sqrt{1-\cdots}}{\cdots}$
default	$A\left(\frac{x\sqrt{-c x^4+a}}{6a c^2\left(x^4-\frac{a}{c}\right)^2}+\frac{5x}{12a^2\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{5\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{12a^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}\right)+B\left(\frac{x^3\sqrt{-c x^4+a}}{6a c^2\left(x^4-\frac{a}{c}\right)^2}+\frac{\cdots}{4a^2\cdots}\right.$

input
```
int((C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/a/c^2*B*x^3+1/6/a/c^3*(A*c+C*a)*x)*(-c*x^4+a)^(1/2)/(x^4-a/c)^2+2*c*(
1/8/a^2/c*B*x^3+1/24/a^2/c^2*(5*A*c-C*a)*x)/(-(x^4-a/c)*c)^(1/2)+1/12/a^2/
c*(5*A*c-C*a)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(
1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/
2),I)+1/4/a^(3/2)*B/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*
(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/
2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{5/2}}\,dx = -\frac{3\,(Bc^3x^8-2\,Bac^2x^4+Ba^2c)\sqrt{a}\left(\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(\frac{c}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)+((Cac^2-(5\,A+3\,B)c^3)x^8-2\,(Ca^2c-$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*(B*c^3*x^8 - 2*B*a*c^2*x^4 + B*a^2*c)*sqrt(a)*(c/a)^(3/4)*ellipti
c_e(arcsin(x*(c/a)^(1/4)), -1) + ((C*a*c^2 - (5*A + 3*B)*c^3)*x^8 - 2*(C*a
^2*c - (5*A + 3*B)*a*c^2)*x^4 + C*a^3 - (5*A + 3*B)*a^2*c)*sqrt(a)*(c/a)^(
3/4)*elliptic_f(arcsin(x*(c/a)^(1/4)), -1) + (3*B*c^3*x^7 - 5*B*a*c^2*x^3
- (C*a*c^2 - 5*A*c^3)*x^5 - (C*a^2*c + 7*A*a*c^2)*x)*sqrt(-c*x^4 + a))/(a^
2*c^4*x^8 - 2*a^3*c^3*x^4 + a^4*c^2)
```

Sympy [A] (verification not implemented)

Time = 28.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{5/2}}\,dx = \frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/(-c*x**4+a)**(5/2),x)
```

output
```
A*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a*
*(5/2)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), c*x**4*ex
p_polar(2*I*pi)/a)/(4*a**(5/2)*gamma(7/4)) + C*x**5*gamma(5/4)*hyper((5/4,
 5/2), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*a**(5/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{(a - cx^4)^{5/2}}\, dx = \int \frac{Cx^4 + Bx^2 + A}{(-cx^4 + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(-c*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{(a - cx^4)^{5/2}}\, dx = \int \frac{Cx^4 + Bx^2 + A}{(-cx^4 + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(-c*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{5/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{(a-c\,x^4)^{5/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a - c*x^4)^(5/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a - c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{5/2}}\,dx=\frac{\sqrt{-c\,x^4+a}\,x+4\left(\int \frac{\sqrt{-c\,x^4+a}}{-c^3x^{12}+3a\,c^2x^8-3a^2c\,x^4+a^3}dx\right)a^3-8\left(\int \frac{\sqrt{-c\,x^4+a}}{-c^3x^{12}+3a\,c^2x^8-3a^2c\,x^4+a^3}\right.}{}$$

input
```
int((C*x^4+B*x^2+A)/(-c*x^4+a)^(5/2),x)
```

output
```
(sqrt(a - c*x**4)*x + 4*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x**4 + 3*a*c
**2*x**8 - c**3*x**12),x)*a**3 - 8*int(sqrt(a - c*x**4)/(a**3 - 3*a**2*c*x
**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a**2*c*x**4 + 4*int(sqrt(a - c*x**4)/
(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*a*c**2*x**8 + 5*int
((sqrt(a - c*x**4)*x**2)/(a**3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**1
2),x)*a**2*b - 10*int((sqrt(a - c*x**4)*x**2)/(a**3 - 3*a**2*c*x**4 + 3*a*
c**2*x**8 - c**3*x**12),x)*a*b*c*x**4 + 5*int((sqrt(a - c*x**4)*x**2)/(a**
3 - 3*a**2*c*x**4 + 3*a*c**2*x**8 - c**3*x**12),x)*b*c**2*x**8)/(5*(a**2 -
 2*a*c*x**4 + c**2*x**8))
```

3.57 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)(a-cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 592

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)(a-cx^4)^{5/2}}\,dx = \frac{x((Ac+aC)d-aBe+(Bcd-(Ac+aC)e)x^2)}{6a(cd^2-ae^2)(a-cx^4)^{3/2}}$$
$$+\frac{x(Acd(5cd^2-11ae^2)-a(Cd-Be)(cd^2+5ae^2)+3(Bcd(cd^2-3ae^2)-Ace(cd^2-3ae^2)+aCe(cd^2+\ldots}{12a^2(cd^2-ae^2)^2\sqrt{a-cx^4}}$$
$$-\frac{(Bcd(cd^2-3ae^2)-Ace(cd^2-3ae^2)+aCe(cd^2+ae^2))\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{5/4}c^{3/4}(cd^2-ae^2)^2\sqrt{a-cx^4}}$$
$$+\frac{(5Ac^2d^2-3a^2Ce^2+\sqrt{a}c^{3/2}d(3Bd+2Ae)+a^{3/2}\sqrt{c}e(2Cd-5Be)-ac(Cd^2-e(4Bd-9Ae)))\sqrt{1-\ldots}}{12a^{7/4}c^{3/4}(\sqrt{c}d+\sqrt{a}e)(cd^2-ae^2)\sqrt{a-cx^4}}$$
$$+\frac{\sqrt[4]{a}e^2(Cd^2-Bde+Ae^2)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d(cd^2-ae^2)^2\sqrt{a-cx^4}}$$

output
```
1/6*x*((A*c+C*a)*d-B*a*e+(B*c*d-(A*c+C*a)*e)*x^2)/a/(-a*e^2+c*d^2)/(-c*x^4
+a)^(3/2)+1/12*x*(A*c*d*(-11*a*e^2+5*c*d^2)-a*(-B*e+C*d)*(5*a*e^2+c*d^2)+3
*(B*c*d*(-3*a*e^2+c*d^2)-A*c*e*(-3*a*e^2+c*d^2)+a*C*e*(a*e^2+c*d^2))*x^2)/
a^2/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)-1/4*(B*c*d*(-3*a*e^2+c*d^2)-A*c*e*(-
3*a*e^2+c*d^2)+a*C*e*(a*e^2+c*d^2))*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/
a^(1/4),I)/a^(5/4)/c^(3/4)/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)+1/12*(5*A*c^2
*d^2-3*a^2*C*e^2+a^(1/2)*c^(3/2)*d*(2*A*e+3*B*d)+a^(3/2)*c^(1/2)*e*(-5*B*e
+2*C*d)-a*c*(C*d^2-e*(-9*A*e+4*B*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*
x/a^(1/4),I)/a^(7/4)/c^(3/4)/(c^(1/2)*d+a^(1/2)*e)/(-a*e^2+c*d^2)/(-c*x^4+
a)^(1/2)+a^(1/4)*e^2*(A*e^2-B*d*e+C*d^2)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1
/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4)/d/(-a*e^2+c*d^2)^2/(-c*x^4+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.32 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)(a-cx^4)^{5/2}}\,dx =$$

$$-\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{c}d(2a(cd^2-ae^2)\,x(Bcdx^2+Ac(d-ex^2)+a(Cd-Be-Cex^2))+x(a-cx^4)\,(3Bc^2d^3x^2+a^2}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)*(a - c*x^4)^(5/2)),x]
```

output
```
-1/12*(Sqrt[-(Sqrt[c]/Sqrt[a])]*Sqrt[c]*d*(2*a*(c*d^2 - a*e^2)*x*(B*c*d*x^
2 + A*c*(d - e*x^2) + a*(C*d - B*e - C*e*x^2)) + x*(a - c*x^4)*(3*B*c^2*d^
3*x^2 + a^2*e^2*(-5*C*d + 5*B*e + 3*C*e*x^2) + a*c*d*(B*e*(d - 9*e*x^2) -
C*d*(d - 3*e*x^2)) + A*c*(c*d^2*(5*d - 3*e*x^2) + a*e^2*(-11*d + 9*e*x^2))
)) + I*(a - c*x^4)*Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*d*(B*c*d*(c*d^2 - 3*a*e^
2) + a*C*e*(c*d^2 + a*e^2) + A*c*e*(-(c*d^2) + 3*a*e^2))*EllipticE[I*ArcSi
nh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - d*(Sqrt[c]*d - Sqrt[a]*e)*(5*A*c^2*d
^2 - 3*a^2*C*e^2 + Sqrt[a]*c^(3/2)*d*(3*B*d + 2*A*e) + a^(3/2)*Sqrt[c]*e*(
2*C*d - 5*B*e) - a*c*(C*d^2 + e*(-4*B*d + 9*A*e)))*EllipticF[I*ArcSinh[Sqr
t[-(Sqrt[c]/Sqrt[a])]*x], -1] - 12*a^2*Sqrt[c]*e^2*(C*d^2 + e*(-(B*d) + A*
e))*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a
])]*x], -1]))/(a^(5/2)*(-(Sqrt[c]/Sqrt[a]))^(3/2)*d*(c*d^2 - a*e^2)^2*(a -
 c*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^{5/2}(d+ex^2)}dx$$

$$\Big\downarrow \ 2259$$

$$\int \left(\frac{e^2(Ae^2-Bde+Cd^2)}{\sqrt{a-cx^4}(d+ex^2)(cd^2-ae^2)^2}+\frac{c(d-ex^2)(Ae^2-Bde+Cd^2)}{\sqrt{a-cx^4}(cx^4-a)(cd^2-ae^2)^2}+\frac{x^2(Bcd-e(aC+Ac))-aBe+aCd}{(a-cx^4)^{5/2}(cd^2-ae^2)}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)\left(\frac{5\sqrt{c}(-aBe+aCd+Acd)}{\sqrt{a}}-3e(aC+Ac)+3Bcd\right)}{12a^{5/4}c^{3/4}\sqrt{a-cx^4}\,(cd^2-ae^2)}-\frac{\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)(-aCe-Ace+Bcd)}{4a^{5/4}c^{3/4}\sqrt{a-cx^4}\,(cd^2-ae^2)}-\frac{\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(\sqrt{c}d-\sqrt{a}e)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)(Ae^2-Bde+Cd^2)}{2a^{3/4}\sqrt{a-cx^4}\,(cd^2-ae^2)^2}+\frac{x(3x^2(Bcd-e(aC+Ac))+5(-aBe+aCd+Acd))}{12a^2\sqrt{a-cx^4}\,(cd^2-ae^2)}-\frac{\sqrt[4]{c}e\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)(Ae^2-Bde+Cd^2)}{2\sqrt[4]{a}\sqrt{a-cx^4}\,(cd^2-ae^2)^2}+\frac{\sqrt[4]{a}e^2\sqrt{1-\frac{cx^4}{a}}(Ae^2-Bde+Cd^2)\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}\,(cd^2-ae^2)^2}-\frac{cx(d-ex^2)\,(Ae^2-Bde+Cd^2)}{2a\sqrt{a-cx^4}\,(cd^2-ae^2)^2}+\frac{x(x^2(Bcd-e(aC+Ac))-aBe+aCd+Acd)}{6a\,(a-cx^4)^{3/2}\,(cd^2-ae^2)}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)*(a - c*x^4)^(5/2)),x]
```

output
```
(x*(A*c*d + a*C*d - a*B*e + (B*c*d - (A*c + a*C)*e)*x^2))/(6*a*(c*d^2 - a*
e^2)*(a - c*x^4)^(3/2)) - (c*(C*d^2 - B*d*e + A*e^2)*x*(d - e*x^2))/(2*a*(
c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) + (x*(5*(A*c*d + a*C*d - a*B*e) + 3*(B*c
*d - (A*c + a*C)*e)*x^2))/(12*a^2*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) - ((B*c
*d - A*c*e - a*C*e)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/
4)], -1])/(4*a^(5/4)*c^(3/4)*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) - (c^(1/4)*e
*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/
a^(1/4)], -1])/(2*a^(1/4)*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) - (c^(1/4)*(S
qrt[c]*d - Sqrt[a]*e)*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*Elliptic
F[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*(c*d^2 - a*e^2)^2*Sqrt[a -
c*x^4]) + ((3*B*c*d - 3*(A*c + a*C)*e + (5*Sqrt[c]*(A*c*d + a*C*d - a*B*e)
)/Sqrt[a])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(12*a^(5/4)*c^(3/4)*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) + (a^(1/4)*e^2*(C*d^
2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d
)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*(c*d^2 - a*e^2)^2*Sqrt[a
- c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1220 vs. $2(528) = 1056$.

Time = 0.95 (sec) , antiderivative size = 1221, normalized size of antiderivative = 2.06

method	result	size
default	Expression too large to display	1221
elliptic	Expression too large to display	2188

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/e^2*(B*e*(1/6/a*x/c^2*(-c*x^4+a)^(1/2)/(x^4-a/c)^2+5/12/a^2*x/(-(x^4-a/c
)*c)^(1/2)+5/12/a^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*
(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2
))^(1/2),I))+C*e*(1/6/a*x^3/c^2*(-c*x^4+a)^(1/2)/(x^4-a/c)^2+1/4/a^2*x^3/(
-(x^4-a/c)*c)^(1/2)+1/4/a^(3/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(
1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(Ellipt
icF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))-
C*d*(1/6/a*x/c^2*(-c*x^4+a)^(1/2)/(x^4-a/c)^2+5/12/a^2*x/(-(x^4-a/c)*c)^(1
/2)+5/12/a^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1
/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2
),I)))+(A*e^2-B*d*e+C*d^2)/e^2*((1/6/c/a*e/(a*e^2-c*d^2)*x^3-1/6/c/a*d/(a*
e^2-c*d^2)*x)*(-c*x^4+a)^(1/2)/(x^4-a/c)^2+2*c*(1/8*e*(3*a*e^2-c*d^2)/a^2/
(a*e^2-c*d^2)^2*x^3-1/24*d*(11*a*e^2-5*c*d^2)/a^2/(a*e^2-c*d^2)^2*x)/(-(x^
4-a/c)*c)^(1/2)-11/12*c*d/a/(a*e^2-c*d^2)^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(
1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ell
ipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)*e^2+5/12*c^2*d^3/a^2/(a*e^2-c*d^2)^2/(
c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2
))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+3/4*c^(1/
2)*e^3/a^(1/2)/(a*e^2-c*d^2)^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1
/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)(a - cx^4)^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(5/2),x, algorithm="fricas"
)
```

output

```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)\left(a-cx^4\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)/(-c*x**4+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)\left(a-cx^4\right)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(-cx^4+a\right)^{\frac{5}{2}}\left(ex^2+d\right)}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-c*x^4 + a)^(5/2)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)\left(a-cx^4\right)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(-cx^4+a\right)^{\frac{5}{2}}\left(ex^2+d\right)}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-c*x^4 + a)^(5/2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)\left(a-cx^4\right)^{5/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\left(a-c\,x^4\right)^{5/2}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(5/2)*(d + e*x^2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(5/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)\left(a-cx^4\right)^{5/2}}\,dx=\left(\int \frac{\sqrt{-c\,x^4+a}}{-c^3e\,x^{14}-c^3d\,x^{12}+3a\,c^2e\,x^{10}+3a\,c^2d\,x^8-3a^2ce\,x^6-3a^2cd\,x^4+a^3e}\right.$$
$$+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^4}{-c^3e\,x^{14}-c^3d\,x^{12}+3a\,c^2e\,x^{10}+3a\,c^2d\,x^8-3a^2ce\,x^6-3a^2cd\,x^4+a^3e\,x^2+a^3d}dx\right)c$$
$$+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c^3e\,x^{14}-c^3d\,x^{12}+3a\,c^2e\,x^{10}+3a\,c^2d\,x^8-3a^2ce\,x^6-3a^2cd\,x^4+a^3e\,x^2+a^3d}dx\right)b$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)/(-c*x^4+a)^(5/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a**3*d + a**3*e*x**2 - 3*a**2*c*d*x**4 - 3*a**2*c*e*
x**6 + 3*a*c**2*d*x**8 + 3*a*c**2*e*x**10 - c**3*d*x**12 - c**3*e*x**14),x
)*a + int((sqrt(a - c*x**4)*x**4)/(a**3*d + a**3*e*x**2 - 3*a**2*c*d*x**4
- 3*a**2*c*e*x**6 + 3*a*c**2*d*x**8 + 3*a*c**2*e*x**10 - c**3*d*x**12 - c*
*3*e*x**14),x)*c + int((sqrt(a - c*x**4)*x**2)/(a**3*d + a**3*e*x**2 - 3*a
**2*c*d*x**4 - 3*a**2*c*e*x**6 + 3*a*c**2*d*x**8 + 3*a*c**2*e*x**10 - c**3
*d*x**12 - c**3*e*x**14),x)*b
```

3.58 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2(a-cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 889

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2(a-cx^4)^{5/2}}\,dx = \frac{x((Ac+aC)d-aBe+(Bcd-(Ac+aC)e)x^2)}{6a(cd^2-ae^2)(d+ex^2)(a-cx^4)^{3/2}}$$
$$-\frac{e(Bcd^3-2Acd^2e-5aCd^2e+4aBde^2-3aAe^3)x}{6ad(cd^2-ae^2)^2(d+ex^2)\sqrt{a-cx^4}}$$
$$+\frac{x(d(Ac(5c^2d^4-18acd^2e^2-17a^2e^4)-a(5a^2Ce^4+4acde^2(6Cd-7Be)+c^2d^3(Cd-2Be)))+3c(B(c^2d^5 \cdots}{12a^2d(cd^2-ae^2)^3\sqrt{a-c \cdots}}$$
$$-\frac{\sqrt[4]{c}(B(c^2d^5-6acd^3e^2-5a^2de^4)+2(aCd^2e(cd^2+4ae^2)-A(c^2d^4e-5acd^2e^3-a^2e^5)))\sqrt{1-\frac{cx^4}{a}}E\left(\mathrm{arc}\cdots\right.}{4a^{5/4}d(cd^2-ae^2)^3\sqrt{a-cx^4}}$$
$$+\frac{(5Ac^{5/2}d^4+5a^{5/2}Cde^3+\sqrt{a}c^2d^3(3Bd-Ae)-ac^{3/2}d^2(Cd^2-e(5Bd-19Ae))+a^{3/2}cde(5Cd^2-e(13B\cdots}{12a^{7/4}\sqrt[4]{c}d\left(\sqrt{c}d-\sqrt{a}e\right)^2\left(\sqrt{c}d\cdots\right.}$$
$$+\frac{\sqrt[4]{a}e^2(cd^2(7Cd^2-e(9Bd-11Ae))+ae^2(3Cd^2-e(Bd+Ae)))\sqrt{1-\frac{cx^4}{a}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}\cdots}{\sqrt[4]{a}\cdots}\right.\right.}{2\sqrt[4]{c}d^2(cd^2-ae^2)^3\sqrt{a-cx^4}}$$

output
```
1/6*x*((A*c+C*a)*d-B*a*e+(B*c*d-(A*c+C*a)*e)*x^2)/a/(-a*e^2+c*d^2)/(e*x^2+
d)/(-c*x^4+a)^(3/2)-1/6*e*(-3*A*a*e^3-2*A*c*d^2*e+4*B*a*d*e^2+B*c*d^3-5*C*
a*d^2*e)*x/a/d/(-a*e^2+c*d^2)^2/(e*x^2+d)/(-c*x^4+a)^(1/2)+1/12*x*(d*(A*c*
(-17*a^2*e^4-18*a*c*d^2*e^2+5*c^2*d^4)-a*(5*a^2*C*e^4+4*a*c*d*e^2*(-7*B*e+
6*C*d)+c^2*d^3*(-2*B*e+C*d)))+3*c*(B*(-5*a^2*d*e^4-6*a*c*d^3*e^2+c^2*d^5)+
2*a*C*d^2*e*(4*a*e^2+c*d^2)-2*A*(-a^2*e^5-5*a*c*d^2*e^3+c^2*d^4*e))*x^2)/a
^2/d/(-a*e^2+c*d^2)^3/(-c*x^4+a)^(1/2)-1/4*c^(1/4)*(B*(-5*a^2*d*e^4-6*a*c*
d^3*e^2+c^2*d^5)+2*a*C*d^2*e*(4*a*e^2+c*d^2)-2*A*(-a^2*e^5-5*a*c*d^2*e^3+c
^2*d^4*e))*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(5/4)/d/(-a*
e^2+c*d^2)^3/(-c*x^4+a)^(1/2)+1/12*(5*A*c^(5/2)*d^4+5*a^(5/2)*C*d*e^3+a^(1
/2)*c^2*d^3*(-A*e+3*B*d)-a*c^(3/2)*d^2*(C*d^2-e*(-19*A*e+5*B*d))+a^(3/2)*c
*d*e*(5*C*d^2-e*(-11*A*e+13*B*d))-a^2*c^(1/2)*e^2*(19*C*d^2-3*e*(-2*A*e+5*
B*d)))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(7/4)/c^(1/4)/d/
(c^(1/2)*d-a^(1/2)*e)^2/(c^(1/2)*d+a^(1/2)*e)^3/(-c*x^4+a)^(1/2)+1/2*a^(1/
4)*e^2*(c*d^2*(7*C*d^2-e*(-11*A*e+9*B*d))+a*e^2*(3*C*d^2-e*(A*e+B*d)))*(1-
c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/c^(1/4
)/d^2/(-a*e^2+c*d^2)^3/(-c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.69 (sec) , antiderivative size = 782, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2(a-cx^4)^{5/2}}\,dx = \frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}dx\left(-6a^2e^4(Cd^2+e(-Bd+Ae))(a-cx^4)^2+2ad(cd^2-ae^2)(d+\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^2*(a - c*x^4)^(5/2)),x]
```

output
```
(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*x*(-6*a^2*e^4*(C*d^2 + e*(-(B*d) + A*e))*(a -
c*x^4)^2 + 2*a*d*(c*d^2 - a*e^2)*(d + e*x^2)*(a^2*C*e^2 + B*c^2*d^2*x^2 +
A*c*(a*e^2 + c*d*(d - 2*e*x^2)) + a*c*(C*d*(d - 2*e*x^2) + B*e*(-2*d + e*x
^2))) - d*(d + e*x^2)*(a - c*x^4)*(5*a^3*C*e^4 - 3*B*c^3*d^4*x^2 + A*c*(11
*a^2*e^4 + 6*a*c*d*e^2*(3*d - 5*e*x^2) + c^2*d^3*(-5*d + 6*e*x^2)) + a^2*c
*e^2*(18*C*d*(d - e*x^2) + B*e*(-22*d + 9*e*x^2)) + a*c^2*d^2*(C*d*(d - 6*
e*x^2) + 2*B*e*(-d + 9*e*x^2)))) + I*(d + e*x^2)*(a - c*x^4)*Sqrt[1 - (c*x
^4)/a]*(3*Sqrt[a]*Sqrt[c]*d*(2*a*C*d^2*e*(c*d^2 + 4*a*e^2) + B*(c^2*d^5 -
6*a*c*d^3*e^2 - 5*a^2*d*e^4) + 2*A*(-(c^2*d^4*e) + 5*a*c*d^2*e^3 + a^2*e^5
))*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + d*(-(Sqrt[c]*d)
+ Sqrt[a]*e)*(5*A*c^(5/2)*d^4 + 5*a^(5/2)*C*d*e^3 + Sqrt[a]*c^2*d^3*(3*B*d
 - A*e) + a^2*Sqrt[c]*e^2*(-19*C*d^2 + 3*e*(5*B*d - 2*A*e)) + a^(3/2)*c*d*
e*(5*C*d^2 + e*(-13*B*d + 11*A*e)) - a*c^(3/2)*d^2*(C*d^2 + e*(-5*B*d + 19
*A*e)))*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + 6*a^2*e^2*(
c*(-7*C*d^4 + d^2*e*(9*B*d - 11*A*e)) + a*e^2*(-3*C*d^2 + e*(B*d + A*e)))*
EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*
x], -1]))/(12*a^2*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*(c*d^2 - a*e^2)^3*(d + e*x^
2)*(a - c*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 2.58 (sec) , antiderivative size = 1340, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + Bx^2 + Cx^4}{(a - cx^4)^{5/2} (d + ex^2)^2} \, dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{e^2 (Ae^2 - Bde + Cd^2)}{\sqrt{a - cx^4} (d + ex^2)^2 (cd^2 - ae^2)^2} + \frac{-cx^2 (2de(aC + Ac) - B(ae^2 + cd^2)) + Ac(ae^2 + cd^2) + a(aCe^2 + cd}{(a - cx^4)^{5/2} (cd^2 - ae^2)^2} \right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(Cd^2-Bed+Ae^2)\,x\sqrt{a-cx^4}e^4}{2d\,(cd^2-ae^2)^3\,(ex^2+d)}-$$
$$\frac{a^{3/4}\sqrt[4]{c}(Cd^2-Bed+Ae^2)\,\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)e^3}{2d\,(cd^2-ae^2)^3\,\sqrt{a-cx^4}}-$$
$$\frac{\sqrt[4]{a}\sqrt[4]{c}(Cd^2-Bed+Ae^2)\,\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{2d\,(\sqrt{c}d+\sqrt{a}e)\,(cd^2-ae^2)^2\,\sqrt{a-cx^4}}+$$
$$\frac{\sqrt[4]{a}(3cd^2-ae^2)\,(Cd^2-Bed+Ae^2)\,\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{2\sqrt[4]{c}d^2\,(cd^2-ae^2)^3\,\sqrt{a-cx^4}}+$$
$$\frac{\sqrt[4]{a}(2cCd^3-ce(3Bd-4Ae)d+ae^2(2Cd-Be))\,\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)e^2}{\sqrt[4]{c}d\,(cd^2-ae^2)^3\,\sqrt{a-cx^4}}-$$
$$\frac{\sqrt[4]{c}(2cCd^3-ce(3Bd-4Ae)d+ae^2(2Cd-Be))\,\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)e}{2\sqrt[4]{a}\,(cd^2-ae^2)^3\,\sqrt{a-cx^4}}+$$
$$\frac{\sqrt[4]{c}(2(Ac+aC)de-B(cd^2+ae^2))\,\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{5/4}\,(cd^2-ae^2)^2\,\sqrt{a-cx^4}}+$$
$$\frac{\left(\frac{(Ac+aC)(5cd^2-6\sqrt{a}\sqrt{c}ed+5ae^2)}{\sqrt{a}}+B(3c^{3/2}d^2-10\sqrt{a}ced+3a\sqrt{c}e^2)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{5/4}\sqrt[4]{c}\,(cd^2-ae^2)^2\,\sqrt{a-cx^4}}-$$
$$\frac{\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\,(a(2Cd-Be)e^2-\sqrt{a}\sqrt{c}(Cd^2-e(Bd-Ae))\,e+c(Cd^3-de(2Bd-3Ae)))\,\sqrt{1-\frac{cx^4}{a}}\,\text{Elliptic}}{2a^{3/4}\,(cd^2-ae^2)^3\,\sqrt{a-cx^4}}$$
$$\frac{cx(a(3Cd^2-e(2Bd-Ae))\,e^2-(2cCd^3-ce(3Bd-4Ae)d+ae^2(2Cd-Be))\,x^2e+c(Cd^4-d^2e(2Bd-3Ae)}{2a\,(cd^2-ae^2)^3\,\sqrt{a-cx^4}}$$
$$\frac{x(5(Ac(cd^2+ae^2)+a(aCe^2+cd(Cd-2Be)))-3c(2(Ac+aC)de-B(cd^2+ae^2))\,x^2)}{12a^2\,(cd^2-ae^2)^2\,\sqrt{a-cx^4}}+$$
$$\frac{x(-c(2(Ac+aC)de-B(cd^2+ae^2))\,x^2+Ac(cd^2+ae^2)+a(aCe^2+cd(Cd-2Be)))}{6a\,(cd^2-ae^2)^2\,(a-cx^4)^{3/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^2*(a - c*x^4)^(5/2)),x]
```

output
```
(x*(A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d - 2*B*e)) - c*(2*(A*c + a*
C)*d*e - B*(c*d^2 + a*e^2))*x^2))/(6*a*(c*d^2 - a*e^2)^2*(a - c*x^4)^(3/2)
) - (c*x*(c*(C*d^4 - d^2*e*(2*B*d - 3*A*e)) + a*e^2*(3*C*d^2 - e*(2*B*d -
A*e)) - e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) + a*e^2*(2*C*d - B*e))*x^2))/
(2*a*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4]) + (x*(5*(A*c*(c*d^2 + a*e^2) + a*(
a*C*e^2 + c*d*(C*d - 2*B*e))) - 3*c*(2*(A*c + a*C)*d*e - B*(c*d^2 + a*e^2)
)*x^2))/(12*a^2*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) - (e^4*(C*d^2 - B*d*e +
 A*e^2)*x*Sqrt[a - c*x^4])/(2*d*(c*d^2 - a*e^2)^3*(d + e*x^2)) - (a^(3/4)*
c^(1/4)*e^3*(C*d^2 - B*d*e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(
c^(1/4)*x)/a^(1/4)], -1])/(2*d*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4]) - (c^(1/
4)*e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) + a*e^2*(2*C*d - B*e))*Sqrt[1 - (c
*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(1/4)*(c*d^2 - a
*e^2)^3*Sqrt[a - c*x^4]) + (c^(1/4)*(2*(A*c + a*C)*d*e - B*(c*d^2 + a*e^2)
)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(4*a^(5/
4)*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*e^2*(C*d^2 - B*d*
e + A*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(2*d*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) + ((((A*c
 + a*C)*(5*c*d^2 - 6*Sqrt[a]*Sqrt[c]*d*e + 5*a*e^2))/Sqrt[a] + B*(3*c^(3/2
)*d^2 - 10*Sqrt[a]*c*d*e + 3*a*Sqrt[c]*e^2))*Sqrt[1 - (c*x^4)/a]*EllipticF
[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(12*a^(5/4)*c^(1/4)*(c*d^2 - a*e^2)^...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2455 vs. 2(807) = 1614.

Time = 1.00 (sec) , antiderivative size = 2456, normalized size of antiderivative = 2.76

method	result	size
default	Expression too large to display	2456
elliptic	Expression too large to display	4267

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
C/e^2*(1/6/a*x/c^2*(-c*x^4+a)^(1/2)/(x^4-a/c)^2+5/12/a^2*x/(-(x^4-a/c)*c)^
(1/2)+5/12/a^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^
(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1
/2),I))+1/e^2*(B*e-2*C*d)*((1/6/c/a*e/(a*e^2-c*d^2)*x^3-1/6/c/a*d/(a*e^2-c
*d^2)*x)*(-c*x^4+a)^(1/2)/(x^4-a/c)^2+2*c*(1/8*e*(3*a*e^2-c*d^2)/a^2/(a*e^
2-c*d^2)^2*x^3-1/24*d*(11*a*e^2-5*c*d^2)/a^2/(a*e^2-c*d^2)^2*x)/(-(x^4-a/c
)*c)^(1/2)-11/12*c*d/a/(a*e^2-c*d^2)^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*
x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
F(x*(c^(1/2)/a^(1/2))^(1/2),I)*e^2+5/12*c^2*d^3/a^2/(a*e^2-c*d^2)^2/(c^(1/
2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1
/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+3/4*c^(1/2)*e^
3/a^(1/2)/(a*e^2-c*d^2)^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^
(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/
a^(1/2))^(1/2),I)-1/4*c^(3/2)*e/a^(3/2)/(a*e^2-c*d^2)^2/(c^(1/2)/a^(1/2))^
(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+
a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)*d^2-3/4*c^(1/2)*e^3/a^(1/2
)/(a*e^2-c*d^2)^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1
+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(c^(1/2)/a^(1/2))
^(1/2),I)+1/4*c^(3/2)*e/a^(3/2)/(a*e^2-c*d^2)^2/(c^(1/2)/a^(1/2))^(1/2)*(1
-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\,(a-cx^4)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(5/2),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\,(a-cx^4)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**2/(-c*x**4+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2\,(a-cx^4)^{5/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(-cx^4+a)^{\frac{5}{2}}(ex^2+d)^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-c*x^4 + a)^(5/2)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2(a-cx^4)^{5/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(-cx^4+a)^{\frac{5}{2}}(ex^2+d)^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-c*x^4 + a)^(5/2)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2(a-cx^4)^{5/2}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(a-c\,x^4)^{5/2}\,(e\,x^2+d)^2}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(5/2)*(d + e*x^2)^2),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(5/2)*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^2(a-cx^4)^{5/2}}\,dx = \left(\int \frac{\sqrt{-c\,x^4}}{-c^3e^2x^{16}-2c^3de\,x^{14}+3a\,c^2e^2x^{12}-c^3d^2x^{12}+6a\,c^2de\,x^{10}-3a^2c\,e^2x^8}\right.$$
$$+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^4}{-c^3e^2x^{16}-2c^3de\,x^{14}+3a\,c^2e^2x^{12}-c^3d^2x^{12}+6a\,c^2de\,x^{10}-3a^2c\,e^2x^8+3a\,c^2d^2x^8-6a^2cde\,x^6+a^3}\right.$$
$$+\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-c^3e^2x^{16}-2c^3de\,x^{14}+3a\,c^2e^2x^{12}-c^3d^2x^{12}+6a\,c^2de\,x^{10}-3a^2c\,e^2x^8+3a\,c^2d^2x^8-6a^2cde\,x^6+a^3}\right.$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^2/(-c*x^4+a)^(5/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a**3*d**2 + 2*a**3*d*e*x**2 + a**3*e**2*x**4 - 3*a**
2*c*d**2*x**4 - 6*a**2*c*d*e*x**6 - 3*a**2*c*e**2*x**8 + 3*a*c**2*d**2*x**
8 + 6*a*c**2*d*e*x**10 + 3*a*c**2*e**2*x**12 - c**3*d**2*x**12 - 2*c**3*d*
e*x**14 - c**3*e**2*x**16),x)*a + int((sqrt(a - c*x**4)*x**4)/(a**3*d**2 +
 2*a**3*d*e*x**2 + a**3*e**2*x**4 - 3*a**2*c*d**2*x**4 - 6*a**2*c*d*e*x**6
 - 3*a**2*c*e**2*x**8 + 3*a*c**2*d**2*x**8 + 6*a*c**2*d*e*x**10 + 3*a*c**2
*e**2*x**12 - c**3*d**2*x**12 - 2*c**3*d*e*x**14 - c**3*e**2*x**16),x)*c +
 int((sqrt(a - c*x**4)*x**2)/(a**3*d**2 + 2*a**3*d*e*x**2 + a**3*e**2*x**4
 - 3*a**2*c*d**2*x**4 - 6*a**2*c*d*e*x**6 - 3*a**2*c*e**2*x**8 + 3*a*c**2*
d**2*x**8 + 6*a*c**2*d*e*x**10 + 3*a*c**2*e**2*x**12 - c**3*d**2*x**12 - 2
*c**3*d*e*x**14 - c**3*e**2*x**16),x)*b
```

3.59 $\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 113

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx=\frac{\left(\sqrt{a}-\sqrt{c}x^2\right)\text{EllipticPi}\left(1-\frac{\sqrt{a}e}{\sqrt{c}d},\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),2\right)}{\sqrt[4]{a}\sqrt[4]{c}d\sqrt{\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}+\sqrt{c}x^2}}\sqrt{a-cx^4}}$$

output
```
(a^(1/2)-c^(1/2)*x^2)*EllipticPi(c^(1/4)*x/a^(1/4)/(1+c^(1/2)*x^2/a^(1/2))
^(1/2),1-a^(1/2)*e/c^(1/2)/d,2^(1/2))/a^(1/4)/c^(1/4)/d/((a^(1/2)-c^(1/2)*
x^2)/(a^(1/2)+c^(1/2)*x^2))^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.32

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx$$
$$=\frac{i\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{c}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)+\left(-\sqrt{c}d+\sqrt{a}e\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},i\text{arcsin}\right.\right.}{\sqrt{c}de\sqrt{a-cx^4}}$$

input
```
Integrate[(1 + (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(I*Sqrt[-(Sqrt[c]/Sqrt[a])]*Sqrt[1 - (c*x^4)/a]*(Sqrt[c]*d*EllipticF[I*Arc
Sinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (-(Sqrt[c]*d) + Sqrt[a]*e)*Ellipti
cPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]
))/(Sqrt[c]*d*e*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1787, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}{\sqrt{a-cx^4}\,(d+ex^2)}dx$$

$$\Big\downarrow \text{ 1787}$$

$$\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}\sqrt{a-\sqrt{a}\sqrt{c}x^2}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{a-\sqrt{a}\sqrt{c}x^2}(ex^2+d)}dx}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 414}$$

$$\frac{\left(a-\sqrt{a}\sqrt{c}x^2\right)\text{EllipticPi}\left(1-\frac{\sqrt{a}e}{\sqrt{c}d},\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),2\right)}{a^{3/4}\sqrt[4]{c}d\sqrt{\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}+\sqrt{c}x^2}}\sqrt{a-cx^4}}$$

input
```
Int[(1 + (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
((a - Sqrt[a]*Sqrt[c]*x^2)*EllipticPi[1 - (Sqrt[a]*e)/(Sqrt[c]*d), ArcTan[
(c^(1/4)*x)/a^(1/4)], 2])/(a^(3/4)*c^(1/4)*d*Sqrt[(Sqrt[a] - Sqrt[c]*x^2)/
(Sqrt[a] + Sqrt[c]*x^2)]*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1787
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d +
e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])   Int[(d + e*x^n)^(p + q)
*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, c, d, e, f, g, n, p
, q, r}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.65

method	result
default	$\frac{\frac{\sqrt{c}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{ed\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}}{\sqrt{a}}$
elliptic	$\frac{\sqrt{(-c\,x^{4}+a)ac}\,\left(\sqrt{a}+\sqrt{c}\,x^{2}\right)\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{c\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-a\,c^{2}x^{4}+a^{2}c}}\right.}{\left(c\,x^{2}\sqrt{-c\,x^{4}+a}+\sqrt{(-c\,x^{4}+a)ac}\right)\sqrt{a}}$

input
```
int((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/a^(1/2)*(c^(1/2)/e/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)
*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/
2))^(1/2),I)-(c^(1/2)*d-a^(1/2)*e)/e/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*
x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
Pi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)
/(c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \frac{\int \frac{\sqrt{a}}{d\sqrt{a-cx^4}+ex^2\sqrt{a-cx^4}}\,dx + \int \frac{\sqrt{c}x^2}{d\sqrt{a-cx^4}+ex^2\sqrt{a-cx^4}}\,dx}{\sqrt{a}}$$

input
```
integrate((1+c**(1/2)*x**2/a**(1/2))/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

output
```
(Integral(sqrt(a)/(d*sqrt(a - c*x**4) + e*x**2*sqrt(a - c*x**4)), x) + Int
egral(sqrt(c)*x**2/(d*sqrt(a - c*x**4) + e*x**2*sqrt(a - c*x**4)), x))/sqr
t(a)
```

Maxima [F]

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((sqrt(c)*x^2/sqrt(a) + 1)/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((sqrt(c)*x^2/sqrt(a) + 1)/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)}\,dx$$

input
```
int(((c^(1/2)*x^2)/a^(1/2) + 1)/((a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int(((c^(1/2)*x^2)/a^(1/2) + 1)/((a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx$$
$$= \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)a}{a}$$

input
```
int((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 -
c*e*x**6),x) + int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6)
,x)*a)/a
```

3.60 $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 114

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \frac{a\left(\frac{c}{a}\right)^{3/4}\left(1-\sqrt{\frac{c}{a}}x^2\right)\text{EllipticPi}\left(1-\frac{e}{\sqrt{\frac{c}{a}}d},\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),2\right)}{cd\sqrt{\frac{1-\sqrt{\frac{c}{a}}x^2}{1+\sqrt{\frac{c}{a}}x^2}}\sqrt{a-cx^4}}$$

output
```
a*(c/a)^(3/4)*(1-(c/a)^(1/2)*x^2)*EllipticPi((c/a)^(1/4)*x/(1+(c/a)^(1/2)*
x^2)^(1/2),1-e/(c/a)^(1/2)/d,2^(1/2))/c/d/((1-(c/a)^(1/2)*x^2)/(1+(c/a)^(1
/2)*x^2))^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.28

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx =$$
$$-\frac{i\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{\frac{c}{a}}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)+\left(-\sqrt{\frac{c}{a}}d+e\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},i\text{arcsinh}\left(\sqrt{-}\right.\right.\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}de\sqrt{a-cx^4}}$$

input
```
Integrate[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
((-I)*Sqrt[1 - (c*x^4)/a]*(Sqrt[c/a]*d*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/
Sqrt[a])]*x], -1] + (-(Sqrt[c/a]*d) + e)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]
*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(Sqrt[-(Sqrt[c]/Sqrt[a]
)]*d*e*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {1787, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{a-cx^4}\,(d+ex^2)}\,dx$$
$$\Big\downarrow \text{1787}$$
$$\frac{\sqrt{x^2\sqrt{\frac{c}{a}}+1}\sqrt{a-ax^2\sqrt{\frac{c}{a}}}\int \frac{\sqrt{\sqrt{\frac{c}{a}}x^2+1}}{\sqrt{a-a\sqrt{\frac{c}{a}}x^2}(ex^2+d)}dx}{\sqrt{a-cx^4}}$$
$$\Big\downarrow \text{414}$$

$$\frac{\left(\frac{c}{a}\right)^{3/4}\left(a-ax^2\sqrt{\frac{c}{a}}\right)\text{EllipticPi}\left(1-\frac{e}{\sqrt{\frac{c}{a}}d},\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),2\right)}{cd\sqrt{\frac{1-x^2\sqrt{\frac{c}{a}}}{x^2\sqrt{\frac{c}{a}}+1}}\sqrt{a-cx^4}}$$

input
```
Int[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
((c/a)^(3/4)*(a - a*Sqrt[c/a]*x^2)*EllipticPi[1 - e/(Sqrt[c/a]*d), ArcTan[
(c/a)^(1/4)*x], 2])/(c*d*Sqrt[(1 - Sqrt[c/a]*x^2)/(1 + Sqrt[c/a]*x^2)]*Sqr
t[a - c*x^4])
```

Defintions of rubi rules used

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1787
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d +
e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])   Int[(d + e*x^n)^(p + q)
*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, c, d, e, f, g, n, p
, q, r}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.65

method	result
default	$\frac{\sqrt{\frac{c}{a}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{\left(\sqrt{\frac{c}{a}}\,d-e\right)\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{ed\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}$
elliptic	$\frac{\sqrt{\frac{\left(-c\,x^{4}+a\right)c}{a}}\,a\left(1+\sqrt{\frac{c}{a}}\,x^{2}\right)\left(\frac{c\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{ae\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-\frac{c^{2}x^{4}}{a}+c}}-\frac{c\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{ae\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-\frac{c^{2}x^{4}}{a}+c}}\right.}{c\,x^{2}\sqrt{-c\,x^{4}+a}+a\sqrt{\frac{\left(-c\,x^{4}+a\right)c}{a}}}$

input
```
int((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
(c/a)^(1/2)/e/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(
1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/
2),I)-((c/a)^(1/2)*d-e)/e/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2)
)^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/
2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a
^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx=\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(-(sqrt(-c*x^4 + a)*x^2*sqrt(c/a) + sqrt(-c*x^4 + a))/(c*e*x^6 + c
*d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{a-cx^4}\,(d+ex^2)}\,dx$$

input
```
integrate((1+(c/a)**(1/2)*x**2)/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((x**2*sqrt(c/a) + 1)/(sqrt(a - c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)}\,dx$$

input
```
int((x^2*(c/a)^(1/2) + 1)/((a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int((x^2*(c/a)^(1/2) + 1)/((a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)a}{a}$$

input
```
int((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 -
c*e*x**6),x) + int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6)
,x)*a)/a
```

3.61 $\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 174

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = -\frac{\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{a}e\sqrt{a-cx^4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a-cx^4}}$$

output
```
-c^(1/4)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(1/4)/e/(-c*x^
4+a)^(1/2)+(c^(1/2)*d+a^(1/2)*e)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*
x^2/a^(1/2))^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)/d,I)/a^
(1/4)/c^(1/4)/d/e/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.57 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.86

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx$$
$$= \frac{i\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{1-\frac{cx^4}{a}}\left(-\sqrt{c}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)+\left(\sqrt{c}d+\sqrt{a}e\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},i\text{arcsin}\right.\right.}{\sqrt{c}de\sqrt{a-cx^4}}$$

input
```
Integrate[(1 - (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(I*Sqrt[-(Sqrt[c]/Sqrt[a])]*Sqrt[1 - (c*x^4)/a]*(-(Sqrt[c]*d*EllipticF[I*A
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]) + (Sqrt[c]*d + Sqrt[a]*e)*Ellipti
cPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]
))/(Sqrt[c]*d*e*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1787, 415, 289, 413, 412, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a-cx^4}\,(d+ex^2)}\,dx$$

$$\Big\downarrow \text{ 1787}$$

$$\frac{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}\int \frac{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}{\sqrt{\sqrt{a}\sqrt{c}x^2+a}\,(ex^2+d)}dx}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 415}$$

$$\frac{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}\left(ex^2+d\right)}dx}{e}-\frac{\sqrt{c}\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}}dx}{\sqrt{a}e}\right)}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 289}$$

$$\frac{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}\left(\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}\left(ex^2+d\right)}dx}{e}-\frac{\sqrt{c}\sqrt{a-cx^4}\int \frac{1}{\sqrt{a-cx^4}}dx}{\sqrt{a}e\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}}\right)}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 413}$$

$$\frac{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}\left(\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}\left(ex^2+d\right)}dx}{e\sqrt{\sqrt{a}\sqrt{c}x^2+a}}-\frac{\sqrt{c}\sqrt{a-cx^4}\int \frac{1}{\sqrt{a-cx^4}}dx}{\sqrt{a}e\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}}\right)}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}\left(\frac{\sqrt[4]{a}\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de\sqrt{\sqrt{a}\sqrt{c}x^2+a}}-\frac{\sqrt{c}\sqrt{a-cx^4}\int \frac{1}{\sqrt{a-cx^4}}dx}{\sqrt{a}e\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}}\right)}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}\left(\frac{\sqrt[4]{a}\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de\sqrt{\sqrt{a}\sqrt{c}x^2+a}}-\frac{\sqrt{c}\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a}e\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}}\right)}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+a}\left(\frac{\sqrt[4]{a}\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de\sqrt{\sqrt{a}\sqrt{c}x^2+a}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{\sqrt[4]{a}e\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}\sqrt{\sqrt{a}\sqrt{c}x^2+}}\right.}{\sqrt{a-cx^4}}$$

input
```
Int[(1 - (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(Sqrt[1 - (Sqrt[c]*x^2)/Sqrt[a]]*Sqrt[a + Sqrt[a]*Sqrt[c]*x^2]*(-((c^(1/4)
*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(a^(1/4)*
e*Sqrt[1 - (Sqrt[c]*x^2)/Sqrt[a]]*Sqrt[a + Sqrt[a]*Sqrt[c]*x^2])) + (a^(1/
4)*((Sqrt[c]*d)/Sqrt[a] + e)*Sqrt[1 + (Sqrt[c]*x^2)/Sqrt[a]]*EllipticPi[-(
(Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e*S
qrt[a + Sqrt[a]*Sqrt[c]*x^2])))/Sqrt[a - c*x^4]
```

Defintions of rubi rules used

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 415
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[d/b   Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x],
x] + Simp[(b*c - a*d)/b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1787
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d +
e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])   Int[(d + e*x^n)^(p + q)
*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, c, d, e, f, g, n, p
, q, r}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07

method	result
default	$-\frac{\frac{\sqrt{c}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{e d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}}{\sqrt{a}}$
elliptic	$\frac{\sqrt{\left(-c\,x^{4}+a\right)a c}\,\left(-\sqrt{a}+\sqrt{c}\,x^{2}\right)\left(\frac{\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{c\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},\right.}{e\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-a\,c^{2}x^{4}+a^{2}c}}\right.}{\left(c\,x^{2}\sqrt{-c\,x^{4}+a}-\sqrt{\left(-c\,x^{4}+a\right)a c}\right)\sqrt{a}}$

input
```
int((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/a^(1/2)*(c^(1/2)/e/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2
)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1
/2))^(1/2),I)-(c^(1/2)*d+a^(1/2)*e)/e/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)
*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ellipti
cPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2
)/(c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm=
"fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = -\frac{\int \left(-\frac{\sqrt{a}}{d\sqrt{a-cx^4}+ex^2\sqrt{a-cx^4}}\right)dx + \int \frac{\sqrt{c}x^2}{d\sqrt{a-cx^4}+ex^2\sqrt{a-cx^4}}\,dx}{\sqrt{a}}$$

input
```
integrate((1-c**(1/2)*x**2/a**(1/2))/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

output
```
-(Integral(-sqrt(a)/(d*sqrt(a - c*x**4) + e*x**2*sqrt(a - c*x**4)), x) + I
ntegral(sqrt(c)*x**2/(d*sqrt(a - c*x**4) + e*x**2*sqrt(a - c*x**4)), x))/s
qrt(a)
```

Maxima [F]

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int -\frac{\frac{\sqrt{c}x^2}{\sqrt{a}}-1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm=
"maxima")
```

output
```
-integrate((sqrt(c)*x^2/sqrt(a) - 1)/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionDegree mismatch inside factorisation over extensionDegree
mismatch
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = -\int \frac{\frac{\sqrt{c}\,x^2}{\sqrt{a}}-1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)}\,dx$$

input
```
int(-((c^(1/2)*x^2)/a^(1/2) - 1)/((a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
-int(((c^(1/2)*x^2)/a^(1/2) - 1)/((a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \frac{-\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)a}{a}$$

input
```
int((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4
 - c*e*x**6),x) + int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x*
*6),x)*a)/a
```

3.62 $\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 179

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = -\frac{\sqrt[4]{a}\sqrt{\frac{c}{a}}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}e\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(\sqrt{\frac{c}{a}}d+e\right)\sqrt{1-\sqrt{\frac{c}{a}}x^2}\sqrt{1+\sqrt{\frac{c}{a}}x^2}\,\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}de\sqrt{a-cx^4}}$$

output
```
-a^(1/4)*(c/a)^(1/2)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(1
/4)/e/(-c*x^4+a)^(1/2)+a^(1/4)*((c/a)^(1/2)*d+e)*(1-(c/a)^(1/2)*x^2)^(1/2)
*(1+(c/a)^(1/2)*x^2)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*e/c^(1/2)
/d,I)/c^(1/4)/d/e/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.82

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx$$
$$= \frac{i\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{\frac{c}{a}}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)-\left(\sqrt{\frac{c}{a}}d+e\right)\text{EllipticPi}\left(-\frac{\sqrt{a}e}{\sqrt{c}d},i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right.\right.\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}de\sqrt{a-cx^4}}$$

input
```
Integrate[(1 - Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(I*Sqrt[1 - (c*x^4)/a]*(Sqrt[c/a]*d*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqr
t[a])]*x], -1] - (Sqrt[c/a]*d + e)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)),
I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e
*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1787, 415, 289, 413, 412, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2\sqrt{\frac{c}{a}}}{\sqrt{a-cx^4}\,(d+ex^2)}\,dx$$
$$\Big\downarrow \; 1787$$
$$\frac{\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}\int \frac{\sqrt{1-\sqrt{\frac{c}{a}}x^2}}{\sqrt{a\sqrt{\frac{c}{a}}x^2+a(ex^2+d)}}dx}{\sqrt{a-cx^4}}$$
$$\Big\downarrow \; 415$$

$$\frac{\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}\left(\frac{\left(d\sqrt{\frac{c}{a}}+e\right)\int\frac{1}{\sqrt{1-\sqrt{\frac{c}{a}}x^2}\sqrt{a\sqrt{\frac{c}{a}}x^2+a}\left(ex^2+d\right)}dx}{e}-\frac{\sqrt{\frac{c}{a}}\int\frac{1}{\sqrt{1-\sqrt{\frac{c}{a}}x^2}\sqrt{a\sqrt{\frac{c}{a}}x^2+a}}dx}{e}\right)}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 289}$$

$$\frac{\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}\left(\frac{\left(d\sqrt{\frac{c}{a}}+e\right)\int\frac{1}{\sqrt{1-\sqrt{\frac{c}{a}}x^2}\sqrt{a\sqrt{\frac{c}{a}}x^2+a}\left(ex^2+d\right)}dx}{e}-\frac{\sqrt{\frac{c}{a}}\sqrt{a-cx^4}\int\frac{1}{\sqrt{a-cx^4}}dx}{e\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}}\right)}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 413}$$

$$\frac{\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}\left(\frac{\sqrt{x^2\sqrt{\frac{c}{a}}+1}\left(d\sqrt{\frac{c}{a}}+e\right)\int\frac{1}{\sqrt{1-\sqrt{\frac{c}{a}}x^2}\sqrt{\sqrt{\frac{c}{a}}x^2+1}\left(ex^2+d\right)}dx}{e\sqrt{ax^2\sqrt{\frac{c}{a}}+a}}-\frac{\sqrt{\frac{c}{a}}\sqrt{a-cx^4}\int\frac{1}{\sqrt{a-cx^4}}dx}{e\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}}\right)}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}\left(\frac{\sqrt{x^2\sqrt{\frac{c}{a}}+1}\left(d\sqrt{\frac{c}{a}}+e\right)\text{EllipticPi}\left(-\frac{e}{\sqrt{\frac{c}{a}}d},\arcsin\left(\sqrt[4]{\frac{c}{a}}x\right),-1\right)}{de\sqrt[4]{\frac{c}{a}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}}-\frac{\sqrt{\frac{c}{a}}\sqrt{a-cx^4}\int\frac{1}{\sqrt{a-cx^4}}dx}{e\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}}\right)}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}\left(\frac{\sqrt{x^2\sqrt{\frac{c}{a}}+1}\left(d\sqrt{\frac{c}{a}}+e\right)\text{EllipticPi}\left(-\frac{e}{\sqrt{\frac{c}{a}}d},\arcsin\left(\sqrt[4]{\frac{c}{a}}x\right),-1\right)}{de\sqrt[4]{\frac{c}{a}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}}-\frac{\sqrt{\frac{c}{a}}\sqrt{1-\frac{cx^4}{a}}\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{e\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}}\right)}{\sqrt{a-cx^4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}\left(\frac{\sqrt{x^2\sqrt{\frac{c}{a}}+1}\left(d\sqrt{\frac{c}{a}}+e\right)\text{EllipticPi}\left(-\frac{e}{\sqrt{\frac{c}{a}}d},\arcsin\left(\sqrt[4]{\frac{c}{a}}x\right),-1\right)}{de\sqrt[4]{\frac{c}{a}}\sqrt{ax^2\sqrt{\frac{c}{a}}+a}}-\frac{\sqrt[4]{a}\sqrt{\frac{c}{a}}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt[4]{c}e\sqrt{1-x^2\sqrt{\frac{c}{a}}}\sqrt{ax^2\sqrt{\cdots}}}\right.}{\sqrt{a-cx^4}}$$

input
```
Int[(1 - Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(Sqrt[1 - Sqrt[c/a]*x^2]*Sqrt[a + a*Sqrt[c/a]*x^2]*(-((a^(1/4)*Sqrt[c/a]*S
qrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*
Sqrt[1 - Sqrt[c/a]*x^2]*Sqrt[a + a*Sqrt[c/a]*x^2])) + ((Sqrt[c/a]*d + e)*S
qrt[1 + Sqrt[c/a]*x^2]*EllipticPi[-(e/(Sqrt[c/a]*d)), ArcSin[(c/a)^(1/4)*x
], -1])/((c/a)^(1/4)*d*e*Sqrt[a + a*Sqrt[c/a]*x^2])))/Sqrt[a - c*x^4]
```

Defintions of rubi rules used

rule 289
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Sim
p[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c + b*d*x^4)^FracPart
[p])   Int[(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*
c + a*d, 0] &&  !IntegerQ[p]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 415
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[d/b   Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x],
x] + Simp[(b*c - a*d)/b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d/c]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1787
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d +
e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])   Int[(d + e*x^n)^(p + q)
*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, c, d, e, f, g, n, p
, q, r}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04

method	result
default	$-\frac{\sqrt{\frac{c}{a}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{e\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{\left(\sqrt{\frac{c}{a}}\,d+e\right)\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, -\frac{\sqrt{a}\,e}{\sqrt{c}\,d}, \frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{e d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}$
elliptic	$\frac{\left(1-\sqrt{\frac{c}{a}}\,x^{2}\right)\sqrt{\frac{\left(-c\,x^{4}+a\right)c}{a}}\,a\left(-\frac{c\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{a e\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-\frac{c^{2}x^{4}}{a}+c}}+\frac{c\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, -\frac{\sqrt{a}\,e}{\sqrt{c}\,d}, \frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{a e\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-\frac{c^{2}x^{4}}{a}+c}}\right)}{-c\,x^{2}\sqrt{-c\,x^{4}+a}+a\sqrt{\frac{\left(-c\,x^{4}+a\right)c}{a}}}$

input
```
int((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-(c/a)^(1/2)/e/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^
(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1
/2),I)+((c/a)^(1/2)*d+e)/e/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2
))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1
/2)/a^(1/2))^(1/2),-a^(1/2)*e/c^(1/2)/d,(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/
a^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int -\frac{x^2\sqrt{\frac{c}{a}}-1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fri
cas")
```

output
```
integral((sqrt(-c*x^4 + a)*x^2*sqrt(c/a) - sqrt(-c*x^4 + a))/(c*e*x^6 + c*
d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = -\int \frac{x^2\sqrt{\frac{c}{a}}}{d\sqrt{a-cx^4}+ex^2\sqrt{a-cx^4}}\,dx - \int \left(-\frac{1}{d\sqrt{a-cx^4}+ex^2\sqrt{a-cx^4}}\right)dx$$

input
```
integrate((1-(c/a)**(1/2)*x**2)/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

output
```
-Integral(x**2*sqrt(c/a)/(d*sqrt(a - c*x**4) + e*x**2*sqrt(a - c*x**4)), x
) - Integral(-1/(d*sqrt(a - c*x**4) + e*x**2*sqrt(a - c*x**4)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int -\frac{x^2\sqrt{\frac{c}{a}}-1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="max
ima")
```

output
```
-integrate((x^2*sqrt(c/a) - 1)/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \int -\frac{x^2\sqrt{\frac{c}{a}}-1}{\sqrt{-cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(-(x^2*sqrt(c/a) - 1)/(sqrt(-c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx = -\int \frac{x^2\sqrt{\frac{c}{a}}-1}{\sqrt{a-c\,x^4}\,(e\,x^2+d)}\,dx$$

input
```
int(-(x^2*(c/a)^(1/2) - 1)/((a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
-int((x^2*(c/a)^(1/2) - 1)/((a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a-cx^4}}\,dx$$
$$= \frac{-\sqrt{c}\sqrt{a}\left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)a}{a}$$

input
```
int((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4
 - c*e*x**6),x) + int(sqrt(a - c*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x*
*6),x)*a)/a
```

3.63 $\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{x\sqrt{1-x^4}}{2(1+x^2)} + \frac{1}{2}E(\arcsin(x)|-1)$$

output
```
x*(-x^4+1)^(1/2)/(2*x^2+2)+1/2*EllipticE(x,I)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{x-x^3+\sqrt{1-x^4}E(\arcsin(x)|-1)}{2\sqrt{1-x^4}}$$

input
```
Integrate[1/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
(x - x^3 + Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[1 - x^4])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1388, 316, 25, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{1}{\sqrt{1-x^2}\,(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 316}$$

$$\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}-\frac{1}{2}\int -\frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx+\frac{\sqrt{1-x^2}x}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{1}{2}E(\arcsin(x)|-1)+\frac{\sqrt{1-x^2}x}{2\sqrt{x^2+1}}$$

input
```
Int[1/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
(x*Sqrt[1 - x^2])/(2*Sqrt[1 + x^2]) + EllipticE[ArcSin[x], -1]/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(27) = 54$.

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.75

method	result	size
risch	$-\frac{x(x^2-1)}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\ \mathrm{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$	88
default	$\frac{(-x^2+1)x}{2\sqrt{(x^2+1)(-x^2+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\ \mathrm{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$	96
elliptic	$\frac{(-x^2+1)x}{2\sqrt{(x^2+1)(-x^2+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\ \mathrm{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$	96

input
```
int(1/(x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(x^2-1)/(-x^4+1)^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1
/2)*EllipticF(x,I)-1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(Ellipt
icF(x,I)-EllipticE(x,I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{(x^2+1)E(\arcsin{(x)}\,|-1)+\sqrt{-x^4+1}x}{2\,(x^2+1)}$$

input
```
integrate(1/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*elliptic_e(arcsin(x), -1) + sqrt(-x^4 + 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-(x-1)\,(x+1)\,(x^2+1)}\,(x^2+1)}\,dx$$

input
```
integrate(1/(x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate(1/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{1}{(x^2+1)\sqrt{1-x^4}}\,dx$$

input
```
int(1/((x^2 + 1)*(1 - x^4)^(1/2)),x)
```

output
```
int(1/((x^2 + 1)*(1 - x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(1+x^2)\sqrt{1-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+1}}{x^6+x^4-x^2-1}dx\right)$$

input
```
int(1/(x^2+1)/(-x^4+1)^(1/2),x)
```

output
```
- int(sqrt( - x**4 + 1)/(x**6 + x**4 - x**2 - 1),x)
```

3.64 $\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 36

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}} \, dx = -\frac{x\sqrt{1-x^4}}{2(1+x^2)} - \frac{1}{2}E(\arcsin(x)|-1) + \text{EllipticF}(\arcsin(x), -1)$$

output
```
-1/2*x*(-x^4+1)^(1/2)/(x^2+1)-1/2*EllipticE(x,I)+EllipticF(x,I)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}} \, dx = \frac{1}{2}\left(-\frac{x}{\sqrt{1-x^4}} + \frac{x^3}{\sqrt{1-x^4}} - E(\arcsin(x)|-1) + 2\,\text{EllipticF}(\arcsin(x), -1)\right)$$

input
```
Integrate[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
(-(x/Sqrt[1 - x^4]) + x^3/Sqrt[1 - x^4] - EllipticE[ArcSin[x], -1] + 2*Ell
ipticF[ArcSin[x], -1])/2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1388, 373, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+1)\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{x^2}{\sqrt{1-x^2}\,(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{373}$$

$$\frac{1}{2}\int \frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}dx-\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{326}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+1}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx\right)-\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{284}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-x^4}}dx-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx\right)-\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt{1-x^4}}dx-E(\arcsin(x)|-1)\right)-\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{2}(2\,\text{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1))-\frac{x\sqrt{1-x^2}}{2\sqrt{x^2+1}}$$

input

```
Int[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
-1/2*(x*Sqrt[1 - x^2])/Sqrt[1 + x^2] + (-EllipticE[ArcSin[x], -1] + 2*Elli
pticF[ArcSin[x], -1])/2
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(30) = 60$.

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.44

method	result	size
risch	$\frac{x(x^2-1)}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{2\sqrt{-x^4+1}}$	88
default	$\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{(-x^2+1)x}{2\sqrt{(x^2+1)(-x^2+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$	96
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{(-x^2+1)x}{2\sqrt{(x^2+1)(-x^2+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$	96

input
```
int(x^2/(x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^2-1)/(-x^4+1)^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/
2)*(EllipticF(x,I)-EllipticE(x,I))+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+
1)^(1/2)*EllipticF(x,I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = -\frac{(x^2+1)E(\arcsin(x)\mid-1)-2\,(x^2+1)F(\arcsin(x)\mid-1)+\sqrt{-x^4+1}x}{2\,(x^2+1)}$$

input
```
integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((x^2 + 1)*elliptic_e(arcsin(x), -1) - 2*(x^2 + 1)*elliptic_f(arcsin(
x), -1) + sqrt(-x^4 + 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-(x-1)(x+1)(x^2+1)}\,(x^2+1)}\,dx$$

input
```
integrate(x**2/(x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx=\int \frac{x^2}{(x^2+1)\,\sqrt{1-x^4}}\,dx$$

input
```
int(x^2/((x^2 + 1)*(1 - x^4)^(1/2)),x)
```

output
```
int(x^2/((x^2 + 1)*(1 - x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx=\frac{-\sqrt{-x^4+1}\,x+\left(\int \frac{\sqrt{-x^4+1}}{x^2+1}dx\right)x^2+\int \frac{\sqrt{-x^4+1}}{x^2+1}dx}{2x^2+2}$$

input
```
int(x^2/(x^2+1)/(-x^4+1)^(1/2),x)
```

output
```
( - sqrt( - x**4 + 1)*x + int(sqrt( - x**4 + 1)/(x**2 + 1),x)*x**2 + int(s
qrt( - x**4 + 1)/(x**2 + 1),x))/(2*(x**2 + 1))
```

3.65 $\int \frac{A+Bx^2}{(1+x^2)\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 48

$$\int \frac{A+Bx^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{(A-B)x\sqrt{1-x^4}}{2(1+x^2)} + \frac{1}{2}(A-B)E(\arcsin(x)|-1) + B\,\text{EllipticF}(\arcsin(x),-1)$$

output
```
(A-B)*x*(-x^4+1)^(1/2)/(2*x^2+2)+1/2*(A-B)*EllipticE(x,I)+B*EllipticF(x,I)
```

Mathematica [A] (verified)

Time = 10.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{(1+x^2)\sqrt{1-x^4}}\,dx = -\frac{(A-B)x(-1+x^2)}{2\sqrt{1-x^4}} + \frac{1}{2}(A-B)E(\arcsin(x)|-1) + B\,\text{EllipticF}(\arcsin(x),-1)$$

input
```
Integrate[(A + B*x^2)/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
-1/2*((A - B)*x*(-1 + x^2))/Sqrt[1 - x^4] + ((A - B)*EllipticE[ArcSin[x],
-1])/2 + B*EllipticF[ArcSin[x], -1]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1388, 402, 25, 399, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(x^2+1)\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{A+Bx^2}{\sqrt{1-x^2}\,(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{402}$$

$$\frac{x\sqrt{1-x^2}(A-B)}{2\sqrt{x^2+1}}-\frac{1}{2}\int -\frac{(A-B)x^2+A+B}{\sqrt{1-x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{(A-B)x^2+A+B}{\sqrt{1-x^2}\sqrt{x^2+1}}dx+\frac{\sqrt{1-x^2}x(A-B)}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{2}\left((A-B)\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx+2B\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+1}}dx\right)+\frac{\sqrt{1-x^2}x(A-B)}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{284}$$

$$\frac{1}{2}\left((A-B)\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx+2B\int \frac{1}{\sqrt{1-x^4}}dx\right)+\frac{\sqrt{1-x^2}x(A-B)}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{2}\left(2B\int \frac{1}{\sqrt{1-x^4}}dx+(A-B)E(\arcsin(x)|-1)\right)+\frac{\sqrt{1-x^2}x(A-B)}{2\sqrt{x^2+1}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{2}((A-B)E(\arcsin(x)|-1)+2B\,\text{EllipticF}(\arcsin(x),-1))+\frac{\sqrt{1-x^2}x(A-B)}{2\sqrt{x^2+1}}$$

input
```
Int[(A + B*x^2)/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
((A - B)*x*Sqrt[1 - x^2])/(2*Sqrt[1 + x^2]) + ((A - B)*EllipticE[ArcSin[x]
, -1] + 2*B*EllipticF[ArcSin[x], -1])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(43) = 86$.

Time = 0.82 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.42

method	result
elliptic	$-\frac{2(-x^2+1)\left(-\frac{A}{4}+\frac{B}{4}\right)x}{\sqrt{(x^2+1)(-x^2+1)}}+\frac{\left(\frac{B}{2}+\frac{A}{2}\right)\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{\sqrt{-x^4+1}}-\frac{\left(\frac{A}{2}-\frac{B}{2}\right)\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x}{\sqrt{-x^4+1}}$
risch	$-\frac{(A-B)x(x^2-1)}{2\sqrt{-x^4+1}}+\frac{A\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{2\sqrt{-x^4+1}}+\frac{B\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{(A-B)\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{Ellip}}{2\sqrt{-x^4}}$
default	$\frac{B\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{\sqrt{-x^4+1}}+(A-B)\left(\frac{(-x^2+1)x}{2\sqrt{(x^2+1)(-x^2+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}}{}\right.$

input
```
int((B*x^2+A)/(x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-x^2+1)*(-1/4*A+1/4*B)*x/((x^2+1)*(-x^2+1))^(1/2)+(1/2*B+1/2*A)*(-x^2+
1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*EllipticF(x,I)-(1/2*A-1/2*B)*(-x^2+1
)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^2}{(1+x^2)\sqrt{1-x^4}}\,dx$$
$$=\frac{\sqrt{-x^4+1}(A-B)x+((A-B)x^2+A-B)E(\arcsin(x)\mid-1)+2(Bx^2+B)F(\arcsin(x)\mid-1)}{2(x^2+1)}$$

input
```
integrate((B*x^2+A)/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-x^4 + 1)*(A - B)*x + ((A - B)*x^2 + A - B)*elliptic_e(arcsin(x)
, -1) + 2*(B*x^2 + B)*elliptic_f(arcsin(x), -1))/(x^2 + 1)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(1+x^2)\sqrt{1-x^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{-(x-1)(x+1)(x^2+1)}(x^2+1)}\,dx$$

input
```
integrate((B*x**2+A)/(x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(1+x^2)\sqrt{1-x^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate((B*x^2+A)/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate((B*x^2+A)/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{B\,x^2+A}{(x^2+1)\,\sqrt{1-x^4}}\,dx$$

input
```
int((A + B*x^2)/((x^2 + 1)*(1 - x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/((x^2 + 1)*(1 - x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{-\sqrt{-x^4+1}\,bx-2\left(\int \frac{\sqrt{-x^4+1}}{x^6+x^4-x^2-1}dx\right)a\,x^2-2\left(\int \frac{\sqrt{-x^4+1}}{x^6+x^4-x^2-1}dx\right)a+\left(\int \frac{\sqrt{-x^4+1}}{x^2+1}dx\right)b\,x^2+\left(\int \frac{\sqrt{-x^4+1}}{x^2+1}dx\right)}{2x^2+2}$$

input
```
int((B*x^2+A)/(x^2+1)/(-x^4+1)^(1/2),x)
```

output
```
( - sqrt( - x**4 + 1)*b*x - 2*int(sqrt( - x**4 + 1)/(x**6 + x**4 - x**2 -
1),x)*a*x**2 - 2*int(sqrt( - x**4 + 1)/(x**6 + x**4 - x**2 - 1),x)*a + int
(sqrt( - x**4 + 1)/(x**2 + 1),x)*b*x**2 + int(sqrt( - x**4 + 1)/(x**2 + 1)
,x)*b)/(2*(x**2 + 1))
```

3.66 $\int \frac{A+Bx^2+Cx^4}{(1+x^2)\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 56

$$\int \frac{A+Bx^2+Cx^4}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{(A-B+C)x\sqrt{1-x^4}}{2(1+x^2)} + \frac{1}{2}(A-B+3C)E(\arcsin(x)|-1) + (B-2C)\,\text{EllipticF}(\arcsin(x),-1)$$

output
```
(A-B+C)*x*(-x^4+1)^(1/2)/(2*x^2+2)+1/2*(A-B+3*C)*EllipticE(x,I)+(B-2*C)*El
lipticF(x,I)
```

Mathematica [A] (verified)

Time = 10.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4}{(1+x^2)\sqrt{1-x^4}}\,dx = -\frac{(A-B+C)x(-1+x^2)}{2\sqrt{1-x^4}} + \frac{1}{2}(A-B+3C)E(\arcsin(x)|-1) + (B-2C)\,\text{EllipticF}(\arcsin(x),-1)$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
-1/2*((A - B + C)*x*(-1 + x^2))/Sqrt[1 - x^4] + ((A - B + 3*C)*EllipticE[A
rcSin[x], -1])/2 + (B - 2*C)*EllipticF[ArcSin[x], -1]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 119 vs. 2(56) = 112.

Time = 0.52 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1388, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(x^2+1)\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{1-x^2}\,(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{A}{\sqrt{1-x^2}\,(x^2+1)^{3/2}}+\frac{Bx^2}{\sqrt{1-x^2}\,(x^2+1)^{3/2}}+\frac{Cx^4}{\sqrt{1-x^2}\,(x^2+1)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}AE(\arcsin(x)|-1)+\frac{A\sqrt{1-x^2}x}{2\sqrt{x^2+1}}+B\,\text{EllipticF}(\arcsin(x),-1)-\frac{1}{2}BE(\arcsin(x)|-1)-2C\,\text{EllipticF}(\arcsin(x),-1)+\frac{3}{2}CE(\arcsin(x)|-1)-\frac{B\sqrt{1-x^2}x}{2\sqrt{x^2+1}}+\frac{C\sqrt{1-x^2}x}{2\sqrt{x^2+1}}$$

input
```
Int[(A + B*x^2 + C*x^4)/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
(A*x*Sqrt[1 - x^2])/(2*Sqrt[1 + x^2]) - (B*x*Sqrt[1 - x^2])/(2*Sqrt[1 + x^
2]) + (C*x*Sqrt[1 - x^2])/(2*Sqrt[1 + x^2]) + (A*EllipticE[ArcSin[x], -1])
/2 - (B*EllipticE[ArcSin[x], -1])/2 + (3*C*EllipticE[ArcSin[x], -1])/2 + B
*EllipticF[ArcSin[x], -1] - 2*C*EllipticF[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(51) = 102.

Time = 1.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.23

method	result
elliptic	$-\frac{2(-x^2+1)\left(-\frac{A}{4}+\frac{B}{4}-\frac{C}{4}\right)x}{\sqrt{(x^2+1)(-x^2+1)}}+\frac{\left(\frac{B}{2}-\frac{C}{2}+\frac{A}{2}\right)\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{\sqrt{-x^4+1}}-\frac{\left(\frac{3C}{2}+\frac{A}{2}-\frac{B}{2}\right)\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)}{\sqrt{-x^4+1}}$
risch	$-\frac{(A-B+C)x(x^2-1)}{2\sqrt{-x^4+1}}+\frac{A\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{2\sqrt{-x^4+1}}+\frac{B\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{2\sqrt{-x^4+1}}-\frac{(A-B+3C)\sqrt{-x^2+1}\,\sqrt{x^2}}{2}$
default	$\frac{B\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\text{EllipticF}(x,i)}{\sqrt{-x^4+1}}-\frac{C\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{\sqrt{-x^4+1}}+(A-B+C)\left(\frac{(-x^2+1)x}{2\sqrt{(x^2+1)(-x^2}}\right.$

input
```
int((C*x^4+B*x^2+A)/(x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-x^2+1)*(-1/4*A+1/4*B-1/4*C)*x/((x^2+1)*(-x^2+1))^(1/2)+(1/2*B-1/2*C+1
/2*A)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*EllipticF(x,I)-(3/2*C+1/
2*A-1/2*B)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-Ell
ipticE(x,I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx^2+Cx^4}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{(-i\,(A-B+3\,C)x^3-i\,(A-B+3\,C)x)E(\arcsin\left(\frac{1}{x}\right)\,|-1)-2\,(-i\,(A+C)x^3-i\,(A+C)x)F(\arcsin}{2\,(x^3+x)}$$

input
```
integrate((C*x^4+B*x^2+A)/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((-I*(A - B + 3*C)*x^3 - I*(A - B + 3*C)*x)*elliptic_e(arcsin(1/x), -1
) - 2*(-I*(A + C)*x^3 - I*(A + C)*x)*elliptic_f(arcsin(1/x), -1) - sqrt(-x
^4 + 1)*(2*C*x^2 + A - B + 3*C))/(x^3 + x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{\sqrt{-(x-1)(x+1)(x^2+1)}\,(x^2+1)}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**2 +
1)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(x^2+1)\,\sqrt{1-x^4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((x^2 + 1)*(1 - x^4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((x^2 + 1)*(1 - x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(1+x^2)\sqrt{1-x^4}}\,dx$$
$$= \frac{-\sqrt{-x^4+1}\,bx - 2\left(\int \frac{\sqrt{-x^4+1}}{x^6+x^4-x^2-1}dx\right) a\,x^2 - 2\left(\int \frac{\sqrt{-x^4+1}}{x^6+x^4-x^2-1}dx\right) a - \left(\int \frac{\sqrt{-x^4+1}}{x^6+x^4-x^2-1}dx\right) b\,x^2 - \left(\int \frac{\sqrt{-x^4}}{x^6+x^4-}\right.}{2x}$$

input
```
int((C*x^4+B*x^2+A)/(x^2+1)/(-x^4+1)^(1/2),x)
```

output
```
( - sqrt( - x**4 + 1)*b*x - 2*int(sqrt( - x**4 + 1)/(x**6 + x**4 - x**2 -
1),x)*a*x**2 - 2*int(sqrt( - x**4 + 1)/(x**6 + x**4 - x**2 - 1),x)*a - int
(sqrt( - x**4 + 1)/(x**6 + x**4 - x**2 - 1),x)*b*x**2 - int(sqrt( - x**4 +
 1)/(x**6 + x**4 - x**2 - 1),x)*b + int((sqrt( - x**4 + 1)*x**4)/(x**6 + x
**4 - x**2 - 1),x)*b*x**2 + int((sqrt( - x**4 + 1)*x**4)/(x**6 + x**4 - x*
*2 - 1),x)*b - 2*int((sqrt( - x**4 + 1)*x**4)/(x**6 + x**4 - x**2 - 1),x)*
c*x**2 - 2*int((sqrt( - x**4 + 1)*x**4)/(x**6 + x**4 - x**2 - 1),x)*c)/(2*
(x**2 + 1))
```

3.67 $\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 453

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{e(21Bcd^2+21Acde-5aBe^2)x\sqrt{a+cx^4}}{21c^2}+\frac{e^2(3Bd+Ae)x^3\sqrt{a+cx^4}}{5c} + \frac{Be^3x^5\sqrt{a+cx^4}}{7c}+\frac{(5Bcd^3+15Acd^2e-9aBde^2-3aAe^3)x\sqrt{a+cx^4}}{5c^{3/2}(\sqrt{a}+\sqrt{c}x^2)} - \frac{\sqrt[4]{a}(5Bcd^3+15Acd^2e-9aBde^2-3aAe^3)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}} + \frac{(105Ac^2d^3+25a^2Be^3-105acde(Bd+Ae)-63a^{3/2}\sqrt{c}e^2(3Bd+Ae)+105\sqrt{a}c^{3/2}d^2(Bd+3Ae))(\sqrt{}}{210\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}}$$

output
```
1/21*e*(21*A*c*d*e-5*B*a*e^2+21*B*c*d^2)*x*(c*x^4+a)^(1/2)/c^2+1/5*e^2*(A*
e+3*B*d)*x^3*(c*x^4+a)^(1/2)/c+1/7*B*e^3*x^5*(c*x^4+a)^(1/2)/c+1/5*(-3*A*a
*e^3+15*A*c*d^2*e-9*B*a*d*e^2+5*B*c*d^3)*x*(c*x^4+a)^(1/2)/c^(3/2)/(a^(1/2
)+c^(1/2)*x^2)-1/5*a^(1/4)*(-3*A*a*e^3+15*A*c*d^2*e-9*B*a*d*e^2+5*B*c*d^3)
*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE
(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(7/4)/(c*x^4+a)^(1/2)+1/2
10*(105*A*c^2*d^3+25*a^2*B*e^3-105*a*c*d*e*(A*e+B*d)-63*a^(3/2)*c^(1/2)*e^
2*(A*e+3*B*d)+105*a^(1/2)*c^(3/2)*d^2*(3*A*e+B*d))*(a^(1/2)+c^(1/2)*x^2)*(
(c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*
x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(9/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.31 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.48

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{ex(a+cx^4)(-25aBe^2+21Ace(5d+ex^2)+3Bc(35d^2+21dex^2+5e^2x^4))+5(21Acd(cd^2-ae^2)+aB}{}$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^3)/Sqrt[a + c*x^4],x]
```

output
```
(e*x*(a + c*x^4)*(-25*a*B*e^2 + 21*A*c*e*(5*d + e*x^2) + 3*B*c*(35*d^2 + 2
1*d*e*x^2 + 5*e^2*x^4)) + 5*(21*A*c*d*(c*d^2 - a*e^2) + a*B*e*(-21*c*d^2 +
 5*a*e^2))*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4
)/a)] + 7*c*(5*B*c*d^3 + 15*A*c*d^2*e - 9*a*B*d*e^2 - 3*a*A*e^3)*x^3*Sqrt[
1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(105*c^2*Sq
rt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{d^2x^2(3Ae+Bd)}{\sqrt{a+cx^4}}+\frac{e^2x^6(Ae+3Bd)}{\sqrt{a+cx^4}}+\frac{3dex^4(Ae+Bd)}{\sqrt{a+cx^4}}+\frac{Ad^3}{\sqrt{a+cx^4}}+\frac{Be^3x^8}{\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{Be^3\sqrt{cx^4+a}x^5}{7c}+\frac{e^2(3Bd+Ae)\sqrt{cx^4+a}x^3}{5c}-\frac{5aBe^3\sqrt{cx^4+a}x}{21c^2}+\frac{de(Bd+Ae)\sqrt{cx^4+a}x}{c}-\frac{3ae^2(3Bd+Ae)\sqrt{cx^4+a}x}{5c^{3/2}\left(\sqrt{c}x^2+\sqrt{a}\right)}+\frac{d^2(Bd+3Ae)\sqrt{cx^4+a}x}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}+\frac{3a^{5/4}e^2(3Bd+Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{cx^4+a}}-\frac{\sqrt[4]{a}d^2(Bd+3Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{cx^4+a}}+\frac{Ad^3\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{cx^4+a}}+\frac{5a^{7/4}Be^3\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{42c^{9/4}\sqrt{cx^4+a}}-\frac{a^{3/4}de(Bd+Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{5/4}\sqrt{cx^4+a}}-\frac{3a^{5/4}e^2(3Bd+Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10c^{7/4}\sqrt{cx^4+a}}+\frac{\sqrt[4]{a}d^2(Bd+3Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{cx^4+a}}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^3)/Sqrt[a + c*x^4],x]
```

output
```
(-5*a*B*e^3*x*Sqrt[a + c*x^4])/(21*c^2) + (d*e*(B*d + A*e)*x*Sqrt[a + c*x^
4])/c + (e^2*(3*B*d + A*e)*x^3*Sqrt[a + c*x^4])/(5*c) + (B*e^3*x^5*Sqrt[a
+ c*x^4])/(7*c) - (3*a*e^2*(3*B*d + A*e)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sq
rt[a] + Sqrt[c]*x^2)) + (d^2*(B*d + 3*A*e)*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sq
rt[a] + Sqrt[c]*x^2)) + (3*a^(5/4)*e^2*(3*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x^4]) - (a^(1/4)*d^2*(B*d + 3*A*
e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (A
*d^3*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a +
c*x^4]) + (5*a^(7/4)*B*e^3*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(42*c^
(9/4)*Sqrt[a + c*x^4]) - (a^(3/4)*d*e*(B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(2*c^(5/4)*Sqrt[a + c*x^4]) - (3*a^(5/4)*e^2*(3*B*d + A*e
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a + c*x^4]) +
(a^(1/4)*d^2*(B*d + 3*A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.73 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.72

method	result
elliptic	$\frac{B e^3 x^5 \sqrt{c x^4+a}}{7c} + \frac{(A e^3+3 B d e^2) x^3 \sqrt{c x^4+a}}{5c} + \frac{\left(3 A d e^2+3 B e d^2-\frac{5 e^3 B a}{7c}\right) x \sqrt{c x^4+a}}{3c} + \frac{\left(A d^3-\frac{\left(3 A d e^2+3 B e d^2-\frac{5 e^3 B a}{7c}\right) a}{3c}\right)}{\sqrt{}}$
risch	$\frac{e x (15 e^2 B x^4 c+21 A c e^2 x^2+63 B c d e x^2+105 A c d e-25 B a e^2+105 B c d^2) \sqrt{c x^4+a}}{105 c^2} - \frac{21 i \sqrt{c} \left(3 A a e^3-15 A c d^2 e+9 B a d e^2-5 B c d^3\right) \sqrt{a} \sqrt{}}{}$
default	$\frac{A d^3 \sqrt{1-\frac{i\sqrt{c}\, x^2}{\sqrt{a}}} \sqrt{1+\frac{i\sqrt{c}\, x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ \sqrt{c x^4+a}} + e^2(Ae+3Bd)\left(\frac{x^3\sqrt{c x^4+a}}{5c} - \frac{3 i a^{\frac{3}{2}} \sqrt{1-\frac{i\sqrt{c}\, x^2}{\sqrt{a}}} \sqrt{1+\frac{i\sqrt{c}\, x^2}{\sqrt{a}}}\ (\mathrm{Elli}}{5 c^{\frac{3}{2}} \sqrt{}}\right.$

input
```
int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*B*e^3*x^5*(c*x^4+a)^(1/2)/c+1/5*(A*e^3+3*B*d*e^2)/c*x^3*(c*x^4+a)^(1/2
)+1/3*(3*A*d*e^2+3*B*e*d^2-5/7*e^3*B/c*a)/c*x*(c*x^4+a)^(1/2)+(A*d^3-1/3*(
3*A*d*e^2+3*B*e*d^2-5/7*e^3*B/c*a)/c*a)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(
1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*El
lipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+I*(3*A*d^2*e+B*d^3-3/5*(A*e^3+3*B*d
*e^2)/c*a)*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/
2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I
*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{21\,(5\,Bacd^3+15\,Aacd^2e-9\,Ba^2de^2-3\,Aa^2e^3)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-(105\,(3\,A+B)ac}{}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(21*(5*B*a*c*d^3 + 15*A*a*c*d^2*e - 9*B*a^2*d*e^2 - 3*A*a^2*e^3)*sqr
t(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) - (105*(3*A + B
)*a*c*d^2*e - (63*A + 25*B)*a^2*e^3 + 105*(B*a*c - A*c^2)*d^3 - 21*(9*B*a^
2 - 5*A*a*c)*d*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/
x), -1) + (15*B*a*c*e^3*x^6 + 105*B*a*c*d^3 + 315*A*a*c*d^2*e - 189*B*a^2*
d*e^2 - 63*A*a^2*e^3 + 21*(3*B*a*c*d*e^2 + A*a*c*e^3)*x^4 + 5*(21*B*a*c*d^
2*e + 21*A*a*c*d*e^2 - 5*B*a^2*e^3)*x^2)*sqrt(c*x^4 + a))/(a*c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.49 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{Ad^3x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{3Ad^2ex^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3Ade^2x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Ae^3x^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)} + \frac{Bd^3x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3Bd^2ex^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{3Bde^2x^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)} + \frac{Be^3x^9\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

output
```
A*d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4
*sqrt(a)*gamma(5/4)) + 3*A*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,)
, c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*A*d*e**2*x**5*gamma
(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma
(9/4)) + A*e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_pola
r(I*pi)/a)/(4*sqrt(a)*gamma(11/4)) + B*d**3*x**3*gamma(3/4)*hyper((1/2, 3/
4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*B*d**2*e*
x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqr
t(a)*gamma(9/4)) + 3*B*d*e**2*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c
*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4)) + B*e**3*x**9*gamma(9/4)*
hyper((1/2, 9/4), (13/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4
))
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^3}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^3}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^3}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(1/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{-25\sqrt{c\,x^4+a}\,ab\,e^3x+105\sqrt{c\,x^4+a}\,acd\,e^2x+21\sqrt{c\,x^4+a}\,ac\,e^3x^3+105\sqrt{c\,x^4+a}\,bc\,d^2ex+63\sqrt{c\,x^4+}}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x)
```

output
```
( - 25*sqrt(a + c*x**4)*a*b*e**3*x + 105*sqrt(a + c*x**4)*a*c*d*e**2*x + 2
1*sqrt(a + c*x**4)*a*c*e**3*x**3 + 105*sqrt(a + c*x**4)*b*c*d**2*e*x + 63*
sqrt(a + c*x**4)*b*c*d*e**2*x**3 + 15*sqrt(a + c*x**4)*b*c*e**3*x**5 + 25*
int(sqrt(a + c*x**4)/(a + c*x**4),x)*a**2*b*e**3 - 105*int(sqrt(a + c*x**4
)/(a + c*x**4),x)*a**2*c*d*e**2 - 105*int(sqrt(a + c*x**4)/(a + c*x**4),x)
*a*b*c*d**2*e + 105*int(sqrt(a + c*x**4)/(a + c*x**4),x)*a*c**2*d**3 - 63*
int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*a**2*c*e**3 - 189*int((sqrt(a
+ c*x**4)*x**2)/(a + c*x**4),x)*a*b*c*d*e**2 + 315*int((sqrt(a + c*x**4)*x
**2)/(a + c*x**4),x)*a*c**2*d**2*e + 105*int((sqrt(a + c*x**4)*x**2)/(a +
c*x**4),x)*b*c**2*d**3)/(105*c**2)
```

3.68 $\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 367

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \frac{e(2Bd+Ae)x\sqrt{a+cx^4}}{3c}+\frac{Be^2x^3\sqrt{a+cx^4}}{5c}+\frac{(5Bcd^2+10Acde-3aBe^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{\sqrt[4]{a}(5Bcd^2+10Acde-3aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}$$
$$+\frac{\left(15Ac^{3/2}d^2-9a^{3/2}Be^2-5a\sqrt{c}e(2Bd+Ae)+15\sqrt{a}cd(Bd+2Ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{Ellip}}{30\sqrt[4]{a}c^{7/4}\sqrt{a+cx^4}}$$

output

```
1/3*e*(A*e+2*B*d)*x*(c*x^4+a)^(1/2)/c+1/5*B*e^2*x^3*(c*x^4+a)^(1/2)/c+1/5*
(10*A*c*d*e-3*B*a*e^2+5*B*c*d^2)*x*(c*x^4+a)^(1/2)/c^(3/2)/(a^(1/2)+c^(1/2
)*x^2)-1/5*a^(1/4)*(10*A*c*d*e-3*B*a*e^2+5*B*c*d^2)*(a^(1/2)+c^(1/2)*x^2)*
((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))/c^(7/4)/(c*x^4+a)^(1/2)+1/30*(15*A*c^(3/2)*d^2-9*a
^(3/2)*B*e^2-5*a*c^(1/2)*e*(A*e+2*B*d)+15*a^(1/2)*c*d*(2*A*e+B*d))*(a^(1/2
)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(7/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{ex(10Bd+5Ae+3Bex^2)(a+cx^4)-5(-3Acd^2+2aBde+aAe^2)x\sqrt{1+\frac{cx^4}{a}}\text{Hypergeometric2F1}\left(\frac{1}{4},\right.}{15c\sqrt{a+cx^4}}$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + c*x^4],x]
```

output
```
(e*x*(10*B*d + 5*A*e + 3*B*e*x^2)*(a + c*x^4) - 5*(-3*A*c*d^2 + 2*a*B*d*e
+ a*A*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4
)/a)] + (5*B*c*d^2 + 10*A*c*d*e - 3*a*B*e^2)*x^3*Sqrt[1 + (c*x^4)/a]*Hyper
geometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(15*c*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$$
$$\Big\downarrow \text{2259}$$
$$\int \left(\frac{ex^4(Ae+2Bd)}{\sqrt{a+cx^4}}+\frac{dx^2(2Ae+Bd)}{\sqrt{a+cx^4}}+\frac{Ad^2}{\sqrt{a+cx^4}}+\frac{Be^2x^6}{\sqrt{a+cx^4}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/4}e(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(Ae+2Bd)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}-\frac{3a^{5/4}Be^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}}+\frac{3a^{5/4}Be^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}+\frac{\sqrt[4]{a}d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(2Ae+Bd)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(2Ae+Bd)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{Ad^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{ex\sqrt{a+cx^4}(Ae+2Bd)}{3c}+\frac{dx\sqrt{a+cx^4}(2Ae+Bd)}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{3aBe^2x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{Be^2x^3\sqrt{a+cx^4}}{5c}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + c*x^4],x]
```

output
```
(e*(2*B*d + A*e)*x*Sqrt[a + c*x^4])/(3*c) + (B*e^2*x^3*Sqrt[a + c*x^4])/(5
*c) - (3*a*B*e^2*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) +
(d*(B*d + 2*A*e)*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (3
*a^(5/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a
+ c*x^4]) - (a^(1/4)*d*(B*d + 2*A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/
2])/(c^(3/4)*Sqrt[a + c*x^4]) + (A*d^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]) - (3*a^(5/4)*B*e^2*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a + c*x^4]) - (a^(3/4)*e*(2*B
*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*c^(5/4)*Sqrt[a + c*x
^4]) + (a^(1/4)*d*(B*d + 2*A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
2*c^(3/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.70

method	result
elliptic	$\frac{B e^2 x^3 \sqrt{c x^4+a}}{5c} + \frac{(A e^2+2Bde) x \sqrt{c x^4+a}}{3c} + \frac{\left(A d^2 - \frac{(A e^2+2Bde) a}{3c}\right) \sqrt{1-\frac{i\sqrt{c}\, x^2}{\sqrt{a}}}\, \sqrt{1+\frac{i\sqrt{c}\, x^2}{\sqrt{a}}}\, \text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, \sqrt{c x^4+a}} + \frac{i\left(2d\right.}{}$
risch	$\frac{ex(3Be x^2+5Ae+10Bd)\sqrt{c x^4+a}}{15c} - \frac{-\frac{i\left(30Acde-9Ba e^2+15Bc d^2\right)\sqrt{a}\, \sqrt{1-\frac{i\sqrt{c}\, x^2}{\sqrt{a}}}\, \sqrt{1+\frac{i\sqrt{c}\, x^2}{\sqrt{a}}}\, \left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right) - \text{EllipticE}\left(x\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, \sqrt{c x^4+a}\, \sqrt{c}}}{}$
default	$\frac{A d^2 \sqrt{1-\frac{i\sqrt{c}\, x^2}{\sqrt{a}}}\, \sqrt{1+\frac{i\sqrt{c}\, x^2}{\sqrt{a}}}\, \text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, \sqrt{c x^4+a}} + e(Ae+2Bd)\left(\frac{x\sqrt{c x^4+a}}{3c} - \frac{a\sqrt{1-\frac{i\sqrt{c}\, x^2}{\sqrt{a}}}\, \sqrt{1+\frac{i\sqrt{c}\, x^2}{\sqrt{a}}}\, \text{EllipticF}\left(\right.}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, \sqrt{c x^4+a}}\right.$

input
```
int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*B*e^2*x^3*(c*x^4+a)^(1/2)/c+1/3*(A*e^2+2*B*d*e)/c*x*(c*x^4+a)^(1/2)+(A
*d^2-1/3*(A*e^2+2*B*d*e)/c*a)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a
^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*
(I*c^(1/2)/a^(1/2))^(1/2),I)+I*(2*d*e*A+B*d^2-3/5*B*e^2/c*a)*a^(1/2)/(I*c^
(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1
/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),
I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.56

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{3\,(5\,Bacd^2 + 10\,Aacde - 3\,Ba^2e^2)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}} E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right) \mid -1\right) - (10\,(3\,A+B)acde + 15\,(Bac -}{}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*(5*B*a*c*d^2 + 10*A*a*c*d*e - 3*B*a^2*e^2)*sqrt(c)*x*(-a/c)^(3/4)*
elliptic_e(arcsin((-a/c)^(1/4)/x), -1) - (10*(3*A + B)*a*c*d*e + 15*(B*a*c
 - A*c^2)*d^2 - (9*B*a^2 - 5*A*a*c)*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f
(arcsin((-a/c)^(1/4)/x), -1) + (3*B*a*c*e^2*x^4 + 15*B*a*c*d^2 + 30*A*a*c*
d*e - 9*B*a^2*e^2 + 5*(2*B*a*c*d*e + A*a*c*e^2)*x^2)*sqrt(c*x^4 + a))/(a*c
^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.88 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.71

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \frac{Ad^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Adex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Ae^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Bd^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Bdex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Be^2x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

output
```
A*d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4
*sqrt(a)*gamma(5/4)) + A*d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x
**4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + A*e**2*x**5*gamma(5/4)*hyp
er((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) +
B*d**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)
/(4*sqrt(a)*gamma(7/4)) + B*d*e*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,),
c*x**4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + B*e**2*x**7*gamma(7/4)*
hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4
))
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^2}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(1/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \frac{5\sqrt{c\,x^4+a}\,a\,e^2x+10\sqrt{c\,x^4+a}\,bdex+3\sqrt{c\,x^4+a}\,b\,e^2x^3-5\left(\int\frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)a^2e^2-10\left(\int\frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)abd}{1}$$

input
```
int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

output
```
(5*sqrt(a + c*x**4)*a*e**2*x + 10*sqrt(a + c*x**4)*b*d*e*x + 3*sqrt(a + c*
x**4)*b*e**2*x**3 - 5*int(sqrt(a + c*x**4)/(a + c*x**4),x)*a**2*e**2 - 10*
int(sqrt(a + c*x**4)/(a + c*x**4),x)*a*b*d*e + 15*int(sqrt(a + c*x**4)/(a
+ c*x**4),x)*a*c*d**2 - 9*int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*a*b*
e**2 + 30*int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*a*c*d*e + 15*int((sq
rt(a + c*x**4)*x**2)/(a + c*x**4),x)*b*c*d**2)/(15*c)
```

3.69 $\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 275

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx = \frac{Bex\sqrt{a+cx^4}}{3c}+\frac{(Bd+Ae)x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}(Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\left(3Acd-aBe+3\sqrt{a}\sqrt{c}(Bd+Ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}$$

output
```
1/3*B*e*x*(c*x^4+a)^(1/2)/c+(A*e+B*d)*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c
^(1/2)*x^2)-a^(1/4)*(A*e+B*d)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^
(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)/c^(3/4)/(c*x^4+a)^(1/2)+1/6*(3*A*c*d-B*a*e+3*a^(1/2)*c^(1/2)*(A*e+B*d))*
(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJac
obiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(5/4)/(c*x^4+a)^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.44

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{Bex(a+cx^4)+(3Acd-aBe)x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+c(Bd+Ae)x^3\sqrt{1+\frac{cx^4}{a}}}{3c\sqrt{a+cx^4}}$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2))/Sqrt[a + c*x^4],x]
```

output
```
(B*e*x*(a + c*x^4) + (3*A*c*d - a*B*e)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometri
c2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + c*(B*d + A*e)*x^3*Sqrt[1 + (c*x^4)/a]*
Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(3*c*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.57, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx$$
$$\Big\downarrow \ 2259$$
$$\int \left(\frac{x^2(Ae+Bd)}{\sqrt{a+cx^4}}+\frac{Ad}{\sqrt{a+cx^4}}+\frac{Bex^4}{\sqrt{a+cx^4}}\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{a^{3/4}Be(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}+\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(Ae+Bd)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(Ae+Bd)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{Ad(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x\sqrt{a+cx^4}(Ae+Bd)}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{Bex\sqrt{a+cx^4}}{3c}$$

input
```
Int[((A + B*x^2)*(d + e*x^2))/Sqrt[a + c*x^4],x]
```

output
```
(B*e*x*Sqrt[a + c*x^4])/(3*c) + ((B*d + A*e)*x*Sqrt[a + c*x^4])/(Sqrt[c]*(
Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (A*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]) - (a^(3/4)*B*e*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*c^(5/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(B*
d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^
4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.74

method	result
elliptic	$\frac{Bex\sqrt{c\,x^4+a}}{3c}+\frac{\left(Ad-\frac{aBe}{3c}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{i(Ae+Bd)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{Elliptic}\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+}}$
default	$\frac{Ad\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{i(Ae+Bd)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$
risch	$\frac{Bex\sqrt{c\,x^4+a}}{3c}+\frac{\frac{3i(Ae+Bd)\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{3Acd\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4}}}{3c}$

input
```
int((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*e*x*(c*x^4+a)^(1/2)/c+(A*d-1/3*a*B*e/c)/(I*c^(1/2)/a^(1/2))^(1/2)*(1
-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1
/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+I*(A*e+B*d)*a^(1/2)/(I*c^(1/2
)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))
^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-E
llipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.46

$$\int \frac{(A+Bx^2)\,(d+ex^2)}{\sqrt{a+cx^4}}\,dx$$
$$=\frac{3\,(Bad+Aae)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)-((3\,A+B)ae+3\,(Ba-Ac)d)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsi}{3\,acx}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*(B*a*d + A*a*e)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/
4)/x), -1) - ((3*A + B)*a*e + 3*(B*a - A*c)*d)*sqrt(c)*x*(-a/c)^(3/4)*elli
ptic_f(arcsin((-a/c)^(1/4)/x), -1) + (B*a*e*x^2 + 3*B*a*d + 3*A*a*e)*sqrt(
c*x^4 + a))/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.61

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx = \frac{Adx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Aex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Bdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Bex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

output
```
A*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + A*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*e
xp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + B*d*x**3*gamma(3/4)*hyper((1/2,
 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + B*e*x**5
*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*gamma(9/4))
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(1/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{\sqrt{c\,x^4+a}\,bex - \left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)abe + 3\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)acd + 3\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)ace + 3\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)b}{3c}$$

input
```
int((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + c*x**4)*b*e*x - int(sqrt(a + c*x**4)/(a + c*x**4),x)*a*b*e + 3*i
nt(sqrt(a + c*x**4)/(a + c*x**4),x)*a*c*d + 3*int((sqrt(a + c*x**4)*x**2)/
(a + c*x**4),x)*a*c*e + 3*int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*b*c*
d)/(3*c)
```

3.70 $\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 227

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx = \frac{Bx\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}$$

output
```
B*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)-a^(1/4)*B*(a^(1/2)+c^(1/
2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)+1/2*(a^(1/2)*B+A*c
^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*In
verseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(3/4)/(c*
x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx = \frac{\sqrt{1+\frac{cx^4}{a}}\left(3Ax\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+Bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)\right)}{3\sqrt{a+cx^4}}$$

input
```
Integrate[(A + B*x^2)/Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[1 + (c*x^4)/a]*(3*A*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
 + B*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[a + c*x^
4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \text{1512}$$

$$\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{\sqrt{a}B\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{B\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{B\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{B\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

input
```
Int[(A + B*x^2)/Sqrt[a + c*x^4],x]
```

output
```
-((B*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c]) + ((A
+ (Sqrt[a]*B)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)
*c^(1/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.74

method	result
default	$\frac{A\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{iB\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$
elliptic	$\frac{A\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{iB\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$

input
```
int((B*x^2+A)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x
^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)
+I*B*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+
I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/
2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx = \frac{Ba\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1) - (Ba-Ac)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1) + \sqrt{cx^4+a}B}{acx}$$

input

```
integrate((B*x^2+A)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output

```
(B*a*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) - (B*a
- A*c)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + sqr
t(c*x^4 + a)*B*a)/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx = \frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input

```
integrate((B*x**2+A)/(c*x**4+a)**(1/2),x)
```

output

```
A*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_p
olar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((A + B*x^2)/(a + c*x^4)^(1/2),x)
```

output
```
int((A + B*x^2)/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx = \left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)a + \left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)b$$

input
```
int((B*x^2+A)/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a + c*x**4),x)*a + int((sqrt(a + c*x**4)*x**2)/(a +
c*x**4),x)*b
```

3.71 $\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 372

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\frac{(Bd-Ae)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2+ae^2}} - \frac{(\sqrt{a}B-A\sqrt{c})(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}(\sqrt{c}d-\sqrt{a}e)\sqrt{a+cx^4}} + \frac{(\sqrt{c}d+\sqrt{a}e)(Bd-Ae)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de(\sqrt{c}d-\sqrt{a}e)\sqrt{a+cx^4}}$$

output
```
-1/2*(-A*e+B*d)*arctan((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/
2))/d^(1/2)/e^(1/2)/(a*e^2+c*d^2)^(1/2)-1/2*(a^(1/2)*B-A*c^(1/2))*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(1/4)/(c^(1/2)*d-a^(1/2)*
e)/(c*x^4+a)^(1/2)+1/4*(c^(1/2)*d+a^(1/2)*e)*(-A*e+B*d)*(a^(1/2)+c^(1/2)*x
^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1
/4)*x/a^(1/4))),-1/4*(c^(1/2)*d-a^(1/2)*e)^2/a^(1/2)/c^(1/2)/d/e,1/2*2^(1/
2))/a^(1/4)/c^(1/4)/d/e/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.37

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\frac{i\sqrt{1+\frac{cx^4}{a}}\left(Bd\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)+(-Bd+Ae)\,\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d},i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right.\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}de\sqrt{a+cx^4}}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
((-I)*Sqrt[1 + (c*x^4)/a]*(B*d*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a
]]*x], -1] + (-(B*d) + A*e)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*Arc
Sinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e*Sq
rt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}\,(d+ex^2)}dx$$

$$\Big\downarrow \text{ 2227}$$

$$\frac{\sqrt{a}(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B-A\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

$$\Big\downarrow \text{ 2221}$$

$$\frac{(Bd-Ae)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{x\sqrt{ae^2+\cdots}}{\sqrt{d}\sqrt{e}\sqrt{a\cdots}}\right.}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}\right.}{\sqrt{c}d-\sqrt{a}e}$$

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B-A\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
-1/2*((Sqrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^
(1/4)*c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4]) + ((B*d - A*e)*(-1/
2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]
*Sqrt[a + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c]*d +
Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e),
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + c*x^4
])))/(Sqrt[c]*d - Sqrt[a]*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.52

method	result
default	$\frac{B\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{(Ae-Bd)\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{ed\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$
elliptic	$\frac{B\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)A}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-$

input
```
int((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/e/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)
*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),
I)+(A*e-B*d)/e/d/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)
*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I*c^(1/2)/a
^(1/2))^(1/2),I/c^(1/2)*a^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/
a^(1/2))^(1/2))
```

Fricas [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + a)*(B*x^2 + A)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d),
x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{a+cx^4}\,(d+ex^2)}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(a + c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{c\,x^4+a}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\left(\int \frac{\sqrt{c\,x^4+a}}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)a+\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)b$$

input
```
int((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6),x)*a + int((sq
rt(a + c*x**4)*x**2)/(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6),x)*b
```

3.72 $\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 641

$$\begin{aligned}
\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx &= \frac{\sqrt{c}(Bd-Ae)x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{e(Bd-Ae)x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)} \\
&- \frac{\left(Bcd^3-3Acd^2e-aBde^2-aAe^3\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\sqrt{e}\left(cd^2+ae^2\right)^{3/2}} \\
&- \frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\left(cd^2+ae^2\right)\sqrt{a+cx^4}} \\
&+ \frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}} \\
&+ \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(Bcd^3-3Acd^2e-aBde^2-aAe^3\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2e\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\sqrt{a+cx^4}}
\end{aligned}$$

output
```
1/2*c^(1/2)*(-A*e+B*d)*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(a^(1/2)+c^(1/2)*
x^2)-1/2*e*(-A*e+B*d)*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)-1/4*(-A*
a*e^3-3*A*c*d^2*e-B*a*d*e^2+B*c*d^3)*arctan((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/
e^(1/2)/(c*x^4+a)^(1/2))/d^(3/2)/e^(1/2)/(a*e^2+c*d^2)^(3/2)-1/2*a^(1/4)*c
^(1/4)*(-A*e+B*d)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/d/(a*e^2+c
*d^2)/(c*x^4+a)^(1/2)+1/2*A*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1
/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*
2^(1/2))/a^(1/4)/d/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+a)^(1/2)+1/8*(c^(1/2)*d+a^
(1/2)*e)*(-A*a*e^3-3*A*c*d^2*e-B*a*d*e^2+B*c*d^3)*(a^(1/2)+c^(1/2)*x^2)*((
c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/
a^(1/4))),-1/4*(c^(1/2)*d-a^(1/2)*e)^2/a^(1/2)/c^(1/2)/d/e,1/2*2^(1/2))/a^
(1/4)/c^(1/4)/d^2/e/(c^(1/2)*d-a^(1/2)*e)/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.31 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.46

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$$
$$= \frac{\frac{de(-Bd+Ae)x(a+cx^4)}{(cd^2+ae^2)(d+ex^2)} - \frac{i\sqrt{1+\frac{cx^4}{a}}\left(i\sqrt{a}\sqrt{c}de(Bd-Ae)E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{c}d(\sqrt{c}d-i\sqrt{a}e)(Bd-Ae)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\right.\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}(cd^2e+ae}}{2d^2\sqrt{a+cx^4}}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

output
```
((d*e*(-(B*d) + A*e)*x*(a + c*x^4))/((c*d^2 + a*e^2)*(d + e*x^2)) - (I*Sqr
t[1 + (c*x^4)/a]*(I*Sqrt[a]*Sqrt[c]*d*e*(B*d - A*e)*EllipticE[I*ArcSinh[Sq
rt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*d*(Sqrt[c]*d - I*Sqrt[a]*e)*(B*d
 - A*e)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + (-(B*c*d^3
) + 3*A*c*d^2*e + a*B*d*e^2 + a*A*e^3)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c
]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt
[a]]*(c*d^2*e + a*e^3)))/(2*d^2*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 600, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {2211, 25, 2233, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 2211}$$

$$-\frac{\int -\frac{ce(Bd-Ae)x^4+2cd(Bd-Ae)x^2+2Acd^2+aAe^2+aBde}{(ex^2+d)\sqrt{cx^4+a}}dx}{2d\left(ae^2+cd^2\right)}-\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{ce(Bd-Ae)x^4+2cd(Bd-Ae)x^2+2Acd^2+aAe^2+aBde}{(ex^2+d)\sqrt{cx^4+a}}dx}{2d\left(ae^2+cd^2\right)}-\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 2233}$$

$$\frac{\frac{\int \frac{ce\left(2Acd^2+\sqrt{a}\sqrt{c}(Bd-Ae)d+\sqrt{c}(\sqrt{c}d+\sqrt{a}e)(Bd-Ae)x^2+ae(Bd+Ae)\right)}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{ce}-\sqrt{a}\sqrt{c}(Bd-Ae)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{2d\left(ae^2+cd^2\right)}-\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2Acd^2+\sqrt{a}\sqrt{c}(Bd-Ae)d+\sqrt{c}(\sqrt{c}d+\sqrt{a}e)(Bd-Ae)x^2+ae(Bd+Ae)}{(ex^2+d)\sqrt{cx^4+a}}dx-\sqrt{c}(Bd-Ae)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{2d\left(ae^2+cd^2\right)}-\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{\int \frac{2Acd^2+\sqrt{a}\sqrt{c}(Bd-Ae)d+\sqrt{c}(\sqrt{c}d+\sqrt{a}e)(Bd-Ae)x^2+ae(Bd+Ae)}{(ex^2+d)\sqrt{cx^4+a}}dx-\sqrt{c}(Bd-Ae)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{2d(ae^2+cd^2)}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d(d+ex^2)(ae^2+cd^2)}$$

$$\Big\downarrow \text{2227}$$

$$\frac{\frac{\sqrt{a}(-aAe^3-aBde^2-3Acd^2e+Bcd^3)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2A\sqrt{c}(ae^2+cd^2)\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\sqrt{c}(Bd-Ae)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{}{(\sqrt{}}}}{}\right.}{2d(ae^2+cd^2)}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d(d+ex^2)(ae^2+cd^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(-aAe^3-aBde^2-3Acd^2e+Bcd^3)\int\frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2A\sqrt{c}(ae^2+cd^2)\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\sqrt{c}(Bd-Ae)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x}}}{\sqrt[4]{c}\sqrt{}}\right.}{2d(ae^2+cd^2)}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d(d+ex^2)(ae^2+cd^2)}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{(-aAe^3-aBde^2-3Acd^2e+Bcd^3)\int\frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\sqrt{c}(Bd-Ae)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\right.}{2d(ae^2+cd^2)}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d(d+ex^2)(ae^2+cd^2)}$$

$$\Big\downarrow \text{2221}$$

$$\frac{\left(-aAe^3-aBde^2-3Acd^2e+Bcd^3\right)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{-}{\sqrt{}}\right.}{2\sqrt{d}\sqrt{e}\sqrt{ae^2}}\right.}{\sqrt{c}d-\sqrt{a}e}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

output
```
-1/2*(e*(B*d - A*e)*x*Sqrt[a + c*x^4])/(d*(c*d^2 + a*e^2)*(d + e*x^2)) + (
-(Sqrt[c]*(B*d - A*e)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a
^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4]))
) + (A*c^(1/4)*(c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a
^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4]) + ((B*c*d^3 - 3*A*c*d^2*e
- a*B*d*e^2 - a*A*e^3)*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 +
 a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2
 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt
[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)
*c^(1/4)*d*e*Sqrt[a + c*x^4])))/(Sqrt[c]*d - Sqrt[a]*e))/(2*d*(c*d^2 + a*e
^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 2211
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))
Int[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(
2*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(
C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.06

method	result
default	$\frac{B\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{ed\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{(Ae-Bd)\left(\frac{e^2x\sqrt{c\,x^4+a}}{2\left(a\,e^2+c\,d^2\right)d\left(e\,x^2+d\right)}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{E}}{2\left(a\,e^2+c\,d^2\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right.}{}$
elliptic	Expression too large to display

input
```
int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/e/d/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/
2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/
2),I/c^(1/2)*a^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1
/2))+(A*e-B*d)/e*(1/2*e^2/(a*e^2+c*d^2)/d*x*(c*x^4+a)^(1/2)/(e*x^2+d)-1/2*
c/(a*e^2+c*d^2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*
(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(
1/2))^(1/2),I)-1/2*I*e*c^(1/2)/(a*e^2+c*d^2)/d*a^(1/2)/(I*c^(1/2)/a^(1/2))
^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*
x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+1/2*I*e*c^(1/2)/(a*e
^2+c*d^2)/d*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1
/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I*c^(1/2)
/a^(1/2))^(1/2),I)+1/2/(a*e^2+c*d^2)/d^2*e^2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-
I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/
2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(1/2)*a^(1/2)/d*e,(-I/a^(1/2
)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I*c^(1/2)
/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^
(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(1/2)*a^(
1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2))*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{a+cx^4}\,(d+ex^2)^2}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(a + c*x**4)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{(d + ex^2)^2 \sqrt{a + cx^4}} \, dx = \int \frac{B\,x^2 + A}{\sqrt{c\,x^4 + a}\,(e\,x^2 + d)^2} \, dx$$

input
```
int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)^2),x)
```

output
```
int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{A + Bx^2}{(d + ex^2)^2 \sqrt{a + cx^4}} \, dx \\
&= \left(\int \frac{\sqrt{c\,x^4 + a}}{c\,e^2x^8 + 2cde\,x^6 + a\,e^2x^4 + c\,d^2x^4 + 2ade\,x^2 + a\,d^2}dx\right) a \\
&\quad + \left(\int \frac{\sqrt{c\,x^4 + a}\,x^2}{c\,e^2x^8 + 2cde\,x^6 + a\,e^2x^4 + c\,d^2x^4 + 2ade\,x^2 + a\,d^2}dx\right) b
\end{aligned}$$

input
```
int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 + c*d**2*x**4 +
2*c*d*e*x**6 + c*e**2*x**8),x)*a + int((sqrt(a + c*x**4)*x**2)/(a*d**2 + 2
*a*d*e*x**2 + a*e**2*x**4 + c*d**2*x**4 + 2*c*d*e*x**6 + c*e**2*x**8),x)*b
```

3.73 $\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 875

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx = \frac{\sqrt{c}(5Bcd^3-9Acd^2e-aBde^2-3aAe^3)\,x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^2(\sqrt{a}+\sqrt{c}x^2)}$$

$$-\frac{e(Bd-Ae)x\sqrt{a+cx^4}}{4d(cd^2+ae^2)(d+ex^2)^2}-\frac{e(5Bcd^3-9Acd^2e-aBde^2-3aAe^3)\,x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^2(d+ex^2)}$$

$$+\frac{(3Ae(5c^2d^4+2acd^2e^2+a^2e^4)-B(3c^2d^5-10acd^3e^2-a^2de^4))\arctan\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}\sqrt{e}(cd^2+ae^2)^{5/2}}$$

$$-\frac{\sqrt[4]{a}\sqrt[4]{c}(5Bcd^3-9Acd^2e-aBde^2-3aAe^3)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2(cd^2+ae^2)^2\sqrt{a+cx^4}}$$

$$+\frac{\sqrt[4]{c}\left(4Acd^2+\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(Bd+3Ae)\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}} \right.\right.}{8\sqrt[4]{a}d^2(\sqrt{c}d-\sqrt{a}e)(cd^2+ae^2)\sqrt{a+cx^4}}$$

$$-\frac{(\sqrt{c}d+\sqrt{a}e)(3Ae(5c^2d^4+2acd^2e^2+a^2e^4)-B(3c^2d^5-10acd^3e^2-a^2de^4))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}}{32\sqrt[4]{a}\sqrt[4]{c}d^3e(\sqrt{c}d-\sqrt{a}e)(cd^2+ae^2)^2\sqrt{a+cx^4}}$$

output
```
1/8*c^(1/2)*(-3*A*a*e^3-9*A*c*d^2*e-B*a*d*e^2+5*B*c*d^3)*x*(c*x^4+a)^(1/2)
/d^2/(a*e^2+c*d^2)^2/(a^(1/2)+c^(1/2)*x^2)-1/4*e*(-A*e+B*d)*x*(c*x^4+a)^(1
/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2-1/8*e*(-3*A*a*e^3-9*A*c*d^2*e-B*a*d*e^2+5*
B*c*d^3)*x*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)+1/16*(3*A*e*(a^2*
e^4+2*a*c*d^2*e^2+5*c^2*d^4)-B*(-a^2*d*e^4-10*a*c*d^3*e^2+3*c^2*d^5))*arct
an((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))/d^(5/2)/e^(1/2)/
(a*e^2+c*d^2)^(5/2)-1/8*a^(1/4)*c^(1/4)*(-3*A*a*e^3-9*A*c*d^2*e-B*a*d*e^2+
5*B*c*d^3)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)
*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/d^2/(a*e^2+c*d^2)
^2/(c*x^4+a)^(1/2)+1/8*c^(1/4)*(4*A*c*d^2+a^(1/2)*c^(1/2)*d*(-A*e+B*d)+a*e
*(3*A*e+B*d))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1
/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/d^2/(
c^(1/2)*d-a^(1/2)*e)/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)-1/32*(c^(1/2)*d+a^(1/2)
*e)*(3*A*e*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)-B*(-a^2*d*e^4-10*a*c*d^3*e^2+
3*c^2*d^5))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2
)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(1/2)*d-a^(1/2)*e)^2
/a^(1/2)/c^(1/2)/d/e,1/2*2^(1/2))/a^(1/4)/c^(1/4)/d^3/e/(c^(1/2)*d-a^(1/2)
*e)/(a*e^2+c*d^2)^2/(c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.27 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx$$

$$= \frac{-\frac{de^2x(a+cx^4)\left(2d(Bd-Ae)(cd^2+ae^2)+(5Bcd^3-9Acd^2e-aBde^2-3aAe^3)(d+ex^2)\right)}{(d+ex^2)^2} - \frac{i\sqrt{1+\frac{cx^4}{a}}\left(-i\sqrt{a}\sqrt{c}de(-5Bcd^3+9Acd^2e+aBde^2+\right.}{}}{}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

output
```
(-((d*e^2*x*(a + c*x^4)*(2*d*(B*d - A*e)*(c*d^2 + a*e^2) + (5*B*c*d^3 - 9*
A*c*d^2*e - a*B*d*e^2 - 3*a*A*e^3)*(d + e*x^2)))/(d + e*x^2)^2) - (I*Sqrt[
1 + (c*x^4)/a]*((-I)*Sqrt[a]*Sqrt[c]*d*e*(-5*B*c*d^3 + 9*A*c*d^2*e + a*B*d
*e^2 + 3*a*A*e^3)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] +
Sqrt[c]*d*(Sqrt[c]*d - I*Sqrt[a]*e)*(A*e*(-7*c*d^2 + (2*I)*Sqrt[a]*Sqrt[c]
*d*e - 3*a*e^2) + B*d*(3*c*d^2 - (2*I)*Sqrt[a]*Sqrt[c]*d*e - a*e^2))*Ellip
ticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + (3*A*e*(5*c^2*d^4 + 2*a
*c*d^2*e^2 + a^2*e^4) + B*(-3*c^2*d^5 + 10*a*c*d^3*e^2 + a^2*d*e^4))*Ellip
ticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x]
, -1]))/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(8*d^3*e*(c*d^2 + a*e^2)^2*Sqrt[a + c*x
^4])
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 788, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2211, 25, 2211, 25, 2233, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{ 2211}$$

$$-\frac{\int -\frac{-ce(Bd-Ae)x^4+4cd(Bd-Ae)x^2+4Acd^2+3aAe^2+aBde}{\left(ex^2+d\right)^2\sqrt{cx^4+a}}dx}{4d\left(ae^2+cd^2\right)}-\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-ce(Bd-Ae)x^4+4cd(Bd-Ae)x^2+4Acd^2+3aAe^2+aBde}{\left(ex^2+d\right)^2\sqrt{cx^4+a}}dx}{4d\left(ae^2+cd^2\right)}-\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 2211}$$

$$\frac{-\frac{\int -\frac{ce\left(5Bcd^3-9Aced^2-aBe^2d-3aAe^3\right)x^4+4cd\left(2Bcd^3-4Aced^2-aBe^2d-aAe^3\right)x^2+aBde\left(7cd^2+ae^2\right)+A\left(8c^2d^4+5ace^2d^2+3a^2e^4\right)}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{2d(ae^2+cd^2)}-\frac{ex\sqrt{a+cx^4}(-\cdots}{\cdots}}{4d\left(ae^2+cd^2\right)}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{ce\left(5Bcd^3-9Aced^2-aBe^2d-3aAe^3\right)x^4+4cd\left(2Bcd^3-4Aced^2-aBe^2d-aAe^3\right)x^2+aBde\left(7cd^2+ae^2\right)+A\left(8c^2d^4+5ace^2d^2+3a^2e^4\right)}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{2d(ae^2+cd^2)}-\frac{ex\sqrt{a+cx^4}(-3aA\cdots}{2d(\cdots}}{4d\left(ae^2+cd^2\right)}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 2233$$

$$\frac{\frac{\frac{\int \frac{ce\left(8Ac^2d^4+\sqrt{a}c^{3/2}(5Bd-9Ae)d^3+ace(7Bd+5Ae)d^2-a^{3/2}\sqrt{c}e^2(Bd+3Ae)d-\left(\left(cd-\sqrt{a}\sqrt{c}e\right)\left(5Bcd^3-9Aced^2-aBe^2d-3aAe^3\right)-4cd\left(2Bcd^3-4Aced^2-aBe^2\cdots\right.\right.\right.}{\left(ex^2+d\right)\sqrt{cx^4+a}}\cdots}{ce}\cdots}{2d(ae^2+cd^2)}\cdots}{4d\left(ae^2\cdots\right.}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{8Ac^2d^4+\sqrt{a}c^{3/2}(5Bd-9Ae)d^3+ace(7Bd+5Ae)d^2-a^{3/2}\sqrt{c}e^2(Bd+3Ae)d-\left(\left(cd-\sqrt{a}\sqrt{c}e\right)\left(5Bcd^3-9Aced^2-aBe^2d-3aAe^3\right)-4cd\left(2Bcd^3-4Aced^2-aBe^2d-\cdots\right.\right.}{\left(ex^2+d\right)\sqrt{cx^4+a}}\cdots}{2d(ae^2+cd^2)}\cdots}{4d\left(ae^2+\cdots\right.}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 1510$$

$$\frac{\int \frac{8Ac^2d^4+\sqrt{a}c^{3/2}(5Bd-9Ae)d^3+ace(7Bd+5Ae)d^2-a^{3/2}\sqrt{c}e^2(Bd+3Ae)d-\left(\left(cd-\sqrt{a}\sqrt{c}e\right)\left(5Bcd^3-9Aced^2-aBe^2d-3aAe^3\right)-4cd\left(2Bcd^3-4Aced^2-aBe^2d-\cdots\right.\right.}{\left(ex^2+d\right)\sqrt{cx^4+a}}\cdots}{2d(ae^2+\cdots}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 2227

$$-\frac{\frac{\sqrt{a}\left(3Ae\left(a^2e^4+2acd^2e^2+5c^2d^4\right)-B\left(-a^2de^4-10acd^3e^2+3c^2d^5\right)\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2\sqrt{c}\left(ae^2+cd^2\right)\left(\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(3Ae+Bd)+4Acd^2\right)\int -}{\sqrt{c}d-\sqrt{a}e}}{2d(ae^2+cd^2)}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\left(3Ae\left(a^2e^4+2acd^2e^2+5c^2d^4\right)-B\left(-a^2de^4-10acd^3e^2+3c^2d^5\right)\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2\sqrt{c}\left(ae^2+cd^2\right)\left(\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(3Ae+Bd)+4Acd^2\right)\int\frac{1}{\sqrt{cx^4+}}}{\sqrt{c}d-\sqrt{a}e}}{2d(ae^2+cd^2)}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 761

$$-\frac{\frac{\left(3Ae\left(a^2e^4+2acd^2e^2+5c^2d^4\right)-B\left(-a^2de^4-10acd^3e^2+3c^2d^5\right)\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^2+cd^2\right)\text{EllipticF}\left(2\arctan\left(\right.\right.}{\sqrt[4]{a}\sqrt{a+cx^4}(\sqrt{c}d-}}{}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

$\Big\downarrow$ 2221

$$-\frac{\left(3Ae\left(a^2e^4+2acd^2e^2+5c^2d^4\right)-B\left(-a^2de^4-10acd^3e^2+3c^2d^5\right)\right)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\right.}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}}\right.}{\sqrt{c}d-\sqrt{a}e}$$

$$\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2+cd^2\right)}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

output
```
-1/4*(e*(B*d - A*e)*x*Sqrt[a + c*x^4])/(d*(c*d^2 + a*e^2)*(d + e*x^2)^2) +
 (-1/2*(e*(5*B*c*d^3 - 9*A*c*d^2*e - a*B*d*e^2 - 3*a*A*e^3)*x*Sqrt[a + c*x
^4])/(d*(c*d^2 + a*e^2)*(d + e*x^2)) + (-(Sqrt[c]*(5*B*c*d^3 - 9*A*c*d^2*e
 - a*B*d*e^2 - 3*a*A*e^3)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2))
+ (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^
4]))) + (c^(1/4)*(c*d^2 + a*e^2)*(4*A*c*d^2 + Sqrt[a]*Sqrt[c]*d*(B*d - A*e
) + a*e*(B*d + 3*A*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*(
Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4]) - ((3*A*e*(5*c^2*d^4 + 2*a*c*d^2*e
^2 + a^2*e^4) - B*(3*c^2*d^5 - 10*a*c*d^3*e^2 - a^2*d*e^4))*(-1/2*((Sqrt[c
]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a +
c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticPi[-1/4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + c*x^4])))/(Sqrt
[c]*d - Sqrt[a]*e))/(2*d*(c*d^2 + a*e^2)))/(4*d*(c*d^2 + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 2211
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Simp[1/(2*d*(q + 1)*(c*d^2 + a*e^2))
Int[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(
2*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(
C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 1591, normalized size of antiderivative = 1.82

method	result	size
default	Expression too large to display	1591
elliptic	Expression too large to display	1982

input
```
int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/e*(1/2*e^2/(a*e^2+c*d^2)/d*x*(c*x^4+a)^(1/2)/(e*x^2+d)-1/2*c/(a*e^2+c*d^
2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*
x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I
)-1/2*I*e*c^(1/2)/(a*e^2+c*d^2)/d*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c
^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*
EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+1/2*I*e*c^(1/2)/(a*e^2+c*d^2)/d*a
^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1
/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/
2),I)+1/2/(a*e^2+c*d^2)/d^2*e^2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2
/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticPi
(x*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(1/2)*a^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1
/2)/(I*c^(1/2)/a^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I*c^(1/2)/a^(1/2))^(1/
2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+
a)^(1/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(1/2)*a^(1/2)/d*e,(-I/
a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2))*c)+(A*e-B*d)/e*(1/4*e^2/
(a*e^2+c*d^2)/d*x*(c*x^4+a)^(1/2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^2)/(a*e
^2+c*d^2)^2/d^2*x*(c*x^4+a)^(1/2)/(e*x^2+d)-1/8*c/d/(a*e^2+c*d^2)^2/(I*c^(
1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/
2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)*a*e^2-7
/8*c^2*d/(a*e^2+c*d^2)^2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{a+cx^4}\,(d+ex^2)^3}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(a + c*x**4)*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)^3}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{c\,x^4+a}\,(e\,x^2+d)^3}\,dx$$

input
```
int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)^3),x)
```

output
```
int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx$$
$$=\left(\int \frac{\sqrt{c\,x^4+a}}{c\,e^3x^{10}+3cd\,e^2x^8+a\,e^3x^6+3c\,d^2e\,x^6+3ad\,e^2x^4+c\,d^3x^4+3a\,d^2e\,x^2+a\,d^3}dx\right)a$$
$$+\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,e^3x^{10}+3cd\,e^2x^8+a\,e^3x^6+3c\,d^2e\,x^6+3ad\,e^2x^4+c\,d^3x^4+3a\,d^2e\,x^2+a\,d^3}dx\right)b$$

input
```
int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*
x**6 + c*d**3*x**4 + 3*c*d**2*e*x**6 + 3*c*d*e**2*x**8 + c*e**3*x**10),x)*
a + int((sqrt(a + c*x**4)*x**2)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**
4 + a*e**3*x**6 + c*d**3*x**4 + 3*c*d**2*e*x**6 + 3*c*d*e**2*x**8 + c*e**3
*x**10),x)*b
```

3.74 $\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 471

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \frac{x(Acd(cd^2-3ae^2)-aBe(3cd^2-ae^2)+c(Bcd^3+3Acd^2e-3aBde^2-aAe}{2ac^2\sqrt{a+cx^4}}$$

$$+\frac{Be^3x\sqrt{a+cx^4}}{3c^2}-\frac{(Bcd^3+3Acd^2e-9aBde^2-3aAe^3)\,x\sqrt{a+cx^4}}{2ac^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$+\frac{(Bcd^3+3Acd^2e-9aBde^2-3aAe^3)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{7/4}\sqrt{a+cx^4}}$$

$$+\frac{\left(3Ac^2d^3-5a^2Be^3+9acde(Bd+Ae)+9a^{3/2}\sqrt{c}e^2(3Bd+Ae)-3\sqrt{a}c^{3/2}d^2(Bd+3Ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}{12a^{5/4}c^{9/4}\sqrt{a+cx^4}}$$

output

```
1/2*x*(A*c*d*(-3*a*e^2+c*d^2)-a*B*e*(-a*e^2+3*c*d^2)+c*(-A*a*e^3+3*A*c*d^2
*e-3*B*a*d*e^2+B*c*d^3)*x^2)/a/c^2/(c*x^4+a)^(1/2)+1/3*B*e^3*x*(c*x^4+a)^(
1/2)/c^2-1/2*(-3*A*a*e^3+3*A*c*d^2*e-9*B*a*d*e^2+B*c*d^3)*x*(c*x^4+a)^(1/2
)/a/c^(3/2)/(a^(1/2)+c^(1/2)*x^2)+1/2*(-3*A*a*e^3+3*A*c*d^2*e-9*B*a*d*e^2+
B*c*d^3)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*E
llipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/c^(7/4)/(c*
x^4+a)^(1/2)+1/12*(3*A*c^2*d^3-5*a^2*B*e^3+9*a*c*d*e*(A*e+B*d)+9*a^(3/2)*c
^(1/2)*e^2*(A*e+3*B*d)-3*a^(1/2)*c^(3/2)*d^2*(3*A*e+B*d))*(a^(1/2)+c^(1/2)
*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c
^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/c^(9/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.47

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \frac{3Acx(cd^3+ae^2(-3d+2ex^2))+aBex(5ae^2+c(-9d^2+18dex^2+2e^2x^4))\ -}{}$$

input

```
Integrate[((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(3/2),x]
```

output

```
(3*A*c*x*(c*d^3 + a*e^2*(-3*d + 2*e*x^2)) + a*B*e*x*(5*a*e^2 + c*(-9*d^2 +
 18*d*e*x^2 + 2*e^2*x^4)) + (a*B*e*(9*c*d^2 - 5*a*e^2) + 3*A*c*d*(c*d^2 +
3*a*e^2))*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)
/a)] + 2*c*(B*c*d^3 + 3*A*c*d^2*e - 9*a*B*d*e^2 - 3*a*A*e^3)*x^3*Sqrt[1 +
(c*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(6*a*c^2*Sqrt[a
 + c*x^4])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 912, normalized size of antiderivative = 1.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2259}$$

$$\int \left(\frac{e(-aBe^2+3Acde+3Bcd^2)}{c^2\sqrt{a+cx^4}}+\frac{cx^2(-aAe^3-3aBde^2+3Acd^2e+Bcd^3)+Acd(cd^2-3ae^2)-aBe(3cd^2-$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^{3/4}B\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e^3}{6c^{9/4}\sqrt{cx^4+a}}+\frac{Bx\sqrt{cx^4+a}e^3}{3c^2}-\frac{\sqrt[4]{a}(3Bd+Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{c^{7/4}\sqrt{cx^4+a}}+\frac{\sqrt[4]{a}(3Bd+Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e^2}{2c^{7/4}\sqrt{cx^4+a}}+\frac{(3Bd+Ae)x\sqrt{cx^4+a}e^2}{c^{3/2}\left(\sqrt{c}x^2+\sqrt{a}\right)}+\frac{\left(3Bcd^2+3Aced-aBe^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}c^{9/4}\sqrt{cx^4+a}}+\frac{\left(Bcd^3+3Aced^2-3aBe^2d-aAe^3\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{7/4}\sqrt{cx^4+a}}+\frac{\left(Ac^2d^3-\sqrt{a}c^{3/2}(Bd+3Ae)d^2-3ace(Bd+Ae)d+a^2Be^3+a^{3/2}\sqrt{c}e^2(3Bd+Ae)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}}{4a^{5/4}c^{9/4}\sqrt{cx^4+a}}\frac{\left(Bcd^3+3Aced^2-3aBe^2d-aAe^3\right)x\sqrt{cx^4+a}}{2ac^{3/2}\left(\sqrt{c}x^2+\sqrt{a}\right)}+\frac{x\left(c\left(Bcd^3+3Aced^2-3aBe^2d-aAe^3\right)x^2+Acd\left(cd^2-3ae^2\right)-aBe\left(3cd^2-ae^2\right)\right)}{2ac^2\sqrt{cx^4+a}}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(3/2),x]
```

output
```
(x*(A*c*d*(c*d^2 - 3*a*e^2) - a*B*e*(3*c*d^2 - a*e^2) + c*(B*c*d^3 + 3*A*c
*d^2*e - 3*a*B*d*e^2 - a*A*e^3)*x^2))/(2*a*c^2*Sqrt[a + c*x^4]) + (B*e^3*x
*Sqrt[a + c*x^4])/(3*c^2) + (e^2*(3*B*d + A*e)*x*Sqrt[a + c*x^4])/(c^(3/2)
*(Sqrt[a] + Sqrt[c]*x^2)) - ((B*c*d^3 + 3*A*c*d^2*e - 3*a*B*d*e^2 - a*A*e^
3)*x*Sqrt[a + c*x^4])/(2*a*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e^2
*(3*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(7/4)*Sqrt[a +
c*x^4]) + ((B*c*d^3 + 3*A*c*d^2*e - 3*a*B*d*e^2 - a*A*e^3)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(7/4)*Sqrt[a + c*x^4]) - (a^(3/4)*B
*e^3*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*c^(9/4)*Sqrt[a + c*x^4])
+ (a^(1/4)*e^2*(3*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c
^(7/4)*Sqrt[a + c*x^4]) + (e*(3*B*c*d^2 + 3*A*c*d*e - a*B*e^2)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(9/4)*Sqrt[a + c*x^4]) + ((A*c^
2*d^3 + a^2*B*e^3 - 3*a*c*d*e*(B*d + A*e) + a^(3/2)*Sqrt[c]*e^2*(3*B*d + A
*e) - Sqrt[a]*c^(3/2)*d^2*(B*d + 3*A*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.13 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.90

method	result
elliptic	$-\frac{2c\left(\frac{\left(Aa\,e^{3}-3Ac\,d^{2}e+3Bad\,e^{2}-Bc\,d^{3}\right)x^{3}}{4c^{2}a}+\frac{\left(3Aacd\,e^{2}-A\,c^{2}d^{3}-a^{2}B\,e^{3}+3Bac\,d^{2}e\right)x}{4a\,c^{3}}\right)}{\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{B\,e^{3}x\sqrt{c\,x^{4}+a}}{3c^{2}}+\frac{\left(\frac{e\left(3Acde-Ba\,e^{2}+3Bc\,d\right.}{c^{2}}\right.}{}$
default	$A\,d^{3}\left(\frac{x}{2a\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+e^{2}(Ae+3Bd)\left(-\frac{x^{3}}{2c\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{3i\sqrt{}}{}\right.$
risch	$\frac{B\,e^{3}x\sqrt{c\,x^{4}+a}}{3c^{2}}+\frac{3c^{2}e^{2}(Ae+3Bd)\left(-\frac{x^{3}}{2c\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{2c^{\frac{3}{2}}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+}{}$

input
```
int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/4/c^2*(A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2-B*c*d^3)/a*x^3+1/4/a/c^3*(3
*A*a*c*d*e^2-A*c^2*d^3-B*a^2*e^3+3*B*a*c*d^2*e)*x)/(c*(a/c+x^4))^(1/2)+1/3
*B*e^3*x*(c*x^4+a)^(1/2)/c^2+(e*(3*A*c*d*e-B*a*e^2+3*B*c*d^2)/c^2-1/2/c^2/
a*(3*A*a*c*d*e^2-A*c^2*d^3-B*a^2*e^3+3*B*a*c*d^2*e)-1/3*B*e^3/c^2*a)/(I*c^
(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1
/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+I*(1/c
*e^2*(A*e+3*B*d)+1/2/c*(A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2-B*c*d^3)/a)*a^(1/2
)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x
^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))
^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx =$$

$$-\frac{3\left((Bac^2d^3+3\,Aac^2d^2e-9\,Ba^2cde^2-3\,Aa^2ce^3)x^5+(Ba^2cd^3+3\,Aa^2cd^2e-9\,Ba^3de^2-3\,Aa^3e^3)x\right)\sqrt{c}}{}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(3*((B*a*c^2*d^3 + 3*A*a*c^2*d^2*e - 9*B*a^2*c*d*e^2 - 3*A*a^2*c*e^3)
*x^5 + (B*a^2*c*d^3 + 3*A*a^2*c*d^2*e - 9*B*a^3*d*e^2 - 3*A*a^3*e^3)*x)*sq
rt(c)*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) - ((9*(A + B)*a*
c^2*d^2*e - (9*A + 5*B)*a^2*c*e^3 + 3*(B*a*c^2 + A*c^3)*d^3 - 9*(3*B*a^2*c
 - A*a*c^2)*d*e^2)*x^5 + (9*(A + B)*a^2*c*d^2*e - (9*A + 5*B)*a^3*e^3 + 3*
(B*a^2*c + A*a*c^2)*d^3 - 9*(3*B*a^3 - A*a^2*c)*d*e^2)*x)*sqrt(c)*(-a/c)^(
3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) - (2*B*a^2*c*e^3*x^6 - 3*B*a^2
*c*d^3 - 9*A*a^2*c*d^2*e + 27*B*a^3*d*e^2 + 9*A*a^3*e^3 + 6*(3*B*a^2*c*d*e
^2 + A*a^2*c*e^3)*x^4 + (3*A*a*c^2*d^3 - 9*B*a^2*c*d^2*e - 9*A*a^2*c*d*e^2
 + 5*B*a^3*e^3)*x^2)*sqrt(c*x^4 + a))/(a^2*c^3*x^5 + a^3*c^2*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**3/(a + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^3}{(c\,x^4+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x)
```

output
```
(5*sqrt(a + c*x**4)*a*b*e**3*x - 9*sqrt(a + c*x**4)*a*c*d*e**2*x + 3*sqrt(
a + c*x**4)*a*c*e**3*x**3 - 9*sqrt(a + c*x**4)*b*c*d**2*e*x + 9*sqrt(a + c
*x**4)*b*c*d*e**2*x**3 + sqrt(a + c*x**4)*b*c*e**3*x**5 - 5*int(sqrt(a + c
*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**3*b*e**3 + 9*int(sqrt(a + c*x
**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**3*c*d*e**2 + 9*int(sqrt(a + c*x
**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**2*b*c*d**2*e - 5*int(sqrt(a + c
*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**2*b*c*e**3*x**4 + 3*int(sqrt(
a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**2*c**2*d**3 + 9*int(sqrt
(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**2*c**2*d*e**2*x**4 + 9*
int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*b*c**2*d**2*e*x*
*4 + 3*int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c**3*d**3
*x**4 - 9*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a
**3*c*e**3 - 27*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8
),x)*a**2*b*c*d*e**2 + 9*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 +
c**2*x**8),x)*a**2*c**2*d**2*e - 9*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a
*c*x**4 + c**2*x**8),x)*a**2*c**2*e**3*x**4 + 3*int((sqrt(a + c*x**4)*x**2
)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*b*c**2*d**3 - 27*int((sqrt(a + c*x*
*4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*b*c**2*d*e**2*x**4 + 9*int(
(sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c**3*d**2*e*x
**4 + 3*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*...
```

3.75 $\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 379

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx = \frac{x(Acd^2-2aBde-aAe^2+(Bcd^2+2Acde-aBe^2)x^2)}{2ac\sqrt{a+cx^4}}$$
$$-\frac{(Bcd^2+2Acde-3aBe^2)x\sqrt{a+cx^4}}{2ac^{3/2}(\sqrt{a}+\sqrt{c}x^2)}$$
$$+\frac{(Bcd^2+2Acde-3aBe^2)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{7/4}\sqrt{a+cx^4}}$$
$$+\frac{(Ac^{3/2}d^2+3a^{3/2}Be^2+a\sqrt{c}e(2Bd+Ae)-\sqrt{a}cd(Bd+2Ae))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\,\mathrm{a}\right.}{4a^{5/4}c^{7/4}\sqrt{a+cx^4}}$$

output
```
1/2*x*(A*c*d^2-2*a*B*d*e-A*a*e^2+(2*A*c*d*e-B*a*e^2+B*c*d^2)*x^2)/a/c/(c*x
^4+a)^(1/2)-1/2*(2*A*c*d*e-3*B*a*e^2+B*c*d^2)*x*(c*x^4+a)^(1/2)/a/c^(3/2)/
(a^(1/2)+c^(1/2)*x^2)+1/2*(2*A*c*d*e-3*B*a*e^2+B*c*d^2)*(a^(1/2)+c^(1/2)*x
^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/
4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/c^(7/4)/(c*x^4+a)^(1/2)+1/4*(A*c^(3/2)
*d^2+3*a^(3/2)*B*e^2+a*c^(1/2)*e*(A*e+2*B*d)-a^(1/2)*c*d*(2*A*e+B*d))*(a^(
1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiA
M(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/c^(7/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.44

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx = \frac{3A(cd^2-ae^2)\,x+6aBex(-d+ex^2)+3(Acd^2+2aBde+aAe^2)\,x\sqrt{1+\frac{cx}{a}}}{}$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(3/2),x]
```

output
```
(3*A*(c*d^2 - a*e^2)*x + 6*a*B*e*x*(-d + e*x^2) + 3*(A*c*d^2 + 2*a*B*d*e +
 a*A*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)
/a)] + 2*(B*c*d^2 + 2*A*c*d*e - 3*a*B*e^2)*x^3*Sqrt[1 + (c*x^4)/a]*Hyperge
ometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(6*a*c*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{x^2(-aBe^2+2Acde+Bcd^2)-aAe^2-2aBde+Acd^2}{c(a+cx^4)^{3/2}}+\frac{e(Ae+2Bd)}{c\sqrt{a+cx^4}}+\frac{Be^2x^2}{c\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-\frac{\sqrt{c}(-aAe^2-2aBde+Acd^2)}{\sqrt{a}}-aBe^2+2Acde+Bcd\right.}{4a^{3/4}c^{7/4}\sqrt{a+cx^4}}$$

$$\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)(-aBe^2+2Acde+Bcd^2)}{2a^{3/4}c^{7/4}\sqrt{a+cx^4}}+\frac{e(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(Ae+2Bd)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}(-aBe^2+2Acde+Bcd^2)}{2ac^{3/2}(\sqrt{a}+\sqrt{c}x^2)}+\frac{x(x^2(-aBe^2+2Acde+Bcd^2)-aAe^2-2aBde+Acd^2)}{2ac\sqrt{a+cx^4}}+\frac{\sqrt[4]{a}Be^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{7/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}Be^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{a+cx^4}}+\frac{Be^2x\sqrt{a+cx^4}}{c^{3/2}(\sqrt{a}+\sqrt{c}x^2)}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(3/2),x]
```

output
```
(x*(A*c*d^2 - 2*a*B*d*e - a*A*e^2 + (B*c*d^2 + 2*A*c*d*e - a*B*e^2)*x^2))/
(2*a*c*Sqrt[a + c*x^4]) + (B*e^2*x*Sqrt[a + c*x^4])/(c^(3/2)*(Sqrt[a] + Sq
rt[c]*x^2)) - ((B*c*d^2 + 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*a*c^(
3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(c^(7/4)*Sqrt[a + c*x^4]) + ((B*c*d^2 + 2*A*c*d*e - a*B*e^2)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(7/4)*Sqrt[a + c*x^
4]) + (a^(1/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(7/4)*S
qrt[a + c*x^4]) + (e*(2*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(2*a^(1/4)*c^(5/4)*Sqrt[a + c*x^4]) - ((B*c*d^2 + 2*A*c*d*e - a*B*e^2 -
(Sqrt[c]*(A*c*d^2 - 2*a*B*d*e - a*A*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(4*a^(3/4)*c^(7/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.85

method	result
elliptic	$-\frac{2c\left(-\frac{\left(2Acde-Ba\,e^2+Bc\,d^2\right)x^3}{4a\,c^2}+\frac{\left(Aa\,e^2-Ac\,d^2+2aBde\right)x}{4c^2a}\right)}{\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{\left(\frac{e(Ae+2Bd)}{c}-\frac{Aa\,e^2-Ac\,d^2+2aBde}{2ac}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{Ellip}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$
default	$A\,d^2\left(\frac{x}{2a\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+e(Ae+2Bd)\left(-\frac{x}{2c\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{\sqrt{1-}}{}\right.$

input
```
int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/4*(2*A*c*d*e-B*a*e^2+B*c*d^2)/a/c^2*x^3+1/4*(A*a*e^2-A*c*d^2+2*B*
a*d*e)/c^2/a*x)/(c*(a/c+x^4))^(1/2)+(e*(A*e+2*B*d)/c-1/2*(A*a*e^2-A*c*d^2+
2*B*a*d*e)/a/c)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*
(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(
1/2))^(1/2),I)+I*(B*e^2/c-1/2*(2*A*c*d*e-B*a*e^2+B*c*d^2)/a/c)*a^(1/2)/(I*
c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^
(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2
),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx =$$
$$-\frac{((Bac^2d^2+2\,Aac^2de-3\,Ba^2ce^2)x^5+(Ba^2cd^2+2\,Aa^2cde-3\,Ba^3e^2)x)\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|\,\cdots\right.}{\cdots}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(((B*a*c^2*d^2 + 2*A*a*c^2*d*e - 3*B*a^2*c*e^2)*x^5 + (B*a^2*c*d^2 +
2*A*a^2*c*d*e - 3*B*a^3*e^2)*x)*sqrt(c)*(-a/c)^(3/4)*elliptic_e(arcsin((-a
/c)^(1/4)/x), -1) - ((2*(A + B)*a*c^2*d*e + (B*a*c^2 + A*c^3)*d^2 - (3*B*a
^2*c - A*a*c^2)*e^2)*x^5 + (2*(A + B)*a^2*c*d*e + (B*a^2*c + A*a*c^2)*d^2
- (3*B*a^3 - A*a^2*c)*e^2)*x)*sqrt(c)*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c
)^(1/4)/x), -1) - (2*B*a^2*c*e^2*x^4 - B*a^2*c*d^2 - 2*A*a^2*c*d*e + 3*B*a
^3*e^2 + (A*a*c^2*d^2 - 2*B*a^2*c*d*e - A*a^2*c*e^2)*x^2)*sqrt(c*x^4 + a))
/(a^2*c^3*x^5 + a^3*c^2*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**2/(a + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^2}{(c\,x^4+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^2}{\left(a+cx^4\right)^{3/2}}\,dx = \frac{-\sqrt{c\,x^4+a}\,a\,e^2x-2\sqrt{c\,x^4+a}\,bdex+\sqrt{c\,x^4+a}\,b\,e^2x^3+\left(\int\frac{\sqrt{c\,x^4+a}}{c^2x^8+2ac\,x^4+a^2}d\right.}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a + c*x**4)*a*e**2*x - 2*sqrt(a + c*x**4)*b*d*e*x + sqrt(a + c*x*
*4)*b*e**2*x**3 + int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*
a**3*e**2 + 2*int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**2
*b*d*e + int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**2*c*d*
*2 + int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**2*c*e**2*x
**4 + 2*int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*b*c*d*e*
x**4 + int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c**2*d**2
*x**4 - 3*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a
**2*b*e**2 + 2*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8)
,x)*a**2*c*d*e + int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**
8),x)*a*b*c*d**2 - 3*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2
*x**8),x)*a*b*c*e**2*x**4 + 2*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x*
*4 + c**2*x**8),x)*a*c**2*d*e*x**4 + int((sqrt(a + c*x**4)*x**2)/(a**2 + 2
*a*c*x**4 + c**2*x**8),x)*b*c**2*d**2*x**4)/(c*(a + c*x**4))
```

3.76 $\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 304

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \frac{x\left(Ad-\frac{aBe}{c}+(Bd+Ae)x^2\right)}{2a\sqrt{a+cx^4}} - \frac{(Bd+Ae)x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{(Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} - \frac{\left(\sqrt{a}(Bd+Ae)-\frac{Acd+aBe}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}$$

output
```
1/2*x*(A*d-a*B*e/c+(A*e+B*d)*x^2)/a/(c*x^4+a)^(1/2)-1/2*(A*e+B*d)*x*(c*x^4
+a)^(1/2)/a/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)+1/2*(A*e+B*d)*(a^(1/2)+c^(1/2)*x
^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/
4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/c^(3/4)/(c*x^4+a)^(1/2)-1/4*(a^(1/2)*(
A*e+B*d)-(A*c*d+B*a*e)/c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+
c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1
/2))/a^(5/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.41

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \frac{3(Acd-aBe)x+3(Acd+aBe)x\sqrt{1+\frac{cx^4}{a}}\ \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{c}{}\right.}{6ac\sqrt{a+cx}}$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(3/2),x]
```

output
```
(3*(A*c*d - a*B*e)*x + 3*(A*c*d + a*B*e)*x*Sqrt[1 + (c*x^4)/a]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + 2*c*(B*d + A*e)*x^3*Sqrt[1 + (c*x^4)
/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(6*a*c*Sqrt[a + c*x^4]
)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{-aBe+cx^2(Ae+Bd)+Acd}{c\,(a+cx^4)^{3/2}}+\frac{Be}{c\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-\sqrt{a}\sqrt{c}(Ae+Bd)-aBe+Acd\right)}{4a^{5/4}c^{5/4}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(Ae+Bd)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}(Ae+Bd)}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{x\left(-aBe+cx^2(Ae+Bd)+Acd\right)}{2ac\sqrt{a+cx^4}}+\frac{Be\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}$$

input
```
Int[((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(3/2),x]
```

output
```
(x*(A*c*d - a*B*e + c*(B*d + A*e)*x^2))/(2*a*c*Sqrt[a + c*x^4]) - ((B*d +
A*e)*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + ((B*d + A*
e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*
x^4]) + (B*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(5/4)*S
qrt[a + c*x^4]) + ((A*c*d - a*B*e - Sqrt[a]*Sqrt[c]*(B*d + A*e))*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(5/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.83

method	result
elliptic	$-\frac{2c\left(-\frac{(Ae+Bd)x^3}{4ca}-\frac{(Acd-Bae)x}{4a\,c^2}\right)}{\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{\left(\frac{Be}{c}+\frac{Acd-Bae}{2ac}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}-\frac{i(Ae+Bd)\sqrt{1-\frac{i\sqrt{c}\,x}{\sqrt{a}}}}{}$
default	$Ad\left(\frac{x}{2a\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}\right)+(Ae+Bd)\left(\frac{x^3}{2a\sqrt{c\left(\frac{a}{c}+x^4\right)}}-\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{}\right.$

input
```
int((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/4*(A*e+B*d)/c/a*x^3-1/4*(A*c*d-B*a*e)/a/c^2*x)/(c*(a/c+x^4))^(1/2
)+(B*e/c+1/2*(A*c*d-B*a*e)/a/c)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2
/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(
x*(I*c^(1/2)/a^(1/2))^(1/2),I)-1/2*I*(A*e+B*d)/a^(1/2)/(I*c^(1/2)/a^(1/2))
^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*
x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x
*(I*c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.62

$$\int\frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx=\frac{((Bc^2d+Ac^2e)x^4+Bacd+Aace)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid-1\right)-((}{}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(((B*c^2*d + A*c^2*e)*x^4 + B*a*c*d + A*a*c*e)*sqrt(a)*(-c/a)^(3/4)*el
liptic_e(arcsin(x*(-c/a)^(1/4)), -1) - (((A + B)*c^2*d + (B*a*c + A*c^2)*e
)*x^4 + (A + B)*a*c*d + (B*a^2 + A*a*c)*e)*sqrt(a)*(-c/a)^(3/4)*elliptic_f
(arcsin(x*(-c/a)^(1/4)), -1) + sqrt(c*x^4 + a)*((B*c^2*d + A*c^2*e)*x^3 +
(A*c^2*d - B*a*c*e)*x))/(a*c^3*x^4 + a^2*c^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.77 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.55

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \frac{Adx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Aex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+a)**(3/2),x)
```

output
```
A*d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a*
*(3/2)*gamma(5/4)) + A*e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), c*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + B*d*x**3*gamma(3/4)*hyper((3/
4, 3/2), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + B*e*x
**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)}{(c\,x^4+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A + Bx^2)(d + ex^2)}{(a + cx^4)^{3/2}}\,dx = \frac{-\sqrt{c\,x^4 + a}\,bex + \left(\int \frac{\sqrt{c\,x^4+a}}{c^2x^8+2ac\,x^4+a^2}dx\right)a^2be + \left(\int \frac{\sqrt{c\,x^4+a}}{c^2x^8+2ac\,x^4+a^2}dx\right)a^2cd + \left(\right.}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a + c*x**4)*b*e*x + int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**
2*x**8),x)*a**2*b*e + int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8)
,x)*a**2*c*d + int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*b
*c*e*x**4 + int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c**2
*d*x**4 + int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a
**2*c*e + int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a
*b*c*d + int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*
c**2*e*x**4 + int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),
x)*b*c**2*d*x**4)/(c*(a + c*x**4))
```

3.77 $\int \frac{A+Bx^2}{(a+cx^4)^{3/2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 262

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}} dx = \frac{x(A+Bx^2)}{2a\sqrt{a+cx^4}} - \frac{Bx\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} - \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}$$

output
```
1/2*x*(B*x^2+A)/a/(c*x^4+a)^(1/2)-1/2*B*x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1/
2)+c^(1/2)*x^2)+1/2*B*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4
)/c^(3/4)/(c*x^4+a)^(1/2)-1/4*(a^(1/2)*B-A*c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*
((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)
*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx = \frac{3Ax+3Ax\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+2Bx^3\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeo}}{6a\sqrt{a+cx^4}}$$

input
```
Integrate[(A + B*x^2)/(a + c*x^4)^(3/2),x]
```

output
```
(3*A*x + 3*A*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x
^4)/a)] + 2*B*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((
c*x^4)/a)])/(6*a*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1493, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1493}$$

$$\frac{x(A+Bx^2)}{2a\sqrt{a+cx^4}}-\frac{\int -\frac{A-Bx^2}{\sqrt{cx^4+a}}dx}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{A-Bx^2}{\sqrt{cx^4+a}}dx}{2a}+\frac{x(A+Bx^2)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{ 1512}$$

$$\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{\sqrt{a}B\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}}{2a}+\frac{x(A+Bx^2)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{B\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}}{2a}+\frac{x(A+Bx^2)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \textbf{761}$$

$$\frac{\frac{B\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}}{2a}+\frac{x(A+Bx^2)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \textbf{1510}$$

$$\frac{\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{B\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a}}{\sqrt{a}+}\right.}{\sqrt{c}}}{2a} \qquad \frac{x(A+Bx^2)}{2a\sqrt{a+cx^4}}$$

input
```
Int[(A + B*x^2)/(a + c*x^4)^(3/2),x]
```

output
```
(x*(A + B*x^2))/(2*a*Sqrt[a + c*x^4]) + ((B*(-((x*Sqrt[a + c*x^4])/(Sqrt[a
] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(
1/4)*Sqrt[a + c*x^4])))/Sqrt[c] + ((A - (Sqrt[a]*B)/Sqrt[c])*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.81

method	result
elliptic	$-\frac{2c\left(-\frac{B\,x^{3}}{4ac}-\frac{Ax}{4ac}\right)}{\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{A\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{iB\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$
default	$A\left(\frac{x}{2a\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+B\left(\frac{x^{3}}{2a\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}-\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\right.}{2\cdots}\right.$

input
```
int((B*x^2+A)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/4/a*B/c*x^3-1/4*A/a/c*x)/(c*(a/c+x^4))^(1/2)+1/2*A/a/(I*c^(1/2)/a
^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1
/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-1/2*I/a^(1/2)
*B/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*
x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2)
)^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.44

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx = \frac{(Bcx^4+Ba)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\middle|-1\right)-((A+B)cx^4+(A+B)a)\sqrt{a}\left(-\cdots\right.}{2\left(ac^2x^4+a^2c\right)}$$

input
```
integrate((B*x^2+A)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((B*c*x^4 + B*a)*sqrt(a)*(-c/a)^(3/4)*elliptic_e(arcsin(x*(-c/a)^(1/4)
), -1) - ((A + B)*c*x^4 + (A + B)*a)*sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsi
n(x*(-c/a)^(1/4)), -1) + (B*c*x^3 + A*c*x)*sqrt(c*x^4 + a))/(a*c^2*x^4 + a
^2*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx = \frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((B*x**2+A)/(c*x**4+a)**(3/2),x)
```

output
```
A*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), c*x**4*exp_
polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx=\int \frac{B\,x^2+A}{(c\,x^4+a)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/(a + c*x^4)^(3/2),x)
```

output
```
int((A + B*x^2)/(a + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx=\left(\int \frac{\sqrt{c\,x^4+a}}{c^2x^8+2ac\,x^4+a^2}dx\right)a+\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c^2x^8+2ac\,x^4+a^2}dx\right)b$$

input
```
int((B*x^2+A)/(c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a + int((sqrt(a +
c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*b
```

3.78 $\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 608

$$\begin{aligned}\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx &= \frac{x(Acd+aBe+c(Bd-Ae)x^2)}{2a(cd^2+ae^2)\sqrt{a+cx^4}}\\
&-\frac{\sqrt{c}(Bd-Ae)x\sqrt{a+cx^4}}{2a(cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{e^{3/2}(Bd-Ae)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}(cd^2+ae^2)^{3/2}}\\
&+\frac{\sqrt[4]{c}(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}(cd^2+ae^2)\sqrt{a+cx^4}}\\
&-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}}\\
&+\frac{e\left(\sqrt{c}d+\sqrt{a}e\right)(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)\sqrt{a+cx^4}}\end{aligned}$$

output
```
1/2*x*(A*c*d+B*a*e+c*(-A*e+B*d)*x^2)/a/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)-1/2*c
^(1/2)*(-A*e+B*d)*x*(c*x^4+a)^(1/2)/a/(a*e^2+c*d^2)/(a^(1/2)+c^(1/2)*x^2)-
1/2*e^(3/2)*(-A*e+B*d)*arctan((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*x^4
+a)^(1/2))/d^(1/2)/(a*e^2+c*d^2)^(3/2)+1/2*c^(1/4)*(-A*e+B*d)*(a^(1/2)+c^(
1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan
(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)-1/
4*(a^(1/2)*B-A*c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*
x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(
5/4)/c^(1/4)/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+a)^(1/2)+1/4*e*(c^(1/2)*d+a^(1/2
)*e)*(-A*e+B*d)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^
(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(1/2)*d-a^(1/2)*
e)^2/a^(1/2)/c^(1/2)/d/e,1/2*2^(1/2))/a^(1/4)/c^(1/4)/d/(c^(1/2)*d-a^(1/2)
*e)/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.09 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx = \frac{A\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cd^2x+aB\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}dex+B\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cd^2x^3-A\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cdex^3-\sqrt{a}\sqrt{c}d(Bd-}{}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)*(a + c*x^4)^(3/2)),x]
```

output
```
(A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^2*x + a*B*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e*x
 + B*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^2*x^3 - A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d
*e*x^3 - Sqrt[a]*Sqrt[c]*d*(B*d - A*e)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*Arc
Sinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + (Sqrt[a]*B - I*A*Sqrt[c])*d*(Sqrt
[c]*d - I*Sqrt[a]*e)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[
c])/Sqrt[a]]*x], -1] + (2*I)*a*B*d*e*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*
Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (2*I
)*a*A*e^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*A
rcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(2*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d
*(c*d^2 + a*e^2)*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\left(a+cx^4\right)^{3/2}\left(d+ex^2\right)}dx$$

$$\Big\downarrow \text{ 2259}$$

$$\int \left(\frac{e(Ae-Bd)}{\sqrt{a+cx^4}\left(d+ex^2\right)\left(ae^2+cd^2\right)}+\frac{aBe+cx^2(Bd-Ae)+Acd}{\left(a+cx^4\right)^{3/2}\left(ae^2+cd^2\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-\sqrt{a}\sqrt{c}(Bd-Ae)+aBe+Acd\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(Bd-Ae)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+cx^4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bd-Ae)\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}d\sqrt{a+cx^4}\left(c^2d^4-a^2e^4\right)}-\frac{\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(Bd-Ae)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)}-\frac{e^{3/2}(Bd-Ae)\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\left(ae^2+cd^2\right)^{3/2}}-\frac{\sqrt{c}x\sqrt{a+cx^4}(Bd-Ae)}{2a\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2+cd^2\right)}+\frac{x\left(aBe+cx^2(Bd-Ae)+Acd\right)}{2a\sqrt{a+cx^4}\left(ae^2+cd^2\right)}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)*(a + c*x^4)^(3/2)),x]
```

output
```
(x*(A*c*d + a*B*e + c*(B*d - A*e)*x^2))/(2*a*(c*d^2 + a*e^2)*Sqrt[a + c*x^
4]) - (Sqrt[c]*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*a*(c*d^2 + a*e^2)*(Sqrt[a
] + Sqrt[c]*x^2)) - (e^(3/2)*(B*d - A*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(S
qrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(2*Sqrt[d]*(c*d^2 + a*e^2)^(3/2)) + (c^(
1/4)*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*d^
2 + a*e^2)*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqr
t[a + c*x^4]) + ((A*c*d + a*B*e - Sqrt[a]*Sqrt[c]*(B*d - A*e))*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)*Sqrt[a +
c*x^4]) + (a^(3/4)*e*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sq
rt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(4*c^(1/4)*d*(c^2*d^4 - a^2*e^4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 564, normalized size of antiderivative = 0.93

method	result
default	$\frac{B\left(\frac{x}{2a\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}\right)}{e}+\frac{(Ae-Bd)\left(-\frac{2c\left(\frac{e\,x^3}{4a\left(a\,e^2+c\,d^2\right)}-\frac{dx}{4a\left(a\,e^2+c\,d^2\right)}\right)}{\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{cd\sqrt{1-\frac{i\sqrt{c}}{\sqrt{}}}}{2c}\right.}{}$
elliptic	$-\frac{2c\left(\frac{(Ae-Bd)x^3}{4a\left(a\,e^2+c\,d^2\right)}-\frac{(Acd+Bae)x}{4a\left(a\,e^2+c\,d^2\right)c}\right)}{\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)Acd}{2a(a\,e^2+c\,d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{Ellipt}}{2(a\,e^2+c\,d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}$

input
```
int((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B/e*(1/2/a*x/(c*(a/c+x^4))^(1/2)+1/2/a/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1
/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*Ell
ipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I))+(A*e-B*d)/e*(-2*c*(1/4/a*e/(a*e^2+c
*d^2)*x^3-1/4/a*d/(a*e^2+c*d^2)*x)/(c*(a/c+x^4))^(1/2)+1/2*c/a*d/(a*e^2+c*
d^2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2
)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2)
,I)+1/2*I*c^(1/2)/a^(1/2)*e/(a*e^2+c*d^2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c
^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*
EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-1/2*I*c^(1/2)/a^(1/2)*e/(a*e^2+c*
d^2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2
)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2)
,I)+1/(a*e^2+c*d^2)*e^2/d/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/
2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I*
c^(1/2)/a^(1/2))^(1/2),I/c^(1/2)*a^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1/2)/(I
*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\,(a+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\,(a+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{(a+cx^4)^{\frac{3}{2}}\,(d+ex^2)}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)/((a + c*x**4)**(3/2)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\,(a+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{(c\,x^4+a)^{3/2}\,(e\,x^2+d)}\,dx$$

input
```
int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)),x)
```

output
```
int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{c\,x^4+a}}{c^2e\,x^{10}+c^2d\,x^8+2ace\,x^6+2acd\,x^4+a^2e\,x^2+a^2d}dx\right)a + \left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c^2e\,x^{10}+c^2d\,x^8+2ace\,x^6+2acd\,x^4+a^2e\,x^2+a^2d}dx\right)b$$

input
```
int((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a**2*d + a**2*e*x**2 + 2*a*c*d*x**4 + 2*a*c*e*x**6 +
 c**2*d*x**8 + c**2*e*x**10),x)*a + int((sqrt(a + c*x**4)*x**2)/(a**2*d +
a**2*e*x**2 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)
*b
```

3.79 $\int \frac{A+Bx^2}{(d+ex^2)^2(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 830

$$
\begin{aligned}
\int \frac{A+Bx^2}{(d+ex^2)^2(a+cx^4)^{3/2}}\,dx &= -\frac{e(Bd-Ae)x}{2d(cd^2+ae^2)(d+ex^2)\sqrt{a+cx^4}} \\
&+ \frac{cx(d(Acd^2+3aBde-2aAe^2)+(Bcd^3-2Acd^2e-2aBde^2+aAe^3)x^2)}{2ad(cd^2+ae^2)^2\sqrt{a+cx^4}} \\
&- \frac{\sqrt{c}(Bcd^3-2Acd^2e-2aBde^2+aAe^3)x\sqrt{a+cx^4}}{2ad(cd^2+ae^2)^2(\sqrt{a}+\sqrt{c}x^2)} \\
&- \frac{e^{3/2}(5Bcd^3-7Acd^2e-aBde^2-aAe^3)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}(cd^2+ae^2)^{5/2}} \\
&+ \frac{\sqrt[4]{c}(Bcd^3-2Acd^2e-2aBde^2+aAe^3)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}d(cd^2+ae^2)^2\sqrt{a+cx^4}} \\
&+ \frac{\sqrt[4]{c}\left(Acd^2-ae(Bd-2Ae)-\sqrt{a}\sqrt{c}d(Bd-Ae)\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{4a^{5/4}d(\sqrt{c}d-\sqrt{a}e)(cd^2+ae^2)\sqrt{a+cx^4}} \\
&+ \frac{e(\sqrt{c}d+\sqrt{a}e)(5Bcd^3-7Acd^2e-aBde^2-aAe^3)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2(\sqrt{c}d-\sqrt{a}e)(cd^2+ae^2)^2\sqrt{a+cx^4}}
\end{aligned}
$$

output
```
-1/2*e*(-A*e+B*d)*x/d/(a*e^2+c*d^2)/(e*x^2+d)/(c*x^4+a)^(1/2)+1/2*c*x*(d*(
-2*A*a*e^2+A*c*d^2+3*B*a*d*e)+(A*a*e^3-2*A*c*d^2*e-2*B*a*d*e^2+B*c*d^3)*x^
2)/a/d/(a*e^2+c*d^2)^2/(c*x^4+a)^(1/2)-1/2*c^(1/2)*(A*a*e^3-2*A*c*d^2*e-2*
B*a*d*e^2+B*c*d^3)*x*(c*x^4+a)^(1/2)/a/d/(a*e^2+c*d^2)^2/(a^(1/2)+c^(1/2)*
x^2)-1/4*e^(3/2)*(-A*a*e^3-7*A*c*d^2*e-B*a*d*e^2+5*B*c*d^3)*arctan((a*e^2+
c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))/d^(3/2)/(a*e^2+c*d^2)^(5/2
)+1/2*c^(1/4)*(A*a*e^3-2*A*c*d^2*e-2*B*a*d*e^2+B*c*d^3)*(a^(1/2)+c^(1/2)*x
^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/
4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/d/(a*e^2+c*d^2)^2/(c*x^4+a)^(1/2)+1/4*
c^(1/4)*(A*c*d^2-a*e*(-2*A*e+B*d)-a^(1/2)*c^(1/2)*d*(-A*e+B*d))*(a^(1/2)+c
^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/d/(c^(1/2)*d-a^(1/2)*e)/(a*e^
2+c*d^2)/(c*x^4+a)^(1/2)+1/8*e*(c^(1/2)*d+a^(1/2)*e)*(-A*a*e^3-7*A*c*d^2*e
-B*a*d*e^2+5*B*c*d^3)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^
2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(1/2)*d-a^
(1/2)*e)^2/a^(1/2)/c^(1/2)/d/e,1/2*2^(1/2))/a^(1/4)/c^(1/4)/d^2/(c^(1/2)*d
-a^(1/2)*e)/(a*e^2+c*d^2)^2/(c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.67 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx^2}{(d+ex^2)^2(a+cx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}d(ae^3(-Bd+Ae)x(a+cx^4)+cdx(d+ex^2)\left(-aAe^2+Bcd^2x^2+Ac\right.}{}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^2*(a + c*x^4)^(3/2)),x]
```

output
```
(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*(a*e^3*(-(B*d) + A*e)*x*(a + c*x^4) + c*d*x*(
d + e*x^2)*(-(a*A*e^2) + B*c*d^2*x^2 + A*c*d*(d - 2*e*x^2) + a*B*e*(2*d -
e*x^2))) - (d + e*x^2)*Sqrt[1 + (c*x^4)/a]*(-(Sqrt[a]*Sqrt[c]*d*(-(B*c*d^3
) + 2*A*c*d^2*e + 2*a*B*d*e^2 - a*A*e^3)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[
c])/Sqrt[a]]*x], -1]) + I*(Sqrt[c]*d*(Sqrt[c]*d - I*Sqrt[a]*e)*(A*c*d^2 +
I*Sqrt[a]*Sqrt[c]*d*(B*d - A*e) + a*e*(2*B*d - A*e))*EllipticF[I*ArcSinh[S
qrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + a*e*(-5*B*c*d^3 + 7*A*c*d^2*e + a*B*d*e
^2 + a*A*e^3)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*S
qrt[c])/Sqrt[a]]*x], -1])))/(2*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*(c*d^3 + a*d*e^
2)^2*(d + e*x^2)*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 1494, normalized size of antiderivative = 1.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{e(Ae-Bd)}{\sqrt{a+cx^4}\,(d+ex^2)^2\,(ae^2+cd^2)}+\frac{e\left(aBe^2+2Acde-Bcd^2\right)}{\sqrt{a+cx^4}\,(d+ex^2)\,(ae^2+cd^2)^2}+\frac{c\left(x^2\left(-aBe^2-2Acde+Bcd^2\right)-aA\right.}{(a+cx^4)^{3/2}\,(ae^2+c\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(Bd-Ae)x\sqrt{cx^4+a}e^3}{2d\left(cd^2+ae^2\right)^2\left(ex^2+d\right)}-\frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{2d\left(cd^2+ae^2\right)^2\sqrt{cx^4+a}}+\frac{\sqrt{c}(Bd-Ae)x\sqrt{cx^4+a}e^2}{2d\left(cd^2+ae^2\right)^2\left(\sqrt{c}x^2+\sqrt{a}\right)}-\frac{(Bd-Ae)\left(3cd^2+ae^2\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+a}}\right)e^{3/2}}{4d^{3/2}\left(cd^2+ae^2\right)^{5/2}}-\frac{\left(Bcd^2-2Aced-aBe^2\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+a}}\right)e^{3/2}}{2\sqrt{d}\left(cd^2+ae^2\right)^{5/2}}-\frac{\sqrt[4]{c}\left(Bcd^2-2Aced-aBe^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^2\sqrt{cx^4+a}}-\frac{\sqrt[4]{c}(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\sqrt{cx^4+a}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)(Bd-Ae)\left(3cd^2+ae^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^2\sqrt{cx^4+a}}$$

$$\frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\left(Bcd^2-2Aced-aBe^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{4\sqrt[4]{c}d\left(cd^2-ae^2\right)\left(cd^2+ae^2\right)^2\sqrt{cx^4+a}}$$

$$\frac{\sqrt[4]{c}\left(Bcd^2-2Aced-aBe^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\left(cd^2+ae^2\right)^2\sqrt{cx^4+a}}-\frac{\sqrt[4]{c}\left(Bcd^2-2Aced-aBe^2-\frac{\sqrt{c}\left(Acd^2+2aBed-aAe^2\right)}{\sqrt{a}}\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}\left(cd^2+ae^2\right)^2\sqrt{cx^4+a}}$$

$$\frac{\sqrt{c}\left(Bcd^2-2Aced-aBe^2\right)x\sqrt{cx^4+a}}{2a\left(cd^2+ae^2\right)^2\left(\sqrt{c}x^2+\sqrt{a}\right)}+\frac{cx\left(Acd^2+2aBed-aAe^2+\left(Bcd^2-2Aced-aBe^2\right)x^2\right)}{2a\left(cd^2+ae^2\right)^2\sqrt{cx^4+a}}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^2*(a + c*x^4)^(3/2)),x]
```

output
```
(c*x*(A*c*d^2 + 2*a*B*d*e - a*A*e^2 + (B*c*d^2 - 2*A*c*d*e - a*B*e^2)*x^2)
)/(2*a*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (Sqrt[c]*e^2*(B*d - A*e)*x*Sqr
t[a + c*x^4])/(2*d*(c*d^2 + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (Sqrt[c]*(
B*c*d^2 - 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*a*(c*d^2 + a*e^2)^2*(
Sqrt[a] + Sqrt[c]*x^2)) - (e^3*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2
+ a*e^2)^2*(d + e*x^2)) - (e^(3/2)*(B*d - A*e)*(3*c*d^2 + a*e^2)*ArcTan[(S
qrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(4*d^(3/2)*(c*d^
2 + a*e^2)^(5/2)) - (e^(3/2)*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*ArcTan[(Sqrt[
c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(2*Sqrt[d]*(c*d^2 +
a*e^2)^(5/2)) - (a^(1/4)*c^(1/4)*e^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*d*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (c^(1/4)*(B*c*d^
2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/
4)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2
+ a*e^2)*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 1384, normalized size of antiderivative = 1.67

method	result	size
default	Expression too large to display	1384
elliptic	Expression too large to display	1664

input
```
int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B/e*(-2*c*(1/4/a*e/(a*e^2+c*d^2)*x^3-1/4/a*d/(a*e^2+c*d^2)*x)/(c*(a/c+x^4)
)^(1/2)+1/2*c/a*d/(a*e^2+c*d^2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2
/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(
x*(I*c^(1/2)/a^(1/2))^(1/2),I)+1/2*I*c^(1/2)/a^(1/2)*e/(a*e^2+c*d^2)/(I*c^
(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1
/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-1/2*I*
c^(1/2)/a^(1/2)*e/(a*e^2+c*d^2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2
/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticE(
x*(I*c^(1/2)/a^(1/2))^(1/2),I)+1/(a*e^2+c*d^2)*e^2/d/(I*c^(1/2)/a^(1/2))^(
1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^
4+a)^(1/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(1/2)*a^(1/2)/d*e,(-
I/a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))+(A*e-B*d)/e*(1/2*e^4/
(a*e^2+c*d^2)^2/d*x*(c*x^4+a)^(1/2)/(e*x^2+d)-2*c*(1/2/a*d*e*c/(a*e^2+c*d^
2)^2*x^3+1/4/a*(a*e^2-c*d^2)/(a*e^2+c*d^2)^2*x)/(c*(a/c+x^4))^(1/2)-1/(I*c
^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(
1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)*e^2*c
/(a*e^2+c*d^2)^2+1/2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(
1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2
)/a^(1/2))^(1/2),I)*c^2/a/(a*e^2+c*d^2)^2*d^2-1/2*I*a^(1/2)/(I*c^(1/2)/a^(
1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2 (a+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2 (a+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{(a+cx^4)^{\frac{3}{2}} (d+ex^2)^2}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)/((a + c*x**4)**(3/2)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2 (a+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}} (ex^2+d)^2}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2 (a+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}(ex^2+d)^2}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2 (a+cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{(c\,x^4+a)^{3/2}\,(e\,x^2+d)^2}\,dx$$

input
```
int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)^2),x)
```

output
```
int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2 (a+cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{c\,x^4+a}}{c^2e^2x^{12}+2c^2de\,x^{10}+2ac\,e^2x^8+c^2d^2x^8+4acde\,x^6+a^2e^2x^4+2ac\,d^2}\right.$$
$$+\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c^2e^2x^{12}+2c^2de\,x^{10}+2ac\,e^2x^8+c^2d^2x^8+4acde\,x^6+a^2e^2x^4+2ac\,d^2x^4+2a^2de\,x^2+a^2d^2}dx\right)b$$

input
```
int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a**2*d**2 + 2*a**2*d*e*x**2 + a**2*e**2*x**4 + 2*a*c
*d**2*x**4 + 4*a*c*d*e*x**6 + 2*a*c*e**2*x**8 + c**2*d**2*x**8 + 2*c**2*d*
e*x**10 + c**2*e**2*x**12),x)*a + int((sqrt(a + c*x**4)*x**2)/(a**2*d**2 +
 2*a**2*d*e*x**2 + a**2*e**2*x**4 + 2*a*c*d**2*x**4 + 4*a*c*d*e*x**6 + 2*a
*c*e**2*x**8 + c**2*d**2*x**8 + 2*c**2*d*e*x**10 + c**2*e**2*x**12),x)*b
```

3.80 $\int \frac{A+Bx^2}{(d+ex^2)^3(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 1142

$$\int \frac{A+Bx^2}{(d+ex^2)^3\,(a+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

output
```
-1/4*e*(-A*e+B*d)*x/d/(a*e^2+c*d^2)/(e*x^2+d)^2/(c*x^4+a)^(1/2)-1/8*e*(-3*
A*a*e^3-13*A*c*d^2*e-B*a*d*e^2+9*B*c*d^3)*x/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)/
(c*x^4+a)^(1/2)+1/8*c*x*(d*(a*B*d*e*(-7*a*e^2+23*c*d^2)+A*(-a^2*e^4-27*a*c
*d^2*e^2+4*c^2*d^4))-(3*A*e*(-a^2*e^4-7*a*c*d^2*e^2+4*c^2*d^4)-B*(a^2*d*e^
4-25*a*c*d^3*e^2+4*c^2*d^5))*x^2)/a/d^2/(a*e^2+c*d^2)^3/(c*x^4+a)^(1/2)+1/
8*c^(1/2)*(3*A*e*(-a^2*e^4-7*a*c*d^2*e^2+4*c^2*d^4)-B*(a^2*d*e^4-25*a*c*d^
3*e^2+4*c^2*d^5))*x*(c*x^4+a)^(1/2)/a/d^2/(a*e^2+c*d^2)^3/(a^(1/2)+c^(1/2)
*x^2)+1/16*e^(3/2)*(3*A*e*(a^2*e^4+2*a*c*d^2*e^2+21*c^2*d^4)-B*(-a^2*d*e^4
-26*a*c*d^3*e^2+35*c^2*d^5))*arctan((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/
(c*x^4+a)^(1/2))/d^(5/2)/(a*e^2+c*d^2)^(7/2)-1/8*c^(1/4)*(3*A*e*(-a^2*e^4-
7*a*c*d^2*e^2+4*c^2*d^4)-B*(a^2*d*e^4-25*a*c*d^3*e^2+4*c^2*d^5))*(a^(1/2)+
c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/d^2/(a*e^2+c*d^2)^3/(c*x^4+a)
^(1/2)+1/8*c^(1/4)*(2*A*c^2*d^4-2*a*c*d^2*e*(-5*A*e+2*B*d)-2*a^(1/2)*c^(3/
2)*d^3*(-2*A*e+B*d)+a^(3/2)*c^(1/2)*d*e^2*(-A*e+3*B*d)+a^2*e^3*(3*A*e+B*d)
)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJ
acobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/d^2/(c^(1/2)*d-a^
(1/2)*e)/(a*e^2+c*d^2)^2/(c*x^4+a)^(1/2)-1/32*e*(c^(1/2)*d+a^(1/2)*e)*(3*A
*e*(a^2*e^4+2*a*c*d^2*e^2+21*c^2*d^4)-B*(-a^2*d*e^4-26*a*c*d^3*e^2+35*c^2*
d^5))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*E...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.91 (sec) , antiderivative size = 630, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx^2}{(d+ex^2)^3 (a+cx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}dx\Big(-2ade^3(Bd-Ae)\left(cd^2+ae^2\right)\left(a+cx^4\right)+ae^3(-13Bcd^3+17Ac\cdots}{}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^3*(a + c*x^4)^(3/2)),x]
```

output
```
(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*x*(-2*a*d*e^3*(B*d - A*e)*(c*d^2 + a*e^2)*(a
+ c*x^4) + a*e^3*(-13*B*c*d^3 + 17*A*c*d^2*e + a*B*d*e^2 + 3*a*A*e^3)*(d +
 e*x^2)*(a + c*x^4) + 4*c*d^2*(d + e*x^2)^2*(B*(-(a^2*e^3) + c^2*d^3*x^2 +
 3*a*c*d*e*(d - e*x^2)) + A*c*(c*d^2*(d - 3*e*x^2) + a*e^2*(-3*d + e*x^2))
)) - (d + e*x^2)^2*Sqrt[1 + (c*x^4)/a]*(Sqrt[a]*Sqrt[c]*d*(3*A*e*(-4*c^2*d
^4 + 7*a*c*d^2*e^2 + a^2*e^4) + B*(4*c^2*d^5 - 25*a*c*d^3*e^2 + a^2*d*e^4)
)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*(Sqrt[c]*d*(Sq
rt[c]*d - I*Sqrt[a]*e)*(4*A*c^2*d^4 + (4*I)*Sqrt[a]*c^(3/2)*d^3*(B*d - 2*A
*e) + 19*a*c*d^2*e*(B*d - A*e) - (2*I)*a^(3/2)*Sqrt[c]*d*e^2*(3*B*d - A*e)
 - a^2*e^3*(B*d + 3*A*e))*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x]
, -1] + a*e*(3*A*e*(21*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + B*(-35*c^2*d^5
 + 26*a*c*d^3*e^2 + a^2*d*e^4))*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I
*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])))/(8*a*Sqrt[(I*Sqrt[c])/Sqrt[a
]]*(c*d^3 + a*d*e^2)^3*(d + e*x^2)^2*Sqrt[a + c*x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2452 vs. $2(1142) = 2284$.

Time = 4.05 (sec) , antiderivative size = 2452, normalized size of antiderivative = 2.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{2259, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\left(a+cx^4\right)^{3/2}\left(d+ex^2\right)^3}\,dx$$

$$\Big\downarrow \text{2259}$$

$$\int \left(\frac{e(Ae-Bd)}{\sqrt{a+cx^4}\left(d+ex^2\right)^3\left(ae^2+cd^2\right)}+\frac{e\left(aBe^2+2Acde-Bcd^2\right)}{\sqrt{a+cx^4}\left(d+ex^2\right)^2\left(ae^2+cd^2\right)^2}+\frac{ce\left(-aAe^3+3aBde^2+3Acd^2e-B\right.}{\sqrt{a+cx^4}\left(d+ex^2\right)\left(ae^2+cd^2\right)^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3(Bd-Ae)\left(3cd^2+ae^2\right)x\sqrt{cx^4+a}e^3}{8d^2\left(cd^2+ae^2\right)^3\left(ex^2+d\right)}-\frac{\left(Bcd^2-2Aced-aBe^2\right)x\sqrt{cx^4+a}e^3}{2d\left(cd^2+ae^2\right)^3\left(ex^2+d\right)}-\frac{(Bd-Ae)x\sqrt{cx^4+a}e^3}{4d\left(cd^2+ae^2\right)^2\left(ex^2+d\right)^2}-$$

$$\frac{3\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\left(3cd^2+ae^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{8d^2\left(cd^2+ae^2\right)^3\sqrt{cx^4+a}}-$$

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\left(Bcd^2-2Aced-aBe^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{2d\left(cd^2+ae^2\right)^3\sqrt{cx^4+a}}+$$

$$\frac{3\sqrt{c}(Bd-Ae)\left(3cd^2+ae^2\right)x\sqrt{cx^4+a}e^2}{8d^2\left(cd^2+ae^2\right)^3\left(\sqrt{c}x^2+\sqrt{a}\right)}+\frac{\sqrt{c}\left(Bcd^2-2Aced-aBe^2\right)x\sqrt{cx^4+a}e^2}{2d\left(cd^2+ae^2\right)^3\left(\sqrt{c}x^2+\sqrt{a}\right)}-$$

$$\frac{\left(3cd^2+ae^2\right)\left(Bcd^2-2Aced-aBe^2\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+a}}\right)e^{3/2}}{4d^{3/2}\left(cd^2+ae^2\right)^{7/2}}-$$

$$\frac{c\left(Bcd^3-3Aced^2-3aBe^2d+aAe^3\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+a}}\right)e^{3/2}}{2\sqrt{d}\left(cd^2+ae^2\right)^{7/2}}-$$

$$\frac{3(Bd-Ae)\left(5c^2d^4+2ace^2d^2+a^2e^4\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+a}}\right)e^{3/2}}{16d^{5/2}\left(cd^2+ae^2\right)^{7/2}}-$$

$$\frac{\sqrt[4]{c}(Bd-Ae)\left(4cd^2-\sqrt{a}\sqrt{c}ed+3ae^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{8\sqrt[4]{a}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^2\sqrt{cx^4+a}}-$$

$$\frac{\sqrt[4]{c}\left(Bcd^2-2Aced-aBe^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^2\sqrt{cx^4+a}}-$$

$$\frac{c^{5/4}\left(Bcd^3-3Aced^2-3aBe^2d+aAe^3\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^3\sqrt{cx^4+a}}+$$

$$\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(3cd^2+ae^2\right)\left(Bcd^2-2Aced-aBe^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\text{arcta}\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^3\sqrt{cx^4+a}}$$

$$\frac{a^{3/4}c^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\left(Bcd^3-3Aced^2-3aBe^2d+aAe^3\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\right.}{4d\left(cd^2-ae^2\right)\left(cd^2+ae^2\right)^3\sqrt{cx^4+a}}$$

$$\frac{3\left(\sqrt{c}d+\sqrt{a}e\right)(Bd-Ae)\left(5c^2d^4+2ace^2d^2+a^2e^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\text{arcta}\right.}{32\sqrt[4]{a}\sqrt[4]{c}d^3\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^3\sqrt{cx^4+a}}$$

$$\frac{c^{5/4}\left(Bcd^3-3Aced^2-3aBe^2d+aAe^3\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\left(cd^2+ae^2\right)^3\sqrt{cx^4+a}}+$$

$$\frac{c^{3/4}\left(Ac^2d^3-\sqrt{a}c^{3/2}(Bd-3Ae)d^2+3ace(Bd-Ae)d-a^2Be^3+a^{3/2}\sqrt{c}e^2(3Bd-Ae)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+}{\left(\sqrt{c}x^2+\right.}}}{4a^{5/4}\left(cd^2+ae^2\right)^3\sqrt{cx^4+a}}$$

$$\frac{c^{3/2}\left(Bcd^3-3Aced^2-3aBe^2d+aAe^3\right)x\sqrt{cx^4+a}}{2a\left(cd^2+ae^2\right)^3\left(\sqrt{c}x^2+\sqrt{a}\right)}+$$

$$cx\left(c\left(Bcd^3-3Aced^2-3aBe^2d+aAe^3\right)x^2+Acd\left(cd^2-3ae^2\right)+aBe\left(3cd^2-ae^2\right)\right)$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^3*(a + c*x^4)^(3/2)),x]
```

output
```
(c*x*(A*c*d*(c*d^2 - 3*a*e^2) + a*B*e*(3*c*d^2 - a*e^2) + c*(B*c*d^3 - 3*A
*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*x^2))/(2*a*(c*d^2 + a*e^2)^3*Sqrt[a + c*
x^4]) + (3*Sqrt[c]*e^2*(B*d - A*e)*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8
*d^2*(c*d^2 + a*e^2)^3*(Sqrt[a] + Sqrt[c]*x^2)) + (Sqrt[c]*e^2*(B*c*d^2 -
2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)^3*(Sqrt[a] +
Sqrt[c]*x^2)) - (c^(3/2)*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*x
*Sqrt[a + c*x^4])/(2*a*(c*d^2 + a*e^2)^3*(Sqrt[a] + Sqrt[c]*x^2)) - (e^3*(
B*d - A*e)*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)^2*(d + e*x^2)^2) - (3*e
^3*(B*d - A*e)*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)
^3*(d + e*x^2)) - (e^3*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/
(2*d*(c*d^2 + a*e^2)^3*(d + e*x^2)) - (e^(3/2)*(3*c*d^2 + a*e^2)*(B*c*d^2
- 2*A*c*d*e - a*B*e^2)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqr
t[a + c*x^4])]])/(4*d^(3/2)*(c*d^2 + a*e^2)^(7/2)) - (c*e^(3/2)*(B*c*d^3 -
3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[
d]*Sqrt[e]*Sqrt[a + c*x^4])]])/(2*Sqrt[d]*(c*d^2 + a*e^2)^(7/2)) - (3*e^(3/
2)*(B*d - A*e)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c*d^2 +
a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])]])/(16*d^(5/2)*(c*d^2 + a*e^2)^
(7/2)) - (3*a^(1/4)*c^(1/4)*e^2*(B*d - A*e)*(3*c*d^2 + a*e^2)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
 + 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 2326, normalized size of antiderivative = 2.04

method	result	size
default	Expression too large to display	2326
elliptic	Expression too large to display	2596

input
```
int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B/e*(1/2*e^4/(a*e^2+c*d^2)^2/d*x*(c*x^4+a)^(1/2)/(e*x^2+d)-2*c*(1/2/a*d*e*
c/(a*e^2+c*d^2)^2*x^3+1/4/a*(a*e^2-c*d^2)/(a*e^2+c*d^2)^2*x)/(c*(a/c+x^4))
^(1/2)-1/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^
(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(
1/2),I)*e^2*c/(a*e^2+c*d^2)^2+1/2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x
^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*Elliptic
F(x*(I*c^(1/2)/a^(1/2))^(1/2),I)*c^2/a/(a*e^2+c*d^2)^2*d^2-1/2*I*a^(1/2)/(
I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/
a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*c^(1/2)*e^3/(a*e^2+c*d^2)^2/d*EllipticF(x*(
I*c^(1/2)/a^(1/2))^(1/2),I)+1/2*I*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c
^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*
c^(1/2)*e^3/(a*e^2+c*d^2)^2/d*EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+I/a
^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1
/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*c^(3/2)*d*e/(a*e^2+c*d^2)^2*Ellipti
cF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-I/a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I
*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2
)*c^(3/2)*d*e/(a*e^2+c*d^2)^2*EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+1/2
*e^4/(a*e^2+c*d^2)^2/d^2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2
))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I*c
^(1/2)/a^(1/2))^(1/2),I/c^(1/2)*a^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))^(1/2)/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3 (a+cx^4)^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3 (a+cx^4)^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**3/(c*x**4+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3 (a+cx^4)^{3/2}}\, dx = \int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}(ex^2+d)^3}\, dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3 (a+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}(ex^2+d)^3}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3 (a+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+a)^{3/2}(ex^2+d)^3}\,dx$$

input
```
int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)^3),x)
```

output
```
int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3 (a+cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{cx^4+}}{c^2e^3x^{14}+3c^2d\,e^2x^{12}+2ac\,e^3x^{10}+3c^2d^2e\,x^{10}+6acd\,e^2x^8+c^2d^3x^8+}\right.$$
$$+\left(\int \frac{\sqrt{cx^4+a}\,x^2}{c^2e^3x^{14}+3c^2d\,e^2x^{12}+2ac\,e^3x^{10}+3c^2d^2e\,x^{10}+6acd\,e^2x^8+c^2d^3x^8+a^2e^3x^6+6ac\,d^2e\,x^6+3a^2d\,e^2x}\right.$$

input
```
int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x)
```

output

```
int(sqrt(a + c*x**4)/(a**2*d**3 + 3*a**2*d**2*e*x**2 + 3*a**2*d*e**2*x**4
+ a**2*e**3*x**6 + 2*a*c*d**3*x**4 + 6*a*c*d**2*e*x**6 + 6*a*c*d*e**2*x**8
 + 2*a*c*e**3*x**10 + c**2*d**3*x**8 + 3*c**2*d**2*e*x**10 + 3*c**2*d*e**2
*x**12 + c**2*e**3*x**14),x)*a + int((sqrt(a + c*x**4)*x**2)/(a**2*d**3 +
3*a**2*d**2*e*x**2 + 3*a**2*d*e**2*x**4 + a**2*e**3*x**6 + 2*a*c*d**3*x**4
 + 6*a*c*d**2*e*x**6 + 6*a*c*d*e**2*x**8 + 2*a*c*e**3*x**10 + c**2*d**3*x*
*8 + 3*c**2*d**2*e*x**10 + 3*c**2*d*e**2*x**12 + c**2*e**3*x**14),x)*b
```

3.81 $\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 232

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2+ae^2}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}\sqrt[4]{c}de\sqrt{a+cx^4}}$$

output
```
-1/2*(c^(1/2)*d/a^(1/2)-e)*arctan((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c
*x^4+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2+c*d^2)^(1/2)+1/4*(c^(1/2)*d+a^(1/2)*
e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Ellipti
cPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*a^(1/2)*(c^(1/2)*d/a^(1/2)-e)^2/
c^(1/2)/d/e,1/2*2^(1/2))/a^(3/4)/c^(1/4)/d/e/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.67

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx$$
$$=\frac{\sqrt{c}\sqrt{1+\frac{cx^4}{a}}\left(\sqrt{c}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)+\left(-\sqrt{c}d+\sqrt{a}e\right)\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d},i\text{arcsinh}\left(\sqrt{\right.\right.\right.}{a\left(\frac{i\sqrt{c}}{\sqrt{a}}\right)^{3/2}de\sqrt{a+cx^4}}$$

input
```
Integrate[(1 + (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(Sqrt[c]*Sqrt[1 + (c*x^4)/a]*(Sqrt[c]*d*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c
])/Sqrt[a]]*x], -1] + (-(Sqrt[c]*d) + Sqrt[a]*e)*EllipticPi[((-I)*Sqrt[a]*
e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(a*((I*Sqrt[
c])/Sqrt[a])^(3/2)*d*e*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}{\sqrt{a+cx^4}\,(d+ex^2)}\,dx$$
$$\Big\downarrow \text{ 2221}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}}-\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}$$

input
```
Int[(1 + (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
-1/2*(((Sqrt[c]*d)/Sqrt[a] - e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sq
rt[e]*Sqrt[a + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) + (((Sqrt[c
]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]
*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a
+ c*x^4])
```

Defintions of rubi rules used

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{\sqrt{c}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}-\frac{(\sqrt{c}\,d-\sqrt{a}\,e)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{e d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}}{\sqrt{a}}$
elliptic	$\frac{\sqrt{(c\,x^4+a)ac}\,(\sqrt{a}+\sqrt{c}\,x^2)\left(\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a\,c^2x^4+a^2c}}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right.}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a\,c^2x^4+a^2c}}\right.}{\left(c\,x^2\sqrt{c\,x^4+a}+\sqrt{(c\,x^4+a)ac}\right)\sqrt{a}}$

input
```
int((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/a^(1/2)*(c^(1/2)/e/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(
1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2
)/a^(1/2))^(1/2),I)-(c^(1/2)*d-a^(1/2)*e)/e/d/(I*c^(1/2)/a^(1/2))^(1/2)*(1
-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1
/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(1/2)*a^(1/2)/d*e,(-I/a^(1/
2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="
fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: catd
ef: division by zero
```

Sympy [F]

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \frac{\int \frac{\sqrt{a}}{d\sqrt{a+cx^4}+ex^2\sqrt{a+cx^4}}\,dx + \int \frac{\sqrt{c}x^2}{d\sqrt{a+cx^4}+ex^2\sqrt{a+cx^4}}\,dx}{\sqrt{a}}$$

input
```
integrate((1+c**(1/2)*x**2/a**(1/2))/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

output
```
(Integral(sqrt(a)/(d*sqrt(a + c*x**4) + e*x**2*sqrt(a + c*x**4)), x) + Int
egral(sqrt(c)*x**2/(d*sqrt(a + c*x**4) + e*x**2*sqrt(a + c*x**4)), x))/sqr
t(a)
```

Maxima [F]

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((sqrt(c)*x^2/sqrt(a) + 1)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}{\sqrt{c\,x^4+a}\,(e\,x^2+d)}\,dx$$

input
```
int(((c^(1/2)*x^2)/a^(1/2) + 1)/((a + c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int(((c^(1/2)*x^2)/a^(1/2) + 1)/((a + c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{c\,x^4+a}}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)a}{a}$$

input
```
int((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(a + c*x**4)*x**2)/(a*d + a*e*x**2 + c*d*x**4 +
c*e*x**6),x) + int(sqrt(a + c*x**4)/(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6)
,x)*a)/a
```

3.82 $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 245

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\frac{\left(\sqrt{\frac{c}{a}}d-e\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2+ae^2}} + \frac{\left(\sqrt{\frac{c}{a}}d+e\right)\left(1+\sqrt{\frac{c}{a}}x^2\right)\sqrt{\frac{\sqrt{\frac{c}{a}}(a+cx^4)}{c\left(\frac{1}{\sqrt[4]{\frac{c}{a}}}+\sqrt[4]{\frac{c}{a}}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{2}\right)}{4\sqrt[4]{\frac{c}{a}}de\sqrt{a+cx^4}}$$

output
```
-1/2*((c/a)^(1/2)*d-e)*arctan((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*x^4
+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2+c*d^2)^(1/2)+1/4*((c/a)^(1/2)*d+e)*(1+(c
/a)^(1/2)*x^2)*((c/a)^(1/2)*(c*x^4+a)/c/(1/(c/a)^(1/4)+(c/a)^(1/4)*x^2)^2)
^(1/2)*EllipticPi(sin(2*arctan((c/a)^(1/4)*x)),-1/4*((c/a)^(1/2)*d-e)^2/(c
/a)^(1/2)/d/e,1/2*2^(1/2))/(c/a)^(1/4)/d/e/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.62

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\frac{i\sqrt{1+\frac{cx^4}{a}}\left(\sqrt{\frac{c}{a}}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)+\left(-\sqrt{\frac{c}{a}}d+e\right)\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d},i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right.\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}de\sqrt{a+cx^4}}$$

input
```
Integrate[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
((-I)*Sqrt[1 + (c*x^4)/a]*(Sqrt[c/a]*d*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]
)/Sqrt[a]]*x], -1] + (-(Sqrt[c/a]*d) + e)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqr
t[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[c])/S
qrt[a]]*d*e*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.93, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{a+cx^4}\,(d+ex^2)}\,dx$$

$$\Big\downarrow \; 2221$$

$$\frac{\left(x^2\sqrt{\frac{c}{a}}+1\right)\sqrt{\frac{a+cx^4}{a\left(x^2\sqrt{\frac{c}{a}}+1\right)^2}}\left(d\sqrt{\frac{c}{a}}+e\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{2}\right)}{4de\sqrt[4]{\frac{c}{a}}\sqrt{a+cx^4}}-\frac{\left(d\sqrt{\frac{c}{a}}-e\right)\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}$$

input
```
Int[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
-1/2*((Sqrt[c/a]*d - e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sq
rt[a + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c/a]*d + e
)*(1 + Sqrt[c/a]*x^2)*Sqrt[(a + c*x^4)/(a*(1 + Sqrt[c/a]*x^2)^2)]*Elliptic
Pi[-1/4*(Sqrt[c/a]*d - e)^2/(Sqrt[c/a]*d*e), 2*ArcTan[(c/a)^(1/4)*x], 1/2]
)/(4*(c/a)^(1/4)*d*e*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.83

method	result
default	$\frac{\sqrt{\frac{c}{a}}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{\left(\sqrt{\frac{c}{a}}\,d-e\right)\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, \frac{i\sqrt{a}\,e}{\sqrt{c}\,d}, \frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{ed\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$
elliptic	$\frac{\sqrt{\frac{(c\,x^{4}+a)c}{a}}\,a\left(1+\sqrt{\frac{c}{a}}\,x^{2}\right)\left(\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{ae\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{\frac{c^{2}x^{4}}{a}+c}}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, \frac{i\sqrt{a}\,e}{\sqrt{c}\,d}, \frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{ae\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{\frac{c^{2}x^{4}}{a}+c}}\right.}{c\,x^{2}\sqrt{c\,x^{4}+a}+a\sqrt{\frac{(c\,x^{4}+a)c}{a}}}$

input
```
int((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c/a)^(1/2)/e/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1
+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/
2))^(1/2),I)-((c/a)^(1/2)*d-e)/e/d/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*
x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*Ellipti
cPi(x*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(1/2)*a^(1/2)/d*e,(-I/a^(1/2)*c^(1/2))
^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{a+cx^4}\,(d+ex^2)}\,dx$$

input
```
integrate((1+(c/a)**(1/2)*x**2)/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((x**2*sqrt(c/a) + 1)/(sqrt(a + c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{c\,x^4+a}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((x^2*(c/a)^(1/2) + 1)/((a + c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
int((x^2*(c/a)^(1/2) + 1)/((a + c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{c\,x^4+a}}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)a}{a}$$

input
```
int((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(a + c*x**4)*x**2)/(a*d + a*e*x**2 + c*d*x**4 +
c*e*x**6),x) + int(sqrt(a + c*x**4)/(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6)
,x)*a)/a
```

3.83 $\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 359

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \frac{(\sqrt{c}d+\sqrt{a}e)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{cd^2+ae^2}} + \frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\,(\sqrt{c}d-\sqrt{a}e)\sqrt{a+cx^4}} - \frac{(\sqrt{c}d+\sqrt{a}e)^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}\sqrt[4]{c}de\,(\sqrt{c}d-\sqrt{a}e)\sqrt{a+cx^4}}$$

output
```
1/2*(c^(1/2)*d+a^(1/2)*e)*arctan((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*
x^4+a)^(1/2))/a^(1/2)/d^(1/2)/e^(1/2)/(a*e^2+c*d^2)^(1/2)+c^(1/4)*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/(c^(1/2)*d-a^(1/2)*e)/(c*x^
4+a)^(1/2)-1/4*(c^(1/2)*d+a^(1/2)*e)^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a
^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-
1/4*(c^(1/2)*d-a^(1/2)*e)^2/a^(1/2)/c^(1/2)/d/e,1/2*2^(1/2))/a^(3/4)/c^(1/
4)/d/e/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.43

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx$$
$$= \frac{\sqrt{c}\sqrt{1+\frac{cx^4}{a}}\left(-\sqrt{c}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)+\left(\sqrt{c}d+\sqrt{a}e\right)\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d},i\text{arcsinh}\left(\sqrt{\right.\right.\right.}{a\left(\frac{i\sqrt{c}}{\sqrt{a}}\right)^{3/2}de\sqrt{a+cx^4}}$$

input
```
Integrate[(1 - (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(Sqrt[c]*Sqrt[1 + (c*x^4)/a]*(-(Sqrt[c]*d*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt
[c])/Sqrt[a]]*x], -1]) + (Sqrt[c]*d + Sqrt[a]*e)*EllipticPi[((-I)*Sqrt[a]*
e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(a*((I*Sqrt[
c])/Sqrt[a])^(3/2)*d*e*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2225, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}\,(d+ex^2)}\,dx$$

$$\Big\downarrow \text{2225}$$

$$\frac{2\sqrt{c}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{c}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

$$\Big\downarrow \ 761$$

$$\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

$$\Big\downarrow \ 2221$$

$$\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(\sqrt{a}e+\sqrt{c}d)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+cx^4}}-\frac{(\sqrt{c}d-\sqrt{a}e)\arctan\left(\frac{x\sqrt{a}}{\sqrt{d}\sqrt{e}}\right.}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}\right.}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

input
```
Int[(1 - (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*(Sqrt[c]*d - Sq
rt[a]*e)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(-1/2*((Sqrt[c]*d - S
qrt[a]*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])
])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-
1/4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)
*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + c*x^4])))/(Sqrt[a]*(Sq
rt[c]*d - Sqrt[a]*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2225
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[2*A*(B/(B*d + A*e))   Int[1/Sqrt[a + c*x^4], x], x] - S
imp[(B*d - A*e)/(B*d + A*e)   Int[(A - B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4])
, x], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c
/a] && EqQ[c*A^2 - a*B^2, 0] && NegQ[B/A]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.57

method	result
default	$-\frac{\frac{\sqrt{c}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{e d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}}{\sqrt{a}}$
elliptic	$\frac{\sqrt{\left(c\,x^{4}+a\right)a c}\,\left(-\sqrt{a}+\sqrt{c}\,x^{2}\right)\left(-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{a\,c^{2}x^{4}+a^{2}c}}+\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\ldots\right.}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{a\,c^{2}x^{4}+a^{2}c}}\right.}{\left(c\,x^{2}\sqrt{c\,x^{4}+a}-\sqrt{\left(c\,x^{4}+a\right)a c}\right)\sqrt{a}}$

input
```
int((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-1/a^(1/2)*(c^(1/2)/e/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^
(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/
2)/a^(1/2))^(1/2),I)-(c^(1/2)*d+a^(1/2)*e)/e/d/(I*c^(1/2)/a^(1/2))^(1/2)*(
1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(
1/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(1/2)*a^(1/2)/d*e,(-I/a^(1
/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\frac{\int \left(-\frac{\sqrt{a}}{d\sqrt{a+cx^4}+ex^2\sqrt{a+cx^4}}\right)dx + \int \frac{\sqrt{c}x^2}{d\sqrt{a+cx^4}+ex^2\sqrt{a+cx^4}}\,dx}{\sqrt{a}}$$

input
```
integrate((1-c**(1/2)*x**2/a**(1/2))/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

output
```
-(Integral(-sqrt(a)/(d*sqrt(a + c*x**4) + e*x**2*sqrt(a + c*x**4)), x) + I
ntegral(sqrt(c)*x**2/(d*sqrt(a + c*x**4) + e*x**2*sqrt(a + c*x**4)), x))/s
qrt(a)
```

Maxima [F]

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int -\frac{\frac{\sqrt{c}x^2}{\sqrt{a}}-1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="
maxima")
```

output
```
-integrate((sqrt(c)*x^2/sqrt(a) - 1)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx=-\int \frac{\frac{\sqrt{c}\,x^2}{\sqrt{a}}-1}{\sqrt{c\,x^4+a}\,(e\,x^2+d)}\,dx$$

input
```
int(-((c^(1/2)*x^2)/a^(1/2) - 1)/((a + c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
-int(((c^(1/2)*x^2)/a^(1/2) - 1)/((a + c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \frac{-\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{c\,x^4+a}}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)a}{a}$$

input
```
int((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(a + c*x**4)*x**2)/(a*d + a*e*x**2 + c*d*x**4
 + c*e*x**6),x) + int(sqrt(a + c*x**4)/(a*d + a*e*x**2 + c*d*x**4 + c*e*x*
*6),x)*a)/a
```

3.84 $\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 373

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \frac{\left(\sqrt{\frac{c}{a}}d+e\right)\arctan\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2+ae^2}} + \frac{a^{3/4}\left(\frac{c}{a}\right)^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{c^{5/4}\left(\sqrt{\frac{c}{a}}d-e\right)\sqrt{a+cx^4}} - \frac{\left(\sqrt{\frac{c}{a}}d+e\right)^2\left(1+\sqrt{\frac{c}{a}}x^2\right)\sqrt{\frac{\sqrt{\frac{c}{a}}(a+cx^4)}{c\left(\frac{1}{\sqrt[4]{\frac{c}{a}}}+\sqrt[4]{\frac{c}{a}}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{2}\right)}{4\sqrt[4]{\frac{c}{a}}d\left(\sqrt{\frac{c}{a}}d-e\right)e\sqrt{a+cx^4}}$$

output
```
1/2*((c/a)^(1/2)*d+e)*arctan((a*e^2+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*x^4+
a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2+c*d^2)^(1/2)+a^(3/4)*(c/a)^(3/2)*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(5/4)/((c/a)^(1/2)*d-e)/(c*x^4+a)
^(1/2)-1/4*((c/a)^(1/2)*d+e)^2*(1+(c/a)^(1/2)*x^2)*((c/a)^(1/2)*(c*x^4+a)/
c/(1/(c/a)^(1/4)+(c/a)^(1/4)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan((c/a)^(
1/4)*x)),-1/4*((c/a)^(1/2)*d-e)^2/(c/a)^(1/2)/d/e,1/2*2^(1/2))/(c/a)^(1/4)
/d/((c/a)^(1/2)*d-e)/e/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.41

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \frac{i\sqrt{1+\frac{cx^4}{a}}\left(\sqrt{\frac{c}{a}}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)-\left(\sqrt{\frac{c}{a}}d+e\right)\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d},i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}de\sqrt{a+cx^4}}$$

input
```
Integrate[(1 - Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(I*Sqrt[1 + (c*x^4)/a]*(Sqrt[c/a]*d*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/S
qrt[a]]*x], -1] - (Sqrt[c/a]*d + e)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d
), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]
]*d*e*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {2225, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2\sqrt{\frac{c}{a}}}{\sqrt{a+cx^4}\,(d+ex^2)}dx$$

$$\Big\downarrow \; 2225$$

$$\frac{2\sqrt{\frac{c}{a}}\int \frac{1}{\sqrt{cx^4+a}}dx}{d\sqrt{\frac{c}{a}}-e}-\frac{\left(d\sqrt{\frac{c}{a}}+e\right)\int \frac{\sqrt{\frac{c}{a}}x^2+1}{(ex^2+d)\sqrt{cx^4+a}}dx}{d\sqrt{\frac{c}{a}}-e}$$

$$\Big\downarrow \; 761$$

$$\frac{\sqrt{\frac{c}{a}}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(d\sqrt{\frac{c}{a}}-e\right)}-\frac{\left(d\sqrt{\frac{c}{a}}+e\right)\int \frac{\sqrt{\frac{c}{a}}x^2+1}{(ex^2+d)\sqrt{cx^4+a}}dx}{d\sqrt{\frac{c}{a}}-e}$$

$$\Big\downarrow \; 2221$$

$$\frac{\sqrt{\frac{c}{a}}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(d\sqrt{\frac{c}{a}}-e\right)}-\frac{\left(d\sqrt{\frac{c}{a}}+e\right)\left(\frac{\left(x^2\sqrt{\frac{c}{a}}+1\right)\sqrt{\frac{a+cx^4}{a\left(x^2\sqrt{\frac{c}{a}}+1\right)^2}}\left(d\sqrt{\frac{c}{a}}+e\right)\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{2}\right)}{4de\sqrt[4]{\frac{c}{a}}\sqrt{a+cx^4}}-\frac{\left(d\sqrt{\frac{c}{a}}-e\right)\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}}\right.}{d\sqrt{\frac{c}{a}}-e}$$

input

```
Int[(1 - Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

output

```
(Sqrt[c/a]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*c^(1/4)*(Sqrt
[c/a]*d - e)*Sqrt[a + c*x^4]) - ((Sqrt[c/a]*d + e)*(-1/2*((Sqrt[c/a]*d - e
)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(Sqrt
[d]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) + ((Sqrt[c/a]*d + e)*(1 + Sqrt[c/a]*x^2)*
Sqrt[(a + c*x^4)/(a*(1 + Sqrt[c/a]*x^2)^2)]*EllipticPi[-1/4*(Sqrt[c/a]*d -
 e)^2/(Sqrt[c/a]*d*e), 2*ArcTan[(c/a)^(1/4)*x], 1/2])/(4*(c/a)^(1/4)*d*e*S
qrt[a + c*x^4])))/(Sqrt[c/a]*d - e)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2225
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[2*A*(B/(B*d + A*e))   Int[1/Sqrt[a + c*x^4], x], x] - S
imp[(B*d - A*e)/(B*d + A*e)   Int[(A - B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4])
, x], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c
/a] && EqQ[c*A^2 - a*B^2, 0] && NegQ[B/A]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.54

method	result
default	$-\frac{\sqrt{\frac{c}{a}}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{\left(\sqrt{\frac{c}{a}}\,d+e\right)\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{ed\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$
elliptic	$\frac{\left(1-\sqrt{\frac{c}{a}}\,x^{2}\right)\sqrt{\frac{\left(c\,x^{4}+a\right)c}{a}}\,a\left(\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\right.}{ae\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{\frac{c^{2}x^{4}}{a}+c}}\right)}{-c\,x^{2}\sqrt{c\,x^{4}+a}+a\sqrt{\frac{\left(c\,x^{4}+a\right)c}{a}}}$

input
```
int((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c/a)^(1/2)/e/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(
1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1
/2))^(1/2),I)+((c/a)^(1/2)*d+e)/e/d/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)
*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icPi(x*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(1/2)*a^(1/2)/d*e,(-I/a^(1/2)*c^(1/2)
)^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\int \frac{x^2\sqrt{\frac{c}{a}}}{d\sqrt{a+cx^4}+ex^2\sqrt{a+cx^4}}\,dx - \int \left(-\frac{1}{d\sqrt{a+cx^4}+ex^2\sqrt{a+cx^4}}\right)dx$$

input
```
integrate((1-(c/a)**(1/2)*x**2)/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

output
```
-Integral(x**2*sqrt(c/a)/(d*sqrt(a + c*x**4) + e*x**2*sqrt(a + c*x**4)), x
) - Integral(-1/(d*sqrt(a + c*x**4) + e*x**2*sqrt(a + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int -\frac{x^2\sqrt{\frac{c}{a}}-1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
-integrate((x^2*sqrt(c/a) - 1)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int -\frac{x^2\sqrt{\frac{c}{a}}-1}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

input
```
integrate((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate(-(x^2*sqrt(c/a) - 1)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\int \frac{x^2\sqrt{\frac{c}{a}}-1}{\sqrt{c\,x^4+a}\,(e\,x^2+d)}\,dx$$

input
```
int(-(x^2*(c/a)^(1/2) - 1)/((a + c*x^4)^(1/2)*(d + e*x^2)),x)
```

output
```
-int((x^2*(c/a)^(1/2) - 1)/((a + c*x^4)^(1/2)*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx$$
$$= \frac{-\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{c\,x^4+a}}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)a}{a}$$

input
```
int((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(a + c*x**4)*x**2)/(a*d + a*e*x**2 + c*d*x**4
 + c*e*x**6),x) + int(sqrt(a + c*x**4)/(a*d + a*e*x**2 + c*d*x**4 + c*e*x*
*6),x)*a)/a
```

3.85 $\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 198

$$\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+2b}x}{\sqrt{a+b+bx^4}}\right)}{2\sqrt{a+2b}} + \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a+b}}\right)\left(\sqrt{a+b}+\sqrt{b}x^2\right)\sqrt{\frac{a+b+bx^4}{\left(\sqrt{a+b}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{4\sqrt[4]{b}\sqrt[4]{a+b}\sqrt{a+b+bx^4}}$$

output
```
1/2*(1+b^(1/2)/(a+b)^(1/2))*arctanh((a+2*b)^(1/2)*x/(b*x^4+a+b)^(1/2))/(a+
2*b)^(1/2)+1/4*(1-b^(1/2)/(a+b)^(1/2))*((a+b)^(1/2)+b^(1/2)*x^2)*((b*x^4+a
+b)/((a+b)^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/(
a+b)^(1/4))),1/4*(b^(1/2)+(a+b)^(1/2))^2/b^(1/2)/(a+b)^(1/2),1/2*2^(1/2))/
b^(1/4)/(a+b)^(1/4)/(b*x^4+a+b)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.97 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$= \frac{i\sqrt{a+b}\left(\frac{a+b+bx^4}{a+b}\right)^{3/2}\left(b^{3/4}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}x\right),-1\right)+(-1)^{3/4}\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}\sqrt[4]{a+b}\left(\sqrt{b}+\sqrt{\right.\right.}{\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}\sqrt[4]{b}\,(a+b+bx^4)^{3/2}}$$

input
```
Integrate[(1 + (Sqrt[b]*x^2)/Sqrt[a + b])/((1 - x^2)*Sqrt[a + b + b*x^4]),
x]
```

output
```
(I*Sqrt[a + b]*((a + b + b*x^4)/(a + b))^(3/2)*(b^(3/4)*EllipticF[I*ArcSin
h[Sqrt[-(Sqrt[b]/Sqrt[-a - b])]*x], -1] + (-1)^(3/4)*Sqrt[-(Sqrt[b]/Sqrt[-
a - b])]*(a + b)^(1/4)*(Sqrt[b] + Sqrt[a + b])*EllipticPi[(I*Sqrt[a + b])/
Sqrt[b], ArcSin[((-1)^(3/4)*b^(1/4)*x)/(a + b)^(1/4)], -1]))/(Sqrt[-(Sqrt[
b]/Sqrt[-a - b])]*b^(1/4)*(a + b + b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = {2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1}{(1-x^2)\sqrt{a+bx^4+b}}\,dx$$
$$\Big\downarrow \; 2223$$

$$\frac{\sqrt[4]{a+b}\left(1-\frac{\sqrt{b}}{\sqrt{a+b}}\right)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{4\sqrt[4]{b}\sqrt{a+bx^4+b}}+\frac{\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\operatorname{arctanh}\left(\frac{x\sqrt{a+2b}}{\sqrt{a+bx^4+b}}\right)}{2\sqrt{a+2b}}$$

input
```
Int[(1 + (Sqrt[b]*x^2)/Sqrt[a + b])/((1 - x^2)*Sqrt[a + b + b*x^4]),x]
```

output
```
((1 + Sqrt[b]/Sqrt[a + b])*ArcTanh[(Sqrt[a + 2*b]*x)/Sqrt[a + b + b*x^4]])
/(2*Sqrt[a + 2*b]) + ((a + b)^(1/4)*(1 - Sqrt[b]/Sqrt[a + b])*(1 + (Sqrt[b
]*x^2)/Sqrt[a + b])*Sqrt[(a + b + b*x^4)/((a + b)*(1 + (Sqrt[b]*x^2)/Sqrt[
a + b])^2)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b]),
2*ArcTan[(b^(1/4)*x)/(a + b)^(1/4)], 1/2])/(4*b^(1/4)*Sqrt[a + b + b*x^4])
```

Defintions of rubi rules used

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.43 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\frac{\sqrt{b}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}}\sqrt{b\,x^4+a+b}}+\left(-\frac{\sqrt{b}}{2}-\frac{\sqrt{a+b}}{2}\right)\left(-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2+2a+2b}{2\sqrt{a+2b}\,\sqrt{b\,x^4+a+b}}\right)}{2\sqrt{a+2b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\,\text{Elli}\cdots}{\sqrt{\frac{i}{\sqrt{\cdots}}}\cdots}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^4+a+b)b(a+b)}\left(\sqrt{a+b}+\sqrt{b}\,x^2\right)\left(-\frac{b\sqrt{1-\frac{i\sqrt{b}\,(a+b)^{\frac{3}{2}}x^2}{a^2+2ba+b^2}}\sqrt{1+\frac{i\sqrt{b}\,(a+b)^{\frac{3}{2}}x^2}{a^2+2ba+b^2}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}\,(a+b)^{\frac{3}{2}}}{a^2+2ba+b^2}},i\right)}{\sqrt{\frac{i\sqrt{b}\,(a+b)^{\frac{3}{2}}}{a^2+2ba+b^2}}\sqrt{b^2a\,x^4+b^3x^4+b\,a^2+2b^2a+b^3}}+\frac{b\sqrt{1-\frac{i\sqrt{b}\,(a+b)^{\frac{3}{2}}x^2}{a^2+2ba+b^2}}\cdots}{\cdots}\cdots\right.}{\left(b\,x^2\sqrt{b\,x^4+a+\cdots}\right.\cdots}$

input
```
int((1+b^(1/2)*x^2/(a+b)^(1/2))/(-x^2+1)/(b*x^4+a+b)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/(a+b)^(1/2)*(b^(1/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+
b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*Elli
pticF(x*(I*b^(1/2)/(a+b)^(1/2))^(1/2),I)+(-1/2*b^(1/2)-1/2*(a+b)^(1/2))*(-
1/2/(a+2*b)^(1/2)*arctanh(1/2*(2*b*x^2+2*a+2*b)/(a+2*b)^(1/2)/(b*x^4+a+b)^
(1/2))+1/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)
*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*EllipticPi(x*(I*b^(
1/2)/(a+b)^(1/2))^(1/2),-I/b^(1/2)*(a+b)^(1/2),(-I*b^(1/2)/(a+b)^(1/2))^(1
/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)))+(1/2*b^(1/2)+1/2*(a+b)^(1/2))*(-1/2/(a
+2*b)^(1/2)*arctanh(1/2*(2*b*x^2+2*a+2*b)/(a+2*b)^(1/2)/(b*x^4+a+b)^(1/2))
-1/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*
b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*EllipticPi(x*(I*b^(1/2)/(
a+b)^(1/2))^(1/2),-I/b^(1/2)*(a+b)^(1/2),(-I*b^(1/2)/(a+b)^(1/2))^(1/2)/(I
*b^(1/2)/(a+b)^(1/2))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \text{Timed out}$$

input
```
integrate((1+b^(1/2)*x^2/(a+b)^(1/2))/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algori
thm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = -\frac{\int \frac{\sqrt{a+b}}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\,dx + \int \frac{\sqrt{b}x^2}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\,dx}{\sqrt{a+b}}$$

input
```
integrate((1+b**(1/2)*x**2/(a+b)**(1/2))/(-x**2+1)/(b*x**4+a+b)**(1/2),x)
```

output
```
-(Integral(sqrt(a + b)/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4 + b)),
 x) + Integral(sqrt(b)*x**2/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4 +
 b)), x))/sqrt(a + b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((1+b^(1/2)*x^2/(a+b)^(1/2))/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((1+b^(1/2)*x^2/(a+b)^(1/2))/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \int -\frac{\frac{\sqrt{b}\,x^2}{\sqrt{a+b}}+1}{(x^2-1)\,\sqrt{b\,x^4+a+b}}\,dx$$

input
```
int(-((b^(1/2)*x^2)/(a + b)^(1/2) + 1)/((x^2 - 1)*(a + b + b*x^4)^(1/2)),x
)
```

output
```
int(-((b^(1/2)*x^2)/(a + b)^(1/2) + 1)/((x^2 - 1)*(a + b + b*x^4)^(1/2)),
x)
```

Reduce [F]

$$\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = -\sqrt{b}\,\sqrt{a+b}\left(\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{ab\,x^6+b^2x^6-ab\,x^4-b^2x^4+a^2x^2+2ab\,x^2+b^2x^2-a^2-2ab-b^2}dx\right)\right.$$
$$-\left(\int \frac{\sqrt{b\,x^4+a+b}}{ab\,x^6+b^2x^6-ab\,x^4-b^2x^4+a^2x^2+2ab\,x^2+b^2x^2-a^2-2ab-b^2}dx\right)a$$
$$\left.-\left(\int \frac{\sqrt{b\,x^4+a+b}}{ab\,x^6+b^2x^6-ab\,x^4-b^2x^4+a^2x^2+2ab\,x^2+b^2x^2-a^2-2ab-b^2}dx\right)b\right)$$

input
```
int((1+b^(1/2)*x^2/(a+b)^(1/2))/(-x^2+1)/(b*x^4+a+b)^(1/2),x)
```

output
```
 - (sqrt(b)*sqrt(a + b)*int((sqrt(a + b*x**4 + b)*x**2)/(a**2*x**2 - a**2
+ a*b*x**6 - a*b*x**4 + 2*a*b*x**2 - 2*a*b + b**2*x**6 - b**2*x**4 + b**2*
x**2 - b**2),x) + int(sqrt(a + b*x**4 + b)/(a**2*x**2 - a**2 + a*b*x**6 -
a*b*x**4 + 2*a*b*x**2 - 2*a*b + b**2*x**6 - b**2*x**4 + b**2*x**2 - b**2),
x)*a + int(sqrt(a + b*x**4 + b)/(a**2*x**2 - a**2 + a*b*x**6 - a*b*x**4 +
2*a*b*x**2 - 2*a*b + b**2*x**6 - b**2*x**4 + b**2*x**2 - b**2),x)*b)
```

3.86 $\int \frac{1+\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$

Optimal result

Integrand size = 72, antiderivative size = 198

$$\int \frac{1+\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx=\frac{\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+2b}x}{\sqrt{a+b+bx^4}}\right)}{2\sqrt{a+2b}}+\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a+b}}\right)\left(\sqrt[4]{a+b}+\frac{\sqrt{b}x^2}{\sqrt[4]{a+b}}\right)\sqrt{\frac{a+b+bx^4}{\left(\sqrt{a+b}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{4\sqrt[4]{b}\sqrt{a+b+bx^4}}$$

output
```
1/2*(1+b^(1/2)/(a+b)^(1/2))*arctanh((a+2*b)^(1/2)*x/(b*x^4+a+b)^(1/2))/(a+
2*b)^(1/2)+1/4*(1-b^(1/2)/(a+b)^(1/2))*((a+b)^(1/4)+b^(1/2)*x^2/(a+b)^(1/4
))*((b*x^4+a+b)/((a+b)^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan
(b^(1/4)*x/(a+b)^(1/4))),1/4*(b^(1/2)+(a+b)^(1/2))^2/b^(1/2)/(a+b)^(1/2),1
/2*2^(1/2))/b^(1/4)/(b*x^4+a+b)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.68 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.05

$$\int \frac{1+\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$= \frac{\left(\frac{a+b+bx^4}{a+b}\right)^{3/2}\left(-ib^{3/4}\left(a+b-\sqrt{b}\sqrt{a+b}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}x\right),-1\right)+\sqrt[4]{-1}a\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}(\right.}{\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}\sqrt[4]{b}\left(\sqrt{b}-\sqrt{a+b}\right)(a+b+bx^4)^{3/2}}$$

input
```
Integrate[(1 + (Sqrt[b]*(a + b - Sqrt[b]*Sqrt[a + b])*x^2)/((a + b)*(-Sqrt
[b] + Sqrt[a + b])))/((1 - x^2)*Sqrt[a + b + b*x^4]),x]
```

output
```
(((a + b + b*x^4)/(a + b))^(3/2)*((-I)*b^(3/4)*(a + b - Sqrt[b]*Sqrt[a + b
])*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[-a - b])]*x], -1] + (-1)^(1/4)*
a*Sqrt[-(Sqrt[b]/Sqrt[-a - b])]*(a + b)^(3/4)*EllipticPi[(I*Sqrt[a + b])/S
qrt[b], ArcSin[((-1)^(3/4)*b^(1/4)*x)/(a + b)^(1/4)], -1]))/(Sqrt[-(Sqrt[b
]/Sqrt[-a - b])]*b^(1/4)*(Sqrt[b] - Sqrt[a + b])*(a + b + b*x^4)^(3/2))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 615 vs. 2(198) = 396.

Time = 1.31 (sec) , antiderivative size = 615, normalized size of antiderivative = 3.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2225, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{\sqrt{b}x^2\left(-\sqrt{b}\sqrt{a+b}+a+b\right)}{(a+b)\left(\sqrt{a+b}-\sqrt{b}\right)}+1}{(1-x^2)\sqrt{a+bx^4+b}}\,dx$$
$$\Big\downarrow \text{2225}$$

$$\frac{2\sqrt{b}\left(-\sqrt{b}\sqrt{a+b}+a+b\right)\int \frac{1}{\sqrt{bx^4+a+b}}dx}{-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}}-\frac{a\sqrt{a+b}\int \frac{\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)}+1}{(1-x^2)\sqrt{bx^4+a+b}}dx}{-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}}$$

$\Big\downarrow$ 761

$$\frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(-\sqrt{b}\sqrt{a+b}+a+b\right)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{\left(-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}\right)\sqrt{a+bx^4+b}}-\frac{a\sqrt{a+b}\int \frac{\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)}+1}{(1-x^2)\sqrt{bx^4+a+b}}dx}{-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}}$$

$\Big\downarrow$ 2223

$$\frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(-\sqrt{b}\sqrt{a+b}+a+b\right)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{\left(-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}\right)\sqrt{a+bx^4+b}}-$$

$$\frac{a\sqrt{a+b}\left(\frac{\left(\frac{\sqrt{b}\left(-\sqrt{b}\sqrt{a+b}+a+b\right)}{(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)}+1\right)\text{arctanh}\left(\frac{x\sqrt{a+2b}}{\sqrt{a+bx^4+b}}\right)}{2\sqrt{a+2b}}-\frac{a\left(\frac{\sqrt{b}x^2\left(-\sqrt{b}\sqrt{a+b}+a+b\right)}{(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2\left(-\sqrt{b}\sqrt{a+b}+a+b\right)}{(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)}+1\right)^2}}\,\text{Ellipti}}{4\sqrt[4]{b}\sqrt{\sqrt{b}-\sqrt{a+b}}\sqrt{-}}\right.}{-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}}$$

input
```
Int[(1 + (Sqrt[b]*(a + b - Sqrt[b]*Sqrt[a + b])*x^2)/((a + b)*(-Sqrt[b] +
Sqrt[a + b])))/((1 - x^2)*Sqrt[a + b + b*x^4]),x]
```

output
```
(b^(1/4)*(a + b)^(1/4)*(a + b - Sqrt[b]*Sqrt[a + b])*(1 + (Sqrt[b]*x^2)/Sq
rt[a + b])*Sqrt[(a + b + b*x^4)/((a + b)*(1 + (Sqrt[b]*x^2)/Sqrt[a + b])^2
)]*EllipticF[2*ArcTan[(b^(1/4)*x)/(a + b)^(1/4)], 1/2])/((2*a*Sqrt[b] + 2*
b^(3/2) - a*Sqrt[a + b] - 2*b*Sqrt[a + b])*Sqrt[a + b + b*x^4]) - (a*Sqrt[
a + b]*(((1 + (Sqrt[b]*(a + b - Sqrt[b]*Sqrt[a + b]))/((a + b)*(Sqrt[b] -
Sqrt[a + b])))*ArcTanh[(Sqrt[a + 2*b]*x)/Sqrt[a + b + b*x^4]])/(2*Sqrt[a +
 2*b]) - (a*(1 + (Sqrt[b]*(a + b - Sqrt[b]*Sqrt[a + b])*x^2)/((a + b)*(Sqr
t[b] - Sqrt[a + b])))*Sqrt[(a + b + b*x^4)/((a + b)*(1 + (Sqrt[b]*(a + b -
 Sqrt[b]*Sqrt[a + b])*x^2)/((a + b)*(Sqrt[b] - Sqrt[a + b])))^2)]*Elliptic
Pi[(a^2 + 8*a*b + 8*b^2 - Sqrt[b]*Sqrt[a + b]*(4*a + 8*b))/(4*Sqrt[b]*(2*a
*Sqrt[b] + 2*b^(3/2) - a*Sqrt[a + b] - 2*b*Sqrt[a + b])), 2*ArcTan[(b^(1/4
)*Sqrt[a + b - Sqrt[b]*Sqrt[a + b]]*x)/(Sqrt[a + b]*Sqrt[Sqrt[b] - Sqrt[a
+ b]])], 1/2])/(4*b^(1/4)*Sqrt[Sqrt[b] - Sqrt[a + b]]*Sqrt[a + b - Sqrt[b]
*Sqrt[a + b]]*Sqrt[a + b + b*x^4])))/(2*a*Sqrt[b] + 2*b^(3/2) - a*Sqrt[a +
 b] - 2*b*Sqrt[a + b])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2225
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[2*A*(B/(B*d + A*e))   Int[1/Sqrt[a + c*x^4], x], x] - S
imp[(B*d - A*e)/(B*d + A*e)   Int[(A - B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4])
, x], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c
/a] && EqQ[c*A^2 - a*B^2, 0] && NegQ[B/A]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.96 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.97

method	result
elliptic	$\frac{\left(1+\frac{\sqrt{b}\left(\sqrt{b}\,\sqrt{a+b}-a-b\right)x^{2}}{(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)}\right)\sqrt{(b\,x^{4}+a+b)b(a+b)}\left(-\frac{b\sqrt{1-\frac{i\sqrt{b}\,(a+b)^{\frac{3}{2}}x^{2}}{a^{2}+2ba+b^{2}}}\,\sqrt{1+\frac{i\sqrt{b}\,(a+b)^{\frac{3}{2}}x^{2}}{a^{2}+2ba+b^{2}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}\,(a+b)^{\frac{3}{2}}}{a^{2}+2ba+b^{2}}},i\right)}{\sqrt{\frac{i\sqrt{b}\,(a+b)^{\frac{3}{2}}}{a^{2}+2ba+b^{2}}}\,\sqrt{b^{2}a\,x^{4}+b^{3}x^{4}+b\,a^{2}+2b^{2}a+b^{3}}}+\frac{b\sqrt{1-\frac{i\sqrt{}}{a}}}{}\right.}{b\,x^{2}\sqrt{}}$
default	$\frac{b^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a+b}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a+b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}}\,\sqrt{b\,x^{4}+a+b}}+\frac{\sqrt{b}\,a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a+b}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a+b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}}\,\sqrt{b\,x^{4}+a+b}}-\frac{b\sqrt{a+b}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a+b}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a+b}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}}\,\sqrt{b\,x^{4}}}$

input
```
int((1+b^(1/2)*(a+b-b^(1/2)*(a+b)^(1/2))*x^2/(a+b)/(-b^(1/2)+(a+b)^(1/2)))
/(-x^2+1)/(b*x^4+a+b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1+b^(1/2)*(b^(1/2)*(a+b)^(1/2)-a-b)*x^2/(a+b)/(b^(1/2)-(a+b)^(1/2)))/(b*x
^2*(b*x^4+a+b)^(1/2)+((b*x^4+a+b)*b*(a+b))^(1/2))*((b*x^4+a+b)*b*(a+b))^(1
/2)*(-b/(I*b^(1/2)*(a+b)^(3/2)/(a^2+2*a*b+b^2))^(1/2)*(1-I*b^(1/2)*(a+b)^(
3/2)/(a^2+2*a*b+b^2)*x^2)^(1/2)*(1+I*b^(1/2)*(a+b)^(3/2)/(a^2+2*a*b+b^2)*x
^2)^(1/2)/(a*b^2*x^4+b^3*x^4+a^2*b+2*a*b^2+b^3)^(1/2)*EllipticF(x*(I*b^(1/
2)*(a+b)^(3/2)/(a^2+2*a*b+b^2))^(1/2),I)+b/(I*b^(1/2)*(a+b)^(3/2)/(a^2+2*a
*b+b^2))^(1/2)*(1-I*b^(1/2)*(a+b)^(3/2)/(a^2+2*a*b+b^2)*x^2)^(1/2)*(1+I*b^
(1/2)*(a+b)^(3/2)/(a^2+2*a*b+b^2)*x^2)^(1/2)/(a*b^2*x^4+b^3*x^4+a^2*b+2*a*
b^2+b^3)^(1/2)*EllipticPi(x*(I*b^(1/2)*(a+b)^(3/2)/(a^2+2*a*b+b^2))^(1/2),
-I/b^(1/2)/(a+b)^(3/2)*(a^2+2*a*b+b^2),(-I*b^(1/2)*(a+b)^(3/2)/(a^2+2*a*b+
b^2))^(1/2)/(I*b^(1/2)*(a+b)^(3/2)/(a^2+2*a*b+b^2))^(1/2))+1/(I*b^(1/2)/(a
+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/(a+b
)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*EllipticPi(x*(I*b^(1/2)/(a+b)^(1/2))^(1/2
),-I/b^(1/2)*(a+b)^(1/2),(-I*b^(1/2)/(a+b)^(1/2))^(1/2)/(I*b^(1/2)/(a+b)^(
1/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \text{Timed out}$$

input
```
integrate((1+b^(1/2)*(a+b-b^(1/2)*(a+b)^(1/2))*x^2/(a+b)/(-b^(1/2)+(a+b)^(
1/2)))/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1+\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx =$$

$$-\frac{\int\left(-\frac{b^{\frac{3}{2}}}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\right)dx+\int\left(-\frac{a\sqrt{b}}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\right)dx+\int\frac{a\sqrt{a+b}}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\,dx+\int\frac{}{x^2\sqrt{a-}}}{(a+b)\left(-\right.}$$

input
```
integrate((1+b**(1/2)*(a+b-b**(1/2)*(a+b)**(1/2))*x**2/(a+b)/(-b**(1/2)+(a
+b)**(1/2)))/(-x**2+1)/(b*x**4+a+b)**(1/2),x)
```

output
```
-(Integral(-b**(3/2)/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4 + b)), x
) + Integral(-a*sqrt(b)/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4 + b))
, x) + Integral(a*sqrt(a + b)/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4
 + b)), x) + Integral(b*sqrt(a + b)/(x**2*sqrt(a + b*x**4 + b) - sqrt(a +
b*x**4 + b)), x) + Integral(b**(3/2)*x**2/(x**2*sqrt(a + b*x**4 + b) - sqr
t(a + b*x**4 + b)), x) + Integral(a*sqrt(b)*x**2/(x**2*sqrt(a + b*x**4 + b
) - sqrt(a + b*x**4 + b)), x) + Integral(-b*x**2*sqrt(a + b)/(x**2*sqrt(a
+ b*x**4 + b) - sqrt(a + b*x**4 + b)), x))/((a + b)*(-sqrt(b) + sqrt(a + b
)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1+\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((1+b^(1/2)*(a+b-b^(1/2)*(a+b)^(1/2))*x^2/(a+b)/(-b^(1/2)+(a+b)^(
1/2)))/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+b^(1/2)*(a+b-b^(1/2)*(a+b)^(1/2))*x^2/(a+b)/(-b^(1/2)+(a+b)^(
1/2)))/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Recursive assumption sageVARa>=(-sa
geVARb) ignoredRecursive assumption sageVARa>=(-sageVARb) ignoredRecursive
 assumpti
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx=\int -\frac{\frac{\sqrt{b}\,x^2\,\left(a+b-\sqrt{b}\,\sqrt{a+b}\right)}{\left(\sqrt{a+b}-\sqrt{b}\right)\,(a+b)}+1}{\left(x^2-1\right)\,\sqrt{b\,x^4+a+b}}\,dx$$

input
```
int(-((b^(1/2)*x^2*(a + b - b^(1/2)*(a + b)^(1/2)))/(((a + b)^(1/2) - b^(1
/2))*(a + b)) + 1)/((x^2 - 1)*(a + b + b*x^4)^(1/2)),x)
```

output
```
int(-((b^(1/2)*x^2*(a + b - b^(1/2)*(a + b)^(1/2)))/(((a + b)^(1/2) - b^(1
/2))*(a + b)) + 1)/((x^2 - 1)*(a + b + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1+\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)}}{(1-x^2)\sqrt{a+b+bx^4}}\,dx=$$
$$-\sqrt{b}\,\sqrt{a+b}\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{ab\,x^6+b^2x^6-ab\,x^4-b^2x^4+a^2x^2+2ab\,x^2+b^2x^2-a^2-2ab-b^2}dx\right)$$
$$-\left(\int \frac{\sqrt{b\,x^4+a+b}}{ab\,x^6+b^2x^6-ab\,x^4-b^2x^4+a^2x^2+2ab\,x^2+b^2x^2-a^2-2ab-b^2}dx\right)a$$
$$-\left(\int \frac{\sqrt{b\,x^4+a+b}}{ab\,x^6+b^2x^6-ab\,x^4-b^2x^4+a^2x^2+2ab\,x^2+b^2x^2-a^2-2ab-b^2}dx\right)b$$

input
```
int((1+b^(1/2)*(a+b-b^(1/2)*(a+b)^(1/2))*x^2/(a+b)/(-b^(1/2)+(a+b)^(1/2)))
/(-x^2+1)/(b*x^4+a+b)^(1/2),x)
```

output
```
 - (sqrt(b)*sqrt(a + b)*int((sqrt(a + b*x**4 + b)*x**2)/(a**2*x**2 - a**2
+ a*b*x**6 - a*b*x**4 + 2*a*b*x**2 - 2*a*b + b**2*x**6 - b**2*x**4 + b**2*
x**2 - b**2),x) + int(sqrt(a + b*x**4 + b)/(a**2*x**2 - a**2 + a*b*x**6 -
a*b*x**4 + 2*a*b*x**2 - 2*a*b + b**2*x**6 - b**2*x**4 + b**2*x**2 - b**2),
x)*a + int(sqrt(a + b*x**4 + b)/(a**2*x**2 - a**2 + a*b*x**6 - a*b*x**4 +
2*a*b*x**2 - 2*a*b + b**2*x**6 - b**2*x**4 + b**2*x**2 - b**2),x)*b)
```

3.87 $\int \frac{-\sqrt{a+b}-\sqrt{b}x^2}{\sqrt{a+b}(-1+x^2)\sqrt{a+b+bx^4}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 202

$$\int \frac{-\sqrt{a+b}-\sqrt{b}x^2}{\sqrt{a+b}\,(-1+x^2)\,\sqrt{a+b+bx^4}}\,dx = \frac{\left(\sqrt{b}+\sqrt{a+b}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+2b}x}{\sqrt{a+b+bx^4}}\right)}{2\sqrt{a+b}\sqrt{a+2b}} - \frac{\left(\sqrt{b}-\sqrt{a+b}\right)\left(\sqrt{a+b}+\sqrt{b}x^2\right)\sqrt{\frac{a+b+bx^4}{\left(\sqrt{a+b}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{4\sqrt[4]{b}(a+b)^{3/4}\sqrt{a+b+bx^4}}$$

output
```
1/2*(b^(1/2)+(a+b)^(1/2))*arctanh((a+2*b)^(1/2)*x/(b*x^4+a+b)^(1/2))/(a+b)
^(1/2)/(a+2*b)^(1/2)-1/4*(b^(1/2)-(a+b)^(1/2))*((a+b)^(1/2)+b^(1/2)*x^2)*(
(b*x^4+a+b)/((a+b)^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(
1/4)*x/(a+b)^(1/4))),1/4*(b^(1/2)+(a+b)^(1/2))^2/b^(1/2)/(a+b)^(1/2),1/2*2
^(1/2))/b^(1/4)/(a+b)^(3/4)/(b*x^4+a+b)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.64 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.95

$$\int \frac{-\sqrt{a+b}-\sqrt{b}x^2}{\sqrt{a+b}\,(-1+x^2)\,\sqrt{a+b+bx^4}}\,dx$$
$$= \frac{i\sqrt{\frac{a+b+bx^4}{a+b}}\left(b^{3/4}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}x\right),-1\right)+(-1)^{3/4}\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}\sqrt[4]{a+b}\left(\sqrt{b}+\sqrt{a+b}\right)\text{Ell}}{\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}\sqrt[4]{b}\sqrt{a+b}\sqrt{a+b+bx^4}}$$

input
```
Integrate[(-Sqrt[a + b] - Sqrt[b]*x^2)/(Sqrt[a + b]*(-1 + x^2)*Sqrt[a + b
+ b*x^4]),x]
```

output
```
(I*Sqrt[(a + b + b*x^4)/(a + b)]*(b^(3/4)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[
b]/Sqrt[-a - b])]*x], -1] + (-1)^(3/4)*Sqrt[-(Sqrt[b]/Sqrt[-a - b])]*(a +
b)^(1/4)*(Sqrt[b] + Sqrt[a + b])*EllipticPi[(I*Sqrt[a + b])/Sqrt[b], ArcSi
n[((-1)^(3/4)*b^(1/4)*x)/(a + b)^(1/4)], -1]))/(Sqrt[-(Sqrt[b]/Sqrt[-a - b
])]*b^(1/4)*Sqrt[a + b]*Sqrt[a + b + b*x^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {27, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-\sqrt{a+b}-\sqrt{b}x^2}{(x^2-1)\,\sqrt{a+b}\sqrt{a+bx^4+b}}\,dx$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{\sqrt{b}x^2+\sqrt{a+b}}{(1-x^2)\sqrt{bx^4+a+b}}dx}{\sqrt{a+b}}$$

$$\Big\downarrow \text{2223}$$

$$\frac{\frac{\left(\sqrt{a+b}+\sqrt{b}\right)\text{arctanh}\left(\frac{x\sqrt{a+2b}}{\sqrt{a+bx^4+b}}\right)}{2\sqrt{a+2b}}-\frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right)\right)}{4\sqrt[4]{b}\sqrt{a+bx^4+b}}}{\sqrt{a+b}}$$

input
```
Int[(-Sqrt[a + b] - Sqrt[b]*x^2)/(Sqrt[a + b]*(-1 + x^2)*Sqrt[a + b + b*x^
4]),x]
```

output
```
(((Sqrt[b] + Sqrt[a + b])*ArcTanh[(Sqrt[a + 2*b]*x)/Sqrt[a + b + b*x^4]])/
(2*Sqrt[a + 2*b]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])*(1 + (Sqrt[b]*x
^2)/Sqrt[a + b])*Sqrt[(a + b + b*x^4)/((a + b)*(1 + (Sqrt[b]*x^2)/Sqrt[a +
 b])^2)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*A
rcTan[(b^(1/4)*x)/(a + b)^(1/4)], 1/2])/(4*b^(1/4)*Sqrt[a + b + b*x^4]))/S
qrt[a + b]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.95 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.15

method	result
default	$\frac{-\frac{\sqrt{b}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}}\sqrt{b\,x^4+a+b}}-\left(-\frac{\sqrt{b}}{2}-\frac{\sqrt{a+b}}{2}\right)\left(-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2+2a+2b}{2\sqrt{a+2b}\,\sqrt{b\,x^4+a+b}}\right)}{2\sqrt{a+2b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\,\mathrm{Ellip}\cdots}{\sqrt{\frac{i\cdots}{\sqrt{a\cdots}}}\cdots}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(a+b)(b\,x^4+a+b)}\,\sqrt{(b\,x^4+a+b)b}\left(\sqrt{a+b}+\sqrt{b}\,x^2\right)\left(-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}},i\right)}{\sqrt{a+b}\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}}\sqrt{b^2x^4+ba+b^2}}+\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\,\mathrm{El}\cdots}{\sqrt{a+b}\sqrt{\cdots}}\cdots\right.}{\left(b\,x^2\sqrt{(a+b)(b\,x^4+a+b)}+\sqrt{\cdots}\right.\cdots}$

input
```
int((-(a+b)^(1/2)-b^(1/2)*x^2)/(a+b)^(1/2)/(x^2-1)/(b*x^4+a+b)^(1/2),x,met
hod=_RETURNVERBOSE)
```

output
```
1/(a+b)^(1/2)*(-b^(1/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+
b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*Elli
pticF(x*(I*b^(1/2)/(a+b)^(1/2))^(1/2),I)-(-1/2*b^(1/2)-1/2*(a+b)^(1/2))*(-
1/2/(a+2*b)^(1/2)*arctanh(1/2*(2*b*x^2+2*a+2*b)/(a+2*b)^(1/2)/(b*x^4+a+b)^
(1/2))+1/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)
*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*EllipticPi(x*(I*b^(
1/2)/(a+b)^(1/2))^(1/2),-I/b^(1/2)*(a+b)^(1/2),(-I*b^(1/2)/(a+b)^(1/2))^(1
/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)))-(1/2*b^(1/2)+1/2*(a+b)^(1/2))*(-1/2/(a
+2*b)^(1/2)*arctanh(1/2*(2*b*x^2+2*a+2*b)/(a+2*b)^(1/2)/(b*x^4+a+b)^(1/2))
-1/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*
b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*EllipticPi(x*(I*b^(1/2)/(
a+b)^(1/2))^(1/2),-I/b^(1/2)*(a+b)^(1/2),(-I*b^(1/2)/(a+b)^(1/2))^(1/2)/(I
*b^(1/2)/(a+b)^(1/2))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{-\sqrt{a+b}-\sqrt{b}x^2}{\sqrt{a+b}\,(-1+x^2)\,\sqrt{a+b+bx^4}}\,dx=\text{Timed out}$$

input
```
integrate((-(a+b)^(1/2)-b^(1/2)*x^2)/(a+b)^(1/2)/(x^2-1)/(b*x^4+a+b)^(1/2)
,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{-\sqrt{a+b}-\sqrt{b}x^2}{\sqrt{a+b}\,(-1+x^2)\,\sqrt{a+b+bx^4}}\,dx$$
$$=-\frac{\int \frac{\sqrt{a+b}}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\,dx+\int \frac{\sqrt{b}x^2}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\,dx}{\sqrt{a+b}}$$

input
```
integrate((-(a+b)**(1/2)-b**(1/2)*x**2)/(a+b)**(1/2)/(x**2-1)/(b*x**4+a+b)
**(1/2),x)
```

output
```
-(Integral(sqrt(a + b)/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4 + b)),
 x) + Integral(sqrt(b)*x**2/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4 +
 b)), x))/sqrt(a + b)
```

Maxima [F]

$$\int \frac{-\sqrt{a+b}-\sqrt{b}x^2}{\sqrt{a+b}\,(-1+x^2)\,\sqrt{a+b+bx^4}}\,dx = \int -\frac{\sqrt{b}x^2+\sqrt{a+b}}{\sqrt{bx^4+a+b}(x^2-1)\sqrt{a+b}}\,dx$$

input
```
integrate((-(a+b)^(1/2)-b^(1/2)*x^2)/(a+b)^(1/2)/(x^2-1)/(b*x^4+a+b)^(1/2)
,x, algorithm="maxima")
```

output
```
-integrate((sqrt(b)*x^2 + sqrt(a + b))/(sqrt(b*x^4 + a + b)*(x^2 - 1)), x)
/sqrt(a + b)
```

Giac [F]

$$\int \frac{-\sqrt{a+b}-\sqrt{b}x^2}{\sqrt{a+b}\,(-1+x^2)\,\sqrt{a+b+bx^4}}\,dx = \int -\frac{\sqrt{b}x^2+\sqrt{a+b}}{\sqrt{bx^4+a+b}(x^2-1)\sqrt{a+b}}\,dx$$

input
```
integrate((-(a+b)^(1/2)-b^(1/2)*x^2)/(a+b)^(1/2)/(x^2-1)/(b*x^4+a+b)^(1/2)
,x, algorithm="giac")
```

output
```
integrate(-(sqrt(b)*x^2 + sqrt(a + b))/(sqrt(b*x^4 + a + b)*(x^2 - 1)*sqrt
(a + b)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-\sqrt{a+b}-\sqrt{b}x^2}{\sqrt{a+b}\,(-1+x^2)\,\sqrt{a+b+bx^4}}\,dx = \int -\frac{\sqrt{a+b}+\sqrt{b}\,x^2}{\sqrt{a+b}\,\left(x^2-1\right)\,\sqrt{b\,x^4+a+b}}\,dx$$

input
```
int(-((a + b)^(1/2) + b^(1/2)*x^2)/((a + b)^(1/2)*(x^2 - 1)*(a + b + b*x^4
)^(1/2)),x)
```

output
```
int(-((a + b)^(1/2) + b^(1/2)*x^2)/((a + b)^(1/2)*(x^2 - 1)*(a + b + b*x^4
)^(1/2)), x)
```

Reduce [F]

$$\int \frac{-\sqrt{a+b}-\sqrt{b}x^2}{\sqrt{a+b}\,(-1+x^2)\,\sqrt{a+b+bx^4}}\,dx = -\sqrt{b}\,\sqrt{a+b}\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{ab\,x^6+b^2x^6-ab\,x^4-b^2x^4+a^2x^2+2ab\,x^2+b^2x^2-a^2-2ab-b^2}dx\right) - \left(\int \frac{\sqrt{b\,x^4+a+b}}{ab\,x^6+b^2x^6-ab\,x^4-b^2x^4+a^2x^2+2ab\,x^2+b^2x^2-a^2-2ab-b^2}dx\right)a - \left(\int \frac{\sqrt{b\,x^4+a+b}}{ab\,x^6+b^2x^6-ab\,x^4-b^2x^4+a^2x^2+2ab\,x^2+b^2x^2-a^2-2ab-b^2}dx\right)b$$

input
```
int((-(a+b)^(1/2)-b^(1/2)*x^2)/(a+b)^(1/2)/(x^2-1)/(b*x^4+a+b)^(1/2),x)
```

output
```
 - (sqrt(b)*sqrt(a + b)*int((sqrt(a + b*x**4 + b)*x**2)/(a**2*x**2 - a**2
+ a*b*x**6 - a*b*x**4 + 2*a*b*x**2 - 2*a*b + b**2*x**6 - b**2*x**4 + b**2*
x**2 - b**2),x) + int(sqrt(a + b*x**4 + b)/(a**2*x**2 - a**2 + a*b*x**6 -
a*b*x**4 + 2*a*b*x**2 - 2*a*b + b**2*x**6 - b**2*x**4 + b**2*x**2 - b**2),
x)*a + int(sqrt(a + b*x**4 + b)/(a**2*x**2 - a**2 + a*b*x**6 - a*b*x**4 +
2*a*b*x**2 - 2*a*b + b**2*x**6 - b**2*x**4 + b**2*x**2 - b**2),x)*b)
```

3.88 $\int \frac{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)+\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$

Optimal result

Integrand size = 68, antiderivative size = 192

$$\int \frac{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)+\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \frac{a\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+2b}x}{\sqrt{a+b+bx^4}}\right)}{2\sqrt{a+2b}} + \frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)^2\left(\sqrt{a+b}+\sqrt{b}x^2\right)\sqrt{\frac{a+b+bx^4}{\left(\sqrt{a+b}+\sqrt{b}x^2\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right)\right.}{4\sqrt[4]{b}\sqrt{a+b+bx^4}}$$

output
```
1/2*a*(a+b)^(1/2)*arctanh((a+2*b)^(1/2)*x/(b*x^4+a+b)^(1/2))/(a+2*b)^(1/2)
+1/4*(a+b)^(1/4)*(b^(1/2)-(a+b)^(1/2))^2*((a+b)^(1/2)+b^(1/2)*x^2)*((b*x^4
+a+b)/((a+b)^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x
/(a+b)^(1/4))),1/4*(b^(1/2)+(a+b)^(1/2))^2/b^(1/2)/(a+b)^(1/2),1/2*2^(1/2)
)/b^(1/4)/(b*x^4+a+b)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.80 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.99

$$\int \frac{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)+\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$= \frac{i\sqrt{\frac{a+b+bx^4}{a+b}}\left(b^{3/4}\left(a+b-\sqrt{b}\sqrt{a+b}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}x\right),-1\right)+(-1)^{3/4}a\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}(a+\right.}{\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}\sqrt[4]{b}\sqrt{a+b+bx^4}}$$

input
```
Integrate[((a + b)*(-Sqrt[b] + Sqrt[a + b]) + Sqrt[b]*(a + b - Sqrt[b]*Sqr
t[a + b])*x^2)/((1 - x^2)*Sqrt[a + b + b*x^4]),x]
```

output
```
(I*Sqrt[(a + b + b*x^4)/(a + b)]*(b^(3/4)*(a + b - Sqrt[b]*Sqrt[a + b])*El
lipticF[I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[-a - b])]*x], -1] + (-1)^(3/4)*a*Sqr
t[-(Sqrt[b]/Sqrt[-a - b])]*(a + b)^(3/4)*EllipticPi[(I*Sqrt[a + b])/Sqrt[b
], ArcSin[((-1)^(3/4)*b^(1/4)*x)/(a + b)^(1/4)], -1]))/(Sqrt[-(Sqrt[b]/Sqr
t[-a - b])]*b^(1/4)*Sqrt[a + b + b*x^4])
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 616 vs. $2(192) = 384$.

Time = 1.12 (sec) , antiderivative size = 616, normalized size of antiderivative = 3.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2225, 25, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b}x^2\left(-\sqrt{b}\sqrt{a+b}+a+b\right)+(a+b)\left(\sqrt{a+b}-\sqrt{b}\right)}{(1-x^2)\sqrt{a+bx^4+b}}\,dx$$
$$\Big\downarrow \; 2225$$

$$-\frac{2\sqrt{b}(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)\left(-\sqrt{b}\sqrt{a+b}+a+b\right)\int \frac{1}{\sqrt{bx^4+a+b}}dx}{-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}}-\frac{a\sqrt{a+b}\int -\frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2+(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)}{(1-x^2)\sqrt{bx^4+a+b}}dx}{-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}}$$

$\Big\downarrow$ 25

$$\frac{a\sqrt{a+b}\int \frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2+(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)}{(1-x^2)\sqrt{bx^4+a+b}}dx}{-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}}-\frac{2\sqrt{b}(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)\left(-\sqrt{b}\sqrt{a+b}+a+b\right)\int \frac{1}{\sqrt{bx^4+a+b}}dx}{-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}}$$

$\Big\downarrow$ 761

$$\frac{a\sqrt{a+b}\int \frac{\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2+(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)}{(1-x^2)\sqrt{bx^4+a+b}}dx}{-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}}-\frac{\sqrt[4]{b}(a+b)^{5/4}\left(\sqrt{b}-\sqrt{a+b}\right)\left(-\sqrt{b}\sqrt{a+b}+a+b\right)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\,\text{EllipticF}\Bigg(2\arctan\Bigg(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\cdots}{\left(-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}\right)\sqrt{a+bx^4+b}}$$

$\Big\downarrow$ 2223

$$\frac{a\sqrt{a+b}\Bigg(\frac{\left(2a\sqrt{b}-\sqrt{a+b}(a+2b)+2b^{3/2}\right)\text{arctanh}\left(\frac{x\sqrt{a+2b}}{\sqrt{a+bx^4+b}}\right)}{2\sqrt{a+2b}}-a(a+b)\sqrt{\sqrt{b}-\sqrt{a+b}}\left(\frac{\sqrt{b}x^2\left(-\sqrt{b}\sqrt{a+b}+a+b\right)}{(a+b)\left(\sqrt{b}-\sqrt{a+b}\right)}+1\right)\sqrt{\frac{a+bx^4\cdots}{(a+b)\Big(\frac{\sqrt{b}x^2\left(-\sqrt{b}\sqrt{\cdots}\right.}{(a+b)\left(\sqrt{b}\cdots\right.}}}\cdots}{-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2\cdots}$$

$$\frac{\sqrt[4]{b}(a+b)^{5/4}\left(\sqrt{b}-\sqrt{a+b}\right)\left(-\sqrt{b}\sqrt{a+b}+a+b\right)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\,\text{EllipticF}\Bigg(2\arctan\Bigg(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\cdots}{\left(-2b\sqrt{a+b}+2a\sqrt{b}-a\sqrt{a+b}+2b^{3/2}\right)\sqrt{a+bx^4+b}}$$

input

```
Int[((a + b)*(-Sqrt[b] + Sqrt[a + b]) + Sqrt[b]*(a + b - Sqrt[b]*Sqrt[a +
b])*x^2)/((1 - x^2)*Sqrt[a + b + b*x^4]),x]
```

output
```
-((b^(1/4)*(a + b)^(5/4)*(Sqrt[b] - Sqrt[a + b])*(a + b - Sqrt[b]*Sqrt[a +
 b])*(1 + (Sqrt[b]*x^2)/Sqrt[a + b])*Sqrt[(a + b + b*x^4)/((a + b)*(1 + (S
qrt[b]*x^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*x)/(a + b)^(1/4)]
, 1/2])/((2*a*Sqrt[b] + 2*b^(3/2) - a*Sqrt[a + b] - 2*b*Sqrt[a + b])*Sqrt[
a + b + b*x^4])) + (a*Sqrt[a + b]*(((2*a*Sqrt[b] + 2*b^(3/2) - Sqrt[a + b]
*(a + 2*b))*ArcTanh[(Sqrt[a + 2*b]*x)/Sqrt[a + b + b*x^4]])/(2*Sqrt[a + 2*
b]) - (a*(a + b)*Sqrt[Sqrt[b] - Sqrt[a + b]]*(1 + (Sqrt[b]*(a + b - Sqrt[b
]*Sqrt[a + b])*x^2)/((a + b)*(Sqrt[b] - Sqrt[a + b])))*Sqrt[(a + b + b*x^4
)/((a + b)*(1 + (Sqrt[b]*(a + b - Sqrt[b]*Sqrt[a + b])*x^2)/((a + b)*(Sqrt
[b] - Sqrt[a + b])))^2)]*EllipticPi[(a^2 + 8*a*b + 8*b^2 - Sqrt[b]*Sqrt[a
+ b]*(4*a + 8*b))/(4*Sqrt[b]*(2*a*Sqrt[b] + 2*b^(3/2) - a*Sqrt[a + b] - 2*
b*Sqrt[a + b])), 2*ArcTan[(b^(1/4)*Sqrt[a + b - Sqrt[b]*Sqrt[a + b]]*x)/(S
qrt[a + b]*Sqrt[Sqrt[b] - Sqrt[a + b]])], 1/2])/(4*b^(1/4)*Sqrt[a + b - Sq
rt[b]*Sqrt[a + b]]*Sqrt[a + b + b*x^4])))/(2*a*Sqrt[b] + 2*b^(3/2) - a*Sqr
t[a + b] - 2*b*Sqrt[a + b])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2225
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[2*A*(B/(B*d + A*e))   Int[1/Sqrt[a + c*x^4], x], x] - S
imp[(B*d - A*e)/(B*d + A*e)   Int[(A - B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4])
, x], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c
/a] && EqQ[c*A^2 - a*B^2, 0] && NegQ[B/A]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.45 (sec) , antiderivative size = 582, normalized size of antiderivative = 3.03

method	result
default	$-\frac{b^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a+b}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a+b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}}\,\sqrt{b\,x^{4}+a+b}}-\frac{\sqrt{b}\,a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a+b}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a+b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}}\,\sqrt{b\,x^{4}+a+b}}+\frac{b\sqrt{a+b}\,\sqrt{1-}}{}$
elliptic	Expression too large to display

input
```
int(((a+b)*(-b^(1/2)+(a+b)^(1/2))+b^(1/2)*(a+b-b^(1/2)*(a+b)^(1/2))*x^2)/(
-x^2+1)/(b*x^4+a+b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-b^(3/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)
*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*EllipticF(x*(I*b^(1
/2)/(a+b)^(1/2))^(1/2),I)-b^(1/2)*a/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(
1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b
)^(1/2)*EllipticF(x*(I*b^(1/2)/(a+b)^(1/2))^(1/2),I)+b*(a+b)^(1/2)/(I*b^(1
/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^
2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*EllipticF(x*(I*b^(1/2)/(a+b)^(1/2))
^(1/2),I)-1/2*a*(a+b)^(1/2)*(-1/2/(a+2*b)^(1/2)*arctanh(1/2*(2*b*x^2+2*a+2
*b)/(a+2*b)^(1/2)/(b*x^4+a+b)^(1/2))-1/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*
b^(1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+
a+b)^(1/2)*EllipticPi(x*(I*b^(1/2)/(a+b)^(1/2))^(1/2),-I/b^(1/2)*(a+b)^(1/
2),(-I*b^(1/2)/(a+b)^(1/2))^(1/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)))+1/2*a*(a
+b)^(1/2)*(-1/2/(a+2*b)^(1/2)*arctanh(1/2*(2*b*x^2+2*a+2*b)/(a+2*b)^(1/2)/
(b*x^4+a+b)^(1/2))+1/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^
(1/2))^(1/2)*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*Ellipti
cPi(x*(I*b^(1/2)/(a+b)^(1/2))^(1/2),-I/b^(1/2)*(a+b)^(1/2),(-I*b^(1/2)/(a+
b)^(1/2))^(1/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)+\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$= \int -\frac{\left(a-\sqrt{a+b}\sqrt{b}+b\right)\sqrt{b}x^2+(a+b)\left(\sqrt{a+b}-\sqrt{b}\right)}{\sqrt{bx^4+a+b}(x^2-1)}\,dx$$

input
```
integrate(((a+b)*(-b^(1/2)+(a+b)^(1/2))+b^(1/2)*(a+b-b^(1/2)*(a+b)^(1/2))*
x^2)/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a + b)*((b*x^2 - a - b)*sqrt(a + b) - ((a + b)*x^2 -
 a - b)*sqrt(b))/(b*x^6 - b*x^4 + (a + b)*x^2 - a - b), x)
```

Sympy [F]

$$\int \frac{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)+\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$= -\int \left(-\frac{b^{\frac{3}{2}}}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\right)dx$$
$$-\int \left(-\frac{a\sqrt{b}}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\right)dx$$
$$-\int \frac{a\sqrt{a+b}}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\,dx-\int \frac{b\sqrt{a+b}}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\,dx$$
$$-\int \frac{b^{\frac{3}{2}}x^2}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\,dx-\int \frac{a\sqrt{b}x^2}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\,dx$$
$$-\int \left(-\frac{bx^2\sqrt{a+b}}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\right)dx$$

input
```
integrate(((a+b)*(-b**(1/2)+(a+b)**(1/2))+b**(1/2)*(a+b-b**(1/2)*(a+b)**(1
/2))*x**2)/(-x**2+1)/(b*x**4+a+b)**(1/2),x)
```

output
```
-Integral(-b**(3/2)/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4 + b)), x)
 - Integral(-a*sqrt(b)/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4 + b)),
 x) - Integral(a*sqrt(a + b)/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4
+ b)), x) - Integral(b*sqrt(a + b)/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b
*x**4 + b)), x) - Integral(b**(3/2)*x**2/(x**2*sqrt(a + b*x**4 + b) - sqrt
(a + b*x**4 + b)), x) - Integral(a*sqrt(b)*x**2/(x**2*sqrt(a + b*x**4 + b)
 - sqrt(a + b*x**4 + b)), x) - Integral(-b*x**2*sqrt(a + b)/(x**2*sqrt(a +
 b*x**4 + b) - sqrt(a + b*x**4 + b)), x)
```

Maxima [F]

$$\int \frac{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)+\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$=\int -\frac{\left(a-\sqrt{a+b}\sqrt{b}+b\right)\sqrt{b}x^2+(a+b)\left(\sqrt{a+b}-\sqrt{b}\right)}{\sqrt{bx^4+a+b}(x^2-1)}\,dx$$

input
```
integrate(((a+b)*(-b^(1/2)+(a+b)^(1/2))+b^(1/2)*(a+b-b^(1/2)*(a+b)^(1/2))*
x^2)/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(((a - sqrt(a + b)*sqrt(b) + b)*sqrt(b)*x^2 + (a + b)*(sqrt(a +
b) - sqrt(b)))/(sqrt(b*x^4 + a + b)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)+\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$=\int -\frac{\left(a-\sqrt{a+b}\sqrt{b}+b\right)\sqrt{b}x^2+(a+b)\left(\sqrt{a+b}-\sqrt{b}\right)}{\sqrt{bx^4+a+b}(x^2-1)}\,dx$$

input
```
integrate(((a+b)*(-b^(1/2)+(a+b)^(1/2))+b^(1/2)*(a+b-b^(1/2)*(a+b)^(1/2))*
x^2)/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="giac")
```

output
```
integrate(-((a - sqrt(a + b)*sqrt(b) + b)*sqrt(b)*x^2 + (a + b)*(sqrt(a +
b) - sqrt(b)))/(sqrt(b*x^4 + a + b)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)+\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \int -\frac{\left(\sqrt{a+b}-\sqrt{b}\right)(a+b)+\sqrt{b}\,x^2\left(a+b-\sqrt{b}\sqrt{a+b}\right)}{(x^2-1)\sqrt{b\,x^4+a+b}}\,dx$$

input
```
int(-(((a + b)^(1/2) - b^(1/2))*(a + b) + b^(1/2)*x^2*(a + b - b^(1/2)*(a
+ b)^(1/2)))/((x^2 - 1)*(a + b + b*x^4)^(1/2)),x)
```

output
```
int(-(((a + b)^(1/2) - b^(1/2))*(a + b) + b^(1/2)*x^2*(a + b - b^(1/2)*(a
+ b)^(1/2)))/((x^2 - 1)*(a + b + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{(a+b)\left(-\sqrt{b}+\sqrt{a+b}\right)+\sqrt{b}\left(a+b-\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx \\
&= -\sqrt{a+b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)a \\
&\quad -\sqrt{a+b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b \\
&\quad +\sqrt{a+b}\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b \\
&\quad +\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)a \\
&\quad +\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b \\
&\quad -\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)a \\
&\quad -\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b
\end{aligned}$$

input
```
int(((a+b)*(-b^(1/2)+(a+b)^(1/2))+b^(1/2)*(a+b-b^(1/2)*(a+b)^(1/2))*x^2)/(
-x^2+1)/(b*x^4+a+b)^(1/2),x)
```

output
```
- sqrt(a + b)*int(sqrt(a + b*x**4 + b)/(a*x**2 - a + b*x**6 - b*x**4 + b*
x**2 - b),x)*a - sqrt(a + b)*int(sqrt(a + b*x**4 + b)/(a*x**2 - a + b*x**6
 - b*x**4 + b*x**2 - b),x)*b + sqrt(a + b)*int((sqrt(a + b*x**4 + b)*x**2)
/(a*x**2 - a + b*x**6 - b*x**4 + b*x**2 - b),x)*b + sqrt(b)*int(sqrt(a + b
*x**4 + b)/(a*x**2 - a + b*x**6 - b*x**4 + b*x**2 - b),x)*a + sqrt(b)*int(
sqrt(a + b*x**4 + b)/(a*x**2 - a + b*x**6 - b*x**4 + b*x**2 - b),x)*b - sq
rt(b)*int((sqrt(a + b*x**4 + b)*x**2)/(a*x**2 - a + b*x**6 - b*x**4 + b*x*
*2 - b),x)*a - sqrt(b)*int((sqrt(a + b*x**4 + b)*x**2)/(a*x**2 - a + b*x**
6 - b*x**4 + b*x**2 - b),x)*b
```

3.89 $\int \frac{\left(a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)\right)\left(1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$

Optimal result

Integrand size = 69, antiderivative size = 301

$$\int \frac{\left(a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)\right)\left(1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \frac{a\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+2b}x}{\sqrt{a+b+bx^4}}\right)}{2\sqrt{a+2b}} + \frac{\sqrt[4]{b}(a+b)^{5/4}\left(1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)\sqrt{\frac{a+b+bx^4}{(a+b)\left(1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{\sqrt{a+b+bx^4}} + \frac{(a+b)^{3/4}\left(\sqrt{b}-\sqrt{a+b}\right)^2\left(1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)\sqrt{\frac{a+b+bx^4}{(a+b)\left(1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right)\right.}{4\sqrt[4]{b}\sqrt{a+b+bx^4}}$$

output
```
1/2*a*(a+b)^(1/2)*arctanh((a+2*b)^(1/2)*x/(b*x^4+a+b)^(1/2))/(a+2*b)^(1/2)
+b^(1/4)*(a+b)^(5/4)*(1+b^(1/2)*x^2/(a+b)^(1/2))*((b*x^4+a+b)/(a+b)/(1+b^(
1/2)*x^2/(a+b)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/(a+b)^(1
/4)),1/2*2^(1/2))/(b*x^4+a+b)^(1/2)+1/4*(a+b)^(3/4)*(b^(1/2)-(a+b)^(1/2))^
2*(1+b^(1/2)*x^2/(a+b)^(1/2))*((b*x^4+a+b)/(a+b)/(1+b^(1/2)*x^2/(a+b)^(1/2
))^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/(a+b)^(1/4))),1/4*(b^(1/2)+(
a+b)^(1/2))^2/b^(1/2)/(a+b)^(1/2),1/2*2^(1/2))/b^(1/4)/(b*x^4+a+b)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.59 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.69

$$\int \frac{\left(a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)\right)\left(1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$= \frac{\sqrt{a+b}\left(\sqrt{b}+\sqrt{a+b}\right)\sqrt{\frac{a+b+bx^4}{a+b}}\left(-ib^{3/4}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}x\right),-1\right)+\sqrt[4]{-1}\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}\sqrt[4]{a}\right.}{\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}\sqrt[4]{b}\sqrt{a+b+bx^4}}$$

input
```
Integrate[((a*Sqrt[a + b] + b*Sqrt[a + b] + Sqrt[b]*(a + b))*(1 - (Sqrt[b]
*x^2)/Sqrt[a + b]))/((1 - x^2)*Sqrt[a + b + b*x^4]),x]
```

output
```
(Sqrt[a + b]*(Sqrt[b] + Sqrt[a + b])*Sqrt[(a + b + b*x^4)/(a + b)]*((-I)*b
^(3/4)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[-a - b])]*x], -1] + (-1)^(1
/4)*Sqrt[-(Sqrt[b]/Sqrt[-a - b])]*(a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])*El
lipticPi[(I*Sqrt[a + b])/Sqrt[b], ArcSin[((-1)^(3/4)*b^(1/4)*x)/(a + b)^(1
/4)], -1]))/(Sqrt[-(Sqrt[b]/Sqrt[-a - b])]*b^(1/4)*Sqrt[a + b + b*x^4])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.058$, Rules used = {27, 2225, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a\sqrt{a+b}+\sqrt{b}(a+b)+b\sqrt{a+b}\right)\left(1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)}{(1-x^2)\sqrt{a+bx^4+b}}\,dx$$

$$\Big\downarrow \; 27$$

$$\sqrt{a+b}\left(\sqrt{b}\sqrt{a+b}+a+b\right)\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{bx^4+a+b}}dx$$

$$\Big\downarrow \text{2225}$$

$$\sqrt{a+b}\left(\sqrt{b}\sqrt{a+b}+a+b\right)\left(\frac{2\sqrt{b}\int\frac{1}{\sqrt{bx^4+a+b}}dx}{\sqrt{a+b}+\sqrt{b}}-\frac{\left(\sqrt{b}-\sqrt{a+b}\right)\int\frac{\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1}{(1-x^2)\sqrt{bx^4+a+b}}dx}{\sqrt{a+b}+\sqrt{b}}\right)$$

$$\Big\downarrow \text{761}$$

$$\sqrt{a+b}\left(\sqrt{b}\sqrt{a+b}+a+b\right)\left(\frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{\left(\sqrt{a+b}+\sqrt{b}\right)\sqrt{a+bx^4+b}}-\right.$$

$$\Big\downarrow \text{2223}$$

$$\sqrt{a+b}\left(\sqrt{b}\sqrt{a+b}+a+b\right)\left(\frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{\left(\sqrt{a+b}+\sqrt{b}\right)\sqrt{a+bx^4+b}}-\right.$$

input
```
Int[((a*Sqrt[a + b] + b*Sqrt[a + b] + Sqrt[b]*(a + b))*(1 - (Sqrt[b]*x^2)/
Sqrt[a + b]))/((1 - x^2)*Sqrt[a + b + b*x^4]),x]
```

output
```
Sqrt[a + b]*(a + b + Sqrt[b]*Sqrt[a + b])*((b^(1/4)*(a + b)^(1/4)*(1 + (Sq
rt[b]*x^2)/Sqrt[a + b])*Sqrt[(a + b + b*x^4)/((a + b)*(1 + (Sqrt[b]*x^2)/S
qrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*x)/(a + b)^(1/4)], 1/2])/((Sqr
t[b] + Sqrt[a + b])*Sqrt[a + b + b*x^4]) - ((Sqrt[b] - Sqrt[a + b])*(((1 +
 Sqrt[b]/Sqrt[a + b])*ArcTanh[(Sqrt[a + 2*b]*x)/Sqrt[a + b + b*x^4]])/(2*S
qrt[a + 2*b]) + ((a + b)^(1/4)*(1 - Sqrt[b]/Sqrt[a + b])*(1 + (Sqrt[b]*x^2
)/Sqrt[a + b])*Sqrt[(a + b + b*x^4)/((a + b)*(1 + (Sqrt[b]*x^2)/Sqrt[a + b
])^2)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*Arc
Tan[(b^(1/4)*x)/(a + b)^(1/4)], 1/2])/(4*b^(1/4)*Sqrt[a + b + b*x^4])))/(S
qrt[b] + Sqrt[a + b]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2225
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[2*A*(B/(B*d + A*e))   Int[1/Sqrt[a + c*x^4], x], x] - S
imp[(B*d - A*e)/(B*d + A*e)   Int[(A - B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4])
, x], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c
/a] && EqQ[c*A^2 - a*B^2, 0] && NegQ[B/A]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.26 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.51

method	result
default	$-\frac{\left(a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}\,(a+b)\right)\left(-\frac{\sqrt{b}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}}\sqrt{b\,x^4+a+b}}+\left(-\frac{\sqrt{b}}{2}+\frac{\sqrt{a+b}}{2}\right)\left(-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2+2a+2b}{2\sqrt{a+2b}\,\sqrt{b\,x^4+c}}\right.}{2\sqrt{a+2b}}\right.\right.}{}$
elliptic	Expression too large to display

input
```
int((a*(a+b)^(1/2)+b*(a+b)^(1/2)+b^(1/2)*(a+b))*(1-b^(1/2)*x^2/(a+b)^(1/2)
)/(-x^2+1)/(b*x^4+a+b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*(a+b)^(1/2)+b*(a+b)^(1/2)+b^(1/2)*(a+b))/(a+b)^(1/2)*(-b^(1/2)/(I*b^(1
/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^
2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*EllipticF(x*(I*b^(1/2)/(a+b)^(1/2))
^(1/2),I)+(-1/2*b^(1/2)+1/2*(a+b)^(1/2))*(-1/2/(a+2*b)^(1/2)*arctanh(1/2*(
2*b*x^2+2*a+2*b)/(a+2*b)^(1/2)/(b*x^4+a+b)^(1/2))-1/(I*b^(1/2)/(a+b)^(1/2)
)^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^
(1/2)/(b*x^4+a+b)^(1/2)*EllipticPi(x*(I*b^(1/2)/(a+b)^(1/2))^(1/2),-I/b^(1
/2)*(a+b)^(1/2),(-I*b^(1/2)/(a+b)^(1/2))^(1/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/
2)))+(1/2*b^(1/2)-1/2*(a+b)^(1/2))*(-1/2/(a+2*b)^(1/2)*arctanh(1/2*(2*b*x^
2+2*a+2*b)/(a+2*b)^(1/2)/(b*x^4+a+b)^(1/2))+1/(I*b^(1/2)/(a+b)^(1/2))^(1/2
)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/
(b*x^4+a+b)^(1/2)*EllipticPi(x*(I*b^(1/2)/(a+b)^(1/2))^(1/2),-I/b^(1/2)*(a
+b)^(1/2),(-I*b^(1/2)/(a+b)^(1/2))^(1/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{\left(a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)\right)\left(1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \int \frac{\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}-1\right)\left(\sqrt{a+b}a+(a+b)\sqrt{b}+\sqrt{a+b}b\right)}{\sqrt{bx^4+a+b}(x^2-1)}\,dx$$

input
```
integrate((a*(a+b)^(1/2)+b*(a+b)^(1/2)+b^(1/2)*(a+b))*(1-b^(1/2)*x^2/(a+b)
^(1/2))/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a + b)*((b*x^2 - a - b)*sqrt(a + b) + ((a + b)*x^2 -
 a - b)*sqrt(b))/(b*x^6 - b*x^4 + (a + b)*x^2 - a - b), x)
```

Sympy [F]

$$\int \frac{\left(a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)\right)\left(1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$= \frac{\left(\int \left(-\frac{\sqrt{a+b}}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\right)dx+\int \frac{\sqrt{b}x^2}{x^2\sqrt{a+bx^4+b}-\sqrt{a+bx^4+b}}\,dx\right)\left(a\sqrt{b}+a\sqrt{a+b}+b^{\frac{3}{2}}+b\sqrt{a+b}\right)}{\sqrt{a+b}}$$

input
```
integrate((a*(a+b)**(1/2)+b*(a+b)**(1/2)+b**(1/2)*(a+b))*(1-b**(1/2)*x**2/
(a+b)**(1/2))/(-x**2+1)/(b*x**4+a+b)**(1/2),x)
```

output
```
(Integral(-sqrt(a + b)/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4 + b)),
 x) + Integral(sqrt(b)*x**2/(x**2*sqrt(a + b*x**4 + b) - sqrt(a + b*x**4 +
 b)), x))*(a*sqrt(b) + a*sqrt(a + b) + b**(3/2) + b*sqrt(a + b))/sqrt(a +
b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)\right)\left(1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a*(a+b)^(1/2)+b*(a+b)^(1/2)+b^(1/2)*(a+b))*(1-b^(1/2)*x^2/(a+b)
^(1/2))/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)\right)\left(1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a*(a+b)^(1/2)+b*(a+b)^(1/2)+b^(1/2)*(a+b))*(1-b^(1/2)*x^2/(a+b)
^(1/2))/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)\right)\left(1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \int \frac{\left(\frac{\sqrt{b}\,x^2}{\sqrt{a+b}}-1\right)\left(a\,\sqrt{a+b}+b\,\sqrt{a+b}+\sqrt{b}\,(a+b)\right)}{(x^2-1)\,\sqrt{b\,x^4+a+b}}\,dx$$

input
```
int((((b^(1/2)*x^2)/(a + b)^(1/2) - 1)*(a*(a + b)^(1/2) + b*(a + b)^(1/2)
+ b^(1/2)*(a + b)))/((x^2 - 1)*(a + b + b*x^4)^(1/2)),x)
```

output
```
int((((b^(1/2)*x^2)/(a + b)^(1/2) - 1)*(a*(a + b)^(1/2) + b*(a + b)^(1/2)
+ b^(1/2)*(a + b)))/((x^2 - 1)*(a + b + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\left(a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)\right)\left(1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$= -\sqrt{a+b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)a$$
$$-\sqrt{a+b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b$$
$$+\sqrt{a+b}\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b$$
$$-\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)a$$
$$-\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b$$
$$+\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)a$$
$$+\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b$$

input
```
int((a*(a+b)^(1/2)+b*(a+b)^(1/2)+b^(1/2)*(a+b))*(1-b^(1/2)*x^2/(a+b)^(1/2)
)/(-x^2+1)/(b*x^4+a+b)^(1/2),x)
```

output
```
- sqrt(a + b)*int(sqrt(a + b*x**4 + b)/(a*x**2 - a + b*x**6 - b*x**4 + b*
x**2 - b),x)*a - sqrt(a + b)*int(sqrt(a + b*x**4 + b)/(a*x**2 - a + b*x**6
 - b*x**4 + b*x**2 - b),x)*b + sqrt(a + b)*int((sqrt(a + b*x**4 + b)*x**2)
/(a*x**2 - a + b*x**6 - b*x**4 + b*x**2 - b),x)*b - sqrt(b)*int(sqrt(a + b
*x**4 + b)/(a*x**2 - a + b*x**6 - b*x**4 + b*x**2 - b),x)*a - sqrt(b)*int(
sqrt(a + b*x**4 + b)/(a*x**2 - a + b*x**6 - b*x**4 + b*x**2 - b),x)*b + sq
rt(b)*int((sqrt(a + b*x**4 + b)*x**2)/(a*x**2 - a + b*x**6 - b*x**4 + b*x*
*2 - b),x)*a + sqrt(b)*int((sqrt(a + b*x**4 + b)*x**2)/(a*x**2 - a + b*x**
6 - b*x**4 + b*x**2 - b),x)*b
```

3.90 $\int \frac{a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)-\sqrt{b}\left(a+b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$

Optimal result

Integrand size = 76, antiderivative size = 301

$$\int \frac{a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)-\sqrt{b}\left(a+b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$= \frac{a\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+2b}x}{\sqrt{a+b+bx^4}}\right)}{2\sqrt{a+2b}}$$
$$+ \frac{\sqrt[4]{b}(a+b)^{5/4}\left(1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)\sqrt{\frac{a+b+bx^4}{(a+b)\left(1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{\sqrt{a+b+bx^4}}$$
$$+ \frac{(a+b)^{3/4}\left(\sqrt{b}-\sqrt{a+b}\right)^2\left(1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)\sqrt{\frac{a+b+bx^4}{(a+b)\left(1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right)\right.}{4\sqrt[4]{b}\sqrt{a+b+bx^4}}$$

output
```
1/2*a*(a+b)^(1/2)*arctanh((a+2*b)^(1/2)*x/(b*x^4+a+b)^(1/2))/(a+2*b)^(1/2)
+b^(1/4)*(a+b)^(5/4)*(1+b^(1/2)*x^2/(a+b)^(1/2))*((b*x^4+a+b)/(a+b)/(1+b^(
1/2)*x^2/(a+b)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/(a+b)^(1
/4)),1/2*2^(1/2))/(b*x^4+a+b)^(1/2)+1/4*(a+b)^(3/4)*(b^(1/2)-(a+b)^(1/2))^
2*(1+b^(1/2)*x^2/(a+b)^(1/2))*((b*x^4+a+b)/(a+b)/(1+b^(1/2)*x^2/(a+b)^(1/2
))^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/(a+b)^(1/4))),1/4*(b^(1/2)+(
a+b)^(1/2))^2/b^(1/2)/(a+b)^(1/2),1/2*2^(1/2))/b^(1/4)/(b*x^4+a+b)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.92 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.63

$$\int \frac{a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)-\sqrt{b}\left(a+b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx =$$
$$-\frac{i\sqrt{\frac{a+b+bx^4}{a+b}}\left(b^{3/4}\left(a+b+\sqrt{b}\sqrt{a+b}\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}x\right),-1\right)-(-1)^{3/4}a\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}(a\right.}{\sqrt{-\frac{\sqrt{b}}{\sqrt{-a-b}}}\sqrt[4]{b}\sqrt{a+b+bx^4}}$$

input
```
Integrate[(a*Sqrt[a + b] + b*Sqrt[a + b] + Sqrt[b]*(a + b) - Sqrt[b]*(a +
b + Sqrt[b]*Sqrt[a + b])*x^2)/((1 - x^2)*Sqrt[a + b + b*x^4]),x]
```

output
```
((-I)*Sqrt[(a + b + b*x^4)/(a + b)]*(b^(3/4)*(a + b + Sqrt[b]*Sqrt[a + b])
*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[-a - b])]*x], -1] - (-1)^(3/4)*a*
Sqrt[-(Sqrt[b]/Sqrt[-a - b])]*(a + b)^(3/4)*EllipticPi[(I*Sqrt[a + b])/Sqr
t[b], ArcSin[((-1)^(3/4)*b^(1/4)*x)/(a + b)^(1/4)], -1]))/(Sqrt[-(Sqrt[b]/
Sqrt[-a - b])]*b^(1/4)*Sqrt[a + b + b*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2225, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-\sqrt{b}x^2\left(\sqrt{b}\sqrt{a+b}+a+b\right)+\sqrt{b}(a+b)+a\sqrt{a+b}+b\sqrt{a+b}}{(1-x^2)\sqrt{a+bx^4+b}}\,dx$$
$$\Big\downarrow \; 2225$$

$$\frac{2\sqrt{b}\sqrt{a+b}\left(\sqrt{b}\sqrt{a+b}+a+b\right)^2\int\frac{1}{\sqrt{bx^4+a+b}}dx}{2b\left(\sqrt{a+b}+\sqrt{b}\right)+a\left(\sqrt{a+b}+2\sqrt{b}\right)}+$$
$$\frac{a\sqrt{a+b}\int\frac{\sqrt{b}\left(a+b+\sqrt{b}\sqrt{a+b}\right)x^2+(a+b)\left(\sqrt{b}+\sqrt{a+b}\right)}{(1-x^2)\sqrt{bx^4+a+b}}dx}{2b\left(\sqrt{a+b}+\sqrt{b}\right)+a\left(\sqrt{a+b}+2\sqrt{b}\right)}$$

$$\Big\downarrow \; 761$$

$$\frac{a\sqrt{a+b}\int\frac{\sqrt{b}\left(a+b+\sqrt{b}\sqrt{a+b}\right)x^2+(a+b)\left(\sqrt{b}+\sqrt{a+b}\right)}{(1-x^2)\sqrt{bx^4+a+b}}dx}{2b\left(\sqrt{a+b}+\sqrt{b}\right)+a\left(\sqrt{a+b}+2\sqrt{b}\right)}+$$
$$\frac{\sqrt[4]{b}(a+b)^{3/4}\left(\sqrt{b}\sqrt{a+b}+a+b\right)^2\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{\left(2b\left(\sqrt{a+b}+\sqrt{b}\right)+a\left(\sqrt{a+b}+2\sqrt{b}\right)\right)\sqrt{a+bx^4+b}}$$

$$\Big\downarrow \; 2223$$

$$\frac{a\sqrt{a+b}\left(\frac{a(a+b)\sqrt{\sqrt{a+b}+\sqrt{b}}\left(\frac{\sqrt{b}x^2\left(\sqrt{b}\sqrt{a+b}+a+b\right)}{(a+b)\left(\sqrt{a+b}+\sqrt{b}\right)}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2\left(\sqrt{b}\sqrt{a+b}+a+b\right)}{(a+b)\left(\sqrt{a+b}+\sqrt{b}\right)}+1\right)^2}}\,\text{EllipticPi}\left(\frac{a^2+8ba+8b^2+\sqrt{b}\sqrt{a+b}(4a+8b)}{4\sqrt{b}\left(2b^{3/2}+2\sqrt{a+b}b+2a\sqrt{b}+a\sqrt{a+b}\right)}\right.}{4\sqrt[4]{b}\sqrt{\sqrt{b}\sqrt{a+b}+a+b}\sqrt{a+bx^4+b}}\right.}{2b\left(\sqrt{a+b}+\sqrt{b}\right)+a\left(\sqrt{a+b}+\right.}$$
$$\frac{\sqrt[4]{b}(a+b)^{3/4}\left(\sqrt{b}\sqrt{a+b}+a+b\right)^2\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right)}{\left(2b\left(\sqrt{a+b}+\sqrt{b}\right)+a\left(\sqrt{a+b}+2\sqrt{b}\right)\right)\sqrt{a+bx^4+b}}$$

input
```
Int[(a*Sqrt[a + b] + b*Sqrt[a + b] + Sqrt[b]*(a + b) - Sqrt[b]*(a + b + Sq
rt[b]*Sqrt[a + b])*x^2)/((1 - x^2)*Sqrt[a + b + b*x^4]),x]
```

output
```
(b^(1/4)*(a + b)^(3/4)*(a + b + Sqrt[b]*Sqrt[a + b])^2*(1 + (Sqrt[b]*x^2)/
Sqrt[a + b])*Sqrt[(a + b + b*x^4)/((a + b)*(1 + (Sqrt[b]*x^2)/Sqrt[a + b])
^2)]*EllipticF[2*ArcTan[(b^(1/4)*x)/(a + b)^(1/4)], 1/2])/((2*b*(Sqrt[b] +
 Sqrt[a + b]) + a*(2*Sqrt[b] + Sqrt[a + b]))*Sqrt[a + b + b*x^4]) + (a*Sqr
t[a + b]*(((2*b*(Sqrt[b] + Sqrt[a + b]) + a*(2*Sqrt[b] + Sqrt[a + b]))*Arc
Tanh[(Sqrt[a + 2*b]*x)/Sqrt[a + b + b*x^4]])/(2*Sqrt[a + 2*b]) + (a*(a + b
)*Sqrt[Sqrt[b] + Sqrt[a + b]]*(1 + (Sqrt[b]*(a + b + Sqrt[b]*Sqrt[a + b])*
x^2)/((a + b)*(Sqrt[b] + Sqrt[a + b])))*Sqrt[(a + b + b*x^4)/((a + b)*(1 +
 (Sqrt[b]*(a + b + Sqrt[b]*Sqrt[a + b])*x^2)/((a + b)*(Sqrt[b] + Sqrt[a +
b])))^2)]*EllipticPi[(a^2 + 8*a*b + 8*b^2 + Sqrt[b]*Sqrt[a + b]*(4*a + 8*b
))/(4*Sqrt[b]*(2*a*Sqrt[b] + 2*b^(3/2) + a*Sqrt[a + b] + 2*b*Sqrt[a + b]))
, 2*ArcTan[(b^(1/4)*Sqrt[a + b + Sqrt[b]*Sqrt[a + b]]*x)/(Sqrt[a + b]*Sqrt
[Sqrt[b] + Sqrt[a + b]])], 1/2])/(4*b^(1/4)*Sqrt[a + b + Sqrt[b]*Sqrt[a +
b]]*Sqrt[a + b + b*x^4])))/(2*b*(Sqrt[b] + Sqrt[a + b]) + a*(2*Sqrt[b] + S
qrt[a + b]))
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2225
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[2*A*(B/(B*d + A*e))   Int[1/Sqrt[a + c*x^4], x], x] - S
imp[(B*d - A*e)/(B*d + A*e)   Int[(A - B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4])
, x], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c
/a] && EqQ[c*A^2 - a*B^2, 0] && NegQ[B/A]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.46 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.93

method	result
default	$\frac{b^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}}\,\sqrt{b\,x^4+a+b}}+\frac{b\sqrt{a+b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a+b}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a+b}}}\,\sqrt{b\,x^4+a+b}}+\frac{\sqrt{b}\,a\sqrt{1-\frac{i\sqrt{}}{\sqrt{}}}}{}$
elliptic	Expression too large to display

input
```
int((a*(a+b)^(1/2)+b*(a+b)^(1/2)+b^(1/2)*(a+b)-b^(1/2)*(a+b+b^(1/2)*(a+b)^
(1/2))*x^2)/(-x^2+1)/(b*x^4+a+b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
b^(3/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)*
(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*EllipticF(x*(I*b^(1/
2)/(a+b)^(1/2))^(1/2),I)+b*(a+b)^(1/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*
b^(1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+
a+b)^(1/2)*EllipticF(x*(I*b^(1/2)/(a+b)^(1/2))^(1/2),I)+b^(1/2)*a/(I*b^(1/
2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^2
/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*EllipticF(x*(I*b^(1/2)/(a+b)^(1/2))^
(1/2),I)-1/2*a*(a+b)^(1/2)*(-1/2/(a+2*b)^(1/2)*arctanh(1/2*(2*b*x^2+2*a+2*
b)/(a+2*b)^(1/2)/(b*x^4+a+b)^(1/2))-1/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b
^(1/2)*x^2/(a+b)^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a
+b)^(1/2)*EllipticPi(x*(I*b^(1/2)/(a+b)^(1/2))^(1/2),-I/b^(1/2)*(a+b)^(1/2
),(-I*b^(1/2)/(a+b)^(1/2))^(1/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)))+1/2*a*(a+
b)^(1/2)*(-1/2/(a+2*b)^(1/2)*arctanh(1/2*(2*b*x^2+2*a+2*b)/(a+2*b)^(1/2)/(
b*x^4+a+b)^(1/2))+1/(I*b^(1/2)/(a+b)^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/(a+b)^(
1/2))^(1/2)*(1+I*b^(1/2)*x^2/(a+b)^(1/2))^(1/2)/(b*x^4+a+b)^(1/2)*Elliptic
Pi(x*(I*b^(1/2)/(a+b)^(1/2))^(1/2),-I/b^(1/2)*(a+b)^(1/2),(-I*b^(1/2)/(a+b
)^(1/2))^(1/2)/(I*b^(1/2)/(a+b)^(1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{a\sqrt{a+b} + b\sqrt{a+b} + \sqrt{b}(a+b) - \sqrt{b}\left(a+b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$= \int \frac{\left(a+\sqrt{a+b}\sqrt{b}+b\right)\sqrt{b}x^2 - \sqrt{a+b}a - (a+b)\sqrt{b} - \sqrt{a+b}b}{\sqrt{bx^4+a+b}(x^2-1)}\,dx$$

input
```
integrate((a*(a+b)^(1/2)+b*(a+b)^(1/2)+b^(1/2)*(a+b)-b^(1/2)*(a+b+b^(1/2)*
(a+b)^(1/2))*x^2)/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a + b)*((b*x^2 - a - b)*sqrt(a + b) + ((a + b)*x^2 -
 a - b)*sqrt(b))/(b*x^6 - b*x^4 + (a + b)*x^2 - a - b), x)
```

Sympy [F]

$$\int \frac{a\sqrt{a+b} + b\sqrt{a+b} + \sqrt{b}(a+b) - \sqrt{b}\left(a+b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$= \int \frac{a\sqrt{b}x^2 - a\sqrt{b} - a\sqrt{a+b} + b^{\frac{3}{2}}x^2 - b^{\frac{3}{2}} + bx^2\sqrt{a+b} - b\sqrt{a+b}}{(x-1)(x+1)\sqrt{a+bx^4+b}}\,dx$$

input
```
integrate((a*(a+b)**(1/2)+b*(a+b)**(1/2)+b**(1/2)*(a+b)-b**(1/2)*(a+b+b**(
1/2)*(a+b)**(1/2))*x**2)/(-x**2+1)/(b*x**4+a+b)**(1/2),x)
```

output
```
Integral((a*sqrt(b)*x**2 - a*sqrt(b) - a*sqrt(a + b) + b**(3/2)*x**2 - b**
(3/2) + b*x**2*sqrt(a + b) - b*sqrt(a + b))/((x - 1)*(x + 1)*sqrt(a + b*x*
*4 + b)), x)
```

Maxima [F]

$$\int \frac{a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)-\sqrt{b}\left(a+b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \int \frac{\left(a+\sqrt{a+b}\sqrt{b}+b\right)\sqrt{b}x^2-\sqrt{a+b}a-(a+b)\sqrt{b}-\sqrt{a+b}b}{\sqrt{bx^4+a+b}(x^2-1)}\,dx$$

input
```
integrate((a*(a+b)^(1/2)+b*(a+b)^(1/2)+b^(1/2)*(a+b)-b^(1/2)*(a+b+b^(1/2)*
(a+b)^(1/2))*x^2)/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((a + sqrt(a + b)*sqrt(b) + b)*sqrt(b)*x^2 - sqrt(a + b)*a - (a
+ b)*sqrt(b) - sqrt(a + b)*b)/(sqrt(b*x^4 + a + b)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)-\sqrt{b}\left(a+b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx = \int \frac{\left(a+\sqrt{a+b}\sqrt{b}+b\right)\sqrt{b}x^2-\sqrt{a+b}a-(a+b)\sqrt{b}-\sqrt{a+b}b}{\sqrt{bx^4+a+b}(x^2-1)}\,dx$$

input
```
integrate((a*(a+b)^(1/2)+b*(a+b)^(1/2)+b^(1/2)*(a+b)-b^(1/2)*(a+b+b^(1/2)*
(a+b)^(1/2))*x^2)/(-x^2+1)/(b*x^4+a+b)^(1/2),x, algorithm="giac")
```

output
```
integrate(((a + sqrt(a + b)*sqrt(b) + b)*sqrt(b)*x^2 - sqrt(a + b)*a - (a
+ b)*sqrt(b) - sqrt(a + b)*b)/(sqrt(b*x^4 + a + b)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)-\sqrt{b}\left(a+b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$=\int -\frac{a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}\,(a+b)-\sqrt{b}\,x^2\left(a+b+\sqrt{b}\,\sqrt{a+b}\right)}{(x^2-1)\sqrt{b\,x^4+a+b}}\,dx$$

input
```
int(-(a*(a + b)^(1/2) + b*(a + b)^(1/2) + b^(1/2)*(a + b) - b^(1/2)*x^2*(a
 + b + b^(1/2)*(a + b)^(1/2)))/((x^2 - 1)*(a + b + b*x^4)^(1/2)),x)
```

output
```
int(-(a*(a + b)^(1/2) + b*(a + b)^(1/2) + b^(1/2)*(a + b) - b^(1/2)*x^2*(a
 + b + b^(1/2)*(a + b)^(1/2)))/((x^2 - 1)*(a + b + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a\sqrt{a+b}+b\sqrt{a+b}+\sqrt{b}(a+b)-\sqrt{b}\left(a+b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b+bx^4}}\,dx$$
$$=-\sqrt{a+b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)a$$
$$-\sqrt{a+b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b$$
$$+\sqrt{a+b}\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b$$
$$-\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)a$$
$$-\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b$$
$$+\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)a$$
$$+\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a+b}\,x^2}{b\,x^6-b\,x^4+a\,x^2+b\,x^2-a-b}dx\right)b$$

input

```
int((a*(a+b)^(1/2)+b*(a+b)^(1/2)+b^(1/2)*(a+b)-b^(1/2)*(a+b+b^(1/2)*(a+b)^
(1/2))*x^2)/(-x^2+1)/(b*x^4+a+b)^(1/2),x)
```

output

```
 - sqrt(a + b)*int(sqrt(a + b*x**4 + b)/(a*x**2 - a + b*x**6 - b*x**4 + b*
x**2 - b),x)*a - sqrt(a + b)*int(sqrt(a + b*x**4 + b)/(a*x**2 - a + b*x**6
 - b*x**4 + b*x**2 - b),x)*b + sqrt(a + b)*int((sqrt(a + b*x**4 + b)*x**2)
/(a*x**2 - a + b*x**6 - b*x**4 + b*x**2 - b),x)*b - sqrt(b)*int(sqrt(a + b
*x**4 + b)/(a*x**2 - a + b*x**6 - b*x**4 + b*x**2 - b),x)*a - sqrt(b)*int(
sqrt(a + b*x**4 + b)/(a*x**2 - a + b*x**6 - b*x**4 + b*x**2 - b),x)*b + sq
rt(b)*int((sqrt(a + b*x**4 + b)*x**2)/(a*x**2 - a + b*x**6 - b*x**4 + b*x*
*2 - b),x)*a + sqrt(b)*int((sqrt(a + b*x**4 + b)*x**2)/(a*x**2 - a + b*x**
6 - b*x**4 + b*x**2 - b),x)*b
```

3.91 $\int (A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 732

$$\int (A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}\,dx = -\frac{(9Bcd^3-24Acd^2e+84aBde^2+64aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{384ce^2x} + \frac{(3Bcd^2+56Acde-12aBe^2)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{192ce} + \frac{1}{48}(9Bd+8Ae)x^3\sqrt{d+ex^2}\sqrt{a-cx^4} + \frac{1}{8}Bex^5\sqrt{d+ex^2}\sqrt{a-cx^4} - \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(9Bcd^3-24Acd^2e+84aBde^2+64aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\ldots\right)}{384e^2\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{\sqrt{a}(3Bcd^3+248Acd^2e+108aBde^2+64aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\ldots\right)}{384\sqrt{c}e\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{(8Acde(cd^2+12ae^2)-B(3c^2d^4-24acd^2e^2-16a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\ldots\right)\right)}{128ce^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/384*(64*A*a*e^3-24*A*c*d^2*e+84*B*a*d*e^2+9*B*c*d^3)*(e*x^2+d)^(1/2)*(-
c*x^4+a)^(1/2)/c/e^2/x+1/192*(56*A*c*d*e-12*B*a*e^2+3*B*c*d^2)*x*(e*x^2+d)
^(1/2)*(-c*x^4+a)^(1/2)/c/e+1/48*(8*A*e+9*B*d)*x^3*(e*x^2+d)^(1/2)*(-c*x^4
+a)^(1/2)+1/8*B*e*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)-1/384*(d+a^(1/2)*e/
c^(1/2))*(64*A*a*e^3-24*A*c*d^2*e+84*B*a*d*e^2+9*B*c*d^3)*(1-a/c/x^4)^(1/2
)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1
-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2
))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/384*a^(1/2)*(64*A*a*e^3+248*A*c*
d^2*e+108*B*a*d*e^2+3*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c
^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)
*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e/(e*x^2+d)^(1/2
)/(-c*x^4+a)^(1/2)+1/128*(8*A*c*d*e*(12*a*e^2+c*d^2)-B*(-16*a^2*e^4-24*a*c
*d^2*e^2+3*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a
^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),
2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a
)^(1/2)
```

Mathematica [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx$$

input
```
Integrate[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4],x]
```

output
```
Integrate[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^4}(A+Bx^2)\left(d+ex^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 2261}$$

$$\int \sqrt{a-cx^4}\left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}dx$$

input
```
Int[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right)\left(e\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{-c\,x^{4}+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}\left(Bx^2+A\right)\left(ex^2+d\right)^{\frac{3}{2}}dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="fricas"
)
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
), x)
```

Sympy [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int (A+Bx^2)\sqrt{a-cx^4}\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}(Bx^2+A)\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}(Bx^2+A)\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \left(B\,x^2+A\right)\,\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2),x)
```

output
```
int((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\frac{-12\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,ab\,e^2x+56\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,acdex+32\sqrt{e\,x^2+d}}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

output
```
( - 12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*e**2*x + 56*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a*c*d*e*x + 32*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*e**2
*x**3 + 3*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*x + 36*sqrt(d + e*x**
2)*sqrt(a - c*x**4)*b*c*d*e*x**3 + 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*
c*e**2*x**5 + 64*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x
**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*e**3 + 84*int((sqrt(d + e*x**2)*sqrt(
a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d*e**2 -
 24*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**
4 - c*e*x**6),x)*a*c**2*d**2*e + 9*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*
x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c**2*d**3 + 24*int((sqrt
(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6)
,x)*a**2*b*e**3 + 176*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d +
a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d*e**2 + 78*int((sqrt(d + e*x**2
)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d
**2*e + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x
**4 - c*e*x**6),x)*a**2*b*d*e**2 + 136*int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d**2*e - 3*int((sqrt
(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a
*b*c*d**3)/(192*c*e)
```

3.92 $\int (A+Bx^2) \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 623

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx = -\frac{(3Bcd^2-6Acde+8aBe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{48ce^2x} + \frac{(Bd+6Ae)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{24e} + \frac{1}{6}Bx^3\sqrt{d+ex^2}\sqrt{a-cx^4} - \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(3Bcd^2-6Acde+8aBe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48e^2\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{\sqrt{a}(Bcd^2+30Acde+8aBe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48\sqrt{c}e\sqrt{d+ex^2}\sqrt{a-cx^4}} - \frac{(Bcd^3-2Acd^2e-4aBde^2-8aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2}{d+}\right.}{16e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/48*(-6*A*c*d*e+8*B*a*e^2+3*B*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/
e^2/x+1/24*(6*A*e+B*d)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e+1/6*B*x^3*(e*x
^2+d)^(1/2)*(-c*x^4+a)^(1/2)-1/48*(d+a^(1/2)*e/c^(1/2))*(-6*A*c*d*e+8*B*a*
e^2+3*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)
*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/4
8*a^(1/2)*(30*A*c*d*e+8*B*a*e^2+B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e
*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/
x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e/(e*x
^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/16*(-8*A*a*e^3-2*A*c*d^2*e-4*B*a*d*e^2+B*c*
d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(
1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+
a^(1/2)*e/c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$$

input
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4],x]
```

output
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}dx$$

input
```
Int[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right)\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}\left(Bx^2+A\right)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int (A + Bx^2) \sqrt{d + ex^2} \sqrt{a - cx^4} \, dx = \int (A + Bx^2) \sqrt{a - cx^4} \sqrt{d + ex^2} \, dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int (A + Bx^2) \sqrt{d + ex^2} \sqrt{a - cx^4} \, dx = \int \sqrt{-cx^4 + a} (Bx^2 + A) \sqrt{ex^2 + d} \, dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int (A + Bx^2) \sqrt{d + ex^2} \sqrt{a - cx^4} \, dx = \int \sqrt{-cx^4 + a} (Bx^2 + A) \sqrt{ex^2 + d} \, dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int \left(B\,x^2+A\right)\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2),x)
```

output
```
int((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$$
$$=\frac{6\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,aex+\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,bdx+4\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,be\,x^3+8\left(\int \frac{\sqrt{e\,x^2+d}}{-ce\,x^6-cd}\right.}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

output
```
(6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*e*x + sqrt(d + e*x**2)*sqrt(a - c*x
**4)*b*d*x + 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*e*x**3 + 8*int((sqrt(d
+ e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)
*a*b*e**2 - 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2
 - c*d*x**4 - c*e*x**6),x)*a*c*d*e + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c*d**2 + 12*int((sqr
t(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6
),x)*a**2*e**2 + 10*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*
e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*d*e + 18*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*d*e - int((sqrt
(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a
*b*d**2)/(24*e)
```

3.93 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 538

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = -\frac{(3Bd-4Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{8e^2x}+\frac{Bx\sqrt{d+ex^2}\sqrt{a-cx^4}}{4e}$$
$$-\frac{c(3Bd-4Ae)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{a}\sqrt{c}(Bd+4Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{(3Bcd^2-4Acde-4aBe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/8*(-4*A*e+3*B*d)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e^2/x+1/4*B*x*(e*x^2+
d)^(1/2)*(-c*x^4+a)^(1/2)/e-1/8*c*(-4*A*e+3*B*d)*(d+a^(1/2)*e/c^(1/2))*(1-
a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ell
ipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c
^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/8*a^(1/2)*c^(1/2)*(
4*A*e+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/
x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/
(d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/8*(-4*A
*c*d*e-4*B*a*e^2+3*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1
/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2
^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*
x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{\sqrt{d+ex^2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,\sqrt{-c\,x^4+a}}{\sqrt{e\,x^2+d}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(1/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,bx - 4\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)ace + 3\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)bcd + 2\left(\int \frac{\sqrt{e}}{-ce}\right.}{4e}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*x - 4*int((sqrt(d + e*x**2)*sqrt(a -
c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*e + 3*int((sqr
t(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6
),x)*b*c*d + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**
2 - c*d*x**4 - c*e*x**6),x)*a*b*e + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*e - int((sqrt(d + e*x**
2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*d)/(4*e
)
```

3.94 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 546

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \frac{\left(\frac{A}{d}-\frac{B}{e}\right)x\sqrt{a-cx^4}}{\sqrt{d+ex^2}} + \frac{(3Bd-2Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{2de^2x}$$
$$+ \frac{\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)(3Bd-2Ae)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2de^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{\sqrt{a}\sqrt{c}(Bd-2Ae)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2de\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{c(3Bd-2Ae)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
(A/d-B/e)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2)+1/2*(-2*A*e+3*B*d)*(e*x^2+d)^
(1/2)*(-c*x^4+a)^(1/2)/d/e^2/x+1/2*c^(1/2)*(c^(1/2)*d+a^(1/2)*e)*(-2*A*e+3
*B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^
(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^
(1/2)*e/c^(1/2)))^(1/2))/d/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/2*a^(1/2
)*c^(1/2)*(-2*A*e+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d
+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2)
,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(
1/2)+1/2*c*(-2*A*e+3*B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2
)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(
1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^
4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,\sqrt{-c\,x^4+a}}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
 + d^2), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(3/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(3/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,ax+2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^6}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx\right)acd$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*x + 2*int((sqrt(d + e*x**2)*sqrt(a -
c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e
*x**6 - c*e**2*x**8),x)*a*c*d*e + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)
*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 -
 c*e**2*x**8),x)*a*c*e**2*x**2 - 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*
x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 -
c*e**2*x**8),x)*b*c*d**2 - 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/
(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2
*x**8),x)*b*c*d*e*x**2 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a
*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x
**8),x)*a*b*d**2 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d**2
+ 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x
)*a*b*d*e*x**2 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d
*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*
d**2 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d*e*x**2 +
a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*d*e*x**2)/
(3*d*(d + e*x**2))
```

3.95 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 584

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx = \frac{\left(\frac{A}{d}-\frac{B}{e}\right)x\sqrt{a-cx^4}}{3\,(d+ex^2)^{3/2}}$$
$$-\frac{(3Bcd^3-aBde^2-2aAe^3)\sqrt{a-cx^4}}{3de^2\,(cd^2-ae^2)\,x\sqrt{d+ex^2}}$$
$$-\frac{\sqrt{c}(3Bcd^3-aBde^2-2aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3d^2e^2\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{a}\sqrt{c}(Bd+2Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3d^2e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{Bc\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/3*(A/d-B/e)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2)-1/3*(-2*A*a*e^3-B*a*d*e^2
+3*B*c*d^3)*(-c*x^4+a)^(1/2)/d/e^2/(-a*e^2+c*d^2)/x/(e*x^2+d)^(1/2)-1/3*c^
(1/2)*(-2*A*a*e^3-B*a*d*e^2+3*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x
^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^
2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^2/e^2/(c^(1/2)
*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/3*a^(1/2)*c^(1/2)*(2*A*e+
B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(
1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(
1/2)*e/c^(1/2)))^(1/2))/d^2/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-B*c*(1-a/c/
x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellipti
cPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^
(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{2261}$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{5/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,\sqrt{-c\,x^{4}+a}}{(e\,x^{2}+d)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x
^4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(5/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(5/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2),x)
```

output

```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*x - 2*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*x**6)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6
 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*b*c*
d**2*e - 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**3 + 3*a*d**2
*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 -
3*c*d*e**2*x**8 - c*e**3*x**10),x)*b*c*d*e**2*x**2 - 2*int((sqrt(d + e*x**
2)*sqrt(a - c*x**4)*x**6)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*
e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10
),x)*b*c*e**3*x**4 - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d**
3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d*
*2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*a*c*d**2*e - 4*int((sqrt(d
+ e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x*
*4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**
3*x**10),x)*a*c*d*e**2*x**2 - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**
4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4
 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*a*c*e**3*x**4 - 3*
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3
*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*
x**8 - c*e**3*x**10),x)*b*c*d**3 - 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4
)*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d...
```

3.96 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 577

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx = \frac{\left(\frac{A}{d}-\frac{B}{e}\right)x\sqrt{a-cx^4}}{5(d+ex^2)^{5/2}} + \frac{(3Bcd^3+2Acd^2e-aBde^2-4aAe^3)\,x\sqrt{a-cx^4}}{15d^2e\,(cd^2-ae^2)\,(d+ex^2)^{3/2}} - \frac{2a(3Bcd^3-8Acd^2e+aBde^2+4aAe^3)\sqrt{a-cx^4}}{15d^2\,(cd^2-ae^2)^2\,x\sqrt{d+ex^2}} - \frac{2a\sqrt{c}(3Bcd^3-8Acd^2e+aBde^2+4aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15d^3\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2-ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{2\sqrt{a}\sqrt{c}(5Acd^2-aBde-4aAe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15d^3(cd^2-ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/5*(A/d-B/e)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^(5/2)+1/15*(-4*A*a*e^3+2*A*c*d^
2*e-B*a*d*e^2+3*B*c*d^3)*x*(-c*x^4+a)^(1/2)/d^2/e/(-a*e^2+c*d^2)/(e*x^2+d)
^(3/2)-2/15*a*(4*A*a*e^3-8*A*c*d^2*e+B*a*d*e^2+3*B*c*d^3)*(-c*x^4+a)^(1/2)
/d^2/(-a*e^2+c*d^2)^2/x/(e*x^2+d)^(1/2)-2/15*a*c^(1/2)*(4*A*a*e^3-8*A*c*d^
2*e+B*a*d*e^2+3*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)
*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/
2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^3/(c^(1/2)*d-a^(1/2)*e)/(-a*
e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+2/15*a^(1/2)*c^(1/2)*(-4*A*a*e
^2+5*A*c*d^2-B*a*d*e)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+
a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),
2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^3/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2
)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(7/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{7/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,\sqrt{-c\,x^{4}+a}}{(e\,x^{2}+d)^{\frac{7}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^4*x^8 + 4*d*e^3*x
^6 + 6*d^2*e^2*x^4 + 4*d^3*e*x^2 + d^4), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(7/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(d + e*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(7/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{(e\,x^2+d)^{7/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(7/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2),x)
```

output

```
( - 5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*d**2*x + 5*sqrt(d + e*x**2)*sq
rt(a - c*x**4)*a*b*d*e*x**3 + 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*e**2
*x**5 - 7*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*e*x**5 - 3*sqrt(d + e*x*
*2)*sqrt(a - c*x**4)*b*c*d**2*x**5 + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*x**10)/(a*d**4 + 4*a*d**3*e*x**2 + 6*a*d**2*e**2*x**4 + 4*a*d*e**3*x**
6 + a*e**4*x**8 - c*d**4*x**4 - 4*c*d**3*e*x**6 - 6*c*d**2*e**2*x**8 - 4*c
*d*e**3*x**10 - c*e**4*x**12),x)*a*b*c*d**3*e**3 + 12*int((sqrt(d + e*x**2
)*sqrt(a - c*x**4)*x**10)/(a*d**4 + 4*a*d**3*e*x**2 + 6*a*d**2*e**2*x**4 +
 4*a*d*e**3*x**6 + a*e**4*x**8 - c*d**4*x**4 - 4*c*d**3*e*x**6 - 6*c*d**2*
e**2*x**8 - 4*c*d*e**3*x**10 - c*e**4*x**12),x)*a*b*c*d**2*e**4*x**2 + 12*
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**10)/(a*d**4 + 4*a*d**3*e*x**2 +
6*a*d**2*e**2*x**4 + 4*a*d*e**3*x**6 + a*e**4*x**8 - c*d**4*x**4 - 4*c*d**
3*e*x**6 - 6*c*d**2*e**2*x**8 - 4*c*d*e**3*x**10 - c*e**4*x**12),x)*a*b*c*
d*e**5*x**4 + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**10)/(a*d**4 + 4*
a*d**3*e*x**2 + 6*a*d**2*e**2*x**4 + 4*a*d*e**3*x**6 + a*e**4*x**8 - c*d**
4*x**4 - 4*c*d**3*e*x**6 - 6*c*d**2*e**2*x**8 - 4*c*d*e**3*x**10 - c*e**4*
x**12),x)*a*b*c*e**6*x**6 - 14*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**1
0)/(a*d**4 + 4*a*d**3*e*x**2 + 6*a*d**2*e**2*x**4 + 4*a*d*e**3*x**6 + a*e*
*4*x**8 - c*d**4*x**4 - 4*c*d**3*e*x**6 - 6*c*d**2*e**2*x**8 - 4*c*d*e**3*
x**10 - c*e**4*x**12),x)*a*c**2*d**4*e**2 - 42*int((sqrt(d + e*x**2)*sq...
```

3.97 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 781

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx = \frac{\left(\frac{A}{d}-\frac{B}{e}\right)x\sqrt{a-cx^4}}{7(d+ex^2)^{7/2}}$$
$$+\frac{(3Bcd^3+4Acd^2e-aBde^2-6aAe^3)\,x\sqrt{a-cx^4}}{35d^2e\,(cd^2-ae^2)\,(d+ex^2)^{5/2}}$$
$$+\frac{2(4Ae(c^2d^4-6acd^2e^2+3a^2e^4)+B(3c^2d^5+3acd^3e^2+2a^2de^4))\,x\sqrt{a-cx^4}}{105d^3e\,(cd^2-ae^2)^2\,(d+ex^2)^{3/2}}$$
$$+\frac{2a(Ae(77c^2d^4-69acd^2e^2+24a^2e^4)-B(21c^2d^5+15acd^3e^2-4a^2de^4))\,\sqrt{a-cx^4}}{105d^3\,(cd^2-ae^2)^3\,x\sqrt{d+ex^2}}$$
$$-\frac{2a\sqrt{c}(21Bc^2d^5-77Ac^2d^4e+15aBcd^3e^2+69aAcd^2e^3-4a^2Bde^4-24a^2Ae^5)\,\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}}{105d^4\left(\sqrt{c}d-\sqrt{a}e\right)^3\left(\sqrt{c}d+\sqrt{a}e\right)^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{2\sqrt{a}\sqrt{c}(4aBde(3cd^2-ae^2)-A(35c^2d^4-51acd^2e^2+24a^2e^4))\,\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\text{arc}\right.}{105d^4\,(cd^2-ae^2)^2\,\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/7*(A/d-B/e)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^(7/2)+1/35*(-6*A*a*e^3+4*A*c*d^
2*e-B*a*d*e^2+3*B*c*d^3)*x*(-c*x^4+a)^(1/2)/d^2/e/(-a*e^2+c*d^2)/(e*x^2+d)
^(5/2)+2/105*(4*A*e*(3*a^2*e^4-6*a*c*d^2*e^2+c^2*d^4)+B*(2*a^2*d*e^4+3*a*c
*d^3*e^2+3*c^2*d^5))*x*(-c*x^4+a)^(1/2)/d^3/e/(-a*e^2+c*d^2)^2/(e*x^2+d)^(
3/2)+2/105*a*(A*e*(24*a^2*e^4-69*a*c*d^2*e^2+77*c^2*d^4)-B*(-4*a^2*d*e^4+1
5*a*c*d^3*e^2+21*c^2*d^5))*(-c*x^4+a)^(1/2)/d^3/(-a*e^2+c*d^2)^3/x/(e*x^2+
d)^(1/2)-2/105*a*c^(1/2)*(-24*A*a^2*e^5+69*A*a*c*d^2*e^3-77*A*c^2*d^4*e-4*
B*a^2*d*e^4+15*B*a*c*d^3*e^2+21*B*c^2*d^5)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*
(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2
)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^4/(c^(1/2)
*d-a^(1/2)*e)^3/(c^(1/2)*d+a^(1/2)*e)^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-2
/105*a^(1/2)*c^(1/2)*(4*a*B*d*e*(-a*e^2+3*c*d^2)-A*(24*a^2*e^4-51*a*c*d^2*
e^2+35*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/
2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/
2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^4/(-a*e^2+c*d^2)^2/(e*x^2+d)^(1/2)/(
-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(9/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{9/2}}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{9/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{-c\,x^4+a}}{(e\,x^2+d)^{\frac{9}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^5*x^10 + 5*d*e^4*
x^8 + 10*d^2*e^3*x^6 + 10*d^3*e^2*x^4 + 5*d^4*e*x^2 + d^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(9/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{(e\,x^2+d)^{9/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(9/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{9/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2),x)
```

output

```
( - 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*e**2*x - 7*sqrt(d + e*x**2)*sq
rt(a - c*x**4)*b*c*d**2*x + 7*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d*e*x*
*3 + 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*e**2*x**5 - 48*int((sqrt(d +
e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d**5*e**2 + 20*a**2*d**4*e**3*x**2
+ 40*a**2*d**3*e**4*x**4 + 40*a**2*d**2*e**5*x**6 + 20*a**2*d*e**6*x**8 +
4*a**2*e**7*x**10 + 21*a*c*d**7 + 105*a*c*d**6*e*x**2 + 206*a*c*d**5*e**2*
x**4 + 190*a*c*d**4*e**3*x**6 + 65*a*c*d**3*e**4*x**8 - 19*a*c*d**2*e**5*x
**10 - 20*a*c*d*e**6*x**12 - 4*a*c*e**7*x**14 - 21*c**2*d**7*x**4 - 105*c*
*2*d**6*e*x**6 - 210*c**2*d**5*e**2*x**8 - 210*c**2*d**4*e**3*x**10 - 105*
c**2*d**3*e**4*x**12 - 21*c**2*d**2*e**5*x**14),x)*a**3*c*d**4*e**5 - 192*
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d**5*e**2 + 20*a**2*d
**4*e**3*x**2 + 40*a**2*d**3*e**4*x**4 + 40*a**2*d**2*e**5*x**6 + 20*a**2*
d*e**6*x**8 + 4*a**2*e**7*x**10 + 21*a*c*d**7 + 105*a*c*d**6*e*x**2 + 206*
a*c*d**5*e**2*x**4 + 190*a*c*d**4*e**3*x**6 + 65*a*c*d**3*e**4*x**8 - 19*a
*c*d**2*e**5*x**10 - 20*a*c*d*e**6*x**12 - 4*a*c*e**7*x**14 - 21*c**2*d**7
*x**4 - 105*c**2*d**6*e*x**6 - 210*c**2*d**5*e**2*x**8 - 210*c**2*d**4*e**
3*x**10 - 105*c**2*d**3*e**4*x**12 - 21*c**2*d**2*e**5*x**14),x)*a**3*c*d*
*3*e**6*x**2 - 288*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d*
*5*e**2 + 20*a**2*d**4*e**3*x**2 + 40*a**2*d**3*e**4*x**4 + 40*a**2*d**2*e
**5*x**6 + 20*a**2*d*e**6*x**8 + 4*a**2*e**7*x**10 + 21*a*c*d**7 + 105*...
```

3.98 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{11/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1002

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{11/2}}\,dx = \frac{\left(\frac{A}{d}-\frac{B}{e}\right)x\sqrt{a-cx^4}}{9\,(d+ex^2)^{9/2}}$$

$$+\frac{(3Bcd^3+6Acd^2e-aBde^2-8aAe^3)\,x\sqrt{a-cx^4}}{63d^2e\,(cd^2-ae^2)\,(d+ex^2)^{7/2}}$$

$$+\frac{2(4Ae(c^2d^4-4acd^2e^2+2a^2e^4)+B(2c^2d^5+acd^3e^2+a^2de^4))\,x\sqrt{a-cx^4}}{105d^3e\,(cd^2-ae^2)^2\,(d+ex^2)^{5/2}}$$

$$+\frac{2(Ae(8c^3d^6-103ac^2d^4e^2+95a^2cd^2e^4-32a^3e^6)+B(4c^3d^7+19ac^2d^5e^2+13a^2cd^3e^4-4a^3de^6))\,x\sqrt{a-}}{315d^4e\,(cd^2-ae^2)^3\,(d+ex^2)^{3/2}}$$

$$+\frac{2a(2Ae(147c^3d^6-174ac^2d^4e^2+123a^2cd^2e^4-32a^3e^6)-B(63c^3d^7+90ac^2d^5e^2-33a^2cd^3e^4+8a^3de^6))}{315d^4\,(cd^2-ae^2)^4\,x\sqrt{d+ex^2}}$$

$$+\frac{2a\sqrt{c}(2Ae(147c^3d^6-174ac^2d^4e^2+123a^2cd^2e^4-32a^3e^6)-B(63c^3d^7+90ac^2d^5e^2-33a^2cd^3e^4+8a^3de^6}{315d^5\left(\sqrt{c}d-\sqrt{a}e\right)^4\left(\sqrt{c}d+\sqrt{a}e\right)^3\sqrt{d+ex^2}\sqrt{a}}$$

$$-\frac{2\sqrt{a}\sqrt{c}(aBde(51c^2d^4-27acd^2e^2+8a^2e^4)-A(105c^3d^6-207ac^2d^4e^2+198a^2cd^2e^4-64a^3e^6))\sqrt{1-\frac{a}{cx}}}{315d^5\,(cd^2-ae^2)^3\,\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/9*(A/d-B/e)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^(9/2)+1/63*(-8*A*a*e^3+6*A*c*d^
2*e-B*a*d*e^2+3*B*c*d^3)*x*(-c*x^4+a)^(1/2)/d^2/e/(-a*e^2+c*d^2)/(e*x^2+d)
^(7/2)+2/105*(4*A*e*(2*a^2*e^4-4*a*c*d^2*e^2+c^2*d^4)+B*(a^2*d*e^4+a*c*d^3
*e^2+2*c^2*d^5))*x*(-c*x^4+a)^(1/2)/d^3/e/(-a*e^2+c*d^2)^2/(e*x^2+d)^(5/2)
+2/315*(A*e*(-32*a^3*e^6+95*a^2*c*d^2*e^4-103*a*c^2*d^4*e^2+8*c^3*d^6)+B*(
-4*a^3*d*e^6+13*a^2*c*d^3*e^4+19*a*c^2*d^5*e^2+4*c^3*d^7))*x*(-c*x^4+a)^(1
/2)/d^4/e/(-a*e^2+c*d^2)^3/(e*x^2+d)^(3/2)+2/315*a*(2*A*e*(-32*a^3*e^6+123
*a^2*c*d^2*e^4-174*a*c^2*d^4*e^2+147*c^3*d^6)-B*(8*a^3*d*e^6-33*a^2*c*d^3*
e^4+90*a*c^2*d^5*e^2+63*c^3*d^7))*(-c*x^4+a)^(1/2)/d^4/(-a*e^2+c*d^2)^4/x/
(e*x^2+d)^(1/2)+2/315*a*c^(1/2)*(2*A*e*(-32*a^3*e^6+123*a^2*c*d^2*e^4-174*
a*c^2*d^4*e^2+147*c^3*d^6)-B*(8*a^3*d*e^6-33*a^2*c*d^3*e^4+90*a*c^2*d^5*e^
2+63*c^3*d^7))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)
*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^5/(c^(1/2)*d-a^(1/2)*e)^4/(c^(1/2)*d+a
^(1/2)*e)^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-2/315*a^(1/2)*c^(1/2)*(a*B*d*
e*(8*a^2*e^4-27*a*c*d^2*e^2+51*c^2*d^4)-A*(-64*a^3*e^6+198*a^2*c*d^2*e^4-2
07*a*c^2*d^4*e^2+105*c^3*d^6))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c
^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)
*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^5/(-a*e^2+c*d^2)^3/(e*
x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{11/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{11/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(11/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(11/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{11/2}}dx$$

$$\Big\downarrow \text{ 2261}$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{11/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(11/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,\sqrt{-c\,x^4+a}}{(e\,x^2+d)^{\frac{11}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(11/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(11/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{11/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(11/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^6*x^12 + 6*d*e^5*
x^10 + 15*d^2*e^4*x^8 + 20*d^3*e^3*x^6 + 15*d^4*e^2*x^4 + 6*d^5*e*x^2 + d^
6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{11/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(11/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(11/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{11/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(11/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{11/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{(e\,x^2+d)^{11/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(11/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{11/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(11/2),x)
```

output
```
( - 48*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*d*e**2*x + 4*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*a*b*e**3*x**3 - 63*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c
*d**3*x + 63*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*e*x**3 + 36*sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*b*c*d*e**2*x**5 + 8*sqrt(d + e*x**2)*sqrt(a -
c*x**4)*b*c*e**3*x**7 + 96*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4
*a**2*d**6*e**2 + 24*a**2*d**5*e**3*x**2 + 60*a**2*d**4*e**4*x**4 + 80*a**
2*d**3*e**5*x**6 + 60*a**2*d**2*e**6*x**8 + 24*a**2*d*e**7*x**10 + 4*a**2*
e**8*x**12 + 7*a*c*d**8 + 42*a*c*d**7*e*x**2 + 101*a*c*d**6*e**2*x**4 + 11
6*a*c*d**5*e**3*x**6 + 45*a*c*d**4*e**4*x**8 - 38*a*c*d**3*e**5*x**10 - 53
*a*c*d**2*e**6*x**12 - 24*a*c*d*e**7*x**14 - 4*a*c*e**8*x**16 - 7*c**2*d**
8*x**4 - 42*c**2*d**7*e*x**6 - 105*c**2*d**6*e**2*x**8 - 140*c**2*d**5*e**
3*x**10 - 105*c**2*d**4*e**4*x**12 - 42*c**2*d**3*e**5*x**14 - 7*c**2*d**2
*e**6*x**16),x)*a**3*b*d**5*e**6 + 480*int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*x**4)/(4*a**2*d**6*e**2 + 24*a**2*d**5*e**3*x**2 + 60*a**2*d**4*e**4*x
**4 + 80*a**2*d**3*e**5*x**6 + 60*a**2*d**2*e**6*x**8 + 24*a**2*d*e**7*x**
10 + 4*a**2*e**8*x**12 + 7*a*c*d**8 + 42*a*c*d**7*e*x**2 + 101*a*c*d**6*e*
*2*x**4 + 116*a*c*d**5*e**3*x**6 + 45*a*c*d**4*e**4*x**8 - 38*a*c*d**3*e**
5*x**10 - 53*a*c*d**2*e**6*x**12 - 24*a*c*d*e**7*x**14 - 4*a*c*e**8*x**16
- 7*c**2*d**8*x**4 - 42*c**2*d**7*e*x**6 - 105*c**2*d**6*e**2*x**8 - 140*c
**2*d**5*e**3*x**10 - 105*c**2*d**4*e**4*x**12 - 42*c**2*d**3*e**5*x**1...
```

3.99 $\int (A+Bx^2)(d+ex^2)^{3/2}(a-cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1007

$$\int (A+Bx^2)(d+ex^2)^{3/2}(a-cx^4)^{3/2}\,dx = \text{Too large to display}$$

output
```
-1/15360*(12*A*e*(128*a^2*e^4-116*a*c*d^2*e^2+15*c^2*d^4)-B*(-2064*a^2*d*e
^4-512*a*c*d^3*e^2+105*c^2*d^5))*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e^4/x+
1/7680*(12*A*c*d*e*(236*a*e^2+5*c*d^2)-B*(240*a^2*e^4-168*a*c*d^2*e^2+35*c
^2*d^4))*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e^3+1/1920*(384*A*a*e^3-12*A
*c*d^2*e+436*B*a*d*e^2+7*B*c*d^3)*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e^2
-1/960*(132*A*c*d*e-140*B*a*e^2+3*B*c*d^2)*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^
(1/2)/e-1/120*c*(12*A*e+13*B*d)*x^7*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)-1/12*
B*c*e*x^9*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)+1/15360*(c^(1/2)*d+a^(1/2)*e)*(
-1536*A*a^2*e^5+1392*A*a*c*d^2*e^3-180*A*c^2*d^4*e-2064*B*a^2*d*e^4-512*B*
a*c*d^3*e^2+105*B*c^2*d^5)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/
2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(
1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e^4/(e*x^2+d)^(1/2)/
(-c*x^4+a)^(1/2)-1/15360*a^(1/2)*(-1536*A*a^2*e^5-8304*A*a*c*d^2*e^3-60*A*
c^2*d^4*e-2544*B*a^2*d*e^4-176*B*a*c*d^3*e^2+35*B*c^2*d^5)*(1-a/c/x^4)^(1/
2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(
1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/
2))/c^(1/2)/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/1024*(12*A*c*d*e*(-48*a
^2*e^4-8*a*c*d^2*e^2+c^2*d^4)-B*(64*a^3*e^6+144*a^2*c*d^2*e^4-36*a*c^2*d^4
*e^2+7*c^3*d^6))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/
2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2...
```

Mathematica [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx$$

input
```
Integrate[(A + B*x^2)*(d + e*x^2)^(3/2)*(a - c*x^4)^(3/2),x]
```

output
```
Integrate[(A + B*x^2)*(d + e*x^2)^(3/2)*(a - c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^{3/2} \left(A+Bx^2\right) \left(d+ex^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 2261}$$

$$\int \left(a-cx^4\right)^{3/2} \left(A+Bx^2\right) \left(d+ex^2\right)^{3/2} dx$$

input
```
Int[(A + B*x^2)*(d + e*x^2)^(3/2)*(a - c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right) \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(-c\,x^{4}+a\right)^{\frac{3}{2}}\, dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}\left(Bx^2+A\right)\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(-(B*c*e*x^8 + (B*c*d + A*c*e)*x^6 + (A*c*d - B*a*e)*x^4 - A*a*d -
 (B*a*d + A*a*e)*x^2)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(A+Bx^2\right)\left(a-cx^4\right)^{\frac{3}{2}}\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(-c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(a - c*x**4)**(3/2)*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}\left(Bx^2+A\right)\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)*(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int (A+Bx^2) \left(d+ex^2\right)^{3/2} \left(a-cx^4\right)^{3/2} dx = \int \left(-cx^4+a\right)^{\frac{3}{2}} \left(Bx^2+A\right) \left(ex^2+d\right)^{\frac{3}{2}} dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)*(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2) \left(d+ex^2\right)^{3/2} \left(a-cx^4\right)^{3/2} dx = \int \left(B\,x^2+A\right) \left(a-c\,x^4\right)^{3/2} \left(e\,x^2+d\right)^{3/2} dx$$

input
```
int((A + B*x^2)*(a - c*x^4)^(3/2)*(d + e*x^2)^(3/2),x)
```

output
```
int((A + B*x^2)*(a - c*x^4)^(3/2)*(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int (A+Bx^2) \left(d+ex^2\right)^{3/2} \left(a-cx^4\right)^{3/2} dx = \text{Too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(3/2),x)
```

output

```
( - 240*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*e**4*x + 2832*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*a**2*c*d*e**3*x + 1536*sqrt(d + e*x**2)*sqrt(a - c*
x**4)*a**2*c*e**4*x**3 + 168*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*d**2*
e**2*x + 1744*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*d*e**3*x**3 + 1120*s
qrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*e**4*x**5 + 60*sqrt(d + e*x**2)*sqr
t(a - c*x**4)*a*c**2*d**3*e*x - 48*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**
2*d**2*e**2*x**3 - 1056*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**2*d*e**3*x*
*5 - 768*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**2*e**4*x**7 - 35*sqrt(d +
e*x**2)*sqrt(a - c*x**4)*b*c**2*d**4*x + 28*sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*b*c**2*d**3*e*x**3 - 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c**2*d**2*
e**2*x**5 - 832*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c**2*d*e**3*x**7 - 640
*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c**2*e**4*x**9 + 1536*int((sqrt(d + e
*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*
*3*c*e**5 + 2064*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x
**2 - c*d*x**4 - c*e*x**6),x)*a**2*b*c*d*e**4 - 1392*int((sqrt(d + e*x**2)
*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c**
2*d**2*e**3 + 512*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*
x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c**2*d**3*e**2 + 180*int((sqrt(d + e*x*
*2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c**
3*d**4*e - 105*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*...
```

3.100 $\int (A+Bx^2)\sqrt{d+ex^2}(a-cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 868

$$\int (A+Bx^2)\sqrt{d+ex^2}(a-cx^4)^{3/2}\,dx =$$

$$-\frac{(10Acde(15cd^2-68ae^2)-B(105c^2d^4-332acd^2e^2-384a^2e^4))\sqrt{d+ex^2}\sqrt{a-cx^4}}{3840ce^4x}$$

$$-\frac{(35Bcd^3-50Acd^2e-108aBde^2-600aAe^3)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{1920e^3}$$

$$+\frac{(7Bcd^2-10Acde+96aBe^2)x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{480e^2}$$

$$-\frac{c(Bd+10Ae)x^5\sqrt{d+ex^2}\sqrt{a-cx^4}}{80e}-\frac{1}{10}Bcx^7\sqrt{d+ex^2}\sqrt{a-cx^4}$$

$$-\frac{(\sqrt{c}d+\sqrt{a}e)(10Acde(15cd^2-68ae^2)-B(105c^2d^4-332acd^2e^2-384a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}}{3840\sqrt{c}e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{\sqrt{a}(10Acde(5cd^2+196ae^2)-B(35c^2d^4-116acd^2e^2-384a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\Big(\text{a}}{3840\sqrt{c}e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{(7Bc^2d^5-10Ac^2d^4e-24aBcd^3e^2+48aAcd^2e^3+48a^2Bde^4+96a^2Ae^5)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{Ellipt}}{256e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/3840*(10*A*c*d*e*(-68*a*e^2+15*c*d^2)-B*(-384*a^2*e^4-332*a*c*d^2*e^2+1
05*c^2*d^4))*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e^4/x-1/1920*(-600*A*a*e^3
-50*A*c*d^2*e-108*B*a*d*e^2+35*B*c*d^3)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)
/e^3+1/480*(-10*A*c*d*e+96*B*a*e^2+7*B*c*d^2)*x^3*(e*x^2+d)^(1/2)*(-c*x^4+
a)^(1/2)/e^2-1/80*c*(10*A*e+B*d)*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e-1/
10*B*c*x^7*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)-1/3840*(c^(1/2)*d+a^(1/2)*e)*(
10*A*c*d*e*(-68*a*e^2+15*c*d^2)-B*(-384*a^2*e^4-332*a*c*d^2*e^2+105*c^2*d^
4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1
/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1
/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/3840
*a^(1/2)*(10*A*c*d*e*(196*a*e^2+5*c*d^2)-B*(-384*a^2*e^4-116*a*c*d^2*e^2+3
5*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)
/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d
/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2
)+1/256*(96*A*a^2*e^5+48*A*a*c*d^2*e^3-10*A*c^2*d^4*e+48*B*a^2*d*e^4-24*B*
a*c*d^3*e^2+7*B*c^2*d^5)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)
*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1
/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^4/(e*x^2+d)^(1/2)/(-c*x^4
+a)^(1/2)
```

Mathematica [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}(a-cx^4)^{3/2}\,dx = \int (A+Bx^2)\sqrt{d+ex^2}(a-cx^4)^{3/2}\,dx$$

input
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*(a - c*x^4)^(3/2),x]
```

output
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*(a - c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^{3/2} \left(A+Bx^2\right) \sqrt{d+ex^2}\,dx$$

$$\Big\downarrow \text{2261}$$

$$\int \left(a-cx^4\right)^{3/2} \left(A+Bx^2\right) \sqrt{d+ex^2}dx$$

input
```
Int[(A + B*x^2)*Sqrt[d + e*x^2]*(a - c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right) \sqrt{e\,x^{2}+d}\,\left(-c\,x^{4}+a\right)^{\frac{3}{2}}\,dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int (A+Bx^2)\,\sqrt{d+ex^2}\big(a-cx^4\big)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}(Bx^2+A)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(-(B*c*x^6 + A*c*x^4 - B*a*x^2 - A*a)*sqrt(-c*x^4 + a)*sqrt(e*x^2
+ d), x)
```

Sympy [F]

$$\int (A+Bx^2)\,\sqrt{d+ex^2}\big(a-cx^4\big)^{3/2}\,dx=\int (A+Bx^2)\,\left(a-cx^4\right)^{\frac{3}{2}}\sqrt{d+ex^2}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(-c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(a - c*x**4)**(3/2)*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int (A+Bx^2)\,\sqrt{d+ex^2}\big(a-cx^4\big)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}(Bx^2+A)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(-cx^4+a\right)^{\frac{3}{2}}\left(Bx^2+A\right)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx=\int \left(B\,x^2+A\right)\,\left(a-c\,x^4\right)^{3/2}\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((A + B*x^2)*(a - c*x^4)^(3/2)*(d + e*x^2)^(1/2),x)
```

output
```
int((A + B*x^2)*(a - c*x^4)^(3/2)*(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(3/2),x)
```

output

```
(600*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*e**3*x + 108*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a*b*d*e**2*x + 384*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*
e**3*x**3 + 50*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**2*e*x - 40*sqrt(d
+ e*x**2)*sqrt(a - c*x**4)*a*c*d*e**2*x**3 - 240*sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*a*c*e**3*x**5 - 35*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**3*x +
 28*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*e*x**3 - 24*sqrt(d + e*x**2
)*sqrt(a - c*x**4)*b*c*d*e**2*x**5 - 192*sqrt(d + e*x**2)*sqrt(a - c*x**4)
*b*c*e**3*x**7 + 384*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a
*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*b*e**4 - 680*int((sqrt(d + e*x**2)*
sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d*
e**3 + 332*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 -
c*d*x**4 - c*e*x**6),x)*a*b*c*d**2*e**2 + 150*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c**2*d**3*e -
 105*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x*
*4 - c*e*x**6),x)*b*c**2*d**4 + 720*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)
*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**3*e**4 + 552*int((sqrt
(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6)
,x)*a**2*b*d*e**3 + 20*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d +
 a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d**2*e**2 - 14*int((sqrt(d + e*
x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*...
```

3.101 $\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 732

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \frac{(105Bcd^3-120Acd^2e-188aBde^2+256aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{384e^4x}$$
$$-\frac{5(7Bcd^2-8Acde-12aBe^2)\,x\sqrt{d+ex^2}\sqrt{a-cx^4}}{192e^3}$$
$$+\frac{c(7Bd-8Ae)x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{48e^2}-\frac{Bcx^5\sqrt{d+ex^2}\sqrt{a-cx^4}}{8e}$$
$$+\frac{c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(105Bcd^3-120Acd^2e-188aBde^2+256aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{}}{\sqrt{c}}}}{\sqrt{2}}\right.\right.}{384e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{\sqrt{a}\sqrt{c}(35Bcd^3-40Acd^2e-68aBde^2-128aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{cx}}}}{\sqrt{2}}\right.\right.}{384e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{(8Acde(5cd^2-12ae^2)-B(35c^2d^4-72acd^2e^2+48a^2e^4))\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\right.}{128e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/384*(256*A*a*e^3-120*A*c*d^2*e-188*B*a*d*e^2+105*B*c*d^3)*(e*x^2+d)^(1/2
)*(-c*x^4+a)^(1/2)/e^4/x-5/192*(-8*A*c*d*e-12*B*a*e^2+7*B*c*d^2)*x*(e*x^2+
d)^(1/2)*(-c*x^4+a)^(1/2)/e^3+1/48*c*(-8*A*e+7*B*d)*x^3*(e*x^2+d)^(1/2)*(-
c*x^4+a)^(1/2)/e^2-1/8*B*c*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e+1/384*c*
(d+a^(1/2)*e/c^(1/2))*(256*A*a*e^3-120*A*c*d^2*e-188*B*a*d*e^2+105*B*c*d^3
)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2
)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2
)*e/c^(1/2)))^(1/2))/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/384*a^(1/2)*c^
(1/2)*(-128*A*a*e^3-40*A*c*d^2*e-68*B*a*d*e^2+35*B*c*d^3)*(1-a/c/x^4)^(1/2
)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1
-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2
))/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/128*(8*A*c*d*e*(-12*a*e^2+5*c*d^
2)-B*(48*a^2*e^4-72*a*c*d^2*e^2+35*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2
)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(
1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^4/(e*
x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/Sqrt[d + e*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{2261}$$

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2)}{\sqrt{d+ex^2}}dx$$

input
```
Int[((A + B*x^2)*(a - c*x^4)^(3/2))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(-c\,x^4+a)^{\frac{3}{2}}}{\sqrt{e\,x^2+d}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx=\int \frac{(A+Bx^2)\,(a-cx^4)^{\frac{3}{2}}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(3/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x**2)*(a - c*x**4)**(3/2)/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(a-cx^4\right)^{3/2}}{\sqrt{d+ex^2}}\,dx=\int \frac{\left(B\,x^2+A\right)\,\left(a-c\,x^4\right)^{3/2}}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(1/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\left(a-cx^4\right)^{3/2}}{\sqrt{d+ex^2}}\,dx=\frac{60\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,ab\,e^2x+40\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,acdex-32\sqrt{e\,x}}{}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(1/2),x)
```

output

```
(60*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*e**2*x + 40*sqrt(d + e*x**2)*sqr
t(a - c*x**4)*a*c*d*e*x - 32*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*e**2*x*
*3 - 35*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*x + 28*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*b*c*d*e*x**3 - 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*
e**2*x**5 - 256*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x*
*2 - c*d*x**4 - c*e*x**6),x)*a**2*c*e**3 + 188*int((sqrt(d + e*x**2)*sqrt(
a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d*e**2 +
 120*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x*
*4 - c*e*x**6),x)*a*c**2*d**2*e - 105*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c**2*d**3 + 72*int((s
qrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x*
*6),x)*a**2*b*e**3 + 16*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d
+ a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d*e**2 - 14*int((sqrt(d + e*x*
*2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c
*d**2*e + 192*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*
d*x**4 - c*e*x**6),x)*a**3*e**3 - 60*int((sqrt(d + e*x**2)*sqrt(a - c*x**4
))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*b*d*e**2 - 40*int((sqrt(
d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*
*2*c*d**2*e + 35*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 -
 c*d*x**4 - c*e*x**6),x)*a*b*c*d**3)/(192*e**3)
```

3.102 $\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 725

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \frac{(Bd-Ae)(cd^2-ae^2)x\sqrt{a-cx^4}}{de^3\sqrt{d+ex^2}}$$
$$-\frac{(105Bcd^3-90Acd^2e-80aBde^2+48aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{48de^4x}$$
$$+\frac{c(11Bd-6Ae)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{24e^3}-\frac{Bcx^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{6e^2}$$
$$-\frac{\sqrt{c}(\sqrt{c}d+\sqrt{a}e)(105Bcd^3-90Acd^2e-80aBde^2+48aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\ldots}}{\sqrt{\ldots}}\right.\right.}{48de^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{a}\sqrt{c}(35Bcd^3-30Acd^2e-32aBde^2+48aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\right.}{48de^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{c(35Bcd^3-30Acd^2e-36aBde^2+24aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\right.}{16e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
(-A*e+B*d)*(-a*e^2+c*d^2)*x*(-c*x^4+a)^(1/2)/d/e^3/(e*x^2+d)^(1/2)-1/48*(4
8*A*a*e^3-90*A*c*d^2*e-80*B*a*d*e^2+105*B*c*d^3)*(e*x^2+d)^(1/2)*(-c*x^4+a
)^(1/2)/d/e^4/x+1/24*c*(-6*A*e+11*B*d)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/
e^3-1/6*B*c*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e^2-1/48*c^(1/2)*(c^(1/2)
*d+a^(1/2)*e)*(48*A*a*e^3-90*A*c*d^2*e-80*B*a*d*e^2+105*B*c*d^3)*(1-a/c/x^
4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE
(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)
))^(1/2))/d/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/48*a^(1/2)*c^(1/2)*(48*
A*a*e^3-30*A*c*d^2*e-32*B*a*d*e^2+35*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/
2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(
1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d/e^3/(e*
x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/16*c*(24*A*a*e^3-30*A*c*d^2*e-36*B*a*d*e^2
+35*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e
)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2
)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx=\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2261}$$

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2)}{(d+ex^2)^{3/2}}\,dx$$

input
```
Int[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(-c\,x^4+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(-(B*c*x^6 + A*c*x^4 - B*a*x^2 - A*a)*sqrt(-c*x^4 + a)*sqrt(e*x^2
+ d)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)\,(a-cx^4)^{\frac{3}{2}}}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(3/2)/(e*x**2+d)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(a - c*x**4)**(3/2)/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(a-c\,x^4\right)^{3/2}}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(3/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(3/2),x)
```

output
```
(12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*e**2*x - 2*sqrt(d + e*x**2)*sqr
t(a - c*x**4)*a*b*d*e*x - 6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*e*x**3
 + 7*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*x**3 - 4*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*b*c*d*e*x**5 + 24*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*
x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 -
c*e**2*x**8),x)*a**2*c*d*e**3 + 24*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*
x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 -
c*e**2*x**8),x)*a**2*c*e**4*x**2 - 36*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6
 - c*e**2*x**8),x)*a*b*c*d**2*e**2 - 36*int((sqrt(d + e*x**2)*sqrt(a - c*x
**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x*
*6 - c*e**2*x**8),x)*a*b*c*d*e**3*x**2 - 30*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*
e*x**6 - c*e**2*x**8),x)*a*c**2*d**3*e - 30*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*
e*x**6 - c*e**2*x**8),x)*a*c**2*d**2*e**2*x**2 + 35*int((sqrt(d + e*x**2)*
sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4
 - 2*c*d*e*x**6 - c*e**2*x**8),x)*b*c**2*d**4 + 35*int((sqrt(d + e*x**2)*sq
rt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 -
2*c*d*e*x**6 - c*e**2*x**8),x)*b*c**2*d**3*e*x**2 + 24*int((sqrt(d + e*...
```

3.103 $\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 740

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \frac{(Bd-Ae)(cd^2-ae^2)\,x\sqrt{a-cx^4}}{3de^3(d+ex^2)^{3/2}}$$

$$-\frac{(9Bcd^3-6Acd^2e-aBde^2-2aAe^3)\,x\sqrt{a-cx^4}}{3d^2e^3\sqrt{d+ex^2}}$$

$$+\frac{(105Bcd^3-60Acd^2e-8aBde^2-16aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{24d^2e^4x}$$

$$-\frac{Bcx\sqrt{d+ex^2}\sqrt{a-cx^4}}{4e^3}$$

$$+\frac{\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)(105Bcd^3-60Acd^2e-8aBde^2-16aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right.\right.\cdots}{24d^2e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$-\frac{\sqrt{a}\sqrt{c}(35Bcd^3-20Acd^2e-8aBde^2-16aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\cdots\right.}{24d^2e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{c(35Bcd^2-20Acde-12aBe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/3*(-A*e+B*d)*(-a*e^2+c*d^2)*x*(-c*x^4+a)^(1/2)/d/e^3/(e*x^2+d)^(3/2)-1/3
*(-2*A*a*e^3-6*A*c*d^2*e-B*a*d*e^2+9*B*c*d^3)*x*(-c*x^4+a)^(1/2)/d^2/e^3/(
e*x^2+d)^(1/2)+1/24*(-16*A*a*e^3-60*A*c*d^2*e-8*B*a*d*e^2+105*B*c*d^3)*(e*
x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/d^2/e^4/x-1/4*B*c*x*(e*x^2+d)^(1/2)*(-c*x^4+
a)^(1/2)/e^3+1/24*c^(1/2)*(c^(1/2)*d+a^(1/2)*e)*(-16*A*a*e^3-60*A*c*d^2*e-
8*B*a*d*e^2+105*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)
*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/
2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^2/e^4/(e*x^2+d)^(1/2)/(-c*x^
4+a)^(1/2)-1/24*a^(1/2)*c^(1/2)*(-16*A*a*e^3-20*A*c*d^2*e-8*B*a*d*e^2+35*B
*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2
)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+
a^(1/2)*e/c^(1/2)))^(1/2))/d^2/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/8*c*
(-20*A*c*d*e-12*B*a*e^2+35*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+
d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)
^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^4/(e*x^2+d)^(1
/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2)}{(d+ex^2)^{5/2}}\,dx$$

$$\downarrow \text{2261}$$

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2)}{(d+ex^2)^{5/2}}dx$$

input
```
Int[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(-c\,x^4+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-(B*c*x^6 + A*c*x^4 - B*a*x^2 - A*a)*sqrt(-c*x^4 + a)*sqrt(e*x^2
+ d)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(A+Bx^2)\,(a-cx^4)^{\frac{3}{2}}}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(3/2)/(e*x**2+d)**(5/2),x)
```

output
```
Integral((A + B*x**2)*(a - c*x**4)**(3/2)/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\int \frac{(B\,x^2+A)\,(a-c\,x^4)^{3/2}}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(5/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(5/2),x)
```

output

```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*d*x + 6*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*a*b*e*x**3 - 5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d*x**5 + 12
*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8)/(a*d**3 + 3*a*d**2*e*x**2 +
3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2
*x**8 - c*e**3*x**10),x)*a*b*c*d**2*e**2 + 24*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*x**8)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6
 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*a*b*
c*d*e**3*x**2 + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8)/(a*d**3 +
3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e
*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*a*b*c*e**4*x**4 + 20*int((sqrt(
d + e*x**2)*sqrt(a - c*x**4)*x**8)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*
x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e
**3*x**10),x)*a*c**2*d**3*e + 40*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x*
*8)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**
4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*a*c**2*d**2*e**2*
x**2 + 20*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8)/(a*d**3 + 3*a*d**2*
e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3
*c*d*e**2*x**8 - c*e**3*x**10),x)*a*c**2*d*e**3*x**4 - 35*int((sqrt(d + e*
x**2)*sqrt(a - c*x**4)*x**8)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 +
 a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3...
```

3.104 $\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 885

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx = \frac{(Bd-Ae)(cd^2-ae^2)x\sqrt{a-cx^4}}{5de^3(d+ex^2)^{5/2}} - \frac{(13Bcd^3-8Acd^2e-aBde^2-4aAe^3)x\sqrt{a-cx^4}}{15d^2e^3(d+ex^2)^{3/2}} + \frac{(45Bc^2d^5-15Ac^2d^4e-31aBcd^3e^2+11aAcd^2e^3-2a^2Bde^4-8a^2Ae^5)x\sqrt{a-cx^4}}{15d^3e^3(cd^2-ae^2)\sqrt{d+ex^2}} - \frac{(105Bc^2d^5-30Ac^2d^4e-77aBcd^3e^2+22aAcd^2e^3-4a^2Bde^4-16a^2Ae^5)\sqrt{d+ex^2}\sqrt{a-cx^4}}{30d^3e^4(cd^2-ae^2)x} + \frac{\sqrt{c}(2Ae(15c^2d^4-11acd^2e^2+8a^2e^4)-B(105c^2d^5-77acd^3e^2-4a^2de^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\Big(a}{30d^3e^4(\sqrt{c}d-\sqrt{a}e)\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{\sqrt{a}\sqrt{c}(35Bcd^3-10Acd^2e+4aBde^2+16aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\right.}{30d^3e^3\sqrt{d+ex^2}\sqrt{a-cx^4}} - \frac{c^2(7Bd-2Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/5*(-A*e+B*d)*(-a*e^2+c*d^2)*x*(-c*x^4+a)^(1/2)/d/e^3/(e*x^2+d)^(5/2)-1/1
5*(-4*A*a*e^3-8*A*c*d^2*e-B*a*d*e^2+13*B*c*d^3)*x*(-c*x^4+a)^(1/2)/d^2/e^3
/(e*x^2+d)^(3/2)+1/15*(-8*A*a^2*e^5+11*A*a*c*d^2*e^3-15*A*c^2*d^4*e-2*B*a^
2*d*e^4-31*B*a*c*d^3*e^2+45*B*c^2*d^5)*x*(-c*x^4+a)^(1/2)/d^3/e^3/(-a*e^2+
c*d^2)/(e*x^2+d)^(1/2)-1/30*(-16*A*a^2*e^5+22*A*a*c*d^2*e^3-30*A*c^2*d^4*e
-4*B*a^2*d*e^4-77*B*a*c*d^3*e^2+105*B*c^2*d^5)*(e*x^2+d)^(1/2)*(-c*x^4+a)^
(1/2)/d^3/e^4/(-a*e^2+c*d^2)/x+1/30*c^(1/2)*(2*A*e*(8*a^2*e^4-11*a*c*d^2*e
^2+15*c^2*d^4)-B*(-4*a^2*d*e^4-77*a*c*d^3*e^2+105*c^2*d^5))*(1-a/c/x^4)^(1
/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*
(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1
/2))/d^3/e^4/(c^(1/2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/30*a
^(1/2)*c^(1/2)*(16*A*a*e^3-10*A*c*d^2*e+4*B*a*d*e^2+35*B*c*d^3)*(1-a/c/x^4
)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(
1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2))
)^(1/2))/d^3/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/2*c^2*(-2*A*e+7*B*d)*(
1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*E
llipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2
)*e/c^(1/2)))^(1/2))/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx=\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(7/2),x]
```

output
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-cx^4\right)^{3/2}\left(A+Bx^2\right)}{\left(d+ex^2\right)^{7/2}}dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{\left(a-cx^4\right)^{3/2}\left(A+Bx^2\right)}{\left(d+ex^2\right)^{7/2}}dx$$

input
```
Int[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{\left(B\,x^2+A\right)\left(-c\,x^4+a\right)^{\frac{3}{2}}}{\left(e\,x^2+d\right)^{\frac{7}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx=\int \frac{\left(-cx^4+a\right)^{\frac{3}{2}}(Bx^2+A)}{\left(ex^2+d\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="fricas"
)
```

output
```
integral(-(B*c*x^6 + A*c*x^4 - B*a*x^2 - A*a)*sqrt(-c*x^4 + a)*sqrt(e*x^2
+ d)/(e^4*x^8 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4 + 4*d^3*e*x^2 + d^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(3/2)/(e*x**2+d)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)\left(a-cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx=\int \frac{\left(-cx^4+a\right)^{\frac{3}{2}}(Bx^2+A)}{\left(ex^2+d\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="maxima"
)
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(7/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx = \int \frac{(B\,x^2+A)\,(a-c\,x^4)^{3/2}}{(e\,x^2+d)^{7/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(7/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(7/2),x)
```

output
```
( - 28*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*d*e*x + 45*sqrt(d + e*x**2
)*sqrt(a - c*x**4)*a**2*c*d**2*x + 18*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*
*2*c*d*e*x**3 - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c*e**2*x**5 + 42*
sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*d**2*x**3 - 15*sqrt(d + e*x**2)*sq
rt(a - c*x**4)*a*c**2*d**2*x**5 - 32*int((sqrt(d + e*x**2)*sqrt(a - c*x**4
)*x**10)/(4*a**2*d**4*e**2 + 16*a**2*d**3*e**3*x**2 + 24*a**2*d**2*e**4*x*
*4 + 16*a**2*d*e**5*x**6 + 4*a**2*e**6*x**8 + 15*a*c*d**6 + 60*a*c*d**5*e*
x**2 + 86*a*c*d**4*e**2*x**4 + 44*a*c*d**3*e**3*x**6 - 9*a*c*d**2*e**4*x**
8 - 16*a*c*d*e**5*x**10 - 4*a*c*e**6*x**12 - 15*c**2*d**6*x**4 - 60*c**2*d
**5*e*x**6 - 90*c**2*d**4*e**2*x**8 - 60*c**2*d**3*e**3*x**10 - 15*c**2*d*
*2*e**4*x**12),x)*a**3*c**2*d**3*e**5 - 96*int((sqrt(d + e*x**2)*sqrt(a -
c*x**4)*x**10)/(4*a**2*d**4*e**2 + 16*a**2*d**3*e**3*x**2 + 24*a**2*d**2*e
**4*x**4 + 16*a**2*d*e**5*x**6 + 4*a**2*e**6*x**8 + 15*a*c*d**6 + 60*a*c*d
**5*e*x**2 + 86*a*c*d**4*e**2*x**4 + 44*a*c*d**3*e**3*x**6 - 9*a*c*d**2*e*
*4*x**8 - 16*a*c*d*e**5*x**10 - 4*a*c*e**6*x**12 - 15*c**2*d**6*x**4 - 60*
c**2*d**5*e*x**6 - 90*c**2*d**4*e**2*x**8 - 60*c**2*d**3*e**3*x**10 - 15*c
**2*d**2*e**4*x**12),x)*a**3*c**2*d**2*e**6*x**2 - 96*int((sqrt(d + e*x**2
)*sqrt(a - c*x**4)*x**10)/(4*a**2*d**4*e**2 + 16*a**2*d**3*e**3*x**2 + 24*
a**2*d**2*e**4*x**4 + 16*a**2*d*e**5*x**6 + 4*a**2*e**6*x**8 + 15*a*c*d**6
 + 60*a*c*d**5*e*x**2 + 86*a*c*d**4*e**2*x**4 + 44*a*c*d**3*e**3*x**6 -...
```

3.105 $\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 933

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx = \frac{(Bd-Ae)(cd^2-ae^2)x\sqrt{a-cx^4}}{7de^3(d+ex^2)^{7/2}}$$
$$-\frac{(17Bcd^3-10Acd^2e-aBde^2-6aAe^3)x\sqrt{a-cx^4}}{35d^2e^3(d+ex^2)^{5/2}}$$
$$+\frac{(71Bc^2d^5-15Ac^2d^4e-55aBcd^3e^2+27aAcd^2e^3-4a^2Bde^4-24a^2Ae^5)x\sqrt{a-cx^4}}{105d^3e^3(cd^2-ae^2)(d+ex^2)^{3/2}}$$
$$+\frac{(48a^2Ae^5(2cd^2-ae^2)+B(105c^3d^7-182ac^2d^5e^2+37a^2cd^3e^4-8a^3de^6))\sqrt{a-cx^4}}{105d^3e^4(cd^2-ae^2)^2x\sqrt{d+ex^2}}$$
$$+\frac{\sqrt{c}(48a^2Ae^5(2cd^2-ae^2)+B(105c^3d^7-182ac^2d^5e^2+37a^2cd^3e^4-8a^3de^6))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E}{105d^4e^4(\sqrt{c}d-\sqrt{a}e)(cd^2-ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{a}\sqrt{c}(12aAe^3(5cd^2-4ae^2)-B(35c^2d^5-31acd^3e^2+8a^2de^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\text{arc}\right.}{105d^4e^3(cd^2-ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{Bc^2\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/7*(-A*e+B*d)*(-a*e^2+c*d^2)*x*(-c*x^4+a)^(1/2)/d/e^3/(e*x^2+d)^(7/2)-1/3
5*(-6*A*a*e^3-10*A*c*d^2*e-B*a*d*e^2+17*B*c*d^3)*x*(-c*x^4+a)^(1/2)/d^2/e^
3/(e*x^2+d)^(5/2)+1/105*(-24*A*a^2*e^5+27*A*a*c*d^2*e^3-15*A*c^2*d^4*e-4*B
*a^2*d*e^4-55*B*a*c*d^3*e^2+71*B*c^2*d^5)*x*(-c*x^4+a)^(1/2)/d^3/e^3/(-a*e
^2+c*d^2)/(e*x^2+d)^(3/2)+1/105*(48*a^2*A*e^5*(-a*e^2+2*c*d^2)+B*(-8*a^3*d
*e^6+37*a^2*c*d^3*e^4-182*a*c^2*d^5*e^2+105*c^3*d^7))*(-c*x^4+a)^(1/2)/d^3
/e^4/(-a*e^2+c*d^2)^2/x/(e*x^2+d)^(1/2)+1/105*c^(1/2)*(48*a^2*A*e^5*(-a*e^
2+2*c*d^2)+B*(-8*a^3*d*e^6+37*a^2*c*d^3*e^4-182*a*c^2*d^5*e^2+105*c^3*d^7)
)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2
)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2
)*e/c^(1/2)))^(1/2))/d^4/e^4/(c^(1/2)*d-a^(1/2)*e)/(-a*e^2+c*d^2)/(e*x^2+d
)^(1/2)/(-c*x^4+a)^(1/2)+1/105*a^(1/2)*c^(1/2)*(12*a*A*e^3*(-4*a*e^2+5*c*d
^2)-B*(8*a^2*d*e^4-31*a*c*d^3*e^2+35*c^2*d^5))*(1-a/c/x^4)^(1/2)*x^3*(a^(1
/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^
(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^4/e^3/
(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+B*c^2*(1-a/c/x^4)^(1/2)*x^
3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^
(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2)
)/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(9/2),x]
```

output
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2)}{(d+ex^2)^{9/2}}dx$$

$$\Big\downarrow \text{2261}$$

$$\int \frac{(a-cx^4)^{3/2}(A+Bx^2)}{(d+ex^2)^{9/2}}dx$$

input
```
Int[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(-c\,x^4+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{9}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="fricas"
)
```

output
```
integral(-(B*c*x^6 + A*c*x^4 - B*a*x^2 - A*a)*sqrt(-c*x^4 + a)*sqrt(e*x^2
+ d)/(e^5*x^10 + 5*d*e^4*x^8 + 10*d^2*e^3*x^6 + 10*d^3*e^2*x^4 + 5*d^4*e*x
^2 + d^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(3/2)/(e*x**2+d)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="maxima"
)
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(9/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(a-c\,x^4\right)^{3/2}}{(e\,x^2+d)^{9/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(9/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\,(a-cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(9/2),x)
```

output

```
( - 12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*e*x + 15*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a**2*c*d*x + 14*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c*
e*x**3 + 14*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*d*x**3 + 4*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*a*b*c*e*x**5 - 5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*
a*c**2*d*x**5 + 112*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**10)/(4*a**2*
d**5*e**2 + 20*a**2*d**4*e**3*x**2 + 40*a**2*d**3*e**4*x**4 + 40*a**2*d**2
*e**5*x**6 + 20*a**2*d*e**6*x**8 + 4*a**2*e**7*x**10 + 5*a*c*d**7 + 25*a*c
*d**6*e*x**2 + 46*a*c*d**5*e**2*x**4 + 30*a*c*d**4*e**3*x**6 - 15*a*c*d**3
*e**4*x**8 - 35*a*c*d**2*e**5*x**10 - 20*a*c*d*e**6*x**12 - 4*a*c*e**7*x**
14 - 5*c**2*d**7*x**4 - 25*c**2*d**6*e*x**6 - 50*c**2*d**5*e**2*x**8 - 50*
c**2*d**4*e**3*x**10 - 25*c**2*d**3*e**4*x**12 - 5*c**2*d**2*e**5*x**14),x
)*a**2*b*c**2*d**4*e**4 + 448*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**10
)/(4*a**2*d**5*e**2 + 20*a**2*d**4*e**3*x**2 + 40*a**2*d**3*e**4*x**4 + 40
*a**2*d**2*e**5*x**6 + 20*a**2*d*e**6*x**8 + 4*a**2*e**7*x**10 + 5*a*c*d**
7 + 25*a*c*d**6*e*x**2 + 46*a*c*d**5*e**2*x**4 + 30*a*c*d**4*e**3*x**6 - 1
5*a*c*d**3*e**4*x**8 - 35*a*c*d**2*e**5*x**10 - 20*a*c*d*e**6*x**12 - 4*a*
c*e**7*x**14 - 5*c**2*d**7*x**4 - 25*c**2*d**6*e*x**6 - 50*c**2*d**5*e**2*
x**8 - 50*c**2*d**4*e**3*x**10 - 25*c**2*d**3*e**4*x**12 - 5*c**2*d**2*e**
5*x**14),x)*a**2*b*c**2*d**3*e**5*x**2 + 672*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*x**10)/(4*a**2*d**5*e**2 + 20*a**2*d**4*e**3*x**2 + 40*a**2*d...
```

3.106 $\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 922

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \frac{(Bd-Ae)(cd^2-ae^2)x\sqrt{a-cx^4}}{9de^3(d+ex^2)^{9/2}}$$

$$-\frac{(21Bcd^3-12Acd^2e-aBde^2-8aAe^3)x\sqrt{a-cx^4}}{63d^2e^3(d+ex^2)^{7/2}}$$

$$+\frac{(35Bc^2d^5-5Ac^2d^4e-29aBcd^3e^2+17aAcd^2e^3-2a^2Bde^4-16a^2Ae^5)x\sqrt{a-cx^4}}{105d^3e^3(cd^2-ae^2)(d+ex^2)^{5/2}}$$

$$-\frac{(2Ae(5c^3d^6-22ac^2d^4e^2+65a^2cd^2e^4-32a^3e^6)+B(35c^3d^7-82ac^2d^5e^2+23a^2cd^3e^4-8a^3de^6))x\sqrt{a-}}{315d^4e^3(cd^2-ae^2)^2(d+ex^2)^{3/2}}$$

$$-\frac{4a^2(21Bc^2d^5-93Ac^2d^4e+15aBcd^3e^2+93aAcd^2e^3-4a^2Bde^4-32a^2Ae^5)\sqrt{a-cx^4}}{315d^4(cd^2-ae^2)^3x\sqrt{d+ex^2}}$$

$$-\frac{4a^2\sqrt{c}(21Bc^2d^5-93Ac^2d^4e+15aBcd^3e^2+93aAcd^2e^3-4a^2Bde^4-32a^2Ae^5)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x}}}{315d^5(\sqrt{c}d-\sqrt{a}e)^3(\sqrt{c}d+\sqrt{a}e)^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$-\frac{4a^{3/2}\sqrt{c}(4aBde(3cd^2-ae^2)-A(45c^2d^4-69acd^2e^2+32a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\text{arc}\right.}{315d^5(cd^2-ae^2)^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/9*(-A*e+B*d)*(-a*e^2+c*d^2)*x*(-c*x^4+a)^(1/2)/d/e^3/(e*x^2+d)^(9/2)-1/6
3*(-8*A*a*e^3-12*A*c*d^2*e-B*a*d*e^2+21*B*c*d^3)*x*(-c*x^4+a)^(1/2)/d^2/e^
3/(e*x^2+d)^(7/2)+1/105*(-16*A*a^2*e^5+17*A*a*c*d^2*e^3-5*A*c^2*d^4*e-2*B*
a^2*d*e^4-29*B*a*c*d^3*e^2+35*B*c^2*d^5)*x*(-c*x^4+a)^(1/2)/d^3/e^3/(-a*e^
2+c*d^2)/(e*x^2+d)^(5/2)-1/315*(2*A*e*(-32*a^3*e^6+65*a^2*c*d^2*e^4-22*a*c
^2*d^4*e^2+5*c^3*d^6)+B*(-8*a^3*d*e^6+23*a^2*c*d^3*e^4-82*a*c^2*d^5*e^2+35
*c^3*d^7))*x*(-c*x^4+a)^(1/2)/d^4/e^3/(-a*e^2+c*d^2)^2/(e*x^2+d)^(3/2)-4/3
15*a^2*(-32*A*a^2*e^5+93*A*a*c*d^2*e^3-93*A*c^2*d^4*e-4*B*a^2*d*e^4+15*B*a
*c*d^3*e^2+21*B*c^2*d^5)*(-c*x^4+a)^(1/2)/d^4/(-a*e^2+c*d^2)^3/x/(e*x^2+d)
^(1/2)-4/315*a^2*c^(1/2)*(-32*A*a^2*e^5+93*A*a*c*d^2*e^3-93*A*c^2*d^4*e-4*
B*a^2*d*e^4+15*B*a*c*d^3*e^2+21*B*c^2*d^5)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*
(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2
)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^5/(c^(1/2)
*d-a^(1/2)*e)^3/(c^(1/2)*d+a^(1/2)*e)^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-4
/315*a^(3/2)*c^(1/2)*(4*a*B*d*e*(-a*e^2+3*c*d^2)-A*(32*a^2*e^4-69*a*c*d^2*
e^2+45*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/
2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/
2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^5/(-a*e^2+c*d^2)^2/(e*x^2+d)^(1/2)/(
-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx=\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(11/2),x]
```

output
```
Integrate[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(11/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-cx^4\right)^{3/2} \left(A+Bx^2\right)}{\left(d+ex^2\right)^{11/2}} \, dx$$

$$\Big\downarrow \ 2261$$

$$\int \frac{\left(a-cx^4\right)^{3/2} \left(A+Bx^2\right)}{\left(d+ex^2\right)^{11/2}}dx$$

input
```
Int[((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(11/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{\left(B\,x^{2}+A\right) \left(-c\,x^{4}+a\right)^{\frac{3}{2}}}{\left(e\,x^{2}+d\right)^{\frac{11}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="fricas
")
```

output
```
integral(-(B*c*x^6 + A*c*x^4 - B*a*x^2 - A*a)*sqrt(-c*x^4 + a)*sqrt(e*x^2
+ d)/(e^6*x^12 + 6*d*e^5*x^10 + 15*d^2*e^4*x^8 + 20*d^3*e^3*x^6 + 15*d^4*e
^2*x^4 + 6*d^5*e*x^2 + d^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(3/2)/(e*x**2+d)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx=\int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="maxima
")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(11/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \int \frac{(-cx^4+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="giac")
```

output
```
integrate((-c*x^4 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \int \frac{(B\,x^2+A)\,(a-c\,x^4)^{3/2}}{(e\,x^2+d)^{11/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(11/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(3/2))/(d + e*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(a-cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(3/2)/(e*x^2+d)^(11/2),x)
```

output
```
( - 54*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*d*e**2*x - 12*sqrt(d + e*x
**2)*sqrt(a - c*x**4)*a**2*b*e**3*x**3 + 33*sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*a**2*c*d**2*e*x + 88*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c*d*e**2*x
**3 - 42*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*d**3*x + 119*sqrt(d + e*x
**2)*sqrt(a - c*x**4)*a*b*c*d**2*e*x**3 + 90*sqrt(d + e*x**2)*sqrt(a - c*x
**4)*a*b*c*d*e**2*x**5 - 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*e**3*x
**7 - 99*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**2*d**2*e*x**5 - 22*sqrt(d
+ e*x**2)*sqrt(a - c*x**4)*a*c**2*d*e**2*x**7 - 77*sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*b*c**2*d**2*e*x**7 - 288*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)
*x**4)/(4*a**2*d**6*e**2 + 24*a**2*d**5*e**3*x**2 + 60*a**2*d**4*e**4*x**4
 + 80*a**2*d**3*e**5*x**6 + 60*a**2*d**2*e**6*x**8 + 24*a**2*d*e**7*x**10
+ 4*a**2*e**8*x**12 + 7*a*c*d**8 + 42*a*c*d**7*e*x**2 + 101*a*c*d**6*e**2*
x**4 + 116*a*c*d**5*e**3*x**6 + 45*a*c*d**4*e**4*x**8 - 38*a*c*d**3*e**5*x
**10 - 53*a*c*d**2*e**6*x**12 - 24*a*c*d*e**7*x**14 - 4*a*c*e**8*x**16 - 7
*c**2*d**8*x**4 - 42*c**2*d**7*e*x**6 - 105*c**2*d**6*e**2*x**8 - 140*c**2
*d**5*e**3*x**10 - 105*c**2*d**4*e**4*x**12 - 42*c**2*d**3*e**5*x**14 - 7*
c**2*d**2*e**6*x**16),x)*a**4*b*d**5*e**6 - 1440*int((sqrt(d + e*x**2)*sqr
t(a - c*x**4)*x**4)/(4*a**2*d**6*e**2 + 24*a**2*d**5*e**3*x**2 + 60*a**2*d
**4*e**4*x**4 + 80*a**2*d**3*e**5*x**6 + 60*a**2*d**2*e**6*x**8 + 24*a**2*
d*e**7*x**10 + 4*a**2*e**8*x**12 + 7*a*c*d**8 + 42*a*c*d**7*e*x**2 + 10...
```

3.107 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 545

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx = -\frac{(5Bd+4Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{8cx}-\frac{Bex\sqrt{d+ex^2}\sqrt{a-cx^4}}{4c}$$
$$-\frac{(5Bd+4Ae)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{(8Acd^2+7aBde+4aAe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8\sqrt{a}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{(3Bcd^2+12Acde+4aBe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8c\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/8*(4*A*e+5*B*d)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/x-1/4*B*e*x*(e*x^2+d
)^(1/2)*(-c*x^4+a)^(1/2)/c-1/8*(4*A*e+5*B*d)*(d+a^(1/2)*e/c^(1/2))*(1-a/c/
x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellipti
cE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/
2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/8*(4*A*a*e^2+8*A*c*d^2+7*B*
a*d*e)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)
^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a
^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/c^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
+1/8*(12*A*c*d*e+4*B*a*e^2+3*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^
2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^
2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/(e*x^2+d)^(1
/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2))/Sqrt[a - c*x^4],x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2))/Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx$$

$$\Big\downarrow \text{ 2261}$$

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2))/Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,(e\,x^{2}+d)^{\frac{3}{2}}}{\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-(B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(-c*x^4 + a)*sqrt(e*x^2 +
d)/(c*x^4 - a), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(A+Bx^2)\,(d+ex^2)^{\frac{3}{2}}}{\sqrt{a-cx^4}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)/sqrt(a - c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(3/2)/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(3/2)/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx = \int \frac{(B\,x^2+A)\,\left(e\,x^2+d\right)^{3/2}}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2))/(a - c*x^4)^(1/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2))/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{\sqrt{a-cx^4}}\,dx = \frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,bex+4\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)ac\,e^2+5\left(\int \frac{\sqrt{e}}{-ce}\right.}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*e*x + 4*int((sqrt(d + e*x**2)*sqrt
(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*e**2 + 5*
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 -
c*e*x**6),x)*b*c*d*e + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d
 + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*e**2 + 8*int((sqrt(d + e*x**2)*s
qrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*d*e +
4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4
- c*e*x**6),x)*b*c*d**2 + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a
*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*d*e + 4*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*d**2)/(4*c)
```

3.108 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 469

$$\begin{aligned}
&\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx \\
&= -\frac{B\sqrt{d+ex^2}\sqrt{a-cx^4}}{2cx} \\
&- \frac{B\left(d+\frac{\sqrt{ae}}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{(2Acd+aBe)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{(Bd+2Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{2\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output
```
-1/2*B*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/x-1/2*B*(d+a^(1/2)*e/c^(1/2))*(1
-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*El
lipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/
c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*(2*A*c*d+B*a*e)*(1-a
/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Elli
pticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^
(1/2)))^(1/2))/a^(1/2)/c^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*(2*A*e
+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^
(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d
+a^(1/2)*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2])/Sqrt[a - c*x^4],x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2])/Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2])/Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2 + A)\,\sqrt{e\,x^2 + d}}{\sqrt{-c\,x^4 + a}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A + Bx^2)\,\sqrt{d + ex^2}}{\sqrt{a - cx^4}}\,dx = \int \frac{(Bx^2 + A)\sqrt{ex^2 + d}}{\sqrt{-cx^4 + a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c*x^4 - a), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)/sqrt(a - c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{\sqrt{-cx^4+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/sqrt(-c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}}{\sqrt{a-c\,x^4}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a - c*x^4)^(1/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a-cx^4}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{-c\,x^4+a}dx\right)b+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-c\,x^4+a}dx\right)a$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a - c*x**4),x)*b + int((sqrt
(d + e*x**2)*sqrt(a - c*x**4))/(a - c*x**4),x)*a
```

3.109 $\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 268

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \frac{A\sqrt{c}\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{a}\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{B\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output

```
A*c^(1/2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x
^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(
d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+B*(1
-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*El
lipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)
*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A + Bx^2}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\, dx = \int \frac{A + Bx^2}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\, dx$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + Bx^2}{\sqrt{a - cx^4}\sqrt{d + ex^2}}\, dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{A + Bx^2}{\sqrt{a - cx^4}\sqrt{d + ex^2}} dx$$

input
```
Int[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^{2}+A}{\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c*e*x^6 + c*d*x^4
- a*e*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(a - c*x**4)*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2)/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^2}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\, dx = \left(\int \frac{\sqrt{e\,x^2 + d}\,\sqrt{-c\,x^4 + a}\,x^2}{-ce\,x^6 - cd\,x^4 + ae\,x^2 + ad}dx\right) b + \left(\int \frac{\sqrt{e\,x^2 + d}\,\sqrt{-c\,x^4 + a}}{-ce\,x^6 - cd\,x^4 + ae\,x^2 + ad}dx\right) a$$

input

```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output

```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 -
c*e*x**6),x)*b + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 -
 c*d*x**4 - c*e*x**6),x)*a
```

3.110 $\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 354

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = -\frac{(Bd-Ae)\sqrt{a-cx^4}}{(cd^2-ae^2)\,x\sqrt{d+ex^2}} - \frac{\sqrt{c}(Bd-Ae)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{A\sqrt{c}\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{a}d\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output

```
-(-A*e+B*d)*(-c*x^4+a)^(1/2)/(-a*e^2+c*d^2)/x/(e*x^2+d)^(1/2)-c^(1/2)*(-A*
e+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)
^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a
^(1/2)*e/c^(1/2)))^(1/2))/d/(c^(1/2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+
a)^(1/2)+A*c^(1/2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(
1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(
1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/d/(e*x^2+d)^(1/2)/(-c*x^4+a)
^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(A + B*x^2)/((d + e*x^2)^(3/2)*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a-cx^4}\,(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{A+Bx^2}{\sqrt{a-cx^4}\,(d+ex^2)^{3/2}}dx$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^2+A}{(e\,x^2+d)^{\frac{3}{2}}\sqrt{-c\,x^4+a}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c*e^2*x^8 + 2*c*d*
e*x^6 - 2*a*d*e*x^2 + (c*d^2 - a*e^2)*x^4 - a*d^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{a-cx^4}\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(3/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(a - c*x**4)*(d + e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2)/((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx\right)b+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx\right)a$$

input
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d**2 + 2*a*d*e*x**2 + a*e*
*2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*b + int((sqrt(d + e
*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**
4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a
```

3.111 $\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 510

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx = \frac{e(Bd-Ae)x\sqrt{a-cx^4}}{3d\,(cd^2-ae^2)\,(d+ex^2)^{3/2}} - \frac{(3Bcd^3-6Acd^2e+aBde^2+2aAe^3)\sqrt{a-cx^4}}{3d\,(cd^2-ae^2)^2\,x\sqrt{d+ex^2}} - \frac{\sqrt{c}(3Bcd^3-6Acd^2e+aBde^2+2aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3d^2\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2-ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{\sqrt{c}(3Acd^2-aBde-2aAe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3\sqrt{a}d^2\,(cd^2-ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/3*e*(-A*e+B*d)*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^(3/2)-1/3*(
2*A*a*e^3-6*A*c*d^2*e+B*a*d*e^2+3*B*c*d^3)*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^
2)^2/x/(e*x^2+d)^(1/2)-1/3*c^(1/2)*(2*A*a*e^3-6*A*c*d^2*e+B*a*d*e^2+3*B*c*
d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(
1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(
1/2)*e/c^(1/2)))^(1/2))/d^2/(c^(1/2)*d-a^(1/2)*e)/(-a*e^2+c*d^2)/(e*x^2+d)
^(1/2)/(-c*x^4+a)^(1/2)+1/3*c^(1/2)*(-2*A*a*e^2+3*A*c*d^2-B*a*d*e)*(1-a/c/
x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellipti
cF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/
2)))^(1/2))/a^(1/2)/d^2/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^(5/2)*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(A + B*x^2)/((d + e*x^2)^(5/2)*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a-cx^4}\,(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{A+Bx^2}{\sqrt{a-cx^4}\,(d+ex^2)^{5/2}}dx$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^(5/2)*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^{2}+A}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^{5/2}\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c*e^3*x^10 + 3*c*d
*e^2*x^8 + (3*c*d^2*e - a*e^3)*x^6 - 3*a*d^2*e*x^2 + (c*d^3 - 3*a*d*e^2)*x
^4 - a*d^3), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{a-cx^4}\,(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(5/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(a - c*x**4)*(d + e*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((A + B*x^2)/((a - c*x^4)^(1/2)*(d + e*x^2)^(5/2)),x)
```

output
```
int((A + B*x^2)/((a - c*x^4)^(1/2)*(d + e*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a-cx^4}}\,dx=\text{Too large to display}$$

input
```
int((B*x^2+A)/(e*x^2+d)^(5/2)/(-c*x^4+a)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*x**3 + 2*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*x**8)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6
 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*b*c*
d**2*e + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8)/(a*d**3 + 3*a*d**2
*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 -
3*c*d*e**2*x**8 - c*e**3*x**10),x)*b*c*d*e**2*x**2 + 2*int((sqrt(d + e*x**
2)*sqrt(a - c*x**4)*x**8)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*
e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10
),x)*b*c*e**3*x**4 + 5*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**
3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d*
*2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*b*c*d**3 + 10*int((sqrt(d +
 e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**
4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3
*x**10),x)*b*c*d**2*e*x**2 + 5*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6
)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4
- 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8 - c*e**3*x**10),x)*b*c*d*e**2*x**4 + 3
*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d
*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4 - 3*c*d**2*e*x**6 - 3*c*d*e**2*x**8
 - c*e**3*x**10),x)*a**2*d**3 + 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/
(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 - c*d**3*x**4...
```

3.112 $\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 669

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}} \, dx = \frac{x(A+Bx^2)(d+ex^2)^{5/2}}{2a\sqrt{a-cx^4}}$$
$$+ \frac{(Bcd^2+2Acde+2aBe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{2ac^2x}$$
$$+ \frac{e(2Bd+Ae)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{2ac} + \frac{Be^2x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{2ac}$$
$$+ \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(Bcd^2+2Acde+2aBe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2ac\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{(Acd(cd^2-3ae^2)-aBe(3cd^2+2ae^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a^{3/2}c^{3/2}\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{e^2(5Bd+2Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2c\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/2*x*(B*x^2+A)*(e*x^2+d)^(5/2)/a/(-c*x^4+a)^(1/2)+1/2*(2*A*c*d*e+2*B*a*e^
2+B*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/c^2/x+1/2*e*(A*e+2*B*d)*x*(e
*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/c+1/2*B*e^2*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a
)^(1/2)/a/c+1/2*(d+a^(1/2)*e/c^(1/2))*(2*A*c*d*e+2*B*a*e^2+B*c*d^2)*(1-a/c
/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellipt
icE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1
/2)))^(1/2))/a/c/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*(A*c*d*(-3*a*e^2+c*d
^2)-a*B*e*(2*a*e^2+3*c*d^2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(
1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2
^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/c^(3/2)/(e*x^2+d)^
(1/2)/(-c*x^4+a)^(1/2)-1/2*e^2*(2*A*e+5*B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2
)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(
1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/(e*x^
2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(5/2))/(a - c*x^4)^(3/2),x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(5/2))/(a - c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(5/2))/(a - c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{5}{2}}}{(-c\,x^4+a)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)\,(d+ex^2)^{\frac{5}{2}}}{(a-cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(5/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(5/2)/(a - c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(5/2)/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(5/2)/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(e\,x^2+d\right)^{5/2}}{\left(a-c\,x^4\right)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(5/2))/(a - c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(5/2))/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a-cx^4)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(5/2)/(-c*x^4+a)^(3/2),x)
```

output
```
(4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*e**4*x + 12*sqrt(d + e*x**2)*sqr
t(a - c*x**4)*a*b*d*e**3*x + 3*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**2*
e**2*x - sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*e**3*x**3 + 3*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*b*c*d**3*e*x - 3*sqrt(d + e*x**2)*sqrt(a - c*x**4)*
b*c*d**2*e**2*x**3 - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8)/(a**2*
d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10
),x)*a**2*c**2*d*e**4 - 5*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8)/(a*
*2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x*
*10),x)*a*b*c**2*d**2*e**3 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**8
)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2
*e*x**10),x)*a*c**3*d*e**4*x**4 + 5*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)
*x**8)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 +
 c**2*e*x**10),x)*b*c**3*d**2*e**3*x**4 - 8*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**2)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*
d*x**8 + c**2*e*x**10),x)*a**4*e**5 - 24*int((sqrt(d + e*x**2)*sqrt(a - c*
x**4)*x**2)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x
**8 + c**2*e*x**10),x)*a**3*b*d*e**4 - 3*int((sqrt(d + e*x**2)*sqrt(a - c*
x**4)*x**2)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x
**8 + c**2*e*x**10),x)*a**3*c*d**2*e**3 + 8*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**2)/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c*...
```

3.113 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 568

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx = \frac{x(A+Bx^2)(d+ex^2)^{3/2}}{2a\sqrt{a-cx^4}} + \frac{(Bd+Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{2acx} + \frac{Bex\sqrt{d+ex^2}\sqrt{a-cx^4}}{2ac}$$
$$+ \frac{(Bd+Ae)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{(Acd^2-2aBde-aAe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a^{3/2}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{Be^2\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{c\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/2*x*(B*x^2+A)*(e*x^2+d)^(3/2)/a/(-c*x^4+a)^(1/2)+1/2*(A*e+B*d)*(e*x^2+d)
^(1/2)*(-c*x^4+a)^(1/2)/a/c/x+1/2*B*e*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a
/c+1/2*(A*e+B*d)*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x
^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^
2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/(e*x^2+d)^(1/2
)/(-c*x^4+a)^(1/2)+1/2*(-A*a*e^2+A*c*d^2-2*B*a*d*e)*(1-a/c/x^4)^(1/2)*x^3*
(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/
2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(
3/2)/c^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-B*e^2*(1-a/c/x^4)^(1/2)*x^3*
(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1
/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/
c/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2))/(a - c*x^4)^(3/2),x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2))/(a - c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2))/(a - c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,(e\,x^{2}+d)^{\frac{3}{2}}}{(-c\,x^{4}+a)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{\frac{3}{2}}}{(a-cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)/(a - c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(3/2)/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(3/2)/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx=\int \frac{(B\,x^2+A)\,\left(e\,x^2+d\right)^{3/2}}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2))/(a - c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2))/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{(a-cx^4)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*e**2*x + 2*sqrt(d + e*x**2)*sqrt(a -
c*x**4)*b*d*e*x + int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a**2*d + a
**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*
a*b*c*d*e**2 - int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a**2*d + a**2
*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*b*c
**2*d*e**2*x**4 - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d +
 a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x
)*a**3*e**3 - 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d + a**
2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*
*2*b*d*e**2 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d + a**
2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*
*2*c*d**2*e + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d + a**
2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*
*2*c*e**3*x**4 + int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d + a*
*2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a
*b*c*d**3 + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d + a**2*
e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*b*
c*d*e**2*x**4 - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d + a
**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*
a*c**2*d**2*e*x**4 - int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2...
```

3.114 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 374

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx = \frac{x(A+Bx^2)\sqrt{d+ex^2}}{2a\sqrt{a-cx^4}} + \frac{B\sqrt{d+ex^2}\sqrt{a-cx^4}}{2acx} + \frac{B\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{(Acd-aBe)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a^{3/2}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/2*x*(B*x^2+A)*(e*x^2+d)^(1/2)/a/(-c*x^4+a)^(1/2)+1/2*B*(e*x^2+d)^(1/2)*(
-c*x^4+a)^(1/2)/a/c/x+1/2*B*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a
^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)
/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/(e*
x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*(A*c*d-B*a*e)*(1-a/c/x^4)^(1/2)*x^3*(a^(
1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c
^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)
/c^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2])/(a - c*x^4)^(3/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2])/(a - c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx$$
$$\Big\downarrow \; 2261$$
$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2])/(a - c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,\sqrt{e\,x^{2}+d}}{(-c\,x^{4}+a)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c^2*x^8 - 2*a*c*x^4
 + a^2), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)/(a - c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/(-c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{(-cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/(-c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}}{(a-c\,x^4)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a - c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a - c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a-cx^4)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,aex+\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,bdx+2\left(\int \frac{\sqrt{e\,x^2}}{c^2e\,x^{10}+c^2d\,x^8-2}\right.}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*e*x + sqrt(d + e*x**2)*sqrt(a - c*x**
4)*b*d*x + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a**2*d + a**2*e
*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a**2*
b*e**2 - int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a**2*d + a**2*e*x**
2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a**2*c*d*
e - int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a**2*d + a**2*e*x**2 - 2
*a*c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*b*c*d**2 - 2
*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a**2*d + a**2*e*x**2 - 2*a*
c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*b*c*e**2*x**4 +
 int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a**2*d + a**2*e*x**2 - 2*a*
c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*c**2*d*e*x**4 +
 int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a**2*d + a**2*e*x**2 - 2*a*
c*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*b*c**2*d**2*x**4
+ int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a**2*d + a**2*e*x**2 - 2*a*c*d*
x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a**3*d*e - int((sqrt(
d + e*x**2)*sqrt(a - c*x**4))/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c
*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a**2*b*d**2 - int((sqrt(d + e*x**
2)*sqrt(a - c*x**4))/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 +
 c**2*d*x**8 + c**2*e*x**10),x)*a**2*c*d*e*x**4 + int((sqrt(d + e*x**2)*sq
rt(a - c*x**4))/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x**6 + c...
```

3.115 $\int \frac{A+Bx^2}{\sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 413

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\left(a-cx^4\right)^{3/2}} \, dx = \frac{(Bd-Ae)\sqrt{d+ex^2}}{2\left(cd^2-ae^2\right)x\sqrt{a-cx^4}}+\frac{(Acd-aBe)x\sqrt{d+ex^2}}{2a\left(cd^2-ae^2\right)\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{c}(Bd-Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{A\sqrt{c}\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a^{3/2}\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/2*(-A*e+B*d)*(e*x^2+d)^(1/2)/(-a*e^2+c*d^2)/x/(-c*x^4+a)^(1/2)+1/2*(A*c*
d-B*a*e)*x*(e*x^2+d)^(1/2)/a/(-a*e^2+c*d^2)/(-c*x^4+a)^(1/2)+1/2*c^(1/2)*(
-A*e+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x
^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(
d+a^(1/2)*e/c^(1/2)))^(1/2))/a/(c^(1/2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x
^4+a)^(1/2)+1/2*A*c^(1/2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2
)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1
/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/(e*x^2+d)^(1/2)/(-c*x
^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx=\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*(a - c*x^4)^(3/2)),x]
```

output
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*(a - c*x^4)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a-cx^4)^{3/2}\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{A+Bx^2}{(a-cx^4)^{3/2}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2)/(Sqrt[d + e*x^2]*(a - c*x^4)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^{2}+A}{\sqrt{e\,x^{2}+d}\,\left(-c\,x^{4}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx=\int \frac{Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c^2*e*x^10 + c^2*d*
x^8 - 2*a*c*e*x^6 - 2*a*c*d*x^4 + a^2*e*x^2 + a^2*d), x)
```

Sympy [F]

$$\int \frac{A + Bx^2}{\sqrt{d + ex^2}\,(a - cx^4)^{3/2}}\,dx = \int \frac{A + Bx^2}{(a - cx^4)^{\frac{3}{2}}\sqrt{d + ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(1/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)/((a - c*x**4)**(3/2)*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2}{\sqrt{d + ex^2}\,(a - cx^4)^{3/2}}\,dx = \int \frac{Bx^2 + A}{(-cx^4 + a)^{\frac{3}{2}}\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((B*x^2 + A)/((-c*x^4 + a)^(3/2)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A + Bx^2}{\sqrt{d + ex^2}\,(a - cx^4)^{3/2}}\,dx = \int \frac{Bx^2 + A}{(-cx^4 + a)^{\frac{3}{2}}\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((-c*x^4 + a)^(3/2)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{(a-c\,x^4)^{3/2}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2)/((a - c*x^4)^(3/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((a - c*x^4)^(3/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a-cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{c^2e\,x^{10}+c^2d\,x^8-2ace\,x^6-2acd\,x^4+a^2e\,x^2+a^2d}dx\right)b + \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{c^2e\,x^{10}+c^2d\,x^8-2ace\,x^6-2acd\,x^4+a^2e\,x^2+a^2d}dx\right)a$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d + a**2*e*x**2 - 2*a*c
*d*x**4 - 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*b + int((sqrt(d +
e*x**2)*sqrt(a - c*x**4))/(a**2*d + a**2*e*x**2 - 2*a*c*d*x**4 - 2*a*c*e*x
**6 + c**2*d*x**8 + c**2*e*x**10),x)*a
```

3.116 $\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 576

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a-cx^4)^{3/2}}\,dx = \frac{e(Bd-Ae)x}{d(cd^2-ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{(Bcd^3-2Acd^2e+3aBde^2-2aAe^3)\sqrt{d+ex^2}}{2d(cd^2-ae^2)^2x\sqrt{a-cx^4}}$$
$$+\frac{c(Acd^2-4aBde+3aAe^2)x\sqrt{d+ex^2}}{2a(cd^2-ae^2)^2\sqrt{a-cx^4}}$$
$$-\frac{\sqrt{c}(2Ae(cd^2+ae^2)-B(cd^3+3ade^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2ad(\sqrt{c}d-\sqrt{a}e)(cd^2-ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{c}(Acd^2+aBde-2aAe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2a^{3/2}d(cd^2-ae^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
e*(-A*e+B*d)*x/d/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*(-2*A
*a*e^3-2*A*c*d^2*e+3*B*a*d*e^2+B*c*d^3)*(e*x^2+d)^(1/2)/d/(-a*e^2+c*d^2)^2
/x/(-c*x^4+a)^(1/2)+1/2*c*(3*A*a*e^2+A*c*d^2-4*B*a*d*e)*x*(e*x^2+d)^(1/2)/
a/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)-1/2*c^(1/2)*(2*A*e*(a*e^2+c*d^2)-B*(3*
a*d*e^2+c*d^3))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2
)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2
)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d/(c^(1/2)*d-a^(1/2)*e)/(-a*e^2+c*d^2
)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*c^(1/2)*(-2*A*a*e^2+A*c*d^2+B*a*d*e
)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2
)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2
)*e/c^(1/2)))^(1/2))/a^(3/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(
1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a-cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a-cx^4)^{3/2}}\,dx$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^(3/2)*(a - c*x^4)^(3/2)),x]
```

output
```
Integrate[(A + B*x^2)/((d + e*x^2)^(3/2)*(a - c*x^4)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a-cx^4)^{3/2}(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2261}$$

$$\int \frac{A+Bx^2}{(a-cx^4)^{3/2}(d+ex^2)^{3/2}}dx$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^(3/2)*(a - c*x^4)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^{2}+A}{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(-c\,x^{4}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^{3/2}\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{Bx^{2}+A}{{\left(-cx^{4}+a\right)}^{\frac{3}{2}}{\left(ex^{2}+d\right)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c^2*e^2*x^12 + 2*c^
2*d*e*x^10 - 4*a*c*d*e*x^6 + (c^2*d^2 - 2*a*c*e^2)*x^8 + 2*a^2*d*e*x^2 - (
2*a*c*d^2 - a^2*e^2)*x^4 + a^2*d^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{A+Bx^2}{\left(a-cx^4\right)^{\frac{3}{2}}\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(3/2)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)/((a - c*x**4)**(3/2)*(d + e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{\left(-cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((B*x^2 + A)/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{\left(-cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((-c*x^4 + a)^(3/2)*(e*x^2 + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\,(a-cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{(a-c\,x^4)^{3/2}\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/((a - c*x^4)^(3/2)*(d + e*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2)/((a - c*x^4)^(3/2)*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\,(a-cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{c^2e^2x^{12}+2c^2de\,x^{10}-2ac\,e^2x^8+c^2d^2x^8-4acde\,x^6+a^2e^2x^4-2ac\,d \cdots}\right.$$

$$+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{c^2e^2x^{12}+2c^2de\,x^{10}-2ac\,e^2x^8+c^2d^2x^8-4acde\,x^6+a^2e^2x^4-2ac\,d^2x^4+2a^2de\,x^2+a^2d^2}dx\right)a$$

input
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(3/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a**2*d**2 + 2*a**2*d*e*x**2
+ a**2*e**2*x**4 - 2*a*c*d**2*x**4 - 4*a*c*d*e*x**6 - 2*a*c*e**2*x**8 + c*
*2*d**2*x**8 + 2*c**2*d*e*x**10 + c**2*e**2*x**12),x)*b + int((sqrt(d + e*
x**2)*sqrt(a - c*x**4))/(a**2*d**2 + 2*a**2*d*e*x**2 + a**2*e**2*x**4 - 2*
a*c*d**2*x**4 - 4*a*c*d*e*x**6 - 2*a*c*e**2*x**8 + c**2*d**2*x**8 + 2*c**2
*d*e*x**10 + c**2*e**2*x**12),x)*a
```

3.117 $\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 476

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx &= -\frac{C\sqrt{d+ex^2}\sqrt{a-cx^4}}{2cex} \\ &- \frac{C\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &+ \frac{(2Ac+aC)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &- \frac{(Cd-2Be)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output
```
-1/2*C*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e/x-1/2*C*(d+a^(1/2)*e/c^(1/2))*
(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*
EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*
e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*(2*A*c+C*a)*(1-a
/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Elli
pticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^
(1/2)))^(1/2))/a^(1/2)/c^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/2*(-2*B*
e+C*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)
^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(
d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{2261}$$
$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^4+B\,x^2+A}{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\, dx = \int \frac{A + Bx^2 + Cx^4}{\sqrt{a - cx^4}\sqrt{d + ex^2}}\, dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt(a - c*x**4)*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\, dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}}\, dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\, dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}}\, dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input

```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output

```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)c + \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)b + \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)a$$

input

```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output

```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 -
c*e*x**6),x)*c + int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x
**2 - c*d*x**4 - c*e*x**6),x)*b + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/
(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a
```

3.118 $\int \frac{2Ae+Cdx^2+2Cex^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 322

$$\int \frac{2Ae+Cdx^2+2Cex^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = -\frac{C\sqrt{d+ex^2}\sqrt{a-cx^4}}{cx} - \frac{C\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{(2Ac+aC)e\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{a}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-C*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/x-C*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4
)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(
1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2))
)^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+(2*A*c+C*a)*e*(1-a/c/x^4)^(1/2)*
x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a
^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))
/a^(1/2)/c^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(2*A*e + C*d*x^2 + 2*C*e*x^4)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),
x]
```

output
```
Integrate[(2*A*e + C*d*x^2 + 2*C*e*x^4)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),
 x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \; 2261$$
$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(2*A*e + C*d*x^2 + 2*C*e*x^4)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{2eC\,x^4 + Cd\,x^2 + 2Ae}{\sqrt{e\,x^2 + d}\,\sqrt{-c\,x^4 + a}}dx$$

input
```
int((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{2\,Cex^4 + Cdx^2 + 2\,Ae}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(2*C*e*x^4 + C*d*x^2 + 2*A*e)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(
c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{a - cx^4}\sqrt{d + ex^2}}\,dx$$

input
```
integrate((2*C*e*x**4+C*d*x**2+2*A*e)/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2)
,x)
```

output
```
Integral((2*A*e + C*d*x**2 + 2*C*e*x**4)/(sqrt(a - c*x**4)*sqrt(d + e*x**2
)), x)
```

Maxima [F]

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{2\,Cex^4 + Cdx^2 + 2\,Ae}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((2*C*e*x^4 + C*d*x^2 + 2*A*e)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d))
, x)
```

Giac [F]

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{2\,Cex^4 + Cdx^2 + 2\,Ae}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((2*C*e*x^4 + C*d*x^2 + 2*A*e)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2Ae+Cdx^2+2Cex^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{2\,C\,e\,x^4+C\,d\,x^2+2\,A\,e}{\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((2*A*e + C*d*x^2 + 2*C*e*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((2*A*e + C*d*x^2 + 2*C*e*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x
)
```

Reduce [F]

$$\begin{aligned}\int \frac{2Ae+Cdx^2+2Cex^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx &= 2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)ce\\ &\quad+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)cd\\ &\quad+2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)ae\end{aligned}$$

input
```
int((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4
- c*e*x**6),x)*c*e + int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a
*e*x**2 - c*d*x**4 - c*e*x**6),x)*c*d + 2*int((sqrt(d + e*x**2)*sqrt(a - c
*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*e
```

3.119 $\int \frac{(A+Bx^2)\sqrt{a+cx^4}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 1110

$$\int \frac{(A+Bx^2)\sqrt{a+cx^4}}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

output
```
1/8*a^(1/2)*d*(-4*A*e+3*B*d)*(c+a/x^4)*(e+d/x^2)*x^3/e^2/(a*e^2+c*d^2)^(1/
2)/(1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/
2)-1/8*(-4*A*e+3*B*d)*(e*x^2+d)^(1/2)*(c*x^4+a)^(1/2)/e^2/x+1/4*B*x*(e*x^2
+d)^(1/2)*(c*x^4+a)^(1/2)/e+1/16*(-4*A*c*d*e+4*B*a*e^2+3*B*c*d^2)*(c+a/x^4
)^(1/2)*(e+d/x^2)^(1/2)*x^3*arctanh(c^(1/2)*(e+d/x^2)^(1/2)/e^(1/2)/(c+a/x
^4)^(1/2))/c^(1/2)/e^(5/2)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2)-1/8*a^(1/4)*(-4
*A*e+3*B*d)*(a*e^2+c*d^2)^(3/4)*(1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))*
(d^2*(c+a/x^4)/(a*e^2+c*d^2)/(1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))^2)^
(1/2)*(e+d/x^2)^(1/2)*x^3*EllipticE(sin(2*arctan(a^(1/4)*(e+d/x^2)^(1/2)/(
a*e^2+c*d^2)^(1/4))),1/2*(2+2*a^(1/2)/(a*e^2+c*d^2)^(1/2)*e)^(1/2))/d/e^2/
(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2)-1/8*a^(1/4)*(a*e^2+c*d^2)^(3/4)*(4*A*c*d*e
-B*(3*c*d^2+2*a*e^2-2*a^(1/2)*e*(a*e^2+c*d^2)^(1/2)))*(1+a^(1/2)*(e+d/x^2)
/(a*e^2+c*d^2)^(1/2))*((c+a/x^4)/(c+a*e^2/d^2)/(1+a^(1/2)*(e+d/x^2)/(a*e^2
+c*d^2)^(1/2))^2)^(1/2)*(e+d/x^2)^(1/2)*x^3*InverseJacobiAM(2*arctan(a^(1/
4)*(e+d/x^2)^(1/2)/(a*e^2+c*d^2)^(1/4)),1/2*(2+2*a^(1/2)/(a*e^2+c*d^2)^(1/
2)*e)^(1/2))/c/d^2/e^2/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2)+1/32*(a*e^2+c*d^2)^
(1/4)*(-4*A*c*d*e+4*B*a*e^2+3*B*c*d^2)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))^2*(
1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))*((c+a/x^4)/(c+a*e^2/d^2)/(1+a^(1/
2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))^2)^(1/2)*(e+d/x^2)^(1/2)*x^3*EllipticPi(
sin(2*arctan(a^(1/4)*(e+d/x^2)^(1/2)/(a*e^2+c*d^2)^(1/4))),1/4*(a^(1/2)...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a+cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a+cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a + c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a + c*x^4])/Sqrt[d + e*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}(A+Bx^2)}{\sqrt{d+ex^2}}dx$$
$$\Big\downarrow \text{ 2261}$$
$$\int \frac{\sqrt{a+cx^4}(A+Bx^2)}{\sqrt{d+ex^2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a + c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{c\,x^4+a}}{\sqrt{e\,x^2+d}}dx$$

input
```
int((B*x^2+A)*(c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A + Bx^2)\sqrt{a + cx^4}}{\sqrt{d + ex^2}}\,dx = \int \frac{\sqrt{cx^4 + a}(Bx^2 + A)}{\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + a)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(A + Bx^2)\sqrt{a + cx^4}}{\sqrt{d + ex^2}}\,dx = \int \frac{(A + Bx^2)\sqrt{a + cx^4}}{\sqrt{d + ex^2}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+a)**(1/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a + c*x**4)/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{(A + Bx^2)\sqrt{a + cx^4}}{\sqrt{d + ex^2}}\,dx = \int \frac{\sqrt{cx^4 + a}(Bx^2 + A)}{\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a+cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a+cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+a}}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((A + B*x^2)*(a + c*x^4)^(1/2))/(d + e*x^2)^(1/2),x)
```

output
```
int(((A + B*x^2)*(a + c*x^4)^(1/2))/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a+cx^4}}{\sqrt{d+ex^2}}\,dx$$
$$=\frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+a}\,bx+4\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+a}\,x^4}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)ace-3\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+a}\,x^4}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)bcd+2\left(\int \frac{\sqrt{e\,x^2+d}}{ce\,x^6+cd\,x}\right.}{4e}$$

input
```
int((B*x^2+A)*(c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a + c*x**4)*b*x + 4*int((sqrt(d + e*x**2)*sqrt(a +
c*x**4)*x**4)/(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6),x)*a*c*e - 3*int((sqr
t(d + e*x**2)*sqrt(a + c*x**4)*x**4)/(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6
),x)*b*c*d + 2*int((sqrt(d + e*x**2)*sqrt(a + c*x**4)*x**2)/(a*d + a*e*x**
2 + c*d*x**4 + c*e*x**6),x)*a*b*e + 4*int((sqrt(d + e*x**2)*sqrt(a + c*x**
4))/(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6),x)*a**2*e - int((sqrt(d + e*x**
2)*sqrt(a + c*x**4))/(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6),x)*a*b*d)/(4*e
)
```

3.120 $\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 660

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx = \frac{B\sqrt{c+\frac{a}{x^4}}\sqrt{e+\frac{d}{x^2}}\,x^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{e+\frac{d}{x^2}}}{\sqrt{e}\sqrt{c+\frac{a}{x^4}}}\right)}{2\sqrt{c}\sqrt{e}\sqrt{d+ex^2}\sqrt{a+cx^4}}$$

$$+\frac{\left(\sqrt{a}Bcd^2+a^{3/2}Be^2-(Acd+aBe)\sqrt{cd^2+ae^2}\right)\left(1+\frac{\sqrt{a}\left(e+\frac{d}{x^2}\right)}{\sqrt{cd^2+ae^2}}\right)\sqrt{\frac{c+\frac{a}{x^4}}{\left(c+\frac{ae^2}{d^2}\right)\left(1+\frac{\sqrt{a}\left(e+\frac{d}{x^2}\right)}{\sqrt{cd^2+ae^2}}\right)^2}}\sqrt{e+\frac{d}{x^2}}\,x^3}{2\sqrt[4]{a}cd^2\sqrt[4]{cd^2+ae^2}\sqrt{d+ex^2}\sqrt{a+cx^4}}$$

$$+\frac{B\sqrt[4]{cd^2+ae^2}\left(\sqrt{a}e-\sqrt{cd^2+ae^2}\right)^2\left(1+\frac{\sqrt{a}\left(e+\frac{d}{x^2}\right)}{\sqrt{cd^2+ae^2}}\right)\sqrt{\frac{c+\frac{a}{x^4}}{\left(c+\frac{ae^2}{d^2}\right)\left(1+\frac{\sqrt{a}\left(e+\frac{d}{x^2}\right)}{\sqrt{cd^2+ae^2}}\right)^2}}\sqrt{e+\frac{d}{x^2}}\,x^3\operatorname{EllipticPi}\left(\right.}{4\sqrt[4]{a}cd^2e\sqrt{d+ex^2}\sqrt{a+cx^4}}$$

output
```
1/2*B*(c+a/x^4)^(1/2)*(e+d/x^2)^(1/2)*x^3*arctanh(c^(1/2)*(e+d/x^2)^(1/2)/
e^(1/2)/(c+a/x^4)^(1/2))/c^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2)+1
/2*(a^(1/2)*B*c*d^2+a^(3/2)*B*e^2-(A*c*d+B*a*e)*(a*e^2+c*d^2)^(1/2))*(1+a^
(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))*((c+a/x^4)/(c+a*e^2/d^2)/(1+a^(1/2)*(
e+d/x^2)/(a*e^2+c*d^2)^(1/2))^2)^(1/2)*(e+d/x^2)^(1/2)*x^3*InverseJacobiAM
(2*arctan(a^(1/4)*(e+d/x^2)^(1/2)/(a*e^2+c*d^2)^(1/4)),1/2*(2+2*a^(1/2)/(a
*e^2+c*d^2)^(1/2)*e)^(1/2))/a^(1/4)/c/d^2/(a*e^2+c*d^2)^(1/4)/(e*x^2+d)^(1
/2)/(c*x^4+a)^(1/2)+1/4*B*(a*e^2+c*d^2)^(1/4)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/
2))^2*(1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))*((c+a/x^4)/(c+a*e^2/d^2)/(
1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))^2)^(1/2)*(e+d/x^2)^(1/2)*x^3*Elli
pticPi(sin(2*arctan(a^(1/4)*(e+d/x^2)^(1/2)/(a*e^2+c*d^2)^(1/4))),1/4*(a^(
1/2)*e+(a*e^2+c*d^2)^(1/2))^2/a^(1/2)/e/(a*e^2+c*d^2)^(1/2),1/2*(2+2*a^(1/
2)/(a*e^2+c*d^2)^(1/2)*e)^(1/2))/a^(1/4)/c/d^2/e/(e*x^2+d)^(1/2)/(c*x^4+a)
^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[a + c*x^4]),x]
```

output
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[a + c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{2261}$$

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[a + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^2 + A}{\sqrt{e\,x^2 + d}\,\sqrt{c\,x^4 + a}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A + Bx^2}{\sqrt{d + ex^2}\sqrt{a + cx^4}}\,dx = \int \frac{Bx^2 + A}{\sqrt{cx^4 + a}\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c*e*x^6 + c*d*x^4 +
a*e*x^2 + a*d), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{a+cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(a + c*x**4)*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{c\,x^4+a}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+a}\,x^2}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)b+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+a}}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)a$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a + c*x**4)*x**2)/(a*d + a*e*x**2 + c*d*x**4 +
c*e*x**6),x)*b + int((sqrt(d + e*x**2)*sqrt(a + c*x**4))/(a*d + a*e*x**2 +
 c*d*x**4 + c*e*x**6),x)*a
```

3.121 $\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1059

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx = \text{Too large to display}$$

output
```
-1/2*a^(1/2)*C*d*(c+a/x^4)*(e+d/x^2)*x^3/c/e/(a*e^2+c*d^2)^(1/2)/(1+a^(1/2
)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2)+1/2*C*(e*
x^2+d)^(1/2)*(c*x^4+a)^(1/2)/c/e/x-1/4*(-2*B*e+C*d)*(c+a/x^4)^(1/2)*(e+d/x
^2)^(1/2)*x^3*arctanh(c^(1/2)*(e+d/x^2)^(1/2)/e^(1/2)/(c+a/x^4)^(1/2))/c^(
1/2)/e^(3/2)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*C*(a*e^2+c*d^2)^(
3/4)*(1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))*(d^2*(c+a/x^4)/(a*e^2+c*d^2
)/(1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))^2)^(1/2)*(e+d/x^2)^(1/2)*x^3*E
llipticE(sin(2*arctan(a^(1/4)*(e+d/x^2)^(1/2)/(a*e^2+c*d^2)^(1/4))),1/2*(2
+2*a^(1/2)/(a*e^2+c*d^2)^(1/2)*e)^(1/2))/c/d/e/(e*x^2+d)^(1/2)/(c*x^4+a)^(
1/2)-1/2*(a^(1/2)*c*d^2*(-B*e+C*d)+a^(3/2)*e^2*(-B*e+C*d)+e*(A*c*d+B*a*e-C
*a*d)*(a*e^2+c*d^2)^(1/2))*(1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))*((c+a
/x^4)/(c+a*e^2/d^2)/(1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))^2)^(1/2)*(e+
d/x^2)^(1/2)*x^3*InverseJacobiAM(2*arctan(a^(1/4)*(e+d/x^2)^(1/2)/(a*e^2+c
*d^2)^(1/4)),1/2*(2+2*a^(1/2)/(a*e^2+c*d^2)^(1/2)*e)^(1/2))/a^(1/4)/c/d^2/
e/(a*e^2+c*d^2)^(1/4)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2)-1/8*(-2*B*e+C*d)*(a*
e^2+c*d^2)^(1/4)*(a^(1/2)*e-(a*e^2+c*d^2)^(1/2))^2*(1+a^(1/2)*(e+d/x^2)/(a
*e^2+c*d^2)^(1/2))*((c+a/x^4)/(c+a*e^2/d^2)/(1+a^(1/2)*(e+d/x^2)/(a*e^2+c*
d^2)^(1/2))^2)^(1/2)*(e+d/x^2)^(1/2)*x^3*EllipticPi(sin(2*arctan(a^(1/4)*(
e+d/x^2)^(1/2)/(a*e^2+c*d^2)^(1/4))),1/4*(a^(1/2)*e+(a*e^2+c*d^2)^(1/2))^2
/a^(1/2)/e/(a*e^2+c*d^2)^(1/2),1/2*(2+2*a^(1/2)/(a*e^2+c*d^2)^(1/2)*e)^...
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+cx^4}\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{2261}$$

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^4+B\,x^2+A}{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + a)*sqrt(e*x^2 + d)/(c*e*x^6 + c*
d*x^4 + a*e*x^2 + a*d), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{\sqrt{a+cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt(a + c*x**4)*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{c\,x^4+a}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx &= \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+a}\,x^4}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)c\\ &\quad+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+a}\,x^2}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)b\\ &\quad+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+a}}{ce\,x^6+cd\,x^4+ae\,x^2+ad}dx\right)a\end{aligned}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x)
```

output

```
int((sqrt(d + e*x**2)*sqrt(a + c*x**4)*x**4)/(a*d + a*e*x**2 + c*d*x**4 +
c*e*x**6),x)*c + int((sqrt(d + e*x**2)*sqrt(a + c*x**4)*x**2)/(a*d + a*e*x
**2 + c*d*x**4 + c*e*x**6),x)*b + int((sqrt(d + e*x**2)*sqrt(a + c*x**4))/
(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6),x)*a
```

3.122 $\int \frac{2Ae+Cdx^2+2Cex^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 569

$$\int \frac{2Ae+Cdx^2+2Cex^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx = -\frac{\sqrt{a}Cd\left(c+\frac{a}{x^4}\right)\left(e+\frac{d}{x^2}\right)x^3}{c\left(\sqrt{cd^2+ae^2}+\sqrt{a}\left(e+\frac{d}{x^2}\right)\right)\sqrt{d+ex^2}\sqrt{a+cx^4}}+\frac{C\sqrt{d+ex^2}\sqrt{a+cx^4}}{cx}$$
$$+\frac{\sqrt[4]{a}C\sqrt[4]{cd^2+ae^2}\left(\sqrt{cd^2+ae^2}+\sqrt{a}\left(e+\frac{d}{x^2}\right)\right)\sqrt{\frac{d^2\left(c+\frac{a}{x^4}\right)}{\left(\sqrt{cd^2+ae^2}+\sqrt{a}\left(e+\frac{d}{x^2}\right)\right)^2}}\sqrt{e+\frac{d}{x^2}}\,x^3E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt{e}}{\sqrt[4]{cd^2+}}\right.\right.}{cd\sqrt{d+ex^2}\sqrt{a+cx^4}}$$
$$-\frac{\sqrt[4]{cd^2+ae^2}\left(2Ace-aCe+\sqrt{a}C\sqrt{cd^2+ae^2}\right)\left(1+\frac{\sqrt{a}\left(e+\frac{d}{x^2}\right)}{\sqrt{cd^2+ae^2}}\right)\sqrt{\frac{d^2\left(c+\frac{a}{x^4}\right)}{(cd^2+ae^2)\left(1+\frac{\sqrt{a}\left(e+\frac{d}{x^2}\right)}{\sqrt{cd^2+ae^2}}\right)^2}}\sqrt{e+\frac{d}{x^2}}\,x^3\,\mathrm{E}}{2\sqrt[4]{a}cd\sqrt{d+ex^2}\sqrt{a+cx^4}}$$

output
```
-a^(1/2)*C*d*(c+a/x^4)*(e+d/x^2)*x^3/c/((a*e^2+c*d^2)^(1/2)+a^(1/2)*(e+d/x
^2))/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2)+C*(e*x^2+d)^(1/2)*(c*x^4+a)^(1/2)/c/x
+a^(1/4)*C*(a*e^2+c*d^2)^(1/4)*((a*e^2+c*d^2)^(1/2)+a^(1/2)*(e+d/x^2))*(d^
2*(c+a/x^4)/((a*e^2+c*d^2)^(1/2)+a^(1/2)*(e+d/x^2))^2)^(1/2)*(e+d/x^2)^(1/
2)*x^3*EllipticE(sin(2*arctan(a^(1/4)*(e+d/x^2)^(1/2)/(a*e^2+c*d^2)^(1/4))
),1/2*(2+2*a^(1/2)/(a*e^2+c*d^2)^(1/2)*e)^(1/2))/c/d/(e*x^2+d)^(1/2)/(c*x^
4+a)^(1/2)-1/2*(a*e^2+c*d^2)^(1/4)*(2*A*c*e-C*a*e+a^(1/2)*C*(a*e^2+c*d^2)^
(1/2))*(1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))*(d^2*(c+a/x^4)/(a*e^2+c*d
^2)/(1+a^(1/2)*(e+d/x^2)/(a*e^2+c*d^2)^(1/2))^2)^(1/2)*(e+d/x^2)^(1/2)*x^3
*InverseJacobiAM(2*arctan(a^(1/4)*(e+d/x^2)^(1/2)/(a*e^2+c*d^2)^(1/4)),1/2
*(2+2*a^(1/2)/(a*e^2+c*d^2)^(1/2)*e)^(1/2))/a^(1/4)/c/d/(e*x^2+d)^(1/2)/(c
*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx = \int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx$$

input
```
Integrate[(2*A*e + C*d*x^2 + 2*C*e*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + c*x^4]),
x]
```

output
```
Integrate[(2*A*e + C*d*x^2 + 2*C*e*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + c*x^4]),
 x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{a+cx^4}\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{ 2261}$$

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{a+cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(2*A*e + C*d*x^2 + 2*C*e*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{2eC\,x^4 + Cd\,x^2 + 2Ae}{\sqrt{e\,x^2 + d}\,\sqrt{c\,x^4 + a}}dx$$

input
```
int((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x)
```

output
```
int((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d + ex^2}\sqrt{a + cx^4}}\,dx = \int \frac{2\,Cex^4 + Cdx^2 + 2\,Ae}{\sqrt{cx^4 + a}\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x, alg
orithm="fricas")
```

output
```
integral((2*C*e*x^4 + C*d*x^2 + 2*A*e)*sqrt(c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x)
```

Sympy [F]

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx = \int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{a+cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((2*C*e*x**4+C*d*x**2+2*A*e)/(e*x**2+d)**(1/2)/(c*x**4+a)**(1/2),
x)
```

output
```
Integral((2*A*e + C*d*x**2 + 2*C*e*x**4)/(sqrt(a + c*x**4)*sqrt(d + e*x**2
)), x)
```

Maxima [F]

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx = \int \frac{2\,Cex^4 + Cdx^2 + 2\,Ae}{\sqrt{cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((2*C*e*x^4 + C*d*x^2 + 2*A*e)/(sqrt(c*x^4 + a)*sqrt(e*x^2 + d)),
 x)
```

Giac [F]

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d+ex^2}\sqrt{a+cx^4}}\,dx = \int \frac{2\,Cex^4 + Cdx^2 + 2\,Ae}{\sqrt{cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((2*C*e*x^4 + C*d*x^2 + 2*A*e)/(sqrt(c*x^4 + a)*sqrt(e*x^2 + d)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d + ex^2}\sqrt{a + cx^4}}\,dx = \int \frac{2\,C\,e\,x^4 + C\,d\,x^2 + 2\,A\,e}{\sqrt{c\,x^4 + a}\,\sqrt{e\,x^2 + d}}\,dx$$

input
```
int((2*A*e + C*d*x^2 + 2*C*e*x^4)/((a + c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((2*A*e + C*d*x^2 + 2*C*e*x^4)/((a + c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{2Ae + Cdx^2 + 2Cex^4}{\sqrt{d + ex^2}\sqrt{a + cx^4}}\,dx = 2\left(\int \frac{\sqrt{e\,x^2 + d}\,\sqrt{c\,x^4 + a}\,x^4}{ce\,x^6 + cd\,x^4 + ae\,x^2 + ad}dx\right)ce + \left(\int \frac{\sqrt{e\,x^2 + d}\,\sqrt{c\,x^4 + a}\,x^2}{ce\,x^6 + cd\,x^4 + ae\,x^2 + ad}dx\right)cd + 2\left(\int \frac{\sqrt{e\,x^2 + d}\,\sqrt{c\,x^4 + a}}{ce\,x^6 + cd\,x^4 + ae\,x^2 + ad}dx\right)ae$$

input
```
int((2*C*e*x^4+C*d*x^2+2*A*e)/(e*x^2+d)^(1/2)/(c*x^4+a)^(1/2),x)
```

output
```
2*int((sqrt(d + e*x**2)*sqrt(a + c*x**4)*x**4)/(a*d + a*e*x**2 + c*d*x**4
+ c*e*x**6),x)*c*e + int((sqrt(d + e*x**2)*sqrt(a + c*x**4)*x**2)/(a*d + a
*e*x**2 + c*d*x**4 + c*e*x**6),x)*c*d + 2*int((sqrt(d + e*x**2)*sqrt(a + c
*x**4))/(a*d + a*e*x**2 + c*d*x**4 + c*e*x**6),x)*a*e
```

3.123 $\int (A+Bx^2)(d+ex^2)^q (a+cx^4)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 276

$$\begin{aligned}\int (A+Bx^2)(d+ex^2)^q (a+cx^4)\,dx &= -\frac{(3Acde(7+2q)-B(15cd^2+ae^2(35+24q+4q^2)))\,x(d+ex^2)^{1+q}}{e^3(3+2q)(5+2q)(7+2q)} \\ &\quad -\frac{c(5Bd-Ae(7+2q))x^3(d+ex^2)^{1+q}}{e^2(5+2q)(7+2q)}+\frac{Bcx^5(d+ex^2)^{1+q}}{e(7+2q)} \\ &\quad +\frac{(Ae(7+2q)(3cd^2+ae^2(15+16q+4q^2))-Bd(15cd^2+ae^2(35+24q+4q^2)))\,x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)}{e^3(3+2q)(5+2q)(7+2q)}\end{aligned}$$

output
```
-(3*A*c*d*e*(7+2*q)-B*(15*c*d^2+a*e^2*(4*q^2+24*q+35)))*x*(e*x^2+d)^(1+q)/
e^3/(3+2*q)/(5+2*q)/(7+2*q)-c*(5*B*d-A*e*(7+2*q))*x^3*(e*x^2+d)^(1+q)/e^2/
(5+2*q)/(7+2*q)+B*c*x^5*(e*x^2+d)^(1+q)/e/(7+2*q)+(A*e*(7+2*q)*(3*c*d^2+a*
e^2*(4*q^2+16*q+15))-B*d*(15*c*d^2+a*e^2*(4*q^2+24*q+35)))*x*(e*x^2+d)^q*h
ypergeom([1/2, -q],[3/2],-e*x^2/d)/e^3/(3+2*q)/(5+2*q)/(7+2*q)/((1+e*x^2/d
)^q)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.48

$$\int (A+Bx^2)(d+ex^2)^q(a+cx^4)\,dx$$
$$=\frac{1}{105}x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\left(105aA\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)+35aBx^2\,\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)+3cx^4\left(7A\,\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right)+5Bx^2\,\text{Hypergeometric2F1}\left(\frac{7}{2},-q,\frac{9}{2},-\frac{ex^2}{d}\right)\right)\right)$$

input
```
Integrate[(A + B*x^2)*(d + e*x^2)^q*(a + c*x^4),x]
```

output
```
(x*(d + e*x^2)^q*(105*a*A*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)] +
35*a*B*x^2*Hypergeometric2F1[3/2, -q, 5/2, -((e*x^2)/d)] + 3*c*x^4*(7*A*Hy
pergeometric2F1[5/2, -q, 7/2, -((e*x^2)/d)] + 5*B*x^2*Hypergeometric2F1[7/
2, -q, 9/2, -((e*x^2)/d)])))/(105*(1 + (e*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.72, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)(A+Bx^2)(d+ex^2)^q\,dx$$
$$\Big\downarrow \text{2257}$$
$$\int \left(aA(d+ex^2)^q+aBx^2(d+ex^2)^q+Acx^4(d+ex^2)^q+Bcx^6(d+ex^2)^q\right)dx$$

$$\Big\downarrow \text{2009}$$

$$aAx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)+\frac{1}{3}aBx^3(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)+\frac{1}{5}Acx^5(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right)+\frac{1}{7}Bcx^7(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{7}{2},-q,\frac{9}{2},-\frac{ex^2}{d}\right)$$

input
```
Int[(A + B*x^2)*(d + e*x^2)^q*(a + c*x^4),x]
```

output
```
(a*A*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(1 + (
e*x^2)/d)^q + (a*B*x^3*(d + e*x^2)^q*Hypergeometric2F1[3/2, -q, 5/2, -((e*
x^2)/d)])/(3*(1 + (e*x^2)/d)^q) + (A*c*x^5*(d + e*x^2)^q*Hypergeometric2F1
[5/2, -q, 7/2, -((e*x^2)/d)])/(5*(1 + (e*x^2)/d)^q) + (B*c*x^7*(d + e*x^2)
^q*Hypergeometric2F1[7/2, -q, 9/2, -((e*x^2)/d)])/(7*(1 + (e*x^2)/d)^q)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int (B\,x^2+A)\,(e\,x^2+d)^q\,(c\,x^4+a)\,dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^q*(c*x^4+a),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^q*(c*x^4+a),x)
```

Fricas [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^q\left(a+cx^4\right)\,dx=\int \left(cx^4+a\right)\left(Bx^2+A\right)\left(ex^2+d\right)^q\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q*(c*x^4+a),x, algorithm="fricas")
```

output
```
integral((B*c*x^6 + A*c*x^4 + B*a*x^2 + A*a)*(e*x^2 + d)^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.82 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.42

$$\int (A+Bx^2)\left(d+ex^2\right)^q\left(a+cx^4\right)\,dx=Aad^qx{}_2F_1\left(\begin{matrix}\frac{1}{2},-q\\\frac{3}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)+\frac{Acd^qx^5{}_2F_1\left(\begin{matrix}\frac{5}{2},-q\\\frac{7}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{5}+\frac{Bad^qx^3{}_2F_1\left(\begin{matrix}\frac{3}{2},-q\\\frac{5}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3}+\frac{Bcd^qx^7{}_2F_1\left(\begin{matrix}\frac{7}{2},-q\\\frac{9}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{7}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**q*(c*x**4+a),x)
```

output
```
A*a*d**q*x*hyper((1/2, -q), (3/2,), e*x**2*exp_polar(I*pi)/d) + A*c*d**q*x
**5*hyper((5/2, -q), (7/2,), e*x**2*exp_polar(I*pi)/d)/5 + B*a*d**q*x**3*h
yper((3/2, -q), (5/2,), e*x**2*exp_polar(I*pi)/d)/3 + B*c*d**q*x**7*hyper(
(7/2, -q), (9/2,), e*x**2*exp_polar(I*pi)/d)/7
```

Maxima [F]

$$\int (A+Bx^2)\,(d+ex^2)^q\,(a+cx^4)\;dx=\int (cx^4+a)\,(Bx^2+A)\,(ex^2+d)^q\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q*(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^q, x)
```

Giac [F]

$$\int (A+Bx^2)\,(d+ex^2)^q\,(a+cx^4)\;dx=\int (cx^4+a)\,(Bx^2+A)\,(ex^2+d)^q\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q*(c*x^4+a),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2)\,(d+ex^2)^q\,(a+cx^4)\;dx=\int \left(B\,x^2+A\right)\,\left(c\,x^4+a\right)\,\left(e\,x^2+d\right)^q\,dx$$

input
```
int((A + B*x^2)*(a + c*x^4)*(d + e*x^2)^q,x)
```

output
```
int((A + B*x^2)*(a + c*x^4)*(d + e*x^2)^q, x)
```

Reduce [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^q \left(a+cx^4\right)\, dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^q*(c*x^4+a),x)
```

output
```
(8*(d + e*x**2)**q*a**2*e**3*q**3*x + 60*(d + e*x**2)**q*a**2*e**3*q**2*x
+ 142*(d + e*x**2)**q*a**2*e**3*q*x + 105*(d + e*x**2)**q*a**2*e**3*x + 8*
(d + e*x**2)**q*a*b*d*e**2*q**3*x + 48*(d + e*x**2)**q*a*b*d*e**2*q**2*x +
 70*(d + e*x**2)**q*a*b*d*e**2*q*x + 8*(d + e*x**2)**q*a*b*e**3*q**3*x**3
+ 52*(d + e*x**2)**q*a*b*e**3*q**2*x**3 + 94*(d + e*x**2)**q*a*b*e**3*q*x*
*3 + 35*(d + e*x**2)**q*a*b*e**3*x**3 - 12*(d + e*x**2)**q*a*c*d**2*e*q**2
*x - 42*(d + e*x**2)**q*a*c*d**2*e*q*x + 8*(d + e*x**2)**q*a*c*d*e**2*q**3
*x**3 + 32*(d + e*x**2)**q*a*c*d*e**2*q**2*x**3 + 14*(d + e*x**2)**q*a*c*d
*e**2*q*x**3 + 8*(d + e*x**2)**q*a*c*e**3*q**3*x**5 + 44*(d + e*x**2)**q*a
*c*e**3*q**2*x**5 + 62*(d + e*x**2)**q*a*c*e**3*q*x**5 + 21*(d + e*x**2)**
q*a*c*e**3*x**5 + 30*(d + e*x**2)**q*b*c*d**3*q*x - 20*(d + e*x**2)**q*b*c
*d**2*e*q**2*x**3 - 10*(d + e*x**2)**q*b*c*d**2*e*q*x**3 + 8*(d + e*x**2)*
*q*b*c*d*e**2*q**3*x**5 + 16*(d + e*x**2)**q*b*c*d*e**2*q**2*x**5 + 6*(d +
 e*x**2)**q*b*c*d*e**2*q*x**5 + 8*(d + e*x**2)**q*b*c*e**3*q**3*x**7 + 36*
(d + e*x**2)**q*b*c*e**3*q**2*x**7 + 46*(d + e*x**2)**q*b*c*e**3*q*x**7 +
15*(d + e*x**2)**q*b*c*e**3*x**7 + 256*int((d + e*x**2)**q/(16*d*q**4 + 12
8*d*q**3 + 344*d*q**2 + 352*d*q + 105*d + 16*e*q**4*x**2 + 128*e*q**3*x**2
 + 344*e*q**2*x**2 + 352*e*q*x**2 + 105*e*x**2),x)*a**2*d*e**3*q**8 + 3968
*int((d + e*x**2)**q/(16*d*q**4 + 128*d*q**3 + 344*d*q**2 + 352*d*q + 105*
d + 16*e*q**4*x**2 + 128*e*q**3*x**2 + 344*e*q**2*x**2 + 352*e*q*x**2 +...
```

3.124 $\int (A+Bx^2)(d+ex^2)^q \, dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int (A+Bx^2)(d+ex^2)^q \, dx = \frac{Bx(d+ex^2)^{1+q}}{e(3+2q)} - \frac{(Bd-Ae(3+2q))x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} \text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{e(3+2q)}$$

output
```
B*x*(e*x^2+d)^(1+q)/e/(3+2*q)-(B*d-A*e*(3+2*q))*x*(e*x^2+d)^q*hypergeom([1
/2, -q],[3/2],-e*x^2/d)/e/(3+2*q)/((1+e*x^2/d)^q)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int (A+Bx^2)(d+ex^2)^q \, dx = \frac{x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} \left(B(d+ex^2)\left(1+\frac{ex^2}{d}\right)^q + (-Bd+Ae(3+2q)) \text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\right.\right.}{e(3+2q)}$$

input
```
Integrate[(A + B*x^2)*(d + e*x^2)^q,x]
```

output
```
(x*(d + e*x^2)^q*(B*(d + e*x^2)*(1 + (e*x^2)/d)^q + (-(B*d) + A*e*(3 + 2*q
))*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)]))/(e*(3 + 2*q)*(1 + (e*x^
2)/d)^q)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^q\,dx$$

$$\Big\downarrow \text{299}$$

$$\left(A-\frac{Bd}{2eq+3e}\right)\int \left(ex^2+d\right)^q dx+\frac{Bx\left(d+ex^2\right)^{q+1}}{e(2q+3)}$$

$$\Big\downarrow \text{238}$$

$$\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(A-\frac{Bd}{2eq+3e}\right)\int \left(\frac{ex^2}{d}+1\right)^q dx+\frac{Bx\left(d+ex^2\right)^{q+1}}{e(2q+3)}$$

$$\Big\downarrow \text{237}$$

$$x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(A-\frac{Bd}{2eq+3e}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)+\frac{Bx\left(d+ex^2\right)^{q+1}}{e(2q+3)}$$

input
```
Int[(A + B*x^2)*(d + e*x^2)^q,x]
```

output
```
(B*x*(d + e*x^2)^(1 + q))/(e*(3 + 2*q)) + ((A - (B*d)/(3*e + 2*e*q))*x*(d
+ e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(1 + (e*x^2)/d)^
q
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

Maple [F]

$$\int (B\,x^{2}+A)\,(e\,x^{2}+d)^{q}\,dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^q,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^q,x)
```

Fricas [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^q\,dx=\int \left(Bx^2+A\right)\left(ex^2+d\right)^q\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x^2 + d)^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int (A+Bx^2)\left(d+ex^2\right)^q\,dx=Ad^qx\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-q\\ \frac{3}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)+\frac{Bd^qx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-q\\ \frac{5}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**q,x)
```

output
```
A*d**q*x*hyper((1/2, -q), (3/2,), e*x**2*exp_polar(I*pi)/d) + B*d**q*x**3*
hyper((3/2, -q), (5/2,), e*x**2*exp_polar(I*pi)/d)/3
```

Maxima [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^q\,dx=\int \left(Bx^2+A\right)\left(ex^2+d\right)^q\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q, x)
```

Giac [F]

$$\int (A+Bx^2)\,(d+ex^2)^q\,dx=\int (Bx^2+A)\,(ex^2+d)^q\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2)\,(d+ex^2)^q\,dx=\int (B\,x^2+A)\,\left(e\,x^2+d\right)^q\,dx$$

input
```
int((A + B*x^2)*(d + e*x^2)^q,x)
```

output
```
int((A + B*x^2)*(d + e*x^2)^q, x)
```

Reduce [F]

$$\int (A+Bx^2)\,(d+ex^2)^q\,dx$$
$$=\frac{2(e\,x^2+d)^q\,aeqx+3(e\,x^2+d)^q\,aex+2(e\,x^2+d)^q\,bdqx+2(e\,x^2+d)^q\,beq\,x^3+(e\,x^2+d)^q\,be\,x^3+16\Big(\int}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^q,x)
```

output
```
(2*(d + e*x**2)**q*a*e*q*x + 3*(d + e*x**2)**q*a*e*x + 2*(d + e*x**2)**q*b
*d*q*x + 2*(d + e*x**2)**q*b*e*q*x**3 + (d + e*x**2)**q*b*e*x**3 + 16*int(
(d + e*x**2)**q/(4*d*q**2 + 8*d*q + 3*d + 4*e*q**2*x**2 + 8*e*q*x**2 + 3*e
*x**2),x)*a*d*e*q**4 + 56*int((d + e*x**2)**q/(4*d*q**2 + 8*d*q + 3*d + 4*
e*q**2*x**2 + 8*e*q*x**2 + 3*e*x**2),x)*a*d*e*q**3 + 60*int((d + e*x**2)**
q/(4*d*q**2 + 8*d*q + 3*d + 4*e*q**2*x**2 + 8*e*q*x**2 + 3*e*x**2),x)*a*d*
e*q**2 + 18*int((d + e*x**2)**q/(4*d*q**2 + 8*d*q + 3*d + 4*e*q**2*x**2 +
8*e*q*x**2 + 3*e*x**2),x)*a*d*e*q - 8*int((d + e*x**2)**q/(4*d*q**2 + 8*d*
q + 3*d + 4*e*q**2*x**2 + 8*e*q*x**2 + 3*e*x**2),x)*b*d**2*q**3 - 16*int((
d + e*x**2)**q/(4*d*q**2 + 8*d*q + 3*d + 4*e*q**2*x**2 + 8*e*q*x**2 + 3*e*
x**2),x)*b*d**2*q**2 - 6*int((d + e*x**2)**q/(4*d*q**2 + 8*d*q + 3*d + 4*e
*q**2*x**2 + 8*e*q*x**2 + 3*e*x**2),x)*b*d**2*q)/(e*(4*q**2 + 8*q + 3))
```

3.125 $\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4} dx = \frac{\left(A-\frac{\sqrt{-a}B}{\sqrt{c}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},-\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)}{2a} + \frac{\left(A+\frac{\sqrt{-a}B}{\sqrt{c}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)}{2a}$$

output
```
1/2*(A-(-a)^(1/2)*B/c^(1/2))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-c^(1/2)*
x^2/(-a)^(1/2),-e*x^2/d)/a/((1+e*x^2/d)^q)+1/2*(A+(-a)^(1/2)*B/c^(1/2))*x*
(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,c^(1/2)*x^2/(-a)^(1/2),-e*x^2/d)/a/((1+e
*x^2/d)^q)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4),x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4), x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4}\,dx$$

$$\Big\downarrow \text{2257}$$

$$\int \left(\frac{(\sqrt{-a}B-A\sqrt{c})(d+ex^2)^q}{2\sqrt{-a}\sqrt{c}(\sqrt{-a}+\sqrt{c}x^2)}-\frac{(\sqrt{-a}B+A\sqrt{c})(d+ex^2)^q}{2\sqrt{-a}\sqrt{c}(\sqrt{-a}-\sqrt{c}x^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(A-\frac{\sqrt{-a}B}{\sqrt{c}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a}+\frac{x\left(\frac{\sqrt{-a}B}{\sqrt{c}}+A\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4),x]
```

output
```
((A - (Sqrt[-a]*B)/Sqrt[c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, -((S
qrt[c]*x^2)/Sqrt[-a]), -((e*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^q) + ((A + (Sqr
t[-a]*B)/Sqrt[c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (Sqrt[c]*x^2)/
Sqrt[-a], -((e*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^q)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^q}{c\,x^4+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^q}{a+cx^4}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+a}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^q}{a+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**q/(c*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^q}{a+cx^4}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+a}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^q}{a+cx^4}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+a}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^q}{c\,x^4+a}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4}\,dx = \left(\int \frac{(e\,x^2+d)^q}{c\,x^4+a}dx\right)a + \left(\int \frac{(e\,x^2+d)^q\,x^2}{c\,x^4+a}dx\right)b$$

input
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x)
```

output
```
int((d + e*x**2)**q/(a + c*x**4),x)*a + int(((d + e*x**2)**q*x**2)/(a + c*
x**4),x)*b
```

3.126 $\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 449

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+cx^4)^2}\,dx = \frac{x(d+ex^2)^{1+q}(Acd+aBe+c(Bd-Ae)x^2)}{4a(cd^2+ae^2)(a+cx^4)}$$
$$+\frac{\left(\sqrt{c}(A(3cd^2+ae^2(3-2q))+2aBdeq)-\sqrt{-a}(B(cd^2+ae^2(1-2q))-2Acdeq)\right)x(d+ex^2)^q\left(1+\frac{e}{}\right.}{8a^2\sqrt{c}(cd^2+ae^2)}$$
$$+\frac{\left(\sqrt{c}(A(3cd^2+ae^2(3-2q))+2aBdeq)+\sqrt{-a}(B(cd^2+ae^2(1-2q))-2Acdeq)\right)x(d+ex^2)^q\left(1+\frac{e}{}\right.}{8a^2\sqrt{c}(cd^2+ae^2)}$$
$$-\frac{e(Bd-Ae)(1+2q)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{4a(cd^2+ae^2)}$$

output
```
1/4*x*(e*x^2+d)^(1+q)*(A*c*d+B*a*e+c*(-A*e+B*d)*x^2)/a/(a*e^2+c*d^2)/(c*x^
4+a)+1/8*(c^(1/2)*(A*(3*c*d^2+a*e^2*(3-2*q))+2*a*B*d*e*q)-(-a)^(1/2)*(B*(c
*d^2+a*e^2*(1-2*q))-2*A*c*d*e*q))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-c^(
1/2)*x^2/(-a)^(1/2),-e*x^2/d)/a^2/c^(1/2)/(a*e^2+c*d^2)/((1+e*x^2/d)^q)+1/
8*(c^(1/2)*(A*(3*c*d^2+a*e^2*(3-2*q))+2*a*B*d*e*q)+(-a)^(1/2)*(B*(c*d^2+a*
e^2*(1-2*q))-2*A*c*d*e*q))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,c^(1/2)*x^2
/(-a)^(1/2),-e*x^2/d)/a^2/c^(1/2)/(a*e^2+c*d^2)/((1+e*x^2/d)^q)-1/4*e*(-A*
e+B*d)*(1+2*q)*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/a/(a*e^2+
c*d^2)/((1+e*x^2/d)^q)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+cx^4)^2}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+cx^4)^2}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4)^2,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \text{2257}$$

$$\int \left(\frac{A(d+ex^2)^q}{(a+cx^4)^2}+\frac{Bx^2(d+ex^2)^q}{(a+cx^4)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$A\int \frac{(ex^2+d)^q}{(cx^4+a)^2}dx+B\int \frac{x^2(ex^2+d)^q}{(cx^4+a)^2}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^q}{(c\,x^4+a)^2}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a)^2,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a)^2,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^q}{(a+cx^4)^2}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^q}{(cx^4+a)^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x^2 + d)^q/(c^2*x^8 + 2*a*c*x^4 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**q/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+cx^4)^2}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{(cx^4+a)^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a)^2, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+cx^4)^2}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{(cx^4+a)^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+cx^4)^2}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^q}{(c\,x^4+a)^2}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4)^2,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+cx^4)^2}\,dx = \left(\int \frac{(e\,x^2+d)^q}{c^2x^8+2ac\,x^4+a^2}dx\right)a+\left(\int \frac{(e\,x^2+d)^q\,x^2}{c^2x^8+2ac\,x^4+a^2}dx\right)b$$

input
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a)^2,x)
```

output
```
int((d + e*x**2)**q/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a + int(((d + e*x**
2)**q*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*b
```

3.127 $\int (d+ex^2)^q (a+cx^4) (A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 425

$$\int (d+ex^2)^q (a+cx^4) (A+Bx^2+Cx^4)\,dx =$$

$$-\frac{(ae^2(63+32q+4q^2)(3Cd-Be(5+2q))+3cd(35Cd^2-e(9+2q)(5Bd-Ae(7+2q))))\,x(d+ex^2)^{}}{e^4(3+2q)(5+2q)(7+2q)(9+2q)}$$

$$+\frac{(aCe^2(63+32q+4q^2)+c(35Cd^2-e(9+2q)(5Bd-Ae(7+2q))))\,x^3(d+ex^2)^{1+q}}{e^3(5+2q)(7+2q)(9+2q)}$$

$$-\frac{c(7Cd-Be(9+2q))x^5(d+ex^2)^{1+q}}{e^2(7+2q)(9+2q)}+\frac{cCx^7(d+ex^2)^{1+q}}{e(9+2q)}$$

$$+\frac{(ae^2(63+32q+4q^2)(3Cd^2-e(5+2q)(Bd-Ae(3+2q)))+3cd^2(35Cd^2-e(9+2q)(5Bd-Ae(7+}{e^4(3+2q)(5+2q)(7+2q)(9+2}$$

output
```
-(a*e^2*(4*q^2+32*q+63)*(3*C*d-B*e*(5+2*q))+3*c*d*(35*C*d^2-e*(9+2*q)*(5*B
*d-A*e*(7+2*q))))*x*(e*x^2+d)^(1+q)/e^4/(3+2*q)/(5+2*q)/(7+2*q)/(9+2*q)+(a
*C*e^2*(4*q^2+32*q+63)+c*(35*C*d^2-e*(9+2*q)*(5*B*d-A*e*(7+2*q))))*x^3*(e*
x^2+d)^(1+q)/e^3/(5+2*q)/(7+2*q)/(9+2*q)-c*(7*C*d-B*e*(9+2*q))*x^5*(e*x^2+
d)^(1+q)/e^2/(7+2*q)/(9+2*q)+c*C*x^7*(e*x^2+d)^(1+q)/e/(9+2*q)+(a*e^2*(4*q
^2+32*q+63)*(3*C*d^2-e*(5+2*q)*(B*d-A*e*(3+2*q)))+3*c*d^2*(35*C*d^2-e*(9+2
*q)*(5*B*d-A*e*(7+2*q))))*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d
)/e^4/(3+2*q)/(5+2*q)/(7+2*q)/(9+2*q)/((1+e*x^2/d)^q)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.38

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right) \left(A+Bx^2+Cx^4\right) \, dx$$
$$= \frac{1}{315}x\left(d+ex^2\right)^q \left(1+\frac{ex^2}{d}\right)^{-q} \left(315aA\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right) + 105aBx^2\,\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right) + 63(Ac+aC)x^4\,\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right) + 5cx^6\left(9B\,\text{Hypergeometric2F1}\left(\frac{7}{2},-q,\frac{9}{2},-\frac{ex^2}{d}\right) + 7Cx^2\,\text{Hypergeometric2F1}\left(\frac{9}{2},-q,\frac{11}{2},-\frac{ex^2}{d}\right)\right)\right)$$

input
```
Integrate[(d + e*x^2)^q*(a + c*x^4)*(A + B*x^2 + C*x^4),x]
```

output
```
(x*(d + e*x^2)^q*(315*a*A*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)] +
105*a*B*x^2*Hypergeometric2F1[3/2, -q, 5/2, -((e*x^2)/d)] + 63*(A*c + a*C)
*x^4*Hypergeometric2F1[5/2, -q, 7/2, -((e*x^2)/d)] + 5*c*x^6*(9*B*Hypergeo
metric2F1[7/2, -q, 9/2, -((e*x^2)/d)] + 7*C*x^2*Hypergeometric2F1[9/2, -q,
 11/2, -((e*x^2)/d)])))/(315*(1 + (e*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.60, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right) \left(A+Bx^2+Cx^4\right) \left(d+ex^2\right)^q \, dx$$

$$\Big\downarrow \text{ 2256}$$

$$\int \left(x^4(aC+Ac)\left(d+ex^2\right)^q+aA\left(d+ex^2\right)^q+aBx^2\left(d+ex^2\right)^q+Bcx^6\left(d+ex^2\right)^q+cCx^8\left(d+ex^2\right)^q\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(aC+Ac)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right)+aAx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)+\frac{1}{3}aBx^3\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)+\frac{1}{7}Bcx^7\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{7}{2},-q,\frac{9}{2},-\frac{ex^2}{d}\right)+\frac{1}{9}cCx^9\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{9}{2},-q,\frac{11}{2},-\frac{ex^2}{d}\right)$$

input
```
Int[(d + e*x^2)^q*(a + c*x^4)*(A + B*x^2 + C*x^4),x]
```

output
```
(a*A*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(1 + (
e*x^2)/d)^q + (a*B*x^3*(d + e*x^2)^q*Hypergeometric2F1[3/2, -q, 5/2, -((e*
x^2)/d)])/(3*(1 + (e*x^2)/d)^q) + ((A*c + a*C)*x^5*(d + e*x^2)^q*Hypergeom
etric2F1[5/2, -q, 7/2, -((e*x^2)/d)])/(5*(1 + (e*x^2)/d)^q) + (B*c*x^7*(d
+ e*x^2)^q*Hypergeometric2F1[7/2, -q, 9/2, -((e*x^2)/d)])/(7*(1 + (e*x^2)/
d)^q) + (c*C*x^9*(d + e*x^2)^q*Hypergeometric2F1[9/2, -q, 11/2, -((e*x^2)/
d)])/(9*(1 + (e*x^2)/d)^q)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)^{q} \left(c\,x^{4}+a\right) \left(C\,x^{4}+B\,x^{2}+A\right)dx$$

input
```
int((e*x^2+d)^q*(c*x^4+a)*(C*x^4+B*x^2+A),x)
```

output
```
int((e*x^2+d)^q*(c*x^4+a)*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \left(d+ex^2\right)^q \left(a+cx^4\right) \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(cx^4+a\right)\left(ex^2+d\right)^q\,dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+a)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*c*x^8 + B*c*x^6 + (C*a + A*c)*x^4 + B*a*x^2 + A*a)*(e*x^2 + d)
^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 50.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.42

$$\int (d+ex^2)^q (a+cx^4) (A+Bx^2+Cx^4) \, dx$$
$$= Aad^q x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -q \\ \frac{3}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right) + \frac{Acd^q x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -q \\ \frac{7}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{5} + \frac{Bad^q x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -q \\ \frac{5}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{3} + \frac{Bcd^q x^7 {}_2F_1\left(\begin{matrix} \frac{7}{2}, -q \\ \frac{9}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{7} + \frac{Cad^q x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -q \\ \frac{7}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{5} + \frac{Ccd^q x^9 {}_2F_1\left(\begin{matrix} \frac{9}{2}, -q \\ \frac{11}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{9}$$

input
```
integrate((e*x**2+d)**q*(c*x**4+a)*(C*x**4+B*x**2+A),x)
```

output
```
A*a*d**q*x*hyper((1/2, -q), (3/2,), e*x**2*exp_polar(I*pi)/d) + A*c*d**q*x
**5*hyper((5/2, -q), (7/2,), e*x**2*exp_polar(I*pi)/d)/5 + B*a*d**q*x**3*h
yper((3/2, -q), (5/2,), e*x**2*exp_polar(I*pi)/d)/3 + B*c*d**q*x**7*hyper(
(7/2, -q), (9/2,), e*x**2*exp_polar(I*pi)/d)/7 + C*a*d**q*x**5*hyper((5/2,
 -q), (7/2,), e*x**2*exp_polar(I*pi)/d)/5 + C*c*d**q*x**9*hyper((9/2, -q),
 (11/2,), e*x**2*exp_polar(I*pi)/d)/9
```

Maxima [F]

$$\int (d+ex^2)^q (a+cx^4) (A+Bx^2+Cx^4) \, dx = \int (Cx^4+Bx^2+A)(cx^4+a)(ex^2+d)^q \, dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+a)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + a)*(e*x^2 + d)^q, x)
```

Giac [F]

$$\int (d+ex^2)^q (a+cx^4) (A+Bx^2+Cx^4) \, dx = \int (Cx^4+Bx^2+A)(cx^4+a)(ex^2+d)^q \, dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+a)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + a)*(e*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^q (a+cx^4) (A+Bx^2+Cx^4) \, dx = \int (c\,x^4+a) \left(e\,x^2+d\right)^q (C\,x^4+B\,x^2+A) \, dx$$

input
```
int((a + c*x^4)*(d + e*x^2)^q*(A + B*x^2 + C*x^4),x)
```

output
```
int((a + c*x^4)*(d + e*x^2)^q*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (d+ex^2)^q (a+cx^4) (A+Bx^2+Cx^4) \, dx = \text{too large to display}$$

input
```
int((e*x^2+d)^q*(c*x^4+a)*(C*x^4+B*x^2+A),x)
```

output

```
(16*(d + e*x**2)**q*a**2*e**4*q**4*x + 192*(d + e*x**2)**q*a**2*e**4*q**3*
x + 824*(d + e*x**2)**q*a**2*e**4*q**2*x + 1488*(d + e*x**2)**q*a**2*e**4*
q*x + 945*(d + e*x**2)**q*a**2*e**4*x + 16*(d + e*x**2)**q*a*b*d*e**3*q**4
*x + 168*(d + e*x**2)**q*a*b*d*e**3*q**3*x + 572*(d + e*x**2)**q*a*b*d*e**
3*q**2*x + 630*(d + e*x**2)**q*a*b*d*e**3*q*x + 16*(d + e*x**2)**q*a*b*e**
4*q**4*x**3 + 176*(d + e*x**2)**q*a*b*e**4*q**3*x**3 + 656*(d + e*x**2)**q
*a*b*e**4*q**2*x**3 + 916*(d + e*x**2)**q*a*b*e**4*q*x**3 + 315*(d + e*x**
2)**q*a*b*e**4*x**3 - 48*(d + e*x**2)**q*a*c*d**2*e**2*q**3*x - 384*(d + e
*x**2)**q*a*c*d**2*e**2*q**2*x - 756*(d + e*x**2)**q*a*c*d**2*e**2*q*x + 3
2*(d + e*x**2)**q*a*c*d*e**3*q**4*x**3 + 272*(d + e*x**2)**q*a*c*d*e**3*q*
*3*x**3 + 632*(d + e*x**2)**q*a*c*d*e**3*q**2*x**3 + 252*(d + e*x**2)**q*a
*c*d*e**3*q*x**3 + 32*(d + e*x**2)**q*a*c*e**4*q**4*x**5 + 320*(d + e*x**2
)**q*a*c*e**4*q**3*x**5 + 1040*(d + e*x**2)**q*a*c*e**4*q**2*x**5 + 1200*(
d + e*x**2)**q*a*c*e**4*q*x**5 + 378*(d + e*x**2)**q*a*c*e**4*x**5 + 60*(d
 + e*x**2)**q*b*c*d**3*e*q**2*x + 270*(d + e*x**2)**q*b*c*d**3*e*q*x - 40*
(d + e*x**2)**q*b*c*d**2*e**2*q**3*x**3 - 200*(d + e*x**2)**q*b*c*d**2*e**
2*q**2*x**3 - 90*(d + e*x**2)**q*b*c*d**2*e**2*q*x**3 + 16*(d + e*x**2)**q
*b*c*d*e**3*q**4*x**5 + 104*(d + e*x**2)**q*b*c*d*e**3*q**3*x**5 + 156*(d
+ e*x**2)**q*b*c*d*e**3*q**2*x**5 + 54*(d + e*x**2)**q*b*c*d*e**3*q*x**5 +
 16*(d + e*x**2)**q*b*c*e**4*q**4*x**7 + 144*(d + e*x**2)**q*b*c*e**4*q...
```

3.128 $\int (d+ex^2)^q (A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 161

$$\int (d+ex^2)^q (A+Bx^2+Cx^4)\,dx = -\frac{(3Cd-Be(5+2q))x(d+ex^2)^{1+q}}{e^2(3+2q)(5+2q)}+\frac{Cx^3(d+ex^2)^{1+q}}{e(5+2q)} + \frac{(3Cd^2-e(5+2q)(Bd-Ae(3+2q)))\,x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{e^2(3+2q)(5+2q)}$$

output
```
-(3*C*d-B*e*(5+2*q))*x*(e*x^2+d)^(1+q)/e^2/(3+2*q)/(5+2*q)+C*x^3*(e*x^2+d)
^(1+q)/e/(5+2*q)+(3*C*d^2-e*(5+2*q)*(B*d-A*e*(3+2*q)))*x*(e*x^2+d)^q*hyper
geom([1/2, -q],[3/2],-e*x^2/d)/e^2/(3+2*q)/(5+2*q)/((1+e*x^2/d)^q)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.63

$$\int \left(d+ex^2\right)^q \left(A+Bx^2+Cx^4\right) \, dx = \frac{1}{15}x\left(d+ex^2\right)^q \left(1+\frac{ex^2}{d}\right)^{-q} \left(15A\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)+5Bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)+3Cx^4\,\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right)\right)$$

input
```
Integrate[(d + e*x^2)^q*(A + B*x^2 + C*x^4),x]
```

output
```
(x*(d + e*x^2)^q*(15*A*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)] + 5*B
*x^2*Hypergeometric2F1[3/2, -q, 5/2, -((e*x^2)/d)] + 3*C*x^4*Hypergeometri
c2F1[5/2, -q, 7/2, -((e*x^2)/d)]))/(15*(1 + (e*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1473, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2+Cx^4\right) \left(d+ex^2\right)^q \, dx$$

$$\Big\downarrow \text{1473}$$

$$\frac{\int \left(ex^2+d\right)^q \left(Ae(2q+5)-(3Cd-Be(2q+5))x^2\right) dx}{e(2q+5)}+\frac{Cx^3\left(d+ex^2\right)^{q+1}}{e(2q+5)}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{(3Cd^2-e(2q+5)(Bd-Ae(2q+3)))\int (ex^2+d)^q dx}{e(2q+3)}-\frac{x(d+ex^2)^{q+1}(3Cd-Be(2q+5))}{e(2q+3)}}{e(2q+5)}+\frac{Cx^3(d+ex^2)^{q+1}}{e(2q+5)}$$

$\Big\downarrow$ 238

$$\frac{\frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}(3Cd^2-e(2q+5)(Bd-Ae(2q+3)))\int \left(\frac{ex^2}{d}+1\right)^q dx}{e(2q+3)}-\frac{x(d+ex^2)^{q+1}(3Cd-Be(2q+5))}{e(2q+3)}}{e(2q+5)}+\frac{Cx^3(d+ex^2)^{q+1}}{e(2q+5)}$$

$\Big\downarrow$ 237

$$\frac{\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)(3Cd^2-e(2q+5)(Bd-Ae(2q+3)))}{e(2q+3)}-\frac{x(d+ex^2)^{q+1}(3Cd-Be(2q+5))}{e(2q+3)}}{e(2q+5)}+\frac{Cx^3(d+ex^2)^{q+1}}{e(2q+5)}$$

input
```
Int[(d + e*x^2)^q*(A + B*x^2 + C*x^4),x]
```

output
```
(C*x^3*(d + e*x^2)^(1 + q))/(e*(5 + 2*q)) + (-(((3*C*d - B*e*(5 + 2*q))*x*
(d + e*x^2)^(1 + q))/(e*(3 + 2*q))) + ((3*C*d^2 - e*(5 + 2*q)*(B*d - A*e*(
3 + 2*q)))*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/
(e*(3 + 2*q)*(1 + (e*x^2)/d)^q))/(e*(5 + 2*q))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)^{q} \left(C\,x^{4}+B\,x^{2}+A\right)dx$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A),x)
```

output
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \left(d+ex^2\right)^q \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(ex^2+d\right)^q\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.61 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.51

$$\int (d+ex^2)^q (A+Bx^2+Cx^4)\,dx = Ad^q x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-q\\ \frac{3}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right) + \frac{Bd^q x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-q\\ \frac{5}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3} + \frac{Cd^q x^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},-q\\ \frac{7}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{5}$$

input
```
integrate((e*x**2+d)**q*(C*x**4+B*x**2+A),x)
```

output
```
A*d**q*x*hyper((1/2, -q), (3/2,), e*x**2*exp_polar(I*pi)/d) + B*d**q*x**3*
hyper((3/2, -q), (5/2,), e*x**2*exp_polar(I*pi)/d)/3 + C*d**q*x**5*hyper((
5/2, -q), (7/2,), e*x**2*exp_polar(I*pi)/d)/5
```

Maxima [F]

$$\int (d+ex^2)^q (A+Bx^2+Cx^4)\,dx = \int (Cx^4+Bx^2+A)(ex^2+d)^q\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q, x)
```

Giac [F]

$$\int (d+ex^2)^q (A+Bx^2+Cx^4)\,dx = \int (Cx^4+Bx^2+A)(ex^2+d)^q\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^q (A+Bx^2+Cx^4)\,dx = \int (e\,x^2+d)^q\,(C\,x^4+B\,x^2+A)\,dx$$

input
```
int((d + e*x^2)^q*(A + B*x^2 + C*x^4),x)
```

output
```
int((d + e*x^2)^q*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (d+ex^2)^q (A+Bx^2+Cx^4)\,dx = \text{Too large to display}$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A),x)
```

output

```
(4*(d + e*x**2)**q*a*e**2*q**2*x + 16*(d + e*x**2)**q*a*e**2*q*x + 15*(d +
 e*x**2)**q*a*e**2*x + 4*(d + e*x**2)**q*b*d*e*q**2*x + 10*(d + e*x**2)**q
*b*d*e*q*x + 4*(d + e*x**2)**q*b*e**2*q**2*x**3 + 12*(d + e*x**2)**q*b*e**
2*q*x**3 + 5*(d + e*x**2)**q*b*e**2*x**3 - 6*(d + e*x**2)**q*c*d**2*q*x +
4*(d + e*x**2)**q*c*d*e*q**2*x**3 + 2*(d + e*x**2)**q*c*d*e*q*x**3 + 4*(d
+ e*x**2)**q*c*e**2*q**2*x**5 + 8*(d + e*x**2)**q*c*e**2*q*x**5 + 3*(d + e
*x**2)**q*c*e**2*x**5 + 64*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*
d*q + 15*d + 8*e*q**3*x**2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*
a*d*e**2*q**6 + 544*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 1
5*d + 8*e*q**3*x**2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**
2*q**5 + 1760*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d +
8*e*q**3*x**2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q**4
 + 2672*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q*
*3*x**2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q**3 + 186
0*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**3*x**
2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q**2 + 450*int((
d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**3*x**2 + 36*
e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q - 32*int((d + e*x**2)
**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**3*x**2 + 36*e*q**2*x**2
 + 46*e*q*x**2 + 15*e*x**2),x)*b*d**2*e*q**5 - 224*int((d + e*x**2)**q/...
```

3.129 $\int \frac{(d+ex^2)^q(A+Bx^2+Cx^4)}{a+cx^4} dx$

Optimal result

Integrand size = 31, antiderivative size = 234

$$\int \frac{(d+ex^2)^q(A+Bx^2+Cx^4)}{a+cx^4} dx = -\frac{(\sqrt{-a}B\sqrt{c}-Ac+aC)\,x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},-\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)}{2ac} + \frac{(\sqrt{-a}B\sqrt{c}+Ac-aC)\,x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},\frac{\sqrt{c}x^2}{\sqrt{-a}}\right)}{2ac} + \frac{Cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c}$$

output
```
-1/2*((-a)^(1/2)*B*c^(1/2)-A*c+a*C)*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-c
^(1/2)*x^2/(-a)^(1/2),-e*x^2/d)/a/c/((1+e*x^2/d)^q)+1/2*((-a)^(1/2)*B*c^(1
/2)+A*c-a*C)*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,c^(1/2)*x^2/(-a)^(1/2),-e
*x^2/d)/a/c/((1+e*x^2/d)^q)+C*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x
^2/d)/c/((1+e*x^2/d)^q)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^q(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \int \frac{(d+ex^2)^q(A+Bx^2+Cx^4)}{a+cx^4}\,dx$$

input
```
Integrate[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + c*x^4),x]
```

output
```
Integrate[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + c*x^4), x]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2+Cx^4)(d+ex^2)^q}{a+cx^4}\,dx$$

$$\Big\downarrow \text{2257}$$

$$\int \left(\frac{(d+ex^2)^q(-aC+Ac+Bcx^2)}{c(a+cx^4)}+\frac{C(d+ex^2)^q}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(\sqrt{-a}B\sqrt{c}+aC-Ac\right)\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2ac}+\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(\sqrt{-a}B\sqrt{c}-aC+Ac\right)\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2ac}+\frac{Cx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c}$$

input
```
Int[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + c*x^4),x]
```

output
```
-1/2*((Sqrt[-a]*B*Sqrt[c] - A*c + a*C)*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q
, 3/2, -((Sqrt[c]*x^2)/Sqrt[-a]), -((e*x^2)/d)])/(a*c*(1 + (e*x^2)/d)^q) +
 ((Sqrt[-a]*B*Sqrt[c] + A*c - a*C)*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/
2, (Sqrt[c]*x^2)/Sqrt[-a], -((e*x^2)/d)])/(2*a*c*(1 + (e*x^2)/d)^q) + (C*x
*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(c*(1 + (e*x
^2)/d)^q)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(e\,x^2+d)^q\,(C\,x^4+B\,x^2+A)}{c\,x^4+a}dx$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a),x)
```

output
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q\,(A+Bx^2+Cx^4)}{a+cx^4}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{cx^4+a}\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+cx^4} \, dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**q*(C*x**4+B*x**2+A)/(c*x**4+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+cx^4} \, dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{cx^4+a} \, dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+cx^4} \, dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{cx^4+a} \, dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+cx^4}\,dx=\int \frac{(e\,x^2+d)^q\,(C\,x^4+B\,x^2+A)}{c\,x^4+a}\,dx$$

input
```
int(((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + c*x^4),x)
```

output
```
int(((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + c*x^4), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+cx^4}\,dx$$
$$=\frac{(e\,x^2+d)^q\,x+4\left(\int \frac{(e\,x^2+d)^q}{2ceq\,x^6+ce\,x^6+2cdq\,x^4+cd\,x^4+2aeq\,x^2+ae\,x^2+2adq+ad}dx\right)ad\,q^2+2\left(\int \frac{(e\,x^2+d)^q}{2ceq\,x^6+ce\,x^6+2cdq\,x^4+cd\,x^4+2\cdots}\right.}{}$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a),x)
```

output

```
((d + e*x**2)**q*x + 4*int((d + e*x**2)**q/(2*a*d*q + a*d + 2*a*e*q*x**2 +
 a*e*x**2 + 2*c*d*q*x**4 + c*d*x**4 + 2*c*e*q*x**6 + c*e*x**6),x)*a*d*q**2
 + 2*int((d + e*x**2)**q/(2*a*d*q + a*d + 2*a*e*q*x**2 + a*e*x**2 + 2*c*d*
q*x**4 + c*d*x**4 + 2*c*e*q*x**6 + c*e*x**6),x)*a*d*q + 4*int(((d + e*x**2
)**q*x**4)/(2*a*d*q + a*d + 2*a*e*q*x**2 + a*e*x**2 + 2*c*d*q*x**4 + c*d*x
**4 + 2*c*e*q*x**6 + c*e*x**6),x)*b*e*q**2 + 4*int(((d + e*x**2)**q*x**4)/
(2*a*d*q + a*d + 2*a*e*q*x**2 + a*e*x**2 + 2*c*d*q*x**4 + c*d*x**4 + 2*c*e
*q*x**6 + c*e*x**6),x)*b*e*q + int(((d + e*x**2)**q*x**4)/(2*a*d*q + a*d +
 2*a*e*q*x**2 + a*e*x**2 + 2*c*d*q*x**4 + c*d*x**4 + 2*c*e*q*x**6 + c*e*x*
*6),x)*b*e + 4*int(((d + e*x**2)**q*x**4)/(2*a*d*q + a*d + 2*a*e*q*x**2 +
a*e*x**2 + 2*c*d*q*x**4 + c*d*x**4 + 2*c*e*q*x**6 + c*e*x**6),x)*c*d*q**2
+ 2*int(((d + e*x**2)**q*x**4)/(2*a*d*q + a*d + 2*a*e*q*x**2 + a*e*x**2 +
2*c*d*q*x**4 + c*d*x**4 + 2*c*e*q*x**6 + c*e*x**6),x)*c*d*q + 4*int(((d +
e*x**2)**q*x**2)/(2*a*d*q + a*d + 2*a*e*q*x**2 + a*e*x**2 + 2*c*d*q*x**4 +
 c*d*x**4 + 2*c*e*q*x**6 + c*e*x**6),x)*b*d*q**2 + 4*int(((d + e*x**2)**q*
x**2)/(2*a*d*q + a*d + 2*a*e*q*x**2 + a*e*x**2 + 2*c*d*q*x**4 + c*d*x**4 +
 2*c*e*q*x**6 + c*e*x**6),x)*b*d*q + int(((d + e*x**2)**q*x**2)/(2*a*d*q +
 a*d + 2*a*e*q*x**2 + a*e*x**2 + 2*c*d*q*x**4 + c*d*x**4 + 2*c*e*q*x**6 +
c*e*x**6),x)*b*d)/(2*q + 1)
```

3.130 $\int \frac{(d+ex^2)^q(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 512

$$\int \frac{(d+ex^2)^q(A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx$$

$$= \frac{x(d+ex^2)^{1+q}(Acd-aCd+aBe+(Bcd-Ace+aCe)x^2)}{4a(cd^2+ae^2)(a+cx^4)}$$

$$+ \frac{\left(Ac(3cd^2+ae^2(3-2q)) - \sqrt{-a}\sqrt{c}(B(cd^2+ae^2(1-2q)) - 2(Ac-aC)deq) + a(aCe^2(1+2q)+cd(\right.}{8a^2c(cd^2+ae^2)}$$

$$+ \frac{\left(Ac(3cd^2+ae^2(3-2q)) + \sqrt{-a}\sqrt{c}(B(cd^2+ae^2(1-2q)) - 2(Ac-aC)deq) + a(aCe^2(1+2q)+cd(\right.}{8a^2c(cd^2+ae^2)}$$

$$- \frac{e(Bcd-Ace+aCe)(1+2q)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{4ac(cd^2+ae^2)}$$

output
```
1/4*x*(e*x^2+d)^(1+q)*(A*c*d-C*a*d+B*a*e+(-A*c*e+B*c*d+C*a*e)*x^2)/a/(a*e^
2+c*d^2)/(c*x^4+a)+1/8*(A*c*(3*c*d^2+a*e^2*(3-2*q))-(-a)^(1/2)*c^(1/2)*(B*
(c*d^2+a*e^2*(1-2*q))-2*(A*c-C*a)*d*e*q)+a*(a*C*e^2*(1+2*q)+c*d*(2*B*e*q+C
*d)))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-c^(1/2)*x^2/(-a)^(1/2),-e*x^2/d
)/a^2/c/(a*e^2+c*d^2)/((1+e*x^2/d)^q)+1/8*(A*c*(3*c*d^2+a*e^2*(3-2*q))+(-a
)^(1/2)*c^(1/2)*(B*(c*d^2+a*e^2*(1-2*q))-2*(A*c-C*a)*d*e*q)+a*(a*C*e^2*(1+
2*q)+c*d*(2*B*e*q+C*d)))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,c^(1/2)*x^2/(
-a)^(1/2),-e*x^2/d)/a^2/c/(a*e^2+c*d^2)/((1+e*x^2/d)^q)-1/4*e*(-A*c*e+B*c*
d+C*a*e)*(1+2*q)*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/a/c/(a*
e^2+c*d^2)/((1+e*x^2/d)^q)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+cx^4)^2} \, dx = \int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+cx^4)^2} \, dx$$

input
```
Integrate[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x]
```

output
```
Integrate[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + c*x^4)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2+Cx^4)(d+ex^2)^q}{(a+cx^4)^2} \, dx$$

$$\Big\downarrow \; 2257$$

$$\int \left(\frac{(d+ex^2)^q (-aC+Ac+Bcx^2)}{c(a+cx^4)^2}+\frac{C(d+ex^2)^q}{c(a+cx^4)}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{(Ac-aC)\int \frac{(ex^2+d)^q}{(cx^4+a)^2}dx}{c}+B\int \frac{x^2(ex^2+d)^q}{(cx^4+a)^2}dx+\frac{Cx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2ac}+\frac{Cx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2ac}$$

input
```
Int[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(e\,x^2+d)^q\,(C\,x^4+B\,x^2+A)}{(c\,x^4+a)^2}dx$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x)
```

output
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{(cx^4+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c^2*x^8 + 2*a*c*x^4 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**q*(C*x**4+B*x**2+A)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+cx^4)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{(cx^4+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a)^2, x)
```

Giac [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+cx^4)^2} dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{(cx^4+a)^2} dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+cx^4)^2} dx = \int \frac{(e\,x^2+d)^q\,(C\,x^4+B\,x^2+A)}{(c\,x^4+a)^2} dx$$

input
```
int(((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + c*x^4)^2,x)
```

output
```
int(((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+cx^4)^2} dx = \text{Too large to display}$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+a)^2,x)
```

output

```
( - (d + e*x**2)**q*b*e*x - (d + e*x**2)**q*c*d*x + int((d + e*x**2)**q/(a
**2*d + a**2*e*x**2 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x
**10),x)*a**2*b*d*e + 4*int((d + e*x**2)**q/(a**2*d + a**2*e*x**2 + 2*a*c*
d*x**4 + 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a**2*c*d**2 + int((
d + e*x**2)**q/(a**2*d + a**2*e*x**2 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + c**2*
d*x**8 + c**2*e*x**10),x)*a*b*c*d*e*x**4 + 4*int((d + e*x**2)**q/(a**2*d +
 a**2*e*x**2 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x
)*a*c**2*d**2*x**4 + 2*int(((d + e*x**2)**q*x**6)/(a**2*d + a**2*e*x**2 +
2*a*c*d*x**4 + 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*b*c*e**2*q
 - 3*int(((d + e*x**2)**q*x**6)/(a**2*d + a**2*e*x**2 + 2*a*c*d*x**4 + 2*a*
c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*a*b*c*e**2 + 2*int(((d + e*x**2)
**q*x**6)/(a**2*d + a**2*e*x**2 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + c**2*d*x**
8 + c**2*e*x**10),x)*a*c**2*d*e*q + 2*int(((d + e*x**2)**q*x**6)/(a**2*d +
 a**2*e*x**2 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x
)*b*c**2*e**2*q*x**4 - 3*int(((d + e*x**2)**q*x**6)/(a**2*d + a**2*e*x**2
 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*b*c**2*e**2
*x**4 + 2*int(((d + e*x**2)**q*x**6)/(a**2*d + a**2*e*x**2 + 2*a*c*d*x**4
 + 2*a*c*e*x**6 + c**2*d*x**8 + c**2*e*x**10),x)*c**3*d*e*q*x**4 + 2*int(((
d + e*x**2)**q*x**2)/(a**2*d + a**2*e*x**2 + 2*a*c*d*x**4 + 2*a*c*e*x**6 +
 c**2*d*x**8 + c**2*e*x**10),x)*a**2*b*e**2*q + int(((d + e*x**2)**q*x*...
```

3.131 $\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 261

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx = \frac{a(Bd-Ae)\left(d+ex^2\right)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|2-\frac{bd}{ae}\right)}{d^{3/2}\sqrt{e}(bd-2ae)\sqrt{ad^2+bd^2x^2+e(bd-ae)x^4}} + \frac{(Abd-aBd-aAe)\left(d+ex^2\right)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),2-\frac{bd}{ae}\right)}{d^{3/2}\sqrt{e}(bd-2ae)\sqrt{ad^2+bd^2x^2+e(bd-ae)x^4}}$$

output
```
a*(-A*e+B*d)*(e*x^2+d)*((a*d+(-a*e+b*d)*x^2)/a/(e*x^2+d))^(1/2)*EllipticE(
e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),(2-b*d/a/e)^(1/2))/d^(3/2)/e^(1/2)/(-2
*a*e+b*d)/(a*d^2+b*d^2*x^2+e*(-a*e+b*d)*x^4)^(1/2)+(-A*a*e+A*b*d-B*a*d)*(e
*x^2+d)*((a*d+(-a*e+b*d)*x^2)/a/(e*x^2+d))^(1/2)*InverseJacobiAM(arctan(e^
(1/2)*x/d^(1/2)),(2-b*d/a/e)^(1/2))/d^(3/2)/e^(1/2)/(-2*a*e+b*d)/(a*d^2+b*
d^2*x^2+e*(-a*e+b*d)*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.60 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)\,x^4}}\,dx$$
$$= \frac{\sqrt{\frac{e}{d}}(Bd-Ae)x(bdx^2+a(d-ex^2))+iad(Bd-Ae)\sqrt{1+\frac{bx^2}{a}-\frac{ex^2}{d}}\sqrt{1+\frac{ex^2}{d}}E\left(i\text{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)|-1+\right.}{d^2\sqrt{\frac{e}{d}}(bd-2ae)\sqrt{(d+ex^2)\,(bdx^2}}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)*Sqrt[a*d^2 + b*d^2*x^2 + (b*d*e - a*e^2
)*x^4]),x]
```

output
```
(Sqrt[e/d]*(B*d - A*e)*x*(b*d*x^2 + a*(d - e*x^2)) + I*a*d*(B*d - A*e)*Sqr
t[1 + (b*x^2)/a - (e*x^2)/d]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[
e/d]*x], -1 + (b*d)/(a*e)] - I*A*d*(b*d - 2*a*e)*Sqrt[1 + (b*x^2)/a - (e*x
^2)/d]*Sqrt[1 + (e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[e/d]*x], -1 + (b*d)/(a
*e)])/(d^2*Sqrt[e/d]*(b*d - 2*a*e)*Sqrt[(d + e*x^2)*(b*d*x^2 + a*(d - e*x^
2))])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1395, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{x^4(bde-ae^2)+ad^2+bd^2x^2}}\,dx$$
$$\Big\downarrow \ 1395$$
$$\frac{\sqrt{d+ex^2}\sqrt{x^2(bd-ae)+ad}\int \frac{Bx^2+A}{(ex^2+d)^{3/2}\sqrt{(bd-ae)x^2+ad}}dx}{\sqrt{ex^4(bd-ae)+ad^2+bd^2x^2}}$$

$$\Big\downarrow \text{400}$$

$$\frac{\sqrt{d+ex^2}\sqrt{x^2(bd-ae)+ad}\left(\frac{(-aAe-aBd+Abd)\int \frac{1}{\sqrt{ex^2+d}\sqrt{(bd-ae)x^2+ad}}dx}{d(bd-2ae)}+\frac{(Bd-Ae)\int \frac{\sqrt{(bd-ae)x^2+ad}}{\left(ex^2+d\right)^{3/2}}dx}{d(bd-2ae)}\right)}{\sqrt{ex^4(bd-ae)+ad^2+bd^2x^2}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{d+ex^2}\sqrt{x^2(bd-ae)+ad}\left(\frac{(-aAe-aBd+Abd)\int \frac{1}{\sqrt{ex^2+d}\sqrt{(bd-ae)x^2+ad}}dx}{d(bd-2ae)}+\frac{(Bd-Ae)\sqrt{x^2(bd-ae)+ad}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|2-\frac{bd}{ae}\right)}{d^{3/2}\sqrt{e}\sqrt{d+ex^2}(bd-2ae)\sqrt{\frac{x^2(bd-ae)+ad}{a\left(d+ex^2\right)}}}\right.}{\sqrt{ex^4(bd-ae)+ad^2+bd^2x^2}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{d+ex^2}\sqrt{x^2(bd-ae)+ad}\left(\frac{\sqrt{x^2(bd-ae)+ad}(-aAe-aBd+Abd)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),2-\frac{bd}{ae}\right)}{ad^{3/2}\sqrt{e}\sqrt{d+ex^2}(bd-2ae)\sqrt{\frac{x^2(bd-ae)+ad}{a\left(d+ex^2\right)}}}+\frac{(Bd-Ae)\sqrt{x^2(bd-ae)+ad}E\left(\right.}{d^{3/2}\sqrt{e}\sqrt{d+ex^2}(bd-2ae)}\right.}{\sqrt{ex^4(bd-ae)+ad^2+bd^2x^2}}$$

input

```
Int[(A + B*x^2)/((d + e*x^2)*Sqrt[a*d^2 + b*d^2*x^2 + (b*d*e - a*e^2)*x^4]
),x]
```

output

```
(Sqrt[d + e*x^2]*Sqrt[a*d + (b*d - a*e)*x^2]*(((B*d - A*e)*Sqrt[a*d + (b*d
 - a*e)*x^2]*EllipticE[ArcTan[(Sqrt[e]*x)/Sqrt[d]], 2 - (b*d)/(a*e)])/(d^(
3/2)*Sqrt[e]*(b*d - 2*a*e)*Sqrt[d + e*x^2]*Sqrt[(a*d + (b*d - a*e)*x^2)/(a
*(d + e*x^2))]) + ((A*b*d - a*B*d - a*A*e)*Sqrt[a*d + (b*d - a*e)*x^2]*Ell
ipticF[ArcTan[(Sqrt[e]*x)/Sqrt[d]], 2 - (b*d)/(a*e)])/(a*d^(3/2)*Sqrt[e]*(
b*d - 2*a*e)*Sqrt[d + e*x^2]*Sqrt[(a*d + (b*d - a*e)*x^2)/(a*(d + e*x^2))]
)))/Sqrt[a*d^2 + b*d^2*x^2 + e*(b*d - a*e)*x^4]
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(252) = 504$.

Time = 2.37 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.14

method	result
elliptic	$\frac{(-a\,e^2x^2+bde\,x^2+ade)x(Ae-Bd)}{d^2e(2ae-bd)\sqrt{\left(x^2+\frac{d}{e}\right)(-a\,e^2x^2+bde\,x^2+ade)}}+\frac{\left(\frac{B}{e}+\frac{Ae-Bd}{de}-\frac{a(Ae-Bd)}{d(2ae-bd)}\right)\sqrt{1-\frac{(ae-bd)x^2}{da}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ae-bd}{da}},\sqrt{-\right.}{\sqrt{\frac{ae-bd}{da}}\,\sqrt{-a\,e^2x^4+bde\,x^4+b\,d^2x^2+a\,d^2}}$
default	$\frac{B\sqrt{1-\frac{(ae-bd)x^2}{da}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ae-bd}{da}},\sqrt{-1+\frac{bde}{-a\,e^2+bde}}\right)}{e\sqrt{\frac{ae-bd}{da}}\,\sqrt{-a\,e^2x^4+bde\,x^4+b\,d^2x^2+a\,d^2}}+\frac{(Ae-Bd)\left(\frac{\left(-a\,e^2x^2+bde\,x^2+ade\right)x}{d^2(2ae-bd)\sqrt{\left(x^2+\frac{d}{e}\right)\left(-a\,e^2x^2+bde\,x^2+ade\right)}}+\right.}{}$

input
```
int((B*x^2+A)/(e*x^2+d)/(a*d^2+b*d^2*x^2+(-a*e^2+b*d*e)*x^4)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
(-a*e^2*x^2+b*d*e*x^2+a*d*e)/d^2/e/(2*a*e-b*d)*x*(A*e-B*d)/((x^2+d/e)*(-a*
e^2*x^2+b*d*e*x^2+a*d*e))^(1/2)+(B/e+1/d/e*(A*e-B*d)-a/d/(2*a*e-b*d)*(A*e-
B*d))/(1/d*(a*e-b*d)/a)^(1/2)*(1-1/d*(a*e-b*d)/a*x^2)^(1/2)*(1+e*x^2/d)^(1
/2)/(-a*e^2*x^4+b*d*e*x^4+b*d^2*x^2+a*d^2)^(1/2)*EllipticF(x*(1/d*(a*e-b*d
)/a)^(1/2),(-1+b*d*e/(-a*e^2+b*d*e))^(1/2))-2*(-(a*e-b*d)*(A*e-B*d)/d^2/(2
*a*e-b*d)-1/2*(-2*a*e^2+2*b*d*e)/d^2/e/(2*a*e-b*d)*(A*e-B*d)-1/e*(-a*e^2+b
*d*e)*(A*e-B*d)/d^2/(2*a*e-b*d))*a*d^2/(1/d*(a*e-b*d)/a)^(1/2)*(1-1/d*(a*e
-b*d)/a*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(-a*e^2*x^4+b*d*e*x^4+b*d^2*x^2+a*d^2
)^(1/2)/(b*d^2+d*(2*a*e-b*d))*(EllipticF(x*(1/d*(a*e-b*d)/a)^(1/2),(-1+b*d
*e/(-a*e^2+b*d*e))^(1/2))-EllipticE(x*(1/d*(a*e-b*d)/a)^(1/2),(-1+b*d*e/(-
a*e^2+b*d*e))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(252) = 504$.

Time = 0.10 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.98

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx =$$

$$-\frac{(Bb^2d^4 - Aa^2de^3 - (2\,Bab + Ab^2)d^3e + (Ba^2 + 2\,Aab)d^2e^2 + (Bb^2d^3e - Aa^2e^4 - (2\,Bab + Ab^2)d^2e^2}{}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(a*d^2+b*d^2*x^2+(-a*e^2+b*d*e)*x^4)^(1/2),x
, algorithm="fricas")
```

output
```
-((B*b^2*d^4 - A*a^2*d*e^3 - (2*B*a*b + A*b^2)*d^3*e + (B*a^2 + 2*A*a*b)*d
^2*e^2 + (B*b^2*d^3*e - A*a^2*e^4 - (2*B*a*b + A*b^2)*d^2*e^2 + (B*a^2 + 2
*A*a*b)*d*e^3)*x^2)*sqrt(a*d^2)*sqrt(-(b*d - a*e)/(a*d))*elliptic_e(arcsin
(x*sqrt(-(b*d - a*e)/(a*d))), a*e/(b*d - a*e)) - (B*b^2*d^4 - A*a^2*d*e^3
+ (B*a^2 - (A + 2*B)*a*b - A*b^2)*d^3*e + ((A + B)*a^2 + 2*A*a*b)*d^2*e^2
+ (B*b^2*d^3*e - A*a^2*e^4 + (B*a^2 - (A + 2*B)*a*b - A*b^2)*d^2*e^2 + ((A
 + B)*a^2 + 2*A*a*b)*d*e^3)*x^2)*sqrt(a*d^2)*sqrt(-(b*d - a*e)/(a*d))*elli
ptic_f(arcsin(x*sqrt(-(b*d - a*e)/(a*d))), a*e/(b*d - a*e)) - (B*a*b*d^3*e
 + A*a^2*d*e^3 - (B*a^2 + A*a*b)*d^2*e^2)*sqrt(b*d^2*x^2 + (b*d*e - a*e^2)
*x^4 + a*d^2)*x)/(a*b^2*d^6*e - 3*a^2*b*d^5*e^2 + 2*a^3*d^4*e^3 + (a*b^2*d
^5*e^2 - 3*a^2*b*d^4*e^3 + 2*a^3*d^3*e^4)*x^2)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{-(d+ex^2)(-ad+aex^2-bdx^2)}\,(d+ex^2)}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)/(a*d**2+b*d**2*x**2+(-a*e**2+b*d*e)*x**4)*
*(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(-(d + e*x**2)*(-a*d + a*e*x**2 - b*d*x**2))*(d
 + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(a*d^2+b*d^2*x^2+(-a*e^2+b*d*e)*x^4)^(1/2),x
, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{bd^2x^2+(bde-ae^2)x^4+ad^2}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(a*d^2+b*d^2*x^2+(-a*e^2+b*d*e)*x^4)^(1/2),x
, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(b*d^2*x^2 + (b*d*e - a*e^2)*x^4 + a*d^2)*(e*x^
2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx = \int \frac{B\,x^2+A}{(e\,x^2+d)\,\sqrt{a\,d^2-x^4\,(a\,e^2-b\,d\,e)+b\,d^2\,x^2}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)*(a*d^2 - x^4*(a*e^2 - b*d*e) + b*d^2*x^2)^(1/
2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)*(a*d^2 - x^4*(a*e^2 - b*d*e) + b*d^2*x^2)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}\,x^2}{-a\,e^3x^6+bd\,e^2x^6-ad\,e^2x^4+2b\,d^2e\,x^4+a\,d^2e\,x^2+b\,d^3x^2+a\,d^3}dx\right)b + \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}}{-a\,e^3x^6+bd\,e^2x^6-ad\,e^2x^4+2b\,d^2e\,x^4+a\,d^2e\,x^2+b\,d^3x^2+a\,d^3}dx\right)a$$

input

```
int((B*x^2+A)/(e*x^2+d)/(a*d^2+b*d^2*x^2+(-a*e^2+b*d*e)*x^4)^(1/2),x)
```

output

```
int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*x**2)/(a*d**3 + a*d*
*2*e*x**2 - a*d*e**2*x**4 - a*e**3*x**6 + b*d**3*x**2 + 2*b*d**2*e*x**4 +
b*d*e**2*x**6),x)*b + int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2
))/(a*d**3 + a*d**2*e*x**2 - a*d*e**2*x**4 - a*e**3*x**6 + b*d**3*x**2 + 2
*b*d**2*e*x**4 + b*d*e**2*x**6),x)*a
```

3.132 $\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 292

$$\begin{aligned}\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx &= -\frac{\sqrt{a}B\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sqrt{1+\frac{(bd-ae)x^2}{ad}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-bd+ae}\,x}{\sqrt{a}\sqrt{d}}\right),\frac{ae}{bd-ae}\right)}{e\sqrt{-bd+ae}\sqrt{ad^2+bd^2x^2+e(bd-ae)x^4}}\\&\quad+\frac{\sqrt{a}(Bd+Ae)\sqrt{1+\frac{ex^2}{d}}\sqrt{1+\frac{(bd-ae)x^2}{ad}}\,\text{EllipticPi}\left(-\frac{ae}{bd-ae},\arcsin\left(\frac{\sqrt{-bd+ae}\,x}{\sqrt{a}\sqrt{d}}\right),\frac{ae}{bd-ae}\right)}{\sqrt{d}e\sqrt{-bd+ae}\sqrt{ad^2+bd^2x^2+e(bd-ae)x^4}}\end{aligned}$$

output
```
-a^(1/2)*B*d^(1/2)*(1+e*x^2/d)^(1/2)*(1+(-a*e+b*d)*x^2/a/d)^(1/2)*Elliptic
F((a*e-b*d)^(1/2)*x/a^(1/2)/d^(1/2),(a*e/(-a*e+b*d))^(1/2))/e/(a*e-b*d)^(1
/2)/(a*d^2+b*d^2*x^2+e*(-a*e+b*d)*x^4)^(1/2)+a^(1/2)*(A*e+B*d)*(1+e*x^2/d)
^(1/2)*(1+(-a*e+b*d)*x^2/a/d)^(1/2)*EllipticPi((a*e-b*d)^(1/2)*x/a^(1/2)/d
^(1/2),-a*e/(-a*e+b*d),(a*e/(-a*e+b*d))^(1/2))/d^(1/2)/e/(a*e-b*d)^(1/2)/(
a*d^2+b*d^2*x^2+e*(-a*e+b*d)*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.69 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx$$
$$= \frac{i\sqrt{1+\frac{bx^2}{a}-\frac{ex^2}{d}}\sqrt{1+\frac{ex^2}{d}}\left(Bd\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{e}{d}}x\right),-1+\frac{bd}{ae}\right)-(Bd+Ae)\,\text{EllipticPi}\left(-1,i\text{arcsin}\right.\right.}{d^2\left(\frac{e}{d}\right)^{3/2}\sqrt{(d+ex^2)\left(bdx^2+a(d-ex^2)\right)}}$$

input
```
Integrate[(A + B*x^2)/((d - e*x^2)*Sqrt[a*d^2 + b*d^2*x^2 + (b*d*e - a*e^2
)*x^4]),x]
```

output
```
(I*Sqrt[1 + (b*x^2)/a - (e*x^2)/d]*Sqrt[1 + (e*x^2)/d]*(B*d*EllipticF[I*Ar
cSinh[Sqrt[e/d]*x], -1 + (b*d)/(a*e)] - (B*d + A*e)*EllipticPi[-1, I*ArcSi
nh[Sqrt[e/d]*x], -1 + (b*d)/(a*e)]))/(d^2*(e/d)^(3/2)*Sqrt[(d + e*x^2)*(b*
d*x^2 + a*(d - e*x^2))])
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 721 vs. 2(292) = 584.

Time = 1.26 (sec) , antiderivative size = 721, normalized size of antiderivative = 2.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.078$, Rules used = {2226, 27, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{x^4(bde-ae^2)+ad^2+bd^2x^2}}\,dx$$

$$\Big\downarrow \; 2226$$

$$\frac{\left(A\left(-\sqrt{a}e\sqrt{bd-ae}-ae^{3/2}+bd\sqrt{e}\right)-\sqrt{a}Bd\sqrt{bd-ae}+aBd\sqrt{e}\right)\int \frac{1}{\sqrt{e(bd-ae)x^4+bd^2x^2+ad^2}}dx}{d\sqrt{e}(bd-2ae)}-\frac{\sqrt{a}\left(\sqrt{a}\sqrt{e}-\sqrt{bd-ae}\right)(Ae+Bd)\int \frac{\sqrt{e}\sqrt{bd-ae}x^2+\sqrt{a}d}{\sqrt{a}d(d-ex^2)\sqrt{e(bd-ae)x^4+bd^2x^2+ad^2}}dx}{\sqrt{e}(bd-2ae)}$$

$\Big\downarrow$ 27

$$\frac{\left(A\left(-\sqrt{a}e\sqrt{bd-ae}-ae^{3/2}+bd\sqrt{e}\right)-\sqrt{a}Bd\sqrt{bd-ae}+aBd\sqrt{e}\right)\int \frac{1}{\sqrt{e(bd-ae)x^4+bd^2x^2+ad^2}}dx}{d\sqrt{e}(bd-2ae)}-\frac{\left(\sqrt{a}\sqrt{e}-\sqrt{bd-ae}\right)(Ae+Bd)\int \frac{\sqrt{e}\sqrt{bd-ae}x^2+\sqrt{a}d}{(d-ex^2)\sqrt{e(bd-ae)x^4+bd^2x^2+ad^2}}dx}{d\sqrt{e}(bd-2ae)}$$

$\Big\downarrow$ 1416

$$\frac{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{ex^4(bd-ae)+ad^2+bd^2x^2}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\left(A\left(-\sqrt{a}e\sqrt{bd-ae}-ae^{3/2}+bd\sqrt{e}\right)-\sqrt{a}Bd\sqrt{bd-ae}+aBd\sqrt{e}\right.}{2\sqrt[4]{a}d^{3/2}e^{3/4}(bd-2ae)\sqrt[4]{bd-ae}\sqrt{ex^4(bd-ae)+ad^2+}}$$
$$-\frac{\left(\sqrt{a}\sqrt{e}-\sqrt{bd-ae}\right)(Ae+Bd)\int \frac{\sqrt{e}\sqrt{bd-ae}x^2+\sqrt{a}d}{(d-ex^2)\sqrt{e(bd-ae)x^4+bd^2x^2+ad^2}}dx}{d\sqrt{e}(bd-2ae)}$$

$\Big\downarrow$ 2222

$$\frac{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{ex^4(bd-ae)+ad^2+bd^2x^2}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\left(A\left(-\sqrt{a}e\sqrt{bd-ae}-ae^{3/2}+bd\sqrt{e}\right)-\sqrt{a}Bd\sqrt{bd-ae}+aBd\sqrt{e}\right.}{2\sqrt[4]{a}d^{3/2}e^{3/4}(bd-2ae)\sqrt[4]{bd-ae}\sqrt{ex^4(bd-ae)+ad^2+}}$$
$$\frac{\left(\sqrt{a}\sqrt{e}-\sqrt{bd-ae}\right)(Ae+Bd)\left(\frac{\left(\sqrt{a}\sqrt{e}-\sqrt{bd-ae}\right)\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{ex^4(bd-ae)+ad^2+bd^2x^2}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticPi}\left(\frac{1}{4}\left(\frac{bd}{\sqrt{a}\sqrt{e}\sqrt{bd-ae}}+2\right)\right.}{4\sqrt[4]{a}\sqrt{d}e^{3/4}\sqrt[4]{bd-ae}\sqrt{ex^4(bd-ae)+ad^2+bd}}\right.}{d\sqrt{e}(bd-2ae)}$$

input
```
Int[(A + B*x^2)/((d - e*x^2)*Sqrt[a*d^2 + b*d^2*x^2 + (b*d*e - a*e^2)*x^4]
),x]
```

output
```
((a*B*d*Sqrt[e] - Sqrt[a]*B*d*Sqrt[b*d - a*e] + A*(b*d*Sqrt[e] - a*e^(3/2)
 - Sqrt[a]*e*Sqrt[b*d - a*e]))*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)*S
qrt[(a*d^2 + b*d^2*x^2 + e*(b*d - a*e)*x^4)/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d
- a*e]*x^2)^2]*EllipticF[2*ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*x)/(a^(1/4)*S
qrt[d])], (2 - (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(2*a^(1/4)*d^(
3/2)*e^(3/4)*(b*d - 2*a*e)*(b*d - a*e)^(1/4)*Sqrt[a*d^2 + b*d^2*x^2 + e*(b
*d - a*e)*x^4]) - ((B*d + A*e)*(Sqrt[a]*Sqrt[e] - Sqrt[b*d - a*e])*(((Sqrt
[a]*Sqrt[e] + Sqrt[b*d - a*e])*ArcTanh[(Sqrt[2]*Sqrt[b]*d*x)/Sqrt[a*d^2 +
b*d^2*x^2 + e*(b*d - a*e)*x^4]])/(2*Sqrt[2]*Sqrt[b]*d*Sqrt[e]) + ((Sqrt[a]
*Sqrt[e] - Sqrt[b*d - a*e])*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)*Sqrt
[(a*d^2 + b*d^2*x^2 + e*(b*d - a*e)*x^4)/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a
*e]*x^2)^2]*EllipticPi[(2 + (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4, 2*
ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*x)/(a^(1/4)*Sqrt[d])], (2 - (b*d)/(Sqrt[
a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(4*a^(1/4)*Sqrt[d]*e^(3/4)*(b*d - a*e)^(1
/4)*Sqrt[a*d^2 + b*d^2*x^2 + e*(b*d - a*e)*x^4])))/(d*Sqrt[e]*(b*d - 2*a*e
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.98

method	result
default	$\frac{(Ae+Bd)\sqrt{-\frac{e\,x^2}{d}+1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{ae-bd}{da}},\frac{ae}{ae-bd},\frac{\sqrt{-\frac{e}{d}}}{\sqrt{\frac{ae-bd}{da}}}\right)}{ed\sqrt{\frac{e}{d}-\frac{b}{a}}\,\sqrt{-a\,e^2x^4+bde\,x^4+b\,d^2x^2+a\,d^2}}-\frac{B\sqrt{1-\frac{(ae-bd)x^2}{da}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{ae-\cdots}{da}}\cdots\right.}{e\sqrt{\frac{ae-bd}{da}}\,\sqrt{-a\,e^2x^4+bde\,x^4+b\,d^2\cdots}}$
elliptic	$-\frac{B\sqrt{-\frac{e\,x^2}{d}+1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{ae-bd}{da}},\sqrt{-1+\frac{bde}{-a\,e^2+bde}}\right)}{e\sqrt{\frac{e}{d}-\frac{b}{a}}\,\sqrt{-a\,e^2x^4+bde\,x^4+b\,d^2x^2+a\,d^2}}+\frac{\sqrt{-\frac{e\,x^2}{d}+1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{ae-bd}{da}},\cdots\right.}{d\sqrt{\frac{e}{d}-\frac{b}{a}}\,\sqrt{-a\,e^2x^4+bde\,x^4+b\,d^2x^2+\cdots}}$

input
```
int((B*x^2+A)/(-e*x^2+d)/(a*d^2+b*d^2*x^2+(-a*e^2+b*d*e)*x^4)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
(A*e+B*d)/e/d/(1/d*e-b/a)^(1/2)*(-e*x^2/d+1+b/a*x^2)^(1/2)*(1+e*x^2/d)^(1/
2)/(-a*e^2*x^4+b*d*e*x^4+b*d^2*x^2+a*d^2)^(1/2)*EllipticPi(x*(1/d*(a*e-b*d
)/a)^(1/2),a*e/(a*e-b*d),(-1/d*e)^(1/2)/(1/d*(a*e-b*d)/a)^(1/2))-B/e/(1/d*
(a*e-b*d)/a)^(1/2)*(1-1/d*(a*e-b*d)/a*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(-a*e^2
*x^4+b*d*e*x^4+b*d^2*x^2+a*d^2)^(1/2)*EllipticF(x*(1/d*(a*e-b*d)/a)^(1/2),
(-1+b*d*e/(-a*e^2+b*d*e))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)\,x^4}}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)/(a*d^2+b*d^2*x^2+(-a*e^2+b*d*e)*x^4)^(1/2),
x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\begin{aligned}&\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)\,x^4}}\,dx\\&=-\int \frac{A}{-d\sqrt{ad^2-ae^2x^4+bd^2x^2+bdex^4}+ex^2\sqrt{ad^2-ae^2x^4+bd^2x^2+bdex^4}}\,dx\\&\quad-\int \frac{Bx^2}{-d\sqrt{ad^2-ae^2x^4+bd^2x^2+bdex^4}+ex^2\sqrt{ad^2-ae^2x^4+bd^2x^2+bdex^4}}\,dx\end{aligned}$$

input
```
integrate((B*x**2+A)/(-e*x**2+d)/(a*d**2+b*d**2*x**2+(-a*e**2+b*d*e)*x**4)
**(1/2),x)
```

output
```
-Integral(A/(-d*sqrt(a*d**2 - a*e**2*x**4 + b*d**2*x**2 + b*d*e*x**4) + e*
x**2*sqrt(a*d**2 - a*e**2*x**4 + b*d**2*x**2 + b*d*e*x**4)), x) - Integral
(B*x**2/(-d*sqrt(a*d**2 - a*e**2*x**4 + b*d**2*x**2 + b*d*e*x**4) + e*x**2
*sqrt(a*d**2 - a*e**2*x**4 + b*d**2*x**2 + b*d*e*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx^2}{(d - ex^2)\sqrt{ad^2 + bd^2x^2 + (bde - ae^2)\,x^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)/(a*d^2+b*d^2*x^2+(-a*e^2+b*d*e)*x^4)^(1/2),
x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{A + Bx^2}{(d - ex^2)\sqrt{ad^2 + bd^2x^2 + (bde - ae^2)\,x^4}}\,dx = \int -\frac{Bx^2 + A}{\sqrt{bd^2x^2 + (bde - ae^2)x^4 + ad^2}(ex^2 - d)}\,dx$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)/(a*d^2+b*d^2*x^2+(-a*e^2+b*d*e)*x^4)^(1/2),
x, algorithm="giac")
```

output
```
integrate(-(B*x^2 + A)/(sqrt(b*d^2*x^2 + (b*d*e - a*e^2)*x^4 + a*d^2)*(e*x
^2 - d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx = \int \frac{B\,x^2+A}{\left(d-e\,x^2\right)\,\sqrt{a\,d^2-x^4\,\left(a\,e^2-b\,d\,e\right)+b\,d^2\,x^2}}\,dx$$

input
```
int((A + B*x^2)/((d - e*x^2)*(a*d^2 - x^4*(a*e^2 - b*d*e) + b*d^2*x^2)^(1/
2)),x)
```

output
```
int((A + B*x^2)/((d - e*x^2)*(a*d^2 - x^4*(a*e^2 - b*d*e) + b*d^2*x^2)^(1/
2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx^2}{(d-ex^2)\sqrt{ad^2+bd^2x^2+(bde-ae^2)x^4}}\,dx &= \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}\,x^2}{a\,e^3x^6-bd\,e^2x^6-ad\,e^2x^4-a\,d^2e\,x^2+b\,d^3x^2+a\,d^3}dx\right)b \\ &\quad+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}}{a\,e^3x^6-bd\,e^2x^6-ad\,e^2x^4-a\,d^2e\,x^2+b\,d^3x^2+a\,d^3}dx\right)a\end{aligned}$$

input
```
int((B*x^2+A)/(-e*x^2+d)/(a*d^2+b*d^2*x^2+(-a*e^2+b*d*e)*x^4)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*x**2)/(a*d**3 - a*d*
*2*e*x**2 - a*d*e**2*x**4 + a*e**3*x**6 + b*d**3*x**2 - b*d*e**2*x**6),x)*
b + int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2))/(a*d**3 - a*d**
2*e*x**2 - a*d*e**2*x**4 + a*e**3*x**6 + b*d**3*x**2 - b*d*e**2*x**6),x)*a
```

3.133 $\int \frac{\sqrt{2+3x^2+x^4}}{\left(1+x^2\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(1+x^2\right)^2} \, dx=\frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{2+3x^2+x^4}}$$

output
```
2^(1/2)*(x^2+1)*((x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1
/2))/(x^4+3*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(1+x^2\right)^2} \, dx =\frac{2x+x^3+i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[Sqrt[2 + 3*x^2 + x^4]/(1 + x^2)^2,x]
```

output
```
(2*x + x^3 + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]],
 2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sq
rt[2 + 3*x^2 + x^4]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1395, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4+3x^2+2}}{\left(x^2+1\right)^2}\,dx$$

$$\Big\downarrow \text{ 1395}$$

$$\frac{\sqrt{x^4+3x^2+2}\int \frac{\sqrt{x^2+2}}{\left(x^2+1\right)^{3/2}}dx}{\sqrt{x^2+1}\sqrt{x^2+2}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{2}\sqrt{x^4+3x^2+2}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\left(x^2+1\right)\sqrt{\frac{x^2+2}{x^2+1}}}$$

input
```
Int[Sqrt[2 + 3*x^2 + x^4]/(1 + x^2)^2,x]
```

output
```
(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]*EllipticE[ArcTan[x], 1/2])/((1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

method	result	size
risch	$\frac{x\left(x^{2}+2\right)}{\sqrt{x^{4}+3 x^{2}+2}}-\frac{i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(\frac{i x \sqrt{2}}{2}, \sqrt{2}\right)-\operatorname{EllipticE}\left(\frac{i x \sqrt{2}}{2}, \sqrt{2}\right)\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}$	80
default	$\frac{\left(x^{2}+2\right) x}{\sqrt{\left(x^{2}+2\right)\left(x^{2}+1\right)}}-\frac{i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(\frac{i x \sqrt{2}}{2}, \sqrt{2}\right)-\operatorname{EllipticE}\left(\frac{i x \sqrt{2}}{2}, \sqrt{2}\right)\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}$	81
elliptic	$\frac{\left(x^{2}+2\right) x}{\sqrt{\left(x^{2}+2\right)\left(x^{2}+1\right)}}-\frac{i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(\frac{i x \sqrt{2}}{2}, \sqrt{2}\right)-\operatorname{EllipticE}\left(\frac{i x \sqrt{2}}{2}, \sqrt{2}\right)\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}$	81

input
```
int((x^4+3*x^2+2)^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/
(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*
2^(1/2),2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{2+3x^2+x^4}}{(1+x^2)^2}\,dx =$$
$$-\frac{\sqrt{2}\sqrt{-\frac{1}{2}}(x^2+1)E(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)\mid 2) - \sqrt{2}\sqrt{-\frac{1}{2}}(x^2+1)F(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)\mid 2) - 2\sqrt{x^4+3x^2+}}{2(x^2+1)}$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*sqrt(-1/2)*(x^2 + 1)*elliptic_e(arcsin(sqrt(-1/2)*x), 2) - s
qrt(2)*sqrt(-1/2)*(x^2 + 1)*elliptic_f(arcsin(sqrt(-1/2)*x), 2) - 2*sqrt(x
^4 + 3*x^2 + 2)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{(x^2+1)(x^2+2)}}{(x^2+1)^2}\,dx$$

input
```
integrate((x**4+3*x**2+2)**(1/2)/(x**2+1)**2,x)
```

output
```
Integral(sqrt((x**2 + 1)*(x**2 + 2))/(x**2 + 1)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4+3x^2+2}}{(x^2+1)^2}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)/(x^2 + 1)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{(x^2+1)^2}\,dx$$

input
```
integrate((x^4+3*x^2+2)^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 + 3*x^2 + 2)/(x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+3x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4+3\,x^2+2}}{(x^2+1)^2}\,dx$$

input
```
int((3*x^2 + x^4 + 2)^(1/2)/(x^2 + 1)^2,x)
```

output
```
int((3*x^2 + x^4 + 2)^(1/2)/(x^2 + 1)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{2+3x^2+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4+3x^2+2}}{x^4+2x^2+1}dx$$

input
```
int((x^4+3*x^2+2)^(1/2)/(x^2+1)^2,x)
```

output
```
int(sqrt(x**4 + 3*x**2 + 2)/(x**4 + 2*x**2 + 1),x)
```

3.134 $\int \frac{(2+x^2)^2}{(2+3x^2+x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{(2+x^2)^2}{(2+3x^2+x^4)^{3/2}} \, dx = \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}$$

output
```
2^(1/2)*(x^2+1)*((x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1
/2))/(x^4+3*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int \frac{(2+x^2)^2}{(2+3x^2+x^4)^{3/2}} \, dx = \frac{2x+x^3+i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|\,2\right)-i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}}{\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(2 + x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(2*x + x^3 + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]],
 2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sq
rt[2 + 3*x^2 + x^4]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1395, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+2)^2}{(x^4+3x^2+2)^{3/2}}\,dx$$
$$\Big\downarrow \; 1395$$
$$\frac{\sqrt{x^2+1}\sqrt{x^2+2}\int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}}dx}{\sqrt{x^4+3x^2+2}}$$
$$\Big\downarrow \; 313$$
$$\frac{\sqrt{2}(x^2+2)\,E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

input
```
Int[(2 + x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

output
```
(Sqrt[2]*(2 + x^2)*EllipticE[ArcTan[x], 1/2])/(Sqrt[(2 + x^2)/(1 + x^2)]*S
qrt[2 + 3*x^2 + x^4])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
 + e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
 + q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

method	result	s
risch	$\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	8
elliptic	$\frac{(x^2+2)x}{\sqrt{(x^2+2)(x^2+1)}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	8
default	$-\frac{2\left(-\frac{3}{2}x^3-2x\right)}{\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}-\frac{8\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{8\left(x^3+\frac{3}{2}x\right)}{\sqrt{x^4+3x^2+2}}$	1

input
```
int((x^2+2)^2/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/
(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*
2^(1/2),2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{(2+x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{-\frac{1}{2}}(x^2+1)E(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)|2) - \sqrt{2}\sqrt{-\frac{1}{2}}(x^2+1)F(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)|2) - 2\sqrt{x^4+3x^2+2}x}{2(x^2+1)}$$

input
```
integrate((x^2+2)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*sqrt(-1/2)*(x^2 + 1)*elliptic_e(arcsin(sqrt(-1/2)*x), 2) - s
qrt(2)*sqrt(-1/2)*(x^2 + 1)*elliptic_f(arcsin(sqrt(-1/2)*x), 2) - 2*sqrt(x
^4 + 3*x^2 + 2)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{(2+x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+2)^2}{((x^2+1)(x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((x**2+2)**2/(x**4+3*x**2+2)**(3/2),x)
```

output
```
Integral((x**2 + 2)**2/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(2+x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx = \int \frac{(x^2+2)^2}{(x^4+3x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+2)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 2)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(2+x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx=\int \frac{(x^2+2)^2}{(x^4+3\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+2)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 2)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx=\int \frac{(x^2+2)^2}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

input
```
int((x^2 + 2)^2/(3*x^2 + x^4 + 2)^(3/2),x)
```

output
```
int((x^2 + 2)^2/(3*x^2 + x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(2+x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx=\int \frac{\sqrt{x^4+3x^2+2}}{x^4+2x^2+1}dx$$

input
```
int((x^2+2)^2/(x^4+3*x^2+2)^(3/2),x)
```

output
```
int(sqrt(x**4 + 3*x**2 + 2)/(x**4 + 2*x**2 + 1),x)
```

3.135 $\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 48

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}$$

output
```
2^(1/2)*(x^2+1)*((x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1
/2))/(x^4+3*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{2x+x^3+i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|\,2\right)-i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(2 + x^2)/((1 + x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
(2*x + x^3 + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]],
 2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sq
rt[2 + 3*x^2 + x^4]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1395, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2}{(x^2+1)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \ 1395$$

$$\frac{\sqrt{x^2+1}\sqrt{x^2+2}\int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}}dx}{\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \ 313$$

$$\frac{\sqrt{2}(x^2+2)\,E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

input
```
Int[(2 + x^2)/((1 + x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
(Sqrt[2]*(2 + x^2)*EllipticE[ArcTan[x], 1/2])/(Sqrt[(2 + x^2)/(1 + x^2)]*S
qrt[2 + 3*x^2 + x^4])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

method	result	size
risch	$\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}} - \frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-\operatorname{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	80
default	$\frac{(x^2+2)x}{\sqrt{(x^2+2)(x^2+1)}} - \frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-\operatorname{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	81
elliptic	$\frac{(x^2+2)x}{\sqrt{(x^2+2)(x^2+1)}} - \frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-\operatorname{EllipticE}\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	81

input
```
int((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/
(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))-EllipticE(1/2*I*x*
2^(1/2),2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = -\frac{\sqrt{2}\sqrt{-\frac{1}{2}}(x^2+1)E(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)|2) - \sqrt{2}\sqrt{-\frac{1}{2}}(x^2+1)F(\arcsin\left(\sqrt{-\frac{1}{2}}x\right)|2) - 2\sqrt{x^4+3x^2+}}{2(x^2+1)}$$

input
```
integrate((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*sqrt(-1/2)*(x^2 + 1)*elliptic_e(arcsin(sqrt(-1/2)*x), 2) - s
qrt(2)*sqrt(-1/2)*(x^2 + 1)*elliptic_f(arcsin(sqrt(-1/2)*x), 2) - 2*sqrt(x
^4 + 3*x^2 + 2)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{x^2+2}{\sqrt{(x^2+1)(x^2+2)}(x^2+1)}\,dx$$

input
```
integrate((x**2+2)/(x**2+1)/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral((x**2 + 2)/(sqrt((x**2 + 1)*(x**2 + 2))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{x^2+2}{\sqrt{x^4+3x^2+2}(x^2+1)}\,dx$$

input
```
integrate((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 + 2)/(sqrt(x^4 + 3*x^2 + 2)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{x^2+2}{\sqrt{x^4+3\,x^2+2}(x^2+1)}\,dx$$

input
```
integrate((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 + 2)/(sqrt(x^4 + 3*x^2 + 2)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{x^2+2}{(x^2+1)\,\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int((x^2 + 2)/((x^2 + 1)*(3*x^2 + x^4 + 2)^(1/2)),x)
```

output
```
int((x^2 + 2)/((x^2 + 1)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{\sqrt{x^4+3x^2+2}}{x^4+2x^2+1}dx$$

input
```
int((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x)
```

output
```
int(sqrt(x**4 + 3*x**2 + 2)/(x**4 + 2*x**2 + 1),x)
```

3.136 $\int \frac{4+x^2-2x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 67

$$\int \frac{4+x^2-2x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx = 3\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}$$

output
```
3*arctan(x/(x^4+x^2+1)^(1/2))+1/2*(x^2+1)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)*El
lipticE(sin(2*arctan(x)),1/2)/(x^4+x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.60 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.37

$$\int \frac{4+x^2-2x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx = \frac{\frac{x+x^3+x^5}{1+x^2} - 5(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left((-1)^{5/6}x\right),(-1)^{2/3}\right) + \sqrt[3]{-1}\sqrt{1+}}{}$$

input
```
Integrate[(4 + x^2 - 2*x^4)/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

output
```
((x + x^3 + x^5)/(1 + x^2) - 5*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1
- (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1
/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSin
h[(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/
3)]) + 12*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*Ell
ipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(2*Sqrt[1 + x^2
+ x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2210, 25, 2230, 27, 1509, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^4+x^2+4}{(x^2+1)^2\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{2210}$$

$$\frac{x\sqrt{x^4+x^2+1}}{2(x^2+1)}-\frac{1}{2}\int -\frac{-x^4-6x^2+7}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{-x^4-6x^2+7}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{2230}$$

$$\frac{1}{2}\left(\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx+\int \frac{6(1-x^2)}{(x^2+1)\sqrt{x^4+x^2+1}}dx\right)+\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx+6\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx\right)+\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{2}\left(6\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{\sqrt{x^4+x^2+1}x}{x^2+1}\right)+\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{2212}$$

$$\frac{1}{2}\left(6\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{\sqrt{x^4+x^2+1}x}{x^2+1}\right)+\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(6\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{\sqrt{x^4+x^2+1}x}{x^2+1}\right)+\frac{\sqrt{x^4+x^2+1}x}{2(x^2+1)}$$

input
```
Int[(4 + x^2 - 2*x^4)/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

output
```
(x*Sqrt[1 + x^2 + x^4])/(2*(1 + x^2)) + (-((x*Sqrt[1 + x^2 + x^4])/(1 + x^
2)) + 6*ArcTan[x/Sqrt[1 + x^2 + x^4]] + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1
 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2230
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-C/e^2   Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]
 + Simp[1/e^2   Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2
 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && Eq
Q[c*d^2 - a*e^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 329, normalized size of antiderivative = 4.91

method	result
risch	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-\operatorname{EllipticE}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+}}{2}\right.\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{5\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4}}$
default	$-\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}}}{\sqrt{}}$

input
```
int((-2*x^4+x^2+4)/(x^2+1)^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)+2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^
(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+
I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1
/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))-5
/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*
3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/
2),1/2*(-2+2*I*3^(1/2))^(1/2))+6/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2
*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2
)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/
2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(58) = 116$.

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.90

$$\int \frac{4+x^2-2x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx = -\frac{2\sqrt{-3}(x^2+1)\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}F\left(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}\right) \mid \frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)+\left(x^2-\sqrt{-3}(x^2+1)+1\right)\sqrt{\frac{1}{2}}}{4\,(x}$$

input
```
integrate((-2*x^4+x^2+4)/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/4*(2*sqrt(-3)*(x^2 + 1)*sqrt(1/2*sqrt(-3) - 1/2)*elliptic_f(arcsin(x*sq
rt(1/2*sqrt(-3) - 1/2)), 1/2*sqrt(-3) - 1/2) + (x^2 - sqrt(-3)*(x^2 + 1) +
 1)*sqrt(1/2*sqrt(-3) - 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-3) - 1/2))
, 1/2*sqrt(-3) - 1/2) - 12*(x^2 + 1)*arctan(x/sqrt(x^4 + x^2 + 1)) - 2*sqr
t(x^4 + x^2 + 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\begin{aligned}
&\int \frac{4+x^2-2x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx \\
&= -\int\left(-\frac{x^2}{x^4\sqrt{x^4+x^2+1}+2x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\right)dx \\
&\quad -\int \frac{2x^4}{x^4\sqrt{x^4+x^2+1}+2x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\,dx \\
&\quad -\int\left(-\frac{4}{x^4\sqrt{x^4+x^2+1}+2x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\right)dx
\end{aligned}$$

input
```
integrate((-2*x**4+x**2+4)/(x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

output
```
-Integral(-x**2/(x**4*sqrt(x**4 + x**2 + 1) + 2*x**2*sqrt(x**4 + x**2 + 1)
 + sqrt(x**4 + x**2 + 1)), x) - Integral(2*x**4/(x**4*sqrt(x**4 + x**2 + 1
) + 2*x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x) - Integral(-
4/(x**4*sqrt(x**4 + x**2 + 1) + 2*x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 +
 x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{4+x^2-2x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx = \int -\frac{2\,x^4-x^2-4}{\sqrt{x^4+x^2+1}(x^2+1)^2}\,dx$$

input
```
integrate((-2*x^4+x^2+4)/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima"
)
```

output
```
-integrate((2*x^4 - x^2 - 4)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

Giac [F]

$$\int \frac{4+x^2-2x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx = \int -\frac{2\,x^4-x^2-4}{\sqrt{x^4+x^2+1}(x^2+1)^2}\,dx$$

input
```
integrate((-2*x^4+x^2+4)/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(2*x^4 - x^2 - 4)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+x^2-2x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx=\int \frac{-2\,x^4+x^2+4}{(x^2+1)^2\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((x^2 - 2*x^4 + 4)/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((x^2 - 2*x^4 + 4)/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{4+x^2-2x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx=4\left(\int \frac{\sqrt{x^4+x^2+1}}{x^8+3x^6+4x^4+3x^2+1}dx\right)-2\left(\int \frac{\sqrt{x^4+x^2+1}\,x^4}{x^8+3x^6+4x^4+3x^2+1}dx\right)+\int \frac{\sqrt{x^4+x^2+1}\,x^2}{x^8+3x^6+4x^4+3x^2+1}dx$$

input
```
int((-2*x^4+x^2+4)/(x^2+1)^2/(x^4+x^2+1)^(1/2),x)
```

output
```
4*int(sqrt(x**4 + x**2 + 1)/(x**8 + 3*x**6 + 4*x**4 + 3*x**2 + 1),x) - 2*i
nt((sqrt(x**4 + x**2 + 1)*x**4)/(x**8 + 3*x**6 + 4*x**4 + 3*x**2 + 1),x) +
 int((sqrt(x**4 + x**2 + 1)*x**2)/(x**8 + 3*x**6 + 4*x**4 + 3*x**2 + 1),x)
```

3.137 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 108

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{Bx\sqrt{ad+(bd+ae)x^2+bex^4}}{2b\sqrt{d+ex^2}} + \frac{(2Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\right)}{2b^{3/2}}$$

output
```
1/2*B*x*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/b/(e*x^2+d)^(1/2)+1/2*(2*A*b-B*a
)*arctanh(b^(1/2)*x*(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2))/b^(
3/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{\sqrt{d+ex^2}\left(\sqrt{b}Bx(a+bx^2)+2(2Ab-aB)\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)\right)}{2b^{3/2}\sqrt{(a+bx^2)(d+ex^2)}}$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2])/Sqrt[a*d + (b*d + a*e)*x^2 + b*e*x
^4],x]
```

output
```
(Sqrt[d + e*x^2]*(Sqrt[b]*B*x*(a + b*x^2) + 2*(2*A*b - a*B)*Sqrt[a + b*x^2
]*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])]))/(2*b^(3/2)*Sqrt[(a +
 b*x^2)*(d + e*x^2)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1395, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{x^2(ae+bd)+ad+bex^4}}dx$$

$$\Big\downarrow \text{1395}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \frac{Bx^2+A}{\sqrt{bx^2+a}}dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{(2Ab-aB)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{Bx\sqrt{a+bx^2}}{2b}\right)}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{(2Ab-aB)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{Bx\sqrt{a+bx^2}}{2b}\right)}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{(2Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}\right)}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2])/Sqrt[a*d + (b*d + a*e)*x^2 + b*e*x^4],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[d + e*x^2]*((B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b -
a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/Sqrt[a*d + (b*d +
 a*e)*x^2 + b*e*x^4]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\sqrt{(e\,x^2+d)(b\,x^2+a)}\,\left(Bx\sqrt{b}\,\sqrt{b\,x^2+a}+2A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)b-B\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)a\right)}{2b^{\frac{3}{2}}\sqrt{e\,x^2+d}\,\sqrt{b\,x^2+a}}$	97
risch	$\frac{Bx(b\,x^2+a)\sqrt{e\,x^2+d}}{2b\sqrt{(e\,x^2+d)(b\,x^2+a)}}+\frac{(2Ab-Ba)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)\sqrt{b\,x^2+a}\,\sqrt{e\,x^2+d}}{2b^{\frac{3}{2}}\sqrt{(e\,x^2+d)(b\,x^2+a)}}$	107

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
1/2*((e*x^2+d)*(b*x^2+a))^(1/2)/b^(3/2)*(B*x*b^(1/2)*(b*x^2+a)^(1/2)+2*A*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))*b-B*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*a)/(e*x^2+d
)^(1/2)/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.62

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$$
$$= \left[\frac{2\sqrt{bex^4+(bd+ae)x^2+ad}\sqrt{ex^2+d}Bbx-((Ba-2\,Ab)ex^2+(Ba-2\,Ab)d)\sqrt{b}\log\left(\frac{2\,bex^4+(2\,bd+ae)x^2}{}\right.}{4\,(b^2ex^2+b^2d)}\right.$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x, a
lgorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(e*x^2 + d)*B*b*x - ((B*
a - 2*A*b)*e*x^2 + (B*a - 2*A*b)*d)*sqrt(b)*log((2*b*e*x^4 + (2*b*d + a*e)
*x^2 + 2*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(e*x^2 + d)*sqrt(b)*x +
 a*d)/(e*x^2 + d)))/(b^2*e*x^2 + b^2*d), 1/2*(sqrt(b*e*x^4 + (b*d + a*e)*x
^2 + a*d)*sqrt(e*x^2 + d)*B*b*x + ((B*a - 2*A*b)*e*x^2 + (B*a - 2*A*b)*d)*
sqrt(-b)*arctan(sqrt(e*x^2 + d)*sqrt(-b)*x/sqrt(b*e*x^4 + (b*d + a*e)*x^2
+ a*d)))/(b^2*e*x^2 + b^2*d)]
```

Sympy [F]

$$\int \frac{(A + Bx^2)\sqrt{d + ex^2}}{\sqrt{ad + (bd + ae)x^2 + bex^4}}\,dx = \int \frac{(A + Bx^2)\sqrt{d + ex^2}}{\sqrt{(a + bx^2)(d + ex^2)}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)/(a*d+(a*e+b*d)*x**2+b*e*x**4)**(1/2
),x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)/sqrt((a + b*x**2)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{(A + Bx^2)\sqrt{d + ex^2}}{\sqrt{ad + (bd + ae)x^2 + bex^4}}\,dx = \int \frac{(Bx^2 + A)\sqrt{ex^2 + d}}{\sqrt{bex^4 + (bd + ae)x^2 + ad}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d
), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{\sqrt{bex^4+(bd+ae)x^2+ad}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}}{\sqrt{b\,e\,x^4+(a\,e+b\,d)\,x^2+a\,d}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a*d + x^2*(a*e + b*d) + b*e*x^4)^(1/2
),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a*d + x^2*(a*e + b*d) + b*e*x^4)^(1/2
), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.37

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{\sqrt{b\,x^2+a}\,bx+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a}{2b}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x)
```

output

```
(sqrt(a + b*x**2)*b*x + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a)/(2*b)
```

3.138 $\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{de+(d^2+e^2)x^2+dex^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 143

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx = \frac{B\operatorname{arctanh}\left(\frac{\sqrt{d}x\sqrt{d+ex^2}}{\sqrt{de+(d^2+e^2)x^2+dex^4}}\right)}{\sqrt{d}e} - \frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2}x\sqrt{d+ex^2}}{\sqrt{d}\sqrt{de+(d^2+e^2)x^2+dex^4}}\right)}{\sqrt{d}e\sqrt{d^2-e^2}}$$

output
```
B*arctanh(d^(1/2)*x*(e*x^2+d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2))/d^(
1/2)/e-(-A*e+B*d)*arctanh((d^2-e^2)^(1/2)*x*(e*x^2+d)^(1/2)/d^(1/2)/(d*e+(
d^2+e^2)*x^2+d*e*x^4)^(1/2))/d^(1/2)/e/(d^2-e^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx = \frac{\sqrt{e+dx^2}\sqrt{d+ex^2}\left((Bd-Ae)\sqrt{-d^2+e^2}\arctan\left(\frac{d^{3/2}+\sqrt{d}ex^2-ex\sqrt{e+dx^2}}{\sqrt{d}\sqrt{-d^2+e^2}}\right)+B(d^2-e^2)\log\left(-\sqrt{d}x+\sqrt{e}\right.\right.}{\sqrt{d}e(-d+e)(d+e)\sqrt{d^2x^2+e^2x^2+de\,(1+x^4)}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[d*e + (d^2 + e^2)*x^2 + d*e*x^
4]),x]
```

output
```
(Sqrt[e + d*x^2]*Sqrt[d + e*x^2]*((B*d - A*e)*Sqrt[-d^2 + e^2]*ArcTan[(d^(
3/2) + Sqrt[d]*e*x^2 - e*x*Sqrt[e + d*x^2])/(Sqrt[d]*Sqrt[-d^2 + e^2])] +
B*(d^2 - e^2)*Log[-(Sqrt[d]*x) + Sqrt[e + d*x^2]]))/(Sqrt[d]*e*(-d + e)*(d
 + e)*Sqrt[d^2*x^2 + e^2*x^2 + d*e*(1 + x^4)])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1395, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{x^2\left(d^2+e^2\right)+dex^4+de}}dx$$

$$\Big\downarrow \ 1395$$

$$\frac{\sqrt{dx^2+e}\sqrt{d+ex^2}\int \frac{Bx^2+A}{\sqrt{dx^2+e}(ex^2+d)}dx}{\sqrt{x^2\left(d^2+e^2\right)+dex^4+de}}$$

$$\Big\downarrow \ 398$$

$$\frac{\sqrt{dx^2+e}\sqrt{d+ex^2}\left(\frac{B\int \frac{1}{\sqrt{dx^2+e}}dx}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{dx^2+e}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{x^2\left(d^2+e^2\right)+dex^4+de}}$$

$$\Big\downarrow \ 224$$

$$\frac{\sqrt{dx^2+e}\sqrt{d+ex^2}\left(\frac{B\int \frac{1}{1-\frac{dx^2}{dx^2+e}}d\frac{x}{\sqrt{dx^2+e}}}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{dx^2+e}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{x^2\left(d^2+e^2\right)+dex^4+de}}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{dx^2+e}\sqrt{d+ex^2}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2+e}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\int\frac{1}{\sqrt{dx^2+e}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{x^2\left(d^2+e^2\right)+dex^4+de}}$$

$$\Big\downarrow \ 291$$

$$\frac{\sqrt{dx^2+e}\sqrt{d+ex^2}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2+e}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\int\frac{1}{d-\frac{\left(d^2-e^2\right)x^2}{dx^2+e}}d\frac{x}{\sqrt{dx^2+e}}}{e}\right)}{\sqrt{x^2\left(d^2+e^2\right)+dex^4+de}}$$

$$\Big\downarrow \ 221$$

$$\frac{\sqrt{dx^2+e}\sqrt{d+ex^2}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2+e}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{x\sqrt{d^2-e^2}}{\sqrt{d}\sqrt{dx^2+e}}\right)}{\sqrt{d}e\sqrt{d^2-e^2}}\right)}{\sqrt{x^2\left(d^2+e^2\right)+dex^4+de}}$$

input
```
Int[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[d*e + (d^2 + e^2)*x^2 + d*e*x^4]),x]
```

output
```
(Sqrt[e + d*x^2]*Sqrt[d + e*x^2]*((B*ArcTanh[(Sqrt[d]*x)/Sqrt[e + d*x^2]])
/(Sqrt[d]*e) - ((B*d - A*e)*ArcTanh[(Sqrt[d^2 - e^2]*x)/(Sqrt[d]*Sqrt[e +
d*x^2])])/(Sqrt[d]*e*Sqrt[d^2 - e^2])))/Sqrt[d*e + (d^2 + e^2)*x^2 + d*e*x
^4]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(123) = 246$.

Time = 0.40 (sec) , antiderivative size = 842, normalized size of antiderivative = 5.89

method	result
default	$-\frac{\sqrt{d e\,x^4+d^2x^2+e^2x^2+d e}\left(2 A d\sqrt{-\frac{d^2-e^2}{e}}\,\sqrt{-d e}\,\ln\left(\frac{\sqrt{d}\,\sqrt{d\,x^2+e}+d x}{\sqrt{d}}\right)e-2 B\,d^2\sqrt{-\frac{d^2-e^2}{e}}\,\sqrt{-d e}\,\ln\left(\frac{\sqrt{d}\,\sqrt{d\,x^2+e}+d x}{\sqrt{d}}\right)+A\,d^{\frac{5}{2}}\ln\right.}{}$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
-1/2/(e*x^2+d)^(1/2)*(d*e*x^4+d^2*x^2+e^2*x^2+d*e)^(1/2)/d^(1/2)*(2*A*d*(-
(d^2-e^2)/e)^(1/2)*(-d*e)^(1/2)*ln((d^(1/2)*(d*x^2+e)^(1/2)+d*x)/d^(1/2))*
e-2*B*d^2*(-(d^2-e^2)/e)^(1/2)*(-d*e)^(1/2)*ln((d^(1/2)*(d*x^2+e)^(1/2)+d*
x)/d^(1/2))+A*d^(5/2)*ln(2*((-(d^2-e^2)/e)^(1/2)*(d*x^2+e)^(1/2)*e+(-d*e)^
(1/2)*d*x+e^2)/(e*x-(-d*e)^(1/2)))*e-A*d^(1/2)*ln(2*((-(d^2-e^2)/e)^(1/2)*
(d*x^2+e)^(1/2)*e+(-d*e)^(1/2)*d*x+e^2)/(e*x-(-d*e)^(1/2)))*e^3-A*d^(5/2)*
ln(-2*((-d*e)^(1/2)*d*x-(-(d^2-e^2)/e)^(1/2)*(d*x^2+e)^(1/2)*e-e^2)/(e*x+(
-d*e)^(1/2)))*e+A*d^(1/2)*ln(-2*((-d*e)^(1/2)*d*x-(-(d^2-e^2)/e)^(1/2)*(d*
x^2+e)^(1/2)*e-e^2)/(e*x+(-d*e)^(1/2)))*e^3-2*A*(-(d^2-e^2)/e)^(1/2)*(-d*e
)^(1/2)*ln((d^(1/2)*(-1/d*(-d*x+(-d*e)^(1/2))*(d*x+(-d*e)^(1/2)))^(1/2)+d*
x)/d^(1/2))*d*e-B*d^(7/2)*ln(2*((-(d^2-e^2)/e)^(1/2)*(d*x^2+e)^(1/2)*e+(-d
*e)^(1/2)*d*x+e^2)/(e*x-(-d*e)^(1/2)))+B*d^(3/2)*ln(2*((-(d^2-e^2)/e)^(1/2
)*(d*x^2+e)^(1/2)*e+(-d*e)^(1/2)*d*x+e^2)/(e*x-(-d*e)^(1/2)))*e^2+B*d^(7/2
)*ln(-2*((-d*e)^(1/2)*d*x-(-(d^2-e^2)/e)^(1/2)*(d*x^2+e)^(1/2)*e-e^2)/(e*x
+(-d*e)^(1/2)))-B*d^(3/2)*ln(-2*((-d*e)^(1/2)*d*x-(-(d^2-e^2)/e)^(1/2)*(d*
x^2+e)^(1/2)*e-e^2)/(e*x+(-d*e)^(1/2)))*e^2+2*B*(-(d^2-e^2)/e)^(1/2)*(-d*e
)^(1/2)*ln((d^(1/2)*(-1/d*(-d*x+(-d*e)^(1/2))*(d*x+(-d*e)^(1/2)))^(1/2)+d*
x)/d^(1/2))*e^2)/(d*x^2+e)^(1/2)/e/(-d*e)^(1/2)/(d+e)/(d-e)/(-(d^2-e^2)/e)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 804, normalized size of antiderivative = 5.62

$$\int \frac{A + Bx^2}{\sqrt{d + ex^2}\sqrt{de + (d^2 + e^2)\, x^2 + dex^4}}\, dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2),x, a
lgorithm="fricas")
```

output
```
[-1/2*(sqrt(d^3 - d*e^2)*(B*d - A*e)*log((2*d^3*x^2 + (2*d^2*e - e^3)*x^4
+ d^2*e + 2*sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)*sqrt(d^3 - d*e^2)*sqrt(e
*x^2 + d)*x)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (B*d^2 - B*e^2)*sqrt(d)*log((2
*d*e*x^4 + (2*d^2 + e^2)*x^2 + 2*sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)*sqr
t(e*x^2 + d)*sqrt(d)*x + d*e)/(e*x^2 + d)))/(d^3*e - d*e^3), 1/2*(2*sqrt(-
d^3 + d*e^2)*(B*d - A*e)*arctan(sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)*sqrt
(-d^3 + d*e^2)*sqrt(e*x^2 + d)*x/(d^2*e*x^4 + d^2*e + (d^3 + d*e^2)*x^2))
+ (B*d^2 - B*e^2)*sqrt(d)*log((2*d*e*x^4 + (2*d^2 + e^2)*x^2 + 2*sqrt(d*e*
x^4 + (d^2 + e^2)*x^2 + d*e)*sqrt(e*x^2 + d)*sqrt(d)*x + d*e)/(e*x^2 + d))
)/(d^3*e - d*e^3), -1/2*(2*(B*d^2 - B*e^2)*sqrt(-d)*arctan(sqrt(e*x^2 + d)
*sqrt(-d)*x/sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)) + sqrt(d^3 - d*e^2)*(B*
d - A*e)*log((2*d^3*x^2 + (2*d^2*e - e^3)*x^4 + d^2*e + 2*sqrt(d*e*x^4 + (
d^2 + e^2)*x^2 + d*e)*sqrt(d^3 - d*e^2)*sqrt(e*x^2 + d)*x)/(e^2*x^4 + 2*d*
e*x^2 + d^2)))/(d^3*e - d*e^3), (sqrt(-d^3 + d*e^2)*(B*d - A*e)*arctan(sqr
t(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)*sqrt(-d^3 + d*e^2)*sqrt(e*x^2 + d)*x/(d
^2*e*x^4 + d^2*e + (d^3 + d*e^2)*x^2)) - (B*d^2 - B*e^2)*sqrt(-d)*arctan(s
qrt(e*x^2 + d)*sqrt(-d)*x/sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)))/(d^3*e -
 d*e^3)]
```

Sympy [F]

$$\int \frac{A + Bx^2}{\sqrt{d + ex^2}\sqrt{de + (d^2 + e^2)\,x^2 + dex^4}}\,dx = \int \frac{A + Bx^2}{\sqrt{(d + ex^2)\,(dx^2 + e)}\sqrt{d + ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(1/2)/(d*e+(d**2+e**2)*x**2+d*e*x**4)**(1
/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt((d + e*x**2)*(d*x**2 + e))*sqrt(d + e*x**2)),
x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{dex^4+(d^2+e^2)x^2+de}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)*sqrt(e*x^2 +
d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2),x, a
lgorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{e\,x^2+d}\,\sqrt{d\,e\,x^4+\left(d^2+e^2\right)\,x^2+d\,e}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^(1/2)*(x^2*(d^2 + e^2) + d*e + d*e*x^4)^(1/2)
),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^(1/2)*(x^2*(d^2 + e^2) + d*e + d*e*x^4)^(1/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 709, normalized size of antiderivative = 4.96

$$\int \frac{A + Bx^2}{\sqrt{d + ex^2}\sqrt{de + (d^2 + e^2)\, x^2 + dex^4}}\, dx$$
$$= \frac{\sqrt{d}\left(-\sqrt{d^2 - e^2}\log\left(2\sqrt{d^2 - e^2}\, d + 2\sqrt{d}\,\sqrt{d\,x^2 + e}\, ex + 2d^2 + 2de\, x^2\right) ae + \sqrt{d^2 - e^2}\log\left(2\sqrt{d^2 - e^2}\, d + \right.\right.}{}$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2),x)
```

output
```
(sqrt(d)*( - sqrt(d**2 - e**2)*log(2*sqrt(d**2 - e**2)*d + 2*sqrt(d)*sqrt(
d*x**2 + e)*e*x + 2*d**2 + 2*d*e*x**2)*a*e + sqrt(d**2 - e**2)*log(2*sqrt(
d**2 - e**2)*d + 2*sqrt(d)*sqrt(d*x**2 + e)*e*x + 2*d**2 + 2*d*e*x**2)*b*d
 + sqrt(d**2 - e**2)*log(( - sqrt(e)*sqrt( - d**2 + e**2)*sqrt(d**2 - e**2
)*d + sqrt(e)*sqrt( - d**2 + e**2)*d**2 - sqrt(e)*sqrt( - d**2 + e**2)*e**
2 + sqrt(d*x**2 + e)*d**2*e - sqrt(d*x**2 + e)*e**3 + sqrt(d)*d**2*e*x - s
qrt(d)*e**3*x)/(sqrt(e)*d**2 - sqrt(e)*e**2))*a*e - sqrt(d**2 - e**2)*log(
( - sqrt(e)*sqrt( - d**2 + e**2)*sqrt(d**2 - e**2)*d + sqrt(e)*sqrt( - d**
2 + e**2)*d**2 - sqrt(e)*sqrt( - d**2 + e**2)*e**2 + sqrt(d*x**2 + e)*d**2
*e - sqrt(d*x**2 + e)*e**3 + sqrt(d)*d**2*e*x - sqrt(d)*e**3*x)/(sqrt(e)*d
**2 - sqrt(e)*e**2))*b*d + sqrt(d**2 - e**2)*log((sqrt(e)*sqrt( - d**2 + e
**2)*sqrt(d**2 - e**2)*d - sqrt(e)*sqrt( - d**2 + e**2)*d**2 + sqrt(e)*sqr
t( - d**2 + e**2)*e**2 + sqrt(d*x**2 + e)*d**2*e - sqrt(d*x**2 + e)*e**3 +
 sqrt(d)*d**2*e*x - sqrt(d)*e**3*x)/(sqrt(e)*d**2 - sqrt(e)*e**2))*a*e - s
qrt(d**2 - e**2)*log((sqrt(e)*sqrt( - d**2 + e**2)*sqrt(d**2 - e**2)*d - s
qrt(e)*sqrt( - d**2 + e**2)*d**2 + sqrt(e)*sqrt( - d**2 + e**2)*e**2 + sqr
t(d*x**2 + e)*d**2*e - sqrt(d*x**2 + e)*e**3 + sqrt(d)*d**2*e*x - sqrt(d)*
e**3*x)/(sqrt(e)*d**2 - sqrt(e)*e**2))*b*d + 2*log((sqrt(d*x**2 + e) + sqr
t(d)*x)/sqrt(e))*b*d**2 - 2*log((sqrt(d*x**2 + e) + sqrt(d)*x)/sqrt(e))*b*
e**2))/(2*d*e*(d**2 - e**2))
```

3.139 $\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-de+(d^2+e^2)x^2-dex^4}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 148

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx = -\frac{B\operatorname{arctanh}\left(\frac{\sqrt{d}x\sqrt{d-ex^2}}{\sqrt{-de+(d^2+e^2)x^2-dex^4}}\right)}{\sqrt{d}e}+\frac{(Bd+Ae)\arctan\left(\frac{\sqrt{d^2-e^2}x\sqrt{d-ex^2}}{\sqrt{d}\sqrt{-de+(d^2+e^2)x^2-dex^4}}\right)}{\sqrt{d}e\sqrt{d^2-e^2}}$$

output
```
-B*arctanh(d^(1/2)*x*(-e*x^2+d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2))/
d^(1/2)/e+(A*e+B*d)*arctanh((d^2-e^2)^(1/2)*x*(-e*x^2+d)^(1/2)/d^(1/2)/(-d
*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2))/d^(1/2)/e/(d^2-e^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx = \frac{\sqrt{-e+dx^2}\sqrt{d-ex^2}\left((Bd+Ae)\sqrt{-d^2+e^2}\arctan\left(\frac{d^{3/2}-\sqrt{d}ex^2+ex\sqrt{-e+dx^2}}{\sqrt{d}\sqrt{-d^2+e^2}}\right)+B(d^2-e^2)\log\left(-\sqrt{d}x+\right.\right.}{\sqrt{d}(d-e)e(d+e)\sqrt{d^2x^2+e^2x^2-de\,(1+x^4)}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d - e*x^2]*Sqrt[-(d*e) + (d^2 + e^2)*x^2 - d*e
*x^4]),x]
```

output
```
(Sqrt[-e + d*x^2]*Sqrt[d - e*x^2]*((B*d + A*e)*Sqrt[-d^2 + e^2]*ArcTan[(d^
(3/2) - Sqrt[d]*e*x^2 + e*x*Sqrt[-e + d*x^2])/(Sqrt[d]*Sqrt[-d^2 + e^2])]
+ B*(d^2 - e^2)*Log[-(Sqrt[d]*x) + Sqrt[-e + d*x^2]]))/(Sqrt[d]*(d - e)*e*
(d + e)*Sqrt[d^2*x^2 + e^2*x^2 - d*e*(1 + x^4)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1395, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{x^2(d^2+e^2)-dex^4-de}}dx$$

$$\Big\downarrow \text{ 1395}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d-ex^2}\int \frac{Bx^2+A}{\sqrt{dx^2-e}(d-ex^2)}dx}{\sqrt{x^2(d^2+e^2)-dex^4-de}}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{dx^2-e}\left(d-ex^2\right)}dx}{e}-\frac{B\int \frac{1}{\sqrt{dx^2-e}}dx}{e}\right)}{\sqrt{x^2(d^2+e^2)-dex^4-de}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{dx^2-e}\left(d-ex^2\right)}dx}{e}-\frac{B\int \frac{1}{1-\frac{dx^2}{dx^2-e}}d\frac{x}{\sqrt{dx^2-e}}}{e}\right)}{\sqrt{x^2(d^2+e^2)-dex^4-de}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{dx^2-e}\left(d-ex^2\right)}dx}{e}-\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2-e}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2\left(d^2+e^2\right)-dex^4-de}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{d-\frac{\left(d^2-e^2\right)x^2}{dx^2-e}}d\frac{x}{\sqrt{dx^2-e}}}{e}-\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2-e}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2\left(d^2+e^2\right)-dex^4-de}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\text{arctanh}\left(\frac{x\sqrt{d^2-e^2}}{\sqrt{d}\sqrt{dx^2-e}}\right)}{\sqrt{d}e\sqrt{d^2-e^2}}-\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2-e}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2\left(d^2+e^2\right)-dex^4-de}}$$

input
```
Int[(A + B*x^2)/(Sqrt[d - e*x^2]*Sqrt[-(d*e) + (d^2 + e^2)*x^2 - d*e*x^4])
,x]
```

output
```
(Sqrt[-e + d*x^2]*Sqrt[d - e*x^2]*(-((B*ArcTanh[(Sqrt[d]*x)/Sqrt[-e + d*x^
2]])/(Sqrt[d]*e)) + ((B*d + A*e)*ArcTanh[(Sqrt[d^2 - e^2]*x)/(Sqrt[d]*Sqrt
[-e + d*x^2])])/(Sqrt[d]*e*Sqrt[d^2 - e^2])))/Sqrt[-(d*e) + (d^2 + e^2)*x^
2 - d*e*x^4]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(128) = 256$.

Time = 0.39 (sec) , antiderivative size = 829, normalized size of antiderivative = 5.60

method	result
default	$\frac{\sqrt{-de\,x^4+d^2x^2+e^2x^2-de}\left(B\,d^{\frac{7}{2}}\ln\left(\frac{2\sqrt{de}\,dx+2\sqrt{d\,x^2-e}\,\sqrt{\frac{d^2-e^2}{e}}\,e-2e^2}{ex-\sqrt{de}}\right)-B\,d^{\frac{7}{2}}\ln\left(-\frac{2\left(\sqrt{de}\,dx-\sqrt{d\,x^2-e}\,\sqrt{\frac{d^2-e^2}{e}}\,e+e^2\right)}{ex+\sqrt{de}}\right)+A\,d \right.}{}$

input
```
int((B*x^2+A)/(-e*x^2+d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
1/2/(-e*x^2+d)^(1/2)*(-d*e*x^4+d^2*x^2+e^2*x^2-d*e)^(1/2)/d^(1/2)*(B*d^(7/
2)*ln(2*((d*e)^(1/2)*d*x+(d*x^2-e)^(1/2)*((d^2-e^2)/e)^(1/2)*e-e^2)/(e*x-(
d*e)^(1/2)))-B*d^(7/2)*ln(-2*((d*e)^(1/2)*d*x-(d*x^2-e)^(1/2)*((d^2-e^2)/e
)^(1/2)*e+e^2)/(e*x+(d*e)^(1/2)))+A*d^(5/2)*ln(2*((d*e)^(1/2)*d*x+(d*x^2-e
)^(1/2)*((d^2-e^2)/e)^(1/2)*e-e^2)/(e*x-(d*e)^(1/2)))*e-A*d^(5/2)*ln(-2*((
d*e)^(1/2)*d*x-(d*x^2-e)^(1/2)*((d^2-e^2)/e)^(1/2)*e+e^2)/(e*x+(d*e)^(1/2)
))*e-B*d^(3/2)*ln(2*((d*e)^(1/2)*d*x+(d*x^2-e)^(1/2)*((d^2-e^2)/e)^(1/2)*e
-e^2)/(e*x-(d*e)^(1/2)))*e^2+B*d^(3/2)*ln(-2*((d*e)^(1/2)*d*x-(d*x^2-e)^(1
/2)*((d^2-e^2)/e)^(1/2)*e+e^2)/(e*x+(d*e)^(1/2)))*e^2-A*d^(1/2)*ln(2*((d*e
)^(1/2)*d*x+(d*x^2-e)^(1/2)*((d^2-e^2)/e)^(1/2)*e-e^2)/(e*x-(d*e)^(1/2)))*
e^3+A*d^(1/2)*ln(-2*((d*e)^(1/2)*d*x-(d*x^2-e)^(1/2)*((d^2-e^2)/e)^(1/2)*e
+e^2)/(e*x+(d*e)^(1/2)))*e^3+2*A*(d*e)^(1/2)*ln((d^(1/2)*(-1/d*(d*x+(d*e)^
(1/2))*(-d*x+(d*e)^(1/2)))^(1/2)+d*x)/d^(1/2))*((d^2-e^2)/e)^(1/2)*d*e-2*A
*d*(d*e)^(1/2)*ln((d^(1/2)*(d*x^2-e)^(1/2)+d*x)/d^(1/2))*((d^2-e^2)/e)^(1/
2)*e+2*B*(d*e)^(1/2)*ln((d^(1/2)*(-1/d*(d*x+(d*e)^(1/2))*(-d*x+(d*e)^(1/2)
))^(1/2)+d*x)/d^(1/2))*((d^2-e^2)/e)^(1/2)*e^2-2*B*d^2*(d*e)^(1/2)*ln((d^(
1/2)*(d*x^2-e)^(1/2)+d*x)/d^(1/2))*((d^2-e^2)/e)^(1/2))/(d*x^2-e)^(1/2)/e/
(d*e)^(1/2)/(d-e)/(d+e)/((d^2-e^2)/e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 888, normalized size of antiderivative = 6.00

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-de+\left(d^2+e^2\right)x^2-dex^4}}\,dx=\text{Too large to display}$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2),x,
 algorithm="fricas")
```

output
```
[1/2*(sqrt(d^3 - d*e^2)*(B*d + A*e)*log(-(2*d^3*x^2 - (2*d^2*e - e^3)*x^4
- d^2*e + 2*sqrt(-d*e*x^4 + (d^2 + e^2)*x^2 - d*e)*sqrt(d^3 - d*e^2)*sqrt(
-e*x^2 + d)*x)/(e^2*x^4 - 2*d*e*x^2 + d^2)) + (B*d^2 - B*e^2)*sqrt(d)*log(
-(2*d*e*x^4 - (2*d^2 + e^2)*x^2 + 2*sqrt(-d*e*x^4 + (d^2 + e^2)*x^2 - d*e)
*sqrt(-e*x^2 + d)*sqrt(d)*x + d*e)/(e*x^2 - d)))/(d^3*e - d*e^3), 1/2*(2*s
qrt(-d^3 + d*e^2)*(B*d + A*e)*arctan(sqrt(-d*e*x^4 + (d^2 + e^2)*x^2 - d*e
)*sqrt(-d^3 + d*e^2)*sqrt(-e*x^2 + d)*x/(d^2*e*x^4 + d^2*e - (d^3 + d*e^2)
*x^2)) + (B*d^2 - B*e^2)*sqrt(d)*log(-(2*d*e*x^4 - (2*d^2 + e^2)*x^2 + 2*s
qrt(-d*e*x^4 + (d^2 + e^2)*x^2 - d*e)*sqrt(-e*x^2 + d)*sqrt(d)*x + d*e)/(e
*x^2 - d)))/(d^3*e - d*e^3), -1/2*(2*(B*d^2 - B*e^2)*sqrt(-d)*arctan(sqrt(
-d*e*x^4 + (d^2 + e^2)*x^2 - d*e)*sqrt(-e*x^2 + d)*sqrt(-d)*x/(d*e*x^4 - (
d^2 + e^2)*x^2 + d*e)) - sqrt(d^3 - d*e^2)*(B*d + A*e)*log(-(2*d^3*x^2 - (
2*d^2*e - e^3)*x^4 - d^2*e + 2*sqrt(-d*e*x^4 + (d^2 + e^2)*x^2 - d*e)*sqrt
(d^3 - d*e^2)*sqrt(-e*x^2 + d)*x)/(e^2*x^4 - 2*d*e*x^2 + d^2)))/(d^3*e - d
*e^3), (sqrt(-d^3 + d*e^2)*(B*d + A*e)*arctan(sqrt(-d*e*x^4 + (d^2 + e^2)*
x^2 - d*e)*sqrt(-d^3 + d*e^2)*sqrt(-e*x^2 + d)*x/(d^2*e*x^4 + d^2*e - (d^3
 + d*e^2)*x^2)) - (B*d^2 - B*e^2)*sqrt(-d)*arctan(sqrt(-d*e*x^4 + (d^2 + e
^2)*x^2 - d*e)*sqrt(-e*x^2 + d)*sqrt(-d)*x/(d*e*x^4 - (d^2 + e^2)*x^2 + d*
e)))/(d^3*e - d*e^3)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{-\left(-d+ex^2\right)\left(dx^2-e\right)}\sqrt{d-ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(-e*x**2+d)**(1/2)/(-d*e+(d**2+e**2)*x**2-d*e*x**4)**
(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(-(-d + e*x**2)*(d*x**2 - e))*sqrt(d - e*x**2))
, x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-dex^4+(d^2+e^2)x^2-de}\sqrt{-ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-d*e*x^4 + (d^2 + e^2)*x^2 - d*e)*sqrt(-e*x^2
+ d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2),x,
 algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{d-e\,x^2}\,\sqrt{-d\,e\,x^4+\left(d^2+e^2\right)\,x^2-d\,e}}\,dx$$

input
```
int((A + B*x^2)/((d - e*x^2)^(1/2)*(x^2*(d^2 + e^2) - d*e - d*e*x^4)^(1/2)
),x)
```

output
```
int((A + B*x^2)/((d - e*x^2)^(1/2)*(x^2*(d^2 + e^2) - d*e - d*e*x^4)^(1/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.78

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx$$
$$= \frac{\sqrt{d}\left(\sqrt{d^2-e^2}\log\left(2\sqrt{d^2-e^2}\,d+2\sqrt{d}\,\sqrt{d\,x^2-e}\,ex-2d^2+2de\,x^2\right)ae+\sqrt{d^2-e^2}\log\left(2\sqrt{d^2-e^2}\,d+\right.\right.}{}$$

input
```
int((B*x^2+A)/(-e*x^2+d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2),x)
```

output
```
(sqrt(d)*(sqrt(d**2 - e**2)*log(2*sqrt(d**2 - e**2)*d + 2*sqrt(d)*sqrt(d*x
**2 - e)*e*x - 2*d**2 + 2*d*e*x**2)*a*e + sqrt(d**2 - e**2)*log(2*sqrt(d**
2 - e**2)*d + 2*sqrt(d)*sqrt(d*x**2 - e)*e*x - 2*d**2 + 2*d*e*x**2)*b*d -
sqrt(d**2 - e**2)*log(( - sqrt(e)*sqrt(d**2 - e**2) + sqrt(d*x**2 - e)*e +
 sqrt(d)*e*x - sqrt(e)*d)/sqrt(e))*a*e - sqrt(d**2 - e**2)*log(( - sqrt(e)
*sqrt(d**2 - e**2) + sqrt(d*x**2 - e)*e + sqrt(d)*e*x - sqrt(e)*d)/sqrt(e)
)*b*d - sqrt(d**2 - e**2)*log((sqrt(e)*sqrt(d**2 - e**2) + sqrt(d*x**2 - e
)*e + sqrt(d)*e*x + sqrt(e)*d)/sqrt(e))*a*e - sqrt(d**2 - e**2)*log((sqrt(
e)*sqrt(d**2 - e**2) + sqrt(d*x**2 - e)*e + sqrt(d)*e*x + sqrt(e)*d)/sqrt(
e))*b*d - 2*log((sqrt(d*x**2 - e) + sqrt(d)*x)/sqrt(e))*b*d**2 + 2*log((sq
rt(d*x**2 - e) + sqrt(d)*x)/sqrt(e))*b*e**2))/(2*d*e*(d**2 - e**2))
```

3.140 $\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-de+(d^2+e^2)x^2-dex^4}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 150

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx = \frac{B\arctan\left(\frac{\sqrt{d}x\sqrt{-d+ex^2}}{\sqrt{-de+(d^2+e^2)x^2-dex^4}}\right)}{\sqrt{d}e} - \frac{(Bd+Ae)\arctan\left(\frac{\sqrt{d^2-e^2}x\sqrt{-d+ex^2}}{\sqrt{d}\sqrt{-de+(d^2+e^2)x^2-dex^4}}\right)}{\sqrt{d}e\sqrt{d^2-e^2}}$$

output
```
B*arctan(d^(1/2)*x*(e*x^2-d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2))/d^(
1/2)/e-(A*e+B*d)*arctan((d^2-e^2)^(1/2)*x*(e*x^2-d)^(1/2)/d^(1/2)/(-d*e+(d
^2+e^2)*x^2-d*e*x^4)^(1/2))/d^(1/2)/e/(d^2-e^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.78 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx = \frac{-\frac{(Bd+Ae)\sqrt{e-dx^2}\sqrt{-d+ex^2}\text{arctanh}\left(\frac{\sqrt{-d^2+e^2}x}{\sqrt{d}\sqrt{e-dx^2}}\right)}{\sqrt{-d^2+e^2}\sqrt{d^2x^2+e^2x^2-de(1+x^4)}} - iB\log\left(-2i\sqrt{d}x - \frac{2\sqrt{d^2x^2+e^2x^2-de(1+x^4)}}{\sqrt{-d+ex^2}}\right)}{\sqrt{d}e}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[-d + e*x^2]*Sqrt[-(d*e) + (d^2 + e^2)*x^2 - d*
e*x^4]),x]
```

output
```
(-(((B*d + A*e)*Sqrt[e - d*x^2]*Sqrt[-d + e*x^2]*ArcTanh[(Sqrt[-d^2 + e^2]
*x)/(Sqrt[d]*Sqrt[e - d*x^2])])/(Sqrt[-d^2 + e^2]*Sqrt[d^2*x^2 + e^2*x^2 -
 d*e*(1 + x^4)])) - I*B*Log[(-2*I)*Sqrt[d]*x - (2*Sqrt[d^2*x^2 + e^2*x^2 -
 d*e*(1 + x^4)])/Sqrt[-d + e*x^2]])/(Sqrt[d]*e)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {1395, 25, 398, 224, 216, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{ex^2-d}\sqrt{x^2\left(d^2+e^2\right)-dex^4-de}}dx$$

$$\Big\downarrow \text{1395}$$

$$\frac{\sqrt{e-dx^2}\sqrt{ex^2-d}\int -\frac{Bx^2+A}{\sqrt{e-dx^2}(d-ex^2)}dx}{\sqrt{x^2\left(d^2+e^2\right)-dex^4-de}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{ex^2-d}\int \frac{Bx^2+A}{\sqrt{e-dx^2}(d-ex^2)}dx}{\sqrt{x^2\left(d^2+e^2\right)-dex^4-de}}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{e-dx^2}\left(d-ex^2\right)}dx}{e}-\frac{B\int \frac{1}{\sqrt{e-dx^2}}dx}{e}\right)}{\sqrt{x^2\left(d^2+e^2\right)-dex^4-de}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{e-dx^2}(d-ex^2)}dx}{e}-\frac{B\int \frac{1}{\frac{dx^2}{e-dx^2}+1}d\frac{x}{\sqrt{e-dx^2}}}{e}\right)}{\sqrt{x^2(d^2+e^2)-dex^4-de}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{e-dx^2}(d-ex^2)}dx}{e}-\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{e-dx^2}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2(d^2+e^2)-dex^4-de}}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{d-\frac{(e^2-d^2)x^2}{e-dx^2}}d\frac{x}{\sqrt{e-dx^2}}}{e}-\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{e-dx^2}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2(d^2+e^2)-dex^4-de}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\arctan\left(\frac{x\sqrt{d^2-e^2}}{\sqrt{d}\sqrt{e-dx^2}}\right)}{\sqrt{d}e\sqrt{d^2-e^2}}-\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{e-dx^2}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2(d^2+e^2)-dex^4-de}}$$

input
```
Int[(A + B*x^2)/(Sqrt[-d + e*x^2]*Sqrt[-(d*e) + (d^2 + e^2)*x^2 - d*e*x^4]
),x]
```

output
```
-((Sqrt[e - d*x^2]*Sqrt[-d + e*x^2]*(-((B*ArcTan[(Sqrt[d]*x)/Sqrt[e - d*x^
2]])/(Sqrt[d]*e)) + ((B*d + A*e)*ArcTan[(Sqrt[d^2 - e^2]*x)/(Sqrt[d]*Sqrt[
e - d*x^2])])/(Sqrt[d]*e*Sqrt[d^2 - e^2])))/Sqrt[-(d*e) + (d^2 + e^2)*x^2
- d*e*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(130) = 260$.

Time = 0.09 (sec) , antiderivative size = 805, normalized size of antiderivative = 5.37

method	result
default	$\frac{\sqrt{e\,x^2-d}\,\sqrt{-de\,x^4+d^2x^2+e^2x^2-de}\,\sqrt{d}\left(-2A\sqrt{de}\,\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{-d\,x^2+e}}\right)\sqrt{-\frac{d^2-e^2}{e}}\,de-2B\sqrt{de}\,\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{-d\,x^2+e}}\right)\sqrt{-\frac{d^2-e^2}{e}}\,d^2\cdots\right.}{}$

input
```
int((B*x^2+A)/(e*x^2-d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
1/2*(e*x^2-d)^(1/2)*(-d*e*x^4+d^2*x^2+e^2*x^2-d*e)^(1/2)*d^(1/2)*(-2*A*(d*
e)^(1/2)*arctan(d^(1/2)*x/(-d*x^2+e)^(1/2))*(-(d^2-e^2)/e)^(1/2)*d*e-2*B*(
d*e)^(1/2)*arctan(d^(1/2)*x/(-d*x^2+e)^(1/2))*(-(d^2-e^2)/e)^(1/2)*d^2+2*A
*(d*e)^(1/2)*arctan(d^(1/2)*x/(1/d*(d*x+(d*e)^(1/2))*(-d*x+(d*e)^(1/2)))^(
1/2))*(-(d^2-e^2)/e)^(1/2)*d*e+A*ln(2*((d*e)^(1/2)*d*x-(-d*x^2+e)^(1/2)*(-
(d^2-e^2)/e)^(1/2)*e-e^2)/(-e*x+(d*e)^(1/2)))*d^(5/2)*e-A*ln(2*((d*e)^(1/2
)*d*x-(-d*x^2+e)^(1/2)*(-(d^2-e^2)/e)^(1/2)*e-e^2)/(-e*x+(d*e)^(1/2)))*d^(
1/2)*e^3-A*ln(2*((d*e)^(1/2)*d*x+(-d*x^2+e)^(1/2)*(-(d^2-e^2)/e)^(1/2)*e+e
^2)/(e*x+(d*e)^(1/2)))*d^(5/2)*e+A*ln(2*((d*e)^(1/2)*d*x+(-d*x^2+e)^(1/2)*
(-(d^2-e^2)/e)^(1/2)*e+e^2)/(e*x+(d*e)^(1/2)))*d^(1/2)*e^3+2*B*(d*e)^(1/2)
*arctan(d^(1/2)*x/(1/d*(d*x+(d*e)^(1/2))*(-d*x+(d*e)^(1/2)))^(1/2))*(-(d^2
-e^2)/e)^(1/2)*e^2+B*ln(2*((d*e)^(1/2)*d*x-(-d*x^2+e)^(1/2)*(-(d^2-e^2)/e)
^(1/2)*e-e^2)/(-e*x+(d*e)^(1/2)))*d^(7/2)-B*ln(2*((d*e)^(1/2)*d*x-(-d*x^2+
e)^(1/2)*(-(d^2-e^2)/e)^(1/2)*e-e^2)/(-e*x+(d*e)^(1/2)))*d^(3/2)*e^2-B*ln(
2*((d*e)^(1/2)*d*x+(-d*x^2+e)^(1/2)*(-(d^2-e^2)/e)^(1/2)*e+e^2)/(e*x+(d*e)
^(1/2)))*d^(7/2)+B*ln(2*((d*e)^(1/2)*d*x+(-d*x^2+e)^(1/2)*(-(d^2-e^2)/e)^(
1/2)*e+e^2)/(e*x+(d*e)^(1/2)))*d^(3/2)*e^2)/(-e*x^2+d)/(-d*x^2+e)^(1/2)/(d
*e)^(3/2)/(d-e)/(d+e)/(-(d^2-e^2)/e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 896, normalized size of antiderivative = 5.97

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2),x,
algorithm="fricas")
```

output
```
[-1/2*(sqrt(-d^3 + d*e^2)*(B*d + A*e)*log(-(2*d^3*x^2 - (2*d^2*e - e^3)*x^
4 - d^2*e - 2*sqrt(-d*e*x^4 + (d^2 + e^2)*x^2 - d*e)*sqrt(-d^3 + d*e^2)*sq
rt(e*x^2 - d)*x)/(e^2*x^4 - 2*d*e*x^2 + d^2)) + (B*d^2 - B*e^2)*sqrt(-d)*l
og(-(2*d*e*x^4 - (2*d^2 + e^2)*x^2 - 2*sqrt(-d*e*x^4 + (d^2 + e^2)*x^2 - d
*e)*sqrt(e*x^2 - d)*sqrt(-d)*x + d*e)/(e*x^2 - d)))/(d^3*e - d*e^3), -1/2*
(2*(B*d^2 - B*e^2)*sqrt(d)*arctan(sqrt(-d*e*x^4 + (d^2 + e^2)*x^2 - d*e)*s
qrt(e*x^2 - d)*sqrt(d)*x/(d*e*x^4 - (d^2 + e^2)*x^2 + d*e)) + sqrt(-d^3 +
d*e^2)*(B*d + A*e)*log(-(2*d^3*x^2 - (2*d^2*e - e^3)*x^4 - d^2*e - 2*sqrt(
-d*e*x^4 + (d^2 + e^2)*x^2 - d*e)*sqrt(-d^3 + d*e^2)*sqrt(e*x^2 - d)*x)/(e
^2*x^4 - 2*d*e*x^2 + d^2)))/(d^3*e - d*e^3), 1/2*(2*sqrt(d^3 - d*e^2)*(B*d
 + A*e)*arctan(sqrt(-d*e*x^4 + (d^2 + e^2)*x^2 - d*e)*sqrt(d^3 - d*e^2)*sq
rt(e*x^2 - d)*x/(d^2*e*x^4 + d^2*e - (d^3 + d*e^2)*x^2)) - (B*d^2 - B*e^2)
*sqrt(-d)*log(-(2*d*e*x^4 - (2*d^2 + e^2)*x^2 - 2*sqrt(-d*e*x^4 + (d^2 + e
^2)*x^2 - d*e)*sqrt(e*x^2 - d)*sqrt(-d)*x + d*e)/(e*x^2 - d)))/(d^3*e - d*
e^3), (sqrt(d^3 - d*e^2)*(B*d + A*e)*arctan(sqrt(-d*e*x^4 + (d^2 + e^2)*x^
2 - d*e)*sqrt(d^3 - d*e^2)*sqrt(e*x^2 - d)*x/(d^2*e*x^4 + d^2*e - (d^3 + d
*e^2)*x^2)) - (B*d^2 - B*e^2)*sqrt(d)*arctan(sqrt(-d*e*x^4 + (d^2 + e^2)*x
^2 - d*e)*sqrt(e*x^2 - d)*sqrt(d)*x/(d*e*x^4 - (d^2 + e^2)*x^2 + d*e)))/(d
^3*e - d*e^3)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{-\left(-d+ex^2\right)\left(dx^2-e\right)}\sqrt{-d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2-d)**(1/2)/(-d*e+(d**2+e**2)*x**2-d*e*x**4)**(
1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(-(-d + e*x**2)*(d*x**2 - e))*sqrt(-d + e*x**2)
), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx$$
$$=\int \frac{Bx^2+A}{\sqrt{-dex^4+(d^2+e^2)x^2-de}\sqrt{ex^2-d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-d*e*x^4 + (d^2 + e^2)*x^2 - d*e)*sqrt(e*x^2 -
 d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-de+(d^2+e^2)\,x^2-dex^4}}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2),x,
algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-de+(d^2+e^2)x^2-dex^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{e\,x^2-d}\,\sqrt{-d\,e\,x^4+(d^2+e^2)\,x^2-d\,e}}\,dx$$

input
```
int((A + B*x^2)/((e*x^2 - d)^(1/2)*(x^2*(d^2 + e^2) - d*e - d*e*x^4)^(1/2)
),x)
```

output
```
int((A + B*x^2)/((e*x^2 - d)^(1/2)*(x^2*(d^2 + e^2) - d*e - d*e*x^4)^(1/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{-de+(d^2+e^2)x^2-dex^4}}\,dx = \frac{\sqrt{d}\left(asin\left(\frac{\sqrt{d}\,x}{\sqrt{e}}\right)b\,d^2-asin\left(\frac{\sqrt{d}\,x}{\sqrt{e}}\right)b\,e^2-\sqrt{d^2-e^2}\,atan\left(\frac{\tan\left(\frac{asin\left(\frac{\sqrt{d}\,x}{\sqrt{e}}\right)}{2}\right)d-e}{\sqrt{d^2-e^2}}\right)ae-\sqrt{d^2-e^2}\,atan\left(\frac{\tan\left(\right.}{}\right.}{de\,(d^2}$$

input
```
int((B*x^2+A)/(e*x^2-d)^(1/2)/(-d*e+(d^2+e^2)*x^2-d*e*x^4)^(1/2),x)
```

output
```
(sqrt(d)*(asin((sqrt(d)*x)/sqrt(e))*b*d**2 - asin((sqrt(d)*x)/sqrt(e))*b*e
**2 - sqrt(d**2 - e**2)*atan((tan(asin((sqrt(d)*x)/sqrt(e))/2)*d - e)/sqrt
(d**2 - e**2))*a*e - sqrt(d**2 - e**2)*atan((tan(asin((sqrt(d)*x)/sqrt(e))
/2)*d - e)/sqrt(d**2 - e**2))*b*d - sqrt(d**2 - e**2)*atan((tan(asin((sqrt
(d)*x)/sqrt(e))/2)*d + e)/sqrt(d**2 - e**2))*a*e - sqrt(d**2 - e**2)*atan(
(tan(asin((sqrt(d)*x)/sqrt(e))/2)*d + e)/sqrt(d**2 - e**2))*b*d))/(d*e*(d*
*2 - e**2))
```

3.141 $\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{de+(d^2+e^2)x^2+dex^4}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 149

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx = -\frac{B\arctan\left(\frac{\sqrt{d}x\sqrt{-d-ex^2}}{\sqrt{de+(d^2+e^2)x^2+dex^4}}\right)}{\sqrt{d}e}+\frac{(Bd-Ae)\arctan\left(\frac{\sqrt{d^2-e^2}x\sqrt{-d-ex^2}}{\sqrt{d}\sqrt{de+(d^2+e^2)x^2+dex^4}}\right)}{\sqrt{d}e\sqrt{d^2-e^2}}$$

output
```
-B*arctan(d^(1/2)*x*(-e*x^2-d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2))/d^
(1/2)/e+(-A*e+B*d)*arctan((d^2-e^2)^(1/2)*x*(-e*x^2-d)^(1/2)/d^(1/2)/(d*e+
(d^2+e^2)*x^2+d*e*x^4)^(1/2))/d^(1/2)/e/(d^2-e^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.71 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx = \frac{\frac{(Bd-Ae)\sqrt{-e-dx^2}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{d^2-e^2}x}{\sqrt{d}\sqrt{-e-dx^2}}\right)}{\sqrt{d^2-e^2}\sqrt{d^2x^2+e^2x^2+de(1+x^4)}}+iB\log\left(-2i\sqrt{d}x-\frac{2\sqrt{d^2x^2+e^2x^2+de(1+x^4)}}{\sqrt{-d-ex^2}}\right)}{\sqrt{d}e}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[-d - e*x^2]*Sqrt[d*e + (d^2 + e^2)*x^2 + d*e*x
^4]),x]
```

output
```
(((B*d - A*e)*Sqrt[-e - d*x^2]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d^2 - e^2]*x)
/(Sqrt[d]*Sqrt[-e - d*x^2])])/(Sqrt[d^2 - e^2]*Sqrt[d^2*x^2 + e^2*x^2 + d*
e*(1 + x^4)]) + I*B*Log[(-2*I)*Sqrt[d]*x - (2*Sqrt[d^2*x^2 + e^2*x^2 + d*e
*(1 + x^4)])/Sqrt[-d - e*x^2]])/(Sqrt[d]*e)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.149$, Rules used = {1395, 25, 398, 224, 216, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{x^2\left(d^2+e^2\right)+dex^4+de}}dx$$

$$\Big\downarrow \text{1395}$$

$$\frac{\sqrt{-dx^2-e}\sqrt{-d-ex^2}\int -\frac{Bx^2+A}{\sqrt{-dx^2-e}\left(ex^2+d\right)}dx}{\sqrt{x^2\left(d^2+e^2\right)+dex^4+de}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{-d-ex^2}\int \frac{Bx^2+A}{\sqrt{-dx^2-e}\left(ex^2+d\right)}dx}{\sqrt{x^2\left(d^2+e^2\right)+dex^4+de}}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{-d-ex^2}\left(\frac{B\int \frac{1}{\sqrt{-dx^2-e}}dx}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{-dx^2-e}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{x^2\left(d^2+e^2\right)+dex^4+de}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{-d-ex^2}\left(\frac{B\int \frac{1}{\frac{dx^2}{-dx^2-e}+1}d\frac{x}{\sqrt{-dx^2-e}}}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{-dx^2-e}(ex^2+d)}dx}{e}\right)}{\sqrt{x^2(d^2+e^2)+dex^4+de}}$$

$$\Big\downarrow \; 216$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{-d-ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{-dx^2-e}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{-dx^2-e}(ex^2+d)}dx}{e}\right)}{\sqrt{x^2(d^2+e^2)+dex^4+de}}$$

$$\Big\downarrow \; 291$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{-d-ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{-dx^2-e}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\int \frac{1}{d-\frac{(e^2-d^2)x^2}{-dx^2-e}}d\frac{x}{\sqrt{-dx^2-e}}}{e}\right)}{\sqrt{x^2(d^2+e^2)+dex^4+de}}$$

$$\Big\downarrow \; 218$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{-d-ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{-dx^2-e}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\arctan\left(\frac{x\sqrt{d^2-e^2}}{\sqrt{d}\sqrt{-dx^2-e}}\right)}{\sqrt{d}e\sqrt{d^2-e^2}}\right)}{\sqrt{x^2(d^2+e^2)+dex^4+de}}$$

input
```
Int[(A + B*x^2)/(Sqrt[-d - e*x^2]*Sqrt[d*e + (d^2 + e^2)*x^2 + d*e*x^4]),x
]
```

output
```
-((Sqrt[-e - d*x^2]*Sqrt[-d - e*x^2]*((B*ArcTan[(Sqrt[d]*x)/Sqrt[-e - d*x^
2]])/(Sqrt[d]*e) - ((B*d - A*e)*ArcTan[(Sqrt[d^2 - e^2]*x)/(Sqrt[d]*Sqrt[-
e - d*x^2])])/(Sqrt[d]*e*Sqrt[d^2 - e^2])))/Sqrt[d*e + (d^2 + e^2)*x^2 + d
*e*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 836 vs. 2(129) = 258.

Time = 0.09 (sec) , antiderivative size = 837, normalized size of antiderivative = 5.62

method	result
default	$\frac{\sqrt{-e\,x^2-d}\,\sqrt{d e\,x^4+d^2x^2+e^2x^2+d e}\,\sqrt{d}\,\left(2A\sqrt{-d e}\,\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{-d\,x^2-e}}\right)d\sqrt{\frac{d^2-e^2}{e}}\,e-2B\sqrt{-d e}\,\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{-d\,x^2-e}}\right)d^2\sqrt{\frac{d^2-e^2}{e}}-2\right.}{}$

input
```
int((B*x^2+A)/(-e*x^2-d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
1/2*(-e*x^2-d)^(1/2)*(d*e*x^4+d^2*x^2+e^2*x^2+d*e)^(1/2)*d^(1/2)*(2*A*(-d*
e)^(1/2)*arctan(d^(1/2)*x/(-d*x^2-e)^(1/2))*d*((d^2-e^2)/e)^(1/2)*e-2*B*(-
d*e)^(1/2)*arctan(d^(1/2)*x/(-d*x^2-e)^(1/2))*d^2*((d^2-e^2)/e)^(1/2)-2*A*
(-d*e)^(1/2)*arctan(d^(1/2)*x/(1/d*(-d*x+(-d*e)^(1/2))*(d*x+(-d*e)^(1/2)))
^(1/2))*((d^2-e^2)/e)^(1/2)*d*e+A*ln(2*((-d*e)^(1/2)*d*x-(-d*x^2-e)^(1/2)*
((d^2-e^2)/e)^(1/2)*e+e^2)/(-e*x+(-d*e)^(1/2)))*d^(5/2)*e-A*ln(2*((-d*e)^(
1/2)*d*x-(-d*x^2-e)^(1/2)*((d^2-e^2)/e)^(1/2)*e+e^2)/(-e*x+(-d*e)^(1/2)))*
d^(1/2)*e^3-A*ln(2*((-d*e)^(1/2)*d*x+(-d*x^2-e)^(1/2)*((d^2-e^2)/e)^(1/2)*
e-e^2)/(e*x+(-d*e)^(1/2)))*d^(5/2)*e+A*ln(2*((-d*e)^(1/2)*d*x+(-d*x^2-e)^(
1/2)*((d^2-e^2)/e)^(1/2)*e-e^2)/(e*x+(-d*e)^(1/2)))*d^(1/2)*e^3+2*B*(-d*e)
^(1/2)*arctan(d^(1/2)*x/(1/d*(-d*x+(-d*e)^(1/2))*(d*x+(-d*e)^(1/2)))^(1/2)
)*((d^2-e^2)/e)^(1/2)*e^2-B*ln(2*((-d*e)^(1/2)*d*x-(-d*x^2-e)^(1/2)*((d^2-
e^2)/e)^(1/2)*e+e^2)/(-e*x+(-d*e)^(1/2)))*d^(7/2)+B*ln(2*((-d*e)^(1/2)*d*x
-(-d*x^2-e)^(1/2)*((d^2-e^2)/e)^(1/2)*e+e^2)/(-e*x+(-d*e)^(1/2)))*d^(3/2)*
e^2+B*ln(2*((-d*e)^(1/2)*d*x+(-d*x^2-e)^(1/2)*((d^2-e^2)/e)^(1/2)*e-e^2)/(
e*x+(-d*e)^(1/2)))*d^(7/2)-B*ln(2*((-d*e)^(1/2)*d*x+(-d*x^2-e)^(1/2)*((d^2
-e^2)/e)^(1/2)*e-e^2)/(e*x+(-d*e)^(1/2)))*d^(3/2)*e^2)/(e*x^2+d)/(-d*x^2-e
)^(1/2)/(-d*e)^(3/2)/(d+e)/(d-e)/((d^2-e^2)/e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 830, normalized size of antiderivative = 5.57

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2),x,
algorithm="fricas")
```

output
```
[1/2*(sqrt(-d^3 + d*e^2)*(B*d - A*e)*log((2*d^3*x^2 + (2*d^2*e - e^3)*x^4
+ d^2*e - 2*sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)*sqrt(-d^3 + d*e^2)*sqrt(
-e*x^2 - d)*x)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (B*d^2 - B*e^2)*sqrt(-d)*log
((2*d*e*x^4 + (2*d^2 + e^2)*x^2 - 2*sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)*
sqrt(-e*x^2 - d)*sqrt(-d)*x + d*e)/(e*x^2 + d)))/(d^3*e - d*e^3), -1/2*(2*
(B*d^2 - B*e^2)*sqrt(d)*arctan(sqrt(-e*x^2 - d)*sqrt(d)*x/sqrt(d*e*x^4 + (
d^2 + e^2)*x^2 + d*e)) - sqrt(-d^3 + d*e^2)*(B*d - A*e)*log((2*d^3*x^2 + (
2*d^2*e - e^3)*x^4 + d^2*e - 2*sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)*sqrt(
-d^3 + d*e^2)*sqrt(-e*x^2 - d)*x)/(e^2*x^4 + 2*d*e*x^2 + d^2)))/(d^3*e - d
*e^3), 1/2*(2*sqrt(d^3 - d*e^2)*(B*d - A*e)*arctan(sqrt(d*e*x^4 + (d^2 + e
^2)*x^2 + d*e)*sqrt(d^3 - d*e^2)*sqrt(-e*x^2 - d)*x/(d^2*e*x^4 + d^2*e + (
d^3 + d*e^2)*x^2)) - (B*d^2 - B*e^2)*sqrt(-d)*log((2*d*e*x^4 + (2*d^2 + e^
2)*x^2 - 2*sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)*sqrt(-e*x^2 - d)*sqrt(-d)
*x + d*e)/(e*x^2 + d)))/(d^3*e - d*e^3), (sqrt(d^3 - d*e^2)*(B*d - A*e)*ar
ctan(sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)*sqrt(d^3 - d*e^2)*sqrt(-e*x^2 -
 d)*x/(d^2*e*x^4 + d^2*e + (d^3 + d*e^2)*x^2)) - (B*d^2 - B*e^2)*sqrt(d)*a
rctan(sqrt(-e*x^2 - d)*sqrt(d)*x/sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)))/(
d^3*e - d*e^3)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{(d+ex^2)\,(dx^2+e)}\sqrt{-d-ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(-e*x**2-d)**(1/2)/(d*e+(d**2+e**2)*x**2+d*e*x**4)**(
1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt((d + e*x**2)*(d*x**2 + e))*sqrt(-d - e*x**2)),
 x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx$$
$$=\int \frac{Bx^2+A}{\sqrt{dex^4+(d^2+e^2)x^2+de}\sqrt{-ex^2-d}}\,dx$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(d*e*x^4 + (d^2 + e^2)*x^2 + d*e)*sqrt(-e*x^2 -
 d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2),x,
algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{-e\,x^2-d}\,\sqrt{d\,e\,x^4+\left(d^2+e^2\right)\,x^2+d\,e}}\,dx$$

input
```
int((A + B*x^2)/((- d - e*x^2)^(1/2)*(x^2*(d^2 + e^2) + d*e + d*e*x^4)^(1/
2)),x)
```

output
```
int((A + B*x^2)/((- d - e*x^2)^(1/2)*(x^2*(d^2 + e^2) + d*e + d*e*x^4)^(1/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 710, normalized size of antiderivative = 4.77

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{de+(d^2+e^2)\,x^2+dex^4}}\,dx = \frac{\sqrt{d}\,i\left(\sqrt{d^2-e^2}\log\left(2\sqrt{d^2-e^2}\,d+2\sqrt{d}\,\sqrt{d\,x^2+e}\,ex+2d^2+2de\,x^2\right)ae-\sqrt{d^2-e^2}\log\left(2\sqrt{d^2-e^2}\,d+\right.\right.}{}$$

input
```
int((B*x^2+A)/(-e*x^2-d)^(1/2)/(d*e+(d^2+e^2)*x^2+d*e*x^4)^(1/2),x)
```

output

```
(sqrt(d)*i*(sqrt(d**2 - e**2)*log(2*sqrt(d**2 - e**2)*d + 2*sqrt(d)*sqrt(d
*x**2 + e)*e*x + 2*d**2 + 2*d*e*x**2)*a*e - sqrt(d**2 - e**2)*log(2*sqrt(d
**2 - e**2)*d + 2*sqrt(d)*sqrt(d*x**2 + e)*e*x + 2*d**2 + 2*d*e*x**2)*b*d
- sqrt(d**2 - e**2)*log(( - sqrt(e)*sqrt( - d**2 + e**2)*sqrt(d**2 - e**2)
*d + sqrt(e)*sqrt( - d**2 + e**2)*d**2 - sqrt(e)*sqrt( - d**2 + e**2)*e**2
 + sqrt(d*x**2 + e)*d**2*e - sqrt(d*x**2 + e)*e**3 + sqrt(d)*d**2*e*x - sq
rt(d)*e**3*x)/(sqrt(e)*d**2 - sqrt(e)*e**2))*a*e + sqrt(d**2 - e**2)*log((
 - sqrt(e)*sqrt( - d**2 + e**2)*sqrt(d**2 - e**2)*d + sqrt(e)*sqrt( - d**2
 + e**2)*d**2 - sqrt(e)*sqrt( - d**2 + e**2)*e**2 + sqrt(d*x**2 + e)*d**2*
e - sqrt(d*x**2 + e)*e**3 + sqrt(d)*d**2*e*x - sqrt(d)*e**3*x)/(sqrt(e)*d*
*2 - sqrt(e)*e**2))*b*d - sqrt(d**2 - e**2)*log((sqrt(e)*sqrt( - d**2 + e*
*2)*sqrt(d**2 - e**2)*d - sqrt(e)*sqrt( - d**2 + e**2)*d**2 + sqrt(e)*sqrt
( - d**2 + e**2)*e**2 + sqrt(d*x**2 + e)*d**2*e - sqrt(d*x**2 + e)*e**3 +
sqrt(d)*d**2*e*x - sqrt(d)*e**3*x)/(sqrt(e)*d**2 - sqrt(e)*e**2))*a*e + sq
rt(d**2 - e**2)*log((sqrt(e)*sqrt( - d**2 + e**2)*sqrt(d**2 - e**2)*d - sq
rt(e)*sqrt( - d**2 + e**2)*d**2 + sqrt(e)*sqrt( - d**2 + e**2)*e**2 + sqrt
(d*x**2 + e)*d**2*e - sqrt(d*x**2 + e)*e**3 + sqrt(d)*d**2*e*x - sqrt(d)*e
**3*x)/(sqrt(e)*d**2 - sqrt(e)*e**2))*b*d - 2*log((sqrt(d*x**2 + e) + sqrt
(d)*x)/sqrt(e))*b*d**2 + 2*log((sqrt(d*x**2 + e) + sqrt(d)*x)/sqrt(e))*b*e
**2))/(2*d*e*(d**2 - e**2))
```

3.142 $\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-de+(d^2-e^2)x^2+dex^4}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 145

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-de+(d^2-e^2)\,x^2+dex^4}}\,dx = \frac{B\operatorname{arctanh}\left(\frac{\sqrt{d}x\sqrt{d+ex^2}}{\sqrt{-de+(d^2-e^2)x^2+dex^4}}\right)}{\sqrt{d}e} - \frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{\sqrt{d^2+e^2}x\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-de+(d^2-e^2)x^2+dex^4}}\right)}{\sqrt{d}e\sqrt{d^2+e^2}}$$

output
```
B*arctanh(d^(1/2)*x*(e*x^2+d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2))/d^
(1/2)/e-(-A*e+B*d)*arctanh((d^2+e^2)^(1/2)*x*(e*x^2+d)^(1/2)/d^(1/2)/(-d*e
+(d^2-e^2)*x^2+d*e*x^4)^(1/2))/d^(1/2)/e/(d^2+e^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-de+(d^2-e^2)\,x^2+dex^4}}\,dx = \frac{\sqrt{-e+dx^2}\sqrt{d+ex^2}\left((-Bd+Ae)\operatorname{arctanh}\left(\frac{d^{3/2}+\sqrt{d}ex^2-ex\sqrt{-e+dx^2}}{\sqrt{d}\sqrt{d^2+e^2}}\right) - B\sqrt{d^2+e^2}\log\left(-\sqrt{d}x+\sqrt{-e+d\right.\right.}{\sqrt{d}e\sqrt{d^2+e^2}\sqrt{d^2x^2-e^2x^2+de\,(-1+x^4)}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[-(d*e) + (d^2 - e^2)*x^2 + d*e
*x^4]),x]
```

output
```
(Sqrt[-e + d*x^2]*Sqrt[d + e*x^2]*((-(B*d) + A*e)*ArcTanh[(d^(3/2) + Sqrt[
d]*e*x^2 - e*x*Sqrt[-e + d*x^2])/(Sqrt[d]*Sqrt[d^2 + e^2])] - B*Sqrt[d^2 +
 e^2]*Log[-(Sqrt[d]*x) + Sqrt[-e + d*x^2]]))/(Sqrt[d]*e*Sqrt[d^2 + e^2]*Sq
rt[d^2*x^2 - e^2*x^2 + d*e*(-1 + x^4)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {1395, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{x^2\left(d^2-e^2\right)+dex^4-de}}dx$$

$$\Big\downarrow \text{1395}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d+ex^2}\int \frac{Bx^2+A}{\sqrt{dx^2-e}(ex^2+d)}dx}{\sqrt{x^2\left(d^2-e^2\right)+dex^4-de}}$$

$$\Big\downarrow \text{398}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d+ex^2}\left(\frac{B\int \frac{1}{\sqrt{dx^2-e}}dx}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{dx^2-e}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{x^2\left(d^2-e^2\right)+dex^4-de}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d+ex^2}\left(\frac{B\int \frac{1}{1-\frac{dx^2}{dx^2-e}}d\frac{x}{\sqrt{dx^2-e}}}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{dx^2-e}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{x^2\left(d^2-e^2\right)+dex^4-de}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d+ex^2}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2-e}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\int\frac{1}{\sqrt{dx^2-e}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{x^2\left(d^2-e^2\right)+dex^4-de}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d+ex^2}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2-e}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\int\frac{1}{d-\frac{\left(d^2+e^2\right)x^2}{dx^2-e}}d\frac{x}{\sqrt{dx^2-e}}}{e}\right)}{\sqrt{x^2\left(d^2-e^2\right)+dex^4-de}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{dx^2-e}\sqrt{d+ex^2}\left(\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2-e}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{x\sqrt{d^2+e^2}}{\sqrt{d}\sqrt{dx^2-e}}\right)}{\sqrt{d}e\sqrt{d^2+e^2}}\right)}{\sqrt{x^2\left(d^2-e^2\right)+dex^4-de}}$$

input
```
Int[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[-(d*e) + (d^2 - e^2)*x^2 + d*e*x^4])
,x]
```

output
```
(Sqrt[-e + d*x^2]*Sqrt[d + e*x^2]*((B*ArcTanh[(Sqrt[d]*x)/Sqrt[-e + d*x^2]
])/(Sqrt[d]*e) - ((B*d - A*e)*ArcTanh[(Sqrt[d^2 + e^2]*x)/(Sqrt[d]*Sqrt[-e
 + d*x^2])])/(Sqrt[d]*e*Sqrt[d^2 + e^2])))/Sqrt[-(d*e) + (d^2 - e^2)*x^2 +
 d*e*x^4]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(125) = 250$.

Time = 0.21 (sec) , antiderivative size = 858, normalized size of antiderivative = 5.92

method	result
default	$\frac{\sqrt{de\,x^4+d^2x^2-e^2x^2-de}\,\sqrt{d}\left(2A\sqrt{-\frac{d^2+e^2}{e}}\,\sqrt{-de}\,d\ln\left(\frac{\sqrt{d}\,\sqrt{d\,x^2-e}+dx}{\sqrt{d}}\right)e-2B\sqrt{-\frac{d^2+e^2}{e}}\,\sqrt{-de}\,d^2\ln\left(\frac{\sqrt{d}\,\sqrt{d\,x^2-e}+dx}{\sqrt{d}}\right)-2A\sqrt{-\right.}{}$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
1/2/(e*x^2+d)^(1/2)*(d*e*x^4+d^2*x^2-e^2*x^2-d*e)^(1/2)*d^(1/2)*(2*A*(-(d^
2+e^2)/e)^(1/2)*(-d*e)^(1/2)*d*ln((d^(1/2)*(d*x^2-e)^(1/2)+d*x)/d^(1/2))*e
-2*B*(-(d^2+e^2)/e)^(1/2)*(-d*e)^(1/2)*d^2*ln((d^(1/2)*(d*x^2-e)^(1/2)+d*x
)/d^(1/2))-2*A*(-(d^2+e^2)/e)^(1/2)*(-d*e)^(1/2)*ln((d^(1/2)*(-1/d*(d*x+(d
*e)^(1/2))*(-d*x+(d*e)^(1/2)))^(1/2)+d*x)/d^(1/2))*d*e+A*d^(5/2)*ln(2*((-(
d^2+e^2)/e)^(1/2)*(d*x^2-e)^(1/2)*e+(-d*e)^(1/2)*d*x-e^2)/(e*x-(-d*e)^(1/2
)))*e+A*d^(1/2)*ln(2*((-(d^2+e^2)/e)^(1/2)*(d*x^2-e)^(1/2)*e+(-d*e)^(1/2)*
d*x-e^2)/(e*x-(-d*e)^(1/2)))*e^3-A*d^(5/2)*ln(-2*((-d*e)^(1/2)*d*x-(-(d^2+
e^2)/e)^(1/2)*(d*x^2-e)^(1/2)*e+e^2)/(e*x+(-d*e)^(1/2)))*e-A*d^(1/2)*ln(-2
*((-d*e)^(1/2)*d*x-(-(d^2+e^2)/e)^(1/2)*(d*x^2-e)^(1/2)*e+e^2)/(e*x+(-d*e)
^(1/2)))*e^3-2*B*(-(d^2+e^2)/e)^(1/2)*(-d*e)^(1/2)*ln((d^(1/2)*(-1/d*(d*x+
(d*e)^(1/2))*(-d*x+(d*e)^(1/2)))^(1/2)+d*x)/d^(1/2))*e^2-B*d^(7/2)*ln(2*((
-(d^2+e^2)/e)^(1/2)*(d*x^2-e)^(1/2)*e+(-d*e)^(1/2)*d*x-e^2)/(e*x-(-d*e)^(1
/2)))-B*d^(3/2)*ln(2*((-(d^2+e^2)/e)^(1/2)*(d*x^2-e)^(1/2)*e+(-d*e)^(1/2)*
d*x-e^2)/(e*x-(-d*e)^(1/2)))*e^2+B*d^(7/2)*ln(-2*((-d*e)^(1/2)*d*x-(-(d^2+
e^2)/e)^(1/2)*(d*x^2-e)^(1/2)*e+e^2)/(e*x+(-d*e)^(1/2)))+B*d^(3/2)*ln(-2*(
(-d*e)^(1/2)*d*x-(-(d^2+e^2)/e)^(1/2)*(d*x^2-e)^(1/2)*e+e^2)/(e*x+(-d*e)^(
1/2)))*e^2)/(d*x^2-e)^(1/2)/(-d*e)^(1/2)/((-d*e)^(1/2)*d+e*(d*e)^(1/2))/((
-d*e)^(1/2)*d-e*(d*e)^(1/2))/(-(d^2+e^2)/e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.87

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-de+(d^2-e^2)\,x^2+dex^4}}\,dx$$
$$=\left[-\frac{\sqrt{d^3+de^2}(Bd-Ae)\log\left(-\frac{2\,d^3x^2+(2\,d^2e+e^3)x^4-d^2e+2\,\sqrt{dex^4+(d^2-e^2)x^2-de}\sqrt{d^3+de^2}\sqrt{ex^2+dx}}{e^2x^4+2\,dex^2+d^2}\right)-(Bd^2+Be^2)\cdot}{2\,(d^3e+de^3)}\right.$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2),x,
algorithm="fricas")
```

output
```
[-1/2*(sqrt(d^3 + d*e^2)*(B*d - A*e)*log(-(2*d^3*x^2 + (2*d^2*e + e^3)*x^4
 - d^2*e + 2*sqrt(d*e*x^4 + (d^2 - e^2)*x^2 - d*e)*sqrt(d^3 + d*e^2)*sqrt(
e*x^2 + d)*x)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (B*d^2 + B*e^2)*sqrt(d)*log(-
(2*d*e*x^4 + (2*d^2 - e^2)*x^2 + 2*sqrt(d*e*x^4 + (d^2 - e^2)*x^2 - d*e)*s
qrt(e*x^2 + d)*sqrt(d)*x - d*e)/(e*x^2 + d)))/(d^3*e + d*e^3), (sqrt(-d^3
- d*e^2)*(B*d - A*e)*arctan(sqrt(d*e*x^4 + (d^2 - e^2)*x^2 - d*e)*sqrt(-d^
3 - d*e^2)*sqrt(e*x^2 + d)*x/(d^2*e*x^4 - d^2*e + (d^3 - d*e^2)*x^2)) - (B
*d^2 + B*e^2)*sqrt(-d)*arctan(sqrt(e*x^2 + d)*sqrt(-d)*x/sqrt(d*e*x^4 + (d
^2 - e^2)*x^2 - d*e)))/(d^3*e + d*e^3)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-de+(d^2-e^2)\,x^2+dex^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{(d+ex^2)\,(dx^2-e)}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(1/2)/(-d*e+(d**2-e**2)*x**2+d*e*x**4)**(
1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt((d + e*x**2)*(d*x**2 - e))*sqrt(d + e*x**2)),
x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-de+(d^2-e^2)\,x^2+dex^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{dex^4+(d^2-e^2)x^2-de}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(d*e*x^4 + (d^2 - e^2)*x^2 - d*e)*sqrt(e*x^2 +
d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-de+(d^2-e^2)x^2+dex^4}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2),x,
algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-de+(d^2-e^2)x^2+dex^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{e\,x^2+d}\,\sqrt{d\,e\,x^4+(d^2-e^2)\,x^2-d\,e}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^(1/2)*(x^2*(d^2 - e^2) - d*e + d*e*x^4)^(1/2)
),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^(1/2)*(x^2*(d^2 - e^2) - d*e + d*e*x^4)^(1/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.71

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{-de+(d^2-e^2)x^2+dex^4}}\,dx$$
$$= \frac{\sqrt{d}\left(-\sqrt{d^2+e^2}\log\left(2\sqrt{d^2+e^2}\,d+2\sqrt{d}\,\sqrt{d\,x^2-e}\,ex+2d^2+2de\,x^2\right)ae+\sqrt{d^2+e^2}\log\left(2\sqrt{d^2+e^2}\,d+\right.\right.}{}$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2),x)
```

output
```
(sqrt(d)*( - sqrt(d**2 + e**2)*log(2*sqrt(d**2 + e**2)*d + 2*sqrt(d)*sqrt(
d*x**2 - e)*e*x + 2*d**2 + 2*d*e*x**2)*a*e + sqrt(d**2 + e**2)*log(2*sqrt(
d**2 + e**2)*d + 2*sqrt(d)*sqrt(d*x**2 - e)*e*x + 2*d**2 + 2*d*e*x**2)*b*d
 + sqrt(d**2 + e**2)*log(( - sqrt(e)*sqrt(d**2 + e**2)*i + sqrt(d*x**2 - e
)*e + sqrt(d)*e*x + sqrt(e)*d*i)/sqrt(e))*a*e - sqrt(d**2 + e**2)*log(( -
sqrt(e)*sqrt(d**2 + e**2)*i + sqrt(d*x**2 - e)*e + sqrt(d)*e*x + sqrt(e)*d
*i)/sqrt(e))*b*d + sqrt(d**2 + e**2)*log((sqrt(e)*sqrt(d**2 + e**2)*i + sq
rt(d*x**2 - e)*e + sqrt(d)*e*x - sqrt(e)*d*i)/sqrt(e))*a*e - sqrt(d**2 + e
**2)*log((sqrt(e)*sqrt(d**2 + e**2)*i + sqrt(d*x**2 - e)*e + sqrt(d)*e*x -
 sqrt(e)*d*i)/sqrt(e))*b*d + 2*log((sqrt(d*x**2 - e) + sqrt(d)*x)/sqrt(e))
*b*d**2 + 2*log((sqrt(d*x**2 - e) + sqrt(d)*x)/sqrt(e))*b*e**2))/(2*d*e*(d
**2 + e**2))
```

3.143 $\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{de+(d^2-e^2)x^2-dex^4}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 146

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{de+(d^2-e^2)\,x^2-dex^4}}\,dx = -\frac{B\operatorname{arctanh}\left(\frac{\sqrt{d}x\sqrt{d-ex^2}}{\sqrt{de+(d^2-e^2)x^2-dex^4}}\right)}{\sqrt{d}e} + \frac{(Bd+Ae)\operatorname{arctanh}\left(\frac{\sqrt{d^2+e^2}x\sqrt{d-ex^2}}{\sqrt{d}\sqrt{de+(d^2-e^2)x^2-dex^4}}\right)}{\sqrt{d}e\sqrt{d^2+e^2}}$$

output
```
-B*arctanh(d^(1/2)*x*(-e*x^2+d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2))/d
^(1/2)/e+(A*e+B*d)*arctanh((d^2+e^2)^(1/2)*x*(-e*x^2+d)^(1/2)/d^(1/2)/(d*e
+(d^2-e^2)*x^2-d*e*x^4)^(1/2))/d^(1/2)/e/(d^2+e^2)^(1/2)
```

Mathematica [A] (verified)

Time = 11.71 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{de+(d^2-e^2)\,x^2-dex^4}}\,dx = \frac{\frac{(Bd+Ae)\sqrt{d^2x^2-e^2x^2+d(e-ex^4)}\operatorname{arctanh}\left(\frac{\sqrt{d^2+e^2}x}{\sqrt{d}\sqrt{e+dx^2}}\right)}{\sqrt{d^2+e^2}\sqrt{e+dx^2}\sqrt{d-ex^2}} + B\left(\log\left(d-ex^2\right) - \log\left(d^2x - dex^3 + \sqrt{d}\sqrt{d-ex^2}\sqrt{de-\right.\right.}{\sqrt{d}e}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d - e*x^2]*Sqrt[d*e + (d^2 - e^2)*x^2 - d*e*x^
4]),x]
```

output
```
(((B*d + A*e)*Sqrt[d^2*x^2 - e^2*x^2 + d*(e - e*x^4)]*ArcTanh[(Sqrt[d^2 +
e^2]*x)/(Sqrt[d]*Sqrt[e + d*x^2])])/(Sqrt[d^2 + e^2]*Sqrt[e + d*x^2]*Sqrt[
d - e*x^2]) + B*(Log[d - e*x^2] - Log[d^2*x - d*e*x^3 + Sqrt[d]*Sqrt[d - e
*x^2]*Sqrt[d*e + d^2*x^2 - e^2*x^2 - d*e*x^4]]))/(Sqrt[d]*e)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1395, 398, 224, 219, 291, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}dx$$

$$\Big\downarrow \text{1395}$$

$$\frac{\sqrt{dx^2+e}\sqrt{d-ex^2}\int \frac{Bx^2+A}{\sqrt{dx^2+e}(d-ex^2)}dx}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

$$\Big\downarrow \text{398}$$

$$\frac{\sqrt{dx^2+e}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{dx^2+e}\left(d-ex^2\right)}dx}{e}-\frac{B\int \frac{1}{\sqrt{dx^2+e}}dx}{e}\right)}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{dx^2+e}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{dx^2+e}\left(d-ex^2\right)}dx}{e}-\frac{B\int \frac{1}{1-\frac{dx^2}{dx^2+e}}d\frac{x}{\sqrt{dx^2+e}}}{e}\right)}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{dx^2+e}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{dx^2+e}\left(d-ex^2\right)}dx}{e}-\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2+e}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\sqrt{dx^2+e}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\int \frac{1}{d-\frac{\left(d^2+e^2\right)x^2}{dx^2+e}}d\frac{x}{\sqrt{dx^2+e}}}{e}-\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2+e}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\sqrt{dx^2+e}\sqrt{d-ex^2}\left(\frac{(Ae+Bd)\text{arctan}\left(\frac{x\sqrt{d^2+e^2}}{\sqrt{d}\sqrt{dx^2+e}}\right)}{\sqrt{d}e\sqrt{d^2+e^2}}-\frac{B\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{dx^2+e}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

input
```
Int[(A + B*x^2)/(Sqrt[d - e*x^2]*Sqrt[d*e + (d^2 - e^2)*x^2 - d*e*x^4]),x]
```

output
```
(Sqrt[e + d*x^2]*Sqrt[d - e*x^2]*(-((B*ArcTanh[(Sqrt[d]*x)/Sqrt[e + d*x^2]
])/(Sqrt[d]*e)) + ((B*d + A*e)*ArcTan[(Sqrt[d^2 + e^2]*x)/(Sqrt[d]*Sqrt[e
 + d*x^2])])/(Sqrt[d]*e*Sqrt[d^2 + e^2])))/Sqrt[d*e + (d^2 - e^2)*x^2 - d*
e*x^4]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(126) = 252$.

Time = 0.19 (sec) , antiderivative size = 807, normalized size of antiderivative = 5.53

method	result
default	$-\frac{\sqrt{-de\,x^4+d^2x^2-e^2x^2+de}\,\sqrt{d}\left(2A\sqrt{\frac{d^2+e^2}{e}}\,d\sqrt{de}\,\ln\left(\frac{\sqrt{d}\,\sqrt{d\,x^2+e}+dx}{\sqrt{d}}\right)e+2B\sqrt{\frac{d^2+e^2}{e}}\,d^2\sqrt{de}\,\ln\left(\frac{\sqrt{d}\,\sqrt{d\,x^2+e}+dx}{\sqrt{d}}\right)-2A\sqrt{\frac{d^2+e^2}{e}}\right.}{}$

input
```
int((B*x^2+A)/(-e*x^2+d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
-1/2*(-d*e*x^4+d^2*x^2-e^2*x^2+d*e)^(1/2)*d^(1/2)*(2*A*((d^2+e^2)/e)^(1/2)
*d*(d*e)^(1/2)*ln((d^(1/2)*(d*x^2+e)^(1/2)+d*x)/d^(1/2))*e+2*B*((d^2+e^2)/
e)^(1/2)*d^2*(d*e)^(1/2)*ln((d^(1/2)*(d*x^2+e)^(1/2)+d*x)/d^(1/2))-2*A*((d
^2+e^2)/e)^(1/2)*(d*e)^(1/2)*ln((d^(1/2)*(-1/d*(-d*x+(-d*e)^(1/2))*(d*x+(-
d*e)^(1/2)))^(1/2)+d*x)/d^(1/2))*d*e-A*d^(5/2)*ln(2*((d*x^2+e)^(1/2)*((d^2
+e^2)/e)^(1/2)*e+(d*e)^(1/2)*d*x+e^2)/(e*x-(d*e)^(1/2)))*e-A*d^(1/2)*ln(2*
((d*x^2+e)^(1/2)*((d^2+e^2)/e)^(1/2)*e+(d*e)^(1/2)*d*x+e^2)/(e*x-(d*e)^(1/
2)))*e^3+A*d^(5/2)*ln(-2*((d*e)^(1/2)*d*x-(d*x^2+e)^(1/2)*((d^2+e^2)/e)^(1
/2)*e-e^2)/(e*x+(d*e)^(1/2)))*e+A*d^(1/2)*ln(-2*((d*e)^(1/2)*d*x-(d*x^2+e)
^(1/2)*((d^2+e^2)/e)^(1/2)*e-e^2)/(e*x+(d*e)^(1/2)))*e^3+2*B*((d^2+e^2)/e)
^(1/2)*(d*e)^(1/2)*ln((d^(1/2)*(-1/d*(-d*x+(-d*e)^(1/2))*(d*x+(-d*e)^(1/2)
))^(1/2)+d*x)/d^(1/2))*e^2-B*d^(7/2)*ln(2*((d*x^2+e)^(1/2)*((d^2+e^2)/e)^(
1/2)*e+(d*e)^(1/2)*d*x+e^2)/(e*x-(d*e)^(1/2)))-B*d^(3/2)*ln(2*((d*x^2+e)^(
1/2)*((d^2+e^2)/e)^(1/2)*e+(d*e)^(1/2)*d*x+e^2)/(e*x-(d*e)^(1/2)))*e^2+B*d
^(7/2)*ln(-2*((d*e)^(1/2)*d*x-(d*x^2+e)^(1/2)*((d^2+e^2)/e)^(1/2)*e-e^2)/(
e*x+(d*e)^(1/2)))+B*d^(3/2)*ln(-2*((d*e)^(1/2)*d*x-(d*x^2+e)^(1/2)*((d^2+e
^2)/e)^(1/2)*e-e^2)/(e*x+(d*e)^(1/2)))*e^2)/(-e*x^2+d)^(1/2)/(d*x^2+e)^(1/
2)/((d*e)^(1/2)*d-(-d*e)^(1/2)*e)/((-d*e)^(1/2)*e+(d*e)^(1/2)*d)/(d*e)^(1/
2)/((d^2+e^2)/e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 446, normalized size of antiderivative = 3.05

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{de+(d^2-e^2)\,x^2-dex^4}}\,dx$$
$$= \left[\frac{\sqrt{d^3+de^2}(Bd+Ae)\log\left(\frac{2\,d^3x^2-(2\,d^2e+e^3)x^4+d^2e+2\,\sqrt{-dex^4+(d^2-e^2)x^2+de}\sqrt{d^3+de^2}\sqrt{-ex^2+d}x}{e^2x^4-2\,dex^2+d^2}\right)+(Bd^2+Be^2)\sqrt{}}{2\,(d^3e+de^3)} \right.$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2),x,
algorithm="fricas")
```

output
```
[1/2*(sqrt(d^3 + d*e^2)*(B*d + A*e)*log((2*d^3*x^2 - (2*d^2*e + e^3)*x^4 +
 d^2*e + 2*sqrt(-d*e*x^4 + (d^2 - e^2)*x^2 + d*e)*sqrt(d^3 + d*e^2)*sqrt(-
e*x^2 + d)*x)/(e^2*x^4 - 2*d*e*x^2 + d^2)) + (B*d^2 + B*e^2)*sqrt(d)*log((
2*d*e*x^4 - (2*d^2 - e^2)*x^2 + 2*sqrt(-d*e*x^4 + (d^2 - e^2)*x^2 + d*e)*s
qrt(-e*x^2 + d)*sqrt(d)*x - d*e)/(e*x^2 - d)))/(d^3*e + d*e^3), (sqrt(-d^3
 - d*e^2)*(B*d + A*e)*arctan(sqrt(-d*e*x^4 + (d^2 - e^2)*x^2 + d*e)*sqrt(-
d^3 - d*e^2)*sqrt(-e*x^2 + d)*x/(d^2*e*x^4 - d^2*e - (d^3 - d*e^2)*x^2)) -
 (B*d^2 + B*e^2)*sqrt(-d)*arctan(sqrt(-d*e*x^4 + (d^2 - e^2)*x^2 + d*e)*sq
rt(-e*x^2 + d)*sqrt(-d)*x/(d*e*x^4 - (d^2 - e^2)*x^2 - d*e)))/(d^3*e + d*e
^3)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{de+(d^2-e^2)\,x^2-dex^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{-\left(-d+ex^2\right)\left(dx^2+e\right)}\sqrt{d-ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(-e*x**2+d)**(1/2)/(d*e+(d**2-e**2)*x**2-d*e*x**4)**(
1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(-(-d + e*x**2)*(d*x**2 + e))*sqrt(d - e*x**2))
, x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{de+(d^2-e^2)\,x^2-dex^4}}\,dx$$
$$= \int \frac{Bx^2+A}{\sqrt{-dex^4+(d^2-e^2)x^2+de}\sqrt{-ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-d*e*x^4 + (d^2 - e^2)*x^2 + d*e)*sqrt(-e*x^2
+ d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{de+(d^2-e^2)\,x^2-dex^4}}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)/(-e*x^2+d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2),x,
algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{de+(d^2-e^2)\,x^2-dex^4}}\,dx$$
$$=\int \frac{B\,x^2+A}{\sqrt{d-e\,x^2}\,\sqrt{-d\,e\,x^4+(d^2-e^2)\,x^2+d\,e}}\,dx$$

input
```
int((A + B*x^2)/((d - e*x^2)^(1/2)*(d*e + x^2*(d^2 - e^2) - d*e*x^4)^(1/2)
),x)
```

output
```
int((A + B*x^2)/((d - e*x^2)^(1/2)*(d*e + x^2*(d^2 - e^2) - d*e*x^4)^(1/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.53

$$\int \frac{A+Bx^2}{\sqrt{d-ex^2}\sqrt{de+(d^2-e^2)x^2-dex^4}}\,dx$$
$$= \frac{\sqrt{d}\left(\sqrt{d^2+e^2}\log\left(2\sqrt{d^2+e^2}\,d+2\sqrt{d}\,\sqrt{d\,x^2+e}\,ex-2d^2+2de\,x^2\right)ae+\sqrt{d^2+e^2}\log\left(2\sqrt{d^2+e^2}\,d+2\right.}{}$$

input
```
int((B*x^2+A)/(-e*x^2+d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2),x)
```

output
```
(sqrt(d)*(sqrt(d**2 + e**2)*log(2*sqrt(d**2 + e**2)*d + 2*sqrt(d)*sqrt(d*x
**2 + e)*e*x - 2*d**2 + 2*d*e*x**2)*a*e + sqrt(d**2 + e**2)*log(2*sqrt(d**
2 + e**2)*d + 2*sqrt(d)*sqrt(d*x**2 + e)*e*x - 2*d**2 + 2*d*e*x**2)*b*d -
sqrt(d**2 + e**2)*log(( - sqrt(e)*sqrt(d**2 + e**2) + sqrt(d*x**2 + e)*e +
 sqrt(d)*e*x - sqrt(e)*d)/sqrt(e))*a*e - sqrt(d**2 + e**2)*log(( - sqrt(e)
*sqrt(d**2 + e**2) + sqrt(d*x**2 + e)*e + sqrt(d)*e*x - sqrt(e)*d)/sqrt(e)
)*b*d - sqrt(d**2 + e**2)*log((sqrt(e)*sqrt(d**2 + e**2) + sqrt(d*x**2 + e
)*e + sqrt(d)*e*x + sqrt(e)*d)/sqrt(e))*a*e - sqrt(d**2 + e**2)*log((sqrt(
e)*sqrt(d**2 + e**2) + sqrt(d*x**2 + e)*e + sqrt(d)*e*x + sqrt(e)*d)/sqrt(
e))*b*d - 2*log((sqrt(d*x**2 + e) + sqrt(d)*x)/sqrt(e))*b*d**2 - 2*log((sq
rt(d*x**2 + e) + sqrt(d)*x)/sqrt(e))*b*e**2))/(2*d*e*(d**2 + e**2))
```

3.144 $\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{de+(d^2-e^2)x^2-dex^4}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 148

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{de+(d^2-e^2)x^2-dex^4}}\,dx = \frac{B\arctan\left(\frac{\sqrt{d}x\sqrt{-d+ex^2}}{\sqrt{de+(d^2-e^2)x^2-dex^4}}\right)}{\sqrt{d}e} - \frac{(Bd+Ae)\arctan\left(\frac{\sqrt{d^2+e^2}x\sqrt{-d+ex^2}}{\sqrt{d}\sqrt{de+(d^2-e^2)x^2-dex^4}}\right)}{\sqrt{d}e\sqrt{d^2+e^2}}$$

output
```
B*arctan(d^(1/2)*x*(e*x^2-d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2))/d^(1
/2)/e-(A*e+B*d)*arctan((d^2+e^2)^(1/2)*x*(e*x^2-d)^(1/2)/d^(1/2)/(d*e+(d^2
-e^2)*x^2-d*e*x^4)^(1/2))/d^(1/2)/e/(d^2+e^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.71 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{de+(d^2-e^2)x^2-dex^4}}\,dx = \frac{-\frac{(Bd+Ae)\sqrt{-e-dx^2}\sqrt{-d+ex^2}\,\text{arctanh}\left(\frac{\sqrt{-d^2-e^2}x}{\sqrt{d}\sqrt{-e-dx^2}}\right)}{\sqrt{-d^2-e^2}\sqrt{d^2x^2-e^2x^2+d(e-ex^4)}} - iB\log\left(-2i\sqrt{d}x - \frac{2\sqrt{de+d^2x^2-e^2x^2-dex^4}}{\sqrt{-d+ex^2}}\right)}{\sqrt{d}e}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[-d + e*x^2]*Sqrt[d*e + (d^2 - e^2)*x^2 - d*e*x
^4]),x]
```

output
```
(-(((B*d + A*e)*Sqrt[-e - d*x^2]*Sqrt[-d + e*x^2]*ArcTanh[(Sqrt[-d^2 - e^2
]*x)/(Sqrt[d]*Sqrt[-e - d*x^2])])/(Sqrt[-d^2 - e^2]*Sqrt[d^2*x^2 - e^2*x^2
 + d*(e - e*x^4)])) - I*B*Log[(-2*I)*Sqrt[d]*x - (2*Sqrt[d*e + d^2*x^2 - e
^2*x^2 - d*e*x^4])/Sqrt[-d + e*x^2]])/(Sqrt[d]*e)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1395, 25, 398, 224, 216, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{ex^2-d}\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}dx$$

$$\Big\downarrow \text{1395}$$

$$\frac{\sqrt{-dx^2-e}\sqrt{ex^2-d}\int -\frac{Bx^2+A}{\sqrt{-dx^2-e}(d-ex^2)}dx}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{ex^2-d}\int \frac{Bx^2+A}{\sqrt{-dx^2-e}(d-ex^2)}dx}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{-dx^2-e}\left(d-ex^2\right)}dx}{e}-\frac{B\int \frac{1}{\sqrt{-dx^2-e}}dx}{e}\right)}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{-dx^2-e}\left(d-ex^2\right)}dx}{e}-\frac{B\int \frac{1}{\frac{dx^2}{-dx^2-e}+1}d\frac{x}{\sqrt{-dx^2-e}}}{e}\right)}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

$$\Big\downarrow \textbf{216}$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{\sqrt{-dx^2-e}\left(d-ex^2\right)}dx}{e}-\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{-dx^2-e}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

$$\Big\downarrow \textbf{291}$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\int \frac{1}{d-\frac{\left(-d^2-e^2\right)x^2}{-dx^2-e}}d\frac{x}{\sqrt{-dx^2-e}}}{e}-\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{-dx^2-e}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

$$\Big\downarrow \textbf{218}$$

$$-\frac{\sqrt{-dx^2-e}\sqrt{ex^2-d}\left(\frac{(Ae+Bd)\arctan\left(\frac{x\sqrt{d^2+e^2}}{\sqrt{d}\sqrt{-dx^2-e}}\right)}{\sqrt{d}e\sqrt{d^2+e^2}}-\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{-dx^2-e}}\right)}{\sqrt{d}e}\right)}{\sqrt{x^2\left(d^2-e^2\right)-dex^4+de}}$$

input
```
Int[(A + B*x^2)/(Sqrt[-d + e*x^2]*Sqrt[d*e + (d^2 - e^2)*x^2 - d*e*x^4]),x
]
```

output
```
-((Sqrt[-e - d*x^2]*Sqrt[-d + e*x^2]*(-((B*ArcTan[(Sqrt[d]*x)/Sqrt[-e - d*
x^2]])/(Sqrt[d]*e)) + ((B*d + A*e)*ArcTan[(Sqrt[d^2 + e^2]*x)/(Sqrt[d]*Sqr
t[-e - d*x^2])])/(Sqrt[d]*e*Sqrt[d^2 + e^2])))/Sqrt[d*e + (d^2 - e^2)*x^2
- d*e*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(128) = 256$.

Time = 0.08 (sec) , antiderivative size = 824, normalized size of antiderivative = 5.57

method	result
default	$-\frac{\sqrt{-de\,x^4+d^2x^2-e^2x^2+de}\left(-2A\sqrt{de}\,\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{-d\,x^2-e}}\right)\sqrt{-\frac{d^2+e^2}{e}}\,de-2B\sqrt{de}\,\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{-d\,x^2-e}}\right)\sqrt{-\frac{d^2+e^2}{e}}\,d^2+2A\sqrt{de}\,\mathrm{a}\right.}{}$

input
```
int((B*x^2+A)/(e*x^2-d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
-1/2*(-d*e*x^4+d^2*x^2-e^2*x^2+d*e)^(1/2)*(-2*A*(d*e)^(1/2)*arctan(d^(1/2)
*x/(-d*x^2-e)^(1/2))*(-(d^2+e^2)/e)^(1/2)*d*e-2*B*(d*e)^(1/2)*arctan(d^(1/
2)*x/(-d*x^2-e)^(1/2))*(-(d^2+e^2)/e)^(1/2)*d^2+2*A*(d*e)^(1/2)*arctan(d^(
1/2)*x/(1/d*(-d*x+(-d*e)^(1/2))*(d*x+(-d*e)^(1/2)))^(1/2))*(-(d^2+e^2)/e)^
(1/2)*d*e+A*ln(-2*((d*e)^(1/2)*d*x-(-d*x^2-e)^(1/2)*(-(d^2+e^2)/e)^(1/2)*e
+e^2)/(e*x-(d*e)^(1/2)))*d^(5/2)*e+A*ln(-2*((d*e)^(1/2)*d*x-(-d*x^2-e)^(1/
2)*(-(d^2+e^2)/e)^(1/2)*e+e^2)/(e*x-(d*e)^(1/2)))*d^(1/2)*e^3-A*ln(2*((d*e
)^(1/2)*d*x+(-d*x^2-e)^(1/2)*(-(d^2+e^2)/e)^(1/2)*e-e^2)/(e*x+(d*e)^(1/2))
)*d^(5/2)*e-A*ln(2*((d*e)^(1/2)*d*x+(-d*x^2-e)^(1/2)*(-(d^2+e^2)/e)^(1/2)*
e-e^2)/(e*x+(d*e)^(1/2)))*d^(1/2)*e^3-2*B*(d*e)^(1/2)*arctan(d^(1/2)*x/(1/
d*(-d*x+(-d*e)^(1/2))*(d*x+(-d*e)^(1/2)))^(1/2))*(-(d^2+e^2)/e)^(1/2)*e^2+
B*ln(-2*((d*e)^(1/2)*d*x-(-d*x^2-e)^(1/2)*(-(d^2+e^2)/e)^(1/2)*e+e^2)/(e*x
-(d*e)^(1/2)))*d^(7/2)+B*ln(-2*((d*e)^(1/2)*d*x-(-d*x^2-e)^(1/2)*(-(d^2+e^
2)/e)^(1/2)*e+e^2)/(e*x-(d*e)^(1/2)))*d^(3/2)*e^2-B*ln(2*((d*e)^(1/2)*d*x+
(-d*x^2-e)^(1/2)*(-(d^2+e^2)/e)^(1/2)*e-e^2)/(e*x+(d*e)^(1/2)))*d^(7/2)-B*
ln(2*((d*e)^(1/2)*d*x+(-d*x^2-e)^(1/2)*(-(d^2+e^2)/e)^(1/2)*e-e^2)/(e*x+(d
*e)^(1/2)))*d^(3/2)*e^2)*d^(1/2)/(e*x^2-d)^(1/2)/(-d*x^2-e)^(1/2)/((d*e)^(
1/2)*d-(-d*e)^(1/2)*e)/((-d*e)^(1/2)*e+(d*e)^(1/2)*d)/(d*e)^(1/2)/(-(d^2+e
^2)/e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 450, normalized size of antiderivative = 3.04

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{de+(d^2-e^2)\,x^2-dex^4}}\,dx$$
$$= \left[-\frac{\sqrt{-d^3-de^2}(Bd+Ae)\log\left(\frac{2\,d^3x^2-(2\,d^2e+e^3)x^4+d^2e-2\,\sqrt{-dex^4+(d^2-e^2)x^2+de}\sqrt{-d^3-de^2}\sqrt{ex^2-d}x}{e^2x^4-2\,dex^2+d^2}\right)+(Bd^2+Be}{2\,(d^3e+de^3)}\right.$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2),x, a
lgorithm="fricas")
```

output
```
[-1/2*(sqrt(-d^3 - d*e^2)*(B*d + A*e)*log((2*d^3*x^2 - (2*d^2*e + e^3)*x^4
 + d^2*e - 2*sqrt(-d*e*x^4 + (d^2 - e^2)*x^2 + d*e)*sqrt(-d^3 - d*e^2)*sqr
t(e*x^2 - d)*x)/(e^2*x^4 - 2*d*e*x^2 + d^2)) + (B*d^2 + B*e^2)*sqrt(-d)*lo
g((2*d*e*x^4 - (2*d^2 - e^2)*x^2 - 2*sqrt(-d*e*x^4 + (d^2 - e^2)*x^2 + d*e
)*sqrt(e*x^2 - d)*sqrt(-d)*x - d*e)/(e*x^2 - d)))/(d^3*e + d*e^3), (sqrt(d
^3 + d*e^2)*(B*d + A*e)*arctan(sqrt(-d*e*x^4 + (d^2 - e^2)*x^2 + d*e)*sqrt
(d^3 + d*e^2)*sqrt(e*x^2 - d)*x/(d^2*e*x^4 - d^2*e - (d^3 - d*e^2)*x^2)) -
 (B*d^2 + B*e^2)*sqrt(d)*arctan(sqrt(-d*e*x^4 + (d^2 - e^2)*x^2 + d*e)*sqr
t(e*x^2 - d)*sqrt(d)*x/(d*e*x^4 - (d^2 - e^2)*x^2 - d*e)))/(d^3*e + d*e^3)
]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{de+(d^2-e^2)\,x^2-dex^4}}\,dx$$
$$= \int \frac{A+Bx^2}{\sqrt{-(-d+ex^2)\,(dx^2+e)}\sqrt{-d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2-d)**(1/2)/(d*e+(d**2-e**2)*x**2-d*e*x**4)**(1
/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(-(-d + e*x**2)*(d*x**2 + e))*sqrt(-d + e*x**2)
), x)
```

Maxima [F]

$$\int \frac{A + Bx^2}{\sqrt{-d + ex^2}\sqrt{de + (d^2 - e^2)\, x^2 - dex^4}}\, dx = \int \frac{Bx^2 + A}{\sqrt{-dex^4 + (d^2 - e^2)x^2 + de}\sqrt{ex^2 - d}}\, dx$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-d*e*x^4 + (d^2 - e^2)*x^2 + d*e)*sqrt(e*x^2 -
 d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{\sqrt{-d + ex^2}\sqrt{de + (d^2 - e^2)\, x^2 - dex^4}}\, dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2-d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2),x, a
lgorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{\sqrt{-d + ex^2}\sqrt{de + (d^2 - e^2)\, x^2 - dex^4}}\, dx = \int \frac{B\,x^2 + A}{\sqrt{e\,x^2 - d}\,\sqrt{-d\,e\,x^4 + (d^2 - e^2)\,x^2 + d\,e}}\, dx$$

input
```
int((A + B*x^2)/((e*x^2 - d)^(1/2)*(d*e + x^2*(d^2 - e^2) - d*e*x^4)^(1/2)
),x)
```

output
```
int((A + B*x^2)/((e*x^2 - d)^(1/2)*(d*e + x^2*(d^2 - e^2) - d*e*x^4)^(1/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.37

$$\int \frac{A+Bx^2}{\sqrt{-d+ex^2}\sqrt{de+(d^2-e^2)\,x^2-dex^4}}\,dx$$
$$=\frac{\sqrt{d}\,i\left(-2asinh\left(\frac{\sqrt{d}\,x}{\sqrt{e}}\right)b\,d^2-2asinh\left(\frac{\sqrt{d}\,x}{\sqrt{e}}\right)b\,e^2+\sqrt{d^2+e^2}\log\left(2\sqrt{d^2+e^2}\,d+2\sqrt{d}\,\sqrt{d\,x^2+e}\,ex-2d^2+\right.\right.}{}$$

input
```
int((B*x^2+A)/(e*x^2-d)^(1/2)/(d*e+(d^2-e^2)*x^2-d*e*x^4)^(1/2),x)
```

output
```
(sqrt(d)*i*( - 2*asinh((sqrt(d)*x)/sqrt(e))*b*d**2 - 2*asinh((sqrt(d)*x)/s
qrt(e))*b*e**2 + sqrt(d**2 + e**2)*log(2*sqrt(d**2 + e**2)*d + 2*sqrt(d)*s
qrt(d*x**2 + e)*e*x - 2*d**2 + 2*d*e*x**2)*a*e + sqrt(d**2 + e**2)*log(2*s
qrt(d**2 + e**2)*d + 2*sqrt(d)*sqrt(d*x**2 + e)*e*x - 2*d**2 + 2*d*e*x**2)
*b*d - sqrt(d**2 + e**2)*log(( - sqrt(e)*sqrt(d**2 + e**2) + sqrt(d*x**2 +
 e)*e + sqrt(d)*e*x - sqrt(e)*d)/sqrt(e))*a*e - sqrt(d**2 + e**2)*log(( -
sqrt(e)*sqrt(d**2 + e**2) + sqrt(d*x**2 + e)*e + sqrt(d)*e*x - sqrt(e)*d)/
sqrt(e))*b*d - sqrt(d**2 + e**2)*log((sqrt(e)*sqrt(d**2 + e**2) + sqrt(d*x
**2 + e)*e + sqrt(d)*e*x + sqrt(e)*d)/sqrt(e))*a*e - sqrt(d**2 + e**2)*log
((sqrt(e)*sqrt(d**2 + e**2) + sqrt(d*x**2 + e)*e + sqrt(d)*e*x + sqrt(e)*d
)/sqrt(e))*b*d))/(2*d*e*(d**2 + e**2))
```

3.145 $\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-de+(d^2-e^2)x^2+dex^4}}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 151

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-de+(d^2-e^2)x^2+dex^4}}\,dx = -\frac{B\arctan\left(\frac{\sqrt{d}x\sqrt{-d-ex^2}}{\sqrt{-de+(d^2-e^2)x^2+dex^4}}\right)}{\sqrt{d}e}+\frac{(Bd-Ae)\arctan\left(\frac{\sqrt{d^2+e^2}x\sqrt{-d-ex^2}}{\sqrt{d}\sqrt{-de+(d^2-e^2)x^2+dex^4}}\right)}{\sqrt{d}e\sqrt{d^2+e^2}}$$

output

```
-B*arctan(d^(1/2)*x*(-e*x^2-d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2))/d
^(1/2)/e+(-A*e+B*d)*arctan((d^2+e^2)^(1/2)*x*(-e*x^2-d)^(1/2)/d^(1/2)/(-d*
e+(d^2-e^2)*x^2+d*e*x^4)^(1/2))/d^(1/2)/e/(d^2+e^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.48 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-de+(d^2-e^2)x^2+dex^4}}\,dx = \frac{\frac{(Bd-Ae)\sqrt{e-dx^2}\sqrt{-d-ex^2}\arctan\left(\frac{\sqrt{d^2+e^2}x}{\sqrt{d}\sqrt{e-dx^2}}\right)}{\sqrt{d^2+e^2}\sqrt{d^2x^2-e^2x^2+de(-1+x^4)}}+iB\log\left(-2i\sqrt{d}x-\frac{2\sqrt{d^2x^2-e^2x^2+de(-1+x^4)}}{\sqrt{-d-ex^2}}\right)}{\sqrt{d}e}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[-d - e*x^2]*Sqrt[-(d*e) + (d^2 - e^2)*x^2 + d*
e*x^4]),x]
```

output
```
(((B*d - A*e)*Sqrt[e - d*x^2]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d^2 + e^2]*x)/
(Sqrt[d]*Sqrt[e - d*x^2])])/(Sqrt[d^2 + e^2]*Sqrt[d^2*x^2 - e^2*x^2 + d*e*
(-1 + x^4)]) + I*B*Log[(-2*I)*Sqrt[d]*x - (2*Sqrt[d^2*x^2 - e^2*x^2 + d*e*
(-1 + x^4)])/Sqrt[-d - e*x^2]])/(Sqrt[d]*e)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {1395, 25, 398, 224, 216, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{x^2(d^2-e^2)+dex^4-de}}dx$$

$$\Big\downarrow \text{1395}$$

$$\frac{\sqrt{e-dx^2}\sqrt{-d-ex^2}\int -\frac{Bx^2+A}{\sqrt{e-dx^2}(ex^2+d)}dx}{\sqrt{x^2(d^2-e^2)+dex^4-de}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{-d-ex^2}\int \frac{Bx^2+A}{\sqrt{e-dx^2}(ex^2+d)}dx}{\sqrt{x^2(d^2-e^2)+dex^4-de}}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{-d-ex^2}\left(\frac{B\int \frac{1}{\sqrt{e-dx^2}}dx}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{e-dx^2}(ex^2+d)}dx}{e}\right)}{\sqrt{x^2(d^2-e^2)+dex^4-de}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{-d-ex^2}\left(\frac{B\int \frac{1}{\frac{dx^2}{e-dx^2}+1}d\frac{x}{\sqrt{e-dx^2}}}{e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{e-dx^2}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{x^2\left(d^2-e^2\right)+dex^4-de}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{-d-ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{e-dx^2}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\int \frac{1}{\sqrt{e-dx^2}\left(ex^2+d\right)}dx}{e}\right)}{\sqrt{x^2\left(d^2-e^2\right)+dex^4-de}}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{-d-ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{e-dx^2}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\int \frac{1}{d-\frac{\left(-d^2-e^2\right)x^2}{e-dx^2}}d\frac{x}{\sqrt{e-dx^2}}}{e}\right)}{\sqrt{x^2\left(d^2-e^2\right)+dex^4-de}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\sqrt{e-dx^2}\sqrt{-d-ex^2}\left(\frac{B\arctan\left(\frac{\sqrt{d}x}{\sqrt{e-dx^2}}\right)}{\sqrt{d}e}-\frac{(Bd-Ae)\arctan\left(\frac{x\sqrt{d^2+e^2}}{\sqrt{d}\sqrt{e-dx^2}}\right)}{\sqrt{d}e\sqrt{d^2+e^2}}\right)}{\sqrt{x^2\left(d^2-e^2\right)+dex^4-de}}$$

input
```
Int[(A + B*x^2)/(Sqrt[-d - e*x^2]*Sqrt[-(d*e) + (d^2 - e^2)*x^2 + d*e*x^4]
),x]
```

output
```
-((Sqrt[e - d*x^2]*Sqrt[-d - e*x^2]*((B*ArcTan[(Sqrt[d]*x)/Sqrt[e - d*x^2]
])/(Sqrt[d]*e) - ((B*d - A*e)*ArcTan[(Sqrt[d^2 + e^2]*x)/(Sqrt[d]*Sqrt[e -
 d*x^2])])/(Sqrt[d]*e*Sqrt[d^2 + e^2])))/Sqrt[-(d*e) + (d^2 - e^2)*x^2 + d
*e*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(131) = 262$.

Time = 0.09 (sec) , antiderivative size = 807, normalized size of antiderivative = 5.34

method	result
default	$-\frac{\sqrt{d e\,x^{4}+d^{2} x^{2}-e^{2} x^{2}-d e}\,\left(2 A \sqrt{-d e}\,\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{-d\,x^{2}+e}}\right) d \sqrt{\frac{d^{2}+e^{2}}{e}}\,e-2 B \sqrt{-d e}\,\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{-d\,x^{2}+e}}\right) d^{2} \sqrt{\frac{d^{2}+e^{2}}{e}}-2 A \sqrt{-d e}\,\text{arc}\right.}{}$

input
```
int((B*x^2+A)/(-e*x^2-d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-1/2*(d*e*x^4+d^2*x^2-e^2*x^2-d*e)^(1/2)*(2*A*(-d*e)^(1/2)*arctan(d^(1/2)*
x/(-d*x^2+e)^(1/2))*d*((d^2+e^2)/e)^(1/2)*e-2*B*(-d*e)^(1/2)*arctan(d^(1/2
)*x/(-d*x^2+e)^(1/2))*d^2*((d^2+e^2)/e)^(1/2)-2*A*(-d*e)^(1/2)*arctan(d^(1
/2)*x/(1/d*(d*x+(d*e)^(1/2))*(-d*x+(d*e)^(1/2)))^(1/2))*((d^2+e^2)/e)^(1/2
)*d*e+A*ln(-2*((-d*e)^(1/2)*d*x-(-d*x^2+e)^(1/2)*((d^2+e^2)/e)^(1/2)*e-e^2
)/(e*x-(-d*e)^(1/2)))*d^(5/2)*e+A*ln(-2*((-d*e)^(1/2)*d*x-(-d*x^2+e)^(1/2)
*((d^2+e^2)/e)^(1/2)*e-e^2)/(e*x-(-d*e)^(1/2)))*d^(1/2)*e^3-A*ln(2*((-d*e)
^(1/2)*d*x+(-d*x^2+e)^(1/2)*((d^2+e^2)/e)^(1/2)*e+e^2)/(e*x+(-d*e)^(1/2)))
*d^(5/2)*e-A*ln(2*((-d*e)^(1/2)*d*x+(-d*x^2+e)^(1/2)*((d^2+e^2)/e)^(1/2)*e
+e^2)/(e*x+(-d*e)^(1/2)))*d^(1/2)*e^3-2*B*(-d*e)^(1/2)*arctan(d^(1/2)*x/(1
/d*(d*x+(d*e)^(1/2))*(-d*x+(d*e)^(1/2)))^(1/2))*((d^2+e^2)/e)^(1/2)*e^2-B*
ln(-2*((-d*e)^(1/2)*d*x-(-d*x^2+e)^(1/2)*((d^2+e^2)/e)^(1/2)*e-e^2)/(e*x-(
-d*e)^(1/2)))*d^(7/2)-B*ln(-2*((-d*e)^(1/2)*d*x-(-d*x^2+e)^(1/2)*((d^2+e^2
)/e)^(1/2)*e-e^2)/(e*x-(-d*e)^(1/2)))*d^(3/2)*e^2+B*ln(2*((-d*e)^(1/2)*d*x
+(-d*x^2+e)^(1/2)*((d^2+e^2)/e)^(1/2)*e+e^2)/(e*x+(-d*e)^(1/2)))*d^(7/2)+B
*ln(2*((-d*e)^(1/2)*d*x+(-d*x^2+e)^(1/2)*((d^2+e^2)/e)^(1/2)*e+e^2)/(e*x+(
-d*e)^(1/2)))*d^(3/2)*e^2)/(-e*x^2-d)^(1/2)/(-d*x^2+e)^(1/2)/((-d*e)^(1/2)
*d+e*(d*e)^(1/2))/((-d*e)^(1/2)*d-e*(d*e)^(1/2))*d^(1/2)/(-d*e)^(1/2)/((d^
2+e^2)/e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.83

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-de+(d^2-e^2)x^2+dex^4}}\,dx$$
$$= \left[\frac{\sqrt{-d^3-de^2}(Bd-Ae)\log\left(-\frac{2d^3x^2+(2d^2e+e^3)x^4-d^2e-2\sqrt{dex^4+(d^2-e^2)x^2-de}\sqrt{-d^3-de^2}\sqrt{-ex^2-d}x}{e^2x^4+2dex^2+d^2}\right)-(Bd^2+Be}{2(d^3e+de^3)}\right.$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2),x,
 algorithm="fricas")
```

output
```
[1/2*(sqrt(-d^3 - d*e^2)*(B*d - A*e)*log(-(2*d^3*x^2 + (2*d^2*e + e^3)*x^4
 - d^2*e - 2*sqrt(d*e*x^4 + (d^2 - e^2)*x^2 - d*e)*sqrt(-d^3 - d*e^2)*sqrt
(-e*x^2 - d)*x)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (B*d^2 + B*e^2)*sqrt(-d)*lo
g(-(2*d*e*x^4 + (2*d^2 - e^2)*x^2 - 2*sqrt(d*e*x^4 + (d^2 - e^2)*x^2 - d*e
)*sqrt(-e*x^2 - d)*sqrt(-d)*x - d*e)/(e*x^2 + d)))/(d^3*e + d*e^3), (sqrt(
d^3 + d*e^2)*(B*d - A*e)*arctan(sqrt(d*e*x^4 + (d^2 - e^2)*x^2 - d*e)*sqrt
(d^3 + d*e^2)*sqrt(-e*x^2 - d)*x/(d^2*e*x^4 - d^2*e + (d^3 - d*e^2)*x^2))
- (B*d^2 + B*e^2)*sqrt(d)*arctan(sqrt(-e*x^2 - d)*sqrt(d)*x/sqrt(d*e*x^4 +
 (d^2 - e^2)*x^2 - d*e)))/(d^3*e + d*e^3)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-de+(d^2-e^2)x^2+dex^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{(d+ex^2)(dx^2-e)}\sqrt{-d-ex^2}}\,dx$$

input
```
integrate((B*x**2+A)/(-e*x**2-d)**(1/2)/(-d*e+(d**2-e**2)*x**2+d*e*x**4)**
(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt((d + e*x**2)*(d*x**2 - e))*sqrt(-d - e*x**2)),
 x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-de+(d^2-e^2)\,x^2+dex^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{dex^4+(d^2-e^2)x^2-de}\sqrt{-ex^2-d}}\,dx$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(d*e*x^4 + (d^2 - e^2)*x^2 - d*e)*sqrt(-e*x^2 -
 d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-de+(d^2-e^2)\,x^2+dex^4}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(-e*x^2-d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2),x,
 algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-de+(d^2-e^2)\,x^2+dex^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{-e\,x^2-d}\,\sqrt{d\,e\,x^4+\left(d^2-e^2\right)\,x^2-d\,e}}\,dx$$

input
```
int((A + B*x^2)/((- d - e*x^2)^(1/2)*(x^2*(d^2 - e^2) - d*e + d*e*x^4)^(1/
2)),x)
```

output
```
int((A + B*x^2)/((- d - e*x^2)^(1/2)*(x^2*(d^2 - e^2) - d*e + d*e*x^4)^(1/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.61

$$\int \frac{A+Bx^2}{\sqrt{-d-ex^2}\sqrt{-de+(d^2-e^2)\,x^2+dex^4}}\,dx$$
$$= \frac{\sqrt{d}\,i\left(\sqrt{d^2+e^2}\log\left(2\sqrt{d^2+e^2}\,d+2\sqrt{d}\,\sqrt{d\,x^2-e}\,ex+2d^2+2de\,x^2\right)ae-\sqrt{d^2+e^2}\log\left(2\sqrt{d^2+e^2}\,d+\right.\right.}{}$$

input
```
int((B*x^2+A)/(-e*x^2-d)^(1/2)/(-d*e+(d^2-e^2)*x^2+d*e*x^4)^(1/2),x)
```

output
```
(sqrt(d)*i*(sqrt(d**2 + e**2)*log(2*sqrt(d**2 + e**2)*d + 2*sqrt(d)*sqrt(d
*x**2 - e)*e*x + 2*d**2 + 2*d*e*x**2)*a*e - sqrt(d**2 + e**2)*log(2*sqrt(d
**2 + e**2)*d + 2*sqrt(d)*sqrt(d*x**2 - e)*e*x + 2*d**2 + 2*d*e*x**2)*b*d
- sqrt(d**2 + e**2)*log(( - sqrt(e)*sqrt(d**2 + e**2)*i + sqrt(d*x**2 - e)
*e + sqrt(d)*e*x + sqrt(e)*d*i)/sqrt(e))*a*e + sqrt(d**2 + e**2)*log(( - s
qrt(e)*sqrt(d**2 + e**2)*i + sqrt(d*x**2 - e)*e + sqrt(d)*e*x + sqrt(e)*d*
i)/sqrt(e))*b*d - sqrt(d**2 + e**2)*log((sqrt(e)*sqrt(d**2 + e**2)*i + sqr
t(d*x**2 - e)*e + sqrt(d)*e*x - sqrt(e)*d*i)/sqrt(e))*a*e + sqrt(d**2 + e*
*2)*log((sqrt(e)*sqrt(d**2 + e**2)*i + sqrt(d*x**2 - e)*e + sqrt(d)*e*x -
sqrt(e)*d*i)/sqrt(e))*b*d - 2*log((sqrt(d*x**2 - e) + sqrt(d)*x)/sqrt(e))*
b*d**2 - 2*log((sqrt(d*x**2 - e) + sqrt(d)*x)/sqrt(e))*b*e**2))/(2*d*e*(d*
*2 + e**2))
```

3.146 $\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 445

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx =$$

$$-\frac{(35a^3Ce^2-64b^3d(Bd+2Ae)-40a^2be(2Cd+Be)+48ab^2(Cd^2+e(2Bd+Ae)))\,x\sqrt{ad+(bd+ae)x^2}}{128b^4\sqrt{d+ex^2}}$$

$$+\frac{(35a^2Ce^2-40abe(2Cd+Be)+48b^2(Cd^2+e(2Bd+Ae)))\,x^3\sqrt{ad+(bd+ae)x^2+bex^4}}{192b^3\sqrt{d+ex^2}}$$

$$-\frac{e(7aCe-8b(2Cd+Be))x^5\sqrt{ad+(bd+ae)x^2+bex^4}}{48b^2\sqrt{d+ex^2}}$$

$$+\frac{Ce^2x^7\sqrt{ad+(bd+ae)x^2+bex^4}}{8b\sqrt{d+ex^2}}$$

$$+\frac{(16Ab^2(8b^2d^2-8abde+3a^2e^2)-a(64b^3Bd^2-35a^3Ce^2+40a^2be(2Cd+Be)-48ab^2d(Cd+2Be)))\,\mathrm{arc}}{128b^{9/2}}$$

output
```
-1/128*(35*a^3*C*e^2-64*b^3*d*(2*A*e+B*d)-40*a^2*b*e*(B*e+2*C*d)+48*a*b^2*
(C*d^2+e*(A*e+2*B*d)))*x*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/b^4/(e*x^2+d)^(
1/2)+1/192*(35*a^2*C*e^2-40*a*b*e*(B*e+2*C*d)+48*b^2*(C*d^2+e*(A*e+2*B*d))
)*x^3*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/b^3/(e*x^2+d)^(1/2)-1/48*e*(7*C*a*
e-8*b*(B*e+2*C*d))*x^5*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/b^2/(e*x^2+d)^(1/
2)+1/8*C*e^2*x^7*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/b/(e*x^2+d)^(1/2)+1/128
*(16*A*b^2*(3*a^2*e^2-8*a*b*d*e+8*b^2*d^2)-a*(64*b^3*B*d^2-35*a^3*C*e^2+40
*a^2*b*e*(B*e+2*C*d)-48*a*b^2*d*(2*B*e+C*d)))*arctanh(b^(1/2)*x*(e*x^2+d)^
(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{\sqrt{d+ex^2}\left(-\sqrt{b}x(a+bx^2)\left(105a^3Ce^2-10a^2be(24Cd+12Be+7C\right.\right.}{}$$

input
```
Integrate[((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/Sqrt[a*d + (b*d + a*e)*x
^2 + b*e*x^4],x]
```

output
```
(Sqrt[d + e*x^2]*(-(Sqrt[b]*x*(a + b*x^2)*(105*a^3*C*e^2 - 10*a^2*b*e*(24*
C*d + 12*B*e + 7*C*e*x^2) - 16*b^3*(6*A*e*(4*d + e*x^2) + 4*B*(3*d^2 + 3*d
*e*x^2 + e^2*x^4) + C*x^2*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4)) + 8*a*b^2*(2*e*
(18*B*d + 9*A*e + 5*B*e*x^2) + C*(18*d^2 + 20*d*e*x^2 + 7*e^2*x^4)))) - 3*
(16*A*b^2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2) + a*(-64*b^3*B*d^2 + 35*a^3*
C*e^2 - 40*a^2*b*e*(2*C*d + B*e) + 48*a*b^2*d*(C*d + 2*B*e)))*Sqrt[a + b*x
^2]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]))/(384*b^(9/2)*Sqrt[(a + b*x^2)*(d
 + e*x^2)])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1395, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{\sqrt{x^2(ae+bd)+ad+bex^4}}dx$$

$$\Big\downarrow \text{1395}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \frac{(ex^2+d)^2(Cx^4+Bx^2+A)}{\sqrt{bx^2+a}}dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{2256}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \left(\frac{Ce^2x^8}{\sqrt{bx^2+a}}+\frac{e(2Cd+Be)x^6}{\sqrt{bx^2+a}}+\frac{(Cd^2+e(2Bd+Ae))x^4}{\sqrt{bx^2+a}}+\frac{d(Bd+2Ae)x^2}{\sqrt{bx^2+a}}+\frac{Ad^2}{\sqrt{bx^2+a}}\right)dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{35a^4Ce^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{9/2}}-\frac{5a^3e\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Be+2Cd)}{16b^{7/2}}-\frac{35a^3Ce^2x\sqrt{a+bx^2}}{128b^4}+\frac{3a^2\text{arctanh}\left(\right.}{}\right.$$

input

```
Int[((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/Sqrt[a*d + (b*d + a*e)*x^2 + b
*e*x^4],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[d + e*x^2]*((-35*a^3*C*e^2*x*Sqrt[a + b*x^2])/(128*b
^4) + (d*(B*d + 2*A*e)*x*Sqrt[a + b*x^2])/(2*b) + (5*a^2*e*(2*C*d + B*e)*x
*Sqrt[a + b*x^2])/(16*b^3) - (3*a*(C*d^2 + e*(2*B*d + A*e))*x*Sqrt[a + b*x
^2])/(8*b^2) + (35*a^2*C*e^2*x^3*Sqrt[a + b*x^2])/(192*b^3) - (5*a*e*(2*C*
d + B*e)*x^3*Sqrt[a + b*x^2])/(24*b^2) + ((C*d^2 + e*(2*B*d + A*e))*x^3*Sq
rt[a + b*x^2])/(4*b) - (7*a*C*e^2*x^5*Sqrt[a + b*x^2])/(48*b^2) + (e*(2*C*
d + B*e)*x^5*Sqrt[a + b*x^2])/(6*b) + (C*e^2*x^7*Sqrt[a + b*x^2])/(8*b) +
(A*d^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] + (35*a^4*C*e^2*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(9/2)) - (a*d*(B*d + 2*A*e)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)) - (5*a^3*e*(2*C*d + B*e)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2)) + (3*a^2*(C*d^2 + e*(2*B*d +
 A*e))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))))/Sqrt[a*d + (b*d
 + a*e)*x^2 + b*e*x^4]
```

Defintions of rubi rules used

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{x\left(-48C\,e^{2}x^{6}b^{3}-64B\,b^{3}e^{2}x^{4}+56Ca\,b^{2}e^{2}x^{4}-128C\,b^{3}de\,x^{4}-96A\,b^{3}e^{2}x^{2}+80Ba\,b^{2}e^{2}x^{2}-192B\,b^{3}de\,x^{2}-70C\,a^{2}b\,e^{2}x^{2}+160Ca\,b^{2}\right.}{384b^{4}}$
default	$\frac{\sqrt{(e\,x^{2}+d)(b\,x^{2}+a)}\left(70C\,a^{2}b^{\frac{3}{2}}e^{2}x^{3}\sqrt{b\,x^{2}+a}-144Aa\,b^{\frac{5}{2}}e^{2}x\sqrt{b\,x^{2}+a}+384A\,b^{\frac{7}{2}}dex\sqrt{b\,x^{2}+a}+120B\,a^{2}b^{\frac{3}{2}}e^{2}x\sqrt{b\,x^{2}+a}-144Ca\,b^{\frac{5}{2}}\right.}{}$

input
```
int((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-1/384*x*(-48*C*b^3*e^2*x^6-64*B*b^3*e^2*x^4+56*C*a*b^2*e^2*x^4-128*C*b^3*
d*e*x^4-96*A*b^3*e^2*x^2+80*B*a*b^2*e^2*x^2-192*B*b^3*d*e*x^2-70*C*a^2*b*e
^2*x^2+160*C*a*b^2*d*e*x^2-96*C*b^3*d^2*x^2+144*A*a*b^2*e^2-384*A*b^3*d*e-
120*B*a^2*b*e^2+288*B*a*b^2*d*e-192*B*b^3*d^2+105*C*a^3*e^2-240*C*a^2*b*d*
e+144*C*a*b^2*d^2)*(b*x^2+a)/b^4/((e*x^2+d)*(b*x^2+a))^(1/2)*(e*x^2+d)^(1/
2)+1/128*(48*A*a^2*b^2*e^2-128*A*a*b^3*d*e+128*A*b^4*d^2-40*B*a^3*b*e^2+96
*B*a^2*b^2*d*e-64*B*a*b^3*d^2+35*C*a^4*e^2-80*C*a^3*b*d*e+48*C*a^2*b^2*d^2
)/b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*(b*x^2+a)^(1/2)/((e*x^2+d)*(b*x^2+
a))^(1/2)*(e*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 955, normalized size of antiderivative = 2.15

$$\int \frac{(d+ex^2)^{5/2}\,(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="fricas")
```

output
```
[1/768*(3*(16*(3*C*a^2*b^2 - 4*B*a*b^3 + 8*A*b^4)*d^3 - 16*(5*C*a^3*b - 6*
B*a^2*b^2 + 8*A*a*b^3)*d^2*e + (35*C*a^4 - 40*B*a^3*b + 48*A*a^2*b^2)*d*e^
2 + (16*(3*C*a^2*b^2 - 4*B*a*b^3 + 8*A*b^4)*d^2*e - 16*(5*C*a^3*b - 6*B*a^
2*b^2 + 8*A*a*b^3)*d*e^2 + (35*C*a^4 - 40*B*a^3*b + 48*A*a^2*b^2)*e^3)*x^2
)*sqrt(b)*log((2*b*e*x^4 + (2*b*d + a*e)*x^2 + 2*sqrt(b*e*x^4 + (b*d + a*e
)*x^2 + a*d)*sqrt(e*x^2 + d)*sqrt(b)*x + a*d)/(e*x^2 + d)) + 2*(48*C*b^4*e
^2*x^7 + 8*(16*C*b^4*d*e - (7*C*a*b^3 - 8*B*b^4)*e^2)*x^5 + 2*(48*C*b^4*d^
2 - 16*(5*C*a*b^3 - 6*B*b^4)*d*e + (35*C*a^2*b^2 - 40*B*a*b^3 + 48*A*b^4)*
e^2)*x^3 - 3*(16*(3*C*a*b^3 - 4*B*b^4)*d^2 - 16*(5*C*a^2*b^2 - 6*B*a*b^3 +
 8*A*b^4)*d*e + (35*C*a^3*b - 40*B*a^2*b^2 + 48*A*a*b^3)*e^2)*x)*sqrt(b*e*
x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(e*x^2 + d))/(b^5*e*x^2 + b^5*d), -1/384*
(3*(16*(3*C*a^2*b^2 - 4*B*a*b^3 + 8*A*b^4)*d^3 - 16*(5*C*a^3*b - 6*B*a^2*b
^2 + 8*A*a*b^3)*d^2*e + (35*C*a^4 - 40*B*a^3*b + 48*A*a^2*b^2)*d*e^2 + (16
*(3*C*a^2*b^2 - 4*B*a*b^3 + 8*A*b^4)*d^2*e - 16*(5*C*a^3*b - 6*B*a^2*b^2 +
 8*A*a*b^3)*d*e^2 + (35*C*a^4 - 40*B*a^3*b + 48*A*a^2*b^2)*e^3)*x^2)*sqrt(
-b)*arctan(sqrt(e*x^2 + d)*sqrt(-b)*x/sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d
)) - (48*C*b^4*e^2*x^7 + 8*(16*C*b^4*d*e - (7*C*a*b^3 - 8*B*b^4)*e^2)*x^5
+ 2*(48*C*b^4*d^2 - 16*(5*C*a*b^3 - 6*B*b^4)*d*e + (35*C*a^2*b^2 - 40*B*a*
b^3 + 48*A*b^4)*e^2)*x^3 - 3*(16*(3*C*a*b^3 - 4*B*b^4)*d^2 - 16*(5*C*a^2*b
^2 - 6*B*a*b^3 + 8*A*b^4)*d*e + (35*C*a^3*b - 40*B*a^2*b^2 + 48*A*a*b^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(5/2)*(C*x**4+B*x**2+A)/(a*d+(a*e+b*d)*x**2+b*e*x**4
)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{\sqrt{bex^4+(bd+ae)x^2+ad}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(5/2)/sqrt(b*e*x^4 + (b*d + a*e)
*x^2 + a*d), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{\sqrt{bex^4+(bd+ae)x^2+ad}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(5/2)/sqrt(b*e*x^4 + (b*d + a*e)
*x^2 + a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{(e\,x^2+d)^{5/2}\,(C\,x^4+B\,x^2+A)}{\sqrt{b\,e\,x^4+(a\,e+b\,d)\,x^2+a\,d}}\,dx$$

input
```
int(((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a*d + x^2*(a*e + b*d) + b*e*x
^4)^(1/2),x)
```

output
```
int(((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a*d + x^2*(a*e + b*d) + b*e*x
^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{-105\sqrt{b\,x^2+a}\,a^3bc\,e^2x-24\sqrt{b\,x^2+a}\,a^2b^3e^2x+240\sqrt{b\,x^2+a}\,a^2b}{}$$

input
```
int((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**3*b*c*e**2*x - 24*sqrt(a + b*x**2)*a**2*b**3*e
**2*x + 240*sqrt(a + b*x**2)*a**2*b**2*c*d*e*x + 70*sqrt(a + b*x**2)*a**2*
b**2*c*e**2*x**3 + 96*sqrt(a + b*x**2)*a*b**4*d*e*x + 16*sqrt(a + b*x**2)*
a*b**4*e**2*x**3 - 144*sqrt(a + b*x**2)*a*b**3*c*d**2*x - 160*sqrt(a + b*x
**2)*a*b**3*c*d*e*x**3 - 56*sqrt(a + b*x**2)*a*b**3*c*e**2*x**5 + 192*sqrt
(a + b*x**2)*b**5*d**2*x + 192*sqrt(a + b*x**2)*b**5*d*e*x**3 + 64*sqrt(a
+ b*x**2)*b**5*e**2*x**5 + 96*sqrt(a + b*x**2)*b**4*c*d**2*x**3 + 128*sqrt
(a + b*x**2)*b**4*c*d*e*x**5 + 48*sqrt(a + b*x**2)*b**4*c*e**2*x**7 + 105*
sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*c*e**2 + 24*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*e**2 - 240*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c*d*e - 96*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**3*d*e + 144*sqrt(b)*log(
(sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*d**2 + 192*sqrt(b)*log
((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**4*d**2)/(384*b**5)
```

3.147 $\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 284

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{(5a^2Ce+8b^2(Bd+Ae)-6ab(Cd+Be))\,x\sqrt{ad+(bd+ae)x^2+b}}{16b^3\sqrt{d+ex^2}}$$
$$-\frac{(5aCe-6b(Cd+Be))x^3\sqrt{ad+(bd+ae)x^2+bex^4}}{24b^2\sqrt{d+ex^2}}+\frac{Cex^5\sqrt{ad+(bd+ae)x^2+bex^4}}{6b\sqrt{d+ex^2}}$$
$$+\frac{(8Ab^2(2bd-ae)-a(8b^2Bd+5a^2Ce-6ab(Cd+Be)))\,\text{arctanh}\left(\frac{\sqrt{b}x\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\right)}{16b^{7/2}}$$

output

```
1/16*(5*a^2*C*e+8*b^2*(A*e+B*d)-6*a*b*(B*e+C*d))*x*(a*d+(a*e+b*d)*x^2+b*e*
x^4)^(1/2)/b^3/(e*x^2+d)^(1/2)-1/24*(5*C*a*e-6*b*(B*e+C*d))*x^3*(a*d+(a*e+
b*d)*x^2+b*e*x^4)^(1/2)/b^2/(e*x^2+d)^(1/2)+1/6*C*e*x^5*(a*d+(a*e+b*d)*x^2
+b*e*x^4)^(1/2)/b/(e*x^2+d)^(1/2)+1/16*(8*A*b^2*(-a*e+2*b*d)-a*(8*b^2*B*d+
5*a^2*C*e-6*a*b*(B*e+C*d)))*arctanh(b^(1/2)*x*(e*x^2+d)^(1/2)/(a*d+(a*e+b*
d)*x^2+b*e*x^4)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{\sqrt{d+ex^2}\left(\sqrt{b}x(a+bx^2)\left(15a^2Ce-2ab(9Cd+9Be+5Cex^2)+4\right.\right.}{}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/Sqrt[a*d + (b*d + a*e)*x
^2 + b*e*x^4],x]
```

output
```
(Sqrt[d + e*x^2]*(Sqrt[b]*x*(a + b*x^2)*(15*a^2*C*e - 2*a*b*(9*C*d + 9*B*e
 + 5*C*e*x^2) + 4*b^2*(6*B*d + 6*A*e + 3*C*d*x^2 + 3*B*e*x^2 + 2*C*e*x^4))
 - 3*(8*A*b^2*(2*b*d - a*e) + a*(-8*b^2*B*d - 5*a^2*C*e + 6*a*b*(C*d + B*e
)))*Sqrt[a + b*x^2]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]))/(48*b^(7/2)*Sqrt
[(a + b*x^2)*(d + e*x^2)])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1395, 2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{\sqrt{x^2(ae+bd)+ad+bex^4}}\,dx$$

$$\Big\downarrow \ 1395$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \frac{(ex^2+d)(Cx^4+Bx^2+A)}{\sqrt{bx^2+a}}dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \ 2256$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \left(\frac{Cex^6}{\sqrt{bx^2+a}}+\frac{(Cd+Be)x^4}{\sqrt{bx^2+a}}+\frac{(Bd+Ae)x^2}{\sqrt{bx^2+a}}+\frac{Ad}{\sqrt{bx^2+a}}\right)dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$\downarrow$ 2009

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(-\frac{5a^3Ce\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Be+Cd)}{8b^{5/2}}+\frac{5a^2Cex\sqrt{a+bx^2}}{16b^3}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right.}{\sqrt{x^2(ae}}$$

input
```
Int[((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/Sqrt[a*d + (b*d + a*e)*x^2 + b
*e*x^4],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[d + e*x^2]*((5*a^2*C*e*x*Sqrt[a + b*x^2])/(16*b^3) +
 ((B*d + A*e)*x*Sqrt[a + b*x^2])/(2*b) - (3*a*(C*d + B*e)*x*Sqrt[a + b*x^2
])/(8*b^2) - (5*a*C*e*x^3*Sqrt[a + b*x^2])/(24*b^2) + ((C*d + B*e)*x^3*Sqr
t[a + b*x^2])/(4*b) + (C*e*x^5*Sqrt[a + b*x^2])/(6*b) + (A*d*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (5*a^3*C*e*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(16*b^(7/2)) - (a*(B*d + A*e)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(2*b^(3/2)) + (3*a^2*(C*d + B*e)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(8*b^(5/2))))/Sqrt[a*d + (b*d + a*e)*x^2 + b*e*x^4]
```

Defintions of rubi rules used

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.76

method	result
risch	$\frac{x\left(8eC\,x^4b^2+12B\,b^2e\,x^2-10Cabe\,x^2+12C\,b^2d\,x^2+24Ae\,b^2-18Babe+24b^2Bd+15a^2Ce-18Cabd\right)\left(b\,x^2+a\right)\sqrt{e\,x^2+d}}{48b^3\sqrt{\left(e\,x^2+d\right)\left(b\,x^2+a\right)}}-\frac{(8Aa\,b^2e\cdots}{}$
default	$-\frac{\sqrt{\left(e\,x^2+d\right)\left(b\,x^2+a\right)}\,\left(-8C\,b^{\frac{5}{2}}e\,x^5\sqrt{b\,x^2+a}-12B\,b^{\frac{5}{2}}e\,x^3\sqrt{b\,x^2+a}+10Ca\,b^{\frac{3}{2}}e\,x^3\sqrt{b\,x^2+a}-12C\,b^{\frac{5}{2}}d\,x^3\sqrt{b\,x^2+a}-24A\,b^{\frac{5}{2}}ex\sqrt{b\,x^2\cdots}\right.}{}$

input
```
int((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
1/48*x*(8*C*b^2*e*x^4+12*B*b^2*e*x^2-10*C*a*b*e*x^2+12*C*b^2*d*x^2+24*A*b^
2*e-18*B*a*b*e+24*B*b^2*d+15*C*a^2*e-18*C*a*b*d)*(b*x^2+a)/b^3/((e*x^2+d)*
(b*x^2+a))^(1/2)*(e*x^2+d)^(1/2)-1/16*(8*A*a*b^2*e-16*A*b^3*d-6*B*a^2*b*e+
8*B*a*b^2*d+5*C*a^3*e-6*C*a^2*b*d)/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*(
b*x^2+a)^(1/2)/((e*x^2+d)*(b*x^2+a))^(1/2)*(e*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex^2)^{3/2}\,(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \left[-\frac{3\,(2\,(3\,Ca^2b-4\,Bab^2+8\,Ab^3)d^2-(5\,Ca^3-6\,Ba^2b+8\,Aab^2)d\cdots}{}\right.$$

$$-\frac{3\,(2\,(3\,Ca^2b-4\,Bab^2+8\,Ab^3)d^2-(5\,Ca^3-6\,Ba^2b+8\,Aab^2)de+(2\,(3\,Ca^2b-4\,Bab^2+8\,Ab^3)de-(5\cdots}{}$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="fricas")
```

output
```
[-1/96*(3*(2*(3*C*a^2*b - 4*B*a*b^2 + 8*A*b^3)*d^2 - (5*C*a^3 - 6*B*a^2*b
+ 8*A*a*b^2)*d*e + (2*(3*C*a^2*b - 4*B*a*b^2 + 8*A*b^3)*d*e - (5*C*a^3 - 6
*B*a^2*b + 8*A*a*b^2)*e^2)*x^2)*sqrt(b)*log((2*b*e*x^4 + (2*b*d + a*e)*x^2
 - 2*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(e*x^2 + d)*sqrt(b)*x + a*d
)/(e*x^2 + d)) - 2*(8*C*b^3*e*x^5 + 2*(6*C*b^3*d - (5*C*a*b^2 - 6*B*b^3)*e
)*x^3 - 3*(2*(3*C*a*b^2 - 4*B*b^3)*d - (5*C*a^2*b - 6*B*a*b^2 + 8*A*b^3)*e
)*x)*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(e*x^2 + d))/(b^4*e*x^2 + b
^4*d), -1/48*(3*(2*(3*C*a^2*b - 4*B*a*b^2 + 8*A*b^3)*d^2 - (5*C*a^3 - 6*B*
a^2*b + 8*A*a*b^2)*d*e + (2*(3*C*a^2*b - 4*B*a*b^2 + 8*A*b^3)*d*e - (5*C*a
^3 - 6*B*a^2*b + 8*A*a*b^2)*e^2)*x^2)*sqrt(-b)*arctan(sqrt(e*x^2 + d)*sqrt
(-b)*x/sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)) - (8*C*b^3*e*x^5 + 2*(6*C*b^
3*d - (5*C*a*b^2 - 6*B*b^3)*e)*x^3 - 3*(2*(3*C*a*b^2 - 4*B*b^3)*d - (5*C*a
^2*b - 6*B*a*b^2 + 8*A*b^3)*e)*x)*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sq
rt(e*x^2 + d))/(b^4*e*x^2 + b^4*d)]
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{(d+ex^2)^{\frac{3}{2}}(A+Bx^2+Cx^4)}{\sqrt{(a+bx^2)(d+ex^2)}}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(C*x**4+B*x**2+A)/(a*d+(a*e+b*d)*x**2+b*e*x**4
)**(1/2),x)
```

output
```
Integral((d + e*x**2)**(3/2)*(A + B*x**2 + C*x**4)/sqrt((a + b*x**2)*(d +
e*x**2)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{\sqrt{bex^4+(bd+ae)x^2+ad}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(3/2)/sqrt(b*e*x^4 + (b*d + a*e)
*x^2 + a*d), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{\sqrt{bex^4+(bd+ae)x^2+ad}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(3/2)/sqrt(b*e*x^4 + (b*d + a*e)
*x^2 + a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,(C\,x^4+B\,x^2+A)}{\sqrt{b\,e\,x^4+(a\,e+b\,d)\,x^2+a\,d}}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a*d + x^2*(a*e + b*d) + b*e*x
^4)^(1/2),x)
```

output
```
int(((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a*d + x^2*(a*e + b*d) + b*e*x
^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{15\sqrt{b\,x^2+a}\,a^2bcex+6\sqrt{b\,x^2+a}\,a\,b^3ex-18\sqrt{b\,x^2+a}\,a\,b^2cdx-1}{}$$

input
```
int((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x)
```

output
```
(15*sqrt(a + b*x**2)*a**2*b*c*e*x + 6*sqrt(a + b*x**2)*a*b**3*e*x - 18*sqr
t(a + b*x**2)*a*b**2*c*d*x - 10*sqrt(a + b*x**2)*a*b**2*c*e*x**3 + 24*sqrt
(a + b*x**2)*b**4*d*x + 12*sqrt(a + b*x**2)*b**4*e*x**3 + 12*sqrt(a + b*x*
*2)*b**3*c*d*x**3 + 8*sqrt(a + b*x**2)*b**3*c*e*x**5 - 15*sqrt(b)*log((sqr
t(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*c*e - 6*sqrt(b)*log((sqrt(a + b*x
**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*e + 18*sqrt(b)*log((sqrt(a + b*x**2)
+ sqrt(b)*x)/sqrt(a))*a**2*b*c*d + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt
(b)*x)/sqrt(a))*a*b**3*d)/(48*b**4)
```

3.148 $\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 173

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{(4bB-3aC)x\sqrt{ad+(bd+ae)x^2+bex^4}}{8b^2\sqrt{d+ex^2}}+\frac{Cx^3\sqrt{ad+(bd+ae)x^2+bex^4}}{4b\sqrt{d+ex^2}}+\frac{(8Ab^2-a(4bB-3aC))\operatorname{arctanh}\left(\frac{\sqrt{b}x\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\right)}{8b^{5/2}}$$

output
```
1/8*(4*B*b-3*C*a)*x*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/b^2/(e*x^2+d)^(1/2)+
1/4*C*x^3*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/b/(e*x^2+d)^(1/2)+1/8*(8*A*b^2
-a*(4*B*b-3*C*a))*arctanh(b^(1/2)*x*(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e
*x^4)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$$
$$= \frac{\sqrt{d+ex^2}\left(\sqrt{b}x(a+bx^2)\,(4bB-3aC+2bCx^2)+2(8Ab^2+a(-4bB+3aC))\,\sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a}}\right.\right.}{8b^{5/2}\sqrt{(a+bx^2)\,(d+ex^2)}}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(A + B*x^2 + C*x^4))/Sqrt[a*d + (b*d + a*e)*x^2
 + b*e*x^4],x]
```

output
```
(Sqrt[d + e*x^2]*(Sqrt[b]*x*(a + b*x^2)*(4*b*B - 3*a*C + 2*b*C*x^2) + 2*(8
*A*b^2 + a*(-4*b*B + 3*a*C))*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a]
 + Sqrt[a + b*x^2])]))/(8*b^(5/2)*Sqrt[(a + b*x^2)*(d + e*x^2)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {1395, 1473, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{\sqrt{x^2(ae+bd)+ad+bex^4}}\,dx$$
$$\Big\downarrow \; 1395$$
$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \frac{Cx^4+Bx^2+A}{\sqrt{bx^2+a}}dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$
$$\Big\downarrow \; 1473$$
$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{\int \frac{(4bB-3aC)x^2+4Ab}{\sqrt{bx^2+a}}dx}{4b}+\frac{Cx^3\sqrt{a+bx^2}}{4b}\right)}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{\frac{\left(8Ab^2-a(4bB-3aC)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{x\sqrt{a+bx^2}(4bB-3aC)}{2b}}{4b}+\frac{Cx^3\sqrt{a+bx^2}}{4b}\right)}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{\frac{\left(8Ab^2-a(4bB-3aC)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{x\sqrt{a+bx^2}(4bB-3aC)}{2b}}{4b}+\frac{Cx^3\sqrt{a+bx^2}}{4b}\right)}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(8Ab^2-a(4bB-3aC)\right)}{2b^{3/2}}+\frac{x\sqrt{a+bx^2}(4bB-3aC)}{2b}}{4b}+\frac{Cx^3\sqrt{a+bx^2}}{4b}\right)}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

input
```
Int[(Sqrt[d + e*x^2]*(A + B*x^2 + C*x^4))/Sqrt[a*d + (b*d + a*e)*x^2 + b*e
*x^4],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[d + e*x^2]*((C*x^3*Sqrt[a + b*x^2])/(4*b) + (((4*b*B
 - 3*a*C)*x*Sqrt[a + b*x^2])/(2*b) + ((8*A*b^2 - a*(4*b*B - 3*a*C))*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(4*b)))/Sqrt[a*d + (b*d + a*e
)*x^2 + b*e*x^4]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.76

method	result
risch	$\frac{x\left(2C\,x^{2}b+4Bb-3Ca\right)\left(b\,x^{2}+a\right)\sqrt{e\,x^{2}+d}}{8b^{2}\sqrt{\left(e\,x^{2}+d\right)\left(b\,x^{2}+a\right)}}+\frac{\left(8A\,b^{2}-4abB+3C\,a^{2}\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)\sqrt{b\,x^{2}+a}\,\sqrt{e\,x^{2}+d}}{8b^{\frac{5}{2}}\sqrt{\left(e\,x^{2}+d\right)\left(b\,x^{2}+a\right)}}$
default	$\frac{\sqrt{\left(e\,x^{2}+d\right)\left(b\,x^{2}+a\right)}\left(2C\,b^{\frac{3}{2}}x^{3}\sqrt{b\,x^{2}+a}+4B\,b^{\frac{3}{2}}\sqrt{b\,x^{2}+a}\,x-3Cax\sqrt{b}\,\sqrt{b\,x^{2}+a}+8A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)b^{2}-4B\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\cdots\right.\right.}{8b^{\frac{5}{2}}\sqrt{e\,x^{2}+d}\,\sqrt{b\,x^{2}+a}}$

input
```
int((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
1/8*x*(2*C*b*x^2+4*B*b-3*C*a)*(b*x^2+a)/b^2/((e*x^2+d)*(b*x^2+a))^(1/2)*(e
*x^2+d)^(1/2)+1/8*(8*A*b^2-4*B*a*b+3*C*a^2)/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)
^(1/2))*(b*x^2+a)^(1/2)/((e*x^2+d)*(b*x^2+a))^(1/2)*(e*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$$
$$= \left[\frac{((3\,Ca^2-4\,Bab+8\,Ab^2)ex^2+(3\,Ca^2-4\,Bab+8\,Ab^2)d)\sqrt{b}\log\left(\frac{2\,bex^4+(2\,bd+ae)x^2+2\,\sqrt{bex^4+(bd+ae)x^2+ad}\sqrt{}}{ex^2+d}\right.}{16\,(b^3ex^2+b^3d)}\right.$$
$$-\frac{((3\,Ca^2-4\,Bab+8\,Ab^2)ex^2+(3\,Ca^2-4\,Bab+8\,Ab^2)d)\sqrt{-b}\arctan\left(\frac{\sqrt{ex^2+d}\sqrt{-b}x}{\sqrt{bex^4+(bd+ae)x^2+ad}}\right)-(2\,Cb^2}{8\,(b^3ex^2+b^3d)}$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="fricas")
```

output
```
[1/16*(((3*C*a^2 - 4*B*a*b + 8*A*b^2)*e*x^2 + (3*C*a^2 - 4*B*a*b + 8*A*b^2
)*d)*sqrt(b)*log((2*b*e*x^4 + (2*b*d + a*e)*x^2 + 2*sqrt(b*e*x^4 + (b*d +
a*e)*x^2 + a*d)*sqrt(e*x^2 + d)*sqrt(b)*x + a*d)/(e*x^2 + d)) + 2*(2*C*b^2
*x^3 - (3*C*a*b - 4*B*b^2)*x)*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(e
*x^2 + d))/(b^3*e*x^2 + b^3*d), -1/8*(((3*C*a^2 - 4*B*a*b + 8*A*b^2)*e*x^2
 + (3*C*a^2 - 4*B*a*b + 8*A*b^2)*d)*sqrt(-b)*arctan(sqrt(e*x^2 + d)*sqrt(-
b)*x/sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)) - (2*C*b^2*x^3 - (3*C*a*b - 4*
B*b^2)*x)*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(e*x^2 + d))/(b^3*e*x^
2 + b^3*d)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{\sqrt{(a+bx^2)\,(d+ex^2)}}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(C*x**4+B*x**2+A)/(a*d+(a*e+b*d)*x**2+b*e*x**4
)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x**2)*(A + B*x**2 + C*x**4)/sqrt((a + b*x**2)*(d + e*x
**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{ex^2+d}}{\sqrt{bex^4+(bd+ae)x^2+ad}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(e*x^2 + d)/sqrt(b*e*x^4 + (b*d + a*e)*x
^2 + a*d), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{ex^2+d}}{\sqrt{bex^4+(bd+ae)x^2+ad}}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(e*x^2 + d)/sqrt(b*e*x^4 + (b*d + a*e)*x
^2 + a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx=\int \frac{\sqrt{e\,x^2+d}\,(C\,x^4+B\,x^2+A)}{\sqrt{b\,e\,x^4+(a\,e+b\,d)\,x^2+a\,d}}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(A + B*x^2 + C*x^4))/(a*d + x^2*(a*e + b*d) + b*e*x
^4)^(1/2),x)
```

output
```
int(((d + e*x^2)^(1/2)*(A + B*x^2 + C*x^4))/(a*d + x^2*(a*e + b*d) + b*e*x
^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$$
$$=\frac{-3\sqrt{b\,x^2+a}\,abcx+4\sqrt{b\,x^2+a}\,b^3x+2\sqrt{b\,x^2+a}\,b^2c\,x^3+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2c+4\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}-}{\sqrt{a}}\right.}{8b^3}$$

input
```
int((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a*b*c*x + 4*sqrt(a + b*x**2)*b**3*x + 2*sqrt(a + b*
x**2)*b**2*c*x**3 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*
a**2*c + 4*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2)/(8*
b**3)
```

3.149 $\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 213

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{Cx\sqrt{ad+(bd+ae)x^2+bex^4}}{2be\sqrt{d+ex^2}} - \frac{(2bCd-2bBe+aCe)\text{arctanh}\left(\frac{\sqrt{b}x\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\right)}{2b^{3/2}e^2} + \frac{(Cd^2-Bde+Ae^2)\,\text{arctanh}\left(\frac{\sqrt{bd-ae}x\sqrt{d+ex^2}}{\sqrt{d}\sqrt{ad+(bd+ae)x^2+bex^4}}\right)}{\sqrt{d}e^2\sqrt{bd-ae}}$$

output
```
1/2*C*x*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/b/e/(e*x^2+d)^(1/2)-1/2*(-2*B*b*
e+C*a*e+2*C*b*d)*arctanh(b^(1/2)*x*(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*
x^4)^(1/2))/b^(3/2)/e^2+(A*e^2-B*d*e+C*d^2)*arctanh((-a*e+b*d)^(1/2)*x*(e*
x^2+d)^(1/2)/d^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2))/d^(1/2)/e^2/(-a*e+
b*d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.59 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.28

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$$
$$= \frac{\sqrt{d+ex^2}\left(\sqrt{b}Cex(a+bx^2)+\frac{2\sqrt{b}(-i\sqrt{a}\sqrt{e}+\sqrt{bd-ae})\sqrt{-bd+2ae-2i\sqrt{a}\sqrt{e}\sqrt{bd-ae}}(Cd^2+e(-Bd+Ae))\sqrt{a+bx^2}\arctan\left(\frac{\sqrt{-bd}}{}\right.}{d^{3/2}\sqrt{bd-ae}}\right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a*d + (b*d + a*e)*x^2
+ b*e*x^4]),x]
```

output
```
(Sqrt[d + e*x^2]*(Sqrt[b]*C*e*x*(a + b*x^2) + (2*Sqrt[b]*((-I)*Sqrt[a]*Sqr
t[e] + Sqrt[b*d - a*e])*Sqrt[-(b*d) + 2*a*e - (2*I)*Sqrt[a]*Sqrt[e]*Sqrt[b
*d - a*e]]*(C*d^2 + e*(-(B*d) + A*e))*Sqrt[a + b*x^2]*ArcTan[(Sqrt[-(b*d)
+ 2*a*e - (2*I)*Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]]*x)/(Sqrt[d]*(Sqrt[a] - Sq
rt[a + b*x^2]))])/(d^(3/2)*Sqrt[b*d - a*e]) + (2*Sqrt[b]*(I*Sqrt[a]*Sqrt[e
] + Sqrt[b*d - a*e])*Sqrt[-(b*d) + 2*a*e + (2*I)*Sqrt[a]*Sqrt[e]*Sqrt[b*d
- a*e]]*(C*d^2 + e*(-(B*d) + A*e))*Sqrt[a + b*x^2]*ArcTan[(Sqrt[-(b*d) + 2
*a*e + (2*I)*Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]]*x)/(Sqrt[d]*(Sqrt[a] - Sqrt[
a + b*x^2]))])/(d^(3/2)*Sqrt[b*d - a*e]) + 2*(2*b*C*d - 2*b*B*e + a*C*e)*S
qrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/(Sqrt[a] - Sqrt[a + b*x^2])]))/(2*b^(3/
2)*e^2*Sqrt[(a + b*x^2)*(d + e*x^2)])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1395, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{x^2(ae+bd)+ad+bex^4}}dx$$

$$\Big\downarrow \text{ 1395}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \frac{Cx^4+Bx^2+A}{\sqrt{bx^2+a}(ex^2+d)}dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{ 7276}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \left(\frac{Cx^2}{e\sqrt{bx^2+a}}-\frac{Cd-Be}{e^2\sqrt{bx^2+a}}+\frac{Cd^2-Bed+Ae^2}{e^2\sqrt{bx^2+a}(ex^2+d)}\right)dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{(Ae^2-Bde+Cd^2)\text{arctanh}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{\sqrt{d}e^2\sqrt{bd-ae}}-\frac{aC\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}e}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Cd-Be)}{\sqrt{b}e^2}+Cx\right.}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a*d + (b*d + a*e)*x^2 + b*e*
x^4]),x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[d + e*x^2]*((C*x*Sqrt[a + b*x^2])/(2*b*e) - (a*C*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)*e) - ((C*d - B*e)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*e^2) + ((C*d^2 - B*d*e + A*e^2)*ArcTa
nh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2])])/(Sqrt[d]*e^2*Sqrt[b*d -
 a*e])))/Sqrt[a*d + (b*d + a*e)*x^2 + b*e*x^4]
```

Defintions of rubi rules used

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(185) = 370$.

Time = 0.29 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.18

method	result
risch	$\frac{Cx\left(b\,x^2+a\right)\sqrt{e\,x^2+d}}{2eb\sqrt{\left(e\,x^2+d\right)\left(b\,x^2+a\right)}}+\frac{\left(\frac{\left(2Bbe-Cae-2Cbd\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{e\sqrt{b}}-\frac{b\left(A\,e^2-Bde+C\,d^2\right)\ln\left(\frac{\frac{2ae-2bd}{e}+\frac{2b\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}+2\sqrt{\frac{ae-b}{e}}}{x}\right)}{\sqrt{-de}\,e\sqrt{\frac{ae-bd}{e}}}\right)}{}$
default	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
1/2*C/e/b*x*(b*x^2+a)/((e*x^2+d)*(b*x^2+a))^(1/2)*(e*x^2+d)^(1/2)+1/2/e/b*
(1/e*(2*B*b*e-C*a*e-2*C*b*d)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-b*(A*e^
2-B*d*e+C*d^2)/(-d*e)^(1/2)/e/((a*e-b*d)/e)^(1/2)*ln((2*(a*e-b*d)/e+2*b*(-
d*e)^(1/2)/e*(x-(-d*e)^(1/2)/e)+2*((a*e-b*d)/e)^(1/2)*((x-(-d*e)^(1/2)/e)^
2*b+2*b*(-d*e)^(1/2)/e*(x-(-d*e)^(1/2)/e)+(a*e-b*d)/e)^(1/2))/(x-(-d*e)^(1
/2)/e))+b*(A*e^2-B*d*e+C*d^2)/(-d*e)^(1/2)/e/((a*e-b*d)/e)^(1/2)*ln((2*(a*
e-b*d)/e-2*b*(-d*e)^(1/2)/e*(x+(-d*e)^(1/2)/e)+2*((a*e-b*d)/e)^(1/2)*((x+(
-d*e)^(1/2)/e)^2*b-2*b*(-d*e)^(1/2)/e*(x+(-d*e)^(1/2)/e)+(a*e-b*d)/e)^(1/2
))/(x+(-d*e)^(1/2)/e)))*(b*x^2+a)^(1/2)/((e*x^2+d)*(b*x^2+a))^(1/2)*(e*x^2
+d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(185) = 370$.

Time = 0.70 (sec) , antiderivative size = 1716, normalized size of antiderivative = 8.06

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\sqrt{ad + (bd + ae)x^2 + bex^4}}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="fricas")
```

output
```
[1/4*(2*(C*b^2*d^2*e - C*a*b*d*e^2)*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*
sqrt(e*x^2 + d)*x + 2*(C*b^2*d^3 - B*b^2*d^2*e + A*b^2*d*e^2 + (C*b^2*d^2*
e - B*b^2*d*e^2 + A*b^2*e^3)*x^2)*sqrt(b*d^2 - a*d*e)*log((2*b*d^2*x^2 + (
2*b*d*e - a*e^2)*x^4 + a*d^2 + 2*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqr
t(b*d^2 - a*d*e)*sqrt(e*x^2 + d)*x)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (2*C*b^
2*d^4 - (C*a*b + 2*B*b^2)*d^3*e - (C*a^2 - 2*B*a*b)*d^2*e^2 + (2*C*b^2*d^3
*e - (C*a*b + 2*B*b^2)*d^2*e^2 - (C*a^2 - 2*B*a*b)*d*e^3)*x^2)*sqrt(b)*log
((2*b*e*x^4 + (2*b*d + a*e)*x^2 + 2*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*
sqrt(e*x^2 + d)*sqrt(b)*x + a*d)/(e*x^2 + d)))/(b^3*d^3*e^2 - a*b^2*d^2*e^
3 + (b^3*d^2*e^3 - a*b^2*d*e^4)*x^2), 1/4*(2*(C*b^2*d^2*e - C*a*b*d*e^2)*s
qrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(e*x^2 + d)*x - 4*(C*b^2*d^3 - B*
b^2*d^2*e + A*b^2*d*e^2 + (C*b^2*d^2*e - B*b^2*d*e^2 + A*b^2*e^3)*x^2)*sqr
t(-b*d^2 + a*d*e)*arctan(sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(-b*d^2
 + a*d*e)*sqrt(e*x^2 + d)*x/(b*d*e*x^4 + a*d^2 + (b*d^2 + a*d*e)*x^2)) - (
2*C*b^2*d^4 - (C*a*b + 2*B*b^2)*d^3*e - (C*a^2 - 2*B*a*b)*d^2*e^2 + (2*C*b
^2*d^3*e - (C*a*b + 2*B*b^2)*d^2*e^2 - (C*a^2 - 2*B*a*b)*d*e^3)*x^2)*sqrt(
b)*log((2*b*e*x^4 + (2*b*d + a*e)*x^2 + 2*sqrt(b*e*x^4 + (b*d + a*e)*x^2 +
 a*d)*sqrt(e*x^2 + d)*sqrt(b)*x + a*d)/(e*x^2 + d)))/(b^3*d^3*e^2 - a*b^2*
d^2*e^3 + (b^3*d^2*e^3 - a*b^2*d*e^4)*x^2), 1/2*((C*b^2*d^2*e - C*a*b*d*e^
2)*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(e*x^2 + d)*x + (2*C*b^2*d...
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)\,(d+ex^2)}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(a*d+(a*e+b*d)*x**2+b*e*x**4
)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt((a + b*x**2)*(d + e*x**2))*sqrt(d + e
*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{bex^4+(bd+ae)x^2+ad}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(
e*x^2 + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.70

$$\begin{aligned}&\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx\\&=\frac{\sqrt{bx^2+a}Cx}{2\,be}-\frac{(Cd^2-Bde+Ae^2)\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2e+2\,bd-ae}{2\,\sqrt{-b^2d^2+abde}}\right)}{\sqrt{-b^2d^2+abde}e^2}\\&\quad+\frac{(2\,Cbd+Cae-2\,Bbe)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}e^2}\end{aligned}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*C*x/(b*e) - (C*d^2 - B*d*e + A*e^2)*sqrt(b)*arctan(1/2
*((sqrt(b)*x - sqrt(b*x^2 + a))^2*e + 2*b*d - a*e)/sqrt(-b^2*d^2 + a*b*d*e
))/(sqrt(-b^2*d^2 + a*b*d*e)*e^2) + 1/2*(2*C*b*d + C*a*e - 2*B*b*e)*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/(b^(3/2)*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{\sqrt{e\,x^2+d}\,\sqrt{b\,e\,x^4+\left(a\,e+b\,d\right)\,x^2+a\,d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(1/2)*(a*d + x^2*(a*e + b*d) + b*e*x^
4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(1/2)*(a*d + x^2*(a*e + b*d) + b*e*x^
4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.60

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{-2\sqrt{d}\,\sqrt{ae-bd}\,atan\left(\frac{\sqrt{ae-bd}-\sqrt{e}\,\sqrt{b\,x^2+a}-\sqrt{e}\,\sqrt{b}\,x}{\sqrt{d}\,\sqrt{b}}\right)a\,b^2e^2+2\sqrt{d}\,\sqrt{ae-bd}\,atan\left(\frac{\sqrt{ae-bd}-\sqrt{e}\,\sqrt{b\,x^2+a}-\sqrt{e}\,\sqrt{b}\,x}{\sqrt{d}\,\sqrt{b}}\right)}{}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x)
```

output
```
( - 2*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x
**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a*b**2*e**2 + 2*sqrt(d)*sqrt(
a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt
(b)*x)/(sqrt(d)*sqrt(b)))*b**3*d*e - 2*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(
a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b
)))*b**2*c*d**2 - 2*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) + sqrt(e
)*sqrt(a + b*x**2) + sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a*b**2*e**2 + 2
*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) + sqrt(e)*sqrt(a + b*x**2)
+ sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*b**3*d*e - 2*sqrt(d)*sqrt(a*e - b*
d)*atan((sqrt(a*e - b*d) + sqrt(e)*sqrt(a + b*x**2) + sqrt(e)*sqrt(b)*x)/(
sqrt(d)*sqrt(b)))*b**2*c*d**2 + sqrt(a + b*x**2)*a*b*c*d*e**2*x - sqrt(a +
 b*x**2)*b**2*c*d**2*e*x - sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt
(a))*a**2*c*d*e**2 + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))
*a*b**2*d*e**2 - sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c
*d**2*e - 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**3*d**2*
e + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**2*c*d**3)/(2*
b**2*d*e**2*(a*e - b*d))
```

3.150 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 246

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = -\frac{(Cd^2-Bde+Ae^2)\,x\sqrt{ad+(bd+ae)x^2+bex^4}}{2de(bd-ae)\,(d+ex^2)^{3/2}} + \frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x\sqrt{d+ex^2}}{\sqrt{ad+(bd+ae)x^2+bex^4}}\right)}{\sqrt{b}e^2} - \frac{(2b(Cd^3-Ade^2)-ae(3Cd^2-e(Bd+Ae)))\operatorname{arctanh}\left(\frac{\sqrt{bd-ae}x\sqrt{d+ex^2}}{\sqrt{d}\sqrt{ad+(bd+ae)x^2+bex^4}}\right)}{2d^{3/2}e^2(bd-ae)^{3/2}}$$

output
```
-1/2*(A*e^2-B*d*e+C*d^2)*x*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/d/e/(-a*e+b*d
)/(e*x^2+d)^(3/2)+C*arctanh(b^(1/2)*x*(e*x^2+d)^(1/2)/(a*d+(a*e+b*d)*x^2+b
*e*x^4)^(1/2))/b^(1/2)/e^2-1/2*(2*b*(-A*d*e^2+C*d^3)-a*e*(3*C*d^2-e*(A*e+B
*d)))*arctanh((-a*e+b*d)^(1/2)*x*(e*x^2+d)^(1/2)/d^(1/2)/(a*d+(a*e+b*d)*x^
2+b*e*x^4)^(1/2))/d^(3/2)/e^2/(-a*e+b*d)^(3/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{-\frac{e(Cd^2+e(-Bd+Ae))x(a+bx^2)}{d(bd-ae)} - \frac{(2b(Cd^3-Ade^2)+ae(-3Cd^2+e(Bd+Ae}{}}{2e^2\sqrt{d+}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*Sqrt[a*d + (b*d + a*e)*x^
2 + b*e*x^4]),x]
```

output
```
(-((e*(C*d^2 + e*(-(B*d) + A*e))*x*(a + b*x^2))/(d*(b*d - a*e))) - ((2*b*(
C*d^3 - A*d*e^2) + a*e*(-3*C*d^2 + e*(B*d + A*e)))*Sqrt[a + b*x^2]*(d + e*
x^2)*ArcTan[(-(e*x*Sqrt[a + b*x^2]) + Sqrt[b]*(d + e*x^2))/(Sqrt[d]*Sqrt[-
(b*d) + a*e])])/(d^(3/2)*(-(b*d) + a*e)^(3/2)) - (2*C*Sqrt[a + b*x^2]*(d +
 e*x^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/(2*e^2*Sqrt[d + e*x^
2]*Sqrt[(a + b*x^2)*(d + e*x^2)])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1395, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}\sqrt{x^2(ae+bd)+ad+bex^4}}\,dx$$

$$\Big\downarrow \text{1395}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \frac{Cx^4+Bx^2+A}{\sqrt{bx^2+a}(ex^2+d)^2}dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int\left(\frac{C}{e^2\sqrt{bx^2+a}}+\frac{Be-2Cd}{e^2\sqrt{bx^2+a}(ex^2+d)}+\frac{Cd^2-Bed+Ae^2}{e^2\sqrt{bx^2+a}(ex^2+d)^2}\right)dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{(2bd-ae)(Ae^2-Bde+Cd^2)\text{arctanh}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{2d^{3/2}e^2(bd-ae)^{3/2}}-\frac{x\sqrt{a+bx^2}(Ae^2-Bde+Cd^2)}{2de(d+ex^2)(bd-ae)}-\frac{(2Cd-Be)\text{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{d}\sqrt{}}\right.}{\sqrt{d}e^2\sqrt{bd-ae}}\right.}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*Sqrt[a*d + (b*d + a*e)*x^2 + b*
e*x^4]),x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[d + e*x^2]*(-1/2*((C*d^2 - B*d*e + A*e^2)*x*Sqrt[a +
 b*x^2])/(d*e*(b*d - a*e)*(d + e*x^2)) + (C*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(Sqrt[b]*e^2) - ((2*C*d - B*e)*ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d
]*Sqrt[a + b*x^2])])/(Sqrt[d]*e^2*Sqrt[b*d - a*e]) + ((2*b*d - a*e)*(C*d^2
 - B*d*e + A*e^2)*ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2])])/
(2*d^(3/2)*e^2*(b*d - a*e)^(3/2))))/Sqrt[a*d + (b*d + a*e)*x^2 + b*e*x^4]
```

Defintions of rubi rules used

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3430 vs. 2(218) = 436.

Time = 0.11 (sec) , antiderivative size = 3431, normalized size of antiderivative = 13.95

method	result	size
default	Expression too large to display	3431

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
-1/4*(3*A*ln(2*((b*x^2+a)^(1/2)*((a*e-b*d)/e)^(1/2)*e-(-d*e)^(1/2)*b*x+a*e
)/(e*x+(-d*e)^(1/2)))*a*b^(3/2)*d*e^4*x^2-3*A*ln(2*((b*x^2+a)^(1/2)*((a*e-
b*d)/e)^(1/2)*e+(-d*e)^(1/2)*b*x+a*e)/(e*x-(-d*e)^(1/2)))*a*b^(3/2)*d*e^4*
x^2+B*ln(2*((b*x^2+a)^(1/2)*((a*e-b*d)/e)^(1/2)*e-(-d*e)^(1/2)*b*x+a*e)/(e
*x+(-d*e)^(1/2)))*a*b^(3/2)*d^2*e^3*x^2-5*C*ln(2*((b*x^2+a)^(1/2)*((a*e-b*
d)/e)^(1/2)*e-(-d*e)^(1/2)*b*x+a*e)/(e*x+(-d*e)^(1/2)))*a*b^(3/2)*d^3*e^2*
x^2+5*C*ln(2*((b*x^2+a)^(1/2)*((a*e-b*d)/e)^(1/2)*e+(-d*e)^(1/2)*b*x+a*e)/
(e*x-(-d*e)^(1/2)))*a*b^(3/2)*d^3*e^2*x^2-B*ln(2*((b*x^2+a)^(1/2)*((a*e-b*
d)/e)^(1/2)*e-(-d*e)^(1/2)*b*x+a*e)/(e*x+(-d*e)^(1/2)))*a^2*d*e^4*x^2*b^(1
/2)+B*ln(2*((b*x^2+a)^(1/2)*((a*e-b*d)/e)^(1/2)*e+(-d*e)^(1/2)*b*x+a*e)/(e
*x-(-d*e)^(1/2)))*a^2*d*e^4*x^2*b^(1/2)+3*C*ln(2*((b*x^2+a)^(1/2)*((a*e-b*
d)/e)^(1/2)*e-(-d*e)^(1/2)*b*x+a*e)/(e*x+(-d*e)^(1/2)))*a^2*d^2*e^3*x^2*b^
(1/2)-3*C*ln(2*((b*x^2+a)^(1/2)*((a*e-b*d)/e)^(1/2)*e+(-d*e)^(1/2)*b*x+a*e
)/(e*x-(-d*e)^(1/2)))*a^2*d^2*e^3*x^2*b^(1/2)-4*A*ln(((-(-b*x+(-b*a)^(1/2)
)/b*(b*x+(-b*a)^(1/2)))^(1/2)*b^(1/2)+b*x)/b^(1/2))*b^2*d^2*e^2*((a*e-b*d)
/e)^(1/2)*(-d*e)^(1/2)+4*A*ln(((b*x^2+a)^(1/2)*b^(1/2)+b*x)/b^(1/2))*b^2*d
^2*e^2*((a*e-b*d)/e)^(1/2)*(-d*e)^(1/2)-B*ln(2*((b*x^2+a)^(1/2)*((a*e-b*d)
/e)^(1/2)*e+(-d*e)^(1/2)*b*x+a*e)/(e*x-(-d*e)^(1/2)))*a*b^(3/2)*d^3*e^2-4*
C*ln(((-(-b*x+(-b*a)^(1/2))/b*(b*x+(-b*a)^(1/2)))^(1/2)*b^(1/2)+b*x)/b^(1/
2))*a^2*d^2*e^2*((a*e-b*d)/e)^(1/2)*(-d*e)^(1/2)-2*A*ln(2*((b*x^2+a)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(217) = 434$.

Time = 2.01 (sec) , antiderivative size = 2501, normalized size of antiderivative = 10.17

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)^{3/2} \sqrt{ad + (bd + ae)x^2 + bex^4}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="fricas")
```

output
```
[-1/4*(2*(C*b^2*d^4*e - A*a*b*d*e^4 - (C*a*b + B*b^2)*d^3*e^2 + (B*a*b + A
*b^2)*d^2*e^3)*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(e*x^2 + d)*x + (
2*C*b^2*d^5 - 3*C*a*b*d^4*e + A*a*b*d^2*e^3 + (B*a*b - 2*A*b^2)*d^3*e^2 +
(2*C*b^2*d^3*e^2 - 3*C*a*b*d^2*e^3 + A*a*b*e^5 + (B*a*b - 2*A*b^2)*d*e^4)*
x^4 + 2*(2*C*b^2*d^4*e - 3*C*a*b*d^3*e^2 + A*a*b*d*e^4 + (B*a*b - 2*A*b^2)
*d^2*e^3)*x^2)*sqrt(b*d^2 - a*d*e)*log((2*b*d^2*x^2 + (2*b*d*e - a*e^2)*x^
4 + a*d^2 + 2*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(b*d^2 - a*d*e)*sq
rt(e*x^2 + d)*x)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 2*(C*b^2*d^6 - 2*C*a*b*d^5
*e + C*a^2*d^4*e^2 + (C*b^2*d^4*e^2 - 2*C*a*b*d^3*e^3 + C*a^2*d^2*e^4)*x^4
 + 2*(C*b^2*d^5*e - 2*C*a*b*d^4*e^2 + C*a^2*d^3*e^3)*x^2)*sqrt(b)*log((2*b
*e*x^4 + (2*b*d + a*e)*x^2 + 2*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(
e*x^2 + d)*sqrt(b)*x + a*d)/(e*x^2 + d)))/(b^3*d^6*e^2 - 2*a*b^2*d^5*e^3 +
 a^2*b*d^4*e^4 + (b^3*d^4*e^4 - 2*a*b^2*d^3*e^5 + a^2*b*d^2*e^6)*x^4 + 2*(
b^3*d^5*e^3 - 2*a*b^2*d^4*e^4 + a^2*b*d^3*e^5)*x^2), -1/2*((C*b^2*d^4*e -
A*a*b*d*e^4 - (C*a*b + B*b^2)*d^3*e^2 + (B*a*b + A*b^2)*d^2*e^3)*sqrt(b*e*
x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(e*x^2 + d)*x - (2*C*b^2*d^5 - 3*C*a*b*d^
4*e + A*a*b*d^2*e^3 + (B*a*b - 2*A*b^2)*d^3*e^2 + (2*C*b^2*d^3*e^2 - 3*C*a
*b*d^2*e^3 + A*a*b*e^5 + (B*a*b - 2*A*b^2)*d*e^4)*x^4 + 2*(2*C*b^2*d^4*e -
 3*C*a*b*d^3*e^2 + A*a*b*d*e^4 + (B*a*b - 2*A*b^2)*d^2*e^3)*x^2)*sqrt(-b*d
^2 + a*d*e)*arctan(sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(-b*d^2 + ...
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)^{3/2}\sqrt{ad + (bd + ae)x^2 + bex^4}}\,dx = \int \frac{A + Bx^2 + Cx^4}{\sqrt{(a + bx^2)(d + ex^2)}\,(d + ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(3/2)/(a*d+(a*e+b*d)*x**2+b*e*x**4
)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt((a + b*x**2)*(d + e*x**2))*(d + e*x**
2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)^{3/2}\sqrt{ad + (bd + ae)x^2 + bex^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{bex^4 + (bd + ae)x^2 + ad}(ex^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*(e*x^
2 + d)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(217) = 434$.

Time = 0.38 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.91

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{\left(2\,Cb^{\frac{3}{2}}d^3-3\,Ca\sqrt{b}d^2e+Ba\sqrt{b}de^2-2\,Ab^{\frac{3}{2}}de^2+Aa\sqrt{b}e^3\right)\cdots}{2\,(bd^2e^2-ade^3)\sqrt{-b^2d^2+\cdots}} - \frac{C\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}e^2} - \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Cb^{\frac{3}{2}}d^3-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ca\sqrt{b}d^2e-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Bb^{\frac{3}{2}}d^2e+\left(\sqrt{b}x\cdots\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4e+4\left(\sqrt{b}x-\sqrt{b\cdots}\right.\right.}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="giac")
```

output
```
1/2*(2*C*b^(3/2)*d^3 - 3*C*a*sqrt(b)*d^2*e + B*a*sqrt(b)*d*e^2 - 2*A*b^(3/
2)*d*e^2 + A*a*sqrt(b)*e^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*e
+ 2*b*d - a*e)/sqrt(-b^2*d^2 + a*b*d*e))/((b*d^2*e^2 - a*d*e^3)*sqrt(-b^2*
d^2 + a*b*d*e)) - C*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/(sqrt(b)*e^2) -
 (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*b^(3/2)*d^3 - (sqrt(b)*x - sqrt(b*x^
2 + a))^2*C*a*sqrt(b)*d^2*e - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*b^(3/2)*
d^2*e + (sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b)*d*e^2 + 2*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*A*b^(3/2)*d*e^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*
sqrt(b)*e^3 + C*a^2*sqrt(b)*d^2*e - B*a^2*sqrt(b)*d*e^2 + A*a^2*sqrt(b)*e^
3)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*e + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2
*b*d - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*e + a^2*e)*(b*d^2*e^2 - a*d*e^3
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{{\left(e\,x^2+d\right)}^{3/2}\,\sqrt{b\,e\,x^4+\left(a\,e+b\,d\right)\,x^2+a\,d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a*d + x^2*(a*e + b*d) + b*e*x^
4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a*d + x^2*(a*e + b*d) + b*e*x^
4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1328, normalized size of antiderivative = 5.40

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x)
```

output
```
( - sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**
2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**2*b*d*e**3 - sqrt(d)*sqrt(a*
e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b
)*x)/(sqrt(d)*sqrt(b)))*a**2*b*e**4*x**2 + sqrt(d)*sqrt(a*e - b*d)*atan((s
qrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sq
rt(b)))*a*b**2*d**2*e**2 + sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) -
 sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a*b**2*d
*e**3*x**2 + 3*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqr
t(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a*b*c*d**3*e + 3*sqr
t(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sq
rt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a*b*c*d**2*e**2*x**2 - 2*sqrt(d)*sqrt(
a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt
(b)*x)/(sqrt(d)*sqrt(b)))*b**2*c*d**4 - 2*sqrt(d)*sqrt(a*e - b*d)*atan((sq
rt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqr
t(b)))*b**2*c*d**3*e*x**2 - sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d)
+ sqrt(e)*sqrt(a + b*x**2) + sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**2*b*
d*e**3 - sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) + sqrt(e)*sqrt(a +
b*x**2) + sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**2*b*e**4*x**2 + sqrt(d)
*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) + sqrt(e)*sqrt(a + b*x**2) + sqrt(e
)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a*b**2*d**2*e**2 + sqrt(d)*sqrt(a*e - b...
```

3.151 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 297

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = -\frac{(Cd^2-Bde+Ae^2)\,x\sqrt{ad+(bd+ae)x^2+bex^4}}{4de(bd-ae)\,(d+ex^2)^{5/2}} + \frac{(2bd(Cd^2+e(Bd-3Ae))-ae(5Cd^2-e(Bd+3Ae)))\,x\sqrt{ad+(bd+ae)x^2+bex^4}}{8d^2e(bd-ae)^2\,(d+ex^2)^{3/2}} - \frac{(ad(4bBd-3aCd-aBe)-A(8b^2d^2-8abde+3a^2e^2))\,\operatorname{arctanh}\left(\frac{\sqrt{bd-ae}\,x\sqrt{d+ex^2}}{\sqrt{d}\sqrt{ad+(bd+ae)x^2+bex^4}}\right)}{8d^{5/2}(bd-ae)^{5/2}}$$

output
```
-1/4*(A*e^2-B*d*e+C*d^2)*x*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/d/e/(-a*e+b*d
)/(e*x^2+d)^(5/2)+1/8*(2*b*d*(C*d^2+e*(-3*A*e+B*d))-a*e*(5*C*d^2-e*(3*A*e+
B*d)))*x*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/d^2/e/(-a*e+b*d)^2/(e*x^2+d)^(3
/2)-1/8*(a*d*(-B*a*e+4*B*b*d-3*C*a*d)-A*(3*a^2*e^2-8*a*b*d*e+8*b^2*d^2))*a
rctanh((-a*e+b*d)^(1/2)*x*(e*x^2+d)^(1/2)/d^(1/2)/(a*d+(a*e+b*d)*x^2+b*e*x
^4)^(1/2))/d^(5/2)/(-a*e+b*d)^(5/2)
```

Mathematica [A] (verified)

Time = 12.03 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{\frac{4d^2(2Cd-Be)x(d+ex^2)\left(e(a+bx^2)-\frac{(2bd-ae)(d+ex^2)\operatorname{arctanh}\left(\sqrt{\frac{(bd-ae)x^2}{d(a+bx^2)}}\right)}{d\sqrt{\frac{(bd-ae)x^2}{d(a+bx^2)}}}\right)}{bd-ae}}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*Sqrt[a*d + (b*d + a*e)*x^
2 + b*e*x^4]),x]
```

output
```
((4*d^2*(2*C*d - B*e)*x*(d + e*x^2)*(e*(a + b*x^2) - ((2*b*d - a*e)*(d + e
*x^2)*ArcTanh[Sqrt[((b*d - a*e)*x^2)/(d*(a + b*x^2))]])/(d*Sqrt[((b*d - a*
e)*x^2)/(d*(a + b*x^2))])))/(b*d - a*e) - ((C*d^2 + e*(-(B*d) + A*e))*x*(d
*e*(a + b*x^2)*(2*b*d*(4*d + 3*e*x^2) - a*e*(5*d + 3*e*x^2)) - ((8*b^2*d^2
 - 8*a*b*d*e + 3*a^2*e^2)*(d + e*x^2)^2*ArcTanh[Sqrt[((b*d - a*e)*x^2)/(d*
(a + b*x^2))]])/Sqrt[((b*d - a*e)*x^2)/(d*(a + b*x^2))]))/(b*d - a*e)^2 +
(8*C*d^(5/2)*Sqrt[a + b*x^2]*(d + e*x^2)^2*ArcTanh[(Sqrt[b*d - a*e]*x)/(Sq
rt[d]*Sqrt[a + b*x^2])])/Sqrt[b*d - a*e])/(8*d^3*e^2*(d + e*x^2)^(3/2)*Sqr
t[(a + b*x^2)*(d + e*x^2)])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.49, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1395, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\sqrt{x^2(ae+bd)+ad+bex^4}}\,dx$$

$$\Big\downarrow \; 1395$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \frac{Cx^4+Bx^2+A}{\sqrt{bx^2+a}(ex^2+d)^3}dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \left(\frac{C}{e^2\sqrt{bx^2+a}(ex^2+d)}+\frac{Be-2Cd}{e^2\sqrt{bx^2+a}(ex^2+d)^2}+\frac{Cd^2-Bed+Ae^2}{e^2\sqrt{bx^2+a}(ex^2+d)^3}\right)dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{(3a^2e^2-8abde+8b^2d^2)(Ae^2-Bde+Cd^2)\text{arctanh}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{8d^{5/2}e^2(bd-ae)^{5/2}}-\frac{3x\sqrt{a+bx^2}(2bd-ae)(Ae^2-Bde+Cd^2)}{8d^2e(d+ex^2)(bd-ae)^2}-\frac{x\sqrt{}}{}\right.}{\sqrt{x^2(ae+bd)+a}}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*Sqrt[a*d + (b*d + a*e)*x^2 + b*
e*x^4]),x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[d + e*x^2]*(-1/4*((C*d^2 - B*d*e + A*e^2)*x*Sqrt[a +
 b*x^2])/(d*e*(b*d - a*e)*(d + e*x^2)^2) + ((2*C*d - B*e)*x*Sqrt[a + b*x^2
])/(2*d*e*(b*d - a*e)*(d + e*x^2)) - (3*(2*b*d - a*e)*(C*d^2 - B*d*e + A*e
^2)*x*Sqrt[a + b*x^2])/(8*d^2*e*(b*d - a*e)^2*(d + e*x^2)) + (C*ArcTanh[(S
qrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2])])/(Sqrt[d]*e^2*Sqrt[b*d - a*e]
) - ((2*b*d - a*e)*(2*C*d - B*e)*ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt
[a + b*x^2])])/(2*d^(3/2)*e^2*(b*d - a*e)^(3/2)) + ((8*b^2*d^2 - 8*a*b*d*e
 + 3*a^2*e^2)*(C*d^2 - B*d*e + A*e^2)*ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]
*Sqrt[a + b*x^2])])/(8*d^(5/2)*e^2*(b*d - a*e)^(5/2))))/Sqrt[a*d + (b*d +
a*e)*x^2 + b*e*x^4]
```

Defintions of rubi rules used

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
 + e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
 + q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && !(EqQ[q,
1] && EqQ[n, 2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5778 vs. 2(271) = 542.

Time = 0.21 (sec) , antiderivative size = 5779, normalized size of antiderivative = 19.46

method	result	size
default	Expression too large to display	5779

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(269) = 538.

Time = 0.14 (sec) , antiderivative size = 1439, normalized size of antiderivative = 4.85

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="fricas")
```

output
```
[1/16*((3*A*a^2*d^3*e^2 + (3*A*a^2*e^5 + (3*C*a^2 - 4*B*a*b + 8*A*b^2)*d^2
*e^3 + (B*a^2 - 8*A*a*b)*d*e^4)*x^6 + (3*C*a^2 - 4*B*a*b + 8*A*b^2)*d^5 +
(B*a^2 - 8*A*a*b)*d^4*e + 3*(3*A*a^2*d*e^4 + (3*C*a^2 - 4*B*a*b + 8*A*b^2)
*d^3*e^2 + (B*a^2 - 8*A*a*b)*d^2*e^3)*x^4 + 3*(3*A*a^2*d^2*e^3 + (3*C*a^2
- 4*B*a*b + 8*A*b^2)*d^4*e + (B*a^2 - 8*A*a*b)*d^3*e^2)*x^2)*sqrt(b*d^2 -
a*d*e)*log((2*b*d^2*x^2 + (2*b*d*e - a*e^2)*x^4 + a*d^2 + 2*sqrt(b*e*x^4 +
 (b*d + a*e)*x^2 + a*d)*sqrt(b*d^2 - a*d*e)*sqrt(e*x^2 + d)*x)/(e^2*x^4 +
2*d*e*x^2 + d^2)) + 2*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*((2*C*b^2*d^5
- 3*A*a^2*d*e^4 - (7*C*a*b - 2*B*b^2)*d^4*e + (5*C*a^2 - B*a*b - 6*A*b^2)*
d^3*e^2 - (B*a^2 - 9*A*a*b)*d^2*e^3)*x^3 - (5*A*a^2*d^2*e^3 + (3*C*a*b - 4
*B*b^2)*d^5 - (3*C*a^2 - 5*B*a*b - 8*A*b^2)*d^4*e - (B*a^2 + 13*A*a*b)*d^3
*e^2)*x)*sqrt(e*x^2 + d))/(b^3*d^9 - 3*a*b^2*d^8*e + 3*a^2*b*d^7*e^2 - a^3
*d^6*e^3 + (b^3*d^6*e^3 - 3*a*b^2*d^5*e^4 + 3*a^2*b*d^4*e^5 - a^3*d^3*e^6)
*x^6 + 3*(b^3*d^7*e^2 - 3*a*b^2*d^6*e^3 + 3*a^2*b*d^5*e^4 - a^3*d^4*e^5)*x
^4 + 3*(b^3*d^8*e - 3*a*b^2*d^7*e^2 + 3*a^2*b*d^6*e^3 - a^3*d^5*e^4)*x^2),
 -1/8*((3*A*a^2*d^3*e^2 + (3*A*a^2*e^5 + (3*C*a^2 - 4*B*a*b + 8*A*b^2)*d^2
*e^3 + (B*a^2 - 8*A*a*b)*d*e^4)*x^6 + (3*C*a^2 - 4*B*a*b + 8*A*b^2)*d^5 +
(B*a^2 - 8*A*a*b)*d^4*e + 3*(3*A*a^2*d*e^4 + (3*C*a^2 - 4*B*a*b + 8*A*b^2)
*d^3*e^2 + (B*a^2 - 8*A*a*b)*d^2*e^3)*x^4 + 3*(3*A*a^2*d^2*e^3 + (3*C*a^2
- 4*B*a*b + 8*A*b^2)*d^4*e + (B*a^2 - 8*A*a*b)*d^3*e^2)*x^2)*sqrt(-b*d^...
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{\sqrt{(a+bx^2)(d+ex^2)}\,(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(5/2)/(a*d+(a*e+b*d)*x**2+b*e*x**4
)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt((a + b*x**2)*(d + e*x**2))*(d + e*x**
2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{bex^4+(bd+ae)x^2+ad}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*(e*x^
2 + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{bex^4+(bd+ae)x^2+ad}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*(e*x^
2 + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{(e\,x^2+d)^{5/2}\,\sqrt{b\,e\,x^4+(a\,e+b\,d)\,x^2+a\,d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a*d + x^2*(a*e + b*d) + b*e*x^
4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a*d + x^2*(a*e + b*d) + b*e*x^
4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 3209, normalized size of antiderivative = 10.80

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x)
```

output
```
( - 6*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x
**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**4*d**2*e**4 - 12*sqrt(d)*s
qrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*
sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**4*d*e**5*x**2 - 6*sqrt(d)*sqrt(a*e - b*d)
*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sq
rt(d)*sqrt(b)))*a**4*e**6*x**4 + 26*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e
 - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))
*a**3*b*d**3*e**3 + 52*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqr
t(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**3*b*d**2*
e**4*x**2 + 26*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqr
t(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**3*b*d*e**5*x**4 -
 6*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2
) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**3*c*d**4*e**2 - 12*sqrt(d)*sq
rt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*s
qrt(b)*x)/(sqrt(d)*sqrt(b)))*a**3*c*d**3*e**3*x**2 - 6*sqrt(d)*sqrt(a*e -
b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)
/(sqrt(d)*sqrt(b)))*a**3*c*d**2*e**4*x**4 - 36*sqrt(d)*sqrt(a*e - b*d)*ata
n((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d
)*sqrt(b)))*a**2*b**2*d**4*e**2 - 72*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*
e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(...
```

3.152 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{7/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 464

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{7/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx =$$

$$-\frac{(Cd^2-Bde+Ae^2)\,x\sqrt{ad+(bd+ae)x^2+bex^4}}{6de(bd-ae)\,(d+ex^2)^{7/2}}$$

$$+\frac{(2bd(Cd^2+e(2Bd-5Ae))-ae(7Cd^2-e(Bd+5Ae)))\,x\sqrt{ad+(bd+ae)x^2+bex^4}}{24d^2e(bd-ae)^2\,(d+ex^2)^{5/2}}$$

$$+\frac{(4b^2d^2(Cd^2+e(2Bd-11Ae))-3a^2e^2(Cd^2+e(Bd+5Ae))-2abde(8Cd^2-e(5Bd+22Ae)))\,x\sqrt{ad+}}{48d^3e(bd-ae)^3\,(d+ex^2)^{3/2}}$$

$$+\frac{(A(16b^3d^3-24ab^2d^2e+18a^2bde^2-5a^3e^3)-ad(8b^2Bd^2+a^2e(Cd+Be)-2abd(3Cd+2Be)))\,\text{arctanh}}{16d^{7/2}(bd-ae)^{7/2}}$$

output
```
-1/6*(A*e^2-B*d*e+C*d^2)*x*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/d/e/(-a*e+b*d
)/(e*x^2+d)^(7/2)+1/24*(2*b*d*(C*d^2+e*(-5*A*e+2*B*d))-a*e*(7*C*d^2-e*(5*A
*e+B*d)))*x*(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2)/d^2/e/(-a*e+b*d)^2/(e*x^2+d)
^(5/2)+1/48*(4*b^2*d^2*(C*d^2+e*(-11*A*e+2*B*d))-3*a^2*e^2*(C*d^2+e*(5*A*e
+B*d))-2*a*b*d*e*(8*C*d^2-e*(22*A*e+5*B*d)))*x*(a*d+(a*e+b*d)*x^2+b*e*x^4)
^(1/2)/d^3/e/(-a*e+b*d)^3/(e*x^2+d)^(3/2)+1/16*(A*(-5*a^3*e^3+18*a^2*b*d*e
^2-24*a*b^2*d^2*e+16*b^3*d^3)-a*d*(8*b^2*B*d^2+a^2*e*(B*e+C*d)-2*a*b*d*(2*
B*e+3*C*d)))*arctanh((-a*e+b*d)^(1/2)*x*(e*x^2+d)^(1/2)/d^(1/2)/(a*d+(a*e+
b*d)*x^2+b*e*x^4)^(1/2))/d^(7/2)/(-a*e+b*d)^(7/2)
```

Mathematica [A] (verified)

Time = 13.08 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{7/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \frac{x\left(-24Cd^3(bd-ae)^2\left(d+ex^2\right)^2\left(e(a+bx^2)-\frac{(2bd-ae)(d-\cdots}{\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^(7/2)*Sqrt[a*d + (b*d + a*e)*x^
2 + b*e*x^4]),x]
```

output
```
(x*(-24*C*d^3*(b*d - a*e)^2*(d + e*x^2)^2*(e*(a + b*x^2) - ((2*b*d - a*e)*
(d + e*x^2)*ArcTanh[Sqrt[((b*d - a*e)*x^2)/(d*(a + b*x^2))]])/(d*Sqrt[((b*
d - a*e)*x^2)/(d*(a + b*x^2))])) + 6*d*(b*d - a*e)*(2*C*d - B*e)*(d + e*x^
2)*(d*e*(a + b*x^2)*(2*b*d*(4*d + 3*e*x^2) - a*e*(5*d + 3*e*x^2)) - ((8*b^
2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*(d + e*x^2)^2*ArcTanh[Sqrt[((b*d - a*e)*x^2
)/(d*(a + b*x^2))]])/Sqrt[((b*d - a*e)*x^2)/(d*(a + b*x^2))]) - (C*d^2 + e
*(-(B*d) + A*e))*(d*e*(a + b*x^2)*(4*b^2*d^2*(18*d^2 + 27*d*e*x^2 + 11*e^2
*x^4) + a^2*e^2*(33*d^2 + 40*d*e*x^2 + 15*e^2*x^4) - 2*a*b*d*e*(45*d^2 + 5
9*d*e*x^2 + 22*e^2*x^4)) - (3*(2*b*d - a*e)*(8*b^2*d^2 - 8*a*b*d*e + 5*a^2
*e^2)*(d + e*x^2)^3*ArcTanh[Sqrt[((b*d - a*e)*x^2)/(d*(a + b*x^2))]])/Sqrt
[((b*d - a*e)*x^2)/(d*(a + b*x^2))])))/(48*d^4*e^2*(b*d - a*e)^3*(d + e*x^
2)^(5/2)*Sqrt[(a + b*x^2)*(d + e*x^2)])
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 658, normalized size of antiderivative = 1.42, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1395, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{7/2}\sqrt{x^2(ae+bd)+ad+bex^4}}dx$$

$$\Big\downarrow \text{1395}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \frac{Cx^4+Bx^2+A}{\sqrt{bx^2+a}(ex^2+d)^4}dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\int \left(\frac{C}{e^2\sqrt{bx^2+a}(ex^2+d)^2}+\frac{Be-2Cd}{e^2\sqrt{bx^2+a}(ex^2+d)^3}+\frac{Cd^2-Bed+Ae^2}{e^2\sqrt{bx^2+a}(ex^2+d)^4}\right)dx}{\sqrt{x^2(ae+bd)+ad+bex^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a+bx^2}\sqrt{d+ex^2}\left(\frac{(2bd-ae)(5a^2e^2-8abde+8b^2d^2)(Ae^2-Bde+Cd^2)\text{arctanh}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{16d^{7/2}e^2(bd-ae)^{7/2}}-\frac{x\sqrt{a+bx^2}(15a^2e^2-44abde+44b^2d^2)}{48d^3e(d+ex^2)(bd-ae}\right.}{}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^(7/2)*Sqrt[a*d + (b*d + a*e)*x^2 + b*
e*x^4]),x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[d + e*x^2]*(-1/6*((C*d^2 - B*d*e + A*e^2)*x*Sqrt[a +
 b*x^2])/(d*e*(b*d - a*e)*(d + e*x^2)^3) + ((2*C*d - B*e)*x*Sqrt[a + b*x^2
])/(4*d*e*(b*d - a*e)*(d + e*x^2)^2) - (5*(2*b*d - a*e)*(C*d^2 - B*d*e + A
*e^2)*x*Sqrt[a + b*x^2])/(24*d^2*e*(b*d - a*e)^2*(d + e*x^2)^2) - (C*x*Sqr
t[a + b*x^2])/(2*d*e*(b*d - a*e)*(d + e*x^2)) + (3*(2*b*d - a*e)*(2*C*d -
B*e)*x*Sqrt[a + b*x^2])/(8*d^2*e*(b*d - a*e)^2*(d + e*x^2)) - ((44*b^2*d^2
 - 44*a*b*d*e + 15*a^2*e^2)*(C*d^2 - B*d*e + A*e^2)*x*Sqrt[a + b*x^2])/(48
*d^3*e*(b*d - a*e)^3*(d + e*x^2)) + (C*(2*b*d - a*e)*ArcTanh[(Sqrt[b*d - a
*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2])])/(2*d^(3/2)*e^2*(b*d - a*e)^(3/2)) - ((2
*C*d - B*e)*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*ArcTanh[(Sqrt[b*d - a*e]*x
)/(Sqrt[d]*Sqrt[a + b*x^2])])/(8*d^(5/2)*e^2*(b*d - a*e)^(5/2)) + ((2*b*d
- a*e)*(8*b^2*d^2 - 8*a*b*d*e + 5*a^2*e^2)*(C*d^2 - B*d*e + A*e^2)*ArcTanh
[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2])])/(16*d^(7/2)*e^2*(b*d - a*
e)^(7/2))))/Sqrt[a*d + (b*d + a*e)*x^2 + b*e*x^4]
```

Defintions of rubi rules used

rule 1395
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(
x_)^(n_))^(q_.), x_Symbol] :> Simp[(a + b*x^n + c*x^(2*n))^FracPart[p]/((d
+ e*x^n)^FracPart[p]*(a/d + c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p
+ q)*(a/d + (c/e)*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q,
1] && EqQ[n, 2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 10035 vs. $2(432) = 864$.

Time = 0.46 (sec) , antiderivative size = 10036, normalized size of antiderivative = 21.63

method	result	size
default	Expression too large to display	10036

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(7/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. $2(432) = 864$.

Time = 0.23 (sec) , antiderivative size = 2520, normalized size of antiderivative = 5.43

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{7/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(7/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="fricas")
```

output
```
[-1/96*(3*(5*A*a^3*d^4*e^3 + (5*A*a^3*e^7 - 2*(3*C*a^2*b - 4*B*a*b^2 + 8*A
*b^3)*d^3*e^4 + (C*a^3 - 4*B*a^2*b + 24*A*a*b^2)*d^2*e^5 + (B*a^3 - 18*A*a
^2*b)*d*e^6)*x^8 - 2*(3*C*a^2*b - 4*B*a*b^2 + 8*A*b^3)*d^7 + (C*a^3 - 4*B*
a^2*b + 24*A*a*b^2)*d^6*e + (B*a^3 - 18*A*a^2*b)*d^5*e^2 + 4*(5*A*a^3*d*e^
6 - 2*(3*C*a^2*b - 4*B*a*b^2 + 8*A*b^3)*d^4*e^3 + (C*a^3 - 4*B*a^2*b + 24*
A*a*b^2)*d^3*e^4 + (B*a^3 - 18*A*a^2*b)*d^2*e^5)*x^6 + 6*(5*A*a^3*d^2*e^5
- 2*(3*C*a^2*b - 4*B*a*b^2 + 8*A*b^3)*d^5*e^2 + (C*a^3 - 4*B*a^2*b + 24*A*
a*b^2)*d^4*e^3 + (B*a^3 - 18*A*a^2*b)*d^3*e^4)*x^4 + 4*(5*A*a^3*d^3*e^4 -
2*(3*C*a^2*b - 4*B*a*b^2 + 8*A*b^3)*d^6*e + (C*a^3 - 4*B*a^2*b + 24*A*a*b^
2)*d^5*e^2 + (B*a^3 - 18*A*a^2*b)*d^4*e^3)*x^2)*sqrt(b*d^2 - a*d*e)*log((2
*b*d^2*x^2 + (2*b*d*e - a*e^2)*x^4 + a*d^2 + 2*sqrt(b*e*x^4 + (b*d + a*e)*
x^2 + a*d)*sqrt(b*d^2 - a*d*e)*sqrt(e*x^2 + d)*x)/(e^2*x^4 + 2*d*e*x^2 + d
^2)) - 2*sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*((4*C*b^3*d^6*e + 15*A*a^3*
d*e^6 - 4*(5*C*a*b^2 - 2*B*b^3)*d^5*e^2 + (13*C*a^2*b + 2*B*a*b^2 - 44*A*b
^3)*d^4*e^3 + (3*C*a^3 - 13*B*a^2*b + 88*A*a*b^2)*d^3*e^4 + (3*B*a^3 - 59*
A*a^2*b)*d^2*e^5)*x^5 + 2*(6*C*b^3*d^7 + 20*A*a^3*d^2*e^5 - (31*C*a*b^2 -
12*B*b^3)*d^6*e + (29*C*a^2*b - 5*B*a*b^2 - 54*A*b^3)*d^5*e^2 - (4*C*a^3 +
 11*B*a^2*b - 113*A*a*b^2)*d^4*e^3 + (4*B*a^3 - 79*A*a^2*b)*d^3*e^4)*x^3 +
 3*(11*A*a^3*d^3*e^4 - 2*(3*C*a*b^2 - 4*B*b^3)*d^7 + (7*C*a^2*b - 12*B*a*b
^2 - 24*A*b^3)*d^6*e - (C*a^3 - 5*B*a^2*b - 54*A*a*b^2)*d^5*e^2 - (B*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)^{7/2} \sqrt{ad + (bd + ae)x^2 + bex^4}} \, dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(7/2)/(a*d+(a*e+b*d)*x**2+b*e*x**4
)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{7/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{bex^4+(bd+ae)x^2+ad}(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(7/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*(e*x^
2 + d)^(7/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{7/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{bex^4+(bd+ae)x^2+ad}(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(7/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(b*e*x^4 + (b*d + a*e)*x^2 + a*d)*(e*x^
2 + d)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{7/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{(e\,x^2+d)^{7/2}\,\sqrt{b\,e\,x^4+(a\,e+b\,d)\,x^2+a\,d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(7/2)*(a*d + x^2*(a*e + b*d) + b*e*x^
4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(7/2)*(a*d + x^2*(a*e + b*d) + b*e*x^
4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 5903, normalized size of antiderivative = 12.72

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{7/2}\sqrt{ad+(bd+ae)x^2+bex^4}}\,dx = \text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(7/2)/(a*d+(a*e+b*d)*x^2+b*e*x^4)^(1/2),x)
```

output
```
( - 15*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*
x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**5*d**3*e**6 - 45*sqrt(d)*
sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)
*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**5*d**2*e**7*x**2 - 45*sqrt(d)*sqrt(a*e -
 b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x
)/(sqrt(d)*sqrt(b)))*a**5*d*e**8*x**4 - 15*sqrt(d)*sqrt(a*e - b*d)*atan((s
qrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sq
rt(b)))*a**5*e**9*x**6 + 81*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d)
- sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**4*b*
d**4*e**5 + 243*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sq
rt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**4*b*d**3*e**6*x*
*2 + 243*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a +
b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**4*b*d**2*e**7*x**4 + 81
*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2)
- sqrt(e)*sqrt(b)*x)/(sqrt(d)*sqrt(b)))*a**4*b*d*e**8*x**6 - 3*sqrt(d)*sqr
t(a*e - b*d)*atan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sq
rt(b)*x)/(sqrt(d)*sqrt(b)))*a**4*c*d**5*e**4 - 9*sqrt(d)*sqrt(a*e - b*d)*a
tan((sqrt(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt
(d)*sqrt(b)))*a**4*c*d**4*e**5*x**2 - 9*sqrt(d)*sqrt(a*e - b*d)*atan((sqrt
(a*e - b*d) - sqrt(e)*sqrt(a + b*x**2) - sqrt(e)*sqrt(b)*x)/(sqrt(d)*sq...
```

3.153 $\int \frac{1-\sqrt{3}-2x^2}{\sqrt{1+x^2}(1-x^2+x^4)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 54

$$\int \frac{1-\sqrt{3}-2x^2}{\sqrt{1+x^2}\,(1-x^2+x^4)}\,dx=-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}x\sqrt{1+x^2}}{1+\left(1-\sqrt{3}\right)x^2}\right)}{\sqrt[4]{3}}$$

output
```
-1/3*2^(1/2)*arctan((-3+2*3^(1/2))^(1/2)*x*(x^2+1)^(1/2)/(1+(1-3^(1/2))*x^
2))*3^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.65

$$\int \frac{1-\sqrt{3}-2x^2}{\sqrt{1+x^2}\,(1-x^2+x^4)}\,dx=\text{RootSum}\Big[1-8\#1^2+30\#1^4-8\#1^6+\#1^8\&,\frac{\log\left(-x+\sqrt{1+x^2}-\#1\right)-4\log\left(-x+\sqrt{1+x^2}-\#1\right)\#1^2+2\sqrt{3}\log\left(-x+\sqrt{1+x^2}-\#1\right.}{-2+15\#1^2-6\#1^4+\#1^6}$$

input
```
Integrate[(1 - Sqrt[3] - 2*x^2)/(Sqrt[1 + x^2]*(1 - x^2 + x^4)),x]
```

output
```
RootSum[1 - 8*#1^2 + 30*#1^4 - 8*#1^6 + #1^8 & , (Log[-x + Sqrt[1 + x^2] -
 #1] - 4*Log[-x + Sqrt[1 + x^2] - #1]*#1^2 + 2*Sqrt[3]*Log[-x + Sqrt[1 + x
^2] - #1]*#1^2 + Log[-x + Sqrt[1 + x^2] - #1]*#1^4)/(-2 + 15*#1^2 - 6*#1^4
 + #1^6) & ]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.57, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2-\sqrt{3}+1}{\sqrt{x^2+1}\,(x^4-x^2+1)}dx$$

$$\Big\downarrow \text{2256}$$

$$\int \left(-\frac{2-2i}{\left(2x^2-i\sqrt{3}-1\right)\sqrt{x^2+1}}-\frac{2+2i}{\left(2x^2+i\sqrt{3}-1\right)\sqrt{x^2+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(2+2i)\sqrt{\frac{-\sqrt{3}+i}{-\sqrt{3}+3i}}\operatorname{arctanh}\left(\frac{x}{\sqrt{\frac{-\sqrt{3}+i}{-\sqrt{3}+3i}}\sqrt{x^2+1}}\right)}{-\sqrt{3}+i}+\frac{i\sqrt{2}\operatorname{arctanh}\left(\frac{x}{\sqrt{\frac{\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{x^2+1}}\right)}{\sqrt[4]{3}}$$

input
```
Int[(1 - Sqrt[3] - 2*x^2)/(Sqrt[1 + x^2]*(1 - x^2 + x^4)),x]
```

output
```
((2 + 2*I)*Sqrt[(I - Sqrt[3])/(3*I - Sqrt[3])]*ArcTanh[x/(Sqrt[(I - Sqrt[3
])/(3*I - Sqrt[3])]*Sqrt[1 + x^2])])/(I - Sqrt[3]) + (I*Sqrt[2]*ArcTanh[x/
(Sqrt[(I + Sqrt[3])/(3*I + Sqrt[3])]*Sqrt[1 + x^2])])/3^(1/4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

method	result
default	$\sum_{\_R=\mathrm{RootOf}(\_Z^4-8\_Z^3+30\_Z^2-8\_Z+1)} \frac{\left(\_R^2-4\_R+1+2\sqrt{3}\,\_R\right)\ln\left(\left(\sqrt{x^2+1}-x\right)^2-\_R\right)}{\_R^3-6\_R^2+15\_R-2}$
pseudoelliptic	$\frac{\left(\sqrt{3}-1\right)\left(-\arctan\left(\frac{\sqrt{2\sqrt{3}+3}\,x-2\sqrt{x^2+1}}{x\sqrt{-3+2\sqrt{3}}}\right)+\arctan\left(\frac{\sqrt{2\sqrt{3}+3}\,x+2\sqrt{x^2+1}}{x\sqrt{-3+2\sqrt{3}}}\right)\right)}{\sqrt{-3+2\sqrt{3}}}$
trager	$\mathrm{RootOf}\left(\_Z^2+6\sqrt{3}\right)\ln\left(\frac{123\,\mathrm{RootOf}\left(\_Z^2+6\sqrt{3}\right)x^4+396\sqrt{x^2+1}\,x^3-71x^4\sqrt{3}\,\mathrm{RootOf}\left(\_Z^2+6\sqrt{3}\right)+105\,\mathrm{RootOf}\left(\_Z^2+6\sqrt{3}\right.}{}\right.$

input
```
int((1-3^(1/2)-2*x^2)/(x^2+1)^(1/2)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(_R^3-6*_R^2+15*_R-2)*(_R^2-4*_R+1+2*3^(1/2)*_R)*ln(((x^2+1)^(1/2)-x
)^2-_R),_R=RootOf(_Z^4-8*_Z^3+30*_Z^2-8*_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \frac{1-\sqrt{3}-2x^2}{\sqrt{1+x^2}\,(1-x^2+x^4)}\,dx = 3^{\frac{1}{4}}\sqrt{\frac{2}{3}}\arctan\left(\frac{3^{\frac{1}{4}}\sqrt{\frac{2}{3}}\left(2\,x^4-2\,x^2-\left(2\,x^3-\sqrt{3}x-x\right)\sqrt{x^2+1}+2\right)}{2\,(2\,x^2-1)}\right)$$

input
```
integrate((1-3^(1/2)-2*x^2)/(x^2+1)^(1/2)/(x^4-x^2+1),x, algorithm="fricas
")
```

output
```
3^(1/4)*sqrt(2/3)*arctan(1/2*3^(1/4)*sqrt(2/3)*(2*x^4 - 2*x^2 - (2*x^3 - s
qrt(3)*x - x)*sqrt(x^2 + 1) + 2)/(2*x^2 - 1))
```

Sympy [F]

$$\int \frac{1-\sqrt{3}-2x^2}{\sqrt{1+x^2}\,(1-x^2+x^4)}\,dx = -\int \frac{\sqrt{3}}{x^4\sqrt{x^2+1}-x^2\sqrt{x^2+1}+\sqrt{x^2+1}}\,dx - \int \frac{2x^2}{x^4\sqrt{x^2+1}-x^2\sqrt{x^2+1}+\sqrt{x^2+1}}\,dx - \int \left(-\frac{1}{x^4\sqrt{x^2+1}-x^2\sqrt{x^2+1}+\sqrt{x^2+1}}\right)dx$$

input
```
integrate((1-3**(1/2)-2*x**2)/(x**2+1)**(1/2)/(x**4-x**2+1),x)
```

output
```
-Integral(sqrt(3)/(x**4*sqrt(x**2 + 1) - x**2*sqrt(x**2 + 1) + sqrt(x**2 +
 1)), x) - Integral(2*x**2/(x**4*sqrt(x**2 + 1) - x**2*sqrt(x**2 + 1) + sq
rt(x**2 + 1)), x) - Integral(-1/(x**4*sqrt(x**2 + 1) - x**2*sqrt(x**2 + 1)
 + sqrt(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-2x^2}{\sqrt{1+x^2}\,(1-x^2+x^4)}\,dx = \int -\frac{2\,x^2+\sqrt{3}-1}{(x^4-x^2+1)\sqrt{x^2+1}}\,dx$$

input
```
integrate((1-3^(1/2)-2*x^2)/(x^2+1)^(1/2)/(x^4-x^2+1),x, algorithm="maxima
")
```

output
```
-integrate((2*x^2 + sqrt(3) - 1)/((x^4 - x^2 + 1)*sqrt(x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(41) = 82$.

Time = 0.21 (sec) , antiderivative size = 368, normalized size of antiderivative = 6.81

$$\int \frac{1-\sqrt{3}-2x^2}{\sqrt{1+x^2}\,(1-x^2+x^4)}\,dx = \text{Too large to display}$$

input
```
integrate((1-3^(1/2)-2*x^2)/(x^2+1)^(1/2)/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/24*(2*108^(1/4)*arctan(-(sqrt(6)*3^(1/4) + 3^(1/4)*sqrt(2) + 4)/(sqrt(6)
*3^(1/4) - 3^(1/4)*sqrt(2))) - 2*108^(1/4)*arctan(-(sqrt(6)*3^(1/4) + 3^(1
/4)*sqrt(2) - 4)/(sqrt(6)*3^(1/4) - 3^(1/4)*sqrt(2))) - 3*12^(1/4)*log(1/2
*sqrt(6)*3^(1/4) + 1/2*3^(1/4)*sqrt(2) + sqrt(3) + 1) + 3*12^(1/4)*log(-1/
2*sqrt(6)*3^(1/4) - 1/2*3^(1/4)*sqrt(2) + sqrt(3) + 1))*sgn(x) - 1/12*(3*1
08^(1/4)*abs(sgn(x))*sgn(x) + 108^(1/4))*arctan(-1/3*3^(3/4)*(3^(1/4)*(sqr
t(6) + sqrt(2)) + 4*sqrt(1/x^2 + 1))/(sqrt(6) - sqrt(2)))/sgn(x) + 1/12*(3
*108^(1/4)*abs(sgn(x))*sgn(x) + 108^(1/4))*arctan(-1/3*3^(3/4)*(3^(1/4)*(s
qrt(6) + sqrt(2)) - 4*sqrt(1/x^2 + 1))/(sqrt(6) - sqrt(2)))/sgn(x) - 1/8*(
12^(1/4)*abs(sgn(x))*sgn(x) - 12^(1/4))*log(1/2*(sqrt(6)*3^(1/4) + 3^(1/4)
*sqrt(2))*sqrt(1/x^2 + 1) + sqrt(3) + 1/x^2 + 1)/sgn(x) + 1/8*(12^(1/4)*ab
s(sgn(x))*sgn(x) - 12^(1/4))*log(-1/2*(sqrt(6)*3^(1/4) + 3^(1/4)*sqrt(2))*
sqrt(1/x^2 + 1) + sqrt(3) + 1/x^2 + 1)/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-2x^2}{\sqrt{1+x^2}\,(1-x^2+x^4)}\,dx=\int -\frac{2\,x^2+\sqrt{3}-1}{\sqrt{x^2+1}\,\left(x^4-x^2+1\right)}\,dx$$

input
```
int(-(3^(1/2) + 2*x^2 - 1)/((x^2 + 1)^(1/2)*(x^4 - x^2 + 1)),x)
```

output
```
int(-(3^(1/2) + 2*x^2 - 1)/((x^2 + 1)^(1/2)*(x^4 - x^2 + 1)), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-2x^2}{\sqrt{1+x^2}\,(1-x^2+x^4)}\,dx=-\sqrt{3}\left(\int \frac{1}{\sqrt{x^2+1}\,x^4-\sqrt{x^2+1}\,x^2+\sqrt{x^2+1}}dx\right)-2\left(\int \frac{x^2}{\sqrt{x^2+1}\,x^4-\sqrt{x^2+1}\,x^2+\sqrt{x^2+1}}dx\right)+\int \frac{1}{\sqrt{x^2+1}\,x^4-\sqrt{x^2+1}\,x^2+\sqrt{x^2+1}}dx$$

input
```
int((1-3^(1/2)-2*x^2)/(x^2+1)^(1/2)/(x^4-x^2+1),x)
```

output
```
- sqrt(3)*int(1/(sqrt(x**2 + 1)*x**4 - sqrt(x**2 + 1)*x**2 + sqrt(x**2 +
1)),x) - 2*int(x**2/(sqrt(x**2 + 1)*x**4 - sqrt(x**2 + 1)*x**2 + sqrt(x**2
 + 1)),x) + int(1/(sqrt(x**2 + 1)*x**4 - sqrt(x**2 + 1)*x**2 + sqrt(x**2 +
 1)),x)
```

3.154 $\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1233

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

output
```
1/16*(8*b^2*C*e^2-2*c*e*(4*B*b*e+4*C*a*e+7*C*b*d)+c^2*(5*C*d^2+2*e*(4*A*e+
7*B*d)))*x*(e*x^2+d)^(1/2)/c^3+1/24*(6*B*c*e-6*C*b*e+5*C*c*d)*x*(e*x^2+d)^
(3/2)/c^2+1/6*C*x*(e*x^2+d)^(5/2)/c-(3*b^3*c*C*d*e^2-b^4*C*e^3-c^2*(A*c-C*
a)*e*(-a*e^2+3*c*d^2)+b*c^2*d*(3*A*c*e^2-6*C*a*e^2+C*c*d^2)-b^2*c*e*(A*c*e
^2-3*C*a*e^2+3*C*c*d^2)-B*c*(c^3*d^3-b^3*e^3-3*c^2*d*e*(a*e+b*d)+b*c*e^2*(
2*a*e+3*b*d))+(b^5*C*e^3-b^4*c*e^2*(B*e+3*C*d)+b^2*c^2*(4*a*e^2*(B*e+3*C*d
)-c*d*(C*d^2+3*e*(A*e+B*d)))-b^3*c*e*(5*C*a*e^2-c*(3*C*d^2+e*(A*e+3*B*d)))
-2*c^3*(A*c*d*(-3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d)))+b*
c^2*(B*c*d*(-9*a*e^2+c*d^2)-e*(a*C*(-5*a*e^2+9*c*d^2)-3*A*c*(-a*e^2+c*d^2)
)))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b
-(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/c^4/(b-(-4*a*c+b^2)^(1/2))^(1/
2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)-(3*b^3*c*C*d*e^2-b^4*C*e^3-c^2*(
A*c-C*a)*e*(-a*e^2+3*c*d^2)+b*c^2*d*(3*A*c*e^2-6*C*a*e^2+C*c*d^2)-b^2*c*e*
(A*c*e^2-3*C*a*e^2+3*C*c*d^2)-B*c*(c^3*d^3-b^3*e^3-3*c^2*d*e*(a*e+b*d)+b*c
*e^2*(2*a*e+3*b*d))-(b^5*C*e^3-b^4*c*e^2*(B*e+3*C*d)+b*c^2*(B*c*d*(-9*a*e^
2+c*d^2)-a*C*e*(-5*a*e^2+9*c*d^2)+3*A*c*e*(-a*e^2+c*d^2))+b^2*c^2*(4*a*e^2
*(B*e+3*C*d)-c*d*(C*d^2+3*e*(A*e+B*d)))-b^3*c*e*(5*C*a*e^2-c*(3*C*d^2+e*(A
*e+3*B*d)))-2*c^3*(A*c*d*(-3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)-c*d^2*(3*B*
e+C*d))))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2
)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/c^4/(b+(-4*a*c+b^2)^(...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 63315 vs. 2(1233) = 2466.

Time = 18.18 (sec) , antiderivative size = 63315, normalized size of antiderivative = 51.35

$$\int \frac{(d+ex^2)^{5/2}\,(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 6.10 (sec) , antiderivative size = 1198, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2256}$$

$$\int \left(\frac{(d+ex^2)^{5/2}(-aC+Ac+x^2(Bc-bC))}{c(a+bx^2+cx^4)}+\frac{C(d+ex^2)^{5/2}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5C\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)d^3}{16c\sqrt{e}}+\frac{5Cx\sqrt{ex^2+d}d^2}{16c}+\frac{5Cx\left(ex^2+d\right)^{3/2}d}{24c}+\frac{Cx\left(ex^2+d\right)^{5/2}}{6c}+$$
$$\frac{\left(Bc-bC-\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)ex\left(ex^2+d\right)^{3/2}}{8c^2}+$$
$$\frac{\left(Bc-bC+\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)ex\left(ex^2+d\right)^{3/2}}{8c^2}+$$
$$\frac{\left(Bc-bC-\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)\left(2c^3d^3-3c^2e\left(bd-\sqrt{b^2-4ac}d+2ae\right)d-b^2\left(b-\sqrt{b^2-4ac}\right)e^3+ce^2\left(3db^2\right.\right.}{2c^4\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4\right.}}}$$
$$\frac{\left(Bc-bC+\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)\left(2c^3d^3-3c^2e\left(bd+\sqrt{b^2-4ac}d+2ae\right)d-b^2\left(b+\sqrt{b^2-4ac}\right)e^3+ce^2\left(3db^2\right.\right.}{2c^4\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4\right.}}}$$
$$\frac{\left(Bc-bC-\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)\sqrt{e}\left(15c^2d^2+4b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd-5\sqrt{b^2-4ac}d+4ae\right)\right)\operatorname{arcta}}{16c^4}$$
$$\frac{\left(Bc-bC+\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)\sqrt{e}\left(15c^2d^2+4b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd+5\sqrt{b^2-4ac}d+4ae\right)\right)\operatorname{arcta}}{16c^4}$$
$$\frac{\left(Bc-bC-\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)e\left(7cd-2\left(b-\sqrt{b^2-4ac}\right)e\right)x\sqrt{ex^2+d}}{16c^3}+$$
$$\frac{\left(Bc-bC+\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)e\left(7cd-2\left(b+\sqrt{b^2-4ac}\right)e\right)x\sqrt{ex^2+d}}{16c^3}$$

input
```
Int[((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(5*C*d^2*x*Sqrt[d + e*x^2])/(16*c) + ((B*c - b*C - (b*B*c - b^2*C - 2*c*(A
*c - a*C))/Sqrt[b^2 - 4*a*c])*e*(7*c*d - 2*(b - Sqrt[b^2 - 4*a*c])*e)*x*Sq
rt[d + e*x^2])/(16*c^3) + ((B*c - b*C + (b*B*c - b^2*C - 2*c*(A*c - a*C))/
Sqrt[b^2 - 4*a*c])*e*(7*c*d - 2*(b + Sqrt[b^2 - 4*a*c])*e)*x*Sqrt[d + e*x^
2])/(16*c^3) + (5*C*d*x*(d + e*x^2)^(3/2))/(24*c) + ((B*c - b*C - (b*B*c -
 b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*e*x*(d + e*x^2)^(3/2))/(8*c^2
) + ((B*c - b*C + (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*e*x
*(d + e*x^2)^(3/2))/(8*c^2) + (C*x*(d + e*x^2)^(5/2))/(6*c) + ((B*c - b*C
- (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*(2*c^3*d^3 - b^2*(b
 - Sqrt[b^2 - 4*a*c])*e^3 - 3*c^2*d*e*(b*d - Sqrt[b^2 - 4*a*c]*d + 2*a*e)
+ c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e - a*Sqrt[b^2 - 4*a*c]
*e))*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^4*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((B*c - b*C + (b*B*c - b^2*C - 2*c*(A*c
- a*C))/Sqrt[b^2 - 4*a*c])*(2*c^3*d^3 - b^2*(b + Sqrt[b^2 - 4*a*c])*e^3 -
3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e) + c*e^2*(3*b^2*d + a*Sqrt[b^
2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTan[(Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/
(2*c^4*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]
) + (5*C*d^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(16*c*Sqrt[e]) + ((B...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 954, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x(8C e^2 c^2 x^4+12B c^2 e^2 x^2-12Cbc e^2 x^2+26C c^2 de x^2+24A c^2 e^2-24Bbc e^2+54B c^2 de-24Cac e^2+24b^2 C e^2-54Cbcde+33}{48c^3}$
pseudoelliptic	$-\frac{\left(2\sqrt{\left(-2ae+bd+\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}\right)a}\left(\left((B c^2-2Cbc)a+b(A c^2-bBc+b^2C)\right)e^3-\frac{5cd\left(A c^2-bBc-Cca+b^2C\right)e^2}{2}-\frac{15c^2d^2}{}\right.\right.}{}$
default	Expression too large to display

input
```
int((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOS
E)
```

output
```
1/48*x*(8*C*c^2*e^2*x^4+12*B*c^2*e^2*x^2-12*C*b*c*e^2*x^2+26*C*c^2*d*e*x^2
+24*A*c^2*e^2-24*B*b*c*e^2+54*B*c^2*d*e-24*C*a*c*e^2+24*C*b^2*e^2-54*C*b*c
*d*e+33*C*c^2*d^2)*(e*x^2+d)^(1/2)/c^3-1/16/c^3*((16*A*b*c^2*e^3-40*A*c^3*
d*e^2+16*B*a*c^2*e^3-16*B*b^2*c*e^3+40*B*b*c^2*d*e^2-30*B*c^3*d^2*e-32*C*a
*b*c*e^3+40*C*a*c^2*d*e^2+16*C*b^3*e^3-40*C*b^2*c*d*e^2+30*C*b*c^2*d^2*e-5
*C*c^3*d^3)/c*ln((e*x^2+d)^(1/2)+x*e^(1/2))/e^(1/2)-8/c/(-4*(a*c-1/4*b^2)*
d^2)^(1/2)*((((B*c-2*C*b)*e+3*C*c*d)*c*e^2*a^2+(b*(A*c^2-B*b*c+C*b^2)*e^3-
3*c*d*(A*c^2-B*b*c+C*b^2)*e^2-3*c^2*d^2*(B*c-C*b)*e-C*c^3*d^3)*a+A*c^4*d^3
)*(arctanh(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^
(1/2))*a)^(1/2))*((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)-arcta
n(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a
)^(1/2))*((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))*(-4*(a*c-1/4*
b^2)*d^2)^(1/2)-2*(arctan(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((-2*a*e+b*d+(-4*(a*
c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))*((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))
*a)^(1/2)+arctanh(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((2*a*e-b*d+(-4*(a*c-1/4*b^2
)*d^2)^(1/2))*a)^(1/2))*((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2
))*d*(-C*a^3*c^2*e^3+c*e*((A*c^2-3/2*b*B*c+2*b^2*C)*e^2+3*c*(B*c-3/2*b*C)*
d*e+3*C*c^2*d^2)*a^2+(-1/2*b^2*(A*c^2-B*b*c+C*b^2)*e^3+3/2*b*c*d*(A*c^2-B*
b*c+C*b^2)*e^2-3*c^2*(A*c^2-1/2*b*B*c+1/2*b^2*C)*d^2*e-c^3*d^3*(B*c-1/2*b*
C))*a+1/2*A*b*c^4*d^3))*2^(1/2)/((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(d+ex^2)^{\frac{5}{2}}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a),x)
```

output
```
Integral((d + e*x**2)**(5/2)*(A + B*x**2 + C*x**4)/(a + b*x**2 + c*x**4),
x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="ma
xima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(5/2)/(c*x^4 + b*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}\left(A+Bx^2+Cx^4\right)}{a+bx^2+cx^4}\,dx = \int \frac{{\left(e\,x^2+d\right)}^{5/2}\,\left(C\,x^4+B\,x^2+A\right)}{c\,x^4+b\,x^2+a} \,d x$$

input
```
int(((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
int(((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.06

$$\int \frac{(d+ex^2)^{5/2}\left(A+Bx^2+Cx^4\right)}{a+bx^2+cx^4}\,dx = \frac{33\sqrt{e\,x^2+d}\,d^2ex+26\sqrt{e\,x^2+d}\,d\,e^2x^3+8\sqrt{e\,x^2+d}\,e^3x^5+15\sqrt{e}}{48e}$$

input
```
int((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x)
```

output
```
(33*sqrt(d + e*x**2)*d**2*e*x + 26*sqrt(d + e*x**2)*d*e**2*x**3 + 8*sqrt(d
 + e*x**2)*e**3*x**5 + 15*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*d**3)/(48*e)
```

3.155 $\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 825

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \frac{(3cCd+4Bce-4bCe)x\sqrt{d+ex^2}}{8c^2}$$

$$+\frac{Cx(d+ex^2)^{3/2}}{4c}$$

$$+\frac{\left(2b^2cCde+2c^2(Ac-aC)de-b^3Ce^2-bc(cCd^2+Ace^2-2aCe^2)+Bc(c^2d^2+b^2e^2-ce(2bd+ae))+ \ldots\right.}{c^3\sqrt{b-\ldots}}$$

$$+\frac{\left(2b^2cCde+2c^2(Ac-aC)de-b^3Ce^2-bc(cCd^2+Ace^2-2aCe^2)+Bc(c^2d^2+b^2e^2-ce(2bd+ae))- \ldots\right.}{c^3\sqrt{b+\ldots}}$$

$$+\frac{(8b^2Ce^2-4ce(3bCd+2bBe+2aCe)+c^2(3Cd^2+4e(3Bd+2Ae)))\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^3\sqrt{e}}$$

output
```
1/8*(4*B*c*e-4*C*b*e+3*C*c*d)*x*(e*x^2+d)^(1/2)/c^2+1/4*C*x*(e*x^2+d)^(3/2
)/c+(2*b^2*c*C*d*e+2*c^2*(A*c-C*a)*d*e-b^3*C*e^2-b*c*(A*c*e^2-2*C*a*e^2+C*
c*d^2)+B*c*(c^2*d^2+b^2*e^2-c*e*(a*e+2*b*d))+(b^4*C*e^2-b^3*c*e*(B*e+2*C*d
)-b*c^2*(2*A*c*d*e-3*B*a*e^2+B*c*d^2-6*C*a*d*e)-b^2*c*(4*C*a*e^2-c*(A*e^2+
2*B*d*e+C*d^2))+2*c^2*(A*c*(-a*e^2+c*d^2)+a*(C*a*e^2-c*d*(2*B*e+C*d))))/(-
4*a*c+b^2)^(1/2))*arctan((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b-(-4*a
*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/c^3/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(2*
c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)+(2*b^2*c*C*d*e+2*c^2*(A*c-C*a)*d*e-b^3
*C*e^2-b*c*(A*c*e^2-2*C*a*e^2+C*c*d^2)+B*c*(c^2*d^2+b^2*e^2-c*e*(a*e+2*b*d
))-(b^4*C*e^2-b^3*c*e*(B*e+2*C*d)-b*c^2*(2*A*c*d*e-3*B*a*e^2+B*c*d^2-6*C*a
*d*e)-b^2*c*(4*C*a*e^2-c*(A*e^2+2*B*d*e+C*d^2))+2*c^2*(A*c*(-a*e^2+c*d^2)+
a*(C*a*e^2-c*d*(2*B*e+C*d))))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b+(-4*a*c
+b^2)^(1/2))*e)^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/c^3/
(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)+1/8*(8
*b^2*C*e^2-4*c*e*(2*B*b*e+2*C*a*e+3*C*b*d)+c^2*(3*C*d^2+4*e*(2*A*e+3*B*d))
)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^3/e^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40458 vs. 2(825) = 1650.

Time = 17.47 (sec) , antiderivative size = 40458, normalized size of antiderivative = 49.04

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.78 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{2256}$$

$$\int \left(\frac{(d+ex^2)^{3/2}(-aC+Ac+x^2(Bc-bC))}{c(a+bx^2+cx^4)}+\frac{C(d+ex^2)^{3/2}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3C\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)d^2}{8c\sqrt{e}}+\frac{3Cx\sqrt{ex^2+d}d}{8c}+\frac{Cx(ex^2+d)^{3/2}}{4c}+$$
$$\frac{\left(Bc-bC-\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2cd-}}{\sqrt{b-}}\right.}{2c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$
$$\frac{\left(Bc-bC+\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}d+ae\right)\right)\arctan\left(\frac{\sqrt{2cd-}}{\sqrt{b+}}\right.}{2c^3\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$
$$\frac{\left(Bc-bC-\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)\sqrt{e}\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{4c^3}+$$
$$\frac{\left(Bc-bC+\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)\sqrt{e}\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{4c^3}+$$
$$\frac{\left(Bc-bC-\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)ex\sqrt{ex^2+d}}{4c^2}+$$
$$\frac{\left(Bc-bC+\frac{-Cb^2+Bcb-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)ex\sqrt{ex^2+d}}{4c^2}$$

input
```
Int[((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(3*C*d*x*Sqrt[d + e*x^2])/(8*c) + ((B*c - b*C - (b*B*c - b^2*C - 2*c*(A*c
- a*C))/Sqrt[b^2 - 4*a*c])*e*x*Sqrt[d + e*x^2])/(4*c^2) + ((B*c - b*C + (b
*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*e*x*Sqrt[d + e*x^2])/(4
*c^2) + (C*x*(d + e*x^2)^(3/2))/(4*c) + ((B*c - b*C - (b*B*c - b^2*C - 2*c
*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^
2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c
^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) +
((B*c - b*C + (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*(2*c^2*
d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a
*e))*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]) + (3*C*d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*
x^2]])/(8*c*Sqrt[e]) + ((B*c - b*C - (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqr
t[b^2 - 4*a*c])*Sqrt[e]*(3*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[
e]*x)/Sqrt[d + e*x^2]])/(4*c^3) + ((B*c - b*C + (b*B*c - b^2*C - 2*c*(A*c
- a*C))/Sqrt[b^2 - 4*a*c])*Sqrt[e]*(3*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*Arc
Tanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(4*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 702, normalized size of antiderivative = 0.85

method	result
risch	$\frac{x(2Cecx^2+4Bce-4Cbe+5Ccd)\sqrt{ex^2+d}}{8c^2}+\frac{\frac{\left(8Ac^2e^2-8Bbce^2+12Bc^2de-8Cace^2+8b^2Ce^2-12Cbcde+3Cc^2d^2\right)\ln\left(\sqrt{e}\,x^2+d\right.}{c\sqrt{e}}}{}$
pseudoelliptic	$\frac{-\sqrt{e}\left(\left(-Cce^2a^2+\left(\left(Ac^2-bBc+b^2C\right)e^2+2cd(Bc-bC)e+Cc^2d^2\right)a-Ac^3d^2\right)\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}+\left(\left(\left(2Bc^2-3Cbc\right)e^2+4C\right.\right.\right.}{}$
default	$\frac{C\left(\frac{x\left(ex^2+d\right)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{ex^2+d}}{2}+\frac{d\ln\left(\sqrt{ex^2+d}+x\sqrt{e}\right)}{2\sqrt{e}}\right)}{4}\right)}{c}-\frac{\sqrt{e}\left(\left(-Cce^2a^2+\left(\left(Ac^2-bBc+b^2C\right)e^2+2cd(Bc-bC)e+C\right.\right.\right.}{}$

input
```
int((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOS
E)
```

output
```
1/8*x*(2*C*c*e*x^2+4*B*c*e-4*C*b*e+5*C*c*d)*(e*x^2+d)^(1/2)/c^2+1/8/c^2*((
8*A*c^2*e^2-8*B*b*c*e^2+12*B*c^2*d*e-8*C*a*c*e^2+8*C*b^2*e^2-12*C*b*c*d*e+
3*C*c^2*d^2)/c*ln((e*x^2+d)^(1/2)+x*e^(1/2))/e^(1/2)+4/c/(-4*(a*c-1/4*b^2)
*d^2)^(1/2)*((arctan(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((-2*a*e+b*d+(-4*(a*c-1/4
*b^2)*d^2)^(1/2))*a)^(1/2))*((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(
1/2)-arctanh(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2
)^(1/2))*a)^(1/2))*((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))*(-
C*c*e^2*a^2+(b^2*C*e^2-b*e*(B*e+2*C*d)*c+(A*e^2+2*B*d*e+C*d^2)*c^2)*a-A*c^
3*d^2)*(-4*(a*c-1/4*b^2)*d^2)^(1/2)-(arctan(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((
-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))*((2*a*e-b*d+(-4*(a*c-1/
4*b^2)*d^2)^(1/2))*a)^(1/2)+arctanh(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((2*a*e-b*
d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))*((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d
^2)^(1/2))*a)^(1/2))*((-3*C*b*c*e^2+2*c^2*e*(B*e+2*C*d))*a^2+(b^3*C*e^2-c*
e*(B*e+2*C*d)*b^2+c^2*(A*e^2+2*B*d*e+C*d^2)*b+(-4*A*d*e-2*B*d^2)*c^3)*a+A*
b*c^3*d^2)*d)*2^(1/2)/((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)/
((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(d+ex^2)^{\frac{3}{2}}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a),x)
```

output
```
Integral((d + e*x**2)**(3/2)*(A + B*x**2 + C*x**4)/(a + b*x**2 + c*x**4),
x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="ma
xima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,(C\,x^4+B\,x^2+A)}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
int(((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.07

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \frac{5\sqrt{e\,x^2+d}\,dex+2\sqrt{e\,x^2+d}\,e^2x^3+3\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)d^2}{8e}$$

input
```
int((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x)
```

output
```
(5*sqrt(d + e*x**2)*d*e*x + 2*sqrt(d + e*x**2)*e**2*x**3 + 3*sqrt(e)*log((
sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*d**2)/(8*e)
```

3.156 $\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 38, antiderivative size = 504

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4} dx = \frac{Cx\sqrt{d+ex^2}}{2c}$$

$$-\frac{\left(bcCd-b^2Ce-c(Ac-aC)e-Bc(cd-be)+\frac{b^3Ce-b^2c(Cd+Be)-2c^2(Acd-aCd-aBe)+bc(Bcd+Ace-3aCe)}{\sqrt{b^2-4ac}}\right)\text{arc}}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$-\frac{\left(bcCd-b^2Ce-c(Ac-aC)e-Bc(cd-be)-\frac{b^3Ce-b^2c(Cd+Be)+bc(Bcd+Ace-3aCe)-2c^2(Acd-a(Cd+Be))}{\sqrt{b^2-4ac}}\right)\text{arc}}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

$$+\frac{(cCd+2Bce-2bCe)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}$$

output
```
1/2*C*x*(e*x^2+d)^(1/2)/c-(b*c*C*d-b^2*C*e-c*(A*c-C*a)*e-B*c*(-b*e+c*d)+(b
^3*C*e-b^2*c*(B*e+C*d)-2*c^2*(A*c*d-B*a*e-C*a*d)+b*c*(A*c*e+B*c*d-3*C*a*e)
)/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b-(
-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/c^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)
/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)-(b*c*C*d-b^2*C*e-c*(A*c-C*a)*e-B*c
*(-b*e+c*d)-(b^3*C*e-b^2*c*(B*e+C*d)+b*c*(A*c*e+B*c*d-3*C*a*e)-2*c^2*(A*c*
d-a*(B*e+C*d)))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b+(-4*a*c+b^2)^(1/2))*e
)^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/c^2/(b+(-4*a*c+b^2
)^(1/2))^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)+1/2*(2*B*c*e-2*C*b*e
+C*c*d)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^2/e^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 22359 vs. 2(504) = 1008.

Time = 17.05 (sec) , antiderivative size = 22359, normalized size of antiderivative = 44.36

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 2256}$$

$$\int \left(\frac{\sqrt{d+ex^2}(-aC+Ac+x^2(Bc-bC))}{c(a+bx^2+cx^4)}+\frac{C\sqrt{d+ex^2}}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)}{2c^2}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)}{2c^2}+\frac{Cd\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{e}}+\frac{Cx\sqrt{d+ex^2}}{2c}$$

input
```
Int[(Sqrt[d + e*x^2]*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
(C*x*Sqrt[d + e*x^2])/(2*c) + ((B*c - b*C - (b*B*c - b^2*C - 2*c*(A*c - a*
C))/Sqrt[b^2 - 4*a*c])*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(2*c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((B*c - b*C + (b*B*c -
 b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sq
rt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) +
(C*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[e]) + ((B*c - b*C - (
b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*Sqrt[e]*ArcTanh[(Sqrt[
e]*x)/Sqrt[d + e*x^2]])/(2*c^2) + ((B*c - b*C + (b*B*c - b^2*C - 2*c*(A*c
- a*C))/Sqrt[b^2 - 4*a*c])*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(
2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.97

method	result
risch	$\frac{Cx\sqrt{e\,x^2+d}}{2c}+\frac{\frac{(2Bce-2Cbe+Ccd)\ln\left(\sqrt{e\,x^2+d}+x\sqrt{e}\right)}{c\sqrt{e}}+\frac{\left(\left(\left((-Ccd-e(Bc-bC))a+A\,c^2d\right)\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}+2d\left(-C\,a^2ce+\left(c\left(Bc\right.\right.\right.\right.\right.}{}}{}$
pseudoelliptic	$\frac{\left(\frac{\left(((-Bc+bC)e-Ccd)a+A\,c^2d\right)\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}}{2}+d\left(-C\,a^2ce+\left(c\left(Bc-\frac{bC}{2}\right)d+e\left(A\,c^2-\frac{1}{2}bBc+\frac{1}{2}b^2C\right)\right)a-\frac{Ab\,c^2d}{2}\right)\right)\sqrt{e}\,\sqrt{\left(\right.}}{}$
default	$\frac{C\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(\sqrt{e\,x^2+d}+x\sqrt{e}\right)}{2\sqrt{e}}\right)}{c}+\frac{\left(\left((-Ccd-e(Bc-bC))a+A\,c^2d\right)\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}+2d\left(-C\,a^2ce+\left(c\left(Bc-\frac{bC}{2}\right)d+e\left(A\,c^2\right.\right.\right.\right.}{}$

input
```
int((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOS
E)
```

output
```
1/2*C*x*(e*x^2+d)^(1/2)/c+1/2/c*((2*B*c*e-2*C*b*e+C*c*d)/c*ln((e*x^2+d)^(1
/2)+x*e^(1/2))/e^(1/2)+1/c/(-4*(a*c-1/4*b^2)*d^2)^(1/2)/((-2*a*e+b*d+(-4*(
a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)*((((-C*c*d-e*(B*c-C*b))*a+A*c^2*d)*(-4*(
a*c-1/4*b^2)*d^2)^(1/2)+2*d*(-C*a^2*c*e+(c*(B*c-1/2*b*C)*d+e*(A*c^2-1/2*b*
B*c+1/2*b^2*C))*a-1/2*A*b*c^2*d))*((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2
))*a)^(1/2)*arctanh(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((2*a*e-b*d+(-4*(a*c-1/4*b
^2)*d^2)^(1/2))*a)^(1/2))-arctan(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((-2*a*e+b*d+
(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))*(((-C*c*d-e*(B*c-C*b))*a+A*c^2*d)*
(-4*(a*c-1/4*b^2)*d^2)^(1/2)-2*d*(-C*a^2*c*e+(c*(B*c-1/2*b*C)*d+e*(A*c^2-1
/2*b*B*c+1/2*b^2*C))*a-1/2*A*b*c^2*d))*((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^
(1/2))*a)^(1/2))*2^(1/2)/((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a),x)
```

output
```
Integral(sqrt(d + e*x**2)*(A + B*x**2 + C*x**4)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{ex^2+d}}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="ma
xima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{e\,x^2+d}\,(C\,x^4+B\,x^2+A)}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
int(((d + e*x^2)^(1/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.08

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \frac{\sqrt{e\,x^2+d}\,ex+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)d}{2e}$$

input
```
int((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x)
```

output
```
(sqrt(d + e*x**2)*e*x + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d)
)*d)/(2*e)
```

3.157 $\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 337

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{\left(Bc-bC-\frac{bBc-b^2C-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(Bc-bC+\frac{bBc-b^2C-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{C\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

output
```
(B*c-C*b-(B*b*c-b^2*C-2*c*(A*c-C*a))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b-
(-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2
))/c/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)+(
B*c-C*b+(B*b*c-b^2*C-2*c*(A*c-C*a))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b+(
-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2)
)/c/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)+C*
arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c/e^(1/2)
```

Mathematica [A] (verified)

Time = 11.60 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$= \frac{\frac{\left(Bc-bC+\frac{-bBc+b^2C+2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}}+\frac{\left(Bc-bC+\frac{bBc-b^2C+2c(-Ac+aC)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}{c}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
(((B*c - b*C + (-(b*B*c) + b^2*C + 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c
]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt
[b^2 - 4*a*c])*e]) + ((B*c - b*C + (b*B*c - b^2*C + 2*c*(-(A*c) + a*C))/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[
b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqr
t[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (C*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*
x^2]])/Sqrt[e])/c
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \; 2256$$

$$\int \left(\frac{-aC+Ac+x^2(Bc-bC)}{c\sqrt{d+ex^2}\,(a+bx^2+cx^4)}+\frac{C}{c\sqrt{d+ex^2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\left(-\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{C\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

output
```
((B*c - b*C - (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*S
qrt[d + e*x^2]])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]) + ((B*c - b*C + (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sq
rt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (C*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/(c*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.97

method	result
default	$\frac{C\ln\left(\sqrt{e\,x^2+d}+x\sqrt{e}\right)}{c\sqrt{e}}-\frac{\sqrt{2}\left(-\frac{\left(-Abcd+2aBcd-Cabd+A\sqrt{-d^2\left(4ac-b^2\right)}\,c-C\sqrt{-d^2\left(4ac-b^2\right)}\,a\right)\operatorname{arctanh}\left(\frac{a\sqrt{e\,x^2+\cdots}}{x\sqrt{\left(2ae-bd+\sqrt{-\cdots}\right.}}\right.}{\sqrt{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\cdots\right.}{\cdots}$
pseudoelliptic	$-\frac{-\sqrt{e}\sqrt{\left(-2ae+bd+\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}\right)a}\left((Ac-Ca)\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}-d((-2Bc+bC)a+Abc)\right)\sqrt{2}\operatorname{arctanh}\left(\frac{\cdots}{x\sqrt{\left(2ae-\cdots\right.}}\right.\cdots}{\cdots}$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOS
E)
```

output
```
C/c*ln((e*x^2+d)^(1/2)+x*e^(1/2))/e^(1/2)-1/2/c*2^(1/2)/(-d^2*(4*a*c-b^2))
^(1/2)*(-(-A*b*c*d+2*a*B*c*d-C*a*b*d+A*(-d^2*(4*a*c-b^2))^(1/2)*c-C*(-d^2*
(4*a*c-b^2))^(1/2)*a)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arcta
nh(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1
/2))+(A*b*c*d-2*a*B*c*d+C*a*b*d+A*(-d^2*(4*a*c-b^2))^(1/2)*c-C*(-d^2*(4*a*
c-b^2))^(1/2)*a)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*arctan(a*
(e*x^2+d)^(1/2)/x*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\,(a + bx^2 + cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\,(a + bx^2 + cx^4)}\,dx = \int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\,(a + bx^2 + cx^4)}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x
)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\,(a + bx^2 + cx^4)}\,dx = \int \frac{Cx^4 + Bx^2 + A}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="ma
xima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \frac{C\,x^4+B\,x^2+A}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.07

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{\sqrt{e}\,\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)}{e}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

output
```
(sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d)))/e
```

3.158 $\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 449

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx=\frac{\left(Cd^2-e(Bd-Ae)\right)x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}$$
$$+\frac{\left(Bcd-bCd-Ace+aCe+\frac{b^2Cd+2c(Acd-aCd+aBe)-b(Bcd+Ace+aCe)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd^2-e(bd-ae)\right)}$$
$$+\frac{\left(Bcd-bCd-Ace+aCe-\frac{b^2Cd+2c(Acd-aCd+aBe)-b(Bcd+Ace+aCe)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\left(cd^2-e(bd-ae)\right)}$$

output
```
(C*d^2-e*(-A*e+B*d))*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)+(B*c*d-C*b*d-
A*c*e+C*a*e+(b^2*C*d+2*c*(A*c*d+B*a*e-C*a*d)-b*(A*c*e+B*c*d+C*a*e))/(-4*a*
c+b^2)^(1/2))*arctan((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b-(-4*a*c+b
^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-(b-(
-4*a*c+b^2)^(1/2))*e)^(1/2)/(c*d^2-e*(-a*e+b*d))+(B*c*d-C*b*d-A*c*e+C*a*e-
(b^2*C*d+2*c*(A*c*d+B*a*e-C*a*d)-b*(A*c*e+B*c*d+C*a*e))/(-4*a*c+b^2)^(1/2)
)*arctan((2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(
1/2)/(e*x^2+d)^(1/2))/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^
(1/2))*e)^(1/2)/(c*d^2-e*(-a*e+b*d))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16421 vs. $2(449) = 898$.

Time = 21.51 (sec) , antiderivative size = 16421, normalized size of antiderivative = 36.57

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)^{3/2} (a + bx^2 + cx^4)} \, dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + Bx^2 + Cx^4}{(d + ex^2)^{3/2} (a + bx^2 + cx^4)} \, dx$$

$$\Big\downarrow \text{ 2256}$$

$$\int \left(\frac{-aC + Ac + x^2(Bc - bC)}{c\,(d + ex^2)^{3/2}\,(a + bx^2 + cx^4)} + \frac{C}{c\,(d + ex^2)^{3/2}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}} - bC + Bc\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}} + \frac{2\left(\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}} - bC + Bc\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}} - \frac{ex\left(-\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}} - bC + Bc\right)}{cd\sqrt{d+ex^2}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)} - \frac{ex\left(\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}} - bC + Bc\right)}{cd\sqrt{d+ex^2}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)} + \frac{Cx}{cd\sqrt{d+ex^2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(C*x)/(c*d*Sqrt[d + e*x^2]) - ((B*c - b*C - (b*B*c - b^2*C - 2*c*(A*c - a*
C))/Sqrt[b^2 - 4*a*c])*e*x)/(c*d*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*Sqrt[
d + e*x^2]) - ((B*c - b*C + (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4
*a*c])*e*x)/(c*d*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*Sqrt[d + e*x^2]) + (2
*(B*c - b*C - (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*S
qrt[d + e*x^2]])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*(2*c*d - (b - Sqrt[b^2 - 4*
a*c])*e)^(3/2)) + (2*(B*c - b*C + (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b +
Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\dfrac{-\dfrac{\sqrt{e\,x^2+d}\sqrt{\left(-2ae+bd+\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}\right)a}\left(\left((-Be+Cd)a+A(eb-cd)\right)\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}+2\left(-a^2Ce+\left(\left(-Bc+\frac{bC}{2}\right)d+e\left(Ac+\frac{Bb}{2}\right)\right)\right.\right.}{2}}{-\dfrac{\sqrt{e\,x^2+d}\,c\sqrt{\left(-2ae+bd+\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}\right)a}\left(\left((-Be+Cd)a+A(eb-cd)\right)\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}+2\left(-a^2Ce+\left(\left(-Bc+b\right.\right.\right.\right.}{2}}$
default	$\frac{Cx}{cd\sqrt{e\,x^2+d}}+\frac{}{}$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOS
E)
```

output
```
1/((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)/(e*x^2+d)^(1/2)*(-1/2
*(e*x^2+d)^(1/2)*((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)*(((-B
*e+C*d)*a+A*(b*e-c*d))*(-4*(a*c-1/4*b^2)*d^2)^(1/2)+2*(-a^2*C*e+((-B*c+1/2
*b*C)*d+e*(A*c+1/2*B*b))*a-1/2*A*b*(b*e-c*d))*d)*d*2^(1/2)*arctanh(a*(e*x^
2+d)^(1/2)/x*2^(1/2)/((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))+(
(2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)*(1/2*(e*x^2+d)^(1/2)*(((
-B*e+C*d)*a+A*(b*e-c*d))*(-4*(a*c-1/4*b^2)*d^2)^(1/2)-2*(-a^2*C*e+((-B*c+1
/2*b*C)*d+e*(A*c+1/2*B*b))*a-1/2*A*b*(b*e-c*d))*d)*d*2^(1/2)*arctan(a*(e*x
^2+d)^(1/2)/x*2^(1/2)/((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))
+(A*e^2-B*d*e+C*d^2)*(-4*(a*c-1/4*b^2)*d^2)^(1/2)*((-2*a*e+b*d+(-4*(a*c-1/
4*b^2)*d^2)^(1/2))*a)^(1/2)*x))/(-4*(a*c-1/4*b^2)*d^2)^(1/2)/((-2*a*e+b*d+
(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)/(a*e^2-b*d*e+c*d^2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{Cx^4+Bx^2+A}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="ma
xima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="gi
ac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)}\,dx = \frac{\sqrt{e\,x^2+d}\,ex+\sqrt{e}\,d+\sqrt{e}\,e\,x^2}{de\,(e\,x^2+d)}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

output
```
(sqrt(d + e*x**2)*e*x + sqrt(e)*d + sqrt(e)*e*x**2)/(d*e*(d + e*x**2))
```

3.159 $\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 687

$$\int \frac{A+Bx^2+Cx^4}{\left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)}\,dx=\frac{\left(Cd^2-e(Bd-Ae)\right)x}{3d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)^{3/2}}$$

$$+\frac{\left(cd^2(2Cd^2-e(5Bd-8Ae))+e(bd(Cd^2+2Bde-5Ae^2)-ae(4Cd^2-Bde-2Ae^2))\right)x}{3d^2\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex^2}}$$

$$+\frac{c\Bigl(Bcd^2-bCd^2-2Acde+2aCde+Abe^2-aBe^2+\frac{b^2(Cd^2+Ae^2)-b(2(Ac+aC)de+B(cd^2+ae^2))+2(Ac(cd^2-ae^2)+a(a}{\sqrt{b^2-4ac}}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(cd^2-e(bd-ae}$$

$$+\frac{c\Bigl(Bcd^2-bCd^2-2Acde+2aCde+Abe^2-aBe^2-\frac{b^2(Cd^2+Ae^2)-b(2(Ac+aC)de+B(cd^2+ae^2))+2(Ac(cd^2-ae^2)+a(a}{\sqrt{b^2-4ac}}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,(cd^2-e(bd-ae}$$

output
```
1/3*(C*d^2-e*(-A*e+B*d))*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(3/2)+1/3*(c*d^
2*(2*C*d^2-e*(-8*A*e+5*B*d))+e*(b*d*(-5*A*e^2+2*B*d*e+C*d^2)-a*e*(-2*A*e^2
-B*d*e+4*C*d^2)))*x/d^2/(a*e^2-b*d*e+c*d^2)^2/(e*x^2+d)^(1/2)+c*(B*c*d^2-b
*C*d^2-2*A*c*d*e+2*C*a*d*e+A*b*e^2-B*a*e^2+(b^2*(A*e^2+C*d^2)-b*(2*(A*c+C*
a)*d*e+B*(a*e^2+c*d^2))+2*A*c*(-a*e^2+c*d^2)+2*a*(C*a*e^2-c*d*(-2*B*e+C*d)
))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b-
(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(2
*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(c*d^2-e*(-a*e+b*d))^2+c*(B*c*d^2-b*C
*d^2-2*A*c*d*e+2*C*a*d*e+A*b*e^2-B*a*e^2-(b^2*(A*e^2+C*d^2)-b*(2*(A*c+C*a)
*d*e+B*(a*e^2+c*d^2))+2*A*c*(-a*e^2+c*d^2)+2*a*(C*a*e^2-c*d*(-2*B*e+C*d)))
/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b+(-
4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c
*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)/(c*d^2-e*(-a*e+b*d))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25519 vs. $2(687) = 1374$.

Time = 21.82 (sec) , antiderivative size = 25519, normalized size of antiderivative = 37.15

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{2256}$$

$$\int \left(\frac{-aC+Ac+x^2(Bc-bC)}{c(d+ex^2)^{5/2}(a+bx^2+cx^4)}+\frac{C}{c(d+ex^2)^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4c\left(-\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{5/2}}+\frac{4c\left(\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{5/2}}-\frac{2ex\left(5cd-e\left(b-\sqrt{b^2-4ac}\right)\right)\left(-\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)}{3cd^2\sqrt{d+ex^2}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^2}-\frac{2ex\left(5cd-e\left(\sqrt{b^2-4ac}+b\right)\right)\left(\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)}{3cd^2\sqrt{d+ex^2}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^2}-\frac{ex\left(-\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)}{3cd(d+ex^2)^{3/2}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{ex\left(\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)}{3cd(d+ex^2)^{3/2}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{2Cx}{3cd^2\sqrt{d+ex^2}}+\frac{Cx}{3cd(d+ex^2)^{3/2}}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
(C*x)/(3*c*d*(d + e*x^2)^(3/2)) - ((B*c - b*C - (b*B*c - b^2*C - 2*c*(A*c
- a*C))/Sqrt[b^2 - 4*a*c])*e*x)/(3*c*d*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
*(d + e*x^2)^(3/2)) - ((B*c - b*C + (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt
[b^2 - 4*a*c])*e*x)/(3*c*d*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(d + e*x^2)
^(3/2)) + (2*C*x)/(3*c*d^2*Sqrt[d + e*x^2]) - (2*(B*c - b*C - (b*B*c - b^2
*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*e*(5*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)*x)/(3*c*d^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)^2*Sqrt[d + e*x^2]) -
(2*(B*c - b*C + (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*e*(5*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x)/(3*c*d^2*(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)^2*Sqrt[d + e*x^2]) + (4*c*(B*c - b*C - (b*B*c - b^2*C - 2*c*(A*c -
a*C))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x
)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a
*c]]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)^(5/2)) + (4*c*(B*c - b*C + (b*B*c
 - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + S
qrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(S
qrt[b + Sqrt[b^2 - 4*a*c]]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$-\frac{(e x^2+d)^{\frac{3}{2}}\left(\left((-A c^2+Cca)d^2+2cd(Ab-Ba)e+(aAc-A b^2+abB-C a^2)e^2\right)\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}-3d\left(\left(a\left(Ab-\frac{2Ba}{3}\right)c-\frac{b}{\cdots}\right.\right.\right.}{\cdots}$
default	$\frac{C\left(\frac{x}{3d\left(e x^2+d\right)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{e x^2+d}}\right)}{c}-\frac{c(e x^2+d)^{\frac{3}{2}}\left(\left((-A c^2+Cca)d^2+2cd(Ab-Ba)e+(aAc-A b^2+abB-C a^2)e^2\right)\sqrt{-4(\cdots}\right.}{\cdots}$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOS
E)
```

output
```
-1/2/(e*x^2+d)^(3/2)*((e*x^2+d)^(3/2)*(((-A*c^2+C*a*c)*d^2+2*c*d*(A*b-B*a)
*e+(A*a*c-A*b^2+B*a*b-C*a^2)*e^2)*(-4*(a*c-1/4*b^2)*d^2)^(1/2)-3*d*((a*(A*
b-2/3*B*a)*c-1/3*b*(A*b^2-B*a*b+C*a^2))*e^2-4/3*c*(a*A*c-1/2*A*b^2+1/2*a*b
*B-C*a^2)*d*e-1/3*c*d^2*((A*b-2*B*a)*c+C*a*b)))*((-2*a*e+b*d+(-4*(a*c-1/4*
b^2)*d^2)^(1/2))*a)^(1/2)*d^2*2^(1/2)*arctanh(a*(e*x^2+d)^(1/2)/x*2^(1/2)/
((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))-((2*a*e-b*d+(-4*(a*c-1
/4*b^2)*d^2)^(1/2))*a)^(1/2)*((((-A*c^2+C*a*c)*d^2+2*c*d*(A*b-B*a)*e+(A*a*
c-A*b^2+B*a*b-C*a^2)*e^2)*(-4*(a*c-1/4*b^2)*d^2)^(1/2)+3*d*((a*(A*b-2/3*B*
a)*c-1/3*b*(A*b^2-B*a*b+C*a^2))*e^2-4/3*c*(a*A*c-1/2*A*b^2+1/2*a*b*B-C*a^2
)*d*e-1/3*c*d^2*((A*b-2*B*a)*c+C*a*b)))*(e*x^2+d)^(3/2)*d^2*2^(1/2)*arctan
(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)
^(1/2))+2*(-4*(a*c-1/4*b^2)*d^2)^(1/2)*((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)
^(1/2))*a)^(1/2)*(C*c*d^5-2*c*e*(-1/3*C*x^2+B)*d^4+3*(1/3*(1/3*C*x^2+B)*b+
c*(-5/9*B*x^2+A)-1/3*C*a)*e^2*d^3-2*((-1/3*B*x^2+A)*b-4/3*A*c*x^2+2/3*C*a*
x^2)*e^3*d^2+(-5/3*A*b*x^2+a*(1/3*B*x^2+A))*e^4*d+2/3*A*a*e^5*x^2)*x))/((2
*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)/(-4*(a*c-1/4*b^2)*d^2)^(1/
2)/((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)/(a*e^2-b*d*e+c*d^2)
^2/d^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)}\,dx=\text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(5/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)}\,dx=\int \frac{Cx^4+Bx^2+A}{(cx^4+bx^2+a)(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a),x, algorithm="ma
xima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a),x, algorithm="gi
ac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)}\,dx=\int \frac{C\,x^4+B\,x^2+A}{(e\,x^2+d)^{5/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.12

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)}\,dx=\frac{3\sqrt{e\,x^2+d}\,dex+2\sqrt{e\,x^2+d}\,e^2x^3-2\sqrt{e}\,d^2-4\sqrt{e}\,de\,x^2-2\sqrt{e}\,e^2x^4}{3d^2e\,(e^2x^4+2de\,x^2+d^2)}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a),x)
```

output
```
(3*sqrt(d + e*x**2)*d*e*x + 2*sqrt(d + e*x**2)*e**2*x**3 - 2*sqrt(e)*d**2
- 4*sqrt(e)*d*e*x**2 - 2*sqrt(e)*e**2*x**4)/(3*d**2*e*(d**2 + 2*d*e*x**2 +
 e**2*x**4))
```

3.160 $\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1335

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

output
```
-1/2*e*(A*c^2*(-2*a*e+b*d)-a*(B*c*(-b*e+2*c*d)-C*(6*a*c*e-2*b^2*e+b*c*d)))
*x*(e*x^2+d)^(1/2)/a/c^2/(-4*a*c+b^2)-1/2*(A*b*c-2*B*a*c+C*a*b)*e*x*(e*x^2
+d)^(3/2)/a/c/(-4*a*c+b^2)+1/2*x*(A*(-2*a*c+b^2)-a*(B*b-2*C*a)+(A*b*c-2*B*
a*c+C*a*b)*x^2)*(e*x^2+d)^(5/2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(A*c^3*
(4*a*e*(a*e^2+2*c*d^2)-b*(5*a*d*e^2+c*d^3))+a*(B*c*(2*c^3*d^3+2*b^3*e^3-2*
c^2*d*e*(-7*a*e+2*b*d)-b*c*e^2*(10*a*e+b*d))+C*(7*b^3*c*d*e^2-4*b^4*e^3-2*
b^2*c*e*(-10*a*e^2+c*d^2)+4*a*c^2*e*(-3*a*e^2+4*c*d^2)-b*c^2*d*(33*a*e^2+c
*d^2)))+(A*c^3*(4*a*c*d*(a*e^2+3*c*d^2)-4*a*b*e*(a*e^2+3*c*d^2)-b^2*(-5*a*
d*e^2+c*d^3))+a*(4*b^5*C*e^3-b^4*c*e^2*(2*B*e+7*C*d)-b^3*c*e*(28*C*a*e^2-c
*d*(B*e+2*C*d))+4*a*c^3*(c*d^2*(2*B*e+C*d)-a*e^2*(4*B*e+13*C*d))+b^2*c^2*(
c*d^2*(4*B*e+C*d)+a*e^2*(14*B*e+47*C*d))-4*b*c^2*(a*C*e*(-11*a*e^2+5*c*d^2
)+B*c*d*(4*a*e^2+c*d^2))))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b-(-4*a*c+b^
2)^(1/2))*e)^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/a/c^3/(
-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^
(1/2)-1/2*(A*c^3*(4*a*e*(a*e^2+2*c*d^2)-b*(5*a*d*e^2+c*d^3))+a*(B*c*(2*c^3
*d^3+2*b^3*e^3-2*c^2*d*e*(-7*a*e+2*b*d)-b*c*e^2*(10*a*e+b*d))+C*(7*b^3*c*d
*e^2-4*b^4*e^3-2*b^2*c*e*(-10*a*e^2+c*d^2)+4*a*c^2*e*(-3*a*e^2+4*c*d^2)-b*
c^2*d*(33*a*e^2+c*d^2)))-(A*c^3*(4*a*c*d*(a*e^2+3*c*d^2)-4*a*b*e*(a*e^2+3*
c*d^2)-b^2*(-5*a*d*e^2+c*d^3))+a*(4*b^5*C*e^3-b^4*c*e^2*(2*B*e+7*C*d)-b^3*
c*e*(28*C*a*e^2-c*d*(B*e+2*C*d))+4*a*c^3*(c*d^2*(2*B*e+C*d)-a*e^2*(4*B*...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 63689 vs. $2(1335) = 2670$.

Time = 18.80 (sec) , antiderivative size = 63689, normalized size of antiderivative = 47.71

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 2256}$$

$$\int \left(\frac{(d+ex^2)^{5/2}(-aC+Ac+x^2(Bc-bC))}{c(a+bx^2+cx^4)^2}+\frac{C(d+ex^2)^{5/2}}{c(a+bx^2+cx^4)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Ce\left(7cd-2\left(b-\sqrt{b^2-4ac}\right)e\right)\sqrt{ex^2+d}x}{8c^2\sqrt{b^2-4ac}}-\frac{Ce\left(7cd-2\left(b+\sqrt{b^2-4ac}\right)e\right)\sqrt{ex^2+d}x}{8c^2\sqrt{b^2-4ac}}+$$

$$\frac{C\left(2c^3d^3-3c^2e\left(bd-\sqrt{b^2-4ac}d+2ae\right)d-b^2\left(b-\sqrt{b^2-4ac}\right)e^3+ce^2\left(3db^2-3\sqrt{b^2-4ac}db+3aeb-a\sqrt{b^2}\right.\right.}{c^3\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$\frac{C\left(2c^3d^3-3c^2e\left(bd+\sqrt{b^2-4ac}d+2ae\right)d-b^2\left(b+\sqrt{b^2-4ac}\right)e^3+ce^2\left(3db^2+3\left(\sqrt{b^2-4ac}d+ae\right)b+a\sqrt{b}\right.\right.}{c^3\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

$$\frac{C\sqrt{e}\left(15c^2d^2+4b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd-5\sqrt{b^2-4ac}d+4ae\right)\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{8c^3\sqrt{b^2-4ac}}-$$

$$\frac{C\sqrt{e}\left(15c^2d^2+4b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(5bd+5\sqrt{b^2-4ac}d+4ae\right)\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{8c^3\sqrt{b^2-4ac}}+$$

$$\frac{(Ac-aC)\int\frac{(ex^2+d)^{5/2}}{(cx^4+bx^2+a)^2}dx}{c}+\frac{(Bc-bC)\int\frac{x^2(ex^2+d)^{5/2}}{(cx^4+bx^2+a)^2}dx}{c}$$

input
```
Int[((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 69.18 (sec) , antiderivative size = 1214, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	Expression too large to display	1214
default	Expression too large to display	1714
risch	Expression too large to display	1849

input
```
int((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/2*(1/2*(1/2*(4*a*c-b^2)*e^2*(c*x^4+b*x^2+a)*(2*B*c*e-4*C*b*e+5*C*c*d)*((
2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*a*((-2*a*e+b*d+(-d^2*(4*a*c-b
^2))^(1/2))*a)^(1/2)*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))-e^(1/2)*(c*((2*a*e
-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1
/2))*a)^(1/2)*(-3*C*a^3*c*e^2+((b^2*C-1/2*c*(7*C*x^2+B)*b+c^2*(-2*C*x^4+B*
x^2+A))*e^2+2*c*e*(-1/2*b*C+c*(C*x^2+B))*d+C*c^2*d^2)*a^2+(1/2*b*(2*b^2*C-
c*(-C*x^2+B)*b+A*c^2)*x^2*e^2-c*e*(C*b^2*x^2+c*(-B*x^2+A)*b+2*A*c^2*x^2)*d
-c^2*(1/2*(-C*x^2+B)*b+c*(B*x^2+A))*d^2)*a+1/2*A*b*c^2*d^2*(c*x^2+b))*x*(e
*x^2+d)^(1/2)-1/2*(arctanh(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((2*a*e-b*d+(-d^2*(
4*a*c-b^2))^(1/2))*a)^(1/2))*((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/
2)-arctan(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2
))*a)^(1/2))*((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))*((4*(-B*c^2+2
*C*b*c)*e^3-13*C*c^2*d*e^2)*a^3+((B*b^2*c-2*C*b^3)*e^3+c*(A*c^2-1/2*b*B*c+
7/2*b^2*C)*d*e^2+e*(2*B*c^3-C*b*c^2)*d^2+C*c^3*d^3)*a^2-(A*b*e-3*d*(A*c-1/
6*B*b))*c^3*d^2*a-1/2*A*b^2*c^3*d^3)*(c*x^4+b*x^2+a)*2^(1/2)))*(-d^2*(4*a*
c-b^2))^(1/2)+e^(1/2)*(-3*C*a^4*c^2*e^3+c*((-3/2*b*B*c+3*b^2*C+A*c^2)*e^2+
(-5*C*b*c+7/2*B*c^2)*d*e+4*C*c^2*d^2)*e*a^3+(1/4*b^3*(B*c-2*C*b)*e^3-3/2*b
*c*(A*c^2+1/12*b*B*c-7/12*b^2*C)*d*e^2+2*c^2*(A*c^2-3/4*b*B*c-1/8*b^2*C)*d
^2*e+1/2*c^3*d^3*(B*c-C*b))*a^2+1/4*b*c^3*(A*b*e-4*(A*c-1/8*B*b)*d)*d^2*a+
1/8*A*b^3*c^3*d^3)*(c*x^4+b*x^2+a)*d*2^(1/2)*(arctan(a*(e*x^2+d)^(1/2)/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(5/2)*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="
maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(5/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6900 vs. $2(1271) = 2542$.

Time = 0.96 (sec) , antiderivative size = 6900, normalized size of antiderivative = 5.17

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="
giac")
```

output
```
1/2*sqrt(e*x^2 + d)*C*e^2*x/c^2 - ((sqrt(e)*x - sqrt(e*x^2 + d))^6*C*a*b*c
^3*d^3*sqrt(e) - 2*(sqrt(e)*x - sqrt(e*x^2 + d))^6*B*a*c^4*d^3*sqrt(e) + (
sqrt(e)*x - sqrt(e*x^2 + d))^6*A*b*c^4*d^3*sqrt(e) - 4*(sqrt(e)*x - sqrt(e
*x^2 + d))^6*C*a*b^2*c^2*d^2*e^(3/2) + 8*(sqrt(e)*x - sqrt(e*x^2 + d))^6*C
*a^2*c^3*d^2*e^(3/2) + 4*(sqrt(e)*x - sqrt(e*x^2 + d))^6*B*a*b*c^3*d^2*e^(
3/2) - 8*(sqrt(e)*x - sqrt(e*x^2 + d))^6*A*a*c^4*d^2*e^(3/2) + 5*(sqrt(e)*
x - sqrt(e*x^2 + d))^6*C*a*b^3*c*d*e^(5/2) - 15*(sqrt(e)*x - sqrt(e*x^2 +
d))^6*C*a^2*b*c^2*d*e^(5/2) - 5*(sqrt(e)*x - sqrt(e*x^2 + d))^6*B*a*b^2*c^
2*d*e^(5/2) + 10*(sqrt(e)*x - sqrt(e*x^2 + d))^6*B*a^2*c^3*d*e^(5/2) + 5*(
sqrt(e)*x - sqrt(e*x^2 + d))^6*A*a*b*c^3*d*e^(5/2) - 2*(sqrt(e)*x - sqrt(e
*x^2 + d))^6*C*a*b^4*e^(7/2) + 8*(sqrt(e)*x - sqrt(e*x^2 + d))^6*C*a^2*b^2
*c*e^(7/2) + 2*(sqrt(e)*x - sqrt(e*x^2 + d))^6*B*a*b^3*c*e^(7/2) - 4*(sqrt
(e)*x - sqrt(e*x^2 + d))^6*C*a^3*c^2*e^(7/2) - 6*(sqrt(e)*x - sqrt(e*x^2 +
 d))^6*B*a^2*b*c^2*e^(7/2) - 2*(sqrt(e)*x - sqrt(e*x^2 + d))^6*A*a*b^2*c^2
*e^(7/2) + 4*(sqrt(e)*x - sqrt(e*x^2 + d))^6*A*a^2*c^3*e^(7/2) - 3*(sqrt(e
)*x - sqrt(e*x^2 + d))^4*C*a*b*c^3*d^4*sqrt(e) + 6*(sqrt(e)*x - sqrt(e*x^2
 + d))^4*B*a*c^4*d^4*sqrt(e) - 3*(sqrt(e)*x - sqrt(e*x^2 + d))^4*A*b*c^4*d
^4*sqrt(e) + 10*(sqrt(e)*x - sqrt(e*x^2 + d))^4*C*a*b^2*c^2*d^3*e^(3/2) -
12*(sqrt(e)*x - sqrt(e*x^2 + d))^4*C*a^2*c^3*d^3*e^(3/2) - 14*(sqrt(e)*x -
 sqrt(e*x^2 + d))^4*B*a*b*c^3*d^3*e^(3/2) + 4*(sqrt(e)*x - sqrt(e*x^2 +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}\,(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(e\,x^2+d)^{5/2}\,(C\,x^4+B\,x^2+A)}{(c\,x^4+b\,x^2+a)^2}\,dx$$

input
```
int(((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
int(((d + e*x^2)^(5/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{5/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}dx\right)d^2 + \left(\int \frac{\sqrt{e\,x^2+d}\,x^4}{c\,x^4+b\,x^2+a}dx\right)e^2 + 2\left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c\,x^4+b\,x^2+a}dx\right)de$$

input
```
int((e*x^2+d)^(5/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a + b*x**2 + c*x**4),x)*d**2 + int((sqrt(d + e*x**2)
*x**4)/(a + b*x**2 + c*x**4),x)*e**2 + 2*int((sqrt(d + e*x**2)*x**2)/(a +
b*x**2 + c*x**4),x)*d*e
```

3.161 $\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 867

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = -\frac{(Abc-2aBc+abC)ex\sqrt{d+ex^2}}{2ac(b^2-4ac)}$$

$$+\frac{x(A(b^2-2ac)-a(bB-2aC)+(Abc-2aBc+abC)x^2)(d+ex^2)^{3/2}}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$+\frac{\left(Ac^2(bcd^2-6acde+2abe^2)+a(C(cd-be)(bcd+2b^2e-10ace)-Bc^2(2cd^2-e(3bd-4ae)))-\frac{Ac^2(12c\cdots}{\cdots}\right.}{2ac^2(b^2-4ac)\cdots}$$

$$+\frac{\left(Ac^2(bcd^2-6acde+2abe^2)+a(C(cd-be)(bcd+2b^2e-10ace)-Bc^2(2cd^2-e(3bd-4ae)))+\frac{Ac^2(12c\cdots}{\cdots}\right.}{2ac^2(b^2-4ac)\cdots}$$

$$+\frac{Ce^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}$$

output
```
-1/2*(A*b*c-2*B*a*c+C*a*b)*e*x*(e*x^2+d)^(1/2)/a/c/(-4*a*c+b^2)+1/2*x*(A*(
-2*a*c+b^2)-a*(B*b-2*C*a)+(A*b*c-2*B*a*c+C*a*b)*x^2)*(e*x^2+d)^(3/2)/a/(-4
*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*c^2*(2*a*b*e^2-6*a*c*d*e+b*c*d^2)+a*(C*(-
b*e+c*d)*(-10*a*c*e+2*b^2*e+b*c*d)-B*c^2*(2*c*d^2-e*(-4*a*e+3*b*d)))-(A*c^
2*(12*a*c^2*d^2-8*a*b*c*d*e-b^2*(-2*a*e^2+c*d^2))+a*(b^3*c*C*d*e-2*b^4*C*e
^2-4*b*c^2*(B*a*e^2+B*c*d^2+3*C*a*d*e)-4*a*c^2*(4*C*a*e^2-c*d*(B*e+C*d))+b
^2*c*(14*C*a*e^2+c*d*(3*B*e+C*d))))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b-(
-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2)
)/a/c^2/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(
1/2))*e)^(1/2)+1/2*(A*c^2*(2*a*b*e^2-6*a*c*d*e+b*c*d^2)+a*(C*(-b*e+c*d)*(-
10*a*c*e+2*b^2*e+b*c*d)-B*c^2*(2*c*d^2-e*(-4*a*e+3*b*d)))+(A*c^2*(12*a*c^2
*d^2-8*a*b*c*d*e-b^2*(-2*a*e^2+c*d^2))+a*(b^3*c*C*d*e-2*b^4*C*e^2-4*b*c^2*
(B*a*e^2+B*c*d^2+3*C*a*d*e)-4*a*c^2*(4*C*a*e^2-c*d*(B*e+C*d))+b^2*c*(14*C*
a*e^2+c*d*(3*B*e+C*d))))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b+(-4*a*c+b^2)
^(1/2))*e)^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/a/c^2/(-4
*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1
/2)+C*e^(3/2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39198 vs. 2(867) = 1734.

Time = 17.90 (sec) , antiderivative size = 39198, normalized size of antiderivative = 45.21

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{2256}$$

$$\int \left(\frac{(d+ex^2)^{3/2}(-aC+Ac+x^2(Bc-bC))}{c(a+bx^2+cx^4)^2}+\frac{C(d+ex^2)^{3/2}}{c(a+bx^2+cx^4)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(Ac-aC)\int \frac{(ex^2+d)^{3/2}}{(cx^4+bx^2+a)^2}dx}{c}+\frac{(Bc-bC)\int \frac{x^2(ex^2+d)^{3/2}}{(cx^4+bx^2+a)^2}dx}{c}+\frac{C\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{C\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{C\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{2c^2\sqrt{b^2-4ac}}-\frac{C\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}{2c^2\sqrt{b^2-4ac}}$$

input
```
Int[((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 948, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$-\frac{\left(\left(8C a^3 c e^2+\left(\left(-2Bde-2C d^2\right)c^2+Cbcde-2b^2C e^2\right)a^2+\left(-6Acd+b\left(Ae+Bd\right)\right)c^2da+A b^2c^2d^2\right)\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}+4\left(\left(\cdots\right.\right.}{\cdots}$
default	Expression too large to display

input
```
int((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/4*(((8*C*a^3*c*e^2+((-2*B*d*e-2*C*d^2)*c^2+C*b*c*d*e-2*b^2*C*e^2)*a^2+(
-6*A*c*d+b*(A*e+B*d))*c^2*d*a+A*b^2*c^2*d^2)*(-4*(a*c-1/4*b^2)*d^2)^(1/2)+
4*(((-2*B*e^2-5*C*d*e)*c^2+3*C*b*c*e^2)*a^3+((-3*A*d*e-B*d^2)*c^3+c^2*(A*e
^2+2*B*d*e+C*d^2)*b+1/4*b^2*c*C*d*e-1/2*b^3*C*e^2)*a^2-1/4*b*c^2*d*(-8*A*c
*d+b*(A*e+B*d))*a-1/4*A*b^3*c^2*d^2)*d)*(c*x^4+b*x^2+a)*((-2*a*e+b*d+(-4*(
a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)*2^(1/2)*arctanh(a*(e*x^2+d)^(1/2)/x*2^(1
/2)/((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))-(((8*C*a^3*c*e^2+(
(-2*B*d*e-2*C*d^2)*c^2+C*b*c*d*e-2*b^2*C*e^2)*a^2+(-6*A*c*d+b*(A*e+B*d))*c
^2*d*a+A*b^2*c^2*d^2)*(-4*(a*c-1/4*b^2)*d^2)^(1/2)-4*(((-2*B*e^2-5*C*d*e)*
c^2+3*C*b*c*e^2)*a^3+((-3*A*d*e-B*d^2)*c^3+c^2*(A*e^2+2*B*d*e+C*d^2)*b+1/4
*b^2*c*C*d*e-1/2*b^3*C*e^2)*a^2-1/4*b*c^2*d*(-8*A*c*d+b*(A*e+B*d))*a-1/4*A
*b^3*c^2*d^2)*d)*(c*x^4+b*x^2+a)*2^(1/2)*arctan(a*(e*x^2+d)^(1/2)/x*2^(1/2
)/((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))+2*(8*(a*c-1/4*b^2)*
e^(3/2)*(c*x^4+b*x^2+a)*a*C*arctanh((e*x^2+d)^(1/2)/x/e^(1/2))+c*(((-2*C*d
-2*(C*x^2+B)*e)*c+C*b*e)*a^2+(((2*B*x^2+2*A)*d+2*A*e*x^2)*c^2+c*((-C*x^2+B
)*d+e*(-B*x^2+A))*b+C*b^2*e*x^2)*a-A*b*c*d*(c*x^2+b))*(e*x^2+d)^(1/2)*x)*(
(-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)*(-4*(a*c-1/4*b^2)*d^2)^
(1/2))*((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))/((2*a*e-b*d+(-4
*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)/(-4*(a*c-1/4*b^2)*d^2)^(1/2)/((-2*a*e+
b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)/(4*a*c-b^2)/a/(c*x^4+b*x^2+a...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\left(A+Bx^2+Cx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="
maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4241 vs. $2(811) = 1622$.

Time = 0.91 (sec) , antiderivative size = 4241, normalized size of antiderivative = 4.89

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="
giac")
```

output
```
-1/2*C*e^(3/2)*log((sqrt(e)*x - sqrt(e*x^2 + d))^2)/c^2 - ((sqrt(e)*x - sq
rt(e*x^2 + d))^6*C*a*b*c^2*d^2*sqrt(e) - 2*(sqrt(e)*x - sqrt(e*x^2 + d))^6
*B*a*c^3*d^2*sqrt(e) + (sqrt(e)*x - sqrt(e*x^2 + d))^6*A*b*c^3*d^2*sqrt(e)
 - 3*(sqrt(e)*x - sqrt(e*x^2 + d))^6*C*a*b^2*c*d*e^(3/2) + 6*(sqrt(e)*x -
sqrt(e*x^2 + d))^6*C*a^2*c^2*d*e^(3/2) + 3*(sqrt(e)*x - sqrt(e*x^2 + d))^6
*B*a*b*c^2*d*e^(3/2) - 6*(sqrt(e)*x - sqrt(e*x^2 + d))^6*A*a*c^3*d*e^(3/2)
 + 2*(sqrt(e)*x - sqrt(e*x^2 + d))^6*C*a*b^3*e^(5/2) - 6*(sqrt(e)*x - sqrt
(e*x^2 + d))^6*C*a^2*b*c*e^(5/2) - 2*(sqrt(e)*x - sqrt(e*x^2 + d))^6*B*a*b
^2*c*e^(5/2) + 4*(sqrt(e)*x - sqrt(e*x^2 + d))^6*B*a^2*c^2*e^(5/2) + 2*(sq
rt(e)*x - sqrt(e*x^2 + d))^6*A*a*b*c^2*e^(5/2) - 3*(sqrt(e)*x - sqrt(e*x^2
 + d))^4*C*a*b*c^2*d^3*sqrt(e) + 6*(sqrt(e)*x - sqrt(e*x^2 + d))^4*B*a*c^3
*d^3*sqrt(e) - 3*(sqrt(e)*x - sqrt(e*x^2 + d))^4*A*b*c^3*d^3*sqrt(e) + 7*(
sqrt(e)*x - sqrt(e*x^2 + d))^4*C*a*b^2*c*d^2*e^(3/2) - 6*(sqrt(e)*x - sqrt
(e*x^2 + d))^4*C*a^2*c^2*d^2*e^(3/2) - 11*(sqrt(e)*x - sqrt(e*x^2 + d))^4*
B*a*b*c^2*d^2*e^(3/2) + 4*(sqrt(e)*x - sqrt(e*x^2 + d))^4*A*b^2*c^2*d^2*e^
(3/2) + 6*(sqrt(e)*x - sqrt(e*x^2 + d))^4*A*a*c^3*d^2*e^(3/2) - 4*(sqrt(e)
*x - sqrt(e*x^2 + d))^4*C*a*b^3*d*e^(5/2) + 4*(sqrt(e)*x - sqrt(e*x^2 + d)
)^4*B*a*b^2*c*d*e^(5/2) + 16*(sqrt(e)*x - sqrt(e*x^2 + d))^4*B*a^2*c^2*d*e
^(5/2) - 16*(sqrt(e)*x - sqrt(e*x^2 + d))^4*A*a*b*c^2*d*e^(5/2) + 8*(sqrt(
e)*x - sqrt(e*x^2 + d))^4*C*a^2*b^2*e^(7/2) - 16*(sqrt(e)*x - sqrt(e*x^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx=\int \frac{(e\,x^2+d)^{3/2}\,(C\,x^4+B\,x^2+A)}{(c\,x^4+b\,x^2+a)^2}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
int(((d + e*x^2)^(3/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}dx\right)d + \left(\int \frac{\sqrt{e\,x^2+d}\,x^2}{c\,x^4+b\,x^2+a}dx\right)e$$

input
```
int((e*x^2+d)^(3/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a + b*x**2 + c*x**4),x)*d + int((sqrt(d + e*x**2)*x**
*2)/(a + b*x**2 + c*x**4),x)*e
```

3.162 $\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 588

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \frac{x(A(b^2-2ac)-a(bB-2aC)+(Abc-2aBc+abC)x^2)\sqrt{d+ex^2}}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$+\frac{\left(Ac(bd-4ae)-a(2Bcd-bCd-2bBe+4aCe)+\frac{Ac(b^2d-12acd+4abe)-a(4acCd+b^2(Cd+2Be)-4b(Bcd+aCe))}{\sqrt{b^2-4ac}}\right)}{2a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

$$-\frac{\left(Ac\left(b^2d-12acd-b\sqrt{b^2-4ac}d+4abe+4a\sqrt{b^2-4ac}e\right)+a\left(2Bc\sqrt{b^2-4ac}d-4acCd+4a\sqrt{b^2-4\right.\right.}{2a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2}}}$$

output
```
1/2*x*(A*(-2*a*c+b^2)-a*(B*b-2*C*a)+(A*b*c-2*B*a*c+C*a*b)*x^2)*(e*x^2+d)^(
1/2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*c*(-4*a*e+b*d)-a*(-2*B*b*e+2*B*
c*d+4*C*a*e-C*b*d)+(A*c*(4*a*b*e-12*a*c*d+b^2*d)-a*(4*a*c*C*d+b^2*(2*B*e+C
*d)-4*b*(B*c*d+C*a*e)))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b-(-4*a*c+b^2)^
(1/2))*e)^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/a/(-4*a*c+
b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)-1
/2*(A*c*(b^2*d-12*a*c*d-b*(-4*a*c+b^2)^(1/2)*d+4*a*b*e+4*a*(-4*a*c+b^2)^(1
/2)*e)+a*(2*B*c*(-4*a*c+b^2)^(1/2)*d-4*a*c*C*d+4*a*(-4*a*c+b^2)^(1/2)*C*e-
b^2*(2*B*e+C*d)+b*(4*B*c*d-(-4*a*c+b^2)^(1/2)*C*d-2*B*(-4*a*c+b^2)^(1/2)*e
+4*C*a*e)))*arctan((2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b+(-4*a*c+b^2
)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/a/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2)
)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21061 vs. 2(588) = 1176.

Time = 17.50 (sec) , antiderivative size = 21061, normalized size of antiderivative = 35.82

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2} dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2} dx$$

$$\Big\downarrow \text{2256}$$

$$\int \left(\frac{\sqrt{d+ex^2}(-aC+Ac+x^2(Bc-bC))}{c(a+bx^2+cx^4)^2}+\frac{C\sqrt{d+ex^2}}{c(a+bx^2+cx^4)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Ac-aC)\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)^2}dx}{c}+\frac{(Bc-bC)\int \frac{x^2\sqrt{ex^2+d}}{(cx^4+bx^2+a)^2}dx}{c}+\frac{C\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{C\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[(Sqrt[d + e*x^2]*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 579, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\dfrac{\dfrac{3\sqrt{\left(-2ae+bd+\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}\right)a}\left(cx^4+bx^2+a\right)\left(\left(\left(\frac{Ca^2}{3}+\left(Ac-\frac{Bb}{6}\right)a-\frac{Ab^2}{6}\right)\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}+\frac{4Ca^3e}{3}+\frac{2\left((Bc-bC)d+2e\left(Ac-\cdots\right.\right.}{3}\cdots\right.\right.}{2}\cdots}{\cdots}$
default	$-\dfrac{Cd\sqrt{2}\left(-\dfrac{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\operatorname{arctanh}\left(\dfrac{a\sqrt{ex^2+d}\,\sqrt{2}}{x\sqrt{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}\right)}{\sqrt{\left(2ae-bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)a}}+\dfrac{\left(-2ae+bd+\sqrt{-d^2\left(4ac-b^2\right)}\right)\arctan\left(\dfrac{\cdots}{x\sqrt{\cdots}}\right)}{\sqrt{\left(-2ae+bd+\sqrt{-d^2\cdots}\right.}}\cdots\right.}{2c\sqrt{-d^2\left(4ac-b^2\right)}\cdots}$

input
```
int((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/(-4*(a*c-1/4*b^2)*d^2)^(1/2)/((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*
a)^(1/2)*(3/2*((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)*(c*x^4+b
*x^2+a)*((1/3*C*a^2+(A*c-1/6*B*b)*a-1/6*A*b^2)*(-4*(a*c-1/4*b^2)*d^2)^(1/2
)+4/3*C*a^3*e+2/3*((B*c-C*b)*d+2*e*(A*c-1/2*B*b))*a^2-4/3*b*(A*c-1/8*B*b)*
d*a+1/6*A*d*b^3)*d*2^(1/2)*arctanh(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((2*a*e-b*d
+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))+((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2
)^(1/2))*a)^(1/2)*(-3/2*(c*x^4+b*x^2+a)*d*((1/3*C*a^2+(A*c-1/6*B*b)*a-1/6*
A*b^2)*(-4*(a*c-1/4*b^2)*d^2)^(1/2)-4/3*C*a^3*e+2/3*((-B*c+C*b)*d-2*e*(A*c
-1/2*B*b))*a^2+4/3*b*(A*c-1/8*B*b)*d*a-1/6*A*d*b^3)*2^(1/2)*arctan(a*(e*x^
2+d)^(1/2)/x*2^(1/2)/((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))+
(e*x^2+d)^(1/2)*((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)*(-4*(a
*c-1/4*b^2)*d^2)^(1/2)*(-C*a^2+(1/2*(-C*x^2+B)*b+c*(B*x^2+A))*a-1/2*A*b*(c
*x^2+b))*x))/((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)/a/(4*a*c-b
^2)/(c*x^4+b*x^2+a)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(1/2)*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{ex^2+d}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="
maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a)^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{\sqrt{e\,x^2+d}\,(C\,x^4+B\,x^2+A)}{(c\,x^4+b\,x^2+a)^2}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
int(((d + e*x^2)^(1/2)*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex^2}(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}dx$$

input
```
int((e*x^2+d)^(1/2)*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(sqrt(d + e*x**2)/(a + b*x**2 + c*x**4),x)
```

3.163 $\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal result

Integrand size = 38, antiderivative size = 833

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^2} \, dx$$

$$= \frac{x\sqrt{d+ex^2}(A(b^2cd-2ac^2d-b^3e+3abce)-a(bBcd-2acCd-b^2Be+2aBce+abCe)+c(A(bcd-b}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)}$$

$$+\frac{\left((Ac-aC)(bd-2ae)(cd-be)-a(Bc-bC)\left(2cd^2-e(3bd-4ae)\right)-\frac{Ac(b^3de-12abcde-b^2(cd^2+2ae^2)+4ac(}{}\right.}{2a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-(}}$$

$$+\frac{\left((Ac-aC)(bd-2ae)(cd-be)-a(Bc-bC)\left(2cd^2-e(3bd-4ae)\right)+\frac{Ac(b^3de-12abcde-b^2(cd^2+2ae^2)+4ac(}{}\right.}{2a\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-(}}$$

output
```
1/2*x*(e*x^2+d)^(1/2)*(A*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)-a*(2*B*a*c*e-
B*b^2*e+B*b*c*d+C*a*b*e-2*C*a*c*d)+c*(A*(2*a*c*e-b^2*e+b*c*d)-a*(-B*b*e+2*
B*c*d+2*C*a*e-C*b*d))*x^2)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2
+a)+1/2*((A*c-C*a)*(-2*a*e+b*d)*(-b*e+c*d)-a*(B*c-C*b)*(2*c*d^2-e*(-4*a*e+
3*b*d))-(A*c*(b^3*d*e-12*a*b*c*d*e-b^2*(2*a*e^2+c*d^2)+4*a*c*(4*a*e^2+3*c*
d^2))-a*(2*b^3*C*d*e-4*a*c^2*d*(-B*e+C*d)+4*b*B*c*(a*e^2+c*d^2)-b^2*(2*C*a
*e^2+c*d*(5*B*e+C*d))))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-(b-(-4*a*c+b^2)^
(1/2))*e)^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(1/2))/a/(-4*a*c+
b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(
c*d^2-e*(-a*e+b*d))+1/2*((A*c-C*a)*(-2*a*e+b*d)*(-b*e+c*d)-a*(B*c-C*b)*(2*
c*d^2-e*(-4*a*e+3*b*d))+(A*c*(b^3*d*e-12*a*b*c*d*e-b^2*(2*a*e^2+c*d^2)+4*a
*c*(4*a*e^2+3*c*d^2))-a*(2*b^3*C*d*e-4*a*c^2*d*(-B*e+C*d)+4*b*B*c*(a*e^2+c
*d^2)-b^2*(2*C*a*e^2+c*d*(5*B*e+C*d))))/(-4*a*c+b^2)^(1/2))*arctan((2*c*d-
(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(e*x^2+d)^(
1/2))/a/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(
1/2))*e)^(1/2)/(c*d^2-e*(-a*e+b*d))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37781 vs. $2(833) = 1666$.

Time = 22.16 (sec) , antiderivative size = 37781, normalized size of antiderivative = 45.36

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)^2),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 2256}$$

$$\int \left(\frac{-aC+Ac+x^2(Bc-bC)}{c\sqrt{d+ex^2}\,(a+bx^2+cx^4)^2}+\frac{C}{c\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Ac-aC)\int \frac{1}{\sqrt{ex^2+d}(cx^4+bx^2+a)^2}dx}{c}+\frac{(Bc-bC)\int \frac{x^2}{\sqrt{ex^2+d}(cx^4+bx^2+a)^2}dx}{c}+\frac{2C\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2C\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 872, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{\left(\frac{3\sqrt{\left(2ae-bd+\sqrt{-4\left(ac-\frac{b^2}{4}\right)d^2}\right)a}\left(\frac{2\left(Ccd-e\left(\frac{bC}{2}+c\left(C\,x^2+B\right)\right)\right)a^2}{3}+\left(-\frac{2c\left(\frac{\left(-C\,x^2+B\right)b}{2}+c\left(B\,x^2+A\right)\right)d}{3}+e\left(\frac{B\,b^2}{3}+c\left(\frac{B\,x}{3}\right.\right.\right.\right.}{4}\right.}{}$
default	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/2/(-4*(a*c-1/4*b^2)*d^2)^(1/2)*((3/4*((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)
^(1/2))*a)^(1/2)*(2/3*(C*c*d-e*(1/2*b*C+c*(C*x^2+B)))*a^2+(-2/3*c*(1/2*(-C
*x^2+B)*b+c*(B*x^2+A))*d+e*(1/3*B*b^2+c*(1/3*B*x^2+A)*b+2/3*A*c^2*x^2))*a-
1/3*A*b*(c*x^2+b)*(b*e-c*d))*((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)
^(1/2)*x*(e*x^2+d)^(1/2)+(c*x^4+b*x^2+a)*((1/4*C*c*d^2-1/4*e*(B*c+1/2*b*C)
*d+A*c*e^2)*a^2+1/4*((-1/2*b*B*c+3*A*c^2)*d^2-5/2*b*e*(A*c-1/5*B*b)*d-A*b^
2*e^2)*a+1/8*A*d*b^2*(b*e-c*d))*(arctan(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((-2*a
*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))*((2*a*e-b*d+(-4*(a*c-1/4*b^
2)*d^2)^(1/2))*a)^(1/2)-arctanh(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((2*a*e-b*d+(-
4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))*((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^
(1/2))*a)^(1/2))*2^(1/2))*(-4*(a*c-1/4*b^2)*d^2)^(1/2)+3/2*(1/3*(C*c*d*e-2
*e^2*(B*c-1/2*b*C))*a^3+(-1/3*c*(B*c-C*b)*d^2-1/3*(A*c^2-b*B*c+5/4*b^2*C)*
e*d+A*b*c*e^2)*a^2-1/6*b*((-4*A*c^2+1/2*b*B*c)*d^2+7/2*b*e*(A*c-1/7*B*b)*d
+A*b^2*e^2)*a+1/12*A*b^3*d*(b*e-c*d))*(c*x^4+b*x^2+a)*d*(arctan(a*(e*x^2+d
)^(1/2)/x*2^(1/2)/((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))*((2
*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)+arctanh(a*(e*x^2+d)^(1/2)/
x*2^(1/2)/((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))*((-2*a*e+b*d
+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2))*2^(1/2))/(a*c-1/4*b^2)/(c*x^4+b*x
^2+a)/((2*a*e-b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)/(a*e^2-b*d*e+c*d^
2)/((-2*a*e+b*d+(-4*(a*c-1/4*b^2)*d^2)^(1/2))*a)^(1/2)/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\,(a + bx^2 + cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\,(a + bx^2 + cx^4)^2}\,dx = \int \frac{Cx^4 + Bx^2 + A}{(cx^4 + bx^2 + a)^2\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="
maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*sqrt(e*x^2 + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\,(a + bx^2 + cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^2}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{e\,x^2+d}\,(c\,x^4+b\,x^2+a)^2}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^2),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^2}\,dx=\int \frac{1}{\sqrt{e\,x^2+d}\,a+\sqrt{e\,x^2+d}\,b\,x^2+\sqrt{e\,x^2+d}\,c\,x^4}dx$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*a + sqrt(d + e*x**2)*b*x**2 + sqrt(d + e*x**2)*c*x
**4),x)
```

3.164 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1348

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

output
```
1/2*e*(A*(b^3*d*e^2+b*c*d*(-3*a*e^2+c*d^2)+4*a*c*e*(-2*a*e^2+c*d^2)-b^2*(-
2*a*e^3+2*c*d^2*e))+a*d*(C*(a*b*e^2-12*a*c*d*e+2*b^2*d*e+b*c*d^2)-B*(2*c^2
*d^2+3*b^2*e^2-2*c*e*(5*a*e+b*d))))*x/a/(-4*a*c+b^2)/d/(a*e^2-b*d*e+c*d^2)
^2/(e*x^2+d)^(1/2)+1/2*x*(A*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)-a*(2*B*a*c
*e-B*b^2*e+B*b*c*d+C*a*b*e-2*C*a*c*d)+c*(A*(2*a*c*e-b^2*e+b*c*d)-a*(-B*b*e
+2*B*c*d+2*C*a*e-C*b*d))*x^2)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)
^(1/2)/(c*x^4+b*x^2+a)+1/2*c*(A*(b^3*d*e^2+12*a^2*c*e^3+b*c*d*(a*e^2+c*d^2
)-2*b^2*(2*a*e^3+c*d^2*e))+a*(C*(-4*a^2*e^3+5*a*b*d*e^2+8*a*c*d^2*e-4*b^2*
d^2*e+b*c*d^3)-B*(2*c^2*d^3-2*c*d*e*(-7*a*e+2*b*d)-b*e^2*(2*a*e+b*d)))+(A*
(b^4*d*e^2+b^2*c*d*(-a*e^2+c*d^2)+20*a*b*c*e*(a*e^2+c*d^2)-12*a*c^2*d*(3*a
*e^2+c*d^2)-2*b^3*(2*a*e^3+c*d^2*e))+a*(b^3*d*e*(B*e+4*C*d)+4*a*c*(a*e^2*(
-4*B*e+5*C*d)-c*d^2*(-2*B*e+C*d))-b^2*(a*e^2*(-2*B*e+11*C*d)+c*d^2*(8*B*e+
C*d))-4*b*(a*C*e*(-a*e^2+c*d^2)-B*c*d*(2*a*e^2+c*d^2))))/(-4*a*c+b^2)^(1/2
))*arctan((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^
(1/2)/(e*x^2+d)^(1/2))/a/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-
(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(a*e^2-b*d*e+c*d^2)^2+1/2*c*(A*(b^3*d*e^2+
12*a^2*c*e^3+b*c*d*(a*e^2+c*d^2)-2*b^2*(2*a*e^3+c*d^2*e))+a*(C*(-4*a^2*e^3
+5*a*b*d*e^2+8*a*c*d^2*e-4*b^2*d^2*e+b*c*d^3)-B*(2*c^2*d^3-2*c*d*e*(-7*a*e
+2*b*d)-b*e^2*(2*a*e+b*d)))-(A*(b^4*d*e^2+b^2*c*d*(-a*e^2+c*d^2)+20*a*b*c*
e*(a*e^2+c*d^2)-12*a*c^2*d*(3*a*e^2+c*d^2)-2*b^3*(2*a*e^3+c*d^2*e))+a*(...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 59143 vs. $2(1348) = 2696$.

Time = 23.07 (sec) , antiderivative size = 59143, normalized size of antiderivative = 43.87

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 2256}$$

$$\int \left(\frac{-aC+Ac+x^2(Bc-bC)}{c(d+ex^2)^{3/2}(a+bx^2+cx^4)^2}+\frac{C}{c(d+ex^2)^{3/2}(a+bx^2+cx^4)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Ac-aC)\int \frac{1}{(ex^2+d)^{3/2}(cx^4+bx^2+a)^2}dx}{c}+\frac{(Bc-bC)\int \frac{x^2}{(ex^2+d)^{3/2}(cx^4+bx^2+a)^2}dx}{c}-\frac{C\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}(ae^2-bde+cd^2)}-\frac{C\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}(ae^2-bde+cd^2)}+\frac{Ce^2x}{cd\sqrt{d+ex^2}(ae^2-bde+cd^2)}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 166.52 (sec) , antiderivative size = 1358, normalized size of antiderivative = 1.01

method	result	size
pseudoelliptic	Expression too large to display	1358
default	Expression too large to display	1914

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/(e*x^2+d)^(1/2)*(((arctanh(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((2*a*e-b*d+(-d^
2*(4*a*c-b^2))^(1/2))*a)^(1/2))*((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^
(1/2)-arctan(a*(e*x^2+d)^(1/2)/x*2^(1/2)/((-2*a*e+b*d+(-d^2*(4*a*c-b^2))^(
1/2))*a)^(1/2))*((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2))*(-3/8*c^2*
(1/3*C*a^2+(A*c-1/6*B*b)*a-1/6*A*b^2)*d^3+5/8*(1/5*(2*B*c+C*b)*a^2+b*(A*c-
1/5*B*b)*a-1/5*A*b^3)*c*e*d^2-9/8*e^2*(-5/9*c*a^3*C+(A*c^2+1/6*b*(B*c+C*b)
)*a^2-1/18*a*B*b^3-1/18*A*b^4)*d+(a*c-1/4*b^2)*e^3*a*(-1/2*B*a+A*b))*(c*x^
4+b*x^2+a)*d*2^(1/2)*(e*x^2+d)^(1/2)-((2*a*e-b*d+(-d^2*(4*a*c-b^2))^(1/2))
*a)^(1/2)*(1/4*c^2*(-C*a^2+((B*c-1/2*b*C)*x^2+A*c+1/2*B*b)*a-1/2*A*b*(c*x^
2+b))*d^4-3/4*(1/3*(-C*c*x^2-2*B*c-C*b)*a^2+(1/3*(-B*c^2+1/2*C*b*c)*x^4+1/
3*(A*c^2+1/2*b*B*c)*x^2+b*(A*c+1/3*B*b))*a-1/3*b*A*(-1/2*c*x^2+b)*(c*x^2+b
))*c*e*d^3-1/4*(-5*c*a^3*C+(-6*C*c^2*x^4+(-B*c^2-9/2*C*b*c)*x^2+A*c^2+3/2*
b*(B*c+C*b))*a^2+((2*A*c^3+B*b*c^2+C*b^2*c)*x^4+(3/2*c^2*b*A+1/2*B*b^2*c+C
*b^3)*x^2-2*A*b^2*c-1/2*B*b^3)*a+1/2*A*b^2*(c*x^2+b)*(-2*c*x^2+b))*e^2*d^2
-1/4*e^3*(4*c*(-1/4*C*x^2+B)*a^3+((5*B*c^2+1/2*C*b*c)*x^4+(A*c^2+11/2*b*B*
c+1/2*b^2*C)*x^2-B*b^2)*a^2-2*(3/4*c*(A*c+B*b)*x^2+b*(A*c+3/4*B*b))*b*x^2*
a+1/2*A*b^3*x^2*(c*x^2+b))*d+(a*c-1/4*b^2)*A*e^4*(c*x^4+b*x^2+a)*a)*((-2*a
*e+b*d+(-d^2*(4*a*c-b^2))^(1/2))*a)^(1/2)*x)*(-d^2*(4*a*c-b^2))^(1/2)+3/2*
(1/3*c^2*(1/2*(-B*c+C*b)*a^2+b*(A*c-1/8*B*b)*a-1/8*A*b^3)*d^3-7/12*c*e*(-8
/7*c*a^3*C+1/7*(-2*B*b*c+5*C*b^2)*a^2+b^2*(A*c-1/7*B*b)*a-1/7*A*b^4)*d^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^2}\,dx = \int \frac{Cx^4+Bx^2+A}{(cx^4+bx^2+a)^2(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="
maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)^(3/2)), x
)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^2}\,dx=\int \frac{C\,x^4+B\,x^2+A}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)^2}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^2),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^2}\,dx=\int \frac{1}{\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2+\sqrt{e\,x^2+d}\,bd\,x^2+\sqrt{e\,x^2+d}\,be}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 + sqrt(d + e*x**2)
*b*d*x**2 + sqrt(d + e*x**2)*b*e*x**4 + sqrt(d + e*x**2)*c*d*x**4 + sqrt(d
 + e*x**2)*c*e*x**6),x)
```

3.165 $\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 2548

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\,(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

output
```
1/6*e*(A*(3*b^3*d*e^2+3*b*c*d*(-3*a*e^2+c*d^2)+4*a*c*e*(-2*a*e^2+3*c*d^2)-
b^2*(-2*a*e^3+6*c*d^2*e))-a*d*(B*(6*c^2*d^2+5*b^2*e^2-2*c*e*(7*a*e+3*b*d))
-C*(2*b^2*d*e-20*a*c*d*e+3*b*(a*e^2+c*d^2))))*x/a/(-4*a*c+b^2)/d/(a*e^2-b*
d*e+c*d^2)^2/(e*x^2+d)^(3/2)-1/6*e*(A*(3*b^4*d^2*e^3-2*a*c*e*(-8*a^2*e^4-5
9*a*c*d^2*e^2+9*c^2*d^4)-b*c*d*(64*a^2*e^4-27*a*c*d^2*e^2+3*c^2*d^4)-b^3*(
-16*a*d*e^4+9*c*d^3*e^2)+b^2*(-4*a^2*e^5-40*a*c*d^2*e^3+9*c^2*d^4*e))+a*d*
(B*(6*c^3*d^4-b^2*e^3*(2*a*e+13*b*d)-c^2*d^2*e*(106*a*e+9*b*d)+c*e^2*(8*a^
2*e^2+49*a*b*d*e+31*b^2*d^2))+C*d*(4*b^3*d*e^2+2*a*c*e*(-19*a*e^2+41*c*d^2
)-b*c*d*(25*a*e^2+3*c*d^2)-b^2*(-11*a*e^3+16*c*d^2*e))))*x/a/(-4*a*c+b^2)/
d^2/(a*e^2-b*d*e+c*d^2)^3/(e*x^2+d)^(1/2)+1/2*x*(A*(3*a*b*c*e-2*a*c^2*d-b^
3*e+b^2*c*d)-a*(2*B*a*c*e-B*b^2*e+B*b*c*d+C*a*b*e-2*C*a*c*d)+c*(A*(2*a*c*e
-b^2*e+b*c*d)-a*(-B*b*e+2*B*c*d+2*C*a*e-C*b*d))*x^2)/a/(-4*a*c+b^2)/(a*e^2
-b*d*e+c*d^2)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)-1/2*c*(A*(b^5*d*e^3+b^3*e*(3
*c^2*d^3-6*a*(-4*a*c+b^2)^(1/2)*e^3-c*d*e*(3*(-4*a*c+b^2)^(1/2)*d-8*a*e))-
b^4*e^2*(3*c*d^2-e*((-4*a*c+b^2)^(1/2)*d-6*a*e))+2*a*c^2*(6*c^2*d^4-c*d^2*
e*((-4*a*c+b^2)^(1/2)*d-30*a*e)-a*e^3*(21*(-4*a*c+b^2)^(1/2)*d+16*a*e))-b^
2*c*(c^2*d^4-3*c*d^2*e*((-4*a*c+b^2)^(1/2)*d+3*a*e)-2*a*e^3*(5*(-4*a*c+b^2
)^(1/2)*d+17*a*e))+b*c*(22*a^2*(-4*a*c+b^2)^(1/2)*e^4+a*c*d*e^2*(3*(-4*a*c
+b^2)^(1/2)*d-64*a*e)-c^2*d^3*((-4*a*c+b^2)^(1/2)*d+28*a*e)))+a*(b^4*B*d*e
^3-b^3*e*(3*c*d^2*(B*e+2*C*d)-e^2*(B*(-4*a*c+b^2)^(1/2)*d-3*C*a*d+4*B*a...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 88367 vs. $2(2548) = 5096$.

Time = 23.82 (sec) , antiderivative size = 88367, normalized size of antiderivative = 34.68

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\,(a+bx^2+cx^4)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4)^2),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 2256}$$

$$\int \left(\frac{-aC+Ac+x^2(Bc-bC)}{c(d+ex^2)^{5/2}(a+bx^2+cx^4)^2}+\frac{C}{c(d+ex^2)^{5/2}(a+bx^2+cx^4)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Ac-aC)\int \frac{1}{(ex^2+d)^{5/2}(cx^4+bx^2+a)^2}dx}{c}+\frac{(Bc-bC)\int \frac{x^2}{(ex^2+d)^{5/2}(cx^4+bx^2+a)^2}dx}{c}-\frac{2cC\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}(ae^2-bde+cd^2)}-\frac{2cC\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\arctan\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}(ae^2-bde+cd^2)}+\frac{Cex\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{d\sqrt{d+ex^2}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)(ae^2-bde+cd^2)}+\frac{Cex\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{d\sqrt{d+ex^2}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)(ae^2-bde+cd^2)}+\frac{2Ce^2x}{3cd^2\sqrt{d+ex^2}(ae^2-bde+cd^2)}+\frac{Ce^2x}{3cd(d+ex^2)^{3/2}(ae^2-bde+cd^2)}$$

input
```
Int[(A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F(-1)]

Timed out.

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}dx$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}\,(a+bx^2+cx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(5/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)^2}\,dx = \int \frac{Cx^4+Bx^2+A}{(cx^4+bx^2+a)^2(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="
maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((c*x^4 + b*x^2 + a)^2*(e*x^2 + d)^(5/2)), x
)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="
giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)^2}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(e\,x^2+d)^{5/2}\,(c\,x^4+b\,x^2+a)^2}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4)^2),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(d+ex^2)^{5/2}(a+bx^2+cx^4)^2}\,dx = \int \frac{}{\sqrt{e\,x^2+d}\,a\,d^2+2\sqrt{e\,x^2+d}\,ade\,x^2+\sqrt{e\,x^2+d}\,a\,e^2x^4+\sqrt{e\,x^2+}}$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*a*d**2 + 2*sqrt(d + e*x**2)*a*d*e*x**2 + sqrt(d +
e*x**2)*a*e**2*x**4 + sqrt(d + e*x**2)*b*d**2*x**2 + 2*sqrt(d + e*x**2)*b*
d*e*x**4 + sqrt(d + e*x**2)*b*e**2*x**6 + sqrt(d + e*x**2)*c*d**2*x**4 + 2
*sqrt(d + e*x**2)*c*d*e*x**6 + sqrt(d + e*x**2)*c*e**2*x**8),x)
```

3.166 $\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 547

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\left(Ace+B(cd-be)+\frac{b^2Be-bc(Bd+Ae)+2c(Acd-aBe)}{\sqrt{b^2-4ac}}\right)x\left(1+\frac{ex^2}{d}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{2}{3},1,\frac{3}{2},-\frac{ex^2}{d},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)(d+ex^2)^{2/3}}$$
$$+\frac{\left(Ace+B(cd-be)-\frac{b^2Be-bc(Bd+Ae)+2c(Acd-aBe)}{\sqrt{b^2-4ac}}\right)x\left(1+\frac{ex^2}{d}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{2}{3},1,\frac{3}{2},-\frac{ex^2}{d},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{c\left(b+\sqrt{b^2-4ac}\right)(d+ex^2)^{2/3}}$$
$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}B\left(\sqrt[3]{d}-\sqrt[3]{d+ex^2}\right)\sqrt{\frac{d^{2/3}+\sqrt[3]{d}\sqrt[3]{d+ex^2}+(d+ex^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{d}-\sqrt[3]{d+ex^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{d}-\sqrt[3]{d}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{d}-\sqrt[3]{d}\cdots}\right.\right.}{cx\sqrt{-\frac{\sqrt[3]{d}\left(\sqrt[3]{d}-\sqrt[3]{d+ex^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{d}-\sqrt[3]{d+ex^2}\right)^2}}}$$

output
```
(A*c*e+B*(-b*e+c*d)+(B*b^2*e-b*c*(A*e+B*d)+2*c*(A*c*d-B*a*e))/(-4*a*c+b^2)
^(1/2))*x*(1+e*x^2/d)^(2/3)*AppellF1(1/2,1,2/3,3/2,-2*c*x^2/(b-(-4*a*c+b^2
)^(1/2)),-e*x^2/d)/c/(b-(-4*a*c+b^2)^(1/2))/(e*x^2+d)^(2/3)+(A*c*e+B*(-b*e
+c*d)-(B*b^2*e-b*c*(A*e+B*d)+2*c*(A*c*d-B*a*e))/(-4*a*c+b^2)^(1/2))*x*(1+e
*x^2/d)^(2/3)*AppellF1(1/2,1,2/3,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^
2/d)/c/(b+(-4*a*c+b^2)^(1/2))/(e*x^2+d)^(2/3)-3^(3/4)*(1/2*6^(1/2)-1/2*2^(
1/2))*B*(d^(1/3)-(e*x^2+d)^(1/3))*((d^(2/3)+d^(1/3)*(e*x^2+d)^(1/3)+(e*x^2
+d)^(2/3))/((1-3^(1/2))*d^(1/3)-(e*x^2+d)^(1/3))^2)^(1/2)*EllipticF(((1+3^
(1/2))*d^(1/3)-(e*x^2+d)^(1/3))/((1-3^(1/2))*d^(1/3)-(e*x^2+d)^(1/3)),2*I-
I*3^(1/2))/c/x/(-d^(1/3)*(d^(1/3)-(e*x^2+d)^(1/3))/((1-3^(1/2))*d^(1/3)-(e
*x^2+d)^(1/3))^2)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx=\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(1/3))/(a + b*x^2 + c*x^4),x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(1/3))/(a + b*x^2 + c*x^4), x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.41, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 2256$$

$$\int \left(\frac{\sqrt[3]{d+ex^2}\left(B-\frac{2Ac-bB}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}+b+2cx^2}+\frac{\sqrt[3]{d+ex^2}\left(\frac{2Ac-bB}{\sqrt{b^2-4ac}}+B\right)}{-\sqrt{b^2-4ac}+b+2cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\sqrt[3]{d+ex^2}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\text{AppellF1}\left(\frac{1}{2},1,-\frac{1}{3},\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\left(b-\sqrt{b^2-4ac}\right)\sqrt[3]{\frac{ex^2}{d}+1}}+\frac{x\sqrt[3]{d+ex^2}\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\text{AppellF1}\left(\frac{1}{2},1,-\frac{1}{3},\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\left(\sqrt{b^2-4ac}+b\right)\sqrt[3]{\frac{ex^2}{d}+1}}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(1/3))/(a + b*x^2 + c*x^4),x]
```

output
```
((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^(1/3)*AppellF1[1/2, 1
, -1/3, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/((b - Sqrt
[b^2 - 4*a*c])*(1 + (e*x^2)/d)^(1/3)) + ((B + (b*B - 2*A*c)/Sqrt[b^2 - 4*a
*c])*x*(d + e*x^2)^(1/3)*AppellF1[1/2, 1, -1/3, 3/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), -((e*x^2)/d)])/((b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^(1/
3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,(e\,x^{2}+d)^{\frac{1}{3}}}{c\,x^{4}+b\,x^{2}+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+b*x^2+a),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+b*x^2+a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx=\int \frac{(A+Bx^2)\,\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/3)/(c*x**4+b*x**2+a),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(1/3)/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{1}{3}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(1/3)/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{1}{3}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(1/3)/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{1/3}}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/3))/(a + b*x^2 + c*x^4),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/3))/(a + b*x^2 + c*x^4), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt[3]{d+ex^2}}{a+bx^2+cx^4}\,dx = \left(\int \frac{(e\,x^2+d)^{\frac{1}{3}}}{c\,x^4+b\,x^2+a}dx\right)a + \left(\int \frac{(e\,x^2+d)^{\frac{1}{3}}\,x^2}{c\,x^4+b\,x^2+a}dx\right)b$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/3)/(c*x^4+b*x^2+a),x)
```

output
```
int((d + e*x**2)**(1/3)/(a + b*x**2 + c*x**4),x)*a + int(((d + e*x**2)**(1
/3)*x**2)/(a + b*x**2 + c*x**4),x)*b
```

3.167 $\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 755

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{e(7Ace(15cd-4be)+B(105c^2d^2+24b^2e^2-ce(84bd+25ae)))x\sqrt{a+bx^2+cx^4}}{105c^3}$$
$$+\frac{e^2(21Bcd-6bBe+7Ace)x^3\sqrt{a+bx^2+cx^4}}{35c^2}+\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$
$$+\frac{(7Ace(45c^2d^2+8b^2e^2-3ce(10bd+3ae))+B(105c^3d^3-48b^3e^3-21c^2de(10bd+9ae)+8bce^2(21bd}{105c^{7/2}(\sqrt{a}+\sqrt{c}x^2)}$$
$$-\frac{\sqrt[4]{a}(7Ace(45c^2d^2+8b^2e^2-3ce(10bd+3ae))+B(105c^3d^3-48b^3e^3-21c^2de(10bd+9ae)+8bce^2(21}{105c^{15/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(7Ac(15c^2d^3-15acde^2+4abe^3)-aBe(105c^2d^2+24b^2e^2-ce(84bd+25ae))+\frac{\sqrt{a}(7Ace(45c^2d^2+8b^2e^2-3}{}\right.}{}$$

output
```
1/105*e*(7*A*c*e*(-4*b*e+15*c*d)+B*(105*c^2*d^2+24*b^2*e^2-c*e*(25*a*e+84*
b*d)))*x*(c*x^4+b*x^2+a)^(1/2)/c^3+1/35*e^2*(7*A*c*e-6*B*b*e+21*B*c*d)*x^3
*(c*x^4+b*x^2+a)^(1/2)/c^2+1/7*B*e^3*x^5*(c*x^4+b*x^2+a)^(1/2)/c+1/105*(7*
A*c*e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d))+B*(105*c^3*d^3-48*b^3*e^
3-21*c^2*d*e*(9*a*e+10*b*d)+8*b*c*e^2*(13*a*e+21*b*d)))*x*(c*x^4+b*x^2+a)^
(1/2)/c^(7/2)/(a^(1/2)+c^(1/2)*x^2)-1/105*a^(1/4)*(7*A*c*e*(45*c^2*d^2+8*b
^2*e^2-3*c*e*(3*a*e+10*b*d))+B*(105*c^3*d^3-48*b^3*e^3-21*c^2*d*e*(9*a*e+1
0*b*d)+8*b*c*e^2*(13*a*e+21*b*d)))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/
(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),
1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(15/4)/(c*x^4+b*x^2+a)^(1/2)+1/210*(7*A
*c*(4*a*b*e^3-15*a*c*d*e^2+15*c^2*d^3)-a*B*e*(105*c^2*d^2+24*b^2*e^2-c*e*(
25*a*e+84*b*d))+a^(1/2)*(7*A*c*e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d
))+B*(105*c^3*d^3-48*b^3*e^3-21*c^2*d*e*(9*a*e+10*b*d)+8*b*c*e^2*(13*a*e+2
1*b*d)))/c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*
x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)
/c^(1/2))^(1/2))/a^(1/4)/c^(13/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.53 (sec) , antiderivative size = 4473, normalized size of antiderivative = 5.92

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^3)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
Sqrt[a + b*x^2 + c*x^4]*(-1/105*(e*(-105*B*c^2*d^2 + 84*b*B*c*d*e - 105*A*
c^2*d*e - 24*b^2*B*e^2 + 28*A*b*c*e^2 + 25*a*B*c*e^2)*x)/c^3 + (e^2*(21*B*
c*d - 6*b*B*e + 7*A*c*e)*x^3)/(35*c^2) + (B*e^3*x^5)/(7*c)) + ((((105*I)/2
)*B*c^2*(-b + Sqrt[b^2 - 4*a*c])*d^3*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[
Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(
-b + Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt
[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]))/
(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) - ((
105*I)*b*B*c*(-b + Sqrt[b^2 - 4*a*c])*d^2*e*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[
b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*(EllipticE[I*A
rcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*
a*c])/(-b + Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b
 - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*
c])]))/(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4
]) + (((315*I)/2)*A*c^2*(-b + Sqrt[b^2 - 4*a*c])*d^2*e*Sqrt[1 - (2*c*x^2)/
(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*(El
lipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sq
rt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*S
qrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + S...
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 690, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2207, 2207, 2207, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{\int \frac{e^2(21Bcd-6bBe+7Ace)x^6+e(21Bcd^2+21Aced-5aBe^2)x^4+7cd^2(Bd+3Ae)x^2+7Acd^3}{\sqrt{cx^4+bx^2+a}}dx}{7c}+\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\Big\downarrow$ 2207

$$\frac{\frac{\int \frac{e\left(7Ace(15cd-4be)+B\left(105c^2d^2+24b^2e^2-ce(84bd+25ae)\right)\right)x^4+\left(21Ace\left(5cd^2-ae^2\right)+B\left(35c^2d^3-63ace^2d+18abe^3\right)\right)x^2+35Ac^2d^3}{\sqrt{cx^4+bx^2+a}}dx}{5c}+\frac{e^2x^3\sqrt{a+bx^2+cx^4}}{}}{7c}$$

$$\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\Big\downarrow$ 2207

$$\frac{\frac{\frac{\int \frac{\left(7Ace\left(45c^2d^2+8b^2e^2-3ce(10bd+3ae)\right)+B\left(105c^3d^3-21c^2e(10bd+9ae)d-48b^3e^3+8bce^2(21bd+13ae)\right)\right)x^2+7Ac\left(15c^2d^3-15ace^2d+4abe^3\right)-aBe\left(105c^2d^2+2\right)}{\sqrt{cx^4+bx^2+a}}}{3c}}{5c}}{7c}$$

$$\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\Big\downarrow$ 1511

$$\frac{\sqrt{a}\left(\frac{\sqrt{c}\left(7Ac\left(4abe^3-15acde^2+15c^2d^3\right)-aBe\left(-ce(25ae+84bd)+24b^2e^2+105c^2d^2\right)\right)}{\sqrt{a}}+7Ace\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)+B\left(-21c^2de(9ae+10bd)+8bce^2\right)\right)}{\sqrt{c}}$$

$$\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a}\left(\frac{\sqrt{c}\left(7Ac\left(4abe^3-15acde^2+15c^2d^3\right)-aBe\left(-ce(25ae+84bd)+24b^2e^2+105c^2d^2\right)\right)}{\sqrt{a}}+7Ace\left(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)+B\left(-21c^2de(9ae+10bd)+8bce^2\right)\right)}{\sqrt{c}}$$

$$\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\Big\downarrow$ 1416

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\frac{\sqrt{c}\left(7Ac\left(4abe^3-15acde^2+15c^2d^3\right)-aBe\left(-ce(25ae+84bd)+24b^2e^2+105c^2d^2\right)\right)}{\sqrt{a}}\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}$$

$$\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$

$\Big\downarrow$ 1509

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\frac{\sqrt{c}\left(7Ac\left(4abe^3-15acde^2+15c^2d^3\right)-aBe\left(-ce(25ae+84bd)+24b^2e^2+105c^2d\right.\right.}{\sqrt{a}}\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}$$

$$\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^3)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(B*e^3*x^5*Sqrt[a + b*x^2 + c*x^4])/(7*c) + ((e^2*(21*B*c*d - 6*b*B*e + 7*
A*c*e)*x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) + ((e*(7*A*c*e*(15*c*d - 4*b*e)
+ B*(105*c^2*d^2 + 24*b^2*e^2 - c*e*(84*b*d + 25*a*e)))*x*Sqrt[a + b*x^2 +
 c*x^4])/(3*c) + (-(((7*A*c*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*
a*e)) + B*(105*c^3*d^3 - 48*b^3*e^3 - 21*c^2*d*e*(10*b*d + 9*a*e) + 8*b*c*
e^2*(21*b*d + 13*a*e)))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*
x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a
]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(7
*A*c*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)) + B*(105*c^3*d^3
- 48*b^3*e^3 - 21*c^2*d*e*(10*b*d + 9*a*e) + 8*b*c*e^2*(21*b*d + 13*a*e))
+ (Sqrt[c]*(7*A*c*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a*b*e^3) - a*B*e*(105*c^2
*d^2 + 24*b^2*e^2 - c*e*(84*b*d + 25*a*e))))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b
*x^2 + c*x^4]))/(3*c))/(5*c))/(7*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 10.95 (sec) , antiderivative size = 673, normalized size of antiderivative = 0.89

method	result
elliptic	$\frac{B\,e^{3} x^{5} \sqrt{c\,x^{4}+b\,x^{2}+a}}{7 c}+\frac{\left(A\,e^{3}+3 B d\,e^{2}-\frac{6 e^{3} B b}{7 c}\right) x^{3} \sqrt{c\,x^{4}+b\,x^{2}+a}}{5 c}+\frac{\left(3 A d\,e^{2}+3 B e\,d^{2}-\frac{5 e^{3} B a}{7 c}-\frac{4 \left(A\,e^{3}+3 B d\,e^{2}-\frac{6 e^{3} B b}{7 c}\right) b}{5 c}\right) x \sqrt{}}{3 c}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*B*e^3*x^5*(c*x^4+b*x^2+a)^(1/2)/c+1/5*(A*e^3+3*B*d*e^2-6/7*e^3*B/c*b)/
c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(3*A*d*e^2+3*B*e*d^2-5/7*e^3*B/c*a-4/5*(A*
e^3+3*B*d*e^2-6/7*e^3*B/c*b)/c*b)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(A*d^3-1/3
*(3*A*d*e^2+3*B*e*d^2-5/7*e^3*B/c*a-4/5*(A*e^3+3*B*d*e^2-6/7*e^3*B/c*b)/c*
b)/c*a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(3*A*d^2*e+B*d^3-3/5*(A*e^3+3*B*
d*e^2-6/7*e^3*B/c*b)/c*a-2/3*(3*A*d*e^2+3*B*e*d^2-5/7*e^3*B/c*a-4/5*(A*e^3
+3*B*d*e^2-6/7*e^3*B/c*b)/c*b)/c*b)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(
1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1041, normalized size of antiderivative = 1.38

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
1/210*(sqrt(1/2)*((105*B*a*c^4*d^3 - 105*(2*B*a*b*c^3 - 3*A*a*c^4)*d^2*e +
 21*(8*B*a*b^2*c^2 - (9*B*a^2 + 10*A*a*b)*c^3)*d*e^2 - (48*B*a*b^3*c + 63*
A*a^2*c^3 - 8*(13*B*a^2*b + 7*A*a*b^2)*c^2)*e^3)*x*sqrt((b^2 - 4*a*c)/c^2)
 - (105*B*a*b*c^3*d^3 - 105*(2*B*a*b^2*c^2 - 3*A*a*b*c^3)*d^2*e + 21*(8*B*
a*b^3*c - (9*B*a^2*b + 10*A*a*b^2)*c^2)*d*e^2 - (48*B*a*b^4 + 63*A*a^2*b*c
^2 - 8*(13*B*a^2*b^2 + 7*A*a*b^3)*c)*e^3)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4
*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/
c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) -
sqrt(1/2)*((105*(B*a*c^4 - A*c^5)*d^3 - 105*(2*B*a*b*c^3 - (3*A + B)*a*c^4
)*d^2*e + 21*(8*B*a*b^2*c^2 + 5*A*a*c^4 - (9*B*a^2 + 2*(5*A + 2*B)*a*b)*c^
3)*d*e^2 - (48*B*a*b^3*c + ((63*A + 25*B)*a^2 + 28*A*a*b)*c^3 - 8*(13*B*a^
2*b + (7*A + 3*B)*a*b^2)*c^2)*e^3)*x*sqrt((b^2 - 4*a*c)/c^2) - (105*(B*a*b
*c^3 + A*b*c^4)*d^3 - 105*(2*B*a*b^2*c^2 - (3*A - B)*a*b*c^3)*d^2*e + 21*(
8*B*a*b^3*c - 5*A*a*b*c^3 - (9*B*a^2*b + 2*(5*A - 2*B)*a*b^2)*c^2)*d*e^2 -
 (48*B*a*b^4 + ((63*A - 25*B)*a^2*b - 28*A*a*b^2)*c^2 - 8*(13*B*a^2*b^2 +
(7*A - 3*B)*a*b^3)*c)*e^3)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)
/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x)
, 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(15*B*a*c^4*e
^3*x^6 + 105*B*a*c^4*d^3 + 3*(21*B*a*c^4*d*e^2 - (6*B*a*b*c^3 - 7*A*a*c^4)
*e^3)*x^4 - 105*(2*B*a*b*c^3 - 3*A*a*c^4)*d^2*e + 21*(8*B*a*b^2*c^2 - (...
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**3/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^3}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
( - 53*sqrt(a + b*x**2 + c*x**4)*a*b*c*e**3*x + 105*sqrt(a + b*x**2 + c*x*
*4)*a*c**2*d*e**2*x + 21*sqrt(a + b*x**2 + c*x**4)*a*c**2*e**3*x**3 + 24*s
qrt(a + b*x**2 + c*x**4)*b**3*e**3*x - 84*sqrt(a + b*x**2 + c*x**4)*b**2*c
*d*e**2*x - 18*sqrt(a + b*x**2 + c*x**4)*b**2*c*e**3*x**3 + 105*sqrt(a + b
*x**2 + c*x**4)*b*c**2*d**2*e*x + 63*sqrt(a + b*x**2 + c*x**4)*b*c**2*d*e*
*2*x**3 + 15*sqrt(a + b*x**2 + c*x**4)*b*c**2*e**3*x**5 + 53*int(sqrt(a +
b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**2*b*c*e**3 - 105*int(sqrt(a +
 b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**2*c**2*d*e**2 - 24*int(sqrt(
a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b**3*e**3 + 84*int(sqrt(a
+ b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b**2*c*d*e**2 - 105*int(sqrt
(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b*c**2*d**2*e + 105*int(s
qrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*c**3*d**3 - 63*int((sq
rt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a**2*c**2*e**3 + 16
0*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*b**2*c*e
**3 - 399*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*
b*c**2*d*e**2 + 315*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x
**4),x)*a*c**3*d**2*e - 48*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**
2 + c*x**4),x)*b**4*e**3 + 168*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b
*x**2 + c*x**4),x)*b**3*c*d*e**2 - 210*int((sqrt(a + b*x**2 + c*x**4)*x**2
)/(a + b*x**2 + c*x**4),x)*b**2*c**2*d**2*e + 105*int((sqrt(a + b*x**2 ...
```

3.168 $\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 528

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{e(10Bcd-4bBe+5Ace)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$$+ \frac{(10Ace(3cd-be)+B(15c^2d^2+8b^2e^2-ce(20bd+9ae)))\,x\sqrt{a+bx^2+cx^4}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$- \frac{\sqrt[4]{a}(10Ace(3cd-be)+B(15c^2d^2+8b^2e^2-ce(20bd+9ae)))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\Big(2\arctan\Big(}{15c^{11/4}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{\left(2aBe(5cd-2be)-5Ac(3cd^2-ae^2)-\frac{\sqrt{a}(10Ace(3cd-be)+B(15c^2d^2+8b^2e^2-ce(20bd+9ae)))}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{}}{30\sqrt[4]{a}c^{9/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/15*e*(5*A*c*e-4*B*b*e+10*B*c*d)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/5*B*e^2*x^
3*(c*x^4+b*x^2+a)^(1/2)/c+1/15*(10*A*c*e*(-b*e+3*c*d)+B*(15*c^2*d^2+8*b^2*
e^2-c*e*(9*a*e+20*b*d)))*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+c^(1/2)*
x^2)-1/15*a^(1/4)*(10*A*c*e*(-b*e+3*c*d)+B*(15*c^2*d^2+8*b^2*e^2-c*e*(9*a*
e+20*b*d)))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2
))^(1/2))/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)-1/30*(2*a*B*e*(-2*b*e+5*c*d)-5*A*
c*(-a*e^2+3*c*d^2)-a^(1/2)*(10*A*c*e*(-b*e+3*c*d)+B*(15*c^2*d^2+8*b^2*e^2-
c*e*(9*a*e+20*b*d)))/c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1
/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*
(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/c^(9/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.70 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}ex(a+bx^2+cx^4)\left(5Ace+B(10cd-4be+3cex^2)\right)+i\left(-b+\sqrt{b^2-4ac}\right)\left(10Ace(3cd-be\right.}{}$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*x*(a + b*x^2 + c*x^4)*(5*A*c*e + B*
(10*c*d - 4*b*e + 3*c*e*x^2)) + I*(-b + Sqrt[b^2 - 4*a*c])*(10*A*c*e*(3*c*
d - b*e) + B*(15*c^2*d^2 + 8*b^2*e^2 - c*e*(20*b*d + 9*a*e)))*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])] - I*(-8*b^3*B*e^2 + 2*b^2*e*(10*B*c*d + 5*A*c*e + 4*B*Sqrt[b^2
- 4*a*c]*e) - b*c*(15*B*c*d^2 + B*e*(20*Sqrt[b^2 - 4*a*c]*d - 17*a*e) + 10
*A*e*(3*c*d + Sqrt[b^2 - 4*a*c]*e)) + c*(B*(-9*a*Sqrt[b^2 - 4*a*c]*e^2 + 5
*c*d*(3*Sqrt[b^2 - 4*a*c]*d - 4*a*e)) + 10*A*c*(3*c*d^2 + e*(3*Sqrt[b^2 -
4*a*c]*d - a*e))))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c]
)]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sq
rt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2207, 2207, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{\int \frac{e(10Bcd-4bBe+5Ace)x^4+(5Bcd^2+10Aced-3aBe^2)x^2+5Acd^2}{\sqrt{cx^4+bx^2+a}}dx}{5c}+\frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \text{2207}$$

$$\frac{\frac{\int -\frac{-\left(\left(10Ace(3cd-be)+B\left(15c^2d^2+8b^2e^2-ce(20bd+9ae)\right)\right)x^2\right)+2aBe(5cd-2be)-5Ac\left(3cd^2-ae^2\right)}{\sqrt{cx^4+bx^2+a}}dx}{3c}+\frac{ex\sqrt{a+bx^2+cx^4}(5Ace-4bBe+10Bcd)}{3c}}{5c}+\frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$\big\downarrow$ 25

$$\frac{\frac{ex\sqrt{a+bx^2+cx^4}(5Ace-4bBe+10Bcd)}{3c}-\frac{\int -\frac{\left(\left(10Ace(3cd-be)+B\left(15c^2d^2+8b^2e^2-ce(20bd+9ae)\right)\right)x^2\right)+2aBe(5cd-2be)-5Ac\left(3cd^2-ae^2\right)}{\sqrt{cx^4+bx^2+a}}dx}{3c}}{5c}+\frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$\big\downarrow$ 1511

$$\frac{\frac{ex\sqrt{a+bx^2+cx^4}(5Ace-4bBe+10Bcd)}{3c}-\frac{\frac{\sqrt{a}\left(B\left(-ce(9ae+20bd)+8b^2e^2+15c^2d^2\right)+10Ace(3cd-be)\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(-9a^{3/2}Bce^2+\sqrt{a}\left(10Ace(3\cdots\right.\right.}{\cdots}}{3c}}{5c}$$

$$\frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$\big\downarrow$ 27

$$\frac{\frac{ex\sqrt{a+bx^2+cx^4}(5Ace-4bBe+10Bcd)}{3c}-\frac{\frac{\left(B\left(-ce(9ae+20bd)+8b^2e^2+15c^2d^2\right)+10Ace(3cd-be)\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(-9a^{3/2}Bce^2+\sqrt{a}\left(10Ace(3cd-be\cdots\right.\right.}{\cdots}}{3c}}{5c}$$

$$\frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$\big\downarrow$ 1416

$$\frac{\frac{ex\sqrt{a+bx^2+cx^4}(5Ace-4bBe+10Bcd)}{3c}-\frac{\frac{\left(B\left(-ce(9ae+20bd)+8b^2e^2+15c^2d^2\right)+10Ace(3cd-be)\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{Ellip}\cdots}{\cdots}}{\cdots}}{5c}$$

$$\frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c}$$

$\big\downarrow$ 1509

$$\frac{ex\sqrt{a+bx^2+cx^4}(5Ace-4bBe+10Bcd)}{3c}-\frac{\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)\left(B\left(-ce(9ae+2\cdots\right.\right.}{\sqrt{c}}\cdots$$

$$\frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(B*e^2*x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) + ((e*(10*B*c*d - 4*b*B*e + 5*A*
c*e)*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (((10*A*c*e*(3*c*d - b*e) + B*(15*
c^2*d^2 + 8*b^2*e^2 - c*e*(20*b*d + 9*a*e)))*(-((x*Sqrt[a + b*x^2 + c*x^4]
)/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/S
qrt[c] - ((15*A*c^(5/2)*d^2 - 9*a^(3/2)*B*c*e^2 - a*Sqrt[c]*e*(10*B*c*d -
4*b*B*e + 5*A*c*e) + Sqrt[a]*(10*A*c*e*(3*c*d - b*e) + B*(15*c^2*d^2 - 20*
b*c*d*e + 8*b^2*e^2)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/(3*c))/
(5*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 7.10 (sec) , antiderivative size = 508, normalized size of antiderivative = 0.96

method	result
elliptic	$\frac{B e^2 x^3 \sqrt{c x^4+b x^2+a}}{5c}+\frac{\left(A e^2+2Bde-\frac{4bB e^2}{5c}\right)x\sqrt{c x^4+b x^2+a}}{3c}+\frac{\left(A d^2-\frac{a\left(A e^2+2Bde-\frac{4bB e^2}{5c}\right)}{3c}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}}{4\sqrt{}}$
risch	$\frac{ex\left(3Be x^2 c+5Ace-4Bbe+10Bcd\right)\sqrt{c x^4+b x^2+a}}{15c^2}-\frac{-\left(10Abc e^2-30A c^2 de+9Bac e^2-8b^2 e^2 B+20Bbcde-15B c^2 d^2\right)a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{}\right)}{}}}{}$
default	Expression too large to display

input
```
int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*B*e^2*x^3*(c*x^4+b*x^2+a)^(1/2)/c+1/3*(A*e^2+2*B*d*e-4/5/c*b*B*e^2)/c*
x*(c*x^4+b*x^2+a)^(1/2)+1/4*(A*d^2-1/3*a/c*(A*e^2+2*B*d*e-4/5/c*b*B*e^2))*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*E
llipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(2*d*e*A+B*d^2-3/5*B*e^2/c*a-2/3/c*b*(A*
e^2+2*B*d*e-4/5/c*b*B*e^2))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4
-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
1/30*(sqrt(1/2)*((15*B*a*c^3*d^2 - 10*(2*B*a*b*c^2 - 3*A*a*c^3)*d*e + (8*B
*a*b^2*c - (9*B*a^2 + 10*A*a*b)*c^2)*e^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (15*
B*a*b*c^2*d^2 - 10*(2*B*a*b^2*c - 3*A*a*b*c^2)*d*e + (8*B*a*b^3 - (9*B*a^2
*b + 10*A*a*b^2)*c)*e^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c
)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x),
1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((15*(B
*a*c^3 - A*c^4)*d^2 - 10*(2*B*a*b*c^2 - (3*A + B)*a*c^3)*d*e + (8*B*a*b^2*
c + 5*A*a*c^3 - (9*B*a^2 + 2*(5*A + 2*B)*a*b)*c^2)*e^2)*x*sqrt((b^2 - 4*a*
c)/c^2) - (15*(B*a*b*c^2 + A*b*c^3)*d^2 - 10*(2*B*a*b^2*c - (3*A - B)*a*b*
c^2)*d*e + (8*B*a*b^3 - 5*A*a*b*c^2 - (9*B*a^2*b + 2*(5*A - 2*B)*a*b^2)*c)
*e^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin
(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2
- 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(3*B*a*c^3*e^2*x^4 + 15*B*a*c^3*d^
2 - 10*(2*B*a*b*c^2 - 3*A*a*c^3)*d*e + (8*B*a*b^2*c - (9*B*a^2 + 10*A*a*b)
*c^2)*e^2 + (10*B*a*c^3*d*e - (4*B*a*b*c^2 - 5*A*a*c^3)*e^2)*x^2)*sqrt(c*x
^4 + b*x^2 + a))/(a*c^4*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**2/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^2}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{5\sqrt{c\,x^4+b\,x^2+a}\,ac\,e^2x-4\sqrt{c\,x^4+b\,x^2+a}\,b^2e^2x+10\sqrt{c\,x^4+b\,x^2+a}\,bcdex+3\sqrt{c\,x^4+b\,x^2+a}\,bc\,e^2}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
(5*sqrt(a + b*x**2 + c*x**4)*a*c*e**2*x - 4*sqrt(a + b*x**2 + c*x**4)*b**2
*e**2*x + 10*sqrt(a + b*x**2 + c*x**4)*b*c*d*e*x + 3*sqrt(a + b*x**2 + c*x
**4)*b*c*e**2*x**3 - 5*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4)
,x)*a**2*c*e**2 + 4*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)
*a*b**2*e**2 - 10*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a
*b*c*d*e + 15*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*c**
2*d**2 - 19*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*
a*b*c*e**2 + 30*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4)
,x)*a*c**2*d*e + 8*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x*
*4),x)*b**3*e**2 - 20*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c
*x**4),x)*b**2*c*d*e + 15*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2
 + c*x**4),x)*b*c**2*d**2)/(15*c**2)
```

3.169 $\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 368

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bex\sqrt{a+bx^2+cx^4}}{3c}+\frac{(3Bcd-2bBe+3Ace)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{\sqrt[4]{a}(3Bcd-2bBe+3Ace)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(3Acd-aBe+\frac{\sqrt{a}(3Bcd-2bBe+3Ace)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{6\sqrt[4]{a}c^{5/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/3*B*e*x*(c*x^4+b*x^2+a)^(1/2)/c+1/3*(3*A*c*e-2*B*b*e+3*B*c*d)*x*(c*x^4+b
*x^2+a)^(1/2)/c^(3/2)/(a^(1/2)+c^(1/2)*x^2)-1/3*a^(1/4)*(3*A*c*e-2*B*b*e+3
*B*c*d)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1
/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)+1/6*(3*A*c*d-B*a*e+a^(1/2)*(3*A*c*e-2*
B*b*e+3*B*c*d)/c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^
(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a
^(1/2)/c^(1/2))^(1/2))/a^(1/4)/c^(5/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.25 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{4Bc\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}ex(a+bx^2+cx^4) - i\left(-b+\sqrt{b^2-4ac}\right)(-3Bcd+2bBe-3Ace)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-}{}}}{}$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(4*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*x*(a + b*x^2 + c*x^4) - I*(-b + S
qrt[b^2 - 4*a*c])*(-3*B*c*d + 2*b*B*e - 3*A*c*e)*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] +
I*(-2*b^2*B*e - c*(6*A*c*d + 3*B*Sqrt[b^2 - 4*a*c]*d - 2*a*B*e + 3*A*Sqrt[
b^2 - 4*a*c]*e) + b*(3*B*c*d + 3*A*c*e + 2*B*Sqrt[b^2 - 4*a*c]*e))*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqr
t[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt
[b^2 - 4*a*c])])/(12*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 +
c*x^4])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2207, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \ 2207$$

$$\frac{\int \frac{(3Bcd-2bBe+3Ace)x^2+3Acd-aBe}{\sqrt{cx^4+bx^2+a}}dx}{3c}+\frac{Bex\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \ 1511$$

$$\frac{\frac{\sqrt{a}\left(\frac{\sqrt{c}(3Acd-aBe)}{\sqrt{a}}+3Ace-2bBe+3Bcd\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}(3Ace-2bBe+3Bcd)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}+\frac{Bex\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\sqrt{a}\left(\frac{\sqrt{c}(3Acd-aBe)}{\sqrt{a}}+3Ace-2bBe+3Bcd\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{(3Ace-2bBe+3Bcd)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}+\frac{Bex\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \ 1416$$

$$\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\frac{\sqrt{c}(3Acd-aBe)}{\sqrt{a}}+3Ace-2bBe+3Bcd\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(3Ace-2bBe+3Bcd)\int}{\sqrt{c}}}{3c}+\frac{Bex\sqrt{a+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \ 1509$$

$$\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\frac{\sqrt{c}(3Acd-aBe)}{\sqrt{a}}+3Ace-2bBe+3Bcd\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a}{(\sqrt{}}}}{}\right.}{}}{3c}+\frac{Bex\sqrt{a+bx^2+cx^4}}{3c}$$

input
```
Int[((A + B*x^2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(B*e*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + (-(((3*B*c*d - 2*b*B*e + 3*A*c*e)*
(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)
*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(3*B*c*d - 2*b*B*e + 3*A*c
*e + (Sqrt[c]*(3*A*c*d - a*B*e))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4
]))/(3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{Bex\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\left(Ad-\frac{aBe}{3c}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\sqrt{-4+\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
risch	$\frac{Bex\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{-\frac{(3Ace-2Bbe+3Bcd)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}}{}$
default	$\frac{Ad\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-\frac{(Ae+B}{}$

input
```
int((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*e*x*(c*x^4+b*x^2+a)^(1/2)/c+1/4*(A*d-1/3*a*B*e/c)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2))-1/2*(A*e+B*d-2/3*b/c*B*e)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\sqrt{\frac{1}{2}}\left((3\,Bac^2d-(2\,Babc-3\,Aac^2)e)x\sqrt{\frac{b^2-4\,ac}{c^2}}-(3\,Babcd-(2\,Bab^2-3\,Aabc)e)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E}{}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(1/2)*((3*B*a*c^2*d - (2*B*a*b*c - 3*A*a*c^2)*e)*x*sqrt((b^2 - 4*
a*c)/c^2) - (3*B*a*b*c*d - (2*B*a*b^2 - 3*A*a*b*c)*e)*x)*sqrt(c)*sqrt((c*s
qrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b
^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*
c)/(a*c)) - sqrt(1/2)*((3*(B*a*c^2 - A*c^3)*d - (2*B*a*b*c - (3*A + B)*a*c
^2)*e)*x*sqrt((b^2 - 4*a*c)/c^2) - (3*(B*a*b*c + A*b*c^2)*d - (2*B*a*b^2 -
 (3*A - B)*a*b*c)*e)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*el
liptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*
(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(B*a*c^2*e*x^2 + 3*
B*a*c^2*d - (2*B*a*b*c - 3*A*a*c^2)*e)*sqrt(c*x^4 + b*x^2 + a))/(a*c^3*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(A+Bx^2)\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2+a}\,bex-\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)abe+3\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)acd+3\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)ace-}{3c}$$

input
```
int((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4)*b*e*x - int(sqrt(a + b*x**2 + c*x**4)/(a + b*x*
*2 + c*x**4),x)*a*b*e + 3*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x*
*4),x)*a*c*d + 3*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4
),x)*a*c*e - 2*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),
x)*b**2*e + 3*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x
)*b*c*d)/(3*c)
```

3.170 $\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 284

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
B*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)-a^(1/4)*B*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/(c
*x^4+b*x^2+a)^(1/2)+1/2*(a^(1/2)*B+A*c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^
4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)
*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/c^(3/4)/(c*x^4+b*x^2+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(B\left(-b+\sqrt{b^2-4ac}\right)E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(\right.}{2\sqrt{2}c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+}}$$

input
```
Integrate[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*(-b + Sqrt[b^2 - 4*a*c])*Ellip
ticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1511}$$

$$\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}B\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{B\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{B\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{\sqrt{a}B}{\sqrt{c}}+A\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{B\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

input
```
Int[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
-((B*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^
(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + ((A + (Sqrt[a]*B)/Sqrt[c])*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a
^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.27

method	result
default	$\frac{A\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-\frac{Ba\sqrt{2}\,\sqrt{}}{}$
elliptic	$\frac{A\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-\frac{Ba\sqrt{2}\,\sqrt{}}{}$

input
```
int((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*A*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ellipti
cF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{cx^4+bx^2+a}Bac+\sqrt{\frac{1}{2}}\left(Bacx\sqrt{\frac{b^2-4ac}{c^2}}-Babx\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}}{x}\right) \mid \frac{bc\sqrt{}}{}\right.}{}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(c*x^4 + b*x^2 + a)*B*a*c + sqrt(1/2)*(B*a*c*x*sqrt((b^2 - 4*a*
c)/c^2) - B*a*b*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*ellipti
c_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*
sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((B*a*c - A*c^2)
*x*sqrt((b^2 - 4*a*c)/c^2) - (B*a*b + A*b*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2
- 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*
c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c))
)/(a*c^2*x)
```

Sympy [F]

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}} \, dx = \int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}} \, dx$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}} \, dx = \int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}} \, dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}} \, dx = \int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}} \, dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)a+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)b$$

input
```
int((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a + int((sqrt(a + b
*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*b
```

3.171 $\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 439

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(Bd-Ae)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}}$$
$$-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}}$$

output
```
-1/2*(-A*e+B*d)*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*x^4+
b*x^2+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)-1/2*(a^(1/2)*B-A
*c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^
(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2)
)^(1/2))/a^(1/4)/c^(1/4)/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+b*x^2+a)^(1/2)+1/4*(
c^(1/2)*d+a^(1/2)*e)*(-A*e+B*d)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^
(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1
/4*(c^(1/2)*d-a^(1/2)*e)^2/a^(1/2)/c^(1/2)/d/e,1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))/a^(1/4)/c^(1/4)/d/e/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.63 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(Bd\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(-Bd+\cdots}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2+c\cdots}}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*d*EllipticF[I*ArcSinh[Sqrt[2]*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])] + (-(B*d) + A*e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d),
 I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d
*e*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2226, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{2226}$$

$$\frac{\sqrt{a}(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \; 27$$

$$\frac{(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \; 1416$$

$$\frac{(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

$$\Big\downarrow \; 2220$$

$$\frac{(Bd-Ae)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\ldots}{2\sqrt{d}\sqrt{e}\sqrt{\ldots}}\right.}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*((Sqrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(1/4)*c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt
[a + b*x^2 + c*x^4]) + ((B*d - A*e)*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[
(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])]
)/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*
ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1
/4)*d*e*Sqrt[a + b*x^2 + c*x^4])))/(Sqrt[c]*d - Sqrt[a]*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.82

method	result
default	$\frac{B\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\operatorname{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}+\frac{(Ae-Bd}{}$
elliptic	$\frac{B\sqrt{2}\sqrt{4+\frac{2bx^2}{a}-\frac{2x^2\sqrt{-4ac+b^2}}{a}}\sqrt{4+\frac{2bx^2}{a}+\frac{2x^2\sqrt{-4ac+b^2}}{a}}\operatorname{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}+\frac{\sqrt{2}\sqrt{}}{}$

input
```
int((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*B/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(A*e-B*d)/e/d*2^(1/2)/(-b/a+1/a*(-4*
a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(
1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*El
lipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^
2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)a + \left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)b$$

input
```
int((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*
x**4 + c*e*x**6),x)*a + int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x*
*2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b
```

3.172 $\int \frac{A+Bx^2}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 782

$$\begin{aligned}
&\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx \\
&= \frac{\sqrt{c}(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} \\
&\quad - \frac{(B(cd^3-ade^2)-Ae(3cd^2-e(2bd-ae)))\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}\sqrt{e}\left(cd^2-bde+ae^2\right)^{3/2}} \\
&\quad - \frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} \\
&\quad + \frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}} \\
&\quad + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(B(cd^3-ade^2)-Ae(3cd^2-e(2bd-ae))\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}\ldots}{4\ldots}\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2e\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

output
```
1/2*c^(1/2)*(-A*e+B*d)*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(a^(1
/2)+c^(1/2)*x^2)-1/2*e*(-A*e+B*d)*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c
*d^2)/(e*x^2+d)-1/4*(B*(-a*d*e^2+c*d^3)-A*e*(3*c*d^2-e*(-a*e+2*b*d)))*arct
an((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^(3
/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(3/2)-1/2*a^(1/4)*c^(1/4)*(-A*e+B*d)*(a^(1
/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE
(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/d/(a*e^
2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*A*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*(
(c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^
(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/d/(c^(1/2)*d-a^(
1/2)*e)/(c*x^4+b*x^2+a)^(1/2)+1/8*(c^(1/2)*d+a^(1/2)*e)*(B*(-a*d*e^2+c*d^3
)-A*e*(3*c*d^2-e*(-a*e+2*b*d)))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^
(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1
/4*(c^(1/2)*d-a^(1/2)*e)^2/a^(1/2)/c^(1/2)/d/e,1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))/a^(1/4)/c^(1/4)/d^2/e/(c^(1/2)*d-a^(1/2)*e)/(a*e^2-b*d*e+c*d^2)/(c*x
^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.95 (sec) , antiderivative size = 1853, normalized size of antiderivative = 2.37

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-1/8*I)*((-4*I)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e^2*(B*d - A*e)*x*(a +
 b*x^2 + c*x^4) + Sqrt[2]*B*(b - Sqrt[b^2 - 4*a*c])*d^2*e*Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*(d + e*x^2)*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + Sqrt[2]*A*(-b + Sqrt[b^2 - 4*a*c])
*d*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*(EllipticE[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + 2*Sqrt[2]*B
*c*d^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticF[I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])] - 2*Sqrt[2]*A*c*d^2*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
(d + e*x^2)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x]
, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - 2*Sqrt[2]*B*c*d^3*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (...
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 727, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2210, 25, 2232, 27, 1509, 2226, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{2210}$$

$$-\frac{\int -\frac{ce(Bd-Ae)x^4+2cd(Bd-Ae)x^2+aBde+A(2cd^2-e(2bd-ae))}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{2d(ae^2-bde+cd^2)}-\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d(d+ex^2)(ae^2-bde+cd^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ce(Bd-Ae)x^4+2cd(Bd-Ae)x^2+2Acd^2+aBde-Ae(2bd-ae)}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{2d\left(ae^2-bde+cd^2\right)}-\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 2232

$$\frac{\frac{\int \frac{ce\left(\sqrt{c}(\sqrt{c}d+\sqrt{a}e)(Bd-Ae)x^2+\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(Bd+Ae)+2Ad(cd-be)\right)}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx}{ce}-\sqrt{a}\sqrt{c}(Bd-Ae)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{2d\left(ae^2-bde+cd^2\right)}-\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 27

$$\frac{\int \frac{\sqrt{c}(\sqrt{c}d+\sqrt{a}e)(Bd-Ae)x^2+\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(Bd+Ae)+2Ad(cd-be)}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}(Bd-Ae)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{2d\left(ae^2-bde+cd^2\right)}-\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1509

$$\frac{\int \frac{\sqrt{c}(\sqrt{c}d+\sqrt{a}e)(Bd-Ae)x^2+\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(Bd+Ae)+2Ad(cd-be)}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}(Bd-Ae)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2 a\right.}{\sqrt[4]{c}\sqrt{a+bx^2}}\right.}{2d\left(ae^2-bde+cd^2\right)}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 2226

$$\frac{\frac{\sqrt{a}\left(B\left(cd^3-ade^2\right)-Ae\left(3cd^2-e(2bd-ae)\right)\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2A\sqrt{c}\left(ae^2-bde+cd^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\sqrt{c}(Bd-Ae)\left(\sqrt[4]{}\right.}{2d\left(ae^2-bde+cd^2\right)}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{\left(B\left(cd^3-ade^2\right)-Ae\left(3cd^2-e(2bd-ae)\right)\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2A\sqrt{c}\left(ae^2-bde+cd^2\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\sqrt{c}(Bd-Ae)\left(\frac{\sqrt[4]{a}(\sqrt{a}\ldots}{\ldots}\right.}{2d\left(ae^2-bde+cd^2\right)}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1416

$$\frac{\frac{\left(B\left(cd^3-ade^2\right)-Ae\left(3cd^2-e(2bd-ae)\right)\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\sqrt{c}(Bd-Ae)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\ldots\right.\right.}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.}{2d\left(ae^2-b\ldots\right.}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 2220

$$\frac{\frac{\left(B\left(cd^3-ade^2\right)-Ae\left(3cd^2-e(2bd-ae)\right)\right)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}-\frac{(\sqrt{\ldots}}{\ldots}\right.}{\sqrt{c}d-\sqrt{a}e}}{\ldots}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*(e*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(d*(c*d^2 - b*d*e + a*e^2)*
(d + e*x^2)) + (-(Sqrt[c]*(B*d - A*e)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt
[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))) + (A*c^(
1/4)*(c*d^2 - b*d*e + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2
 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^
2 + c*x^4]) + ((B*(c*d^3 - a*d*e^2) - A*e*(3*c*d^2 - e*(2*b*d - a*e)))*(-1
/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d
]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e +
 a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[c]*d
)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + b*x^2 + c*x^4])))/(Sqr
t[c]*d - Sqrt[a]*e))/(2*d*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 2210

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

rule 2220

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2232

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + b*x^2
 + c*x^4], x], x] + Simp[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d -
 a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
&&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1494 vs. 2(654) = 1308.

Time = 3.07 (sec) , antiderivative size = 1495, normalized size of antiderivative = 1.91

method	result	size
default	Expression too large to display	1495
elliptic	Expression too large to display	2301

input
```
int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/e/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2
*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1
/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))+(A*e-B*d)/e*(1/2*e^2/(a
*e^2-b*d*e+c*d^2)/d*x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)-1/8*c/(a*e^2-b*d*e+c
*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*x^2-2/a*x^2*(-4
*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x
^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/(a*e^2-b*d*e+c*d^
2)/d*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*x^2-2/a*x^2*(-
4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*x^2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*
x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1
/4*c*e/(a*e^2-b*d*e+c*d^2)/d*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)
*(4+2*b/a*x^2-2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*x^2+2/a*x^2*(-4*a
*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*Elliptic
E(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1/2)/(-b/a+1/a*(
-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(...
```

Fricas [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(c*e^2*x^8 + (2*c*d*e + b*e^2
)*x^6 + (c*d^2 + 2*b*d*e + a*e^2)*x^4 + a*d^2 + (b*d^2 + 2*a*d*e)*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/((d + e*x**2)**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{(e\,x^2+d)^2\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx \\
&= \left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,e^2x^8+b\,e^2x^6+2cde\,x^6+a\,e^2x^4+2bde\,x^4+c\,d^2x^4+2ade\,x^2+b\,d^2x^2+a\,d^2}dx\right)a \\
&\quad+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,e^2x^8+b\,e^2x^6+2cde\,x^6+a\,e^2x^4+2bde\,x^4+c\,d^2x^4+2ade\,x^2+b\,d^2x^2+a\,d^2}dx\right)b
\end{aligned}$$

input
```
int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
int(sqrt(a + b*x**2 + c*x**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 + b*d**
2*x**2 + 2*b*d*e*x**4 + b*e**2*x**6 + c*d**2*x**4 + 2*c*d*e*x**6 + c*e**2*
x**8),x)*a + int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d**2 + 2*a*d*e*x**2 +
 a*e**2*x**4 + b*d**2*x**2 + 2*b*d*e*x**4 + b*e**2*x**6 + c*d**2*x**4 + 2*
c*d*e*x**6 + c*e**2*x**8),x)*b
```

3.173 $\int \frac{A+Bx^2}{\left(d+ex^2\right)^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 1125

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

output
```
-1/8*c^(1/2)*(3*A*e*(3*c*d^2-e*(-a*e+2*b*d))-B*d*(5*c*d^2-e*(a*e+2*b*d)))*
x*(c*x^4+b*x^2+a)^(1/2)/d^2/(a*e^2-b*d*e+c*d^2)^2/(a^(1/2)+c^(1/2)*x^2)-1/
4*e*(-A*e+B*d)*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^2+1
/8*e*(3*A*e*(3*c*d^2-e*(-a*e+2*b*d))-B*d*(5*c*d^2-e*(a*e+2*b*d)))*x*(c*x^4
+b*x^2+a)^(1/2)/d^2/(a*e^2-b*d*e+c*d^2)^2/(e*x^2+d)-1/16*(B*d*(3*c^2*d^4-1
0*a*c*d^2*e^2+a*e^3*(-a*e+4*b*d))-A*e*(15*c^2*d^4-2*c*d^2*e*(-3*a*e+10*b*d
)+e^2*(3*a^2*e^2-8*a*b*d*e+8*b^2*d^2)))*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x
/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^(5/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2
)^(5/2)+1/8*a^(1/4)*c^(1/4)*(3*A*e*(3*c*d^2-e*(-a*e+2*b*d))-B*d*(5*c*d^2-e
*(a*e+2*b*d)))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2
)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(
1/2))^(1/2))/d^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)^(1/2)+1/8*c^(1/4)*(
a^(1/2)*c^(1/2)*d*(-A*e+B*d)+a*e*(3*A*e+B*d)+4*A*d*(-b*e+c*d))*(a^(1/2)+c^
(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM
(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/d^2/
(c^(1/2)*d-a^(1/2)*e)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)+1/32*(c^(1
/2)*d+a^(1/2)*e)*(B*d*(3*c^2*d^4-10*a*c*d^2*e^2+a*e^3*(-a*e+4*b*d))-A*e*(1
5*c^2*d^4-2*c*d^2*e*(-3*a*e+10*b*d)+e^2*(3*a^2*e^2-8*a*b*d*e+8*b^2*d^2)))*
(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Elli
pticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(1/2)*d-a^(1/2)*e)^2/a^...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.09 (sec) , antiderivative size = 781, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{-\frac{4de^2x(a+bx^2+cx^4)\left(2d(Bd-Ae)\left(cd^2+e(-bd+ae)\right)+\left(-3Ae\left(3cd^2+e(-2bd+ae)\right)+B\left(5cd^3-de(2bd+ae)\right)\right)(d+ex^2)\right)}{(d+ex^2)^2} - \frac{i\sqrt{2}\sqrt{\frac{b+\sqrt{b^2-4ac+\cdots}}{b+\sqrt{b^2-4\cdots}}}\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-4*d*e^2*x*(a + b*x^2 + c*x^4)*(2*d*(B*d - A*e)*(c*d^2 + e*(-(b*d) + a*e
)) + (-3*A*e*(3*c*d^2 + e*(-2*b*d + a*e)) + B*(5*c*d^3 - d*e*(2*b*d + a*e)
))*(d + e*x^2)))/(d + e*x^2)^2 - (I*Sqrt[2]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*((-b + Sqrt[b^2 - 4*a*c])*d*e*(3*A*e*(3*c*d^2 + e*(-2*b*d + a*e)) + B*
(-5*c*d^3 + d*e*(2*b*d + a*e)))*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sq
rt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + d
*(B*d*(6*c^2*d^3 + c*d*e*(-5*b*d + 5*Sqrt[b^2 - 4*a*c]*d - 6*a*e) - (-b +
Sqrt[b^2 - 4*a*c])*e^2*(2*b*d + a*e)) - A*e*(14*c^2*d^3 - 3*(-b + Sqrt[b^2
 - 4*a*c])*e^2*(2*b*d - a*e) + c*d*e*(-17*b*d + 9*Sqrt[b^2 - 4*a*c]*d + 2*
a*e)))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + 2*(B*(-3*c^2*d^5 + 10*a*c*
d^3*e^2 + a*d*e^3*(-4*b*d + a*e)) + A*e*(15*c^2*d^4 + 2*c*d^2*e*(-10*b*d +
 3*a*e) + e^2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)))*EllipticPi[((b + Sqrt[
b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])])/(32*d^3*e*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + b*x^2 + c*
x^4])
```

Rubi [A] (verified)

Time = 3.59 (sec) , antiderivative size = 985, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2210, 25, 2210, 25, 2232, 27, 1509, 2226, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 2210}$$

$$-\frac{\int -\frac{-ce(Bd-Ae)x^4+2(Bd-Ae)(2cd-be)x^2+4Acd^2+aBde-Ae(4bd-3ae)}{(ex^2+d)^2\sqrt{cx^4+bx^2+a}}dx}{4d\left(ae^2-bde+cd^2\right)}-\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-ce(Bd-Ae)x^4+2(Bd-Ae)(2cd-be)x^2+4Acd^2+aBde-Ae(4bd-3ae)}{(ex^2+d)^2\sqrt{cx^4+bx^2+a}}dx}{4d\left(ae^2-bde+cd^2\right)}-\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2-bde+cd^2\right)}$$

↓ 2210

$$\frac{\frac{ex\sqrt{a+bx^2+cx^4}\left(3Ae\left(3cd^2-e(2bd-ae)\right)-B\left(5cd^3-de(ae+2bd)\right)\right)}{2d(d+ex^2)(ae^2-bde+cd^2)}-\frac{\int -\frac{-ce\left(3Ae\left(3cd^2-e(2bd-ae)\right)-B\left(5cd^3-de(2bd+ae)\right)\right)x^4-2cd\left(Ae\left(8cd^2-e(5b\right.\right.}{}}{}}{4d\left(ae^2-bde+cd^2\right)}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2-bde+cd^2\right)}$$

↓ 25

$$\frac{\frac{\int \frac{-ce\left(3Ae\left(3cd^2-e(2bd-ae)\right)-B\left(5cd^3-de(2bd+ae)\right)\right)x^4-2cd\left(Ae\left(8cd^2-e(5bd-2ae)\right)-B\left(4cd^3-de(bd+2ae)\right)\right)x^2+ade(Bd-Ae)(5cd-2be)+\left(4Acd^2+aBed-\right.}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}}{2d(ae^2-bde+cd^2)}}{4d\left(ae^2-bde+cd^2\right)}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2-bde+cd^2\right)}$$

↓ 2232

$$\sqrt{a}\sqrt{c}\left(3Ae\left(3cd^2-e(2bd-ae)\right)-B\left(5cd^3-de(ae+2bd)\right)\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx+\frac{\int \frac{ce\left(\left((cd-\sqrt{a}\sqrt{c}e)\left(3Ae\left(3cd^2-e(2bd-ae)\right)-B\left(5cd^3-de(2bd+ae)\right)\right)-2\right.\right.}{}}{}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2-bde+cd^2\right)}$$

↓ 27

$$\sqrt{c}\left(3Ae\left(3cd^2-e(2bd-ae)\right)-B\left(5cd^3-de(ae+2bd)\right)\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx+\int \frac{\left((cd-\sqrt{a}\sqrt{c}e)\left(3Ae\left(3cd^2-e(2bd-ae)\right)-B\left(5cd^3-de(2bd+ae)\right)\right)-2cd\left(Ae\left(8c\right.\right.\right.}{}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2-bde+cd^2\right)}$$

↓ 1509

$$\int \frac{\left((cd-\sqrt{a}\sqrt{c}e)\left(3Ae\left(3cd^2-e(2bd-ae)\right)-B\left(5cd^3-de(2bd+ae)\right)\right)-2cd\left(Ae\left(8cd^2-e(5bd-2ae)\right)-B\left(4cd^3-de(bd+2ae)\right)\right)\right)x^2+ade(Bd-Ae)(5cd-2be)+(4Acd^2}{(ex^2+d)\sqrt{cx^4+bx^2+a}}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2-bde+cd^2\right)}$$

$\big\downarrow$ 2226

$$\frac{\sqrt{a}\left(B\left(ade^3(4bd-ae)-10acd^3e^2+3c^2d^5\right)-Ae\left(e^2\left(3a^2e^2-8abde+8b^2d^2\right)-2cd^2e(10bd-3ae)+15c^2d^4\right)\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2\sqrt{c}\left(ae^2-bde+cd^2\right)(}{}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2-bde+cd^2\right)}$$

$\big\downarrow$ 27

$$\frac{\left(B\left(ade^3(4bd-ae)-10acd^3e^2+3c^2d^5\right)-Ae\left(e^2\left(3a^2e^2-8abde+8b^2d^2\right)-2cd^2e(10bd-3ae)+15c^2d^4\right)\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{2\sqrt{c}\left(ae^2-bde+cd^2\right)(\sqrt{a}\sqrt{c}}{}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2-bde+cd^2\right)}$$

$\big\downarrow$ 1416

$$\frac{\left(B\left(ade^3(4bd-ae)-10acd^3e^2+3c^2d^5\right)-Ae\left(e^2\left(3a^2e^2-8abde+8b^2d^2\right)-2cd^2e(10bd-3ae)+15c^2d^4\right)\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{}}}}{}$$

$$\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{4d\left(d+ex^2\right)^2\left(ae^2-bde+cd^2\right)}$$

$\big\downarrow$ 2220

$$\frac{\frac{e(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\sqrt{cx^4+bx^2+a}x}{2d(cd^2-bed+ae^2)(ex^2+d)}+\frac{\sqrt{c}(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\left(\frac{\sqrt[4]{a}(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{\cdots}{(\cdots}}}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{e(Bd-Ae)x\sqrt{cx^4+bx^2+a}}{4d\left(cd^2-bed+ae^2\right)\left(ex^2+d\right)^2}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/4*(e*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(d*(c*d^2 - b*d*e + a*e^2)*
(d + e*x^2)^2) + ((e*(3*A*e*(3*c*d^2 - e*(2*b*d - a*e)) - B*(5*c*d^3 - d*e
*(2*b*d + a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)*(
d + e*x^2)) + (Sqrt[c]*(3*A*e*(3*c*d^2 - e*(2*b*d - a*e)) - B*(5*c*d^3 - d
*e*(2*b*d + a*e)))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2))
 + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqr
t[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])) + (c^(1/4)*(c*d^2 - b*d*e +
a*e^2)*(Sqrt[a]*Sqrt[c]*d*(B*d - A*e) + a*e*(B*d + 3*A*e) + 4*A*d*(c*d - b
*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/
4])/(a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) + ((B*(3*c^2
*d^5 - 10*a*c*d^3*e^2 + a*d*e^3*(4*b*d - a*e)) - A*e*(15*c^2*d^4 - 2*c*d^2
*e*(10*b*d - 3*a*e) + e^2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)))*(-1/2*((Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[
e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2]
) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[
a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + b*x^2 + c*x^4])))/(Sqrt[c]...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2232
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + b*x^2
 + c*x^4], x], x] + Simp[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d -
 a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
&&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4475 vs. $2(989) = 1978$.

Time = 4.90 (sec) , antiderivative size = 4476, normalized size of antiderivative = 3.98

method	result	size
default	Expression too large to display	4476
elliptic	Expression too large to display	5671

input
```
int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/e*(1/2*e^2/(a*e^2-b*d*e+c*d^2)/d*x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)-1/8*c
/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*
x^2-2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*x^2+2/a*x^2*(-4*a*c+b^2)^(1
/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/(a
*e^2-b*d*e+c*d^2)/d*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a
*x^2-2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*x^2+2/a*x^2*(-4*a*c+b^2)^(
1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2))-1/4*c*e/(a*e^2-b*d*e+c*d^2)/d*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^
2)^(1/2))^(1/2)*(4+2*b/a*x^2-2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*x^
2+2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1
/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2*(-4*a*c+b
^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4
+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))*a-1/(a*e^2-b*d*e+c*d^2)/d*e*2^(1/
2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2*(-4*a*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/((d + e*x**2)**3*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^3}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^3}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{(e\,x^2+d)^3\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^3*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^3*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,e^3x^{10}+b\,e^3x^8+3cd\,e^2x^8+a\,e^3x^6+3bd\,e^2x^6+3c\,d^2e\,x^6+3ad\,e^2x^4+3b\,d^2e\,x^4+c\,d^3x^4+3a\,d^2e$$
$$+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,e^3x^{10}+b\,e^3x^8+3cd\,e^2x^8+a\,e^3x^6+3bd\,e^2x^6+3c\,d^2e\,x^6+3ad\,e^2x^4+3b\,d^2e\,x^4+c\,d^3x^4+3a\,d^2$$

input
```
int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4
+ a*e**3*x**6 + b*d**3*x**2 + 3*b*d**2*e*x**4 + 3*b*d*e**2*x**6 + b*e**3*x
**8 + c*d**3*x**4 + 3*c*d**2*e*x**6 + 3*c*d*e**2*x**8 + c*e**3*x**10),x)*a
 + int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*
e**2*x**4 + a*e**3*x**6 + b*d**3*x**2 + 3*b*d**2*e*x**4 + 3*b*d*e**2*x**6
+ b*e**3*x**8 + c*d**3*x**4 + 3*c*d**2*e*x**6 + 3*c*d*e**2*x**8 + c*e**3*x
**10),x)*b
```

3.174 $\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 859

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x(Ac(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2))+aB(ab^2e^3+2ace(3cd^2\cdots}{\cdots}$$

$$+\frac{Be^3x\sqrt{a+bx^2+cx^4}}{3c^2}$$

$$+\frac{(aB(6c^3d^3-8b^3e^3-9c^2de(bd+6ae)+bce^2(18bd+29ae))+3Ac(2ab^2e^3+6ace(cd^2-ae^2)-bcd(cd^2+\cdots}{3ac^{5/2}(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\cdots}$$

$$-\frac{(aB(6c^3d^3-8b^3e^3-9c^2de(bd+6ae)+bce^2(18bd+29ae))+3Ac(2ab^2e^3+6ace(cd^2-ae^2)-bcd(cd^2\cdots}{3a^{3/4}c^{11/4}(b^2-4ac)\sqrt{a+bx^2+c\cdots}}$$

$$-\frac{\left(3Ac^3d^3-5a^2Bce^3-3\sqrt{a}c^{5/2}d^2(Bd+3Ae)+ae(3cd-2be)(3Bcd-4bBe+3Ace)+3a^{3/2}\sqrt{c}e^2(9Bcd\cdots\right.}{6a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)c^{11/4}\sqrt{\cdots}}$$

output
```
x*(A*c*(b^2*c*d^3-2*a*c*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2))+a*B*(a*b
^2*e^3+2*a*c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))-(a*B*(-b*e+2*c*d)*(
c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))+A*c*(a*b^2*e^3+2*a*c*e*(-a*e^2+3*c*d^2)-b
*c*d*(3*a*e^2+c*d^2)))*x^2)/a/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/3*B
*e^3*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/3*(a*B*(6*c^3*d^3-8*b^3*e^3-9*c^2*d*e*(
6*a*e+b*d)+b*c*e^2*(29*a*e+18*b*d))+3*A*c*(2*a*b^2*e^3+6*a*c*e*(-a*e^2+c*d
^2)-b*c*d*(3*a*e^2+c*d^2)))*x*(c*x^4+b*x^2+a)^(1/2)/a/c^(5/2)/(-4*a*c+b^2)
/(a^(1/2)+c^(1/2)*x^2)-1/3*(a*B*(6*c^3*d^3-8*b^3*e^3-9*c^2*d*e*(6*a*e+b*d)
+b*c*e^2*(29*a*e+18*b*d))+3*A*c*(2*a*b^2*e^3+6*a*c*e*(-a*e^2+c*d^2)-b*c*d*
(3*a*e^2+c*d^2)))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*
x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/
c^(1/2))^(1/2))/a^(3/4)/c^(11/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/6*(3
*A*c^3*d^3-5*a^2*B*c*e^3-3*a^(1/2)*c^(5/2)*d^2*(3*A*e+B*d)+a*e*(-2*b*e+3*c
*d)*(3*A*c*e-4*B*b*e+3*B*c*d)+3*a^(3/2)*c^(1/2)*e^2*(3*A*c*e-4*B*b*e+9*B*c
*d))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)
*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/
2))/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.63 (sec) , antiderivative size = 5432, normalized size of antiderivative = 6.32

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 765, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2206, 2207, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2206}$$

$$x\left(-\left(x^2\left(Ac\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)+aB(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\right)\right)+A\ldots$$

$$ac^2\left(b^2-4ac\right)\sqrt{a-\ldots}$$

$$\int \frac{aB\left(4a-\frac{b^2}{c}\right)e^3x^4-\frac{\left(aB\left(2c^3d^3-3c^2e(bd+6ae)d-2b^3e^3+bce^2(6bd+7ae)\right)+Ac\left(2ab^2e^3+6ac\left(cd^2-ae^2\right)e-bcd\left(cd^2+3ae^2\right)\right)\right)x^2}{c^2}+\frac{a\left(ab^2Be^3-bc\left(Bcd^3+3A\ldots\right.\right.}{\ldots}}{\sqrt{cx^4+bx^2+a}}$$

$$a\left(b^2-4ac\right)$$

$$\Big\downarrow \text{ 2207}$$

$$x\left(-\left(x^2\left(Ac\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)+aB(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\right)\right)+A\ldots$$

$$ac^2\left(b^2-4ac\right)\sqrt{a-\ldots}$$

$$\frac{\int \frac{a\left(4ab^2Be^3-3bc\left(Bcd^3+3Aced^2+3aBe^2d+aAe^3\right)+2c\left(aBe\left(9cd^2-5ae^2\right)+3Acd\left(cd^2+3ae^2\right)\right)\right)-\left(aB\left(6c^3d^3-9c^2e(bd+6ae)d-8b^3e^3+bce^2(18bd+29ae)\right)+3A\ldots\right.}{c\sqrt{cx^4+bx^2+a}}}{3c}$$

$$a\left(b^2-4ac\right)$$

$$\Big\downarrow \text{ 27}$$

$$x\left(-\left(x^2\left(Ac\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)+aB(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\right)\right)+A\ldots$$

$$ac^2\left(b^2-4ac\right)\sqrt{a-\ldots}$$

$$\frac{\int \frac{a\left(4ab^2Be^3-3bc\left(Bcd^3+3Aced^2+3aBe^2d+aAe^3\right)+2c\left(aBe\left(9cd^2-5ae^2\right)+3Acd\left(cd^2+3ae^2\right)\right)\right)-\left(aB\left(6c^3d^3-9c^2e(bd+6ae)d-8b^3e^3+bce^2(18bd+29ae)\right)+3A\ldots\right.}{\sqrt{cx^4+bx^2+a}}}{3c^2}$$

$$a\left(b^2-4ac\right)$$

$$\Big\downarrow \text{ 1511}$$

$$x\left(-\left(x^2\left(Ac\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)+aB(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\right)\right)+A$$
$$ac^2\left(b^2-4ac\right)\sqrt{a}-$$

$$\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(3a^{3/2}\sqrt{c}e^2(3Ace-4bBe+9Bcd)-5a^2Bce^3+ae(3cd-2be)(3Ace-4bBe+3Bcd)-3\sqrt{a}c^{5/2}d^2(3Ae+Bd)+3Ac^3d^3\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\sqrt{a}\left(3Ac\left(2\right.\right.}{}$$
$$3c^2$$
$$a\left(b^2\right.$$

$\downarrow$ 27

$$x\left(-\left(x^2\left(Ac\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)+aB(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\right)\right)+A$$
$$ac^2\left(b^2-4ac\right)\sqrt{a}-$$

$$\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(3a^{3/2}\sqrt{c}e^2(3Ace-4bBe+9Bcd)-5a^2Bce^3+ae(3cd-2be)(3Ace-4bBe+3Bcd)-3\sqrt{a}c^{5/2}d^2(3Ae+Bd)+3Ac^3d^3\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\left(3Ac\left(2ab\right.\right.}{}$$
$$3c^2$$
$$a\left(b^2-\right.$$

$\downarrow$ 1416

$$x\left(-\left(x^2\left(Ac\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)+aB(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\right)\right)+A$$
$$ac^2\left(b^2-4ac\right)\sqrt{a}-$$

$$\frac{\left(3Ac\left(2ab^2e^3-bcd\left(3ae^2+cd^2\right)+6ace\left(cd^2-ae^2\right)\right)+aB\left(-9c^2de(6ae+bd)+bce^2(29ae+18bd)-8b^3e^3+6c^3d^3\right)\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right.}{}$$

$\downarrow$ 1509

$$x\left(-\left(x^2\left(Ac\left(ab^2e^3-bcd\left(3ae^2+cd^2\right)+2ace\left(3cd^2-ae^2\right)\right)+aB(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\right)\right)+A$$
$$ac^2\left(b^2-4ac\right)\sqrt{a}-$$

$$\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(3a^{3/2}\sqrt{c}e^2(3Ace-4bBe+9Bcd)-5a^2Bce^3+ae(3cd-2be)(3Ace-4\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(A*c*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2))
+ a*B*(a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2)) -
(a*B*(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)) + A*c*(a*b^2*e^
3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2)))*x^2))/(a*c^2*(b^
2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (-1/3*(a*B*(b^2 - 4*a*c)*e^3*x*Sqrt[
a + b*x^2 + c*x^4])/c^2 + (((a*B*(6*c^3*d^3 - 8*b^3*e^3 - 9*c^2*d*e*(b*d +
 6*a*e) + b*c*e^2*(18*b*d + 29*a*e)) + 3*A*c*(2*a*b^2*e^3 + 6*a*c*e*(c*d^2
 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2)))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt
[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]
+ (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(3*A*c^3*d^3 - 5*a^2*B*c*e^3 - 3*Sqrt[a
]*c^(5/2)*d^2*(B*d + 3*A*e) + a*e*(3*c*d - 2*b*e)*(3*B*c*d - 4*b*B*e + 3*A
*c*e) + 3*a^(3/2)*Sqrt[c]*e^2*(9*B*c*d - 4*b*B*e + 3*A*c*e))*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqr
t[a + b*x^2 + c*x^4]))/(3*c^2))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 12.68 (sec) , antiderivative size = 1141, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	1141
default	Expression too large to display	2436
risch	Expression too large to display	2472

input
```
int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2/c^3*(2*A*a^2*c^2*e^3-A*a*b^2*c*e^3+3*A*a*b*c^2*d*e^2-6*A*a*c^3*d
^2*e+A*b*c^3*d^3-3*B*a^2*b*c*e^3+6*B*a^2*c^2*d*e^2+B*a*b^3*e^3-3*B*a*b^2*c
*d*e^2+3*B*a*b*c^2*d^2*e-2*B*a*c^3*d^3)/a/(4*a*c-b^2)*x^3-1/2/c^3*(A*a^2*b
*c*e^3-6*A*a^2*c^2*d*e^2+3*A*a*b*c^2*d^2*e+2*A*a*c^3*d^3-A*b^2*c^2*d^3+2*B
*a^3*c*e^3-B*a^2*b^2*e^3+3*B*a^2*b*c*d*e^2-6*B*a^2*c^2*d^2*e+B*a*b*c^2*d^3
)/a/(4*a*c-b^2)*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)+1/3*B*e^3*x*(c*x^4+b*x^2+a)
^(1/2)/c^2+1/4*(-e*(A*b*c*e^2-3*A*c^2*d*e+B*a*c*e^2-B*b^2*e^2+3*B*b*c*d*e-
3*B*c^2*d^2)/c^3+1/c^3*(A*a*b*c*e^3-3*A*a*c^2*d*e^2+A*c^3*d^3+B*a^2*c*e^3-
B*a*b^2*e^3+3*B*a*b*c*d*e^2-3*B*a*c^2*d^2*e)/a-1/c^2*(A*a^2*b*c*e^3-6*A*a^
2*c^2*d*e^2+3*A*a*b*c^2*d^2*e+2*A*a*c^3*d^3-A*b^2*c^2*d^3+2*B*a^3*c*e^3-B*
a^2*b^2*e^3+3*B*a^2*b*c*d*e^2-6*B*a^2*c^2*d^2*e+B*a*b*c^2*d^3)/a/(4*a*c-b^
2)-1/3*B*e^3/c^2*a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(e^2/c^2*(A*c*e-B*b*
e+3*B*c*d)+1/c^2*(2*A*a^2*c^2*e^3-A*a*b^2*c*e^3+3*A*a*b*c^2*d*e^2-6*A*a*c^
3*d^2*e+A*b*c^3*d^3-3*B*a^2*b*c*e^3+6*B*a^2*c^2*d*e^2+B*a*b^3*e^3-3*B*a*b^
2*c*d*e^2+3*B*a*b*c^2*d^2*e-2*B*a*c^3*d^3)/a/(4*a*c-b^2)-2/3*B/c^2*e^3*b)*
a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2623 vs. $2(780) = 1560$.

Time = 0.12 (sec) , antiderivative size = 2623, normalized size of antiderivative = 3.05

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
-1/6*(sqrt(1/2)*((3*(2*B*a*b - A*b^2)*c^4*d^3 - 9*(B*a*b^2*c^3 - 2*A*a*b*c
^4)*d^2*e + 9*(2*B*a*b^3*c^2 - (6*B*a^2*b + A*a*b^2)*c^3)*d*e^2 - (8*B*a*b
^4*c + 18*A*a^2*b*c^3 - (29*B*a^2*b^2 + 6*A*a*b^3)*c^2)*e^3)*x^5 + (3*(2*B
*a*b^2 - A*b^3)*c^3*d^3 - 9*(B*a*b^3*c^2 - 2*A*a*b^2*c^3)*d^2*e + 9*(2*B*a
*b^4*c - (6*B*a^2*b^2 + A*a*b^3)*c^2)*d*e^2 - (8*B*a*b^5 + 18*A*a^2*b^2*c^
2 - (29*B*a^2*b^3 + 6*A*a*b^4)*c)*e^3)*x^3 + (3*(2*B*a^2*b - A*a*b^2)*c^3*
d^3 - 9*(B*a^2*b^2*c^2 - 2*A*a^2*b*c^3)*d^2*e + 9*(2*B*a^2*b^3*c - (6*B*a^
3*b + A*a^2*b^2)*c^2)*d*e^2 - (8*B*a^2*b^4 + 18*A*a^3*b*c^2 - (29*B*a^3*b^
2 + 6*A*a^2*b^3)*c)*e^3)*x - ((3*(2*B*a - A*b)*c^5*d^3 - 9*(B*a*b*c^4 - 2*
A*a*c^5)*d^2*e + 9*(2*B*a*b^2*c^3 - (6*B*a^2 + A*a*b)*c^4)*d*e^2 - (8*B*a*
b^3*c^2 + 18*A*a^2*c^4 - (29*B*a^2*b + 6*A*a*b^2)*c^3)*e^3)*x^5 + (3*(2*B*
a*b - A*b^2)*c^4*d^3 - 9*(B*a*b^2*c^3 - 2*A*a*b*c^4)*d^2*e + 9*(2*B*a*b^3*
c^2 - (6*B*a^2*b + A*a*b^2)*c^3)*d*e^2 - (8*B*a*b^4*c + 18*A*a^2*b*c^3 - (
29*B*a^2*b^2 + 6*A*a*b^3)*c^2)*e^3)*x^3 + (3*(2*B*a^2 - A*a*b)*c^4*d^3 - 9
*(B*a^2*b*c^3 - 2*A*a^2*c^4)*d^2*e + 9*(2*B*a^2*b^2*c^2 - (6*B*a^3 + A*a^2
*b)*c^3)*d*e^2 - (8*B*a^2*b^3*c + 18*A*a^3*c^3 - (29*B*a^3*b + 6*A*a^2*b^2
)*c^2)*e^3)*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)
/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2)
- b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + sqrt(
1/2)*((3*(2*A*b*c^5 - (2*B*a*b - (A - B)*b^2)*c^4)*d^3 + 9*(B*a*b^2*c^3...
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)\,(d+ex^2)^3}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**3/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(e\,x^2+d\right)^3}{\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(11*sqrt(a + b*x**2 + c*x**4)*a*b*c*e**3*x - 9*sqrt(a + b*x**2 + c*x**4)*a
*c**2*d*e**2*x + 3*sqrt(a + b*x**2 + c*x**4)*a*c**2*e**3*x**3 - 8*sqrt(a +
 b*x**2 + c*x**4)*b**3*e**3*x + 18*sqrt(a + b*x**2 + c*x**4)*b**2*c*d*e**2
*x - 4*sqrt(a + b*x**2 + c*x**4)*b**2*c*e**3*x**3 - 9*sqrt(a + b*x**2 + c*
x**4)*b*c**2*d**2*e*x + 9*sqrt(a + b*x**2 + c*x**4)*b*c**2*d*e**2*x**3 + s
qrt(a + b*x**2 + c*x**4)*b*c**2*e**3*x**5 - 11*int(sqrt(a + b*x**2 + c*x**
4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x
)*a**3*b*c*e**3 + 9*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a
*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**3*c**2*d*e**2 + 8*int(
sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*
b*c*x**6 + c**2*x**8),x)*a**2*b**3*e**3 - 18*int(sqrt(a + b*x**2 + c*x**4)
/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*
a**2*b**2*c*d*e**2 - 11*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 +
 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*b**2*c*e**3*x**2
 + 9*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*
x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*b*c**2*d**2*e + 9*int(sqrt(a + b*x*
*2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c*
*2*x**8),x)*a**2*b*c**2*d*e**2*x**2 - 11*int(sqrt(a + b*x**2 + c*x**4)/(a*
*2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2
*b*c**2*e**3*x**4 + 3*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 ...
```

3.175 $\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 633

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{x\left(c\left(\frac{aB(bcd^2-4acde+abe^2)}{c}-A(b^2d^2-2abde-2a(cd^2-ae^2))\right)-(Ac(bcd^2-4acde+abe^2)-aB(2c^2d^2+\ldots}{ac(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$-\frac{(Ac(bcd^2-4acde+abe^2)-2aB(c^2d^2+b^2e^2-ce(bd+3ae)))x\sqrt{a+bx^2+cx^4}}{ac^{3/2}(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)}$$

$$+\frac{(Ac(bcd^2-4acde+abe^2)-2aB(c^2d^2+b^2e^2-ce(bd+3ae)))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\ldots\right.\right.}{a^{3/4}c^{7/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$-\frac{(Ac^2d^2+3a^{3/2}B\sqrt{c}e^2-\sqrt{a}c^{3/2}d(Bd+2Ae)+ae(2Bcd-2bBe+Ace))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{El}\ldots}{2a^{3/4}(b-2\sqrt{a}\sqrt{c})c^{7/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-x*(c*(a*B*(a*b*e^2-4*a*c*d*e+b*c*d^2)/c-A*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+
c*d^2)))-(A*c*(a*b*e^2-4*a*c*d*e+b*c*d^2)-a*B*(2*c^2*d^2+b^2*e^2-2*c*e*(a*
e+b*d)))*x^2)/a/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-(A*c*(a*b*e^2-4*a*c*d
*e+b*c*d^2)-2*a*B*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d)))*x*(c*x^4+b*x^2+a)^(1/
2)/a/c^(3/2)/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)+(A*c*(a*b*e^2-4*a*c*d*e+b*
c*d^2)-2*a*B*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d)))*(a^(1/2)+c^(1/2)*x^2)*((c*
x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)
*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/c^(7/4)/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)^(1/2)-1/2*(A*c^2*d^2+3*a^(3/2)*B*c^(1/2)*e^2-a^(1/2)*c^(3
/2)*d*(2*A*e+B*d)+a*e*(A*c*e-2*B*b*e+2*B*c*d))*(a^(1/2)+c^(1/2)*x^2)*((c*x
^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4
)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b-2*a^(1/2)*c^(1/2)
)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.85 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(-aB(abe^2+2c^2d^2x^2+b^2e^2x^2+bcd(d-2ex^2)-2ace(2d}{}$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-(a*B*(a*b*e^2 + 2*c^2*d^2*x^2 +
b^2*e^2*x^2 + b*c*d*(d - 2*e*x^2) - 2*a*c*e*(2*d + e*x^2))) + A*c*(b^2*d^2
 + 2*a^2*e^2 + b*c*d^2*x^2 + a*b*e*(-2*d + e*x^2) - 2*a*c*d*(d + 2*e*x^2))
) - I*(-b + Sqrt[b^2 - 4*a*c])*(-(A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2)) + 2
*a*B*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))*Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*
x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(
2*a*B*(b^2*(-b + Sqrt[b^2 - 4*a*c])*e^2 + c^2*d*(Sqrt[b^2 - 4*a*c]*d - 4*a
*e) + c*e*(b^2*d - b*Sqrt[b^2 - 4*a*c]*d + 4*a*b*e - 3*a*Sqrt[b^2 - 4*a*c]
*e)) + A*c*(b^2*(c*d^2 + a*e^2) - b*Sqrt[b^2 - 4*a*c]*(c*d^2 + a*e^2) - 4*
a*c*(c*d^2 - Sqrt[b^2 - 4*a*c]*d*e + a*e^2)))*Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*
x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(4*a
*c^2*(-b^2 + 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4
])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2206, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 2206}$$

$$-\frac{\int \frac{(Ac(bcd^2-4aced+abe^2)-2aB(c^2d^2+b^2e^2-ce(bd+3ae)))x^2+a(ae(4Bcd-bBe+2Ace)+cd(2Acd-b(Bd+2Ae)))}{c\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}-$$

$$\frac{x\Big(c\Big(\frac{aB(abe^2-4acde+bcd^2)}{c}-A(-2abde-2a(cd^2-ae^2)+b^2d^2)\Big)-x^2(Ac(abe^2-4acde+bcd^2)-aB(-2ce(ae}{ac(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{(Ac(bcd^2-4aced+abe^2)-2aB(c^2d^2+b^2e^2-ce(bd+3ae)))x^2+a(ae(4Bcd-bBe+2Ace)+cd(2Acd-b(Bd+2Ae)))}{\sqrt{cx^4+bx^2+a}}dx}{ac(b^2-4ac)}-$$

$$\frac{x\left(c\left(\frac{aB(abe^2-4acde+bcd^2)}{c}-A(-2abde-2a(cd^2-ae^2)+b^2d^2)\right)-x^2\left(Ac(abe^2-4acde+bcd^2)-aB(-2ce(ae\right.\right.}{ac(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 1511

$$-\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(3a^{3/2}B\sqrt{c}e^2+ae(Ace-2bBe+2Bcd)-\sqrt{a}c^{3/2}d(2Ae+Bd)+Ac^2d^2)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}(Ac(abe^2-4acde+bcd^2)-2aB(}{}}{ac(b^2-4ac)}$$

$$\frac{x\left(c\left(\frac{aB(abe^2-4acde+bcd^2)}{c}-A(-2abde-2a(cd^2-ae^2)+b^2d^2)\right)-x^2\left(Ac(abe^2-4acde+bcd^2)-aB(-2ce(ae\right.\right.}{ac(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 27

$$-\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(3a^{3/2}B\sqrt{c}e^2+ae(Ace-2bBe+2Bcd)-\sqrt{a}c^{3/2}d(2Ae+Bd)+Ac^2d^2)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{(Ac(abe^2-4acde+bcd^2)-2aB(-ce}{}}{ac(b^2-4ac)}$$

$$\frac{x\left(c\left(\frac{aB(abe^2-4acde+bcd^2)}{c}-A(-2abde-2a(cd^2-ae^2)+b^2d^2)\right)-x^2\left(Ac(abe^2-4acde+bcd^2)-aB(-2ce(ae\right.\right.}{ac(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 1416

$$-\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)(3a^{3/2}B\sqrt{c}e^2+ae(Ace-2bBe+2Bcd)-\sqrt{a}c^{3/2}d(2A}{2c^{3/4}\sqrt{a+bx^2+cx^4}}}{ac(b^2-4ac)}$$

$$\frac{x\left(c\left(\frac{aB(abe^2-4acde+bcd^2)}{c}-A(-2abde-2a(cd^2-ae^2)+b^2d^2)\right)-x^2\left(Ac(abe^2-4acde+bcd^2)-aB(-2ce(ae\right.\right.}{ac(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$\downarrow$ 1509

$$-\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)(3a^{3/2}B\sqrt{c}e^2+ae(Ace-2bBe+2Bcd)-\sqrt{a}c^{3/2}d(2A}{2c^{3/4}\sqrt{a+bx^2+cx^4}}$$

$$\frac{x\left(c\left(\frac{aB(abe^2-4acde+bcd^2)}{c}-A(-2abde-2a(cd^2-ae^2)+b^2d^2)\right)-x^2\left(Ac(abe^2-4acde+bcd^2)-aB(-2ce(ae\right.\right.}{ac(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-((x*(c*((a*B*(b*c*d^2 - 4*a*c*d*e + a*b*e^2))/c - A*(b^2*d^2 - 2*a*b*d*e
- 2*a*(c*d^2 - a*e^2))) - (A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - a*B*(2*c^
2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)))*x^2))/(a*c*(b^2 - 4*a*c)*Sqrt[a + b*
x^2 + c*x^4])) - (-(((A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - 2*a*B*(c^2*d^2
 + b^2*e^2 - c*e*(b*d + 3*a*e)))*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] +
 Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a
^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(A*c^2*d^2 + 3*a^(3/2)*B*Sqrt[c]*e^2 - Sqrt
[a]*c^(3/2)*d*(B*d + 2*A*e) + a*e*(2*B*c*d - 2*b*B*e + A*c*e))*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*S
qrt[a + b*x^2 + c*x^4]))/(a*c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 799, normalized size of antiderivative = 1.26

method	result
elliptic	$-\frac{2c\left(\frac{\left(Aabc\,e^{2}-4Aa\,c^{2}de+Ab\,c^{2}d^{2}+2B\,a^{2}c\,e^{2}-Ba\,b^{2}e^{2}+2Bbcdea-2Ba\,c^{2}d^{2}\right)x^{3}}{2c^{2}a\left(4ac-b^{2}\right)}+\frac{\left(2A\,a^{2}c\,e^{2}-2Aabcde-2A\,c^{2}d^{2}a+A\,b^{2}c\,d^{2}-B\,a^{2}b\,e^{2}+4B\right.}{2c^{2}\left(4ac-b^{2}\right)a}\right.}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}$
default	Expression too large to display

input
```
int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2/c^2*(A*a*b*c*e^2-4*A*a*c^2*d*e+A*b*c^2*d^2+2*B*a^2*c*e^2-B*a*b^2
*e^2+2*B*a*b*c*d*e-2*B*a*c^2*d^2)/a/(4*a*c-b^2)*x^3+1/2*(2*A*a^2*c*e^2-2*A
*a*b*c*d*e-2*A*a*c^2*d^2+A*b^2*c*d^2-B*a^2*b*e^2+4*B*a^2*c*d*e-B*a*b*c*d^2
)/c^2/(4*a*c-b^2)/a*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)+1/4*(e*(A*c*e-B*b*e+2*B
*c*d)/c^2-1/c^2*(A*a*c*e^2-A*c^2*d^2-B*a*b*e^2+2*B*a*c*d*e)/a+1/a*(2*A*a^2
*c*e^2-2*A*a*b*c*d*e-2*A*a*c^2*d^2+A*b^2*c*d^2-B*a^2*b*e^2+4*B*a^2*c*d*e-B
*a*b*c*d^2)/(4*a*c-b^2)/c)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*
(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(B*e^2/c+1/a*
(A*a*b*c*e^2-4*A*a*c^2*d*e+A*b*c^2*d^2+2*B*a^2*c*e^2-B*a*b^2*e^2+2*B*a*b*c
*d*e-2*B*a*c^2*d^2)/(4*a*c-b^2)/c)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1669 vs. $2(564) = 1128$.

Time = 0.10 (sec) , antiderivative size = 1669, normalized size of antiderivative = 2.64

$$\int \frac{(A + Bx^2)(d + ex^2)^2}{(a + bx^2 + cx^4)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output

```
-1/2*(sqrt(1/2)*(((2*B*a*b - A*b^2)*c^3*d^2 - 2*(B*a*b^2*c^2 - 2*A*a*b*c^3
)*d*e + (2*B*a*b^3*c - (6*B*a^2*b + A*a*b^2)*c^2)*e^2)*x^5 + ((2*B*a*b^2 -
 A*b^3)*c^2*d^2 - 2*(B*a*b^3*c - 2*A*a*b^2*c^2)*d*e + (2*B*a*b^4 - (6*B*a^
2*b^2 + A*a*b^3)*c)*e^2)*x^3 + ((2*B*a^2*b - A*a*b^2)*c^2*d^2 - 2*(B*a^2*b
^2*c - 2*A*a^2*b*c^2)*d*e + (2*B*a^2*b^3 - (6*B*a^3*b + A*a^2*b^2)*c)*e^2)
*x - (((2*B*a - A*b)*c^4*d^2 - 2*(B*a*b*c^3 - 2*A*a*c^4)*d*e + (2*B*a*b^2*
c^2 - (6*B*a^2 + A*a*b)*c^3)*e^2)*x^5 + ((2*B*a*b - A*b^2)*c^3*d^2 - 2*(B*
a*b^2*c^2 - 2*A*a*b*c^3)*d*e + (2*B*a*b^3*c - (6*B*a^2*b + A*a*b^2)*c^2)*e
^2)*x^3 + ((2*B*a^2 - A*a*b)*c^3*d^2 - 2*(B*a^2*b*c^2 - 2*A*a^2*c^3)*d*e +
 (2*B*a^2*b^2*c - (6*B*a^3 + A*a^2*b)*c^2)*e^2)*x)*sqrt((b^2 - 4*a*c)/c^2)
)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1
/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c
)/c^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*(((2*A*b*c^4 - (2*B*a*b - (A - B)
*b^2)*c^3)*d^2 + 2*(B*a*b^2*c^2 - (2*(A - B)*a*b + A*b^2)*c^3)*d*e - (2*B*
a*b^3*c - 2*A*a*b*c^3 - (6*B*a^2*b + (A - B)*a*b^2)*c^2)*e^2)*x^5 + ((2*A*
b^2*c^3 - (2*B*a*b^2 - (A - B)*b^3)*c^2)*d^2 + 2*(B*a*b^3*c - (2*(A - B)*a
*b^2 + A*b^3)*c^2)*d*e - (2*B*a*b^4 - 2*A*a*b^2*c^2 - (6*B*a^2*b^2 + (A -
B)*a*b^3)*c)*e^2)*x^3 + ((2*A*a*b*c^3 - (2*B*a^2*b - (A - B)*a*b^2)*c^2)*d
^2 + 2*(B*a^2*b^2*c - (2*(A - B)*a^2*b + A*a*b^2)*c^2)*d*e - (2*B*a^2*b^3
- 2*A*a^2*b*c^2 - (6*B*a^3*b + (A - B)*a^2*b^2)*c)*e^2)*x + (((2*A*c^5 ...
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input

```
integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+b*x**2+a)**(3/2),x)
```

output

```
Integral((A + B*x**2)*(d + e*x**2)**2/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^2}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4)*a*c*e**2*x + 2*sqrt(a + b*x**2 + c*x**4)*b**
2*e**2*x - 2*sqrt(a + b*x**2 + c*x**4)*b*c*d*e*x + sqrt(a + b*x**2 + c*x**
4)*b*c*e**2*x**3 + int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*
c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**3*c*e**2 - 2*int(sqrt(a
 + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x*
*6 + c**2*x**8),x)*a**2*b**2*e**2 + 2*int(sqrt(a + b*x**2 + c*x**4)/(a**2
+ 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*b*
c*d*e + int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b*
*2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*b*c*e**2*x**2 + int(sqrt(a + b*x
**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c
**2*x**8),x)*a**2*c**2*d**2 + int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*
x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*c**2*e**2*
x**4 - 2*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b
**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b**3*e**2*x**2 + 2*int(sqrt(a + b*
x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 +
c**2*x**8),x)*a*b**2*c*d*e*x**2 - 2*int(sqrt(a + b*x**2 + c*x**4)/(a**2 +
2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b**2*c*
e**2*x**4 + int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4
+ b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b*c**2*d**2*x**2 + 2*int(sqrt(a
 + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c...
```

3.176 $\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 481

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{x(aB(bd-2ae)-A(b^2d-2acd-abe)-(Ac(bd-2ae)-aB(2cd-be))x^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{(Ac(bd-2ae)-aB(2cd-be))x\sqrt{a+bx^2+cx^4}}{a\sqrt{c}(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)} + \frac{(Ac(bd-2ae)-aB(2cd-be))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}c^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{(\sqrt{a}B-A\sqrt{c})(\sqrt{c}d-\sqrt{a}e)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}(b-2\sqrt{a}\sqrt{c})c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-x*(a*B*(-2*a*e+b*d)-A*(-a*b*e-2*a*c*d+b^2*d)-(A*c*(-2*a*e+b*d)-a*B*(-b*e+
2*c*d))*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-(A*c*(-2*a*e+b*d)-a*B*(-
b*e+2*c*d))*x*(c*x^4+b*x^2+a)^(1/2)/a/c^(1/2)/(-4*a*c+b^2)/(a^(1/2)+c^(1/2
)*x^2)+(A*c*(-2*a*e+b*d)-a*B*(-b*e+2*c*d))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b
*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/c^(3/4)/(-4*a*c+b^2)/(c*x
^4+b*x^2+a)^(1/2)+1/2*(a^(1/2)*B-A*c^(1/2))*(c^(1/2)*d-a^(1/2)*e)*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacob
iAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(
b-2*a^(1/2)*c^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.77 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(aB(-2ae+2cdx^2+b(d-ex^2))+A(-b^2d+b(ae-cdx^2)+\cdots}{\cdots}$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a*B*(-2*a*e + 2*c*d*x^2 + b*(d - e
*x^2)) + A*(-(b^2*d) + b*(a*e - c*d*x^2) + 2*a*c*(d + e*x^2))) + I*(-b + S
qrt[b^2 - 4*a*c])*(A*c*(b*d - 2*a*e) + a*B*(-2*c*d + b*e))*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 -
4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])] - I*(A*c*(-(b^2*d) + 4*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*Sqrt[b^
2 - 4*a*c]*e) + a*B*(b*(-b + Sqrt[b^2 - 4*a*c])*e + c*(-2*Sqrt[b^2 - 4*a*c
]*d + 4*a*e)))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*E
llipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(4*a*c*(-b^2 + 4*a*c)*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2206, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 2206}$$

$$-\frac{\int -\frac{a(bBd-2Acd+Abe-2aBe)-(Ac(bd-2ae)-aB(2cd-be))x^2}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}-\frac{x\left(-A\left(-abe-2acd+b^2d\right)-\left(x^2(Ac(bd-2ae)-aB(2cd-be))\right)+aB(bd-2ae)\right)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{a(bBd-2Acd+Abe-2aBe)-(Ac(bd-2ae)-aB(2cd-be))x^2}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}-\frac{x\left(-A\left(-abe-2acd+b^2d\right)-\left(x^2(Ac(bd-2ae)-aB(2cd-be))\right)+aB(bd-2ae)\right)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}B-A\sqrt{c})(\sqrt{c}d-\sqrt{a}e)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\sqrt{a}(-2aAce+abBe-2aBcd+Abcd)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{a(b^2-4ac)}-\frac{x\left(-A\left(-abe-2acd+b^2d\right)-\left(x^2(Ac(bd-2ae)-aB(2cd-be))\right)+aB(bd-2ae)\right)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}B-A\sqrt{c})(\sqrt{c}d-\sqrt{a}e)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{(-2aAce+abBe-2aBcd+Abcd)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{a(b^2-4ac)}-\frac{x\left(-A\left(-abe-2acd+b^2d\right)-\left(x^2(Ac(bd-2ae)-aB(2cd-be))\right)+aB(bd-2ae)\right)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\frac{(-2aAce+abBe-2aBcd+Abcd)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(2\,\text{arcta}\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(-A\left(-abe-2acd+b^2d\right)-\left(x^2(Ac(bd-2ae)-aB(2cd-be))\right)+aB(bd-2ae)\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1509

$$\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d-\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}+\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\ldots}}{\ldots}\right.}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(-A\left(-abe-2acd+b^2d\right)-\left(x^2(Ac(bd-2ae)-aB(2cd-be))\right)+aB(bd-2ae)\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-((x*(a*B*(b*d - 2*a*e) - A*(b^2*d - 2*a*c*d - a*b*e) - (A*c*(b*d - 2*a*e)
 - a*B*(2*c*d - b*e))*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) + (
((A*b*c*d - 2*a*B*c*d + a*b*B*e - 2*a*A*c*e)*(-((x*Sqrt[a + b*x^2 + c*x^4]
)/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/S
qrt[c] + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[a]*B - A*Sqrt[c])*(Sqrt[c]
*d - Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.23

method	result
elliptic	$-\frac{2c\left(-\frac{(2aAce-Abcd-Babe+2aBcd)x^3}{2ca\left(4ac-b^2\right)}-\frac{\left(Aabe+2aAcd-A\,b^2d-2B\,a^2e+Babd\right)x}{2ca\left(4ac-b^2\right)}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{Be}{c}+\frac{Acd-Bae}{ac}-\frac{Aabe+2aAcd-A\,b^2d-2B\,a^2e+}{a\left(4ac-b^2\right)}\right.}{}$
default	Expression too large to display

input
```
int((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/2/c*(2*A*a*c*e-A*b*c*d-B*a*b*e+2*B*a*c*d)/a/(4*a*c-b^2)*x^3-1/2/c
*(A*a*b*e+2*A*a*c*d-A*b^2*d-2*B*a^2*e+B*a*b*d)/a/(4*a*c-b^2)*x)/((x^4+b/c*
x^2+a/c)*c)^(1/2)+1/4*(B*e/c+(A*c*d-B*a*e)/a/c-(A*a*b*e+2*A*a*c*d-A*b^2*d-
2*B*a^2*e+B*a*b*d)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*(2*A*a
*c*e-A*b*c*d-B*a*b*e+2*B*a*c*d)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ellipti
cF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(409) = 818$.

Time = 0.09 (sec) , antiderivative size = 1031, normalized size of antiderivative = 2.14

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*(((2*B*a*b - A*b^2)*c^2*d - (B*a*b^2*c - 2*A*a*b*c^2)*e)*x^
4 + (2*B*a^2*b - A*a*b^2)*c*d + ((2*B*a*b^2 - A*b^3)*c*d - (B*a*b^3 - 2*A*
a*b^2*c)*e)*x^2 - (B*a^2*b^2 - 2*A*a^2*b*c)*e - (((2*B*a^2 - A*a*b)*c^2*d
- (B*a^2*b*c - 2*A*a^2*c^2)*e)*x^4 + (2*B*a^3 - A*a^2*b)*c*d + ((2*B*a^2*b
 - A*a*b^2)*c*d - (B*a^2*b^2 - 2*A*a^2*b*c)*e)*x^2 - (B*a^3*b - 2*A*a^3*c)
*e)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/
a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)),
 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*(((B*a
*b^2*c - (2*(A - B)*a*b + A*b^2)*c^2)*d + (2*A*a*b*c^2 - (2*B*a^2*b - (A -
 B)*a*b^2)*c)*e)*x^4 + ((B*a*b^3 - (2*(A - B)*a*b^2 + A*b^3)*c)*d - (2*B*a
^2*b^2 - (A - B)*a*b^3 - 2*A*a*b^2*c)*e)*x^2 + (B*a^2*b^2 - (2*(A - B)*a^2
*b + A*a*b^2)*c)*d - (2*B*a^3*b - (A - B)*a^2*b^2 - 2*A*a^2*b*c)*e + (((B*
a^2*b*c - (2*(A + B)*a^2 - A*a*b)*c^2)*d - (2*A*a^2*c^2 + (2*B*a^3 - (A +
B)*a^2*b)*c)*e)*x^4 + ((B*a^2*b^2 - (2*(A + B)*a^2*b - A*a*b^2)*c)*d - (2*
B*a^3*b - (A + B)*a^2*b^2 + 2*A*a^2*b*c)*e)*x^2 + (B*a^3*b - (2*(A + B)*a^
3 - A*a^2*b)*c)*d - (2*B*a^4 - (A + B)*a^3*b + 2*A*a^3*c)*e)*sqrt((b^2 - 4
*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arc
sin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b
^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - 2*sqrt(c*x^4 + b*x^2 + a)*(((2*B*
a^2 - A*a*b)*c^2*d - (B*a^2*b*c - 2*A*a^2*c^2)*e)*x^3 - ((2*B*a^3 - A*a...
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A + Bx^2)(d + ex^2)}{(a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{(Bx^2 + A)(ex^2 + d)}{(cx^4 + bx^2 + a)^{\frac{3}{2}}} \, dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A + Bx^2)(d + ex^2)}{(a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{(Bx^2 + A)(ex^2 + d)}{(cx^4 + bx^2 + a)^{\frac{3}{2}}} \, dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + Bx^2)(d + ex^2)}{(a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{(B\,x^2 + A)\,(e\,x^2 + d)}{(c\,x^4 + b\,x^2 + a)^{3/2}} \, dx$$

input
```
int(((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-\sqrt{c\,x^4+b\,x^2+a}\,bex+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx\right)a^2be+\left(\int \frac{}{c^2x^8}\right.}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4)*b*e*x + int(sqrt(a + b*x**2 + c*x**4)/(a**2
+ 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*b*
e + int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x
**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*c*d + int(sqrt(a + b*x**2 + c*x**4)/
(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a
*b**2*e*x**2 + int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x*
*4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b*c*d*x**2 + int(sqrt(a + b*
x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 +
c**2*x**8),x)*a*b*c*e*x**4 + int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x
**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*c**2*d*x**4 +
int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**
2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*c*e + int((sqrt(a + b*x**2 + c*x*
*4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*
x**8),x)*a*b*c*d + int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2
 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b*c*e*x**2 + int(
(sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x*
*4 + 2*b*c*x**6 + c**2*x**8),x)*a*c**2*e*x**4 + int((sqrt(a + b*x**2 + c*x
**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2
*x**8),x)*b**2*c*d*x**2 + int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a
*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*b*c**2*d*...
```

3.177 $\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 398

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{(Ab-2aB)\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{(Ab-2aB)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
x*(A*b^2-B*a*b-2*A*a*c+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(
1/2)-(A*b-2*B*a)*c^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+c
^(1/2)*x^2)+(A*b-2*B*a)*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^
(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1
/2*(a^(1/2)*B-A*c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c
^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/c^(1/4)/(c*x^4+b*x^2
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.49 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(aB(b+2cx^2)-A(b^2-2ac+bcx^2))+i(Ab-2aB)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{}}{}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-1/4*(4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*
a*c + b*c*x^2)) + I*(A*b - 2*a*B)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 -
4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])] - I*(-2*a*B*Sqrt[b^2 - 4*a*c] + A*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a
*c]))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1492, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \ 1492$$

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\int -\frac{a(bB-2Ac)-(Ab-2aB)cx^2}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{a(bB-2Ac)-(Ab-2aB)cx^2}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}+\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \ 1511$$

$$\frac{\sqrt{a}\sqrt{c}(Ab-2aB)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx-\sqrt{a}\left(\sqrt{c}(Ab-2aB)-\sqrt{a}(bB-2Ac)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}+$$
$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{c}(Ab-2aB)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{a}\left(\sqrt{c}(Ab-2aB)-\sqrt{a}(bB-2Ac)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}+$$
$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \ 1416$$

$$\frac{\sqrt{c}(Ab-2aB)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}(Ab-2aB)-\sqrt{a}(bB-2Ac)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}}{a\left(b^2-4ac\right)}$$
$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \ 1509$$

$$\frac{\sqrt{c}(Ab-2aB)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+b}{\left(\sqrt{a}-\right.}}}{}}{a\left(b^2-4ac\right)}$$
$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[
a + b*x^2 + c*x^4]) + ((A*b - 2*a*B)*Sqrt[c]*(-((x*Sqrt[a + b*x^2 + c*x^4]
)/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])) -
(a^(1/4)*((A*b - 2*a*B)*Sqrt[c] - Sqrt[a]*(b*B - 2*A*c))*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a
+ b*x^2 + c*x^4]))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.28

method	result
elliptic	$-\frac{2c\left(\frac{(Ab-2Ba)x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2aAc-A\,b^2+abB\right)x}{2ca\left(4ac-b^2\right)}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{A}{a}-\frac{2aAc-A\,b^2+abB}{a\left(4ac-b^2\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \text{Ellip}}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2}}$
default	$A\left(-\frac{2c\left(\frac{b\,x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^2}{a\left(4ac-b^2\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \text{EllipticF}\left(\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\right.$

input
```
int((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2*(A*b-2*B*a)/a/(4*a*c-b^2)*x^3-1/2*(2*A*a*c-A*b^2+B*a*b)/c/a/(4*a
*c-b^2)*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)+1/4*(A/a-(2*A*a*c-A*b^2+B*a*b)/a/(4
*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2
+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(A*b-2*B*a)*c/(4*a*c-b^2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\Big((2\,Bab-Ab^2)c^2x^4+(2\,Bab^2-Ab^3)cx^2+(2\,Ba^2b-Aab^2)c-((2\,Ba^2-}{}$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*((2*B*a*b - A*b^2)*c^2*x^4 + (2*B*a*b^2 - A*b^3)*c*x^2 + (2
*B*a^2*b - A*a*b^2)*c - ((2*B*a^2 - A*a*b)*c^2*x^4 + (2*B*a^2*b - A*a*b^2)
*c*x^2 + (2*B*a^3 - A*a^2*b)*c)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*s
qrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt(
(b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*
c)/(a*c)) - sqrt(1/2)*(B*a^2*b^2 + (B*a*b^2*c - (2*(A - B)*a*b + A*b^2)*c^
2)*x^4 + (B*a*b^3 - (2*(A - B)*a*b^2 + A*b^3)*c)*x^2 - (2*(A - B)*a^2*b +
A*a*b^2)*c + (B*a^3*b + (B*a^2*b*c - (2*(A + B)*a^2 - A*a*b)*c^2)*x^4 + (B
*a^2*b^2 - (2*(A + B)*a^2*b - A*a*b^2)*c)*x^2 - (2*(A + B)*a^3 - A*a^2*b)*
c)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a
)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)),
1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - 2*((2*B*a^2 - A*a
*b)*c^2*x^3 + (2*A*a^2*c^2 + (B*a^2*b - A*a*b^2)*c)*x)*sqrt(c*x^4 + b*x^2
+ a))/(a^3*b^2*c - 4*a^4*c^2 + (a^2*b^2*c^2 - 4*a^3*c^3)*x^4 + (a^2*b^3*c
- 4*a^3*b*c^2)*x^2)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{B\,x^2+A}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((A + B*x^2)/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx=\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx\right)a+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx\right)b$$

input
```
int((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4
+ 2*b*c*x**6 + c**2*x**8),x)*a + int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**
2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*b
```

3.178 $\int \frac{A+Bx^2}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 882

$$\int \frac{A+Bx^2}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{x(aB(bcd-b^2e+2ace)-A(b^2cd-2ac^2d-b^3e+3abce)+c(aB(2cd-be)-A(bcd-b^2e+2ace))x^2)}{a(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt{c}(aB(2cd-be)-A(bcd-b^2e+2ace))x\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)(cd^2-bde+ae^2)(\sqrt{a}+\sqrt{c}x^2)}$$

$$-\frac{e^{3/2}(Bd-Ae)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}(cd^2-bde+ae^2)^{3/2}}$$

$$-\frac{\sqrt[4]{c}(aB(2cd-be)-A(bcd-b^2e+2ace))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right.}{a^{3/4}(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$

$$+\frac{(\sqrt{a}B-A\sqrt{c})\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}(b-2\sqrt{a}\sqrt{c})(\sqrt{c}d-\sqrt{a}e)\sqrt{a+bx^2+cx^4}}$$

$$+\frac{e(\sqrt{c}d+\sqrt{a}e)(Bd-Ae)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}d(\sqrt{c}d-\sqrt{a}e)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$

output
```
-x*(a*B*(2*a*c*e-b^2*e+b*c*d)-A*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)+c*(a*B
*(-b*e+2*c*d)-A*(2*a*c*e-b^2*e+b*c*d))*x^2)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*
d^2)/(c*x^4+b*x^2+a)^(1/2)+c^(1/2)*(a*B*(-b*e+2*c*d)-A*(2*a*c*e-b^2*e+b*c*
d))*x*(c*x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(a^(1/2)+c^
(1/2)*x^2)-1/2*e^(3/2)*(-A*e+B*d)*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/
2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^(1/2)/(a*e^2-b*d*e+c*d^2)^(3/2)-c^(1/4
)*(a*B*(-b*e+2*c*d)-A*(2*a*c*e-b^2*e+b*c*d))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4
+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/
a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-4*a*c+b^2)/(a*e^2-b*d
*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(a^(1/2)*B-A*c^(1/2))*c^(1/4)*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacob
iAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(
b-2*a^(1/2)*c^(1/2))/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+b*x^2+a)^(1/2)+1/4*e*(c^
(1/2)*d+a^(1/2)*e)*(-A*e+B*d)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1
/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4
*(c^(1/2)*d-a^(1/2)*e)^2/a^(1/2)/c^(1/2)/d/e,1/2*(2-b/a^(1/2)/c^(1/2))^(1/
2))/a^(1/4)/c^(1/4)/d/(c^(1/2)*d-a^(1/2)*e)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x
^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.46 (sec) , antiderivative size = 1736, normalized size of antiderivative = 1.97

$$\int \frac{A+Bx^2}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-4*A*b^2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d^2*x + 4*a*b*B*c*Sqrt[c/(b +
Sqrt[b^2 - 4*a*c])]*d^2*x + 8*a*A*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d^2*
x + 4*A*b^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x - 4*a*b^2*B*Sqrt[c/(b +
Sqrt[b^2 - 4*a*c])]*d*e*x - 12*a*A*b*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e
*x + 8*a^2*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x - 4*A*b*c^2*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*d^2*x^3 + 8*a*B*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*d^2*x^3 + 4*A*b^2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x^3 - 4*a*b*B*c*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x^3 - 8*a*A*c^2*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*d*e*x^3 - I*(-b + Sqrt[b^2 - 4*a*c])*d*(a*B*(2*c*d - b*e) + A*(-(
b*c*d) + b^2*e - 2*a*c*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2
- 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
 (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*d*(a*B*(b*(b - Sqrt[
b^2 - 4*a*c])*e + 2*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e)) + A*(-(b^3*e) + b*c*(
-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) - 2*a*c*
(2*c*d + Sqrt[b^2 - 4*a*c]*e)))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (2*I)*a*b^2*B*d*e
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(...
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 1045, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2258}$$

$$\int \left(\frac{e(Ae-Bd)}{(d+ex^2)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}+\frac{aBe-Abe+cx^2(Bd-Ae)+Acd}{(a+bx^2+cx^4)^{3/2}\,(ae^2-bde+cd^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/4}e(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\frac{\sqrt{c}d}{\sqrt{a}}+\right.}{4\sqrt[4]{c}d\left(cd^2-ae^2\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}$$

$$\frac{e^{3/2}(Bd-Ae)\arctan\left(\frac{\sqrt{cd^2-bed+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+bx^2+a}}\right)}{2\sqrt{d}\left(cd^2-bed+ae^2\right)^{3/2}}-$$

$$\frac{\sqrt[4]{c}\left(aB(2cd-be)-A\left(-eb^2+cdb+2ace\right)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}-$$

$$\frac{\sqrt[4]{c}\left(aBe-\sqrt{a}\sqrt{c}(Bd-Ae)+A(cd-be)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{}}\right.\right.}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}$$

$$\frac{\sqrt[4]{c}e(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}+$$

$$\frac{\sqrt{c}\left(aB(2cd-be)-A\left(-eb^2+cdb+2ace\right)\right)x\sqrt{cx^4+bx^2+a}}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)}-$$

$$\frac{x\left(c\left(aB(2cd-be)-A\left(-eb^2+cdb+2ace\right)\right)x^2+abc(Bd-Ae)-\left(b^2-2ac\right)(Acd-Abe+aBe)\right)}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-((x*(a*b*c*(B*d - A*e) - (b^2 - 2*a*c)*(A*c*d - A*b*e + a*B*e) + c*(a*B*(
2*c*d - b*e) - A*(b*c*d - b^2*e + 2*a*c*e))*x^2))/(a*(b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (Sqrt[c]*(a*B*(2*c*d - b*e) -
 A*(b*c*d - b^2*e + 2*a*c*e))*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)) - (e^(3/2)*(B*d - A*e)*Ar
cTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x
^4])])/(2*Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^(3/2)) - (c^(1/4)*(a*B*(2*c*d -
b*e) - A*(b*c*d - b^2*e + 2*a*c*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*(c*d^2 - b*d*e +
 a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(Sqrt[
c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1
/4)*(a*B*e - Sqrt[a]*Sqrt[c]*(B*d - A*e) + A*(c*d - b*e))*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2
*Sqrt[a]*Sqrt[c])*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (a^(3
/4)*e*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3240 vs. 2(752) = 1504.

Time = 1.16 (sec) , antiderivative size = 3241, normalized size of antiderivative = 3.67

method	result	size
default	Expression too large to display	3241
elliptic	Expression too large to display	4597

input
```
int((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B/e*(-2*c*(1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^
4+b/c*x^2+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2
*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+(A*e-B*d)/e*(-2*c*(1/2*(2
*a*c*e-b^2*e+b*c*d)/a/(4*a*c-b^2)/(a*e^2-b*d*e+c*d^2)*x^3+1/2*(3*a*b*c*e-2
*a*c^2*d-b^3*e+b^2*c*d)/a/(4*a*c-b^2)/(a*e^2-b*d*e+c*d^2)/c*x)/((x^4+b/c*x
^2+a/c)*c)^(1/2)-1/4*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*
x^2-2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*x^2+2/a*x^2*(-4*a*c+b^2)^(1
/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a/(a*e^2-b
*d*e+c*d^2)*e*b+1/4*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*x
^2-2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b/a*x^2+2/a*x^2*(-4*a*c+b^2)^(1/
2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{Bx^2+A}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{B\,x^2+A}{(e\,x^2+d)\,(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx=\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2e\,x^{10}+2bce\,x^8+c^2d\,x^8+2ace\,x^6+b^2e\,x^6+2bcd\,x^6+2abe\,x^4\right.$$

$$+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c^2e\,x^{10}+2bce\,x^8+c^2d\,x^8+2ace\,x^6+b^2e\,x^6+2bcd\,x^6+2abe\,x^4+2acd\,x^4+b^2d\,x^4+a^2e\,x^2+2abd\right.$$

input
```
int((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

output

```
int(sqrt(a + b*x**2 + c*x**4)/(a**2*d + a**2*e*x**2 + 2*a*b*d*x**2 + 2*a*b
*e*x**4 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + b**2*d*x**4 + b**2*e*x**6 + 2*b*c*
d*x**6 + 2*b*c*e*x**8 + c**2*d*x**8 + c**2*e*x**10),x)*a + int((sqrt(a + b
*x**2 + c*x**4)*x**2)/(a**2*d + a**2*e*x**2 + 2*a*b*d*x**2 + 2*a*b*e*x**4
+ 2*a*c*d*x**4 + 2*a*c*e*x**6 + b**2*d*x**4 + b**2*e*x**6 + 2*b*c*d*x**6 +
 2*b*c*e*x**8 + c**2*d*x**8 + c**2*e*x**10),x)*b
```

3.179 $\int \frac{A+Bx^2}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 1247

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

output
```
-1/2*e*(-A*e+B*d)*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2)-
1/2*x*(a*(-A*e+B*d)*(b*e+2*c*d)*(2*a*c*e-b^2*e+b*c*d)-2*(c*(-2*a*c+b^2)*d-
b*(-3*a*c+b^2)*e)*(2*a*B*d*e+A*(c*d^2-e*(a*e+b*d)))+c*(a*(-A*e+B*d)*(-b*e+
2*c*d)*(b*e+2*c*d)-2*(2*a*c*e-b^2*e+b*c*d)*(2*a*B*d*e+A*(c*d^2-e*(a*e+b*d)
)))*x^2)/a/(-4*a*c+b^2)/d/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)^(1/2)+1/2*
c^(1/2)*(a*(-A*e+B*d)*(-b*e+2*c*d)*(b*e+2*c*d)-2*(2*a*c*e-b^2*e+b*c*d)*(2*
a*B*d*e+A*(c*d^2-e*(a*e+b*d))))*x*(c*x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/d/(
a*e^2-b*d*e+c*d^2)^2/(a^(1/2)+c^(1/2)*x^2)+1/4*e^(3/2)*(A*e*(7*c*d^2-e*(-a
*e+4*b*d))-B*d*(5*c*d^2-e*(a*e+2*b*d)))*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x
/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)^(5/2)-
1/2*c^(1/4)*(a*(-A*e+B*d)*(-b*e+2*c*d)*(b*e+2*c*d)-2*(2*a*c*e-b^2*e+b*c*d)
*(2*a*B*d*e+A*(c*d^2-e*(a*e+b*d))))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)
/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4)))
,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-4*a*c+b^2)/d/(a*e^2-b*d*e+c*d^
2)^2/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(a*c^(1/2)*e*(-2*A*e+B*d)-A*c^(1/2)
*d*(-b*e+c*d)+a^(1/2)*(-A*e+B*d)*(-b*e+c*d))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4
+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*
x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/
d/(c^(1/2)*d-a^(1/2)*e)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)-1/8*e*(c
^(1/2)*d+a^(1/2)*e)*(A*e*(7*c*d^2-e*(-a*e+4*b*d))-B*d*(5*c*d^2-e*(a*e+2...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.92 (sec) , antiderivative size = 8031, normalized size of antiderivative = 6.44

$$\int \frac{A+Bx^2}{(d+ex^2)^2(a+bx^2+cx^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 3.48 (sec) , antiderivative size = 2112, normalized size of antiderivative = 1.69, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)^2(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \ 2258$$

$$\int \left(\frac{A\left(-ce(ae+2bd)+b^2e^2+c^2d^2\right)-cx^2\left(Ae(2cd-be)-B\left(cd^2-ae^2\right)\right)+aBe(2cd-be)}{(a+bx^2+cx^4)^{3/2}(ae^2-bde+cd^2)^2}+\frac{}{(d+ex^2)^2\sqrt{a+}}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{(Bd-Ae)x\sqrt{cx^4+bx^2+a}e^3}{2d\left(cd^2-bed+ae^2\right)^2\left(ex^2+d\right)}-\frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)e^2}{2d\left(cd^2-bed+ae^2\right)^2\sqrt{cx^4+bx^2+a}}+\frac{\sqrt{c}(Bd-Ae)x\sqrt{cx^4+bx^2+a}e^2}{2d\left(cd^2-bed+ae^2\right)^2\left(\sqrt{c}x^2+\sqrt{a}\right)}-\frac{(Bd-Ae)\left(3cd^2-e(2bd-ae)\right)\arctan\left(\frac{\sqrt{cd^2-bed+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+bx^2+a}}\right)e^{3/2}}{4d^{3/2}\left(cd^2-bed+ae^2\right)^{5/2}}+\frac{\left(Ae(2cd-be)-B\left(cd^2-ae^2\right)\right)\arctan\left(\frac{\sqrt{cd^2-bed+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+bx^2+a}}\right)e^{3/2}}{2\sqrt{d}\left(cd^2-bed+ae^2\right)^{5/2}}+\frac{\sqrt[4]{c}\left(Ae(2cd-be)-B\left(cd^2-ae^2\right)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)e}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bed+ae^2\right)^2\sqrt{cx^4+bx^2+a}}-\frac{\sqrt[4]{c}(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)e}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)(Bd-Ae)\left(3cd^2-e(2bd-ae)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\right.\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bed+ae^2\right)^2\sqrt{cx^4+bx^2+a}}$$

$$\frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2\left(Ae(2cd-be)-B\left(cd^2-ae^2\right)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\right.\right.}{4\sqrt[4]{c}d\left(cd^2-ae^2\right)\left(cd^2-bed+ae^2\right)^2\sqrt{cx^4+bx^2+a}}$$

$$\frac{\sqrt[4]{c}\left(aB\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)+A\left(-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de\right)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\right.}}}{a^{3/4}\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^2\sqrt{cx^4+bx^2+a}}$$

$$\frac{\sqrt[4]{c}\left(a^{3/2}B\sqrt{c}e^2+a(2Bcd-bBe-Ace)e+A(cd-be)^2-\sqrt{a}\sqrt{c}\left(Bcd^2-Ae(2cd-be)\right)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+b}{\left(\sqrt{c}x^2\right.}}}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\left(cd^2-bed+ae^2\right)^2\sqrt{cx^4+bx^2+a}}$$

$$\frac{\sqrt{c}\left(aB\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)+A\left(-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de\right)\right)x\sqrt{cx^4+bx^2+a}}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^2\left(\sqrt{c}x^2+\sqrt{a}\right)}+\frac{x\left(-c\left(aB\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)+A\left(-e^2b^3+2cdeb^2-c\left(cd^2-3ae^2\right)b-4ac^2de\right)\right)x^2+abc\left(Ae(2cd-b\right.\right.}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)^2\sqrt{cx^4}}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(x*(a*b*c*(A*e*(2*c*d - b*e) - B*(c*d^2 - a*e^2)) + (b^2 - 2*a*c)*(a*B*e*(
2*c*d - b*e) + A*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e))) - c*(a*B*(2*c^2*
d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + A*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^
2 - b*c*(c*d^2 - 3*a*e^2)))*x^2))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)
^2*Sqrt[a + b*x^2 + c*x^4]) + (Sqrt[c]*e^2*(B*d - A*e)*x*Sqrt[a + b*x^2 +
c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (Sqrt[c]
*(a*B*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + A*(2*b^2*c*d*e - 4*a*c^2
*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2)))*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^
2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (e^3*(B*d
- A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^
2)) - (e^(3/2)*(B*d - A*e)*(3*c*d^2 - e*(2*b*d - a*e))*ArcTan[(Sqrt[c*d^2
- b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])]])/(4*d^(3/2)
*(c*d^2 - b*d*e + a*e^2)^(5/2)) + (e^(3/2)*(A*e*(2*c*d - b*e) - B*(c*d^2 -
 a*e^2))*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a +
b*x^2 + c*x^4])]])/(2*Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^(5/2)) - (a^(1/4)*c^(
1/4)*e^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^4]
) - (c^(1/4)*(a*B*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + A*(2*b^2*c*d
*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2)))*(Sqrt[a] + Sqrt[c]...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8275 vs. 2(1105) = 2210.

Time = 2.17 (sec) , antiderivative size = 8276, normalized size of antiderivative = 6.64

method	result	size
default	Expression too large to display	8276
elliptic	Expression too large to display	9725

input
```
int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2(a+bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2(a+bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + Bx^2}{(d + ex^2)^2 (a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{Bx^2 + A}{(cx^4 + bx^2 + a)^{\frac{3}{2}} (ex^2 + d)^2} \, dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{A + Bx^2}{(d + ex^2)^2 (a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{Bx^2 + A}{(cx^4 + bx^2 + a)^{\frac{3}{2}} (ex^2 + d)^2} \, dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{(d + ex^2)^2 (a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{B\,x^2 + A}{(e\,x^2 + d)^2\,(c\,x^4 + b\,x^2 + a)^{3/2}} \, dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\,(a+bx^2+cx^4)^{3/2}}\,dx = \left(\int \frac{}{c^2e^2x^{12}+2bc\,e^2x^{10}+2c^2de\,x^{10}+2ac\,e^2x^8+b^2e^2x^8+4bcde\,x^8 \cdots}\right.$$
$$+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+\cdots}}{c^2e^2x^{12}+2bc\,e^2x^{10}+2c^2de\,x^{10}+2ac\,e^2x^8+b^2e^2x^8+4bcde\,x^8+c^2d^2x^8+2ab\,e^2x^6+4acde\,x^6+2b^2\cdots}\right.$$

input
```
int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a**2*d**2 + 2*a**2*d*e*x**2 + a**2*e**2*x**
4 + 2*a*b*d**2*x**2 + 4*a*b*d*e*x**4 + 2*a*b*e**2*x**6 + 2*a*c*d**2*x**4 +
 4*a*c*d*e*x**6 + 2*a*c*e**2*x**8 + b**2*d**2*x**4 + 2*b**2*d*e*x**6 + b**
2*e**2*x**8 + 2*b*c*d**2*x**6 + 4*b*c*d*e*x**8 + 2*b*c*e**2*x**10 + c**2*d
**2*x**8 + 2*c**2*d*e*x**10 + c**2*e**2*x**12),x)*a + int((sqrt(a + b*x**2
 + c*x**4)*x**2)/(a**2*d**2 + 2*a**2*d*e*x**2 + a**2*e**2*x**4 + 2*a*b*d**
2*x**2 + 4*a*b*d*e*x**4 + 2*a*b*e**2*x**6 + 2*a*c*d**2*x**4 + 4*a*c*d*e*x*
*6 + 2*a*c*e**2*x**8 + b**2*d**2*x**4 + 2*b**2*d*e*x**6 + b**2*e**2*x**8 +
 2*b*c*d**2*x**6 + 4*b*c*d*e*x**8 + 2*b*c*e**2*x**10 + c**2*d**2*x**8 + 2*
c**2*d*e*x**10 + c**2*e**2*x**12),x)*b
```

3.180 $\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 273

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4a^{3/4}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}$$

output
```
-1/2*(c^(1/2)*d/a^(1/2)-e)*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/2)/e^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+1/4*(
c^(1/2)*d+a^(1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/
2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*a^(1/2)*
(c^(1/2)*d/a^(1/2)-e)^2/c^(1/2)/d/e,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/
4)/c^(1/4)/d/e/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.90 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.15

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\sqrt{c}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(-\sqrt{c}d\right.}{\sqrt{2}\sqrt{a}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2}}$$

input
```
Integrate[(1 + (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]
),x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Sqrt[c]*d*EllipticF[I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt
[b^2 - 4*a*c])] + (-(Sqrt[c]*d) + Sqrt[a]*e)*EllipticPi[((b + Sqrt[b^2 - 4
*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[a]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 2220$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}-\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

input
```
Int[(1 + (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*(((Sqrt[c]*d)/Sqrt[a] - e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sq
rt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d
*e + a*e^2]) + (((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqr
t[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.36

method	result
default	$\frac{\frac{\sqrt{c}\,\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{2}}{}}{\sqrt{a}}$
elliptic	$\sqrt{\left(c\,x^4+b\,x^2+a\right)ac}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^2}{2a}-\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\sqrt{1+\frac{b\,x^2}{2a}+\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\mathrm{EllipticPi}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},-\frac{2a}{\left(-b+\sqrt{-4\right.}}\right.}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\right.$

input
```
int((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
1/a^(1/2)*(1/4*c^(1/2)/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-(c^(1/2)*d-a^(1/2)*
e)/e/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^
2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(
1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1 + \frac{\sqrt{c}x^2}{\sqrt{a}}}{(d + ex^2)\sqrt{a + bx^2 + cx^4}}\,dx = \frac{\int \frac{\sqrt{a}}{d\sqrt{a+bx^2+cx^4}+ex^2\sqrt{a+bx^2+cx^4}}\,dx + \int \frac{\sqrt{c}x^2}{d\sqrt{a+bx^2+cx^4}+ex^2\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}}$$

input
```
integrate((1+c**(1/2)*x**2/a**(1/2))/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x
)
```

output
```
(Integral(sqrt(a)/(d*sqrt(a + b*x**2 + c*x**4) + e*x**2*sqrt(a + b*x**2 +
c*x**4)), x) + Integral(sqrt(c)*x**2/(d*sqrt(a + b*x**2 + c*x**4) + e*x**2
*sqrt(a + b*x**2 + c*x**4)), x))/sqrt(a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1 + \frac{\sqrt{c}x^2}{\sqrt{a}}}{(d + ex^2)\sqrt{a + bx^2 + cx^4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\frac{\sqrt{c}\,x^2}{\sqrt{a}}+1}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(((c^(1/2)*x^2)/a^(1/2) + 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(((c^(1/2)*x^2)/a^(1/2) + 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)a}{a}$$

input
```
int((1+c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*
d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x) + int(sqrt(a + b*x**2 + c*x**4
)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a)/a
```

3.181 $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 288

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(\sqrt{\frac{c}{a}}d-e\right)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} + \frac{\left(\sqrt{\frac{c}{a}}d+e\right)\left(1+\sqrt{\frac{c}{a}}x^2\right)\sqrt{\frac{\sqrt{\frac{c}{a}}(a+bx^2+cx^4)}{c\left(\frac{1}{\sqrt[4]{\frac{c}{a}}}+\sqrt[4]{\frac{c}{a}}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right)}{4\sqrt[4]{\frac{c}{a}}de\sqrt{a+bx^2+cx^4}}$$

output
```
-1/2*((c/a)^(1/2)*d-e)*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/
(c*x^4+b*x^2+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+1/4*((c/a
)^(1/2)*d+e)*(1+(c/a)^(1/2)*x^2)*((c/a)^(1/2)*(c*x^4+b*x^2+a)/c/(1/(c/a)^(
1/4)+(c/a)^(1/4)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan((c/a)^(1/4)*x)),-1/
4*((c/a)^(1/2)*d-e)^2/(c/a)^(1/2)/d/e,1/2*(2-b*(c/a)^(1/2)/c)^(1/2))/(c/a)
^(1/4)/d/e/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.57 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.08

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\sqrt{\frac{c}{a}}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+\left(-\sqrt{\frac{c}{a}}\right.\right.}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2+}}$$

input
```
Integrate[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Sqrt[c/a]*d*EllipticF[I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])] + (-(Sqrt[c/a]*d) + e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c
])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.94, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{ 2220}$$

$$\frac{\left(x^2\sqrt{\frac{c}{a}}+1\right)\sqrt{\frac{a+bx^2+cx^4}{a\left(x^2\sqrt{\frac{c}{a}}+1\right)^2}}\left(d\sqrt{\frac{c}{a}}+e\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right)}{4de\sqrt[4]{\frac{c}{a}}\sqrt{a+bx^2+cx^4}}-\frac{\left(d\sqrt{\frac{c}{a}}-e\right)\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

input
```
Int[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*((Sqrt[c/a]*d - e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sq
rt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e
^2]) + ((Sqrt[c/a]*d + e)*(1 + Sqrt[c/a]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*
(1 + Sqrt[c/a]*x^2)^2)]*EllipticPi[-1/4*(Sqrt[c/a]*d - e)^2/(Sqrt[c/a]*d*e
), 2*ArcTan[(c/a)^(1/4)*x], (2 - (b*Sqrt[c/a])/c)/4])/(4*(c/a)^(1/4)*d*e*S
qrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.29

method	result
default	$\frac{\sqrt{\frac{c}{a}}\,\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{\left(\sqrt{\frac{c}{a}}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{\frac{\left(c\,x^{4}+b\,x^{2}+a\right)c}{a}}\,a\left(1+\sqrt{\frac{c}{a}}\,x^{2}\right)\left(\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^{2}}{2a}-\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{1+\frac{b\,x^{2}}{2a}+\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\operatorname{EllipticPi}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},-\frac{2ae\cdots}{\left(-b+\sqrt{-4\cdots}\right.}\cdots\right)}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\cdots\right.}{\cdots}$

input
```
int((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/4*(c/a)^(1/2)/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-((c/a)^(1/2)*d-e)/e/d*2^(1
/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2*(-4*a*c+b
^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4
+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((1+(c/a)**(1/2)*x**2)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((x**2*sqrt(c/a) + 1)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^2\sqrt{\frac{c}{a}}+1}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((x^2*(c/a)^(1/2) + 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((x^2*(c/a)^(1/2) + 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)a}{a}$$

input
```
int((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*
d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x) + int(sqrt(a + b*x**2 + c*x**4
)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a)/a
```

3.182 $\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 426

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4a^{3/4}\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}}$$

output

```
1/2*(c^(1/2)*d+a^(1/2)*e)*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/2)/e^(1/
2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2
)+c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^
(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2)
)^(1/2))/a^(1/4)/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+b*x^2+a)^(1/2)-1/4*(c^(1/2)*
d+a^(1/2)*e)^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2
)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(1/2)*d-a^(
1/2)*e)^2/a^(1/2)/c^(1/2)/d/e,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/c^(
1/4)/d/e/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.69 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.74

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(-\sqrt{c}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(\sqrt{cd}\right.}{\sqrt{2}\sqrt{a}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2}}$$

input
```
Integrate[(1 - (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]
),x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(-(Sqrt[c]*d*EllipticF[I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])]) + (Sqrt[c]*d + Sqrt[a]*e)*EllipticPi[((b + Sqrt[b^2 - 4
*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[a]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {2224, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 2224}$$

$$\frac{2\sqrt{c}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$\Big\downarrow$ 27

$$\frac{2\sqrt{c}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

$\Big\downarrow$ 1416

$$\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

$\Big\downarrow$ 2220

$$\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(\sqrt{a}e+\sqrt{c}d)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}-\frac{(\sqrt{c}d-\sqrt{a}e)\text{ ar}}{2\sqrt{d}\sqrt{}}\right.}{\sqrt{a}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

input
```
Int[(1 - (Sqrt[c]*x^2)/Sqrt[a])/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt
[c]*d + Sqrt[a]*e)*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 - b*d
*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e
]*Sqrt[c*d^2 - b*d*e + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/
4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a +
b*x^2 + c*x^4])))/(Sqrt[a]*(Sqrt[c]*d - Sqrt[a]*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2224
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[2*A*(B/(B*d + A*e))   Int[1/Sqrt[a + b*x
^2 + c*x^4], x], x] - Simp[(B*d - A*e)/(B*d + A*e)   Int[(A - B*x^2)/((d +
e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x]
&& NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && NegQ[B/A]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\frac{\sqrt{c}\,\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-\frac{(\sqrt{c}\,d+\sqrt{a}\,e)\cdots}{\cdots}}{\sqrt{\cdots}}$
elliptic	$\frac{\sqrt{(c\,x^4+b\,x^2+a)ac}\,(-\sqrt{a}+\sqrt{c}\,x^2)\left(\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^2}{2a}-\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\sqrt{1+\frac{b\,x^2}{2a}+\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\mathrm{EllipticPi}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},-\frac{2\cdots}{\left(-b+\sqrt{\cdots}\right.}\cdots\right)}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\cdots\right.}{\cdots}$

input
```
int((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
-1/a^(1/2)*(1/4*c^(1/2)/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(
-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-(c^(1/2)*d+a^(1/2)
*e)/e/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x
^2*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^
(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\int\left(-\frac{\sqrt{a}}{d\sqrt{a+bx^2+cx^4}+ex^2\sqrt{a+bx^2+cx^4}}\right)dx + \int\frac{\sqrt{c}x^2}{d\sqrt{a+bx^2+cx^4}+ex^2\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}}$$

input
```
integrate((1-c**(1/2)*x**2/a**(1/2))/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x
)
```

output
```
-(Integral(-sqrt(a)/(d*sqrt(a + b*x**2 + c*x**4) + e*x**2*sqrt(a + b*x**2
+ c*x**4)), x) + Integral(sqrt(c)*x**2/(d*sqrt(a + b*x**2 + c*x**4) + e*x*
*2*sqrt(a + b*x**2 + c*x**4)), x))/sqrt(a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\int \frac{\frac{\sqrt{c}\,x^2}{\sqrt{a}}-1}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(-((c^(1/2)*x^2)/a^(1/2) - 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x
)
```

output
```
-int(((c^(1/2)*x^2)/a^(1/2) - 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),
x)
```

Reduce [F]

$$\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{-\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)a}{a}$$

input
```
int((1-c^(1/2)*x^2/a^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 +
 b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x) + int(sqrt(a + b*x**2 + c*x
**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a)/a
```

3.183 $\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 442

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{\frac{c}{a}}d+e\right)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} + \frac{a^{3/4}\left(\frac{c}{a}\right)^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{5/4}\left(\sqrt{\frac{c}{a}}d-e\right)\sqrt{a+bx^2+cx^4}} - \frac{\left(\sqrt{\frac{c}{a}}d+e\right)^2\left(1+\sqrt{\frac{c}{a}}x^2\right)\sqrt{\frac{\sqrt{\frac{c}{a}}(a+bx^2+cx^4)}{c\left(\frac{1}{\sqrt[4]{\frac{c}{a}}}+\sqrt[4]{\frac{c}{a}}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right)}{4\sqrt[4]{\frac{c}{a}}d\left(\sqrt{\frac{c}{a}}d-e\right)e\sqrt{a+bx^2+cx^4}}$$

output
```
1/2*((c/a)^(1/2)*d+e)*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(
c*x^4+b*x^2+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+a^(3/4)*(c
/a)^(3/2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^
(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2)
)^(1/2))/c^(5/4)/((c/a)^(1/2)*d-e)/(c*x^4+b*x^2+a)^(1/2)-1/4*((c/a)^(1/2)*
d+e)^2*(1+(c/a)^(1/2)*x^2)*((c/a)^(1/2)*(c*x^4+b*x^2+a)/c/(1/(c/a)^(1/4)+(
c/a)^(1/4)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan((c/a)^(1/4)*x)),-1/4*((c/
a)^(1/2)*d-e)^2/(c/a)^(1/2)/d/e,1/2*(2-b*(c/a)^(1/2)/c)^(1/2))/(c/a)^(1/4)
/d/((c/a)^(1/2)*d-e)/e/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.71

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\sqrt{\frac{c}{a}}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\left(\sqrt{\frac{c}{a}}d+\right.\right.}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[(1 - Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Sqrt[c/a]*d*EllipticF[I*ArcSinh[Sqrt
[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])] - (Sqrt[c/a]*d + e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/
(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2224, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2\sqrt{\frac{c}{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{2224}$$

$$\frac{2\sqrt{\frac{c}{a}}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{d\sqrt{\frac{c}{a}}-e}-\frac{\left(d\sqrt{\frac{c}{a}}+e\right)\int \frac{\sqrt{\frac{c}{a}}x^2+1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{d\sqrt{\frac{c}{a}}-e}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\sqrt{\frac{c}{a}}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(d\sqrt{\frac{c}{a}}-e\right)}-\frac{\left(d\sqrt{\frac{c}{a}}+e\right)\int \frac{\sqrt{\frac{c}{a}}x^2+1}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{d\sqrt{\frac{c}{a}}-e}$$

$$\Big\downarrow \text{2220}$$

$$\frac{\sqrt{\frac{c}{a}}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(d\sqrt{\frac{c}{a}}-e\right)}-\frac{\left(d\sqrt{\frac{c}{a}}+e\right)\left(\frac{\left(x^2\sqrt{\frac{c}{a}}+1\right)\sqrt{\frac{a+bx^2+cx^4}{a\left(x^2\sqrt{\frac{c}{a}}+1\right)^2}}\left(d\sqrt{\frac{c}{a}}+e\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right)}{4de\sqrt[4]{\frac{c}{a}}\sqrt{a+bx^2+cx^4}}-\frac{\left(d\sqrt{\frac{c}{a}}-e\right)\arctan\left(\right.}{2\sqrt{d}\sqrt{e}\sqrt{ae}}\right.}{d\sqrt{\frac{c}{a}}-e}$$

input
```
Int[(1 - Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[c/a]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt
[c]))/4])/(a^(1/4)*c^(1/4)*(Sqrt[c/a]*d - e)*Sqrt[a + b*x^2 + c*x^4]) - ((
Sqrt[c/a]*d + e)*(-1/2*((Sqrt[c/a]*d - e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e
^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c
*d^2 - b*d*e + a*e^2]) + ((Sqrt[c/a]*d + e)*(1 + Sqrt[c/a]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(a*(1 + Sqrt[c/a]*x^2)^2)]*EllipticPi[-1/4*(Sqrt[c/a]*d - e
)^2/(Sqrt[c/a]*d*e), 2*ArcTan[(c/a)^(1/4)*x], (2 - (b*Sqrt[c/a])/c)/4])/(4
*(c/a)^(1/4)*d*e*Sqrt[a + b*x^2 + c*x^4])))/(Sqrt[c/a]*d - e)
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2224
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[2*A*(B/(B*d + A*e))   Int[1/Sqrt[a + b*x
^2 + c*x^4], x], x] - Simp[(B*d - A*e)/(B*d + A*e)   Int[(A - B*x^2)/((d +
e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x]
&& NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && NegQ[B/A]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.83

method	result
default	$-\frac{\sqrt{\frac{c}{a}}\,\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+\frac{(\sqrt{}$
elliptic	$\frac{\left(1-\sqrt{\frac{c}{a}}\,x^2\right)\sqrt{\frac{\left(c\,x^4+b\,x^2+a\right)c}{a}}\,a\left(-\frac{c\sqrt{2}\,\sqrt{4+\frac{2b\,x^2}{a}-\frac{2x^2\sqrt{-4ac+b^2}}{a}}\,\sqrt{4+\frac{2b\,x^2}{a}+\frac{2x^2\sqrt{-4ac+b^2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+}}{}\right.}{4ae\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{\frac{c^2x^4}{a}+\frac{bc\,x^2}{a}+c}}\right.}{}$

input
```
int((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-1/4*(c/a)^(1/2)/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+((c/a)^(1/2)*d+e)/e/d*2^(
1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2*(-4*a*c+
b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^
4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\int \frac{x^2\sqrt{\frac{c}{a}}}{d\sqrt{a+bx^2+cx^4}+ex^2\sqrt{a+bx^2+cx^4}}\,dx - \int \left(-\frac{1}{d\sqrt{a+bx^2+cx^4}+ex^2\sqrt{a+bx^2+cx^4}}\right)dx$$

input
```
integrate((1-(c/a)**(1/2)*x**2)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
-Integral(x**2*sqrt(c/a)/(d*sqrt(a + b*x**2 + c*x**4) + e*x**2*sqrt(a + b*
x**2 + c*x**4)), x) - Integral(-1/(d*sqrt(a + b*x**2 + c*x**4) + e*x**2*sq
rt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int -\frac{x^2\sqrt{\frac{c}{a}}-1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="maxima")
```

output
```
-integrate((x^2*sqrt(c/a) - 1)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int -\frac{x^2\sqrt{\frac{c}{a}}-1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate(-(x^2*sqrt(c/a) - 1)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\int \frac{x^2\sqrt{\frac{c}{a}}-1}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(-(x^2*(c/a)^(1/2) - 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
-int((x^2*(c/a)^(1/2) - 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{-\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)a}{a}$$

input
```
int((1-(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 +
 b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x) + int(sqrt(a + b*x**2 + c*x
**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a)/a
```

3.184 $\int \frac{A+Bx^2}{(d+ex^2)\sqrt{bx^2+c\left(\frac{a}{c}+x^4\right)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 439

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{bx^2+c\left(\frac{a}{c}+x^4\right)}}\,dx = -\frac{(Bd-Ae)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} - \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}}$$

output
```
-1/2*(-A*e+B*d)*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/(c*x^4+
b*x^2+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)-1/2*(a^(1/2)*B-A
*c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^
(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2)
)^(1/2))/a^(1/4)/c^(1/4)/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+b*x^2+a)^(1/2)+1/4*(
c^(1/2)*d+a^(1/2)*e)*(-A*e+B*d)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^
(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1
/4*(c^(1/2)*d-a^(1/2)*e)^2/a^(1/2)/c^(1/2)/d/e,1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))/a^(1/4)/c^(1/4)/d/e/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{bx^2+c\left(\frac{a}{c}+x^4\right)}}\,dx =$$
$$-\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(Bd\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(-Bd+\right.}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2+c}}$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)*Sqrt[b*x^2 + c*(a/c + x^4)]),x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*d*EllipticF[I*ArcSinh[Sqrt[2]*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])] + (-(B*d) + A*e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d),
 I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d
*e*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2091, 2226, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{c\left(\frac{a}{c}+x^4\right)+bx^2}}dx$$

$$\Big\downarrow \text{ 2091}$$

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \textbf{2226}$$

$$\frac{\sqrt{a}(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \textbf{1416}$$

$$\frac{(Bd-Ae)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

$$\Big\downarrow \textbf{2220}$$

$$\frac{(Bd-Ae)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\text{arctan}\ldots}{2\sqrt{d}\sqrt{e}\sqrt{\ldots}}\right.}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

input
```
Int[(A + B*x^2)/((d + e*x^2)*Sqrt[b*x^2 + c*(a/c + x^4)]),x]
```

output
```
-1/2*((Sqrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(1/4)*c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt
[a + b*x^2 + c*x^4]) + ((B*d - A*e)*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[
(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])]
)/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*
ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1
/4)*d*e*Sqrt[a + b*x^2 + c*x^4])))/(Sqrt[c]*d - Sqrt[a]*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.82

method	result
default	$\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+\frac{(Ae-Bd}{}$
elliptic	$\frac{B\sqrt{2}\,\sqrt{4+\frac{2b\,x^2}{a}-\frac{2x^2\sqrt{-4ac+b^2}}{a}}\,\sqrt{4+\frac{2b\,x^2}{a}+\frac{2x^2\sqrt{-4ac+b^2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+\frac{\sqrt{2}\,\sqrt{}}{}$

input
```
int((B*x^2+A)/(e*x^2+d)/(b*x^2+c*(a/c+x^4))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*B/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(A*e-B*d)/e/d*2^(1/2)/(-b/a+1/a*(-4*
a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(
1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*El
lipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^
2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)\sqrt{bx^2+c\left(\frac{a}{c}+x^4\right)}}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(b*x^2+c*(a/c+x^4))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)\sqrt{bx^2+c\left(\frac{a}{c}+x^4\right)}}\,dx=\int \frac{A+Bx^2}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)/(b*x**2+c*(a/c+x**4))**(1/2),x)
```

output
```
Integral((A + B*x**2)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)\sqrt{bx^2+c\left(\frac{a}{c}+x^4\right)}}\,dx=\int \frac{Bx^2+A}{\sqrt{bx^2+\left(x^4+\frac{a}{c}\right)c}\,(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(b*x^2+c*(a/c+x^4))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((B*x^2 + A)/(sqrt(b*x^2 + (x^4 + a/c)*c)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{bx^2+c\left(\frac{a}{c}+x^4\right)}}\,dx = \int \frac{Bx^2+A}{\sqrt{bx^2+\left(x^4+\frac{a}{c}\right)c}(ex^2+d)}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)/(b*x^2+c*(a/c+x^4))^(1/2),x, algorithm="giac
")
```

output
```
integrate((B*x^2 + A)/(sqrt(b*x^2 + (x^4 + a/c)*c)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{bx^2+c\left(\frac{a}{c}+x^4\right)}}\,dx = \int \frac{B\,x^2+A}{\left(e\,x^2+d\right)\,\sqrt{c\,\left(\frac{a}{c}+x^4\right)+b\,x^2}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)*(c*(a/c + x^4) + b*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)*(c*(a/c + x^4) + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}
&\int \frac{A+Bx^2}{(d+ex^2)\sqrt{bx^2+c\left(\frac{a}{c}+x^4\right)}}\,dx \\
&= \left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)a \\
&\quad + \left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)b
\end{aligned}$$

input
```
int((B*x^2+A)/(e*x^2+d)/(b*x^2+c*(a/c+x^4))^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*
x**4 + c*e*x**6),x)*a + int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x*
*2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b
```

3.185 $\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 106

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{631(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{2525(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

output
```
631/4*(x^2+1)*((x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*2^(1/2
))*2^(1/2)/(x^4+3*x^2+2)^(1/2)-2525/28*(x^2+2)*EllipticPi(x/(x^2+1)^(1/2),
2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\left(441\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)+505\,\text{EllipticPi}\left(\frac{10}{7},i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)\right)}{7\sqrt{2+3x^2+x^4}}$$

input
```
Integrate[(946 + 315*x^2)/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
((-1/7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(441*EllipticF[I*ArcSinh[x/Sqrt[2]],
 2] + 505*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4
]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2218, 27, 1412, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{315x^2+946}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 2218}$$

$$\frac{631}{2}\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{2525}{8}\int \frac{4(x^2+1)}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{631}{2}\int \frac{1}{\sqrt{x^4+3x^2+2}}dx-\frac{2525}{2}\int \frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 1412}$$

$$\frac{631(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{2525}{2}\int \frac{x^2+1}{(5x^2+7)\sqrt{x^4+3x^2+2}}dx$$

$$\Big\downarrow \text{ 1786}$$

$$\frac{631(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{2525\sqrt{x^2+1}\sqrt{x^2+2}\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}(5x^2+7)}dx}{2\sqrt{x^4+3x^2+2}}$$

$$\Big\downarrow \text{ 414}$$

$$\frac{631(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{2525(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

input
```
Int[(946 + 315*x^2)/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

output
```
(631*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqr
t[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1
/2])/(14*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1412
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

rule 2218
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Sqrt[b^2 - 4*a*c]}, Simp[(2*a*B - A
*(b + q))/(2*a*e - d*(b + q))   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Sim
p[(B*d - A*e)/(2*a*e - d*(b + q))   Int[(2*a + (b + q)*x^2)/((d + e*x^2)*Sq
rt[a + b*x^2 + c*x^4]), x], x] /; RationalQ[q]] /; FreeQ[{a, b, c, d, e, A,
 B}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*A^
2 - b*A*B + a*B^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{63 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(\frac{i x \sqrt{2}}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}-\frac{505 i \sqrt{2}\, \sqrt{1+\frac{x^{2}}{2}}\, \sqrt{x^{2}+1}\, \operatorname{EllipticPi}\left(\frac{i x \sqrt{2}}{2}, \frac{10}{7}, \sqrt{2}\right)}{7 \sqrt{x^{4}+3 x^{2}+2}}$	93
elliptic	$-\frac{63 i \sqrt{2}\, \sqrt{2 x^{2}+4}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(\frac{i x \sqrt{2}}{2}, \sqrt{2}\right)}{2 \sqrt{x^{4}+3 x^{2}+2}}-\frac{505 i \sqrt{2}\, \sqrt{1+\frac{x^{2}}{2}}\, \sqrt{x^{2}+1}\, \operatorname{EllipticPi}\left(\frac{i x \sqrt{2}}{2}, \frac{10}{7}, \sqrt{2}\right)}{7 \sqrt{x^{4}+3 x^{2}+2}}$	93

input
```
int((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-63/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
F(1/2*I*x*2^(1/2),2^(1/2))-505/7*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)
/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx=\int \frac{315\,x^2+946}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

input
```
integrate((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(x^4 + 3*x^2 + 2)*(315*x^2 + 946)/(5*x^6 + 22*x^4 + 31*x^2 +
14), x)
```

Sympy [F]

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx=\int \frac{315x^2+946}{\sqrt{(x^2+1)\,(x^2+2)}\,(5x^2+7)}\,dx$$

input
```
integrate((315*x**2+946)/(5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

output
```
Integral((315*x**2 + 946)/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx=\int \frac{315\,x^2+946}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

input
```
integrate((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((315*x^2 + 946)/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx=\int \frac{315\,x^2+946}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

input
```
integrate((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((315*x^2 + 946)/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{315\,x^2+946}{(5\,x^2+7)\,\sqrt{x^4+3\,x^2+2}}\,dx$$

input
```
int((315*x^2 + 946)/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)),x)
```

output
```
int((315*x^2 + 946)/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = 946\left(\int \frac{\sqrt{x^4+3x^2+2}}{5x^6+22x^4+31x^2+14}dx\right) + 315\left(\int \frac{\sqrt{x^4+3x^2+2}\,x^2}{5x^6+22x^4+31x^2+14}dx\right)$$

input
```
int((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x)
```

output
```
946*int(sqrt(x**4 + 3*x**2 + 2)/(5*x**6 + 22*x**4 + 31*x**2 + 14),x) + 315
*int((sqrt(x**4 + 3*x**2 + 2)*x**2)/(5*x**6 + 22*x**4 + 31*x**2 + 14),x)
```

3.186 $\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 273

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(\sqrt{c}d-\sqrt{a}e)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} + \frac{(\sqrt{c}d+\sqrt{a}e)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}$$

output

```
-1/2*(c^(1/2)*d-a^(1/2)*e)*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/2)/e^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+1/4*(
c^(1/2)*d+a^(1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/
2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*a^(1/2)*
(c^(1/2)*d/a^(1/2)-e)^2/c^(1/2)/d/e,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/
4)/c^(1/4)/d/e/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx =$$
$$-\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\sqrt{c}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(-\sqrt{c}d\right.}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2+}}$$

input
```
Integrate[(Sqrt[a] + Sqrt[c]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Sqrt[c]*d*EllipticF[I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt
[b^2 - 4*a*c])] + (-(Sqrt[c]*d) + Sqrt[a]*e)*EllipticPi[((b + Sqrt[b^2 - 4
*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \text{ 2220}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

input
```
Int[(Sqrt[a] + Sqrt[c]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt
[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e
 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[c]
*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.34

method	result
default	$\frac{\sqrt{c}\,\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}} - \frac{(\sqrt{c}\,d-}{}$
elliptic	$\frac{\sqrt{a\left(c\,x^4+b\,x^2+a\right)}\,\sqrt{c\left(c\,x^4+b\,x^2+a\right)}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(\frac{c\sqrt{2}\,\sqrt{4+\frac{2b\,x^2}{a}-\frac{2x^2\sqrt{-4ac+b^2}}{a}}\,\sqrt{4+\frac{2b\,x^2}{a}+\frac{2x^2\sqrt{-4ac+b^2}}{a}}\ \text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{-b+\sqrt{}}}{2}\right.}{4e\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{c^2x^4+bc\,x^2+ac}}\right.}{}$

input

```
int((a^(1/2)+c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURN
VERBOSE)
```

output

```
1/4*c^(1/2)/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*
x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-(c^(1/2)*d-a^(1/2)*e)/e/d*2^(1
/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2*(-4*a*c+b
^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4
+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\,\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input

```
integrate((a^(1/2)+c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a} + \sqrt{c}x^2}{(d + ex^2)\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{\sqrt{a} + \sqrt{c}x^2}{(d + ex^2)\sqrt{a + bx^2 + cx^4}}\,dx$$

input
```
integrate((a**(1/2)+c**(1/2)*x**2)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((sqrt(a) + sqrt(c)*x**2)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4))
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a} + \sqrt{c}x^2}{(d + ex^2)\sqrt{a + bx^2 + cx^4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^(1/2)+c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((a^(1/2)+c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{a}+\sqrt{c}\,x^2}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((a^(1/2) + c^(1/2)*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((a^(1/2) + c^(1/2)*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx\\&=\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)\\&\quad+\sqrt{c}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)\end{aligned}$$

input
```
int((a^(1/2)+c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(a + b*x**2 + c*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**
4 + c*d*x**4 + c*e*x**6),x) + sqrt(c)*int((sqrt(a + b*x**2 + c*x**4)*x**2)
/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)
```

3.187 $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 288

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(\sqrt{\frac{c}{a}}d-e\right)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} + \frac{\left(\sqrt{\frac{c}{a}}d+e\right)\left(1+\sqrt{\frac{c}{a}}x^2\right)\sqrt{\frac{\sqrt{\frac{c}{a}}(a+bx^2+cx^4)}{c\left(\frac{1}{\sqrt[4]{\frac{c}{a}}}+\sqrt[4]{\frac{c}{a}}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right)}{4\sqrt[4]{\frac{c}{a}}de\sqrt{a+bx^2+cx^4}}$$

output
```
-1/2*((c/a)^(1/2)*d-e)*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/2)/e^(1/2)/
(c*x^4+b*x^2+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+1/4*((c/a
)^(1/2)*d+e)*(1+(c/a)^(1/2)*x^2)*((c/a)^(1/2)*(c*x^4+b*x^2+a)/c/(1/(c/a)^(
1/4)+(c/a)^(1/4)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan((c/a)^(1/4)*x)),-1/
4*((c/a)^(1/2)*d-e)^2/(c/a)^(1/2)/d/e,1/2*(2-b*(c/a)^(1/2)/c)^(1/2))/(c/a)
^(1/4)/d/e/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.08

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\sqrt{\frac{c}{a}}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+\left(-\sqrt{\frac{c}{a}}\right.\right.}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2+}}$$

input
```
Integrate[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Sqrt[c/a]*d*EllipticF[I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])] + (-(Sqrt[c/a]*d) + e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c
])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.94, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\frac{c}{a}}+1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{ 2220}$$

$$\frac{\left(x^2\sqrt{\frac{c}{a}}+1\right)\sqrt{\frac{a+bx^2+cx^4}{a\left(x^2\sqrt{\frac{c}{a}}+1\right)^2}}\left(d\sqrt{\frac{c}{a}}+e\right)\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right)}{4de\sqrt[4]{\frac{c}{a}}\sqrt{a+bx^2+cx^4}}-\frac{\left(d\sqrt{\frac{c}{a}}-e\right)\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

input

```
Int[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output

```
-1/2*((Sqrt[c/a]*d - e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sq
rt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e
^2]) + ((Sqrt[c/a]*d + e)*(1 + Sqrt[c/a]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*
(1 + Sqrt[c/a]*x^2)^2)]*EllipticPi[-1/4*(Sqrt[c/a]*d - e)^2/(Sqrt[c/a]*d*e
), 2*ArcTan[(c/a)^(1/4)*x], (2 - (b*Sqrt[c/a])/c)/4])/(4*(c/a)^(1/4)*d*e*S
qrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 2220

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.29

method	result
default	$\frac{\sqrt{\frac{c}{a}}\,\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{\left(\sqrt{\frac{c}{a}}\right.}{}$
elliptic	$\frac{\sqrt{\frac{\left(c\,x^{4}+b\,x^{2}+a\right)c}{a}}\,a\left(1+\sqrt{\frac{c}{a}}\,x^{2}\right)\left(\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^{2}}{2a}-\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\sqrt{1+\frac{b\,x^{2}}{2a}+\frac{x^{2}\sqrt{-4ac+b^{2}}}{2a}}\,\operatorname{EllipticPi}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},-\frac{2ae}{\left(-b+\sqrt{-4a}\right.}\right.}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\right.}{}$

input
```
int((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/4*(c/a)^(1/2)/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-((c/a)^(1/2)*d-e)/e/d*2^(1
/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2*(-4*a*c+b
^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4
+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^2\sqrt{\frac{c}{a}}+1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((1+(c/a)**(1/2)*x**2)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((x**2*sqrt(c/a) + 1)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

input
```
integrate((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate((x^2*sqrt(c/a) + 1)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((x^2*(c/a)^(1/2) + 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((x^2*(c/a)^(1/2) + 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)a}{a}$$

input
```
int((1+(c/a)^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*
d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x) + int(sqrt(a + b*x**2 + c*x**4
)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a)/a
```

3.188 $\int \frac{2+3\sqrt{2}+2(3+\sqrt{2})x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 148

$$\int \frac{2+3\sqrt{2}+2\left(3+\sqrt{2}\right)x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{7\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{2\sqrt{15}} + \frac{\left(3+\sqrt{2}\right)^2\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}\left(12-11\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

output
```
-7/30*arctan(1/3*15^(1/2)*x/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/24*(3+2^(1/2
))^2*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*EllipticPi(
sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*2^(3/4)/
(2*x^4+2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

$$\int \frac{2+3\sqrt{2}+2(3+\sqrt{2})\,x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \frac{(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(3(3+\sqrt{2})\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{1-i}x\right),i\right)-7\,\text{EllipticPi}\left(\frac{1}{3}\right.\right.}{6\sqrt{1+2x^2+2x^4}}$$

input
```
Integrate[(2 + 3*Sqrt[2] + 2*(3 + Sqrt[2])*x^2)/((3 + 2*x^2)*Sqrt[1 + 2*x^
2 + 2*x^4]),x]
```

output
```
((1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*(3*(3 + Sqrt[2]
)*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 7*EllipticPi[1/3 + I/3, I*ArcSi
nh[Sqrt[1 - I]*x], I]))/(6*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2(3+\sqrt{2})\,x^2+3\sqrt{2}+2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

$$\Big\downarrow \text{2220}$$

$$\frac{(3+\sqrt{2})^2(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticPi}\left(\frac{1}{24}(12-11\sqrt{2}),2\arctan\left(\sqrt[4]{2}x\right),\frac{1}{4}(2-\sqrt{2})\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{7\arctan\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{15}}$$

input
```
Int[(2 + 3*Sqrt[2] + 2*(3 + Sqrt[2])*x^2)/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*
x^4]),x]
```

output
```
(-7*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(2*Sqrt[15]) + ((3 + Sq
rt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*E
llipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12
*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Defintions of rubi rules used

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.34

method	result
default	$\frac{\sqrt{2}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\operatorname{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\operatorname{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-$
elliptic	$\frac{3\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{2}}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-$

input
```
int((2+3*2^(1/2)+2*(3+2^(1/2))*x^2)/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2^(1/2)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+
1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+3/(-1+I)^(1/2
)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(
x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-7/3/(-1+I)^(1/2)*(1+x^2-I*x^2)^(
1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1
/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Fricas [F]

$$\int \frac{2+3\sqrt{2}+2\left(3+\sqrt{2}\right)x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{2\,x^2\left(\sqrt{2}+3\right)+3\,\sqrt{2}+2}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

input
```
integrate((2+3*2^(1/2)+2*(3+2^(1/2))*x^2)/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),
x, algorithm="fricas")
```

output
```
integral(sqrt(2*x^4 + 2*x^2 + 1)*(6*x^2 + sqrt(2)*(2*x^2 + 3) + 2)/(4*x^6
+ 10*x^4 + 8*x^2 + 3), x)
```

Sympy [F]

$$\int \frac{2+3\sqrt{2}+2\left(3+\sqrt{2}\right)x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{2\sqrt{2}x^2+6x^2+2+3\sqrt{2}}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

input
```
integrate((2+3*2**(1/2)+2*(3+2**(1/2))*x**2)/(2*x**2+3)/(2*x**4+2*x**2+1)*
*(1/2),x)
```

output
```
Integral((2*sqrt(2)*x**2 + 6*x**2 + 2 + 3*sqrt(2))/((2*x**2 + 3)*sqrt(2*x*
*4 + 2*x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{2+3\sqrt{2}+2\left(3+\sqrt{2}\right)x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{2\,x^2\left(\sqrt{2}+3\right)+3\,\sqrt{2}+2}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)}\,dx$$

input
```
integrate((2+3*2^(1/2)+2*(3+2^(1/2))*x^2)/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((2*x^2*(sqrt(2) + 3) + 3*sqrt(2) + 2)/(sqrt(2*x^4 + 2*x^2 + 1)*(
2*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{2+3\sqrt{2}+2\left(3+\sqrt{2}\right)x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{2\,x^2\left(\sqrt{2}+3\right)+3\,\sqrt{2}+2}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)}\,dx$$

input
```
integrate((2+3*2^(1/2)+2*(3+2^(1/2))*x^2)/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),
x, algorithm="giac")
```

output
```
integrate((2*x^2*(sqrt(2) + 3) + 3*sqrt(2) + 2)/(sqrt(2*x^4 + 2*x^2 + 1)*(
2*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3\sqrt{2}+2\left(3+\sqrt{2}\right)x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx=\int \frac{3\,\sqrt{2}+2\,x^2\,\left(\sqrt{2}+3\right)+2}{\left(2\,x^2+3\right)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

input
```
int((3*2^(1/2) + 2*x^2*(2^(1/2) + 3) + 2)/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)
^(1/2)),x)
```

output
```
int((3*2^(1/2) + 2*x^2*(2^(1/2) + 3) + 2)/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{2+3\sqrt{2}+2\left(3+\sqrt{2}\right)x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx = 3\sqrt{2}\left(\int \frac{\sqrt{2x^4+2x^2+1}}{4x^6+10x^4+8x^2+3}dx\right) + 2\sqrt{2}\left(\int \frac{\sqrt{2x^4+2x^2+1}\,x^2}{4x^6+10x^4+8x^2+3}dx\right) + 2\left(\int \frac{\sqrt{2x^4+2x^2+1}}{4x^6+10x^4+8x^2+3}dx\right) + 6\left(\int \frac{\sqrt{2x^4+2x^2+1}\,x^2}{4x^6+10x^4+8x^2+3}dx\right)$$

input
```
int((2+3*2^(1/2)+2*(3+2^(1/2))*x^2)/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

output
```
3*sqrt(2)*int(sqrt(2*x**4 + 2*x**2 + 1)/(4*x**6 + 10*x**4 + 8*x**2 + 3),x)
 + 2*sqrt(2)*int((sqrt(2*x**4 + 2*x**2 + 1)*x**2)/(4*x**6 + 10*x**4 + 8*x*
*2 + 3),x) + 2*int(sqrt(2*x**4 + 2*x**2 + 1)/(4*x**6 + 10*x**4 + 8*x**2 +
3),x) + 6*int((sqrt(2*x**4 + 2*x**2 + 1)*x**2)/(4*x**6 + 10*x**4 + 8*x**2
+ 3),x)
```

3.189 $\int \frac{a+b+\sqrt{b}\sqrt{a+b}-\left(b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx$

Optimal result

Integrand size = 64, antiderivative size = 397

$$\int \frac{a+b+\sqrt{b}\sqrt{a+b}-\left(b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx = \frac{1}{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{a+b-2bx^2+bx^4}}\right)$$
$$+\frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(\sqrt{b}+\sqrt{a+b}\right)\left(a+b+\sqrt{b}\sqrt{a+b}\right)\left(1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2bx^2+bx^4}{(a+b)\left(1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\cdots\right.\right.}{\left(a+2\left(b+\sqrt{b}\sqrt{a+b}\right)\right)\sqrt{a+b-2bx^2+bx^4}\cdots}$$
$$+\frac{a^2\left(\sqrt{a+b}+\sqrt{b}x^2\right)\sqrt{\frac{a+b-2bx^2+bx^4}{\left(\sqrt{a+b}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}+\sqrt{a+b}\right)^2}{4\sqrt{b}\sqrt{a+b}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\left(1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right)}{4\sqrt[4]{b}\sqrt[4]{a+b}\left(a+2\left(b+\sqrt{b}\sqrt{a+b}\right)\right)\sqrt{a+b-2bx^2+bx^4}}$$

output
```
1/2*a^(1/2)*arctanh(a^(1/2)*x/(b*x^4-2*b*x^2+a+b)^(1/2))+b^(1/4)*(a+b)^(1/
4)*(b^(1/2)+(a+b)^(1/2))*(a+b+b^(1/2)*(a+b)^(1/2))*(1+b^(1/2)*x^2/(a+b)^(1
/2))*((b*x^4-2*b*x^2+a+b)/(a+b)/(1+b^(1/2)*x^2/(a+b)^(1/2))^2)^(1/2)*Inver
seJacobiAM(2*arctan(b^(1/4)*x/(a+b)^(1/4)),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(
1/2))/(a+2*b+2*b^(1/2)*(a+b)^(1/2))/(b*x^4-2*b*x^2+a+b)^(1/2)+1/4*a^2*((a+
b)^(1/2)+b^(1/2)*x^2)*((b*x^4-2*b*x^2+a+b)/((a+b)^(1/2)+b^(1/2)*x^2)^2)^(1
/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/(a+b)^(1/4))),1/4*(b^(1/2)+(a+b)^(1/
2))^2/b^(1/2)/(a+b)^(1/2),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))/b^(1/4)/(a+
b)^(1/4)/(a+2*b+2*b^(1/2)*(a+b)^(1/2))/(b*x^4-2*b*x^2+a+b)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.72 (sec) , antiderivative size = 3113, normalized size of antiderivative = 7.84

$$\int \frac{a+b+\sqrt{b}\sqrt{a+b}-\left(b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b + Sqrt[b]*Sqrt[a + b] - (b + Sqrt[b]*Sqrt[a + b])*x^2)/((
1 - x^2)*Sqrt[a + b - 2*b*x^2 + b*x^4]),x]
```

output
```
((-I)*b*Sqrt[1 - (Sqrt[b]*x^2)/((-I)*Sqrt[a] + Sqrt[b])]*Sqrt[1 - (Sqrt[b]
*x^2)/(I*Sqrt[a] + Sqrt[b])]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[b]/((-I)*Sqrt
[a] + Sqrt[b]))]*x], ((-I)*Sqrt[a] + Sqrt[b])/(I*Sqrt[a] + Sqrt[b])])/(Sqr
t[-(Sqrt[b]/((-I)*Sqrt[a] + Sqrt[b]))]*Sqrt[a + b*(-1 + x^2)^2]) - (I*Sqrt
[b]*Sqrt[a + b]*Sqrt[1 - (Sqrt[b]*x^2)/((-I)*Sqrt[a] + Sqrt[b])]*Sqrt[1 -
(Sqrt[b]*x^2)/(I*Sqrt[a] + Sqrt[b])]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[b]/((
-I)*Sqrt[a] + Sqrt[b]))]*x], ((-I)*Sqrt[a] + Sqrt[b])/(I*Sqrt[a] + Sqrt[b]
)])/(Sqrt[-(Sqrt[b]/((-I)*Sqrt[a] + Sqrt[b]))]*Sqrt[a + b*(-1 + x^2)^2]) -
 (a*(Sqrt[1 - (I*Sqrt[a])/Sqrt[b]] + Sqrt[1 + (I*Sqrt[a])/Sqrt[b]])*(-Sqrt
[1 - (I*Sqrt[a])/Sqrt[b]] + x)^2*Sqrt[(Sqrt[((-I)*Sqrt[a] + Sqrt[b])/Sqrt[
b]]*(-Sqrt[1 + (I*Sqrt[a])/Sqrt[b]] + x))/((Sqrt[1 - (I*Sqrt[a])/Sqrt[b]]
+ Sqrt[1 + (I*Sqrt[a])/Sqrt[b]])*(-Sqrt[1 - (I*Sqrt[a])/Sqrt[b]] + x))]*Sq
rt[(Sqrt[((-I)*Sqrt[a] + Sqrt[b])/Sqrt[b]]*(Sqrt[1 + (I*Sqrt[a])/Sqrt[b]]
+ x))/((Sqrt[1 - (I*Sqrt[a])/Sqrt[b]] - Sqrt[1 + (I*Sqrt[a])/Sqrt[b]])*(-S
qrt[1 - (I*Sqrt[a])/Sqrt[b]] + x))]*Sqrt[((Sqrt[((-I)*Sqrt[a] + Sqrt[b])/S
qrt[b]] - Sqrt[(I*Sqrt[a] + Sqrt[b])/Sqrt[b]])*(Sqrt[((-I)*Sqrt[a] + Sqrt[
b])/Sqrt[b]] + x))/((Sqrt[((-I)*Sqrt[a] + Sqrt[b])/Sqrt[b]] + Sqrt[(I*Sqrt
[a] + Sqrt[b])/Sqrt[b]])*(Sqrt[((-I)*Sqrt[a] + Sqrt[b])/Sqrt[b]] - x))]*((
1 + Sqrt[1 - (I*Sqrt[a])/Sqrt[b]])*EllipticF[ArcSin[Sqrt[((Sqrt[((-I)*Sqrt
[a] + Sqrt[b])/Sqrt[b]] - Sqrt[(I*Sqrt[a] + Sqrt[b])/Sqrt[b]])*(Sqrt[((...
```

Rubi [A] (warning: unable to verify)

Time = 1.08 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.49, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2224, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-\left(x^2\left(\sqrt{b}\sqrt{a+b}+b\right)\right)+\sqrt{b}\sqrt{a+b}+a+b}{(1-x^2)\sqrt{a+bx^4-2bx^2+b}}dx$$

$$\Big\downarrow \text{ 2224}$$

$$\frac{2\sqrt{b}\left(\sqrt{a+b}+\sqrt{b}\right)\left(\sqrt{b}\sqrt{a+b}+a+b\right)\int\frac{1}{\sqrt{bx^4-2bx^2+a+b}}dx}{2\left(\sqrt{b}\sqrt{a+b}+b\right)+a}+\frac{a\int\frac{\sqrt{b}\left(\sqrt{b}+\sqrt{a+b}\right)x^2+a+b+\sqrt{b}\sqrt{a+b}}{(1-x^2)\sqrt{bx^4-2bx^2+a+b}}dx}{2\left(\sqrt{b}\sqrt{a+b}+b\right)+a}$$

$\downarrow$ 1416

$$\frac{a\int\frac{\sqrt{b}\left(\sqrt{b}+\sqrt{a+b}\right)x^2+a+b+\sqrt{b}\sqrt{a+b}}{(1-x^2)\sqrt{bx^4-2bx^2+a+b}}dx}{2\left(\sqrt{b}\sqrt{a+b}+b\right)+a}+\frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(\sqrt{a+b}+\sqrt{b}\right)\left(\sqrt{b}\sqrt{a+b}+a+b\right)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4-2bx^2+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right.}{\left(2\left(\sqrt{b}\sqrt{a+b}+b\right)+a\right)\sqrt{a+bx^4-2bx^2+b}}$$

$\downarrow$ 2222

$$\frac{a\left(\frac{a\sqrt{\sqrt{b}\sqrt{a+b}+a+b}\left(\frac{\sqrt{b}x^2\left(\sqrt{a+b}+\sqrt{b}\right)}{\sqrt{b}\sqrt{a+b}+a+b}+1\right)\sqrt{\frac{a+bx^4-2bx^2+b}{(a+b)\left(\frac{\sqrt{b}x^2\left(\sqrt{a+b}+\sqrt{b}\right)}{\sqrt{b}\sqrt{a+b}+a+b}+1\right)^2}}\text{EllipticPi}\left(\frac{\left(a+2\left(b+\sqrt{a+b}\sqrt{b}\right)\right)^2}{4\sqrt{b}\left(\sqrt{b}+\sqrt{a+b}\right)\left(a+b+\sqrt{b}\sqrt{a+b}\right)},2\arctan\left(\frac{\sqrt[4]{b}\sqrt{\sqrt{b}+}}{\sqrt{a+b+\sqrt{}}}\right.\right.}{4\sqrt[4]{b}\sqrt{\sqrt{a+b}+\sqrt{b}}\sqrt{a+bx^4-2bx^2+b}}\right.}{2\left(\sqrt{b}\sqrt{a+b}+b\right)+a}$$

$$\frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(\sqrt{a+b}+\sqrt{b}\right)\left(\sqrt{b}\sqrt{a+b}+a+b\right)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+bx^4-2bx^2+b}{(a+b)\left(\frac{\sqrt{b}x^2}{\sqrt{a+b}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right),\frac{1}{2}\right.}{\left(2\left(\sqrt{b}\sqrt{a+b}+b\right)+a\right)\sqrt{a+bx^4-2bx^2+b}}$$

input
```
Int[(a + b + Sqrt[b]*Sqrt[a + b] - (b + Sqrt[b]*Sqrt[a + b])*x^2)/((1 - x^
2)*Sqrt[a + b - 2*b*x^2 + b*x^4]),x]
```

output

```
(b^(1/4)*(a + b)^(1/4)*(Sqrt[b] + Sqrt[a + b])*(a + b + Sqrt[b]*Sqrt[a + b
])*(1 + (Sqrt[b]*x^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*x^2 + b*x^4)/((a + b)
*(1 + (Sqrt[b]*x^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*x)/(a + b
)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/((a + 2*(b + Sqrt[b]*Sqrt[a + b]))
*Sqrt[a + b - 2*b*x^2 + b*x^4]) + (a*(((a + 2*(b + Sqrt[b]*Sqrt[a + b]))*A
rcTanh[(Sqrt[a]*x)/Sqrt[a + b - 2*b*x^2 + b*x^4]])/(2*Sqrt[a]) + (a*Sqrt[a
 + b + Sqrt[b]*Sqrt[a + b]]*(1 + (Sqrt[b]*(Sqrt[b] + Sqrt[a + b])*x^2)/(a
+ b + Sqrt[b]*Sqrt[a + b]))*Sqrt[(a + b - 2*b*x^2 + b*x^4)/((a + b)*(1 + (
Sqrt[b]*(Sqrt[b] + Sqrt[a + b])*x^2)/(a + b + Sqrt[b]*Sqrt[a + b]))^2)]*El
lipticPi[(a + 2*(b + Sqrt[b]*Sqrt[a + b]))^2/(4*Sqrt[b]*(Sqrt[b] + Sqrt[a
+ b])*(a + b + Sqrt[b]*Sqrt[a + b])), 2*ArcTan[(b^(1/4)*Sqrt[Sqrt[b] + Sqr
t[a + b]]*x)/Sqrt[a + b + Sqrt[b]*Sqrt[a + b]]], (1 + (Sqrt[b]*(a + b + Sq
rt[b]*Sqrt[a + b]))/((a + b)*(Sqrt[b] + Sqrt[a + b])))/2])/(4*b^(1/4)*Sqrt
[Sqrt[b] + Sqrt[a + b]]*Sqrt[a + b - 2*b*x^2 + b*x^4])))/(a + 2*(b + Sqrt[
b]*Sqrt[a + b]))
```

Defintions of rubi rules used

rule 1416

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2222

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2224

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[2*A*(B/(B*d + A*e))   Int[1/Sqrt[a + b*x
^2 + c*x^4], x], x] - Simp[(B*d - A*e)/(B*d + A*e)   Int[(A - B*x^2)/((d +
e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x]
&& NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && NegQ[B/A]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.76 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.70

method	result
default	$\frac{b\sqrt{1-\frac{\left(i\sqrt{b}\,\sqrt{a}+b\right)x^{2}}{a+b}}\,\sqrt{1+\frac{\left(i\sqrt{b}\,\sqrt{a}-b\right)x^{2}}{a+b}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}\,\sqrt{a}+b}{a+b}},\sqrt{-1-\frac{2\left(i\sqrt{b}\,\sqrt{a}-b\right)}{a+b}}\right)}{\sqrt{\frac{i\sqrt{b}\,\sqrt{a}+b}{a+b}}\,\sqrt{b\,x^{4}-2b\,x^{2}+a+b}}+\frac{\sqrt{b}\,\sqrt{a+b}\,\sqrt{1-\frac{\left(i\sqrt{b}\,\sqrt{a}+b\right)x^{2}}{a+b}}\,\sqrt{1+\frac{(}{}}}{\sqrt{\frac{iv}{}}}$
elliptic	$\frac{\sqrt{(b\,x^{4}-2b\,x^{2}+a+b)\,b(a+b)}\,\left(\sqrt{a+b}\,\sqrt{b}\,x^{2}+b\,x^{2}-\sqrt{b}\,\sqrt{a+b}-a-b\right)\left(\frac{b(a+b)\sqrt{1-\frac{\left(i b^{\frac{3}{2}}\sqrt{a}+b^{2}\right)x^{2}}{b(a+b)}}\,\sqrt{1+\frac{\left(i b^{\frac{3}{2}}\sqrt{a}-b^{2}\right)x^{2}}{b(a+b)}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{3}{}}\right.}{\sqrt{\frac{i b^{\frac{3}{2}}\sqrt{a}+b^{2}}{b(a+b)}}\,\sqrt{a\,b^{2}x^{4}+b^{3}x^{4}-2a\,b^{2}x^{2}-}}\right.}{\sqrt{b\,x^{4}-2b\,x^{2}+a+b}\,ab\,x^{2}+\sqrt{b\,x^{4}-2b\,x^{2}+a+b}\,b^{2}}$

input

```
int((a+b+b^(1/2)*(a+b)^(1/2)-(b+b^(1/2)*(a+b)^(1/2))*x^2)/(-x^2+1)/(b*x^4-
2*b*x^2+a+b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
b/((I*b^(1/2)*a^(1/2)+b)/(a+b))^(1/2)*(1-(I*b^(1/2)*a^(1/2)+b)/(a+b)*x^2)^
(1/2)*(1+(I*b^(1/2)*a^(1/2)-b)/(a+b)*x^2)^(1/2)/(b*x^4-2*b*x^2+a+b)^(1/2)*
EllipticF(x*((I*b^(1/2)*a^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*b^(1/2)*a^(1/2)-b
)/(a+b))^(1/2))+b^(1/2)*(a+b)^(1/2)/((I*b^(1/2)*a^(1/2)+b)/(a+b))^(1/2)*(1
-(I*b^(1/2)*a^(1/2)+b)/(a+b)*x^2)^(1/2)*(1+(I*b^(1/2)*a^(1/2)-b)/(a+b)*x^2
)^(1/2)/(b*x^4-2*b*x^2+a+b)^(1/2)*EllipticF(x*((I*b^(1/2)*a^(1/2)+b)/(a+b)
)^(1/2),(-1-2*(I*b^(1/2)*a^(1/2)-b)/(a+b))^(1/2))-1/2*a*(-1/2/a^(1/2)*arct
anh(a^(1/2)/(b*x^4-2*b*x^2+a+b)^(1/2))-1/((I*b^(1/2)*a^(1/2)+b)/(a+b))^(1/
2)*(1-(I*b^(1/2)*a^(1/2)+b)/(a+b)*x^2)^(1/2)*(1+(I*b^(1/2)*a^(1/2)-b)/(a+b
)*x^2)^(1/2)/(b*x^4-2*b*x^2+a+b)^(1/2)*EllipticPi(x*((I*b^(1/2)*a^(1/2)+b)
/(a+b))^(1/2),1/(I*b^(1/2)*a^(1/2)+b)*(a+b),(-(I*b^(1/2)*a^(1/2)-b)/(a+b))
^(1/2)/((I*b^(1/2)*a^(1/2)+b)/(a+b))^(1/2)))+1/2*a*(-1/2/a^(1/2)*arctanh(a
^(1/2)/(b*x^4-2*b*x^2+a+b)^(1/2))+1/((I*b^(1/2)*a^(1/2)+b)/(a+b))^(1/2)*(1
-(I*b^(1/2)*a^(1/2)+b)/(a+b)*x^2)^(1/2)*(1+(I*b^(1/2)*a^(1/2)-b)/(a+b)*x^2
)^(1/2)/(b*x^4-2*b*x^2+a+b)^(1/2)*EllipticPi(x*((I*b^(1/2)*a^(1/2)+b)/(a+b
))^(1/2),1/(I*b^(1/2)*a^(1/2)+b)*(a+b),(-(I*b^(1/2)*a^(1/2)-b)/(a+b))^(1/2
)/((I*b^(1/2)*a^(1/2)+b)/(a+b))^(1/2)))
```

Fricas [F]

$$\int \frac{a+b+\sqrt{b}\sqrt{a+b}-\left(b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx = \int \frac{\left(\sqrt{a+b}\sqrt{b}+b\right)x^2-a-\sqrt{a+b}\sqrt{b}-b}{\sqrt{bx^4-2\,bx^2+a+b}(x^2-1)}\,dx$$

input
```
integrate((a+b+b^(1/2)*(a+b)^(1/2)-(b+b^(1/2)*(a+b)^(1/2))*x^2)/(-x^2+1)/(
b*x^4-2*b*x^2+a+b)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 - 2*b*x^2 + a + b)*(b*x^2 + (x^2 - 1)*sqrt(a + b)*sqrt
(b) - a - b)/(b*x^6 - 3*b*x^4 + (a + 3*b)*x^2 - a - b), x)
```

Sympy [F]

$$\int \frac{a+b+\sqrt{b}\sqrt{a+b}-\left(b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx = \int \frac{-a+\sqrt{b}x^2\sqrt{a+b}-\sqrt{b}\sqrt{a+b}+bx^2-b}{(x-1)(x+1)\sqrt{a+bx^4-2bx^2+b}}\,dx$$

input
```
integrate((a+b+b**(1/2)*(a+b)**(1/2)-(b+b**(1/2)*(a+b)**(1/2))*x**2)/(-x**
2+1)/(b*x**4-2*b*x**2+a+b)**(1/2),x)
```

output
```
Integral((-a + sqrt(b)*x**2*sqrt(a + b) - sqrt(b)*sqrt(a + b) + b*x**2 - b
)/((x - 1)*(x + 1)*sqrt(a + b*x**4 - 2*b*x**2 + b)), x)
```

Maxima [F]

$$\int \frac{a+b+\sqrt{b}\sqrt{a+b}-\left(b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx = \int \frac{\left(\sqrt{a+b}\sqrt{b}+b\right)x^2-a-\sqrt{a+b}\sqrt{b}-b}{\sqrt{bx^4-2\,bx^2+a+b}(x^2-1)}\,dx$$

input
```
integrate((a+b+b^(1/2)*(a+b)^(1/2)-(b+b^(1/2)*(a+b)^(1/2))*x^2)/(-x^2+1)/(
b*x^4-2*b*x^2+a+b)^(1/2),x, algorithm="maxima")
```

output
```
integrate(((sqrt(a + b)*sqrt(b) + b)*x^2 - a - sqrt(a + b)*sqrt(b) - b)/(s
qrt(b*x^4 - 2*b*x^2 + a + b)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{a+b+\sqrt{b}\sqrt{a+b}-\left(b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx = \int \frac{\left(\sqrt{a+b}\sqrt{b}+b\right)x^2-a-\sqrt{a+b}\sqrt{b}-b}{\sqrt{bx^4-2\,bx^2+a+b}(x^2-1)}\,dx$$

input
```
integrate((a+b+b^(1/2)*(a+b)^(1/2)-(b+b^(1/2)*(a+b)^(1/2))*x^2)/(-x^2+1)/(
b*x^4-2*b*x^2+a+b)^(1/2),x, algorithm="giac")
```

output
```
integrate(((sqrt(a + b)*sqrt(b) + b)*x^2 - a - sqrt(a + b)*sqrt(b) - b)/(s
qrt(b*x^4 - 2*b*x^2 + a + b)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b+\sqrt{b}\sqrt{a+b}-\left(b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx = \int -\frac{a+b-x^2\left(b+\sqrt{b}\,\sqrt{a+b}\right)+\sqrt{b}\,\sqrt{a+b}}{(x^2-1)\,\sqrt{b\,x^4-2\,b\,x^2+a+b}}\,dx$$

input
```
int(-(a + b - x^2*(b + b^(1/2)*(a + b)^(1/2)) + b^(1/2)*(a + b)^(1/2))/((x
^2 - 1)*(a + b - 2*b*x^2 + b*x^4)^(1/2)),x)
```

output
```
int(-(a + b - x^2*(b + b^(1/2)*(a + b)^(1/2)) + b^(1/2)*(a + b)^(1/2))/((x
^2 - 1)*(a + b - 2*b*x^2 + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b+\sqrt{b}\sqrt{a+b}-\left(b+\sqrt{b}\sqrt{a+b}\right)x^2}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx$$
$$=\int \frac{a+b+\sqrt{b}\,\sqrt{a+b}-\left(b+\sqrt{b}\,\sqrt{a+b}\right)x^2}{(-x^2+1)\sqrt{b\,x^4-2b\,x^2+a+b}}dx$$

input
```
int((a+b+b^(1/2)*(a+b)^(1/2)-(b+b^(1/2)*(a+b)^(1/2))*x^2)/(-x^2+1)/(b*x^4-
2*b*x^2+a+b)^(1/2),x)
```

output
```
int((a+b+b^(1/2)*(a+b)^(1/2)-(b+b^(1/2)*(a+b)^(1/2))*x^2)/(-x^2+1)/(b*x^4-
2*b*x^2+a+b)^(1/2),x)
```

3.190 $\int (1+2x^2)^3 (4-7x^2+x^4) \sqrt{2+5x^2+3x^4} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 257

$$
\begin{aligned}
\int (1+2x^2)^3 (4-7x^2+x^4) \sqrt{2+5x^2+3x^4} \, dx &= \frac{34898492x(2+3x^2)}{32837805\sqrt{2+5x^2+3x^4}} + \frac{x(23671285+19259523x^2)\sqrt{2+5x^2+3x^4}}{10945935} \\
&\quad + \frac{54101x(2+5x^2+3x^4)^{3/2}}{243243} + \frac{21742x^3(2+5x^2+3x^4)^{3/2}}{34749} - \frac{2116x^5(2+5x^2+3x^4)^{3/2}}{1287} \\
&\quad + \frac{8}{39}x^7(2+5x^2+3x^4)^{3/2} - \frac{34898492\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{32837805\sqrt{2+5x^2+3x^4}} + \frac{3048673\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}}{2189187\sqrt{2}}
\end{aligned}
$$

output
```
34898492/32837805*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/10945935*x*(19259523
*x^2+23671285)*(3*x^4+5*x^2+2)^(1/2)+54101/243243*x*(3*x^4+5*x^2+2)^(3/2)+
21742/34749*x^3*(3*x^4+5*x^2+2)^(3/2)-2116/1287*x^5*(3*x^4+5*x^2+2)^(3/2)+
8/39*x^7*(3*x^4+5*x^2+2)^(3/2)-34898492/32837805*2^(1/2)*(x^2+1)*((3*x^2+2
)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^
(1/2)+3048673/2189187*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJac
obiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.60

$$\int (1+2x^2)^3 (4-7x^2+x^4) \sqrt{2+5x^2+3x^4}\, dx$$
$$= \frac{3x(57080750+232961291x^2+322377915x^4+33267609x^6-393594255x^8-395181990x^{10}-94609620}{}$$

input
```
Integrate[(1 + 2*x^2)^3*(4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(3*x*(57080750 + 232961291*x^2 + 322377915*x^4 + 33267609*x^6 - 393594255*
x^8 - 395181990*x^10 - 94609620*x^12 + 20207880*x^14) - (34898492*I)*Sqrt[
3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] +
(4411762*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt
[3/2]*x], 2/3])/(32837805*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2207, 2207, 2207, 27, 2207, 1490, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2+1)^3 (x^4-7x^2+4) \sqrt{3x^4+5x^2+2}\, dx$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{1}{39}\int \sqrt{3x^4+5x^2+2}(-2116x^8-1906x^6+273x^4+663x^2+156)\, dx + \frac{8}{39}(3x^4+5x^2+2)^{3/2} x^7$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{1}{39}\left(\frac{1}{33}\int \sqrt{3x^4+5x^2+2}\left(21742x^6+30169x^4+21879x^2+5148\right)dx-\frac{2116}{33}x^5\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{8}{39}\left(3x^4+5x^2+2\right)^{3/2}x^7$$

$$\Big\downarrow \; 2207$$

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{1}{27}\int 3\sqrt{3x^4+5x^2+2}\left(54101x^4+153427x^2+46332\right)dx+\frac{21742}{27}\left(3x^4+5x^2+2\right)^{3/2}x^3\right)-\frac{2116}{33}x^5(3\right.$$
$$\frac{8}{39}\left(3x^4+5x^2+2\right)^{3/2}x^7$$

$$\Big\downarrow \; 27$$

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{1}{9}\int \sqrt{3x^4+5x^2+2}\left(54101x^4+153427x^2+46332\right)dx+\frac{21742}{27}\left(3x^4+5x^2+2\right)^{3/2}x^3\right)-\frac{2116}{33}x^5(3x^4\right.$$
$$\frac{8}{39}\left(3x^4+5x^2+2\right)^{3/2}x^7$$

$$\Big\downarrow \; 2207$$

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\int \left(2139947x^2+864770\right)\sqrt{3x^4+5x^2+2}dx+\frac{54101}{21}x\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{21742}{27}(3x^4+5x^2\right.\right.$$
$$\frac{8}{39}\left(3x^4+5x^2+2\right)^{3/2}x^7$$

$$\Big\downarrow \; 1490$$

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\left(\frac{1}{45}\int \frac{2\left(17449246x^2+15243365\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(19259523x^2+23671285\right)\right)+\frac{541}{2}\right.\right.\right.$$
$$\frac{8}{39}\left(3x^4+5x^2+2\right)^{3/2}x^7$$

$$\Big\downarrow \; 27$$

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\left(\frac{2}{45}\int \frac{17449246x^2+15243365}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(19259523x^2+23671285\right)\right)+\frac{54101}{21}x\right.\right.\right.$$
$$\frac{8}{39}\left(3x^4+5x^2+2\right)^{3/2}x^7$$

$$\Big\downarrow \; 1503$$

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\left(\frac{2}{45}\left(15243365\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx+17449246\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{45}x\sqrt{3x^4+5x^2+\right.\right.\right.\right.\right.$$
$$\frac{8}{39}\left(3x^4+5x^2+2\right)^{3/2}x^7$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\left(\frac{2}{45}\left(17449246\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{15243365(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)\right.\right.\right.\right.$$
$$\frac{8}{39}\left(3x^4+5x^2+2\right)^{3/2}x^7$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\left(\frac{2}{45}\left(\frac{15243365(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+17449246\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\right.\right.\right.\right.\right.\right.$$
$$\frac{8}{39}\left(3x^4+5x^2+2\right)^{3/2}x^7$$

input
```
Int[(1 + 2*x^2)^3*(4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(8*x^7*(2 + 5*x^2 + 3*x^4)^(3/2))/39 + ((-2116*x^5*(2 + 5*x^2 + 3*x^4)^(3/
2))/33 + ((21742*x^3*(2 + 5*x^2 + 3*x^4)^(3/2))/27 + ((54101*x*(2 + 5*x^2
+ 3*x^4)^(3/2))/21 + ((x*(23671285 + 19259523*x^2)*Sqrt[2 + 5*x^2 + 3*x^4]
)/45 + (2*(17449246*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2
]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqr
t[2 + 5*x^2 + 3*x^4])) + (15243365*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*E
llipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])))/45)/21)/9)/
33)/39
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 16.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.56

method	result
risch	$\frac{x(6735960x^{10}-42763140x^8-64946070x^6+5554125x^4+45129708x^2+28540375)\sqrt{3x^4+5x^2+2}}{10945935}-\frac{3048673i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{Ellip}}{2189187\sqrt{3x^4+5x^2}}$
default	$\frac{5708075x\sqrt{3x^4+5x^2+2}}{2189187}-\frac{3048673i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2189187\sqrt{3x^4+5x^2+2}}+\frac{34898492i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{Ellip}\right.}{32837805\sqrt{3x^4+5x^2+2}}$
elliptic	$\frac{5708075x\sqrt{3x^4+5x^2+2}}{2189187}-\frac{3048673i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2189187\sqrt{3x^4+5x^2+2}}+\frac{34898492i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{Ellip}\right.}{32837805\sqrt{3x^4+5x^2+2}}$

input
```
int((2*x^2+1)^3*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/10945935*x*(6735960*x^10-42763140*x^8-64946070*x^6+5554125*x^4+45129708*
x^2+28540375)*(3*x^4+5*x^2+2)^(1/2)-3048673/2189187*I*(x^2+1)^(1/2)*(6*x^2
+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+34898492/328378
05*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/
2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.36

$$\int \left(1+2x^2\right)^3 \left(4-7x^2+x^4\right) \sqrt{2+5x^2+3x^4}\, dx =$$
$$-\frac{69796984\sqrt{3}\sqrt{-\frac{2}{3}}xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 206987269\sqrt{3}\sqrt{-\frac{2}{3}}xF(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 3\,(2020788}{}$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/98513415*(69796984*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3)/x)
, 3/2) - 206987269*sqrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/x), 3
/2) - 3*(20207880*x^12 - 128289420*x^10 - 194838210*x^8 + 16662375*x^6 + 1
35389124*x^4 + 85621125*x^2 + 34898492)*sqrt(3*x^4 + 5*x^2 + 2))/x
```

Sympy [F]

$$\int \left(1+2x^2\right)^3 \left(4-7x^2+x^4\right) \sqrt{2+5x^2+3x^4}\, dx$$
$$= \int \sqrt{(x^2+1)\,(3x^2+2)}\left(2x^2+1\right)^3 \left(x^4-7x^2+4\right)\, dx$$

input
```
integrate((2*x**2+1)**3*(x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt((x**2 + 1)*(3*x**2 + 2))*(2*x**2 + 1)**3*(x**4 - 7*x**2 + 4)
, x)
```

Maxima [F]

$$\int (1+2x^2)^3 (4-7x^2+x^4) \sqrt{2+5x^2+3x^4}\, dx$$
$$= \int \sqrt{3\,x^4+5\,x^2+2}(x^4-7\,x^2+4)\,(2\,x^2+1)^3\, dx$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1)^3, x)
```

Giac [F]

$$\int (1+2x^2)^3 (4-7x^2+x^4) \sqrt{2+5x^2+3x^4}\, dx$$
$$= \int \sqrt{3\,x^4+5\,x^2+2}(x^4-7\,x^2+4)\,(2\,x^2+1)^3\, dx$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x^2)^3 (4-7x^2+x^4) \sqrt{2+5x^2+3x^4}\, dx$$
$$= \int (2\,x^2+1)^3\,(x^4-7\,x^2+4)\,\sqrt{3\,x^4+5\,x^2+2}\, dx$$

input
```
int((2*x^2 + 1)^3*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int((2*x^2 + 1)^3*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \left(1+2x^2\right)^3 \left(4-7x^2+x^4\right) \sqrt{2+5x^2+3x^4}\,dx$$
$$= \frac{8\sqrt{3x^4+5x^2+2}\,x^{11}}{13} - \frac{1676\sqrt{3x^4+5x^2+2}\,x^9}{429} - \frac{68726\sqrt{3x^4+5x^2+2}\,x^7}{11583} + \frac{123425\sqrt{3x^4+5x^2+2}\,x^5}{243243} + \frac{385724\sqrt{3x^4+5x^2+2}\,x^3}{93555} + \frac{5708075\sqrt{3x^4+5x^2+2}\,x}{2189187} + \frac{6097346\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)}{2189187} + \frac{34898492\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)}{10945935}$$

input
```
int((2*x^2+1)^3*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(6735960*sqrt(3*x**4 + 5*x**2 + 2)*x**11 - 42763140*sqrt(3*x**4 + 5*x**2 +
 2)*x**9 - 64946070*sqrt(3*x**4 + 5*x**2 + 2)*x**7 + 5554125*sqrt(3*x**4 +
 5*x**2 + 2)*x**5 + 45129708*sqrt(3*x**4 + 5*x**2 + 2)*x**3 + 28540375*sqr
t(3*x**4 + 5*x**2 + 2)*x + 30486730*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4
+ 5*x**2 + 2),x) + 34898492*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 +
 5*x**2 + 2),x))/10945935
```

3.191 $\int (1+2x^2)^2 (4-7x^2+x^4) \sqrt{2+5x^2+3x^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 234

$$\begin{aligned}
&\int (1+2x^2)^2 (4-7x^2+x^4) \sqrt{2+5x^2+3x^4}\,dx \\
&= \frac{1291552x(2+3x^2)}{841995\sqrt{2+5x^2+3x^4}} + \frac{x(88715-195687x^2)\sqrt{2+5x^2+3x^4}}{280665} \\
&\quad + \frac{5893x(2+5x^2+3x^4)^{3/2}}{6237} - \frac{952}{891}x^3(2+5x^2+3x^4)^{3/2} \\
&\quad + \frac{4}{33}x^5(2+5x^2+3x^4)^{3/2} - \frac{1291552\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{841995\sqrt{2+5x^2+3x^4}} \\
&\quad + \frac{100715\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{56133\sqrt{2+5x^2+3x^4}}
\end{aligned}$$

output

```
1291552/841995*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/280665*x*(-195687*x^2+8
8715)*(3*x^4+5*x^2+2)^(1/2)+5893/6237*x*(3*x^4+5*x^2+2)^(3/2)-952/891*x^3*
(3*x^4+5*x^2+2)^(3/2)+4/33*x^5*(3*x^4+5*x^2+2)^(3/2)-1291552/841995*2^(1/2
)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2
))/(3*x^4+5*x^2+2)^(1/2)+100715/56133*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^
(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.64

$$\int (1+2x^2)^2 (4-7x^2+x^4) \sqrt{2+5x^2+3x^4}\, dx$$
$$= \frac{3x(1238170+4156381x^2+3238035x^4-3046671x^6-5350995x^8-1678320x^{10}+306180x^{12})-129155}{841995\sqrt{}}$$

input
```
Integrate[(1 + 2*x^2)^2*(4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(3*x*(1238170 + 4156381*x^2 + 3238035*x^4 - 3046671*x^6 - 5350995*x^8 - 16
78320*x^10 + 306180*x^12) - (1291552*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x
^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (284402*I)*Sqrt[3]*Sqrt[1 + x
^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(841995*Sqrt[2
 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2207, 2207, 27, 2207, 1490, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2+1)^2 (x^4-7x^2+4) \sqrt{3x^4+5x^2+2}\, dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{33}\int \sqrt{3x^4+5x^2+2}(-952x^6-403x^4+297x^2+132)\, dx+\frac{4}{33}(3x^4+5x^2+2)^{3/2}x^5$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{33}\left(\frac{1}{27}\int 3\sqrt{3x^4+5x^2+2}\left(5893x^4+4577x^2+1188\right)dx-\frac{952}{27}x^3\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{4}{33}\left(3x^4+5x^2+2\right)^{3/2}x^5$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{33}\left(\frac{1}{9}\int \sqrt{3x^4+5x^2+2}\left(5893x^4+4577x^2+1188\right)dx-\frac{952}{27}x^3\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{4}{33}\left(3x^4+5x^2+2\right)^{3/2}x^5$$

$$\Big\downarrow \text{ 2207}$$

$$\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\int \left(13162-21743x^2\right)\sqrt{3x^4+5x^2+2}dx+\frac{5893}{21}x\left(3x^4+5x^2+2\right)^{3/2}\right)-\frac{952}{27}x^3\left(3x^4+5x^2+2\right)^{3/2}\right)\frac{4}{33}\left(3x^4+5x^2+2\right)^{3/2}x^5$$

$$\Big\downarrow \text{ 1490}$$

$$\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\left(\frac{1}{45}\int \frac{2\left(645776x^2+503575\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(88715-195687x^2\right)\right)+\frac{5893}{21}x\left(3x^4+5x^2\right.\right.\right.\frac{4}{33}\left(3x^4+5x^2+2\right)^{3/2}x^5$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\left(\frac{2}{45}\int \frac{645776x^2+503575}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(88715-195687x^2\right)\right)+\frac{5893}{21}x\left(3x^4+5x^2+2\right)\right.\right.\frac{4}{33}\left(3x^4+5x^2+2\right)^{3/2}x^5$$

$$\Big\downarrow \text{ 1503}$$

$$\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\left(\frac{2}{45}\left(503575\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+645776\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(88715\right.\right.\right.\right.\frac{4}{33}\left(3x^4+5x^2+2\right)^{3/2}x^5$$

$$\Big\downarrow \text{ 1413}$$

$$\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\left(\frac{2}{45}\left(645776\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{503575\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+\frac{1}{45}x\sqrt{\right.\right.\right.\frac{4}{33}\left(3x^4+5x^2+2\right)^{3/2}x^5$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{33}\left(\frac{1}{9}\left(\frac{1}{21}\left(\frac{2}{45}\left(\frac{503575(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+645776\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1\right.\right.\right.\right.\right.$$
$$\frac{4}{33}\left(3x^4+5x^2+2\right)^{3/2}x^5$$

input
```
Int[(1 + 2*x^2)^2*(4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(4*x^5*(2 + 5*x^2 + 3*x^4)^(3/2))/33 + ((-952*x^3*(2 + 5*x^2 + 3*x^4)^(3/2
))/27 + ((5893*x*(2 + 5*x^2 + 3*x^4)^(3/2))/21 + ((x*(88715 - 195687*x^2)*
Sqrt[2 + 5*x^2 + 3*x^4])/45 + (2*(645776*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^
2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[Arc
Tan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (503575*(1 + x^2)*Sqrt[(2 +
3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*
x^4])))/45)/21)/9)/33
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 10.84 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.60

method	result
risch	$\frac{x(102060x^8-729540x^6-635805x^4+530478x^2+619085)\sqrt{3x^4+5x^2+2}}{280665}-\frac{100715i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{56133\sqrt{3x^4+5x^2+2}}+\frac{129155}{}$
default	$\frac{123817x\sqrt{3x^4+5x^2+2}}{56133}-\frac{100715i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{56133\sqrt{3x^4+5x^2+2}}+\frac{1291552i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{Elliptic}\right.}{841995\sqrt{3x^4+5x^2+2}}$
elliptic	$\frac{123817x\sqrt{3x^4+5x^2+2}}{56133}-\frac{100715i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{56133\sqrt{3x^4+5x^2+2}}+\frac{1291552i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{Elliptic}\right.}{841995\sqrt{3x^4+5x^2+2}}$

input
```
int((2*x^2+1)^2*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/280665*x*(102060*x^8-729540*x^6-635805*x^4+530478*x^2+619085)*(3*x^4+5*x
^2+2)^(1/2)-100715/56133*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(
1/2)*EllipticF(I*x,1/2*6^(1/2))+1291552/841995*I*(x^2+1)^(1/2)*(6*x^2+4)^(
1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6
^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.37

$$\int (1+2x^2)^2 (4-7x^2+x^4) \sqrt{2+5x^2+3x^4}\, dx =$$

$$-\frac{2583104\sqrt{3}\sqrt{-\frac{2}{3}}xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 7115279\sqrt{3}\sqrt{-\frac{2}{3}}xF(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 3(306180x^{10}}{2525985x}$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/2525985*(2583104*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3)/x),
3/2) - 7115279*sqrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/x), 3/2)
- 3*(306180*x^10 - 2188620*x^8 - 1907415*x^6 + 1591434*x^4 + 1857255*x^2 +
 1291552)*sqrt(3*x^4 + 5*x^2 + 2))/x
```

Sympy [F]

$$\begin{aligned}\int (1+2x^2)^2 (4-7x^2+x^4) \sqrt{2+5x^2+3x^4}\, dx \\ = \int \sqrt{(x^2+1)(3x^2+2)}(2x^2+1)^2 (x^4-7x^2+4)\ dx\end{aligned}$$

input
```
integrate((2*x**2+1)**2*(x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt((x**2 + 1)*(3*x**2 + 2))*(2*x**2 + 1)**2*(x**4 - 7*x**2 + 4)
, x)
```

Maxima [F]

$$\int \left(1+2x^2\right)^2 \left(4-7x^2+x^4\right) \sqrt{2+5x^2+3x^4}\,dx = \int \sqrt{3\,x^4+5\,x^2+2}\left(x^4-7\,x^2+4\right)\left(2\,x^2+1\right)^2\,dx$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1)^2, x)
```

Giac [F]

$$\int \left(1+2x^2\right)^2 \left(4-7x^2+x^4\right) \sqrt{2+5x^2+3x^4}\,dx = \int \sqrt{3\,x^4+5\,x^2+2}\left(x^4-7\,x^2+4\right)\left(2\,x^2+1\right)^2\,dx$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+2x^2\right)^2 \left(4-7x^2+x^4\right) \sqrt{2+5x^2+3x^4}\,dx = \int \left(2\,x^2+1\right)^2 \left(x^4-7\,x^2+4\right) \sqrt{3\,x^4+5\,x^2+2}\,dx$$

input
```
int((2*x^2 + 1)^2*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int((2*x^2 + 1)^2*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \left(1+2x^2\right)^2 \left(4-7x^2+x^4\right) \sqrt{2+5x^2+3x^4}\,dx = \frac{4\sqrt{3x^4+5x^2+2}\,x^9}{11} - \frac{772\sqrt{3x^4+5x^2+2}\,x^7}{297} - \frac{14129\sqrt{3x^4+5x^2+2}\,x^5}{6237} + \frac{58942\sqrt{3x^4+5x^2+2}\,x^3}{31185} + \frac{123817\sqrt{3x^4+5x^2+2}\,x}{56133} + \frac{201430\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)}{56133} + \frac{1291552\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)}{280665}$$

input
```
int((2*x^2+1)^2*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(102060*sqrt(3*x**4 + 5*x**2 + 2)*x**9 - 729540*sqrt(3*x**4 + 5*x**2 + 2)*
x**7 - 635805*sqrt(3*x**4 + 5*x**2 + 2)*x**5 + 530478*sqrt(3*x**4 + 5*x**2
 + 2)*x**3 + 619085*sqrt(3*x**4 + 5*x**2 + 2)*x + 1007150*int(sqrt(3*x**4
+ 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x) + 1291552*int((sqrt(3*x**4 + 5*x**2
 + 2)*x**2)/(3*x**4 + 5*x**2 + 2),x))/280665
```

3.192 $\int (1+2x^2)(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 211

$$\begin{aligned}\int (1+2x^2)(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}\,dx &= \frac{7484x(2+3x^2)}{5103\sqrt{2+5x^2+3x^4}}+\frac{x(5935+5121x^2)\sqrt{2+5x^2+3x^4}}{1701}-\frac{137}{189}x(2+5x^2+3x^4)^{3/2}\\ &\quad+\frac{2}{27}x^3(2+5x^2+3x^4)^{3/2}-\frac{7484\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\middle|-\frac{1}{2}\right)}{5103\sqrt{2+5x^2+3x^4}}\\ &\quad+\frac{3335\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{1701\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
7484/5103*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/1701*x*(5121*x^2+5935)*(3*x^
4+5*x^2+2)^(1/2)-137/189*x*(3*x^4+5*x^2+2)^(3/2)+2/27*x^3*(3*x^4+5*x^2+2)^
(3/2)-7484/5103*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2
+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+3335/1701*2^(1/2)*(x^2+1)*(
(3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5
*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

$$\int (1+2x^2)(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}\,dx$$
$$= \frac{3x(6938+15761x^2+309x^4-16965x^6-7317x^8+1134x^{10})-7484i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\Big(i\operatorname{arcsinh}\Big(}{5103\sqrt{2+5x^2+3x^4}}$$

input

```
Integrate[(1 + 2*x^2)*(4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output

```
(3*x*(6938 + 15761*x^2 + 309*x^4 - 16965*x^6 - 7317*x^8 + 1134*x^10) - (74
84*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*
x], 2/3] + (814*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSi
nh[Sqrt[3/2]*x], 2/3])/(5103*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2207, 27, 2207, 27, 1490, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2+1)(x^4-7x^2+4)\sqrt{3x^4+5x^2+2}\,dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{27}\int 3(-137x^4+5x^2+36)\sqrt{3x^4+5x^2+2}dx+\frac{2}{27}(3x^4+5x^2+2)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int (-137x^4+5x^2+36)\sqrt{3x^4+5x^2+2}dx+\frac{2}{27}(3x^4+5x^2+2)^{3/2}x^3$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{9}\left(\frac{1}{21}\int 5\left(569x^2+206\right)\sqrt{3x^4+5x^2+2}dx-\frac{137}{21}x\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{2}{27}\left(3x^4+5x^2+2\right)^{3/2}x^3$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\left(\frac{5}{21}\int \left(569x^2+206\right)\sqrt{3x^4+5x^2+2}dx-\frac{137}{21}x\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{2}{27}\left(3x^4+5x^2+2\right)^{3/2}x^3$$

$$\Big\downarrow \; 1490$$

$$\frac{1}{9}\left(\frac{5}{21}\left(\frac{1}{45}\int \frac{2\left(3742x^2+3335\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(5121x^2+5935\right)\right)-\frac{137}{21}x\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{2}{27}\left(3x^4+5x^2+2\right)^{3/2}x^3$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\left(\frac{5}{21}\left(\frac{2}{45}\int \frac{3742x^2+3335}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(5121x^2+5935\right)\right)-\frac{137}{21}x\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{2}{27}\left(3x^4+5x^2+2\right)^{3/2}x^3$$

$$\Big\downarrow \; 1503$$

$$\frac{1}{9}\left(\frac{5}{21}\left(\frac{2}{45}\left(3335\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+3742\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(5121x^2+5935\right)\right. \right. \frac{2}{27}\left(3x^4+5x^2+2\right)^{3/2}x^3$$

$$\Big\downarrow \; 1413$$

$$\frac{1}{9}\left(\frac{5}{21}\left(\frac{2}{45}\left(3742\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{3335\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+\frac{1}{45}x\sqrt{3x^4+5x\right.\right. \frac{2}{27}\left(3x^4+5x^2+2\right)^{3/2}x^3$$

$$\Big\downarrow \; 1456$$

$$\frac{1}{9}\left(\frac{5}{21}\left(\frac{2}{45}\left(\frac{3335(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+3742\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}}{3\sqrt{3x^4+}}\right.\right.\right.\right.$$

$$\frac{2}{27}\left(3x^4+5x^2+2\right)^{3/2}x^3$$

input
```
Int[(1 + 2*x^2)*(4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(2*x^3*(2 + 5*x^2 + 3*x^4)^(3/2))/27 + ((-137*x*(2 + 5*x^2 + 3*x^4)^(3/2))
/21 + (5*((x*(5935 + 5121*x^2)*Sqrt[2 + 5*x^2 + 3*x^4])/45 + (2*(3742*((x*
(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*
x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) +
 (3335*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(
Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])))/45))/21)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 7.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.64

method	result
risch	$\frac{x(378x^6-3069x^4-792x^2+3469)\sqrt{3x^4+5x^2+2}}{1701}-\frac{3335i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{1701\sqrt{3x^4+5x^2+2}}+\frac{7484i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ (\text{EllipticF}\ldots}{5103\sqrt{3x^4\ldots}$
default	$-\frac{88x^3\sqrt{3x^4+5x^2+2}}{189}+\frac{3469x\sqrt{3x^4+5x^2+2}}{1701}-\frac{3335i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{1701\sqrt{3x^4+5x^2+2}}+\frac{7484i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ (\text{EllipticF}\ldots}{5103\sqrt{3x^4\ldots}$
elliptic	$-\frac{88x^3\sqrt{3x^4+5x^2+2}}{189}+\frac{3469x\sqrt{3x^4+5x^2+2}}{1701}-\frac{3335i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{1701\sqrt{3x^4+5x^2+2}}+\frac{7484i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ (\text{EllipticF}\ldots}{5103\sqrt{3x^4\ldots}$

input
```
int((2*x^2+1)*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1701*x*(378*x^6-3069*x^4-792*x^2+3469)*(3*x^4+5*x^2+2)^(1/2)-3335/1701*I
*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(
1/2))+7484/5103*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(Ell
ipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int (1+2x^2)(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}\,dx =$$

$$-\frac{14968\sqrt{3}\sqrt{-\frac{2}{3}}xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 44983\sqrt{3}\sqrt{-\frac{2}{3}}xF(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 3\,(1134\,x^8 - 9207\,x}{15309\,x}$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="fric
as")
```

output
```
-1/15309*(14968*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3)/x), 3/2)
 - 44983*sqrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/x), 3/2) - 3*(1
134*x^8 - 9207*x^6 - 2376*x^4 + 10407*x^2 + 7484)*sqrt(3*x^4 + 5*x^2 + 2))
/x
```

Sympy [F]

$$\int (1+2x^2)(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}\,dx = \int \sqrt{(x^2+1)(3x^2+2)}(2x^2+1)(x^4-7x^2+4)\,dx$$

input
```
integrate((2*x**2+1)*(x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt((x**2 + 1)*(3*x**2 + 2))*(2*x**2 + 1)*(x**4 - 7*x**2 + 4), x
)
```

Maxima [F]

$$\int (1+2x^2)\left(4-7x^2+x^4\right)\sqrt{2+5x^2+3x^4}\,dx = \int \sqrt{3\,x^4+5\,x^2+2}\left(x^4-7\,x^2+4\right)\left(2\,x^2+1\right)dx$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1), x)
```

Giac [F]

$$\int (1+2x^2)\left(4-7x^2+x^4\right)\sqrt{2+5x^2+3x^4}\,dx = \int \sqrt{3\,x^4+5\,x^2+2}\left(x^4-7\,x^2+4\right)\left(2\,x^2+1\right)dx$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x^2)\left(4-7x^2+x^4\right)\sqrt{2+5x^2+3x^4}\,dx = \int \left(2\,x^2+1\right)\,\left(x^4-7\,x^2+4\right)\,\sqrt{3\,x^4+5\,x^2+2}\,dx$$

input
```
int((2*x^2 + 1)*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int((2*x^2 + 1)*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int (1+2x^2)(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}\,dx = \frac{2\sqrt{3x^4+5x^2+2}\,x^7}{9} - \frac{341\sqrt{3x^4+5x^2+2}\,x^5}{189} - \frac{88\sqrt{3x^4+5x^2+2}\,x^3}{189} + \frac{3469\sqrt{3x^4+5x^2+2}\,x}{1701} + \frac{6670\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)}{1701} + \frac{7484\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)}{1701}$$

input
```
int((2*x^2+1)*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(378*sqrt(3*x**4 + 5*x**2 + 2)*x**7 - 3069*sqrt(3*x**4 + 5*x**2 + 2)*x**5
- 792*sqrt(3*x**4 + 5*x**2 + 2)*x**3 + 3469*sqrt(3*x**4 + 5*x**2 + 2)*x +
6670*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x) + 7484*int((sq
rt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2 + 2),x))/1701
```

3.193 $\int (4-7x^2+x^4)\sqrt{2+5x^2+3x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 188

$$\begin{aligned}\int (4-7x^2+x^4)\sqrt{2+5x^2+3x^4}\,dx &= \frac{8488x(2+3x^2)}{2835\sqrt{2+5x^2+3x^4}}+\frac{1}{945}x(395-1503x^2)\sqrt{2+5x^2+3x^4}\\&\quad+\frac{1}{21}x(2+5x^2+3x^4)^{3/2}-\frac{8488\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{2835\sqrt{2+5x^2+3x^4}}\\&\quad+\frac{659\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{189\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
8488/2835*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/945*x*(-1503*x^2+395)*(3*x^4
+5*x^2+2)^(1/2)+1/21*x*(3*x^4+5*x^2+2)^(3/2)-8488/2835*2^(1/2)*(x^2+1)*((3
*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x
^2+2)^(1/2)+659/189*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacob
iAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.56 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.74

$$\int (4-7x^2+x^4)\sqrt{2+5x^2+3x^4}\,dx$$
$$=\frac{3x(970-131x^2-4665x^4-3159x^6+405x^8)-8488i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)+18}{2835\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[(4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(3*x*(970 - 131*x^2 - 4665*x^4 - 3159*x^6 + 405*x^8) - (8488*I)*Sqrt[3]*Sq
rt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (1898
*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x]
, 2/3])/(2835*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2207, 1490, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^4-7x^2+4)\sqrt{3x^4+5x^2+2}\,dx$$
$$\Big\downarrow \ 2207$$
$$\frac{1}{21}\int (82-167x^2)\sqrt{3x^4+5x^2+2}dx+\frac{1}{21}x(3x^4+5x^2+2)^{3/2}$$
$$\Big\downarrow \ 1490$$

$$\frac{1}{21}\left(\frac{1}{45}\int \frac{2(4244x^2+3295)}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{45}x\sqrt{3x^4+5x^2+2}(395-1503x^2)\right)+\frac{1}{21}x(3x^4+5x^2+2)^{3/2}$$

$\big\downarrow$ 27

$$\frac{1}{21}\left(\frac{2}{45}\int \frac{4244x^2+3295}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{45}x\sqrt{3x^4+5x^2+2}(395-1503x^2)\right)+\frac{1}{21}x(3x^4+5x^2+2)^{3/2}$$

$\big\downarrow$ 1503

$$\frac{1}{21}\left(\frac{2}{45}\left(3295\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+4244\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{45}x\sqrt{3x^4+5x^2+2}(395-1503x^2)\right)+\frac{1}{21}x(3x^4+5x^2+2)^{3/2}$$

$\big\downarrow$ 1413

$$\frac{1}{21}\left(\frac{2}{45}\left(4244\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{3295(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+\frac{1}{45}x\sqrt{3x^4+5x^2+\right.$$
$$\frac{1}{21}x(3x^4+5x^2+2)^{3/2}$$

$\big\downarrow$ 1456

$$\frac{1}{21}\left(\frac{2}{45}\left(\frac{3295(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+4244\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E(a}{3\sqrt{3x^4+5x^2}}\right.\right.\right.$$
$$\frac{1}{21}x(3x^4+5x^2+2)^{3/2}$$

input
```
Int[(4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(x*(2 + 5*x^2 + 3*x^4)^(3/2))/21 + ((x*(395 - 1503*x^2)*Sqrt[2 + 5*x^2 + 3
*x^4])/45 + (2*(4244*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[
2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sq
rt[2 + 5*x^2 + 3*x^4])) + (3295*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Elli
pticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])))/45)/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 3.92 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.69

method	result
risch	$\frac{x(135x^4-1278x^2+485)\sqrt{3x^4+5x^2+2}}{945}-\frac{659i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{189\sqrt{3x^4+5x^2+2}}+\frac{8488i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\right.}{2835\sqrt{3x^4+5x^2+2}}$
default	$\frac{x^5\sqrt{3x^4+5x^2+2}}{7}-\frac{142x^3\sqrt{3x^4+5x^2+2}}{105}+\frac{97x\sqrt{3x^4+5x^2+2}}{189}-\frac{659i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{189\sqrt{3x^4+5x^2+2}}+\frac{8488i\sqrt{x^2+1}\,\sqrt{6x}}{}$
elliptic	$\frac{x^5\sqrt{3x^4+5x^2+2}}{7}-\frac{142x^3\sqrt{3x^4+5x^2+2}}{105}+\frac{97x\sqrt{3x^4+5x^2+2}}{189}-\frac{659i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{189\sqrt{3x^4+5x^2+2}}+\frac{8488i\sqrt{x^2+1}\,\sqrt{6x}}{}$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/945*x*(135*x^4-1278*x^2+485)*(3*x^4+5*x^2+2)^(1/2)-659/189*I*(x^2+1)^(1/
2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+8488/2
835*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1
/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41

$$\int \left(4-7x^2+x^4\right)\sqrt{2+5x^2+3x^4}\,dx =$$

$$-\frac{16976\sqrt{3}\sqrt{-\frac{2}{3}}xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2})-46631\sqrt{3}\sqrt{-\frac{2}{3}}xF(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2})-3\left(405\,x^6-3834\,x^4\right.}{8505\,x}$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/8505*(16976*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3)/x), 3/2)
- 46631*sqrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/x), 3/2) - 3*(40
5*x^6 - 3834*x^4 + 1455*x^2 + 8488)*sqrt(3*x^4 + 5*x^2 + 2))/x
```

Sympy [F]

$$\int (4-7x^2+x^4)\sqrt{2+5x^2+3x^4}\,dx = \int \sqrt{(x^2+1)(3x^2+2)}\,(x^4-7x^2+4)\;dx$$

input
```
integrate((x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral(sqrt((x**2 + 1)*(3*x**2 + 2))*(x**4 - 7*x**2 + 4), x)
```

Maxima [F]

$$\int (4-7x^2+x^4)\sqrt{2+5x^2+3x^4}\,dx = \int \sqrt{3\,x^4+5\,x^2+2}\,(x^4-7\,x^2+4)\;dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4), x)
```

Giac [F]

$$\int \left(4-7x^2+x^4\right)\sqrt{2+5x^2+3x^4}\,dx=\int \sqrt{3\,x^4+5\,x^2+2}\left(x^4-7\,x^2+4\right)dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(4-7x^2+x^4\right)\sqrt{2+5x^2+3x^4}\,dx=\int \left(x^4-7\,x^2+4\right)\sqrt{3\,x^4+5\,x^2+2}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \left(4-7x^2+x^4\right)\sqrt{2+5x^2+3x^4}\,dx=\frac{\sqrt{3x^4+5x^2+2}\,x^5}{7}-\frac{142\sqrt{3x^4+5x^2+2}\,x^3}{105}+\frac{97\sqrt{3x^4+5x^2+2}\,x}{189}+\frac{1318\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)}{189}+\frac{8488\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)}{945}$$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(135*sqrt(3*x**4 + 5*x**2 + 2)*x**5 - 1278*sqrt(3*x**4 + 5*x**2 + 2)*x**3
+ 485*sqrt(3*x**4 + 5*x**2 + 2)*x + 6590*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*
x**4 + 5*x**2 + 2),x) + 8488*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4
+ 5*x**2 + 2),x))/945
```

3.194 $\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{1+2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 249

$$\begin{aligned}\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{1+2x^2}\,dx &= \frac{1879x(2+3x^2)}{1080\sqrt{2+5x^2+3x^4}}-\frac{43}{36}x\sqrt{2+5x^2+3x^4}\\ &\quad+\frac{1}{10}x^3\sqrt{2+5x^2+3x^4}\\ &\quad-\frac{1879(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{540\sqrt{2}\sqrt{2+5x^2+3x^4}}\\ &\quad+\frac{95(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{36\sqrt{2}\sqrt{2+5x^2+3x^4}}\\ &\quad+\frac{31(1+x^2)\operatorname{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{4\sqrt{3}\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output

```
1879/1080*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-43/36*x*(3*x^4+5*x^2+2)^(1/2)+
1/10*x^3*(3*x^4+5*x^2+2)^(1/2)-1879/1080*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1
))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+95
/72*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/
2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+31/12*(x^2+1)*EllipticPi(x*6^(1/2)/(6*x
^2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))^(1/2)/(3*x^4+5*x
^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.73

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{1+2x^2}\,dx = \frac{-5160x-12468x^3-6660x^5+648x^7-3758i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-2327i\sqrt{3}\sqrt{}}{2160\sqrt{}}$$

input
```
Integrate[((4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4])/(1 + 2*x^2),x]
```

output
```
(-5160*x - 12468*x^3 - 6660*x^5 + 648*x^7 - (3758*I)*Sqrt[3]*Sqrt[1 + x^2]
*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (2327*I)*Sqrt[3]
*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (1
395*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticPi[4/3, I*ArcSinh[Sqr
t[3/2]*x], 2/3])/(2160*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4-7x^2+4)\sqrt{3x^4+5x^2+2}}{2x^2+1}\,dx$$

$$\Big\downarrow \text{2258}$$

$$\int \left(-\frac{35x^4}{4\sqrt{3x^4+5x^2+2}}-\frac{49x^2}{8\sqrt{3x^4+5x^2+2}}+\frac{31}{16(2x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{97}{16\sqrt{3x^4+5x^2+2}}+\frac{3x^6}{2\sqrt{3x^4+5}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{9\sqrt{3x^4+5x^2+2}}+\frac{79(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{36\sqrt{2}\sqrt{3x^4+5x^2+2}}-\frac{73\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{135\sqrt{3x^4+5x^2+2}}-\frac{259(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{108\sqrt{2}\sqrt{3x^4+5x^2+2}}+\frac{31(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{4\sqrt{3}\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}-\frac{43}{36}\sqrt{3x^4+5x^2+2}x+\frac{1879(3x^2+2)x}{1080\sqrt{3x^4+5x^2+2}}+\frac{1}{10}\sqrt{3x^4+5x^2+2}x^3$$

input
```
Int[((4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4])/(1 + 2*x^2),x]
```

output
```
(1879*x*(2 + 3*x^2))/(1080*Sqrt[2 + 5*x^2 + 3*x^4]) - (43*x*Sqrt[2 + 5*x^2
 + 3*x^4])/36 + (x^3*Sqrt[2 + 5*x^2 + 3*x^4])/10 - (259*(1 + x^2)*Sqrt[(2
+ 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(108*Sqrt[2]*Sqrt[2 + 5*x^
2 + 3*x^4]) - (73*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[
ArcTan[x], -1/2])/(135*Sqrt[2 + 5*x^2 + 3*x^4]) + (79*(1 + x^2)*Sqrt[(2 +
3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(36*Sqrt[2]*Sqrt[2 + 5*x^2 +
 3*x^4]) + (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcT
an[x], -1/2])/(9*Sqrt[2 + 5*x^2 + 3*x^4]) + (31*(1 + x^2)*EllipticPi[-1/3,
 ArcTan[Sqrt[3/2]*x], 1/3])/(4*Sqrt[3]*Sqrt[(1 + x^2)/(2 + 3*x^2)]*Sqrt[2
+ 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 5.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.69

method	result
elliptic	$\frac{x^3\sqrt{3x^4+5x^2+2}}{10} - \frac{43x\sqrt{3x^4+5x^2+2}}{36} - \frac{10739i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{4320\sqrt{3x^4+5x^2+2}} - \frac{1879i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right.}{1080\sqrt{3x^4+5x^2+2}}$ …
risch	$\frac{x(18x^2-215)\sqrt{3x^4+5x^2+2}}{180} - \frac{1217i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{288\sqrt{3x^4+5x^2+2}} + \frac{1879i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\ldots\right.}{1080\sqrt{3x^4+5x^2+2}}$ …
default	$-\frac{43x\sqrt{3x^4+5x^2+2}}{36} - \frac{101i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{288\sqrt{3x^4+5x^2+2}} - \frac{1153i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right.\right.}{540\sqrt{3x^4+5x^2+2}}$ …

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/10*x^3*(3*x^4+5*x^2+2)^(1/2)-43/36*x*(3*x^4+5*x^2+2)^(1/2)-10739/4320*I*
(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1
/2))-1879/1080*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*Ellip
ticE(I*x,1/2*6^(1/2))-31/16*I*(x^2+1)^(1/2)*(1+3/2*x^2)^(1/2)/(3*x^4+5*x^2
+2)^(1/2)*EllipticPi(I*x,2,1/2*I*(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{1+2x^2}\,dx = \int \frac{\sqrt{3\,x^4+5\,x^2+2}(x^4-7\,x^2+4)}{2\,x^2+1}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1),x, algorithm="fric
as")
```

output
```
integral(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{1+2x^2}\,dx = \int \frac{\sqrt{(x^2+1)(3x^2+2)}(x^4-7x^2+4)}{2x^2+1}\,dx$$

input
```
integrate((x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(1/2)/(2*x**2+1),x)
```

output
```
Integral(sqrt((x**2 + 1)*(3*x**2 + 2))*(x**4 - 7*x**2 + 4)/(2*x**2 + 1), x
)
```

Maxima [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{1+2x^2}\,dx = \int \frac{\sqrt{3\,x^4+5\,x^2+2}(x^4-7\,x^2+4)}{2\,x^2+1}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1),x, algorithm="maxi
ma")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{1+2x^2}\,dx = \int \frac{\sqrt{3\,x^4+5\,x^2+2}(x^4-7\,x^2+4)}{2\,x^2+1}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1),x, algorithm="giac
")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{1+2x^2}\,dx = \int \frac{(x^4-7\,x^2+4)\,\sqrt{3\,x^4+5\,x^2+2}}{2\,x^2+1}\,dx$$

input
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2))/(2*x^2 + 1),x)
```

output
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2))/(2*x^2 + 1), x)
```

Reduce [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{1+2x^2}\,dx = \frac{\sqrt{3x^4+5x^2+2}\,x^3}{10} - \frac{43\sqrt{3x^4+5x^2+2}\,x}{36} + \frac{187\left(\int \frac{\sqrt{3x^4+5x^2+2}}{6x^6+13x^4+9x^2+2}dx\right)}{18} + \frac{1879\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^4}{6x^6+13x^4+9x^2+2}dx\right)}{180} + \frac{1991\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{6x^6+13x^4+9x^2+2}dx\right)}{90}$$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1),x)
```

output
```
(18*sqrt(3*x**4 + 5*x**2 + 2)*x**3 - 215*sqrt(3*x**4 + 5*x**2 + 2)*x + 187
0*int(sqrt(3*x**4 + 5*x**2 + 2)/(6*x**6 + 13*x**4 + 9*x**2 + 2),x) + 1879*
int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(6*x**6 + 13*x**4 + 9*x**2 + 2),x) +
3982*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(6*x**6 + 13*x**4 + 9*x**2 + 2),
x))/180
```

3.195 $\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 256

$$\begin{aligned}\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^2}\,dx &= -\frac{547x(2+3x^2)}{144\sqrt{2+5x^2+3x^4}}+\frac{1}{12}x\sqrt{2+5x^2+3x^4}\\ &\quad+\frac{31x\sqrt{2+5x^2+3x^4}}{8\,(1+2x^2)}+\frac{547(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{72\sqrt{2}\sqrt{2+5x^2+3x^4}}\\ &\quad-\frac{367(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{24\sqrt{2}\sqrt{2+5x^2+3x^4}}\\ &\quad+\frac{41\sqrt{3}(1+x^2)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{8\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
-547/144*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/12*x*(3*x^4+5*x^2+2)^(1/2)+31
*x*(3*x^4+5*x^2+2)^(1/2)/(16*x^2+8)+547/144*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^
2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
-367/48*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x
),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+41/8*(x^2+1)*EllipticPi(x*6^(1/2)/(
6*x^2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))^(1/2)/(3*x^4+
5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.46 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.84

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^2}\,dx = \frac{12x(190+483x^2+305x^4+12x^6)+1094i\sqrt{3}\sqrt{1+x^2}(1+2x^2)\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-733}{}$$

input
```
Integrate[((4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4])/(1 + 2*x^2)^2,x]
```

output
```
(12*x*(190 + 483*x^2 + 305*x^4 + 12*x^6) + (1094*I)*Sqrt[3]*Sqrt[1 + x^2]*
(1 + 2*x^2)*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (733*
I)*Sqrt[3]*Sqrt[1 + x^2]*(1 + 2*x^2)*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[S
qrt[3/2]*x], 2/3] - (369*I)*Sqrt[3]*Sqrt[1 + x^2]*(1 + 2*x^2)*Sqrt[2 + 3*x
^2]*EllipticPi[4/3, I*ArcSinh[Sqrt[3/2]*x], 2/3])/(288*(1 + 2*x^2)*Sqrt[2
+ 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4-7x^2+4)\sqrt{3x^4+5x^2+2}}{(2x^2+1)^2}\,dx$$

$$\Big\downarrow \; 2258$$

$$\int \left(\frac{3x^4}{4\sqrt{3x^4+5x^2+2}}-\frac{19x^2}{4\sqrt{3x^4+5x^2+2}}+\frac{27}{4(2x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{31}{16(2x^2+1)^2\sqrt{3x^4+5x^2+2}}-\frac{}{16}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{367(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{24\sqrt{2}\sqrt{3x^4+5x^2+2}}+\frac{547(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{72\sqrt{2}\sqrt{3x^4+5x^2+2}}+\frac{41\sqrt{3}(x^2+1)\operatorname{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{8\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{31\sqrt{3x^4+5x^2+2}x}{8(2x^2+1)}+\frac{1}{12}\sqrt{3x^4+5x^2+2}x-\frac{547(3x^2+2)x}{144\sqrt{3x^4+5x^2+2}}$$

input
```
Int[((4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4])/(1 + 2*x^2)^2,x]
```

output
```
(-547*x*(2 + 3*x^2))/(144*Sqrt[2 + 5*x^2 + 3*x^4]) + (x*Sqrt[2 + 5*x^2 + 3
*x^4])/12 + (31*x*Sqrt[2 + 5*x^2 + 3*x^4])/(8*(1 + 2*x^2)) + (547*(1 + x^2
)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(72*Sqrt[2]*Sqrt
[2 + 5*x^2 + 3*x^4]) - (367*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Elliptic
F[ArcTan[x], -1/2])/(24*Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) + (41*Sqrt[3]*(1
+ x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(8*Sqrt[(1 + x^2)/(2 +
3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 7.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.70

method	result
elliptic	$\frac{31x\sqrt{3x^4+5x^2+2}}{8(2x^2+1)}+\frac{x\sqrt{3x^4+5x^2+2}}{12}-\frac{1105i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{576\sqrt{3x^4+5x^2+2}}+\frac{547i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)}{144\sqrt{3x^4+5x^2+2}}$
risch	$\frac{\sqrt{3x^4+5x^2+2}\,x(4x^2+95)}{48x^2+24}+\frac{361i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{192\sqrt{3x^4+5x^2+2}}-\frac{547i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix\right.\right.}{144\sqrt{3x^4+5x^2+2}}$
default	$\frac{x\sqrt{3x^4+5x^2+2}}{12}-\frac{395i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{192\sqrt{3x^4+5x^2+2}}+\frac{5i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{36\sqrt{3x^4+5x^2+2}}+3$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)^2,x,method=_RETURNVERBOS
E)
```

output
```
31/8*x*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)+1/12*x*(3*x^4+5*x^2+2)^(1/2)-1105/5
76*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2
*6^(1/2))+547/144*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*El
lipticE(I*x,1/2*6^(1/2))-123/32*I*(x^2+1)^(1/2)*(1+3/2*x^2)^(1/2)/(3*x^4+5
*x^2+2)^(1/2)*EllipticPi(I*x,2,1/2*I*(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^2}\,dx=\int \frac{\sqrt{3\,x^4+5\,x^2+2}(x^4-7\,x^2+4)}{(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)^2,x, algorithm="fr
icas")
```

output
```
integral(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(4*x^4 + 4*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^2}\,dx = \int \frac{\sqrt{(x^2+1)(3x^2+2)}(x^4-7x^2+4)}{(2x^2+1)^2}\,dx$$

input
```
integrate((x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(1/2)/(2*x**2+1)**2,x)
```

output
```
Integral(sqrt((x**2 + 1)*(3*x**2 + 2))*(x**4 - 7*x**2 + 4)/(2*x**2 + 1)**2
, x)
```

Maxima [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^2}\,dx = \int \frac{\sqrt{3\,x^4+5\,x^2+2}(x^4-7\,x^2+4)}{(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)^2,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1)^2, x)
```

Giac [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^2}\,dx = \int \frac{\sqrt{3\,x^4+5\,x^2+2}(x^4-7\,x^2+4)}{(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^2}\,dx = \int \frac{(x^4-7\,x^2+4)\,\sqrt{3\,x^4+5\,x^2+2}}{(2\,x^2+1)^2}\,dx$$

input
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2))/(2*x^2 + 1)^2,x)
```

output
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2))/(2*x^2 + 1)^2, x)
```

Reduce [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^2}\,dx = \frac{3\sqrt{3x^4+5x^2+2}\,x^3-50\sqrt{3x^4+5x^2+2}\,x+488\left(\int \frac{\sqrt{3x^4+5x^2+2}}{12x^8+32x^6+31x^4+13x^2+2}dx\right)x^2+244\left(\int \frac{\sqrt{3x^4+5x^2}}{12x^8+32x^6+31x^4}\right.}{}$$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)^2,x)
```

output
```
(3*sqrt(3*x**4 + 5*x**2 + 2)*x**3 - 50*sqrt(3*x**4 + 5*x**2 + 2)*x + 488*i
nt(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x
)*x**2 + 244*int(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32*x**6 + 31*x**4 +
13*x**2 + 2),x) - 186*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(12*x**8 + 32*x
**6 + 31*x**4 + 13*x**2 + 2),x)*x**2 - 93*int((sqrt(3*x**4 + 5*x**2 + 2)*x
**6)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x) + 780*int((sqrt(3*x**4
 + 5*x**2 + 2)*x**2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x)*x**2 +
 390*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(12*x**8 + 32*x**6 + 31*x**4 + 1
3*x**2 + 2),x))/(18*(2*x**2 + 1))
```

3.196 $\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 265

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^3}\,dx = -\frac{145x(2+3x^2)}{64\sqrt{2+5x^2+3x^4}}+\frac{31x\sqrt{2+5x^2+3x^4}}{16(1+2x^2)^2}+\frac{153x\sqrt{2+5x^2+3x^4}}{32(1+2x^2)}+\frac{145(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{32\sqrt{2}\sqrt{2+5x^2+3x^4}}-\frac{79(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{32\sqrt{2}\sqrt{2+5x^2+3x^4}}-\frac{95(1+x^2)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{32\sqrt{3}\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}$$

output
```
-145/64*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+31/16*x*(3*x^4+5*x^2+2)^(1/2)/(2
*x^2+1)^2+153*x*(3*x^4+5*x^2+2)^(1/2)/(64*x^2+32)+145/64*2^(1/2)*(x^2+1)*(
(3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5
*x^2+2)^(1/2)-79/64*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacob
iAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)-95/96*(x^2+1)*EllipticP
i(x*6^(1/2)/(6*x^2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))^
(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.80

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^3}\,dx = \frac{\frac{744x(2+5x^2+3x^4)}{(1+2x^2)^2}+\frac{1836x(2+5x^2+3x^4)}{1+2x^2}+870i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-269i\sqrt{3}\sqrt{1+x^2}}{384\sqrt{2+5}}$$

input
```
Integrate[((4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4])/(1 + 2*x^2)^3,x]
```

output
```
((744*x*(2 + 5*x^2 + 3*x^4))/(1 + 2*x^2)^2 + (1836*x*(2 + 5*x^2 + 3*x^4))/
(1 + 2*x^2) + (870*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*Ar
cSinh[Sqrt[3/2]*x], 2/3] - (269*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*E
llipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (95*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[
2 + 3*x^2]*EllipticPi[4/3, I*ArcSinh[Sqrt[3/2]*x], 2/3])/(384*Sqrt[2 + 5*x
^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4-7x^2+4)\sqrt{3x^4+5x^2+2}}{(2x^2+1)^3}\,dx$$

$$\Big\downarrow \text{2258}$$

$$\int \left(\frac{3x^2}{8\sqrt{3x^4+5x^2+2}}+\frac{15}{8(2x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{27}{4(2x^2+1)^2\sqrt{3x^4+5x^2+2}}+\frac{31}{16(2x^2+1)^3\sqrt{3x^4+5x$$

$$\Big\downarrow \text{2009}$$

$$-\frac{79(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{32\sqrt{2}\sqrt{3x^4+5x^2+2}}+\frac{145(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{32\sqrt{2}\sqrt{3x^4+5x^2+2}}-\frac{11\sqrt{3}(x^2+1)\operatorname{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{961(x^2+1)\operatorname{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{32\sqrt{3}\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{153\sqrt{3x^4+5x^2+2}x}{32(2x^2+1)}+\frac{31\sqrt{3x^4+5x^2+2}x}{16(2x^2+1)^2}-\frac{145(3x^2+2)x}{64\sqrt{3x^4+5x^2+2}}$$

input

```
Int[((4 - 7*x^2 + x^4)*Sqrt[2 + 5*x^2 + 3*x^4])/(1 + 2*x^2)^3,x]
```

output

```
(-145*x*(2 + 3*x^2))/(64*Sqrt[2 + 5*x^2 + 3*x^4]) + (31*x*Sqrt[2 + 5*x^2 +
 3*x^4])/(16*(1 + 2*x^2)^2) + (153*x*Sqrt[2 + 5*x^2 + 3*x^4])/(32*(1 + 2*x
^2)) + (145*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/
2])/(32*Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) - (79*(1 + x^2)*Sqrt[(2 + 3*x^2)/
(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(32*Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]
) + (961*(1 + x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(32*Sqrt[3]
*Sqrt[(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4]) - (11*Sqrt[3]*(1 + x
^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(Sqrt[(1 + x^2)/(2 + 3*x^2
)]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 10.58 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.69

method	result
risch	$\frac{\sqrt{3x^4+5x^2+2}\,x\,(306x^2+215)}{32(2x^2+1)^2}+\frac{601i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{256\sqrt{3x^4+5x^2+2}}-\frac{145i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\right.}{64\sqrt{3x^4+5x^2+2}}$
default	$\frac{31x\sqrt{3x^4+5x^2+2}}{16(2x^2+1)^2}+\frac{153x\sqrt{3x^4+5x^2+2}}{32(2x^2+1)}+\frac{21i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{256\sqrt{3x^4+5x^2+2}}+\frac{145i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)}{64\sqrt{3x^4+5x^2+2}}$
elliptic	$\frac{31x\sqrt{3x^4+5x^2+2}}{16(2x^2+1)^2}+\frac{153x\sqrt{3x^4+5x^2+2}}{32(2x^2+1)}+\frac{21i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{256\sqrt{3x^4+5x^2+2}}+\frac{145i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)}{64\sqrt{3x^4+5x^2+2}}$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)^3,x,method=_RETURNVERBOS
E)
```

output
```
1/32*(3*x^4+5*x^2+2)^(1/2)*x*(306*x^2+215)/(2*x^2+1)^2+601/256*I*(x^2+1)^(
1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))-145/
64*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/
2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))+95/128*I*(x^2+1)^(1/2)*(1+3/2*x^2)^
(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticPi(I*x,2,1/2*I*(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^3}\,dx=\int \frac{\sqrt{3\,x^4+5\,x^2+2}(x^4-7\,x^2+4)}{(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)^3,x, algorithm="fr
icas")
```

output
```
integral(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(8*x^6 + 12*x^4 + 6*x^2
 + 1), x)
```

Sympy [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^3}\,dx=\int \frac{\sqrt{(x^2+1)\,(3x^2+2)}(x^4-7x^2+4)}{(2x^2+1)^3}\,dx$$

input
```
integrate((x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(1/2)/(2*x**2+1)**3,x)
```

output
```
Integral(sqrt((x**2 + 1)*(3*x**2 + 2))*(x**4 - 7*x**2 + 4)/(2*x**2 + 1)**3
, x)
```

Maxima [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^3}\,dx=\int \frac{\sqrt{3\,x^4+5\,x^2+2}(x^4-7\,x^2+4)}{(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)^3,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1)^3, x)
```

Giac [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^3}\,dx = \int \frac{\sqrt{3\,x^4+5\,x^2+2}(x^4-7\,x^2+4)}{(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)^3,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^3}\,dx = \int \frac{(x^4-7\,x^2+4)\,\sqrt{3\,x^4+5\,x^2+2}}{(2\,x^2+1)^3}\,dx$$

input
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2))/(2*x^2 + 1)^3,x)
```

output
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(1/2))/(2*x^2 + 1)^3, x)
```

Reduce [F]

$$\int \frac{(4-7x^2+x^4)\sqrt{2+5x^2+3x^4}}{(1+2x^2)^3}\,dx$$
$$= \frac{-149\sqrt{3x^4+5x^2+2}\,x^3-190\sqrt{3x^4+5x^2+2}\,x+720\left(\int \frac{\sqrt{3x^4+5x^2+2}}{24x^{10}+76x^8+94x^6+57x^4+17x^2+2}dx\right)x^4+720\left(\int \frac{}{24x} \cdots\right.}{}$$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)^3,x)
```

output

```
( - 149*sqrt(3*x**4 + 5*x**2 + 2)*x**3 - 190*sqrt(3*x**4 + 5*x**2 + 2)*x +
 720*int(sqrt(3*x**4 + 5*x**2 + 2)/(24*x**10 + 76*x**8 + 94*x**6 + 57*x**4
 + 17*x**2 + 2),x)*x**4 + 720*int(sqrt(3*x**4 + 5*x**2 + 2)/(24*x**10 + 76
*x**8 + 94*x**6 + 57*x**4 + 17*x**2 + 2),x)*x**2 + 180*int(sqrt(3*x**4 + 5
*x**2 + 2)/(24*x**10 + 76*x**8 + 94*x**6 + 57*x**4 + 17*x**2 + 2),x) + 291
2*int(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2
),x)*x**4 + 2912*int(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32*x**6 + 31*x**
4 + 13*x**2 + 2),x)*x**2 + 728*int(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32
*x**6 + 31*x**4 + 13*x**2 + 2),x) - 2028*int((sqrt(3*x**4 + 5*x**2 + 2)*x*
*6)/(24*x**10 + 76*x**8 + 94*x**6 + 57*x**4 + 17*x**2 + 2),x)*x**4 - 2028*
int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(24*x**10 + 76*x**8 + 94*x**6 + 57*x*
*4 + 17*x**2 + 2),x)*x**2 - 507*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(24*x
**10 + 76*x**8 + 94*x**6 + 57*x**4 + 17*x**2 + 2),x) + 2184*int((sqrt(3*x*
*4 + 5*x**2 + 2)*x**6)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x)*x**4
 + 2184*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(12*x**8 + 32*x**6 + 31*x**4
+ 13*x**2 + 2),x)*x**2 + 546*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(12*x**8
 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x) - 2184*int((sqrt(3*x**4 + 5*x**2 +
2)*x**2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x)*x**4 - 2184*int((s
qrt(3*x**4 + 5*x**2 + 2)*x**2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2)
,x)*x**2 - 546*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(12*x**8 + 32*x**6 ...
```

3.197 $\int (1+2x^2)^3 (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 285

$$\int (1+2x^2)^3 (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2} \, dx = \frac{1152397882x(2+3x^2)}{558242685\sqrt{2+5x^2+3x^4}} + \frac{2x(202977940+101236329x^2)\sqrt{2+5x^2+3x^4}}{186080895} + \frac{x(4895329+3917921x^2)(2+5x^2+3x^4)^{3/2}}{4135131} + \frac{2609x(2+5x^2+3x^4)^{5/2}}{11583} + \frac{1706x^3(2+5x^2+3x^4)^{5/2}}{5967} - \frac{908}{765}x^5(2+5x^2+3x^4)^{5/2} + \frac{8}{51}x^7(2+5x^2+3x^4)^{5/2} - \frac{1152397882\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{558242685\sqrt{2+5x^2+3x^4}} + 9$$

output
```
1152397882/558242685*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+2/186080895*x*(1012
36329*x^2+202977940)*(3*x^4+5*x^2+2)^(1/2)+1/4135131*x*(3917921*x^2+489532
9)*(3*x^4+5*x^2+2)^(3/2)+2609/11583*x*(3*x^4+5*x^2+2)^(5/2)+1706/5967*x^3*
(3*x^4+5*x^2+2)^(5/2)-908/765*x^5*(3*x^4+5*x^2+2)^(5/2)+8/51*x^7*(3*x^4+5*
x^2+2)^(5/2)-1152397882/558242685*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2
)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+94891466/
37216179*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(
x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.58

$$\int (1+2x^2)^3 (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2} dx = \frac{3x(2028379660+10486174576x^2+23128188873x^4+23400228114x^6-4137291324x^8-3}{}$$

input
```
Integrate[(1 + 2*x^2)^3*(4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(3*x*(2028379660 + 10486174576*x^2 + 23128188873*x^4 + 23400228114*x^6 - 4
137291324*x^8 - 39846191490*x^10 - 44404657731*x^12 - 20236507830*x^14 - 2
022808788*x^16 + 788107320*x^18) - (1152397882*I)*Sqrt[3]*Sqrt[1 + x^2]*Sq
rt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (203483222*I)*Sqrt[
3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(
558242685*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {2207, 2207, 27, 2207, 2207, 1490, 27, 1490, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2+1)^3 (x^4-7x^2+4) (3x^4+5x^2+2)^{3/2} dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{51}\int (3x^4+5x^2+2)^{3/2} (-2724x^8-2458x^6+357x^4+867x^2+204) dx + \frac{8}{51}(3x^4+5x^2+2)^{5/2} x^7$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{51}\left(\frac{1}{45}\int 15\left(3x^4+5x^2+2\right)^{3/2}\left(1706x^6+2887x^4+2601x^2+612\right)dx-\frac{908}{15}x^5\left(3x^4+5x^2+2\right)^{5/2}\right)+$$
$$\frac{8}{51}\left(3x^4+5x^2+2\right)^{5/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{51}\left(\frac{1}{3}\int \left(3x^4+5x^2+2\right)^{3/2}\left(1706x^6+2887x^4+2601x^2+612\right)dx-\frac{908}{15}x^5\left(3x^4+5x^2+2\right)^{5/2}\right)+$$
$$\frac{8}{51}\left(3x^4+5x^2+2\right)^{5/2}x^7$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{51}\left(\frac{1}{3}\left(\frac{1}{39}\int \left(3x^4+5x^2+2\right)^{3/2}\left(44353x^4+91203x^2+23868\right)dx+\frac{1706}{39}\left(3x^4+5x^2+2\right)^{5/2}x^3\right)-\frac{908}{15}x^5\left(3x^4\right.$$
$$\frac{8}{51}\left(3x^4+5x^2+2\right)^{5/2}x^7$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{51}\left(\frac{1}{3}\left(\frac{1}{39}\left(\frac{1}{33}\int \left(1679109x^2+698938\right)\left(3x^4+5x^2+2\right)^{3/2}dx+\frac{44353}{33}x\left(3x^4+5x^2+2\right)^{5/2}\right)+\frac{1706}{39}\left(3x^4+5\right.\right.\right.$$
$$\frac{8}{51}\left(3x^4+5x^2+2\right)^{5/2}x^7$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{51}\left(\frac{1}{3}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{1}{63}\int 6\left(11248481x^2+9782369\right)\sqrt{3x^4+5x^2+2}dx+\frac{1}{21}x\left(3917921x^2+4895329\right)\left(3x^4+5x^2\right.\right.\right.\right.\right.$$
$$\frac{8}{51}\left(3x^4+5x^2+2\right)^{5/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{51}\left(\frac{1}{3}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\int \left(11248481x^2+9782369\right)\sqrt{3x^4+5x^2+2}dx+\frac{1}{21}x\left(3917921x^2+4895329\right)\left(3x^4+5x^2+\right.\right.\right.\right.\right.$$
$$\frac{8}{51}\left(3x^4+5x^2+2\right)^{5/2}x^7$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{51}\left(\frac{1}{3}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{45}\int \frac{576198941x^2+474457330}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(101236329x^2+202977940\right)\right)\right.\right.\right.\right.\right.$$
$$\frac{8}{51}\left(3x^4+5x^2+2\right)^{5/2}x^7$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{51}\left(\frac{1}{3}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{45}\left(474457330\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx+576198941\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{45}x\sqrt{3x^4+\right.\right.\right.\right.\right.\right.$$
$$\frac{8}{51}\left(3x^4+5x^2+2\right)^{5/2}x^7$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{51}\left(\frac{1}{3}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{45}\left(576198941\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{237228665\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan\right.}{\sqrt{3x^4+5x^2+2}}\right.\right.\right.\right.\right.\right.$$
$$\frac{8}{51}\left(3x^4+5x^2+2\right)^{5/2}x^7$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{51}\left(\frac{1}{3}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{45}\left(\frac{237228665\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+576198941\left(\frac{x(3x^2+}{3\sqrt{3x^4+5}}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{8}{51}\left(3x^4+5x^2+2\right)^{5/2}x^7$$

input
```
Int[(1 + 2*x^2)^3*(4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(8*x^7*(2 + 5*x^2 + 3*x^4)^(5/2))/51 + ((-908*x^5*(2 + 5*x^2 + 3*x^4)^(5/2
))/15 + ((1706*x^3*(2 + 5*x^2 + 3*x^4)^(5/2))/39 + ((44353*x*(2 + 5*x^2 +
3*x^4)^(5/2))/33 + ((x*(4895329 + 3917921*x^2)*(2 + 5*x^2 + 3*x^4)^(3/2))/
21 + (2*((x*(202977940 + 101236329*x^2)*Sqrt[2 + 5*x^2 + 3*x^4])/45 + (576
198941*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*S
qrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 +
 3*x^4])) + (237228665*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Ellip
ticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4])/45))/21)/33)/39)/3)/51
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 16.06 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.54

method	result
risch	$\frac{x(262702440x^{14}-1112106996x^{12}-5067125910x^{10}-5614938063x^{8}-545749785x^{6}+3273777909x^{4}+2707612713x^{2}+1014189830)\sqrt{}}{186080895}$
default	$\frac{363753101x^{5}\sqrt{3x^{4}+5x^{2}+2}}{20675655}+\frac{23141989x^{3}\sqrt{3x^{4}+5x^{2}+2}}{1590435}+\frac{202837966x\sqrt{3x^{4}+5x^{2}+2}}{37216179}+\frac{1152397882i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,(\text{Ellipt}}{558242685\sqrt{3}}$
elliptic	$\frac{363753101x^{5}\sqrt{3x^{4}+5x^{2}+2}}{20675655}+\frac{23141989x^{3}\sqrt{3x^{4}+5x^{2}+2}}{1590435}+\frac{202837966x\sqrt{3x^{4}+5x^{2}+2}}{37216179}+\frac{1152397882i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,(\text{Ellipt}}{558242685\sqrt{3}}$

input
```
int((2*x^2+1)^3*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/186080895*x*(262702440*x^14-1112106996*x^12-5067125910*x^10-5614938063*x
^8-545749785*x^6+3273777909*x^4+2707612713*x^2+1014189830)*(3*x^4+5*x^2+2)
^(1/2)-94891466/37216179*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(
1/2)*EllipticF(I*x,1/2*6^(1/2))+1152397882/558242685*I*(x^2+1)^(1/2)*(6*x^
2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x
,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.36

$$\int \left(1+2x^2\right)^3 \left(4-7x^2+x^4\right) \left(2+5x^2+3x^4\right)^{3/2} dx =$$
$$-\frac{2304795764\sqrt{3}\sqrt{-\frac{2}{3}}xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) | \frac{3}{2}) - 6574911734\sqrt{3}\sqrt{-\frac{2}{3}}xF(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) | \frac{3}{2}) - 3\,(788107\cdots}{\cdots}$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="fr
icas")
```

output
```
-1/1674728055*(2304795764*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3
)/x), 3/2) - 6574911734*sqrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/
x), 3/2) - 3*(788107320*x^16 - 3336320988*x^14 - 15201377730*x^12 - 168448
14189*x^10 - 1637249355*x^8 + 9821333727*x^6 + 8122838139*x^4 + 3042569490
*x^2 + 1152397882)*sqrt(3*x^4 + 5*x^2 + 2))/x
```

Sympy [F]

$$\int \left(1+2x^2\right)^3 \left(4-7x^2+x^4\right) \left(2+5x^2+3x^4\right)^{3/2} dx = \int \left(\left(x^2+1\right)\left(3x^2+2\right)\right)^{\frac{3}{2}} \left(2x^2+1\right)^3 \left(x^4-7x^2+4\right) dx$$

input
```
integrate((2*x**2+1)**3*(x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral(((x**2 + 1)*(3*x**2 + 2))**(3/2)*(2*x**2 + 1)**3*(x**4 - 7*x**2 +
 4), x)
```

Maxima [F]

$$\int (1+2x^2)^3 (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2} dx = \int (3x^4+5x^2+2)^{\frac{3}{2}} (x^4-7x^2+4)(2x^2+1)^3 dx$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1)^3, x)
```

Giac [F]

$$\int (1+2x^2)^3 (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2} dx = \int (3x^4+5x^2+2)^{\frac{3}{2}} (x^4-7x^2+4)(2x^2+1)^3 dx$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x^2)^3 (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2} dx = \int (2x^2+1)^3 (x^4-7x^2+4) (3x^4+5x^2+2)^{3/2} dx$$

input
```
int((2*x^2 + 1)^3*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int((2*x^2 + 1)^3*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int (1+2x^2)^3 (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2} \, dx = \frac{24\sqrt{3x^4+5x^2+2}\,x^{15}}{17} - \frac{508\sqrt{3x^4+5x^2+2}\,x^{13}}{85} - \frac{354\sqrt{3x^4+5x^2+2}\,x^{11}}{13} - \frac{1100321\sqrt{3x^4+5x^2+2}\,x^{9}}{36465} - \frac{577513\sqrt{3x^4+5x^2+2}\,x^{7}}{196911} + \frac{363753101\sqrt{3x^4+5x^2+2}\,x^{5}}{20675655} + \frac{23141989\sqrt{3x^4+5x^2+2}\,x^{3}}{1590435} + \frac{202837966\sqrt{3x^4+5x^2+2}\,x}{37216179} + \frac{189782932\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2} dx\right)}{37216179} + \frac{1152397882\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2} dx\right)}{186080895}$$

input
```
int((2*x^2+1)^3*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
(262702440*sqrt(3*x**4 + 5*x**2 + 2)*x**15 - 1112106996*sqrt(3*x**4 + 5*x*
*2 + 2)*x**13 - 5067125910*sqrt(3*x**4 + 5*x**2 + 2)*x**11 - 5614938063*sq
rt(3*x**4 + 5*x**2 + 2)*x**9 - 545749785*sqrt(3*x**4 + 5*x**2 + 2)*x**7 +
3273777909*sqrt(3*x**4 + 5*x**2 + 2)*x**5 + 2707612713*sqrt(3*x**4 + 5*x**
2 + 2)*x**3 + 1014189830*sqrt(3*x**4 + 5*x**2 + 2)*x + 948914660*int(sqrt(
3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x) + 1152397882*int((sqrt(3*x**
4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2 + 2),x))/186080895
```

3.198 $\int (1+2x^2)^2 (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 262

$$\int (1+2x^2)^2 (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2}\,dx = \frac{15107402x(2+3x^2)}{6567561\sqrt{2+5x^2+3x^4}} + \frac{2x(2823722+1514979x^2)\sqrt{2+5x^2+3x^4}}{2189187} + \frac{x(57469-63175x^2)(2+5x^2+3x^4)^{3/2}}{243243} + \frac{6067x(2+5x^2+3x^4)^{5/2}}{11583} - \frac{256}{351}x^3(2+5x^2+3x^4)^{5/2} + \frac{4}{45}x^5(2+5x^2+3x^4)^{5/2} - \frac{15107402\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{6567561\sqrt{2+5x^2+3x^4}} + \frac{624}{}$$

output
```
15107402/6567561*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+2/2189187*x*(1514979*x^
2+2823722)*(3*x^4+5*x^2+2)^(1/2)+1/243243*x*(-63175*x^2+57469)*(3*x^4+5*x^
2+2)^(3/2)+6067/11583*x*(3*x^4+5*x^2+2)^(5/2)-256/351*x^3*(3*x^4+5*x^2+2)^
(5/2)+4/45*x^5*(3*x^4+5*x^2+2)^(5/2)-15107402/6567561*2^(1/2)*(x^2+1)*((3*
x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^
2+2)^(1/2)+6244958/2189187*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*Inver
seJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.61

$$\int (1+2x^2)^2 (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2} dx = \frac{3x(112685380+491968300x^2+794467761x^4+339980580x^6-716368428x^8-118218933}{}$$

input
```
Integrate[(1 + 2*x^2)^2*(4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(3*x*(112685380 + 491968300*x^2 + 794467761*x^4 + 339980580*x^6 - 71636842
8*x^8 - 1182189330*x^10 - 651494907*x^12 - 84199500*x^14 + 26270244*x^16)
- (75537010*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[S
qrt[3/2]*x], 2/3] + (13087430*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Ell
ipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(32837805*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {2207, 27, 2207, 2207, 1490, 27, 1490, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2+1)^2 (x^4-7x^2+4) (3x^4+5x^2+2)^{3/2} dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{45}\int 5(3x^4+5x^2+2)^{3/2} (-256x^6-107x^4+81x^2+36) dx + \frac{4}{45}(3x^4+5x^2+2)^{5/2} x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int (3x^4+5x^2+2)^{3/2} (-256x^6-107x^4+81x^2+36) dx + \frac{4}{45}(3x^4+5x^2+2)^{5/2} x^5$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{9}\left(\frac{1}{39}\int \left(3x^4+5x^2+2\right)^{3/2}\left(6067x^4+4695x^2+1404\right)dx-\frac{256}{39}x^3\left(3x^4+5x^2+2\right)^{5/2}\right)+\frac{4}{45}\left(3x^4+5x^2+2\right)^{5/2}x^5$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{9}\left(\frac{1}{39}\left(\frac{1}{33}\int \left(34198-27075x^2\right)\left(3x^4+5x^2+2\right)^{3/2}dx+\frac{6067}{33}x\left(3x^4+5x^2+2\right)^{5/2}\right)-\frac{256}{39}x^3\left(3x^4+5x^2+2\right)^{5}\right.$$
$$\frac{4}{45}\left(3x^4+5x^2+2\right)^{5/2}x^5$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{9}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{1}{63}\int 6\left(841655x^2+660689\right)\sqrt{3x^4+5x^2+2}dx+\frac{1}{21}x\left(57469-63175x^2\right)\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{6}{}\right.\right.$$
$$\frac{4}{45}\left(3x^4+5x^2+2\right)^{5/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\int \left(841655x^2+660689\right)\sqrt{3x^4+5x^2+2}dx+\frac{1}{21}x\left(57469-63175x^2\right)\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{60}{3}\right.\right.$$
$$\frac{4}{45}\left(3x^4+5x^2+2\right)^{5/2}x^5$$

$$\Big\downarrow \text{1490}$$

$$\frac{1}{9}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{45}\int \frac{5\left(7553701x^2+6244958\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{9}x\sqrt{3x^4+5x^2+2}\left(1514979x^2+2823722\right)\right)+\frac{1}{21}x\left(574\right.\right.\right.\right.$$
$$\frac{4}{45}\left(3x^4+5x^2+2\right)^{5/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{9}\int \frac{7553701x^2+6244958}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{9}x\sqrt{3x^4+5x^2+2}\left(1514979x^2+2823722\right)\right)+\frac{1}{21}x\left(57469-\right.\right.\right.\right.$$
$$\frac{4}{45}\left(3x^4+5x^2+2\right)^{5/2}x^5$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{9}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{9}\left(6244958\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx+7553701\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{9}x\sqrt{3x^4+5x^2+2}(15\right.\right.\right.\right.\right.$$
$$\frac{4}{45}\left(3x^4+5x^2+2\right)^{5/2}x^5$$

$\downarrow$ 1413

$$\frac{1}{9}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{9}\left(7553701\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{3122479\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)\right.\right.\right.\right.$$
$$\frac{4}{45}\left(3x^4+5x^2+2\right)^{5/2}x^5$$

$\downarrow$ 1456

$$\frac{1}{9}\left(\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{9}\left(\frac{3122479\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+7553701\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\right.\right.\right.\right.\right.\right.$$
$$\frac{4}{45}\left(3x^4+5x^2+2\right)^{5/2}x^5$$

input
```
Int[(1 + 2*x^2)^2*(4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(4*x^5*(2 + 5*x^2 + 3*x^4)^(5/2))/45 + ((-256*x^3*(2 + 5*x^2 + 3*x^4)^(5/2
))/39 + ((6067*x*(2 + 5*x^2 + 3*x^4)^(5/2))/33 + ((x*(57469 - 63175*x^2)*(
2 + 5*x^2 + 3*x^4)^(3/2))/21 + (2*((x*(2823722 + 1514979*x^2)*Sqrt[2 + 5*x
^2 + 3*x^4])/9 + (7553701*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (
Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/
(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (3122479*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)
/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4])/9))/21)/3
3)/39)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 10.86 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.57

method	result
risch	$\frac{x(8756748x^{12}-42661080x^{10}-151901001x^{8}-112454055x^{6}+49901283x^{4}+105127425x^{2}+56342690)\sqrt{3x^4+5x^2+2}}{10945935} - \frac{6244958i\sqrt{\ldots}}{\ldots}$
default	$\frac{5544587x^{5}\sqrt{3x^4+5x^2+2}}{1216215}+\frac{179705x^{3}\sqrt{3x^4+5x^2+2}}{18711}+\frac{11268538x\sqrt{3x^4+5x^2+2}}{2189187}-\frac{6244958i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\ldots\right.}{2189187\sqrt{3x^4+5x^2+2}}$
elliptic	$\frac{5544587x^{5}\sqrt{3x^4+5x^2+2}}{1216215}+\frac{179705x^{3}\sqrt{3x^4+5x^2+2}}{18711}+\frac{11268538x\sqrt{3x^4+5x^2+2}}{2189187}-\frac{6244958i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\ldots\right.}{2189187\sqrt{3x^4+5x^2+2}}$

input
```
int((2*x^2+1)^2*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/10945935*x*(8756748*x^12-42661080*x^10-151901001*x^8-112454055*x^6+49901
283*x^4+105127425*x^2+56342690)*(3*x^4+5*x^2+2)^(1/2)-6244958/2189187*I*(x
^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2
))+15107402/6567561*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*
(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.37

$$\int \left(1+2 x^2\right)^2 \left(4-7 x^2+x^4\right) \left(2+5 x^2+3 x^4\right)^{3/2} \, dx = -\frac{151074020 \sqrt{3} \sqrt{-\frac{2}{3}} x E(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 432097130 \sqrt{3} \sqrt{-\frac{2}{3}} x F(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 3 (26270244}{}$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="fr
icas")
```

output
```
-1/98513415*(151074020*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3)/x
), 3/2) - 432097130*sqrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/x),
3/2) - 3*(26270244*x^14 - 127983240*x^12 - 455703003*x^10 - 337362165*x^8
+ 149703849*x^6 + 315382275*x^4 + 169028070*x^2 + 75537010)*sqrt(3*x^4 + 5
*x^2 + 2))/x
```

Sympy [F]

$$\int \left(1+2 x^2\right)^2 \left(4-7 x^2+x^4\right) \left(2+5 x^2+3 x^4\right)^{3/2} \, dx = \int \left(\left(x^{2}+1\right) \left(3 x^{2}+2\right)\right)^{\frac{3}{2}} \left(2 x^{2}+1\right)^{2} \left(x^{4}-7 x^{2}+4\right)\, dx$$

input
```
integrate((2*x**2+1)**2*(x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral(((x**2 + 1)*(3*x**2 + 2))**(3/2)*(2*x**2 + 1)**2*(x**4 - 7*x**2 +
 4), x)
```

Maxima [F]

$$\int \left(1+2x^2\right)^2 \left(4-7x^2+x^4\right) \left(2+5x^2+3x^4\right)^{3/2} \, dx = \int \left(3\,x^4+5\,x^2+2\right)^{\frac{3}{2}} \left(x^4-7\,x^2+4\right) \left(2\,x^2+1\right)^2 \, dx$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1)^2, x)
```

Giac [F]

$$\int \left(1+2x^2\right)^2 \left(4-7x^2+x^4\right) \left(2+5x^2+3x^4\right)^{3/2} \, dx = \int \left(3\,x^4+5\,x^2+2\right)^{\frac{3}{2}} \left(x^4-7\,x^2+4\right) \left(2\,x^2+1\right)^2 \, dx$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+2x^2\right)^2 \left(4-7x^2+x^4\right) \left(2+5x^2+3x^4\right)^{3/2} \, dx = \int \left(2\,x^2+1\right)^2 \left(x^4-7\,x^2+4\right) \left(3\,x^4+5\,x^2+2\right)^{3/2} \, dx$$

input
```
int((2*x^2 + 1)^2*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int((2*x^2 + 1)^2*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \left(1+2x^2\right)^2 \left(4-7x^2+x^4\right) \left(2+5x^2+3x^4\right)^{3/2} dx = \frac{4\sqrt{3x^4+5x^2+2}\,x^{13}}{5} - \frac{152\sqrt{3x^4+5x^2+2}\,x^{11}}{39} - \frac{29767\sqrt{3x^4+5x^2+2}\,x^9}{2145} - \frac{118999\sqrt{3x^4+5x^2+2}\,x^7}{11583} + \frac{5544587\sqrt{3x^4+5x^2+2}\,x^5}{1216215} + \frac{179705\sqrt{3x^4+5x^2+2}\,x^3}{18711} + \frac{11268538\sqrt{3x^4+5x^2+2}\,x}{2189187} + \frac{12489916\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)}{2189187} + \frac{15107402\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)}{2189187}$$

input
```
int((2*x^2+1)^2*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
(8756748*sqrt(3*x**4 + 5*x**2 + 2)*x**13 - 42661080*sqrt(3*x**4 + 5*x**2 +
 2)*x**11 - 151901001*sqrt(3*x**4 + 5*x**2 + 2)*x**9 - 112454055*sqrt(3*x*
*4 + 5*x**2 + 2)*x**7 + 49901283*sqrt(3*x**4 + 5*x**2 + 2)*x**5 + 10512742
5*sqrt(3*x**4 + 5*x**2 + 2)*x**3 + 56342690*sqrt(3*x**4 + 5*x**2 + 2)*x +
62449580*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x) + 75537010
*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2 + 2),x))/10945935
```

3.199 $\int (1+2x^2)(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 239

$$\int (1+2x^2)(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}\,dx = \frac{9842698x(2+3x^2)}{3648645\sqrt{2+5x^2+3x^4}} + \frac{2x(1708960+836181x^2)\sqrt{2+5x^2+3x^4}}{1216215} + \frac{x(49801+43169x^2)(2+5x^2+3x^4)^{3/2}}{27027} - \frac{587x(2+5x^2+3x^4)^{5/2}}{1287} + \frac{2}{39}x^3(2+5x^2+3x^4)^{5/2} - \frac{9842698\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3648645\sqrt{2+5x^2+3x^4}} + \frac{809714\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{Ellip}}{243243\sqrt{2+5x^2}}$$

output
```
9842698/3648645*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+2/1216215*x*(836181*x^2+
1708960)*(3*x^4+5*x^2+2)^(1/2)+1/27027*x*(43169*x^2+49801)*(3*x^4+5*x^2+2)
^(3/2)-587/1287*x*(3*x^4+5*x^2+2)^(5/2)+2/39*x^3*(3*x^4+5*x^2+2)^(5/2)-984
2698/3648645*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)
^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+809714/243243*2^(1/2)*(x^2+1)*
((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+
5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.65

$$\int (1+2x^2)(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}\,dx = \frac{3x(11362300+40241704x^2+409951545x^4-13462434x^6-57295080x^8-40001850x^{10}-6\cdots}{\cdots}$$

input
```
Integrate[(1 + 2*x^2)*(4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(3*x*(11362300 + 40241704*x^2 + 40951545*x^4 - 13462434*x^6 - 57295080*x^8
 - 40001850*x^10 - 6557355*x^12 + 1683990*x^14) - (9842698*I)*Sqrt[3]*Sqrt
[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (174555
8*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x
], 2/3])/(3648645*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2207, 2207, 1490, 27, 1490, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2+1)(x^4-7x^2+4)(3x^4+5x^2+2)^{3/2}\,dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{39}\int (-587x^4+27x^2+156)(3x^4+5x^2+2)^{3/2}\,dx+\frac{2}{39}(3x^4+5x^2+2)^{5/2}x^3$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{39}\left(\frac{1}{33}\int \left(18501x^2+6322\right)\left(3x^4+5x^2+2\right)^{3/2}dx-\frac{587}{33}x\left(3x^4+5x^2+2\right)^{5/2}\right)+\frac{2}{39}\left(3x^4+5x^2+2\right)^{5/2}x^3$$

$\big\downarrow$ 1490

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{1}{63}\int 6\left(92909x^2+82961\right)\sqrt{3x^4+5x^2+2}dx+\frac{1}{21}x\left(43169x^2+49801\right)\left(3x^4+5x^2+2\right)^{3/2}\right)-\frac{587}{33}x(3\right.$$
$$\frac{2}{39}\left(3x^4+5x^2+2\right)^{5/2}x^3$$

$\big\downarrow$ 27

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\int \left(92909x^2+82961\right)\sqrt{3x^4+5x^2+2}dx+\frac{1}{21}x\left(43169x^2+49801\right)\left(3x^4+5x^2+2\right)^{3/2}\right)-\frac{587}{33}x(3x\right.$$
$$\frac{2}{39}\left(3x^4+5x^2+2\right)^{5/2}x^3$$

$\big\downarrow$ 1490

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{45}\int \frac{4921349x^2+4048570}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(836181x^2+1708960\right)\right)+\frac{1}{21}x\left(43169x^2+4\right.\right.\right.$$
$$\frac{2}{39}\left(3x^4+5x^2+2\right)^{5/2}x^3$$

$\big\downarrow$ 1503

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{45}\left(4048570\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+4921349\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{45}x\sqrt{3x^4+5x^2+2}(836\right.\right.\right.$$
$$\frac{2}{39}\left(3x^4+5x^2+2\right)^{5/2}x^3$$

$\big\downarrow$ 1413

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{45}\left(4921349\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{2024285\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)+\right.\right.\right.$$
$$\frac{2}{39}\left(3x^4+5x^2+2\right)^{5/2}x^3$$

$\big\downarrow$ 1456

$$\frac{1}{39}\left(\frac{1}{33}\left(\frac{2}{21}\left(\frac{1}{45}\left(\frac{2024285\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+4921349\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\right.\right.\right.\right.\right.$$

$$\frac{2}{39}\left(3x^4+5x^2+2\right)^{5/2}x^3$$

input
```
Int[(1 + 2*x^2)*(4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(2*x^3*(2 + 5*x^2 + 3*x^4)^(5/2))/39 + ((-587*x*(2 + 5*x^2 + 3*x^4)^(5/2))
/33 + ((x*(49801 + 43169*x^2)*(2 + 5*x^2 + 3*x^4)^(3/2))/21 + (2*((x*(1708
960 + 836181*x^2)*Sqrt[2 + 5*x^2 + 3*x^4])/45 + (4921349*((x*(2 + 3*x^2))/
(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2
)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (2024285*Sqr
t[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqr
t[2 + 5*x^2 + 3*x^4])/45))/21)/33)/39
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 7.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.61

method	result
risch	$\frac{x(561330x^{10}-3121335x^8-8505945x^6-2840895x^4+5917977x^2+5681150)\sqrt{3x^4+5x^2+2}}{1216215}-\frac{809714i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(i\right.}{243243\sqrt{3x^4+5x^2+2}}$
default	$-\frac{9001x^7\sqrt{3x^4+5x^2+2}}{1287}-\frac{63131x^5\sqrt{3x^4+5x^2+2}}{27027}+\frac{50581x^3\sqrt{3x^4+5x^2+2}}{10395}+\frac{1136230x\sqrt{3x^4+5x^2+2}}{243243}-\frac{809714i\sqrt{x^2+1}\,\sqrt{6}}{243243}$
elliptic	$-\frac{9001x^7\sqrt{3x^4+5x^2+2}}{1287}-\frac{63131x^5\sqrt{3x^4+5x^2+2}}{27027}+\frac{50581x^3\sqrt{3x^4+5x^2+2}}{10395}+\frac{1136230x\sqrt{3x^4+5x^2+2}}{243243}-\frac{809714i\sqrt{x^2+1}\,\sqrt{6}}{243243}$

input
```
int((2*x^2+1)*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1216215*x*(561330*x^10-3121335*x^8-8505945*x^6-2840895*x^4+5917977*x^2+5
681150)*(3*x^4+5*x^2+2)^(1/2)-809714/243243*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2
)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+9842698/3648645*I*(x^2+
1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))
-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.38

$$\int (1+2x^2)(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}\,dx =$$

$$-\frac{19685396\sqrt{3}\sqrt{-\frac{2}{3}}xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2}) - 56122526\sqrt{3}\sqrt{-\frac{2}{3}}xF(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2}) - 3\,(1683990\,x^{12}}{109459}$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="fric
as")
```

output
```
-1/10945935*(19685396*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3)/x)
, 3/2) - 56122526*sqrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/x), 3/
2) - 3*(1683990*x^12 - 9364005*x^10 - 25517835*x^8 - 8522685*x^6 + 1775393
1*x^4 + 17043450*x^2 + 9842698)*sqrt(3*x^4 + 5*x^2 + 2))/x
```

Sympy [F]

$$\int (1+2x^2)(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}\,dx = \int ((x^2+1)(3x^2+2))^{\frac{3}{2}}\cdot(2x^2+1)(x^4-7x^2+4)\,dx$$

input
```
integrate((2*x**2+1)*(x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral(((x**2 + 1)*(3*x**2 + 2))**(3/2)*(2*x**2 + 1)*(x**4 - 7*x**2 + 4)
, x)
```

Maxima [F]

$$\int (1+2x^2)\left(4-7x^2+x^4\right)\left(2+5x^2+3x^4\right)^{3/2}\,dx = \int \left(3\,x^4+5\,x^2+2\right)^{\frac{3}{2}}\left(x^4-7\,x^2+4\right)\left(2\,x^2+1\right)dx$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1), x)
```

Giac [F]

$$\int (1+2x^2)\left(4-7x^2+x^4\right)\left(2+5x^2+3x^4\right)^{3/2}\,dx = \int \left(3\,x^4+5\,x^2+2\right)^{\frac{3}{2}}\left(x^4-7\,x^2+4\right)\left(2\,x^2+1\right)dx$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac
")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)*(2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x^2)\left(4-7x^2+x^4\right)\left(2+5x^2+3x^4\right)^{3/2}\,dx = \int \left(2\,x^2+1\right)\,\left(x^4-7\,x^2+4\right)\,\left(3\,x^4+5\,x^2+2\right)^{3/2}\,dx$$

input
```
int((2*x^2 + 1)*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int((2*x^2 + 1)*(x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int (1+2x^2)(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}\,dx = \frac{6\sqrt{3x^4+5x^2+2}\,x^{11}}{13} - \frac{367\sqrt{3x^4+5x^2+2}\,x^9}{143} - \frac{9001\sqrt{3x^4+5x^2+2}\,x^7}{1287} - \frac{63131\sqrt{3x^4+5x^2+2}\,x^5}{27027} + \frac{50581\sqrt{3x^4+5x^2+2}\,x^3}{10395} + \frac{1136230\sqrt{3x^4+5x^2+2}\,x}{243243} + \frac{1619428\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)}{243243} + \frac{9842698\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)}{1216215}$$

input
```
int((2*x^2+1)*(x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
(561330*sqrt(3*x**4 + 5*x**2 + 2)*x**11 - 3121335*sqrt(3*x**4 + 5*x**2 + 2
)*x**9 - 8505945*sqrt(3*x**4 + 5*x**2 + 2)*x**7 - 2840895*sqrt(3*x**4 + 5*
x**2 + 2)*x**5 + 5917977*sqrt(3*x**4 + 5*x**2 + 2)*x**3 + 5681150*sqrt(3*x
**4 + 5*x**2 + 2)*x + 8097140*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x*
*2 + 2),x) + 9842698*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2
 + 2),x))/1216215
```

3.200 $\int (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 216

$$\begin{aligned}\int (4-7x^2+x^4) (2+5x^2+3x^4)^{3/2} \, dx &= \frac{104954x(2+3x^2)}{31185\sqrt{2+5x^2+3x^4}} \\ &+ \frac{2x(19910+10863x^2)\sqrt{2+5x^2+3x^4}}{10395} - \frac{1}{231}x(15+203x^2) (2+5x^2+3x^4)^{3/2} \\ &+ \frac{1}{33}x(2+5x^2+3x^4)^{5/2} - \frac{104954\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x) \middle| -\frac{1}{2}\right)}{31185\sqrt{2+5x^2+3x^4}} \\ &+ \frac{8686\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x), -\frac{1}{2}\right)}{2079\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
104954/31185*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+2/10395*x*(10863*x^2+19910)
*(3*x^4+5*x^2+2)^(1/2)-1/231*x*(203*x^2+15)*(3*x^4+5*x^2+2)^(3/2)+1/33*x*(
3*x^4+5*x^2+2)^(5/2)-104954/31185*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2
)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+8686/2079
*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I
*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.69

$$\int (4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}\,dx = \frac{3x(79460+211412x^2+79005x^4-196992x^6-192240x^8-39690x^{10}+8505x^{12})-104954}{31}$$

input
```
Integrate[(4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(3*x*(79460 + 211412*x^2 + 79005*x^4 - 196992*x^6 - 192240*x^8 - 39690*x^1
0 + 8505*x^12) - (104954*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Elliptic
E[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (18094*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 +
3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(31185*Sqrt[2 + 5*x^2 + 3*x
^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2207, 1490, 27, 1490, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^4-7x^2+4)(3x^4+5x^2+2)^{3/2}\,dx$$

$$\Big\downarrow \; 2207$$

$$\frac{1}{33}\int (130-261x^2)(3x^4+5x^2+2)^{3/2}\,dx+\frac{1}{33}x(3x^4+5x^2+2)^{5/2}$$

$$\Big\downarrow \; 1490$$

$$\frac{1}{33}\left(\frac{1}{63}\int 18\left(1207x^2+925\right)\sqrt{3x^4+5x^2+2}dx-\frac{1}{7}x\left(203x^2+15\right)\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{1}{33}x\left(3x^4+5x^2+2\right)^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{33}\left(\frac{2}{7}\int \left(1207x^2+925\right)\sqrt{3x^4+5x^2+2}dx-\frac{1}{7}x\left(203x^2+15\right)\left(3x^4+5x^2+2\right)^{3/2}\right)+\frac{1}{33}x\left(3x^4+5x^2+2\right)^{5/2}$$

$$\Big\downarrow \; 1490$$

$$\frac{1}{33}\left(\frac{2}{7}\left(\frac{1}{45}\int \frac{52477x^2+43430}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(10863x^2+19910\right)\right)-\frac{1}{7}x\left(203x^2+15\right)\left(3x^4+5x^2+\right.\right.$$
$$\frac{1}{33}x\left(3x^4+5x^2+2\right)^{5/2}$$

$$\Big\downarrow \; 1503$$

$$\frac{1}{33}\left(\frac{2}{7}\left(\frac{1}{45}\left(43430\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+52477\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{45}x\sqrt{3x^4+5x^2+2}\left(10863x^2+199\right.\right.\right.$$
$$\frac{1}{33}x\left(3x^4+5x^2+2\right)^{5/2}$$

$$\Big\downarrow \; 1413$$

$$\frac{1}{33}\left(\frac{2}{7}\left(\frac{1}{45}\left(52477\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{21715\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)+\frac{1}{45}x\sqrt{3x^4}\right.\right.$$
$$\frac{1}{33}x\left(3x^4+5x^2+2\right)^{5/2}$$

$$\Big\downarrow \; 1456$$

$$\frac{1}{33}\left(\frac{2}{7}\left(\frac{1}{45}\left(\frac{21715\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+52477\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{}}{3\sqrt{3}}\right.\right.\right.\right.$$
$$\frac{1}{33}x\left(3x^4+5x^2+2\right)^{5/2}$$

input
```
Int[(4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(x*(2 + 5*x^2 + 3*x^4)^(5/2))/33 + (-1/7*(x*(15 + 203*x^2)*(2 + 5*x^2 + 3*
x^4)^(3/2)) + (2*((x*(19910 + 10863*x^2)*Sqrt[2 + 5*x^2 + 3*x^4])/45 + (52
477*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt
[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*
x^4])) + (21715*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[Ar
cTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4])/45))/7)/33
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 3.95 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.65

method	result
risch	$\frac{x(2835x^8-17955x^6-36045x^4+6381x^2+39730)\sqrt{3x^4+5x^2+2}}{10395} - \frac{8686i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix, \frac{\sqrt{6}}{2}\right)}{2079\sqrt{3x^4+5x^2+2}} + \frac{104954i\sqrt{x^2+1}\,\sqrt{}}{}$
default	$\frac{3x^9\sqrt{3x^4+5x^2+2}}{11} - \frac{19x^7\sqrt{3x^4+5x^2+2}}{11} - \frac{267x^5\sqrt{3x^4+5x^2+2}}{77} + \frac{709x^3\sqrt{3x^4+5x^2+2}}{1155} + \frac{7946x\sqrt{3x^4+5x^2+2}}{2079} - \frac{8686i\sqrt{x^2}}{}$
elliptic	$\frac{3x^9\sqrt{3x^4+5x^2+2}}{11} - \frac{19x^7\sqrt{3x^4+5x^2+2}}{11} - \frac{267x^5\sqrt{3x^4+5x^2+2}}{77} + \frac{709x^3\sqrt{3x^4+5x^2+2}}{1155} + \frac{7946x\sqrt{3x^4+5x^2+2}}{2079} - \frac{8686i\sqrt{x^2}}{}$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10395*x*(2835*x^8-17955*x^6-36045*x^4+6381*x^2+39730)*(3*x^4+5*x^2+2)^(1
/2)-8686/2079*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*Ellipt
icF(I*x,1/2*6^(1/2))+104954/31185*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5
*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.40

$$\int \left(4-7x^2+x^4\right) \left(2+5x^2+3x^4\right)^{3/2} \, dx =$$

$$-\frac{209908\sqrt{3}\sqrt{-\frac{2}{3}}xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 600778\sqrt{3}\sqrt{-\frac{2}{3}}xF(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 3\,(8505\,x^{10} - 53865}{93555\,x}$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/93555*(209908*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3)/x), 3/2
) - 600778*sqrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/x), 3/2) - 3*
(8505*x^10 - 53865*x^8 - 108135*x^6 + 19143*x^4 + 119190*x^2 + 104954)*sqr
t(3*x^4 + 5*x^2 + 2))/x
```

Sympy [F]

$$\int \left(4-7x^2+x^4\right) \left(2+5x^2+3x^4\right)^{3/2} \, dx = \int \left(\left(x^{2} + 1\right) \left(3x^{2} + 2\right)\right)^{\frac{3}{2}} \left(x^{4} - 7x^{2} + 4\right)\, dx$$

input
```
integrate((x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral(((x**2 + 1)*(3*x**2 + 2))**(3/2)*(x**4 - 7*x**2 + 4), x)
```

Maxima [F]

$$\int \left(4-7x^2+x^4\right) \left(2+5x^2+3x^4\right)^{3/2} \, dx = \int \left(3\,x^{4} + 5\,x^{2} + 2\right)^{\frac{3}{2}} \left(x^{4} - 7\,x^{2} + 4\right) \,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4), x)
```

Giac [F]

$$\int (4-7x^2+x^4)\left(2+5x^2+3x^4\right)^{3/2}\,dx=\int \left(3\,x^4+5\,x^2+2\right)^{\frac{3}{2}}\left(x^4-7\,x^2+4\right)dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (4-7x^2+x^4)\left(2+5x^2+3x^4\right)^{3/2}\,dx=\int \left(x^4-7\,x^2+4\right)\left(3\,x^4+5\,x^2+2\right)^{3/2}dx$$

input
```
int((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int (4-7x^2+x^4)\left(2+5x^2+3x^4\right)^{3/2}\,dx=\frac{3\sqrt{3x^4+5x^2+2}\,x^9}{11}-\frac{19\sqrt{3x^4+5x^2+2}\,x^7}{11}-\frac{267\sqrt{3x^4+5x^2+2}\,x^5}{77}+\frac{709\sqrt{3x^4+5x^2+2}\,x^3}{1155}+\frac{7946\sqrt{3x^4+5x^2+2}\,x}{2079}+\frac{17372\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)}{2079}+\frac{104954\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)}{10395}$$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2),x)
```

output

```
(2835*sqrt(3*x**4 + 5*x**2 + 2)*x**9 - 17955*sqrt(3*x**4 + 5*x**2 + 2)*x**
7 - 36045*sqrt(3*x**4 + 5*x**2 + 2)*x**5 + 6381*sqrt(3*x**4 + 5*x**2 + 2)*
x**3 + 39730*sqrt(3*x**4 + 5*x**2 + 2)*x + 86860*int(sqrt(3*x**4 + 5*x**2
+ 2)/(3*x**4 + 5*x**2 + 2),x) + 104954*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2
)/(3*x**4 + 5*x**2 + 2),x))/10395
```

3.201 $\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{1+2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 295

$$\begin{aligned}\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{1+2x^2}\,dx &= \frac{246517x(2+3x^2)}{54432\sqrt{2+5x^2+3x^4}} \\ &+ \frac{15823x\sqrt{2+5x^2+3x^4}}{9072} - \frac{845}{504}x^3\sqrt{2+5x^2+3x^4} - \frac{305}{252}x^5\sqrt{2+5x^2+3x^4} \\ &+ \frac{1}{6}x^7\sqrt{2+5x^2+3x^4} - \frac{246517(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{27216\sqrt{2}\sqrt{2+5x^2+3x^4}} \\ &+ \frac{95929(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{9072\sqrt{2}\sqrt{2+5x^2+3x^4}} \\ &+ \frac{31(1+x^2)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{16\sqrt{3}\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
246517/54432*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+15823/9072*x*(3*x^4+5*x^2+2
)^(1/2)-845/504*x^3*(3*x^4+5*x^2+2)^(1/2)-305/252*x^5*(3*x^4+5*x^2+2)^(1/2
)+1/6*x^7*(3*x^4+5*x^2+2)^(1/2)-246517/54432*2^(1/2)*(x^2+1)*((3*x^2+2)/(x
^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2
)+95929/18144*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(ar
ctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+31/48*(x^2+1)*EllipticPi(x*6^
(1/2)/(6*x^2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))^(1/2)/
(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.39 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.65

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{1+2x^2}\,dx = \frac{379752x+584340x^3-606492x^5-1170072x^7-304560x^9+5\cdots}{}$$

input
```
Integrate[((4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2))/(1 + 2*x^2),x]
```

output
```
(379752*x + 584340*x^3 - 606492*x^5 - 1170072*x^7 - 304560*x^9 + 54432*x^1
1 - (493034*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[S
qrt[3/2]*x], 2/3] + (56587*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Ellipt
icF[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (17577*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2
+ 3*x^2]*EllipticPi[4/3, I*ArcSinh[Sqrt[3/2]*x], 2/3])/(108864*Sqrt[2 + 5*
x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4-7x^2+4)(3x^4+5x^2+2)^{3/2}}{2x^2+1}\,dx$$

$\big\downarrow$ 2258

$$\int \left(-\frac{479x^4}{16\sqrt{3x^4+5x^2+2}}+\frac{671x^2}{32\sqrt{3x^4+5x^2+2}}+\frac{31}{64(2x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{993}{64\sqrt{3x^4+5x^2+2}}+\frac{9x}{2\sqrt{3x^4+}}\right.$$

$\big\downarrow$ 2009

$$\frac{404\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{567\sqrt{3x^4+5x^2+2}}+\frac{27667(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{3024\sqrt{2}\sqrt{3x^4+5x^2+2}}-\frac{66046\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{8505\sqrt{3x^4+5x^2+2}}+\frac{41947(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{6480\sqrt{2}\sqrt{3x^4+5x^2+2}}+\frac{31(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{16\sqrt{3}\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{15823\sqrt{3x^4+5x^2+2}x}{9072}+\frac{246517(3x^2+2)x}{54432\sqrt{3x^4+5x^2+2}}+\frac{1}{6}\sqrt{3x^4+5x^2+2}x^7-\frac{305}{252}\sqrt{3x^4+5x^2+2}x^5-\frac{845}{504}\sqrt{3x^4+5x^2+2}x^3$$

input

```
Int[((4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2))/(1 + 2*x^2),x]
```

output

```
(246517*x*(2 + 3*x^2))/(54432*Sqrt[2 + 5*x^2 + 3*x^4]) + (15823*x*Sqrt[2 +
 5*x^2 + 3*x^4])/9072 - (845*x^3*Sqrt[2 + 5*x^2 + 3*x^4])/504 - (305*x^5*S
qrt[2 + 5*x^2 + 3*x^4])/252 + (x^7*Sqrt[2 + 5*x^2 + 3*x^4])/6 + (41947*(1
+ x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(6480*Sqrt[
2]*Sqrt[2 + 5*x^2 + 3*x^4]) - (66046*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1
 + x^2)]*EllipticE[ArcTan[x], -1/2])/(8505*Sqrt[2 + 5*x^2 + 3*x^4]) + (276
67*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(3024
*Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) + (404*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2
)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(567*Sqrt[2 + 5*x^2 + 3*x^4]) + (
31*(1 + x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(16*Sqrt[3]*Sqrt[
(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 5.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.62

method	result
risch	$\frac{x(1512x^6-10980x^4-15210x^2+15823)\sqrt{3x^4+5x^2+2}}{9072} - \frac{436447i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix, \frac{\sqrt{6}}{2}\right)}{72576\sqrt{3x^4+5x^2+2}} + \frac{246517i\sqrt{x^2+1}\,\sqrt{6x^2+4}}{54}$
elliptic	$\frac{x^7\sqrt{3x^4+5x^2+2}}{6} - \frac{305x^5\sqrt{3x^4+5x^2+2}}{252} - \frac{845x^3\sqrt{3x^4+5x^2+2}}{504} + \frac{15823x\sqrt{3x^4+5x^2+2}}{9072} - \frac{323273i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{Ellipti}}{217728\sqrt{3x^4+5x^2+2}}$
default	$-\frac{305x^5\sqrt{3x^4+5x^2+2}}{252} - \frac{845x^3\sqrt{3x^4+5x^2+2}}{504} + \frac{15823x\sqrt{3x^4+5x^2+2}}{9072} + \frac{466033i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix, \frac{\sqrt{6}}{2}\right)}{362880\sqrt{3x^4+5x^2+2}} - \frac{471}{}$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/9072*x*(1512*x^6-10980*x^4-15210*x^2+15823)*(3*x^4+5*x^2+2)^(1/2)-436447
/72576*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x
,1/2*6^(1/2))+246517/54432*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)
^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))-31/64*I*(x^
2+1)^(1/2)*(1+3/2*x^2)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticPi(I*x,2,1/2*I*
(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{1+2x^2}\,dx = \int \frac{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(x^4-7\,x^2+4)}{2\,x^2+1}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1),x, algorithm="fric
as")
```

output
```
integral((3*x^8 - 16*x^6 - 21*x^4 + 6*x^2 + 8)*sqrt(3*x^4 + 5*x^2 + 2)/(2*
x^2 + 1), x)
```

Sympy [F]

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{1+2x^2}\,dx = \int \frac{((x^2+1)(3x^2+2))^{\frac{3}{2}}(x^4-7x^2+4)}{2x^2+1}\,dx$$

input
```
integrate((x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(3/2)/(2*x**2+1),x)
```

output
```
Integral(((x**2 + 1)*(3*x**2 + 2))**(3/2)*(x**4 - 7*x**2 + 4)/(2*x**2 + 1)
, x)
```

Maxima [F]

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{1+2x^2}\,dx = \int \frac{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(x^4-7\,x^2+4)}{2\,x^2+1}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1),x, algorithm="maxi
ma")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{1+2x^2}\,dx = \int \frac{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(x^4-7\,x^2+4)}{2\,x^2+1}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1),x, algorithm="giac
")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{1+2x^2}\,dx = \int \frac{(x^4-7\,x^2+4)\,(3\,x^4+5\,x^2+2)^{3/2}}{2\,x^2+1}\,dx$$

input
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2))/(2*x^2 + 1),x)
```

output
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2))/(2*x^2 + 1), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{1+2x^2}\,dx &= \frac{\sqrt{3x^4+5x^2+2}\,x^7}{6} \\ &- \frac{305\sqrt{3x^4+5x^2+2}\,x^5}{252} - \frac{845\sqrt{3x^4+5x^2+2}\,x^3}{504} \\ &+ \frac{15823\sqrt{3x^4+5x^2+2}\,x}{9072} + \frac{56753\left(\int \frac{\sqrt{3x^4+5x^2+2}}{6x^6+13x^4+9x^2+2}dx\right)}{4536} \\ &+ \frac{246517\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^4}{6x^6+13x^4+9x^2+2}dx\right)}{9072} + \frac{170741\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{6x^6+13x^4+9x^2+2}dx\right)}{4536}\end{aligned}$$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1),x)
```

output
```
(1512*sqrt(3*x**4 + 5*x**2 + 2)*x**7 - 10980*sqrt(3*x**4 + 5*x**2 + 2)*x**
5 - 15210*sqrt(3*x**4 + 5*x**2 + 2)*x**3 + 15823*sqrt(3*x**4 + 5*x**2 + 2)
*x + 113506*int(sqrt(3*x**4 + 5*x**2 + 2)/(6*x**6 + 13*x**4 + 9*x**2 + 2),
x) + 246517*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(6*x**6 + 13*x**4 + 9*x**
2 + 2),x) + 341482*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(6*x**6 + 13*x**4
+ 9*x**2 + 2),x))/9072
```

3.202 $\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^2} dx$

Optimal result

Integrand size = 36, antiderivative size = 302

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^2} dx = \frac{61199x(2+3x^2)}{20160\sqrt{2+5x^2+3x^4}} - \frac{265}{336}x\sqrt{2+5x^2+3x^4} - \frac{32}{35}x^3\sqrt{2+5x^2+3x^4} + \frac{3}{28}x^5\sqrt{2+5x^2+3x^4} + \frac{31x\sqrt{2+5x^2+3x^4}}{32(1+2x^2)} - \frac{61199(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{10080\sqrt{2}\sqrt{2+5x^2+3x^4}} + \frac{2719(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{672\sqrt{2}\sqrt{2+5x^2+3x^4}} + \frac{371(1+x^2)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{32\sqrt{3}\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}$$

output
```
61199/20160*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-265/336*x*(3*x^4+5*x^2+2)^(1
/2)-32/35*x^3*(3*x^4+5*x^2+2)^(1/2)+3/28*x^5*(3*x^4+5*x^2+2)^(1/2)+31*x*(3
*x^4+5*x^2+2)^(1/2)/(64*x^2+32)-61199/20160*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^
2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
+2719/1344*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arcta
n(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+371/96*(x^2+1)*EllipticPi(x*6^(1
/2)/(6*x^2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))^(1/2)/(3
*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.52 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.75

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^2} \, dx = \frac{12x(1210-13719x^2-51613x^4-52596x^6-13752x^8+2160x^{1}}{}$$

input
```
Integrate[((4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2))/(1 + 2*x^2)^2,x]
```

output
```
(12*x*(1210 - 13719*x^2 - 51613*x^4 - 52596*x^6 - 13752*x^8 + 2160*x^10) -
 (122398*I)*Sqrt[3]*Sqrt[1 + x^2]*(1 + 2*x^2)*Sqrt[2 + 3*x^2]*EllipticE[I*
ArcSinh[Sqrt[3/2]*x], 2/3] - (48847*I)*Sqrt[3]*Sqrt[1 + x^2]*(1 + 2*x^2)*S
qrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (38955*I)*Sqrt[3]*
Sqrt[1 + x^2]*(1 + 2*x^2)*Sqrt[2 + 3*x^2]*EllipticPi[4/3, I*ArcSinh[Sqrt[3
/2]*x], 2/3])/(40320*(1 + 2*x^2)*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.58, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4-7x^2+4)(3x^4+5x^2+2)^{3/2}}{(2x^2+1)^2}\,dx$$

$$\Big\downarrow \ 2258$$

$$\int \left(-\frac{389x^4}{16\sqrt{3x^4+5x^2+2}}-\frac{45x^2}{16\sqrt{3x^4+5x^2+2}}+\frac{29}{8(2x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{31}{64(2x^2+1)^2\sqrt{3x^4+5x^2+2}}+\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{14\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{9\sqrt{3x^4+5x^2+2}}+\frac{14429(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{2016\sqrt{2}\sqrt{3x^4+5x^2+2}}+\frac{4297\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{945\sqrt{3x^4+5x^2+2}}-\frac{13103(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{864\sqrt{2}\sqrt{3x^4+5x^2+2}}-\frac{31\sqrt{3}(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{32\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{29(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{2\sqrt{3}\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{31\sqrt{3x^4+5x^2+2}x}{32(2x^2+1)}-\frac{265}{336}\sqrt{3x^4+5x^2+2}x+\frac{61199(3x^2+2)x}{20160\sqrt{3x^4+5x^2+2}}+\frac{3}{28}\sqrt{3x^4+5x^2+2}x^5-\frac{32}{35}\sqrt{3x^4+5x^2+2}x^3$$

input
```
Int[((4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2))/(1 + 2*x^2)^2,x]
```

output
```
(61199*x*(2 + 3*x^2))/(20160*Sqrt[2 + 5*x^2 + 3*x^4]) - (265*x*Sqrt[2 + 5*
x^2 + 3*x^4])/336 - (32*x^3*Sqrt[2 + 5*x^2 + 3*x^4])/35 + (3*x^5*Sqrt[2 +
5*x^2 + 3*x^4])/28 + (31*x*Sqrt[2 + 5*x^2 + 3*x^4])/(32*(1 + 2*x^2)) - (13
103*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(864
*Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) + (4297*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^
2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(945*Sqrt[2 + 5*x^2 + 3*x^4]) +
(14429*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(
2016*Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) - (14*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*
x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(9*Sqrt[2 + 5*x^2 + 3*x^4]) +
(29*(1 + x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(2*Sqrt[3]*Sqrt[
(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4]) - (31*Sqrt[3]*(1 + x^2)*El
lipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(32*Sqrt[(1 + x^2)/(2 + 3*x^2)]*
Sqrt[2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 6.89 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.64

method	result
risch	$\frac{\sqrt{3x^4+5x^2+2}\,x(720x^6-5784x^4-8372x^2+605)}{6720x^2+3360}-\frac{34249i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{5376\sqrt{3x^4+5x^2+2}}+\frac{61199i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \left(\text{Ellipti}\right.}{20160\sqrt{3}}$
elliptic	$\frac{31x\sqrt{3x^4+5x^2+2}}{32(2x^2+1)}+\frac{3x^5\sqrt{3x^4+5x^2+2}}{28}-\frac{32x^3\sqrt{3x^4+5x^2+2}}{35}-\frac{265x\sqrt{3x^4+5x^2+2}}{336}-\frac{268939i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix\right.}{80640\sqrt{3x^4+5x^2+2}}$
default	$\frac{3x^5\sqrt{3x^4+5x^2+2}}{28}-\frac{32x^3\sqrt{3x^4+5x^2+2}}{35}-\frac{265x\sqrt{3x^4+5x^2+2}}{336}-\frac{94553i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{26880\sqrt{3x^4+5x^2+2}}+\frac{23i\sqrt{x^2+1}\,\sqrt{6}}{}$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1)^2,x,method=_RETURNVERBOS
E)
```

output
```
1/3360*(3*x^4+5*x^2+2)^(1/2)*x*(720*x^6-5784*x^4-8372*x^2+605)/(2*x^2+1)-3
4249/5376*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(
I*x,1/2*6^(1/2))+61199/20160*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+
2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))-371/128*I
*(x^2+1)^(1/2)*(1+3/2*x^2)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticPi(I*x,2,1/
2*I*(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^2}\,dx=\int \frac{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(x^4-7\,x^2+4)}{(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1)^2,x, algorithm="fr
icas")
```

output
```
integral((3*x^8 - 16*x^6 - 21*x^4 + 6*x^2 + 8)*sqrt(3*x^4 + 5*x^2 + 2)/(4*
x^4 + 4*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^2}\,dx=\int \frac{((x^2+1)(3x^2+2))^{\frac{3}{2}}(x^4-7x^2+4)}{(2x^2+1)^2}\,dx$$

input
```
integrate((x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(3/2)/(2*x**2+1)**2,x)
```

output
```
Integral(((x**2 + 1)*(3*x**2 + 2))**(3/2)*(x**4 - 7*x**2 + 4)/(2*x**2 + 1)
**2, x)
```

Maxima [F]

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^2}\,dx = \int \frac{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(x^4-7\,x^2+4)}{(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1)^2,x, algorithm="ma
xima")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1)^2, x)
```

Giac [F]

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^2}\,dx = \int \frac{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(x^4-7\,x^2+4)}{(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1)^2,x, algorithm="gi
ac")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^2}\,dx = \int \frac{(x^4-7\,x^2+4)\,(3\,x^4+5\,x^2+2)^{3/2}}{(2\,x^2+1)^2}\,dx$$

input
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2))/(2*x^2 + 1)^2,x)
```

output
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2))/(2*x^2 + 1)^2, x)
```

Reduce [F]

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^2}\,dx = \frac{180\sqrt{3x^4+5x^2+2}\,x^7 - 1446\sqrt{3x^4+5x^2+2}\,x^5 - 2093\sqrt{3x^4+}}{}$$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1)^2,x)
```

output
```
(180*sqrt(3*x**4 + 5*x**2 + 2)*x**7 - 1446*sqrt(3*x**4 + 5*x**2 + 2)*x**5
- 2093*sqrt(3*x**4 + 5*x**2 + 2)*x**3 + 8308*sqrt(3*x**4 + 5*x**2 + 2)*x -
 6352*int(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2
 + 2),x)*x**2 - 3176*int(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32*x**6 + 31
*x**4 + 13*x**2 + 2),x) - 36682*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(12*x
**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x)*x**2 - 18341*int((sqrt(3*x**4 +
5*x**2 + 2)*x**6)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x) + 12780*i
nt((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2
 + 2),x)*x**2 + 6390*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(12*x**8 + 32*x*
*6 + 31*x**4 + 13*x**2 + 2),x))/(840*(2*x**2 + 1))
```

3.203 $\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 309

$$\begin{aligned}\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^3}\,dx &= -\frac{3917x(2+3x^2)}{1280\sqrt{2+5x^2+3x^4}} \\ &\quad -\frac{13}{16}x\sqrt{2+5x^2+3x^4}+\frac{3}{40}x^3\sqrt{2+5x^2+3x^4}+\frac{31x\sqrt{2+5x^2+3x^4}}{64(1+2x^2)^2} \\ &\quad +\frac{649x\sqrt{2+5x^2+3x^4}}{128(1+2x^2)}+\frac{3917(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{640\sqrt{2}\sqrt{2+5x^2+3x^4}} \\ &\quad -\frac{1783(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\mathrm{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{128\sqrt{2}\sqrt{2+5x^2+3x^4}} \\ &\quad +\frac{2617(1+x^2)\,\mathrm{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{128\sqrt{3}\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
-3917/1280*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-13/16*x*(3*x^4+5*x^2+2)^(1/2)
+3/40*x^3*(3*x^4+5*x^2+2)^(1/2)+31/64*x*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)^2+
649*x*(3*x^4+5*x^2+2)^(1/2)/(256*x^2+128)+3917/1280*2^(1/2)*(x^2+1)*((3*x^
2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+
2)^(1/2)-1783/256*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiA
M(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+2617/384*(x^2+1)*Elliptic
Pi(x*6^(1/2)/(6*x^2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))
^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.66

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^3}\,dx = \frac{\frac{12x(6070+24091x^2+27619x^4+4318x^6-4704x^8+576x^{10})}{(1+2x^2)^2}+23502i\sqrt{3}\sqrt{1+x^2}$$

input
```
Integrate[((4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2))/(1 + 2*x^2)^3,x]
```

output
```
((12*x*(6070 + 24091*x^2 + 27619*x^4 + 4318*x^6 - 4704*x^8 + 576*x^10))/(1
 + 2*x^2)^2 + (23502*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*
ArcSinh[Sqrt[3/2]*x], 2/3] - (27097*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^
2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (13085*I)*Sqrt[3]*Sqrt[1 + x^2
]*Sqrt[2 + 3*x^2]*EllipticPi[4/3, I*ArcSinh[Sqrt[3/2]*x], 2/3])/(7680*Sqrt
[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^4-7x^2+4)\,(3x^4+5x^2+2)^{3/2}}{(2x^2+1)^3}\,dx$$

$$\Big\downarrow \; 2258$$

$$\int \left(-\frac{93x^4}{16\sqrt{3x^4+5x^2+2}}-\frac{37x^2}{4\sqrt{3x^4+5x^2+2}}+\frac{555}{64\,(2x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{29}{8\,(2x^2+1)^2\sqrt{3x^4+5x^2+2}}+\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{1783(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{128\sqrt{2}\sqrt{3x^4+5x^2+2}}+\frac{3917(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{640\sqrt{2}\sqrt{3x^4+5x^2+2}}+\frac{69\sqrt{3}(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{16\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{961(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{128\sqrt{3}\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{649\sqrt{3x^4+5x^2+2}\,x}{128\,(2x^2+1)}+\frac{31\sqrt{3x^4+5x^2+2}\,x}{64\,(2x^2+1)^2}-\frac{13}{16}\sqrt{3x^4+5x^2+2}\,x-\frac{3917(3x^2+2)\,x}{1280\sqrt{3x^4+5x^2+2}}+\frac{3}{40}\sqrt{3x^4+5x^2+2}\,x^3$$

input
```
Int[((4 - 7*x^2 + x^4)*(2 + 5*x^2 + 3*x^4)^(3/2))/(1 + 2*x^2)^3,x]
```

output
```
(-3917*x*(2 + 3*x^2))/(1280*Sqrt[2 + 5*x^2 + 3*x^4]) - (13*x*Sqrt[2 + 5*x^
2 + 3*x^4])/16 + (3*x^3*Sqrt[2 + 5*x^2 + 3*x^4])/40 + (31*x*Sqrt[2 + 5*x^2
 + 3*x^4])/(64*(1 + 2*x^2)^2) + (649*x*Sqrt[2 + 5*x^2 + 3*x^4])/(128*(1 +
2*x^2)) + (3917*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
 -1/2])/(640*Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) - (1783*(1 + x^2)*Sqrt[(2 +
3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(128*Sqrt[2]*Sqrt[2 + 5*x^2
+ 3*x^4]) + (961*(1 + x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(12
8*Sqrt[3]*Sqrt[(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4]) + (69*Sqrt[
3]*(1 + x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(16*Sqrt[(1 + x^2
)/(2 + 3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 10.69 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.62

method	result
risch	$\frac{\sqrt{3x^4+5x^2+2}\,x\left(192x^6-1888x^4+4458x^2+3035\right)}{640\left(2x^2+1\right)^2}-\frac{719i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{1024\sqrt{3x^4+5x^2+2}}-\frac{3917i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \left(\text{EllipticF}\ldots\right.}{1280\sqrt{3x^4\ldots}}$
default	$\frac{31x\sqrt{3x^4+5x^2+2}}{64\left(2x^2+1\right)^2}+\frac{649x\sqrt{3x^4+5x^2+2}}{128\left(2x^2+1\right)}-\frac{19263i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{5120\sqrt{3x^4+5x^2+2}}+\frac{3917i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticE}\left(ix,\ldots\right.}{1280\sqrt{3x^4+5x^2+2}}$
elliptic	$\frac{31x\sqrt{3x^4+5x^2+2}}{64\left(2x^2+1\right)^2}+\frac{649x\sqrt{3x^4+5x^2+2}}{128\left(2x^2+1\right)}-\frac{19263i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{5120\sqrt{3x^4+5x^2+2}}+\frac{3917i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticE}\left(ix,\ldots\right.}{1280\sqrt{3x^4+5x^2+2}}$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1)^3,x,method=_RETURNVERBOS
E)
```

output
```
1/640*(3*x^4+5*x^2+2)^(1/2)*x*(192*x^6-1888*x^4+4458*x^2+3035)/(2*x^2+1)^2
-719/1024*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(
I*x,1/2*6^(1/2))-3917/1280*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)
^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))-2617/512*I*
(x^2+1)^(1/2)*(1+3/2*x^2)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticPi(I*x,2,1/2
*I*(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{(4-7x^2+x^4)\,(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^3}\,dx = \int \frac{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(x^4-7\,x^2+4)}{(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1)^3,x, algorithm="fr
icas")
```

output
```
integral((3*x^8 - 16*x^6 - 21*x^4 + 6*x^2 + 8)*sqrt(3*x^4 + 5*x^2 + 2)/(8*
x^6 + 12*x^4 + 6*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{(4-7x^2+x^4)\,(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^3}\,dx = \int \frac{((x^2+1)\,(3x^2+2))^{\frac{3}{2}}\,(x^4-7x^2+4)}{(2x^2+1)^3}\,dx$$

input
```
integrate((x**4-7*x**2+4)*(3*x**4+5*x**2+2)**(3/2)/(2*x**2+1)**3,x)
```

output
```
Integral(((x**2 + 1)*(3*x**2 + 2))**(3/2)*(x**4 - 7*x**2 + 4)/(2*x**2 + 1)
**3, x)
```

Maxima [F]

$$\int \frac{(4-7x^2+x^4)\,(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^3}\,dx = \int \frac{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(x^4-7\,x^2+4)}{(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1)^3,x, algorithm="ma
xima")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1)^3, x)
```

Giac [F]

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^3}\,dx = \int \frac{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(x^4-7\,x^2+4)}{(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1)^3,x, algorithm="gi
ac")
```

output
```
integrate((3*x^4 + 5*x^2 + 2)^(3/2)*(x^4 - 7*x^2 + 4)/(2*x^2 + 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^3}\,dx = \int \frac{(x^4-7\,x^2+4)\,(3\,x^4+5\,x^2+2)^{3/2}}{(2\,x^2+1)^3}\,dx$$

input
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2))/(2*x^2 + 1)^3,x)
```

output
```
int(((x^4 - 7*x^2 + 4)*(5*x^2 + 3*x^4 + 2)^(3/2))/(2*x^2 + 1)^3, x)
```

Reduce [F]

$$\int \frac{(4-7x^2+x^4)(2+5x^2+3x^4)^{3/2}}{(1+2x^2)^3}\,dx = \frac{396\sqrt{3x^4+5x^2+2}\,x^7-3894\sqrt{3x^4+5x^2+2}\,x^5-33697\sqrt{3x^4}}{}$$

input
```
int((x^4-7*x^2+4)*(3*x^4+5*x^2+2)^(3/2)/(2*x^2+1)^3,x)
```

output

```
(396*sqrt(3*x**4 + 5*x**2 + 2)*x**7 - 3894*sqrt(3*x**4 + 5*x**2 + 2)*x**5
- 33697*sqrt(3*x**4 + 5*x**2 + 2)*x**3 - 68576*sqrt(3*x**4 + 5*x**2 + 2)*x
 + 205344*int(sqrt(3*x**4 + 5*x**2 + 2)/(24*x**10 + 76*x**8 + 94*x**6 + 57
*x**4 + 17*x**2 + 2),x)*x**4 + 205344*int(sqrt(3*x**4 + 5*x**2 + 2)/(24*x*
*10 + 76*x**8 + 94*x**6 + 57*x**4 + 17*x**2 + 2),x)*x**2 + 51336*int(sqrt(
3*x**4 + 5*x**2 + 2)/(24*x**10 + 76*x**8 + 94*x**6 + 57*x**4 + 17*x**2 + 2
),x) + 427744*int(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32*x**6 + 31*x**4 +
 13*x**2 + 2),x)*x**4 + 427744*int(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32
*x**6 + 31*x**4 + 13*x**2 + 2),x)*x**2 + 106936*int(sqrt(3*x**4 + 5*x**2 +
 2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x) - 65724*int((sqrt(3*x**
4 + 5*x**2 + 2)*x**6)/(24*x**10 + 76*x**8 + 94*x**6 + 57*x**4 + 17*x**2 +
2),x)*x**4 - 65724*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(24*x**10 + 76*x**
8 + 94*x**6 + 57*x**4 + 17*x**2 + 2),x)*x**2 - 16431*int((sqrt(3*x**4 + 5*
x**2 + 2)*x**6)/(24*x**10 + 76*x**8 + 94*x**6 + 57*x**4 + 17*x**2 + 2),x)
+ 320808*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(12*x**8 + 32*x**6 + 31*x**4
 + 13*x**2 + 2),x)*x**4 + 320808*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(12*
x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x)*x**2 + 80202*int((sqrt(3*x**4 +
 5*x**2 + 2)*x**6)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x) - 320808
*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x*
*2 + 2),x)*x**4 - 320808*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(12*x**8 ...
```

3.204 $\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 229

$$\begin{aligned}\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx &= \frac{299197x(2+3x^2)}{76545\sqrt{2+5x^2+3x^4}} - \frac{3359x\sqrt{2+5x^2+3x^4}}{5103} \\ &+ \frac{5602x^3\sqrt{2+5x^2+3x^4}}{2835} \\ &- \frac{1508}{567}x^5\sqrt{2+5x^2+3x^4} + \frac{8}{27}x^7\sqrt{2+5x^2+3x^4} \\ &- \frac{299197\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{76545\sqrt{2+5x^2+3x^4}} \\ &+ \frac{13565\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{5103\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
299197/76545*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-3359/5103*x*(3*x^4+5*x^2+2)
^(1/2)+5602/2835*x^3*(3*x^4+5*x^2+2)^(1/2)-1508/567*x^5*(3*x^4+5*x^2+2)^(1
/2)+8/27*x^7*(3*x^4+5*x^2+2)^(1/2)-299197/76545*2^(1/2)*(x^2+1)*((3*x^2+2)
/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(
1/2)+13565/5103*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(
arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.63

$$\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{3x(-33590+16861x^2+65985x^4-172926x^6-165780x^8+22680x^{10})-299197i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}\,I}{76545\sqrt{2+5x^2+3}}$$

input
```
Integrate[((1 + 2*x^2)^3*(4 - 7*x^2 + x^4))/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(3*x*(-33590 + 16861*x^2 + 65985*x^4 - 172926*x^6 - 165780*x^8 + 22680*x^1
0) - (299197*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[
Sqrt[3/2]*x], 2/3] + (163547*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Elli
pticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(76545*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2207, 2207, 2207, 27, 2207, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2+1)^3(x^4-7x^2+4)}{\sqrt{3x^4+5x^2+2}}\,dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{27}\int \frac{-1508x^8-1354x^6+189x^4+459x^2+108}{\sqrt{3x^4+5x^2+2}}dx+\frac{8}{27}\sqrt{3x^4+5x^2+2}\,x^7$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{27}\left(\frac{1}{21}\int \frac{16806x^6+19049x^4+9639x^2+2268}{\sqrt{3x^4+5x^2+2}}dx-\frac{1508}{21}x^5\sqrt{3x^4+5x^2+2}\right)+\frac{8}{27}\sqrt{3x^4+5x^2+2}x^7$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{27}\left(\frac{1}{21}\left(\frac{1}{15}\int \frac{3(-16795x^4+14583x^2+11340)}{\sqrt{3x^4+5x^2+2}}dx+\frac{5602}{5}\sqrt{3x^4+5x^2+2}x^3\right)-\frac{1508}{21}x^5\sqrt{3x^4+5x^2+2}\right)+\frac{8}{27}\sqrt{3x^4+5x^2+2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27}\left(\frac{1}{21}\left(\frac{1}{5}\int \frac{-16795x^4+14583x^2+11340}{\sqrt{3x^4+5x^2+2}}dx+\frac{5602}{5}\sqrt{3x^4+5x^2+2}x^3\right)-\frac{1508}{21}x^5\sqrt{3x^4+5x^2+2}\right)+\frac{8}{27}\sqrt{3x^4+5x^2+2}x^7$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{27}\left(\frac{1}{21}\left(\frac{1}{5}\left(\frac{1}{9}\int \frac{299197x^2+135650}{\sqrt{3x^4+5x^2+2}}dx-\frac{16795}{9}x\sqrt{3x^4+5x^2+2}\right)+\frac{5602}{5}\sqrt{3x^4+5x^2+2}x^3\right)-\frac{1508}{21}x^5\sqrt{3x\right.\right.$$
$$\frac{8}{27}\sqrt{3x^4+5x^2+2}x^7$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{27}\left(\frac{1}{21}\left(\frac{1}{5}\left(\frac{1}{9}\left(135650\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+299197\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{16795}{9}x\sqrt{3x^4+5x^2+2}\right)+\right.\right.\right.$$
$$\frac{8}{27}\sqrt{3x^4+5x^2+2}x^7$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{27}\left(\frac{1}{21}\left(\frac{1}{5}\left(\frac{1}{9}\left(299197\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{67825\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)-\frac{1679}{9}\right.\right.\right.\right.$$
$$\frac{8}{27}\sqrt{3x^4+5x^2+2}x^7$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{27}\left(\frac{1}{21}\left(\frac{1}{5}\left(\frac{1}{9}\left(\frac{67825\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+299197\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+\right.\right.\right.\right.\right.$$

$$\frac{8}{27}\sqrt{3x^4+5x^2+2}x^7$$

input
```
Int[((1 + 2*x^2)^3*(4 - 7*x^2 + x^4))/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(8*x^7*Sqrt[2 + 5*x^2 + 3*x^4])/27 + ((-1508*x^5*Sqrt[2 + 5*x^2 + 3*x^4])/
21 + ((5602*x^3*Sqrt[2 + 5*x^2 + 3*x^4])/5 + ((-16795*x*Sqrt[2 + 5*x^2 + 3
*x^4])/9 + (299197*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]
*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt
[2 + 5*x^2 + 3*x^4])) + (67825*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2
)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4])/9)/5)/21)/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 15.97 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.59

method	result
risch	$\frac{x(7560x^6-67860x^4+50418x^2-16795)\sqrt{3x^4+5x^2+2}}{25515}-\frac{13565i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{5103\sqrt{3x^4+5x^2+2}}+\frac{299197i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ (}{765}$
default	$-\frac{13565i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{5103\sqrt{3x^4+5x^2+2}}+\frac{299197i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{76545\sqrt{3x^4+5x^2+2}}-\frac{3359x\sqrt{3x}}{510}$
elliptic	$-\frac{13565i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{5103\sqrt{3x^4+5x^2+2}}+\frac{299197i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{76545\sqrt{3x^4+5x^2+2}}-\frac{3359x\sqrt{3x}}{510}$

input
```
int((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/25515*x*(7560*x^6-67860*x^4+50418*x^2-16795)*(3*x^4+5*x^2+2)^(1/2)-13565
/5103*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,
1/2*6^(1/2))+299197/76545*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^
(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.36

$$\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = -\frac{598394\sqrt{3}\sqrt{-\frac{2}{3}}\,xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 1208819\sqrt{3}\sqrt{-\frac{2}{3}}\,xF(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 3\,(22680\,x^8 - 20}{229635\,x}$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/229635*(598394*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3)/x), 3/
2) - 1208819*sqrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/x), 3/2) -
3*(22680*x^8 - 203580*x^6 + 151254*x^4 - 50385*x^2 + 299197)*sqrt(3*x^4 +
5*x^2 + 2))/x
```

Sympy [F]

$$\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(2x^2+1)^3(x^4-7x^2+4)}{\sqrt{(x^2+1)(3x^2+2)}}\,dx$$

input
```
integrate((2*x**2+1)**3*(x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((2*x**2 + 1)**3*(x**4 - 7*x**2 + 4)/sqrt((x**2 + 1)*(3*x**2 + 2))
, x)
```

Maxima [F]

$$\int \frac{(1+2x^2)^3 (4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)^3}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^3/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{(1+2x^2)^3 (4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)^3}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^3/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2)^3 (4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(2\,x^2+1)^3\,(x^4-7\,x^2+4)}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int(((2*x^2 + 1)^3*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(((2*x^2 + 1)^3*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{8\sqrt{3x^4+5x^2+2}\,x^7}{27} - \frac{1508\sqrt{3x^4+5x^2+2}\,x^5}{567} + \frac{5602\sqrt{3x^4+5x^2+2}\,x^3}{2835} - \frac{3359\sqrt{3x^4+5x^2+2}\,x}{5103} + \frac{27130\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)}{5103} + \frac{299197\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)}{25515}$$

input
```
int((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(7560*sqrt(3*x**4 + 5*x**2 + 2)*x**7 - 67860*sqrt(3*x**4 + 5*x**2 + 2)*x**
5 + 50418*sqrt(3*x**4 + 5*x**2 + 2)*x**3 - 16795*sqrt(3*x**4 + 5*x**2 + 2)
*x + 135650*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x) + 29919
7*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2 + 2),x))/25515
```

3.205 $\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 206

$$\begin{aligned}\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx &= -\frac{2771x(2+3x^2)}{945\sqrt{2+5x^2+3x^4}}+\frac{187}{63}x\sqrt{2+5x^2+3x^4}\\ &\quad-\frac{208}{105}x^3\sqrt{2+5x^2+3x^4}+\frac{4}{21}x^5\sqrt{2+5x^2+3x^4}\\ &\quad+\frac{2771\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{945\sqrt{2+5x^2+3x^4}}\\ &\quad-\frac{61\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{63\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output

```
-2771/945*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+187/63*x*(3*x^4+5*x^2+2)^(1/2)
-208/105*x^3*(3*x^4+5*x^2+2)^(1/2)+4/21*x^5*(3*x^4+5*x^2+2)^(1/2)+2771/945
*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I
*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)-61/63*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^
(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{5610x+10281x^3-585x^5-4716x^7+540x^9+2771i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-216}{945\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[((1 + 2*x^2)^2*(4 - 7*x^2 + x^4))/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(5610*x + 10281*x^3 - 585*x^5 - 4716*x^7 + 540*x^9 + (2771*I)*Sqrt[3]*Sqrt
[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (2161*I
)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x],
2/3])/(945*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2207, 2207, 27, 2207, 25, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2+1)^2(x^4-7x^2+4)}{\sqrt{3x^4+5x^2+2}}\,dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{21}\int \frac{-624x^6-271x^4+189x^2+84}{\sqrt{3x^4+5x^2+2}}dx+\frac{4}{21}\sqrt{3x^4+5x^2+2}x^5$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{21}\left(\frac{1}{15}\int \frac{9(935x^4+731x^2+140)}{\sqrt{3x^4+5x^2+2}}dx-\frac{208}{5}x^3\sqrt{3x^4+5x^2+2}\right)+\frac{4}{21}\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 27

$$\frac{1}{21}\left(\frac{3}{5}\int\frac{935x^4+731x^2+140}{\sqrt{3x^4+5x^2+2}}dx-\frac{208}{5}x^3\sqrt{3x^4+5x^2+2}\right)+\frac{4}{21}\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 2207

$$\frac{1}{21}\left(\frac{3}{5}\left(\frac{1}{9}\int-\frac{2771x^2+610}{\sqrt{3x^4+5x^2+2}}dx+\frac{935}{9}\sqrt{3x^4+5x^2+2}x\right)-\frac{208}{5}x^3\sqrt{3x^4+5x^2+2}\right)+\frac{4}{21}\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 25

$$\frac{1}{21}\left(\frac{3}{5}\left(\frac{935}{9}x\sqrt{3x^4+5x^2+2}-\frac{1}{9}\int\frac{2771x^2+610}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{208}{5}x^3\sqrt{3x^4+5x^2+2}\right)+\frac{4}{21}\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 1503

$$\frac{1}{21}\left(\frac{3}{5}\left(\frac{1}{9}\left(-610\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx-2771\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{935}{9}\sqrt{3x^4+5x^2+2}x\right)-\frac{208}{5}x^3\sqrt{3x^4\right.$$
$$\frac{4}{21}\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 1413

$$\frac{1}{21}\left(\frac{3}{5}\left(\frac{1}{9}\left(-2771\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx-\frac{305\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)+\frac{935}{9}\sqrt{3x^4+\right.\right.$$
$$\frac{4}{21}\sqrt{3x^4+5x^2+2}x^5$$

$\Big\downarrow$ 1456

$$\frac{1}{21}\left(\frac{3}{5}\left(\frac{1}{9}\left(-\frac{305\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}-2771\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2}{x^2}}}{3\sqrt{3x^4}}\right.\right.\right.\right.$$
$$\frac{4}{21}\sqrt{3x^4+5x^2+2}x^5$$

input
```
Int[((1 + 2*x^2)^2*(4 - 7*x^2 + x^4))/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(4*x^5*Sqrt[2 + 5*x^2 + 3*x^4])/21 + ((-208*x^3*Sqrt[2 + 5*x^2 + 3*x^4])/5
 + (3*((935*x*Sqrt[2 + 5*x^2 + 3*x^4])/9 + (-2771*((x*(2 + 3*x^2))/(3*Sqrt
[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Elli
pticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) - (305*Sqrt[2]*(1 + x
^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2
 + 3*x^4])/9))/5)/21
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 11.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

method	result
risch	$\frac{x(60x^4-624x^2+935)\sqrt{3x^4+5x^2+2}}{315}+\frac{61i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{63\sqrt{3x^4+5x^2+2}}-\frac{2771i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{Ell}\right.}{945\sqrt{3x^4+5x^2+2}}$
default	$\frac{61i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{63\sqrt{3x^4+5x^2+2}}-\frac{2771i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{945\sqrt{3x^4+5x^2+2}}+\frac{187x\sqrt{3x^4+5x^2+2}}{63}$
elliptic	$\frac{61i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{63\sqrt{3x^4+5x^2+2}}-\frac{2771i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{945\sqrt{3x^4+5x^2+2}}+\frac{187x\sqrt{3x^4+5x^2+2}}{63}$

input
```
int((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/315*x*(60*x^4-624*x^2+935)*(3*x^4+5*x^2+2)^(1/2)+61/63*I*(x^2+1)^(1/2)*(
6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))-2771/945*I
*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^
(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.37

$$\int \frac{(1+2x^2)^2\,(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx$$
$$=\frac{5542\sqrt{3}\sqrt{-\frac{2}{3}}xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2})-8287\sqrt{3}\sqrt{-\frac{2}{3}}xF(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2})+3\,(180\,x^6-1872\,x^4+28}{2835\,x}$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fr
icas")
```

output
```
1/2835*(5542*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3)/x), 3/2) -
8287*sqrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/x), 3/2) + 3*(180*x
^6 - 1872*x^4 + 2805*x^2 - 2771)*sqrt(3*x^4 + 5*x^2 + 2))/x
```

Sympy [F]

$$\int \frac{(1+2x^2)^2 (4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(2x^2+1)^2 (x^4-7x^2+4)}{\sqrt{(x^2+1)(3x^2+2)}}\,dx$$

input
```
integrate((2*x**2+1)**2*(x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((2*x**2 + 1)**2*(x**4 - 7*x**2 + 4)/sqrt((x**2 + 1)*(3*x**2 + 2))
, x)
```

Maxima [F]

$$\int \frac{(1+2x^2)^2 (4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)^2}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^2/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{(1+2x^2)^2 (4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)^2}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^2/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2)^2 (4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(2\,x^2+1)^2\,(x^4-7\,x^2+4)}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int(((2*x^2 + 1)^2*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(((2*x^2 + 1)^2*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1+2x^2)^2 (4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{4\sqrt{3x^4+5x^2+2}\,x^5}{21} - \frac{208\sqrt{3x^4+5x^2+2}\,x^3}{105} + \frac{187\sqrt{3x^4+5x^2+2}\,x}{63} - \frac{122\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)}{63} - \frac{2771\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)}{315}$$

input
```
int((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(60*sqrt(3*x**4 + 5*x**2 + 2)*x**5 - 624*sqrt(3*x**4 + 5*x**2 + 2)*x**3 +
935*sqrt(3*x**4 + 5*x**2 + 2)*x - 610*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**
4 + 5*x**2 + 2),x) - 2771*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5
*x**2 + 2),x))/315
```

3.206 $\int \frac{(1+2x^2)(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 183

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}} \, dx = \frac{2377x(2+3x^2)}{405\sqrt{2+5x^2+3x^4}} - \frac{47}{27}x\sqrt{2+5x^2+3x^4} + \frac{2}{15}x^3\sqrt{2+5x^2+3x^4} - \frac{2377\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{405\sqrt{2+5x^2+3x^4}} + \frac{101\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{27\sqrt{2+5x^2+3x^4}}$$

output
```
2377/405*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-47/27*x*(3*x^4+5*x^2+2)^(1/2)+2
/15*x^3*(3*x^4+5*x^2+2)^(1/2)-2377/405*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))
^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+101/
27*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2
*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.74

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{3x(-470-1139x^2-615x^4+54x^6)-2377i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\,\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)+1367i\sqrt{3}\sqrt{1}}{405\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[((1 + 2*x^2)*(4 - 7*x^2 + x^4))/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(3*x*(-470 - 1139*x^2 - 615*x^4 + 54*x^6) - (2377*I)*Sqrt[3]*Sqrt[1 + x^2]
*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (1367*I)*Sqrt[3]
*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(40
5*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2207, 2207, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2+1)(x^4-7x^2+4)}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{15}\int \frac{-235x^4+3x^2+60}{\sqrt{3x^4+5x^2+2}}dx+\frac{2}{15}\sqrt{3x^4+5x^2+2}x^3$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{15}\left(\frac{1}{9}\int \frac{2377x^2+1010}{\sqrt{3x^4+5x^2+2}}dx-\frac{235}{9}x\sqrt{3x^4+5x^2+2}\right)+\frac{2}{15}\sqrt{3x^4+5x^2+2}x^3$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{15}\left(\frac{1}{9}\left(1010\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx+2377\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{235}{9}x\sqrt{3x^4+5x^2+2}\right)+\frac{2}{15}\sqrt{3x^4+5x^2+2}x^3$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{15}\left(\frac{1}{9}\left(2377\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{505\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)-\frac{235}{9}x\sqrt{3x^4+5x^2+2}\right)+\frac{2}{15}\sqrt{3x^4+5x^2+2}x^3$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{15}\left(\frac{1}{9}\left(\frac{505\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+2377\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\ldots\right)}{3\sqrt{3x^4+5x^2+2}}\right)\right)\ldots\right)+\frac{2}{15}\sqrt{3x^4+5x^2+2}x^3$$

input
```
Int[((1 + 2*x^2)*(4 - 7*x^2 + x^4))/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(2*x^3*Sqrt[2 + 5*x^2 + 3*x^4])/15 + ((-235*x*Sqrt[2 + 5*x^2 + 3*x^4])/9 +
 (2377*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*S
qrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 +
 3*x^4])) + (505*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[A
rcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4])/9)/15
```

Defintions of rubi rules used

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 6.83 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.68

method	result
risch	$\frac{x(18x^2-235)\sqrt{3x^4+5x^2+2}}{135}-\frac{101i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{27\sqrt{3x^4+5x^2+2}}+\frac{2377i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\right.}{405\sqrt{3x^4+5x^2+2}}$
default	$\frac{2377i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{405\sqrt{3x^4+5x^2+2}}-\frac{101i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{27\sqrt{3x^4+5x^2+2}}-\frac{47x\sqrt{3x^4+5x^2+2}}{27}$
elliptic	$\frac{2377i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{405\sqrt{3x^4+5x^2+2}}-\frac{101i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{27\sqrt{3x^4+5x^2+2}}-\frac{47x\sqrt{3x^4+5x^2+2}}{27}$

input
```
int((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/135*x*(18*x^2-235)*(3*x^4+5*x^2+2)^(1/2)-101/27*I*(x^2+1)^(1/2)*(6*x^2+4
)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+2377/405*I*(x^2+1
)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-
EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx =$$
$$-\frac{4754\sqrt{3}\sqrt{-\frac{2}{3}}xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 9299\sqrt{3}\sqrt{-\frac{2}{3}}xF(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right) \mid \frac{3}{2}) - 3\,(54\,x^4 - 705\,x^2 + 23}{1215\,x}$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fric
as")
```

output
```
-1/1215*(4754*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3)/x), 3/2) -
 9299*sqrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/x), 3/2) - 3*(54*x
^4 - 705*x^2 + 2377)*sqrt(3*x^4 + 5*x^2 + 2))/x
```

Sympy [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(2x^2+1)(x^4-7x^2+4)}{\sqrt{(x^2+1)(3x^2+2)}}\,dx$$

input
```
integrate((2*x**2+1)*(x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((2*x**2 + 1)*(x**4 - 7*x**2 + 4)/sqrt((x**2 + 1)*(3*x**2 + 2)), x
)
```

Maxima [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac
")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{(2\,x^2+1)\,(x^4-7\,x^2+4)}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int(((2*x^2 + 1)*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int(((2*x^2 + 1)*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{2\sqrt{3x^4+5x^2+2}\,x^3}{15} - \frac{47\sqrt{3x^4+5x^2+2}\,x}{27} + \frac{202\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)}{27} + \frac{2377\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)}{135}$$

input
```
int((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(18*sqrt(3*x**4 + 5*x**2 + 2)*x**3 - 235*sqrt(3*x**4 + 5*x**2 + 2)*x + 101
0*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 + 5*x**2 + 2),x) + 2377*int((sqrt(
3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2 + 2),x))/135
```

3.207 $\int \frac{4-7x^2+x^4}{\sqrt{2+5x^2+3x^4}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 160

$$\begin{aligned}\int \frac{4-7x^2+x^4}{\sqrt{2+5x^2+3x^4}} \, dx &= -\frac{73x(2+3x^2)}{27\sqrt{2+5x^2+3x^4}}+\frac{1}{9}x\sqrt{2+5x^2+3x^4} \\ &\quad+\frac{73\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{27\sqrt{2+5x^2+3x^4}} \\ &\quad+\frac{17\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{9\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output

```
-73/27*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/9*x*(3*x^4+5*x^2+2)^(1/2)+73/27
*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I
*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+17/9*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(
1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int \frac{4-7x^2+x^4}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{6x+15x^3+9x^5+73i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-107i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{Elliptic}}{27\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[(4 - 7*x^2 + x^4)/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(6*x + 15*x^3 + 9*x^5 + (73*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Ellip
ticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (107*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 +
 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(27*Sqrt[2 + 5*x^2 + 3*x^4
])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2207, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-7x^2+4}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{9}\int \frac{34-73x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{1}{9}\sqrt{3x^4+5x^2+2}x$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{9}\left(34\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx-73\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{9}\sqrt{3x^4+5x^2+2}x$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{9}\left(\frac{17\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}-73\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{9}\sqrt{3x^4+5x^2+2}x$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{9}\left(\frac{17\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}-73\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E(\arctan(x}{3\sqrt{3x^4+5x^2+2}}\right.\right.$$
$$\frac{1}{9}\sqrt{3x^4+5x^2+2}x$$

input
```
Int[(4 - 7*x^2 + x^4)/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

output
```
(x*Sqrt[2 + 5*x^2 + 3*x^4])/9 + (-73*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 +
3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[
x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (17*Sqrt[2]*(1 + x^2)*Sqrt[(2 +
3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4])/9
```

Defintions of rubi rules used

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

method	result	si
default	$\frac{x\sqrt{3x^4+5x^2+2}}{9}-\frac{17i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{9\sqrt{3x^4+5x^2+2}}-\frac{73i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{27\sqrt{3x^4+5x^2+2}}$	11
risch	$\frac{x\sqrt{3x^4+5x^2+2}}{9}-\frac{17i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{9\sqrt{3x^4+5x^2+2}}-\frac{73i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{27\sqrt{3x^4+5x^2+2}}$	11
elliptic	$\frac{x\sqrt{3x^4+5x^2+2}}{9}-\frac{17i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{9\sqrt{3x^4+5x^2+2}}-\frac{73i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{27\sqrt{3x^4+5x^2+2}}$	11

input
```
int((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*x*(3*x^4+5*x^2+2)^(1/2)-17/9*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*
x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))-73/27*I*(x^2+1)^(1/2)*(6*x^2+4)^(1
/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.42

$$\int \frac{4-7x^2+x^4}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{146\,\sqrt{3}\sqrt{-\frac{2}{3}}xE(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2})+7\,\sqrt{3}\sqrt{-\frac{2}{3}}xF(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2})+3\,\sqrt{3\,x^4+5\,x^2+2}(3\,x^2-73}{81\,x}$$

input
```
integrate((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/81*(146*sqrt(3)*sqrt(-2/3)*x*elliptic_e(arcsin(sqrt(-2/3)/x), 3/2) + 7*s
qrt(3)*sqrt(-2/3)*x*elliptic_f(arcsin(sqrt(-2/3)/x), 3/2) + 3*sqrt(3*x^4 +
 5*x^2 + 2)*(3*x^2 - 73))/x
```

Sympy [F]

$$\int \frac{4-7x^2+x^4}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7x^2+4}{\sqrt{(x^2+1)\,(3x^2+2)}}\,dx$$

input
```
integrate((x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((x**4 - 7*x**2 + 4)/sqrt((x**2 + 1)*(3*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{4-7x^2+x^4}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7\,x^2+4}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^4 - 7*x^2 + 4)/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Giac [F]

$$\int \frac{4-7x^2+x^4}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7\,x^2+4}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^4 - 7*x^2 + 4)/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4-7x^2+x^4}{\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7\,x^2+4}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)/(5*x^2 + 3*x^4 + 2)^(1/2),x)
```

output
```
int((x^4 - 7*x^2 + 4)/(5*x^2 + 3*x^4 + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{4-7x^2+x^4}{\sqrt{2+5x^2+3x^4}}\,dx = \frac{\sqrt{3x^4+5x^2+2}\,x}{9} + \frac{34\left(\int \frac{\sqrt{3x^4+5x^2+2}}{3x^4+5x^2+2}dx\right)}{9} - \frac{73\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{3x^4+5x^2+2}dx\right)}{9}$$

input
```
int((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(sqrt(3*x**4 + 5*x**2 + 2)*x + 34*int(sqrt(3*x**4 + 5*x**2 + 2)/(3*x**4 +
5*x**2 + 2),x) - 73*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(3*x**4 + 5*x**2
+ 2),x))/9
```

3.208 $\int \frac{4-7x^2+x^4}{(1+2x^2)\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 201

$$\begin{aligned}\int \frac{4-7x^2+x^4}{(1+2x^2)\sqrt{2+5x^2+3x^4}}\,dx &= \frac{x(2+3x^2)}{6\sqrt{2+5x^2+3x^4}}\\ &\quad-\frac{(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{2}\sqrt{2+5x^2+3x^4}}\\ &\quad-\frac{27(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}\\ &\quad+\frac{31(1+x^2)\operatorname{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{3}\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output

```
1/6*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-1/6*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+
1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)-2
7/2*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/
2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+31/3*(x^2+1)*EllipticPi(x*6^(1/2)/(6*x^
2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))^(1/2)/(3*x^4+5*x^
2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.53

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\sqrt{2+5x^2+3x^4}}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+3x^2}\left(2E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-17\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)+31\,\text{EllipticPi}\left(\frac{4}{3},\right.\right.}{4\sqrt{6+15x^2+9x^4}}$$

input
```
Integrate[(4 - 7*x^2 + x^4)/((1 + 2*x^2)*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
((-1/4*I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2*EllipticE[I*ArcSinh[Sqrt[3/2]*x
], 2/3] - 17*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3] + 31*EllipticPi[4/3, I
*ArcSinh[Sqrt[3/2]*x], 2/3]))/Sqrt[6 + 15*x^2 + 9*x^4]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.33, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2234, 1503, 1413, 1456, 1538, 27, 1413, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-7x^2+4}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{2234}$$

$$\frac{31}{4}\int \frac{1}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx-\frac{1}{4}\int \frac{15-2x^2}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{4}\left(2\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx-15\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{31}{4}\int \frac{1}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{4}\left(2\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx-\frac{15\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+$$
$$\frac{31}{4}\int \frac{1}{\left(2x^2+1\right)\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{1456}$$

$$\frac{31}{4}\int \frac{1}{\left(2x^2+1\right)\sqrt{3x^4+5x^2+2}}dx+$$
$$\frac{1}{4}\left(2\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{3x^4+5x^2+2}}\right)-\frac{15\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.$$

$$\Big\downarrow \text{1538}$$

$$\frac{31}{4}\left(\int \frac{2\left(3x^2+2\right)}{\left(2x^2+1\right)\sqrt{3x^4+5x^2+2}}dx-3\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx\right)+$$
$$\frac{1}{4}\left(2\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{3x^4+5x^2+2}}\right)-\frac{15\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{31}{4}\left(2\int \frac{3x^2+2}{\left(2x^2+1\right)\sqrt{3x^4+5x^2+2}}dx-3\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx\right)+$$
$$\frac{1}{4}\left(2\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{3x^4+5x^2+2}}\right)-\frac{15\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.$$

$$\Big\downarrow \text{1413}$$

$$\frac{31}{4}\left(2\int \frac{3x^2+2}{\left(2x^2+1\right)\sqrt{3x^4+5x^2+2}}dx-\frac{3\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+$$
$$\frac{1}{4}\left(2\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{3x^4+5x^2+2}}\right)-\frac{15\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.$$

$$\Big\downarrow \text{1786}$$

$$\frac{31}{4}\left(\frac{2\sqrt{x^2+1}\sqrt{3x^2+2}\int \frac{\sqrt{3x^2+2}}{\sqrt{x^2+1}\left(2x^2+1\right)}dx}{\sqrt{3x^4+5x^2+2}}-\frac{3\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+$$
$$\frac{1}{4}\left(2\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{3x^4+5x^2+2}}\right)-\frac{15\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.$$

$$\Big\downarrow \text{414}$$

$$\frac{31}{4}\left(\frac{4(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{3}\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}-\frac{3(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+$$
$$\frac{1}{4}\left(2\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{3x^4+5x^2+2}}\right)-\frac{15(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\right.}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.$$

input
```
Int[(4 - 7*x^2 + x^4)/((1 + 2*x^2)*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
(2*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[
(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x
^4])) - (15*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/
2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]))/4 + (31*((-3*(1 + x^2)*Sqrt[(2 + 3*
x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^
4]) + (4*(1 + x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(Sqrt[3]*Sq
rt[(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1538

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/(2*c*d - e*(b - q)))   I
nt[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[e/(2*c*d - e*(b - q))   Int[(b
- q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !LtQ[c, 0]
```

rule 1786

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

rule 2234

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.68

method	result
elliptic	$\frac{49i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{24\sqrt{3x^4+5x^2+2}}-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)}{6\sqrt{3x^4+5x^2+2}}-\frac{31i\sqrt{x^2+1}\,\sqrt{1+\frac{3x^2}{2}}\,\operatorname{EllipticPi}\left(ix,2,\frac{i\sqrt{-3}\,\sqrt{2}}{2}\right.}{4\sqrt{3x^4+5x^2+2}}$
default	$\frac{15i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{8\sqrt{3x^4+5x^2+2}}+\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{6\sqrt{3x^4+5x^2+2}}-\frac{31i\sqrt{x^2+1}\,\sqrt{1+\frac{3x^2}{2}}\,\mathrm{Ell}}{4\sqrt{3x^4+}}$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
49/24*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,
1/2*6^(1/2))-1/6*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*Ell
ipticE(I*x,1/2*6^(1/2))-31/4*I*(x^2+1)^(1/2)*(1+3/2*x^2)^(1/2)/(3*x^4+5*x^
2+2)^(1/2)*EllipticPi(I*x,2,1/2*I*(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{x^4-7\,x^2+4}{\sqrt{3\,x^4+5\,x^2+2}(2\,x^2+1)}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(6*x^6 + 13*x^4 + 9*x^2
 + 2), x)
```

Sympy [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7x^2+4}{\sqrt{(x^2+1)(3x^2+2)}\,(2x^2+1)}\,dx$$

input
```
integrate((x**4-7*x**2+4)/(2*x**2+1)/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((x**4 - 7*x**2 + 4)/(sqrt((x**2 + 1)*(3*x**2 + 2))*(2*x**2 + 1)),
 x)
```

Maxima [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7x^2+4}{\sqrt{3x^4+5x^2+2}(2x^2+1)}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((x^4 - 7*x^2 + 4)/(sqrt(3*x^4 + 5*x^2 + 2)*(2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7x^2+4}{\sqrt{3x^4+5x^2+2}(2x^2+1)}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(1/2),x, algorithm="giac
")
```

output
```
integrate((x^4 - 7*x^2 + 4)/(sqrt(3*x^4 + 5*x^2 + 2)*(2*x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{x^4-7\,x^2+4}{(2\,x^2+1)\,\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)*(5*x^2 + 3*x^4 + 2)^(1/2)),x)
```

output
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)*(5*x^2 + 3*x^4 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\sqrt{2+5x^2+3x^4}}\,dx=4\left(\int \frac{\sqrt{3x^4+5x^2+2}}{6x^6+13x^4+9x^2+2}dx\right)+\int \frac{\sqrt{3x^4+5x^2+2}\,x^4}{6x^6+13x^4+9x^2+2}dx-7\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{6x^6+13x^4+9x^2+2}dx\right)$$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
4*int(sqrt(3*x**4 + 5*x**2 + 2)/(6*x**6 + 13*x**4 + 9*x**2 + 2),x) + int((
sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(6*x**6 + 13*x**4 + 9*x**2 + 2),x) - 7*int
((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(6*x**6 + 13*x**4 + 9*x**2 + 2),x)
```

3.209 $\int \frac{4-7x^2+x^4}{(1+2x^2)^2\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 235

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2\sqrt{2+5x^2+3x^4}}\,dx = -\frac{31x(2+3x^2)}{4\sqrt{2+5x^2+3x^4}}+\frac{31x\sqrt{2+5x^2+3x^4}}{2(1+2x^2)} + \frac{31(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{2\sqrt{2}\sqrt{2+5x^2+3x^4}} + \frac{71(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+5x^2+3x^4}} - \frac{125(1+x^2)\operatorname{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{2\sqrt{3}\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}$$

output

```
-31/4*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+31*x*(3*x^4+5*x^2+2)^(1/2)/(4*x^2+
2)+31/4*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2
),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+71/4*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^
2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
-125/6*(x^2+1)*EllipticPi(x*6^(1/2)/(6*x^2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1
/2)/((x^2+1)/(3*x^2+2))^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.48 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.80

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2\sqrt{2+5x^2+3x^4}}\,dx = \frac{\frac{372x(2+5x^2+3x^4)}{1+2x^2}+186i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-95i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}}{24\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[(4 - 7*x^2 + x^4)/((1 + 2*x^2)^2*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
((372*x*(2 + 5*x^2 + 3*x^4))/(1 + 2*x^2) + (186*I)*Sqrt[3]*Sqrt[1 + x^2]*S
qrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (95*I)*Sqrt[3]*Sqr
t[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (125*I
)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticPi[4/3, I*ArcSinh[Sqrt[3/2
]*x], 2/3])/(24*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {2210, 2234, 25, 1503, 1413, 1456, 1538, 27, 1413, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-7x^2+4}{(2x^2+1)^2\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 2210}$$

$$\frac{31x\sqrt{3x^4+5x^2+2}}{2(2x^2+1)}-\frac{1}{2}\int \frac{93x^4+92x^2+54}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{ 2234}$$

$$\frac{1}{2}\left(\frac{1}{4}\int -\frac{186x^2+91}{\sqrt{3x^4+5x^2+2}}dx-\frac{125}{4}\int \frac{1}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx\right)+\frac{31\sqrt{3x^4+5x^2+2}x}{2(2x^2+1)}$$

$\big\downarrow$ 25

$$\frac{1}{2}\left(-\frac{125}{4}\int \frac{1}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx-\frac{1}{4}\int \frac{186x^2+91}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{31\sqrt{3x^4+5x^2+2}x}{2(2x^2+1)}$$

$\big\downarrow$ 1503

$$\frac{1}{2}\left(\frac{1}{4}\left(-91\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx-186\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{125}{4}\int \frac{1}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx\right)+\frac{31\sqrt{3x^4+5x^2+2}x}{2(2x^2+1)}$$

$\big\downarrow$ 1413

$$\frac{1}{2}\left(\frac{1}{4}\left(-186\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx-\frac{91(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)-\frac{125}{4}\int \frac{1}{(2x^2+1)\sqrt{3x^4}}\right. \frac{31\sqrt{3x^4+5x^2+2}x}{2(2x^2+1)}$$

$\big\downarrow$ 1456

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{91(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}-186\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E(\text{arcta}}{3\sqrt{3x^4+5x^2+2}}\right.\right.\right. \frac{31\sqrt{3x^4+5x^2+2}x}{2(2x^2+1)}$$

$\big\downarrow$ 1538

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{91(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}-186\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E(\text{arcta}}{3\sqrt{3x^4+5x^2+2}}\right.\right.\right. \frac{31\sqrt{3x^4+5x^2+2}x}{2(2x^2+1)}$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{91\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}-186\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\text{arcta}\right.}{3\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$

$$\frac{31\sqrt{3x^4+5x^2+2}x}{2\left(2x^2+1\right)}$$

$\downarrow$ 1413

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{91\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}-186\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\text{arcta}\right.}{3\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$

$$\frac{31\sqrt{3x^4+5x^2+2}x}{2\left(2x^2+1\right)}$$

$\downarrow$ 1786

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{91\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}-186\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\text{arcta}\right.}{3\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$

$$\frac{31\sqrt{3x^4+5x^2+2}x}{2\left(2x^2+1\right)}$$

$\downarrow$ 414

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{91\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}-186\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\text{arcta}\right.}{3\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$

$$\frac{31\sqrt{3x^4+5x^2+2}x}{2\left(2x^2+1\right)}$$

input
```
Int[(4 - 7*x^2 + x^4)/((1 + 2*x^2)^2*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
(31*x*Sqrt[2 + 5*x^2 + 3*x^4])/(2*(1 + 2*x^2)) + ((-186*((x*(2 + 3*x^2))/(
3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) - (91*(1 + x^2)
*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 +
 5*x^2 + 3*x^4]))/4 - (125*((-3*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Elli
pticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) + (4*(1 + x^2)*E
llipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(Sqrt[3]*Sqrt[(1 + x^2)/(2 + 3*
x^2)]*Sqrt[2 + 5*x^2 + 3*x^4])))/4)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1538
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/(2*c*d - e*(b - q)))   I
nt[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[e/(2*c*d - e*(b - q))   Int[(b
- q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !LtQ[c, 0]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

Maple [A] (verified)

Time = 6.92 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.69

method	result
default	$-\frac{33 i \sqrt{x^2+1}\, \sqrt{6 x^2+4}\, \operatorname{EllipticF}\left(i x, \frac{\sqrt{6}}{2}\right)}{16 \sqrt{3 x^4+5 x^2+2}}+\frac{31 x \sqrt{3 x^4+5 x^2+2}}{2\left(2 x^2+1\right)}+\frac{31 i \sqrt{x^2+1}\, \sqrt{6 x^2+4}\, \operatorname{EllipticE}\left(i x, \frac{\sqrt{6}}{2}\right)}{4 \sqrt{3 x^4+5 x^2+2}}+\frac{125 i \sqrt{x^2+1}\, \sqrt{1+\frac{3}{}}}{8\sqrt{}}$
elliptic	$-\frac{33 i \sqrt{x^2+1}\, \sqrt{6 x^2+4}\, \operatorname{EllipticF}\left(i x, \frac{\sqrt{6}}{2}\right)}{16 \sqrt{3 x^4+5 x^2+2}}+\frac{31 x \sqrt{3 x^4+5 x^2+2}}{2\left(2 x^2+1\right)}+\frac{31 i \sqrt{x^2+1}\, \sqrt{6 x^2+4}\, \operatorname{EllipticE}\left(i x, \frac{\sqrt{6}}{2}\right)}{4 \sqrt{3 x^4+5 x^2+2}}+\frac{125 i \sqrt{x^2+1}\, \sqrt{1+\frac{3}{}}}{8\sqrt{}}$
risch	$\frac{31 x \sqrt{3 x^4+5 x^2+2}}{2\left(2 x^2+1\right)}+\frac{91 i \sqrt{x^2+1}\, \sqrt{6 x^2+4}\, \operatorname{EllipticF}\left(i x, \frac{\sqrt{6}}{2}\right)}{16 \sqrt{3 x^4+5 x^2+2}}-\frac{31 i \sqrt{x^2+1}\, \sqrt{6 x^2+4}\, \left(\operatorname{EllipticF}\left(i x, \frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(i x, \frac{\sqrt{6}}{2}\right)\right)}{4 \sqrt{3 x^4+5 x^2+2}}+$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-33/16*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x
,1/2*6^(1/2))+31/2*x*(3*x^4+5*x^2+2)^(1/2)/(2*x^2+1)+31/4*I*(x^2+1)^(1/2)*
(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticE(I*x,1/2*6^(1/2))+125/8*I*(
x^2+1)^(1/2)*(1+3/2*x^2)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticPi(I*x,2,1/2*
I*(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7\,x^2+4}{\sqrt{3\,x^4+5\,x^2+2}(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(12*x^8 + 32*x^6 + 31*x
^4 + 13*x^2 + 2), x)
```

Sympy [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7x^2+4}{\sqrt{(x^2+1)(3x^2+2)}(2x^2+1)^2}\,dx$$

input
```
integrate((x**4-7*x**2+4)/(2*x**2+1)**2/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((x**4 - 7*x**2 + 4)/(sqrt((x**2 + 1)*(3*x**2 + 2))*(2*x**2 + 1)**
2), x)
```

Maxima [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7x^2+4}{\sqrt{3x^4+5x^2+2}(2x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((x^4 - 7*x^2 + 4)/(sqrt(3*x^4 + 5*x^2 + 2)*(2*x^2 + 1)^2), x)
```

Giac [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7x^2+4}{\sqrt{3x^4+5x^2+2}(2x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((x^4 - 7*x^2 + 4)/(sqrt(3*x^4 + 5*x^2 + 2)*(2*x^2 + 1)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7\,x^2+4}{(2\,x^2+1)^2\,\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)^2*(5*x^2 + 3*x^4 + 2)^(1/2)),x)
```

output
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)^2*(5*x^2 + 3*x^4 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2\sqrt{2+5x^2+3x^4}}\,dx = 4\left(\int \frac{\sqrt{3x^4+5x^2+2}}{12x^8+32x^6+31x^4+13x^2+2}dx\right) + \int \frac{\sqrt{3x^4+5x^2+2}\,x^4}{12x^8+32x^6+31x^4+13x^2+2}dx - 7\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{12x^8+32x^6+31x^4+13x^2+2}dx\right)$$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
4*int(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2
),x) + int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(12*x**8 + 32*x**6 + 31*x**4 +
 13*x**2 + 2),x) - 7*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(12*x**8 + 32*x*
*6 + 31*x**4 + 13*x**2 + 2),x)
```

3.210 $\int \frac{4-7x^2+x^4}{(1+2x^2)^3\sqrt{2+5x^2+3x^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 265

$$\begin{aligned}\int \frac{4-7x^2+x^4}{(1+2x^2)^3\sqrt{2+5x^2+3x^4}}\,dx &= \frac{343x(2+3x^2)}{16\sqrt{2+5x^2+3x^4}}+\frac{31x\sqrt{2+5x^2+3x^4}}{4(1+2x^2)^2} \\ &\quad -\frac{343x\sqrt{2+5x^2+3x^4}}{8(1+2x^2)} \\ &\quad -\frac{343(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{8\sqrt{2}\sqrt{2+5x^2+3x^4}} \\ &\quad -\frac{567(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{8\sqrt{2}\sqrt{2+5x^2+3x^4}} \\ &\quad +\frac{387\sqrt{3}(1+x^2)\operatorname{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{8\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output

```
343/16*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+31/4*x*(3*x^4+5*x^2+2)^(1/2)/(2*x
^2+1)^2-343*x*(3*x^4+5*x^2+2)^(1/2)/(16*x^2+8)-343/16*2^(1/2)*(x^2+1)*((3*
x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^
2+2)^(1/2)-567/16*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiA
M(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+387/8*(x^2+1)*EllipticPi(
x*6^(1/2)/(6*x^2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))^(1
/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.60 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.80

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3\sqrt{2+5x^2+3x^4}}\,dx$$
$$= \frac{\frac{248x(2+5x^2+3x^4)}{(1+2x^2)^2} - \frac{1372x(2+5x^2+3x^4)}{1+2x^2} - 686i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right) + 281i\sqrt{3}\sqrt{1+x^2}}{32\sqrt{2+5}}$$

input
```
Integrate[(4 - 7*x^2 + x^4)/((1 + 2*x^2)^3*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
((248*x*(2 + 5*x^2 + 3*x^4))/(1 + 2*x^2)^2 - (1372*x*(2 + 5*x^2 + 3*x^4))/
(1 + 2*x^2) - (686*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*Ar
cSinh[Sqrt[3/2]*x], 2/3] + (281*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*E
llipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (387*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt
[2 + 3*x^2]*EllipticPi[4/3, I*ArcSinh[Sqrt[3/2]*x], 2/3])/(32*Sqrt[2 + 5*x
^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {2210, 2210, 27, 2234, 25, 1503, 1413, 1456, 1538, 27, 1413, 1786, 414}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-7x^2+4}{(2x^2+1)^3\sqrt{3x^4+5x^2+2}}\,dx$$

$$\Big\downarrow \text{2210}$$

$$\frac{31x\sqrt{3x^4+5x^2+2}}{4(2x^2+1)^2} - \frac{1}{4}\int \frac{-93x^4-126x^2+46}{(2x^2+1)^2\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{2210}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{3\left(343x^4+374x^2+198\right)}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx-\frac{343x\sqrt{3x^4+5x^2+2}}{2\left(2x^2+1\right)}\right)+\frac{31\sqrt{3x^4+5x^2+2}x}{4\left(2x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{3}{2}\int \frac{343x^4+374x^2+198}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx-\frac{343x\sqrt{3x^4+5x^2+2}}{2\left(2x^2+1\right)}\right)+\frac{31\sqrt{3x^4+5x^2+2}x}{4\left(2x^2+1\right)^2}$$

$$\Big\downarrow \text{2234}$$

$$\frac{1}{4}\left(\frac{3}{2}\left(\frac{387}{4}\int \frac{1}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx-\frac{1}{4}\int -\frac{686x^2+405}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{343x\sqrt{3x^4+5x^2+2}}{2\left(2x^2+1\right)}\right)+\frac{31\sqrt{3x^4+5x^2+2}x}{4\left(2x^2+1\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{3}{2}\left(\frac{387}{4}\int \frac{1}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx+\frac{1}{4}\int \frac{686x^2+405}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{343x\sqrt{3x^4+5x^2+2}}{2\left(2x^2+1\right)}\right)+\frac{31\sqrt{3x^4+5x^2+2}x}{4\left(2x^2+1\right)^2}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{4}\left(\frac{3}{2}\left(\frac{1}{4}\left(405\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+686\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{387}{4}\int \frac{1}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx\right)-34\right.$$
$$\frac{31\sqrt{3x^4+5x^2+2}x}{4\left(2x^2+1\right)^2}$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{4}\left(\frac{3}{2}\left(\frac{1}{4}\left(686\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{405(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+\frac{387}{4}\int \frac{1}{(2x^2+1)\sqrt{3}}\right.\right.$$
$$\frac{31\sqrt{3x^4+5x^2+2}x}{4\left(2x^2+1\right)^2}$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{4}\left(\frac{3}{2}\left(\frac{387}{4}\int \frac{1}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx+\frac{1}{4}\left(\frac{405(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+686\left(\frac{x(3}{3\sqrt{3x^4}}\right.\right.\right.\right.$$

$$\frac{31\sqrt{3x^4+5x^2+2}x}{4(2x^2+1)^2}$$

$$\Big\downarrow \text{1538}$$

$$\frac{1}{4}\left(\frac{3}{2}\left(\frac{387}{4}\left(\int \frac{2(3x^2+2)}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx-3\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{4}\left(\frac{405(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}(a}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$

$$\frac{31\sqrt{3x^4+5x^2+2}x}{4(2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{3}{2}\left(\frac{387}{4}\left(2\int \frac{3x^2+2}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx-3\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{1}{4}\left(\frac{405(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}(}{\sqrt{2}\sqrt{3x^4+5x^2+}}\right.\right.\right.$$

$$\frac{31\sqrt{3x^4+5x^2+2}x}{4(2x^2+1)^2}$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{4}\left(\frac{3}{2}\left(\frac{387}{4}\left(2\int \frac{3x^2+2}{(2x^2+1)\sqrt{3x^4+5x^2+2}}dx-\frac{3(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+\frac{1}{4}\left(\frac{405(x^2+}{}\right.\right.\right.$$

$$\frac{31\sqrt{3x^4+5x^2+2}x}{4(2x^2+1)^2}$$

$$\Big\downarrow \text{1786}$$

$$\frac{1}{4}\left(\frac{3}{2}\left(\frac{387}{4}\left(\frac{2\sqrt{x^2+1}\sqrt{3x^2+2}\int \frac{\sqrt{3x^2+2}}{\sqrt{x^2+1}(2x^2+1)}dx}{\sqrt{3x^4+5x^2+2}}-\frac{3(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+\frac{1}{4}\left(\frac{405(}{}\right.\right.\right.$$

$$\frac{31\sqrt{3x^4+5x^2+2}x}{4(2x^2+1)^2}$$

$$\Big\downarrow \text{414}$$

$$\frac{1}{4}\left(\frac{3}{2}\left(\frac{387}{4}\left(\frac{4(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{3}\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}-\frac{3(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+\right.\right.$$

$$\frac{31\sqrt{3x^4+5x^2+2}x}{4\left(2x^2+1\right)^2}$$

input
```
Int[(4 - 7*x^2 + x^4)/((1 + 2*x^2)^3*Sqrt[2 + 5*x^2 + 3*x^4]),x]
```

output
```
(31*x*Sqrt[2 + 5*x^2 + 3*x^4])/(4*(1 + 2*x^2)^2) + ((-343*x*Sqrt[2 + 5*x^2
 + 3*x^4])/(2*(1 + 2*x^2)) + (3*((686*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 +
 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan
[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (405*(1 + x^2)*Sqrt[(2 + 3*x^2)
/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]))
/4 + (387*((-3*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x],
-1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) + (4*(1 + x^2)*EllipticPi[-1/3, A
rcTan[Sqrt[3/2]*x], 1/3])/(Sqrt[3]*Sqrt[(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5*
x^2 + 3*x^4])))/4))/2)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1538
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/(2*c*d - e*(b - q)))   I
nt[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[e/(2*c*d - e*(b - q))   Int[(b
- q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 !LtQ[c, 0]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

rule 2210
```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*
x^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(
q + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q +
1)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a
, b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && ILtQ[q, -1
]
```

rule 2234
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 +
c*x^4], x], x] + Simp[(C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^
2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

Maple [A] (verified)

Time = 10.77 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{3x^4+5x^2+2}\,x(686x^2+281)}{8(2x^2+1)^2}-\frac{1215i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{64\sqrt{3x^4+5x^2+2}}+\frac{343i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{Ellipt}\right.}{16\sqrt{3x^4+5x^2+2}}$
default	$\frac{31x\sqrt{3x^4+5x^2+2}}{4(2x^2+1)^2}-\frac{343x\sqrt{3x^4+5x^2+2}}{8(2x^2+1)}+\frac{157i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{64\sqrt{3x^4+5x^2+2}}-\frac{343i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)}{16\sqrt{3x^4+5x^2+2}}$
elliptic	$\frac{31x\sqrt{3x^4+5x^2+2}}{4(2x^2+1)^2}-\frac{343x\sqrt{3x^4+5x^2+2}}{8(2x^2+1)}+\frac{157i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{64\sqrt{3x^4+5x^2+2}}-\frac{343i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)}{16\sqrt{3x^4+5x^2+2}}$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)^3/(3*x^4+5*x^2+2)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/8*(3*x^4+5*x^2+2)^(1/2)*x*(686*x^2+281)/(2*x^2+1)^2-1215/64*I*(x^2+1)^(
1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+343/
16*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/
2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))-1161/32*I*(x^2+1)^(1/2)*(1+3/2*x^2)
^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticPi(I*x,2,1/2*I*(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7\,x^2+4}{\sqrt{3\,x^4+5\,x^2+2}(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^3/(3*x^4+5*x^2+2)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(24*x^10 + 76*x^8 + 94*
x^6 + 57*x^4 + 17*x^2 + 2), x)
```

Sympy [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3\sqrt{2+5x^2+3x^4}}\,dx = \int \frac{x^4-7x^2+4}{\sqrt{(x^2+1)\,(3x^2+2)}\,(2x^2+1)^3}\,dx$$

input
```
integrate((x**4-7*x**2+4)/(2*x**2+1)**3/(3*x**4+5*x**2+2)**(1/2),x)
```

output
```
Integral((x**4 - 7*x**2 + 4)/(sqrt((x**2 + 1)*(3*x**2 + 2))*(2*x**2 + 1)**
3), x)
```

Maxima [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{x^4-7\,x^2+4}{\sqrt{3\,x^4+5\,x^2+2}(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^3/(3*x^4+5*x^2+2)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((x^4 - 7*x^2 + 4)/(sqrt(3*x^4 + 5*x^2 + 2)*(2*x^2 + 1)^3), x)
```

Giac [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{x^4-7\,x^2+4}{\sqrt{3\,x^4+5\,x^2+2}(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^3/(3*x^4+5*x^2+2)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((x^4 - 7*x^2 + 4)/(sqrt(3*x^4 + 5*x^2 + 2)*(2*x^2 + 1)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3\sqrt{2+5x^2+3x^4}}\,dx=\int \frac{x^4-7\,x^2+4}{(2\,x^2+1)^3\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)^3*(5*x^2 + 3*x^4 + 2)^(1/2)),x)
```

output
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)^3*(5*x^2 + 3*x^4 + 2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3\sqrt{2+5x^2+3x^4}}\,dx$$
$$= \frac{7\sqrt{3x^4+5x^2+2}\,x^3+14\sqrt{3x^4+5x^2+2}\,x+48\left(\int \frac{\sqrt{3x^4+5x^2+2}}{24x^{10}+76x^8+94x^6+57x^4+17x^2+2}dx\right)x^4+48\left(\int \frac{\sqrt{3}}{24x^{10}+76x^8+}\right.}{}$$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)^3/(3*x^4+5*x^2+2)^(1/2),x)
```

output
```
(7*sqrt(3*x**4 + 5*x**2 + 2)*x**3 + 14*sqrt(3*x**4 + 5*x**2 + 2)*x + 48*in
t(sqrt(3*x**4 + 5*x**2 + 2)/(24*x**10 + 76*x**8 + 94*x**6 + 57*x**4 + 17*x
**2 + 2),x)*x**4 + 48*int(sqrt(3*x**4 + 5*x**2 + 2)/(24*x**10 + 76*x**8 +
94*x**6 + 57*x**4 + 17*x**2 + 2),x)*x**2 + 12*int(sqrt(3*x**4 + 5*x**2 + 2
)/(24*x**10 + 76*x**8 + 94*x**6 + 57*x**4 + 17*x**2 + 2),x) - 112*int(sqrt
(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x)*x**4
- 112*int(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2
 + 2),x)*x**2 - 28*int(sqrt(3*x**4 + 5*x**2 + 2)/(12*x**8 + 32*x**6 + 31*x
**4 + 13*x**2 + 2),x) - 84*int((sqrt(3*x**4 + 5*x**2 + 2)*x**6)/(12*x**8 +
 32*x**6 + 31*x**4 + 13*x**2 + 2),x)*x**4 - 84*int((sqrt(3*x**4 + 5*x**2 +
 2)*x**6)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x)*x**2 - 21*int((sq
rt(3*x**4 + 5*x**2 + 2)*x**6)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),
x) + 12*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(24*x**10 + 76*x**8 + 94*x**6
 + 57*x**4 + 17*x**2 + 2),x)*x**4 + 12*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4
)/(24*x**10 + 76*x**8 + 94*x**6 + 57*x**4 + 17*x**2 + 2),x)*x**2 + 3*int((
sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(24*x**10 + 76*x**8 + 94*x**6 + 57*x**4 +
17*x**2 + 2),x) + 84*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(12*x**8 + 32*x*
*6 + 31*x**4 + 13*x**2 + 2),x)*x**4 + 84*int((sqrt(3*x**4 + 5*x**2 + 2)*x*
*2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x)*x**2 + 21*int((sqrt(3*x
**4 + 5*x**2 + 2)*x**2)/(12*x**8 + 32*x**6 + 31*x**4 + 13*x**2 + 2),x))...
```

3.211 $\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 211

$$\begin{aligned}\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx &= \frac{3437x(2+3x^2)}{135\sqrt{2+5x^2+3x^4}} \\ &- \frac{x(689+1013x^2)}{27\sqrt{2+5x^2+3x^4}} - \frac{68}{27}x\sqrt{2+5x^2+3x^4} + \frac{8}{45}x^3\sqrt{2+5x^2+3x^4} \\ &- \frac{3437\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{135\sqrt{2+5x^2+3x^4}} \\ &+ \frac{293(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{9\sqrt{2}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output

```
3437/135*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-1/27*x*(1013*x^2+689)/(3*x^4+5*
x^2+2)^(1/2)-68/27*x*(3*x^4+5*x^2+2)^(1/2)+8/45*x^3*(3*x^4+5*x^2+2)^(1/2)-
3437/135*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/
2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+293/18*2^(1/2)*(x^2+1)*((3*x^2+2)/
(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.63

$$\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{-4125x-6717x^3-900x^5+72x^7-3437i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcs}\right.}{135\sqrt{2}}$$

input
```
Integrate[((1 + 2*x^2)^3*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(-4125*x - 6717*x^3 - 900*x^5 + 72*x^7 - (3437*I)*Sqrt[3]*Sqrt[1 + x^2]*Sq
rt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (1972*I)*Sqrt[3]*Sq
rt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(135*S
qrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2206, 27, 2207, 27, 2207, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2+1)^3(x^4-7x^2+4)}{(3x^4+5x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2206}$$

$$-\frac{1}{2}\int -\frac{2(72x^6-516x^4+1411x^2+743)}{27\sqrt{3x^4+5x^2+2}}dx-\frac{x(1013x^2+689)}{27\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27}\int \frac{72x^6-516x^4+1411x^2+743}{\sqrt{3x^4+5x^2+2}}dx-\frac{x(1013x^2+689)}{27\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{27}\left(\frac{1}{15}\int \frac{3\left(-3060x^4+6911x^2+3715\right)}{\sqrt{3x^4+5x^2+2}}dx+\frac{24}{5}\sqrt{3x^4+5x^2+2}x^3\right)-\frac{x\left(1013x^2+689\right)}{27\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 27

$$\frac{1}{27}\left(\frac{1}{5}\int \frac{-3060x^4+6911x^2+3715}{\sqrt{3x^4+5x^2+2}}dx+\frac{24}{5}\sqrt{3x^4+5x^2+2}x^3\right)-\frac{x\left(1013x^2+689\right)}{27\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 2207

$$\frac{1}{27}\left(\frac{1}{5}\left(\frac{1}{9}\int \frac{27\left(3437x^2+1465\right)}{\sqrt{3x^4+5x^2+2}}dx-340x\sqrt{3x^4+5x^2+2}\right)+\frac{24}{5}\sqrt{3x^4+5x^2+2}x^3\right)-\frac{x\left(1013x^2+689\right)}{27\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 27

$$\frac{1}{27}\left(\frac{1}{5}\left(3\int \frac{3437x^2+1465}{\sqrt{3x^4+5x^2+2}}dx-340x\sqrt{3x^4+5x^2+2}\right)+\frac{24}{5}\sqrt{3x^4+5x^2+2}x^3\right)-\frac{x\left(1013x^2+689\right)}{27\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1503

$$\frac{1}{27}\left(\frac{1}{5}\left(3\left(1465\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+3437\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-340x\sqrt{3x^4+5x^2+2}\right)+\frac{24}{5}\sqrt{3x^4+5x\right.$$
$$\frac{x\left(1013x^2+689\right)}{27\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1413

$$\frac{1}{27}\left(\frac{1}{5}\left(3\left(3437\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{1465\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)-340x\sqrt{3x^4+5x^2\right.\right.$$
$$\frac{x\left(1013x^2+689\right)}{27\sqrt{3x^4+5x^2+2}}$$

$\downarrow$ 1456

$$\frac{1}{27}\left(\frac{1}{5}\left(3\left(\frac{1465\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+3437\left(\frac{x\left(3x^2+2\right)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}\left(x^2+1\right)\sqrt{\frac{3x^2+2}{x^2+1}}E}{3\sqrt{3x^4+5}}\right.\right.\right.\right.$$
$$\frac{x\left(1013x^2+689\right)}{27\sqrt{3x^4+5x^2+2}}$$

input
```
Int[((1 + 2*x^2)^3*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
-1/27*(x*(689 + 1013*x^2))/Sqrt[2 + 5*x^2 + 3*x^4] + ((24*x^3*Sqrt[2 + 5*x
^2 + 3*x^4])/5 + (-340*x*Sqrt[2 + 5*x^2 + 3*x^4] + 3*(3437*((x*(2 + 3*x^2)
)/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x
^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (1465*(1 +
 x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqr
t[2 + 5*x^2 + 3*x^4])))/5)/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 15.97 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.64

method	result
risch	$\frac{x(24x^6-300x^4-2239x^2-1375)}{45\sqrt{3x^4+5x^2+2}}-\frac{293i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{18\sqrt{3x^4+5x^2+2}}+\frac{3437i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{Ellipt}\right.}{135\sqrt{3x^4+5x^2+2}}$
elliptic	$-\frac{6\left(\frac{1013}{162}x^3+\frac{689}{162}x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{8x^3\sqrt{3x^4+5x^2+2}}{45}-\frac{68x\sqrt{3x^4+5x^2+2}}{27}-\frac{293i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{18\sqrt{3x^4+5x^2+2}}+\frac{3437i\sqrt{x^2+1}\,\sqrt{6}}{}$
default	$-\frac{24\left(-\frac{5}{4}x^3-\frac{13}{12}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{293i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{18\sqrt{3x^4+5x^2+2}}+\frac{3437i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right.}{135\sqrt{3x^4+5x^2+2}}$

input
```
int((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/45*x*(24*x^6-300*x^4-2239*x^2-1375)/(3*x^4+5*x^2+2)^(1/2)-293/18*I*(x^2+
1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+
3437/135*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(
I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.57

$$\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx =$$

$$-\frac{13748\sqrt{3}\sqrt{-\frac{2}{3}}(3x^5+5x^3+2x)E(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2}) - 26933\sqrt{3}\sqrt{-\frac{2}{3}}(3x^5+5x^3+2x)F(\arcsin\left(\right.}{810\,(3x^5+5x^3+2x)}$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fr
icas")
```

output
```
-1/810*(13748*sqrt(3)*sqrt(-2/3)*(3*x^5 + 5*x^3 + 2*x)*elliptic_e(arcsin(s
qrt(-2/3)/x), 3/2) - 26933*sqrt(3)*sqrt(-2/3)*(3*x^5 + 5*x^3 + 2*x)*ellipt
ic_f(arcsin(sqrt(-2/3)/x), 3/2) - 12*(36*x^8 - 450*x^6 + 1797*x^4 + 6530*x
^2 + 3437)*sqrt(3*x^4 + 5*x^2 + 2))/(3*x^5 + 5*x^3 + 2*x)
```

Sympy [F]

$$\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(2x^2+1)^3(x^4-7x^2+4)}{((x^2+1)(3x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((2*x**2+1)**3*(x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((2*x**2 + 1)**3*(x**4 - 7*x**2 + 4)/((x**2 + 1)*(3*x**2 + 2))**(3
/2), x)
```

Maxima [F]

$$\int \frac{(1+2x^2)^3\,(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)^3}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^3/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(1+2x^2)^3\,(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)^3}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^3/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2)^3\,(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(2\,x^2+1)^3\,(x^4-7\,x^2+4)}{(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(((2*x^2 + 1)^3*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int(((2*x^2 + 1)^3*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(1+2x^2)^3 (4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{72\sqrt{3x^4+5x^2+2}\,x^7 - 900\sqrt{3x^4+5x^2+2}\,x^5 + 3594\sqrt{3x^4+5x^2+2}\,x}{}$$

input
```
int((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
(72*sqrt(3*x**4 + 5*x**2 + 2)*x**7 - 900*sqrt(3*x**4 + 5*x**2 + 2)*x**5 +
3594*sqrt(3*x**4 + 5*x**2 + 2)*x**3 + 8665*sqrt(3*x**4 + 5*x**2 + 2)*x - 5
0370*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 +
 4),x)*x**4 - 83950*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x
**4 + 20*x**2 + 4),x)*x**2 - 33580*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 +
 30*x**6 + 37*x**4 + 20*x**2 + 4),x) - 57807*int((sqrt(3*x**4 + 5*x**2 + 2
)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*x**4 - 96345*int((sq
rt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x
)*x**2 - 38538*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37
*x**4 + 20*x**2 + 4),x))/(135*(3*x**4 + 5*x**2 + 2))
```

3.212 $\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 188

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = -\frac{1411x(2+3x^2)}{81\sqrt{2+5x^2+3x^4}}+\frac{x(257+365x^2)}{9\sqrt{2+5x^2+3x^4}} + \frac{4}{27}x\sqrt{2+5x^2+3x^4}+\frac{1411\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{81\sqrt{2+5x^2+3x^4}} - \frac{725(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{27\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output

```
-1411/81*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/9*x*(365*x^2+257)/(3*x^4+5*x^
2+2)^(1/2)+4/27*x*(3*x^4+5*x^2+2)^(1/2)+1411/81*2^(1/2)*(x^2+1)*((3*x^2+2)
/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(
1/2)-725/54*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arct
an(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.68

$$\int \frac{(1+2x^2)^2 (4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{2337x+3345x^3+36x^5+1411i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\right.}{81\sqrt{2+5x^2}}$$

input
```
Integrate[((1 + 2*x^2)^2*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(2337*x + 3345*x^3 + 36*x^5 + (1411*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^
2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (686*I)*Sqrt[3]*Sqrt[1 + x^2]*
Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(81*Sqrt[2 + 5*x^2
 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2206, 27, 2207, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2+1)^2 (x^4-7x^2+4)}{(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{x(365x^2+257)}{9\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int \frac{2(-12x^4+457x^2+239)}{9\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(365x^2+257)}{9\sqrt{3x^4+5x^2+2}}-\frac{1}{9}\int \frac{-12x^4+457x^2+239}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \text{2207}$$

$$\frac{1}{9}\left(\frac{4}{3}x\sqrt{3x^4+5x^2+2}-\frac{1}{9}\int\frac{3(1411x^2+725)}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{x(365x^2+257)}{9\sqrt{3x^4+5x^2+2}}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{4}{3}x\sqrt{3x^4+5x^2+2}-\frac{1}{3}\int\frac{1411x^2+725}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{x(365x^2+257)}{9\sqrt{3x^4+5x^2+2}}$$

$\Big\downarrow$ 1503

$$\frac{1}{9}\left(\frac{1}{3}\left(-725\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx-1411\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)+\frac{4}{3}\sqrt{3x^4+5x^2+2}x\right)+\frac{x(365x^2+257)}{9\sqrt{3x^4+5x^2+2}}$$

$\Big\downarrow$ 1413

$$\frac{1}{9}\left(\frac{1}{3}\left(-1411\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx-\frac{725(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)+\frac{4}{3}\sqrt{3x^4+5x^2+2}x\right)$$
$$\frac{x(365x^2+257)}{9\sqrt{3x^4+5x^2+2}}$$

$\Big\downarrow$ 1456

$$\frac{1}{9}\left(\frac{1}{3}\left(-\frac{725(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}-1411\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E(\text{ar}}{3\sqrt{3x^4+5x^2+}}\right.\right.\right.$$
$$\frac{x(365x^2+257)}{9\sqrt{3x^4+5x^2+2}}$$

input
```
Int[((1 + 2*x^2)^2*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(x*(257 + 365*x^2))/(9*Sqrt[2 + 5*x^2 + 3*x^4]) + ((4*x*Sqrt[2 + 5*x^2 + 3
*x^4])/3 + (-1411*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*
(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[
2 + 5*x^2 + 3*x^4])) - (725*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Elliptic
F[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]))/3)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Maple [A] (verified)

Time = 12.48 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.69

method	result
risch	$\frac{x(12x^4+1115x^2+779)}{27\sqrt{3x^4+5x^2+2}}+\frac{725i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{54\sqrt{3x^4+5x^2+2}}-\frac{1411i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{81\sqrt{3x^4+5x^2+2}}$
elliptic	$-\frac{6\left(-\frac{365}{54}x^3-\frac{257}{54}x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{4x\sqrt{3x^4+5x^2+2}}{27}+\frac{725i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{54\sqrt{3x^4+5x^2+2}}-\frac{1411i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)\right.}{81\sqrt{3x^4+5x^2+2}}$
default	$-\frac{24\left(-\frac{5}{4}x^3-\frac{13}{12}x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{725i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{54\sqrt{3x^4+5x^2+2}}-\frac{1411i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{81\sqrt{3x^4+5x^2+2}}$

input
```
int((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/27*x*(12*x^4+1115*x^2+779)/(3*x^4+5*x^2+2)^(1/2)+725/54*I*(x^2+1)^(1/2)*
(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))-1411/81*I
*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^
(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.62

$$\int \frac{(1+2x^2)^2\,(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx=\frac{5644\,\sqrt{3}\sqrt{-\frac{2}{3}}(3\,x^5+5\,x^3+2\,x)E(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2})-12169\,\sqrt{3}\sqrt{-\frac{2}{3}}}{}$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fr
icas")
```

output
```
1/486*(5644*sqrt(3)*sqrt(-2/3)*(3*x^5 + 5*x^3 + 2*x)*elliptic_e(arcsin(sqr
t(-2/3)/x), 3/2) - 12169*sqrt(3)*sqrt(-2/3)*(3*x^5 + 5*x^3 + 2*x)*elliptic
_f(arcsin(sqrt(-2/3)/x), 3/2) + 12*(18*x^6 - 444*x^4 - 2359*x^2 - 1411)*sq
rt(3*x^4 + 5*x^2 + 2))/(3*x^5 + 5*x^3 + 2*x)
```

Sympy [F]

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(2x^2+1)^2(x^4-7x^2+4)}{((x^2+1)(3x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((2*x**2+1)**2*(x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((2*x**2 + 1)**2*(x**4 - 7*x**2 + 4)/((x**2 + 1)*(3*x**2 + 2))**(3
/2), x)
```

Maxima [F]

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(x^4-7x^2+4)(2x^2+1)^2}{(3x^4+5x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^2/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)^2}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^2/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(2\,x^2+1)^2\,(x^4-7\,x^2+4)}{(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(((2*x^2 + 1)^2*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int(((2*x^2 + 1)^2*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{36\sqrt{3x^4+5x^2+2}\,x^5-888\sqrt{3x^4+5x^2+2}\,x^3-2543\sqrt{3x^4+5x^2+2}\,x}{}$$

input
```
int((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
(36*sqrt(3*x**4 + 5*x**2 + 2)*x**5 - 888*sqrt(3*x**4 + 5*x**2 + 2)*x**3 -
2543*sqrt(3*x**4 + 5*x**2 + 2)*x + 16230*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*
x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*x**4 + 27050*int(sqrt(3*x**4 +
5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*x**2 + 10820*int
(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x) +
 18171*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 +
20*x**2 + 4),x)*x**4 + 30285*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8
+ 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*x**2 + 12114*int((sqrt(3*x**4 + 5*x*
*2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x))/(81*(3*x**4 +
 5*x**2 + 2))
```

3.213 $\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 167

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{151x(2+3x^2)}{9\sqrt{2+5x^2+3x^4}} - \frac{x(113+149x^2)}{3\sqrt{2+5x^2+3x^4}} - \frac{151\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{9\sqrt{2+5x^2+3x^4}} + \frac{119(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output

```
151/9*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-1/3*x*(149*x^2+113)/(3*x^4+5*x^2+2
)^(1/2)-151/9*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1
)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+119/6*2^(1/2)*(x^2+1)*((3*x^2
+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.74

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = -\frac{339x+447x^3+151i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-32i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\right.}{9\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[((1 + 2*x^2)*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
-1/9*(339*x + 447*x^3 + (151*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Elli
pticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (32*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 +
 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/Sqrt[2 + 5*x^2 + 3*x^4]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2206, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2+1)(x^4-7x^2+4)}{(3x^4+5x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \ 2206$$

$$-\frac{1}{2}\int -\frac{2(151x^2+119)}{3\sqrt{3x^4+5x^2+2}}dx-\frac{x(149x^2+113)}{3\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\int \frac{151x^2+119}{\sqrt{3x^4+5x^2+2}}dx-\frac{x(149x^2+113)}{3\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{1503}$$

$$\frac{1}{3}\left(119\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+151\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{x(149x^2+113)}{3\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{1413}$$

$$\frac{1}{3}\left(151\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{119(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)-\frac{x(149x^2+113)}{3\sqrt{3x^4+5x^2+2}}$$

$$\Big\downarrow \text{1456}$$

$$\frac{1}{3}\left(\frac{119(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+151\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E(\arctan(x}{3\sqrt{3x^4+5x^2+2}}\right.\right.$$

$$\frac{x(149x^2+113)}{3\sqrt{3x^4+5x^2+2}}$$

input
```
Int[((1 + 2*x^2)*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
-1/3*(x*(113 + 149*x^2))/Sqrt[2 + 5*x^2 + 3*x^4] + (151*((x*(2 + 3*x^2))/(
3*Sqrt[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (119*(1 + x^2
)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2
+ 5*x^2 + 3*x^4]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 6.91 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{x(149x^2+113)}{3\sqrt{3x^4+5x^2+2}}-\frac{119i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{6\sqrt{3x^4+5x^2+2}}+\frac{151i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{9\sqrt{3x^4+5x^2+2}}$
elliptic	$-\frac{6\left(\frac{149}{18}x^3+\frac{113}{18}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{119i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{6\sqrt{3x^4+5x^2+2}}+\frac{151i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{9\sqrt{3x^4+5x^2+2}}$
default	$-\frac{6\left(x^3+\frac{5}{6}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{119i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{6\sqrt{3x^4+5x^2+2}}+\frac{151i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{9\sqrt{3x^4+5x^2+2}}-$

input
```
int((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(149*x^2+113)/(3*x^4+5*x^2+2)^(1/2)-119/6*I*(x^2+1)^(1/2)*(6*x^2+4)
^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+151/9*I*(x^2+1)^(1
/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-Elli
pticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

$$\int \frac{(1+2x^2)\,(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx =$$

$$-\frac{604\sqrt{3}\sqrt{-\frac{2}{3}}(3\,x^5+5\,x^3+2\,x)E(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2})-1675\sqrt{3}\sqrt{-\frac{2}{3}}(3\,x^5+5\,x^3+2\,x)F(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right.}{54\,(3\,x^5+5\,x^3+2\,x)}$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fric
as")
```

output
```
-1/54*(604*sqrt(3)*sqrt(-2/3)*(3*x^5 + 5*x^3 + 2*x)*elliptic_e(arcsin(sqrt
(-2/3)/x), 3/2) - 1675*sqrt(3)*sqrt(-2/3)*(3*x^5 + 5*x^3 + 2*x)*elliptic_f
(arcsin(sqrt(-2/3)/x), 3/2) - 12*(3*x^4 + 208*x^2 + 151)*sqrt(3*x^4 + 5*x^
2 + 2))/(3*x^5 + 5*x^3 + 2*x)
```

Sympy [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{(2x^2+1)(x^4-7x^2+4)}{((x^2+1)(3x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((2*x**2+1)*(x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((2*x**2 + 1)*(x**4 - 7*x**2 + 4)/((x**2 + 1)*(3*x**2 + 2))**(3/2)
, x)
```

Maxima [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{(x^4-7\,x^2+4)(2\,x^2+1)}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{(x^4-7\,x^2+4)(2\,x^2+1)}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac
")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{(2\,x^2+1)\,(x^4-7\,x^2+4)}{(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int(((2*x^2 + 1)*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int(((2*x^2 + 1)*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{6\sqrt{3x^4+5x^2+2}\,x^3+59\sqrt{3x^4+5x^2+2}\,x-246\left(\int \frac{\sqrt{3x^4+5x^2+2}}{9x^8+30x^6+37x^4+20x^2+4}\right.}{}$$

input
```
int((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
(6*sqrt(3*x**4 + 5*x**2 + 2)*x**3 + 59*sqrt(3*x**4 + 5*x**2 + 2)*x - 246*i
nt(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)
*x**4 - 410*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20
*x**2 + 4),x)*x**2 - 164*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 +
 37*x**4 + 20*x**2 + 4),x) - 81*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x*
*8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*x**4 - 135*int((sqrt(3*x**4 + 5*x
**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*x**2 - 54*int
((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 +
4),x))/(9*(3*x**4 + 5*x**2 + 2))
```

3.214 $\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{41x}{3\sqrt{2+5x^2+3x^4}} + \frac{77\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{2+5x^2+3x^4}} - \frac{63(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
41/3*x/(3*x^4+5*x^2+2)^(1/2)+77/3*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2
)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)-63/2*2^(1
/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1
/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.96

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{3x(65+77x^2)+77i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-14i\sqrt{3}\sqrt{1}}{3\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[(4 - 7*x^2 + x^4)/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(3*x*(65 + 77*x^2) + (77*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Elliptic
E[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (14*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x
^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(3*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2206, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-7x^2+4}{(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \; 2206$$

$$\frac{x(77x^2+65)}{\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int \frac{14(11x^2+9)}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{x(77x^2+65)}{\sqrt{3x^4+5x^2+2}}-7\int \frac{11x^2+9}{\sqrt{3x^4+5x^2+2}}dx$$

$$\Big\downarrow \; 1503$$

$$\frac{x(77x^2+65)}{\sqrt{3x^4+5x^2+2}}-7\left(9\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx+11\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)$$

$\Big\downarrow$ 1413

$$\frac{x(77x^2+65)}{\sqrt{3x^4+5x^2+2}}-7\left(11\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{9(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}\right)$$

$\Big\downarrow$ 1456

$$\frac{x(77x^2+65)}{\sqrt{3x^4+5x^2+2}}-7\left(\frac{9(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+11\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\middle|-\right.}{3\sqrt{3x^4+5x^2+2}}\right.\right.$$

input
```
Int[(4 - 7*x^2 + x^4)/(2 + 5*x^2 + 3*x^4)^(3/2),x]
```

output
```
(x*(65 + 77*x^2))/Sqrt[2 + 5*x^2 + 3*x^4] - 7*(11*((x*(2 + 3*x^2))/(3*Sqrt
[2 + 5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Elli
pticE[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (9*(1 + x^2)*Sqrt[(
2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2
+ 3*x^4]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

method	result
risch	$\frac{x(77x^2+65)}{\sqrt{3x^4+5x^2+2}}+\frac{63i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{77i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^4+5x^2+2}}$
elliptic	$-\frac{6(-\frac{77}{6}x^3-\frac{65}{6}x)}{\sqrt{3x^4+5x^2+2}}+\frac{63i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{77i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^4+5x^2+2}}$
default	$-\frac{6(-\frac{5}{6}x^3-\frac{2}{3}x)}{\sqrt{3x^4+5x^2+2}}+\frac{63i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{77i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\operatorname{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^4+5x^2+2}}-$

input
```
int((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*(77*x^2+65)/(3*x^4+5*x^2+2)^(1/2)+63/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(
3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))-77/3*I*(x^2+1)^(1/2)*(6*x^
2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x
,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.68

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{3/2}}\,dx = -\frac{154\sqrt{2}(3i\,x^4+5i\,x^2+2i)E(\arcsin{(i\,x)}\mid\frac{3}{2})+343\sqrt{2}(-3i\,x^4-5i\,x^2-2i)F(\arcsin{(i\,x)}\mid\frac{3}{2})-6\sqrt{3\,x^4}}{6\,(3\,x^4+5\,x^2+2)}$$

input
```
integrate((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(154*sqrt(2)*(3*I*x^4 + 5*I*x^2 + 2*I)*elliptic_e(arcsin(I*x), 3/2) +
 343*sqrt(2)*(-3*I*x^4 - 5*I*x^2 - 2*I)*elliptic_f(arcsin(I*x), 3/2) - 6*s
qrt(3*x^4 + 5*x^2 + 2)*(77*x^3 + 65*x))/(3*x^4 + 5*x^2 + 2)
```

Sympy [F]

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7x^2+4}{((x^2+1)\,(3x^2+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((x**4 - 7*x**2 + 4)/((x**2 + 1)*(3*x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 - 7*x^2 + 4)/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 - 7*x^2 + 4)/(3*x^4 + 5*x^2 + 2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)/(5*x^2 + 3*x^4 + 2)^(3/2),x)
```

output
```
int((x^4 - 7*x^2 + 4)/(5*x^2 + 3*x^4 + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{3/2}}\,dx = \frac{-\sqrt{3x^4+5x^2+2}\,x+42\left(\int \frac{\sqrt{3x^4+5x^2+2}}{9x^8+30x^6+37x^4+20x^2+4}dx\right)x^4+70\left(\int \frac{\sqrt{3x^4+5x^2+2}}{9x^8+30x^6+37x^4+20x^2+4}dx\right)\cdots}{\cdots}$$

input
```
int((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
( - sqrt(3*x**4 + 5*x**2 + 2)*x + 42*int(sqrt(3*x**4 + 5*x**2 + 2)/(9*x**8
 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*x**4 + 70*int(sqrt(3*x**4 + 5*x**2
+ 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x)*x**2 + 28*int(sqrt(3*x*
*4 + 5*x**2 + 2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x) - 63*int((s
qrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),
x)*x**4 - 105*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x**8 + 30*x**6 + 37*
x**4 + 20*x**2 + 4),x)*x**2 - 42*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(9*x
**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4),x))/(3*(3*x**4 + 5*x**2 + 2))
```

3.215 $\int \frac{4-7x^2+x^4}{(1+2x^2)\left(2+5x^2+3x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 223

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\left(2+5x^2+3x^4\right)^{3/2}}\,dx = \frac{53x(2+3x^2)}{\sqrt{2+5x^2+3x^4}} - \frac{3x(49+53x^2)}{\sqrt{2+5x^2+3x^4}} - \frac{53\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{\sqrt{2+5x^2+3x^4}} + \frac{25(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}} + \frac{124(1+x^2)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{3}\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}$$

output
```
53*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-3*x*(53*x^2+49)/(3*x^4+5*x^2+2)^(1/2)
-53*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/
2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+25/2*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1)
)^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+124
/3*(x^2+1)*EllipticPi(x*6^(1/2)/(6*x^2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1/2)/
((x^2+1)/(3*x^2+2))^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.33 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\left(2+5x^2+3x^4\right)^{3/2}}\,dx =$$
$$-\frac{441x+477x^3+159i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}\,E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)-41i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\right.}{3\sqrt{2+5x^2+3x^4}}$$

input
```
Integrate[(4 - 7*x^2 + x^4)/((1 + 2*x^2)*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output
```
-1/3*(441*x + 477*x^3 + (159*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Elli
pticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (41*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 +
 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (31*I)*Sqrt[3]*Sqrt[1 + x
^2]*Sqrt[2 + 3*x^2]*EllipticPi[4/3, I*ArcSinh[Sqrt[3/2]*x], 2/3])/Sqrt[2 +
 5*x^2 + 3*x^4]
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-7x^2+4}{(2x^2+1)\left(3x^4+5x^2+2\right)^{3/2}}\,dx$$
$$\Big\downarrow \; 2258$$
$$\int \left(\frac{12}{(x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{31}{(2x^2+1)\sqrt{3x^4+5x^2+2}}-\frac{82}{(3x^2+2)\sqrt{3x^4+5x^2+2}}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{93\sqrt{\frac{3x^2+2}{x^2+1}}\left(x^2+1\right)\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}+\frac{59\sqrt{2}\left(3x^2+2\right)\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}-\frac{53\sqrt{2}\left(3x^2+2\right)E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}+\frac{124\left(x^2+1\right)\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{3}\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}-\frac{123x\left(x^2+1\right)}{\sqrt{3x^4+5x^2+2}}+\frac{41x\left(3x^2+2\right)}{\sqrt{3x^4+5x^2+2}}$$

input
```
Int[(4 - 7*x^2 + x^4)/((1 + 2*x^2)*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output
```
(-123*x*(1 + x^2))/Sqrt[2 + 5*x^2 + 3*x^4] + (41*x*(2 + 3*x^2))/Sqrt[2 + 5
*x^2 + 3*x^4] - (53*Sqrt[2]*(2 + 3*x^2)*EllipticE[ArcTan[x], -1/2])/(Sqrt[
(2 + 3*x^2)/(1 + x^2)]*Sqrt[2 + 5*x^2 + 3*x^4]) + (59*Sqrt[2]*(2 + 3*x^2)*
EllipticF[ArcTan[x], -1/2])/(Sqrt[(2 + 3*x^2)/(1 + x^2)]*Sqrt[2 + 5*x^2 +
3*x^4]) - (93*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -
1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) + (124*(1 + x^2)*EllipticPi[-1/3,
ArcTan[Sqrt[3/2]*x], 1/3])/(Sqrt[3]*Sqrt[(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5
*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.72

method	result
elliptic	$-\frac{6\left(\frac{53}{2}x^3+\frac{49}{2}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{6i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4+5x^2+2}}-\frac{53i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4+5x^2+2}}-\frac{31i\sqrt{x^2+1}\,\sqrt{1+\frac{3x^2}{2}}\ \text{El}}{\sqrt{3x^4+}}$
risch	$-\frac{3x(53x^2+49)}{\sqrt{3x^4+5x^2+2}}-\frac{59i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4+5x^2+2}}+\frac{53i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{\sqrt{3x^4+5x^2+2}}-$
default	$\frac{-\frac{225}{8}x^3-\frac{195}{8}x}{\sqrt{3x^4+5x^2+2}}-\frac{131i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{8\sqrt{3x^4+5x^2+2}}+\frac{83i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{8\sqrt{3x^4+5x^2+2}}-$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-6*(53/2*x^3+49/2*x)/(3*x^4+5*x^2+2)^(1/2)-6*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/
2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))-53*I*(x^2+1)^(1/2)*(6*
x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticE(I*x,1/2*6^(1/2))-31*I*(x^2+1)
^(1/2)*(1+3/2*x^2)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticPi(I*x,2,1/2*I*(-3)
^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\left(2+5x^2+3x^4\right)^{3/2}}\,dx=\int \frac{x^4-7\,x^2+4}{\left(3\,x^4+5\,x^2+2\right)^{\frac{3}{2}}\left(2\,x^2+1\right)}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(18*x^10 + 69*x^8 + 104
*x^6 + 77*x^4 + 28*x^2 + 4), x)
```

Sympy [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{x^4-7x^2+4}{((x^2+1)(3x^2+2))^{\frac{3}{2}}\cdot(2x^2+1)}\,dx$$

input
```
integrate((x**4-7*x**2+4)/(2*x**2+1)/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((x**4 - 7*x**2 + 4)/(((x**2 + 1)*(3*x**2 + 2))**(3/2)*(2*x**2 + 1
)), x)
```

Maxima [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(2\,x^2+1)}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((x^4 - 7*x^2 + 4)/((3*x^4 + 5*x^2 + 2)^(3/2)*(2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(2\,x^2+1)}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(3/2),x, algorithm="giac
")
```

output
```
integrate((x^4 - 7*x^2 + 4)/((3*x^4 + 5*x^2 + 2)^(3/2)*(2*x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4-7x^2+x^4}{(1+2x^2)(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(2\,x^2+1)\,(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)*(5*x^2 + 3*x^4 + 2)^(3/2)),x)
```

output
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)*(5*x^2 + 3*x^4 + 2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)(2+5x^2+3x^4)^{3/2}}\,dx = 4\left(\int \frac{\sqrt{3x^4+5x^2+2}}{18x^{10}+69x^8+104x^6+77x^4+28x^2+4}dx\right) + \int \frac{\sqrt{3x^4+5x^2+2}\,x^4}{18x^{10}+69x^8+104x^6+77x^4+28x^2+4}dx - 7\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{18x^{10}+69x^8+104x^6+77x^4+28x^2+4}dx\right)$$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
4*int(sqrt(3*x**4 + 5*x**2 + 2)/(18*x**10 + 69*x**8 + 104*x**6 + 77*x**4 +
 28*x**2 + 4),x) + int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(18*x**10 + 69*x**
8 + 104*x**6 + 77*x**4 + 28*x**2 + 4),x) - 7*int((sqrt(3*x**4 + 5*x**2 + 2
)*x**2)/(18*x**10 + 69*x**8 + 104*x**6 + 77*x**4 + 28*x**2 + 4),x)
```

3.216 $\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 255

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{3/2}}\,dx = \frac{31x}{2(1+2x^2)\sqrt{2+5x^2+3x^4}} - \frac{166x(2+3x^2)}{\sqrt{2+5x^2+3x^4}} + \frac{x(1003+996x^2)}{2\sqrt{2+5x^2+3x^4}} + \frac{166\sqrt{2}(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{2+5x^2+3x^4}} + \frac{231(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}} - \frac{746(1+x^2)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{3}\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}$$

output
```
31/2*x/(2*x^2+1)/(3*x^4+5*x^2+2)^(1/2)-166*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/
2)+1/2*x*(996*x^2+1003)/(3*x^4+5*x^2+2)^(1/2)+166*2^(1/2)*(x^2+1)*((3*x^2+
2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)
^(1/2)+231/2*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arc
tan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)-746/3*(x^2+1)*EllipticPi(x*6^(
1/2)/(6*x^2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))^(1/2)/(
3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{3/2}}\,dx = \frac{3102x+9006x^3+5976x^5+996i\sqrt{3}\sqrt{1+x^2}(1+2x^2)\sqrt{2+3x^2}E\big(}{}$$

input
```
Integrate[(4 - 7*x^2 + x^4)/((1 + 2*x^2)^2*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output
```
(3102*x + 9006*x^3 + 5976*x^5 + (996*I)*Sqrt[3]*Sqrt[1 + x^2]*(1 + 2*x^2)*
Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (339*I)*Sqrt[3]*S
qrt[1 + x^2]*(1 + 2*x^2)*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]*x],
 2/3] + (373*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticPi[4/3, I*Ar
cSinh[Sqrt[3/2]*x], 2/3] + (746*I)*Sqrt[3]*x^2*Sqrt[1 + x^2]*Sqrt[2 + 3*x^
2]*EllipticPi[4/3, I*ArcSinh[Sqrt[3/2]*x], 2/3])/(6*(1 + 2*x^2)*Sqrt[2 + 5
*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.87, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-7x^2+4}{(2x^2+1)^2(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{2258}$$

$$\int \left(-\frac{12}{(x^2+1)\sqrt{3x^4+5x^2+2}}-\frac{140}{(2x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{246}{(3x^2+2)\sqrt{3x^4+5x^2+2}}+\frac{31}{(2x^2+1)^2\sqrt{3x^4+}}\right.$$

$$\Big\downarrow \text{2009}$$

$$
\frac{210\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+\frac{93(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}-\frac{141\sqrt{2}(3x^2+2)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}+\frac{31\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{3x^4+5x^2+2}}+\frac{135\sqrt{2}(3x^2+2)\,E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}-\frac{62\sqrt{3}(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}-\frac{560(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{3}\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{62\sqrt{3x^4+5x^2+2}x}{2x^2+1}+\frac{369(x^2+1)\,x}{\sqrt{3x^4+5x^2+2}}-\frac{154(3x^2+2)\,x}{\sqrt{3x^4+5x^2+2}}
$$

input

```
Int[(4 - 7*x^2 + x^4)/((1 + 2*x^2)^2*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output

```
(369*x*(1 + x^2))/Sqrt[2 + 5*x^2 + 3*x^4] - (154*x*(2 + 3*x^2))/Sqrt[2 + 5
*x^2 + 3*x^4] + (62*x*Sqrt[2 + 5*x^2 + 3*x^4])/(1 + 2*x^2) + (135*Sqrt[2]*
(2 + 3*x^2)*EllipticE[ArcTan[x], -1/2])/(Sqrt[(2 + 3*x^2)/(1 + x^2)]*Sqrt[
2 + 5*x^2 + 3*x^4]) + (31*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*El
lipticE[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4] - (141*Sqrt[2]*(2 + 3*x^
2)*EllipticF[ArcTan[x], -1/2])/(Sqrt[(2 + 3*x^2)/(1 + x^2)]*Sqrt[2 + 5*x^2
 + 3*x^4]) + (93*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x]
, -1/2])/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) + (210*Sqrt[2]*(1 + x^2)*Sqrt[(
2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4]
- (560*(1 + x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(Sqrt[3]*Sqrt
[(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4]) - (62*Sqrt[3]*(1 + x^2)*E
llipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(Sqrt[(1 + x^2)/(2 + 3*x^2)]*Sq
rt[2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 7.06 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.73

method	result
risch	$\frac{x(996x^4+1501x^2+517)}{(2x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{657i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{4\sqrt{3x^4+5x^2+2}}-\frac{166i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\right.\right.}{\sqrt{3x^4+5x^2+2}}$
elliptic	$\frac{62x\sqrt{3x^4+5x^2+2}}{2x^2+1}-\frac{6\left(-\frac{135}{2}x^3-\frac{131}{2}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{7i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{4\sqrt{3x^4+5x^2+2}}+\frac{166i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4+5x^2+2}}+$
default	$-\frac{3\left(-\frac{5}{4}x^3-\frac{13}{12}x\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{9i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{8\sqrt{3x^4+5x^2+2}}-\frac{5i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\text{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{8\sqrt{3x^4+5x^2+2}}+6$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
x*(996*x^4+1501*x^2+517)/(2*x^2+1)/(3*x^4+5*x^2+2)^(1/2)+657/4*I*(x^2+1)^(
1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))-166*
I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(I*x,1/2*6
^(1/2))-EllipticE(I*x,1/2*6^(1/2)))+373/2*I*(x^2+1)^(1/2)*(1+3/2*x^2)^(1/2
)/(3*x^4+5*x^2+2)^(1/2)*EllipticPi(I*x,2,1/2*I*(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2 (2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(36*x^12 + 156*x^10 + 2
77*x^8 + 258*x^6 + 133*x^4 + 36*x^2 + 4), x)
```

Sympy [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2 (2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7x^2+4}{((x^2+1)\,(3x^2+2))^{\frac{3}{2}}\,(2x^2+1)^2}\,dx$$

input
```
integrate((x**4-7*x**2+4)/(2*x**2+1)**2/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((x**4 - 7*x**2 + 4)/(((x**2 + 1)*(3*x**2 + 2))**(3/2)*(2*x**2 + 1
)**2), x)
```

Maxima [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2 (2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((x^4 - 7*x^2 + 4)/((3*x^4 + 5*x^2 + 2)^(3/2)*(2*x^2 + 1)^2), x)
```

Giac [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2 (2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((x^4 - 7*x^2 + 4)/((3*x^4 + 5*x^2 + 2)^(3/2)*(2*x^2 + 1)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2 (2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(2\,x^2+1)^2\,(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)^2*(5*x^2 + 3*x^4 + 2)^(3/2)),x)
```

output
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)^2*(5*x^2 + 3*x^4 + 2)^(3/2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{4-7x^2+x^4}{(1+2x^2)^2 (2+5x^2+3x^4)^{3/2}}\,dx &= 4\left(\int \frac{\sqrt{3x^4+5x^2+2}}{36x^{12}+156x^{10}+277x^8+258x^6+133x^4+36x^2+4}dx\right)\\ &+\int \frac{\sqrt{3x^4+5x^2+2}\,x^4}{36x^{12}+156x^{10}+277x^8+258x^6+133x^4+36x^2+4}dx\\ &-7\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{36x^{12}+156x^{10}+277x^8+258x^6+133x^4+36x^2+4}dx\right)\end{aligned}$$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
4*int(sqrt(3*x**4 + 5*x**2 + 2)/(36*x**12 + 156*x**10 + 277*x**8 + 258*x**
6 + 133*x**4 + 36*x**2 + 4),x) + int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(36*
x**12 + 156*x**10 + 277*x**8 + 258*x**6 + 133*x**4 + 36*x**2 + 4),x) - 7*i
nt((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(36*x**12 + 156*x**10 + 277*x**8 + 258
*x**6 + 133*x**4 + 36*x**2 + 4),x)
```

3.217 $\int \frac{4-7x^2+x^4}{(1+2x^2)^3(2+5x^2+3x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 293

$$\begin{aligned}\int \frac{4-7x^2+x^4}{(1+2x^2)^3(2+5x^2+3x^4)^{3/2}} \, dx &= \frac{31x}{4(1+2x^2)^2\sqrt{2+5x^2+3x^4}} \\ &- \frac{591x}{8(1+2x^2)\sqrt{2+5x^2+3x^4}} + \frac{2363x(2+3x^2)}{4\sqrt{2+5x^2+3x^4}} \\ &- \frac{x(14735+14178x^2)}{8\sqrt{2+5x^2+3x^4}} - \frac{2363(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{2\sqrt{2}\sqrt{2+5x^2+3x^4}} \\ &- \frac{2565(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+5x^2+3x^4}} \\ &+ \frac{6385(1+x^2)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{2\sqrt{3}\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
31/4*x/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(1/2)-591/8*x/(2*x^2+1)/(3*x^4+5*x^2+2)
^(1/2)+2363/4*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)-1/8*x*(14178*x^2+14735)/(3
*x^4+5*x^2+2)^(1/2)-2363/4*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*Ellip
ticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)-2565/4*2^(1/2)*(
x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/
(3*x^4+5*x^2+2)^(1/2)+6385/6*(x^2+1)*EllipticPi(x*6^(1/2)/(6*x^2+4)^(1/2),
-1/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.88 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.66

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3(2+5x^2+3x^4)^{3/2}}\,dx = \frac{-\frac{12x(3816+18575x^2+28913x^4+14178x^6)}{(1+2x^2)^2}-14178i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}E\Big(ix$$

input
```
Integrate[(4 - 7*x^2 + x^4)/((1 + 2*x^2)^3*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output
```
((-12*x*(3816 + 18575*x^2 + 28913*x^4 + 14178*x^6))/(1 + 2*x^2)^2 - (14178
*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticE[I*ArcSinh[Sqrt[3/2]*x]
, 2/3] + (5283*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSin
h[Sqrt[3/2]*x], 2/3] - (6385*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*Elli
pticPi[4/3, I*ArcSinh[Sqrt[3/2]*x], 2/3])/(24*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-7x^2+4}{(2x^2+1)^3(3x^4+5x^2+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2258}$$

$$\int \left(\frac{12}{(x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{468}{(2x^2+1)\sqrt{3x^4+5x^2+2}}-\frac{738}{(3x^2+2)\sqrt{3x^4+5x^2+2}}-\frac{140}{(2x^2+1)^2\sqrt{3x^4+5x^2+2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{912\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}-\frac{465(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{2\sqrt{2}\sqrt{3x^4+5x^2+2}}+\frac{387\sqrt{2}(3x^2+2)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}-\frac{140\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{3x^4+5x^2+2}}-\frac{279(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{2\sqrt{2}\sqrt{3x^4+5x^2+2}}-\frac{381\sqrt{2}(3x^2+2)E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}+\frac{904\sqrt{3}(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{961(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{2\sqrt{3}\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}-\frac{839\sqrt{3x^4+5x^2+2}x}{2(2x^2+1)}+\frac{31\sqrt{3x^4+5x^2+2}x}{(2x^2+1)^2}-\frac{1107(x^2+1)x}{\sqrt{3x^4+5x^2+2}}+\frac{2315(3x^2+2)x}{4\sqrt{3x^4+5x^2+2}}$$

input
```
Int[(4 - 7*x^2 + x^4)/((1 + 2*x^2)^3*(2 + 5*x^2 + 3*x^4)^(3/2)),x]
```

output
```
(-1107*x*(1 + x^2))/Sqrt[2 + 5*x^2 + 3*x^4] + (2315*x*(2 + 3*x^2))/(4*Sqrt
[2 + 5*x^2 + 3*x^4]) + (31*x*Sqrt[2 + 5*x^2 + 3*x^4])/(1 + 2*x^2)^2 - (839
*x*Sqrt[2 + 5*x^2 + 3*x^4])/(2*(1 + 2*x^2)) - (381*Sqrt[2]*(2 + 3*x^2)*Ell
ipticE[ArcTan[x], -1/2])/(Sqrt[(2 + 3*x^2)/(1 + x^2)]*Sqrt[2 + 5*x^2 + 3*x
^4]) - (279*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/
2])/(2*Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) - (140*Sqrt[2]*(1 + x^2)*Sqrt[(2 +
 3*x^2)/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4] + (
387*Sqrt[2]*(2 + 3*x^2)*EllipticF[ArcTan[x], -1/2])/(Sqrt[(2 + 3*x^2)/(1 +
 x^2)]*Sqrt[2 + 5*x^2 + 3*x^4]) - (465*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2
)]*EllipticF[ArcTan[x], -1/2])/(2*Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4]) - (912*
Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/
Sqrt[2 + 5*x^2 + 3*x^4] + (961*(1 + x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]
*x], 1/3])/(2*Sqrt[3]*Sqrt[(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4])
 + (904*Sqrt[3]*(1 + x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(Sqr
t[(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 10.62 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{x\left(14178x^{6}+28913x^{4}+18575x^{2}+3816\right)}{2\left(2x^{2}+1\right)^{2}\sqrt{3x^{4}+5x^{2}+2}}-\frac{8895i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{16\sqrt{3x^{4}+5x^{2}+2}}+\frac{2363i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\left(\operatorname{EllipticF}\left(ix,\right.\right.}{4\sqrt{3x^{4}+5x^{2}+}}$
elliptic	$\frac{31x\sqrt{3x^{4}+5x^{2}+2}}{\left(2x^{2}+1\right)^{2}}-\frac{839x\sqrt{3x^{4}+5x^{2}+2}}{2\left(2x^{2}+1\right)}-\frac{6\left(\frac{381}{2}x^{3}+\frac{377}{2}x\right)}{\sqrt{3x^{4}+5x^{2}+2}}+\frac{557i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{16\sqrt{3x^{4}+5x^{2}+2}}-\frac{2363i\sqrt{x^{2}+1}\,\sqrt{6x}}{4\sqrt{3x}}$
default	$\frac{31x\sqrt{3x^{4}+5x^{2}+2}}{\left(2x^{2}+1\right)^{2}}-\frac{839x\sqrt{3x^{4}+5x^{2}+2}}{2\left(2x^{2}+1\right)}-\frac{93\left(\frac{83}{4}x^{3}+\frac{247}{12}x\right)}{2\sqrt{3x^{4}+5x^{2}+2}}+\frac{557i\sqrt{x^{2}+1}\,\sqrt{6x^{2}+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{16\sqrt{3x^{4}+5x^{2}+2}}-\frac{2363i\sqrt{x^{2}+1}\,\sqrt{6x}}{4\sqrt{3x}}$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)^3/(3*x^4+5*x^2+2)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/2*x*(14178*x^6+28913*x^4+18575*x^2+3816)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(1
/2)-8895/16*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*Elliptic
F(I*x,1/2*6^(1/2))+2363/4*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^
(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))-6385/8*I*(x^
2+1)^(1/2)*(1+3/2*x^2)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticPi(I*x,2,1/2*I*
(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^3/(3*x^4+5*x^2+2)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(72*x^14 + 348*x^12 + 7
10*x^10 + 793*x^8 + 524*x^6 + 205*x^4 + 44*x^2 + 4), x)
```

Sympy [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3(2+5x^2+3x^4)^{3/2}}\,dx = \int \frac{x^4-7x^2+4}{((x^2+1)(3x^2+2))^{\frac{3}{2}}(2x^2+1)^3}\,dx$$

input
```
integrate((x**4-7*x**2+4)/(2*x**2+1)**3/(3*x**4+5*x**2+2)**(3/2),x)
```

output
```
Integral((x**4 - 7*x**2 + 4)/(((x**2 + 1)*(3*x**2 + 2))**(3/2)*(2*x**2 + 1
)**3), x)
```

Maxima [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^3/(3*x^4+5*x^2+2)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((x^4 - 7*x^2 + 4)/((3*x^4 + 5*x^2 + 2)^(3/2)*(2*x^2 + 1)^3), x)
```

Giac [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{3}{2}}(2\,x^2+1)^3}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^3/(3*x^4+5*x^2+2)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((x^4 - 7*x^2 + 4)/((3*x^4 + 5*x^2 + 2)^(3/2)*(2*x^2 + 1)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3(2+5x^2+3x^4)^{3/2}}\,dx=\int \frac{x^4-7\,x^2+4}{(2\,x^2+1)^3\,(3\,x^4+5\,x^2+2)^{3/2}}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)^3*(5*x^2 + 3*x^4 + 2)^(3/2)),x)
```

output
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)^3*(5*x^2 + 3*x^4 + 2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^3\,(2+5x^2+3x^4)^{3/2}}\,dx = 4\left(\int \frac{\sqrt{3x^4+5x^2+2}}{72x^{14}+348x^{12}+710x^{10}+793x^8+524x^6+205x^4+44x^2+4}\right.$$

$$+\int \frac{\sqrt{3x^4+5x^2+2}\,x^4}{72x^{14}+348x^{12}+710x^{10}+793x^8+524x^6+205x^4+44x^2+4}dx$$

$$-7\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{72x^{14}+348x^{12}+710x^{10}+793x^8+524x^6+205x^4+44x^2+4}dx\right)$$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)^3/(3*x^4+5*x^2+2)^(3/2),x)
```

output
```
4*int(sqrt(3*x**4 + 5*x**2 + 2)/(72*x**14 + 348*x**12 + 710*x**10 + 793*x*
*8 + 524*x**6 + 205*x**4 + 44*x**2 + 4),x) + int((sqrt(3*x**4 + 5*x**2 + 2
)*x**4)/(72*x**14 + 348*x**12 + 710*x**10 + 793*x**8 + 524*x**6 + 205*x**4
 + 44*x**2 + 4),x) - 7*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(72*x**14 + 34
8*x**12 + 710*x**10 + 793*x**8 + 524*x**6 + 205*x**4 + 44*x**2 + 4),x)
```

3.218 $\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 195

$$\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = -\frac{x(689+1013x^2)}{81(2+5x^2+3x^4)^{3/2}} - \frac{5803x(2+3x^2)}{162\sqrt{2+5x^2+3x^4}} + \frac{x(15689+17553x^2)}{162\sqrt{2+5x^2+3x^4}} + \frac{5803(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\,\middle|-\frac{1}{2}\right)}{81\sqrt{2}\sqrt{2+5x^2+3x^4}} - \frac{2473(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{27\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
-1/81*x*(1013*x^2+689)/(3*x^4+5*x^2+2)^(3/2)-5803/162*x*(3*x^2+2)/(3*x^4+5
*x^2+2)^(1/2)+1/162*x*(17553*x^2+15689)/(3*x^4+5*x^2+2)^(1/2)+5803/162*2^(
1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(
1/2))/(3*x^4+5*x^2+2)^(1/2)-2473/54*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1
/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{(1+2x^2)^3(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{3x(10000+37175x^2+44944x^4+17553x^6)+5803i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x}}{}$$

input
```
Integrate[((1 + 2*x^2)^3*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(3*x*(10000 + 37175*x^2 + 44944*x^4 + 17553*x^6) + (5803*I)*Sqrt[3]*Sqrt[1
 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticE[I*ArcSinh[Sqrt[3/2]*
x], 2/3] - (857*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^
4)*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(162*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2206, 27, 2206, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2+1)^3(x^4-7x^2+4)}{(3x^4+5x^2+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{2206}$$

$$-\frac{1}{6}\int -\frac{2(216x^6-1548x^4-1845x^2+851)}{27(3x^4+5x^2+2)^{3/2}}dx-\frac{x(1013x^2+689)}{81(3x^4+5x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{81}\int \frac{216x^6-1548x^4-1845x^2+851}{(3x^4+5x^2+2)^{3/2}}dx-\frac{x(1013x^2+689)}{81(3x^4+5x^2+2)^{3/2}}$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{81}\left(\frac{x(17553x^2+15689)}{2\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int\frac{3(5803x^2+4946)}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{x(1013x^2+689)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{1}{81}\left(\frac{x(17553x^2+15689)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\int\frac{5803x^2+4946}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{x(1013x^2+689)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 1503

$$\frac{1}{81}\left(\frac{x(17553x^2+15689)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(4946\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx+5803\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)\right)-\frac{x(1013x^2+689)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 1413

$$\frac{1}{81}\left(\frac{x(17553x^2+15689)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(5803\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{2473\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right.\right.$$
$$\frac{x(1013x^2+689)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

$\Big\downarrow$ 1456

$$\frac{1}{81}\left(\frac{x(17553x^2+15689)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(\frac{2473\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+5803\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}\right.\right.\right.$$
$$\frac{x(1013x^2+689)}{81\left(3x^4+5x^2+2\right)^{3/2}}$$

input
```
Int[((1 + 2*x^2)^3*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
-1/81*(x*(689 + 1013*x^2))/(2 + 5*x^2 + 3*x^4)^(3/2) + ((x*(15689 + 17553*
x^2))/(2*Sqrt[2 + 5*x^2 + 3*x^4]) - (3*(5803*((x*(2 + 3*x^2))/(3*Sqrt[2 +
5*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE
[ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (2473*Sqrt[2]*(1 + x^2)*
Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3
*x^4]))/2)/81
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 16.48 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.69

method	result
risch	$\frac{x(17553x^6+44944x^4+37175x^2+10000)}{54(3x^4+5x^2+2)^{\frac{3}{2}}}+\frac{2473i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{54\sqrt{3x^4+5x^2+2}}-\frac{5803i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)\right.}{162\sqrt{3x^4+5x^2+2}}$
elliptic	$\frac{\left(-\frac{1013}{729}x^3-\frac{689}{729}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(-\frac{5851}{324}x^3-\frac{15689}{972}x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{2473i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{54\sqrt{3x^4+5x^2+2}}-\frac{5803i\sqrt{x^2+1}\,\sqrt{6x^2+4}}{}$
default	$\frac{4\left(\frac{5}{18}x^3+\frac{13}{54}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{24\left(\frac{115}{12}x^3+\frac{145}{18}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{5803i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\mathrm{EllipticE}\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{162\sqrt{3x^4+5x^2+2}}+\frac{2473i}{}$

input
```
int((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
1/54*x*(17553*x^6+44944*x^4+37175*x^2+10000)/(3*x^4+5*x^2+2)^(3/2)+2473/54
*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6
^(1/2))-5803/162*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(El
lipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.77

$$\int \frac{(1+2x^2)^3\,(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx=\frac{11606\,\sqrt{3}\sqrt{-\frac{2}{3}}(9\,x^9+30\,x^7+37\,x^5+20\,x^3+4\,x)E(\arcsin\left(\frac{\sqrt{-\frac{2}{3}}}{x}\right)\mid\frac{3}{2}}{}$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fr
icas")
```

output
```
1/486*(11606*sqrt(3)*sqrt(-2/3)*(9*x^9 + 30*x^7 + 37*x^5 + 20*x^3 + 4*x)*e
lliptic_e(arcsin(sqrt(-2/3)/x), 3/2) - 33863*sqrt(3)*sqrt(-2/3)*(9*x^9 + 3
0*x^7 + 37*x^5 + 20*x^3 + 4*x)*elliptic_f(arcsin(sqrt(-2/3)/x), 3/2) + 6*(
216*x^8 - 19629*x^6 - 51593*x^4 - 43030*x^2 - 11606)*sqrt(3*x^4 + 5*x^2 +
2))/(9*x^9 + 30*x^7 + 37*x^5 + 20*x^3 + 4*x)
```

Sympy [F]

$$\int \frac{(1+2x^2)^3 (4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(2x^2+1)^3 (x^4-7x^2+4)}{((x^2+1)(3x^2+2))^{\frac{5}{2}}}\,dx$$

input
```
integrate((2*x**2+1)**3*(x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral((2*x**2 + 1)**3*(x**4 - 7*x**2 + 4)/((x**2 + 1)*(3*x**2 + 2))**(5
/2), x)
```

Maxima [F]

$$\int \frac{(1+2x^2)^3 (4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)^3}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^3/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{(1+2x^2)^3 (4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)^3}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^3/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2)^3\,(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(2\,x^2+1)^3\,(x^4-7\,x^2+4)}{(3\,x^4+5\,x^2+2)^{5/2}}\,dx$$

input
```
int(((2*x^2 + 1)^3*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(5/2),x)
```

output
```
int(((2*x^2 + 1)^3*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(1+2x^2)^3\,(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{2160\sqrt{3x^4+5x^2+2}\,x^7+26280\sqrt{3x^4+5x^2+2}\,x^5+36700\sqrt{3x^4+5x^2}}{}$$

input
```
int((2*x^2+1)^3*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x)
```

output
```
(2160*sqrt(3*x**4 + 5*x**2 + 2)*x**7 + 26280*sqrt(3*x**4 + 5*x**2 + 2)*x**
5 + 36700*sqrt(3*x**4 + 5*x**2 + 2)*x**3 + 20643*sqrt(3*x**4 + 5*x**2 + 2)
*x - 342414*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8
 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**8 - 1141380*int(sqrt(3*x**4 +
5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x*
*2 + 8),x)*x**6 - 1407702*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x*
*10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**4 - 760920*int(s
qrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186
*x**4 + 60*x**2 + 8),x)*x**2 - 152184*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x*
*12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x) + 47263
5*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x**8 +
305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**8 + 1575450*int((sqrt(3*x**4 + 5*
x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 6
0*x**2 + 8),x)*x**6 + 1943055*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*x**
12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**4 + 1
050300*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x*
*8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**2 + 210060*int((sqrt(3*x**4
+ 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4
 + 60*x**2 + 8),x))/(810*(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4))
```

3.219 $\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 160

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{x(257+365x^2)}{27(2+5x^2+3x^4)^{3/2}} - \frac{733x}{18\sqrt{2+5x^2+3x^4}} - \frac{4171(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\middle|-\frac{1}{2}\right)}{27\sqrt{2}\sqrt{2+5x^2+3x^4}} + \frac{1723(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{9\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output

```
1/27*x*(365*x^2+257)/(3*x^4+5*x^2+2)^(3/2)-733/18*x/(3*x^4+5*x^2+2)^(1/2)-
4171/54*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2
),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+1723/18*2^(1/2)*(x^2+1)*((3*x^2+2)/
(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{-3x(6856+25667x^2+31396x^4+12513x^6)-4171i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3}}{}$$

input
```
Integrate[((1 + 2*x^2)^2*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(-3*x*(6856 + 25667*x^2 + 31396*x^4 + 12513*x^6) - (4171*I)*Sqrt[3]*Sqrt[1
 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticE[I*ArcSinh[Sqrt[3/2]*
x], 2/3] + (725*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^
4)*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(54*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2206, 27, 2206, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2+1)^2(x^4-7x^2+4)}{(3x^4+5x^2+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{x(365x^2+257)}{27(3x^4+5x^2+2)^{3/2}}-\frac{1}{6}\int \frac{2(-36x^4-819x^2+203)}{9(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(365x^2+257)}{27(3x^4+5x^2+2)^{3/2}}-\frac{1}{27}\int \frac{-36x^4-819x^2+203}{(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{27}\left(\frac{1}{2}\int\frac{3(4171x^2+3446)}{\sqrt{3x^4+5x^2+2}}dx-\frac{x(12513x^2+10541)}{2\sqrt{3x^4+5x^2+2}}\right)+\frac{x(365x^2+257)}{27(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{1}{27}\left(\frac{3}{2}\int\frac{4171x^2+3446}{\sqrt{3x^4+5x^2+2}}dx-\frac{x(12513x^2+10541)}{2\sqrt{3x^4+5x^2+2}}\right)+\frac{x(365x^2+257)}{27(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 1503

$$\frac{1}{27}\left(\frac{3}{2}\left(3446\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx+4171\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{x(12513x^2+10541)}{2\sqrt{3x^4+5x^2+2}}\right)+\frac{x(365x^2+257)}{27(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 1413

$$\frac{1}{27}\left(\frac{3}{2}\left(4171\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{1723\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)-\frac{x(12513x^2+105}{2\sqrt{3x^4+5x^2+}}\right.\frac{x(365x^2+257)}{27(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 1456

$$\frac{1}{27}\left(\frac{3}{2}\left(\frac{1723\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+4171\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E}{3\sqrt{3x^4+5.}}\right.\right.\right.\frac{x(365x^2+257)}{27(3x^4+5x^2+2)^{3/2}}$$

input
```
Int[((1 + 2*x^2)^2*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(x*(257 + 365*x^2))/(27*(2 + 5*x^2 + 3*x^4)^(3/2)) + (-1/2*(x*(10541 + 125
13*x^2))/Sqrt[2 + 5*x^2 + 3*x^4] + (3*(4171*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5
*x^2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[
ArcTan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (1723*Sqrt[2]*(1 + x^2)*S
qrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*
x^4]))/2)/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 11.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{x(12513x^6+31396x^4+25667x^2+6856)}{18(3x^4+5x^2+2)^{\frac{3}{2}}}-\frac{1723i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{18\sqrt{3x^4+5x^2+2}}+\frac{4171i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\right.\right.}{54\sqrt{3x^4+5x^2+}}$
elliptic	$\frac{\left(\frac{365}{243}x^3+\frac{257}{243}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\frac{4171}{108}x^3+\frac{10541}{324}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{1723i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{18\sqrt{3x^4+5x^2+2}}+\frac{4171i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{El}\right.}{54}$
default	$\frac{4\left(\frac{5}{18}x^3+\frac{13}{54}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{24\left(\frac{115}{12}x^3+\frac{145}{18}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{1723i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{18\sqrt{3x^4+5x^2+2}}+\frac{4171i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{Ellipt}\right.}{54\sqrt{3}}$

input
```
int((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/18*x*(12513*x^6+31396*x^4+25667*x^2+6856)/(3*x^4+5*x^2+2)^(3/2)-1723/18
*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6
^(1/2))+4171/54*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(Ell
ipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int \frac{(1+2x^2)^2\,(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx =$$
$$-\frac{4171\,\sqrt{2}(-9i\,x^8-30i\,x^6-37i\,x^4-20i\,x^2-4i)E(\arcsin{(i\,x)}\mid\frac{3}{2})+9340\,\sqrt{2}(9i\,x^8+30i\,x^6+37i\,x^4+2}{54\,(9\,x^8+30\,x^6+37\,x^4}$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fr
icas")
```

output
```
-1/54*(4171*sqrt(2)*(-9*I*x^8 - 30*I*x^6 - 37*I*x^4 - 20*I*x^2 - 4*I)*elli
ptic_e(arcsin(I*x), 3/2) + 9340*sqrt(2)*(9*I*x^8 + 30*I*x^6 + 37*I*x^4 + 2
0*I*x^2 + 4*I)*elliptic_f(arcsin(I*x), 3/2) + 3*(12513*x^7 + 31396*x^5 + 2
5667*x^3 + 6856*x)*sqrt(3*x^4 + 5*x^2 + 2))/(9*x^8 + 30*x^6 + 37*x^4 + 20*
x^2 + 4)
```

Sympy [F]

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(2x^2+1)^2(x^4-7x^2+4)}{((x^2+1)(3x^2+2))^{\frac{5}{2}}}\,dx$$

input
```
integrate((2*x**2+1)**2*(x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral((2*x**2 + 1)**2*(x**4 - 7*x**2 + 4)/((x**2 + 1)*(3*x**2 + 2))**(5
/2), x)
```

Maxima [F]

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)^2}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^2/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(x^4-7\,x^2+4)(2\,x^2+1)^2}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)^2/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(2\,x^2+1)^2\,(x^4-7\,x^2+4)}{(3\,x^4+5\,x^2+2)^{5/2}}\,dx$$

input
```
int(((2*x^2 + 1)^2*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(5/2),x)
```

output
```
int(((2*x^2 + 1)^2*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(1+2x^2)^2(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{-360\sqrt{3x^4+5x^2+2}\,x^5+320\sqrt{3x^4+5x^2+2}\,x^3-51\sqrt{3x^4+5x^2+2}\,x}{}$$

input
```
int((2*x^2+1)^2*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x)
```

output

```
( - 360*sqrt(3*x***4 + 5*x**2 + 2)*x***5 + 320*sqrt(3*x***4 + 5*x**2 + 2)*x***
3 - 51*sqrt(3*x***4 + 5*x**2 + 2)*x + 10638*int(sqrt(3*x***4 + 5*x**2 + 2)/(
27*x**12 + 135*x***10 + 279*x***8 + 305*x***6 + 186*x***4 + 60*x***2 + 8),x)*x*
*8 + 35460*int(sqrt(3*x***4 + 5*x**2 + 2)/(27*x**12 + 135*x***10 + 279*x***8
+ 305*x***6 + 186*x***4 + 60*x***2 + 8),x)*x***6 + 43734*int(sqrt(3*x***4 + 5*x
**2 + 2)/(27*x**12 + 135*x***10 + 279*x***8 + 305*x***6 + 186*x***4 + 60*x***2
+ 8),x)*x***4 + 23640*int(sqrt(3*x***4 + 5*x**2 + 2)/(27*x**12 + 135*x***10 +
 279*x***8 + 305*x***6 + 186*x***4 + 60*x***2 + 8),x)*x***2 + 4728*int(sqrt(3*x
**4 + 5*x**2 + 2)/(27*x**12 + 135*x***10 + 279*x***8 + 305*x***6 + 186*x***4 +
 60*x***2 + 8),x) - 1215*int((sqrt(3*x***4 + 5*x**2 + 2)*x***4)/(27*x**12 + 1
35*x***10 + 279*x***8 + 305*x***6 + 186*x***4 + 60*x***2 + 8),x)*x***8 - 4050*in
t((sqrt(3*x***4 + 5*x**2 + 2)*x***4)/(27*x**12 + 135*x***10 + 279*x***8 + 305*
x**6 + 186*x***4 + 60*x***2 + 8),x)*x***6 - 4995*int((sqrt(3*x***4 + 5*x**2 +
2)*x***4)/(27*x**12 + 135*x***10 + 279*x***8 + 305*x***6 + 186*x***4 + 60*x***2
+ 8),x)*x***4 - 2700*int((sqrt(3*x***4 + 5*x**2 + 2)*x***4)/(27*x**12 + 135*x
**10 + 279*x***8 + 305*x***6 + 186*x***4 + 60*x***2 + 8),x)*x***2 - 540*int((sq
rt(3*x***4 + 5*x**2 + 2)*x***4)/(27*x**12 + 135*x***10 + 279*x***8 + 305*x***6
+ 186*x***4 + 60*x***2 + 8),x))/(270*(9*x***8 + 30*x***6 + 37*x***4 + 20*x***2 +
 4))
```

3.220 $\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 160

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = -\frac{x(113+149x^2)}{9(2+5x^2+3x^4)^{3/2}} + \frac{425x}{6\sqrt{2+5x^2+3x^4}} + \frac{2419(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{9\sqrt{2}\sqrt{2+5x^2+3x^4}} - \frac{997(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{3\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
-1/9*x*(149*x^2+113)/(3*x^4+5*x^2+2)^(3/2)+425/6*x/(3*x^4+5*x^2+2)^(1/2)+2
419/18*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2)
,1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)-997/6*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^
2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{3x(4000+14927x^2+18208x^4+7257x^6)+2419i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}(2}{}$$

input

```
Integrate[((1 + 2*x^2)*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output

```
(3*x*(4000 + 14927*x^2 + 18208*x^4 + 7257*x^6) + (2419*I)*Sqrt[3]*Sqrt[1 +
 x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticE[I*ArcSinh[Sqrt[3/2]*x]
, 2/3] - (425*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)
*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(18*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2206, 27, 1492, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2+1)(x^4-7x^2+4)}{(3x^4+5x^2+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{2206}$$

$$-\frac{1}{6}\int -\frac{2(131-441x^2)}{3(3x^4+5x^2+2)^{3/2}}dx-\frac{x(149x^2+113)}{9(3x^4+5x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{131-441x^2}{(3x^4+5x^2+2)^{3/2}}dx-\frac{x(149x^2+113)}{9(3x^4+5x^2+2)^{3/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{9}\left(\frac{x(7257x^2+6113)}{2\sqrt{3x^4+5x^2+2}}-\frac{1}{2}\int\frac{3(2419x^2+1994)}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{x(149x^2+113)}{9(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{x(7257x^2+6113)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\int\frac{2419x^2+1994}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{x(149x^2+113)}{9(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 1503

$$\frac{1}{9}\left(\frac{x(7257x^2+6113)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(1994\int\frac{1}{\sqrt{3x^4+5x^2+2}}dx+2419\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)\right)-\frac{x(149x^2+113)}{9(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 1413

$$\frac{1}{9}\left(\frac{x(7257x^2+6113)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(2419\int\frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{997\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)\right)$$
$$\frac{x(149x^2+113)}{9(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 1456

$$\frac{1}{9}\left(\frac{x(7257x^2+6113)}{2\sqrt{3x^4+5x^2+2}}-\frac{3}{2}\left(\frac{997\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+2419\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}}{}\right.\right.\right.$$
$$\frac{x(149x^2+113)}{9(3x^4+5x^2+2)^{3/2}}$$

input
```
Int[((1 + 2*x^2)*(4 - 7*x^2 + x^4))/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
-1/9*(x*(113 + 149*x^2))/(2 + 5*x^2 + 3*x^4)^(3/2) + ((x*(6113 + 7257*x^2)
)/(2*Sqrt[2 + 5*x^2 + 3*x^4]) - (3*(2419*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^
2 + 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[Arc
Tan[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (997*Sqrt[2]*(1 + x^2)*Sqrt[
(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4]
))/2)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 7.59 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

method	result
risch	$\frac{x\left(7257x^6+18208x^4+14927x^2+4000\right)}{6\left(3x^4+5x^2+2\right)^{\frac{3}{2}}}+\frac{997i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{6\sqrt{3x^4+5x^2+2}}-\frac{2419i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\right.}{18\sqrt{3x^4+5x^2+2}}$
elliptic	$\frac{\left(-\frac{149}{81}x^3-\frac{113}{81}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(-\frac{2419}{36}x^3-\frac{6113}{108}x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{997i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{6\sqrt{3x^4+5x^2+2}}-\frac{2419i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{E}\right.}{18}$
default	$\frac{\left(-\frac{2}{9}x^3-\frac{5}{27}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(-\frac{97}{12}x^3-\frac{245}{36}x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{997i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\operatorname{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{6\sqrt{3x^4+5x^2+2}}-\frac{2419i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(\mathrm{Elliptic}\right.}{18\sqrt{3x^4}}$

input
```
int((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x*(7257*x^6+18208*x^4+14927*x^2+4000)/(3*x^4+5*x^2+2)^(3/2)+997/6*I*(x
^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2
))-2419/18*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(Elliptic
F(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = -\frac{2419\sqrt{2}(9i\,x^8+30i\,x^6+37i\,x^4+20i\,x^2+4i)E(\arcsin{(i\,x)}\mid\frac{3}{2})+5410\sqrt{2}(-9i\,x^8-30i\,x^6-37i\,x^4-2}{18\,(9\,x^8+30\,x^6+37\,x^4}$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fric
as")
```

output
```
-1/18*(2419*sqrt(2)*(9*I*x^8 + 30*I*x^6 + 37*I*x^4 + 20*I*x^2 + 4*I)*ellip
tic_e(arcsin(I*x), 3/2) + 5410*sqrt(2)*(-9*I*x^8 - 30*I*x^6 - 37*I*x^4 - 2
0*I*x^2 - 4*I)*elliptic_f(arcsin(I*x), 3/2) - 3*(7257*x^7 + 18208*x^5 + 14
927*x^3 + 4000*x)*sqrt(3*x^4 + 5*x^2 + 2))/(9*x^8 + 30*x^6 + 37*x^4 + 20*x
^2 + 4)
```

Sympy [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{(2x^2+1)(x^4-7x^2+4)}{((x^2+1)(3x^2+2))^{\frac{5}{2}}}\,dx$$

input
```
integrate((2*x**2+1)*(x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral((2*x**2 + 1)*(x**4 - 7*x**2 + 4)/((x**2 + 1)*(3*x**2 + 2))**(5/2)
, x)
```

Maxima [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx=\int \frac{(x^4-7\,x^2+4)(2\,x^2+1)}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx=\int \frac{(x^4-7\,x^2+4)(2\,x^2+1)}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="giac
")
```

output
```
integrate((x^4 - 7*x^2 + 4)*(2*x^2 + 1)/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx=\int \frac{(2\,x^2+1)\,(x^4-7\,x^2+4)}{(3\,x^4+5\,x^2+2)^{5/2}}\,dx$$

input
```
int(((2*x^2 + 1)*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(5/2),x)
```

output
```
int(((2*x^2 + 1)*(x^4 - 7*x^2 + 4))/(5*x^2 + 3*x^4 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(1+2x^2)(4-7x^2+x^4)}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{-20\sqrt{3x^4+5x^2+2}\,x^3-21\sqrt{3x^4+5x^2+2}\,x+3618\left(\int \frac{\sqrt{3x}}{27x^{12}+135x^{10}+279x}\right.}{}$$

input
```
int((2*x^2+1)*(x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x)
```

output
```
( - 20*sqrt(3*x**4 + 5*x**2 + 2)*x**3 - 21*sqrt(3*x**4 + 5*x**2 + 2)*x + 3
618*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x
**6 + 186*x**4 + 60*x**2 + 8),x)*x**8 + 12060*int(sqrt(3*x**4 + 5*x**2 + 2
)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)
*x**6 + 14874*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x*
*8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**4 + 8040*int(sqrt(3*x**4 + 5
*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**
2 + 8),x)*x**2 + 1608*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10
+ 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x) - 13365*int((sqrt(3*x**
4 + 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x*
*4 + 60*x**2 + 8),x)*x**8 - 44550*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27
*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**6
 - 54945*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*
x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**4 - 29700*int((sqrt(3*x**4
 + 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**
4 + 60*x**2 + 8),x)*x**2 - 5940*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*x
**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x))/(90*(
9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4))
```

3.221 $\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 158

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{x(65+77x^2)}{3(2+5x^2+3x^4)^{3/2}} - \frac{213x}{2\sqrt{2+5x^2+3x^4}} - \frac{1219(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{2}\sqrt{2+5x^2+3x^4}} + \frac{503(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}}$$

output
```
1/3*x*(77*x^2+65)/(3*x^4+5*x^2+2)^(3/2)-213/2*x/(3*x^4+5*x^2+2)^(1/2)-1219
/6*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2
*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+503/2*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1)
)^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \frac{-3x(2008+7515x^2+9172x^4+3657x^6)-1219i\sqrt{3}\sqrt{1+x^2}\sqrt{2+3x^2}(2+5x^2$$

input
```
Integrate[(4 - 7*x^2 + x^4)/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(-3*x*(2008 + 7515*x^2 + 9172*x^4 + 3657*x^6) - (1219*I)*Sqrt[3]*Sqrt[1 +
x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticE[I*ArcSinh[Sqrt[3/2]*x],
 2/3] + (213*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*
EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3])/(6*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2206, 27, 1492, 27, 1503, 1413, 1456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-7x^2+4}{(3x^4+5x^2+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{2206}$$

$$\frac{x(77x^2+65)}{3(3x^4+5x^2+2)^{3/2}}-\frac{1}{6}\int \frac{2(59-231x^2)}{(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(77x^2+65)}{3(3x^4+5x^2+2)^{3/2}}-\frac{1}{3}\int \frac{59-231x^2}{(3x^4+5x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{3(1219x^2+1006)}{\sqrt{3x^4+5x^2+2}}dx-\frac{x(3657x^2+3077)}{2\sqrt{3x^4+5x^2+2}}\right)+\frac{x(77x^2+65)}{3(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1219x^2+1006}{\sqrt{3x^4+5x^2+2}}dx-\frac{x(3657x^2+3077)}{2\sqrt{3x^4+5x^2+2}}\right)+\frac{x(77x^2+65)}{3(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 1503

$$\frac{1}{3}\left(\frac{3}{2}\left(1006\int \frac{1}{\sqrt{3x^4+5x^2+2}}dx+1219\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx\right)-\frac{x(3657x^2+3077)}{2\sqrt{3x^4+5x^2+2}}\right)+\frac{x(77x^2+65)}{3(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 1413

$$\frac{1}{3}\left(\frac{3}{2}\left(1219\int \frac{x^2}{\sqrt{3x^4+5x^2+2}}dx+\frac{503\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}\right)-\frac{x(3657x^2+3077)}{2\sqrt{3x^4+5x^2+2}}\right)$$

$$\frac{x(77x^2+65)}{3(3x^4+5x^2+2)^{3/2}}$$

$\Big\downarrow$ 1456

$$\frac{1}{3}\left(\frac{3}{2}\left(\frac{503\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\operatorname{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+1219\left(\frac{x(3x^2+2)}{3\sqrt{3x^4+5x^2+2}}-\frac{\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E(a}{3\sqrt{3x^4+5x^2}}\right.\right.\right.$$

$$\frac{x(77x^2+65)}{3(3x^4+5x^2+2)^{3/2}}$$

input
```
Int[(4 - 7*x^2 + x^4)/(2 + 5*x^2 + 3*x^4)^(5/2),x]
```

output
```
(x*(65 + 77*x^2))/(3*(2 + 5*x^2 + 3*x^4)^(3/2)) + (-1/2*(x*(3077 + 3657*x^
2))/Sqrt[2 + 5*x^2 + 3*x^4] + (3*(1219*((x*(2 + 3*x^2))/(3*Sqrt[2 + 5*x^2
+ 3*x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticE[ArcTa
n[x], -1/2])/(3*Sqrt[2 + 5*x^2 + 3*x^4])) + (503*Sqrt[2]*(1 + x^2)*Sqrt[(2
 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4]))
/2)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1413
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q)*x^2)/(2*a +
(b - q)*x^2)]/(2*a*Rt[(b - q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /;
 FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1456
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b - q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b - q)/(2*a), 2]*(2*a + (b - q)*x^2)*(Sqrt[(2*a + (b + q
)*x^2)/(2*a + (b - q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b - q)/(2*a), 2]*x], -2*(q/(b - q))], x] /; PosQ[(b - q)/a]] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1503
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[d   Int[1/Sqrt[a + b*x^2 + c*x^4]
, x], x] + Simp[e   Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)
/a] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{x(3657x^6+9172x^4+7515x^2+2008)}{2(3x^4+5x^2+2)^{\frac{3}{2}}}-\frac{503i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{1219i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)-\right.}{6\sqrt{3x^4+5x^2+2}}$
elliptic	$\frac{\left(\frac{77}{27}x^3+\frac{65}{27}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\frac{1219}{12}x^3+\frac{3077}{36}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{503i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{1219i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{Ellipti}\right.}{6\sqrt{3x^4}}$
default	$\frac{\left(\frac{5}{27}x^3+\frac{4}{27}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(\frac{20}{3}x^3+\frac{101}{18}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{503i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{1219i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\text{EllipticF}\right.}{6\sqrt{3x^4+5}}$

input
```
int((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(3657*x^6+9172*x^4+7515*x^2+2008)/(3*x^4+5*x^2+2)^(3/2)-503/2*I*(x^
2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2)
)+1219/6*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*(EllipticF(
I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{5/2}}\,dx =$$
$$-\frac{1219\sqrt{2}(-9i\,x^8-30i\,x^6-37i\,x^4-20i\,x^2-4i)E(\arcsin(i\,x)\mid\frac{3}{2})+2728\sqrt{2}(9i\,x^8+30i\,x^6+37i\,x^4+2}{6\,(9\,x^8+30\,x^6+37\,x^4+}$$

input
```
integrate((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(1219*sqrt(2)*(-9*I*x^8 - 30*I*x^6 - 37*I*x^4 - 20*I*x^2 - 4*I)*ellip
tic_e(arcsin(I*x), 3/2) + 2728*sqrt(2)*(9*I*x^8 + 30*I*x^6 + 37*I*x^4 + 20
*I*x^2 + 4*I)*elliptic_f(arcsin(I*x), 3/2) + 3*(3657*x^7 + 9172*x^5 + 7515
*x^3 + 2008*x)*sqrt(3*x^4 + 5*x^2 + 2))/(9*x^8 + 30*x^6 + 37*x^4 + 20*x^2
+ 4)
```

Sympy [F]

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7x^2+4}{((x^2+1)\,(3x^2+2))^{\frac{5}{2}}}\,dx$$

input
```
integrate((x**4-7*x**2+4)/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral((x**4 - 7*x**2 + 4)/((x**2 + 1)*(3*x**2 + 2))**(5/2), x)
```

Maxima [F]

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((x^4 - 7*x^2 + 4)/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
integrate((x^4 - 7*x^2 + 4)/(3*x^4 + 5*x^2 + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4-7x^2+x^4}{(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{5/2}}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)/(5*x^2 + 3*x^4 + 2)^(5/2),x)
```

output
```
int((x^4 - 7*x^2 + 4)/(5*x^2 + 3*x^4 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{4-7x^2+x^4}{\left(2+5x^2+3x^4\right)^{5/2}}\,dx = \frac{7\sqrt{3x^4+5x^2+2}\,x+234\left(\int \frac{\sqrt{3x^4+5x^2+2}}{27x^{12}+135x^{10}+279x^8+305x^6+186x^4+60x^2+8}dx\right)x^8+780\left(\right.}{}$$

input
```
int((x^4-7*x^2+4)/(3*x^4+5*x^2+2)^(5/2),x)
```

output
```
(7*sqrt(3*x**4 + 5*x**2 + 2)*x + 234*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**
12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**8 + 7
80*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x*
*6 + 186*x**4 + 60*x**2 + 8),x)*x**6 + 962*int(sqrt(3*x**4 + 5*x**2 + 2)/(
27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x*
*4 + 520*int(sqrt(3*x**4 + 5*x**2 + 2)/(27*x**12 + 135*x**10 + 279*x**8 +
305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**2 + 104*int(sqrt(3*x**4 + 5*x**2
+ 2)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8)
,x) + 1035*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 27
9*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**8 + 3450*int((sqrt(3*x**
4 + 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x*
*4 + 60*x**2 + 8),x)*x**6 + 4255*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*
x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**4
+ 2300*int((sqrt(3*x**4 + 5*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x*
*8 + 305*x**6 + 186*x**4 + 60*x**2 + 8),x)*x**2 + 460*int((sqrt(3*x**4 + 5
*x**2 + 2)*x**4)/(27*x**12 + 135*x**10 + 279*x**8 + 305*x**6 + 186*x**4 +
60*x**2 + 8),x))/(10*(9*x**8 + 30*x**6 + 37*x**4 + 20*x**2 + 4))
```

3.222 $\int \frac{4-7x^2+x^4}{(1+2x^2)(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 253

$$\int \frac{4-7x^2+x^4}{(1+2x^2)(2+5x^2+3x^4)^{5/2}}\,dx = -\frac{x(49+53x^2)}{(2+5x^2+3x^4)^{3/2}} - \frac{395x(2+3x^2)}{2\sqrt{2+5x^2+3x^4}} + \frac{x(889+1185x^2)}{2\sqrt{2+5x^2+3x^4}} + \frac{395(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}} - \frac{915(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}} + \frac{496(1+x^2)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{3}\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}$$

output
```
-x*(53*x^2+49)/(3*x^4+5*x^2+2)^(3/2)-395/2*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/
2)+1/2*x*(1185*x^2+889)/(3*x^4+5*x^2+2)^(1/2)+395/2*2^(1/2)*(x^2+1)*((3*x^
2+2)/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+
2)^(1/2)-915/2*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(a
rctan(x),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+496/3*(x^2+1)*EllipticPi(x*6
^(1/2)/(6*x^2+4)^(1/2),-1/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))^(1/2)
/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.95

$$\int \frac{4-7x^2+x^4}{(1+2x^2)(2+5x^2+3x^4)^{5/2}}\,dx = \frac{-294x-318x^3+2667x(2+5x^2+3x^4)+3555x^3(2+5x^2+3x^4)+}{}$$

input
```
Integrate[(4 - 7*x^2 + x^4)/((1 + 2*x^2)*(2 + 5*x^2 + 3*x^4)^(5/2)),x]
```

output
```
(-294*x - 318*x^3 + 2667*x*(2 + 5*x^2 + 3*x^4) + 3555*x^3*(2 + 5*x^2 + 3*x
^4) + (1185*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*E
llipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (99*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[
2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3] - (2
48*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticPi
[4/3, I*ArcSinh[Sqrt[3/2]*x], 2/3])/(6*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.81, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-7x^2+4}{(2x^2+1)(3x^4+5x^2+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{2258}$$

$$\int \left(-\frac{87}{(x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{124}{(2x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{75}{(3x^2+2)\sqrt{3x^4+5x^2+2}}-\frac{12}{(x^2+1)^2\sqrt{3x^4+5}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{186\sqrt{2}\sqrt{\frac{3x^2+2}{x^2+1}}\,(x^2+1)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}-\frac{42\sqrt{2}(3x^2+2)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}-\frac{459(3x^2+2)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}+\frac{160\sqrt{2}(3x^2+2)\,E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}+\frac{75(3x^2+2)\,E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}+\frac{496(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{3}\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{471x(x^2+1)}{2\sqrt{3x^4+5x^2+2}}-\frac{123x(x^2+1)}{(3x^2+2)\sqrt{3x^4+5x^2+2}}-\frac{157x(3x^2+2)}{2\sqrt{3x^4+5x^2+2}}+\frac{4x(3x^2+2)}{\sqrt{3x^4+5x^2+2}\,(x^2+1)}$$

input
```
Int[(4 - 7*x^2 + x^4)/((1 + 2*x^2)*(2 + 5*x^2 + 3*x^4)^(5/2)),x]
```

output
```
(471*x*(1 + x^2))/(2*Sqrt[2 + 5*x^2 + 3*x^4]) - (123*x*(1 + x^2))/((2 + 3*
x^2)*Sqrt[2 + 5*x^2 + 3*x^4]) - (157*x*(2 + 3*x^2))/(2*Sqrt[2 + 5*x^2 + 3*
x^4]) + (4*x*(2 + 3*x^2))/((1 + x^2)*Sqrt[2 + 5*x^2 + 3*x^4]) + (75*(2 + 3
*x^2)*EllipticE[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Sqr
t[2 + 5*x^2 + 3*x^4]) + (160*Sqrt[2]*(2 + 3*x^2)*EllipticE[ArcTan[x], -1/2
])/(Sqrt[(2 + 3*x^2)/(1 + x^2)]*Sqrt[2 + 5*x^2 + 3*x^4]) - (459*(2 + 3*x^2
)*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Sqrt[2
+ 5*x^2 + 3*x^4]) - (42*Sqrt[2]*(2 + 3*x^2)*EllipticF[ArcTan[x], -1/2])/(S
qrt[(2 + 3*x^2)/(1 + x^2)]*Sqrt[2 + 5*x^2 + 3*x^4]) - (186*Sqrt[2]*(1 + x^
2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2
+ 3*x^4] + (496*(1 + x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/2]*x], 1/3])/(Sqr
t[3]*Sqrt[(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.78

method	result
elliptic	$\frac{\left(-\frac{53}{9}x^3-\frac{49}{9}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(-\frac{395}{4}x^3-\frac{889}{12}x\right)}{\sqrt{3x^4+5x^2+2}}+\frac{74i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4+5x^2+2}}+\frac{395i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticE}}{2\sqrt{3x^4+5x^2+2}}$
default	$-\frac{15\left(\frac{5}{18}x^3+\frac{13}{54}x\right)\sqrt{3x^4+5x^2+2}}{4\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}+\frac{\frac{1725}{8}x^3+\frac{725}{4}x}{\sqrt{3x^4+5x^2+2}}+\frac{3695i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \mathrm{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{24\sqrt{3x^4+5x^2+2}}-\frac{1919i\sqrt{x^2+1}\,\sqrt{6x^2+4}\left(\mathrm{Ellip}\right.}{24\sqrt{3}}$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-53/9*x^3-49/9*x)*(3*x^4+5*x^2+2)^(1/2)/(x^4+5/3*x^2+2/3)^2-6*(-395/4*x^3
-889/12*x)/(3*x^4+5*x^2+2)^(1/2)+74*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4
+5*x^2+2)^(1/2)*EllipticF(I*x,1/2*6^(1/2))+395/2*I*(x^2+1)^(1/2)*(6*x^2+4)
^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticE(I*x,1/2*6^(1/2))-124*I*(x^2+1)^(1/2
)*(1+3/2*x^2)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticPi(I*x,2,1/2*I*(-3)^(1/2
)*2^(1/2))
```

Fricas [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\left(2+5x^2+3x^4\right)^{5/2}}\,dx=\int \frac{x^4-7\,x^2+4}{\left(3\,x^4+5\,x^2+2\right)^{\frac{5}{2}}\left(2\,x^2+1\right)}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(54*x^14 + 297*x^12 + 6
93*x^10 + 889*x^8 + 677*x^6 + 306*x^4 + 76*x^2 + 8), x)
```

Sympy [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7x^2+4}{((x^2+1)(3x^2+2))^{\frac{5}{2}}\cdot(2x^2+1)}\,dx$$

input
```
integrate((x**4-7*x**2+4)/(2*x**2+1)/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral((x**4 - 7*x**2 + 4)/(((x**2 + 1)*(3*x**2 + 2))**(5/2)*(2*x**2 + 1
)), x)
```

Maxima [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}(2\,x^2+1)}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="maxi
ma")
```

output
```
integrate((x^4 - 7*x^2 + 4)/((3*x^4 + 5*x^2 + 2)^(5/2)*(2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}(2\,x^2+1)}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(5/2),x, algorithm="giac
")
```

output
```
integrate((x^4 - 7*x^2 + 4)/((3*x^4 + 5*x^2 + 2)^(5/2)*(2*x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\left(2+5x^2+3x^4\right)^{5/2}}\,dx=\int \frac{x^4-7\,x^2+4}{\left(2\,x^2+1\right)\,\left(3\,x^4+5\,x^2+2\right)^{5/2}}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)*(5*x^2 + 3*x^4 + 2)^(5/2)),x)
```

output
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)*(5*x^2 + 3*x^4 + 2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)\left(2+5x^2+3x^4\right)^{5/2}}\,dx=4\left(\int \frac{\sqrt{3x^4+5x^2+2}}{54x^{14}+297x^{12}+693x^{10}+889x^8+677x^6+306x^4+76x^2+8}\right.$$
$$+\int \frac{\sqrt{3x^4+5x^2+2}\,x^4}{54x^{14}+297x^{12}+693x^{10}+889x^8+677x^6+306x^4+76x^2+8}dx$$
$$\left.-7\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{54x^{14}+297x^{12}+693x^{10}+889x^8+677x^6+306x^4+76x^2+8}dx\right)\right)$$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)/(3*x^4+5*x^2+2)^(5/2),x)
```

output
```
4*int(sqrt(3*x**4 + 5*x**2 + 2)/(54*x**14 + 297*x**12 + 693*x**10 + 889*x*
*8 + 677*x**6 + 306*x**4 + 76*x**2 + 8),x) + int((sqrt(3*x**4 + 5*x**2 + 2
)*x**4)/(54*x**14 + 297*x**12 + 693*x**10 + 889*x**8 + 677*x**6 + 306*x**4
 + 76*x**2 + 8),x) - 7*int((sqrt(3*x**4 + 5*x**2 + 2)*x**2)/(54*x**14 + 29
7*x**12 + 693*x**10 + 889*x**8 + 677*x**6 + 306*x**4 + 76*x**2 + 8),x)
```

3.223 $\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 290

$$\begin{aligned}\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{5/2}}\,dx &= -\frac{x(49+53x^2)}{(1+2x^2)(2+5x^2+3x^4)^{3/2}} \\ &+ \frac{152x}{(1+2x^2)\sqrt{2+5x^2+3x^4}} - \frac{967x(2+3x^2)}{2\sqrt{2+5x^2+3x^4}} \\ &+ \frac{x(3533+2901x^2)}{2\sqrt{2+5x^2+3x^4}} + \frac{967(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}} \\ &+ \frac{3075(1+x^2)\sqrt{\frac{2+3x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+5x^2+3x^4}} \\ &- \frac{1656\sqrt{3}(1+x^2)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{\frac{1+x^2}{2+3x^2}}\sqrt{2+5x^2+3x^4}}\end{aligned}$$

output
```
-x*(53*x^2+49)/(2*x^2+1)/(3*x^4+5*x^2+2)^(3/2)+152*x/(2*x^2+1)/(3*x^4+5*x^
2+2)^(1/2)-967/2*x*(3*x^2+2)/(3*x^4+5*x^2+2)^(1/2)+1/2*x*(2901*x^2+3533)/(
3*x^4+5*x^2+2)^(1/2)+967/2*2^(1/2)*(x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*Ellip
ticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))/(3*x^4+5*x^2+2)^(1/2)+3075/2*2^(1/2)*(
x^2+1)*((3*x^2+2)/(x^2+1))^(1/2)*InverseJacobiAM(arctan(x),1/2*I*2^(1/2))/
(3*x^4+5*x^2+2)^(1/2)-1656*(x^2+1)*EllipticPi(x*6^(1/2)/(6*x^2+4)^(1/2),-1
/3,1/3*3^(1/2))*3^(1/2)/((x^2+1)/(3*x^2+2))^(1/2)/(3*x^4+5*x^2+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.50 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.92

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{5/2}}\,dx = \frac{262x+270x^3+2665x(2+5x^2+3x^4)+2157x^3(2+5x^2+3x^4)+\cdots}{}$$

input
```
Integrate[(4 - 7*x^2 + x^4)/((1 + 2*x^2)^2*(2 + 5*x^2 + 3*x^4)^(5/2)),x]
```

output
```
(262*x + 270*x^3 + 2665*x*(2 + 5*x^2 + 3*x^4) + 2157*x^3*(2 + 5*x^2 + 3*x^
4) + (496*x*(2 + 5*x^2 + 3*x^4)^2)/(1 + 2*x^2) + (967*I)*Sqrt[3]*Sqrt[1 +
x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticE[I*ArcSinh[Sqrt[3/2]*x],
 2/3] - (533*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*
EllipticF[I*ArcSinh[Sqrt[3/2]*x], 2/3] + (828*I)*Sqrt[3]*Sqrt[1 + x^2]*Sqr
t[2 + 3*x^2]*(2 + 5*x^2 + 3*x^4)*EllipticPi[4/3, I*ArcSinh[Sqrt[3/2]*x], 2
/3])/(2*(2 + 5*x^2 + 3*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2258, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-7x^2+4}{(2x^2+1)^2(3x^4+5x^2+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2258}$$

$$\int \left(\frac{111}{(x^2+1)\sqrt{3x^4+5x^2+2}}-\frac{1056}{(2x^2+1)\sqrt{3x^4+5x^2+2}}+\frac{1251}{(3x^2+2)\sqrt{3x^4+5x^2+2}}+\frac{12}{(x^2+1)^2\sqrt{3x^4+5x^2}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1770\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{3x^4+5x^2+2}}+\frac{42\sqrt{2}(3x^2+2)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}-\frac{549(3x^2+2)\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}+\frac{124\sqrt{2}(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{3x^4+5x^2+2}}-\frac{266\sqrt{2}(3x^2+2)\,E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}+\frac{1251(3x^2+2)\,E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{\sqrt{2}\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{3x^4+5x^2+2}}-\frac{1656\sqrt{3}(x^2+1)\,\text{EllipticPi}\left(-\frac{1}{3},\arctan\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{3}\right)}{\sqrt{\frac{x^2+1}{3x^2+2}}\sqrt{3x^4+5x^2+2}}+\frac{248\sqrt{3x^4+5x^2+2}x}{2x^2+1}+\frac{3015(x^2+1)x}{2\sqrt{3x^4+5x^2+2}}-\frac{4(3x^2+2)x}{(x^2+1)\sqrt{3x^4+5x^2+2}}-\frac{1253(3x^2+2)x}{2\sqrt{3x^4+5x^2+2}}+\frac{369(x^2+1)x}{(3x^2+2)\sqrt{3x^4+5x^2+2}}$$

input
```
Int[(4 - 7*x^2 + x^4)/((1 + 2*x^2)^2*(2 + 5*x^2 + 3*x^4)^(5/2)),x]
```

output
```
(3015*x*(1 + x^2))/(2*Sqrt[2 + 5*x^2 + 3*x^4]) + (369*x*(1 + x^2))/((2 + 3
*x^2)*Sqrt[2 + 5*x^2 + 3*x^4]) - (1253*x*(2 + 3*x^2))/(2*Sqrt[2 + 5*x^2 +
3*x^4]) - (4*x*(2 + 3*x^2))/((1 + x^2)*Sqrt[2 + 5*x^2 + 3*x^4]) + (248*x*S
qrt[2 + 5*x^2 + 3*x^4])/(1 + 2*x^2) + (1251*(2 + 3*x^2)*EllipticE[ArcTan[x
], -1/2])/(Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]*Sqrt[2 + 5*x^2 + 3*x^4]) -
(266*Sqrt[2]*(2 + 3*x^2)*EllipticE[ArcTan[x], -1/2])/(Sqrt[(2 + 3*x^2)/(1
+ x^2)]*Sqrt[2 + 5*x^2 + 3*x^4]) + (124*Sqrt[2]*(1 + x^2)*Sqrt[(2 + 3*x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], -1/2])/Sqrt[2 + 5*x^2 + 3*x^4] - (549*(2
+ 3*x^2)*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]*
Sqrt[2 + 5*x^2 + 3*x^4]) + (42*Sqrt[2]*(2 + 3*x^2)*EllipticF[ArcTan[x], -1
/2])/(Sqrt[(2 + 3*x^2)/(1 + x^2)]*Sqrt[2 + 5*x^2 + 3*x^4]) + (1770*Sqrt[2]
*(1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2])/Sqrt[2
+ 5*x^2 + 3*x^4] - (1656*Sqrt[3]*(1 + x^2)*EllipticPi[-1/3, ArcTan[Sqrt[3/
2]*x], 1/3])/(Sqrt[(1 + x^2)/(2 + 3*x^2)]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2258
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 5.85 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.68

method	result
risch	$\frac{x(17406x^8+58911x^6+72950x^4+39013x^2+7576)}{2(2x^2+1)(3x^4+5x^2+2)^{\frac{3}{2}}}+\frac{651i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{967i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \left(\text{EllipticF}\cdots\right.}{2\sqrt{3x^4+\cdots}}$
elliptic	$\frac{744x\sqrt{3x^4+5x^2+2}}{6x^2+3}+\frac{\left(15x^3+\frac{131}{9}x\right)\sqrt{3x^4+5x^2+2}}{\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{6\left(-\frac{719}{4}x^3-\frac{2665}{12}x\right)}{\sqrt{3x^4+5x^2+2}}-\frac{158i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{\sqrt{3x^4+5x^2+2}}+\frac{967i\cdots}{\cdots}$
default	$\frac{\left(\frac{5}{18}x^3+\frac{13}{54}x\right)\sqrt{3x^4+5x^2+2}}{4\left(x^4+\frac{5}{3}x^2+\frac{2}{3}\right)^2}-\frac{3\left(\frac{115}{12}x^3+\frac{145}{18}x\right)}{2\sqrt{3x^4+5x^2+2}}-\frac{3907i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \text{EllipticF}\left(ix,\frac{\sqrt{6}}{2}\right)}{24\sqrt{3x^4+5x^2+2}}+\frac{115i\sqrt{x^2+1}\,\sqrt{6x^2+4}\ \left(\text{EllipticF}\cdots\right.}{24\sqrt{3x^4\cdots}}$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
1/2*x*(17406*x^8+58911*x^6+72950*x^4+39013*x^2+7576)/(2*x^2+1)/(3*x^4+5*x^
2+2)^(3/2)+651/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*Ell
ipticF(I*x,1/2*6^(1/2))-967/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2
+2)^(1/2)*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))+1242*I*(
x^2+1)^(1/2)*(1+3/2*x^2)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*EllipticPi(I*x,2,1/2*
I*(-3)^(1/2)*2^(1/2))
```

Fricas [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(3*x^4 + 5*x^2 + 2)*(x^4 - 7*x^2 + 4)/(108*x^16 + 648*x^14 +
1683*x^12 + 2471*x^10 + 2243*x^8 + 1289*x^6 + 458*x^4 + 92*x^2 + 8), x)
```

Sympy [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7x^2+4}{((x^2+1)(3x^2+2))^{\frac{5}{2}}(2x^2+1)^2}\,dx$$

input
```
integrate((x**4-7*x**2+4)/(2*x**2+1)**2/(3*x**4+5*x**2+2)**(5/2),x)
```

output
```
Integral((x**4 - 7*x**2 + 4)/(((x**2 + 1)*(3*x**2 + 2))**(5/2)*(2*x**2 + 1
)**2), x)
```

Maxima [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((x^4 - 7*x^2 + 4)/((3*x^4 + 5*x^2 + 2)^(5/2)*(2*x^2 + 1)^2), x)
```

Giac [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(3\,x^4+5\,x^2+2)^{\frac{5}{2}}(2\,x^2+1)^2}\,dx$$

input
```
integrate((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((x^4 - 7*x^2 + 4)/((3*x^4 + 5*x^2 + 2)^(5/2)*(2*x^2 + 1)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{5/2}}\,dx = \int \frac{x^4-7\,x^2+4}{(2\,x^2+1)^2\,(3\,x^4+5\,x^2+2)^{5/2}}\,dx$$

input
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)^2*(5*x^2 + 3*x^4 + 2)^(5/2)),x)
```

output
```
int((x^4 - 7*x^2 + 4)/((2*x^2 + 1)^2*(5*x^2 + 3*x^4 + 2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{4-7x^2+x^4}{(1+2x^2)^2(2+5x^2+3x^4)^{5/2}}\,dx=4\left(\int \frac{\sqrt{3x^4+5x^2+2}}{108x^{16}+648x^{14}+1683x^{12}+2471x^{10}+2243x^8+1289x^6+4}\right.$$

$$+\int \frac{\sqrt{3x^4+5x^2+2}\,x^4}{108x^{16}+648x^{14}+1683x^{12}+2471x^{10}+2243x^8+1289x^6+458x^4+92x^2+8}dx$$

$$\left.-7\left(\int \frac{\sqrt{3x^4+5x^2+2}\,x^2}{108x^{16}+648x^{14}+1683x^{12}+2471x^{10}+2243x^8+1289x^6+458x^4+92x^2+8}dx\right)\right)$$

input
```
int((x^4-7*x^2+4)/(2*x^2+1)^2/(3*x^4+5*x^2+2)^(5/2),x)
```

output
```
4*int(sqrt(3*x**4 + 5*x**2 + 2)/(108*x**16 + 648*x**14 + 1683*x**12 + 2471
*x**10 + 2243*x**8 + 1289*x**6 + 458*x**4 + 92*x**2 + 8),x) + int((sqrt(3*
x**4 + 5*x**2 + 2)*x**4)/(108*x**16 + 648*x**14 + 1683*x**12 + 2471*x**10
+ 2243*x**8 + 1289*x**6 + 458*x**4 + 92*x**2 + 8),x) - 7*int((sqrt(3*x**4
+ 5*x**2 + 2)*x**2)/(108*x**16 + 648*x**14 + 1683*x**12 + 2471*x**10 + 224
3*x**8 + 1289*x**6 + 458*x**4 + 92*x**2 + 8),x)
```

3.224 $\int (A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1241

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

output
```
1/384*(8*A*c*e*(3*c^2*d^2-3*b^2*e^2+8*c*e*(a*e+b*d))-B*(9*c^3*d^3-15*b^3*e
^3-3*c^2*d*e*(28*a*e+3*b*d)+b*c*e^2*(52*a*e+31*b*d)))*(e*x^2+d)^(1/2)*(c*x
^4+b*x^2+a)^(1/2)/c^3/e^2/x+1/192*(8*A*c*e*(b*e+7*c*d)+B*(3*c^2*d^2-5*b^2*
e^2+2*c*e*(6*a*e+5*b*d)))*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e+1/
48*(8*A*c*e+B*b*e+9*B*c*d)*x^3*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c+1/8
*B*e*x^5*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)-1/768*(-4*a*c+b^2)^(1/2)*(8
*A*c*e*(3*c^2*d^2-3*b^2*e^2+8*c*e*(a*e+b*d))-B*(9*c^3*d^3-15*b^3*e^3-3*c^2
*d*e*(28*a*e+3*b*d)+b*c*e^2*(52*a*e+31*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+
b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d
-2*a*e))^(1/2))*2^(1/2)/c^3/e^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*
a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/384*(-4*a*c+b^2)^(1/2)*(8*A*c*e*(31*c^
2*d^2+3*b^2*e^2-2*c*e*(4*a*e+5*b*d))+B*(3*c^3*d^3-15*b^3*e^3+b*c*e^2*(52*a
*e+41*b*d)-c^2*d*e*(108*a*e+29*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1
/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/
2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(
1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^3/e/(e*x^2+d)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)-1/64*(-4*a*c+b^2)^(1/2)*(8*A*c*e*(c^3*d^3-b^3*e^
3-3*c^2*d*e*(4*a*e+b*d)+b*c*e^2*(4*a*e+3*b*d))-B*(3*c^4*d^4-5*b^4*e^4-4*c^
3*d^2*e*(-6*a*e+b*d)+12*b^2*c*e^3*(2*a*e+b*d)-2*c^2*e^2*(8*a^2*e^2+24*a...
```

Mathematica [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
Integrate[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
Integrate[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2) \left(d+ex^2\right)^{3/2} \sqrt{a+bx^2+cx^4} \, dx$$
$$\Big\downarrow \text{ 2260}$$
$$\int (A+Bx^2) \left(d+ex^2\right)^{3/2} \sqrt{a+bx^2+cx^4} dx$$

input
```
Int[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right) \left(e\,x^{2}+d\right)^{\frac{3}{2}} \sqrt{c\,x^{4}+b\,x^{2}+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \left(A+Bx^2\right)\left(d+ex^2\right)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}\left(Bx^2+A\right)\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}\left(Bx^2+A\right)\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \left(B\,x^2+A\right)\,\left(e\,x^2+d\right)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
(20*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*c*e**2*x + 56*sqrt(d +
e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*c**2*d*e*x + 32*sqrt(d + e*x**2)*sqrt(
a + b*x**2 + c*x**4)*a*c**2*e**2*x**3 - 5*sqrt(d + e*x**2)*sqrt(a + b*x**2
 + c*x**4)*b**3*e**2*x + 10*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**
2*c*d*e*x + 4*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*c*e**2*x**3
+ 3*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c**2*d**2*x + 36*sqrt(d +
 e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c**2*d*e*x**3 + 24*sqrt(d + e*x**2)*s
qrt(a + b*x**2 + c*x**4)*b*c**2*e**2*x**5 + 64*int((sqrt(d + e*x**2)*sqrt(
a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**
4 + c*e*x**6),x)*a**2*c**2*e**3 - 76*int((sqrt(d + e*x**2)*sqrt(a + b*x**2
 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x*
*6),x)*a*b**2*c*e**3 + 148*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)
*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*b
*c**2*d*e**2 + 24*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a
*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*c**3*d**2*
e + 15*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x*
*2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b**4*e**3 - 31*int((sqr
t(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 +
 b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b**3*c*d*e**2 + 9*int((sqrt(d + e*x**2
)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4...
```

3.225 $\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 964

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{(6Ace(cd+be)-B(3c^2d^2+3b^2e^2-2ce(bd+4ae)))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{48c^2e^2x}$$
$$+ \frac{(Bcd+bBe+6Ace)x\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{24ce} + \frac{1}{6}Bx^3\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}$$
$$- \frac{\sqrt{b^2-4ac}(6Ace(cd+be)-B(3c^2d^2+3b^2e^2-2ce(bd+4ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}x\sqrt{d+ex^2}E\left(\arcsin\left(\right.\right.}{48\sqrt{2}c^2e^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$
$$- \frac{\sqrt{b^2-4ac}(6Ace(5cd-be)+B(c^2d^2+3b^2e^2-4ce(bd+2ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3}{24\sqrt{2}c^2e\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$
$$+ \frac{\sqrt{b^2-4ac}(B(cd-be)(c^2d^2-b^2e^2+4ace^2)-2Ace(c^2d^2+b^2e^2-2ce(bd+2ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{}}{4\sqrt{2}c^2\left(b+\sqrt{b^2-4ac}\right)e^2\sqrt{d+ex^2}}$$

output
```
1/48*(6*A*c*e*(b*e+c*d)-B*(3*c^2*d^2+3*b^2*e^2-2*c*e*(4*a*e+b*d)))*(e*x^2+
d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e^2/x+1/24*(6*A*c*e+B*b*e+B*c*d)*x*(e*x
^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e+1/6*B*x^3*(e*x^2+d)^(1/2)*(c*x^4+b*x
^2+a)^(1/2)-1/96*(-4*a*c+b^2)^(1/2)*(6*A*c*e*(b*e+c*d)-B*(3*c^2*d^2+3*b^2*
e^2-2*c*e*(4*a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d
)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(
1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2
)/c^2/e^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x
^2+a)^(1/2)-1/48*(-4*a*c+b^2)^(1/2)*(6*A*c*e*(-b*e+5*c*d)+B*(c^2*d^2+3*b^2
*e^2-4*c*e*(2*a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/
x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x
^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(
-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^2/e/(e*x^2+d)^(1/2)/(c*x^4+b*
x^2+a)^(1/2)+1/8*(-4*a*c+b^2)^(1/2)*(B*(-b*e+c*d)*(4*a*c*e^2-b^2*e^2+c^2*d
^2)-2*A*c*e*(c^2*d^2+b^2*e^2-2*c*e*(2*a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a
*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*E
llipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b
^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*
c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^2/(b+(-4*a*c+b^2)^(1/2))/e^2/(e*x^
2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx=\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 2260}$$

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}dx$$

input
```
Int[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \left(B\,x^2+A\right)\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx=\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx = \int (B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
(6*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*c*e*x + sqrt(d + e*x**2)*s
qrt(a + b*x**2 + c*x**4)*b**2*e*x + sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x
**4)*b*c*d*x + 4*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c*e*x**3 + 1
4*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 +
b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*b*c*e**2 + 6*int((sqrt(d +
 e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*
x**4 + c*d*x**4 + c*e*x**6),x)*a*c**2*d*e - 3*int((sqrt(d + e*x**2)*sqrt(a
 + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4
 + c*e*x**6),x)*b**3*e**2 + 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x*
*4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*
b**2*c*d*e - 3*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d
+ a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b*c**2*d**2 + 1
2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 +
b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a**2*c*e**2 - 2*int((sqrt(d
+ e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e
*x**4 + c*d*x**4 + c*e*x**6),x)*a*b**2*e**2 + 22*int((sqrt(d + e*x**2)*sqr
t(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x
**4 + c*e*x**6),x)*a*b*c*d*e - 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c
*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),
x)*b**3*d*e - 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(...
```

3.226 $\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 805

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{(3Bcd-bBe-4Ace)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{8ce^2x} + \frac{Bx\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{4e}$$

$$+ \frac{\sqrt{b^2-4ac}(3Bcd-bBe-4Ace)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right)}{8\sqrt{2}ce^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{\sqrt{b^2-4ac}(Bcd-bBe+4Ace)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{4\sqrt{2}ce\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{\sqrt{b^2-4ac}(4Ace(cd-be)-B(3c^2d^2-b^2e^2-2ce(bd-2ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3}{2\sqrt{2}c\left(b+\sqrt{b^2-4ac}\right)e^2\sqrt{d+ex^2}\sqrt{a+bx^2}}$$

output
```
-1/8*(-4*A*c*e-B*b*e+3*B*c*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e^2/
x+1/4*B*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/e+1/16*(-4*a*c+b^2)^(1/2)*
(-4*A*c*e-B*b*e+3*B*c*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+
d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^
(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/
2)/c/e^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^
2+a)^(1/2)-1/8*(-4*a*c+b^2)^(1/2)*(4*A*c*e-B*b*e+B*c*d)*(-a*(c+a/x^4+b/x^2
)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2
)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/
2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/
c/e/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/4*(-4*a*c+b^2)^(1/2)*(4*A*c*e*
(-b*e+c*d)-B*(3*c^2*d^2-b^2*e^2-2*c*e*(-2*a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(
-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x
^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a
*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-
4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c/(b+(-4*a*c+b^2)^(1/2))/e^2/(e*
x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[d + e*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx$$

$$\downarrow \text{2260}$$

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{c\,x^4+b\,x^2+a}}{\sqrt{e\,x^2+d}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a + b*x**2 + c*x**4)/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2+a}}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2)^(1/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,bx+4\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,x^4}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)ace+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,x^4}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+}\right.}{}$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*x + 4*int((sqrt(d + e*x**2)*
sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*
d*x**4 + c*e*x**6),x)*a*c*e + int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x*
*4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*
b**2*e - 3*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*
e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b*c*d + 6*int((sqrt
(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 +
b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*b*e - 2*int((sqrt(d + e*x**2)*sqrt(a
+ b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4
+ c*e*x**6),x)*b**2*d + 4*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))
/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a**2*e -
int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a*d + a*e*x**2 + b*d*x**
2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*b*d)/(4*e)
```

3.227 $\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 754

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}\,dx = -\frac{(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{de\sqrt{d+ex^2}} + \frac{(3Bd-2Ae)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{2de^2x} - \frac{\sqrt{b^2-4ac}(3Bd-2Ae)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right)}{2\sqrt{2}de^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt{b^2-4ac}(Bd-2Ae)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right)}{\sqrt{2}de\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{2}\sqrt{b^2-4ac}(3Bcd-bBe-2Ace)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\operatorname{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\cdots\right.\right.}{\left(b+\sqrt{b^2-4ac}\right)e^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
-(-A*e+B*d)*x*(c*x^4+b*x^2+a)^(1/2)/d/e/(e*x^2+d)^(1/2)+1/2*(-2*A*e+3*B*d)
*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/d/e^2/x-1/4*(-4*a*c+b^2)^(1/2)*(-2*
A*e+3*B*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*Ellip
ticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c
+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/d/e^2/(-a*(
e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/2
*(-4*a*c+b^2)^(1/2)*(-2*A*e+B*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(
-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+
(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*
d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/d/e/(e*x^2+d)^(1/2)/(c*
x^4+b*x^2+a)^(1/2)-2^(1/2)*(-4*a*c+b^2)^(1/2)*(-2*A*c*e-B*b*e+3*B*c*d)*(-a
*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))
*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/
2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^
2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/(b+(-4*a*c+b^2)^(1/2))
/e^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2260$$

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{c\,x^4+b\,x^2+a}}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*
d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d)**(3/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2+a}}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2)^(3/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{3/2}}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output

```
(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*c*x + sqrt(d + e*x**2)*sqrt(
a + b*x**2 + c*x**4)*b**2*x - 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4)*x**6)/(a*b*d**2*e + 2*a*b*d*e**2*x**2 + a*b*e**3*x**4 + 3*a*c*d**3 +
 6*a*c*d**2*e*x**2 + 3*a*c*d*e**2*x**4 + b**2*d**2*e*x**2 + 2*b**2*d*e**2*
x**4 + b**2*e**3*x**6 + 3*b*c*d**3*x**2 + 7*b*c*d**2*e*x**4 + 5*b*c*d*e**2
*x**6 + b*c*e**3*x**8 + 3*c**2*d**3*x**4 + 6*c**2*d**2*e*x**6 + 3*c**2*d*e
**2*x**8),x)*a*b*c**2*d*e**2 - 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c
*x**4)*x**6)/(a*b*d**2*e + 2*a*b*d*e**2*x**2 + a*b*e**3*x**4 + 3*a*c*d**3
+ 6*a*c*d**2*e*x**2 + 3*a*c*d*e**2*x**4 + b**2*d**2*e*x**2 + 2*b**2*d*e**2
*x**4 + b**2*e**3*x**6 + 3*b*c*d**3*x**2 + 7*b*c*d**2*e*x**4 + 5*b*c*d*e**
2*x**6 + b*c*e**3*x**8 + 3*c**2*d**3*x**4 + 6*c**2*d**2*e*x**6 + 3*c**2*d*
e**2*x**8),x)*a*b*c**2*e**3*x**2 - 6*int((sqrt(d + e*x**2)*sqrt(a + b*x**2
 + c*x**4)*x**6)/(a*b*d**2*e + 2*a*b*d*e**2*x**2 + a*b*e**3*x**4 + 3*a*c*d
**3 + 6*a*c*d**2*e*x**2 + 3*a*c*d*e**2*x**4 + b**2*d**2*e*x**2 + 2*b**2*d*
e**2*x**4 + b**2*e**3*x**6 + 3*b*c*d**3*x**2 + 7*b*c*d**2*e*x**4 + 5*b*c*d
*e**2*x**6 + b*c*e**3*x**8 + 3*c**2*d**3*x**4 + 6*c**2*d**2*e*x**6 + 3*c**
2*d*e**2*x**8),x)*a*c**3*d**2*e - 6*int((sqrt(d + e*x**2)*sqrt(a + b*x**2
+ c*x**4)*x**6)/(a*b*d**2*e + 2*a*b*d*e**2*x**2 + a*b*e**3*x**4 + 3*a*c*d*
*3 + 6*a*c*d**2*e*x**2 + 3*a*c*d*e**2*x**4 + b**2*d**2*e*x**2 + 2*b**2*d*e
**2*x**4 + b**2*e**3*x**6 + 3*b*c*d**3*x**2 + 7*b*c*d**2*e*x**4 + 5*b*c...
```

3.228 $\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 837

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}\,dx = -\frac{(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{3de(d+ex^2)^{3/2}}$$
$$+\frac{(Ae^2(bd-2ae)-Bd(3cd^2-e(2bd-ae)))\sqrt{a+bx^2+cx^4}}{3de^2(cd^2-bde+ae^2)x\sqrt{d+ex^2}}$$
$$-\frac{\sqrt{b^2-4ac}(Ae^2(bd-2ae)-Bd(3cd^2-e(2bd-ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3\sqrt{2}d^2e^2(cd^2-bde+ae^2)\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(Bd+2Ae)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{}}{bd+\sqrt{}}\right.}{3d^2e\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{2\sqrt{2}Bc\sqrt{b^2-4ac}\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{}{bd}\right.}{\left(b+\sqrt{b^2-4ac}\right)e^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/3*(-A*e+B*d)*x*(c*x^4+b*x^2+a)^(1/2)/d/e/(e*x^2+d)^(3/2)+1/3*(A*e^2*(-2
*a*e+b*d)-B*d*(3*c*d^2-e*(-a*e+2*b*d)))*(c*x^4+b*x^2+a)^(1/2)/d/e^2/(a*e^2
-b*d*e+c*d^2)/x/(e*x^2+d)^(1/2)-1/6*(-4*a*c+b^2)^(1/2)*(A*e^2*(-2*a*e+b*d)
-B*d*(3*c*d^2-e*(-a*e+2*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(
e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))
*2^(1/2)/d^2/e^2/(a*e^2-b*d*e+c*d^2)/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))
*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/3*2^(1/2)*(-4*a*c+b^2)^(1/2)*(2*A
*e+B*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+
b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)
*d-2*a*e))^(1/2))/d^2/e/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+2*2^(1/2)*B*
c*(-4*a*c+b^2)^(1/2)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)
/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)
/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1
/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2)
)/(b+(-4*a*c+b^2)^(1/2))/e^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}\,dx$$

$$\downarrow 2260$$

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{c\,x^4+b\,x^2+a}}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(5/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d)**(5/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2+a}}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2)^(5/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{5/2}}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(5/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*c*x - sqrt(d + e*x**2)*sq
rt(a + b*x**2 + c*x**4)*b**2*x + int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c
*x**4)*x**6)/(a*b*d**3*e + 3*a*b*d**2*e**2*x**2 + 3*a*b*d*e**3*x**4 + a*b*
e**4*x**6 - 3*a*c*d**4 - 9*a*c*d**3*e*x**2 - 9*a*c*d**2*e**2*x**4 - 3*a*c*
d*e**3*x**6 + b**2*d**3*e*x**2 + 3*b**2*d**2*e**2*x**4 + 3*b**2*d*e**3*x**
6 + b**2*e**4*x**8 - 3*b*c*d**4*x**2 - 8*b*c*d**3*e*x**4 - 6*b*c*d**2*e**2
*x**6 + b*c*e**4*x**10 - 3*c**2*d**4*x**4 - 9*c**2*d**3*e*x**6 - 9*c**2*d*
*2*e**2*x**8 - 3*c**2*d*e**3*x**10),x)*b**3*c*d**2*e**2 + 2*int((sqrt(d +
e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**6)/(a*b*d**3*e + 3*a*b*d**2*e**2*x**2
 + 3*a*b*d*e**3*x**4 + a*b*e**4*x**6 - 3*a*c*d**4 - 9*a*c*d**3*e*x**2 - 9*
a*c*d**2*e**2*x**4 - 3*a*c*d*e**3*x**6 + b**2*d**3*e*x**2 + 3*b**2*d**2*e*
*2*x**4 + 3*b**2*d*e**3*x**6 + b**2*e**4*x**8 - 3*b*c*d**4*x**2 - 8*b*c*d*
*3*e*x**4 - 6*b*c*d**2*e**2*x**6 + b*c*e**4*x**10 - 3*c**2*d**4*x**4 - 9*c
**2*d**3*e*x**6 - 9*c**2*d**2*e**2*x**8 - 3*c**2*d*e**3*x**10),x)*b**3*c*d
*e**3*x**2 + int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**6)/(a*b*d*
*3*e + 3*a*b*d**2*e**2*x**2 + 3*a*b*d*e**3*x**4 + a*b*e**4*x**6 - 3*a*c*d*
*4 - 9*a*c*d**3*e*x**2 - 9*a*c*d**2*e**2*x**4 - 3*a*c*d*e**3*x**6 + b**2*d
**3*e*x**2 + 3*b**2*d**2*e**2*x**4 + 3*b**2*d*e**3*x**6 + b**2*e**4*x**8 -
 3*b*c*d**4*x**2 - 8*b*c*d**3*e*x**4 - 6*b*c*d**2*e**2*x**6 + b*c*e**4*x**
10 - 3*c**2*d**4*x**4 - 9*c**2*d**3*e*x**6 - 9*c**2*d**2*e**2*x**8 - 3*...
```

3.229 $\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 810

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}\,dx = -\frac{(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{5de(d+ex^2)^{5/2}}$$

$$+\frac{(Ae(2cd^2-e(3bd-4ae))+Bd(3cd^2-e(2bd-ae)))\,x\sqrt{a+bx^2+cx^4}}{15d^2e(cd^2-bde+ae^2)(d+ex^2)^{3/2}}$$

$$-\frac{(b^2d^2(2Bd+3Ae)-bd(5Acd^2+2aBde+13aAe^2)-2a(3Bcd^3-8Acd^2e-aBde^2-4aAe^3))\sqrt{a+bx^2}}{15d^2(cd^2-bde+ae^2)^2\,x\sqrt{d+ex^2}}$$

$$+\frac{\sqrt{b^2-4ac}(b^2d^2(2Bd+3Ae)-bd(5Acd^2+2aBde+13aAe^2)-2a(3Bcd^3-8Acd^2e-aBde^2-4aAe^3))}{15\sqrt{2}d^3(cd^2-bde+ae^2)^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(bBd^2-10Acd^2+9Abde-2aBde-8aAe^2)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{Ellipti}}{15d^3(cd^2-bde+ae^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/5*(-A*e+B*d)*x*(c*x^4+b*x^2+a)^(1/2)/d/e/(e*x^2+d)^(5/2)+1/15*(A*e*(2*c
*d^2-e*(-4*a*e+3*b*d))+B*d*(3*c*d^2-e*(-a*e+2*b*d)))*x*(c*x^4+b*x^2+a)^(1/
2)/d^2/e/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(3/2)-1/15*(b^2*d^2*(3*A*e+2*B*d)-b
*d*(13*A*a*e^2+5*A*c*d^2+2*B*a*d*e)-2*a*(-4*A*a*e^3-8*A*c*d^2*e-B*a*d*e^2+
3*B*c*d^3))*(c*x^4+b*x^2+a)^(1/2)/d^2/(a*e^2-b*d*e+c*d^2)^2/x/(e*x^2+d)^(1
/2)+1/30*(-4*a*c+b^2)^(1/2)*(b^2*d^2*(3*A*e+2*B*d)-b*d*(13*A*a*e^2+5*A*c*d
^2+2*B*a*d*e)-2*a*(-4*A*a*e^3-8*A*c*d^2*e-B*a*d*e^2+3*B*c*d^3))*(-a*(c+a/x
^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^
2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-
4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/d^3/(a*e^2-b*d*e+c*d^2)^2/(-a*(e
+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/15
*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-8*A*a*e^2+9*A*b*d*e-10*A*c*d^2-2*B*a*d*e+B*b
*d^2)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^
2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d
-2*a*e))^(1/2))/d^3/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(7/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}\,dx$$

$$\Big\downarrow 2260$$

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{c\,x^4+b\,x^2+a}}{(e\,x^2+d)^{\frac{7}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(7/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(7/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^4*x^8 + 4*
d*e^3*x^6 + 6*d^2*e^2*x^4 + 4*d^3*e*x^2 + d^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(e*x^2 + d)^(7/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(e*x^2 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2+a}}{(e\,x^2+d)^{7/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2)^(7/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{7/2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2+a}}{(e\,x^2+d)^{\frac{7}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(7/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(7/2),x)
```

3.230 $\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1240

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}\,dx = \text{Too large to display}$$

output
```
-1/7*(-A*e+B*d)*x*(c*x^4+b*x^2+a)^(1/2)/d/e/(e*x^2+d)^(7/2)+1/35*(A*e*(4*c
*d^2-e*(-6*a*e+5*b*d))+B*d*(3*c*d^2-e*(-a*e+2*b*d)))*x*(c*x^4+b*x^2+a)^(1/
2)/d^2/e/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(5/2)+1/105*(2*B*d*(3*c^2*d^4-c*d^2
*e*(3*a*e+4*b*d)+e^2*(2*a^2*e^2-3*a*b*d*e+3*b^2*d^2))+A*e*(8*c^2*d^4-3*c*d
^2*e*(-16*a*e+9*b*d)+e^2*(24*a^2*e^2-43*a*b*d*e+15*b^2*d^2)))*x*(c*x^4+b*x
^2+a)^(1/2)/d^3/e/(a*e^2-b*d*e+c*d^2)^2/(e*x^2+d)^(3/2)+1/105*(3*b^3*d^3*e
*(5*A*e+2*B*d)-b^2*d^2*(103*A*a*e^3+42*A*c*d^2*e+9*B*a*d*e^2+14*B*c*d^3)+b
*d*(a*B*d*e*(19*a*e^2+c*d^2)+A*(128*a^2*e^4+237*a*c*d^2*e^2+35*c^2*d^4))-2
*a*(A*e*(24*a^2*e^4+69*a*c*d^2*e^2+77*c^2*d^4)-B*(-4*a^2*d*e^4-15*a*c*d^3*
e^2+21*c^2*d^5)))*(c*x^4+b*x^2+a)^(1/2)/d^3/(a*e^2-b*d*e+c*d^2)^3/x/(e*x^2
+d)^(1/2)-1/210*(-4*a*c+b^2)^(1/2)*(3*b^3*d^3*e*(5*A*e+2*B*d)-b^2*d^2*(103
*A*a*e^3+42*A*c*d^2*e+9*B*a*d*e^2+14*B*c*d^3)+b*d*(a*B*d*e*(19*a*e^2+c*d^2
)+A*(128*a^2*e^4+237*a*c*d^2*e^2+35*c^2*d^4))-2*a*(A*e*(24*a^2*e^4+69*a*c*
d^2*e^2+77*c^2*d^4)-B*(-4*a^2*d*e^4-15*a*c*d^3*e^2+21*c^2*d^5)))*(-a*(c+a/
x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x
^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(
-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/d^4/(a*e^2-b*d*e+c*d^2)^3/(-a*(
e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/1
05*2^(1/2)*(-4*a*c+b^2)^(1/2)*(3*b^2*d^2*e*(20*A*e+B*d)+8*a*B*d*e*(a*e^2+3
*c*d^2)-b*d*(104*A*a*e^3+126*A*c*d^2*e+15*B*a*d*e^2+7*B*c*d^3)+2*A*(24*...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(9/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}\,dx$$
$$\Big\downarrow \; 2260$$
$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{c\,x^4+b\,x^2+a}}{(e\,x^2+d)^{\frac{9}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(9/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(9/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(9/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^5*x^10 + 5
*d*e^4*x^8 + 10*d^2*e^3*x^6 + 10*d^3*e^2*x^4 + 5*d^4*e*x^2 + d^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(9/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(e*x^2 + d)^(9/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(9/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(e*x^2 + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2+a}}{(e\,x^2+d)^{9/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2)^(9/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a+bx^2+cx^4}}{(d+ex^2)^{9/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2+a}}{(e\,x^2+d)^{\frac{9}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(9/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)^(9/2),x)
```

3.231 $\int (A+Bx^2)\sqrt{d+ex^2}(a+bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1564

$$\int (A+Bx^2)\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{3/2}\,dx = \text{Too large to display}$$

output
```
1/3840*(10*A*c*e*(15*c^3*d^3-9*b^3*e^3-c^2*d*e*(-68*a*e+31*b*d)+3*b*c*e^2*
(20*a*e+3*b*d))-B*(105*c^4*d^4-45*b^4*e^4-2*c^3*d^2*e*(-166*a*e+95*b*d)+30
*b^2*c*e^3*(10*a*e+b*d)+12*c^2*e^2*(-32*a^2*e^2-18*a*b*d*e+3*b^2*d^2)))*(e
*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^3/e^4/x-1/1920*(10*A*c*e*(5*c^2*d^2-
3*b^2*e^2-10*c*e*(6*a*e+b*d))-B*(35*c^3*d^3-15*b^3*e^3-c^2*d*e*(-108*a*e+6
1*b*d)+3*b*c*e^2*(28*a*e+3*b*d)))*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/
c^2/e^3+1/480*(10*A*c*e*(9*b*e+c*d)-B*(7*c^2*d^2-3*b^2*e^2-12*c*e*(8*a*e+b
*d)))*x^3*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e^2+1/80*(10*A*c*e+11*B*
b*e+B*c*d)*x^5*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/e+1/10*B*c*x^7*(e*x^2
+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)-1/7680*(-4*a*c+b^2)^(1/2)*(10*A*c*e*(15*c^
3*d^3-9*b^3*e^3-c^2*d*e*(-68*a*e+31*b*d)+3*b*c*e^2*(20*a*e+3*b*d))-B*(105*
c^4*d^4-45*b^4*e^4-2*c^3*d^2*e*(-166*a*e+95*b*d)+30*b^2*c*e^3*(10*a*e+b*d)
+12*c^2*e^2*(-32*a^2*e^2-18*a*b*d*e+3*b^2*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a
*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2
)*d-2*a*e))^(1/2))*2^(1/2)/c^3/e^4/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d
-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/3840*(-4*a*c+b^2)^(1/2)*(10*A*c*e*(
5*c^3*d^3-9*b^3*e^3+15*b*c*e^2*(4*a*e+b*d)-c^2*d*e*(196*a*e+11*b*d))-B*(35
*c^4*d^4-45*b^4*e^4-4*c^3*d^2*e*(-29*a*e+17*b*d)+60*b^2*c*e^3*(5*a*e+b*d)+
6*c^2*e^2*(-64*a^2*e^2-64*a*b*d*e+3*b^2*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*...
```

Mathematica [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}(a+bx^2+cx^4)^{3/2}\,dx=\int (A+Bx^2)\sqrt{d+ex^2}(a+bx^2+cx^4)^{3/2}\,dx$$

input
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2\right) \sqrt{d+ex^2} \left(a+bx^2+cx^4\right)^{3/2} \, dx$$
$$\Big\downarrow \ 2260$$
$$\int \left(A+Bx^2\right) \sqrt{d+ex^2} \left(a+bx^2+cx^4\right)^{3/2}dx$$

input
```
Int[(A + B*x^2)*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right) \sqrt{e\,x^{2}+d}\,\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(Bx^2+A\right)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((B*c*x^6 + (B*b + A*c)*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(c*x^4 +
b*x^2 + a)*sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(A+Bx^2\right)\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(Bx^2+A\right)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(Bx^2+A\right)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(B\,x^2+A\right)\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

input
```
int((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(B\,x^2+A\right)\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

3.232 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1243

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

output
```
1/384*(8*A*c*e*(32*a*c*e^2+3*b^2*e^2-22*b*c*d*e+15*c^2*d^2)-B*(105*c^3*d^3
+9*b^3*e^3-c^2*d*e*(-188*a*e+145*b*d)+15*b*c*e^2*(-4*a*e+b*d)))*(e*x^2+d)^
(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e^4/x-1/192*(8*A*c*e*(-7*b*e+5*c*d)-B*(60*
a*c*e^2+3*b^2*e^2-46*b*c*d*e+35*c^2*d^2))*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a
)^(1/2)/c/e^3-1/48*(-8*A*c*e-9*B*b*e+7*B*c*d)*x^3*(e*x^2+d)^(1/2)*(c*x^4+b
*x^2+a)^(1/2)/e^2+1/8*B*c*x^5*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/e-1/76
8*(-4*a*c+b^2)^(1/2)*(8*A*c*e*(32*a*c*e^2+3*b^2*e^2-22*b*c*d*e+15*c^2*d^2)
-B*(105*c^3*d^3+9*b^3*e^3-c^2*d*e*(-188*a*e+145*b*d)+15*b*c*e^2*(-4*a*e+b*
d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1
/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^
(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^2/e^4/(-a*(e+d/
x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/384*(
-4*a*c+b^2)^(1/2)*(B*(35*c^3*d^3+9*b^3*e^3-c^2*d*e*(-68*a*e+53*b*d)+3*b*c*
e^2*(-20*a*e+3*b*d))-8*A*c*e*(5*c^2*d^2+3*b^2*e^2-8*c*e*(2*a*e+b*d)))*(-a*
(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*
d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(
1/2))*2^(1/2)/c^2/e^3/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/64*(-4*a*c+b
^2)^(1/2)*(8*A*c*e*(-b*e+c*d)*(5*c^2*d^2-b^2*e^2-4*c*e*(-3*a*e+b*d))-B*(35
*c^4*d^4+3*b^4*e^4+4*b^2*c*e^3*(-6*a*e+b*d)-12*c^3*d^2*e*(-6*a*e+5*b*d)...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx=\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[d + e*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{ 2260}$$

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}dx$$

input
```
Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{\sqrt{e\,x^2+d}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(1/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="fr
icas")
```

output
```
integral((B*c*x^6 + (B*b + A*c)*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(c*x^4 +
b*x^2 + a)/sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{(A+Bx^2)(a+bx^2+cx^4)^{\frac{3}{2}}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x**2)*(a + b*x**2 + c*x**4)**(3/2)/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{3/2}}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(1/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^{3/2}}{\sqrt{d+ex^2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(1/2),x)
```

output
```
(116*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*c*e**2*x - 40*sqrt(d +
 e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*c**2*d*e*x + 32*sqrt(d + e*x**2)*sqrt
(a + b*x**2 + c*x**4)*a*c**2*e**2*x**3 + 3*sqrt(d + e*x**2)*sqrt(a + b*x**
2 + c*x**4)*b**3*e**2*x - 46*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*
*2*c*d*e*x + 36*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*c*e**2*x**
3 + 35*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c**2*d**2*x - 28*sqrt(
d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c**2*d*e*x**3 + 24*sqrt(d + e*x**2
)*sqrt(a + b*x**2 + c*x**4)*b*c**2*e**2*x**5 + 256*int((sqrt(d + e*x**2)*s
qrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d
*x**4 + c*e*x**6),x)*a**2*c**2*e**3 + 84*int((sqrt(d + e*x**2)*sqrt(a + b*
x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*
e*x**6),x)*a*b**2*c*e**3 - 364*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x
**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)
*a*b*c**2*d*e**2 + 120*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**
4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*c**3*
d**2*e - 9*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*
e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b**4*e**3 - 15*int(
(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x*
*2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b**3*c*d*e**2 + 145*int((sqrt(d +
e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*...
```

3.233 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1097

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

output
```
-(-A*e+B*d)*(a*e^2-b*d*e+c*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/d/e^3/(e*x^2+d)^(1
/2)-1/48*(6*A*c*e*(15*c*d^2-e*(-8*a*e+13*b*d))-B*d*(105*c^2*d^2+3*b^2*e^2-
20*c*e*(-4*a*e+5*b*d)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/d/e^4/x-1/
24*(-6*A*c*e-7*B*b*e+11*B*c*d)*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/e^3
+1/6*B*c*x^3*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/e^2+1/96*(-4*a*c+b^2)^(
1/2)*(6*A*c*e*(15*c*d^2-e*(-8*a*e+13*b*d))-B*d*(105*c^2*d^2+3*b^2*e^2-20*c
*e*(-4*a*e+5*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1
/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)
*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c/
d/e^4/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a
)^(1/2)+1/48*(-4*a*c+b^2)^(1/2)*(B*d*(32*a*c*e^2+3*b^2*e^2-38*b*c*d*e+35*c
^2*d^2)-6*A*c*e*(5*c*d^2-e*(-8*a*e+5*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^
2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*Ellipt
icF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+
b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c/d/e^3/(e*x
^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/8*(-4*a*c+b^2)^(1/2)*(6*A*c*e*(4*a*c*e
^2+b^2*e^2-6*b*c*d*e+5*c^2*d^2)-B*(35*c^3*d^3+b^3*e^3+3*b*c*e^2*(-4*a*e+3*
b*d)-9*c^2*d*e*(-4*a*e+5*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-
a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+
(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx=\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{(d+ex^2)^{3/2}}\,dx$$
$$\Big\downarrow \ 2260$$
$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{(d+ex^2)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(3/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="fr
icas")
```

output
```
integral((B*c*x^6 + (B*b + A*c)*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(c*x^4 +
b*x^2 + a)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)(a+bx^2+cx^4)^{\frac{3}{2}}}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{3/2}}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(3/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(3/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(3/2),x)
```

3.234 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1035

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \text{Too large to display}$$

output
```
-1/3*(-A*e+B*d)*(a*e^2-b*d*e+c*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/d/e^3/(e*x^2+d
)^(3/2)+1/3*(B*d*(9*c*d^2-e*(-a*e+5*b*d))-2*A*e*(3*c*d^2-e*(a*e+b*d)))*x*(
c*x^4+b*x^2+a)^(1/2)/d^2/e^3/(e*x^2+d)^(1/2)-1/24*(B*d*(105*c*d^2-e*(-8*a*
e+55*b*d))-4*A*e*(15*c*d^2-4*e*(a*e+b*d)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)
^(1/2)/d^2/e^4/x+1/4*B*c*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/e^3+1/48*
(-4*a*c+b^2)^(1/2)*(B*d*(105*c*d^2-e*(-8*a*e+55*b*d))-4*A*e*(15*c*d^2-4*e*
(a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*Elli
pticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*
c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/d^2/e^4/(-
a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-
1/24*(-4*a*c+b^2)^(1/2)*(B*d*(35*c*d^2-e*(-8*a*e+23*b*d))-4*A*e*(5*c*d^2-2
*e*(2*a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b
+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b
^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/d^2/e^3/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)-1/4*(-4*a*c+b^2)^(1/2)*(4*A*c*e*(-3*b*e+5*c*d)-B*(35*c^2*d^2+3*b^2*
e^2-6*c*e*(-2*a*e+5*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+
d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*
a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b
^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{5/2}}dx$$
$$\Big\downarrow \text{ 2260}$$
$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{5/2}}dx$$

input
```
Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(5/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(5/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{5/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(5/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{(d+ex^2)^{5/2}}\,dx=\int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(5/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(5/2),x)
```

3.235 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1311

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx = \text{Too large to display}$$

output
```
-1/5*(-A*e+B*d)*(a*e^2-b*d*e+c*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/d/e^3/(e*x^2+d
)^(5/2)+1/15*(B*d*(13*c*d^2-e*(-a*e+7*b*d))-2*A*e*(4*c*d^2-e*(2*a*e+b*d)))
*x*(c*x^4+b*x^2+a)^(1/2)/d^2/e^3/(e*x^2+d)^(3/2)+1/15*(A*e*(15*c^2*d^4-c*d
^2*e*(-11*a*e+10*b*d)-e^2*(-8*a^2*e^2+3*a*b*d*e+2*b^2*d^2))-B*d*(45*c^2*d^
4-c*d^2*e*(-31*a*e+50*b*d)+e^2*(-2*a^2*e^2-3*a*b*d*e+8*b^2*d^2)))*x*(c*x^4
+b*x^2+a)^(1/2)/d^3/e^3/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)-1/30*(2*A*e*(1
5*c^2*d^4-c*d^2*e*(-11*a*e+10*b*d)-e^2*(-8*a^2*e^2+3*a*b*d*e+2*b^2*d^2))-B
*d*(105*c^2*d^4-c*d^2*e*(-77*a*e+115*b*d)+2*e^2*(-2*a^2*e^2-3*a*b*d*e+8*b^
2*d^2)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/d^3/e^4/(a*e^2-b*d*e+c*d^2)
/x+1/60*(-4*a*c+b^2)^(1/2)*(2*A*e*(15*c^2*d^4-c*d^2*e*(-11*a*e+10*b*d)-e^2
*(-8*a^2*e^2+3*a*b*d*e+2*b^2*d^2))-B*d*(105*c^2*d^4-c*d^2*e*(-77*a*e+115*b
*d)+2*e^2*(-2*a^2*e^2-3*a*b*d*e+8*b^2*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b
^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-
2*a*e))^(1/2))*2^(1/2)/d^3/e^4/(a*e^2-b*d*e+c*d^2)/(-a*(e+d/x^2)/((b+(-4*a
*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/30*(-4*a*c+b^2)^(1/
2)*(B*d*(35*c*d^2-4*e*(a*e+2*b*d))-2*A*e*(5*c*d^2+e*(8*a*e+b*d)))*(-a*(c+a
/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*
a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx=\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(7/2),x]
```

output
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(7/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{7/2}} \, dx$$
$$\Big\downarrow \text{ 2260}$$
$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{7/2}}dx$$

input
```
Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(7/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{7}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(7/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(7/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx=\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}(Bx^2+A)}{\left(ex^2+d\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="fr
icas")
```

output
```
integral((B*c*x^6 + (B*b + A*c)*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(c*x^4 +
b*x^2 + a)*sqrt(e*x^2 + d)/(e^4*x^8 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4 + 4*d^3*
e*x^2 + d^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{\left(d+ex^2\right)^{7/2}}\,dx=\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}(Bx^2+A)}{\left(ex^2+d\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(7/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{3/2}}{(e\,x^2+d)^{7/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(7/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{7/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{7}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(7/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(7/2),x)
```

3.236 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1429

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx = \text{Too large to display}$$

output
```
-1/7*(-A*e+B*d)*(a*e^2-b*d*e+c*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/d/e^3/(e*x^2+d
)^(7/2)+1/35*(B*d*(17*c*d^2-e*(-a*e+9*b*d))-2*A*e*(5*c*d^2-e*(3*a*e+b*d)))
*x*(c*x^4+b*x^2+a)^(1/2)/d^2/e^3/(e*x^2+d)^(5/2)+1/105*(3*A*e*(5*c^2*d^4-c
*d^2*e*(-9*a*e+2*b*d)-e^2*(-8*a^2*e^2+5*a*b*d*e+2*b^2*d^2))-B*d*(71*c^2*d^
4-c*d^2*e*(-55*a*e+76*b*d)+e^2*(-4*a^2*e^2-a*b*d*e+8*b^2*d^2)))*x*(c*x^4+b
*x^2+a)^(1/2)/d^3/e^3/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(3/2)-1/105*(6*A*e^4*(
-2*a*e+b*d)*(b^2*d^2+4*a*b*d*e-4*a*(a*e^2+2*c*d^2))+B*d*(105*c^3*d^6-7*c^2
*d^4*e*(-26*a*e+25*b*d)+c*d^2*e^2*(37*a^2*e^2-120*a*b*d*e+56*b^2*d^2)+e^3*
(8*a^3*e^3-5*a^2*b*d*e^2-5*a*b^2*d^2*e+8*b^3*d^3)))*(c*x^4+b*x^2+a)^(1/2)/
d^3/e^4/(a*e^2-b*d*e+c*d^2)^2/x/(e*x^2+d)^(1/2)+1/210*(-4*a*c+b^2)^(1/2)*(
6*A*e^4*(-2*a*e+b*d)*(b^2*d^2+4*a*b*d*e-4*a*(a*e^2+2*c*d^2))+B*d*(105*c^3*
d^6-7*c^2*d^4*e*(-26*a*e+25*b*d)+c*d^2*e^2*(37*a^2*e^2-120*a*b*d*e+56*b^2*
d^2)+e^3*(8*a^3*e^3-5*a^2*b*d*e^2-5*a*b^2*d^2*e+8*b^3*d^3)))*(-a*(c+a/x^4+
b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/
(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a
*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/d^4/e^4/(a*e^2-b*d*e+c*d^2)^2/(-a*(
e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/1
05*2^(1/2)*(-4*a*c+b^2)^(1/2)*(3*A*e^3*(b^2*d^2+16*a*b*d*e-4*a*(4*a*e^2+5*
c*d^2))-B*d*(35*c^2*d^4-c*d^2*e*(-31*a*e+28*b*d)-e^2*(-8*a^2*e^2+a*b*d*e+4
*b^2*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(9/2),x]
```

output
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(9/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{(d+ex^2)^{9/2}}\,dx$$
$$\Big\downarrow \text{2260}$$
$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{(d+ex^2)^{9/2}}dx$$

input
```
Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(9/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{9}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(9/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(9/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx=\int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="ma
xima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(9/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(9/2),x, algorithm="gi
ac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{3/2}}{(e\,x^2+d)^{9/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(9/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(9/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{9/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{9}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(9/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(9/2),x)
```

3.237 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{11/2}} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 1593

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{11/2}} \, dx = \text{Too large to display}$$

output
```
-1/9*(-A*e+B*d)*(a*e^2-b*d*e+c*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/d/e^3/(e*x^2+d
)^(9/2)+1/63*(B*d*(21*c*d^2-e*(-a*e+11*b*d))-2*A*e*(6*c*d^2-e*(4*a*e+b*d))
)*x*(c*x^4+b*x^2+a)^(1/2)/d^2/e^3/(e*x^2+d)^(7/2)+1/315*(A*e*(15*c^2*d^4-c
*d^2*e*(-51*a*e+2*b*d)-e^2*(-48*a^2*e^2+35*a*b*d*e+10*b^2*d^2))-B*d*(105*c
^2*d^4-c*d^2*e*(-87*a*e+110*b*d)+e^2*(-6*a^2*e^2+a*b*d*e+8*b^2*d^2)))*x*(c
*x^4+b*x^2+a)^(1/2)/d^3/e^3/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(5/2)+1/315*(B*d
*(35*c^3*d^6-c^2*d^4*e*(-82*a*e+55*b*d)+c*d^2*e^2*(23*a^2*e^2-52*a*b*d*e+8
*b^2*d^2)+e^3*(8*a^3*e^3-9*a^2*b*d*e^2-3*a*b^2*d^2*e+8*b^3*d^3))+2*A*e*(5*
c^3*d^6-2*c^2*d^4*e*(-11*a*e+2*b*d)-c*d^2*e^2*(-65*a^2*e^2+46*a*b*d*e+4*b^
2*d^2)+e^3*(32*a^3*e^3-54*a^2*b*d*e^2+15*a*b^2*d^2*e+5*b^3*d^3)))*x*(c*x^4
+b*x^2+a)^(1/2)/d^4/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x^2+d)^(3/2)+1/315*(2*b^4
*d^4*(5*A*e+4*B*d)-b^3*d^3*(-25*A*a*e^2+18*A*c*d^2+7*B*a*d*e)-3*a*b^2*d^2*
(81*A*a*e^3+41*A*c*d^2*e+3*B*a*d*e^2+19*B*c*d^3)+4*a*b*d*(a*B*d*e*(8*a*e^2
+15*c*d^2)+A*(82*a^2*e^4+147*a*c*d^2*e^2+36*c^2*d^4))-4*a^2*(A*e*(32*a^2*e
^4+93*a*c*d^2*e^2+93*c^2*d^4)-B*(-4*a^2*d*e^4-15*a*c*d^3*e^2+21*c^2*d^5)))
*(c*x^4+b*x^2+a)^(1/2)/d^4/(a*e^2-b*d*e+c*d^2)^3/x/(e*x^2+d)^(1/2)-1/630*(
-4*a*c+b^2)^(1/2)*(2*b^4*d^4*(5*A*e+4*B*d)-b^3*d^3*(-25*A*a*e^2+18*A*c*d^2
+7*B*a*d*e)-3*a*b^2*d^2*(81*A*a*e^3+41*A*c*d^2*e+3*B*a*d*e^2+19*B*c*d^3)+4
*a*b*d*(a*B*d*e*(8*a*e^2+15*c*d^2)+A*(82*a^2*e^4+147*a*c*d^2*e^2+36*c^2*d^
4))-4*a^2*(A*e*(32*a^2*e^4+93*a*c*d^2*e^2+93*c^2*d^4)-B*(-4*a^2*d*e^4-1...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{\left(d+ex^2\right)^{11/2}}\,dx=\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{\left(d+ex^2\right)^{11/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(11/2),x]
```

output
```
Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(11/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{(d+ex^2)^{11/2}}\,dx$$
$$\Big\downarrow \; 2260$$
$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{(d+ex^2)^{11/2}}dx$$

input
```
Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(11/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{11}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(11/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(11/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="f
ricas")
```

output
```
integral((B*c*x^6 + (B*b + A*c)*x^4 + (B*a + A*b)*x^2 + A*a)*sqrt(c*x^4 +
b*x^2 + a)*sqrt(e*x^2 + d)/(e^6*x^12 + 6*d*e^5*x^10 + 15*d^2*e^4*x^8 + 20*
d^3*e^3*x^6 + 15*d^4*e^2*x^4 + 6*d^5*e*x^2 + d^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="m
axima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(11/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}(Bx^2+A)}{(ex^2+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(11/2),x, algorithm="g
iac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(B*x^2 + A)/(e*x^2 + d)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{3/2}}{(e\,x^2+d)^{11/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(11/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^{3/2}}{(d+ex^2)^{11/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(e\,x^2+d)^{\frac{11}{2}}}dx$$

input
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(11/2),x)
```

output
```
int((B*x^2+A)*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d)^(11/2),x)
```

3.238 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 814

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{(5Bcd-3bBe+4Ace)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{8c^2x}$$

$$+\frac{Bex\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{4c}$$

$$-\frac{\sqrt{b^2-4ac}(5Bcd-3bBe+4Ace)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right)}{8\sqrt{2}c^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt{b^2-4ac}(aBe(7cd-3be)-4Ac(2cd^2-ae^2))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticF}\left(\arcsin\right.}{4\sqrt{2}ac^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt{b^2-4ac}(4Ace(3cd-be)+B(3c^2d^2+3b^2e^2-2ce(3bd+2ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3}{2\sqrt{2}c^2\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex^2}\sqrt{a+bx^2+}}$$

output
```
1/8*(4*A*c*e-3*B*b*e+5*B*c*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/x+
1/4*B*e*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c-1/16*(-4*a*c+b^2)^(1/2)*
(4*A*c*e-3*B*b*e+5*B*c*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2
+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1
/2)/c^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2
+a)^(1/2)+1/8*(-4*a*c+b^2)^(1/2)*(a*B*e*(-3*b*e+7*c*d)-4*A*c*(-a*e^2+2*c*d
^2))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2
)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-
2*a*e))^(1/2))*2^(1/2)/a/c^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/4*(-4
*a*c+b^2)^(1/2)*(4*A*c*e*(-b*e+3*c*d)+B*(3*c^2*d^2+3*b^2*e^2-2*c*e*(2*a*e+
3*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c
+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)
*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^
2/(b+(-4*a*c+b^2)^(1/2))/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2))/Sqrt[a + b*x^2 + c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx$$

↓ 2260

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2))/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}}{\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{\frac{3}{2}}}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{3/2}}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx=\frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,bex+4\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,x^4}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)ac\,e}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*e*x + 4*int((sqrt(d + e*x**2
)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 +
c*d*x**4 + c*e*x**6),x)*a*c*e**2 - 3*int((sqrt(d + e*x**2)*sqrt(a + b*x**2
 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x*
*6),x)*b**2*e**2 + 5*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)
/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b*c*d*e -
 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 +
 b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*b*e**2 + 8*int((sqrt(d +
e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x
**4 + c*d*x**4 + c*e*x**6),x)*a*c*d*e - 2*int((sqrt(d + e*x**2)*sqrt(a + b
*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c
*e*x**6),x)*b**2*d*e + 4*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x
**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b*c*d
**2 - int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a*d + a*e*x**2 + b
*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*b*d*e + 4*int((sqrt(d + e*x
**2)*sqrt(a + b*x**2 + c*x**4))/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*
d*x**4 + c*e*x**6),x)*a*c*d**2)/(4*c)
```

3.239 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 690

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{B\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{2cx}$$
$$-\frac{B\sqrt{b^2-4ac}\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}x\sqrt{d+ex^2}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right)}{2\sqrt{2}c\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt{b^2-4ac}(2Acd-aBe)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{\ldots}{bd+\ldots}\right)}{\sqrt{2}ac\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(Bcd-bBe+2Ace)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3\operatorname{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\ldots\right)}{c\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
1/2*B*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/x-1/4*B*(-4*a*c+b^2)^(1/2)*(
-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+
(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*
d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c/(-a*(e+d/x^2)/((b+(-4
*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/2*(-4*a*c+b^2)^(1
/2)*(2*A*c*d-B*a*e)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/
((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*
c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/c/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)+2^(1/2)*(-4*a*c+b^2)^(1/2)*(2*A*c*e-B*b*e+B*c*d)*(-a*(c+a/x^4+b/x^2)
/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)
*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4
*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+
(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))/(e*x^2+d)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2])/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2])/Sqrt[a + b*x^2 + c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 2260$$

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2])/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{e\,x^2+d}}{\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{\sqrt{a+bx^2+cx^4}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)b+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)a$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4
),x)*b + int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a + b*x**2 + c*
x**4),x)*a
```

3.240 $\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 430

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{2}A\sqrt{b^2-4ac}\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-\cdots}\right)}{a\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}} + \frac{2\sqrt{2}B\sqrt{b^2-4ac}\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3\,\text{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\cdots\right)}{\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
-2^(1/2)*A*(-4*a*c+b^2)^(1/2)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*
(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+
2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(
b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/a/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)+2*2^(1/2)*B*(-4*a*c+b^2)^(1/2)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/
2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/
2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b
+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)
*d-2*a*e))^(1/2))/(b+(-4*a*c+b^2)^(1/2))/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \text{2260}$$
$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^{2}+A}{\sqrt{e\,x^{2}+d}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c*e*x^6 + (c
*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,x^2}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)b+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)a$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*
d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b + int((sqrt(d + e*x**2)*sqrt
(a + b*x**2 + c*x**4))/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 +
c*e*x**6),x)*a
```

3.241 $\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 470

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{(Bd-Ae)\sqrt{a+bx^2+cx^4}}{(cd^2-bde+ae^2)\,x\sqrt{d+ex^2}} - \frac{\sqrt{b^2-4ac}(Bd-Ae)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right)}{\sqrt{2}d\,(cd^2-bde+ae^2)\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{2}A\sqrt{b^2-4ac}\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right)}{ad\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
(-A*e+B*d)*(c*x^4+b*x^2+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/x/(e*x^2+d)^(1/2)-1/2
*(-4*a*c+b^2)^(1/2)*(-A*e+B*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(
e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))
*2^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a
*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-2^(1/2)*A*(-4*a*c+b^2)^(1/2)*(-a*(c+a/x^4
+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e)
)^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/a/
d/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
Integrate[(A + B*x^2)/((d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \; 2260$$
$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^{2}+A}{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \sqrt{c\,x^{4}+b\,x^{2}+a}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c*e^2*x^8 +
(2*c*d*e + b*e^2)*x^6 + (c*d^2 + 2*b*d*e + a*e^2)*x^4 + a*d^2 + (b*d^2 + 2
*a*d*e)*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{(d+ex^2)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/((d + e*x**2)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{(e\,x^2+d)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,e^2x^8+b\,e^2x^6+2cde\,x^6+a\,e^2x^4+2bde\,x^4+c\,d^2x^4+2ade\,x^2 -}\right.$$
$$+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{c\,e^2x^8+b\,e^2x^6+2cde\,x^6+a\,e^2x^4+2bde\,x^4+c\,d^2x^4+2ade\,x^2+b\,d^2x^2+a\,d^2}dx\right)a$$

input
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d**2 + 2*a*d*e*x*
*2 + a*e**2*x**4 + b*d**2*x**2 + 2*b*d*e*x**4 + b*e**2*x**6 + c*d**2*x**4
+ 2*c*d*e*x**6 + c*e**2*x**8),x)*b + int((sqrt(d + e*x**2)*sqrt(a + b*x**2
 + c*x**4))/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 + b*d**2*x**2 + 2*b*d*e*x
**4 + b*e**2*x**6 + c*d**2*x**4 + 2*c*d*e*x**6 + c*e**2*x**8),x)*a
```

3.242 $\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 660

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}\,dx = -\frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{3d(cd^2-bde+ae^2)(d+ex^2)^{3/2}}$$
$$-\frac{(2Ae(3cd^2-e(2bd-ae))-Bd(3cd^2-e(bd+ae)))\sqrt{a+bx^2+cx^4}}{3d(cd^2-bde+ae^2)^2x\sqrt{d+ex^2}}$$
$$+\frac{\sqrt{b^2-4ac}(2Ae(3cd^2-e(2bd-ae))-Bd(3cd^2-e(bd+ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1}}{\ }\right.\right.}{3\sqrt{2}d^2(cd^2-bde+ae^2)^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(aBde+A(3cd^2-e(3bd-2ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticF}\left(\text{arcsin}\right.}{3ad^2(cd^2-bde+ae^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/3*e*(-A*e+B*d)*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^
(3/2)-1/3*(2*A*e*(3*c*d^2-e*(-a*e+2*b*d))-B*d*(3*c*d^2-e*(a*e+b*d)))*(c*x^
4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)^2/x/(e*x^2+d)^(1/2)+1/6*(-4*a*c+b^2
)^(1/2)*(2*A*e*(3*c*d^2-e*(-a*e+2*b*d))-B*d*(3*c*d^2-e*(a*e+b*d)))*(-a*(c+
a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a
/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d
+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/d^2/(a*e^2-b*d*e+c*d^2)^2/(-a
*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1
/3*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a*B*d*e+A*(3*c*d^2-e*(-2*a*e+3*b*d)))*(-a*(
c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d
-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*
2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1
/2))/a/d^2/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^(5/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
Integrate[(A + B*x^2)/((d + e*x^2)^(5/2)*Sqrt[a + b*x^2 + c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{ 2260}$$

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^(5/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^{2}+A}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}\sqrt{c\,x^{4}+b\,x^{2}+a}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c*e^3*x^10 +
 (3*c*d*e^2 + b*e^3)*x^8 + (3*c*d^2*e + 3*b*d*e^2 + a*e^3)*x^6 + (c*d^3 +
3*b*d^2*e + 3*a*d*e^2)*x^4 + a*d^3 + (b*d^3 + 3*a*d^2*e)*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{(d+ex^2)^{\frac{5}{2}}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(5/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/((d + e*x**2)**(5/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{(e\,x^2+d)^{5/2}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{5/2}\sqrt{a+bx^2+cx^4}}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,}}{c\,e^3x^{10}+b\,e^3x^8+3cd\,e^2x^8+a\,e^3x^6+3bd\,e^2x^6+3c\,d^2e\,x^6+3ad}\right.$$
$$+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{c\,e^3x^{10}+b\,e^3x^8+3cd\,e^2x^8+a\,e^3x^6+3bd\,e^2x^6+3c\,d^2e\,x^6+3ad\,e^2x^4+3b\,d^2e\,x^4+c\,d^3x^4+3a\,d^2e\,x}\right.$$

input
```
int((B*x^2+A)/(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d**3 + 3*a*d**2*e
*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 + b*d**3*x**2 + 3*b*d**2*e*x**4 + 3*
b*d*e**2*x**6 + b*e**3*x**8 + c*d**3*x**4 + 3*c*d**2*e*x**6 + 3*c*d*e**2*x
**8 + c*e**3*x**10),x)*b + int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)
)/(a*d**3 + 3*a*d**2*e*x**2 + 3*a*d*e**2*x**4 + a*e**3*x**6 + b*d**3*x**2
+ 3*b*d**2*e*x**4 + 3*b*d*e**2*x**6 + b*e**3*x**8 + c*d**3*x**4 + 3*c*d**2
*e*x**6 + 3*c*d*e**2*x**8 + c*e**3*x**10),x)*a
```

3.243 $\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1091

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

output
```
x*(A*b^2-B*a*b-2*A*a*c+(A*b-2*B*a)*c*x^2)*(e*x^2+d)^(5/2)/a/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)^(1/2)-1/2*(2*A*c*(a*b*e^2-4*a*c*d*e+b*c*d^2)-a*B*(4*c^2*d^2
+3*b^2*e^2-4*c*e*(2*a*e+b*d)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/c^2
/(-4*a*c+b^2)/x-e*(2*A*c*(-a*e+b*d)-a*B*(-b*e+4*c*d))*x*(e*x^2+d)^(1/2)*(c
*x^4+b*x^2+a)^(1/2)/a/c/(-4*a*c+b^2)-(A*b-2*B*a)*e^2*x^3*(e*x^2+d)^(1/2)*(
c*x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)+1/4*(2*A*c*(a*b*e^2-4*a*c*d*e+b*c*d^2)
-a*B*(4*c^2*d^2+3*b^2*e^2-4*c*e*(2*a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+
b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d
-2*a*e))^(1/2))*2^(1/2)/a/c^2/(-4*a*c+b^2)^(1/2)/(-a*(e+d/x^2)/((b+(-4*a*c
+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(3*a*b^2*B*e^3-2*b*
c*(A*a*e^3+3*A*c*d^2*e+3*B*a*d*e^2+B*c*d^3)+4*c*(a*B*e*(-2*a*e^2+3*c*d^2)+
A*c*d*(3*a*e^2+c*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x
^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^
2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-
4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/c^2/(-4*a*c+b^2)^(1/2)/(e*x^2+
d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+2^(1/2)*(-4*a*c+b^2)^(1/2)*e^2*(2*A*c*e-3*B
*b*e+5*B*c*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-
4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(5/2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(5/2))/(a + b*x^2 + c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2260$$

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(5/2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{5}{2}}}{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{\frac{5}{2}}}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(5/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(5/2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(5/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(5/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{5/2}}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(5/2))/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(5/2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
( - 4*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a**2*e**4*x - 5*sqrt(d +
e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*d*e**3*x + sqrt(d + e*x**2)*sqrt(a +
 b*x**2 + c*x**4)*a*b*e**4*x**3 + 3*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x
**4)*a*c*d**2*e**2*x - sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*c*d*e*
*3*x**3 + 3*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*d**2*e**2*x +
3*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*d*e**3*x**3 + 3*sqrt(d +
 e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c*d**3*e*x - 3*sqrt(d + e*x**2)*sqrt(
a + b*x**2 + c*x**4)*b*c*d**2*e**2*x**3 - 6*int((sqrt(d + e*x**2)*sqrt(a +
 b*x**2 + c*x**4)*x**8)/(3*a**2*b**2*d*e**2 + 3*a**2*b**2*e**3*x**2 - 2*a*
*2*b*c*d**2*e - 2*a**2*b*c*d*e**2*x**2 - a**2*c**2*d**3 - a**2*c**2*d**2*e
*x**2 + 6*a*b**3*d*e**2*x**2 + 6*a*b**3*e**3*x**4 - 4*a*b**2*c*d**2*e*x**2
 + 2*a*b**2*c*d*e**2*x**4 + 6*a*b**2*c*e**3*x**6 - 2*a*b*c**2*d**3*x**2 -
6*a*b*c**2*d**2*e*x**4 - 4*a*b*c**2*d*e**2*x**6 - 2*a*c**3*d**3*x**4 - 2*a
*c**3*d**2*e*x**6 + 3*b**4*d*e**2*x**4 + 3*b**4*e**3*x**6 - 2*b**3*c*d**2*
e*x**4 + 4*b**3*c*d*e**2*x**6 + 6*b**3*c*e**3*x**8 - b**2*c**2*d**3*x**4 -
 5*b**2*c**2*d**2*e*x**6 - b**2*c**2*d*e**2*x**8 + 3*b**2*c**2*e**3*x**10
- 2*b*c**3*d**3*x**6 - 4*b*c**3*d**2*e*x**8 - 2*b*c**3*d*e**2*x**10 - c**4
*d**3*x**8 - c**4*d**2*e*x**10),x)*a**2*b**3*c*e**7 + 10*int((sqrt(d + e*x
**2)*sqrt(a + b*x**2 + c*x**4)*x**8)/(3*a**2*b**2*d*e**2 + 3*a**2*b**2*e**
3*x**2 - 2*a**2*b*c*d**2*e - 2*a**2*b*c*d*e**2*x**2 - a**2*c**2*d**3 - ...
```

3.244 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 895

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)(d+ex^2)^{3/2}}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(Ac(bd-2ae)-aB(2cd-be))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{ac(b^2-4ac)x}$$
$$-\frac{(Ab-2aB)ex\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)}$$
$$+\frac{(Ac(bd-2ae)-aB(2cd-be))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right)}{\sqrt{2}ac\sqrt{b^2-4ac}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt{2}(2c(Acd^2+2aBde+aAe^2)-b(Bcd^2+2Acde+aBe^2))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{Ellipt}}{ac\sqrt{b^2-4ac}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{2\sqrt{2}B\sqrt{b^2-4ac}e^2\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{}{b}\right.}{c\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
x*(A*b^2-B*a*b-2*A*a*c+(A*b-2*B*a)*c*x^2)*(e*x^2+d)^(3/2)/a/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)^(1/2)-(A*c*(-2*a*e+b*d)-a*B*(-b*e+2*c*d))*(e*x^2+d)^(1/2)*(
c*x^4+b*x^2+a)^(1/2)/a/c/(-4*a*c+b^2)/x-(A*b-2*B*a)*e*x*(e*x^2+d)^(1/2)*(c
*x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)+1/2*(A*c*(-2*a*e+b*d)-a*B*(-b*e+2*c*d))
*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(
1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2
)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/c/(-4*a*c+b^2)^(1/2
)/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1
/2)+2^(1/2)*(2*c*(A*a*e^2+A*c*d^2+2*B*a*d*e)-b*(2*A*c*d*e+B*a*e^2+B*c*d^2)
)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(
1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))
^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a
*e))^(1/2))/a/c/(-4*a*c+b^2)^(1/2)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+2
*2^(1/2)*B*(-4*a*c+b^2)^(1/2)*e^2*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*
(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(
1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-
4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-
2*a*e))^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2260}$$

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}}{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{(A+Bx^2)\left(d+ex^2\right)^{\frac{3}{2}}}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{3/2}}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*e**2*x + 2*sqrt(d + e*x**2)*
sqrt(a + b*x**2 + c*x**4)*b*d*e*x + int((sqrt(d + e*x**2)*sqrt(a + b*x**2
+ c*x**4)*x**6)/(a**2*b*d*e + a**2*b*e**2*x**2 - a**2*c*d**2 - a**2*c*d*e*
x**2 + 2*a*b**2*d*e*x**2 + 2*a*b**2*e**2*x**4 - 2*a*b*c*d**2*x**2 + 2*a*b*
c*e**2*x**6 - 2*a*c**2*d**2*x**4 - 2*a*c**2*d*e*x**6 + b**3*d*e*x**4 + b**
3*e**2*x**6 - b**2*c*d**2*x**4 + b**2*c*d*e*x**6 + 2*b**2*c*e**2*x**8 - 2*
b*c**2*d**2*x**6 - b*c**2*d*e*x**8 + b*c**2*e**2*x**10 - c**3*d**2*x**8 -
c**3*d*e*x**10),x)*a*b**3*e**4 - 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 +
 c*x**4)*x**6)/(a**2*b*d*e + a**2*b*e**2*x**2 - a**2*c*d**2 - a**2*c*d*e*x
**2 + 2*a*b**2*d*e*x**2 + 2*a*b**2*e**2*x**4 - 2*a*b*c*d**2*x**2 + 2*a*b*c
*e**2*x**6 - 2*a*c**2*d**2*x**4 - 2*a*c**2*d*e*x**6 + b**3*d*e*x**4 + b**3
*e**2*x**6 - b**2*c*d**2*x**4 + b**2*c*d*e*x**6 + 2*b**2*c*e**2*x**8 - 2*b
*c**2*d**2*x**6 - b*c**2*d*e*x**8 + b*c**2*e**2*x**10 - c**3*d**2*x**8 - c
**3*d*e*x**10),x)*a*b**2*c*d*e**3 + int((sqrt(d + e*x**2)*sqrt(a + b*x**2
+ c*x**4)*x**6)/(a**2*b*d*e + a**2*b*e**2*x**2 - a**2*c*d**2 - a**2*c*d*e*
x**2 + 2*a*b**2*d*e*x**2 + 2*a*b**2*e**2*x**4 - 2*a*b*c*d**2*x**2 + 2*a*b*
c*e**2*x**6 - 2*a*c**2*d**2*x**4 - 2*a*c**2*d*e*x**6 + b**3*d*e*x**4 + b**
3*e**2*x**6 - b**2*c*d**2*x**4 + b**2*c*d*e*x**6 + 2*b**2*c*e**2*x**8 - 2*
b*c**2*d**2*x**6 - b*c**2*d*e*x**8 + b*c**2*e**2*x**10 - c**3*d**2*x**8 -
c**3*d*e*x**10),x)*a*b*c**2*d**2*e**2 + int((sqrt(d + e*x**2)*sqrt(a + ...
```

3.245 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 533

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)\sqrt{d+ex^2}}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(Ab-2aB)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)x}$$
$$+\frac{(Ab-2aB)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}\,d}{bd+\sqrt{b^2-4ac}\,d-2ae}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt{2}(bBd-2Acd+Abe-2aBe)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{a\sqrt{b^2-4ac}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
x*(A*b^2-B*a*b-2*A*a*c+(A*b-2*B*a)*c*x^2)*(e*x^2+d)^(1/2)/a/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)^(1/2)-(A*b-2*B*a)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(
-4*a*c+b^2)/x+1/2*(A*b-2*B*a)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e
*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*
2^(1/2)/a/(-4*a*c+b^2)^(1/2)/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e
))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-2^(1/2)*(A*b*e-2*A*c*d-2*B*a*e+B*b*d)*(-a*(
c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d
-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*
2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1
/2))/a/(-4*a*c+b^2)^(1/2)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 2260}$$
$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,\sqrt{e\,x^{2}+d}}{(c\,x^{4}+b\,x^{2}+a)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c^2*x^8 + 2*
b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{3/2}}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output

```
(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*e*x + sqrt(d + e*x**2)*sqrt(
a + b*x**2 + c*x**4)*b*d*x + int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**
4)*x**4)/(a**2*d + a**2*e*x**2 + 2*a*b*d*x**2 + 2*a*b*e*x**4 + 2*a*c*d*x**
4 + 2*a*c*e*x**6 + b**2*d*x**4 + b**2*e*x**6 + 2*b*c*d*x**6 + 2*b*c*e*x**8
 + c**2*d*x**8 + c**2*e*x**10),x)*a**2*b*e**2 + int((sqrt(d + e*x**2)*sqrt
(a + b*x**2 + c*x**4)*x**4)/(a**2*d + a**2*e*x**2 + 2*a*b*d*x**2 + 2*a*b*e
*x**4 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + b**2*d*x**4 + b**2*e*x**6 + 2*b*c*d*
x**6 + 2*b*c*e*x**8 + c**2*d*x**8 + c**2*e*x**10),x)*a**2*c*d*e - int((sqr
t(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2*d + a**2*e*x**2 + 2*a*
b*d*x**2 + 2*a*b*e*x**4 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + b**2*d*x**4 + b**2
*e*x**6 + 2*b*c*d*x**6 + 2*b*c*e*x**8 + c**2*d*x**8 + c**2*e*x**10),x)*a*b
**2*d*e + int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2*d +
a**2*e*x**2 + 2*a*b*d*x**2 + 2*a*b*e*x**4 + 2*a*c*d*x**4 + 2*a*c*e*x**6 +
b**2*d*x**4 + b**2*e*x**6 + 2*b*c*d*x**6 + 2*b*c*e*x**8 + c**2*d*x**8 + c*
*2*e*x**10),x)*a*b**2*e**2*x**2 + int((sqrt(d + e*x**2)*sqrt(a + b*x**2 +
c*x**4)*x**4)/(a**2*d + a**2*e*x**2 + 2*a*b*d*x**2 + 2*a*b*e*x**4 + 2*a*c*
d*x**4 + 2*a*c*e*x**6 + b**2*d*x**4 + b**2*e*x**6 + 2*b*c*d*x**6 + 2*b*c*e
*x**8 + c**2*d*x**8 + c**2*e*x**10),x)*a*b*c*d**2 + int((sqrt(d + e*x**2)*
sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2*d + a**2*e*x**2 + 2*a*b*d*x**2 + 2*a
*b*e*x**4 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + b**2*d*x**4 + b**2*e*x**6 + 2...
```

3.246 $\int \frac{A+Bx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 611

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{3/2}}\,dx = \frac{(aB(2cd-be)-A(bcd-b^2e+2ace))\sqrt{d+ex^2}}{(b^2-4ac)\,(cd^2-bde+ae^2)\,x\sqrt{a+bx^2+cx^4}}$$
$$+\frac{c(b(Bd+Ae)-2(Acd+aBe))x\sqrt{d+ex^2}}{(b^2-4ac)\,(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(aB(2cd-be)-A(bcd-b^2e+2ace))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-\cdots}\right.}{\sqrt{2}a\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt{2}(bB-2Ac)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right.}{a\sqrt{b^2-4ac}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
(a*B*(-b*e+2*c*d)-A*(2*a*c*e-b^2*e+b*c*d))*(e*x^2+d)^(1/2)/(-4*a*c+b^2)/(a
*e^2-b*d*e+c*d^2)/x/(c*x^4+b*x^2+a)^(1/2)+c*(b*(A*e+B*d)-2*A*c*d-2*B*a*e)*
x*(e*x^2+d)^(1/2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)-1
/2*(a*B*(-b*e+2*c*d)-A*(2*a*c*e-b^2*e+b*c*d))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+
b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d
-2*a*e))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-a*(e+d/
x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-2^(1/2)
*(-2*A*c+B*b)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-
4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)
^(1/2)*d-2*a*e))^(1/2))/a/(-4*a*c+b^2)^(1/2)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{3/2}}\,dx$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2260$$

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{3/2}}dx$$

input
```
Int[(A + B*x^2)/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^{2}+A}{\sqrt{e\,x^{2}+d}\,\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c^2*e*x^10 +
 (c^2*d + 2*b*c*e)*x^8 + (2*b*c*d + (b^2 + 2*a*c)*e)*x^6 + (2*a*b*e + (b^2
 + 2*a*c)*d)*x^4 + a^2*d + (2*a*b*d + a^2*e)*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{\sqrt{e\,x^2+d}\,(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+}}{c^2e\,x^{10}+2bce\,x^8+c^2d\,x^8+2ace\,x^6+b^2e\,x^6+2bcd\,x^6+2abe\,x^4}\right.$$
$$+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{c^2e\,x^{10}+2bce\,x^8+c^2d\,x^8+2ace\,x^6+b^2e\,x^6+2bcd\,x^6+2abe\,x^4+2acd\,x^4+b^2d\,x^4+a^2e\,x^2+2abd}\right.$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2*d + a**2*e*x**
2 + 2*a*b*d*x**2 + 2*a*b*e*x**4 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + b**2*d*x**
4 + b**2*e*x**6 + 2*b*c*d*x**6 + 2*b*c*e*x**8 + c**2*d*x**8 + c**2*e*x**10
),x)*b + int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a**2*d + a**2*e
*x**2 + 2*a*b*d*x**2 + 2*a*b*e*x**4 + 2*a*c*d*x**4 + 2*a*c*e*x**6 + b**2*d
*x**4 + b**2*e*x**6 + 2*b*c*d*x**6 + 2*b*c*e*x**8 + c**2*d*x**8 + c**2*e*x
**10),x)*a
```

3.247 $\int \frac{A+Bx^2}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 905

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = -\frac{e(Bd-Ae)x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{(2aBd(c^2d^2+b^2e^2-ce(bd+3ae))-A(b^3de^2+bcd(cd^2-3ae^2)+4ace(cd^2-ae^2)-b^2(2cd^2e-ae^3)))}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)^2x\sqrt{a+bx^2+cx^4}}$$
$$+\frac{c(b^2e(Bd-2Ae)-2c(Acd^2+4aBde-3aAe^2)+b(Bcd^2+2Acde+aBe^2))x\sqrt{d+ex^2}}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(2aBd(c^2d^2+b^2e^2-ce(bd+3ae))-A(b^3de^2+bcd(cd^2-3ae^2)+4ace(cd^2-ae^2)-b^2(2cd^2e-ae^3)))}{\sqrt{2}a\sqrt{b^2-4ac}d\left(cd^2-bde+ae^2\right)^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-}}}$$
$$+\frac{\sqrt{2}(b^2e(Bd-Ae)-bcd(Bd+Ae)+2c(Acd^2-aBde+2aAe^2))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3}{a\sqrt{b^2-4ac}d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}\sqrt{a+bx^2+c}}$$

output
```
-e*(-A*e+B*d)*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2
)+(2*a*B*d*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))-A*(b^3*d*e^2+b*c*d*(-3*a*e^2+
c*d^2)+4*a*c*e*(-a*e^2+c*d^2)-b^2*(-a*e^3+2*c*d^2*e)))*(e*x^2+d)^(1/2)/(-4
*a*c+b^2)/d/(a*e^2-b*d*e+c*d^2)^2/x/(c*x^4+b*x^2+a)^(1/2)+c*(b^2*e*(-2*A*e
+B*d)-2*c*(-3*A*a*e^2+A*c*d^2+4*B*a*d*e)+b*(2*A*c*d*e+B*a*e^2+B*c*d^2))*x*
(e*x^2+d)^(1/2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)^(1/2)-1
/2*(2*a*B*d*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))-A*(b^3*d*e^2+b*c*d*(-3*a*e^2
+c*d^2)+4*a*c*e*(-a*e^2+c*d^2)-b^2*(-a*e^3+2*c*d^2*e)))*(-a*(c+a/x^4+b/x^2
)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^
2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(1/2)/d/(a*e^2-b*d*e+c*d^
2)^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)
^(1/2)+2^(1/2)*(b^2*e*(-A*e+B*d)-b*c*d*(A*e+B*d)+2*c*(2*A*a*e^2+A*c*d^2-B*
a*d*e))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+
b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)
*d-2*a*e))^(1/2))/a/(-4*a*c+b^2)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
Integrate[(A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 2260}$$
$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{3/2}}dx$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^2+A}{(e\,x^2+d)^{\frac{3}{2}}(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c^2*e^2*x^12
 + 2*(c^2*d*e + b*c*e^2)*x^10 + (c^2*d^2 + 4*b*c*d*e + (b^2 + 2*a*c)*e^2)*
x^8 + 2*(b*c*d^2 + a*b*e^2 + (b^2 + 2*a*c)*d*e)*x^6 + (4*a*b*d*e + a^2*e^2
 + (b^2 + 2*a*c)*d^2)*x^4 + a^2*d^2 + 2*(a*b*d^2 + a^2*d*e)*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{(d+ex^2)^{\frac{3}{2}}(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)**(3/2)),
x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx = \left(\int \frac{}{c^2e^2x^{12}+2bc\,e^2x^{10}+2c^2de\,x^{10}+2ac\,e^2x^8+b^2e^2x^8+4bcde\,x^8}\right.$$

$$+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+}}{c^2e^2x^{12}+2bc\,e^2x^{10}+2c^2de\,x^{10}+2ac\,e^2x^8+b^2e^2x^8+4bcde\,x^8+c^2d^2x^8+2ab\,e^2x^6+4acde\,x^6+2b^2}\right.$$

input
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2*d**2 + 2*a**2*
d*e*x**2 + a**2*e**2*x**4 + 2*a*b*d**2*x**2 + 4*a*b*d*e*x**4 + 2*a*b*e**2*
x**6 + 2*a*c*d**2*x**4 + 4*a*c*d*e*x**6 + 2*a*c*e**2*x**8 + b**2*d**2*x**4
 + 2*b**2*d*e*x**6 + b**2*e**2*x**8 + 2*b*c*d**2*x**6 + 4*b*c*d*e*x**8 + 2
*b*c*e**2*x**10 + c**2*d**2*x**8 + 2*c**2*d*e*x**10 + c**2*e**2*x**12),x)*
b + int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a**2*d**2 + 2*a**2*d
*e*x**2 + a**2*e**2*x**4 + 2*a*b*d**2*x**2 + 4*a*b*d*e*x**4 + 2*a*b*e**2*x
**6 + 2*a*c*d**2*x**4 + 4*a*c*d*e*x**6 + 2*a*c*e**2*x**8 + b**2*d**2*x**4
+ 2*b**2*d*e*x**6 + b**2*e**2*x**8 + 2*b*c*d**2*x**6 + 4*b*c*d*e*x**8 + 2*
b*c*e**2*x**10 + c**2*d**2*x**8 + 2*c**2*d*e*x**10 + c**2*e**2*x**12),x)*a
```

3.248 $\int \frac{(A+Bx^2)(d+ex^2)^{7/2}}{(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1562

$$\int \frac{(A+Bx^2)(d+ex^2)^{7/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \text{Too large to display}$$

output
```
1/3*x*(A*b^2-B*a*b-2*A*a*c+(A*b-2*B*a)*c*x^2)*(e*x^2+d)^(7/2)/a/(-4*a*c+b^
2)/(c*x^4+b*x^2+a)^(3/2)+1/3*x*(e*x^2+d)^(5/2)*(a*B*(16*a^2*c*e-8*a*b^2*e+
4*a*b*c*d+b^3*d)+A*(-12*a^2*b*c*e+20*a^2*c^2*d+5*a*b^3*e-17*a*b^2*c*d+2*b^
4*d)+c*(a*B*(-8*a*b*e+12*a*c*d+b^2*d)+2*A*(-4*a^2*c*e+3*a*b^2*e-8*a*b*c*d+
b^3*d))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/3*(2*A*c^2*(-2*a*e
+b*d)*(b^2*d^2+4*a*b*d*e-4*a*(a*e^2+2*c*d^2))+a*B*(3*a*b^3*e^3+b^2*c*d*(a*
e^2+c*d^2)-4*a*b*c*e*(5*a*e^2+6*c*d^2)+4*a*c^2*d*(11*a*e^2+3*c*d^2)))*(e*x
^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/c^2/(-4*a*c+b^2)^2/x+1/3*e*(2*a*B*(1
2*a*b*c*d*e-b^2*(-a*e^2+c*d^2)-12*a*c*(a*e^2+c*d^2))-A*c*(4*b^3*d^2+11*a*b
^2*d*e+4*a^2*c*d*e-8*a*b*(a*e^2+4*c*d^2)))*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+
a)^(1/2)/a^2/c/(-4*a*c+b^2)^2-1/3*e^2*(a*B*(-8*a*b*e+12*a*c*d+b^2*d)+2*A*(
-4*a^2*c*e+3*a*b^2*e-8*a*b*c*d+b^3*d))*x^3*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)
^(1/2)/a^2/(-4*a*c+b^2)^2+1/6*(2*A*c^2*(-2*a*e+b*d)*(b^2*d^2+4*a*b*d*e-4*a
*(a*e^2+2*c*d^2))+a*B*(3*a*b^3*e^3+b^2*c*d*(a*e^2+c*d^2)-4*a*b*c*e*(5*a*e^
2+6*c*d^2)+4*a*c^2*d*(11*a*e^2+3*c*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2)
)^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))
^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a
*e))^(1/2))*2^(1/2)/a^2/c^2/(-4*a*c+b^2)^(3/2)/(-a*(e+d/x^2)/((b+(-4*a*c+b
^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/3*2^(1/2)*(A*c^2*(b^3*d
^3*e-4*a*b*d*e*(8*a*e^2+9*c*d^2)-b^2*(-15*a*d^2*e^2+c*d^4)+4*a*(4*a^2*e...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{7/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{7/2}}{(a+bx^2+cx^4)^{5/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(7/2))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(7/2))/(a + b*x^2 + c*x^4)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{7/2}}{\left(a+bx^2+cx^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{2260}$$

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{7/2}}{\left(a+bx^2+cx^4\right)^{5/2}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(7/2))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{7}{2}}}{(c\,x^4+b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(7/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(7/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{7/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{7}{2}}}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(7/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((B*e^3*x^8 + (3*B*d*e^2 + A*e^3)*x^6 + 3*(B*d^2*e + A*d*e^2)*x^4
+ A*d^3 + (B*d^3 + 3*A*d^2*e)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)
/(c^3*x^12 + 3*b*c^2*x^10 + 3*(b^2*c + a*c^2)*x^8 + (b^3 + 6*a*b*c)*x^6 +
3*a^2*b*x^2 + 3*(a*b^2 + a^2*c)*x^4 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{7/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(7/2)/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{7/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{7}{2}}}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(7/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(7/2)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{7/2}}{(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^{\frac{7}{2}}}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(7/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(7/2)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{7/2}}{(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{7/2}}{(c\,x^4+b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(7/2))/(a + b*x^2 + c*x^4)^(5/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(7/2))/(a + b*x^2 + c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{7/2}}{(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{7}{2}}}{(c\,x^4+b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(7/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(7/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

3.249 $\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 956

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)(d+ex^2)^{5/2}}{3a(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$
$$+\frac{x(d+ex^2)^{3/2}(aB(b^3d+4abcd-6ab^2e+8a^2ce)+A(2b^4d-17ab^2cd+20a^2c^2d+3ab^3e-4a^2bce)+c(a}{3a^2(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(A(2b^3d^2+3ab^2de+20a^2cde-8ab(2cd^2+ae^2))+aB(b^2d^2-16abde+4a(3cd^2+4ae^2)))\sqrt{d+ex^2}\sqrt{a}}{3a^2(b^2-4ac)^2x}$$
$$-\frac{e(aB(b^2d+12acd-8abe)+2Ab(b^2d-8acd+2abe))x\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{3a^2(b^2-4ac)^2}$$
$$+\frac{(A(2b^3d^2+3ab^2de+20a^2cde-8ab(2cd^2+ae^2))+aB(b^2d^2-16abde+4a(3cd^2+4ae^2)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x}\right.}{b^2-4ac}}}{3\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+}}$$
$$+\frac{\sqrt{2}(cd^2-bde+ae^2)(8aB(bd-2ae)+A(b^2d-20acd+8abe))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3\,\mathrm{E}}{3a^2(b^2-4ac)^{3/2}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
1/3*x*(A*b^2-B*a*b-2*A*a*c+(A*b-2*B*a)*c*x^2)*(e*x^2+d)^(5/2)/a/(-4*a*c+b^
2)/(c*x^4+b*x^2+a)^(3/2)+1/3*x*(e*x^2+d)^(3/2)*(a*B*(8*a^2*c*e-6*a*b^2*e+4
*a*b*c*d+b^3*d)+A*(-4*a^2*b*c*e+20*a^2*c^2*d+3*a*b^3*e-17*a*b^2*c*d+2*b^4*
d)+c*(a*B*(-8*a*b*e+12*a*c*d+b^2*d)+2*A*b*(2*a*b*e-8*a*c*d+b^2*d))*x^2)/a^
2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/3*(A*(2*b^3*d^2+3*a*b^2*d*e+20*a^
2*c*d*e-8*a*b*(a*e^2+2*c*d^2))+a*B*(b^2*d^2-16*a*b*d*e+4*a*(4*a*e^2+3*c*d^
2)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)^2/x-1/3*e*(a*B
*(-8*a*b*e+12*a*c*d+b^2*d)+2*A*b*(2*a*b*e-8*a*c*d+b^2*d))*x*(e*x^2+d)^(1/2
)*(c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)^2+1/6*(A*(2*b^3*d^2+3*a*b^2*d*e+2
0*a^2*c*d*e-8*a*b*(a*e^2+2*c*d^2))+a*B*(b^2*d^2-16*a*b*d*e+4*a*(4*a*e^2+3*
c*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*Ellipti
cE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b
^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a^2/(-4*a*c+b
^2)^(3/2)/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x
^2+a)^(1/2)+1/3*2^(1/2)*(a*e^2-b*d*e+c*d^2)*(8*a*B*(-2*a*e+b*d)+A*(8*a*b*e
-20*a*c*d+b^2*d))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((
b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+
b^2)^(1/2)*d-2*a*e))^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/(e*x^2+d)^(1/2)/(c*x^4+
b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(5/2))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(5/2))/(a + b*x^2 + c*x^4)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}\,dx$$
$$\Big\downarrow \ 2260$$
$$\int \frac{(A+Bx^2)\,(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(5/2))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{5}{2}}}{(c\,x^4+b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((B*e^2*x^6 + (2*B*d*e + A*e^2)*x^4 + A*d^2 + (B*d^2 + 2*A*d*e)*x^
2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)/(c^3*x^12 + 3*b*c^2*x^10 + 3*(b
^2*c + a*c^2)*x^8 + (b^3 + 6*a*b*c)*x^6 + 3*a^2*b*x^2 + 3*(a*b^2 + a^2*c)*
x^4 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(5/2)/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(5/2)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{5}{2}}}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(5/2)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{5/2}}{(c\,x^4+b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(5/2))/(a + b*x^2 + c*x^4)^(5/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(5/2))/(a + b*x^2 + c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{5/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{5}{2}}}{(c\,x^4+b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(5/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

3.250 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 845

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)(d+ex^2)^{3/2}}{3a(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$$+\frac{x\sqrt{d+ex^2}(abB(b^2d+4acd-4abe)+A(2b^4d-17ab^2cd+20a^2c^2d+ab^3e+4a^2bce)+c(aB(b^2d+12ac}{3a^2(b^2-4ac)^2\sqrt{a+bx^2+cx^4}}$$

$$-\frac{(aB(b^2d+12acd-8abe)+2A(b^3d-8abcd+ab^2e+4a^2ce))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{3a^2(b^2-4ac)^2x}$$

$$+\frac{(aB(b^2d+12acd-8abe)+2A(b^3d-8abcd+ab^2e+4a^2ce))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+}}{}\right.\right.}{3\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{2}(aB(5b^2de+12acde-8b(cd^2+ae^2))+A(b^3de-20abcde-b^2(cd^2-2ae^2)+4ac(5cd^2+2ae^2)))\sqrt{}}{3a^2(b^2-4ac)^{3/2}\sqrt{d+ex^2}\sqrt{}}$$

output
```
1/3*x*(A*b^2-B*a*b-2*A*a*c+(A*b-2*B*a)*c*x^2)*(e*x^2+d)^(3/2)/a/(-4*a*c+b^
2)/(c*x^4+b*x^2+a)^(3/2)+1/3*x*(e*x^2+d)^(1/2)*(a*b*B*(-4*a*b*e+4*a*c*d+b^
2*d)+A*(4*a^2*b*c*e+20*a^2*c^2*d+a*b^3*e-17*a*b^2*c*d+2*b^4*d)+c*(a*B*(-8*
a*b*e+12*a*c*d+b^2*d)+2*A*(4*a^2*c*e+a*b^2*e-8*a*b*c*d+b^3*d))*x^2)/a^2/(-
4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/3*(a*B*(-8*a*b*e+12*a*c*d+b^2*d)+2*A*
(4*a^2*c*e+a*b^2*e-8*a*b*c*d+b^3*d))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)
/a^2/(-4*a*c+b^2)^2/x+1/6*(a*B*(-8*a*b*e+12*a*c*d+b^2*d)+2*A*(4*a^2*c*e+a*
b^2*e-8*a*b*c*d+b^3*d))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d
)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(
1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2
)/a^2/(-4*a*c+b^2)^(3/2)/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(
1/2)/(c*x^4+b*x^2+a)^(1/2)-1/3*2^(1/2)*(a*B*(5*b^2*d*e+12*a*c*d*e-8*b*(a*e
^2+c*d^2))+A*(b^3*d*e-20*a*b*c*d*e-b^2*(-2*a*e^2+c*d^2)+4*a*c*(2*a*e^2+5*c
*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+
b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)
*d-2*a*e))^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2260}$$

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}}{(c\,x^4+b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fr
icas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*
x^2 + d)/(c^3*x^12 + 3*b*c^2*x^10 + 3*(b^2*c + a*c^2)*x^8 + (b^3 + 6*a*b*c
)*x^6 + 3*a^2*b*x^2 + 3*(a*b^2 + a^2*c)*x^4 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{3/2}}{(c\,x^4+b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(5/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}}{(c\,x^4+b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

3.251 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 965

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)\sqrt{d+ex^2}}{3a(b^2-4ac)(a+bx^2+cx^4)^{3/2}}$$

$$+\frac{(aB(b^3de+12abcde-b^2(cd^2+2ae^2)-4ac(3cd^2+2ae^2))+A(2b^4de-17ab^2cde+4a^2c^2de-b^3(2cd^2+}{3a(b^2-4ac)^2(cd^2-bde+ae^2)x\sqrt{a+bx^2+cx^4}}$$

$$+\frac{c(aB(7b^2de+4acde-8b(cd^2+ae^2))-A(b^2cd^2-b^3de+20abcde-4ac(5cd^2+4ae^2)))x\sqrt{d+ex^2}}{3a(b^2-4ac)^2(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$

$$-\frac{(aB(b^3de+12abcde-b^2(cd^2+2ae^2)-4ac(3cd^2+2ae^2))+A(2b^4de-17ab^2cde+4a^2c^2de-b^3(2cd^2+}{3\sqrt{2}a^2(b^2-4ac)^{3/2}(cd^2-bde+ae^2)\sqrt{}}$$

$$+\frac{\sqrt{2}(2aB(4bcd-b^2e-4ace)+A(b^2cd-20ac^2d-b^3e+12abce))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3}{3a^2(b^2-4ac)^{3/2}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
1/3*x*(A*b^2-B*a*b-2*A*a*c+(A*b-2*B*a)*c*x^2)*(e*x^2+d)^(1/2)/a/(-4*a*c+b^
2)/(c*x^4+b*x^2+a)^(3/2)+1/3*(a*B*(b^3*d*e+12*a*b*c*d*e-b^2*(2*a*e^2+c*d^2
)-4*a*c*(2*a*e^2+3*c*d^2))+A*(2*b^4*d*e-17*a*b^2*c*d*e+4*a^2*c^2*d*e-b^3*(
a*e^2+2*c*d^2)+4*a*b*c*(3*a*e^2+4*c*d^2)))*(e*x^2+d)^(1/2)/a/(-4*a*c+b^2)^
2/(a*e^2-b*d*e+c*d^2)/x/(c*x^4+b*x^2+a)^(1/2)+1/3*c*(a*B*(7*b^2*d*e+4*a*c*
d*e-8*b*(a*e^2+c*d^2))-A*(b^2*c*d^2-b^3*d*e+20*a*b*c*d*e-4*a*c*(4*a*e^2+5*
c*d^2)))*x*(e*x^2+d)^(1/2)/a/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x
^2+a)^(1/2)-1/6*(a*B*(b^3*d*e+12*a*b*c*d*e-b^2*(2*a*e^2+c*d^2)-4*a*c*(2*a*
e^2+3*c*d^2))+A*(2*b^4*d*e-17*a*b^2*c*d*e+4*a^2*c^2*d*e-b^3*(a*e^2+2*c*d^2
)+4*a*b*c*(3*a*e^2+4*c*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e
*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*
2^(1/2)/a^2/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*d^2)/(-a*(e+d/x^2)/((b+(-4*a
*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/3*2^(1/2)*(2*a*B*(-
4*a*c*e-b^2*e+4*b*c*d)+A*(12*a*b*c*e-20*a*c^2*d-b^3*e+b^2*c*d))*(-a*(c+a/x
^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*
e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/
a^2/(-4*a*c+b^2)^(3/2)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{5/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4)^(5/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{5/2}}\,dx$$

$$\Big\downarrow \ 2260$$

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{5/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{e\,x^2+d}}{(c\,x^4+b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(A + Bx^2)\sqrt{d + ex^2}}{(a + bx^2 + cx^4)^{5/2}}\,dx = \int \frac{(Bx^2 + A)\sqrt{ex^2 + d}}{(cx^4 + bx^2 + a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c^3*x^12 + 3
*b*c^2*x^10 + 3*(b^2*c + a*c^2)*x^8 + (b^3 + 6*a*b*c)*x^6 + 3*a^2*b*x^2 +
3*(a*b^2 + a^2*c)*x^4 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + Bx^2)\sqrt{d + ex^2}}{(a + bx^2 + cx^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A + Bx^2)\sqrt{d + ex^2}}{(a + bx^2 + cx^4)^{5/2}}\,dx = \int \frac{(Bx^2 + A)\sqrt{ex^2 + d}}{(cx^4 + bx^2 + a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex^2+d}}{(cx^4+bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}}{(c\,x^4+b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4)^(5/2),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}}{(c\,x^4+b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

3.252 $\int \frac{A+Bx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1344

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{5/2}}\,dx = \text{Too large to display}$$

output
```
-1/3*x*(e*x^2+d)^(1/2)*(a*B*(2*a*c*e-b^2*e+b*c*d)-A*(3*a*b*c*e-2*a*c^2*d-b
^3*e+b^2*c*d)+c*(a*B*(-b*e+2*c*d)-A*(2*a*c*e-b^2*e+b*c*d))*x^2)/a/(-4*a*c+
b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(3/2)-1/3*(a*B*(b^4*d*e^2+b^2*c*d
*(a*e^2+c*d^2)-4*a*b*c*e*(3*a*e^2+4*c*d^2)+4*a*c^2*d*(7*a*e^2+3*c*d^2)-b^3
*(-a*e^3+2*c*d^2*e))+2*A*(b^5*d*e^2+b^3*c*d*(-6*a*e^2+c*d^2)-4*a*b*c^2*d*(
a*e^2+2*c*d^2)-8*a^2*c^2*e*(2*a*e^2+c*d^2)+2*a*b^2*c*e*(7*a*e^2+8*c*d^2)-2
*b^4*(a*e^3+c*d^2*e)))*(e*x^2+d)^(1/2)/a/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2
)^2/x/(c*x^4+b*x^2+a)^(1/2)+1/3*c*(2*a*B*(6*b^2*c*d^2*e-b^3*d*e^2+8*a^2*c*
e^3-4*b*c*d*(2*a*e^2+c*d^2))-A*(b^4*d*e^2+b^2*c*d*(-7*a*e^2+c*d^2)+4*a*b*c
*e*(5*a*e^2+8*c*d^2)-4*a*c^2*d*(9*a*e^2+5*c*d^2)-b^3*(3*a*e^3+2*c*d^2*e)))
*x*(e*x^2+d)^(1/2)/a/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)^
(1/2)+1/6*(a*B*(b^4*d*e^2+b^2*c*d*(a*e^2+c*d^2)-4*a*b*c*e*(3*a*e^2+4*c*d^2
)+4*a*c^2*d*(7*a*e^2+3*c*d^2)-b^3*(-a*e^3+2*c*d^2*e))+2*A*(b^5*d*e^2+b^3*c
*d*(-6*a*e^2+c*d^2)-4*a*b*c^2*d*(a*e^2+2*c*d^2)-8*a^2*c^2*e*(2*a*e^2+c*d^2
)+2*a*b^2*c*e*(7*a*e^2+8*c*d^2)-2*b^4*(a*e^3+c*d^2*e)))*(-a*(c+a/x^4+b/x^2
)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^
2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*d^
2)^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)
^(1/2)-1/3*2^(1/2)*(a*B*(7*b^2*c*d*e+4*a*c^2*d*e+b^3*e^2-4*b*c*(3*a*e^2...
```

Mathematica [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{5/2}}\,dx$$

input
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)^(5/2)),x]
```

output
```
Integrate[(A + B*x^2)/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2260}$$

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{5/2}}dx$$

input
```
Int[(A + B*x^2)/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^2+A}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{5}{2}}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c^3*e*x^14 +
 (c^3*d + 3*b*c^2*e)*x^12 + 3*(b*c^2*d + (b^2*c + a*c^2)*e)*x^10 + (3*(b^2
*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*x^8 + ((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a
^2*c)*e)*x^6 + 3*(a^2*b*e + (a*b^2 + a^2*c)*d)*x^4 + a^3*d + (3*a^2*b*d +
a^3*e)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)^{5/2}}\,dx=\int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{5}{2}}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(5/2)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\int \frac{Bx^2+A}{\left(cx^4+bx^2+a\right)^{\frac{5}{2}}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(5/2)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\int \frac{B\,x^2+A}{\sqrt{e\,x^2+d}\,{\left(c\,x^4+b\,x^2+a\right)}^{5/2}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(5/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\int \frac{B\,x^2+A}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

3.253 $\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 2023

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{5/2}}\,dx = \text{Too large to display}$$

output
```
-1/3*x*(a*B*(2*a*c*e-b^2*e+b*c*d)-A*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)+c*
(a*B*(-b*e+2*c*d)-A*(2*a*c*e-b^2*e+b*c*d))*x^2)/a/(-4*a*c+b^2)/(a*e^2-b*d*
e+c*d^2)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(3/2)-1/3*e*(2*a*B*d*(c^2*d^2+2*b
^2*e^2-c*e*(7*a*e+b*d))-A*(b^3*d*e^2+b*c*d*(-3*a*e^2+c*d^2)+4*a*c*e*(-3*a*
e^2+c*d^2)-b^2*(-3*a*e^3+2*c*d^2*e)))*x/a/(-4*a*c+b^2)/d/(a*e^2-b*d*e+c*d^
2)^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/3*(a*B*d*(b^5*d*e^3+b^3*c*d*e
*(-10*a*e^2+3*c*d^2)+4*a*b*c^2*d*e*(14*a*e^2+5*c*d^2)-b^4*(-7*a*e^4+3*c*d^
2*e^2)-4*a*c^2*(-20*a^2*e^4+15*a*c*d^2*e^2+3*c^2*d^4)-b^2*c*(52*a^2*e^4-3*
a*c*d^2*e^2+c^2*d^4))+A*(2*b^6*d^2*e^3-b^3*c*d*(-50*a^2*e^4-39*a*c*d^2*e^2
+2*c^2*d^4)-a*b^2*c*e*(-24*a^2*e^4+71*a*c*d^2*e^2+47*c^2*d^4)+4*a*b*c^2*d*
(-20*a^2*e^4-3*a*c*d^2*e^2+4*c^2*d^4)+4*a^2*c^2*e*(-12*a^2*e^4+27*a*c*d^2*
e^2+7*c^2*d^4)-b^5*(7*a*d*e^4+6*c*d^3*e^2)+b^4*(-3*a^2*e^5-a*c*d^2*e^3+6*c
^2*d^4*e)))*(e*x^2+d)^(1/2)/a/(-4*a*c+b^2)^2/d/(a*e^2-b*d*e+c*d^2)^3/x/(c*
x^4+b*x^2+a)^(1/2)+1/3*c*(A*(b^5*d*e^3+b^3*c*d*e*(4*a*e^2+3*c*d^2)-4*a*b*c
^2*d*e*(16*a*e^2+11*c*d^2)-3*b^4*(3*a*e^4+c*d^2*e^2)-b^2*c*(-62*a^2*e^4-21
*a*c*d^2*e^2+c^2*d^4)+4*a*c^2*(-22*a^2*e^4+15*a*c*d^2*e^2+5*c^2*d^4))+a*B*
(5*b^4*d*e^3+b^2*c*d*e*(-35*a*e^2+17*c*d^2)-4*a*c^2*d*e*(-31*a*e^2+c*d^2)-
b^3*(-3*a*e^4+6*c*d^2*e^2)-4*b*c*(5*a^2*e^4+6*a*c*d^2*e^2+2*c^2*d^4)))*x*(
e*x^2+d)^(1/2)/a/(-4*a*c+b^2)^2/(a*e^2-b*d*e+c*d^2)^3/(c*x^4+b*x^2+a)^(1/2
)-1/6*(a*B*d*(b^5*d*e^3+b^3*c*d*e*(-10*a*e^2+3*c*d^2)+4*a*b*c^2*d*e*(14...
```

Mathematica [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{5/2}}\,dx$$

input
```
Integrate[(A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(5/2)),x]
```

output
```
Integrate[(A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(5/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{5/2}}\,dx$$
$$\Big\downarrow \; 2260$$
$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{5/2}}dx$$

input
```
Int[(A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{B\,x^2+A}{(e\,x^2+d)^{\frac{3}{2}}\,(c\,x^4+b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\int \frac{Bx^2+A}{\left(cx^4+bx^2+a\right)^{\frac{5}{2}}\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(c^3*e^2*x^16
 + (2*c^3*d*e + 3*b*c^2*e^2)*x^14 + (c^3*d^2 + 6*b*c^2*d*e + 3*(b^2*c + a*
c^2)*e^2)*x^12 + (3*b*c^2*d^2 + 6*(b^2*c + a*c^2)*d*e + (b^3 + 6*a*b*c)*e^
2)*x^10 + (3*(b^2*c + a*c^2)*d^2 + 2*(b^3 + 6*a*b*c)*d*e + 3*(a*b^2 + a^2*
c)*e^2)*x^8 + (3*a^2*b*e^2 + (b^3 + 6*a*b*c)*d^2 + 6*(a*b^2 + a^2*c)*d*e)*
x^6 + a^3*d^2 + (6*a^2*b*d*e + a^3*e^2 + 3*(a*b^2 + a^2*c)*d^2)*x^4 + (3*a
^2*b*d^2 + 2*a^3*d*e)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{5}{2}}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(5/2)*(e*x^2 + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{5}{2}}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(5/2)*(e*x^2 + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{B\,x^2+A}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)^{5/2}}\,dx$$

input
```
int((A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(5/2)),x)
```

output
```
int((A + B*x^2)/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^{3/2}\,(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{B\,x^2+A}{(e\,x^2+d)^{\frac{3}{2}}\,(c\,x^4+b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

output
```
int((B*x^2+A)/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a)^(5/2),x)
```

3.254 $\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 697

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{C\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{2cex}$$

$$-\frac{\sqrt{b^2-4ac}\,C\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}\,d}{bd+\sqrt{b^2-4ac}\,d-2ae}\right)}{2\sqrt{2}ce\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{b^2-4ac}(2Ac-aC)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{}}{bd+\sqrt{b^2}}\right.}{\sqrt{2}ac\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(cCd-2Bce+bCe)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\right.}{c\left(b+\sqrt{b^2-4ac}\right)e\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output

```
1/2*C*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e/x-1/4*(-4*a*c+b^2)^(1/2)*C
*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(
1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2
)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c/e/(-a*(e+d/x^2)/((b
+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/2*(-4*a*c+b^2
)^(1/2)*(2*A*c-C*a)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/
((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*
c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/c/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)-2^(1/2)*(-4*a*c+b^2)^(1/2)*(-2*B*c*e+C*b*e+C*c*d)*(-a*(c+a/x^4+b/x^2
)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2
)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-
4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d
+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))/e/(e*x^2+d)^
(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input

```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output

```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]), x
]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 2260$$

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^4+B\,x^2+A}{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)
), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x
)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}dx$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(d + e*x**2),x)
```

3.255 $\int (A+Bx^2)(d+ex^2)^q(a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 318

$$\int (A+Bx^2)(d+ex^2)^q(a+bx^2+cx^4)\,dx = -\frac{(Ae(7+2q)(3cd-be(5+2q))-B(15cd^2-e(7+2q)(3bd-ae(5+2q))))x(d+ex^2)^{1+q}}{e^3(3+2q)(5+2q)(7+2q)} + \frac{(Ace(7+2q)-B(5cd-be(7+2q)))x^3(d+ex^2)^{1+q}}{e^2(5+2q)(7+2q)} + \frac{Bcx^5(d+ex^2)^{1+q}}{e(7+2q)} + \frac{(Ae(7+2q)(3cd^2-e(5+2q)(bd-ae(3+2q)))-Bd(15cd^2-e(7+2q)(3bd-ae(5+2q))))x(d+e}{e^3(3+2q)(5+2q)(7+2q)}$$

output
```
-(A*e*(7+2*q)*(3*c*d-b*e*(5+2*q))-B*(15*c*d^2-e*(7+2*q)*(3*b*d-a*e*(5+2*q)
)))*x*(e*x^2+d)^(1+q)/e^3/(3+2*q)/(5+2*q)/(7+2*q)+(A*c*e*(7+2*q)-B*(5*c*d-
b*e*(7+2*q)))*x^3*(e*x^2+d)^(1+q)/e^2/(5+2*q)/(7+2*q)+B*c*x^5*(e*x^2+d)^(1
+q)/e/(7+2*q)+(A*e*(7+2*q)*(3*c*d^2-e*(5+2*q)*(b*d-a*e*(3+2*q)))-B*d*(15*c
*d^2-e*(7+2*q)*(3*b*d-a*e*(5+2*q))))*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/
2],-e*x^2/d)/e^3/(3+2*q)/(5+2*q)/(7+2*q)/((1+e*x^2/d)^q)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.44

$$\int (A+Bx^2)(d+ex^2)^q(a+bx^2+cx^4)\,dx$$
$$=\frac{1}{105}x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\left(105aA\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)+35(Ab+aB)x^2\,\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)+21(bB+Ac)x^4\,\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right)+15Bcx^6\,\text{Hypergeometric2F1}\left(\frac{7}{2},-q,\frac{9}{2},-\frac{ex^2}{d}\right)\right)$$

input
```
Integrate[(A + B*x^2)*(d + e*x^2)^q*(a + b*x^2 + c*x^4),x]
```

output
```
(x*(d + e*x^2)^q*(105*a*A*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)] +
35*(A*b + a*B)*x^2*Hypergeometric2F1[3/2, -q, 5/2, -((e*x^2)/d)] + 21*(b*B
 + A*c)*x^4*Hypergeometric2F1[5/2, -q, 7/2, -((e*x^2)/d)] + 15*B*c*x^6*Hyp
ergeometric2F1[7/2, -q, 9/2, -((e*x^2)/d)]))/(105*(1 + (e*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.66, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2)(a+bx^2+cx^4)(d+ex^2)^q\,dx$$
$$\Big\downarrow \; 2256$$
$$\int \left(x^2(aB+Ab)(d+ex^2)^q+aA(d+ex^2)^q+x^4(Ac+bB)(d+ex^2)^q+Bcx^6(d+ex^2)^q\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{1}{3}x^3(aB+Ab)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)+ aAx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)+\frac{1}{5}x^5(Ac+ bB)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right)+ \frac{1}{7}Bcx^7\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{7}{2},-q,\frac{9}{2},-\frac{ex^2}{d}\right)$$

input
```
Int[(A + B*x^2)*(d + e*x^2)^q*(a + b*x^2 + c*x^4),x]
```

output
```
(a*A*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(1 + (
e*x^2)/d)^q + ((A*b + a*B)*x^3*(d + e*x^2)^q*Hypergeometric2F1[3/2, -q, 5/
2, -((e*x^2)/d)])/(3*(1 + (e*x^2)/d)^q) + ((b*B + A*c)*x^5*(d + e*x^2)^q*H
ypergeometric2F1[5/2, -q, 7/2, -((e*x^2)/d)])/(5*(1 + (e*x^2)/d)^q) + (B*c
*x^7*(d + e*x^2)^q*Hypergeometric2F1[7/2, -q, 9/2, -((e*x^2)/d)])/(7*(1 +
(e*x^2)/d)^q)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right)\left(e\,x^{2}+d\right)^{q}\left(c\,x^{4}+b\,x^{2}+a\right)dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^q*(c*x^4+b*x^2+a),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^q*(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^q\left(a+bx^2+cx^4\right)\,dx=\int \left(cx^{4}+bx^{2}+a\right)\left(Bx^{2}+A\right)\left(ex^{2}+d\right)^{q}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((B*c*x^6 + (B*b + A*c)*x^4 + (B*a + A*b)*x^2 + A*a)*(e*x^2 + d)^q
, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.06 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.56

$$\int (A+Bx^2)(d+ex^2)^q(a+bx^2+cx^4)\,dx$$
$$= Aad^q x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-q\\ \frac{3}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)+\frac{Abd^q x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-q\\ \frac{5}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3}$$
$$+\frac{Acd^q x^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},-q\\ \frac{7}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{5}+\frac{Bad^q x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-q\\ \frac{5}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3}$$
$$+\frac{Bbd^q x^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2},-q\\ \frac{7}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{5}+\frac{Bcd^q x^7\,{}_2F_1\left(\begin{matrix}\frac{7}{2},-q\\ \frac{9}{2}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{7}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**q*(c*x**4+b*x**2+a),x)
```

output
```
A*a*d**q*x*hyper((1/2, -q), (3/2,), e*x**2*exp_polar(I*pi)/d) + A*b*d**q*x
**3*hyper((3/2, -q), (5/2,), e*x**2*exp_polar(I*pi)/d)/3 + A*c*d**q*x**5*h
yper((5/2, -q), (7/2,), e*x**2*exp_polar(I*pi)/d)/5 + B*a*d**q*x**3*hyper(
(3/2, -q), (5/2,), e*x**2*exp_polar(I*pi)/d)/3 + B*b*d**q*x**5*hyper((5/2,
 -q), (7/2,), e*x**2*exp_polar(I*pi)/d)/5 + B*c*d**q*x**7*hyper((7/2, -q),
 (9/2,), e*x**2*exp_polar(I*pi)/d)/7
```

Maxima [F]

$$\int (A+Bx^2)(d+ex^2)^q(a+bx^2+cx^4)\,dx=\int (cx^4+bx^2+a)(Bx^2+A)(ex^2+d)^q\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^q, x)
```

Giac [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^q\left(a+bx^2+cx^4\right)\,dx=\int \left(cx^4+bx^2+a\right)\left(Bx^2+A\right)\left(ex^2+d\right)^q\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^q\left(a+bx^2+cx^4\right)\,dx$$
$$=\int \left(B\,x^2+A\right)\,\left(e\,x^2+d\right)^q\,\left(c\,x^4+b\,x^2+a\right)\,dx$$

input
```
int((A + B*x^2)*(d + e*x^2)^q*(a + b*x^2 + c*x^4),x)
```

output
```
int((A + B*x^2)*(d + e*x^2)^q*(a + b*x^2 + c*x^4), x)
```

Reduce [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^q\left(a+bx^2+cx^4\right)\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^q*(c*x^4+b*x^2+a),x)
```

output
```
(8*(d + e*x**2)**q*a**2*e**3*q**3*x + 60*(d + e*x**2)**q*a**2*e**3*q**2*x
+ 142*(d + e*x**2)**q*a**2*e**3*q*x + 105*(d + e*x**2)**q*a**2*e**3*x + 16
*(d + e*x**2)**q*a*b*d*e**2*q**3*x + 96*(d + e*x**2)**q*a*b*d*e**2*q**2*x
+ 140*(d + e*x**2)**q*a*b*d*e**2*q*x + 16*(d + e*x**2)**q*a*b*e**3*q**3*x*
*3 + 104*(d + e*x**2)**q*a*b*e**3*q**2*x**3 + 188*(d + e*x**2)**q*a*b*e**3
*q*x**3 + 70*(d + e*x**2)**q*a*b*e**3*x**3 - 12*(d + e*x**2)**q*a*c*d**2*e
*q**2*x - 42*(d + e*x**2)**q*a*c*d**2*e*q*x + 8*(d + e*x**2)**q*a*c*d*e**2
*q**3*x**3 + 32*(d + e*x**2)**q*a*c*d*e**2*q**2*x**3 + 14*(d + e*x**2)**q*
a*c*d*e**2*q*x**3 + 8*(d + e*x**2)**q*a*c*e**3*q**3*x**5 + 44*(d + e*x**2)
**q*a*c*e**3*q**2*x**5 + 62*(d + e*x**2)**q*a*c*e**3*q*x**5 + 21*(d + e*x*
*2)**q*a*c*e**3*x**5 - 12*(d + e*x**2)**q*b**2*d**2*e*q**2*x - 42*(d + e*x
**2)**q*b**2*d**2*e*q*x + 8*(d + e*x**2)**q*b**2*d*e**2*q**3*x**3 + 32*(d
+ e*x**2)**q*b**2*d*e**2*q**2*x**3 + 14*(d + e*x**2)**q*b**2*d*e**2*q*x**3
 + 8*(d + e*x**2)**q*b**2*e**3*q**3*x**5 + 44*(d + e*x**2)**q*b**2*e**3*q*
*2*x**5 + 62*(d + e*x**2)**q*b**2*e**3*q*x**5 + 21*(d + e*x**2)**q*b**2*e*
*3*x**5 + 30*(d + e*x**2)**q*b*c*d**3*q*x - 20*(d + e*x**2)**q*b*c*d**2*e*
q**2*x**3 - 10*(d + e*x**2)**q*b*c*d**2*e*q*x**3 + 8*(d + e*x**2)**q*b*c*d
*e**2*q**3*x**5 + 16*(d + e*x**2)**q*b*c*d*e**2*q**2*x**5 + 6*(d + e*x**2)
**q*b*c*d*e**2*q*x**5 + 8*(d + e*x**2)**q*b*c*e**3*q**3*x**7 + 36*(d + e*x
**2)**q*b*c*e**3*q**2*x**7 + 46*(d + e*x**2)**q*b*c*e**3*q*x**7 + 15*(d...
```

3.256 $\int (A+Bx^2)(d+ex^2)^q\,dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int (A+Bx^2)(d+ex^2)^q\,dx = \frac{Bx(d+ex^2)^{1+q}}{e(3+2q)} - \frac{(Bd-Ae(3+2q))x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{e(3+2q)}$$

output
```
B*x*(e*x^2+d)^(1+q)/e/(3+2*q)-(B*d-A*e*(3+2*q))*x*(e*x^2+d)^q*hypergeom([1
/2, -q],[3/2],-e*x^2/d)/e/(3+2*q)/((1+e*x^2/d)^q)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int (A+Bx^2)(d+ex^2)^q\,dx = \frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\left(B(d+ex^2)\left(1+\frac{ex^2}{d}\right)^q+(-Bd+Ae(3+2q))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\right.\right.}{e(3+2q)}$$

input
```
Integrate[(A + B*x^2)*(d + e*x^2)^q,x]
```

output
```
(x*(d + e*x^2)^q*(B*(d + e*x^2)*(1 + (e*x^2)/d)^q + (-(B*d) + A*e*(3 + 2*q
))*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)]))/(e*(3 + 2*q)*(1 + (e*x^
2)/d)^q)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^q\,dx$$

$$\Big\downarrow \text{ 299}$$

$$\left(A-\frac{Bd}{2eq+3e}\right)\int \left(ex^2+d\right)^q dx+\frac{Bx\left(d+ex^2\right)^{q+1}}{e(2q+3)}$$

$$\Big\downarrow \text{ 238}$$

$$\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(A-\frac{Bd}{2eq+3e}\right)\int \left(\frac{ex^2}{d}+1\right)^q dx+\frac{Bx\left(d+ex^2\right)^{q+1}}{e(2q+3)}$$

$$\Big\downarrow \text{ 237}$$

$$x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(A-\frac{Bd}{2eq+3e}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)+\frac{Bx\left(d+ex^2\right)^{q+1}}{e(2q+3)}$$

input
```
Int[(A + B*x^2)*(d + e*x^2)^q,x]
```

output
```
(B*x*(d + e*x^2)^(1 + q))/(e*(3 + 2*q)) + ((A - (B*d)/(3*e + 2*e*q))*x*(d
+ e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(1 + (e*x^2)/d)^
q
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right)\left(e\,x^{2}+d\right)^{q}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^q,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^q,x)
```

Fricas [F]

$$\int \left(A+Bx^2\right) \left(d+ex^2\right)^q \, dx = \int \left(Bx^2+A\right)\left(ex^2+d\right)^q \, dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x^2 + d)^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.53 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int \left(A+Bx^2\right) \left(d+ex^2\right)^q \, dx = Ad^q x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -q \\ \frac{3}{2} \end{matrix}\middle| \frac{ex^2e^{i\pi}}{d}\right) + \frac{Bd^q x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -q \\ \frac{5}{2} \end{matrix}\middle| \frac{ex^2e^{i\pi}}{d}\right)}{3}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**q,x)
```

output
```
A*d**q*x*hyper((1/2, -q), (3/2,), e*x**2*exp_polar(I*pi)/d) + B*d**q*x**3*
hyper((3/2, -q), (5/2,), e*x**2*exp_polar(I*pi)/d)/3
```

Maxima [F]

$$\int \left(A+Bx^2\right) \left(d+ex^2\right)^q \, dx = \int \left(Bx^2+A\right)\left(ex^2+d\right)^q \, dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q, x)
```

Giac [F]

$$\int (A+Bx^2) (d+ex^2)^q \, dx = \int (Bx^2+A)(ex^2+d)^q \, dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2) (d+ex^2)^q \, dx = \int (B\,x^2+A) \, (e\,x^2+d)^q \, dx$$

input
```
int((A + B*x^2)*(d + e*x^2)^q,x)
```

output
```
int((A + B*x^2)*(d + e*x^2)^q, x)
```

Reduce [F]

$$\int (A+Bx^2) (d+ex^2)^q \, dx$$
$$= \frac{2(e\,x^2+d)^q\,aeqx+3(e\,x^2+d)^q\,aex+2(e\,x^2+d)^q\,bdqx+2(e\,x^2+d)^q\,beq\,x^3+(e\,x^2+d)^q\,be\,x^3+16\Big(\int}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^q,x)
```

output
```
(2*(d + e*x**2)**q*a*e*q*x + 3*(d + e*x**2)**q*a*e*x + 2*(d + e*x**2)**q*b
*d*q*x + 2*(d + e*x**2)**q*b*e*q*x**3 + (d + e*x**2)**q*b*e*x**3 + 16*int(
(d + e*x**2)**q/(4*d*q**2 + 8*d*q + 3*d + 4*e*q**2*x**2 + 8*e*q*x**2 + 3*e
*x**2),x)*a*d*e*q**4 + 56*int((d + e*x**2)**q/(4*d*q**2 + 8*d*q + 3*d + 4*
e*q**2*x**2 + 8*e*q*x**2 + 3*e*x**2),x)*a*d*e*q**3 + 60*int((d + e*x**2)**
q/(4*d*q**2 + 8*d*q + 3*d + 4*e*q**2*x**2 + 8*e*q*x**2 + 3*e*x**2),x)*a*d*
e*q**2 + 18*int((d + e*x**2)**q/(4*d*q**2 + 8*d*q + 3*d + 4*e*q**2*x**2 +
8*e*q*x**2 + 3*e*x**2),x)*a*d*e*q - 8*int((d + e*x**2)**q/(4*d*q**2 + 8*d*
q + 3*d + 4*e*q**2*x**2 + 8*e*q*x**2 + 3*e*x**2),x)*b*d**2*q**3 - 16*int((
d + e*x**2)**q/(4*d*q**2 + 8*d*q + 3*d + 4*e*q**2*x**2 + 8*e*q*x**2 + 3*e*
x**2),x)*b*d**2*q**2 - 6*int((d + e*x**2)**q/(4*d*q**2 + 8*d*q + 3*d + 4*e
*q**2*x**2 + 8*e*q*x**2 + 3*e*x**2),x)*b*d**2*q)/(e*(4*q**2 + 8*q + 3))
```

3.257 $\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 31, antiderivative size = 218

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4} dx = \frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{b-\sqrt{b^2-4ac}} + \frac{\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{b+\sqrt{b^2-4ac}}$$

output
```
(B-(-2*A*c+B*b)/(-4*a*c+b^2)^(1/2))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2
*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)/(b-(-4*a*c+b^2)^(1/2))/((1+e*x^2/d
)^q)+(B+(-2*A*c+B*b)/(-4*a*c+b^2)^(1/2))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3
/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/(b+(-4*a*c+b^2)^(1/2))/((1+e*
x^2/d)^q)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2256}$$

$$\int \left(\frac{(d+ex^2)^q\left(\frac{2Ac-bB}{\sqrt{b^2-4ac}}+B\right)}{-\sqrt{b^2-4ac}+b+2cx^2}+\frac{(d+ex^2)^q\left(B-\frac{2Ac-bB}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}+b+2cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b-\sqrt{b^2-4ac}}+\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}+b}$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

output
```
((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q
, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/((b - Sqrt[b^2 -
 4*a*c])*(1 + (e*x^2)/d)^q) + ((B + (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*x*(d
+ e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -
((e*x^2)/d)])/((b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,(e\,x^{2}+d)^{q}}{c\,x^{4}+b\,x^{2}+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^q}{a+bx^2+cx^4}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(B\,x^2+A)\,\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \left(\int \frac{\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}dx\right)a+\left(\int \frac{\left(e\,x^2+d\right)^q x^2}{c\,x^4+b\,x^2+a}dx\right)b$$

input
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

output
```
int((d + e*x**2)**q/(a + b*x**2 + c*x**4),x)*a + int(((d + e*x**2)**q*x**2
)/(a + b*x**2 + c*x**4),x)*b
```

3.258 $\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 857

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+bx^2+cx^4)^2}\,dx =$$

$$-\frac{x(d+ex^2)^{1+q}\left(aB(bcd-b^2e+2ace)-A(b^2cd-2ac^2d-b^3e+3abce)+c(aB(2cd-be)-A(bcd-\right.}{2a(b^2-4ac)(cd^2-bde+ae^2)(a+bx^2+cx^4)}$$

$$-\frac{c\Big(2aB(cd^2+e(ae(1-2q)-bd(1-q)))+A(b^2de-b(cd^2+ae^2(1-2q))-4acdeq)+\frac{A(b^3de-12abcde-}{}}{2a(b^2-}$$

$$-\frac{c\Big(2aB(cd^2+e(ae(1-2q)-bd(1-q)))+A(b^2de-b(cd^2+ae^2(1-2q))-4acdeq)-\frac{A(b^3de-12abcde-}{}}{2a(b^2-}$$

$$+\frac{e(aB(2cd-be)-A(bcd-b^2e+2ace))(1+2q)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\right.}{2a(b^2-4ac)(cd^2-bde+ae^2)}$$

output
```
-1/2*x*(e*x^2+d)^(1+q)*(a*B*(2*a*c*e-b^2*e+b*c*d)-A*(3*a*b*c*e-2*a*c^2*d-b
^3*e+b^2*c*d)+c*(a*B*(-b*e+2*c*d)-A*(2*a*c*e-b^2*e+b*c*d))*x^2)/a/(-4*a*c+
b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)-1/2*c*(2*a*B*(c*d^2+e*(a*e*(1-2*q
)-b*d*(1-q)))+A*(b^2*d*e-b*(c*d^2+a*e^2*(1-2*q))-4*a*c*d*e*q)+(A*(b^3*d*e-
12*a*b*c*d*e-b^2*(c*d^2+a*e^2*(1-2*q))+4*a*c*(3*c*d^2+a*e^2*(3-2*q)))-2*a*
B*(2*b*(a*e^2+c*d^2)-b^2*d*e*(2-q)-4*a*c*d*e*q))/(-4*a*c+b^2)^(1/2))*x*(e*
x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)/a
/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/((1+e*x^2/d)^q)-1
/2*c*(2*a*B*(c*d^2+e*(a*e*(1-2*q)-b*d*(1-q)))+A*(b^2*d*e-b*(c*d^2+a*e^2*(1
-2*q))-4*a*c*d*e*q)-(A*(b^3*d*e-12*a*b*c*d*e-b^2*(c*d^2+a*e^2*(1-2*q))+4*a
*c*(3*c*d^2+a*e^2*(3-2*q)))-2*a*B*(2*b*(a*e^2+c*d^2)-b^2*d*e*(2-q)-4*a*c*d
*e*q))/(-4*a*c+b^2)^(1/2))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b
+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/a/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))/(a*e^
2-b*d*e+c*d^2)/((1+e*x^2/d)^q)+1/2*e*(a*B*(-b*e+2*c*d)-A*(2*a*c*e-b^2*e+b*
c*d))*(1+2*q)*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/a/(-4*a*c+
b^2)/(a*e^2-b*d*e+c*d^2)/((1+e*x^2/d)^q)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+bx^2+cx^4)^2}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+bx^2+cx^4)^2}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 2256$$

$$\int \left(\frac{A(d+ex^2)^q}{(a+bx^2+cx^4)^2} + \frac{Bx^2(d+ex^2)^q}{(a+bx^2+cx^4)^2} \right) dx$$

$$\Big\downarrow \text{2009}$$

$$A \int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)^2} dx + B \int \frac{x^2(ex^2+d)^q}{(cx^4+bx^2+a)^2} dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^q}{(c\,x^4+b\,x^2+a)^2} dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(e*x^2 + d)^q/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^
4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**q/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(A + Bx^2)(d + ex^2)^q}{(a + bx^2 + cx^4)^2}\,dx = \int \frac{(Bx^2 + A)(ex^2 + d)^q}{(cx^4 + bx^2 + a)^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + Bx^2)(d + ex^2)^q}{(a + bx^2 + cx^4)^2}\,dx = \int \frac{(B\,x^2 + A)\,(e\,x^2 + d)^q}{(c\,x^4 + b\,x^2 + a)^2}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(A + Bx^2)(d + ex^2)^q}{(a + bx^2 + cx^4)^2}\,dx = \left(\int \frac{(e\,x^2 + d)^q}{c^2x^8 + 2bc\,x^6 + 2ac\,x^4 + b^2x^4 + 2ab\,x^2 + a^2}dx\right)a + \left(\int \frac{(e\,x^2 + d)^q\,x^2}{c^2x^8 + 2bc\,x^6 + 2ac\,x^4 + b^2x^4 + 2ab\,x^2 + a^2}dx\right)b$$

input
```
int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((d + e*x**2)**q/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**
*6 + c**2*x**8),x)*a + int(((d + e*x**2)**q*x**2)/(a**2 + 2*a*b*x**2 + 2*a
*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*b
```

3.259 $\int (d+ex^2)^q (a+bx^2+cx^4)(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 529

$$\int (d+ex^2)^q (a+bx^2+cx^4)(A+Bx^2+Cx^4)\,dx =$$

$$-\frac{(3c(35Cd^3-de(9+2q)(5Bd-Ae(7+2q)))+e(9+2q)\left(ae(7+2q)(3Cd-Be(5+2q))-b(15Cd^2\right.}{e^4(3+2q)(5+2q)(7+2q)(9+2q)}$$

$$-\frac{(e(9+2q)(5bCd-bBe(7+2q)-aCe(7+2q))-c(35Cd^2-e(9+2q)(5Bd-Ae(7+2q))))\,x^3(d+}{e^3(5+2q)(7+2q)(9+2q)}$$

$$-\frac{(7cCd-Bce(9+2q)-bCe(9+2q))x^5(d+ex^2)^{1+q}}{e^2(7+2q)(9+2q)}+\frac{cCx^7(d+ex^2)^{1+q}}{e(9+2q)}$$

$$+\frac{\left(aAe^4(315+286q+84q^2+8q^3)-\frac{d\left(e(9+2q)\left(15bCd^2-be(7+2q)(3Bd-Ae(5+2q))-ae(7+2q)(3Cd-Be(5+2q))\right)-3cd(35Cd\right.}{3+2q}\right.}{e^4(5+2q)(7+2q)(9}$$

output
```
-(3*c*(35*C*d^3-d*e*(9+2*q)*(5*B*d-A*e*(7+2*q)))+e*(9+2*q)*(a*e*(7+2*q)*(3
*C*d-B*e*(5+2*q))-b*(15*C*d^2-e*(7+2*q)*(3*B*d-A*e*(5+2*q)))))*x*(e*x^2+d)
^(1+q)/e^4/(3+2*q)/(5+2*q)/(7+2*q)/(9+2*q)-(e*(9+2*q)*(5*C*b*d-b*B*e*(7+2*
q)-a*C*e*(7+2*q))-c*(35*C*d^2-e*(9+2*q)*(5*B*d-A*e*(7+2*q))))*x^3*(e*x^2+d
)^(1+q)/e^3/(5+2*q)/(7+2*q)/(9+2*q)-(7*C*c*d-B*c*e*(9+2*q)-b*C*e*(9+2*q))*
x^5*(e*x^2+d)^(1+q)/e^2/(7+2*q)/(9+2*q)+c*C*x^7*(e*x^2+d)^(1+q)/e/(9+2*q)+
(a*A*e^4*(8*q^3+84*q^2+286*q+315)-d*(e*(9+2*q)*(15*b*C*d^2-b*e*(7+2*q)*(3*
B*d-A*e*(5+2*q))-a*e*(7+2*q)*(3*C*d-B*e*(5+2*q)))-3*c*d*(35*C*d^2-e*(9+2*q
)*(5*B*d-A*e*(7+2*q))))/(3+2*q))*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-
e*x^2/d)/e^4/(5+2*q)/(7+2*q)/(9+2*q)/((1+e*x^2/d)^q)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.33

$$\begin{aligned}\int (d+ex^2)^q\,(a+bx^2+cx^4)\,(A+Bx^2+Cx^4)\;dx &= \frac{1}{315}x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\left(315aA\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)\right.\\ &\quad+105(Ab+aB)x^2\,\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)\\ &\quad+63(bB+Ac+aC)x^4\,\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right)\\ &\quad+45(Bc+bC)x^6\,\text{Hypergeometric2F1}\left(\frac{7}{2},-q,\frac{9}{2},-\frac{ex^2}{d}\right)\\ &\quad\left.+35cCx^8\,\text{Hypergeometric2F1}\left(\frac{9}{2},-q,\frac{11}{2},-\frac{ex^2}{d}\right)\right)\end{aligned}$$

input
```
Integrate[(d + e*x^2)^q*(a + b*x^2 + c*x^4)*(A + B*x^2 + C*x^4),x]
```

output
```
(x*(d + e*x^2)^q*(315*a*A*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)] +
105*(A*b + a*B)*x^2*Hypergeometric2F1[3/2, -q, 5/2, -((e*x^2)/d)] + 63*(b*
B + A*c + a*C)*x^4*Hypergeometric2F1[5/2, -q, 7/2, -((e*x^2)/d)] + 45*(B*c
 + b*C)*x^6*Hypergeometric2F1[7/2, -q, 9/2, -((e*x^2)/d)] + 35*c*C*x^8*Hyp
ergeometric2F1[9/2, -q, 11/2, -((e*x^2)/d)]))/(315*(1 + (e*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.51, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)\left(A+Bx^2+Cx^4\right)\left(d+ex^2\right)^q\,dx$$

$$\Big\downarrow \text{ 2256}$$

$$\int \left(x^4\left(d+ex^2\right)^q(aC+Ac+bB)+x^2(aB+Ab)\left(d+ex^2\right)^q+aA\left(d+ex^2\right)^q+x^6(bC+Bc)\left(d+ex^2\right)^q+cCx^8$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right)(aC+Ac+bB)+\frac{1}{3}x^3(aB+Ab)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)+aAx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)+\frac{1}{7}x^7(bC+Bc)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{7}{2},-q,\frac{9}{2},-\frac{ex^2}{d}\right)+\frac{1}{9}cCx^9\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{9}{2},-q,\frac{11}{2},-\frac{ex^2}{d}\right)$$

input
```
Int[(d + e*x^2)^q*(a + b*x^2 + c*x^4)*(A + B*x^2 + C*x^4),x]
```

output
```
(a*A*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(1 + (
e*x^2)/d)^q + ((A*b + a*B)*x^3*(d + e*x^2)^q*Hypergeometric2F1[3/2, -q, 5/
2, -((e*x^2)/d)])/(3*(1 + (e*x^2)/d)^q) + ((b*B + A*c + a*C)*x^5*(d + e*x^
2)^q*Hypergeometric2F1[5/2, -q, 7/2, -((e*x^2)/d)])/(5*(1 + (e*x^2)/d)^q)
+ ((B*c + b*C)*x^7*(d + e*x^2)^q*Hypergeometric2F1[7/2, -q, 9/2, -((e*x^2)
/d)])/(7*(1 + (e*x^2)/d)^q) + (c*C*x^9*(d + e*x^2)^q*Hypergeometric2F1[9/2
, -q, 11/2, -((e*x^2)/d)])/(9*(1 + (e*x^2)/d)^q)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)^{q} \left(c\,x^{4}+b\,x^{2}+a\right) \left(C\,x^{4}+B\,x^{2}+A\right)dx$$

input
```
int((e*x^2+d)^q*(c*x^4+b*x^2+a)*(C*x^4+B*x^2+A),x)
```

output
```
int((e*x^2+d)^q*(c*x^4+b*x^2+a)*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \left(d+ex^2\right)^q \left(a+bx^2+cx^4\right) \left(A+Bx^2+Cx^4\right) \, dx$$
$$= \int \left(Cx^{4}+Bx^{2}+A\right)\left(cx^{4}+bx^{2}+a\right)\left(ex^{2}+d\right)^{q}\,dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+b*x^2+a)*(C*x^4+B*x^2+A),x, algorithm="fricas
")
```

output
```
integral((C*c*x^8 + (C*b + B*c)*x^6 + (C*a + B*b + A*c)*x^4 + (B*a + A*b)*
x^2 + A*a)*(e*x^2 + d)^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 60.59 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.51

$$\int (d + ex^2)^q (a + bx^2 + cx^4) (A + Bx^2 + Cx^4) \, dx$$

$$= Aad^q x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -q \\ \frac{3}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right) + \frac{Abd^q x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -q \\ \frac{5}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{3} + \frac{Acd^q x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -q \\ \frac{7}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{5} + \frac{Bad^q x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -q \\ \frac{5}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{3} + \frac{Bbd^q x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -q \\ \frac{7}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{5} + \frac{Bcd^q x^7 {}_2F_1\left(\begin{matrix} \frac{7}{2}, -q \\ \frac{9}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{7} + \frac{Cad^q x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -q \\ \frac{7}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{5} + \frac{Cbd^q x^7 {}_2F_1\left(\begin{matrix} \frac{7}{2}, -q \\ \frac{9}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{7} + \frac{Ccd^q x^9 {}_2F_1\left(\begin{matrix} \frac{9}{2}, -q \\ \frac{11}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{9}$$

input
```
integrate((e*x**2+d)**q*(c*x**4+b*x**2+a)*(C*x**4+B*x**2+A),x)
```

output
```
A*a*d**q*x*hyper((1/2, -q), (3/2,), e*x**2*exp_polar(I*pi)/d) + A*b*d**q*x
**3*hyper((3/2, -q), (5/2,), e*x**2*exp_polar(I*pi)/d)/3 + A*c*d**q*x**5*h
yper((5/2, -q), (7/2,), e*x**2*exp_polar(I*pi)/d)/5 + B*a*d**q*x**3*hyper(
(3/2, -q), (5/2,), e*x**2*exp_polar(I*pi)/d)/3 + B*b*d**q*x**5*hyper((5/2,
 -q), (7/2,), e*x**2*exp_polar(I*pi)/d)/5 + B*c*d**q*x**7*hyper((7/2, -q),
 (9/2,), e*x**2*exp_polar(I*pi)/d)/7 + C*a*d**q*x**5*hyper((5/2, -q), (7/2
,), e*x**2*exp_polar(I*pi)/d)/5 + C*b*d**q*x**7*hyper((7/2, -q), (9/2,), e
*x**2*exp_polar(I*pi)/d)/7 + C*c*d**q*x**9*hyper((9/2, -q), (11/2,), e*x**
2*exp_polar(I*pi)/d)/9
```

Maxima [F]

$$\int (d+ex^2)^q (a+bx^2+cx^4) (A+Bx^2+Cx^4) \, dx = \int (Cx^4+Bx^2+A)(cx^4+bx^2+a)(ex^2+d)^q \, dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+b*x^2+a)*(C*x^4+B*x^2+A),x, algorithm="maxima
")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)*(e*x^2 + d)^q, x)
```

Giac [F]

$$\int (d+ex^2)^q (a+bx^2+cx^4) (A+Bx^2+Cx^4) \, dx = \int (Cx^4+Bx^2+A)(cx^4+bx^2+a)(ex^2+d)^q \, dx$$

input
```
integrate((e*x^2+d)^q*(c*x^4+b*x^2+a)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(c*x^4 + b*x^2 + a)*(e*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^q (a+bx^2+cx^4) (A+Bx^2+Cx^4) \, dx = \int (e\,x^2+d)^q \,(C\,x^4+B\,x^2+A) \, (c\,x^4+b\,x^2+a) \, dx$$

input
```
int((d + e*x^2)^q*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4),x)
```

output
```
int((d + e*x^2)^q*(A + B*x^2 + C*x^4)*(a + b*x^2 + c*x^4), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^q \left(a+bx^2+cx^4\right) \left(A+Bx^2+Cx^4\right) \, dx = \text{too large to display}$$

input
```
int((e*x^2+d)^q*(c*x^4+b*x^2+a)*(C*x^4+B*x^2+A),x)
```

output
```
(16*(d + e*x**2)**q*a**2*e**4*q**4*x + 192*(d + e*x**2)**q*a**2*e**4*q**3*
x + 824*(d + e*x**2)**q*a**2*e**4*q**2*x + 1488*(d + e*x**2)**q*a**2*e**4*
q*x + 945*(d + e*x**2)**q*a**2*e**4*x + 32*(d + e*x**2)**q*a*b*d*e**3*q**4
*x + 336*(d + e*x**2)**q*a*b*d*e**3*q**3*x + 1144*(d + e*x**2)**q*a*b*d*e*
*3*q**2*x + 1260*(d + e*x**2)**q*a*b*d*e**3*q*x + 32*(d + e*x**2)**q*a*b*e
**4*q**4*x**3 + 352*(d + e*x**2)**q*a*b*e**4*q**3*x**3 + 1312*(d + e*x**2)
**q*a*b*e**4*q**2*x**3 + 1832*(d + e*x**2)**q*a*b*e**4*q*x**3 + 630*(d + e
*x**2)**q*a*b*e**4*x**3 - 48*(d + e*x**2)**q*a*c*d**2*e**2*q**3*x - 384*(d
 + e*x**2)**q*a*c*d**2*e**2*q**2*x - 756*(d + e*x**2)**q*a*c*d**2*e**2*q*x
 + 32*(d + e*x**2)**q*a*c*d*e**3*q**4*x**3 + 272*(d + e*x**2)**q*a*c*d*e**
3*q**3*x**3 + 632*(d + e*x**2)**q*a*c*d*e**3*q**2*x**3 + 252*(d + e*x**2)*
*q*a*c*d*e**3*q*x**3 + 32*(d + e*x**2)**q*a*c*e**4*q**4*x**5 + 320*(d + e*
x**2)**q*a*c*e**4*q**3*x**5 + 1040*(d + e*x**2)**q*a*c*e**4*q**2*x**5 + 12
00*(d + e*x**2)**q*a*c*e**4*q*x**5 + 378*(d + e*x**2)**q*a*c*e**4*x**5 - 2
4*(d + e*x**2)**q*b**2*d**2*e**2*q**3*x - 192*(d + e*x**2)**q*b**2*d**2*e*
*2*q**2*x - 378*(d + e*x**2)**q*b**2*d**2*e**2*q*x + 16*(d + e*x**2)**q*b*
*2*d*e**3*q**4*x**3 + 136*(d + e*x**2)**q*b**2*d*e**3*q**3*x**3 + 316*(d +
 e*x**2)**q*b**2*d*e**3*q**2*x**3 + 126*(d + e*x**2)**q*b**2*d*e**3*q*x**3
 + 16*(d + e*x**2)**q*b**2*e**4*q**4*x**5 + 160*(d + e*x**2)**q*b**2*e**4*
q**3*x**5 + 520*(d + e*x**2)**q*b**2*e**4*q**2*x**5 + 600*(d + e*x**2)*...
```

3.260 $\int (d+ex^2)^q (A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 161

$$\int (d+ex^2)^q (A+Bx^2+Cx^4)\,dx = -\frac{(3Cd-Be(5+2q))x(d+ex^2)^{1+q}}{e^2(3+2q)(5+2q)}+\frac{Cx^3(d+ex^2)^{1+q}}{e(5+2q)} + \frac{(3Cd^2-e(5+2q)(Bd-Ae(3+2q)))\,x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{e^2(3+2q)(5+2q)}$$

output
```
-(3*C*d-B*e*(5+2*q))*x*(e*x^2+d)^(1+q)/e^2/(3+2*q)/(5+2*q)+C*x^3*(e*x^2+d)
^(1+q)/e/(5+2*q)+(3*C*d^2-e*(5+2*q)*(B*d-A*e*(3+2*q)))*x*(e*x^2+d)^q*hyper
geom([1/2, -q],[3/2],-e*x^2/d)/e^2/(3+2*q)/(5+2*q)/((1+e*x^2/d)^q)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.63

$$\int (d+ex^2)^q (A+Bx^2+Cx^4)\, dx = \frac{1}{15}x(d+ex^2)^q \left(1+\frac{ex^2}{d}\right)^{-q} \left(15A\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)+5Bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{2},-q,\frac{5}{2},-\frac{ex^2}{d}\right)+3Cx^4\,\text{Hypergeometric2F1}\left(\frac{5}{2},-q,\frac{7}{2},-\frac{ex^2}{d}\right)\right)$$

input
```
Integrate[(d + e*x^2)^q*(A + B*x^2 + C*x^4),x]
```

output
```
(x*(d + e*x^2)^q*(15*A*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)] + 5*B
*x^2*Hypergeometric2F1[3/2, -q, 5/2, -((e*x^2)/d)] + 3*C*x^4*Hypergeometri
c2F1[5/2, -q, 7/2, -((e*x^2)/d)]))/(15*(1 + (e*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1473, 299, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2+Cx^4)(d+ex^2)^q\, dx$$

$$\Big\downarrow \ 1473$$

$$\frac{\int (ex^2+d)^q (Ae(2q+5)-(3Cd-Be(2q+5))x^2)\, dx}{e(2q+5)}+\frac{Cx^3(d+ex^2)^{q+1}}{e(2q+5)}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{(3Cd^2-e(2q+5)(Bd-Ae(2q+3)))\int (ex^2+d)^q dx}{e(2q+3)}-\frac{x(d+ex^2)^{q+1}(3Cd-Be(2q+5))}{e(2q+3)}}{e(2q+5)}+\frac{Cx^3(d+ex^2)^{q+1}}{e(2q+5)}$$

$\big\downarrow$ 238

$$\frac{\frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}(3Cd^2-e(2q+5)(Bd-Ae(2q+3)))\int \left(\frac{ex^2}{d}+1\right)^q dx}{e(2q+3)}-\frac{x(d+ex^2)^{q+1}(3Cd-Be(2q+5))}{e(2q+3)}}{e(2q+5)}+\frac{Cx^3(d+ex^2)^{q+1}}{e(2q+5)}$$

$\big\downarrow$ 237

$$\frac{\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)(3Cd^2-e(2q+5)(Bd-Ae(2q+3)))}{e(2q+3)}-\frac{x(d+ex^2)^{q+1}(3Cd-Be(2q+5))}{e(2q+3)}}{e(2q+5)}+\frac{Cx^3(d+ex^2)^{q+1}}{e(2q+5)}$$

input
```
Int[(d + e*x^2)^q*(A + B*x^2 + C*x^4),x]
```

output
```
(C*x^3*(d + e*x^2)^(1 + q))/(e*(5 + 2*q)) + (-(((3*C*d - B*e*(5 + 2*q))*x*
(d + e*x^2)^(1 + q))/(e*(3 + 2*q))) + ((3*C*d^2 - e*(5 + 2*q)*(B*d - A*e*(
3 + 2*q)))*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/
(e*(3 + 2*q)*(1 + (e*x^2)/d)^q))/(e*(5 + 2*q))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Maple [F]

$$\int \left(e\,x^{2}+d\right)^{q} \left(C\,x^{4}+B\,x^{2}+A\right)dx$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A),x)
```

output
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \left(d+ex^2\right)^q \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(ex^2+d\right)^q\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.78 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.51

$$\int (d+ex^2)^q (A+Bx^2+Cx^4)\, dx = Ad^q x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -q \\ \frac{3}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right) + \frac{Bd^q x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -q \\ \frac{5}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{3} + \frac{Cd^q x^5 {}_2F_1\left(\begin{matrix} \frac{5}{2}, -q \\ \frac{7}{2} \end{matrix}\middle| \frac{ex^2 e^{i\pi}}{d}\right)}{5}$$

input
```
integrate((e*x**2+d)**q*(C*x**4+B*x**2+A),x)
```

output
```
A*d**q*x*hyper((1/2, -q), (3/2,), e*x**2*exp_polar(I*pi)/d) + B*d**q*x**3*
hyper((3/2, -q), (5/2,), e*x**2*exp_polar(I*pi)/d)/3 + C*d**q*x**5*hyper((
5/2, -q), (7/2,), e*x**2*exp_polar(I*pi)/d)/5
```

Maxima [F]

$$\int (d+ex^2)^q (A+Bx^2+Cx^4)\, dx = \int (Cx^4+Bx^2+A)(ex^2+d)^q\, dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q, x)
```

Giac [F]

$$\int (d+ex^2)^q (A+Bx^2+Cx^4) \, dx = \int (Cx^4+Bx^2+A)(ex^2+d)^q \, dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^q (A+Bx^2+Cx^4) \, dx = \int (e\,x^2+d)^q \left(C\,x^4+B\,x^2+A\right) \, dx$$

input
```
int((d + e*x^2)^q*(A + B*x^2 + C*x^4),x)
```

output
```
int((d + e*x^2)^q*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (d+ex^2)^q (A+Bx^2+Cx^4) \, dx = \text{Too large to display}$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A),x)
```

output

```
(4*(d + e*x**2)**q*a*e**2*q**2*x + 16*(d + e*x**2)**q*a*e**2*q*x + 15*(d +
 e*x**2)**q*a*e**2*x + 4*(d + e*x**2)**q*b*d*e*q**2*x + 10*(d + e*x**2)**q
*b*d*e*q*x + 4*(d + e*x**2)**q*b*e**2*q**2*x**3 + 12*(d + e*x**2)**q*b*e**
2*q*x**3 + 5*(d + e*x**2)**q*b*e**2*x**3 - 6*(d + e*x**2)**q*c*d**2*q*x +
4*(d + e*x**2)**q*c*d*e*q**2*x**3 + 2*(d + e*x**2)**q*c*d*e*q*x**3 + 4*(d
+ e*x**2)**q*c*e**2*q**2*x**5 + 8*(d + e*x**2)**q*c*e**2*q*x**5 + 3*(d + e
*x**2)**q*c*e**2*x**5 + 64*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*
d*q + 15*d + 8*e*q**3*x**2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*
a*d*e**2*q**6 + 544*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 1
5*d + 8*e*q**3*x**2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**
2*q**5 + 1760*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d +
8*e*q**3*x**2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q**4
 + 2672*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q*
*3*x**2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q**3 + 186
0*int((d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**3*x**
2 + 36*e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q**2 + 450*int((
d + e*x**2)**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**3*x**2 + 36*
e*q**2*x**2 + 46*e*q*x**2 + 15*e*x**2),x)*a*d*e**2*q - 32*int((d + e*x**2)
**q/(8*d*q**3 + 36*d*q**2 + 46*d*q + 15*d + 8*e*q**3*x**2 + 36*e*q**2*x**2
 + 46*e*q*x**2 + 15*e*x**2),x)*b*d**2*e*q**5 - 224*int((d + e*x**2)**q/...
```

3.261 $\int \frac{(d+ex^2)^q(A+Bx^2+Cx^4)}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 36, antiderivative size = 312

$$\int \frac{(d+ex^2)^q(A+Bx^2+Cx^4)}{a+bx^2+cx^4} dx = \frac{\left(Bc-bC-\frac{bBc-b^2C-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{c\left(b-\sqrt{b^2-4ac}\right)} + \frac{\left(Bc-bC+\frac{bBc-b^2C-2c(Ac-aC)}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-q,1,\frac{3}{2},-\frac{ex^2}{d},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{c\left(b+\sqrt{b^2-4ac}\right)} + \frac{Cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c}$$

output

```
(B*c-C*b-(B*b*c-b^2*C-2*c*(A*c-C*a))/(-4*a*c+b^2)^(1/2))*x*(e*x^2+d)^q*App
ellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)/c/(b-(-4*a*c+
b^2)^(1/2))/((1+e*x^2/d)^q)+(B*c-C*b+(B*b*c-b^2*C-2*c*(A*c-C*a))/(-4*a*c+b
^2)^(1/2))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)),-e*x^2/d)/c/(b+(-4*a*c+b^2)^(1/2))/((1+e*x^2/d)^q)+C*x*(e*x^2+d)^q*hy
pergeom([1/2, -q],[3/2],-e*x^2/d)/c/((1+e*x^2/d)^q)
```

Mathematica [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx$$

input
```
Integrate[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
Integrate[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2256, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2+Cx^4)(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 2256$$

$$\int \left(\frac{(d+ex^2)^q(-aC+Ac+x^2(Bc-bC))}{c(a+bx^2+cx^4)}+\frac{C(d+ex^2)^q}{c}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(-\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}+\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(\frac{-2c(Ac-aC)+b^2(-C)+bBc}{\sqrt{b^2-4ac}}-bC+Bc\right)\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}+\frac{Cx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},-\frac{ex^2}{d}\right)}{c}$$

input
```
Int[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x]
```

output
```
((B*c - b*C - (b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*x*(d +
e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((
e*x^2)/d)])/(c*(b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + ((B*c - b*C +
(b*B*c - b^2*C - 2*c*(A*c - a*C))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*Appel
lF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c
*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (C*x*(d + e*x^2)^q*Hypergeom
etric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(c*(1 + (e*x^2)/d)^q)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(e\,x^2+d)^q\,(C\,x^4+B\,x^2+A)}{c\,x^4+b\,x^2+a}dx$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x)
```

output
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas
")
```

output
```
integral((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**q*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+bx^2+cx^4}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima
")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+bx^2+cx^4} dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{cx^4+bx^2+a} dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+bx^2+cx^4} dx = \int \frac{(e\,x^2+d)^q\,(C\,x^4+B\,x^2+A)}{c\,x^4+b\,x^2+a} dx$$

input
```
int(((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4),x)
```

output
```
int(((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{a+bx^2+cx^4} dx = \frac{(e\,x^2+d)^q\,x+4\left(\int \frac{(e\,x^2+d)^q}{2eq\,x^2+e\,x^2+2dq+d}dx\right)d\,q^2+2\left(\int \frac{(e\,x^2+d)^q}{2eq\,x^2+e\,x^2+2dq+d}dx\right)dq}{2q+1}$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a),x)
```

output
```
((d + e*x**2)**q*x + 4*int((d + e*x**2)**q/(2*d*q + d + 2*e*q*x**2 + e*x**
2),x)*d*q**2 + 2*int((d + e*x**2)**q/(2*d*q + d + 2*e*q*x**2 + e*x**2),x)*
d*q)/(2*q + 1)
```

3.262 $\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 1114

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2} \, dx = \text{Too large to display}$$

output
```
1/2*x*(e*x^2+d)^(1+q)*(A*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)-a*(2*B*a*c*e-
B*b^2*e+B*b*c*d+C*a*b*e-2*C*a*c*d)+c*(A*(2*a*c*e-b^2*e+b*c*d)-a*(-B*b*e+2*
B*c*d+2*C*a*e-C*b*d))*x^2)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2
+a)-1/2*(A*c*(b^2*d*e-b*(c*d^2+a*e^2*(1-2*q))-4*a*c*d*e*q)+a*(2*B*c*(c*d^2
+e*(a*e*(1-2*q)-b*d*(1-q)))-C*(b*(c*d^2+a*e^2*(1-2*q))-b^2*d*e*(1-2*q)-4*a
*c*d*e*q))+(A*c*(b^3*d*e-12*a*b*c*d*e-b^2*(c*d^2+a*e^2*(1-2*q))+4*a*c*(3*c
*d^2+a*e^2*(3-2*q)))+a*(b^2*(c*d*(C*d+2*B*e*(2-q))+a*C*e^2*(1-2*q))-b^3*C*
d*e*(1-2*q)-4*b*c*(B*(a*e^2+c*d^2)+a*C*d*e*(1+2*q))+4*a*c*(a*C*e^2*(1+2*q)
+c*d*(2*B*e*q+C*d))))/(-4*a*c+b^2)^(1/2))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,
3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)/a/(-4*a*c+b^2)/(b-(-4*a*c+b^
2)^(1/2))/(a*e^2-b*d*e+c*d^2)/((1+e*x^2/d)^q)-1/2*(A*c*(b^2*d*e-b*(c*d^2+a
*e^2*(1-2*q))-4*a*c*d*e*q)+a*(2*B*c*(c*d^2+e*(a*e*(1-2*q)-b*d*(1-q)))-C*(b
*(c*d^2+a*e^2*(1-2*q))-b^2*d*e*(1-2*q)-4*a*c*d*e*q))-(A*c*(b^3*d*e-12*a*b*
c*d*e-b^2*(c*d^2+a*e^2*(1-2*q))+4*a*c*(3*c*d^2+a*e^2*(3-2*q)))+a*(b^2*(c*d
*(C*d+2*B*e*(2-q))+a*C*e^2*(1-2*q))-b^3*C*d*e*(1-2*q)-4*b*c*(B*(a*e^2+c*d^
2)+a*C*d*e*(1+2*q))+4*a*c*(a*C*e^2*(1+2*q)+c*d*(2*B*e*q+C*d))))/(-4*a*c+b^
2)^(1/2))*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)),-e*x^2/d)/a/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/((
1+e*x^2/d)^q)-1/2*e*(A*(2*a*c*e-b^2*e+b*c*d)-a*(-B*b*e+2*B*c*d+2*C*a*e-C*b
*d))*(1+2*q)*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/a/(-4*a*...
```

Mathematica [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx=\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx$$

input
```
Integrate[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
Integrate[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2+Cx^4)\,(d+ex^2)^q}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \ 2256$$

$$\int \left(\frac{(d+ex^2)^q\,(-aC+Ac+x^2(Bc-bC))}{c\,(a+bx^2+cx^4)^2}+\frac{C\,(d+ex^2)^q}{c\,(a+bx^2+cx^4)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(Ac-aC)\int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)^2}dx}{c}+\frac{(Bc-bC)\int \frac{x^2(ex^2+d)^q}{(cx^4+bx^2+a)^2}dx}{c}-\frac{2Cx\,(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2Cx\,(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

input
```
Int[((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2256
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(e\,x^2+d)^q\,(C\,x^4+B\,x^2+A)}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^q\,(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fric
as")
```

output
```
integral((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2
*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q\,(A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**q*(C*x**4+B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(ex^2+d)^q}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac
")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(e\,x^2+d)^q\,(C\,x^4+B\,x^2+A)}{(c\,x^4+b\,x^2+a)^2}\,dx$$

input
```
int(((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

output
```
int(((d + e*x^2)^q*(A + B*x^2 + C*x^4))/(a + b*x^2 + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^q (A+Bx^2+Cx^4)}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}dx$$

input
```
int((e*x^2+d)^q*(C*x^4+B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((d + e*x**2)**q/(a + b*x**2 + c*x**4),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
              finalresult={"A",""}
            ,(*ELSE*)
              finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
             ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:  #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file