Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.2-Quartic-trinomial/122-1.2.2.8

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**134**]. This is test number [122].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	35.07 (47)	64.93 (87)
Maple	35.07 (47)	64.93 (87)
Rubi	34.33 (46)	65.67 (88)
Fricas	11.94 (16)	88.06 (118)
Giac	7.46 (10)	92.54 (124)
Reduce	1.49 (2)	98.51 (132)
Mupad	0.75 (1)	99.25 (133)
Maxima	0.00 (0)	100.00 (134)
Sympy	0.00 (0)	100.00 (134)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

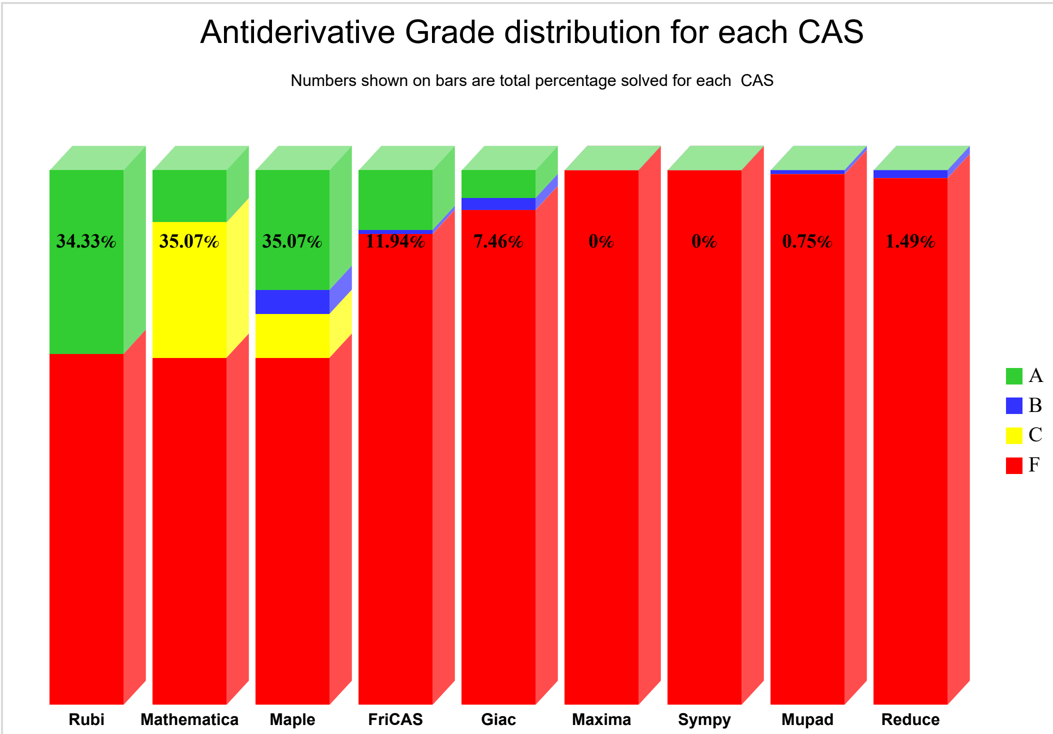
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

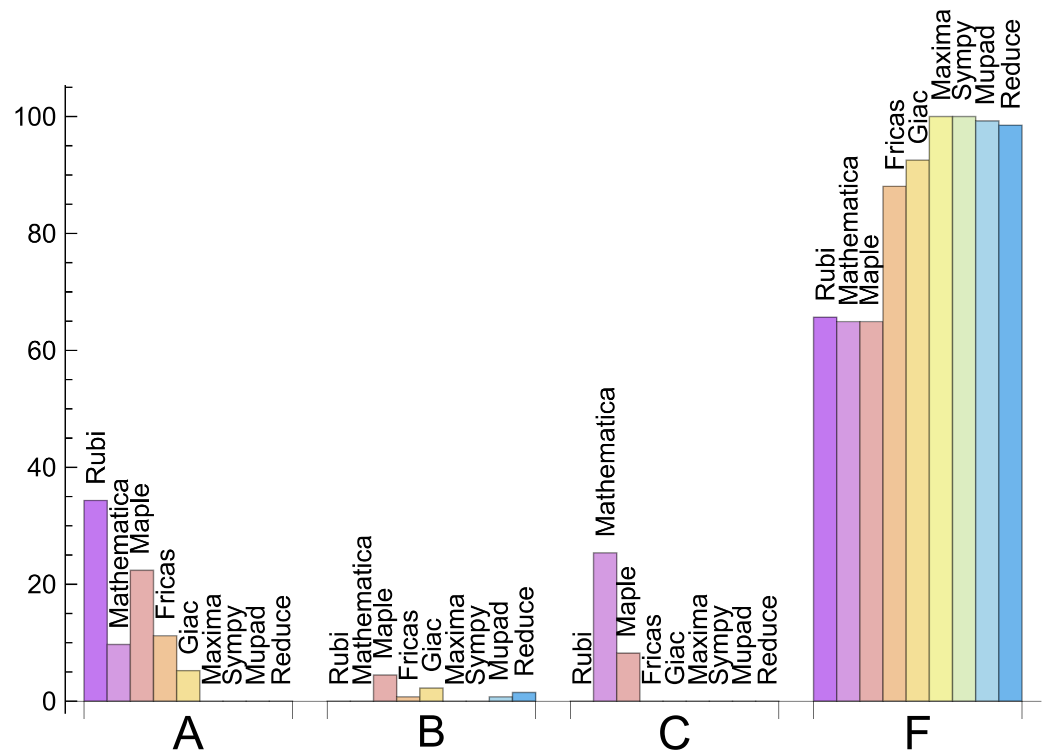
System	% A grade	% B grade	% C grade	% F grade
Rubi	34.328	0.000	0.000	65.672
Maple	22.388	4.478	8.209	64.925
Fricas	11.194	0.746	0.000	88.060
Mathematica	9.701	0.000	25.373	64.925
Giac	5.224	2.239	0.000	92.537
Mupad	0.000	0.746	0.000	99.254
Maxima	0.000	0.000	0.000	100.000
Reduce	0.000	1.493	0.000	98.507
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	87	100.00	0.00	0.00
Maple	87	100.00	0.00	0.00
Rubi	88	100.00	0.00	0.00
Fricas	118	72.03	27.97	0.00
Giac	124	95.16	1.61	3.23
Reduce	132	100.00	0.00	0.00
Mupad	133	0.00	100.00	0.00
Maxima	134	98.51	0.00	1.49
Sympy	134	97.01	2.99	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Giac	0.15
Reduce	0.26
Rubi	0.71
Mupad	0.79
Maple	2.29
Fricas	3.26
Mathematica	7.73
Sympy	-nan(ind)
Maxima	-nan(ind)

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Giac	163.80	1.44	80.50	1.40
Reduce	185.00	2.70	185.00	2.70
Rubi	309.76	1.13	279.00	1.03
Mathematica	326.11	1.17	310.00	1.05
Maple	347.77	1.33	339.00	1.27
Fricas	363.19	2.46	119.00	1.30
Mupad	397.00	5.16	397.00	5.16
Sympy	-nan(ind)	-nan(ind)	nan	nan
Maxima	-nan(ind)	-nan(ind)	nan	nan

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

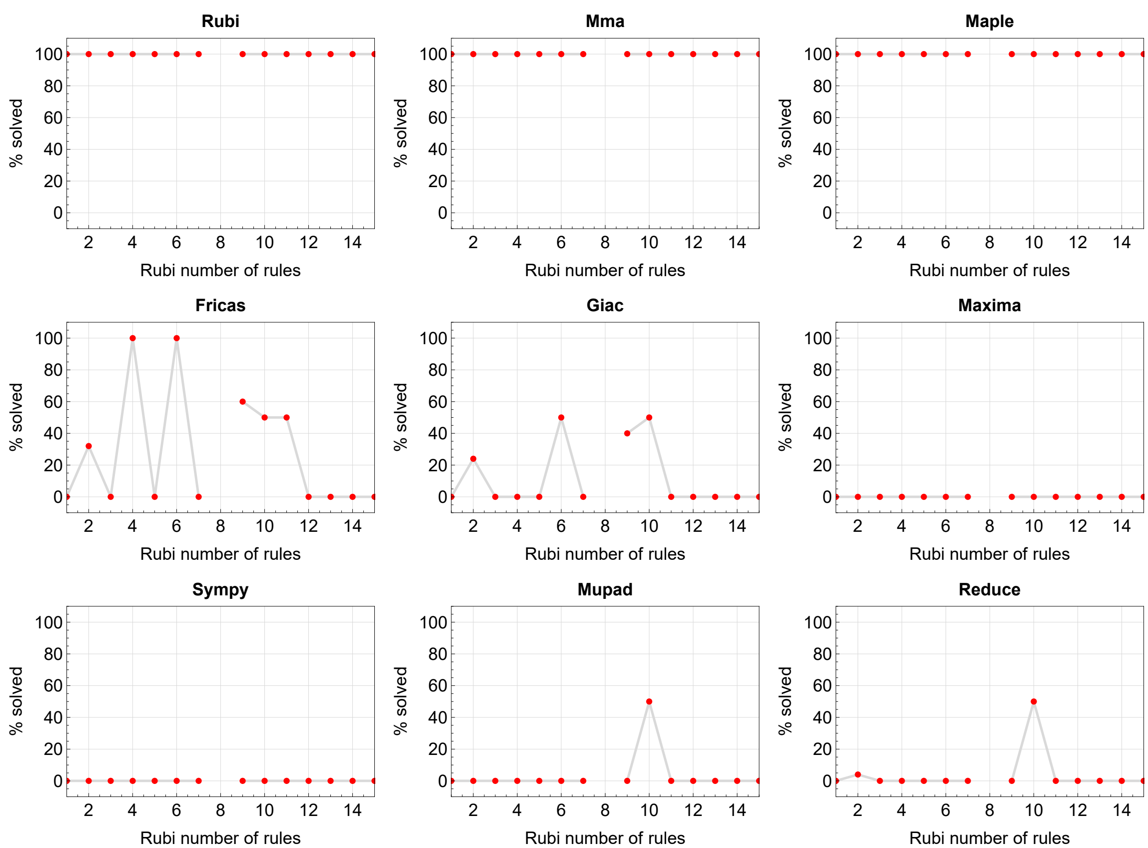


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

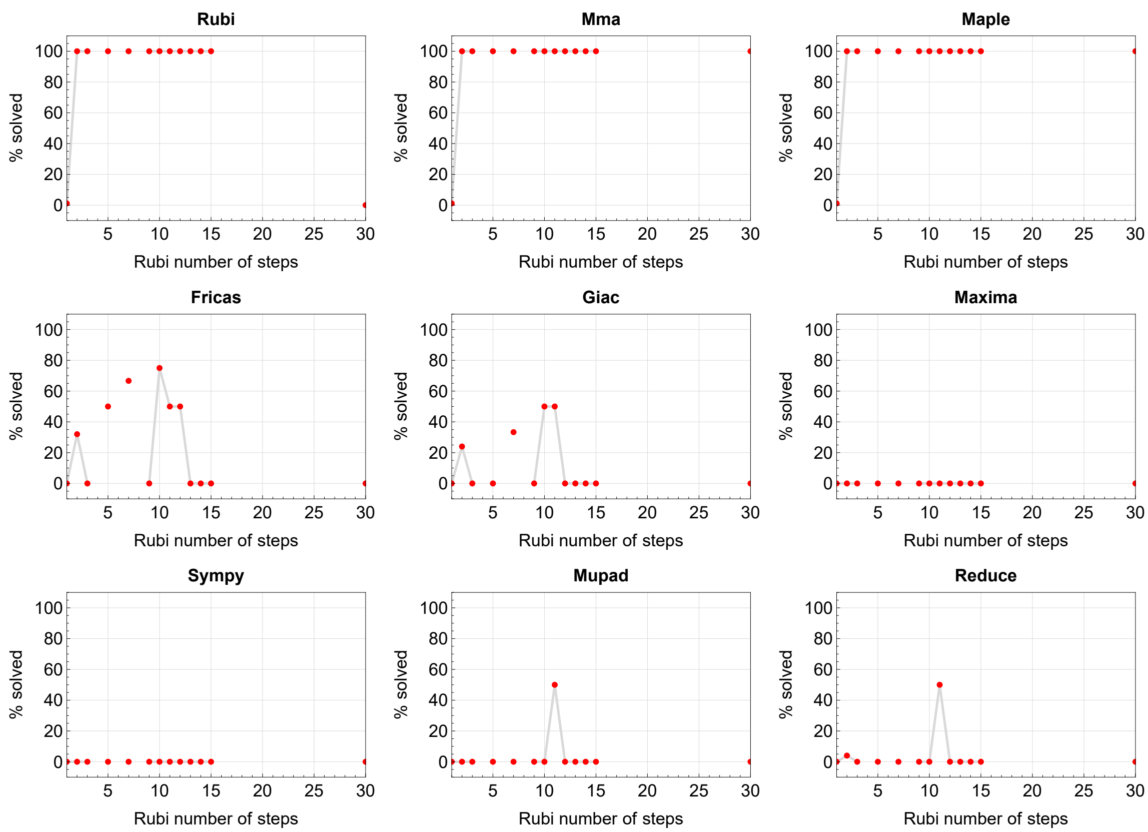


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

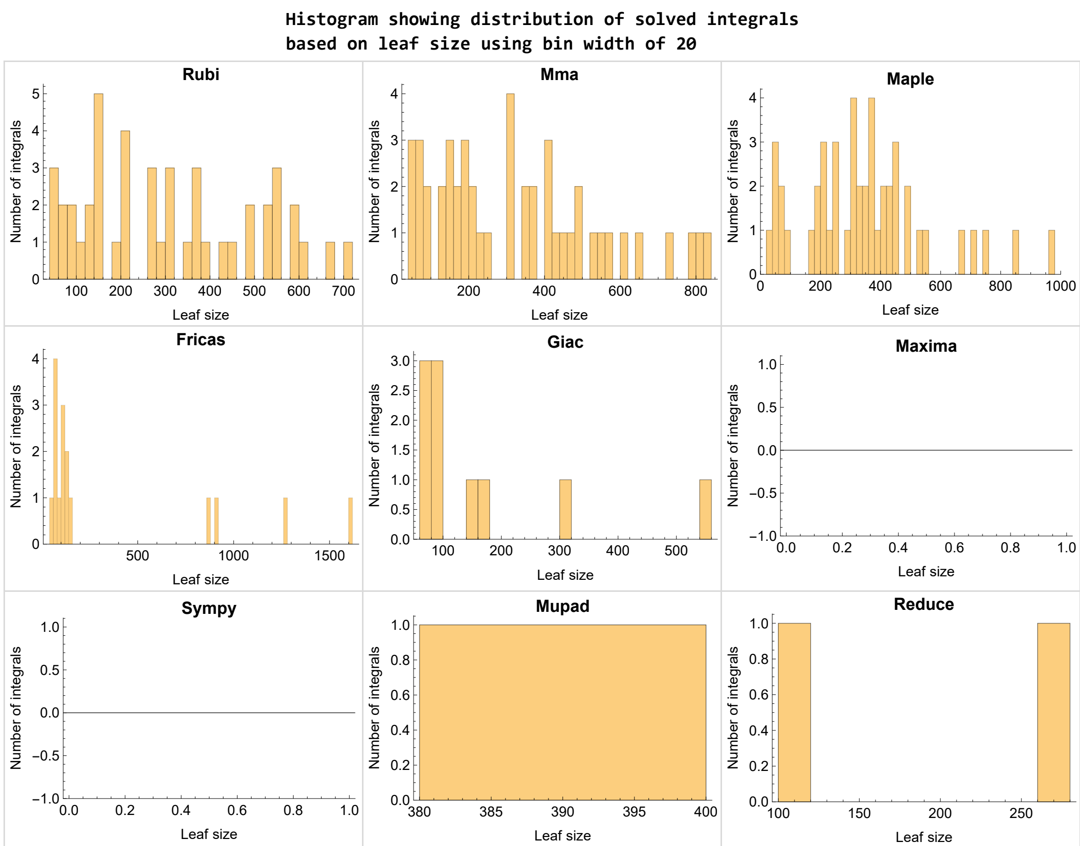


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

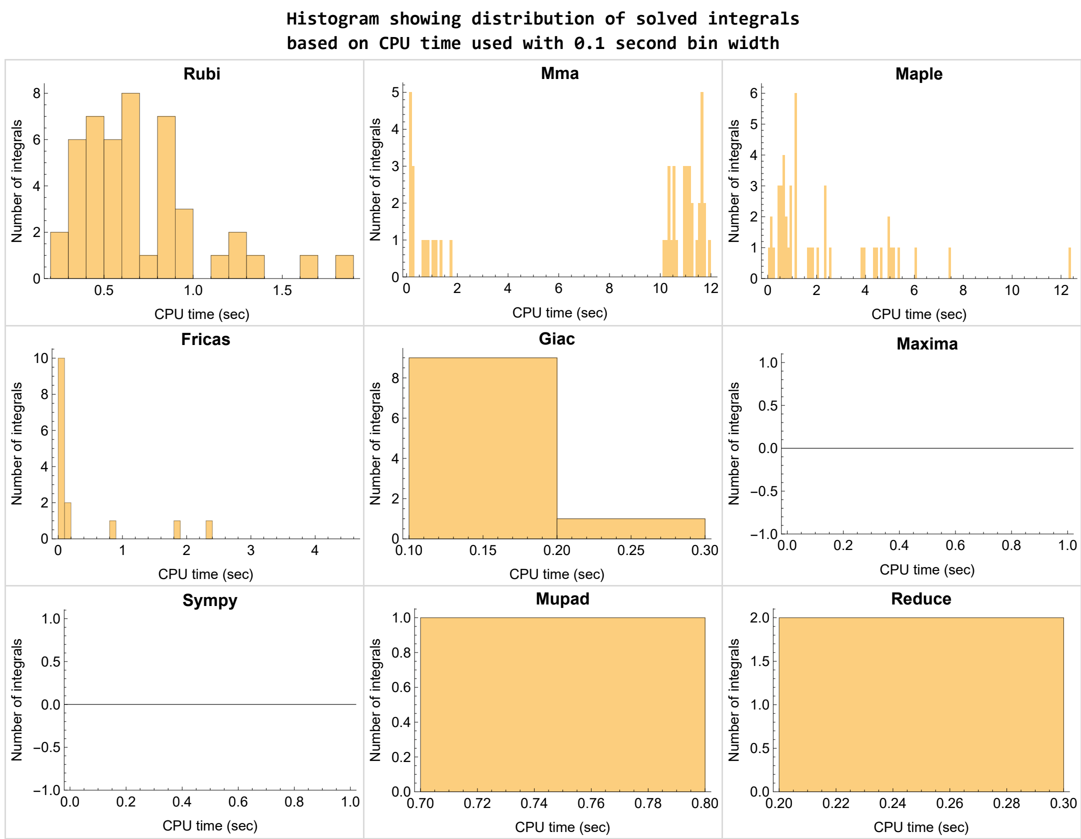


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

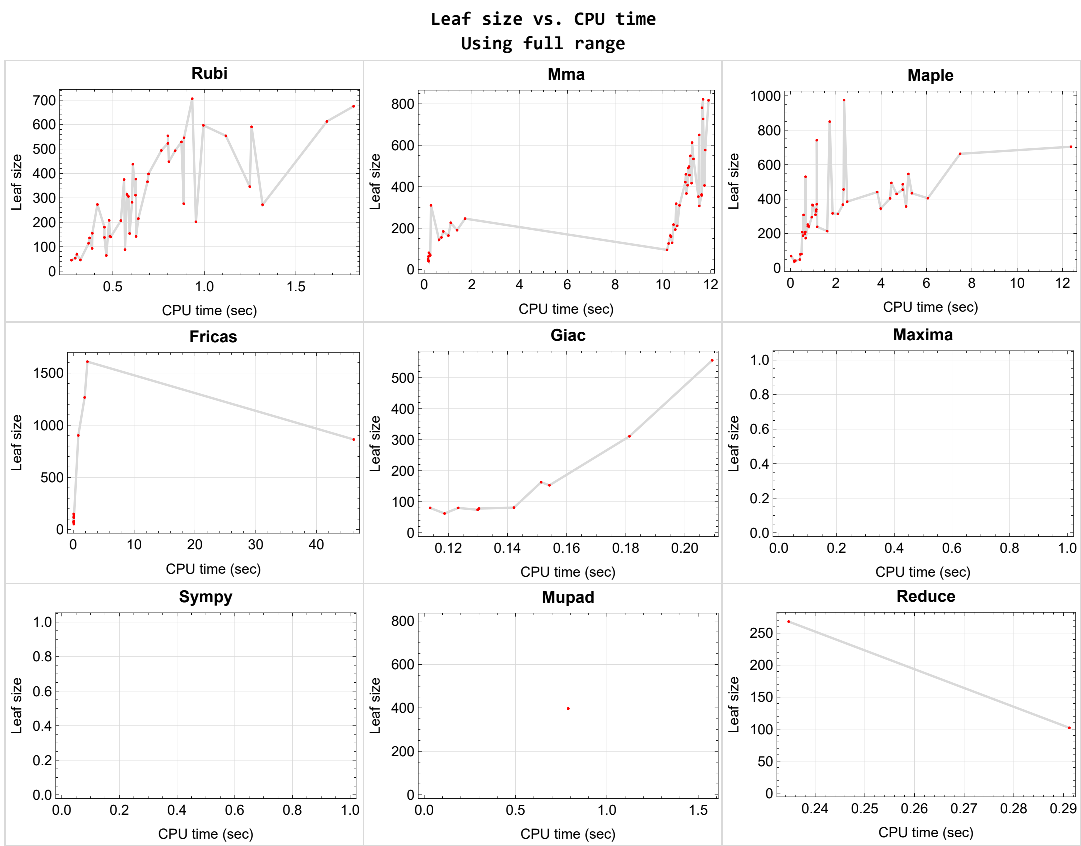


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {90}

Mathematica {33}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

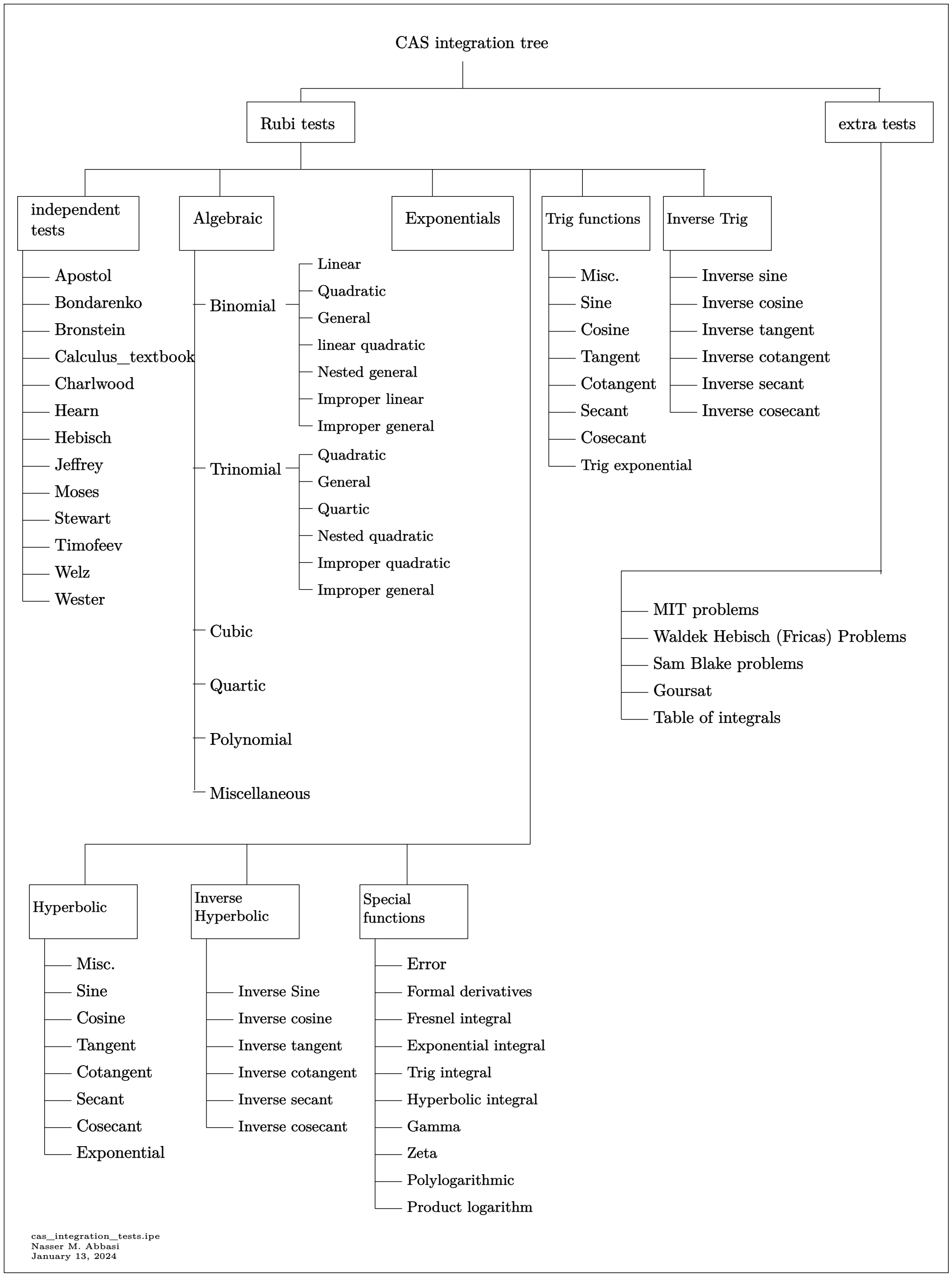
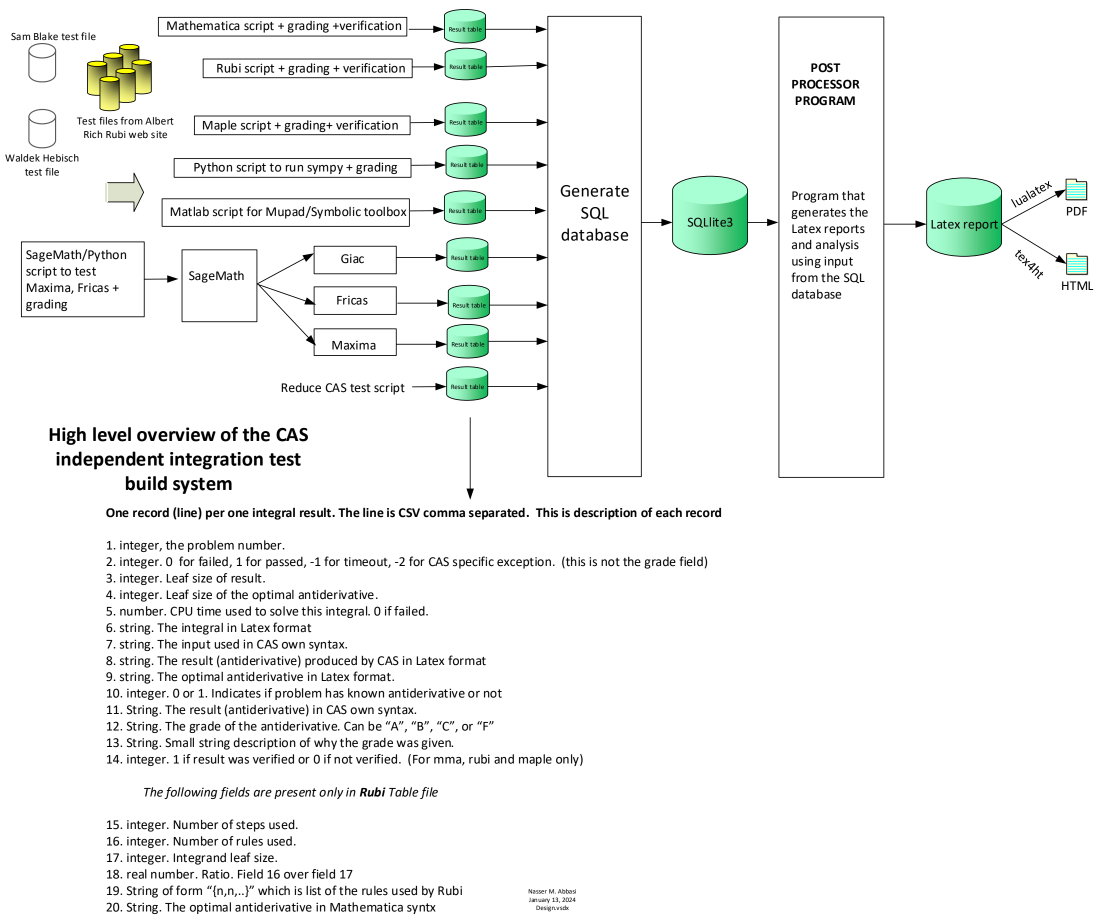


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103 }

B grade { }

C grade { }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 21, 22, 23, 24, 25, 26, 27, 93, 94, 95, 96, 97, 98 }

B grade { }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 28, 29, 30, 31, 32, 33, 90, 91, 92, 99, 100, 101, 102, 103 }

F normal fail { 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade { 3, 14, 15, 16, 17, 18 }

C grade { 28, 29, 30, 31, 32, 33, 99, 100, 101, 102, 103 }

F normal fail { 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 23, 25, 26, 27, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103 }

B grade { 90 }

C grade { }

F normal fail { 7, 8, 11, 13, 17, 20, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 104, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 9, 10, 12, 14, 15, 16, 18, 19, 21, 22, 24, 28, 29, 30, 31, 32, 68, 69, 76, 83, 91, 92, 105, 114, 122, 123, 128 }

F(-2) exception fail { }

Maxima

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { }

F(-2) exception fail { 91, 92 }

Giac

A grade { 25, 26, 90, 93, 94, 95, 96 }

B grade { 27, 97, 98 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { 91, 92 }

F(-2) exception fail { 21, 22, 23, 24 }

Mupad

A grade { }

B grade { 90 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { 14, 15, 16, 17 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 90, 96 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	706	817	434	0	0	0	0	613	0
N.S.	1	1.74	2.02	1.07	0.00	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.934	11.911	5.352	0.000	0.000	0.000	0.000	0.400	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	554	362	368	0	0	0	0	386	0
N.S.	1	1.65	1.08	1.10	0.00	0.00	0.00	0.00	1.15	0.00
time (sec)	N/A	0.801	11.620	2.309	0.000	0.000	0.000	0.000	0.343	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	375	497	456	0	0	0	0	249	0
N.S.	1	1.37	1.81	1.66	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.562	11.093	2.340	0.000	0.000	0.000	0.000	0.290	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	398	490	385	0	0	0	0	253	0
N.S.	1	1.54	1.89	1.49	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	0.695	11.059	2.500	0.000	0.000	0.000	0.000	0.311	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	377	613	404	0	0	0	0	270	0
N.S.	1	1.29	2.09	1.38	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.626	11.220	4.392	0.000	0.000	0.000	0.000	0.417	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	523	781	455	0	0	0	0	462	0
N.S.	1	1.47	2.20	1.28	0.00	0.00	0.00	0.00	1.30	0.00
time (sec)	N/A	0.801	11.635	4.948	0.000	0.000	0.000	0.000	0.502	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	346	352	357	0	0	0	0	389	0
N.S.	1	1.05	1.07	1.08	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	1.248	11.484	5.097	0.000	0.000	0.000	0.000	0.371	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	276	367	430	0	0	0	0	206	0
N.S.	1	1.03	1.37	1.60	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.888	10.983	4.679	0.000	0.000	0.000	0.000	0.294	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	215	193	329	0	0	0	0	89	0
N.S.	1	1.01	0.91	1.55	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.639	10.518	1.128	0.000	0.000	0.000	0.000	0.219	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	129	173	0	0	0	0	86	0
N.S.	1	1.00	0.95	1.27	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.373	10.385	0.669	0.000	0.000	0.000	0.000	0.214	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	318	214	0	0	0	0	86	0
N.S.	1	1.00	1.53	1.03	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.480	10.554	1.618	0.000	0.000	0.000	0.000	0.290	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	314	460	314	0	0	0	0	212	0
N.S.	1	1.14	1.67	1.14	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.578	10.961	2.092	0.000	0.000	0.000	0.000	0.378	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	494	650	441	0	0	0	0	336	0
N.S.	1	1.40	1.84	1.25	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.765	11.519	3.824	0.000	0.000	0.000	0.000	0.454	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	597	577	704	0	0	0	0	1571	0
N.S.	1	1.47	1.42	1.74	0.00	0.00	0.00	0.00	3.88	0.00
time (sec)	N/A	0.995	11.771	12.373	0.000	0.000	0.000	0.000	1.355	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	493	358	975	0	0	0	0	919	0
N.S.	1	1.34	0.98	2.66	0.00	0.00	0.00	0.00	2.50	0.00
time (sec)	N/A	0.841	11.624	2.362	0.000	0.000	0.000	0.000	1.067	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	366	417	850	0	0	0	0	587	0
N.S.	1	1.15	1.31	2.66	0.00	0.00	0.00	0.00	1.84	0.00
time (sec)	N/A	0.689	11.199	1.728	0.000	0.000	0.000	0.000	0.836	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	307	742	0	0	0	0	131	0
N.S.	1	1.00	0.99	2.39	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.624	11.522	1.158	0.000	0.000	0.000	0.000	0.696	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	407	530	0	0	0	0	128	0
N.S.	1	1.00	1.33	1.73	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.585	11.032	0.663	0.000	0.000	0.000	0.000	0.721	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	448	534	486	0	0	0	0	579	0
N.S.	1	1.15	1.38	1.25	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	0.806	11.279	4.950	0.000	0.000	0.000	0.000	0.695	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	546	727	663	0	0	0	0	909	0
N.S.	1	1.19	1.58	1.44	0.00	0.00	0.00	0.00	1.98	0.00
time (sec)	N/A	0.889	11.683	7.480	0.000	0.000	0.000	0.000	0.602	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	272	226	295	0	0	0	0	103	0
N.S.	1	1.07	0.89	1.16	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	1.318	1.113	0.937	0.000	0.000	0.000	0.000	0.261	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	202	184	243	0	0	0	0	103	0
N.S.	1	1.06	0.96	1.27	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.954	0.801	0.761	0.000	0.000	0.000	0.000	0.245	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	142	144	198	0	863	0	0	101	0
N.S.	1	1.03	1.04	1.43	0.00	6.25	0.00	0.00	0.73	0.00
time (sec)	N/A	0.627	0.623	0.630	0.000	46.114	0.000	0.000	0.219	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	164	208	0	0	0	0	98	0
N.S.	1	1.00	1.15	1.45	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.484	1.015	0.518	0.000	0.000	0.000	0.000	0.232	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	155	209	0	902	0	153	97	0
N.S.	1	1.00	1.12	1.51	0.00	6.54	0.00	1.11	0.70	0.00
time (sec)	N/A	0.454	0.724	0.654	0.000	0.828	0.000	0.154	0.280	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	207	190	252	0	1266	0	311	98	0
N.S.	1	1.10	1.01	1.33	0.00	6.70	0.00	1.65	0.52	0.00
time (sec)	N/A	0.544	1.377	0.763	0.000	1.867	0.000	0.181	0.327	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	282	246	309	0	1609	0	556	100	0
N.S.	1	1.11	0.97	1.22	0.00	6.36	0.00	2.20	0.40	0.00
time (sec)	N/A	0.605	1.712	1.102	0.000	2.328	0.000	0.209	0.400	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	702	675	406	405	0	0	0	0	363	0
N.S.	1	0.96	0.58	0.58	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	1.815	11.740	6.059	0.000	0.000	0.000	0.000	0.381	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	591	456	546	0	0	0	0	186	0
N.S.	1	1.02	0.78	0.94	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	1.258	11.111	5.199	0.000	0.000	0.000	0.000	0.315	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	529	211	370	0	0	0	0	121	0
N.S.	1	1.03	0.41	0.72	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.875	10.598	1.156	0.000	0.000	0.000	0.000	0.228	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	554	422	317	0	0	0	0	121	0
N.S.	1	1.04	0.79	0.59	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.118	10.938	1.856	0.000	0.000	0.000	0.000	0.306	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	593	613	549	345	0	0	0	0	201	0
N.S.	1	1.03	0.93	0.58	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.669	11.148	3.979	0.000	0.000	0.000	0.000	0.427	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	0	822	494	0	0	0	0	316	0
N.S.	1	0.00	1.14	0.68	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.000	11.680	4.448	0.000	0.000	0.000	0.000	0.615	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	874	0	0	0	0	0	0	0	1029	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.584	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	737	0	0	0	0	0	0	0	737	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.205	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	623	0	0	0	0	0	0	0	458	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.937	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	530	0	0	0	0	0	0	0	482	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.143	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	524	0	0	0	0	0	0	0	454	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.659	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	551	0	0	0	0	0	0	0	1526	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.77	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.282	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	488	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.081	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	583	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	18.179	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	700	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	105.305	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	874	0	0	0	0	0	0	0	1029	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.051	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	732	0	0	0	0	0	0	0	737	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.886	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	620	0	0	0	0	0	0	0	690	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.662	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	569	0	0	0	0	0	0	0	806	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.238	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	592	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	7.971	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	637	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	16.342	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	582	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	23.670	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	700	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	126.287	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	825	0	0	0	0	0	0	0	32	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	738	0	0	0	0	0	0	0	737	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.274	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	629	0	0	0	0	0	0	0	459	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.991	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	538	0	0	0	0	0	0	0	291	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.731	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	488	0	0	0	0	0	0	0	244	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.898	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	486	0	0	0	0	0	0	0	308	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.311	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	413	0	0	0	0	0	0	0	1063	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.57	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.619	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	492	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.644	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	588	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	18.494	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	673	0	0	0	0	0	0	0	1491	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.22	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.861	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	580	0	0	0	0	0	0	0	1070	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.84	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.567	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	546	0	0	0	0	0	0	0	679	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.227	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	518	0	0	0	0	0	0	0	855	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.65	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.278	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	431	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4.848	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	503	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.617	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	593	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	19.154	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	668	0	0	0	0	0	0	0	461	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.119	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	551	0	0	0	0	0	0	0	234	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.792	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	476	0	0	0	0	0	0	0	151	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.429	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	427	0	0	0	0	0	0	0	131	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.617	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	368	0	0	0	0	0	0	0	254	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.432	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	445	0	0	0	0	0	0	0	1063	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.39	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4.157	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	546	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	7.610	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	662	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	17.122	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	815	0	0	0	0	0	0	0	933	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.864	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	676	0	0	0	0	0	0	0	517	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.295	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	558	0	0	0	0	0	0	0	350	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.951	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	501	0	0	0	0	0	0	0	185	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.085	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	497	0	0	0	0	0	0	0	311	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.074	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	452	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.475	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	561	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	11.869	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	691	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	29.875	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	842	0	0	0	0	0	0	0	1657	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.97	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.885	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	717	0	0	0	0	0	0	0	1418	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.98	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.378	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	658	0	0	0	0	0	0	0	669	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.499	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	547	0	0	0	0	0	0	0	503	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.343	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	529	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.503	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	669	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	10.317	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	840	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	25.513	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	114	81	81	0	150	0	80	268	397
N.S.	1	1.48	1.05	1.05	0.00	1.95	0.00	1.04	3.48	5.16
time (sec)	N/A	0.368	0.199	0.481	0.000	0.066	0.000	0.114	0.235	0.788

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	310	366	0	0	0	0	114	0
N.S.	1	1.00	1.14	1.34	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.416	0.286	0.970	0.000	0.000	0.000	0.000	0.351	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	438	310	364	0	0	0	0	115	0
N.S.	1	1.04	0.74	0.86	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.610	10.694	0.990	0.000	0.000	0.000	0.000	0.336	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	88	72	69	0	81	0	81	67	0
N.S.	1	1.07	0.88	0.84	0.00	0.99	0.00	0.99	0.82	0.00
time (sec)	N/A	0.567	0.247	0.026	0.000	0.079	0.000	0.142	0.264	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	63	49	0	74	0	74	67	0
N.S.	1	1.02	1.00	0.78	0.00	1.17	0.00	1.17	1.06	0.00
time (sec)	N/A	0.465	0.178	0.410	0.000	0.077	0.000	0.130	0.262	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	45	49	37	0	62	0	62	65	0
N.S.	1	0.96	1.04	0.79	0.00	1.32	0.00	1.32	1.38	0.00
time (sec)	N/A	0.274	0.163	0.171	0.000	0.083	0.000	0.119	0.248	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	42	0	78	0	78	102	0
N.S.	1	1.00	0.79	0.79	0.00	1.47	0.00	1.47	1.92	0.00
time (sec)	N/A	0.295	0.179	0.212	0.000	0.081	0.000	0.130	0.291	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	42	0	67	0	80	137	0
N.S.	1	1.00	0.87	0.91	0.00	1.46	0.00	1.74	2.98	0.00
time (sec)	N/A	0.323	0.192	0.163	0.000	0.073	0.000	0.123	0.249	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	68	79	0	119	0	163	67	0
N.S.	1	1.00	0.73	0.85	0.00	1.28	0.00	1.75	0.72	0.00
time (sec)	N/A	0.387	0.255	0.431	0.000	0.082	0.000	0.151	0.227	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	154	159	239	0	124	0	0	111	0
N.S.	1	1.01	1.05	1.57	0.00	0.82	0.00	0.00	0.73	0.00
time (sec)	N/A	0.592	10.339	1.175	0.000	0.109	0.000	0.000	0.198	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	140	126	308	0	119	0	0	69	0
N.S.	1	1.01	0.91	2.23	0.00	0.86	0.00	0.00	0.50	0.00
time (sec)	N/A	0.489	10.238	0.574	0.000	0.110	0.000	0.000	0.179	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	95	188	0	52	0	0	66	0
N.S.	1	1.00	1.38	2.72	0.00	0.75	0.00	0.00	0.96	0.00
time (sec)	N/A	0.304	10.172	0.553	0.000	0.097	0.000	0.000	0.187	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	164	241	0	115	0	0	65	0
N.S.	1	1.00	1.06	1.55	0.00	0.74	0.00	0.00	0.42	0.00
time (sec)	N/A	0.388	10.309	0.812	0.000	0.095	0.000	0.000	0.183	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	217	339	0	130	0	0	124	0
N.S.	1	1.00	1.21	1.88	0.00	0.72	0.00	0.00	0.69	0.00
time (sec)	N/A	0.454	10.444	1.139	0.000	0.095	0.000	0.000	0.217	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1604	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.028	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1229	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4.858	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	964	0	0	0	0	0	0	0	1034	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.786	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	805	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.734	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	760	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	13.685	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	816	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	726	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	958	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1295	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1576	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1241	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4.540	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	993	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.062	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	874	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	15.508	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	882	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1013	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.164	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	954	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1306	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1746	0	0	0	0	0	0	0	36	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	806	0	0	0	0	0	0	0	360	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.879	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	697	0	0	0	0	0	0	0	33	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	634	0	0	0	0	0	0	0	37	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.368	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	483	0	0	0	0	0	0	0	1105	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.29	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.458	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	619	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	44.399	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	851	0	0	0	0	0	0	0	41	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1032	0	0	0	0	0	0	0	1178	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.372	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	819	0	0	0	0	0	0	0	575	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.859	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	722	0	0	0	0	0	0	0	1487	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.636	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	719	0	0	0	0	0	0	0	1508	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.10	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.281	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	626	0	0	0	0	0	0	0	46	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	865	0	0	0	0	0	0	0	46	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1220	0	0	0	0	0	0	0	46	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.027	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [7] had the largest ratio of [.4687500000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.74	32	0.062
2	A	2	2	1.65	32	0.062
3	A	2	2	1.37	29	0.069
4	A	2	2	1.54	32	0.062
5	A	2	2	1.29	32	0.062
6	A	2	2	1.47	32	0.062
7	A	15	15	1.05	32	0.469
8	A	13	13	1.03	32	0.406
9	A	10	10	1.01	32	0.312
10	A	5	5	1.00	29	0.172
11	A	2	2	1.00	32	0.062
12	A	2	2	1.14	32	0.062
13	A	2	2	1.40	32	0.062
14	A	2	2	1.47	32	0.062
15	A	2	2	1.34	32	0.062
16	A	2	2	1.15	32	0.062
17	A	2	2	1.00	32	0.062
18	A	2	2	1.00	29	0.069
19	A	2	2	1.15	32	0.062
20	A	2	2	1.19	32	0.062
21	A	15	14	1.07	36	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	14	13	1.06	36	0.361
23	A	12	11	1.03	34	0.324
24	A	2	2	1.00	36	0.056
25	A	2	2	1.00	36	0.056
26	A	2	2	1.10	36	0.056
27	A	2	2	1.11	36	0.056
28	A	11	11	0.96	36	0.306
29	A	9	9	1.02	36	0.250
30	A	7	7	1.03	33	0.212
31	A	9	9	1.04	36	0.250
32	A	12	12	1.03	36	0.333
33	F	0	0	N/A	0.000	N/A
34	F	0	0	N/A	0.000	N/A
35	F	0	0	N/A	0.000	N/A
36	F	0	0	N/A	0.000	N/A
37	F	0	0	N/A	0.000	N/A
38	F	0	0	N/A	0.000	N/A
39	F	0	0	N/A	0.000	N/A
40	F	0	0	N/A	0.000	N/A
41	F	0	0	N/A	0.000	N/A
42	F	0	0	N/A	0.000	N/A
43	F	0	0	N/A	0.000	N/A
44	F	0	0	N/A	0.000	N/A
45	F	0	0	N/A	0.000	N/A
46	F	0	0	N/A	0.000	N/A
47	F	0	0	N/A	0.000	N/A
48	F	0	0	N/A	0.000	N/A
49	F	0	0	N/A	0.000	N/A
50	F	0	0	N/A	0.000	N/A
51	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	F	0	0	N/A	0.000	N/A
53	F	0	0	N/A	0.000	N/A
54	F	0	0	N/A	0.000	N/A
55	F	0	0	N/A	0.000	N/A
56	F	0	0	N/A	0.000	N/A
57	F	0	0	N/A	0.000	N/A
58	F	0	0	N/A	0.000	N/A
59	F	0	0	N/A	0.000	N/A
60	F	0	0	N/A	0.000	N/A
61	F	0	0	N/A	0.000	N/A
62	F	0	0	N/A	0.000	N/A
63	F	0	0	N/A	0.000	N/A
64	F	0	0	N/A	0.000	N/A
65	F	0	0	N/A	0.000	N/A
66	F	0	0	N/A	0.000	N/A
67	F	0	0	N/A	0.000	N/A
68	F	0	0	N/A	0.000	N/A
69	F	0	0	N/A	0.000	N/A
70	F	0	0	N/A	0.000	N/A
71	F	0	0	N/A	0.000	N/A
72	F	0	0	N/A	0.000	N/A
73	F	0	0	N/A	0.000	N/A
74	F	0	0	N/A	0.000	N/A
75	F	0	0	N/A	0.000	N/A
76	F	0	0	N/A	0.000	N/A
77	F	0	0	N/A	0.000	N/A
78	F	0	0	N/A	0.000	N/A
79	F	0	0	N/A	0.000	N/A
80	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
81	F	0	0	N/A	0.000	N/A
82	F	0	0	N/A	0.000	N/A
83	F	0	0	N/A	0.000	N/A
84	F	0	0	N/A	0.000	N/A
85	F	0	0	N/A	0.000	N/A
86	F	0	0	N/A	0.000	N/A
87	F	0	0	N/A	0.000	N/A
88	F	0	0	N/A	0.000	N/A
89	F	0	0	N/A	0.000	N/A
90	A	11	10	1.48	28	0.357
91	A	1	1	1.00	41	0.024
92	A	3	3	1.04	42	0.071
93	A	10	9	1.07	30	0.300
94	A	10	9	1.02	30	0.300
95	A	7	6	0.96	28	0.214
96	A	2	2	1.00	30	0.067
97	A	2	2	1.00	30	0.067
98	A	2	2	1.00	30	0.067
99	A	10	9	1.01	30	0.300
100	A	7	6	1.01	30	0.200
101	A	5	4	1.00	27	0.148
102	A	2	2	1.00	30	0.067
103	A	2	2	1.00	30	0.067
104	F	0	0	N/A	0.000	N/A
105	F	0	0	N/A	0.000	N/A
106	F	0	0	N/A	0.000	N/A
107	F	0	0	N/A	0.000	N/A
108	F	0	0	N/A	0.000	N/A
109	F	0	0	N/A	0.000	N/A
110	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
111	F	0	0	N/A	0.000	N/A
112	F	0	0	N/A	0.000	N/A
113	F	0	0	N/A	0.000	N/A
114	F	0	0	N/A	0.000	N/A
115	F	0	0	N/A	0.000	N/A
116	F	0	0	N/A	0.000	N/A
117	F	0	0	N/A	0.000	N/A
118	F	0	0	N/A	0.000	N/A
119	F	0	0	N/A	0.000	N/A
120	F	0	0	N/A	0.000	N/A
121	F	0	0	N/A	0.000	N/A
122	F	0	0	N/A	0.000	N/A
123	F	0	0	N/A	0.000	N/A
124	F	0	0	N/A	0.000	N/A
125	F	0	0	N/A	0.000	N/A
126	F	0	0	N/A	0.000	N/A
127	F	0	0	N/A	0.000	N/A
128	F	0	0	N/A	0.000	N/A
129	F	0	0	N/A	0.000	N/A
130	F	0	0	N/A	0.000	N/A
131	F	0	0	N/A	0.000	N/A
132	F	0	0	N/A	0.000	N/A
133	F	0	0	N/A	0.000	N/A
134	F	0	0	N/A	0.000	N/A

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 405

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \frac{(7Bc^3-7Ac^2d-2aBd^2)\,x\sqrt{a-cx^4}}{21cd^3} - \frac{(Bc-Ad)x^3\sqrt{a-cx^4}}{5d^2} + \frac{Bx^5\sqrt{a-cx^4}}{7d}$$
$$+ \frac{a^{3/4}(Bc-Ad)\,(5c^3-2ad^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{3/4}d^4\sqrt{a-cx^4}}$$
$$- \frac{\sqrt[4]{a}\left(21\sqrt{a}\sqrt{c}d(Bc-Ad)\,(5c^3-2ad^2)-5(7Ac^2d(3c^3-2ad^2)-B(21c^6-14ac^3d^2-2a^2d^4))\right)\sqrt{1-\cdots}}{105c^{5/4}d^5\sqrt{a-cx^4}}$$
$$+ \frac{\sqrt[4]{a}c^{3/4}(Bc-Ad)\,(c^3-ad^2)\sqrt{1-\frac{cx^4}{a}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d^5\sqrt{a-cx^4}}$$

output
```
1/21*(-7*A*c^2*d-2*B*a*d^2+7*B*c^3)*x*(-c*x^4+a)^(1/2)/c/d^3-1/5*(-A*d+B*c
)*x^3*(-c*x^4+a)^(1/2)/d^2+1/7*B*x^5*(-c*x^4+a)^(1/2)/d+1/5*a^(3/4)*(-A*d+
B*c)*(-2*a*d^2+5*c^3)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(
3/4)/d^4/(-c*x^4+a)^(1/2)-1/105*a^(1/4)*(21*a^(1/2)*c^(1/2)*d*(-A*d+B*c)*(
-2*a*d^2+5*c^3)-35*A*c^2*d*(-2*a*d^2+3*c^3)+5*B*(-2*a^2*d^4-14*a*c^3*d^2+2
1*c^6))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(5/4)/d^5/(-c*x
^4+a)^(1/2)+a^(1/4)*c^(3/4)*(-A*d+B*c)*(-a*d^2+c^3)*(1-c*x^4/a)^(1/2)*Elli
pticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/2),I)/d^5/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.91 (sec) , antiderivative size = 817, normalized size of antiderivative = 2.02

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$$
$$= \frac{35aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3d^2x - 35aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2d^3x - 10a^2B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d^4x - 21aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2d^3x^3 + 21aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cd^4x^3 -}{}$$

input
```
Integrate[(x^4*(A + B*x^2)*Sqrt[a - c*x^4])/(c + d*x^2),x]
```

output
```
(35*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*d^2*x - 35*a*A*Sqrt[-(Sqrt[c]/Sqrt[a]
)]*c^2*d^3*x - 10*a^2*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^4*x - 21*a*B*Sqrt[-(Sqr
t[c]/Sqrt[a])]*c^2*d^3*x^3 + 21*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^4*x^3 - 3
5*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^4*d^2*x^5 + 35*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c
^3*d^3*x^5 + 25*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^4*x^5 + 21*B*Sqrt[-(Sqrt[
c]/Sqrt[a])]*c^3*d^3*x^7 - 21*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d^4*x^7 - 15*
B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d^4*x^9 + (21*I)*Sqrt[a]*Sqrt[c]*d*(B*c - A
*d)*(-5*c^3 + 2*a*d^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt
[c]/Sqrt[a])]*x], -1] - I*(7*A*Sqrt[c]*d*(15*c^(9/2) + 15*Sqrt[a]*c^3*d -
10*a*c^(3/2)*d^2 - 6*a^(3/2)*d^3) + B*(-105*c^6 - 105*Sqrt[a]*c^(9/2)*d +
70*a*c^3*d^2 + 42*a^(3/2)*c^(3/2)*d^3 + 10*a^2*d^4))*Sqrt[1 - (c*x^4)/a]*E
llipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (105*I)*B*c^6*Sqrt[1
 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/
Sqrt[a])]*x], -1] + (105*I)*A*c^5*d*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt
[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (105*I)*a*B*
c^3*d^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[S
qrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (105*I)*a*A*c^2*d^3*Sqrt[1 - (c*x^4)/a]*
EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x],
-1])/(105*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^5*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.74, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a-cx^4}\left(A+Bx^2\right)}{c+dx^2}dx$$

$$\Big\downarrow \text{2249}$$

$$\int \left(-\frac{c\left(c^3-ad^2\right)(Bc-Ad)}{d^5\sqrt{a-cx^4}}+\frac{x^2\left(c^3-ad^2\right)(Bc-Ad)}{d^4\sqrt{a-cx^4}}-\frac{x^4\left(-aBd^2-Ac^2d+Bc^3\right)}{d^3\sqrt{a-cx^4}}+\frac{aAc^2d^3-aBc^3d^2-A}{d^5\sqrt{a-cx^4}\,(c+}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a^{7/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{3/4}d^2\sqrt{a-cx^4}}+\frac{3a^{7/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{3/4}d^2\sqrt{a-cx^4}}-\frac{a^{3/4}(c^3-ad^2)\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}d^4\sqrt{a-cx^4}}+\frac{a^{3/4}(c^3-ad^2)\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}d^4\sqrt{a-cx^4}}-\frac{a^{5/4}\sqrt{1-\frac{cx^4}{a}}(-aBd^2-Ac^2d+Bc^3)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}d^3\sqrt{a-cx^4}}-\frac{5a^{9/4}B\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{21c^{5/4}d\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}c^{3/4}(c^3-ad^2)\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d^5\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}c^{3/4}(c^3-ad^2)\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d^5\sqrt{a-cx^4}}-\frac{x\sqrt{a-cx^4}\left(Ac^2d-B(c^3-ad^2)\right)}{3cd^3}-\frac{x^3\sqrt{a-cx^4}(Bc-Ad)}{5d^2}+\frac{5aBx\sqrt{a-cx^4}}{21cd}+\frac{Bx^5\sqrt{a-cx^4}}{7d}$$

input
```
Int[(x^4*(A + B*x^2)*Sqrt[a - c*x^4])/(c + d*x^2),x]
```

output
```
(5*a*B*x*Sqrt[a - c*x^4])/(21*c*d) - ((A*c^2*d - B*(c^3 - a*d^2))*x*Sqrt[a
 - c*x^4])/(3*c*d^3) - ((B*c - A*d)*x^3*Sqrt[a - c*x^4])/(5*d^2) + (B*x^5*
Sqrt[a - c*x^4])/(7*d) + (3*a^(7/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*Ellipt
icE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(3/4)*d^2*Sqrt[a - c*x^4]) + (a
^(3/4)*(B*c - A*d)*(c^3 - a*d^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(
1/4)*x)/a^(1/4)], -1])/(c^(3/4)*d^4*Sqrt[a - c*x^4]) - (5*a^(9/4)*B*Sqrt[1
 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(21*c^(5/4)*d*Sq
rt[a - c*x^4]) - (3*a^(7/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcS
in[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(3/4)*d^2*Sqrt[a - c*x^4]) - (a^(1/4)*c
^(3/4)*(B*c - A*d)*(c^3 - a*d^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(
1/4)*x)/a^(1/4)], -1])/(d^5*Sqrt[a - c*x^4]) - (a^(3/4)*(B*c - A*d)*(c^3 -
 a*d^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c
^(3/4)*d^4*Sqrt[a - c*x^4]) - (a^(5/4)*(B*c^3 - A*c^2*d - a*B*d^2)*Sqrt[1
- (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(5/4)*d^3*Sq
rt[a - c*x^4]) + (a^(1/4)*c^(3/4)*(B*c - A*d)*(c^3 - a*d^2)*Sqrt[1 - (c*x^
4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(d^5*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{x(-15B\,x^4 c\,d^2-21Ac\,d^2x^2+21B\,c^2d\,x^2+35A\,c^2d+10Ba\,d^2-35B\,c^3)\sqrt{-c\,x^4+a}}{105c\,d^3}+\frac{-\frac{5\left(14aA\,c^2d^3-21A\,c^5d-2a^2B\,d^4-14aB\,c^3d^2 \cdots\right.}{d^2\sqrt{\cdots}}}{\cdots}$
default	$\frac{c^2(Ad-Bc)\left(\frac{c^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{d^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-c\,x^4+a}}+\frac{\sqrt{c}\,\sqrt{a}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-c\,x^4+a}}-\frac{\sqrt{c}\,\sqrt{a}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\cdots}}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\cdots}\cdots\right.}{\cdots}$
elliptic	Expression too large to display

input
```
int(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/105*x*(-15*B*c*d^2*x^4-21*A*c*d^2*x^2+21*B*c^2*d*x^2+35*A*c^2*d+10*B*a*
d^2-35*B*c^3)*(-c*x^4+a)^(1/2)/c/d^3+1/105/c/d^3*(-5*(14*A*a*c^2*d^3-21*A*
c^5*d-2*B*a^2*d^4-14*B*a*c^3*d^2+21*B*c^6)/d^2/(c^(1/2)/a^(1/2))^(1/2)*(1-
c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*
EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-21*c^(1/2)/d*(2*A*a*d^3-5*A*c^3*d-2
*B*a*c*d^2+5*B*c^4)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2)
)^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/
2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))+105*c^2*(A*a*
d^3-A*c^3*d-B*a*c*d^2+B*c^4)/d^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^
(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(
c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2
)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$$

input
```
integrate(x**4*(B*x**2+A)*(-c*x**4+a)**(1/2)/(d*x**2+c),x)
```

output
```
Integral(x**4*(A + B*x**2)*sqrt(a - c*x**4)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^4}{dx^2+c}\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^4/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^4}{dx^2+c}\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^4/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \int \frac{x^4\,(B\,x^2+A)\,\sqrt{a-c\,x^4}}{d\,x^2+c}\,dx$$

input
```
int((x^4*(A + B*x^2)*(a - c*x^4)^(1/2))/(c + d*x^2),x)
```

output
```
int((x^4*(A + B*x^2)*(a - c*x^4)^(1/2))/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$$
$$= \frac{-10\sqrt{-c\,x^4+a}\,ab\,d^2x-35\sqrt{-c\,x^4+a}\,a\,c^2dx+21\sqrt{-c\,x^4+a}\,ac\,d^2x^3+35\sqrt{-c\,x^4+a}\,b\,c^3x-21\sqrt{-c}}{}$$

input
```
int(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x)
```

output
```
( - 10*sqrt(a - c*x**4)*a*b*d**2*x - 35*sqrt(a - c*x**4)*a*c**2*d*x + 21*s
qrt(a - c*x**4)*a*c*d**2*x**3 + 35*sqrt(a - c*x**4)*b*c**3*x - 21*sqrt(a -
 c*x**4)*b*c**2*d*x**3 + 15*sqrt(a - c*x**4)*b*c*d**2*x**5 + 10*int(sqrt(a
 - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a**2*b*c*d**2 + 35*i
nt(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a**2*c**3*d
 - 35*int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a*b*
c**4 + 42*int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x*
*6),x)*a**2*c*d**3 - 42*int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x**2 - c**2
*x**4 - c*d*x**6),x)*a*b*c**2*d**2 - 105*int((sqrt(a - c*x**4)*x**4)/(a*c
+ a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a*c**4*d + 105*int((sqrt(a - c*x**4)
*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*b*c**5 + 10*int((sqrt(a
- c*x**4)*x**2)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a**2*b*d**3 - 2
8*int((sqrt(a - c*x**4)*x**2)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a
**2*c**2*d**2 + 28*int((sqrt(a - c*x**4)*x**2)/(a*c + a*d*x**2 - c**2*x**4
 - c*d*x**6),x)*a*b*c**3*d)/(105*c*d**3)
```

3.2 $\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 336

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = -\frac{(Bc-Ad)x\sqrt{a-cx^4}}{3d^2}+\frac{Bx^3\sqrt{a-cx^4}}{5d}$$
$$-\frac{a^{3/4}(5Bc^3-5Ac^2d-2aBd^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{3/4}d^3\sqrt{a-cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(5\sqrt{c}(Bc-Ad)\left(3c^3-2ad^2\right)+3\sqrt{a}d(5Bc^3-5Ac^2d-2aBd^2)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{15c^{3/4}d^4\sqrt{a-cx^4}}$$
$$-\frac{\sqrt[4]{a}(Bc-Ad)\left(c^3-ad^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d^4\sqrt{a-cx^4}}$$

output
```
-1/3*(-A*d+B*c)*x*(-c*x^4+a)^(1/2)/d^2+1/5*B*x^3*(-c*x^4+a)^(1/2)/d-1/5*a^
(3/4)*(-5*A*c^2*d-2*B*a*d^2+5*B*c^3)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x
/a^(1/4),I)/c^(3/4)/d^3/(-c*x^4+a)^(1/2)+1/15*a^(1/4)*(5*c^(1/2)*(-A*d+B*c
)*(-2*a*d^2+3*c^3)+3*a^(1/2)*d*(-5*A*c^2*d-2*B*a*d^2+5*B*c^3))*(1-c*x^4/a)
^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(3/4)/d^4/(-c*x^4+a)^(1/2)-a^(1/4)
*(-A*d+B*c)*(-a*d^2+c^3)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a
^(1/2)*d/c^(3/2),I)/c^(1/4)/d^4/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.62 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.08

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$$
$$= \frac{-3i\sqrt{a}d(-5Bc^3+5Ac^2d+2aBd^2)\sqrt{1-\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+i\left(5A\sqrt{c}d\left(3c^3+3\sqrt{a}c^{3/2}d\right.\right.}{}$$

input
```
Integrate[(x^2*(A + B*x^2)*Sqrt[a - c*x^4])/(c + d*x^2),x]
```

output
```
((-3*I)*Sqrt[a]*d*(-5*B*c^3 + 5*A*c^2*d + 2*a*B*d^2)*Sqrt[1 - (c*x^4)/a]*E
llipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*(5*A*Sqrt[c]*d*(3*
c^3 + 3*Sqrt[a]*c^(3/2)*d - 2*a*d^2) + B*(-15*c^(9/2) - 15*Sqrt[a]*c^3*d +
 10*a*c^(3/2)*d^2 + 6*a^(3/2)*d^3))*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSin
h[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*(Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2
*x*(5*B*c - 5*A*d - 3*B*d*x^2)*(-a + c*x^4) - (15*I)*(B*c - A*d)*(-c^3 + a
*d^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqr
t[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(15*Sqrt[-(Sqrt[c]/Sqrt[a])]*Sqrt[c]*d^4*S
qrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.65, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a-cx^4}(A+Bx^2)}{c+dx^2}\,dx$$
$$\Big\downarrow \text{ 2249}$$

$$\int \left(\frac{(c^3-ad^2)(Bc-Ad)}{d^4\sqrt{a-cx^4}} - \frac{x^2(-aBd^2-Ac^2d+Bc^3)}{d^3\sqrt{a-cx^4}} + \frac{-aAcd^3+aBc^2d^2+Ac^4d-Bc^5}{d^4\sqrt{a-cx^4}(c+dx^2)} + \frac{cx^4(Bc-Ad)}{d^2\sqrt{a-cx^4}} - \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}(-aBd^2-Ac^2d+Bc^3)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}d^3\sqrt{a-cx^4}} - \frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}(-aBd^2-Ac^2d+Bc^3)\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}d^3\sqrt{a-cx^4}} + \frac{a^{5/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{c}d^2\sqrt{a-cx^4}} + \frac{3a^{7/4}B\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5c^{3/4}d\sqrt{a-cx^4}} - \frac{3a^{7/4}B\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{3/4}d\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}(c^3-ad^2)\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d^4\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}(c^3-ad^2)\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d^4\sqrt{a-cx^4}} - \frac{x\sqrt{a-cx^4}(Bc-Ad)}{3d^2} + \frac{Bx^3\sqrt{a-cx^4}}{5d}$$

input
```
Int[(x^2*(A + B*x^2)*Sqrt[a - c*x^4])/(c + d*x^2),x]
```

output
```
-1/3*((B*c - A*d)*x*Sqrt[a - c*x^4])/d^2 + (B*x^3*Sqrt[a - c*x^4])/(5*d) -
 (3*a^(7/4)*B*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(5*c^(3/4)*d*Sqrt[a - c*x^4]) - (a^(3/4)*(B*c^3 - A*c^2*d - a*B*d^2)*S
qrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*d^
3*Sqrt[a - c*x^4]) + (3*a^(7/4)*B*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^
(1/4)*x)/a^(1/4)], -1])/(5*c^(3/4)*d*Sqrt[a - c*x^4]) + (a^(5/4)*(B*c - A*
d)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(1
/4)*d^2*Sqrt[a - c*x^4]) + (a^(1/4)*(B*c - A*d)*(c^3 - a*d^2)*Sqrt[1 - (c*
x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d^4*Sqrt[a -
c*x^4]) + (a^(3/4)*(B*c^3 - A*c^2*d - a*B*d^2)*Sqrt[1 - (c*x^4)/a]*Ellipti
cF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*d^3*Sqrt[a - c*x^4]) - (a^(1
/4)*(B*c - A*d)*(c^3 - a*d^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)
/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d^4*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.10

method	result
risch	$\frac{x(3Bx^2d+5Ad-5Bc)\sqrt{-cx^4+a}}{15d^2}+\frac{\frac{5\left(2Aad^3-3Ac^3d-2Bacd^2+3Bc^4\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{d^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}-\frac{3\left(5Ac^2d+2Bad^2\right.}{}}{}$
default	$\frac{Ad\left(\frac{x\sqrt{-cx^4+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}\right)+Bd\left(\frac{x^3\sqrt{-cx^4+a}}{5}-\frac{2a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right.\right.}{5\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}\sqrt{c}}\right.}{d^2}$
elliptic	Expression too large to display

input
```
int(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*B*d*x^2+5*A*d-5*B*c)*(-c*x^4+a)^(1/2)/d^2+1/15/d^2*(5*(2*A*a*d^3
-3*A*c^3*d-2*B*a*c*d^2+3*B*c^4)/d^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2
/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x
*(c^(1/2)/a^(1/2))^(1/2),I)-3/d*(5*A*c^2*d+2*B*a*d^2-5*B*c^3)*a^(1/2)/(c^(
1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^
(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-Ell
ipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))-15*(A*a*d^3-A*c^3*d-B*a*c*d^2+B*c^4)/
d^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a
^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/
2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$$

input
```
integrate(x**2*(B*x**2+A)*(-c*x**4+a)**(1/2)/(d*x**2+c),x)
```

output
```
Integral(x**2*(A + B*x**2)*sqrt(a - c*x**4)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^2}{dx^2+c}\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^2/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^2}{dx^2+c}\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^2/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx=\int \frac{x^2\,(B\,x^2+A)\,\sqrt{a-c\,x^4}}{d\,x^2+c}\,dx$$

input
```
int((x^2*(A + B*x^2)*(a - c*x^4)^(1/2))/(c + d*x^2),x)
```

output
```
int((x^2*(A + B*x^2)*(a - c*x^4)^(1/2))/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$$
$$=\frac{5\sqrt{-c\,x^4+a}\,adx-5\sqrt{-c\,x^4+a}\,bcx+3\sqrt{-c\,x^4+a}\,bd\,x^3-5\left(\int\frac{\sqrt{-c\,x^4+a}}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)a^2cd+5\left(\int\frac{}{-cd}\right.}{}$$

input
```
int(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x)
```

output
```
(5*sqrt(a - c*x**4)*a*d*x - 5*sqrt(a - c*x**4)*b*c*x + 3*sqrt(a - c*x**4)*
b*d*x**3 - 5*int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),
x)*a**2*c*d + 5*int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**
6),x)*a*b*c**2 + 6*int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x**2 - c**2*x**4
 - c*d*x**6),x)*a*b*d**2 + 15*int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x**2
- c**2*x**4 - c*d*x**6),x)*a*c**2*d - 15*int((sqrt(a - c*x**4)*x**4)/(a*c
+ a*d*x**2 - c**2*x**4 - c*d*x**6),x)*b*c**3 + 10*int((sqrt(a - c*x**4)*x*
*2)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a**2*d**2 - 4*int((sqrt(a -
 c*x**4)*x**2)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a*b*c*d)/(15*d**
2)
```

3.3 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 274

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \frac{Bx\sqrt{a-cx^4}}{3d}+\frac{a^{3/4}\sqrt[4]{c}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\left(3Bc^3-3Ac^2d-2aBd^2+3\sqrt{a}\sqrt{c}d(Bc-Ad)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{c}d^3\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}(Bc-Ad)\left(c^3-ad^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{5/4}d^3\sqrt{a-cx^4}}$$

output
```
1/3*B*x*(-c*x^4+a)^(1/2)/d+a^(3/4)*c^(1/4)*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*El
lipticE(c^(1/4)*x/a^(1/4),I)/d^2/(-c*x^4+a)^(1/2)-1/3*a^(1/4)*(3*B*c^3-3*A
*c^2*d-2*B*a*d^2+3*a^(1/2)*c^(1/2)*d*(-A*d+B*c))*(1-c*x^4/a)^(1/2)*Ellipti
cF(c^(1/4)*x/a^(1/4),I)/c^(1/4)/d^3/(-c*x^4+a)^(1/2)+a^(1/4)*(-A*d+B*c)*(-
a*d^2+c^3)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/
2),I)/c^(5/4)/d^3/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.09 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.81

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$$
$$= \frac{aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cd^2x - B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2d^2x^5 - 3i\sqrt{a}c^{3/2}d(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right) - ic(3A}{}$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(c + d*x^2),x]
```

output
```
(a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^2*x - B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d^2
*x^5 - (3*I)*Sqrt[a]*c^(3/2)*d*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[I
*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*c*(3*A*Sqrt[c]*d*(c^(3/2) +
Sqrt[a]*d) + B*(-3*c^3 - 3*Sqrt[a]*c^(3/2)*d + 2*a*d^2))*Sqrt[1 - (c*x^4)/
a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (3*I)*B*c^4*Sqrt
[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c
]/Sqrt[a])]*x], -1] + (3*I)*A*c^3*d*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt
[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (3*I)*a*B*c*
d^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[
-(Sqrt[c]/Sqrt[a])]*x], -1] - (3*I)*a*A*d^3*Sqrt[1 - (c*x^4)/a]*EllipticPi
[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(3*Sq
rt[-(Sqrt[c]/Sqrt[a])]*c*d^3*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}\left(A+Bx^2\right)}{c+dx^2}dx$$

$$\Big\downarrow \text{ 2259}$$

$$\int \left(\frac{aAd^3-aBcd^2-Ac^3d+Bc^4}{d^3\sqrt{a-cx^4}\left(c+dx^2\right)}-\frac{-aBd^2-Ac^2d+Bc^3}{d^3\sqrt{a-cx^4}}+\frac{cx^2(Bc-Ad)}{d^2\sqrt{a-cx^4}}-\frac{Bcx^4}{d\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^{3/4}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d^2\sqrt{a-cx^4}}+\frac{a^{3/4}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2\sqrt{a-cx^4}}-\frac{a^{5/4}B\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{c}d\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\left(c^3-ad^2\right)\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{5/4}d^3\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-aBd^2-Ac^2d+Bc^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d^3\sqrt{a-cx^4}}+\frac{Bx\sqrt{a-cx^4}}{3d}$$

input

```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(c + d*x^2),x]
```

output

```
(B*x*Sqrt[a - c*x^4])/(3*d) + (a^(3/4)*c^(1/4)*(B*c - A*d)*Sqrt[1 - (c*x^4
)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(d^2*Sqrt[a - c*x^4]) - (
a^(5/4)*B*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(3*c^(1/4)*d*Sqrt[a - c*x^4]) - (a^(3/4)*c^(1/4)*(B*c - A*d)*Sqrt[1 - (c*x
^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(d^2*Sqrt[a - c*x^4]) -
 (a^(1/4)*(B*c^3 - A*c^2*d - a*B*d^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin
[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d^3*Sqrt[a - c*x^4]) + (a^(1/4)*(B*c
- A*d)*(c^3 - a*d^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2))
, ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(5/4)*d^3*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(224) = 448$.

Time = 2.34 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.66

method	result
risch	$\frac{Bx\sqrt{-cx^4+a}}{3d} - \frac{-\frac{3\sqrt{c}\,d(Ad-Bc)\sqrt{a}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}+\frac{3Bc^3\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\cdots}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4\cdots}}}{\cdots}$
default	$\frac{B\left(\frac{x\sqrt{-cx^4+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}\right)}{d} + \frac{(Ad-Bc)\left(\frac{c^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{d^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}+\frac{\sqrt{c}\sqrt{a}\cdots}{\cdots}\right.}{\cdots}$
elliptic	Expression too large to display

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x*(-c*x^4+a)^(1/2)/d-1/3/d*(1/d^2*(-3*c^(1/2)*d*(A*d-B*c)*a^(1/2)/(c
^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2)
)^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE
(x*(c^(1/2)/a^(1/2))^(1/2),I))+3*B*c^3/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*
x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
F(x*(c^(1/2)/a^(1/2))^(1/2),I)-3*A*c^2*d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2
)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ellipt
icF(x*(c^(1/2)/a^(1/2))^(1/2),I)-2*B*a*d^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1
/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Elli
pticF(x*(c^(1/2)/a^(1/2))^(1/2),I))-3*(A*a*d^3-A*c^3*d-B*a*c*d^2+B*c^4)/d^
2/c/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a
^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/
2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/(d*x**2+c),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(A + Bx^2)\sqrt{a - cx^4}}{c + dx^2}\,dx = \int \frac{\sqrt{-cx^4 + a}(Bx^2 + A)}{dx^2 + c}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(A + Bx^2)\sqrt{a - cx^4}}{c + dx^2}\,dx = \int \frac{\sqrt{-cx^4 + a}(Bx^2 + A)}{dx^2 + c}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + Bx^2)\sqrt{a - cx^4}}{c + dx^2}\,dx = \int \frac{(B\,x^2 + A)\,\sqrt{a - c\,x^4}}{d\,x^2 + c}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(c + d*x^2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{c+dx^2}\,dx$$
$$= \frac{\sqrt{-c\,x^4+a}\,bx+3\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)a^2d-\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)abc-3\left(\int \frac{\sqrt{-c\,x^4+a}\,x^4}{-cd\,x^6-c^2x^4+ad\,x^2}\right.}{3d}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(d*x^2+c),x)
```

output
```
(sqrt(a - c*x**4)*b*x + 3*int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4
 - c*d*x**6),x)*a**2*d - int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4
- c*d*x**6),x)*a*b*c - 3*int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x**2 - c**
2*x**4 - c*d*x**6),x)*a*c*d + 3*int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x**
2 - c**2*x**4 - c*d*x**6),x)*b*c**2 + 2*int((sqrt(a - c*x**4)*x**2)/(a*c +
 a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a*b*d)/(3*d)
```

3.4 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(c+dx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 259

$$\begin{aligned}\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(c+dx^2)}\,dx &= -\frac{A\sqrt{a-cx^4}}{cx} - \frac{a^{3/4}(Bc+Ad)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}d\sqrt{a-cx^4}}\\ &\quad+\frac{\sqrt[4]{a}\left(c^{3/2}(Bc-Ad)+\sqrt{a}d(Bc+Ad)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}d^2\sqrt{a-cx^4}}\\ &\quad-\frac{\sqrt[4]{a}(Bc-Ad)\left(c^3-ad^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{9/4}d^2\sqrt{a-cx^4}}\end{aligned}$$

output
```
-A*(-c*x^4+a)^(1/2)/c/x-a^(3/4)*(A*d+B*c)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1
/4)*x/a^(1/4),I)/c^(3/4)/d/(-c*x^4+a)^(1/2)+a^(1/4)*(c^(3/2)*(-A*d+B*c)+a^
(1/2)*d*(A*d+B*c))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(3/4
)/d^2/(-c*x^4+a)^(1/2)-a^(1/4)*(-A*d+B*c)*(-a*d^2+c^3)*(1-c*x^4/a)^(1/2)*E
llipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/2),I)/c^(9/4)/d^2/(-c*x^4+a)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.06 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.89

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(c+dx^2)}\,dx$$
$$= \frac{-aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cd^2 + A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2d^2x^4 + i\sqrt{a}c^{3/2}d(Bc+Ad)x\sqrt{1-\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right) - ic^{3/2}}{}$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^2*(c + d*x^2)),x]
```

output
```
(-(a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^2) + A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d^
2*x^4 + I*Sqrt[a]*c^(3/2)*d*(B*c + A*d)*x*Sqrt[1 - (c*x^4)/a]*EllipticE[I*
ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*c^(3/2)*(A*d*(-c^(3/2) + Sqrt
[a]*d) + B*(c^(5/2) + Sqrt[a]*c*d))*x*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcS
inh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*B*c^4*x*Sqrt[1 - (c*x^4)/a]*Ellip
ticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] -
 I*A*c^3*d*x*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcS
inh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*a*B*c*d^2*x*Sqrt[1 - (c*x^4)/a]*E
llipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -
1] + I*a*A*d^3*x*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*
ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d^
2*x*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.54, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}\left(A+Bx^2\right)}{x^2\left(c+dx^2\right)}dx$$
$$\Big\downarrow \text{2249}$$
$$\int \left(-\frac{\left(c^3-ad^2\right)(Bc-Ad)}{cd^2\sqrt{a-cx^4}\left(c+dx^2\right)}+\frac{c(Bc-Ad)}{d^2\sqrt{a-cx^4}}+\frac{aA}{cx^2\sqrt{a-cx^4}}-\frac{Bcx^2}{d\sqrt{a-cx^4}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^{3/4}A\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{3/4}\sqrt{a-cx^4}}-\frac{a^{3/4}A\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{3/4}B\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-cx^4}}-\frac{a^{3/4}B\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}c^{3/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d^2\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\left(c^3-ad^2\right)\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{9/4}d^2\sqrt{a-cx^4}}-\frac{A\sqrt{a-cx^4}}{cx}$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^2*(c + d*x^2)),x]
```

output
```
-((A*Sqrt[a - c*x^4])/(c*x)) - (a^(3/4)*A*Sqrt[1 - (c*x^4)/a]*EllipticE[Ar
cSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) - (a^(3/4)*B*c^(
1/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(d*Sq
rt[a - c*x^4]) + (a^(3/4)*A*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*
x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(3/4)*B*c^(1/4)*Sqrt[1 -
(c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(d*Sqrt[a - c*x^4])
 + (a^(1/4)*c^(3/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1
/4)*x)/a^(1/4)], -1])/(d^2*Sqrt[a - c*x^4]) - (a^(1/4)*(B*c - A*d)*(c^3 -
a*d^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^(1
/4)*x)/a^(1/4)], -1])/(c^(9/4)*d^2*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.49

method	result
risch	$-\frac{A\sqrt{-c\,x^{4}+a}}{cx}-\frac{c\left(-\frac{d(Ad+Bc)\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,\sqrt{c}}+\frac{Acd\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{El}\cdots}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}\cdots}}\cdots\right)}{d^{2}\cdots}$
default	$\frac{A\left(-\frac{\sqrt{-c\,x^{4}+a}}{x}+\frac{2\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right)}{c}-\frac{(Ad-Bc)\left(\frac{c^{2}\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\cdots}{d^{2}\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{\cdots}}\cdots\right)}{\cdots}$
elliptic	$-\frac{A\sqrt{-c\,x^{4}+a}}{cx}-\frac{c\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)A}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{c^{2}\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)B}{d^{2}\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}+\frac{\sqrt{a}\cdots}{\cdots}$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-A*(-c*x^4+a)^(1/2)/c/x-1/c*(c/d^2*(-d*(A*d+B*c)*a^(1/2)/(c^(1/2)/a^(1/2))
^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4
+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(
1/2)/a^(1/2))^(1/2),I))+A*c*d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/
2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1
/2)/a^(1/2))^(1/2),I)-B*c^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2)
)^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2
)/a^(1/2))^(1/2),I))+(A*a*d^3-A*c^3*d-B*a*c*d^2+B*c^4)/d^2/c/(c^(1/2)/a^(1
/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c
*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*d/c^(3/2),(-c^
(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(c+dx^2)}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(c+dx^2)}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**2/(d*x**2+c),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**2*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(c+dx^2)}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(dx^2+c)x^2}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(c+dx^2)}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(dx^2+c)x^2}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(c+dx^2)}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{x^2\,(d\,x^2+c)}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^2*(c + d*x^2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^2*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(c+dx^2)}\,dx$$

$$= \frac{\sqrt{-c\,x^4+a}\,a+2\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^8-c^2x^6+ad\,x^4+ac\,x^2}dx\right)a^2cx+\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)a^2dx+\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^6-c^2x^4+ad}\right.}{cx}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(d*x^2+c),x)
```

output
```
(sqrt(a - c*x**4)*a + 2*int(sqrt(a - c*x**4)/(a*c*x**2 + a*d*x**4 - c**2*x
**6 - c*d*x**8),x)*a**2*c*x + int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*
x**4 - c*d*x**6),x)*a**2*d*x + int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2
*x**4 - c*d*x**6),x)*a*b*c*x + int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x**2
 - c**2*x**4 - c*d*x**6),x)*a*c*d*x - int((sqrt(a - c*x**4)*x**4)/(a*c + a
*d*x**2 - c**2*x**4 - c*d*x**6),x)*b*c**2*x)/(c*x)
```

3.5 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(c+dx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 293

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(c+dx^2)}\,dx = -\frac{A\sqrt{a-cx^4}}{3cx^3} - \frac{(Bc-Ad)\sqrt{a-cx^4}}{c^2x} - \frac{a^{3/4}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{7/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(3\sqrt{a}d(Bc-Ad)-c^{3/2}(3Bc-Ad)\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{7/4}d\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}(Bc-Ad)\left(c^3-ad^2\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{13/4}d\sqrt{a-cx^4}}$$

output

```
-1/3*A*(-c*x^4+a)^(1/2)/c/x^3-(-A*d+B*c)*(-c*x^4+a)^(1/2)/c^2/x-a^(3/4)*(-
A*d+B*c)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(7/4)/(-c*x^4+
a)^(1/2)+1/3*a^(1/4)*(3*a^(1/2)*d*(-A*d+B*c)-c^(3/2)*(-A*d+3*B*c))*(1-c*x^
4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(7/4)/d/(-c*x^4+a)^(1/2)+a^(1/
4)*(-A*d+B*c)*(-a*d^2+c^3)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),
-a^(1/2)*d/c^(3/2),I)/c^(13/4)/d/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.22 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.09

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(c+dx^2)}\,dx$$
$$= \frac{-aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2d-3aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2dx^2+3aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cd^2x^2+A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3dx^4+3B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3dx^6-3A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{}$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^4*(c + d*x^2)),x]
```

output
```
(-(a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d) - 3*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^
2*d*x^2 + 3*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^2*x^2 + A*Sqrt[-(Sqrt[c]/Sqrt
[a])]*c^3*d*x^4 + 3*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*d*x^6 - 3*A*Sqrt[-(Sqrt
[c]/Sqrt[a])]*c^2*d^2*x^6 + (3*I)*Sqrt[a]*c^(3/2)*d*(B*c - A*d)*x^3*Sqrt[1
 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*c^(
3/2)*(A*d*(c^(3/2) - 3*Sqrt[a]*d) - 3*B*(c^(5/2) - Sqrt[a]*c*d))*x^3*Sqrt[
1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (3*I
)*B*c^4*x^3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSi
nh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (3*I)*A*c^3*d*x^3*Sqrt[1 - (c*x^4)/a
]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x]
, -1] + (3*I)*a*B*c*d^2*x^3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c
^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (3*I)*a*A*d^3*x^3*Sq
rt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt
[c]/Sqrt[a])]*x], -1])/(3*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*d*x^3*Sqrt[a - c*x^
4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}\left(A+Bx^2\right)}{x^4\left(c+dx^2\right)}dx$$

$$\Big\downarrow \text{ 2249}$$

$$\int \left(\frac{a(Bc-Ad)}{c^2x^2\sqrt{a-cx^4}}+\frac{\left(c^3-ad^2\right)(Bc-Ad)}{c^2d\sqrt{a-cx^4}\left(c+dx^2\right)}+\frac{aA}{cx^4\sqrt{a-cx^4}}-\frac{Bc}{d\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{7/4}\sqrt{a-cx^4}}-\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{7/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\left(c^3-ad^2\right)\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{13/4}d\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}A\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{\sqrt{a-cx^4}(Bc-Ad)}{c^2x}-\frac{A\sqrt{a-cx^4}}{3cx^3}-\frac{\sqrt[4]{a}Bc^{3/4}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-cx^4}}$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^4*(c + d*x^2)),x]
```

output
```
-1/3*(A*Sqrt[a - c*x^4])/(c*x^3) - ((B*c - A*d)*Sqrt[a - c*x^4])/(c^2*x) -
 (a^(3/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(
1/4)], -1])/(c^(7/4)*Sqrt[a - c*x^4]) + (a^(1/4)*A*Sqrt[1 - (c*x^4)/a]*Ell
ipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(1/4)*Sqrt[a - c*x^4]) - (a^
(1/4)*B*c^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)],
 -1])/(d*Sqrt[a - c*x^4]) + (a^(3/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*Ellip
ticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(7/4)*Sqrt[a - c*x^4]) + (a^(1/4
)*(B*c - A*d)*(c^3 - a*d^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c
^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(13/4)*d*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{\sqrt{-c x^4+a}\,(-3 A d x^2+3 B c x^2+A c)}{3 c^2 x^3}+\frac{c\left(-\frac{3 d (A d-B c)\sqrt{a}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}\,\sqrt{c}}+\frac{Acd \ldots}{\ldots}\ldots\right)}{\ldots}$
default	$\frac{A\left(-\frac{\sqrt{-c x^4+a}}{3 x^3}-\frac{2 c\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}\right)}{c}-\frac{(A d-B c)\left(-\frac{\sqrt{-c x^4+a}}{x}+\frac{2\sqrt{c}\,\sqrt{a}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{Elliptic}\ldots\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-\ldots}}\ldots\right)}{c^2}\ldots$
elliptic	$-\frac{A\sqrt{-c x^4+a}}{3 c x^3}+\frac{(A d-B c)\sqrt{-c x^4+a}}{c^2 x}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)B c}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}\,d}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\ldots}{3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}\ldots}\ldots$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-c*x^4+a)^(1/2)*(-3*A*d*x^2+3*B*c*x^2+A*c)/c^2/x^3+1/3/c^2*(c/d*(-3*
d*(A*d-B*c)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*
(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/
2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))+A*c*d/(c^(1/2
)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/
2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-3*B*c^2/(c^(1/2
)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/
2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))+3*(A*a*d^3-A*c
^3*d-B*a*c*d^2+B*c^4)/d/c/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^
(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)
/a^(1/2))^(1/2),-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/
2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(c+dx^2)}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(c+dx^2)}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**4/(d*x**2+c),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**4*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(A + Bx^2)\sqrt{a - cx^4}}{x^4(c + dx^2)}\, dx = \int \frac{\sqrt{-cx^4 + a}(Bx^2 + A)}{(dx^2 + c)x^4}\, dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{(A + Bx^2)\sqrt{a - cx^4}}{x^4(c + dx^2)}\, dx = \int \frac{\sqrt{-cx^4 + a}(Bx^2 + A)}{(dx^2 + c)x^4}\, dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + Bx^2)\sqrt{a - cx^4}}{x^4(c + dx^2)}\, dx = \int \frac{(B\,x^2 + A)\,\sqrt{a - c\,x^4}}{x^4\,(d\,x^2 + c)}\, dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^4*(c + d*x^2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^4*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(c+dx^2)}\,dx$$
$$= \frac{-\sqrt{-c\,x^4+a}\,a-3\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^8-c^2x^6+ad\,x^4+ac\,x^2}dx\right)a^2d\,x^3+3\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^8-c^2x^6+ad\,x^4+ac\,x^2}dx\right)abc\,x^3-2\left(\int \frac{}{-cd}\right.}{3c\,x^3}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(d*x^2+c),x)
```

output
```
( - sqrt(a - c*x**4)*a - 3*int(sqrt(a - c*x**4)/(a*c*x**2 + a*d*x**4 - c**
2*x**6 - c*d*x**8),x)*a**2*d*x**3 + 3*int(sqrt(a - c*x**4)/(a*c*x**2 + a*d
*x**4 - c**2*x**6 - c*d*x**8),x)*a*b*c*x**3 - 2*int(sqrt(a - c*x**4)/(a*c
+ a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a*c**2*x**3 + int((sqrt(a - c*x**4)*
x**2)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a*c*d*x**3 - 3*int((sqrt(
a - c*x**4)*x**2)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*b*c**2*x**3)/
(3*c*x**3)
```

3.6 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(c+dx^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 355

$$\begin{aligned}
&\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(c+dx^2)}\,dx \\
&= -\frac{A\sqrt{a-cx^4}}{5cx^5}-\frac{(Bc-Ad)\sqrt{a-cx^4}}{3c^2x^3}+\frac{(2Ac^3+5aBcd-5aAd^2)\sqrt{a-cx^4}}{5ac^3x} \\
&\quad+\frac{(2Ac^3+5aBcd-5aAd^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5\sqrt[4]{a}c^{11/4}\sqrt{a-cx^4}} \\
&\quad-\frac{\left(6Ac^3-5\sqrt{a}c^{3/2}(Bc-Ad)+15ad(Bc-Ad)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{15\sqrt[4]{a}c^{11/4}\sqrt{a-cx^4}} \\
&\quad-\frac{\sqrt[4]{a}(Bc-Ad)(c^3-ad^2)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{17/4}\sqrt{a-cx^4}}
\end{aligned}$$

output
```
-1/5*A*(-c*x^4+a)^(1/2)/c/x^5-1/3*(-A*d+B*c)*(-c*x^4+a)^(1/2)/c^2/x^3+1/5*
(-5*A*a*d^2+2*A*c^3+5*B*a*c*d)*(-c*x^4+a)^(1/2)/a/c^3/x+1/5*(-5*A*a*d^2+2*
A*c^3+5*B*a*c*d)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4)/
c^(11/4)/(-c*x^4+a)^(1/2)-1/15*(6*A*c^3-5*a^(1/2)*c^(3/2)*(-A*d+B*c)+15*a*
d*(-A*d+B*c))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(1/4)/c^(
11/4)/(-c*x^4+a)^(1/2)-a^(1/4)*(-A*d+B*c)*(-a*d^2+c^3)*(1-c*x^4/a)^(1/2)*E
llipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/2),I)/c^(17/4)/(-c*x^4+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.64 (sec) , antiderivative size = 781, normalized size of antiderivative = 2.20

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(c+dx^2)}\,dx$$
$$= \frac{-3a^2A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3-5a^2B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3x^2+5a^2A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2dx^2+9aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^4x^4+15a^2B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2dx^4-15a^2}{}$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^6*(c + d*x^2)),x]
```

output
```
(-3*a^2*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3 - 5*a^2*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*
c^3*x^2 + 5*a^2*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d*x^2 + 9*a*A*Sqrt[-(Sqrt[c
]/Sqrt[a])]*c^4*x^4 + 15*a^2*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d*x^4 - 15*a^2
*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^2*x^4 + 5*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^4
*x^6 - 5*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*d*x^6 - 6*A*Sqrt[-(Sqrt[c]/Sqrt[
a])]*c^5*x^8 - 15*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*d*x^8 + 15*a*A*Sqrt[-(S
qrt[c]/Sqrt[a])]*c^2*d^2*x^8 - (3*I)*Sqrt[a]*c^(3/2)*(2*A*c^3 + 5*a*B*c*d
- 5*a*A*d^2)*x^5*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sq
rt[a])]*x], -1] + I*Sqrt[a]*c^(3/2)*(6*A*c^3 + 15*a*d*(B*c - A*d) + 5*Sqrt
[a]*c^(3/2)*(-(B*c) + A*d))*x^5*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sq
rt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (15*I)*a*B*c^4*x^5*Sqrt[1 - (c*x^4)/a]*El
lipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1
] - (15*I)*a*A*c^3*d*x^5*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3
/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (15*I)*a^2*B*c*d^2*x^5*
Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sq
rt[c]/Sqrt[a])]*x], -1] + (15*I)*a^2*A*d^3*x^5*Sqrt[1 - (c*x^4)/a]*Ellipti
cPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(1
5*a*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^4*x^5*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}\left(A+Bx^2\right)}{x^6\left(c+dx^2\right)}\,dx$$

$$\Big\downarrow \ 2249$$

$$\int \left(-\frac{\left(c^3-ad^2\right)(Bc-Ad)}{c^3\sqrt{a-cx^4}\left(c+dx^2\right)}+\frac{aAd^2-aBcd-Ac^3}{c^3x^2\sqrt{a-cx^4}}+\frac{a(Bc-Ad)}{c^2x^4\sqrt{a-cx^4}}+\frac{aA}{cx^6\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}\sqrt{a-cx^4}}-\frac{\sqrt{1-\frac{cx^4}{a}}\left(A\left(c^3-ad^2\right)+aBcd\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{a}c^{11/4}\sqrt{a-cx^4}}+\frac{\sqrt{1-\frac{cx^4}{a}}\left(A\left(c^3-ad^2\right)+aBcd\right)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}c^{11/4}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\left(c^3-ad^2\right)\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{17/4}\sqrt{a-cx^4}}+\frac{3A\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5\sqrt[4]{a}\sqrt{a-cx^4}}-\frac{3A\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5\sqrt[4]{a}\sqrt{a-cx^4}}+\frac{\sqrt{a-cx^4}\left(A\left(c^3-ad^2\right)+aBcd\right)}{ac^3x}-\frac{\sqrt{a-cx^4}(Bc-Ad)}{3c^2x^3}-\frac{3A\sqrt{a-cx^4}}{5ax}-\frac{A\sqrt{a-cx^4}}{5cx^5}$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^6*(c + d*x^2)),x]
```

output
```
-1/5*(A*Sqrt[a - c*x^4])/(c*x^5) - ((B*c - A*d)*Sqrt[a - c*x^4])/(3*c^2*x^
3) - (3*A*Sqrt[a - c*x^4])/(5*a*x) + ((a*B*c*d + A*(c^3 - a*d^2))*Sqrt[a -
 c*x^4])/(a*c^3*x) - (3*A*c^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^
(1/4)*x)/a^(1/4)], -1])/(5*a^(1/4)*Sqrt[a - c*x^4]) + ((a*B*c*d + A*(c^3 -
 a*d^2))*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(
a^(1/4)*c^(11/4)*Sqrt[a - c*x^4]) + (3*A*c^(1/4)*Sqrt[1 - (c*x^4)/a]*Ellip
ticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(5*a^(1/4)*Sqrt[a - c*x^4]) + (a^(1
/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)],
 -1])/(3*c^(5/4)*Sqrt[a - c*x^4]) - ((a*B*c*d + A*(c^3 - a*d^2))*Sqrt[1 -
(c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(a^(1/4)*c^(11/4)*S
qrt[a - c*x^4]) - (a^(1/4)*(B*c - A*d)*(c^3 - a*d^2)*Sqrt[1 - (c*x^4)/a]*E
llipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(17
/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{\sqrt{-c\,x^{4}+a}\,\left(15 A a\,d^{2} x^{4}-6 A\,c^{3} x^{4}-15 B a c d\,x^{4}-5 A a c d\,x^{2}+5 B a\,c^{2} x^{2}+3 A a\,c^{2}\right)}{15 c^{3} a\,x^{5}}-\frac{-\frac{3 \sqrt{c}\,\left(5 A a\,d^{2}-2 A\,c^{3}-5 a B c d\right) \sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}}{}$
default	$\frac{A\left(-\frac{\sqrt{-c\,x^{4}+a}}{5 x^{5}}+\frac{2 c\sqrt{-c\,x^{4}+a}}{5 a x}-\frac{2 c^{\frac{3}{2}} \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)\right)}{5\sqrt{a}\,\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right)}{c}-\frac{\left(A d-B c\right)\left(-\frac{\sqrt{-c\,x^{4}+a}}{3 x^{3}}\right.}{}$
elliptic	Expression too large to display

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-c*x^4+a)^(1/2)*(15*A*a*d^2*x^4-6*A*c^3*x^4-15*B*a*c*d*x^4-5*A*a*c*
d*x^2+5*B*a*c^2*x^2+3*A*a*c^2)/c^3/a/x^5-1/15/a/c^3*(-3*c^(1/2)*(5*A*a*d^2
-2*A*c^3-5*B*a*c*d)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2)
)^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/
2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))-5*B*c^3*a/(c^
(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))
^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+15*(A*a*d^3
-A*c^3*d-B*a*c*d^2+B*c^4)*a/c/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/
2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(
1/2)/a^(1/2))^(1/2),-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a
^(1/2))^(1/2))+5*A*a*c^2*d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))
^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)
/a^(1/2))^(1/2),I))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(d*x^2+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(c+dx^2)}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(c+dx^2)}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**6/(d*x**2+c),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**6*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\,(c+dx^2)}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(dx^2+c)x^6}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\,(c+dx^2)}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(dx^2+c)x^6}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\,(c+dx^2)}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{x^6\,(d\,x^2+c)}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^6*(c + d*x^2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^6*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(c+dx^2)}\,dx$$
$$= \frac{-9\sqrt{-c\,x^4+a}\,ac+25\sqrt{-c\,x^4+a}\,ad\,x^2-15\sqrt{-c\,x^4+a}\,bc\,x^2+30\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^{10}-c^2x^8+ad\,x^6+ac\,x^4}dx\right)a^2cd\,x^5}{}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(d*x^2+c),x)
```

output
```
( - 9*sqrt(a - c*x**4)*a*c + 25*sqrt(a - c*x**4)*a*d*x**2 - 15*sqrt(a - c*
x**4)*b*c*x**2 + 30*int(sqrt(a - c*x**4)/(a*c*x**4 + a*d*x**6 - c**2*x**8
- c*d*x**10),x)*a**2*c*d*x**5 + 75*int(sqrt(a - c*x**4)/(a*c*x**2 + a*d*x*
*4 - c**2*x**6 - c*d*x**8),x)*a**2*d**2*x**5 - 45*int(sqrt(a - c*x**4)/(a*
c*x**2 + a*d*x**4 - c**2*x**6 - c*d*x**8),x)*a*b*c*d*x**5 - 18*int(sqrt(a
- c*x**4)/(a*c*x**2 + a*d*x**4 - c**2*x**6 - c*d*x**8),x)*a*c**3*x**5 + 2*
int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a*c**2*d*x
**5 - 30*int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*b
*c**3*x**5 - 25*int((sqrt(a - c*x**4)*x**2)/(a*c + a*d*x**2 - c**2*x**4 -
c*d*x**6),x)*a*c*d**2*x**5 + 15*int((sqrt(a - c*x**4)*x**2)/(a*c + a*d*x**
2 - c**2*x**4 - c*d*x**6),x)*b*c**2*d*x**5)/(45*c**2*x**5)
```

3.7 $\int \frac{x^6 (A+Bx^2)}{(c+dx^2) \sqrt{a-cx^4}} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 330

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \frac{(Bc-Ad)x\sqrt{a-cx^4}}{3cd^2} - \frac{Bx^3\sqrt{a-cx^4}}{5cd} + \frac{a^{3/4}(5Bc^3-5Ac^2d+3aBd^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}d^3\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\left(5\sqrt{c}(Bc-Ad)\left(3c^3+ad^2\right)+3\sqrt{a}d(5Bc^3-5Ac^2d+3aBd^2)\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{15c^{7/4}d^4\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}c^{7/4}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d^4\sqrt{a-cx^4}}$$

output

```
1/3*(-A*d+B*c)*x*(-c*x^4+a)^(1/2)/c/d^2-1/5*B*x^3*(-c*x^4+a)^(1/2)/c/d+1/5
*a^(3/4)*(-5*A*c^2*d+3*B*a*d^2+5*B*c^3)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4
)*x/a^(1/4),I)/c^(7/4)/d^3/(-c*x^4+a)^(1/2)-1/15*a^(1/4)*(5*c^(1/2)*(-A*d+
B*c)*(a*d^2+3*c^3)+3*a^(1/2)*d*(-5*A*c^2*d+3*B*a*d^2+5*B*c^3))*(1-c*x^4/a)
^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(7/4)/d^4/(-c*x^4+a)^(1/2)+a^(1/4)
*c^(7/4)*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2
)*d/c^(3/2),I)/d^4/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.48 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.07

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx$$
$$= \frac{-3i\sqrt{a}d(5Bc^3-5Ac^2d+3aBd^2)\sqrt{1-\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+i\left(-5A\sqrt{c}d\left(3c^3+3\sqrt{a}c^{3/2}d\right.\right.}{}$$

input
```
Integrate[(x^6*(A + B*x^2))/((c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
((-3*I)*Sqrt[a]*d*(5*B*c^3 - 5*A*c^2*d + 3*a*B*d^2)*Sqrt[1 - (c*x^4)/a]*El
lipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*(-5*A*Sqrt[c]*d*(3*
c^3 + 3*Sqrt[a]*c^(3/2)*d + a*d^2) + B*(15*c^(9/2) + 15*Sqrt[a]*c^3*d + 5*
a*c^(3/2)*d^2 + 9*a^(3/2)*d^3))*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sq
rt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*(Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*x*(
5*B*c - 5*A*d - 3*B*d*x^2)*(a - c*x^4) - (15*I)*c^3*(B*c - A*d)*Sqrt[1 - (
c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt
[a])]*x], -1]))/(15*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^(3/2)*d^4*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {2237, 25, 2237, 2235, 25, 27, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^2)}{\sqrt{a-cx^4}(c+dx^2)}\,dx$$
$$\Big\downarrow \text{2237}$$

$$-\frac{\int -\frac{5cd(Bx^2+A)x^6+B(dx^2+c)(3a-5cx^4)x^2}{(dx^2+c)\sqrt{a-cx^4}}dx}{5cd}-\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\big\downarrow$ 25

$$\frac{\int \frac{5cd(Bx^2+A)x^6+B(dx^2+c)(3a-5cx^4)x^2}{(dx^2+c)\sqrt{a-cx^4}}dx}{5cd}-\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\big\downarrow$ 2237

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{\int \frac{5c(Bc-Ad)\left(dx^2+c\right)\left(a-3cx^4\right)-3cd\left(5cd\left(Bx^2+A\right)x^6+B\left(dx^2+c\right)\left(3a-5cx^4\right)x^2\right)}{\left(dx^2+c\right)\sqrt{a-cx^4}}dx}{3cd}}{5cd}-\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\big\downarrow$ 2235

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{-\frac{15c^5(Bc-Ad)\int \frac{1}{\left(dx^2+c\right)\sqrt{a-cx^4}}dx}{d^2}-\frac{\int -\frac{c\left(5(Bc-Ad)\left(3c^3+ad^2\right)-3d\left(5Bc^3-5Adc^2+3aBd^2\right)x^2\right)}{\sqrt{a-cx^4}}dx}{d^2}}{3cd}}{5cd}-\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\big\downarrow$ 25

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{\frac{\int \frac{c\left(5(Bc-Ad)\left(3c^3+ad^2\right)-3d\left(5Bc^3-5Adc^2+3aBd^2\right)x^2\right)}{\sqrt{a-cx^4}}dx}{d^2}-\frac{15c^5(Bc-Ad)\int \frac{1}{\left(dx^2+c\right)\sqrt{a-cx^4}}dx}{d^2}}{3cd}}{5cd}-\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\big\downarrow$ 27

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{\frac{c\int \frac{5(Bc-Ad)\left(3c^3+ad^2\right)-3d\left(5Bc^3-5Adc^2+3aBd^2\right)x^2}{\sqrt{a-cx^4}}dx}{d^2}-\frac{15c^5(Bc-Ad)\int \frac{1}{\left(dx^2+c\right)\sqrt{a-cx^4}}dx}{d^2}}{3cd}}{5cd}-\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\big\downarrow$ 1513

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{\frac{c\left(\left(5\left(ad^2+3c^3\right)(Bc-Ad)+\frac{3\sqrt{a}d\left(3aBd^2-5Ac^2d+5Bc^3\right)}{\sqrt{c}}\right)\int\frac{1}{\sqrt{a-cx^4}}dx-\frac{3\sqrt{a}d\left(3aBd^2-5Ac^2d+5Bc^3\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{d^2}-\frac{15c\cdots}{\cdots}}{3cd}}{5cd}$$

$$\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\downarrow$ 27

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{\frac{c\left(\left(5\left(ad^2+3c^3\right)(Bc-Ad)+\frac{3\sqrt{a}d\left(3aBd^2-5Ac^2d+5Bc^3\right)}{\sqrt{c}}\right)\int\frac{1}{\sqrt{a-cx^4}}dx-\frac{3d\left(3aBd^2-5Ac^2d+5Bc^3\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{d^2}-\frac{15c^5(B\cdots}{\cdots}}{3cd}}{5cd}$$

$$\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\downarrow$ 765

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{\frac{c\left(\frac{\sqrt{1-\frac{cx^4}{a}}\left(5\left(ad^2+3c^3\right)(Bc-Ad)+\frac{3\sqrt{a}d\left(3aBd^2-5Ac^2d+5Bc^3\right)}{\sqrt{c}}\right)\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-\frac{3d\left(3aBd^2-5Ac^2d+5Bc^3\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}\right)}{d^2}\cdots}{3cd}}{5cd}$$

$$\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\downarrow$ 762

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{\frac{c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5\left(ad^2+3c^3\right)(Bc-Ad)+\frac{3\sqrt{a}d\left(3aBd^2-5Ac^2d+5Bc^3\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3d\left(3aBd^2-5Ac^2d+5\cdots\right.}{\sqrt{\cdots}}\cdots\right.}{d^2}\cdots}{3cd}}{5cd}$$

$$\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\downarrow$ 1390

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{\frac{c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5\left(ad^2+3c^3\right)(Bc-Ad)+\frac{3\sqrt{a}d\left(3aBd^2-5Ac^2d+5Bc^3\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3d\sqrt{1-\frac{cx^4}{a}}\left(3aBd^2-\cdots\right.}{\cdots}\right.}{d^2}}{3cd}}{5cd}$$

$$\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\Big\downarrow$ 1389

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{\frac{c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5\left(ad^2+3c^3\right)(Bc-Ad)+\frac{3\sqrt{a}d\left(3aBd^2-5Ac^2d+5Bc^3\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3\sqrt{a}d\sqrt{1-\frac{cx^4}{a}}\left(3aBd\cdots\right.}{\cdots}\right.}{d^2}}{3cd}}{5cd}$$

$$\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\Big\downarrow$ 327

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{\frac{c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5\left(ad^2+3c^3\right)(Bc-Ad)+\frac{3\sqrt{a}d\left(3aBd^2-5Ac^2d+5Bc^3\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3a^{3/4}d\sqrt{1-\frac{cx^4}{a}}\left(3a\cdots\right.}{\cdots}\right.}{d^2}}{3cd}}{5cd}$$

$$\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\Big\downarrow$ 1543

$$\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{\frac{c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5\left(ad^2+3c^3\right)(Bc-Ad)+\frac{3\sqrt{a}d\left(3aBd^2-5Ac^2d+5Bc^3\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3a^{3/4}d\sqrt{1-\frac{cx^4}{a}}\left(3a\cdots\right.}{\cdots}\right.}{d^2}}{3cd}}{5cd}$$

$$\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

$\Big\downarrow$ 1542

$$\frac{\frac{\frac{5x\sqrt{a-cx^4}(Bc-Ad)}{3d}-\frac{c\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(5\left(ad^2+3c^3\right)(Bc-Ad)+\frac{3\sqrt{a}d\left(3aBd^2-5Ac^2d+5Bc^3\right)}{\sqrt{c}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3a^{3/4}d\sqrt{1-\frac{cx^4}{a}}\left(3aB\cdots\right.}{\cdots}\right.}{d^2}\cdots}{3cd}}{5cd}$$

$$\frac{Bx^3\sqrt{a-cx^4}}{5cd}$$

input
```
Int[(x^6*(A + B*x^2))/((c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
-1/5*(B*x^3*Sqrt[a - c*x^4])/(c*d) + ((5*(B*c - A*d)*x*Sqrt[a - c*x^4])/(3
*d) - ((c*((-3*a^(3/4)*d*(5*B*c^3 - 5*A*c^2*d + 3*a*B*d^2)*Sqrt[1 - (c*x^4
)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4])
 + (a^(1/4)*(5*(B*c - A*d)*(3*c^3 + a*d^2) + (3*Sqrt[a]*d*(5*B*c^3 - 5*A*c
^2*d + 3*a*B*d^2))/Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*
x)/a^(1/4)], -1])/(c^(1/4)*Sqrt[a - c*x^4])))/d^2 - (15*a^(1/4)*c^(15/4)*(
B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(
c^(1/4)*x)/a^(1/4)], -1])/(d^2*Sqrt[a - c*x^4]))/(3*c*d))/(5*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

rule 2237
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> W
ith[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt[a + c*x^4]/(c*
e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q - 3)*Px - Coeff[Px, x,
 q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a
+ c*x^4]), x], x] /; GtQ[q, 4]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 5.10 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{x\left(3B\,x^{2}d+5Ad-5Bc\right)\sqrt{-c\,x^{4}+a}}{15c\,d^{2}}-\frac{-\frac{5\left(Aa\,d^{3}+3A\,c^{3}d-Bac\,d^{2}-3B\,c^{4}\right)\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{d^{2}\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}-\frac{3\left(5A\,c^{2}d-3Ba\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{d^{2}(Ad-Bc)\left(-\frac{x\sqrt{-c\,x^{4}+a}}{3c}+\frac{a\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right)+\frac{\sqrt{c}\,d(Ad-Bc)\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\cdots\right.\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\cdots}{\cdots}$
elliptic	Expression too large to display

input
```
int(x^6*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(3*B*d*x^2+5*A*d-5*B*c)*(-c*x^4+a)^(1/2)/c/d^2-1/15/c/d^2*(-5*(A*a
*d^3+3*A*c^3*d-B*a*c*d^2-3*B*c^4)/d^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x
^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF
(x*(c^(1/2)/a^(1/2))^(1/2),I)-3/d*(5*A*c^2*d-3*B*a*d^2-5*B*c^3)*a^(1/2)/(c
^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2)
)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-E
llipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))+15*c^3*(A*d-B*c)/d^2/(c^(1/2)/a^(1/
2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*
x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*d/c^(3/2),(-c^(
1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx=\int \frac{(Bx^2+A)x^6}{\sqrt{-cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate(x^6*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(B*x^8 + A*x^6)*sqrt(-c*x^4 + a)/(c*d*x^6 + c^2*x^4 - a*d*x^2 -
a*c), x)
```

Sympy [F]

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx=\int \frac{x^6(A+Bx^2)}{\sqrt{a-cx^4}\,(c+dx^2)}\,dx$$

input
```
integrate(x**6*(B*x**2+A)/(d*x**2+c)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(x**6*(A + B*x**2)/(sqrt(a - c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx=\int \frac{(Bx^2+A)x^6}{\sqrt{-cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate(x^6*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^6/(sqrt(-c*x^4 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx=\int \frac{(Bx^2+A)x^6}{\sqrt{-cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate(x^6*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^6/(sqrt(-c*x^4 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx=\int \frac{x^6\,(B\,x^2+A)}{\sqrt{a-c\,x^4}\,(d\,x^2+c)}\,dx$$

input
```
int((x^6*(A + B*x^2))/((a - c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((x^6*(A + B*x^2))/((a - c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx$$
$$=\frac{-5\sqrt{-c\,x^4+a}\,adx+5\sqrt{-c\,x^4+a}\,bcx-3\sqrt{-c\,x^4+a}\,bd\,x^3+5\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)a^2cd-5\left(\int \frac{}{}\right.}{}$$

input
```
int(x^6*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 5*sqrt(a - c*x**4)*a*d*x + 5*sqrt(a - c*x**4)*b*c*x - 3*sqrt(a - c*x**
4)*b*d*x**3 + 5*int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**
6),x)*a**2*c*d - 5*int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*
x**6),x)*a*b*c**2 + 9*int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x**2 - c**2*x
**4 - c*d*x**6),x)*a*b*d**2 - 15*int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x*
*2 - c**2*x**4 - c*d*x**6),x)*a*c**2*d + 15*int((sqrt(a - c*x**4)*x**4)/(a
*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*b*c**3 + 5*int((sqrt(a - c*x**4)*
x**2)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a**2*d**2 + 4*int((sqrt(a
 - c*x**4)*x**2)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a*b*c*d)/(15*c
*d**2)
```

3.8 $\int \frac{x^4(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 268

$$\begin{aligned}\int \frac{x^4(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx &= -\frac{Bx\sqrt{a-cx^4}}{3cd} - \frac{a^{3/4}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}d^2\sqrt{a-cx^4}} \\ &+ \frac{\sqrt[4]{a}\left(3Bc^3-3Ac^2d+aBd^2+3\sqrt{a}\sqrt{c}d(Bc-Ad)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{5/4}d^3\sqrt{a-cx^4}} \\ &- \frac{\sqrt[4]{a}c^{3/4}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d^3\sqrt{a-cx^4}}\end{aligned}$$

output
```
-1/3*B*x*(-c*x^4+a)^(1/2)/c/d-a^(3/4)*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*Ellipti
cE(c^(1/4)*x/a^(1/4),I)/c^(3/4)/d^2/(-c*x^4+a)^(1/2)+1/3*a^(1/4)*(3*B*c^3-
3*A*c^2*d+B*a*d^2+3*a^(1/2)*c^(1/2)*d*(-A*d+B*c))*(1-c*x^4/a)^(1/2)*Ellipt
icF(c^(1/4)*x/a^(1/4),I)/c^(5/4)/d^3/(-c*x^4+a)^(1/2)-a^(1/4)*c^(3/4)*(-A*
d+B*c)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/2),I
)/d^3/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.98 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.37

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx$$
$$= \frac{-aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d^2x + B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cd^2x^5 + 3i\sqrt{a}\sqrt{c}d(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right) - i(-3A$$

input
```
Integrate[(x^4*(A + B*x^2))/((c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(-(a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*x) + B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^2*
x^5 + (3*I)*Sqrt[a]*Sqrt[c]*d*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*
ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*(-3*A*Sqrt[c]*d*(c^(3/2) + Sq
rt[a]*d) + B*(3*c^3 + 3*Sqrt[a]*c^(3/2)*d + a*d^2))*Sqrt[1 - (c*x^4)/a]*El
lipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (3*I)*B*c^3*Sqrt[1 -
(c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqr
t[a])]*x], -1] - (3*I)*A*c^2*d*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d
)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(3*Sqrt[-(Sqrt[c]/
Sqrt[a])]*c*d^3*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2237, 25, 2235, 25, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a-cx^4}\,(c+dx^2)}\,dx$$
$$\Big\downarrow \ 2237$$

$$-\frac{\int -\frac{3cd(Bx^2+A)x^4+B(dx^2+c)(a-3cx^4)}{(dx^2+c)\sqrt{a-cx^4}}dx}{3cd}-\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3cd(Bx^2+A)x^4+B(dx^2+c)(a-3cx^4)}{(dx^2+c)\sqrt{a-cx^4}}dx}{3cd}-\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$$\Big\downarrow \text{2235}$$

$$\frac{-\frac{3c^3(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}-\frac{\int -\frac{3Bc^3-3Adc^2-3d(Bc-Ad)x^2c+aBd^2}{\sqrt{a-cx^4}}dx}{d^2}}{3cd}-\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3Bc^3-3Adc^2-3d(Bc-Ad)x^2c+aBd^2}{\sqrt{a-cx^4}}dx}{d^2}-\frac{3c^3(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}}{3cd}-\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$$\Big\downarrow \text{1513}$$

$$\frac{\frac{(3\sqrt{a}\sqrt{c}d(Bc-Ad)+aBd^2-3Ac^2d+3Bc^3)\int \frac{1}{\sqrt{a-cx^4}}dx-3\sqrt{a}\sqrt{c}d(Bc-Ad)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{d^2}-\frac{3c^3(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}}{3cd}-\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(3\sqrt{a}\sqrt{c}d(Bc-Ad)+aBd^2-3Ac^2d+3Bc^3)\int \frac{1}{\sqrt{a-cx^4}}dx-3\sqrt{c}d(Bc-Ad)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{d^2}-\frac{3c^3(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}}{3cd}-\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{\frac{\sqrt{1-\frac{cx^4}{a}}(3\sqrt{a}\sqrt{c}d(Bc-Ad)+aBd^2-3Ac^2d+3Bc^3)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-3\sqrt{c}d(Bc-Ad)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{d^2}-\frac{3c^3(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}}{3cd}-\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}d(Bc-Ad)+aBd^2-3Ac^2d+3Bc^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-3\sqrt{c}d(Bc-Ad)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{d^2}-\frac{3c^3(Bc-Ad)\int\frac{1}{\left(dx^2+c\right)\sqrt{a}\cdots}}{d^2}\cdots}{3cd}$$

$$\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$\downarrow$ 1390

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}d(Bc-Ad)+aBd^2-3Ac^2d+3Bc^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3\sqrt{c}d\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}}{d^2}-\frac{3c^3(Bc-Ad)\int\frac{\cdots}{\left(dx^2\cdots\right.}}{d^2}\cdots}{3cd}$$

$$\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$\downarrow$ 1389

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}d(Bc-Ad)+aBd^2-3Ac^2d+3Bc^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3\sqrt{a}\sqrt{c}d\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\int\frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{a-cx^4}}}{d^2}-\frac{3c^3(Bc-Ad)\int\cdots}{\cdots}}{3cd}$$

$$\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$\downarrow$ 327

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}d(Bc-Ad)+aBd^2-3Ac^2d+3Bc^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3a^{3/4}\sqrt[4]{c}d\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-cx^4}}}{d^2}-\frac{3c^3(\cdots}{\cdots}}{3cd}$$

$$\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$\downarrow$ 1543

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}d(Bc-Ad)+aBd^2-3Ac^2d+3Bc^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3a^{3/4}\sqrt[4]{c}d\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-cx^4}}}{d^2}-\frac{3c^3\sqrt{\cdots}}{\cdots}}{3cd}$$

$$\frac{Bx\sqrt{a-cx^4}}{3cd}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}d(Bc-Ad)+aBd^2-3Ac^2d+3Bc^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{3a^{3/4}\sqrt[4]{c}d\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-cx^4}}}{d^2}-\frac{3\sqrt[4]{a}\cdots}{\cdots}}{3cd}$$

$$\frac{Bx\sqrt{a-cx^4}}{3cd}$$

input
```
Int[(x^4*(A + B*x^2))/((c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
-1/3*(B*x*Sqrt[a - c*x^4])/(c*d) + (((-3*a^(3/4)*c^(1/4)*d*(B*c - A*d)*Sqr
t[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - c*x^
4] + (a^(1/4)*(3*B*c^3 - 3*A*c^2*d + a*B*d^2 + 3*Sqrt[a]*Sqrt[c]*d*(B*c -
A*d))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(
1/4)*Sqrt[a - c*x^4]))/d^2 - (3*a^(1/4)*c^(7/4)*(B*c - A*d)*Sqrt[1 - (c*x^
4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(d^2*Sqrt[a - c*x^4]))/(3*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

rule 2237
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> W
ith[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt[a + c*x^4]/(c*
e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q - 3)*Px - Coeff[Px, x,
 q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a
+ c*x^4]), x], x] /; GtQ[q, 4]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.60

method	result
default	$\frac{c(Ad-Bc)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,d}{c^{\frac{3}{2}}},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d^3\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{d(Ad-Bc)\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}$
risch	$-\frac{Bx\sqrt{-c\,x^4+a}}{3cd}+\frac{-\frac{3\sqrt{c}\,d(Ad-Bc)\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}+\frac{B\,a\,d^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-}}}{}$
elliptic	$-\frac{Bx\sqrt{-c\,x^4+a}}{3cd}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)cA}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,d^2}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)c^2B}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,d^3}+$

input
```
int(x^4*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c*(A*d-B*c)/d^3/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c
^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^
(1/2),-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))
-1/d^3*(d*(A*d-B*c)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2)
)^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(
x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))+A*c*d
/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1
/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-B*c^2/(
c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2
))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-B*d^2*(-1
/3/c*x*(-c*x^4+a)^(1/2)+1/3*a/c/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(
1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^
(1/2)/a^(1/2))^(1/2),I))
```

Fricas [F]

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \int \frac{(Bx^2+A)x^4}{\sqrt{-cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate(x^4*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(B*x^6 + A*x^4)*sqrt(-c*x^4 + a)/(c*d*x^6 + c^2*x^4 - a*d*x^2 -
a*c), x)
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \int \frac{x^4(A+Bx^2)}{\sqrt{a-cx^4}\,(c+dx^2)}\,dx$$

input
```
integrate(x**4*(B*x**2+A)/(d*x**2+c)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(x**4*(A + B*x**2)/(sqrt(a - c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \int \frac{(Bx^2+A)x^4}{\sqrt{-cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate(x^4*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^4/(sqrt(-c*x^4 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \int \frac{(Bx^2+A)x^4}{\sqrt{-cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate(x^4*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^4/(sqrt(-c*x^4 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \int \frac{x^4\,(B\,x^2+A)}{\sqrt{a-c\,x^4}\,(d\,x^2+c)}\,dx$$

input
```
int((x^4*(A + B*x^2))/((a - c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((x^4*(A + B*x^2))/((a - c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx$$
$$= \frac{-\sqrt{-c\,x^4+a}\,bx+\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)abc+3\left(\int \frac{\sqrt{-c\,x^4+a}\,x^4}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)acd-3\left(\int \frac{\sqrt{-c\,x^4+a}\,x}{-cd\,x^6-c^2x^4+ad}\right.}{3cd}$$

input
```
int(x^4*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a - c*x**4)*b*x + int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**
4 - c*d*x**6),x)*a*b*c + 3*int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x**2 - c
**2*x**4 - c*d*x**6),x)*a*c*d - 3*int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x
**2 - c**2*x**4 - c*d*x**6),x)*b*c**2 + int((sqrt(a - c*x**4)*x**2)/(a*c +
 a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a*b*d)/(3*c*d)
```

3.9 $\int \frac{x^2(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 212

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \frac{a^{3/4}B\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}d\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(Ad-B\left(c+\frac{\sqrt{a}d}{\sqrt{c}}\right)\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d^2\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d^2\sqrt{a-cx^4}}$$

output
```
a^(3/4)*B*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(3/4)/d/(-c*x
^4+a)^(1/2)+a^(1/4)*(A*d-B*(c+a^(1/2)*d/c^(1/2)))*(1-c*x^4/a)^(1/2)*Ellipt
icF(c^(1/4)*x/a^(1/4),I)/c^(1/4)/d^2/(-c*x^4+a)^(1/2)+a^(1/4)*(-A*d+B*c)*(
1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/2),I)/c^(1/4
)/d^2/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.52 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx$$
$$= \frac{i\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}BdE\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-\left(Bc^{3/2}+\sqrt{a}Bd-A\sqrt{c}d\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{}}{\sqrt{}}}\right.\right.\right.}{\sqrt{a}\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2}d^2\sqrt{a-cx^4}}$$

input
```
Integrate[(x^2*(A + B*x^2))/((c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(I*Sqrt[1 - (c*x^4)/a]*(Sqrt[a]*B*d*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqr
t[a])]*x], -1] - (B*c^(3/2) + Sqrt[a]*B*d - A*Sqrt[c]*d)*EllipticF[I*ArcSi
nh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*(B*c - A*d)*EllipticPi[-((Sq
rt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(Sqrt[a]*(
-(Sqrt[c]/Sqrt[a]))^(3/2)*d^2*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2235, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a-cx^4}\,(c+dx^2)}\,dx$$
$$\Big\downarrow \text{ 2235}$$
$$\frac{c(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}-\frac{\int \frac{-Bdx^2+Bc-Ad}{\sqrt{a-cx^4}}dx}{d^2}$$
$$\Big\downarrow \text{ 1513}$$

$$\frac{c(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}-\frac{-\left(\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{c}}+c\right)\right)\int \frac{1}{\sqrt{a-cx^4}}dx\right)-\frac{\sqrt{a}Bd\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-cx^4}}dx}{\sqrt{c}}}{d^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}-\frac{-\left(\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{c}}+c\right)\right)\int \frac{1}{\sqrt{a-cx^4}}dx\right)-\frac{Bd\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{d^2}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{c(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}-\frac{-\frac{\sqrt{1-\frac{cx^4}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{c}}+c\right)\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{a-cx^4}}-\frac{Bd\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}}{d^2}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{c(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}-\frac{-\frac{Bd\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a-cx^4}}dx}{\sqrt{c}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{c}}+c\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{d^2}$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{c(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}-\frac{-\frac{Bd\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{1-\frac{cx^4}{a}}}dx}{\sqrt{c}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{c}}+c\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{d^2}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{c(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2}-\frac{-\frac{\sqrt{a}Bd\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{c}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}}dx}{\sqrt{c}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{c}}+c\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{d^2}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\frac{c(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d^2} - \frac{-\frac{a^{3/4}Bd\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{c}}+c\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{d^2}}{}$$

$$\Big\downarrow \; 1543$$

$$\frac{c\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{1-\frac{cx^4}{a}}}dx}{d^2\sqrt{a-cx^4}} - \frac{-\frac{a^{3/4}Bd\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{c}}+c\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{d^2}$$

$$\Big\downarrow \; 1542$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d^2\sqrt{a-cx^4}} - \frac{-\frac{a^{3/4}Bd\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{c}}+c\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\sqrt{a-cx^4}}}{d^2}$$

input
```
Int[(x^2*(A + B*x^2))/((c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
-((-((a^(3/4)*B*d*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)
], -1])/(c^(3/4)*Sqrt[a - c*x^4])) - (a^(1/4)*(A*d - B*(c + (Sqrt[a]*d)/Sq
rt[c]))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c
^(1/4)*Sqrt[a - c*x^4]))/d^2) + (a^(1/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*E
llipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/
4)*d^2*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.55

method	result
default	$\frac{\frac{A d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{B d\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}-\frac{Bc\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{}}{d^2}$
elliptic	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)A}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)Bc}{d^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{B\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}}{}}}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c}}$

input
```
int(x^2*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(A*d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2
)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),
I)-B*d*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^
(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^
(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))-B*c/(c^(1/2)/a^(1/
2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*
x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))-1/d^2*(A*d-B*c)/(c^(1
/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(
1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*d/c^(3
/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \int \frac{x^2(A+Bx^2)}{\sqrt{a-cx^4}\,(c+dx^2)}\,dx$$

input
```
integrate(x**2*(B*x**2+A)/(d*x**2+c)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2)/(sqrt(a - c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(A + Bx^2)}{(c + dx^2)\sqrt{a - cx^4}}\,dx = \int \frac{(Bx^2 + A)x^2}{\sqrt{-cx^4 + a}(dx^2 + c)}\,dx$$

input
```
integrate(x^2*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^2/(sqrt(-c*x^4 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(A + Bx^2)}{(c + dx^2)\sqrt{a - cx^4}}\,dx = \int \frac{(Bx^2 + A)x^2}{\sqrt{-cx^4 + a}(dx^2 + c)}\,dx$$

input
```
integrate(x^2*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^2/(sqrt(-c*x^4 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A + Bx^2)}{(c + dx^2)\sqrt{a - cx^4}}\,dx = \int \frac{x^2\,(B\,x^2 + A)}{\sqrt{a - c\,x^4}\,(d\,x^2 + c)}\,dx$$

input
```
int((x^2*(A + B*x^2))/((a - c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((x^2*(A + B*x^2))/((a - c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \left(\int \frac{\sqrt{-c\,x^4+a}\,x^4}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)b + \left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)a$$

input
```
int(x^2*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x)
```

output
```
int((sqrt(a - c*x**4)*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*b +
 int((sqrt(a - c*x**4)*x**2)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a
```

3.10 $\int \frac{A+Bx^2}{(c+dx^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 136

$$\int \frac{A+Bx^2}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \frac{\sqrt[4]{a}B\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{5/4}d\sqrt{a-cx^4}}$$

output
```
a^(1/4)*B*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(1/4)/d/(-c*x
^4+a)^(1/2)-a^(1/4)*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1
/4),-a^(1/2)*d/c^(3/2),I)/c^(5/4)/d/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2}{(c+dx^2)\sqrt{a-cx^4}}\,dx = -\frac{i\sqrt{1-\frac{cx^4}{a}}\left(Bc\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right),-1\right)+(-Bc+Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{}}{\sqrt{}}}\right.\right.\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cd\sqrt{a-cx^4}}$$

input
```
Integrate[(A + B*x^2)/((c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
((-I)*Sqrt[1 - (c*x^4)/a]*(B*c*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])
]*x], -1] + (-(B*c) + A*d)*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sq
rt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d*Sqrt[a - c*
x^4])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2229, 765, 762, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a-cx^4}\,(c+dx^2)}dx$$

$$\Big\downarrow \text{2229}$$

$$\frac{B\int \frac{1}{\sqrt{a-cx^4}}dx}{d}-\frac{(Bc-Ad)\int \frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d}$$

$$\Big\downarrow \text{765}$$

$$\frac{B\sqrt{1-\frac{cx^4}{a}}\int\frac{1}{\sqrt{1-\frac{cx^4}{a}}}dx}{d\sqrt{a-cx^4}}-\frac{(Bc-Ad)\int\frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d}$$

$\Big\downarrow$ 762

$$\frac{\sqrt[4]{a}B\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}-\frac{(Bc-Ad)\int\frac{1}{(dx^2+c)\sqrt{a-cx^4}}dx}{d}$$

$\Big\downarrow$ 1543

$$\frac{\sqrt[4]{a}B\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}-\frac{\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\int\frac{1}{(dx^2+c)\sqrt{1-\frac{cx^4}{a}}}dx}{d\sqrt{a-cx^4}}$$

$\Big\downarrow$ 1542

$$\frac{\sqrt[4]{a}B\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}d\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{5/4}d\sqrt{a-cx^4}}$$

input
```
Int[(A + B*x^2)/((c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(a^(1/4)*B*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])
/(c^(1/4)*d*Sqrt[a - c*x^4]) - (a^(1/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*El
lipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(5/4
)*d*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 2229
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[B/e   Int[1/Sqrt[a + c*x^4], x], x] + Simp[(e*A - d*B)/
e   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e, A, B}
, x] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.27

method	result
default	$\frac{B\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}+\frac{(Ad-Bc)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,d}{c^{\frac{3}{2}}},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{dc\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}$
elliptic	$\frac{B\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{\sqrt{a}\,d}{c^{\frac{3}{2}}},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)A}{c\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{}$

input
```
int((B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a
^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+(A*d
-B*c)/d/c/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)
*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),
-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(c+dx^2)\sqrt{a-cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(c+dx^2)\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{a-cx^4}\,(c+dx^2)}\,dx$$

input
```
integrate((B*x**2+A)/(d*x**2+c)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(sqrt(a - c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate((B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate((B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{a-c\,x^4}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2)/((a - c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2)/((a - c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(c+dx^2)\sqrt{a-cx^4}}\,dx = \left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)a + \left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)b$$

input
```
int((B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*a + int((s
qrt(a - c*x**4)*x**2)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*b
```

3.11 $\int \frac{A+Bx^2}{x^2(c+dx^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 208

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx
&= -\frac{A\sqrt{a-cx^4}}{acx}-\frac{A\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}c^{3/4}\sqrt{a-cx^4}}\\
&\quad+\frac{A\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{a}c^{3/4}\sqrt{a-cx^4}}\\
&\quad+\frac{\sqrt[4]{a}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{9/4}\sqrt{a-cx^4}}
\end{aligned}
$$

output

```
-A*(-c*x^4+a)^(1/2)/a/c/x-A*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),
I)/a^(1/4)/c^(3/4)/(-c*x^4+a)^(1/2)+A*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*
x/a^(1/4),I)/a^(1/4)/c^(3/4)/(-c*x^4+a)^(1/2)+a^(1/4)*(-A*d+B*c)*(1-c*x^4/
a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/2),I)/c^(9/4)/(-c*x^
4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.55 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx^2}{x^2(c+dx^2)\sqrt{a-cx^4}}\,dx$$
$$= \frac{-aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c+A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2x^4+i\sqrt{a}Ac^{3/2}x\sqrt{1-\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-i\sqrt{a}Ac^{3/2}x\sqrt{1-\frac{c}{}}}{}$$

input
```
Integrate[(A + B*x^2)/(x^2*(c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(-(a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c) + A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*x^4 +
I*Sqrt[a]*A*c^(3/2)*x*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[
c]/Sqrt[a])]*x], -1] - I*Sqrt[a]*A*c^(3/2)*x*Sqrt[1 - (c*x^4)/a]*EllipticF
[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*a*B*c*x*Sqrt[1 - (c*x^4)/a
]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x]
, -1] + I*a*A*d*x*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I
*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(a*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2
*x*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2\sqrt{a-cx^4}(c+dx^2)}\,dx$$
$$\Big\downarrow \text{ 2249}$$
$$\int \left(\frac{Bc-Ad}{c\sqrt{a-cx^4}(c+dx^2)}+\frac{A}{cx^2\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{9/4}\sqrt{a-cx^4}}+\frac{A\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{a}c^{3/4}\sqrt{a-cx^4}}-\frac{A\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}c^{3/4}\sqrt{a-cx^4}}-\frac{A\sqrt{a-cx^4}}{acx}$$

input
```
Int[(A + B*x^2)/(x^2*(c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
-((A*Sqrt[a - c*x^4])/(a*c*x)) - (A*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(
c^(1/4)*x)/a^(1/4)], -1])/(a^(1/4)*c^(3/4)*Sqrt[a - c*x^4]) + (A*Sqrt[1 -
(c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(a^(1/4)*c^(3/4)*Sq
rt[a - c*x^4]) + (a^(1/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sq
rt[a]*d)/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(9/4)*Sqrt[a - c*x
^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03

method	result
default	$\frac{A\left(-\frac{\sqrt{-c\,x^4+a}}{a x}+\frac{\sqrt{c}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)}{c}-\frac{(Ad-Bc)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{Ellipt}}{c^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c}}$
risch	$-\frac{A\sqrt{-c\,x^4+a}}{acx}-\frac{-\frac{A\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}+\frac{(Ad-Bc)a\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{Ellip}}{c\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c}}}{ac}$
elliptic	$-\frac{A\sqrt{-c\,x^4+a}}{acx}+\frac{A\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{a}\,\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}-\frac{A\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{a}\,\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}-\frac{d\sqrt{1-}}{}$

input
```
int((B*x^2+A)/x^2/(d*x^2+c)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A/c*(-1/a*(-c*x^4+a)^(1/2)/x+c^(1/2)/a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^
(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(E
llipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),
I)))-(A*d-B*c)/c^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(
1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2
))^(1/2),-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/
2))
```

Fricas [F]

$$\int \frac{A+Bx^2}{x^2\,(c+dx^2)\,\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}(dx^2+c)x^2}\,dx$$

input
```
integrate((B*x^2+A)/x^2/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)*(B*x^2 + A)/(c*d*x^8 + c^2*x^6 - a*d*x^4 - a*c*
x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2}{x^2\sqrt{a-cx^4}\left(c+dx^2\right)}\,dx$$

input
```
integrate((B*x**2+A)/x**2/(d*x**2+c)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**2*sqrt(a - c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}(dx^2+c)x^2}\,dx$$

input
```
integrate((B*x^2+A)/x^2/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}(dx^2+c)x^2}\,dx$$

input
```
integrate((B*x^2+A)/x^2/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\int \frac{B\,x^2+A}{x^2\,\sqrt{a-c\,x^4}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2)/(x^2*(a - c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2)/(x^2*(a - c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^8-c^2x^6+ad\,x^4+ac\,x^2}dx\right)a+\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^6-c^2x^4+ad\,x^2+ac}dx\right)b$$

input
```
int((B*x^2+A)/x^2/(d*x^2+c)/(-c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a*c*x**2 + a*d*x**4 - c**2*x**6 - c*d*x**8),x)*a + i
nt(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x**6),x)*b
```

3.12 $\int \frac{A+Bx^2}{x^4(c+dx^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 275

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx = -\frac{A\sqrt{a-cx^4}}{3acx^3}-\frac{(Bc-Ad)\sqrt{a-cx^4}}{ac^2x}-\frac{(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}c^{7/4}\sqrt{a-cx^4}} + \frac{\left(Ac^{3/2}+3\sqrt{a}(Bc-Ad)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3a^{3/4}c^{7/4}\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}d(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{13/4}\sqrt{a-cx^4}}$$

output
```
-1/3*A*(-c*x^4+a)^(1/2)/a/c/x^3-(-A*d+B*c)*(-c*x^4+a)^(1/2)/a/c^2/x-(-A*d+
B*c)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4)/c^(7/4)/(-c*
x^4+a)^(1/2)+1/3*(A*c^(3/2)+3*a^(1/2)*(-A*d+B*c))*(1-c*x^4/a)^(1/2)*Ellipt
icF(c^(1/4)*x/a^(1/4),I)/a^(3/4)/c^(7/4)/(-c*x^4+a)^(1/2)-a^(1/4)*d*(-A*d+
B*c)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/2),I)/
c^(13/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.96 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.67

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx$$
$$= \frac{-aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2-3aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2x^2+3aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cdx^2+A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3x^4+3B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3x^6-3A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2dx^6}{}$$

input
```
Integrate[(A + B*x^2)/(x^4*(c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(-(a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2) - 3*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*
x^2 + 3*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d*x^2 + A*Sqrt[-(Sqrt[c]/Sqrt[a])]*
c^3*x^4 + 3*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*x^6 - 3*A*Sqrt[-(Sqrt[c]/Sqrt[a
])]*c^2*d*x^6 + (3*I)*Sqrt[a]*c^(3/2)*(B*c - A*d)*x^3*Sqrt[1 - (c*x^4)/a]*
EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*c^(3/2)*(A*c^(3/2
) + 3*Sqrt[a]*(B*c - A*d))*x^3*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqr
t[-(Sqrt[c]/Sqrt[a])]*x], -1] + (3*I)*a*B*c*d*x^3*Sqrt[1 - (c*x^4)/a]*Elli
pticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]
- (3*I)*a*A*d^2*x^3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)),
 I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(3*a*Sqrt[-(Sqrt[c]/Sqrt[a])]
*c^3*x^3*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4\sqrt{a-cx^4}\left(c+dx^2\right)}\,dx$$

$$\Big\downarrow \text{2249}$$

$$\int \left(-\frac{d(Bc-Ad)}{c^2\sqrt{a-cx^4}\,(c+dx^2)}+\frac{Bc-Ad}{c^2x^2\sqrt{a-cx^4}}+\frac{A}{cx^4\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{A\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3a^{3/4}\sqrt[4]{c}\sqrt{a-cx^4}}+\frac{\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{a}c^{7/4}\sqrt{a-cx^4}}-\frac{\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}c^{7/4}\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}d\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{13/4}\sqrt{a-cx^4}}-\frac{\sqrt{a-cx^4}(Bc-Ad)}{ac^2x}-\frac{A\sqrt{a-cx^4}}{3acx^3}$$

input
```
Int[(A + B*x^2)/(x^4*(c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
-1/3*(A*Sqrt[a - c*x^4])/(a*c*x^3) - ((B*c - A*d)*Sqrt[a - c*x^4])/(a*c^2*
x) - ((B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)
], -1])/(a^(1/4)*c^(7/4)*Sqrt[a - c*x^4]) + (A*Sqrt[1 - (c*x^4)/a]*Ellipti
cF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*a^(3/4)*c^(1/4)*Sqrt[a - c*x^4]) +
 ((B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(a^(1/4)*c^(7/4)*Sqrt[a - c*x^4]) - (a^(1/4)*d*(B*c - A*d)*Sqrt[1 - (c
*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -
1])/(c^(13/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.14

method	result
default	$\frac{A\left(-\frac{\sqrt{-c\,x^4+a}}{3a\,x^3}+\frac{c\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)}{c}-\frac{(Ad-Bc)\left(-\frac{\sqrt{-c\,x^4+a}}{ax}+\frac{\sqrt{c}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\right.\right.}{\sqrt{a}\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c}}\right.}{c^2}$
risch	$-\frac{\sqrt{-c\,x^4+a}\,(-3Ad\,x^2+3Bc\,x^2+Ac)}{3c^2a\,x^3}+\frac{\frac{A\,c^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}+\frac{3B\,c^{\frac{3}{2}}\sqrt{a}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x}}}{}$
elliptic	$-\frac{A\sqrt{-c\,x^4+a}}{3ac\,x^3}+\frac{(Ad-Bc)\sqrt{-c\,x^4+a}}{a\,c^2x}+\frac{A\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(\right.}{c^{\frac{3}{2}}\sqrt{a}\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+}}$

input
```
int((B*x^2+A)/x^4/(d*x^2+c)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A/c*(-1/3/a*(-c*x^4+a)^(1/2)/x^3+1/3*c/a/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2
)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ellipt
icF(x*(c^(1/2)/a^(1/2))^(1/2),I))-(A*d-B*c)/c^2*(-1/a*(-c*x^4+a)^(1/2)/x+c
^(1/2)/a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^
(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(
1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))+d*(A*d-B*c)/c^3/(c^(1/2)/
a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)
/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*d/c^(3/2),
(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)/x^4/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2}{x^4\sqrt{a-cx^4}\left(c+dx^2\right)}\,dx$$

input
```
integrate((B*x**2+A)/x**4/(d*x**2+c)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**4*sqrt(a - c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-cx^4+a}(dx^2+c)x^4}\,dx$$

input
```
integrate((B*x^2+A)/x^4/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{-cx^4+a}(dx^2+c)x^4}\,dx$$

input
```
integrate((B*x^2+A)/x^4/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx = \int \frac{B\,x^2+A}{x^4\,\sqrt{a-c\,x^4}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2)/(x^4*(a - c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2)/(x^4*(a - c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{x^4(c+dx^2)\sqrt{a-cx^4}}\,dx$$
$$= \frac{-\sqrt{-c\,x^4+a}-3\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^8-c^2x^6+ad\,x^4+ac\,x^2}dx\right)ad\,x^3+3\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^8-c^2x^6+ad\,x^4+ac\,x^2}dx\right)bc\,x^3+\left(\int \frac{\sqrt{-c}}{-cd\,x^6-c^2x}\right.}{3c\,x^3}$$

input
```
int((B*x^2+A)/x^4/(d*x^2+c)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a - c*x**4) - 3*int(sqrt(a - c*x**4)/(a*c*x**2 + a*d*x**4 - c**2*
x**6 - c*d*x**8),x)*a*d*x**3 + 3*int(sqrt(a - c*x**4)/(a*c*x**2 + a*d*x**4
 - c**2*x**6 - c*d*x**8),x)*b*c*x**3 + int(sqrt(a - c*x**4)/(a*c + a*d*x**
2 - c**2*x**4 - c*d*x**6),x)*c**2*x**3 + int((sqrt(a - c*x**4)*x**2)/(a*c
+ a*d*x**2 - c**2*x**4 - c*d*x**6),x)*c*d*x**3)/(3*c*x**3)
```

3.13 $\int \frac{A+Bx^2}{x^6(c+dx^2)\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 353

$$\begin{aligned}&\int \frac{A+Bx^2}{x^6(c+dx^2)\sqrt{a-cx^4}}\,dx\\&=-\frac{A\sqrt{a-cx^4}}{5acx^5}-\frac{(Bc-Ad)\sqrt{a-cx^4}}{3ac^2x^3}-\frac{(3Ac^3-5aBcd+5aAd^2)\sqrt{a-cx^4}}{5a^2c^3x}\\&\quad-\frac{(3Ac^3-5aBcd+5aAd^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5a^{5/4}c^{11/4}\sqrt{a-cx^4}}\\&\quad+\frac{\left(9Ac^3+5\sqrt{a}c^{3/2}(Bc-Ad)-15ad(Bc-Ad)\right)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{15a^{5/4}c^{11/4}\sqrt{a-cx^4}}\\&\quad+\frac{\sqrt[4]{a}d^2(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{17/4}\sqrt{a-cx^4}}\end{aligned}$$

output
```
-1/5*A*(-c*x^4+a)^(1/2)/a/c/x^5-1/3*(-A*d+B*c)*(-c*x^4+a)^(1/2)/a/c^2/x^3-
1/5*(5*A*a*d^2+3*A*c^3-5*B*a*c*d)*(-c*x^4+a)^(1/2)/a^2/c^3/x-1/5*(5*A*a*d^
2+3*A*c^3-5*B*a*c*d)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(5
/4)/c^(11/4)/(-c*x^4+a)^(1/2)+1/15*(9*A*c^3+5*a^(1/2)*c^(3/2)*(-A*d+B*c)-1
5*a*d*(-A*d+B*c))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(5/4)
/c^(11/4)/(-c*x^4+a)^(1/2)+a^(1/4)*d^2*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*Ellipt
icPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/2),I)/c^(17/4)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.52 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.84

$$\int \frac{A+Bx^2}{x^6\,(c+dx^2)\,\sqrt{a-cx^4}}\,dx$$
$$= \frac{-3a^2A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3-5a^2B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3x^2+5a^2A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2dx^2-6aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^4x^4+15a^2B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2dx^4-15a}{}$$

input
```
Integrate[(A + B*x^2)/(x^6*(c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
(-3*a^2*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3 - 5*a^2*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*
c^3*x^2 + 5*a^2*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d*x^2 - 6*a*A*Sqrt[-(Sqrt[c
]/Sqrt[a])]*c^4*x^4 + 15*a^2*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d*x^4 - 15*a^2
*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^2*x^4 + 5*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^4
*x^6 - 5*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*d*x^6 + 9*A*Sqrt[-(Sqrt[c]/Sqrt[
a])]*c^5*x^8 - 15*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*d*x^8 + 15*a*A*Sqrt[-(S
qrt[c]/Sqrt[a])]*c^2*d^2*x^8 - (3*I)*Sqrt[a]*c^(3/2)*(-3*A*c^3 + 5*a*B*c*d
 - 5*a*A*d^2)*x^5*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/S
qrt[a])]*x], -1] + I*Sqrt[a]*c^(3/2)*(-9*A*c^3 + 15*a*d*(B*c - A*d) + 5*Sq
rt[a]*c^(3/2)*(-(B*c) + A*d))*x^5*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (15*I)*a^2*B*c*d^2*x^5*Sqrt[1 - (c*x^4)
/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*
x], -1] + (15*I)*a^2*A*d^3*x^5*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d
)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(15*a^2*Sqrt[-(Sqr
t[c]/Sqrt[a])]*c^4*x^5*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.40, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^6\sqrt{a-cx^4}\,(c+dx^2)}dx$$

$$\Big\downarrow \text{2249}$$

$$\int \left(\frac{d^2(Bc-Ad)}{c^3\sqrt{a-cx^4}\,(c+dx^2)}-\frac{d(Bc-Ad)}{c^3x^2\sqrt{a-cx^4}}+\frac{Bc-Ad}{c^2x^4\sqrt{a-cx^4}}+\frac{A}{cx^6\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3a^{3/4}c^{5/4}\sqrt{a-cx^4}}+\frac{3A\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{5a^{5/4}\sqrt{a-cx^4}}-\frac{3A\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5a^{5/4}\sqrt{a-cx^4}}-\frac{3A\sqrt{a-cx^4}}{5a^2x}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{17/4}\sqrt{a-cx^4}}-\frac{d\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{a}c^{11/4}\sqrt{a-cx^4}}+\frac{d\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}c^{11/4}\sqrt{a-cx^4}}+\frac{d\sqrt{a-cx^4}(Bc-Ad)}{ac^3x}-\frac{\sqrt{a-cx^4}(Bc-Ad)}{3ac^2x^3}-\frac{A\sqrt{a-cx^4}}{5acx^5}$$

input
```
Int[(A + B*x^2)/(x^6*(c + d*x^2)*Sqrt[a - c*x^4]),x]
```

output
```
-1/5*(A*Sqrt[a - c*x^4])/(a*c*x^5) - ((B*c - A*d)*Sqrt[a - c*x^4])/(3*a*c^
2*x^3) - (3*A*Sqrt[a - c*x^4])/(5*a^2*x) + (d*(B*c - A*d)*Sqrt[a - c*x^4])
/(a*c^3*x) - (3*A*c^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)
/a^(1/4)], -1])/(5*a^(5/4)*Sqrt[a - c*x^4]) + (d*(B*c - A*d)*Sqrt[1 - (c*x
^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(a^(1/4)*c^(11/4)*Sqrt[
a - c*x^4]) + (3*A*c^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x
)/a^(1/4)], -1])/(5*a^(5/4)*Sqrt[a - c*x^4]) + ((B*c - A*d)*Sqrt[1 - (c*x^
4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*a^(3/4)*c^(5/4)*Sqrt[
a - c*x^4]) - (d*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)
*x)/a^(1/4)], -1])/(a^(1/4)*c^(11/4)*Sqrt[a - c*x^4]) + (a^(1/4)*d^2*(B*c
- A*d)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^(1
/4)*x)/a^(1/4)], -1])/(c^(17/4)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{\sqrt{-c\,x^4+a}\,(15Aa\,d^2x^4+9A\,c^3x^4-15Bacd\,x^4-5Aacd\,x^2+5Ba\,c^2x^2+3Aa\,c^2)}{15c^3a^2x^5}-\frac{-\frac{3\sqrt{c}\left(5Aa\,d^2+3A\,c^3-5aBcd\right)\sqrt{a}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}}{}$
default	$\frac{A\left(-\frac{\sqrt{-c\,x^4+a}}{5a\,x^5}-\frac{3c\sqrt{-c\,x^4+a}}{5a^2x}+\frac{3c^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-c\,x^4+a}}\right)}{c}-\frac{(Ad-Bc)\left(-\frac{\sqrt{-c\,x^4+a}}{3a\,x^3}\right.}{}$
elliptic	$-\frac{A\sqrt{-c\,x^4+a}}{5ac\,x^5}+\frac{(Ad-Bc)\sqrt{-c\,x^4+a}}{3c^2a\,x^3}-\frac{(5Aa\,d^2+3A\,c^3-5aBcd)\sqrt{-c\,x^4+a}}{5a^2c^3x}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3ca\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-c\,x^4+a}}$

input
```
int((B*x^2+A)/x^6/(d*x^2+c)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-c*x^4+a)^(1/2)*(15*A*a*d^2*x^4+9*A*c^3*x^4-15*B*a*c*d*x^4-5*A*a*c*
d*x^2+5*B*a*c^2*x^2+3*A*a*c^2)/c^3/a^2/x^5-1/15/a^2/c^3*(-3*c^(1/2)*(5*A*a
*d^2+3*A*c^3-5*B*a*c*d)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(
1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c
^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))-5*B*c^3*a
/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1
/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+5*A*a*c
^2*d/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/
a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+15*
a^2*d^2*(A*d-B*c)/c/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*
(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/
2))^(1/2),-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1
/2)))
```

Fricas [F]

$$\int \frac{A+Bx^2}{x^6\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}(dx^2+c)x^6}\,dx$$

input
```
integrate((B*x^2+A)/x^6/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c*x^4 + a)*(B*x^2 + A)/(c*d*x^12 + c^2*x^10 - a*d*x^8 - a*
c*x^6), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^6\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2}{x^6\sqrt{a-cx^4}\,\left(c+dx^2\right)}\,dx$$

input
```
integrate((B*x**2+A)/x**6/(d*x**2+c)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)/(x**6*sqrt(a - c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^6\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}(dx^2+c)x^6}\,dx$$

input
```
integrate((B*x^2+A)/x^6/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^6\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{-cx^4+a}(dx^2+c)x^6}\,dx$$

input
```
integrate((B*x^2+A)/x^6/(d*x^2+c)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(-c*x^4 + a)*(d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^6\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx=\int \frac{B\,x^2+A}{x^6\,\sqrt{a-c\,x^4}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2)/(x^6*(a - c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2)/(x^6*(a - c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{x^6\left(c+dx^2\right)\sqrt{a-cx^4}}\,dx = \frac{-3\sqrt{-c\,x^4+a}\,a-5\sqrt{-c\,x^4+a}\,b\,x^2-15\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^{10}-c^2x^8+ad\,x^6+ac\,x^4}dx\right)a^2d\,x^5-15\left(\int \frac{\sqrt{-c\,x^4+a}}{-cd\,x^8-c^2x^6+ad\,x^4+ac}\right.}{}$$

input
```
int((B*x^2+A)/x^6/(d*x^2+c)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 3*sqrt(a - c*x**4)*a - 5*sqrt(a - c*x**4)*b*x**2 - 15*int(sqrt(a - c*x
**4)/(a*c*x**4 + a*d*x**6 - c**2*x**8 - c*d*x**10),x)*a**2*d*x**5 - 15*int
(sqrt(a - c*x**4)/(a*c*x**2 + a*d*x**4 - c**2*x**6 - c*d*x**8),x)*a*b*d*x*
*5 + 9*int(sqrt(a - c*x**4)/(a*c*x**2 + a*d*x**4 - c**2*x**6 - c*d*x**8),x
)*a*c**2*x**5 + 9*int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 - c*d*x
**6),x)*a*c*d*x**5 + 5*int(sqrt(a - c*x**4)/(a*c + a*d*x**2 - c**2*x**4 -
c*d*x**6),x)*b*c**2*x**5 + 5*int((sqrt(a - c*x**4)*x**2)/(a*c + a*d*x**2 -
 c**2*x**4 - c*d*x**6),x)*b*c*d*x**5)/(15*a*c*x**5)
```

3.14 $\int \frac{x^8(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 405

$$\int \frac{x^8(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \frac{ax\left(A-\frac{aBd}{c^2}+\left(B-\frac{Ad}{c}\right)x^2\right)}{2(c^3-ad^2)\sqrt{a-cx^4}}+\frac{Bx\sqrt{a-cx^4}}{3c^2d}$$
$$+\frac{a^{3/4}(Bc-Ad)(2c^3-3ad^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2c^{7/4}d^2(c^3-ad^2)\sqrt{a-cx^4}}$$
$$-\frac{\sqrt[4]{a}\left(6Bc^6-6Ac^5d+2aBc^3d^2+3aAc^2d^3-5a^2Bd^4+3\sqrt{a}\sqrt{c}d(Bc-Ad)(2c^3-3ad^2)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{Ellipt}}{6c^{9/4}d^3(c^3-ad^2)\sqrt{a-cx^4}}$$
$$+\frac{\sqrt[4]{a}c^{11/4}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d^3(c^3-ad^2)\sqrt{a-cx^4}}$$

output
```
1/2*a*x*(A-a*B*d/c^2+(B-A*d/c)*x^2)/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)+1/3*B*x*
(-c*x^4+a)^(1/2)/c^2/d+1/2*a^(3/4)*(-A*d+B*c)*(-3*a*d^2+2*c^3)*(1-c*x^4/a)
^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(7/4)/d^2/(-a*d^2+c^3)/(-c*x^4+a)^
(1/2)-1/6*a^(1/4)*(6*B*c^6-6*A*c^5*d+2*a*B*c^3*d^2+3*a*A*c^2*d^3-5*a^2*B*d
^4+3*a^(1/2)*c^(1/2)*d*(-A*d+B*c)*(-3*a*d^2+2*c^3))*(1-c*x^4/a)^(1/2)*Elli
pticF(c^(1/4)*x/a^(1/4),I)/c^(9/4)/d^3/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)+a^(1/
4)*c^(11/4)*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(
1/2)*d/c^(3/2),I)/d^3/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.77 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.42

$$\int \frac{x^8(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \frac{-2aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3d^2x-3aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2d^3x+5a^2B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d^4x-3aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2d^3}{}$$

input
```
Integrate[(x^8*(A + B*x^2))/((c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(-2*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*d^2*x - 3*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])
]*c^2*d^3*x + 5*a^2*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^4*x - 3*a*B*Sqrt[-(Sqrt[c
]/Sqrt[a])]*c^2*d^3*x^3 + 3*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^4*x^3 + 2*B*S
qrt[-(Sqrt[c]/Sqrt[a])]*c^4*d^2*x^5 - 2*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^4
*x^5 - (3*I)*Sqrt[a]*Sqrt[c]*d*(B*c - A*d)*(-2*c^3 + 3*a*d^2)*Sqrt[1 - (c*
x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*(-c^(3/2)
 + Sqrt[a]*d)*(-3*A*Sqrt[c]*d*(2*c^3 + 4*Sqrt[a]*c^(3/2)*d + 3*a*d^2) + B*
(6*c^(9/2) + 12*Sqrt[a]*c^3*d + 14*a*c^(3/2)*d^2 + 5*a^(3/2)*d^3))*Sqrt[1
- (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (6*I)*
B*c^6*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqr
t[-(Sqrt[c]/Sqrt[a])]*x], -1] - (6*I)*A*c^5*d*Sqrt[1 - (c*x^4)/a]*Elliptic
Pi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(6*
Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d^3*(-c^3 + a*d^2)*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(A+Bx^2)}{(a-cx^4)^{3/2}(c+dx^2)}dx$$

$$\Big\downarrow \text{2249}$$

$$\int \left(\frac{a^2(-aBd+cx^2(Bc-Ad)+Ac^2)}{c^2(c^3-ad^2)(a-cx^4)^{3/2}}-\frac{c^4(Bc-Ad)}{d^3(ad^2-c^3)\sqrt{a-cx^4}(c+dx^2)}+\frac{Ac^2d-B(ad^2+c^3)}{c^2d^3\sqrt{a-cx^4}}+\frac{x^2(Bc-A}{cd^2\sqrt{a-c}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{7/4}d^2\sqrt{a-cx^4}}+\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{7/4}d^2\sqrt{a-cx^4}}+\frac{a^{5/4}\sqrt{1-\frac{cx^4}{a}}(\sqrt{a}B+A\sqrt{c})\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2c^{9/4}(\sqrt{a}d+c^{3/2})\sqrt{a-cx^4}}-\frac{a^{7/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2c^{7/4}(c^3-ad^2)\sqrt{a-cx^4}}-\frac{a^{5/4}B\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3c^{9/4}d\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}c^{11/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d^3(c^3-ad^2)\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(Ac^2d-B(ad^2+c^3))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{9/4}d^3\sqrt{a-cx^4}}+\frac{ax(-aBd+cx^2(Bc-Ad)+Ac^2)}{2c^2(c^3-ad^2)\sqrt{a-cx^4}}+\frac{Bx\sqrt{a-cx^4}}{3c^2d}$$

input
```
Int[(x^8*(A + B*x^2))/((c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(a*x*(A*c^2 - a*B*d + c*(B*c - A*d)*x^2))/(2*c^2*(c^3 - a*d^2)*Sqrt[a - c*
x^4]) + (B*x*Sqrt[a - c*x^4])/(3*c^2*d) + (a^(3/4)*(B*c - A*d)*Sqrt[1 - (c
*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(7/4)*d^2*Sqrt[a -
 c*x^4]) - (a^(7/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1
/4)*x)/a^(1/4)], -1])/(2*c^(7/4)*(c^3 - a*d^2)*Sqrt[a - c*x^4]) - (a^(5/4)
*B*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(9
/4)*d*Sqrt[a - c*x^4]) + (a^(5/4)*(Sqrt[a]*B + A*Sqrt[c])*Sqrt[1 - (c*x^4)
/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(9/4)*(c^(3/2) + Sqrt
[a]*d)*Sqrt[a - c*x^4]) - (a^(3/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*Ellipti
cF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(7/4)*d^2*Sqrt[a - c*x^4]) + (a^(1
/4)*(A*c^2*d - B*(c^3 + a*d^2))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1
/4)*x)/a^(1/4)], -1])/(c^(9/4)*d^3*Sqrt[a - c*x^4]) + (a^(1/4)*c^(11/4)*(B
*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c
^(1/4)*x)/a^(1/4)], -1])/(d^3*(c^3 - a*d^2)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(349) = 698$.

Time = 12.37 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.74

method	result
risch	$\frac{Bx\sqrt{-cx^4+a}}{3c^2d} - \frac{-\frac{3\sqrt{c}\,d(Ad-Bc)\sqrt{a}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4+a}}+\frac{3Bc^3\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\ \ldots}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{-cx^4 \ldots}}}{\ldots}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int(x^8*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x*(-c*x^4+a)^(1/2)/c^2/d-1/3/c^2/d*(1/d^2*(-3*c^(1/2)*d*(A*d-B*c)*a^
(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2
/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-E
llipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))+3*B*c^3/(c^(1/2)/a^(1/2))^(1/2)*(1-
c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*
EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-3*A*c^2*d/(c^(1/2)/a^(1/2))^(1/2)*(
1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2
)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+4*B*a*d^2/(c^(1/2)/a^(1/2))^(1/2)
*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1
/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))+3*a^2*d/(a*d^2-c^3)*(2*c*(-1/4
*(A*d-B*c)/a*x^3+1/4*(A*c^2-B*a*d)/a/c*x)/(-(x^4-a/c)*c)^(1/2)+1/2*(A*c^2-
B*a*d)/a/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*
x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)
-1/2*(A*d-B*c)*c^(1/2)/a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1
/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^
(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))-3*c^5*(A*
d-B*c)/d^2/(a*d^2-c^3)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/
2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^
(1/2))^(1/2),-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))
^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^8*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(B*x**2+A)/(d*x**2+c)/(-c*x**4+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^8(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^8}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate(x^8*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^8/((-c*x^4 + a)^(3/2)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^8(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^8}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate(x^8*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^8/((-c*x^4 + a)^(3/2)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \int \frac{x^8\,(B\,x^2+A)}{(a-c\,x^4)^{3/2}\,(d\,x^2+c)}\,dx$$

input
```
int((x^8*(A + B*x^2))/((a - c*x^4)^(3/2)*(c + d*x^2)),x)
```

output
```
int((x^8*(A + B*x^2))/((a - c*x^4)^(3/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^8(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^8*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x)
```

output
```
(9*sqrt(a - c*x**4)*a**2*d**2*x - 4*sqrt(a - c*x**4)*a*b*c*d*x - 3*sqrt(a
- c*x**4)*a*c**2*d*x**3 + 3*sqrt(a - c*x**4)*b*c**3*x**3 - sqrt(a - c*x**4
)*b*c**2*d*x**5 - 9*int(sqrt(a - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*
x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**4*c*d**2 + 4*int(sqr
t(a - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*
x**8 + c**2*d*x**10),x)*a**3*b*c**2*d + 9*int(sqrt(a - c*x**4)/(a**2*c + a
**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a
**3*c**2*d**2*x**4 - 4*int(sqrt(a - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c*
*2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**2*b*c**3*d*x**4 -
 9*int((sqrt(a - c*x**4)*x**6)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a
*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**3*c*d**3 + 9*int((sqrt(a - c*x
**4)*x**6)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**
8 + c**2*d*x**10),x)*a**2*b*c**2*d**2 - 3*int((sqrt(a - c*x**4)*x**6)/(a**
2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**1
0),x)*a**2*c**4*d + 9*int((sqrt(a - c*x**4)*x**6)/(a**2*c + a**2*d*x**2 -
2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**2*c**2*d**3
*x**4 + 3*int((sqrt(a - c*x**4)*x**6)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**
4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a*b*c**5 - 9*int((sqrt(a -
 c*x**4)*x**6)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3
*x**8 + c**2*d*x**10),x)*a*b*c**3*d**2*x**4 + 3*int((sqrt(a - c*x**4)*x...
```

3.15 $\int \frac{x^6(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 367

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \frac{x\left(a(Bc-Ad)+c^2\left(A-\frac{aBd}{c^2}\right)x^2\right)}{2c\,(c^3-ad^2)\sqrt{a-cx^4}} - \frac{a^{3/4}(2Bc^3+Ac^2d-3aBd^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2c^{7/4}d\,(c^3-ad^2)\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(2Bc^3+4\sqrt{a}Bc^{3/2}d-2Ac^2d+3aBd^2-\sqrt{a}A\sqrt{c}d^2\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2c^{7/4}d^2\left(c^{3/2}+\sqrt{a}d\right)\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}c^{7/4}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d^2\,(c^3-ad^2)\sqrt{a-cx^4}}$$

output
```
1/2*x*(a*(-A*d+B*c)+c^2*(A-a*B*d/c^2)*x^2)/c/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)
-1/2*a^(3/4)*(A*c^2*d-3*B*a*d^2+2*B*c^3)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/
4)*x/a^(1/4),I)/c^(7/4)/d/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)+1/2*a^(1/4)*(2*B*c
^3+4*a^(1/2)*B*c^(3/2)*d-2*A*c^2*d+3*B*a*d^2-a^(1/2)*A*c^(1/2)*d^2)*(1-c*x
^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(7/4)/d^2/(c^(3/2)+a^(1/2)*d)
/(-c*x^4+a)^(1/2)-a^(1/4)*c^(7/4)*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*EllipticPi(
c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/2),I)/d^2/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.62 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.98

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \frac{i\sqrt{a}d(-2Bc^3-Ac^2d+3aBd^2)\sqrt{1-\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-i(}{}$$

input
```
Integrate[(x^6*(A + B*x^2))/((c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(I*Sqrt[a]*d*(-2*B*c^3 - A*c^2*d + 3*a*B*d^2)*Sqrt[1 - (c*x^4)/a]*Elliptic
E[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*(-c^(3/2) + Sqrt[a]*d)*(-
(A*Sqrt[c]*d*(2*c^(3/2) + Sqrt[a]*d)) + B*(2*c^3 + 4*Sqrt[a]*c^(3/2)*d + 3
*a*d^2))*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*
x], -1] + Sqrt[c]*(Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*x*(-(A*c^2*x^2) + a*(-(B*c
) + A*d + B*d*x^2)) - (2*I)*c^3*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticPi
[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/(2*S
qrt[-(Sqrt[c]/Sqrt[a])]*c^(3/2)*d^2*(-c^3 + a*d^2)*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^2)}{(a-cx^4)^{3/2}(c+dx^2)}\,dx$$

$$\Big\downarrow \text{2249}$$

$$\int \left(\frac{c^3(Bc-Ad)}{d^2(ad^2-c^3)\sqrt{a-cx^4}(c+dx^2)}+\frac{a\left(x^2(Ac^2-aBd)+a(Bc-Ad)\right)}{c(c^3-ad^2)(a-cx^4)^{3/2}}+\frac{Bc-Ad}{cd^2\sqrt{a-cx^4}}-\frac{Bx^2}{cd\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}B+A\sqrt{c}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2c^{7/4}\left(\sqrt{a}d+c^{3/2}\right)\sqrt{a-cx^4}}-\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(Ac^2-aBd\right)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2c^{7/4}\left(c^3-ad^2\right)\sqrt{a-cx^4}}+\frac{a^{3/4}B\sqrt{1-\frac{cx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{7/4}d\sqrt{a-cx^4}}-\frac{a^{3/4}B\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{7/4}d\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{5/4}d^2\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}c^{7/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d^2\left(c^3-ad^2\right)\sqrt{a-cx^4}}+\frac{x\left(x^2\left(Ac^2-aBd\right)+a(Bc-Ad)\right)}{2c\left(c^3-ad^2\right)\sqrt{a-cx^4}}$$

input
```
Int[(x^6*(A + B*x^2))/((c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(x*(a*(B*c - A*d) + (A*c^2 - a*B*d)*x^2))/(2*c*(c^3 - a*d^2)*Sqrt[a - c*x^
4]) - (a^(3/4)*B*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)]
, -1])/(c^(7/4)*d*Sqrt[a - c*x^4]) - (a^(3/4)*(A*c^2 - a*B*d)*Sqrt[1 - (c*
x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(7/4)*(c^3 - a*d^
2)*Sqrt[a - c*x^4]) + (a^(3/4)*B*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(
1/4)*x)/a^(1/4)], -1])/(c^(7/4)*d*Sqrt[a - c*x^4]) + (a^(3/4)*(Sqrt[a]*B +
 A*Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1]
)/(2*c^(7/4)*(c^(3/2) + Sqrt[a]*d)*Sqrt[a - c*x^4]) + (a^(1/4)*(B*c - A*d)
*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(5/4)*
d^2*Sqrt[a - c*x^4]) - (a^(1/4)*c^(7/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*El
lipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(d^2*(c
^3 - a*d^2)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(307) = 614$.

Time = 2.36 (sec) , antiderivative size = 975, normalized size of antiderivative = 2.66

method	result	size
default	Expression too large to display	975
elliptic	Expression too large to display	1087

input
```
int(x^6*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(d^2*(A*d-B*c)*(1/2/c*x/(-(x^4-a/c)*c)^(1/2)-1/2/c/(c^(1/2)/a^(1/2))
^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4
+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))-c*d*(A*d-B*c)*(1/2/a*x^3
/(-(x^4-a/c)*c)^(1/2)+1/2/a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a
^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(Elli
pticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))
)+A*c^2*d*(1/2/a*x/(-(x^4-a/c)*c)^(1/2)+1/2/a/(c^(1/2)/a^(1/2))^(1/2)*(1-c
^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*E
llipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))+B*d^3*(1/2/c*x^3/(-(x^4-a/c)*c)^(1/
2)+3/2/c^(3/2)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/
2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/2)/a^
(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))-B*c^3*(1/2/a*x/(-
(x^4-a/c)*c)^(1/2)+1/2/a/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(
1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a
^(1/2))^(1/2),I)))-c^3*(A*d-B*c)/d^4*(2*c*(1/4/a*d/(a*d^2-c^3)*x^3-1/4*c/a
/(a*d^2-c^3)*x)/(-(x^4-a/c)*c)^(1/2)-1/2*c^2/a/(a*d^2-c^3)/(c^(1/2)/a^(1/2
))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x
^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/2*c^(1/2)/a^(1/2)*d/(
a*d^2-c^3)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2
)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^6*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\left(a-cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(B*x**2+A)/(d*x**2+c)/(-c*x**4+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\left(a-cx^4\right)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^6}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate(x^6*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^6/((-c*x^4 + a)^(3/2)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\left(a-cx^4\right)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^6}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate(x^6*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^6/((-c*x^4 + a)^(3/2)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\,(a-cx^4)^{3/2}}\,dx = \int \frac{x^6\,(B\,x^2+A)}{(a-c\,x^4)^{3/2}\,(d\,x^2+c)}\,dx$$

input
```
int((x^6*(A + B*x^2))/((a - c*x^4)^(3/2)*(c + d*x^2)),x)
```

output
```
int((x^6*(A + B*x^2))/((a - c*x^4)^(3/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^6(A+Bx^2)}{(c+dx^2)\,(a-cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^6*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x)
```

output
```
(3*sqrt(a - c*x**4)*a*b*d*x - sqrt(a - c*x**4)*b*c**2*x**3 - 3*int(sqrt(a
- c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8
 + c**2*d*x**10),x)*a**3*b*c*d + 3*int(sqrt(a - c*x**4)/(a**2*c + a**2*d*x
**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**2*b*c
**2*d*x**4 - 3*int((sqrt(a - c*x**4)*x**6)/(a**2*c + a**2*d*x**2 - 2*a*c**
2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**2*b*c*d**2 + int((
sqrt(a - c*x**4)*x**6)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**
6 + c**3*x**8 + c**2*d*x**10),x)*a**2*c**3*d - int((sqrt(a - c*x**4)*x**6)
/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d
*x**10),x)*a*b*c**4 + 3*int((sqrt(a - c*x**4)*x**6)/(a**2*c + a**2*d*x**2
- 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a*b*c**2*d**
2*x**4 - int((sqrt(a - c*x**4)*x**6)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4
 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a*c**4*d*x**4 + int((sqrt(a
 - c*x**4)*x**6)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c*
*3*x**8 + c**2*d*x**10),x)*b*c**5*x**4 - 3*int((sqrt(a - c*x**4)*x**2)/(a*
*2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**
10),x)*a**3*b*d**2 + 3*int((sqrt(a - c*x**4)*x**2)/(a**2*c + a**2*d*x**2 -
 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**2*b*c**3 +
 3*int((sqrt(a - c*x**4)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a
*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**2*b*c*d**2*x**4 - 3*int((sq...
```

3.16 $\int \frac{x^4(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 319

$$\begin{aligned}\int \frac{x^4(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx &= \frac{x\left(Ac-\frac{aBd}{c}+(Bc-Ad)x^2\right)}{2(c^3-ad^2)\sqrt{a-cx^4}} \\ &\quad -\frac{a^{3/4}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2c^{3/4}(c^3-ad^2)\sqrt{a-cx^4}} \\ &\quad -\frac{\sqrt[4]{a}(2Bc^{3/2}+\sqrt{a}Bd-A\sqrt{c}d)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2c^{5/4}d(c^{3/2}+\sqrt{a}d)\sqrt{a-cx^4}} \\ &\quad +\frac{\sqrt[4]{a}c^{3/4}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d(c^3-ad^2)\sqrt{a-cx^4}}\end{aligned}$$

output
```
1/2*x*(A*c-a*B*d/c+(-A*d+B*c)*x^2)/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)-1/2*a^(3/
4)*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/c^(3/4)/(-a
*d^2+c^3)/(-c*x^4+a)^(1/2)-1/2*a^(1/4)*(2*B*c^(3/2)+a^(1/2)*B*d-A*c^(1/2)*
d)*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/c^(5/4)/d/(c^(3/2)+a^(
1/2)*d)/(-c*x^4+a)^(1/2)+a^(1/4)*c^(3/4)*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*Elli
pticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/2),I)/d/(-a*d^2+c^3)/(-c*x^4+a)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.20 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.31

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \frac{-A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2dx+aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}d^2x-B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2dx^3+A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cd^2x^3-i\sqrt{a}\sqrt{}}{}$$

input
```
Integrate[(x^4*(A + B*x^2))/((c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(-(A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d*x) + a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*
x - B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d*x^3 + A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d^
2*x^3 - I*Sqrt[a]*Sqrt[c]*d*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*Ar
cSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*(-c^(3/2) + Sqrt[a]*d)*(2*B*c^(
3/2) + Sqrt[a]*B*d - A*Sqrt[c]*d)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (2*I)*B*c^3*Sqrt[1 - (c*x^4)/a]*Ellipti
cPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (
2*I)*A*c^2*d*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcS
inh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(2*Sqrt[-(Sqrt[c]/Sqrt[a])]*(-(c^4*d
) + a*c*d^3)*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2)}{(a-cx^4)^{3/2}(c+dx^2)}\,dx$$

$$\Big\downarrow \ 2249$$

$$\int \left(-\frac{c^2(Bc-Ad)}{d\left(ad^2-c^3\right)\sqrt{a-cx^4}\left(c+dx^2\right)}+\frac{a\left(-aBd+cx^2(Bc-Ad)+Ac^2\right)}{c\left(c^3-ad^2\right)\left(a-cx^4\right)^{3/2}}-\frac{B}{cd\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2c^{3/4}\left(c^3-ad^2\right)\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}B+A\sqrt{c}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2c^{5/4}\left(\sqrt{a}d+c^{3/2}\right)\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}c^{3/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{d\left(c^3-ad^2\right)\sqrt{a-cx^4}}+\frac{x\left(-aBd+cx^2(Bc-Ad)+Ac^2\right)}{2c\left(c^3-ad^2\right)\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}B\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{5/4}d\sqrt{a-cx^4}}$$

input
```
Int[(x^4*(A + B*x^2))/((c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(x*(A*c^2 - a*B*d + c*(B*c - A*d)*x^2))/(2*c*(c^3 - a*d^2)*Sqrt[a - c*x^4]
) - (a^(3/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/
a^(1/4)], -1])/(2*c^(3/4)*(c^3 - a*d^2)*Sqrt[a - c*x^4]) - (a^(1/4)*B*Sqrt
[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(5/4)*d*Sqr
t[a - c*x^4]) + (a^(1/4)*(Sqrt[a]*B + A*Sqrt[c])*Sqrt[1 - (c*x^4)/a]*Ellip
ticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(5/4)*(c^(3/2) + Sqrt[a]*d)*Sq
rt[a - c*x^4]) + (a^(1/4)*c^(3/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*Elliptic
Pi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(d*(c^3 - a*d
^2)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 849 vs. 2(261) = 522.

Time = 1.73 (sec) , antiderivative size = 850, normalized size of antiderivative = 2.66

method	result
default	$\frac{c^2(Ad-Bc)\left(\frac{2c\left(\frac{d\,x^3}{4a\left(a\,d^2-c^3\right)}-\frac{cx}{4a\left(a\,d^2-c^3\right)}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}-\frac{c^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\left(a\,d^2-c^3\right)\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}+\frac{\sqrt{c}\,d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\right.}{2\sqrt{a}\left(a\,d^2-c^3\right)\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}}{d^3}$
elliptic	$\frac{2c\left(\frac{(Ad-Bc)x^3}{4c\left(a\,d^2-c^3\right)}-\frac{\left(A\,c^2-Bad\right)x}{4c^2\left(a\,d^2-c^3\right)}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)B}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,cd}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\right.}{2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\left(a\,d^2-c^3\right)}$

input
```
int(x^4*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
c^2*(A*d-B*c)/d^3*(2*c*(1/4/a*d/(a*d^2-c^3)*x^3-1/4*c/a/(a*d^2-c^3)*x)/(-(
x^4-a/c)*c)^(1/2)-1/2*c^2/a/(a*d^2-c^3)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)
*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ellipti
cF(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/2*c^(1/2)/a^(1/2)*d/(a*d^2-c^3)/(c^(1/2)
/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2
)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-1/2*c^(1/2)/a^(1
/2)*d/(a*d^2-c^3)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1
+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(c^(1/2)/a^(1/2))
^(1/2),I)+1/(a*d^2-c^3)*d^2/c/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/
2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(
1/2)/a^(1/2))^(1/2),-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a
^(1/2))^(1/2)))-1/d^3*(-d*(A*d-B*c)*(1/2/a*x^3/(-(x^4-a/c)*c)^(1/2)+1/2/a^
(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2
/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1
/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))+A*c*d*(1/2/a*x/(-(x^4-a/c)
*c)^(1/2)+1/2/a/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c
^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(
1/2),I))-B*c^2*(1/2/a*x/(-(x^4-a/c)*c)^(1/2)+1/2/a/(c^(1/2)/a^(1/2))^(1/2)
*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1
/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))-B*d^2*(1/2/c*x/(-(x^4-a/c)*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)\,(a-cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(B*x**2+A)/(d*x**2+c)/(-c*x**4+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^4}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate(x^4*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^4/((-c*x^4 + a)^(3/2)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^4}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate(x^4*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^4/((-c*x^4 + a)^(3/2)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)\,(a-cx^4)^{3/2}}\,dx = \int \frac{x^4\,(B\,x^2+A)}{(a-c\,x^4)^{3/2}\,(d\,x^2+c)}\,dx$$

input
```
int((x^4*(A + B*x^2))/((a - c*x^4)^(3/2)*(c + d*x^2)),x)
```

output
```
int((x^4*(A + B*x^2))/((a - c*x^4)^(3/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2)}{(c+dx^2)\,(a-cx^4)^{3/2}}\,dx = \frac{\sqrt{-c\,x^4+a}\,ax-\left(\int \frac{\sqrt{-c\,x^4+a}}{c^2d\,x^{10}+c^3x^8-2acd\,x^6-2a\,c^2x^4+a^2d\,x^2+a^2c}dx\right)a^3c+\left(\int \frac{}{c^2d\,x^{10}+}\right.}{}$$

input
```
int(x^4*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x)
```

output
```
(sqrt(a - c*x**4)*a*x - int(sqrt(a - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c
**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**3*c + int(sqrt(a
 - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**
8 + c**2*d*x**10),x)*a**2*c**2*x**4 - int((sqrt(a - c*x**4)*x**6)/(a**2*c
+ a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x
)*a**2*c*d + int((sqrt(a - c*x**4)*x**6)/(a**2*c + a**2*d*x**2 - 2*a*c**2*
x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a*b*c**2 + int((sqrt(a
- c*x**4)*x**6)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**
3*x**8 + c**2*d*x**10),x)*a*c**2*d*x**4 - int((sqrt(a - c*x**4)*x**6)/(a**
2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**1
0),x)*b*c**3*x**4 - int((sqrt(a - c*x**4)*x**2)/(a**2*c + a**2*d*x**2 - 2*
a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**3*d + int((sq
rt(a - c*x**4)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6
+ c**3*x**8 + c**2*d*x**10),x)*a**2*c*d*x**4)/(c**2*(a - c*x**4))
```

3.17 $\int \frac{x^2(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 311

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \frac{x(a(Bc-Ad)+(Ac^2-aBd)x^2)}{2a(c^3-ad^2)\sqrt{a-cx^4}} - \frac{(Ac^2-aBd)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}c^{3/4}(c^3-ad^2)\sqrt{a-cx^4}} + \frac{(\sqrt{a}B+A\sqrt{c})\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{a}c^{3/4}(c^{3/2}+\sqrt{a}d)\sqrt{a-cx^4}} - \frac{\sqrt[4]{a}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}(c^3-ad^2)\sqrt{a-cx^4}}$$

output
```
1/2*x*(a*(-A*d+B*c)+(A*c^2-B*a*d)*x^2)/a/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)-1/2
*(A*c^2-B*a*d)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4)/c^
(3/4)/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)+1/2*(a^(1/2)*B+A*c^(1/2))*(1-c*x^4/a)^
(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(1/4)/c^(3/4)/(c^(3/2)+a^(1/2)*d)/(
-c*x^4+a)^(1/2)-a^(1/4)*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/
a^(1/4),-a^(1/2)*d/c^(3/2),I)/c^(1/4)/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.52 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.99

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx =$$

$$-\frac{i\sqrt{a}(-Ac^2+aBd)\sqrt{1-\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-i\sqrt{a}(\sqrt{a}B+A\sqrt{c})\left(-c^{3/2}+\sqrt{a}d\right)\sqrt{1-\frac{c}{}}}{}$$

input
```
Integrate[(x^2*(A + B*x^2))/((c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
-1/2*(I*Sqrt[a]*(-(A*c^2) + a*B*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh
[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*Sqrt[a]*(Sqrt[a]*B + A*Sqrt[c])*(-c^
(3/2) + Sqrt[a]*d)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/
Sqrt[a])]*x], -1] + Sqrt[c]*(Sqrt[-(Sqrt[c]/Sqrt[a])]*x*(-(A*c^2*x^2) + a*
(-(B*c) + A*d + B*d*x^2)) - (2*I)*a*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1]))/
(a^(3/2)*(-(Sqrt[c]/Sqrt[a]))^(3/2)*(-c^3 + a*d^2)*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2)}{(a-cx^4)^{3/2}(c+dx^2)}\,dx$$

$$\Big\downarrow \text{ 2249}$$

$$\int \left(\frac{x^2(Ac^2-aBd)+a(Bc-Ad)}{(c^3-ad^2)(a-cx^4)^{3/2}}-\frac{c(Bc-Ad)}{(c^3-ad^2)\sqrt{a-cx^4}(c+dx^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}B+A\sqrt{c}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{a}c^{3/4}\left(\sqrt{a}d+c^{3/2}\right)\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{c}\,(c^3-ad^2)\sqrt{a-cx^4}}-\frac{\sqrt{1-\frac{cx^4}{a}}(Ac^2-aBd)\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}c^{3/4}\,(c^3-ad^2)\sqrt{a-cx^4}}+\frac{x(x^2(Ac^2-aBd)+a(Bc-Ad))}{2a\,(c^3-ad^2)\sqrt{a-cx^4}}$$

input
```
Int[(x^2*(A + B*x^2))/((c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(x*(a*(B*c - A*d) + (A*c^2 - a*B*d)*x^2))/(2*a*(c^3 - a*d^2)*Sqrt[a - c*x^
4]) - ((A*c^2 - a*B*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^
(1/4)], -1])/(2*a^(1/4)*c^(3/4)*(c^3 - a*d^2)*Sqrt[a - c*x^4]) + ((Sqrt[a]
*B + A*Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)],
 -1])/(2*a^(1/4)*c^(3/4)*(c^(3/2) + Sqrt[a]*d)*Sqrt[a - c*x^4]) - (a^(1/4)
*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin
[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*(c^3 - a*d^2)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(255) = 510$.

Time = 1.16 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.39

method	result
default	$\frac{Ad\left(\frac{x}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)+Bd\left(\frac{x^3}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right.}{2\sqrt{a}\,\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right.}{d^2}$
elliptic	$\frac{2c\left(-\frac{\left(A\,c^2-Bad\right)x^3}{4ac\left(a\,d^2-c^3\right)}+\frac{(Ad-Bc)x}{4c\left(a\,d^2-c^3\right)}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)Ad}{2\left(a\,d^2-c^3\right)\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\right.}{2\left(a\,d^2-c^3\right)\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+}}$

input
```
int(x^2*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(A*d*(1/2/a*x/(-(x^4-a/c)*c)^(1/2)+1/2/a/(c^(1/2)/a^(1/2))^(1/2)*(1-
c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*
EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I))+B*d*(1/2/a*x^3/(-(x^4-a/c)*c)^(1/2
)+1/2/a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(
1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(
1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))-B*c*(1/2/a*x/(-(x^
4-a/c)*c)^(1/2)+1/2/a/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2
)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1
/2))^(1/2),I)))-c/d^2*(A*d-B*c)*(2*c*(1/4/a*d/(a*d^2-c^3)*x^3-1/4*c/a/(a*d
^2-c^3)*x)/(-(x^4-a/c)*c)^(1/2)-1/2*c^2/a/(a*d^2-c^3)/(c^(1/2)/a^(1/2))^(1
/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)
^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/2*c^(1/2)/a^(1/2)*d/(a*d^2
-c^3)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2
/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-1/
2*c^(1/2)/a^(1/2)*d/(a*d^2-c^3)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(
1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(c^
(1/2)/a^(1/2))^(1/2),I)+1/(a*d^2-c^3)*d^2/c/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(
1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ell
ipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1
/2)/(c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx=\int \frac{(Bx^2+A)x^2}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate(x^2*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((B*x^4 + A*x^2)*sqrt(-c*x^4 + a)/(c^2*d*x^10 + c^3*x^8 - 2*a*c*d*
x^6 - 2*a*c^2*x^4 + a^2*d*x^2 + a^2*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(B*x**2+A)/(d*x**2+c)/(-c*x**4+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx=\int \frac{(Bx^2+A)x^2}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate(x^2*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^2/((-c*x^4 + a)^(3/2)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^2}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate(x^2*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^2/((-c*x^4 + a)^(3/2)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \int \frac{x^2\,(B\,x^2+A)}{(a-c\,x^4)^{3/2}\,(d\,x^2+c)}\,dx$$

input
```
int((x^2*(A + B*x^2))/((a - c*x^4)^(3/2)*(c + d*x^2)),x)
```

output
```
int((x^2*(A + B*x^2))/((a - c*x^4)^(3/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2)}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{-c\,x^4+a}\,x^4}{c^2d\,x^{10}+c^3x^8-2acd\,x^6-2a\,c^2x^4+a^2d\,x^2+a^2c}dx\right)b + \left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{c^2d\,x^{10}+c^3x^8-2acd\,x^6-2a\,c^2x^4+a^2d\,x^2+a^2c}dx\right)a$$

input
```
int(x^2*(B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x)
```

output
```
int((sqrt(a - c*x**4)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*
d*x**6 + c**3*x**8 + c**2*d*x**10),x)*b + int((sqrt(a - c*x**4)*x**2)/(a**
2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**1
0),x)*a
```

3.18 $\int \frac{A+Bx^2}{(c+dx^2)(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 306

$$\begin{aligned}\int \frac{A+Bx^2}{(c+dx^2)(a-cx^4)^{3/2}}\,dx &= \frac{x(Ac^2-aBd+c(Bc-Ad)x^2)}{2a(c^3-ad^2)\sqrt{a-cx^4}} \\ &- \frac{\sqrt[4]{c}(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}(c^3-ad^2)\sqrt{a-cx^4}} \\ &+ \frac{(\sqrt{a}B+A\sqrt{c})\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\sqrt[4]{c}\left(c^{3/2}+\sqrt{a}d\right)\sqrt{a-cx^4}} \\ &+ \frac{\sqrt[4]{a}d(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{5/4}(c^3-ad^2)\sqrt{a-cx^4}}\end{aligned}$$

output
```
1/2*x*(A*c^2-B*a*d+c*(-A*d+B*c)*x^2)/a/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)-1/2*c
^(1/4)*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/a^(1/4)
/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)+1/2*(a^(1/2)*B+A*c^(1/2))*(1-c*x^4/a)^(1/2)
*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(3/4)/c^(1/4)/(c^(3/2)+a^(1/2)*d)/(-c*x^
4+a)^(1/2)+a^(1/4)*d*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(
1/4),-a^(1/2)*d/c^(3/2),I)/c^(5/4)/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.03 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx^2}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \frac{-A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3x + aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cdx - B\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3x^3 + A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2dx^3 - i\sqrt{a}c^{3/2}}{}$$

input
```
Integrate[(A + B*x^2)/((c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(-(A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*x) + a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d*x
- B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*x^3 + A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d*x^
3 - I*Sqrt[a]*c^(3/2)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*(Sqrt[a]*B + A*Sqrt[c])*c*(-c^(3/2) +
 Sqrt[a]*d)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a]
)]*x], -1] + (2*I)*a*B*c*d*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^
(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (2*I)*a*A*d^2*Sqrt[1
- (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/S
qrt[a])]*x], -1])/(2*a*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*(-c^3 + a*d^2)*Sqrt[a -
c*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a-cx^4)^{3/2}(c+dx^2)}\,dx$$

$$\Big\downarrow \; 2259$$

$$\int \left(\frac{-aBd+cx^2(Bc-Ad)+Ac^2}{(c^3-ad^2)(a-cx^4)^{3/2}} - \frac{d(Ad-Bc)}{(c^3-ad^2)\sqrt{a-cx^4}(c+dx^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}B+A\sqrt{c}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\sqrt[4]{c}\left(\sqrt{a}d+c^{3/2}\right)\sqrt{a-cx^4}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}\left(c^3-ad^2\right)\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}d\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{5/4}\left(c^3-ad^2\right)\sqrt{a-cx^4}}+\frac{x\left(-aBd+cx^2(Bc-Ad)+Ac^2\right)}{2a\left(c^3-ad^2\right)\sqrt{a-cx^4}}$$

input
```
Int[(A + B*x^2)/((c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(x*(A*c^2 - a*B*d + c*(B*c - A*d)*x^2))/(2*a*(c^3 - a*d^2)*Sqrt[a - c*x^4]
) - (c^(1/4)*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/
a^(1/4)], -1])/(2*a^(1/4)*(c^3 - a*d^2)*Sqrt[a - c*x^4]) + ((Sqrt[a]*B + A
*Sqrt[c])*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(2*a^(3/4)*c^(1/4)*(c^(3/2) + Sqrt[a]*d)*Sqrt[a - c*x^4]) + (a^(1/4)*d*(B*
c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^
(1/4)*x)/a^(1/4)], -1])/(c^(5/4)*(c^3 - a*d^2)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(250) = 500$.

Time = 0.66 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.73

method	result
default	$\frac{B\left(\frac{x}{2a\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)}{d}+\frac{(Ad-Bc)\left(\frac{2c\left(\frac{d\,x^3}{4a\left(a\,d^2-c^3\right)}-\frac{cx}{4a\left(a\,d^2-c^3\right)}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}-\frac{c^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{\ldots}}{2a\left(a\,d^2\ldots\right.}\ldots\right.}{\ldots}$
elliptic	$\frac{2c\left(\frac{(Ad-Bc)x^3}{4a\left(a\,d^2-c^3\right)}-\frac{\left(A\,c^2-Bad\right)x}{4a\left(a\,d^2-c^3\right)c}\right)}{\sqrt{-\left(x^4-\frac{a}{c}\right)c}}-\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)A\,c^2}{2a(a\,d^2-c^3)\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}+\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\ldots}\right.}{2(a\,d^2-c^3)\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a\ldots}}$

input
```
int((B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B/d*(1/2/a*x/(-(x^4-a/c)*c)^(1/2)+1/2/a/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)
*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*Ellipti
cF(x*(c^(1/2)/a^(1/2))^(1/2),I))+(A*d-B*c)/d*(2*c*(1/4/a*d/(a*d^2-c^3)*x^3
-1/4*c/a/(a*d^2-c^3)*x)/(-(x^4-a/c)*c)^(1/2)-1/2*c^2/a/(a*d^2-c^3)/(c^(1/2
)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/
2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/2*c^(1/2)/a^(
1/2)*d/(a*d^2-c^3)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(
1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2)
)^(1/2),I)-1/2*c^(1/2)/a^(1/2)*d/(a*d^2-c^3)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^
(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*El
lipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)+1/(a*d^2-c^3)*d^2/c/(c^(1/2)/a^(1/2))
^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4
+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*d/c^(3/2),(-c^(1/2
)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{(a-cx^4)^{\frac{3}{2}}(c+dx^2)}\,dx$$

input
```
integrate((B*x**2+A)/(d*x**2+c)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)/((a - c*x**4)**(3/2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate((B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((-c*x^4 + a)^(3/2)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

input
```
integrate((B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((-c*x^4 + a)^(3/2)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{(a-c\,x^4)^{3/2}\,(d\,x^2+c)}\,dx$$

input
```
int((A + B*x^2)/((a - c*x^4)^(3/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2)/((a - c*x^4)^(3/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{(c+dx^2)(a-cx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{-c\,x^4+a}}{c^2d\,x^{10}+c^3x^8-2acd\,x^6-2a\,c^2x^4+a^2d\,x^2+a^2c}dx\right)a + \left(\int \frac{\sqrt{-c\,x^4+a}\,x^2}{c^2d\,x^{10}+c^3x^8-2acd\,x^6-2a\,c^2x^4+a^2d\,x^2+a^2c}dx\right)b$$

input
```
int((B*x^2+A)/(d*x^2+c)/(-c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6
+ c**3*x**8 + c**2*d*x**10),x)*a + int((sqrt(a - c*x**4)*x**2)/(a**2*c + a
**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*b
```

3.19 $\int \frac{A+Bx^2}{x^2(c+dx^2)(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 388

$$\begin{aligned}\int \frac{A+Bx^2}{x^2(c+dx^2)(a-cx^4)^{3/2}}\,dx &= \frac{Ac^2-aBd+c(Bc-Ad)x^2}{2a(c^3-ad^2)x\sqrt{a-cx^4}} \\ &- \frac{(3Ac^3-aBcd-2aAd^2)\sqrt{a-cx^4}}{2a^2c(c^3-ad^2)x} \\ &- \frac{(3Ac^3-aBcd-2aAd^2)\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{5/4}c^{3/4}(c^3-ad^2)\sqrt{a-cx^4}} \\ &+ \frac{\left(3Ac^{3/2}+\sqrt{a}(Bc+2Ad)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{5/4}c^{3/4}\left(c^{3/2}+\sqrt{a}d\right)\sqrt{a-cx^4}} \\ &- \frac{\sqrt[4]{a}d^2(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{9/4}(c^3-ad^2)\sqrt{a-cx^4}}\end{aligned}$$

output
```
1/2*(A*c^2-B*a*d+c*(-A*d+B*c)*x^2)/a/(-a*d^2+c^3)/x/(-c*x^4+a)^(1/2)-1/2*(
-2*A*a*d^2+3*A*c^3-B*a*c*d)*(-c*x^4+a)^(1/2)/a^2/c/(-a*d^2+c^3)/x-1/2*(-2*
A*a*d^2+3*A*c^3-B*a*c*d)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),I)/
a^(5/4)/c^(3/4)/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)+1/2*(3*A*c^(3/2)+a^(1/2)*(2*
A*d+B*c))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x/a^(1/4),I)/a^(5/4)/c^(3/4)
/(c^(3/2)+a^(1/2)*d)/(-c*x^4+a)^(1/2)-a^(1/4)*d^2*(-A*d+B*c)*(1-c*x^4/a)^(
1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^(3/2),I)/c^(9/4)/(-a*d^2+c^
3)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.28 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^2}{x^2(c+dx^2)(a-cx^4)^{3/2}}\,dx = \frac{2aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^4 - 2a^2A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}cd^2 - aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^4x^2 + aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^3dx^2 - 3A}{}$$

input
```
Integrate[(A + B*x^2)/(x^2*(c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(2*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^4 - 2*a^2*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d
^2 - a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^4*x^2 + a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c
^3*d*x^2 - 3*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^5*x^4 + a*B*Sqrt[-(Sqrt[c]/Sqrt[
a])]*c^3*d*x^4 + 2*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d^2*x^4 + I*Sqrt[a]*c^
(3/2)*(-3*A*c^3 + a*B*c*d + 2*a*A*d^2)*x*Sqrt[1 - (c*x^4)/a]*EllipticE[I*A
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*Sqrt[a]*c^(3/2)*(-c^(3/2) + Sq
rt[a]*d)*(3*A*c^(3/2) + Sqrt[a]*(B*c + 2*A*d))*x*Sqrt[1 - (c*x^4)/a]*Ellip
ticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (2*I)*a^2*B*c*d^2*x*Sqrt
[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c
]/Sqrt[a])]*x], -1] + (2*I)*a^2*A*d^3*x*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((
Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(2*a^2*Sq
rt[-(Sqrt[c]/Sqrt[a])]*c^2*(-c^3 + a*d^2)*x*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2 (a-cx^4)^{3/2} (c+dx^2)} \, dx$$

$$\Big\downarrow \text{2249}$$

$$\int \left(-\frac{d^2(Bc-Ad)}{c (c^3-ad^2) \sqrt{a-cx^4} (c+dx^2)}+\frac{cx^2 (Ac^2-aBd)+ac(Bc-Ad)}{a (c^3-ad^2) (a-cx^4)^{3/2}}+\frac{A}{acx^2\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(\sqrt{a}B+A\sqrt{c})\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{5/4}\left(\sqrt{a}d+c^{3/2}\right)\sqrt{a-cx^4}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}(Ac^2-aBd)\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{5/4}(c^3-ad^2)\sqrt{a-cx^4}}+\frac{A\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{a^{5/4}c^{3/4}\sqrt{a-cx^4}}-\frac{A\sqrt{1-\frac{cx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{a^{5/4}c^{3/4}\sqrt{a-cx^4}}+\frac{cx\left(x^2(Ac^2-aBd)+a(Bc-Ad)\right)}{2a^2(c^3-ad^2)\sqrt{a-cx^4}}-\frac{A\sqrt{a-cx^4}}{a^2cx}-\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{9/4}(c^3-ad^2)\sqrt{a-cx^4}}$$

input
```
Int[(A + B*x^2)/(x^2*(c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(c*x*(a*(B*c - A*d) + (A*c^2 - a*B*d)*x^2))/(2*a^2*(c^3 - a*d^2)*Sqrt[a -
c*x^4]) - (A*Sqrt[a - c*x^4])/(a^2*c*x) - (A*Sqrt[1 - (c*x^4)/a]*EllipticE
[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(a^(5/4)*c^(3/4)*Sqrt[a - c*x^4]) - (c^
(1/4)*(A*c^2 - a*B*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(
1/4)], -1])/(2*a^(5/4)*(c^3 - a*d^2)*Sqrt[a - c*x^4]) + (A*Sqrt[1 - (c*x^4
)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(a^(5/4)*c^(3/4)*Sqrt[a -
 c*x^4]) + ((Sqrt[a]*B + A*Sqrt[c])*c^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticF[
ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(5/4)*(c^(3/2) + Sqrt[a]*d)*Sqrt[a
- c*x^4]) - (a^(1/4)*d^2*(B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqr
t[a]*d)/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(9/4)*(c^3 - a*d^2)
*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{A\sqrt{-c\,x^4+a}}{c\,a^2 x}-\frac{-\frac{A\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}+\frac{a^2(Ad-Bc)d^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{\left(a\,d^2-c^3\right)c\sqrt{}}}{}$
default	$\frac{A\left(\frac{c\,x^3}{2a^2\sqrt{-\left(x^4-\frac{a}{c}\right)c}}-\frac{\sqrt{-c\,x^4+a}}{a^2 x}+\frac{3\sqrt{c}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{2a^{\frac{3}{2}}\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)}{c}-\frac{(Ad-Bc)\left(\frac{2c\left(\frac{d}{4a(a}\right.}{}\right.}{}$
elliptic	Expression too large to display

input
```
int((B*x^2+A)/x^2/(d*x^2+c)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(-c*x^4+a)^(1/2)/c/a^2/x-1/a^2/c*(-A*c^(1/2)*a^(1/2)/(c^(1/2)/a^(1/2))^
(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+
a)^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1
/2))^(1/2),I))+a^2*(A*d-B*c)*d^2/(a*d^2-c^3)/c/(c^(1/2)/a^(1/2))^(1/2)*(1-
c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*
EllipticPi(x*(c^(1/2)/a^(1/2))^(1/2),-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))
^(1/2)/(c^(1/2)/a^(1/2))^(1/2))-a*c^2/(a*d^2-c^3)*(2*c*(-1/4*(A*c^2-B*a*d)
/a/c*x^3+1/4*(A*d-B*c)/c*x)/(-(x^4-a/c)*c)^(1/2)+(1/2*A*d-1/2*B*c)/(c^(1/2
)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))^(1/
2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-1/2*(A*c^2-B*a*
d)/a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2
)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2
))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x^2+A)/x^2/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx = \int \frac{A+Bx^2}{x^2\left(a-cx^4\right)^{\frac{3}{2}}\left(c+dx^2\right)}\,dx$$

input
```
integrate((B*x**2+A)/x**2/(d*x**2+c)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)/(x**2*(a - c*x**4)**(3/2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)x^2}\,dx$$

input
```
integrate((B*x^2+A)/x^2/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((-c*x^4 + a)^(3/2)*(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)x^2}\,dx$$

input
```
integrate((B*x^2+A)/x^2/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((-c*x^4 + a)^(3/2)*(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{B\,x^2+A}{x^2\,{\left(a-c\,x^4\right)}^{3/2}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2)/(x^2*(a - c*x^4)^(3/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2)/(x^2*(a - c*x^4)^(3/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{x^2\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx = \frac{-\sqrt{-c\,x^4+a}-\left(\int \frac{\sqrt{-c\,x^4+a}}{c^2d\,x^{10}+c^3x^8-2acd\,x^6-2a\,c^2x^4+a^2d\,x^2+a^2c}dx\right)a^2dx+\left(\int \frac{}{c^2d}\right.}{}$$

input

```
int((B*x^2+A)/x^2/(d*x^2+c)/(-c*x^4+a)^(3/2),x)
```

output

```
( - sqrt(a - c*x**4) - int(sqrt(a - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c*
*2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**2*d*x + int(sqrt(
a - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x*
*8 + c**2*d*x**10),x)*a*b*c*x + int(sqrt(a - c*x**4)/(a**2*c + a**2*d*x**2
 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a*c*d*x**5
- int(sqrt(a - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**
6 + c**3*x**8 + c**2*d*x**10),x)*b*c**2*x**5 + 3*int((sqrt(a - c*x**4)*x**
4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2
*d*x**10),x)*a*c*d*x - 3*int((sqrt(a - c*x**4)*x**4)/(a**2*c + a**2*d*x**2
 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*c**2*d*x**5
 + 3*int((sqrt(a - c*x**4)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2
*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a*c**2*x - 3*int((sqrt(a - c*x*
*4)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8
 + c**2*d*x**10),x)*c**3*x**5)/(c*x*(a - c*x**4))
```

3.20 $\int \frac{A+Bx^2}{x^4(c+dx^2)(a-cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 460

$$\begin{aligned}\int \frac{A+Bx^2}{x^4(c+dx^2)(a-cx^4)^{3/2}}\,dx &= \frac{Ac^2-aBd+c(Bc-Ad)x^2}{2a(c^3-ad^2)x^3\sqrt{a-cx^4}} \\ &- \frac{(5Ac^3-3aBcd-2aAd^2)\sqrt{a-cx^4}}{6a^2c(c^3-ad^2)x^3} - \frac{(Bc-Ad)(3c^3-2ad^2)\sqrt{a-cx^4}}{2a^2c^2(c^3-ad^2)x} \\ &- \frac{(Bc-Ad)(3c^3-2ad^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{5/4}c^{7/4}(c^3-ad^2)\sqrt{a-cx^4}} \\ &+ \frac{\left(5Ac^3+\sqrt{a}c^{3/2}(9Bc-4Ad)+6ad(Bc-Ad)\right)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{6a^{7/4}c^{7/4}\left(c^{3/2}+\sqrt{a}d\right)\sqrt{a-cx^4}} \\ &+ \frac{\sqrt[4]{a}d^3(Bc-Ad)\sqrt{1-\frac{cx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{13/4}(c^3-ad^2)\sqrt{a-cx^4}}\end{aligned}$$

output
```
1/2*(A*c^2-B*a*d+c*(-A*d+B*c)*x^2)/a/(-a*d^2+c^3)/x^3/(-c*x^4+a)^(1/2)-1/6
*(-2*A*a*d^2+5*A*c^3-3*B*a*c*d)*(-c*x^4+a)^(1/2)/a^2/c/(-a*d^2+c^3)/x^3-1/
2*(-A*d+B*c)*(-2*a*d^2+3*c^3)*(-c*x^4+a)^(1/2)/a^2/c^2/(-a*d^2+c^3)/x-1/2*
(-A*d+B*c)*(-2*a*d^2+3*c^3)*(1-c*x^4/a)^(1/2)*EllipticE(c^(1/4)*x/a^(1/4),
I)/a^(5/4)/c^(7/4)/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)+1/6*(5*A*c^3+a^(1/2)*c^(3
/2)*(-4*A*d+9*B*c)+6*a*d*(-A*d+B*c))*(1-c*x^4/a)^(1/2)*EllipticF(c^(1/4)*x
/a^(1/4),I)/a^(7/4)/c^(7/4)/(c^(3/2)+a^(1/2)*d)/(-c*x^4+a)^(1/2)+a^(1/4)*d
^3*(-A*d+B*c)*(1-c*x^4/a)^(1/2)*EllipticPi(c^(1/4)*x/a^(1/4),-a^(1/2)*d/c^
(3/2),I)/c^(13/4)/(-a*d^2+c^3)/(-c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.68 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx = \frac{2aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^5-2a^2A\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^2d^2+6aB\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^5x^2-6aA\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}c^4dx^2 -}{}$$

input
```
Integrate[(A + B*x^2)/(x^4*(c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(2*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^5 - 2*a^2*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2
*d^2 + 6*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^5*x^2 - 6*a*A*Sqrt[-(Sqrt[c]/Sqrt[
a])]*c^4*d*x^2 - 6*a^2*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^2*d^2*x^2 + 6*a^2*A*Sq
rt[-(Sqrt[c]/Sqrt[a])]*c*d^3*x^2 - 5*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^6*x^4 +
3*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^4*d*x^4 + 2*a*A*Sqrt[-(Sqrt[c]/Sqrt[a])]*
c^3*d^2*x^4 - 9*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^6*x^6 + 9*A*Sqrt[-(Sqrt[c]/Sq
rt[a])]*c^5*d*x^6 + 6*a*B*Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*d^2*x^6 - 6*a*A*Sqr
t[-(Sqrt[c]/Sqrt[a])]*c^2*d^3*x^6 + (3*I)*Sqrt[a]*c^(3/2)*(B*c - A*d)*(-3*
c^3 + 2*a*d^2)*x^3*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/
Sqrt[a])]*x], -1] - I*c^(3/2)*(-c^(3/2) + Sqrt[a]*d)*(5*A*c^3 + Sqrt[a]*c^
(3/2)*(9*B*c - 4*A*d) + 6*a*d*(B*c - A*d))*x^3*Sqrt[1 - (c*x^4)/a]*Ellipti
cF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (6*I)*a^2*B*c*d^3*x^3*Sqrt
[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c
]/Sqrt[a])]*x], -1] - (6*I)*a^2*A*d^4*x^3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-
((Sqrt[a]*d)/c^(3/2)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(6*a^2*
Sqrt[-(Sqrt[c]/Sqrt[a])]*c^3*(-c^3 + a*d^2)*x^3*Sqrt[a - c*x^4])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4\left(a-cx^4\right)^{3/2}\left(c+dx^2\right)}dx$$

$$\Big\downarrow \text{2249}$$

$$\int \left(\frac{Bc-Ad}{ac^2x^2\sqrt{a-cx^4}}+\frac{c\left(-aBd+cx^2(Bc-Ad)+Ac^2\right)}{a\left(c^3-ad^2\right)\left(a-cx^4\right)^{3/2}}+\frac{d^3(Bc-Ad)}{c^2\left(c^3-ad^2\right)\sqrt{a-cx^4}\left(c+dx^2\right)}+\frac{A}{acx^4\sqrt{a-cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{a^{5/4}c^{7/4}\sqrt{a-cx^4}}+\frac{c^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\sqrt{a}B+A\sqrt{c}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{7/4}\left(\sqrt{a}d+c^{3/2}\right)\sqrt{a-cx^4}}-\frac{\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{a^{5/4}c^{7/4}\sqrt{a-cx^4}}-\frac{c^{5/4}\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)E\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{5/4}\left(c^3-ad^2\right)\sqrt{a-cx^4}}+\frac{A\sqrt{1-\frac{cx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{3a^{7/4}\sqrt[4]{c}\sqrt{a-cx^4}}-\frac{\sqrt{a-cx^4}(Bc-Ad)}{a^2c^2x}+\frac{cx\left(-aBd+cx^2(Bc-Ad)+Ac^2\right)}{2a^2\left(c^3-ad^2\right)\sqrt{a-cx^4}}-\frac{A\sqrt{a-cx^4}}{3a^2cx^3}+\frac{\sqrt[4]{a}d^3\sqrt{1-\frac{cx^4}{a}}(Bc-Ad)\text{EllipticPi}\left(-\frac{\sqrt{a}d}{c^{3/2}},\arcsin\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)}{c^{13/4}\left(c^3-ad^2\right)\sqrt{a-cx^4}}$$

input
```
Int[(A + B*x^2)/(x^4*(c + d*x^2)*(a - c*x^4)^(3/2)),x]
```

output
```
(c*x*(A*c^2 - a*B*d + c*(B*c - A*d)*x^2))/(2*a^2*(c^3 - a*d^2)*Sqrt[a - c*
x^4]) - (A*Sqrt[a - c*x^4])/(3*a^2*c*x^3) - ((B*c - A*d)*Sqrt[a - c*x^4])/
(a^2*c^2*x) - ((B*c - A*d)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x
)/a^(1/4)], -1])/(a^(5/4)*c^(7/4)*Sqrt[a - c*x^4]) - (c^(5/4)*(B*c - A*d)*
Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*a^(5/4)
*(c^3 - a*d^2)*Sqrt[a - c*x^4]) + (A*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(3*a^(7/4)*c^(1/4)*Sqrt[a - c*x^4]) + ((Sqrt[a]
*B + A*Sqrt[c])*c^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a
^(1/4)], -1])/(2*a^(7/4)*(c^(3/2) + Sqrt[a]*d)*Sqrt[a - c*x^4]) + ((B*c -
A*d)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(a^(5
/4)*c^(7/4)*Sqrt[a - c*x^4]) + (a^(1/4)*d^3*(B*c - A*d)*Sqrt[1 - (c*x^4)/a
]*EllipticPi[-((Sqrt[a]*d)/c^(3/2)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^
(13/4)*(c^3 - a*d^2)*Sqrt[a - c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 7.48 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.44

method	result
risch	$-\frac{\sqrt{-c x^4+a}\,(-3Ad x^2+3Bc x^2+Ac)}{3c^2a^2x^3}+\frac{\frac{A c^2\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}+\frac{3B c^{\frac{3}{2}}\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x}}}{}$
default	$\frac{A\left(-\frac{\sqrt{-c x^4+a}}{3a^2x^3}+\frac{cx}{2a^2\sqrt{-\left(x^4-\frac{a}{c}\right)c}}+\frac{5c\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{6a^2\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c x^4+a}}\right)}{c}-\frac{(Ad-Bc)\left(\frac{c x^3}{2a^2\sqrt{-\left(x^4-\frac{a}{c}\right)c}}-\frac{\sqrt{-c x^4+a}}{a^2x}+\frac{3}{}\right.}{}$
elliptic	Expression too large to display

input
```
int((B*x^2+A)/x^4/(d*x^2+c)/(-c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-c*x^4+a)^(1/2)*(-3*A*d*x^2+3*B*c*x^2+A*c)/c^2/a^2/x^3+1/3/a^2/c^2*(
A*c^2/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2
/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)+3*
B*c^(3/2)*a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1
+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2)
)^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I))-3*A*c^(1/2)*d*a^(1/2)/(
c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2
))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-Elliptic
E(x*(c^(1/2)/a^(1/2))^(1/2),I))-3*a*c^3/(a*d^2-c^3)*(2*c*(-1/4*(A*d-B*c)/a
*x^3+1/4*(A*c^2-B*a*d)/a/c*x)/(-(x^4-a/c)*c)^(1/2)+1/2*(A*c^2-B*a*d)/a/(c^
(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1/2)*x^2/a^(1/2))
^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(c^(1/2)/a^(1/2))^(1/2),I)-1/2*(A*d-B*
c)*c^(1/2)/a^(1/2)/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(
1+c^(1/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*(EllipticF(x*(c^(1/2)/a^(1/2
))^(1/2),I)-EllipticE(x*(c^(1/2)/a^(1/2))^(1/2),I)))+3*a^2*d^3*(A*d-B*c)/(
a*d^2-c^3)/c/(c^(1/2)/a^(1/2))^(1/2)*(1-c^(1/2)*x^2/a^(1/2))^(1/2)*(1+c^(1
/2)*x^2/a^(1/2))^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(c^(1/2)/a^(1/2))^(1/
2),-a^(1/2)*d/c^(3/2),(-c^(1/2)/a^(1/2))^(1/2)/(c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)x^4}\,dx$$

input
```
integrate((B*x^2+A)/x^4/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)/(c^2*d*x^14 + c^3*x^12 - 2*a*c*d*x^1
0 - 2*a*c^2*x^8 + a^2*d*x^6 + a^2*c*x^4), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{A+Bx^2}{x^4\left(a-cx^4\right)^{\frac{3}{2}}\left(c+dx^2\right)}\,dx$$

input
```
integrate((B*x**2+A)/x**4/(d*x**2+c)/(-c*x**4+a)**(3/2),x)
```

output
```
Integral((A + B*x**2)/(x**4*(a - c*x**4)**(3/2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{(-cx^4+a)^{\frac{3}{2}}(dx^2+c)x^4}\,dx$$

input
```
integrate((B*x^2+A)/x^4/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((-c*x^4 + a)^(3/2)*(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{\left(-cx^4+a\right)^{\frac{3}{2}}(dx^2+c)x^4}\,dx$$

input
```
integrate((B*x^2+A)/x^4/(d*x^2+c)/(-c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((-c*x^4 + a)^(3/2)*(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx=\int \frac{B\,x^2+A}{x^4\,\left(a-c\,x^4\right)^{3/2}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2)/(x^4*(a - c*x^4)^(3/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2)/(x^4*(a - c*x^4)^(3/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2}{x^4\left(c+dx^2\right)\left(a-cx^4\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((B*x^2+A)/x^4/(d*x^2+c)/(-c*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a - c*x**4)*a - 3*sqrt(a - c*x**4)*b*x**2 - 3*int(sqrt(a - c*x**4
)/(a**2*c*x**2 + a**2*d*x**4 - 2*a*c**2*x**6 - 2*a*c*d*x**8 + c**3*x**10 +
 c**2*d*x**12),x)*a**3*d*x**3 + 3*int(sqrt(a - c*x**4)/(a**2*c*x**2 + a**2
*d*x**4 - 2*a*c**2*x**6 - 2*a*c*d*x**8 + c**3*x**10 + c**2*d*x**12),x)*a**
2*c*d*x**7 - 3*int(sqrt(a - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4
- 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**2*b*d*x**3 + 5*int(sqrt(a
 - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**
8 + c**2*d*x**10),x)*a**2*c**2*x**3 + 3*int(sqrt(a - c*x**4)/(a**2*c + a**
2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a*b
*c*d*x**7 - 5*int(sqrt(a - c*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 -
 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a*c**3*x**7 + 9*int((sqrt(a -
 c*x**4)*x**4)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3
*x**8 + c**2*d*x**10),x)*a*b*c*d*x**3 - 9*int((sqrt(a - c*x**4)*x**4)/(a**
2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**1
0),x)*b*c**2*d*x**7 + 5*int((sqrt(a - c*x**4)*x**2)/(a**2*c + a**2*d*x**2
- 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a**2*c*d*x**
3 + 9*int((sqrt(a - c*x**4)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 -
2*a*c*d*x**6 + c**3*x**8 + c**2*d*x**10),x)*a*b*c**2*x**3 - 5*int((sqrt(a
- c*x**4)*x**2)/(a**2*c + a**2*d*x**2 - 2*a*c**2*x**4 - 2*a*c*d*x**6 + c**
3*x**8 + c**2*d*x**10),x)*a*c**2*d*x**7 - 9*int((sqrt(a - c*x**4)*x**2)...
```

3.21 $\int \frac{x^5(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 254

$$\begin{aligned}\int \frac{x^5(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx &= \frac{(3c^3C-3Bc^2d+3Acd^2-2aCd^2)\sqrt{a+cx^4}}{6c^2d^3}-\frac{(cC-Bd)x^2\sqrt{a+cx^4}}{4cd^2}\\ &\quad+\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{(2c^4C-2Bc^3d+2Ac^2d^2-acCd^2+aBd^3)\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4c^{3/2}d^4}\\ &\quad-\frac{c^2(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{c^3+ad^2}\sqrt{a+cx^4}}\right)}{2d^4\sqrt{c^3+ad^2}}\end{aligned}$$

output
```
1/6*(3*A*c*d^2-3*B*c^2*d-2*C*a*d^2+3*C*c^3)*(c*x^4+a)^(1/2)/c^2/d^3-1/4*(-
B*d+C*c)*x^2*(c*x^4+a)^(1/2)/c/d^2+1/6*C*x^4*(c*x^4+a)^(1/2)/c/d-1/4*(2*A*
c^2*d^2+B*a*d^3-2*B*c^3*d-C*a*c*d^2+2*C*c^4)*arctanh(c^(1/2)*x^2/(c*x^4+a)
^(1/2))/c^(3/2)/d^4-1/2*c^2*(A*d^2-B*c*d+C*c^2)*arctanh((-c^2*x^2+a*d)/(a*
d^2+c^3)^(1/2)/(c*x^4+a)^(1/2))/d^4/(a*d^2+c^3)^(1/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.89

$$\int \frac{x^5(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx$$

$$= \frac{d\sqrt{a+cx^4}(6c^3C-4aCd^2-3c^2d(2B+Cx^2)+cd^2(6A+3Bx^2+2Cx^4)) - \frac{12c^4(c^2C-Bcd+Ad^2)\arctan\left(\frac{c^{3/2}+\cdots}{\cdots}\right)}{\sqrt{-c^3-ad^2}}\cdots}{12c^2d^4}$$

input
```
Integrate[(x^5*(A + B*x^2 + C*x^4))/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(d*Sqrt[a + c*x^4]*(6*c^3*C - 4*a*C*d^2 - 3*c^2*d*(2*B + C*x^2) + c*d^2*(6
*A + 3*B*x^2 + 2*C*x^4)) - (12*c^4*(c^2*C - B*c*d + A*d^2)*ArcTan[(c^(3/2)
 + Sqrt[c]*d*x^2 - d*Sqrt[a + c*x^4])/Sqrt[-c^3 - a*d^2]])/Sqrt[-c^3 - a*d
^2] + 3*Sqrt[c]*(2*c^4*C - 2*B*c^3*d + 2*A*c^2*d^2 - a*c*C*d^2 + a*B*d^3)*
Log[-(Sqrt[c]*x^2) + Sqrt[a + c*x^4]])/(12*c^2*d^4)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2237, 27, 2237, 27, 2237, 27, 2253, 2239, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(A+Bx^2+Cx^4\right)}{\sqrt{a+cx^4}\left(c+dx^2\right)}dx$$

$$\Big\downarrow \ 2237$$

$$\frac{\int -\frac{2\left(Cx^3\left(dx^2+c\right)\left(3cx^4+2a\right)-3cdx^5\left(Cx^4+Bx^2+A\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{6cd}+\frac{Cx^4\sqrt{a+cx^4}}{6cd}$$

$$\Big\downarrow \ 27$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\int \frac{Cx^3\left(dx^2+c\right)\left(3cx^4+2a\right)-3cdx^5\left(Cx^4+Bx^2+A\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{3cd}$$

$$\Big\downarrow \text{ 2237}$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{\int -\frac{2\left(3c(cC-Bd)x\left(dx^2+c\right)\left(2cx^4+a\right)-2cd\left(Cx^3\left(dx^2+c\right)\left(3cx^4+2a\right)-3cdx^5\left(Cx^4+Bx^2+A\right)\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{4cd}+\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}}{3cd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}-\frac{\int \frac{3c(cC-Bd)x\left(dx^2+c\right)\left(2cx^4+a\right)-2cd\left(Cx^3\left(dx^2+c\right)\left(3cx^4+2a\right)-3cdx^5\left(Cx^4+Bx^2+A\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{2cd}}{3cd}$$

$$\Big\downarrow \text{ 2237}$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}-\frac{\frac{\int -\frac{2\left(2c^2\left(3Cc^3-3Bdc^2+3Ad^2c-2aCd^2\right)x^3\left(dx^2+c\right)-d\left(3c^2(cC-Bd)x\left(dx^2+c\right)\left(2cx^4+a\right)-2c^2d\left(Cx^3\left(dx^2+c\right)\left(3cx^4+2a\right)-3cdx\right.\right.\right.}{\left(dx^2+c\right)\sqrt{cx^4+a}}}{2cd}}{2cd}}{3cd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}-\frac{\frac{\sqrt{a+cx^4}\left(-2aCd^2+3Acd^2-3Bc^2d+3c^3C\right)}{d}-\frac{\int \frac{2c^2\left(3Cc^3-3Bdc^2+3Ad^2c-2aCd^2\right)x^3\left(dx^2+c\right)-d\left(3c^2(cC-Bd)x\left(dx^2+c\right)\left(2cx^4+\right.\right.}{\left(dx^2+c\right)\sqrt{cx^4+a}}}{cd}}{2cd}}{3cd}$$

$$\Big\downarrow \text{ 2253}$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}-\frac{\frac{\sqrt{a+cx^4}\left(-2aCd^2+3Acd^2-3Bc^2d+3c^3C\right)}{d}-\frac{\int \frac{x\left(3c^2\left(2Cc^4-2Bdc^3+2Ad^2c^2-aCd^2c+aBd^3\right)x^2-3ac^3d(cC-Bd)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{cd}}{2cd}}{3cd}$$

$$\Big\downarrow \text{ 2239}$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}-\frac{\frac{\sqrt{a+cx^4}\left(-2aCd^2+3Acd^2-3Bc^2d+3c^3C\right)}{d}-\frac{\int -\frac{3c^2\left(acd(cC-Bd)-\left(2Cc^4-2Bdc^3+2Ad^2c^2-aCd^2c+aBd^3\right)x^2\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{2cd}}{2cd}}{3cd}$$

$$\Big\downarrow \; 27$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}-\frac{\frac{3c\int \frac{acd(cC-Bd)-\left(2Cc^4-2Bdc^3+2Ad^2c^2-aCd^2c+aBd^3\right)x^2}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{2d}+\frac{\sqrt{a+cx^4}\left(-2aCd^2+3Acd^2-3Bc^2d+3c^3C\right)}{d}}{2cd}}{3cd}$$

$$\Big\downarrow \; 719$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}-\frac{\frac{3c\left(\frac{2c^3\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{d}-\frac{\left(aBd^3-acCd^2+2Ac^2d^2-2Bc^3d+2c^4C\right)\int \frac{1}{\sqrt{cx^4+a}}dx^2}{d}\right)}{2d}+\frac{\sqrt{a+cx^4}\left(-2aC\cdots\right.}{\cdots}}{2cd}}{3cd}$$

$$\Big\downarrow \; 224$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}-\frac{\frac{3c\left(\frac{2c^3\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{d}-\frac{\left(aBd^3-acCd^2+2Ac^2d^2-2Bc^3d+2c^4C\right)\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}}{d}\right)}{2d}+\frac{\sqrt{a+cx^4}\left(-\cdots\right.}{\cdots}}{2cd}}{3cd}$$

$$\Big\downarrow \; 219$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}-\frac{\frac{3c\left(\frac{2c^3\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(aBd^3-acCd^2+2Ac^2d^2-2Bc^3d+2c^4C\right)}{\sqrt{c}d}\right)}{2d}+\frac{\sqrt{a+cx\cdots}}{\cdots}}{2cd}}{3cd}$$

$$\Big\downarrow \; 488$$

$$\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}-\frac{\frac{3c\left(-\frac{2c^3\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{-x^4+c^3+ad^2}d\frac{ad-c^2x^2}{\sqrt{cx^4+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(aBd^3-acCd^2+2Ac^2d^2-2Bc^3d+2c^4C\right)}{\sqrt{c}d}\right)}{2d}+\frac{\sqrt{\cdots}}{\cdots}}{2cd}}{3cd}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{Cx^4\sqrt{a+cx^4}}{6cd}-\frac{\frac{3x^2\sqrt{a+cx^4}(cC-Bd)}{4d}-\frac{3c\left(-\frac{2c^3\text{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{ad^2+c^3}\sqrt{a+cx^4}}\right)(Ad^2-Bcd+c^2C)}{d\sqrt{ad^2+c^3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)(aBd^3-acCd^2+2Ac^2d^2-2Bc^3d+2c^4C)}{\sqrt{c}d}\right)}{2d}}{2cd}}{3cd}$$

input
```
Int[(x^5*(A + B*x^2 + C*x^4))/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(C*x^4*Sqrt[a + c*x^4])/(6*c*d) - ((3*(c*C - B*d)*x^2*Sqrt[a + c*x^4])/(4*
d) - (((3*c^3*C - 3*B*c^2*d + 3*A*c*d^2 - 2*a*C*d^2)*Sqrt[a + c*x^4])/d +
(3*c*(-(((2*c^4*C - 2*B*c^3*d + 2*A*c^2*d^2 - a*c*C*d^2 + a*B*d^3)*ArcTanh
[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(Sqrt[c]*d)) - (2*c^3*(c^2*C - B*c*d + A*
d^2)*ArcTanh[(a*d - c^2*x^2)/(Sqrt[c^3 + a*d^2]*Sqrt[a + c*x^4])])/(d*Sqrt
[c^3 + a*d^2])))/(2*d))/(2*c*d))/(3*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2237
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> W
ith[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt[a + c*x^4]/(c*
e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q - 3)*Px - Coeff[Px, x,
 q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a
+ c*x^4]), x], x] /; GtQ[q, 4]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x]
```

rule 2239
```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x)^q*(a + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && PolyQ[Px, x^2]
```

rule 2253
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> With[{m = Expon[Px, x, Min]}, Int[x^m*ExpandToSum[Px/x^m, x]*(d + e*x^2
)^q*(a + c*x^4)^p, x] /; GtQ[m, 0] &&  !MatchQ[Px, x^m*(u_.)]] /; FreeQ[{a,
 c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.16

method	result
risch	$\frac{(2Ccd^2x^4+3Bcd^2x^2-3Cc^2dx^2+6Acd^2-6dc^2B-4Cad^2+6Cc^3)\sqrt{cx^4+a}}{12c^2d^3} - \frac{\frac{(2Ac^2d^2+Bad^3-2Bc^3d-Cacd^2+2Cc^4)\ln(\sqrt{c}x^2+\cdots}{2d\sqrt{c}}}{\cdots}$
default	$-\frac{c^2(Ad^2-Bcd+Cc^2)\ln\left(\frac{\frac{2ad^2+2c^3}{d^2}-\frac{2c^2(x^2+\frac{c}{d})}{d}+2\sqrt{\frac{ad^2+c^3}{d^2}}\sqrt{(x^2+\frac{c}{d})^2c-\frac{2c^2(x^2+\frac{c}{d})}{d}+\frac{ad^2+c^3}{d^2}}}{x^2+\frac{c}{d}}\right)}{2d^5\sqrt{\frac{ad^2+c^3}{d^2}}} - \frac{-d^2(Bd-Cc)\left(\frac{x^2\sqrt{cx^4\cdots}}{4c}\cdots\right.}{\cdots}$
elliptic	$\frac{d^2(Bd-Cc)\left(\frac{x^2\sqrt{cx^4+a}}{2c}-\frac{a\ln(\sqrt{c}x^2+\sqrt{cx^4+a})}{2c^{\frac{3}{2}}}\right)+\frac{d(Ad^2-Bcd+Cc^2)\sqrt{cx^4+a}}{c}+Cd^3\left(\frac{x^4\sqrt{cx^4+a}}{3c}-\frac{2a\sqrt{cx^4+a}}{3c^2}\right)+dc^{\frac{3}{2}}B\ln(\sqrt{c}x^2\cdots}{2d^4}$

input
```
int(x^5*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*C*c*d^2*x^4+3*B*c*d^2*x^2-3*C*c^2*d*x^2+6*A*c*d^2-6*B*c^2*d-4*C*a*
d^2+6*C*c^3)*(c*x^4+a)^(1/2)/c^2/d^3-1/2/d^3/c*(1/2*(2*A*c^2*d^2+B*a*d^3-2
*B*c^3*d-C*a*c*d^2+2*C*c^4)/d*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))/c^(1/2)+c^3*
(A*d^2-B*c*d+C*c^2)/d^2/((a*d^2+c^3)/d^2)^(1/2)*ln((2*(a*d^2+c^3)/d^2-2*c^
2/d*(x^2+c/d)+2*((a*d^2+c^3)/d^2)^(1/2)*((x^2+c/d)^2*c-2*c^2/d*(x^2+c/d)+(
a*d^2+c^3)/d^2)^(1/2))/(x^2+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^5*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^5(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx=\int \frac{x^5(A+Bx^2+Cx^4)}{\sqrt{a+cx^4}\,(c+dx^2)}\,dx$$

input
```
integrate(x**5*(C*x**4+B*x**2+A)/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral(x**5*(A + B*x**2 + C*x**4)/(sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^5(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^5}{\sqrt{cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate(x^5*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^5/(sqrt(c*x^4 + a)*(d*x^2 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^5*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx=\int \frac{x^5\,(C\,x^4+B\,x^2+A)}{\sqrt{c\,x^4+a}\,(d\,x^2+c)}\,dx$$

input
```
int((x^5*(A + B*x^2 + C*x^4))/((a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((x^5*(A + B*x^2 + C*x^4))/((a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^5(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx=\left(\int \frac{x^9}{\sqrt{c\,x^4+a}\,c+\sqrt{c\,x^4+a}\,d\,x^2}dx\right)c+\left(\int \frac{x^7}{\sqrt{c\,x^4+a}\,c+\sqrt{c\,x^4+a}\,d\,x^2}dx\right)b+\left(\int \frac{x^5}{\sqrt{c\,x^4+a}\,c+\sqrt{c\,x^4+a}\,d\,x^2}dx\right)a$$

input
```
int(x^5*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output
```
int(x**9/(sqrt(a + c*x**4)*c + sqrt(a + c*x**4)*d*x**2),x)*c + int(x**7/(s
qrt(a + c*x**4)*c + sqrt(a + c*x**4)*d*x**2),x)*b + int(x**5/(sqrt(a + c*x
**4)*c + sqrt(a + c*x**4)*d*x**2),x)*a
```

3.22 $\int \frac{x^3(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 191

$$\int \frac{x^3(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = -\frac{(cC-Bd)\sqrt{a+cx^4}}{2cd^2}+\frac{Cx^2\sqrt{a+cx^4}}{4cd} + \frac{(2c^3C-2Bc^2d+2Acd^2-aCd^2)\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4c^{3/2}d^3} + \frac{c(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{c^3+ad^2}\sqrt{a+cx^4}}\right)}{2d^3\sqrt{c^3+ad^2}}$$

output
```
-1/2*(-B*d+C*c)*(c*x^4+a)^(1/2)/c/d^2+1/4*C*x^2*(c*x^4+a)^(1/2)/c/d+1/4*(2
*A*c*d^2-2*B*c^2*d-C*a*d^2+2*C*c^3)*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/c
^(3/2)/d^3+1/2*c*(A*d^2-B*c*d+C*c^2)*arctanh((-c^2*x^2+a*d)/(a*d^2+c^3)^(1
/2)/(c*x^4+a)^(1/2))/d^3/(a*d^2+c^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

$$\int \frac{x^3(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx$$
$$= \frac{(-2cC+2Bd+Cdx^2)\sqrt{a+cx^4}}{4cd^2}$$
$$+ \frac{c(c^2C-Bcd+Ad^2)\arctan\left(\frac{c^{3/2}+\sqrt{c}dx^2-d\sqrt{a+cx^4}}{\sqrt{-c^3-ad^2}}\right)}{d^3\sqrt{-c^3-ad^2}}$$
$$+ \frac{(-2c^3C+2Bc^2d-2Acd^2+aCd^2)\log\left(-\sqrt{c}x^2+\sqrt{a+cx^4}\right)}{4c^{3/2}d^3}$$

input
```
Integrate[(x^3*(A + B*x^2 + C*x^4))/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
((-2*c*C + 2*B*d + C*d*x^2)*Sqrt[a + c*x^4])/(4*c*d^2) + (c*(c^2*C - B*c*d
 + A*d^2)*ArcTan[(c^(3/2) + Sqrt[c]*d*x^2 - d*Sqrt[a + c*x^4])/Sqrt[-c^3 -
 a*d^2]])/(d^3*Sqrt[-c^3 - a*d^2]) + ((-2*c^3*C + 2*B*c^2*d - 2*A*c*d^2 +
a*C*d^2)*Log[-(Sqrt[c]*x^2) + Sqrt[a + c*x^4]])/(4*c^(3/2)*d^3)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {2237, 27, 2237, 27, 2253, 2239, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx^2+Cx^4)}{\sqrt{a+cx^4}\,(c+dx^2)}\,dx$$
$$\Big\downarrow \ 2237$$
$$\frac{\int -\frac{2(Cx(dx^2+c)(2cx^4+a)-2cdx^3(Cx^4+Bx^2+A))}{(dx^2+c)\sqrt{cx^4+a}}dx}{4cd}+\frac{Cx^2\sqrt{a+cx^4}}{4cd}$$
$$\Big\downarrow \ 27$$

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\int \frac{Cx(dx^2+c)(2cx^4+a)-2cdx^3(Cx^4+Bx^2+A)}{(dx^2+c)\sqrt{cx^4+a}}dx}{2cd}$$

$\downarrow$ 2237

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\frac{\int -\frac{2\left(2c^2(cC-Bd)x^3\left(dx^2+c\right)-cd\left(Cx\left(dx^2+c\right)\left(2cx^4+a\right)-2cdx^3\left(Cx^4+Bx^2+A\right)\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{2cd}+\frac{\sqrt{a+cx^4}(cC-Bd)}{d}}{2cd}$$

$\downarrow$ 27

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\frac{\sqrt{a+cx^4}(cC-Bd)}{d}-\frac{\int \frac{2c^2(cC-Bd)x^3\left(dx^2+c\right)-cd\left(Cx\left(dx^2+c\right)\left(2cx^4+a\right)-2cdx^3\left(Cx^4+Bx^2+A\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{cd}}{2cd}$$

$\downarrow$ 2253

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\frac{\sqrt{a+cx^4}(cC-Bd)}{d}-\frac{\int \frac{x\left(c\left(2Cc^3-2Bdc^2+2Ad^2c-aCd^2\right)x^2-ac^2Cd\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{cd}}{2cd}$$

$\downarrow$ 2239

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\frac{\sqrt{a+cx^4}(cC-Bd)}{d}-\frac{\int -\frac{c\left(acCd-\left(2Cc^3-2Bdc^2+2Ad^2c-aCd^2\right)x^2\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{2cd}}{2cd}$$

$\downarrow$ 25

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\frac{\int \frac{c\left(acCd-\left(2Cc^3-2Bdc^2+2Ad^2c-aCd^2\right)x^2\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{2cd}+\frac{\sqrt{a+cx^4}(cC-Bd)}{d}}{2cd}$$

$\downarrow$ 27

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\frac{\int \frac{acCd-\left(2Cc^3-2Bdc^2+2Ad^2c-aCd^2\right)x^2}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{2d}+\frac{\sqrt{a+cx^4}(cC-Bd)}{d}}{2cd}$$

$\downarrow$ 719

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\frac{\frac{2c^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{d}-\frac{\left(-aCd^2+2Acd^2-2Bc^2d+2c^3C\right)\int \frac{1}{\sqrt{cx^4+a}}dx^2}{d}}{2d}+\frac{\sqrt{a+cx^4}(cC-Bd)}{d}}{2cd}$$

$$\Big\downarrow \ 224$$

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\frac{\frac{2c^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{d}-\frac{\left(-aCd^2+2Acd^2-2Bc^2d+2c^3C\right)\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}}{d}}{2d}+\frac{\sqrt{a+cx^4}(cC-Bd)}{d}}{2cd}$$

$$\Big\downarrow \ 219$$

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\frac{\frac{2c^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(-aCd^2+2Acd^2-2Bc^2d+2c^3C\right)}{\sqrt{c}d}}{2d}+\frac{\sqrt{a+cx^4}(cC-Bd)}{d}}{2cd}$$

$$\Big\downarrow \ 488$$

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\frac{-\frac{2c^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{-x^4+c^3+ad^2}d\frac{ad-c^2x^2}{\sqrt{cx^4+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(-aCd^2+2Acd^2-2Bc^2d+2c^3C\right)}{\sqrt{c}d}}{2d}+\frac{\sqrt{a+cx^4}(cC-Bd)}{d}}{2cd}$$

$$\Big\downarrow \ 219$$

$$\frac{Cx^2\sqrt{a+cx^4}}{4cd}-\frac{\frac{-\frac{2c^2\text{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{ad^2+c^3}\sqrt{a+cx^4}}\right)\left(Ad^2-Bcd+c^2C\right)}{d\sqrt{ad^2+c^3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(-aCd^2+2Acd^2-2Bc^2d+2c^3C\right)}{\sqrt{c}d}}{2d}+\frac{\sqrt{a+cx^4}(cC-Bd)}{d}}{2cd}$$

input

```
Int[(x^3*(A + B*x^2 + C*x^4))/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output

```
(C*x^2*Sqrt[a + c*x^4])/(4*c*d) - ((((c*C - B*d)*Sqrt[a + c*x^4])/d + (-(((
2*c^3*C - 2*B*c^2*d + 2*A*c*d^2 - a*C*d^2)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a +
c*x^4]])/(Sqrt[c]*d)) - (2*c^2*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - c^2*
x^2)/(Sqrt[c^3 + a*d^2]*Sqrt[a + c*x^4])])/(d*Sqrt[c^3 + a*d^2]))/(2*d))/(
2*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2237
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> W
ith[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt[a + c*x^4]/(c*
e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q - 3)*Px - Coeff[Px, x,
 q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a
+ c*x^4]), x], x] /; GtQ[q, 4]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x]
```

rule 2239
```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x)^q*(a + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && PolyQ[Px, x^2]
```

rule 2253
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> With[{m = Expon[Px, x, Min]}, Int[x^m*ExpandToSum[Px/x^m, x]*(d + e*x^2
)^q*(a + c*x^4)^p, x] /; GtQ[m, 0] &&  !MatchQ[Px, x^m*(u_.)]] /; FreeQ[{a,
 c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.27

method	result
risch	$\frac{(Cd\,x^2+2Bd-2Cc)\sqrt{c\,x^4+a}}{4c\,d^2}+\frac{\frac{\left(2Ac\,d^2-2d\,c^2B-Ca\,d^2+2C\,c^3\right)\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{2d\sqrt{c}}+\frac{c^2\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{2a\,d^2+2c^3}{d^2}-\frac{2c^2\left(x^2+\frac{c}{d}\right.}{d}\ldots\right.}{d^2\ldots}}{2c\,d^2}$ …
default	$\frac{\frac{d(Bd-Cc)\sqrt{c\,x^4+a}}{2c}+\frac{\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}+C\,d^2\left(\frac{x^2\sqrt{c\,x^4+a}}{4c}-\frac{a\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{4c^{\frac{3}{2}}}\right)}{d^3}+\frac{c\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\ldots\right.}{\ldots}$ …
elliptic	$\frac{\frac{A\,d^2\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{\sqrt{c}}+C\,c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)+\frac{d(Bd-Cc)\sqrt{c\,x^4+a}}{c}+C\,d^2\left(\frac{x^2\sqrt{c\,x^4+a}}{2c}-\frac{a\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{2c^{\frac{3}{2}}}\right)-B\sqrt{c}\,d\ln\left(\ldots\right.}{2d^3}$ …

input
```
int(x^3*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(C*d*x^2+2*B*d-2*C*c)*(c*x^4+a)^(1/2)/c/d^2+1/2/c/d^2*(1/2*(2*A*c*d^2-
2*B*c^2*d-C*a*d^2+2*C*c^3)/d*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))/c^(1/2)+c^2*(
A*d^2-B*c*d+C*c^2)/d^2/((a*d^2+c^3)/d^2)^(1/2)*ln((2*(a*d^2+c^3)/d^2-2*c^2
/d*(x^2+c/d)+2*((a*d^2+c^3)/d^2)^(1/2)*((x^2+c/d)^2*c-2*c^2/d*(x^2+c/d)+(a
*d^2+c^3)/d^2)^(1/2))/(x^2+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \int \frac{x^3(A+Bx^2+Cx^4)}{\sqrt{a+cx^4}\,(c+dx^2)}\,dx$$

input
```
integrate(x**3*(C*x**4+B*x**2+A)/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral(x**3*(A + B*x**2 + C*x**4)/(sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^3}{\sqrt{cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate(x^3*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^3/(sqrt(c*x^4 + a)*(d*x^2 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \int \frac{x^3\,(C\,x^4+B\,x^2+A)}{\sqrt{c\,x^4+a}\,(d\,x^2+c)}\,dx$$

input
```
int((x^3*(A + B*x^2 + C*x^4))/((a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((x^3*(A + B*x^2 + C*x^4))/((a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \left(\int \frac{x^7}{\sqrt{c\,x^4+a}\,c+\sqrt{c\,x^4+a}\,d\,x^2}dx\right)c + \left(\int \frac{x^5}{\sqrt{c\,x^4+a}\,c+\sqrt{c\,x^4+a}\,d\,x^2}dx\right)b + \left(\int \frac{x^3}{\sqrt{c\,x^4+a}\,c+\sqrt{c\,x^4+a}\,d\,x^2}dx\right)a$$

input
```
int(x^3*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output
```
int(x**7/(sqrt(a + c*x**4)*c + sqrt(a + c*x**4)*d*x**2),x)*c + int(x**5/(s
qrt(a + c*x**4)*c + sqrt(a + c*x**4)*d*x**2),x)*b + int(x**3/(sqrt(a + c*x
**4)*c + sqrt(a + c*x**4)*d*x**2),x)*a
```

3.23 $\int \frac{x(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 138

$$\int \frac{x(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \frac{C\sqrt{a+cx^4}}{2cd} - \frac{(cC-Bd)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}d^2} - \frac{(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{c^3+ad^2}\sqrt{a+cx^4}}\right)}{2d^2\sqrt{c^3+ad^2}}$$

output
```
1/2*C*(c*x^4+a)^(1/2)/c/d-1/2*(-B*d+C*c)*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/
2))/c^(1/2)/d^2-1/2*(A*d^2-B*c*d+C*c^2)*arctanh((-c^2*x^2+a*d)/(a*d^2+c^3)
^(1/2)/(c*x^4+a)^(1/2))/d^2/(a*d^2+c^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{x(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \frac{\frac{Cd\sqrt{a+cx^4}}{c} - \frac{2(c^2C-Bcd+Ad^2)\arctan\left(\frac{c^{3/2}+\sqrt{c}dx^2-d\sqrt{a+cx^4}}{\sqrt{-c^3-ad^2}}\right)}{\sqrt{-c^3-ad^2}} + \frac{(cC-Bd)\log\left(-\sqrt{c}x^2+\sqrt{a+cx^4}\right)}{\sqrt{c}}}{2d^2}$$

input
```
Integrate[(x*(A + B*x^2 + C*x^4))/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
((C*d*Sqrt[a + c*x^4])/c - (2*(c^2*C - B*c*d + A*d^2)*ArcTan[(c^(3/2) + Sq
rt[c]*d*x^2 - d*Sqrt[a + c*x^4])/Sqrt[-c^3 - a*d^2]])/Sqrt[-c^3 - a*d^2] +
 ((c*C - B*d)*Log[-(Sqrt[c]*x^2) + Sqrt[a + c*x^4]])/Sqrt[c])/(2*d^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2237, 27, 2253, 2239, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx^2+Cx^4\right)}{\sqrt{a+cx^4}\left(c+dx^2\right)}dx$$

$$\Big\downarrow \text{2237}$$

$$\frac{\int -\frac{2\left(cCx^3\left(dx^2+c\right)-cdx\left(Cx^4+Bx^2+A\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{2cd}+\frac{C\sqrt{a+cx^4}}{2cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{C\sqrt{a+cx^4}}{2cd}-\frac{\int \frac{cCx^3\left(dx^2+c\right)-cdx\left(Cx^4+Bx^2+A\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{cd}$$

$$\Big\downarrow \text{2253}$$

$$\frac{C\sqrt{a+cx^4}}{2cd}-\frac{\int \frac{x\left(c(cC-Bd)x^2-Acd\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{cd}$$

$$\Big\downarrow \text{2239}$$

$$\frac{C\sqrt{a+cx^4}}{2cd}-\frac{\int -\frac{c\left(Ad-(cC-Bd)x^2\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{2cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{c\left(Ad-(cC-Bd)x^2\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx^2}{2cd}+\frac{C\sqrt{a+cx^4}}{2cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Ad-(cC-Bd)x^2}{(dx^2+c)\sqrt{cx^4+a}}dx^2}{2d}+\frac{C\sqrt{a+cx^4}}{2cd}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{(Ad^2-Bcd+c^2C)\int \frac{1}{(dx^2+c)\sqrt{cx^4+a}}dx^2}{d}-\frac{(cC-Bd)\int \frac{1}{\sqrt{cx^4+a}}dx^2}{d}}{2d}+\frac{C\sqrt{a+cx^4}}{2cd}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(Ad^2-Bcd+c^2C)\int \frac{1}{(dx^2+c)\sqrt{cx^4+a}}dx^2}{d}-\frac{(cC-Bd)\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}}{d}}{2d}+\frac{C\sqrt{a+cx^4}}{2cd}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(Ad^2-Bcd+c^2C)\int \frac{1}{(dx^2+c)\sqrt{cx^4+a}}dx^2}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)(cC-Bd)}{\sqrt{c}d}}{2d}+\frac{C\sqrt{a+cx^4}}{2cd}$$

$$\Big\downarrow \text{488}$$

$$\frac{-\frac{(Ad^2-Bcd+c^2C)\int \frac{1}{-x^4+c^3+ad^2}d\frac{ad-c^2x^2}{\sqrt{cx^4+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)(cC-Bd)}{\sqrt{c}d}}{2d}+\frac{C\sqrt{a+cx^4}}{2cd}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{\text{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{ad^2+c^3}\sqrt{a+cx^4}}\right)(Ad^2-Bcd+c^2C)}{d\sqrt{ad^2+c^3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)(cC-Bd)}{\sqrt{c}d}}{2d}+\frac{C\sqrt{a+cx^4}}{2cd}$$

input
```
Int[(x*(A + B*x^2 + C*x^4))/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(C*Sqrt[a + c*x^4])/(2*c*d) + (-((((c*C - B*d)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a
 + c*x^4]])/(Sqrt[c]*d)) - ((c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - c^2*x^2
)/(Sqrt[c^3 + a*d^2]*Sqrt[a + c*x^4])])/(d*Sqrt[c^3 + a*d^2]))/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2237
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> W
ith[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt[a + c*x^4]/(c*
e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q - 3)*Px - Coeff[Px, x,
 q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a
+ c*x^4]), x], x] /; GtQ[q, 4]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x]
```

rule 2239
```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x)^q*(a + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && PolyQ[Px, x^2]
```

rule 2253
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> With[{m = Expon[Px, x, Min]}, Int[x^m*ExpandToSum[Px/x^m, x]*(d + e*x^2
)^q*(a + c*x^4)^p, x] /; GtQ[m, 0] &&  !MatchQ[Px, x^m*(u_.)]] /; FreeQ[{a,
 c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.43

method	result
default	$\frac{\frac{(Bd-Cc)\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}+\frac{Cd\sqrt{c\,x^4+a}}{2c}}{d^2}-\frac{\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2c^3}{d^2}-\frac{2c^2\left(x^2+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+c^3}{d^2}}\sqrt{\left(x^2+\frac{c}{d}\right)^2c-\frac{2c^2\left(x^2+\frac{c}{d}\right)}{d}\cdots}}{x^2+\frac{c}{d}}\cdots\right.}{2d^3\sqrt{\frac{a\,d^2+c^3}{d^2}}\cdots}$
risch	$\frac{C\sqrt{c\,x^4+a}}{2cd}+\frac{\frac{(Bd-Cc)\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{2d\sqrt{c}}-\frac{\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2c^3}{d^2}-\frac{2c^2\left(x^2+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+c^3}{d^2}}\sqrt{\left(x^2+\frac{c}{d}\right)^2c-\frac{2c^2\left(x^2+\frac{c}{d}\right)}{d}+\cdots}}{x^2+\frac{c}{d}}\cdots\right.}{2d^2\sqrt{\frac{a\,d^2+c^3}{d^2}}\cdots}}{d}$
elliptic	$\frac{\frac{Bd\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{\sqrt{c}}+\frac{Cd\sqrt{c\,x^4+a}}{c}-C\sqrt{c}\,\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{2d^2}-\frac{\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2c^3}{d^2}-\frac{2c^2\left(x^2+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+c^3}{d^2}}\cdots}{x^2+\cdots}\cdots\right.}{2d^3\sqrt{\frac{a\,d^2+c^3}{d^2}}\cdots}$

input
```
int(x*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(1/2*(B*d-C*c)*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))/c^(1/2)+1/2*C*d/c*(c*
x^4+a)^(1/2))-1/2*(A*d^2-B*c*d+C*c^2)/d^3/((a*d^2+c^3)/d^2)^(1/2)*ln((2*(a
*d^2+c^3)/d^2-2*c^2/d*(x^2+c/d)+2*((a*d^2+c^3)/d^2)^(1/2)*((x^2+c/d)^2*c-2
*c^2/d*(x^2+c/d)+(a*d^2+c^3)/d^2)^(1/2))/(x^2+c/d))
```

Fricas [A] (verification not implemented)

Time = 46.11 (sec) , antiderivative size = 863, normalized size of antiderivative = 6.25

$$\int \frac{x(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/4*((C*c^4 - B*c^3*d + C*a*c*d^2 - B*a*d^3)*sqrt(c)*log(-2*c*x^4 - 2*sq
rt(c*x^4 + a)*sqrt(c)*x^2 - a) - (C*c^3 - B*c^2*d + A*c*d^2)*sqrt(c^3 + a*
d^2)*log((2*a*c^2*d*x^2 - (2*c^4 + a*c*d^2)*x^4 - a*c^3 - 2*a^2*d^2 - 2*sq
rt(c*x^4 + a)*(c^2*x^2 - a*d)*sqrt(c^3 + a*d^2))/(d^2*x^4 + 2*c*d*x^2 + c^
2)) - 2*(C*c^3*d + C*a*d^3)*sqrt(c*x^4 + a))/(c^4*d^2 + a*c*d^4), 1/4*(2*(
C*c^4 - B*c^3*d + C*a*c*d^2 - B*a*d^3)*sqrt(-c)*arctan(sqrt(c*x^4 + a)*sqr
t(-c)/(c*x^2)) + (C*c^3 - B*c^2*d + A*c*d^2)*sqrt(c^3 + a*d^2)*log((2*a*c^
2*d*x^2 - (2*c^4 + a*c*d^2)*x^4 - a*c^3 - 2*a^2*d^2 - 2*sqrt(c*x^4 + a)*(c
^2*x^2 - a*d)*sqrt(c^3 + a*d^2))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 2*(C*c^3*d
 + C*a*d^3)*sqrt(c*x^4 + a))/(c^4*d^2 + a*c*d^4), -1/4*(2*(C*c^3 - B*c^2*d
 + A*c*d^2)*sqrt(-c^3 - a*d^2)*arctan(sqrt(c*x^4 + a)*(c^2*x^2 - a*d)*sqrt
(-c^3 - a*d^2)/((c^4 + a*c*d^2)*x^4 + a*c^3 + a^2*d^2)) + (C*c^4 - B*c^3*d
 + C*a*c*d^2 - B*a*d^3)*sqrt(c)*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x
^2 - a) - 2*(C*c^3*d + C*a*d^3)*sqrt(c*x^4 + a))/(c^4*d^2 + a*c*d^4), -1/2
*((C*c^3 - B*c^2*d + A*c*d^2)*sqrt(-c^3 - a*d^2)*arctan(sqrt(c*x^4 + a)*(c
^2*x^2 - a*d)*sqrt(-c^3 - a*d^2)/((c^4 + a*c*d^2)*x^4 + a*c^3 + a^2*d^2))
- (C*c^4 - B*c^3*d + C*a*c*d^2 - B*a*d^3)*sqrt(-c)*arctan(sqrt(c*x^4 + a)*
sqrt(-c)/(c*x^2)) - (C*c^3*d + C*a*d^3)*sqrt(c*x^4 + a))/(c^4*d^2 + a*c*d^
4)]
```

Sympy [F]

$$\int \frac{x(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \int \frac{x(A+Bx^2+Cx^4)}{\sqrt{a+cx^4}\,(c+dx^2)}\,dx$$

input
```
integrate(x*(C*x**4+B*x**2+A)/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral(x*(A + B*x**2 + C*x**4)/(sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx=\int \frac{(Cx^4+Bx^2+A)x}{\sqrt{cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate(x*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x/(sqrt(c*x^4 + a)*(d*x^2 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \int \frac{x\,(C\,x^4+B\,x^2+A)}{\sqrt{c\,x^4+a}\,(d\,x^2+c)}\,dx$$

input
```
int((x*(A + B*x^2 + C*x^4))/((a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((x*(A + B*x^2 + C*x^4))/((a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \left(\int \frac{x^5}{\sqrt{c\,x^4+a}\,c+\sqrt{c\,x^4+a}\,d\,x^2}dx\right)c + \left(\int \frac{x^3}{\sqrt{c\,x^4+a}\,c+\sqrt{c\,x^4+a}\,d\,x^2}dx\right)b + \left(\int \frac{x}{\sqrt{c\,x^4+a}\,c+\sqrt{c\,x^4+a}\,d\,x^2}dx\right)a$$

input
```
int(x*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output
```
int(x**5/(sqrt(a + c*x**4)*c + sqrt(a + c*x**4)*d*x**2),x)*c + int(x**3/(s
qrt(a + c*x**4)*c + sqrt(a + c*x**4)*d*x**2),x)*b + int(x/(sqrt(a + c*x**4
)*c + sqrt(a + c*x**4)*d*x**2),x)*a
```

3.24 $\int \frac{A+Bx^2+Cx^4}{x(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 143

$$\int \frac{A+Bx^2+Cx^4}{x(c+dx^2)\sqrt{a+cx^4}}\,dx = \frac{C\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}d} + \frac{(c^2C-Bcd+Ad^2)\,\mathrm{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{c^3+ad^2}\sqrt{a+cx^4}}\right)}{2cd\sqrt{c^3+ad^2}} - \frac{A\,\mathrm{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}c}$$

output
```
1/2*C*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/c^(1/2)/d+1/2*(A*d^2-B*c*d+C*c^
2)*arctanh((-c^2*x^2+a*d)/(a*d^2+c^3)^(1/2)/(c*x^4+a)^(1/2))/c/d/(a*d^2+c^
3)^(1/2)-1/2*A*arctanh((c*x^4+a)^(1/2)/a^(1/2))/a^(1/2)/c
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^2+Cx^4}{x(c+dx^2)\sqrt{a+cx^4}}\,dx$$

$$=\frac{\frac{2A\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\frac{2(c^2C-Bcd+Ad^2)\arctan\left(\frac{c^{3/2}+\sqrt{c}dx^2-d\sqrt{a+cx^4}}{\sqrt{-c^3-ad^2}}\right)}{\sqrt{-c^3-ad^2}}-\sqrt{c}C\log\left(-\sqrt{c}x^2+\sqrt{a+cx^4}\right)}{d}}{2c}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
((2*A*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + c*x^4])/Sqrt[a]])/Sqrt[a] + ((2*(c^2
*C - B*c*d + A*d^2)*ArcTan[(c^(3/2) + Sqrt[c]*d*x^2 - d*Sqrt[a + c*x^4])/S
qrt[-c^3 - a*d^2]])/Sqrt[-c^3 - a*d^2] - Sqrt[c]*C*Log[-(Sqrt[c]*x^2) + Sq
rt[a + c*x^4]])/d)/(2*c)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x\sqrt{a+cx^4}(c+dx^2)}\,dx$$

$$\Big\downarrow \text{2249}$$

$$\int \left(-\frac{x(Ad^2-Bcd+c^2C)}{cd\sqrt{a+cx^4}(c+dx^2)}+\frac{A}{cx\sqrt{a+cx^4}}+\frac{Cx}{d\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{ad^2+c^3}\sqrt{a+cx^4}}\right)(Ad^2-Bcd+c^2C)}{2cd\sqrt{ad^2+c^3}}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}c}+\frac{C\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}d}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(C*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]*d) + ((c^2*C - B*c*d
 + A*d^2)*ArcTanh[(a*d - c^2*x^2)/(Sqrt[c^3 + a*d^2]*Sqrt[a + c*x^4])])/(2
*c*d*Sqrt[c^3 + a*d^2]) - (A*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]])/(2*Sqrt[a]*
c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.45

method	result
default	$-\frac{A\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{2c\sqrt{a}}+\frac{C\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{2d\sqrt{c}}+\frac{\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2c^3}{d^2}-\frac{2c^2\left(x^2+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+c^3}{d^2}}\,\sqrt{\left(x^2+\frac{c}{d}\right)^2}}{x^2+\frac{c}{d}}\right)}{2c\,d^2\sqrt{\frac{a\,d^2+c^3}{d^2}}}$
elliptic	$-\frac{A\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{2c\sqrt{a}}+\frac{C\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{2d\sqrt{c}}+\frac{\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2c^3}{d^2}-\frac{2c^2\left(x^2+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+c^3}{d^2}}\,\sqrt{\left(x^2+\frac{c}{d}\right)^2}}{x^2+\frac{c}{d}}\right)}{2c\,d^2\sqrt{\frac{a\,d^2+c^3}{d^2}}}$

input
```
int((C*x^4+B*x^2+A)/x/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/c/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^4+a)^(1/2))/x^2)+1/2*C/d*ln(c^(1/2
)*x^2+(c*x^4+a)^(1/2))/c^(1/2)+1/2*(A*d^2-B*c*d+C*c^2)/c/d^2/((a*d^2+c^3)/
d^2)^(1/2)*ln((2*(a*d^2+c^3)/d^2-2*c^2/d*(x^2+c/d)+2*((a*d^2+c^3)/d^2)^(1/
2)*((x^2+c/d)^2*c-2*c^2/d*(x^2+c/d)+(a*d^2+c^3)/d^2)^(1/2))/(x^2+c/d))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/x/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x\sqrt{a+cx^4}\left(c+dx^2\right)}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x*sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}(dx^2+c)x}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx^2+Cx^4}{x\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((C*x^4+B*x^2+A)/x/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x\,\sqrt{c\,x^4+a}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x*(a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x*(a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{x\,(c + dx^2)\,\sqrt{a + cx^4}}\,dx = \left(\int \frac{x^3}{\sqrt{c\,x^4 + a}\,c + \sqrt{c\,x^4 + a}\,d\,x^2}dx\right)c + \left(\int \frac{x}{\sqrt{c\,x^4 + a}\,c + \sqrt{c\,x^4 + a}\,d\,x^2}dx\right)b + \left(\int \frac{1}{\sqrt{c\,x^4 + a}\,cx + \sqrt{c\,x^4 + a}\,d\,x^3}dx\right)a$$

input
```
int((C*x^4+B*x^2+A)/x/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output
```
int(x**3/(sqrt(a + c*x**4)*c + sqrt(a + c*x**4)*d*x**2),x)*c + int(x/(sqrt
(a + c*x**4)*c + sqrt(a + c*x**4)*d*x**2),x)*b + int(1/(sqrt(a + c*x**4)*c
*x + sqrt(a + c*x**4)*d*x**3),x)*a
```

3.25 $\int \frac{A+Bx^2+Cx^4}{x^3(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 138

$$\int \frac{A+Bx^2+Cx^4}{x^3(c+dx^2)\sqrt{a+cx^4}}\,dx = -\frac{A\sqrt{a+cx^4}}{2acx^2} - \frac{(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{c^3+ad^2}\sqrt{a+cx^4}}\right)}{2c^2\sqrt{c^3+ad^2}} - \frac{(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}c^2}$$

output

```
-1/2*A*(c*x^4+a)^(1/2)/a/c/x^2-1/2*(A*d^2-B*c*d+C*c^2)*arctanh((-c^2*x^2+a
*d)/(a*d^2+c^3)^(1/2)/(c*x^4+a)^(1/2))/c^2/(a*d^2+c^3)^(1/2)-1/2*(-A*d+B*c
)*arctanh((c*x^4+a)^(1/2)/a^(1/2))/a^(1/2)/c^2
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2+Cx^4}{x^3(c+dx^2)\sqrt{a+cx^4}}\,dx = -\frac{\frac{Ac\sqrt{a+cx^4}}{ax^2}+\frac{2(c^2C-Bcd+Ad^2)\arctan\left(\frac{c^{3/2}+\sqrt{c}dx^2-d\sqrt{a+cx^4}}{\sqrt{-c^3-ad^2}}\right)}{\sqrt{-c^3-ad^2}}-\frac{2(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}}{2c^2}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^3*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
-1/2*((A*c*Sqrt[a + c*x^4])/(a*x^2) + (2*(c^2*C - B*c*d + A*d^2)*ArcTan[(c
^(3/2) + Sqrt[c]*d*x^2 - d*Sqrt[a + c*x^4])/Sqrt[-c^3 - a*d^2]])/Sqrt[-c^3
 - a*d^2] - (2*(B*c - A*d)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + c*x^4])/Sqrt[a]
])/Sqrt[a])/c^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^3\sqrt{a+cx^4}(c+dx^2)}\,dx$$

$$\Big\downarrow \text{2249}$$

$$\int \left(\frac{x(Ad^2-Bcd+c^2C)}{c^2\sqrt{a+cx^4}(c+dx^2)}+\frac{Bc-Ad}{c^2x\sqrt{a+cx^4}}+\frac{A}{cx^3\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)(Bc-Ad)}{2\sqrt{a}c^2}-\frac{\text{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{ad^2+c^3}\sqrt{a+cx^4}}\right)(Ad^2-Bcd+c^2C)}{2c^2\sqrt{ad^2+c^3}}-\frac{A\sqrt{a+cx^4}}{2acx^2}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^3*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
-1/2*(A*Sqrt[a + c*x^4])/(a*c*x^2) - ((c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d
 - c^2*x^2)/(Sqrt[c^3 + a*d^2]*Sqrt[a + c*x^4])])/(2*c^2*Sqrt[c^3 + a*d^2]
) - ((B*c - A*d)*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]])/(2*Sqrt[a]*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.51

method	result
default	$-\frac{A\sqrt{c\,x^4+a}}{2ac\,x^2}+\frac{(Ad-Bc)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{2c^2\sqrt{a}}-\frac{(A\,d^2-Bcd+C\,c^2)\ln\left(\frac{\frac{2a\,d^2+2c^3}{d^2}-\frac{2c^2\left(x^2+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+c^3}{d^2}}\sqrt{\left(x^2+\frac{c}{d}\right)^2c-\frac{2c}{}}}{x^2+\frac{c}{d}}\right.}{2c^2d\sqrt{\frac{a\,d^2+c^3}{d^2}}}$
elliptic	$-\frac{A\sqrt{c\,x^4+a}}{2ac\,x^2}+\frac{(Ad-Bc)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{2c^2\sqrt{a}}-\frac{(A\,d^2-Bcd+C\,c^2)\ln\left(\frac{\frac{2a\,d^2+2c^3}{d^2}-\frac{2c^2\left(x^2+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+c^3}{d^2}}\sqrt{\left(x^2+\frac{c}{d}\right)^2c-\frac{2c}{}}}{x^2+\frac{c}{d}}\right.}{2c^2d\sqrt{\frac{a\,d^2+c^3}{d^2}}}$
risch	$-\frac{A\sqrt{c\,x^4+a}}{2ac\,x^2}-\frac{-\frac{(Ad-Bc)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{2c\sqrt{a}}+\frac{\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2c^3}{d^2}-\frac{2c^2\left(x^2+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+c^3}{d^2}}\sqrt{\left(x^2+\frac{c}{d}\right)^2c-\frac{2c^2(x^2}{d}}}{x^2+\frac{c}{d}}\right.}{2cd\sqrt{\frac{a\,d^2+c^3}{d^2}}}}{c}$

input
```
int((C*x^4+B*x^2+A)/x^3/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A*(c*x^4+a)^(1/2)/a/c/x^2+1/2*(A*d-B*c)/c^2/a^(1/2)*ln((2*a+2*a^(1/2)
*(c*x^4+a)^(1/2))/x^2)-1/2*(A*d^2-B*c*d+C*c^2)/c^2/d/((a*d^2+c^3)/d^2)^(1/
2)*ln((2*(a*d^2+c^3)/d^2-2*c^2/d*(x^2+c/d)+2*((a*d^2+c^3)/d^2)^(1/2)*((x^2
+c/d)^2*c-2*c^2/d*(x^2+c/d)+(a*d^2+c^3)/d^2)^(1/2))/(x^2+c/d))
```

Fricas [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 902, normalized size of antiderivative = 6.54

$$\int \frac{A+Bx^2+Cx^4}{x^3\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/x^3/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
[1/4*((C*a*c^2 - B*a*c*d + A*a*d^2)*sqrt(c^3 + a*d^2)*x^2*log((2*a*c^2*d*x
^2 - (2*c^4 + a*c*d^2)*x^4 - a*c^3 - 2*a^2*d^2 - 2*sqrt(c*x^4 + a)*(c^2*x^
2 - a*d)*sqrt(c^3 + a*d^2))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - (B*c^4 - A*c^3*
d + B*a*c*d^2 - A*a*d^3)*sqrt(a)*x^2*log(-(c*x^4 + 2*sqrt(c*x^4 + a)*sqrt(
a) + 2*a)/x^4) - 2*(A*c^4 + A*a*c*d^2)*sqrt(c*x^4 + a))/((a*c^5 + a^2*c^2*
d^2)*x^2), -1/4*(2*(C*a*c^2 - B*a*c*d + A*a*d^2)*sqrt(-c^3 - a*d^2)*x^2*ar
ctan(sqrt(c*x^4 + a)*(c^2*x^2 - a*d)*sqrt(-c^3 - a*d^2)/((c^4 + a*c*d^2)*x
^4 + a*c^3 + a^2*d^2)) + (B*c^4 - A*c^3*d + B*a*c*d^2 - A*a*d^3)*sqrt(a)*x
^2*log(-(c*x^4 + 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(A*c^4 + A*a*c*
d^2)*sqrt(c*x^4 + a))/((a*c^5 + a^2*c^2*d^2)*x^2), 1/4*(2*(B*c^4 - A*c^3*d
 + B*a*c*d^2 - A*a*d^3)*sqrt(-a)*x^2*arctan(sqrt(c*x^4 + a)*sqrt(-a)/a) +
(C*a*c^2 - B*a*c*d + A*a*d^2)*sqrt(c^3 + a*d^2)*x^2*log((2*a*c^2*d*x^2 - (
2*c^4 + a*c*d^2)*x^4 - a*c^3 - 2*a^2*d^2 - 2*sqrt(c*x^4 + a)*(c^2*x^2 - a*
d)*sqrt(c^3 + a*d^2))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 2*(A*c^4 + A*a*c*d^2)
*sqrt(c*x^4 + a))/((a*c^5 + a^2*c^2*d^2)*x^2), -1/2*((C*a*c^2 - B*a*c*d +
A*a*d^2)*sqrt(-c^3 - a*d^2)*x^2*arctan(sqrt(c*x^4 + a)*(c^2*x^2 - a*d)*sqr
t(-c^3 - a*d^2)/((c^4 + a*c*d^2)*x^4 + a*c^3 + a^2*d^2)) - (B*c^4 - A*c^3*
d + B*a*c*d^2 - A*a*d^3)*sqrt(-a)*x^2*arctan(sqrt(c*x^4 + a)*sqrt(-a)/a) +
 (A*c^4 + A*a*c*d^2)*sqrt(c*x^4 + a))/((a*c^5 + a^2*c^2*d^2)*x^2)]
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^3\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^3\sqrt{a+cx^4}\left(c+dx^2\right)}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**3/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**3*sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^3\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}(dx^2+c)x^3}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^3/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2+Cx^4}{x^3\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\frac{A}{\left(\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2-a\right)\sqrt{c}}+\frac{\left(Cc^2-Bcd+Ad^2\right)\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)d+c^{\frac{3}{2}}}{\sqrt{-c^3-ad^2}}\right)}{\sqrt{-c^3-ad^2}c^2}+\frac{(Bc-Ad)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}c^2}$$

input
```
integrate((C*x^4+B*x^2+A)/x^3/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
A/(((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)*sqrt(c)) + (C*c^2 - B*c*d + A*d
^2)*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + a))*d + c^(3/2))/sqrt(-c^3 - a*d^
2))/(sqrt(-c^3 - a*d^2)*c^2) + (B*c - A*d)*arctan(-(sqrt(c)*x^2 - sqrt(c*x
^4 + a))/sqrt(-a))/(sqrt(-a)*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^3(c+dx^2)\sqrt{a+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^3\,\sqrt{c\,x^4+a}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^3*(a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^3*(a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{x^3(c+dx^2)\sqrt{a+cx^4}}\,dx&=\left(\int \frac{x}{\sqrt{c\,x^4+a}\,c+\sqrt{c\,x^4+a}\,d\,x^2}dx\right)c\\&\quad+\left(\int \frac{1}{\sqrt{c\,x^4+a}\,c\,x^3+\sqrt{c\,x^4+a}\,d\,x^5}dx\right)a\\&\quad+\left(\int \frac{1}{\sqrt{c\,x^4+a}\,cx+\sqrt{c\,x^4+a}\,d\,x^3}dx\right)b\end{aligned}$$

input
```
int((C*x^4+B*x^2+A)/x^3/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output
```
int(x/(sqrt(a + c*x**4)*c + sqrt(a + c*x**4)*d*x**2),x)*c + int(1/(sqrt(a
+ c*x**4)*c*x**3 + sqrt(a + c*x**4)*d*x**5),x)*a + int(1/(sqrt(a + c*x**4)
*c*x + sqrt(a + c*x**4)*d*x**3),x)*b
```

3.26 $\int \frac{A+Bx^2+Cx^4}{x^5(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 189

$$\int \frac{A+Bx^2+Cx^4}{x^5(c+dx^2)\sqrt{a+cx^4}}\,dx = -\frac{A\sqrt{a+cx^4}}{4acx^4}-\frac{(Bc-Ad)\sqrt{a+cx^4}}{2ac^2x^2} + \frac{d(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{c^3+ad^2}\sqrt{a+cx^4}}\right)}{2c^3\sqrt{c^3+ad^2}} - \frac{(2ac(cC-Bd)-A(c^3-2ad^2))\operatorname{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{4a^{3/2}c^3}$$

output
```
-1/4*A*(c*x^4+a)^(1/2)/a/c/x^4-1/2*(-A*d+B*c)*(c*x^4+a)^(1/2)/a/c^2/x^2+1/
2*d*(A*d^2-B*c*d+C*c^2)*arctanh((-c^2*x^2+a*d)/(a*d^2+c^3)^(1/2)/(c*x^4+a)
^(1/2))/c^3/(a*d^2+c^3)^(1/2)-1/4*(2*a*c*(-B*d+C*c)-A*(-2*a*d^2+c^3))*arct
anh((c*x^4+a)^(1/2)/a^(1/2))/a^(3/2)/c^3
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2+Cx^4}{x^5\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx = -\frac{\frac{c\sqrt{a+cx^4}\left(2Bcx^2+A\left(c-2dx^2\right)\right)}{ax^4}-\frac{4d\left(c^2C-Bcd+Ad^2\right)\arctan\left(\frac{c^{3/2}+\sqrt{c}dx^2-d\sqrt{a+cx^4}}{\sqrt{-c^3-ad^2}}\right)}{\sqrt{-c^3-ad^2}}+\frac{2\left(2ac(-cC+Bd)+A\left(c^3-2ad^2\right)\right)\text{arctanh}}{a^{3/2}}}{4c^3}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^5*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
-1/4*((c*Sqrt[a + c*x^4]*(2*B*c*x^2 + A*(c - 2*d*x^2)))/(a*x^4) - (4*d*(c^
2*C - B*c*d + A*d^2)*ArcTan[(c^(3/2) + Sqrt[c]*d*x^2 - d*Sqrt[a + c*x^4])/
Sqrt[-c^3 - a*d^2]])/Sqrt[-c^3 - a*d^2] + (2*(2*a*c*(-(c*C) + B*d) + A*(c^
3 - 2*a*d^2))*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + c*x^4])/Sqrt[a]])/a^(3/2))/c
^3
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^5\sqrt{a+cx^4}\left(c+dx^2\right)}\,dx$$

$$\Big\downarrow \; 2249$$

$$\int \left(\frac{Bc-Ad}{c^2x^3\sqrt{a+cx^4}}+\frac{Ad^2-Bcd+c^2C}{c^3x\sqrt{a+cx^4}}-\frac{dx\left(Ad^2-Bcd+c^2C\right)}{c^3\sqrt{a+cx^4}\left(c+dx^2\right)}+\frac{A}{cx^5\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{A\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{2\sqrt{a}c^3}+\frac{d\text{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{ad^2+c^3}\sqrt{a+cx^4}}\right)(Ad^2-Bcd+c^2C)}{2c^3\sqrt{ad^2+c^3}}-\frac{\sqrt{a+cx^4}(Bc-Ad)}{2ac^2x^2}-\frac{A\sqrt{a+cx^4}}{4acx^4}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^5*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
-1/4*(A*Sqrt[a + c*x^4])/(a*c*x^4) - ((B*c - A*d)*Sqrt[a + c*x^4])/(2*a*c^
2*x^2) + (d*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - c^2*x^2)/(Sqrt[c^3 + a*
d^2]*Sqrt[a + c*x^4])])/(2*c^3*Sqrt[c^3 + a*d^2]) + (A*ArcTanh[Sqrt[a + c*
x^4]/Sqrt[a]])/(4*a^(3/2)) - ((c^2*C - B*c*d + A*d^2)*ArcTanh[Sqrt[a + c*x
^4]/Sqrt[a]])/(2*Sqrt[a]*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{\sqrt{c\,x^4+a}\,(-2Ad\,x^2+2Bc\,x^2+Ac)}{4c^2a\,x^4}-\frac{\frac{(2Aa\,d^2-A\,c^3-2aBcd+2Ca\,c^2)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{2c\sqrt{a}}-\frac{a\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2c^3}{d^2}-\cdots}{\cdots}\right)}{\cdots}}{2c^2a}$
default	$\frac{A\left(-\frac{\sqrt{c\,x^4+a}}{4a\,x^4}+\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{4a^{\frac{3}{2}}}\right)}{c}+\frac{(Ad-Bc)\sqrt{c\,x^4+a}}{2c^2x^2a}-\frac{(A\,d^2-Bcd+C\,c^2)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{2c^3\sqrt{a}}+\frac{(A\,d^2-Bcd+C\cdots}{\cdots}$
elliptic	$\frac{A\left(-\frac{\sqrt{c\,x^4+a}}{2a\,x^4}+\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{2a^{\frac{3}{2}}}\right)}{2c}+\frac{(Ad-Bc)\sqrt{c\,x^4+a}}{2c^2x^2a}-\frac{(A\,d^2-Bcd+C\,c^2)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^4+a}}{x^2}\right)}{2c^3\sqrt{a}}+\frac{(A\,d^2-Bcd+C\cdots}{\cdots}$

input
```
int((C*x^4+B*x^2+A)/x^5/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^4+a)^(1/2)*(-2*A*d*x^2+2*B*c*x^2+A*c)/c^2/a/x^4-1/2/c^2/a*(1/2*(
2*A*a*d^2-A*c^3-2*B*a*c*d+2*C*a*c^2)/c/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^4+a)
^(1/2))/x^2)-a*(A*d^2-B*c*d+C*c^2)/c/((a*d^2+c^3)/d^2)^(1/2)*ln((2*(a*d^2+
c^3)/d^2-2*c^2/d*(x^2+c/d)+2*((a*d^2+c^3)/d^2)^(1/2)*((x^2+c/d)^2*c-2*c^2/
d*(x^2+c/d)+(a*d^2+c^3)/d^2)^(1/2))/(x^2+c/d)))
```

Fricas [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 1266, normalized size of antiderivative = 6.70

$$\int \frac{A+Bx^2+Cx^4}{x^5\,(c+dx^2)\,\sqrt{a+cx^4}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/x^5/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
[1/8*(2*(C*a^2*c^2*d - B*a^2*c*d^2 + A*a^2*d^3)*sqrt(c^3 + a*d^2)*x^4*log(
(2*a*c^2*d*x^2 - (2*c^4 + a*c*d^2)*x^4 - a*c^3 - 2*a^2*d^2 + 2*sqrt(c*x^4
+ a)*(c^2*x^2 - a*d)*sqrt(c^3 + a*d^2))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + (2*
C*a*c^5 - A*c^6 - 2*B*a*c^4*d - 2*B*a^2*c*d^3 + 2*A*a^2*d^4 + (2*C*a^2*c^2
 + A*a*c^3)*d^2)*sqrt(a)*x^4*log(-(c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a
)/x^4) - 2*(A*a*c^5 + A*a^2*c^2*d^2 + 2*(B*a*c^5 - A*a*c^4*d + B*a^2*c^2*d
^2 - A*a^2*c*d^3)*x^2)*sqrt(c*x^4 + a))/((a^2*c^6 + a^3*c^3*d^2)*x^4), 1/8
*(4*(C*a^2*c^2*d - B*a^2*c*d^2 + A*a^2*d^3)*sqrt(-c^3 - a*d^2)*x^4*arctan(
sqrt(c*x^4 + a)*(c^2*x^2 - a*d)*sqrt(-c^3 - a*d^2)/((c^4 + a*c*d^2)*x^4 +
a*c^3 + a^2*d^2)) + (2*C*a*c^5 - A*c^6 - 2*B*a*c^4*d - 2*B*a^2*c*d^3 + 2*A
*a^2*d^4 + (2*C*a^2*c^2 + A*a*c^3)*d^2)*sqrt(a)*x^4*log(-(c*x^4 - 2*sqrt(c
*x^4 + a)*sqrt(a) + 2*a)/x^4) - 2*(A*a*c^5 + A*a^2*c^2*d^2 + 2*(B*a*c^5 -
A*a*c^4*d + B*a^2*c^2*d^2 - A*a^2*c*d^3)*x^2)*sqrt(c*x^4 + a))/((a^2*c^6 +
 a^3*c^3*d^2)*x^4), 1/4*((2*C*a*c^5 - A*c^6 - 2*B*a*c^4*d - 2*B*a^2*c*d^3
+ 2*A*a^2*d^4 + (2*C*a^2*c^2 + A*a*c^3)*d^2)*sqrt(-a)*x^4*arctan(sqrt(c*x^
4 + a)*sqrt(-a)/a) + (C*a^2*c^2*d - B*a^2*c*d^2 + A*a^2*d^3)*sqrt(c^3 + a*
d^2)*x^4*log((2*a*c^2*d*x^2 - (2*c^4 + a*c*d^2)*x^4 - a*c^3 - 2*a^2*d^2 +
2*sqrt(c*x^4 + a)*(c^2*x^2 - a*d)*sqrt(c^3 + a*d^2))/(d^2*x^4 + 2*c*d*x^2
+ c^2)) - (A*a*c^5 + A*a^2*c^2*d^2 + 2*(B*a*c^5 - A*a*c^4*d + B*a^2*c^2*d^
2 - A*a^2*c*d^3)*x^2)*sqrt(c*x^4 + a))/((a^2*c^6 + a^3*c^3*d^2)*x^4), 1...
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^5\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^5\sqrt{a+cx^4}\left(c+dx^2\right)}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**5/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**5*sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^5(c+dx^2)\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}(dx^2+c)x^5}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^5/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)*x^5), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx^2+Cx^4}{x^5(c+dx^2)\sqrt{a+cx^4}}\,dx=-\frac{(Cc^2d-Bcd^2+Ad^3)\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)d+c^{\frac{3}{2}}}{\sqrt{-c^3-ad^2}}\right)}{\sqrt{-c^3-ad^2}c^3}+\frac{(2\,Cac^2-Ac^3-2\,Bacd+2\,Aad^2)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}ac^3}+\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^3Ac^2+2\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2Bac^{\frac{3}{2}}-2\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2Aa\sqrt{c}d+\left(\sqrt{c}x^2-\right.}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2-a\right)^2ac^2}$$

input
```
integrate((C*x^4+B*x^2+A)/x^5/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
-(C*c^2*d - B*c*d^2 + A*d^3)*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + a))*d +
c^(3/2))/sqrt(-c^3 - a*d^2))/(sqrt(-c^3 - a*d^2)*c^3) + 1/2*(2*C*a*c^2 - A
*c^3 - 2*B*a*c*d + 2*A*a*d^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + a))/sqrt
(-a))/(sqrt(-a)*a*c^3) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + a))^3*A*c^2 + 2*
(sqrt(c)*x^2 - sqrt(c*x^4 + a))^2*B*a*c^(3/2) - 2*(sqrt(c)*x^2 - sqrt(c*x^
4 + a))^2*A*a*sqrt(c)*d + (sqrt(c)*x^2 - sqrt(c*x^4 + a))*A*a*c^2 - 2*B*a^
2*c^(3/2) + 2*A*a^2*sqrt(c)*d)/(((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)^2*
a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^5\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^5\,\sqrt{c\,x^4+a}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^5*(a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^5*(a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{x^5\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=&\left(\int \frac{1}{\sqrt{c\,x^4+a}\,c\,x^5+\sqrt{c\,x^4+a}\,d\,x^7}dx\right)a\\&+\left(\int \frac{1}{\sqrt{c\,x^4+a}\,c\,x^3+\sqrt{c\,x^4+a}\,d\,x^5}dx\right)b\\&+\left(\int \frac{1}{\sqrt{c\,x^4+a}\,cx+\sqrt{c\,x^4+a}\,d\,x^3}dx\right)c\end{aligned}$$

input
```
int((C*x^4+B*x^2+A)/x^5/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output
```
int(1/(sqrt(a + c*x**4)*c*x**5 + sqrt(a + c*x**4)*d*x**7),x)*a + int(1/(sq
rt(a + c*x**4)*c*x**3 + sqrt(a + c*x**4)*d*x**5),x)*b + int(1/(sqrt(a + c*
x**4)*c*x + sqrt(a + c*x**4)*d*x**3),x)*c
```

3.27 $\int \frac{A+Bx^2+Cx^4}{x^7(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 253

$$\begin{aligned}
&\int \frac{A+Bx^2+Cx^4}{x^7(c+dx^2)\sqrt{a+cx^4}}\,dx \\
&= -\frac{A\sqrt{a+cx^4}}{6acx^6} - \frac{(Bc-Ad)\sqrt{a+cx^4}}{4ac^2x^4} - \frac{(3ac(cC-Bd)-A(2c^3-3ad^2))\sqrt{a+cx^4}}{6a^2c^3x^2} \\
&\quad - \frac{d^2(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{c^3+ad^2}\sqrt{a+cx^4}}\right)}{2c^4\sqrt{c^3+ad^2}} \\
&\quad + \frac{(Bc^4-Ac^3d+2ac^2Cd-2aBcd^2+2aAd^3)\,\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{4a^{3/2}c^4}
\end{aligned}$$

output
```
-1/6*A*(c*x^4+a)^(1/2)/a/c/x^6-1/4*(-A*d+B*c)*(c*x^4+a)^(1/2)/a/c^2/x^4-1/
6*(3*a*c*(-B*d+C*c)-A*(-3*a*d^2+2*c^3))*(c*x^4+a)^(1/2)/a^2/c^3/x^2-1/2*d^
2*(A*d^2-B*c*d+C*c^2)*arctanh((-c^2*x^2+a*d)/(a*d^2+c^3)^(1/2)/(c*x^4+a)^(
1/2))/c^4/(a*d^2+c^3)^(1/2)+1/4*(2*A*a*d^3-A*c^3*d-2*B*a*c*d^2+B*c^4+2*C*a
*c^2*d)*arctanh((c*x^4+a)^(1/2)/a^(1/2))/a^(3/2)/c^4
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4}{x^7\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx$$
$$= \frac{\frac{c\sqrt{a+cx^4}\left(4Ac^3x^4-a\left(3cx^2\left(Bc+2cCx^2-2Bdx^2\right)+A\left(2c^2-3cdx^2+6d^2x^4\right)\right)\right)}{a^2x^6}-\frac{12d^2\left(c^2C-Bcd+Ad^2\right)\arctan\left(\frac{c^{3/2}+\sqrt{c}dx^2-d\sqrt{a+cx^4}}{\sqrt{-c^3-ad^2}}\right)}{\sqrt{-c^3-ad^2}}+\frac{6}{}}{12c^4}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^7*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
((c*Sqrt[a + c*x^4]*(4*A*c^3*x^4 - a*(3*c*x^2*(B*c + 2*c*C*x^2 - 2*B*d*x^2
) + A*(2*c^2 - 3*c*d*x^2 + 6*d^2*x^4))))/(a^2*x^6) - (12*d^2*(c^2*C - B*c*
d + A*d^2)*ArcTan[(c^(3/2) + Sqrt[c]*d*x^2 - d*Sqrt[a + c*x^4])/Sqrt[-c^3
- a*d^2]])/Sqrt[-c^3 - a*d^2] + (6*(-(B*c^4) + A*c^3*d - 2*a*c^2*C*d + 2*a
*B*c*d^2 - 2*a*A*d^3)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + c*x^4])/Sqrt[a]])/a^
(3/2))/(12*c^4)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^7\sqrt{a+cx^4}\left(c+dx^2\right)}\,dx$$

$$\Big\downarrow \text{2249}$$

$$\int \left(\frac{Bc-Ad}{c^2x^5\sqrt{a+cx^4}}-\frac{d\left(Ad^2-Bcd+c^2C\right)}{c^4x\sqrt{a+cx^4}}+\frac{d^2x\left(Ad^2-Bcd+c^2C\right)}{c^4\sqrt{a+cx^4}\left(c+dx^2\right)}+\frac{Ad^2-Bcd+c^2C}{c^3x^3\sqrt{a+cx^4}}+\frac{A}{cx^7\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)(Bc-Ad)}{4a^{3/2}c}+\frac{A\sqrt{a+cx^4}}{3a^2x^2}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{2\sqrt{a}c^4}-\frac{d^2\operatorname{arctanh}\left(\frac{ad-c^2x^2}{\sqrt{ad^2+c^3}\sqrt{a+cx^4}}\right)(Ad^2-Bcd+c^2C)}{2c^4\sqrt{ad^2+c^3}}-\frac{\sqrt{a+cx^4}(Bc-Ad)}{4ac^2x^4}-\frac{\sqrt{a+cx^4}(Ad^2-Bcd+c^2C)}{2ac^3x^2}-\frac{A\sqrt{a+cx^4}}{6acx^6}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^7*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
-1/6*(A*Sqrt[a + c*x^4])/(a*c*x^6) - ((B*c - A*d)*Sqrt[a + c*x^4])/(4*a*c^
2*x^4) + (A*Sqrt[a + c*x^4])/(3*a^2*x^2) - ((c^2*C - B*c*d + A*d^2)*Sqrt[a
 + c*x^4])/(2*a*c^3*x^2) - (d^2*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - c^2
*x^2)/(Sqrt[c^3 + a*d^2]*Sqrt[a + c*x^4])])/(2*c^4*Sqrt[c^3 + a*d^2]) + ((
B*c - A*d)*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]])/(4*a^(3/2)*c) + (d*(c^2*C - B
*c*d + A*d^2)*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]])/(2*Sqrt[a]*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{\sqrt{c\,x^{4}+a}\,\left(6Aa\,d^{2}x^{4}-4A\,c^{3}x^{4}-6Bacd\,x^{4}+6Ca\,c^{2}x^{4}-3Aacd\,x^{2}+3Ba\,c^{2}x^{2}+2Aa\,c^{2}\right)}{12c^{3}a^{2}x^{6}}+\frac{\frac{\left(2Aa\,d^{3}-A\,c^{3}d-2Bac\,d^{2}+B\,c^{4}+2Ca\,c^{2}d\right)}{2c\sqrt{a}}}{}$
default	$-\frac{A\sqrt{c\,x^{4}+a}\,\left(-2c\,x^{4}+a\right)}{6c\,x^{6}a^{2}}-\frac{\left(Ad-Bc\right)\left(-\frac{\sqrt{c\,x^{4}+a}}{4a\,x^{4}}+\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}\right)}{c^{2}}-\frac{\left(A\,d^{2}-Bcd+C\,c^{2}\right)\sqrt{c\,x^{4}+a}}{2c^{3}x^{2}a}-\frac{d\left(A\,d^{2}-Bcd\right.}{}$
elliptic	$\frac{A\left(-\frac{\sqrt{c\,x^{4}+a}}{3a\,x^{6}}+\frac{2c\sqrt{c\,x^{4}+a}}{3a^{2}x^{2}}\right)}{2c}-\frac{\left(Ad-Bc\right)\left(-\frac{\sqrt{c\,x^{4}+a}}{2a\,x^{4}}+\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}\right)}{2c^{2}}-\frac{\left(A\,d^{2}-Bcd+C\,c^{2}\right)\sqrt{c\,x^{4}+a}}{2c^{3}x^{2}a}-\frac{d\left(A\,d^{2}-B\right.}{}$

input
```
int((C*x^4+B*x^2+A)/x^7/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(c*x^4+a)^(1/2)*(6*A*a*d^2*x^4-4*A*c^3*x^4-6*B*a*c*d*x^4+6*C*a*c^2*x
^4-3*A*a*c*d*x^2+3*B*a*c^2*x^2+2*A*a*c^2)/c^3/a^2/x^6+1/2/c^3/a*(1/2*(2*A*
a*d^3-A*c^3*d-2*B*a*c*d^2+B*c^4+2*C*a*c^2*d)/c/a^(1/2)*ln((2*a+2*a^(1/2)*(
c*x^4+a)^(1/2))/x^2)-d*a*(A*d^2-B*c*d+C*c^2)/c/((a*d^2+c^3)/d^2)^(1/2)*ln(
(2*(a*d^2+c^3)/d^2-2*c^2/d*(x^2+c/d)+2*((a*d^2+c^3)/d^2)^(1/2)*((x^2+c/d)^
2*c-2*c^2/d*(x^2+c/d)+(a*d^2+c^3)/d^2)^(1/2))/(x^2+c/d)))
```

Fricas [A] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 1609, normalized size of antiderivative = 6.36

$$\int \frac{A+Bx^2+Cx^4}{x^7\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/x^7/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
[1/24*(6*(C*a^2*c^2*d^2 - B*a^2*c*d^3 + A*a^2*d^4)*sqrt(c^3 + a*d^2)*x^6*l
og((2*a*c^2*d*x^2 - (2*c^4 + a*c*d^2)*x^4 - a*c^3 - 2*a^2*d^2 - 2*sqrt(c*x
^4 + a)*(c^2*x^2 - a*d)*sqrt(c^3 + a*d^2))/(d^2*x^4 + 2*c*d*x^2 + c^2)) +
3*(B*c^7 - B*a*c^4*d^2 - 2*B*a^2*c*d^4 + 2*A*a^2*d^5 + (2*C*a^2*c^2 + A*a*
c^3)*d^3 + (2*C*a*c^5 - A*c^6)*d)*sqrt(a)*x^6*log(-(c*x^4 + 2*sqrt(c*x^4 +
 a)*sqrt(a) + 2*a)/x^4) - 2*(2*A*a*c^6 + 2*A*a^2*c^3*d^2 + 2*(3*C*a*c^6 -
2*A*c^7 - 3*B*a*c^5*d - 3*B*a^2*c^2*d^3 + 3*A*a^2*c*d^4 + (3*C*a^2*c^3 + A
*a*c^4)*d^2)*x^4 + 3*(B*a*c^6 - A*a*c^5*d + B*a^2*c^3*d^2 - A*a^2*c^2*d^3)
*x^2)*sqrt(c*x^4 + a))/((a^2*c^7 + a^3*c^4*d^2)*x^6), -1/24*(12*(C*a^2*c^2
*d^2 - B*a^2*c*d^3 + A*a^2*d^4)*sqrt(-c^3 - a*d^2)*x^6*arctan(sqrt(c*x^4 +
 a)*(c^2*x^2 - a*d)*sqrt(-c^3 - a*d^2)/((c^4 + a*c*d^2)*x^4 + a*c^3 + a^2*
d^2)) - 3*(B*c^7 - B*a*c^4*d^2 - 2*B*a^2*c*d^4 + 2*A*a^2*d^5 + (2*C*a^2*c^
2 + A*a*c^3)*d^3 + (2*C*a*c^5 - A*c^6)*d)*sqrt(a)*x^6*log(-(c*x^4 + 2*sqrt
(c*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(2*A*a*c^6 + 2*A*a^2*c^3*d^2 + 2*(3*C*
a*c^6 - 2*A*c^7 - 3*B*a*c^5*d - 3*B*a^2*c^2*d^3 + 3*A*a^2*c*d^4 + (3*C*a^2
*c^3 + A*a*c^4)*d^2)*x^4 + 3*(B*a*c^6 - A*a*c^5*d + B*a^2*c^3*d^2 - A*a^2*
c^2*d^3)*x^2)*sqrt(c*x^4 + a))/((a^2*c^7 + a^3*c^4*d^2)*x^6), -1/12*(3*(B*
c^7 - B*a*c^4*d^2 - 2*B*a^2*c*d^4 + 2*A*a^2*d^5 + (2*C*a^2*c^2 + A*a*c^3)*
d^3 + (2*C*a*c^5 - A*c^6)*d)*sqrt(-a)*x^6*arctan(sqrt(c*x^4 + a)*sqrt(-a)/
a) - 3*(C*a^2*c^2*d^2 - B*a^2*c*d^3 + A*a^2*d^4)*sqrt(c^3 + a*d^2)*x^6*...
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^7\,(c+dx^2)\,\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^7\sqrt{a+cx^4}\,(c+dx^2)}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**7/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**7*sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^7 (c + dx^2) \sqrt{a + cx^4}} \, dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{cx^4 + a}(dx^2 + c)x^7} \, dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^7/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)*x^7), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(226) = 452$.

Time = 0.21 (sec) , antiderivative size = 556, normalized size of antiderivative = 2.20

$$\int \frac{A + Bx^2 + Cx^4}{x^7 (c + dx^2) \sqrt{a + cx^4}} \, dx = \frac{(Cc^2d^2 - Bcd^3 + Ad^4) \arctan \left(-\frac{\left(\sqrt{c}x^2 - \sqrt{cx^4+a}\right)d + c^{\frac{3}{2}}}{\sqrt{-c^3 - ad^2}} \right)}{\sqrt{-c^3 - ad^2}c^4} - \frac{(Bc^4 + 2\,Cac^2d - Ac^3d - 2\,Bacd^2 + 2\,Aad^3) \arctan \left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4+a}}{\sqrt{-a}} \right)}{2\sqrt{-a}ac^4} + \frac{3\left(\sqrt{c}x^2 - \sqrt{cx^4 + a}\right)^5 Bc^3 - 3\left(\sqrt{c}x^2 - \sqrt{cx^4 + a}\right)^5 Ac^2d + 6\left(\sqrt{c}x^2 - \sqrt{cx^4 + a}\right)^4 Cac^{\frac{5}{2}} - 6\left(\sqrt{c}x^2 - \right.}{}$$

input
```
integrate((C*x^4+B*x^2+A)/x^7/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
(C*c^2*d^2 - B*c*d^3 + A*d^4)*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + a))*d +
 c^(3/2))/sqrt(-c^3 - a*d^2))/(sqrt(-c^3 - a*d^2)*c^4) - 1/2*(B*c^4 + 2*C*
a*c^2*d - A*c^3*d - 2*B*a*c*d^2 + 2*A*a*d^3)*arctan(-(sqrt(c)*x^2 - sqrt(c
*x^4 + a))/sqrt(-a))/(sqrt(-a)*a*c^4) + 1/6*(3*(sqrt(c)*x^2 - sqrt(c*x^4 +
 a))^5*B*c^3 - 3*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^5*A*c^2*d + 6*(sqrt(c)*x^
2 - sqrt(c*x^4 + a))^4*C*a*c^(5/2) - 6*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^4*B
*a*c^(3/2)*d + 6*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^4*A*a*sqrt(c)*d^2 - 12*(s
qrt(c)*x^2 - sqrt(c*x^4 + a))^2*C*a^2*c^(5/2) + 12*(sqrt(c)*x^2 - sqrt(c*x
^4 + a))^2*A*a*c^(7/2) + 12*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^2*B*a^2*c^(3/2
)*d - 12*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^2*A*a^2*sqrt(c)*d^2 - 3*(sqrt(c)*
x^2 - sqrt(c*x^4 + a))*B*a^2*c^3 + 3*(sqrt(c)*x^2 - sqrt(c*x^4 + a))*A*a^2
*c^2*d + 6*C*a^3*c^(5/2) - 4*A*a^2*c^(7/2) - 6*B*a^3*c^(3/2)*d + 6*A*a^3*s
qrt(c)*d^2)/(((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)^3*a*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^7(c+dx^2)\sqrt{a+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^7\,\sqrt{c\,x^4+a}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^7*(a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^7*(a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^7(c+dx^2)\sqrt{a+cx^4}}\,dx=\left(\int \frac{1}{\sqrt{c\,x^4+a}\,c\,x^7+\sqrt{c\,x^4+a}\,d\,x^9}dx\right)a+\left(\int \frac{1}{\sqrt{c\,x^4+a}\,c\,x^5+\sqrt{c\,x^4+a}\,d\,x^7}dx\right)b+\left(\int \frac{1}{\sqrt{c\,x^4+a}\,c\,x^3+\sqrt{c\,x^4+a}\,d\,x^5}dx\right)c$$

input
```
int((C*x^4+B*x^2+A)/x^7/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output

```
int(1/(sqrt(a + c*x**4)*c*x**7 + sqrt(a + c*x**4)*d*x**9),x)*a + int(1/(sq
rt(a + c*x**4)*c*x**5 + sqrt(a + c*x**4)*d*x**7),x)*b + int(1/(sqrt(a + c*
x**4)*c*x**3 + sqrt(a + c*x**4)*d*x**5),x)*c
```

3.28 $\int \frac{x^4(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 702

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = -\frac{(cC-Bd)x\sqrt{a+cx^4}}{3cd^2} + \frac{Cx^3\sqrt{a+cx^4}}{5cd} + \frac{(5c^3C-5Bc^2d+5Acd^2-3aCd^2)\,x\sqrt{a+cx^4}}{5c^{3/2}d^3\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$+ \frac{c^{3/2}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{c^3+ad^2}x}{\sqrt{c}\sqrt{d}\sqrt{a+cx^4}}\right)}{2d^{7/2}\sqrt{c^3+ad^2}}$$

$$- \frac{\sqrt[4]{a}(5c^3C-5Bc^2d+5Acd^2-3aCd^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}d^3\sqrt{a+cx^4}}$$

$$+ \frac{\sqrt[4]{a}\big(30c^{9/2}C-30Bc^{7/2}d-10\sqrt{a}c^3Cd+10\sqrt{a}Bc^2d^2+30Ac^{5/2}d^2-14ac^{3/2}Cd^2+5aB\sqrt{c}d^3-15\sqrt{a}}{30c^{7/4}d^3\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+}}$$

$$- \frac{c^{3/4}\left(c^{3/2}+\sqrt{a}d\right)\left(c^2C-Bcd+Ad^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(c^{3/2}-\sqrt{a}d\right)^2}{4\sqrt{a}c^{3/2}d}, 2\arctan\right.}{4\sqrt[4]{a}d^4\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}$$

output
```
-1/3*(-B*d+C*c)*x*(c*x^4+a)^(1/2)/c/d^2+1/5*C*x^3*(c*x^4+a)^(1/2)/c/d+1/5*
(5*A*c*d^2-5*B*c^2*d-3*C*a*d^2+5*C*c^3)*x*(c*x^4+a)^(1/2)/c^(3/2)/d^3/(a^(
1/2)+c^(1/2)*x^2)+1/2*c^(3/2)*(A*d^2-B*c*d+C*c^2)*arctan((a*d^2+c^3)^(1/2)
*x/c^(1/2)/d^(1/2)/(c*x^4+a)^(1/2))/d^(7/2)/(a*d^2+c^3)^(1/2)-1/5*a^(1/4)*
(5*A*c*d^2-5*B*c^2*d-3*C*a*d^2+5*C*c^3)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(
a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*2^(1/2))/c^(7/4)/d^3/(c*x^4+a)^(1/2)+1/30*a^(1/4)*(30*c^(9/2)*C-30*B*c^
(7/2)*d-10*a^(1/2)*c^3*C*d+10*a^(1/2)*B*c^2*d^2+30*A*c^(5/2)*d^2-14*a*c^(3
/2)*C*d^2+5*a*B*c^(1/2)*d^3-15*a^(1/2)*A*c*d^3+9*a^(3/2)*C*d^3)*(a^(1/2)+c
^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(7/4)/d^3/(c^(3/2)-a^(1/2)*d)/(c*x^
4+a)^(1/2)-1/4*c^(3/4)*(c^(3/2)+a^(1/2)*d)*(A*d^2-B*c*d+C*c^2)*(a^(1/2)+c^
(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arct
an(c^(1/4)*x/a^(1/4))),-1/4*(c^(3/2)-a^(1/2)*d)^2/a^(1/2)/c^(3/2)/d,1/2*2^
(1/2))/a^(1/4)/d^4/(c^(3/2)-a^(1/2)*d)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.74 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.58

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx$$
$$= \frac{-3\sqrt{a}d(-5c^3C+5Bc^2d-5Acd^2+3aCd^2)\sqrt{1+\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\left(15ic^{9/2}C-15iBc\right.}{}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(-3*Sqrt[a]*d*(-5*c^3*C + 5*B*c^2*d - 5*A*c*d^2 + 3*a*C*d^2)*Sqrt[1 + (c*x
^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + ((15*I)*c^(
9/2)*C - (15*I)*B*c^(7/2)*d - 15*Sqrt[a]*c^3*C*d + 15*Sqrt[a]*B*c^2*d^2 +
(15*I)*A*c^(5/2)*d^2 - (5*I)*a*c^(3/2)*C*d^2 + (5*I)*a*B*Sqrt[c]*d^3 - 15*
Sqrt[a]*A*c*d^3 + 9*a^(3/2)*C*d^3)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh
[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*(-(Sqrt[(I*Sqrt[c])/Sqrt[a]]*
d^2*x*(5*c*C - 5*B*d - 3*C*d*x^2)*(a + c*x^4)) - (15*I)*c^2*(c^2*C - B*c*d
 + A*d^2)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*d)/c^(3/2), I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(15*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(3/
2)*d^4*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 675, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {2237, 25, 2237, 25, 2233, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+cx^4}\,(c+dx^2)}\,dx$$

$$\Big\downarrow \text{2237}$$

$$\frac{\int -\frac{Cx^2(dx^2+c)(5cx^4+3a)-5cdx^4(Cx^4+Bx^2+A)}{(dx^2+c)\sqrt{cx^4+a}}dx}{5cd}+\frac{Cx^3\sqrt{a+cx^4}}{5cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{Cx^3\sqrt{a+cx^4}}{5cd}-\frac{\int \frac{Cx^2(dx^2+c)(5cx^4+3a)-5cdx^4(Cx^4+Bx^2+A)}{(dx^2+c)\sqrt{cx^4+a}}dx}{5cd}$$

$$\Big\downarrow \text{2237}$$

$$\frac{Cx^3\sqrt{a+cx^4}}{5cd}-\frac{\frac{\int -\frac{5c(cC-Bd)\left(dx^2+c\right)\left(3cx^4+a\right)-3cd\left(Cx^2\left(dx^2+c\right)\left(5cx^4+3a\right)-5cdx^4\left(Cx^4+Bx^2+A\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{3cd}+\frac{5x\sqrt{a+cx^4}(cC-Bd)}{3d}}{5cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{Cx^3\sqrt{a+cx^4}}{5cd}-\frac{\frac{5x\sqrt{a+cx^4}(cC-Bd)}{3d}-\frac{\int\frac{5c(cC-Bd)\left(dx^2+c\right)\left(3cx^4+a\right)-3cd\left(Cx^2\left(dx^2+c\right)\left(5cx^4+3a\right)-5cdx^4\left(Cx^4+Bx^2+A\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{3cd}}{5cd}$$

$\downarrow$ 2233

$$\frac{Cx^3\sqrt{a+cx^4}}{5cd}-\frac{\frac{5x\sqrt{a+cx^4}(cC-Bd)}{3d}-\frac{\frac{\int\frac{c\left(\sqrt{a}c^{3/2}\left(5\sqrt{a}\sqrt{c}d(cC-Bd)+3\left(5Cc^3-5Bdc^2+5Ad^2c-3aCd^2\right)\right)-\left(ac(4cC+5Bd)d^2+3\left(c^2-\sqrt{a}\sqrt{c}d\right)\left(5Cc^3-5Bdc^2+5Ad^2c-3aC\right.\right.\right.}{\left(dx^2+c\right)\sqrt{cx^4+a}}}{cd}}{3cd}}{5cd}$$

$\downarrow$ 27

$$\frac{Cx^3\sqrt{a+cx^4}}{5cd}-\frac{\frac{5x\sqrt{a+cx^4}(cC-Bd)}{3d}-\frac{\frac{\int\frac{\sqrt{a}c^{3/2}\left(5\sqrt{a}\sqrt{c}d(cC-Bd)+3\left(5Cc^3-5Bdc^2+5Ad^2c-3aCd^2\right)\right)-\left(ac(4cC+5Bd)d^2+3\left(c^2-\sqrt{a}\sqrt{c}d\right)\left(5Cc^3-5Bdc^2+5Ad^2c-3aCd\right.\right.}{\left(dx^2+c\right)\sqrt{cx^4+a}}}{d}}{3cd}}{5cd}$$

$\downarrow$ 1510

$$\frac{Cx^3\sqrt{a+cx^4}}{5cd}-\frac{\frac{5x\sqrt{a+cx^4}(cC-Bd)}{3d}-\frac{\frac{\int\frac{\sqrt{a}c^{3/2}\left(5\sqrt{a}\sqrt{c}d(cC-Bd)+3\left(5Cc^3-5Bdc^2+5Ad^2c-3aCd^2\right)\right)-\left(ac(4cC+5Bd)d^2+3\left(c^2-\sqrt{a}\sqrt{c}d\right)\left(5Cc^3-5Bdc^2+5Ad^2c-3aCd\right.\right.}{\left(dx^2+c\right)\sqrt{cx^4+a}}}{d}}{3cd}}{5cd}$$

$\downarrow$ 2227

$$\frac{Cx^3\sqrt{a+cx^4}}{5cd}-\frac{\frac{5x\sqrt{a+cx^4}(cC-Bd)}{3d}-\frac{\frac{\sqrt{a}\sqrt{c}\left(9a^{3/2}Cd^3-5\sqrt{a}cd\left(3Ad^2-2Bcd+2c^2C\right)-a\sqrt{c}d^2(14cC-5Bd)+30c^{5/2}\left(Ad^2-Bcd+c^2C\right)\right)\int\frac{1}{\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}-\frac{15\sqrt{a}c^4\left(Ad^2-Bc\right.}{}}{d}}{5cd}$$

$\downarrow$ 27

$$\frac{Cx^3\sqrt{a+cx^4}}{5cd}-$$

$$\frac{5x\sqrt{a+cx^4}(cC-Bd)}{3d} - \frac{\frac{\frac{\sqrt{a}\sqrt{c}\left(9a^{3/2}Cd^3-5\sqrt{a}cd\left(3Ad^2-2Bcd+2c^2C\right)-a\sqrt{c}d^2(14cC-5Bd)+30c^{5/2}\left(Ad^2-Bcd+c^2C\right)\right)\int\frac{1}{\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d} - \frac{15c^4\left(Ad^2-Bcd+c\right.}{}}{d}}{5cd}$$

$\downarrow$ 761

$$\frac{Cx^3\sqrt{a+cx^4}}{5cd}-$$

$$\frac{5x\sqrt{a+cx^4}(cC-Bd)}{3d} - \frac{\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(9a^{3/2}Cd^3-5\sqrt{a}cd\left(3Ad^2-2Bcd+2c^2C\right)-a\sqrt{c}d^2(14cC-5\right.}{2\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}}{d}$$

$\downarrow$ 2221

$$\frac{Cx^3\sqrt{a+cx^4}}{5cd}-$$

$$\frac{5x\sqrt{a+cx^4}(cC-Bd)}{3d} - \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(9a^{3/2}Cd^3-5\sqrt{a}cd\left(3Ad^2-2Bcd+2c^2C\right)-a\sqrt{c}d^2(14cC-5\right.}{2\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4))/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(C*x^3*Sqrt[a + c*x^4])/(5*c*d) - ((5*(c*C - B*d)*x*Sqrt[a + c*x^4])/(3*d)
 - ((-3*Sqrt[c]*(5*c^3*C - 5*B*c^2*d + 5*A*c*d^2 - 3*a*C*d^2)*(-((x*Sqrt[a
 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/d + ((a^(1/4)*c^(1/4)*(9*a^(3/2)
*C*d^3 - a*Sqrt[c]*d^2*(14*c*C - 5*B*d) + 30*c^(5/2)*(c^2*C - B*c*d + A*d^
2) - 5*Sqrt[a]*c*d*(2*c^2*C - 2*B*c*d + 3*A*d^2))*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(2*(c^(3/2) - Sqrt[a]*d)*Sqrt[a + c*x^4]) - (15*c^4*(c^2*
C - B*c*d + A*d^2)*(-1/2*((c^(3/2) - Sqrt[a]*d)*ArcTan[(Sqrt[c^3 + a*d^2]*
x)/(Sqrt[c]*Sqrt[d]*Sqrt[a + c*x^4])])/(Sqrt[c]*Sqrt[d]*Sqrt[c^3 + a*d^2])
 + ((c^(3/2) + Sqrt[a]*d)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*(c^(3/2)/Sqrt[a] - d)^2)/(c^(
3/2)*d), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(5/4)*d*Sqrt[a
+ c*x^4])))/(c^(3/2) - Sqrt[a]*d))/d)/(3*c*d))/(5*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2237
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> W
ith[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt[a + c*x^4]/(c*
e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q - 3)*Px - Coeff[Px, x,
 q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a
+ c*x^4]), x], x] /; GtQ[q, 4]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.06 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.58

method	result
risch	$\frac{x(3Cd\,x^2+5Bd-5Cc)\sqrt{c\,x^4+a}}{15c\,d^2}+\frac{-\frac{5\left(3A\,c^2d^2+Ba\,d^3-3B\,c^3d-Cac\,d^2+3C\,c^4\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{d^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{3i\left(5Ac\right.}{}}{}$
default	$\frac{c\left(A\,d^2-Bcd+C\,c^2\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,d}{c^{\frac{3}{2}}},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{\frac{C\,c^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}}{}$
elliptic	Expression too large to display

input
```
int(x^4*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*C*d*x^2+5*B*d-5*C*c)/c*(c*x^4+a)^(1/2)/d^2+1/15/d^2/c*(-5*(3*A*c
^2*d^2+B*a*d^3-3*B*c^3*d-C*a*c*d^2+3*C*c^4)/d^2/(I*c^(1/2)/a^(1/2))^(1/2)*
(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^
(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+3*I/d*(5*A*c*d^2-5*B*c^2*d-
3*C*a*d^2+5*C*c^3)*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1
/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(Ellipt
icF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I
))+15*c^2*(A*d^2-B*c*d+C*c^2)/d^2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x
^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*Elliptic
Pi(x*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(3/2)*a^(1/2)*d,(-I/a^(1/2)*c^(1/2))^(1
/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a+cx^4}\,(c+dx^2)}\,dx$$

input
```
integrate(x**4*(C*x**4+B*x**2+A)/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral(x**4*(A + B*x**2 + C*x**4)/(sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{\sqrt{cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(sqrt(c*x^4 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{\sqrt{cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(sqrt(c*x^4 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \int \frac{x^4\,(C\,x^4+B\,x^2+A)}{\sqrt{c\,x^4+a}\,(d\,x^2+c)}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4))/((a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4))/((a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx$$
$$= \frac{5\sqrt{c\,x^4+a}\,bdx - 5\sqrt{c\,x^4+a}\,c^2x + 3\sqrt{c\,x^4+a}\,cd\,x^3 - 5\left(\int \frac{\sqrt{c\,x^4+a}}{cd\,x^6+c^2x^4+ad\,x^2+ac}dx\right)abcd + 5\left(\int \frac{\sqrt{c\,x^4+}}{cd\,x^6+c^2x^4+c}\right.}{}$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output

```
(5*sqrt(a + c*x**4)*b*d*x - 5*sqrt(a + c*x**4)*c**2*x + 3*sqrt(a + c*x**4)
*c*d*x**3 - 5*int(sqrt(a + c*x**4)/(a*c + a*d*x**2 + c**2*x**4 + c*d*x**6)
,x)*a*b*c*d + 5*int(sqrt(a + c*x**4)/(a*c + a*d*x**2 + c**2*x**4 + c*d*x**
6),x)*a*c**3 + 6*int((sqrt(a + c*x**4)*x**4)/(a*c + a*d*x**2 + c**2*x**4 +
 c*d*x**6),x)*a*c*d**2 - 15*int((sqrt(a + c*x**4)*x**4)/(a*c + a*d*x**2 +
c**2*x**4 + c*d*x**6),x)*b*c**2*d + 15*int((sqrt(a + c*x**4)*x**4)/(a*c +
a*d*x**2 + c**2*x**4 + c*d*x**6),x)*c**4 - 5*int((sqrt(a + c*x**4)*x**2)/(
a*c + a*d*x**2 + c**2*x**4 + c*d*x**6),x)*a*b*d**2 - 4*int((sqrt(a + c*x**
4)*x**2)/(a*c + a*d*x**2 + c**2*x**4 + c*d*x**6),x)*a*c**2*d)/(15*c*d**2)
```

3.29 $\int \frac{x^2(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 581

$$
\begin{aligned}
&\int \frac{x^2(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx \\
&= \frac{Cx\sqrt{a+cx^4}}{3cd} - \frac{(cC-Bd)x\sqrt{a+cx^4}}{\sqrt{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{c}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{c^3+ad^2}x}{\sqrt{c}\sqrt{d}\sqrt{a+cx^4}}\right)}{2d^{5/2}\sqrt{c^3+ad^2}} \\
&\quad + \frac{\sqrt[4]{a}(cC-Bd)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}d^2\sqrt{a+cx^4}} \\
&\quad - \frac{\sqrt[4]{a}\left(6c^3C-6Bc^2d-2\sqrt{a}c^{3/2}Cd+3\sqrt{a}B\sqrt{c}d^2+3Acd^2-aCd^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{Elliptic}}{6c^{5/4}d^2\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}} \\
&\quad + \frac{\left(c^{3/2}+\sqrt{a}d\right)(c^2C-Bcd+Ad^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(c^{3/2}-\sqrt{a}d\right)^2}{4\sqrt{a}c^{3/2}d}, 2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}d^3\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}
\end{aligned}
$$

output
```
1/3*C*x*(c*x^4+a)^(1/2)/c/d-(-B*d+C*c)*x*(c*x^4+a)^(1/2)/c^(1/2)/d^2/(a^(1
/2)+c^(1/2)*x^2)-1/2*c^(1/2)*(A*d^2-B*c*d+C*c^2)*arctan((a*d^2+c^3)^(1/2)*
x/c^(1/2)/d^(1/2)/(c*x^4+a)^(1/2))/d^(5/2)/(a*d^2+c^3)^(1/2)+a^(1/4)*(-B*d
+C*c)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Elli
pticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/d^2/(c*x^4+a)^
(1/2)-1/6*a^(1/4)*(6*C*c^3-6*B*c^2*d-2*a^(1/2)*c^(3/2)*C*d+3*a^(1/2)*B*c^(
1/2)*d^2+3*A*c*d^2-C*a*d^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1
/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))
/c^(5/4)/d^2/(c^(3/2)-a^(1/2)*d)/(c*x^4+a)^(1/2)+1/4*(c^(3/2)+a^(1/2)*d)*(
A*d^2-B*c*d+C*c^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^
2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(3/2)-a^(1/2)
*d)^2/a^(1/2)/c^(3/2)/d,1/2*2^(1/2))/a^(1/4)/c^(1/4)/d^3/(c^(3/2)-a^(1/2)*
d)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.11 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.78

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx$$
$$= \frac{a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}Cd^2x+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cCd^2x^5-3\sqrt{a}\sqrt{c}d(cC-Bd)\sqrt{1+\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\left(-3ic^3C+3\right.}{}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*C*d^2*x + Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*C*d^2*x
^5 - 3*Sqrt[a]*Sqrt[c]*d*(c*C - B*d)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + ((-3*I)*c^3*C + (3*I)*B*c^2*d + 3*S
qrt[a]*c^(3/2)*C*d - 3*Sqrt[a]*B*Sqrt[c]*d^2 - (3*I)*A*c*d^2 + I*a*C*d^2)*
Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]
+ (3*I)*c^3*C*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*d)/c^(3/2), I*A
rcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (3*I)*B*c^2*d*Sqrt[1 + (c*x^4)/
a]*EllipticPi[((-I)*Sqrt[a]*d)/c^(3/2), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]
]*x], -1] + (3*I)*A*c*d^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*d)/
c^(3/2), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(3*Sqrt[(I*Sqrt[c])/
Sqrt[a]]*c*d^3*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2237, 25, 2233, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(A+Bx^2+Cx^4\right)}{\sqrt{a+cx^4} \left(c+dx^2\right)} \, dx$$

$$\Big\downarrow \text{2237}$$

$$\frac{\int -\frac{C\left(dx^2+c\right)\left(3cx^4+a\right)-3cdx^2\left(Cx^4+Bx^2+A\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{3cd}+\frac{Cx\sqrt{a+cx^4}}{3cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{Cx\sqrt{a+cx^4}}{3cd}-\frac{\int \frac{C\left(dx^2+c\right)\left(3cx^4+a\right)-3cdx^2\left(Cx^4+Bx^2+A\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{3cd}$$

$$\Big\downarrow \text{2233}$$

$$\frac{Cx\sqrt{a+cx^4}}{3cd}-\frac{\frac{\int \frac{c\left(\sqrt{a}c\left(\sqrt{a}Cd+3\sqrt{c}(cC-Bd)\right)-\left((3Ac-aC)d^2+3\sqrt{c}\left(c^{3/2}-\sqrt{a}d\right)(cC-Bd)\right)x^2\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{cd}-\frac{3\sqrt{a}\sqrt{c}(cC-Bd)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{d}}{3cd}$$

$$\Big\downarrow \; 27$$

$$\frac{Cx\sqrt{a+cx^4}}{3cd}-\frac{\frac{\int \frac{\sqrt{a}c(\sqrt{a}Cd+3\sqrt{c}(cC-Bd))-\left((3Ac-aC)d^2+3\sqrt{c}\left(c^{3/2}-\sqrt{a}d\right)(cC-Bd)\right)x^2}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{d}-\frac{3\sqrt{c}(cC-Bd)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{d}}{3cd}$$

$$\Big\downarrow \; 1510$$

$$\frac{Cx\sqrt{a+cx^4}}{3cd}-\frac{\frac{\int \frac{\sqrt{a}c(\sqrt{a}Cd+3\sqrt{c}(cC-Bd))-\left((3Ac-aC)d^2+3\sqrt{c}\left(c^{3/2}-\sqrt{a}d\right)(cC-Bd)\right)x^2}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{d}-\frac{3\sqrt{c}(cC-Bd)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\cdots\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\cdots\right.}{d}\cdots}{3cd}$$

$$\Big\downarrow \; 2227$$

$$\frac{Cx\sqrt{a+cx^4}}{3cd}-\frac{\frac{\frac{\sqrt{a}\left(3\sqrt{a}B\sqrt{c}d^2-2\sqrt{a}c^{3/2}Cd-aCd^2+3Acd^2-6Bc^2d+6c^3C\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}-\frac{3\sqrt{a}c^2\left(Ad^2-Bcd+c^2C\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}}{d}-\frac{3\sqrt{c}(cC-Bd)\left(\frac{\sqrt[4]{a}(\cdots}{\cdots}\cdots\right.}{\cdots}}{3cd}$$

$$\Big\downarrow \; 27$$

$$\frac{Cx\sqrt{a+cx^4}}{3cd}-\frac{\frac{\frac{\sqrt{a}\left(3\sqrt{a}B\sqrt{c}d^2-2\sqrt{a}c^{3/2}Cd-aCd^2+3Acd^2-6Bc^2d+6c^3C\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}-\frac{3c^2\left(Ad^2-Bcd+c^2C\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}}{d}-\frac{3\sqrt{c}(cC-Bd)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{3cd}$$

$$\Big\downarrow \; 761$$

$$\frac{Cx\sqrt{a+cx^4}}{3cd}-\frac{\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(3\sqrt{a}B\sqrt{c}d^2-2\sqrt{a}c^{3/2}Cd-aCd^2+3Acd^2-6Bc^2d+6c^3C\right)}{2\sqrt[4]{c}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}-\frac{3c^2\left(Ad^2-Bcd+c^2C\right)\int \frac{\sqrt{\cdots}}{\left(dx^2\cdots\right.}\cdots}{c^{3/2}-\sqrt{a}d}\cdots}{d}\cdots}{3cd}$$

$$\Big\downarrow \text{2221}$$

$$\frac{Cx\sqrt{a+cx^4}}{3cd}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(3\sqrt{a}B\sqrt{c}d^2-2\sqrt{a}c^{3/2}Cd-aCd^2+3Acd^2-6Bc^2d+6c^3C\right)}{2\sqrt[4]{c}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}-\frac{3c^2\left(Ad^2-Bcd+c^2C\right)\left(\frac{\left(\sqrt{a}d-\cdots\right.}{\cdots}\right.}{\cdots}}{d}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(C*x*Sqrt[a + c*x^4])/(3*c*d) - ((-3*Sqrt[c]*(c*C - B*d)*(-((x*Sqrt[a + c*
x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/d + ((a^(1/4)*(6*c^3*C - 6*B*c^2*d -
2*Sqrt[a]*c^(3/2)*C*d + 3*Sqrt[a]*B*Sqrt[c]*d^2 + 3*A*c*d^2 - a*C*d^2)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(1/4)*(c^(3/2) - Sqrt[a]*d)*Sqr
t[a + c*x^4]) - (3*c^2*(c^2*C - B*c*d + A*d^2)*(-1/2*((c^(3/2) - Sqrt[a]*d
)*ArcTan[(Sqrt[c^3 + a*d^2]*x)/(Sqrt[c]*Sqrt[d]*Sqrt[a + c*x^4])])/(Sqrt[c
]*Sqrt[d]*Sqrt[c^3 + a*d^2]) + ((c^(3/2) + Sqrt[a]*d)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*(
c^(3/2)/Sqrt[a] - d)^2)/(c^(3/2)*d), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(4*a^(1/4)*c^(5/4)*d*Sqrt[a + c*x^4])))/(c^(3/2) - Sqrt[a]*d))/d)/(3*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2237
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> W
ith[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt[a + c*x^4]/(c*
e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q - 3)*Px - Coeff[Px, x,
 q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a
+ c*x^4]), x], x] /; GtQ[q, 4]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.20 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.94

method	result
default	$\frac{A d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{C c^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{i d(Bd-Cc)\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{}$ $\frac{3 i d(Bd-Cc)\sqrt{c}\,\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{3C c^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x}}$
risch	$\frac{Cx\sqrt{c\,x^4+a}}{3cd}+$
elliptic	Expression too large to display

input
```
int(x^2*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(A*d^2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+
I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2
))^(1/2),I)+C*c^2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2
)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a
^(1/2))^(1/2),I)+I*d*(B*d-C*c)*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1
/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(
1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1
/2))^(1/2),I))+C*d^2*(1/3/c*x*(c*x^4+a)^(1/2)-1/3*a/c/(I*c^(1/2)/a^(1/2))^
(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x
^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I))-B*c*d/(I*c^(1/2)/a^(
1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2
)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I))-(A*d^2-B*c*d+C
*c^2)/d^3/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c
^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))
^(1/2),I/c^(3/2)*a^(1/2)*d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^
(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a+cx^4}\,(c+dx^2)}\,dx$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4)/(sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(A + Bx^2 + Cx^4)}{(c + dx^2)\sqrt{a + cx^4}}\,dx = \int \frac{(Cx^4 + Bx^2 + A)x^2}{\sqrt{cx^4 + a}(dx^2 + c)}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(sqrt(c*x^4 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2(A + Bx^2 + Cx^4)}{(c + dx^2)\sqrt{a + cx^4}}\,dx = \int \frac{(Cx^4 + Bx^2 + A)x^2}{\sqrt{cx^4 + a}(dx^2 + c)}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(sqrt(c*x^4 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A + Bx^2 + Cx^4)}{(c + dx^2)\sqrt{a + cx^4}}\,dx = \int \frac{x^2\,(C\,x^4 + B\,x^2 + A)}{\sqrt{c\,x^4 + a}\,(d\,x^2 + c)}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/((a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/((a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \frac{\sqrt{c\,x^4+a}\,x-\left(\int \frac{\sqrt{c\,x^4+a}}{cd\,x^6+c^2x^4+ad\,x^2+ac}dx\right)ac+3\left(\int \frac{\sqrt{c\,x^4+a}\,x^4}{cd\,x^6+c^2x^4+ad\,x^2+ac}dx\right)bd-3\left(\int \frac{\sqrt{c\,x^4+a}\,x^4}{cd\,x^6+c^2x^4+ad\,x^2+ac}dx\right)c^2}{3d}$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + c*x**4)*x - int(sqrt(a + c*x**4)/(a*c + a*d*x**2 + c**2*x**4 + c
*d*x**6),x)*a*c + 3*int((sqrt(a + c*x**4)*x**4)/(a*c + a*d*x**2 + c**2*x**
4 + c*d*x**6),x)*b*d - 3*int((sqrt(a + c*x**4)*x**4)/(a*c + a*d*x**2 + c**
2*x**4 + c*d*x**6),x)*c**2 + 2*int((sqrt(a + c*x**4)*x**2)/(a*c + a*d*x**2
 + c**2*x**4 + c*d*x**6),x)*a*d)/(3*d)
```

3.30 $\int \frac{A+Bx^2+Cx^4}{(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 516

$$\int \frac{A+Bx^2+Cx^4}{(c+dx^2)\sqrt{a+cx^4}}\,dx$$

$$= \frac{Cx\sqrt{a+cx^4}}{\sqrt{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{c^3+ad^2}x}{\sqrt{c}\sqrt{d}\sqrt{a+cx^4}}\right)}{2\sqrt{c}d^{3/2}\sqrt{c^3+ad^2}}$$

$$- \frac{\sqrt[4]{a}C\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}d\sqrt{a+cx^4}}$$

$$+ \frac{\left(Acd-aCd+\sqrt{a}\sqrt{c}(2cC-Bd)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}d\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}$$

$$- \frac{\left(c^{3/2}+\sqrt{a}d\right)(c^2C-Bcd+Ad^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(c^{3/2}-\sqrt{a}d\right)^2}{4\sqrt{a}c^{3/2}d},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{4\sqrt[4]{a}c^{5/4}d^2\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}$$

output
```
C*x*(c*x^4+a)^(1/2)/c^(1/2)/d/(a^(1/2)+c^(1/2)*x^2)+1/2*(A*d^2-B*c*d+C*c^2
)*arctan((a*d^2+c^3)^(1/2)*x/c^(1/2)/d^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)/d^(3
/2)/(a*d^2+c^3)^(1/2)-a^(1/4)*C*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+
c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/
2))/c^(3/4)/d/(c*x^4+a)^(1/2)+1/2*(A*c*d-C*a*d+a^(1/2)*c^(1/2)*(-B*d+2*C*c
))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Inverse
JacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(3/4)/d/(c^(3/
2)-a^(1/2)*d)/(c*x^4+a)^(1/2)-1/4*(c^(3/2)+a^(1/2)*d)*(A*d^2-B*c*d+C*c^2)*
(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi
(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(3/2)-a^(1/2)*d)^2/a^(1/2)/c^(3/
2)/d,1/2*2^(1/2))/a^(1/4)/c^(5/4)/d^2/(c^(3/2)-a^(1/2)*d)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.60 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^2+Cx^4}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \frac{\sqrt{1+\frac{cx^4}{a}}\left(\sqrt{a}\sqrt{c}CdE\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+i\left(\left(c^2C-Bcd+i\sqrt{a}\sqrt{c}Cd\right)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\right.\right.\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cd^2\sqrt{a+cx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(Sqrt[1 + (c*x^4)/a]*(Sqrt[a]*Sqrt[c]*C*d*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt
[c])/Sqrt[a]]*x], -1] + I*((c^2*C - B*c*d + I*Sqrt[a]*Sqrt[c]*C*d)*Ellipti
cF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (c^2*C - B*c*d + A*d^2)*E
llipticPi[((-I)*Sqrt[a]*d)/c^(3/2), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x]
, -1])))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^2*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2233, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a+cx^4}\,(c+dx^2)}\,dx$$

$$\Big\downarrow \text{2233}$$

$$\frac{\int \frac{\sqrt{c}(\sqrt{c}(\sqrt{a}\sqrt{c}C+Ad)-(Cc^{3/2}-Bd\sqrt{c}-\sqrt{a}Cd)x^2)}{(dx^2+c)\sqrt{cx^4+a}}dx}{cd}-\frac{\sqrt{a}C\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c}(\sqrt{a}\sqrt{c}C+Ad)-(Cc^{3/2}-Bd\sqrt{c}-\sqrt{a}Cd)x^2}{(dx^2+c)\sqrt{cx^4+a}}dx}{\sqrt{c}d}-\frac{C\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}d}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\int \frac{\sqrt{c}(\sqrt{a}\sqrt{c}C+Ad)-(Cc^{3/2}-Bd\sqrt{c}-\sqrt{a}Cd)x^2}{(dx^2+c)\sqrt{cx^4+a}}dx}{\sqrt{c}d}-\frac{C\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}d}$$

$$\Big\downarrow \text{2227}$$

$$\frac{\frac{(\sqrt{a}\sqrt{c}(2cC-Bd)-aCd+Acd)\int \frac{1}{\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}-\frac{\sqrt{a}\sqrt{c}(Ad^2-Bcd+c^2C)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}}{\sqrt{c}d}-\frac{C\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\left(\sqrt{a}\sqrt{c}(2cC-Bd)-aCd+Acd\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}-\frac{\sqrt{c}\left(Ad^2-Bcd+c^2C\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}}{\sqrt{c}d}-\frac{C\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}d}$$

$\Big\downarrow$ 761

$$\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}\sqrt{c}(2cC-Bd)-aCd+Acd\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}-\frac{\sqrt{c}\left(Ad^2-Bcd+c^2C\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}}{\sqrt{c}d}-\frac{C\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}d}$$

$\Big\downarrow$ 2221

$$\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}\sqrt{c}(2cC-Bd)-aCd+Acd\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}-\frac{\sqrt{c}\left(Ad^2-Bcd+c^2C\right)\left(\frac{\left(\sqrt{a}d+c^{3/2}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\cdots}{\cdots}\right.}{\cdots}}{\sqrt{c}d}-\frac{C\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}d}$$

input
```
Int[(A + B*x^2 + C*x^4)/((c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
-((C*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/(Sqrt[c]*d)) +
(((A*c*d - a*C*d + Sqrt[a]*Sqrt[c]*(2*c*C - B*d))*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(c^(3/2) - Sqrt[a]*d)*Sqrt[a + c*x^4])
 - (Sqrt[c]*(c^2*C - B*c*d + A*d^2)*(-1/2*((c^(3/2) - Sqrt[a]*d)*ArcTan[(S
qrt[c^3 + a*d^2]*x)/(Sqrt[c]*Sqrt[d]*Sqrt[a + c*x^4])])/(Sqrt[c]*Sqrt[d]*S
qrt[c^3 + a*d^2]) + ((c^(3/2) + Sqrt[a]*d)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*(c^(3/2)/Sqr
t[a] - d)^2)/(c^(3/2)*d), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*
c^(5/4)*d*Sqrt[a + c*x^4])))/(c^(3/2) - Sqrt[a]*d))/(Sqrt[c]*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.72

method	result
default	$\frac{\frac{B d \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\, x^{4}+a}}+\frac{i C d \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\, x^{4}+a}\, \sqrt{c}}-\frac{C c \sqrt{1-\cdots}}{\cdots}}{d^{2}}$
elliptic	$\frac{\sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right) B}{d \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\, x^{4}+a}}-\frac{\sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right) C c}{d^{2} \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\, x^{4}+a}}+\frac{i C \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1\cdots}}{d \sqrt{\frac{i \sqrt{\cdots}}{\sqrt{\cdots}}}}$

input
```
int((C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(B*d/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*
c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))
^(1/2),I)+I*C*d*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2)
)^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF
(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))-
C*c/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)
*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),
I))+(A*d^2-B*c*d+C*c^2)/d^2/c/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a
^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x
*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(3/2)*a^(1/2)*d,(-I/a^(1/2)*c^(1/2))^(1/2)/
(I*c^(1/2)/a^(1/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{\sqrt{a+cx^4}\,(c+dx^2)}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(c+dx^2)\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(c+dx^2)\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}(dx^2+c)}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(c+dx^2)\sqrt{a+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{c\,x^4+a}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(c+dx^2)\sqrt{a+cx^4}}\,dx = \left(\int \frac{\sqrt{c\,x^4+a}}{cd\,x^6+c^2x^4+ad\,x^2+ac}dx\right)a + \left(\int \frac{\sqrt{c\,x^4+a}\,x^4}{cd\,x^6+c^2x^4+ad\,x^2+ac}dx\right)c + \left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{cd\,x^6+c^2x^4+ad\,x^2+ac}dx\right)b$$

input
```
int((C*x^4+B*x^2+A)/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a*c + a*d*x**2 + c**2*x**4 + c*d*x**6),x)*a + int((s
qrt(a + c*x**4)*x**4)/(a*c + a*d*x**2 + c**2*x**4 + c*d*x**6),x)*c + int((
sqrt(a + c*x**4)*x**2)/(a*c + a*d*x**2 + c**2*x**4 + c*d*x**6),x)*b
```

3.31 $\int \frac{A+Bx^2+Cx^4}{x^2(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 534

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx$$
$$= -\frac{A\sqrt{a+cx^4}}{acx}+\frac{Ax\sqrt{a+cx^4}}{a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\left(c^2C-Bcd+Ad^2\right)\arctan\left(\frac{\sqrt{c^3+ad^2}x}{\sqrt{c}\sqrt{d}\sqrt{a+cx^4}}\right)}{2c^{3/2}\sqrt{d}\sqrt{c^3+ad^2}}$$
$$-\frac{A\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/4}\sqrt{a+cx^4}}$$
$$+\frac{\left(Ac^{3/2}-a\sqrt{c}C+\sqrt{a}(Bc-2Ad)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}$$
$$+\frac{\left(c^{3/2}+\sqrt{a}d\right)\left(c^2C-Bcd+Ad^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(c^{3/2}-\sqrt{a}d\right)^2}{4\sqrt{a}c^{3/2}d},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{4\sqrt[4]{a}c^{9/4}d\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}$$

output
```
-A*(c*x^4+a)^(1/2)/a/c/x+A*x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1/2)+c^(1/2)*x^
2)-1/2*(A*d^2-B*c*d+C*c^2)*arctan((a*d^2+c^3)^(1/2)*x/c^(1/2)/d^(1/2)/(c*x
^4+a)^(1/2))/c^(3/2)/d^(1/2)/(a*d^2+c^3)^(1/2)-A*(a^(1/2)+c^(1/2)*x^2)*((c
*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*2^(1/2))/a^(3/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*(A*c^(3/2)-a*c^(1
/2)*C+a^(1/2)*(-2*A*d+B*c))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1
/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))
/a^(3/4)/c^(3/4)/(c^(3/2)-a^(1/2)*d)/(c*x^4+a)^(1/2)+1/4*(c^(3/2)+a^(1/2)*
d)*(A*d^2-B*c*d+C*c^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x
^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(3/2)-a^(
1/2)*d)^2/a^(1/2)/c^(3/2)/d,1/2*2^(1/2))/a^(1/4)/c^(9/4)/d/(c^(3/2)-a^(1/2
)*d)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.94 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2+Cx^4}{x^2(c+dx^2)\sqrt{a+cx^4}}\,dx$$
$$= \frac{-aA\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cd - A\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c^2dx^4 + \sqrt{a}Ac^{3/2}dx\sqrt{1+\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right) - i\sqrt{a}c^{3/2}\left(\sqrt{a}\sqrt{c}C - \right.}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(-(a*A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d) - A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^2*d*
x^4 + Sqrt[a]*A*c^(3/2)*d*x*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(
I*Sqrt[c])/Sqrt[a]]*x], -1] - I*Sqrt[a]*c^(3/2)*(Sqrt[a]*Sqrt[c]*C - I*A*d
)*x*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
-1] + I*a*c^2*C*x*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*d)/c^(3/2),
 I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a*B*c*d*x*Sqrt[1 + (c*x^4
)/a]*EllipticPi[((-I)*Sqrt[a]*d)/c^(3/2), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[
a]]*x], -1] + I*a*A*d^2*x*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*d)/
c^(3/2), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(a*Sqrt[(I*Sqrt[c])/
Sqrt[a]]*c^2*d*x*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2245, 25, 2233, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+cx^4}\left(c+dx^2\right)}dx$$

$$\Big\downarrow \text{ 2245}$$

$$-\frac{\int -\frac{Acdx^4+c(Ac+aC)x^2+a(Bc-Ad)}{(dx^2+c)\sqrt{cx^4+a}}dx}{ac}-\frac{A\sqrt{a+cx^4}}{acx}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{Acdx^4+c(Ac+aC)x^2+a(Bc-Ad)}{(dx^2+c)\sqrt{cx^4+a}}dx}{ac}-\frac{A\sqrt{a+cx^4}}{acx}$$

$$\Big\downarrow \text{ 2233}$$

$$\frac{\frac{\int \frac{\sqrt{a}cd\left(\sqrt{c}(\sqrt{a}\sqrt{c}C+Ad)x^2+Ac^{3/2}+\sqrt{a}(Bc-Ad)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{cd}-\sqrt{a}A\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{ac}-\frac{A\sqrt{a+cx^4}}{acx}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a}\int \frac{\sqrt{c}(\sqrt{a}\sqrt{c}C+Ad)x^2+Ac^{3/2}+\sqrt{a}(Bc-Ad)}{(dx^2+c)\sqrt{cx^4+a}}dx-A\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{ac}-\frac{A\sqrt{a+cx^4}}{acx}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{\sqrt{a}\int \frac{\sqrt{c}(\sqrt{a}\sqrt{c}C+Ad)x^2+Ac^{3/2}+\sqrt{a}(Bc-Ad)}{(dx^2+c)\sqrt{cx^4+a}}dx-A\sqrt{c}\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{ac}$$

$$\frac{A\sqrt{a+cx^4}}{acx}$$

$$\Big\downarrow \text{ 2227}$$

$$\frac{\sqrt{a}\left(\frac{\sqrt{c}\left(\sqrt{a}(Bc-2Ad)-a\sqrt{c}C+Ac^{3/2}\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}+\frac{a\left(Ad^2-Bcd+c^2C\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}\right)-A\sqrt{c}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a\cdots}{\left(\sqrt{a}+\cdots\right.}}\cdots}{\sqrt[4]{\cdots}}\cdots\right.}{ac}\cdots$$

$$\frac{A\sqrt{a+cx^4}}{acx}$$

$\downarrow$ 27

$$\frac{\sqrt{a}\left(\frac{\sqrt{c}\left(\sqrt{a}(Bc-2Ad)-a\sqrt{c}C+Ac^{3/2}\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}+\frac{\sqrt{a}\left(Ad^2-Bcd+c^2C\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}\right)-A\sqrt{c}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+\cdots}{\left(\sqrt{a}+\cdots\right.}}\cdots}{\sqrt[4]{\cdots}}\cdots\right.}{ac}\cdots$$

$$\frac{A\sqrt{a+cx^4}}{acx}$$

$\downarrow$ 761

$$\frac{\sqrt{a}\left(\frac{\sqrt{a}\left(Ad^2-Bcd+c^2C\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}(Bc-2Ad)-a\sqrt{c}C+\cdots\right.}{2\sqrt[4]{a}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}\cdots\right.}{ac}\cdots$$

$$\frac{A\sqrt{a+cx^4}}{acx}$$

$\downarrow$ 2221

$$\frac{\sqrt{a}\left(\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\sqrt{a}(Bc-2Ad)-a\sqrt{c}C+Ac^{3/2}\right)}{2\sqrt[4]{a}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}+\frac{\sqrt{a}\left(Ad^2-Bcd+c^2C\right)\left(\frac{\left(\sqrt{a}d+c^{3/2}\right)\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots$$

$$\frac{A\sqrt{a+cx^4}}{acx}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
-((A*Sqrt[a + c*x^4])/(a*c*x)) + (-(A*Sqrt[c]*(-((x*Sqrt[a + c*x^4])/(Sqrt
[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c
^(1/4)*Sqrt[a + c*x^4]))) + Sqrt[a]*((c^(1/4)*(A*c^(3/2) - a*Sqrt[c]*C + S
qrt[a]*(B*c - 2*A*d))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*
(c^(3/2) - Sqrt[a]*d)*Sqrt[a + c*x^4]) + (Sqrt[a]*(c^2*C - B*c*d + A*d^2)*
(-1/2*((c^(3/2) - Sqrt[a]*d)*ArcTan[(Sqrt[c^3 + a*d^2]*x)/(Sqrt[c]*Sqrt[d]
*Sqrt[a + c*x^4])])/(Sqrt[c]*Sqrt[d]*Sqrt[c^3 + a*d^2]) + ((c^(3/2) + Sqrt
[a]*d)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticPi[-1/4*(Sqrt[a]*(c^(3/2)/Sqrt[a] - d)^2)/(c^(3/2)*d), 2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(5/4)*d*Sqrt[a + c*x^4])))/(c^(3/2
) - Sqrt[a]*d)))/(a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2245
```
Int[((Px_)*(x_)^(m_))/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_
Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 2], C = Coeff[Px, x,
 4]}, Simp[A*x^(m + 1)*(Sqrt[a + c*x^4]/(a*d*(m + 1))), x] + Simp[1/(a*d*(m
 + 1))   Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + c*x^4]))*Simp[a*B*d*(m + 1) -
 A*a*e*(m + 1) + (a*C*d*(m + 1) - A*c*d*(m + 3))*x^2 - A*c*e*(m + 3)*x^4, x
], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2, 2] && ILtQ[m/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.59

method	result
default	$\frac{C\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{A\left(-\frac{\sqrt{c\,x^{4}+a}}{ax}+\frac{i\sqrt{c}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\cdots}\right.\right.}{\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right.}{c}$
risch	$-\frac{A\sqrt{c\,x^{4}+a}}{acx}+\frac{\frac{iA\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{Cca\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\cdots}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}}{ac}$
elliptic	$-\frac{A\sqrt{c\,x^{4}+a}}{acx}+\frac{C\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{iA\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}-\frac{iA\sqrt{\cdots}}{\cdots}$

input
```
int((C*x^4+B*x^2+A)/x^2/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
C/d/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)
*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),
I)+A/c*(-1/a*(c*x^4+a)^(1/2)/x+I*c^(1/2)/a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)
*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)
^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^
(1/2))^(1/2),I)))-(A*d^2-B*c*d+C*c^2)/c^2/d/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I
*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2
)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),I/c^(3/2)*a^(1/2)*d,(-I/a^(1/2)*c
^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{x^2 (c + dx^2) \sqrt{a + cx^4}} \, dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^2 (c + dx^2) \sqrt{a + cx^4}} \, dx = \int \frac{A + Bx^2 + Cx^4}{x^2 \sqrt{a + cx^4} \, (c + dx^2)} \, dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**2*sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^2 (c + dx^2) \sqrt{a + cx^4}} \, dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{cx^4 + a}(dx^2 + c)x^2} \, dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}(dx^2+c)x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^2\,\sqrt{c\,x^4+a}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\left(\int \frac{\sqrt{c\,x^4+a}}{cd\,x^8+c^2x^6+ad\,x^4+ac\,x^2}dx\right)a+\left(\int \frac{\sqrt{c\,x^4+a}}{cd\,x^6+c^2x^4+ad\,x^2+ac}dx\right)b+\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{cd\,x^6+c^2x^4+ad\,x^2+ac}dx\right)c$$

input
```
int((C*x^4+B*x^2+A)/x^2/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output

```
int(sqrt(a + c*x**4)/(a*c*x**2 + a*d*x**4 + c**2*x**6 + c*d*x**8),x)*a + i
nt(sqrt(a + c*x**4)/(a*c + a*d*x**2 + c**2*x**4 + c*d*x**6),x)*b + int((sq
rt(a + c*x**4)*x**2)/(a*c + a*d*x**2 + c**2*x**4 + c*d*x**6),x)*c
```

3.32 $\int \frac{A+Bx^2+Cx^4}{x^4(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 593

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx = -\frac{A\sqrt{a+cx^4}}{3acx^3}-\frac{(Bc-Ad)\sqrt{a+cx^4}}{ac^2x}$$
$$+\frac{(Bc-Ad)x\sqrt{a+cx^4}}{ac^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt{d}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{c^3+ad^2}x}{\sqrt{c}\sqrt{d}\sqrt{a+cx^4}}\right)}{2c^{5/2}\sqrt{c^3+ad^2}}$$
$$-\frac{(Bc-Ad)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{7/4}\sqrt{a+cx^4}}$$
$$-\frac{\left(Ac^3-\sqrt{a}c^{3/2}(3Bc-2Ad)-3a(c^2C-2Bcd+2Ad^2)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\operatorname{arcta}\right.}{6a^{5/4}c^{7/4}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}$$
$$-\frac{\left(c^{3/2}+\sqrt{a}d\right)\left(c^2C-Bcd+Ad^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(c^{3/2}-\sqrt{a}d\right)^2}{4\sqrt{a}c^{3/2}d},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{4\sqrt[4]{a}c^{13/4}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}$$

output
```
-1/3*A*(c*x^4+a)^(1/2)/a/c/x^3-(-A*d+B*c)*(c*x^4+a)^(1/2)/a/c^2/x+(-A*d+B*
c)*x*(c*x^4+a)^(1/2)/a/c^(3/2)/(a^(1/2)+c^(1/2)*x^2)+1/2*d^(1/2)*(A*d^2-B*
c*d+C*c^2)*arctan((a*d^2+c^3)^(1/2)*x/c^(1/2)/d^(1/2)/(c*x^4+a)^(1/2))/c^(
5/2)/(a*d^2+c^3)^(1/2)-(-A*d+B*c)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2
)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(
1/2))/a^(3/4)/c^(7/4)/(c*x^4+a)^(1/2)-1/6*(A*c^3-a^(1/2)*c^(3/2)*(-2*A*d+3
*B*c)-3*a*(2*A*d^2-2*B*c*d+C*c^2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/
2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2
^(1/2))/a^(5/4)/c^(7/4)/(c^(3/2)-a^(1/2)*d)/(c*x^4+a)^(1/2)-1/4*(c^(3/2)+a
^(1/2)*d)*(A*d^2-B*c*d+C*c^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^
(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(3
/2)-a^(1/2)*d)^2/a^(1/2)/c^(3/2)/d,1/2*2^(1/2))/a^(1/4)/c^(13/4)/(c^(3/2)-
a^(1/2)*d)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.15 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2+Cx^4}{x^4(c+dx^2)\sqrt{a+cx^4}}\,dx$$

$$= \frac{-aA\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c^2 - 3aB\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c^2x^2 + 3aA\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cdx^2 - A\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c^3x^4 - 3B\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c^3x^6 + 3A\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c^2dx^6 + 3\sqrt{a}c^3}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(-(a*A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^2) - 3*a*B*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^
2*x^2 + 3*a*A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d*x^2 - A*Sqrt[(I*Sqrt[c])/Sqrt[
a]]*c^3*x^4 - 3*B*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^3*x^6 + 3*A*Sqrt[(I*Sqrt[c])
/Sqrt[a]]*c^2*d*x^6 + 3*Sqrt[a]*c^(3/2)*(B*c - A*d)*x^3*Sqrt[1 + (c*x^4)/a
]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - c^(3/2)*((-I)*A*
c^(3/2) + 3*Sqrt[a]*(B*c - A*d))*x^3*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (3*I)*a*c^2*C*x^3*Sqrt[1 + (c*x^4)/
a]*EllipticPi[((-I)*Sqrt[a]*d)/c^(3/2), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]
]*x], -1] + (3*I)*a*B*c*d*x^3*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]
*d)/c^(3/2), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (3*I)*a*A*d^2*x
^3*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*d)/c^(3/2), I*ArcSinh[Sqrt
[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(3*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^3*x^3*Sqrt
[a + c*x^4])
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2245, 25, 2245, 25, 2233, 25, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a+cx^4}\left(c+dx^2\right)}dx$$

$$\Big\downarrow \text{2245}$$

$$-\frac{\int -\frac{-Acdx^4-c(Ac-3aC)x^2+3a(Bc-Ad)}{x^2(dx^2+c)\sqrt{cx^4+a}}dx}{3ac}-\frac{A\sqrt{a+cx^4}}{3acx^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-Acdx^4-c(Ac-3aC)x^2+3a(Bc-Ad)}{x^2(dx^2+c)\sqrt{cx^4+a}}dx}{3ac}-\frac{A\sqrt{a+cx^4}}{3acx^3}$$

$$\Big\downarrow \text{2245}$$

$$\frac{-\frac{\int -\frac{3acd(Bc-Ad)x^4+ac^2(3Bc-4Ad)x^2+a\left(3ac(cC-Bd)-A\left(c^3-3ad^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}(Bc-Ad)}{cx}}{3ac}-\frac{A\sqrt{a+cx^4}}{3acx^3}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{3acd(Bc-Ad)x^4+ac^2(3Bc-4Ad)x^2+a\left(3ac(cC-Bd)-A\left(c^3-3ad^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}(Bc-Ad)}{cx}}{3ac}-\frac{A\sqrt{a+cx^4}}{3acx^3}$$

$\downarrow$ 2233

$$\frac{\frac{\frac{\int -\frac{acd\left(Ac^3-3\sqrt{a}(Bc-Ad)c^{3/2}+d\left(Ac^{3/2}-\sqrt{a}(3Bc-3Ad)\right)x^2\sqrt{c}-3a\left(Cc^2-Bdc+Ad^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{cd}-3a^{3/2}\sqrt{c}(Bc-Ad)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}(Bc-Ad)}{cx}}{3ac}$$

$$\frac{A\sqrt{a+cx^4}}{3acx^3}$$

$\downarrow$ 25

$$\frac{\frac{-3a^{3/2}\sqrt{c}(Bc-Ad)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx-\frac{\int \frac{acd\left(Ac^3-3\sqrt{a}(Bc-Ad)c^{3/2}+d\left(Ac^{3/2}-\sqrt{a}(3Bc-3Ad)\right)x^2\sqrt{c}-3a\left(Cc^2-Bdc+Ad^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{cd}}{ac}-\frac{3\sqrt{a+cx^4}(Bc-Ad)}{cx}}{3ac}$$

$$\frac{A\sqrt{a+cx^4}}{3acx^3}$$

$\downarrow$ 27

$$\frac{\frac{-a\int\frac{Ac^3-3\sqrt{a}(Bc-Ad)c^{3/2}+d\left(Ac^{3/2}-\sqrt{a}(3Bc-3Ad)\right)x^2\sqrt{c}-3a\left(Cc^2-Bdc+Ad^2\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx-3a\sqrt{c}(Bc-Ad)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}(Bc-Ad)}{cx}}{3ac}-$$

$$\frac{A\sqrt{a+cx^4}}{3acx^3}$$

$\downarrow$ 1510

$$\frac{\frac{-a\int\frac{Ac^3-3\sqrt{a}(Bc-Ad)c^{3/2}+d\left(Ac^{3/2}-\sqrt{a}(3Bc-3Ad)\right)x^2\sqrt{c}-3a\left(Cc^2-Bdc+Ad^2\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx-3a\sqrt{c}(Bc-Ad)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{ac}}{3ac}$$

$$\frac{A\sqrt{a+cx^4}}{3acx^3}$$

$\downarrow$ 2227

$$\frac{-a\left(\frac{3a^{3/2}d\left(Ad^2-Bcd+c^2C\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}+\frac{\sqrt{c}\left(-\sqrt{a}c^{3/2}(3Bc-2Ad)-3a\left(2Ad^2-2Bcd+c^2C\right)+Ac^3\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}\right)-3a\sqrt{c}(Bc-Ad)\left(\frac{\sqrt[4]{a}\cdots}{\cdots}\right.}{\frac{ac}{3ac}}$$

$$\frac{A\sqrt{a+cx^4}}{3acx^3}$$

$\downarrow$ 27

$$\frac{-a\left(\frac{3ad\left(Ad^2-Bcd+c^2C\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}+\frac{\sqrt{c}\left(-\sqrt{a}c^{3/2}(3Bc-2Ad)-3a\left(2Ad^2-2Bcd+c^2C\right)+Ac^3\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}\right)-3a\sqrt{c}(Bc-Ad)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\cdots\right.}{\cdots}\right.}{\frac{ac}{3ac}}$$

$$\frac{A\sqrt{a+cx^4}}{3acx^3}$$

$\downarrow$ 761

$$\frac{-a\left(\frac{3ad\left(Ad^2-Bcd+c^2C\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{c^{3/2}-\sqrt{a}d}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-\sqrt{a}c^{3/2}(3Bc-2Ad)-3a\left(2Ad^2-2B\cdots\right.\right.}{2\sqrt[4]{a}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}\cdots\right.}{\frac{ac}{3ac}}$$

$$\frac{A\sqrt{a+cx^4}}{3acx^3}$$

$\downarrow$ 2221

$$\frac{-a\left(\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-\sqrt{a}c^{3/2}(3Bc-2Ad)-3a\left(2Ad^2-2Bcd+c^2C\right)+Ac^3\right)}{2\sqrt[4]{a}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}+\frac{3ad\left(Ad^2-Bcd+c^2C\right)\left(\frac{\left(\sqrt{a}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{A\sqrt{a+cx^4}}{3acx^3}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^4*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
-1/3*(A*Sqrt[a + c*x^4])/(a*c*x^3) + ((-3*(B*c - A*d)*Sqrt[a + c*x^4])/(c*
x) + (-3*a*Sqrt[c]*(B*c - A*d)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x
^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a +
 c*x^4])) - a*((c^(1/4)*(A*c^3 - Sqrt[a]*c^(3/2)*(3*B*c - 2*A*d) - 3*a*(c^
2*C - 2*B*c*d + 2*A*d^2))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1
/4)*(c^(3/2) - Sqrt[a]*d)*Sqrt[a + c*x^4]) + (3*a*d*(c^2*C - B*c*d + A*d^2
)*(-1/2*((c^(3/2) - Sqrt[a]*d)*ArcTan[(Sqrt[c^3 + a*d^2]*x)/(Sqrt[c]*Sqrt[
d]*Sqrt[a + c*x^4])]])/(Sqrt[c]*Sqrt[d]*Sqrt[c^3 + a*d^2]) + ((c^(3/2) + Sq
rt[a]*d)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticPi[-1/4*(Sqrt[a]*(c^(3/2)/Sqrt[a] - d)^2)/(c^(3/2)*d), 2*ArcTan
[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(5/4)*d*Sqrt[a + c*x^4])))/(c^(3
/2) - Sqrt[a]*d)))/(a*c))/(3*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2245
```
Int[((Px_)*(x_)^(m_))/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_
Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 2], C = Coeff[Px, x,
 4]}, Simp[A*x^(m + 1)*(Sqrt[a + c*x^4]/(a*d*(m + 1))), x] + Simp[1/(a*d*(m
 + 1))   Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + c*x^4]))*Simp[a*B*d*(m + 1) -
 A*a*e*(m + 1) + (a*C*d*(m + 1) - A*c*d*(m + 3))*x^2 - A*c*e*(m + 3)*x^4, x
], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2, 2] && ILtQ[m/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.58

method	result
default	$\frac{A\left(-\frac{\sqrt{c\,x^4+a}}{3a\,x^3}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)}{c}-\frac{(Ad-Bc)\left(-\frac{\sqrt{c\,x^4+a}}{ax}+\frac{i\sqrt{c}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x \cdots\right.\right.}{\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c \cdots}}\right.\cdots}{c^2}$
risch	$-\frac{\sqrt{c\,x^4+a}\,(-3Ad\,x^2+3Bc\,x^2+Ac)}{3c^2a\,x^3}-\frac{\frac{A\,c^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{3iB\,c^{\frac{3}{2}}\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{Ellipti}\cdots\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c \cdots}}\cdots}{\cdots}$
elliptic	$-\frac{A\sqrt{c\,x^4+a}}{3ac\,x^3}+\frac{(Ad-Bc)\sqrt{c\,x^4+a}}{a\,c^2x}-\frac{A\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\cdots}{\sqrt{c}\,\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+\cdots}}\cdots$

input
```
int((C*x^4+B*x^2+A)/x^4/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A/c*(-1/3/a*(c*x^4+a)^(1/2)/x^3-1/3*c/a/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(
1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*El
lipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I))-(A*d-B*c)/c^2*(-1/a*(c*x^4+a)^(1/2
)/x+I*c^(1/2)/a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^
(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I*c^(1
/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I)))+(A*d^2-B*
c*d+C*c^2)/c^3/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(
1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I*c^(1/2)/a^(
1/2))^(1/2),I/c^(3/2)*a^(1/2)*d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1
/2))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a+cx^4}\left(c+dx^2\right)}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**4*sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}(dx^2+c)x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}(dx^2+c)x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^4\,\sqrt{c\,x^4+a}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx = \frac{-\sqrt{c\,x^4+a}-3\left(\int \frac{\sqrt{c\,x^4+a}}{cd\,x^8+c^2x^6+ad\,x^4+ac\,x^2}dx\right)ad\,x^3+3\left(\int \frac{\sqrt{c\,x^4+a}}{cd\,x^8+c^2x^6+ad\,x^4+ac\,x^2}dx\right)bc\,x^3+2\left(\int \frac{\sqrt{c\,x^4+a}}{cd\,x^6+c^2x^4+ad}\right.}{3c\,x^3}$$

input
```
int((C*x^4+B*x^2+A)/x^4/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a + c*x**4) - 3*int(sqrt(a + c*x**4)/(a*c*x**2 + a*d*x**4 + c**2*
x**6 + c*d*x**8),x)*a*d*x**3 + 3*int(sqrt(a + c*x**4)/(a*c*x**2 + a*d*x**4
 + c**2*x**6 + c*d*x**8),x)*b*c*x**3 + 2*int(sqrt(a + c*x**4)/(a*c + a*d*x
**2 + c**2*x**4 + c*d*x**6),x)*c**2*x**3 - int((sqrt(a + c*x**4)*x**2)/(a*
c + a*d*x**2 + c**2*x**4 + c*d*x**6),x)*c*d*x**3)/(3*c*x**3)
```

3.33 $\int \frac{A+Bx^2+Cx^4}{x^6(c+dx^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 723

$$
\begin{aligned}
&\int \frac{A+Bx^2+Cx^4}{x^6(c+dx^2)\sqrt{a+cx^4}}\,dx \\
&= -\frac{A\sqrt{a+cx^4}}{5acx^5} - \frac{(Bc-Ad)\sqrt{a+cx^4}}{3ac^2x^3} - \frac{(5ac(cC-Bd)-A(3c^3-5ad^2))\sqrt{a+cx^4}}{5a^2c^3x} \\
&\quad + \frac{(5ac(cC-Bd)-A(3c^3-5ad^2))\,x\sqrt{a+cx^4}}{5a^2c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad - \frac{d^{3/2}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{c^3+ad^2}\,x}{\sqrt{c}\sqrt{d}\sqrt{a+cx^4}}\right)}{2c^{7/2}\sqrt{c^3+ad^2}} \\
&\quad - \frac{(5ac(cC-Bd)-A(3c^3-5ad^2))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5a^{7/4}c^{11/4}\sqrt{a+cx^4}} \\
&\quad - \frac{\left(9Ac^{9/2}+\sqrt{a}c^3(5Bc-14Ad)+30a^{3/2}d(c^2C-Bcd+Ad^2)-5ac^{3/2}(3c^2C-2Bcd+2Ad^2)\right)\left(\sqrt{a}+\right.}{30a^{7/4}c^{11/4}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}} \\
&\quad + \frac{d\left(c^{3/2}+\sqrt{a}d\right)\left(c^2C-Bcd+Ad^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(c^{3/2}-\sqrt{a}d\right)^2}{4\sqrt{a}c^{3/2}d},2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{4\sqrt[4]{a}c^{17/4}\left(c^{3/2}-\sqrt{a}d\right)\sqrt{a+cx^4}}
\end{aligned}
$$

output
```
-1/5*A*(c*x^4+a)^(1/2)/a/c/x^5-1/3*(-A*d+B*c)*(c*x^4+a)^(1/2)/a/c^2/x^3-1/
5*(5*a*c*(-B*d+C*c)-A*(-5*a*d^2+3*c^3))*(c*x^4+a)^(1/2)/a^2/c^3/x+1/5*(5*a
*c*(-B*d+C*c)-A*(-5*a*d^2+3*c^3))*x*(c*x^4+a)^(1/2)/a^2/c^(5/2)/(a^(1/2)+c
^(1/2)*x^2)-1/2*d^(3/2)*(A*d^2-B*c*d+C*c^2)*arctan((a*d^2+c^3)^(1/2)*x/c^(
1/2)/d^(1/2)/(c*x^4+a)^(1/2))/c^(7/2)/(a*d^2+c^3)^(1/2)-1/5*(5*a*c*(-B*d+C
*c)-A*(-5*a*d^2+3*c^3))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*
x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(7
/4)/c^(11/4)/(c*x^4+a)^(1/2)-1/30*(9*A*c^(9/2)+a^(1/2)*c^3*(-14*A*d+5*B*c)
+30*a^(3/2)*d*(A*d^2-B*c*d+C*c^2)-5*a*c^(3/2)*(2*A*d^2-2*B*c*d+3*C*c^2))*(
a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(7/4)/c^(11/4)/(c^(3/2)-a^
(1/2)*d)/(c*x^4+a)^(1/2)+1/4*d*(c^(3/2)+a^(1/2)*d)*(A*d^2-B*c*d+C*c^2)*(a^
(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(si
n(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(3/2)-a^(1/2)*d)^2/a^(1/2)/c^(3/2)/
d,1/2*2^(1/2))/a^(1/4)/c^(17/4)/(c^(3/2)-a^(1/2)*d)/(c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.68 (sec) , antiderivative size = 822, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx$$

$$= \frac{-3a^2A\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c^3 - 5a^2B\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c^3x^2 + 5a^2A\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c^2dx^2 + 6aA\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c^4x^4 - 15a^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c^3Cx^4 + 15a^2B\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}c}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
(-3*a^2*A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^3 - 5*a^2*B*Sqrt[(I*Sqrt[c])/Sqrt[a]
]*c^3*x^2 + 5*a^2*A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^2*d*x^2 + 6*a*A*Sqrt[(I*Sq
rt[c])/Sqrt[a]]*c^4*x^4 - 15*a^2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^3*C*x^4 + 15*
a^2*B*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^2*d*x^4 - 15*a^2*A*Sqrt[(I*Sqrt[c])/Sqrt
[a]]*c*d^2*x^4 - 5*a*B*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^4*x^6 + 5*a*A*Sqrt[(I*S
qrt[c])/Sqrt[a]]*c^3*d*x^6 + 9*A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^5*x^8 - 15*a*
Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^4*C*x^8 + 15*a*B*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^3
*d*x^8 - 15*a*A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^2*d^2*x^8 + 3*Sqrt[a]*c^(3/2)*
(5*a*c*(c*C - B*d) + A*(-3*c^3 + 5*a*d^2))*x^5*Sqrt[1 + (c*x^4)/a]*Ellipti
cE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - Sqrt[a]*c^(3/2)*(-9*A*c^3
 - (5*I)*Sqrt[a]*c^(3/2)*(B*c - A*d) + 15*a*(c^2*C - B*c*d + A*d^2))*x^5*S
qrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] +
 (15*I)*a^2*c^2*C*d*x^5*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*d)/c^
(3/2), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (15*I)*a^2*B*c*d^2*x^
5*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*d)/c^(3/2), I*ArcSinh[Sqrt[
(I*Sqrt[c])/Sqrt[a]]*x], -1] + (15*I)*a^2*A*d^3*x^5*Sqrt[1 + (c*x^4)/a]*El
lipticPi[((-I)*Sqrt[a]*d)/c^(3/2), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
 -1])/(15*a^2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^4*x^5*Sqrt[a + c*x^4])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a+cx^4}\,(c+dx^2)}\,dx$$

$$\Big\downarrow \; 2245$$

$$-\frac{\int -\frac{-3Acdx^4-c(3Ac-5aC)x^2+5a(Bc-Ad)}{x^4(dx^2+c)\sqrt{cx^4+a}}dx}{5ac}-\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-3Acdx^4-c(3Ac-5aC)x^2+5a(Bc-Ad)}{x^4(dx^2+c)\sqrt{cx^4+a}}dx}{5ac}-\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$$\Big\downarrow \; 2245$$

$$\frac{-\frac{\int \frac{5acd(Bc-Ad)x^4+ac^2(5Bc+4Ad)x^2+3a\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)}{x^2\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{3ac}-\frac{5\sqrt{a+cx^4}(Bc-Ad)}{3cx^3}}{5ac}-\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 2245

$$-\frac{-\frac{\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\right.}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(B\right.}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\right.}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(B\right.}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\right.}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(B\cdots\right.}{\cdots}}{3ac}}{5ac}\cdots$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{-\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\cdots\right.}{\cdots}}{3ac}}{5ac}\cdots$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(B\cdots\right.}{\cdots}}{3ac}}{5ac}\cdots$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{-\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\cdots\right.}{\cdots}}{3ac}}{5ac}\cdots$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(B\cdots\right.}{\cdots}}{3ac}}{5ac}\cdots$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\cdots\right.}{\cdots}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(B\cdots\right.}{\cdots}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\cdots\right.}{\cdots}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(B\cdots\right.}{\cdots}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\cdots\right.}{\cdots}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}(5ad(B}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\downarrow$ 25

$$-\frac{\frac{-\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}(5ad(Bc}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\downarrow$ 25

$$-\frac{\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}(5ad(B}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\downarrow$ 25

$$-\frac{\frac{-\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}(5ad(Bc}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\downarrow$ 25

$$-\frac{\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}(5ad(B}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\downarrow$ 25

$$-\frac{-\frac{-\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\right.}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(B\right.}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{-\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\right.}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(B\right.}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{-\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\right.}{}}{3ac}}{5ac}$$

$$\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\frac{\int -\frac{-3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(9Ac^3-15aCc^2+20aBdc-20aAd^2\right)x^2+5a^2\left(Adc^3-3aCdc^2-3aAd^3-B\left(c^4-3acd^2\right)\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(B\cdots\right.}{\cdots}}{3ac}}{5ac}-\frac{A\sqrt{a+cx^4}}{5acx^5}$$

$\downarrow$ 25

$$-\frac{-\frac{-\frac{\int -\frac{3acd\left((3Ac-5aC)c^2+5ad(Bc-Ad)\right)x^4-ac^2\left(5ac(3cC-4Bd)-A\left(9c^3-20ad^2\right)\right)x^2+5a^2\left(Bc^4-Adc^3+3aCdc^2-3aBd^2c+3aAd^3\right)}{\left(dx^2+c\right)\sqrt{cx^4+a}}dx}{ac}-\frac{3\sqrt{a+cx^4}\left(5ad(Bc\cdots\right.}{\cdots}}{3ac}}{5ac}-\frac{A\sqrt{a+cx^4}}{5acx^5}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^6*(c + d*x^2)*Sqrt[a + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2245
```
Int[((Px_)*(x_)^(m_))/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_
Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 2], C = Coeff[Px, x,
 4]}, Simp[A*x^(m + 1)*(Sqrt[a + c*x^4]/(a*d*(m + 1))), x] + Simp[1/(a*d*(m
 + 1))   Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + c*x^4]))*Simp[a*B*d*(m + 1) -
 A*a*e*(m + 1) + (a*C*d*(m + 1) - A*c*d*(m + 3))*x^2 - A*c*e*(m + 3)*x^4, x
], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2, 2] && ILtQ[m/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.45 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\sqrt{c\,x^{4}+a}\,\left(15 A a\,d^{2} x^{4}-9 A\,c^{3} x^{4}-15 B a c d\,x^{4}+15 C a\,c^{2} x^{4}-5 A a c d\,x^{2}+5 B a\,c^{2} x^{2}+3 A a\,c^{2}\right)}{15 c^{3} a^{2} x^{5}}+\frac{3 i \sqrt{c}\,\left(5 A a\,d^{2}-3 A\,c^{3}-5 a B c d+5 C a\,c^{2}\right)}{}$
default	$\frac{A\left(-\frac{\sqrt{c\,x^{4}+a}}{5 a\,x^{5}}+\frac{3 c\sqrt{c\,x^{4}+a}}{5 a^{2} x}-\frac{3 i\,c^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5 a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)}{c}-\frac{\left(A d-B c\right)\left(-\frac{\sqrt{c\,x^{4}+a}}{3 a\,x^{3}}-\right.}{}$
elliptic	Expression too large to display

input
```
int((C*x^4+B*x^2+A)/x^6/(d*x^2+c)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(c*x^4+a)^(1/2)*(15*A*a*d^2*x^4-9*A*c^3*x^4-15*B*a*c*d*x^4+15*C*a*c^
2*x^4-5*A*a*c*d*x^2+5*B*a*c^2*x^2+3*A*a*c^2)/c^3/a^2/x^5+1/15/a^2/c^3*(3*I
*c^(1/2)*(5*A*a*d^2-3*A*c^3-5*B*a*c*d+5*C*a*c^2)*a^(1/2)/(I*c^(1/2)/a^(1/2
))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(
c*x^4+a)^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^
(1/2)/a^(1/2))^(1/2),I))-5*B*c^3*a/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*
x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*Ellipti
cF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+5*A*a*c^2*d/(I*c^(1/2)/a^(1/2))^(1/2)*(1
-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1
/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-15*d*a^2*(A*d^2-B*c*d+C*c^2)/
c/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x
^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),I
/c^(3/2)*a^(1/2)*d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}(dx^2+c)x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="fric
as")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + a)/(c*d*x^12 + c^2*x^10 + a*d*x^
8 + a*c*x^6), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a+cx^4}\left(c+dx^2\right)}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**6/(d*x**2+c)/(c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**6*sqrt(a + c*x**4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}(dx^2+c)x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+a}(dx^2+c)x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(d*x^2+c)/(c*x^4+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + a)*(d*x^2 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^6\,\sqrt{c\,x^4+a}\,\left(d\,x^2+c\right)}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^6*(a + c*x^4)^(1/2)*(c + d*x^2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^6*(a + c*x^4)^(1/2)*(c + d*x^2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\left(c+dx^2\right)\sqrt{a+cx^4}}\,dx$$
$$=\frac{-3\sqrt{c\,x^4+a}\,a-5\sqrt{c\,x^4+a}\,b\,x^2-15\left(\int \frac{\sqrt{c\,x^4+a}}{cd\,x^{10}+c^2x^8+ad\,x^6+ac\,x^4}dx\right)a^2d\,x^5-15\left(\int \frac{\sqrt{c\,x^4+a}}{cd\,x^8+c^2x^6+ad\,x^4+ac\,x^2}dx\right)}{}$$

input
```
int((C*x^4+B*x^2+A)/x^6/(d*x^2+c)/(c*x^4+a)^(1/2),x)
```

output
```
( - 3*sqrt(a + c*x**4)*a - 5*sqrt(a + c*x**4)*b*x**2 - 15*int(sqrt(a + c*x
**4)/(a*c*x**4 + a*d*x**6 + c**2*x**8 + c*d*x**10),x)*a**2*d*x**5 - 15*int
(sqrt(a + c*x**4)/(a*c*x**2 + a*d*x**4 + c**2*x**6 + c*d*x**8),x)*a*b*d*x*
*5 + 6*int(sqrt(a + c*x**4)/(a*c*x**2 + a*d*x**4 + c**2*x**6 + c*d*x**8),x
)*a*c**2*x**5 - 9*int(sqrt(a + c*x**4)/(a*c + a*d*x**2 + c**2*x**4 + c*d*x
**6),x)*a*c*d*x**5 - 5*int(sqrt(a + c*x**4)/(a*c + c**2*x**4 +
c*d*x**6),x)*b*c**2*x**5 - 5*int((sqrt(a + c*x**4)*x**2)/(a*c + a*d*x**2 +
 c**2*x**4 + c*d*x**6),x)*b*c*d*x**5)/(15*a*c*x**5)
```

3.34 $\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 874

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$$
$$= \frac{(50Acde(3cd^2-4ae^2)-B(105c^2d^4-92acd^2e^2+256a^2e^4))\sqrt{d+ex^2}\sqrt{a-cx^4}}{3840c^2e^4x}$$
$$+ \frac{(35Bcd^3-50Acd^2e-28aBde^2-120aAe^3)\,x\sqrt{d+ex^2}\sqrt{a-cx^4}}{1920ce^3}$$
$$- \frac{(7Bcd^2-10Acde+16aBe^2)\,x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{480ce^2}$$
$$+ \frac{(Bd+10Ae)x^5\sqrt{d+ex^2}\sqrt{a-cx^4}}{80e} + \frac{1}{10}Bx^7\sqrt{d+ex^2}\sqrt{a-cx^4}$$
$$+ \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(50Acde(3cd^2-4ae^2)-B(105c^2d^4-92acd^2e^2+256a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\Big(}{3840ce^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{\sqrt{a}(10Acde(5cd^2-44ae^2)-B(35c^2d^4-36acd^2e^2+256a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\Big(}{3840c^{3/2}e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{(2Ae(5c^2d^4-8acd^2e^2+16a^2e^4)-B(7c^2d^5-8acd^3e^2-16a^2de^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{Elliptic}}{256ce^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/3840*(50*A*c*d*e*(-4*a*e^2+3*c*d^2)-B*(256*a^2*e^4-92*a*c*d^2*e^2+105*c^
2*d^4))*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c^2/e^4/x+1/1920*(-120*A*a*e^3-50
*A*c*d^2*e-28*B*a*d*e^2+35*B*c*d^3)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e
^3-1/480*(-10*A*c*d*e+16*B*a*e^2+7*B*c*d^2)*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)
^(1/2)/c/e^2+1/80*(10*A*e+B*d)*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e+1/10
*B*x^7*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)+1/3840*(d+a^(1/2)*e/c^(1/2))*(50*A
*c*d*e*(-4*a*e^2+3*c*d^2)-B*(256*a^2*e^4-92*a*c*d^2*e^2+105*c^2*d^4))*(1-a
/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Elli
pticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^
(1/2)))^(1/2))/c/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/3840*a^(1/2)*(10*A
*c*d*e*(-44*a*e^2+5*c*d^2)-B*(256*a^2*e^4-36*a*c*d^2*e^2+35*c^2*d^4))*(1-a
/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Elli
pticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^
(1/2)))^(1/2))/c^(3/2)/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/256*(2*A*e*(
16*a^2*e^4-8*a*c*d^2*e^2+5*c^2*d^4)-B*(-16*a^2*d*e^4-8*a*c*d^3*e^2+7*c^2*d
^5))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(
1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+
a^(1/2)*e/c^(1/2)))^(1/2))/c/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$$

input
```
Integrate[x^4*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4],x]
```

output
```
Integrate[x^4*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt{a - cx^4} \left(A + Bx^2\right) \sqrt{d + ex^2} \, dx$$

$$\Big\downarrow \text{ 2251}$$

$$\int x^4 \sqrt{a - cx^4} \left(A + Bx^2\right) \sqrt{d + ex^2} dx$$

input
```
Int[x^4*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int x^{4} \left(B \,x^{2}+A \right) \sqrt{e \,x^{2}+d}\, \sqrt{-c \,x^{4}+a}dx$$

input
```
int(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

output
```
int(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}x^4\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="fri
cas")
```

output
```
integral((B*x^6 + A*x^4)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int x^4(A+Bx^2)\sqrt{a-cx^4}\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**4*(B*x**2+A)*(e*x**2+d)**(1/2)*(-c*x**4+a)**(1/2),x)
```

output
```
Integral(x**4*(A + B*x**2)*sqrt(a - c*x**4)*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}x^4\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)*x^4, x)
```

Giac [F]

$$\int x^4\left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int\sqrt{-cx^4+a}\left(Bx^2+A\right)\sqrt{ex^2+d}x^4\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int x^4\,\left(B\,x^2+A\right)\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}\,dx$$

input
```
int(x^4*(A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2),x)
```

output
```
int(x^4*(A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^4\left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\text{Too large to display}$$

input
```
int(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

output
```
( - 120*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*e**3*x - 28*sqrt(d + e*x**2
)*sqrt(a - c*x**4)*a*b*d*e**2*x - 64*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b
*e**3*x**3 - 50*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**2*e*x + 40*sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*a*c*d*e**2*x**3 + 240*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*a*c*e**3*x**5 + 35*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**3*x
- 28*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*e*x**3 + 24*sqrt(d + e*x**
2)*sqrt(a - c*x**4)*b*c*d*e**2*x**5 + 192*sqrt(d + e*x**2)*sqrt(a - c*x**4
)*b*c*e**3*x**7 + 256*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d +
a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*b*e**4 + 200*int((sqrt(d + e*x**2)
*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d
*e**3 - 92*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 -
c*d*x**4 - c*e*x**6),x)*a*b*c*d**2*e**2 - 150*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c**2*d**3*e +
 105*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x*
*4 - c*e*x**6),x)*b*c**2*d**4 + 240*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)
*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**3*e**4 + 248*int((sqrt
(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6)
,x)*a**2*b*d*e**3 - 20*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d +
 a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d**2*e**2 + 14*int((sqrt(d + e*
x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*...
```

3.35 $\int x^2(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 737

$$\int x^2(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$$
$$= \frac{(15Bcd^3-24Acd^2e-20aBde^2-64aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{384ce^3x}$$
$$- \frac{(5Bcd^2-8Acde+12aBe^2)\,x\sqrt{d+ex^2}\sqrt{a-cx^4}}{192ce^2}$$
$$+ \frac{(Bd+8Ae)x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{48e} + \frac{1}{8}Bx^5\sqrt{d+ex^2}\sqrt{a-cx^4}$$
$$- \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(8Ae(3cd^2+8ae^2)-5B(3cd^3-4ade^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\right.}{384e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{\sqrt{a}(5Bcd^3-8Acd^2e-44aBde^2-64aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\right.}{384\sqrt{c}e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{(8Acde(cd^2-4ae^2)-B(5c^2d^4-8acd^2e^2+16a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\right.}{128ce^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/384*(-64*A*a*e^3-24*A*c*d^2*e-20*B*a*d*e^2+15*B*c*d^3)*(e*x^2+d)^(1/2)*(
-c*x^4+a)^(1/2)/c/e^3/x-1/192*(-8*A*c*d*e+12*B*a*e^2+5*B*c*d^2)*x*(e*x^2+d
)^(1/2)*(-c*x^4+a)^(1/2)/c/e^2+1/48*(8*A*e+B*d)*x^3*(e*x^2+d)^(1/2)*(-c*x^
4+a)^(1/2)/e+1/8*B*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)-1/384*(d+a^(1/2)*e
/c^(1/2))*(8*A*e*(8*a*e^2+3*c*d^2)-5*B*(-4*a*d*e^2+3*c*d^3))*(1-a/c/x^4)^(
1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2
*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(
1/2))/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/384*a^(1/2)*(-64*A*a*e^3-8*A*
c*d^2*e-44*B*a*d*e^2+5*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(
c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2
)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e^2/(e*x^2+d)^(
1/2)/(-c*x^4+a)^(1/2)-1/128*(8*A*c*d*e*(-4*a*e^2+c*d^2)-B*(16*a^2*e^4-8*a*
c*d^2*e^2+5*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+
a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2)
,2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/e^3/(e*x^2+d)^(1/2)/(-c*x^4+
a)^(1/2)
```

Mathematica [F]

$$\int x^2(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int x^2(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$$

input
```
Integrate[x^2*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4],x]
```

output
```
Integrate[x^2*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}\,dx$$

$$\Big\downarrow \; 2251$$

$$\int x^2\sqrt{a-cx^4}\left(A+Bx^2\right)\sqrt{d+ex^2}dx$$

input
```
Int[x^2*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int x^2\left(B\,x^2+A\right)\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}dx$$

input
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

output
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int x^2\left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int\sqrt{-cx^4+a}\left(Bx^2+A\right)\sqrt{ex^2+d}x^2\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="fri
cas")
```

output
```
integral((B*x^4 + A*x^2)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int x^2(A+Bx^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int x^2(A+Bx^2)\,\sqrt{a-cx^4}\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**2*(B*x**2+A)*(e*x**2+d)**(1/2)*(-c*x**4+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2)*sqrt(a - c*x**4)*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int x^2(A+Bx^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}x^2\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)*x^2, x)
```

Giac [F]

$$\int x^2(A+Bx^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}x^2\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (A+Bx^2) \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx = \int x^2 \left(B\,x^2+A\right)\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}\,dx$$

input
```
int(x^2*(A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2),x)
```

output
```
int(x^2*(A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2 (A+Bx^2) \sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$$
$$= \frac{-12\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,ab\,e^2x+8\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,acdex+32\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,ac\,e^2x^3-5}{}$$

input
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

output
```
( - 12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*e**2*x + 8*sqrt(d + e*x**2)*s
qrt(a - c*x**4)*a*c*d*e*x + 32*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*e**2*
x**3 - 5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*x + 4*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*b*c*d*e*x**3 + 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*
e**2*x**5 + 64*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**
2 - c*d*x**4 - c*e*x**6),x)*a**2*c*e**3 + 20*int((sqrt(d + e*x**2)*sqrt(a
- c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d*e**2 + 2
4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4
- c*e*x**6),x)*a*c**2*d**2*e - 15*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x
**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c**2*d**3 + 24*int((sqrt(
d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),
x)*a**2*b*e**3 + 80*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*
e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d*e**2 - 2*int((sqrt(d + e*x**2)*s
qrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d**2
*e + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4
 - c*e*x**6),x)*a**2*b*d*e**2 - 8*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/
(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d**2*e + 5*int((sqrt(d +
e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*
d**3)/(192*c*e**2)
```

3.36 $\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 623

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx = -\frac{(3Bcd^2-6Acde+8aBe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{48ce^2x}$$
$$+\frac{(Bd+6Ae)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{24e}+\frac{1}{6}Bx^3\sqrt{d+ex^2}\sqrt{a-cx^4}$$
$$-\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(3Bcd^2-6Acde+8aBe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{a}(Bcd^2+30Acde+8aBe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48\sqrt{c}e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{(Bcd^3-2Acd^2e-4aBde^2-8aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\ldots\right.}{16e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/48*(-6*A*c*d*e+8*B*a*e^2+3*B*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/
e^2/x+1/24*(6*A*e+B*d)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e+1/6*B*x^3*(e*x
^2+d)^(1/2)*(-c*x^4+a)^(1/2)-1/48*(d+a^(1/2)*e/c^(1/2))*(-6*A*c*d*e+8*B*a*
e^2+3*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)
*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/4
8*a^(1/2)*(30*A*c*d*e+8*B*a*e^2+B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e
*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/
x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e/(e*x
^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/16*(-8*A*a*e^3-2*A*c*d^2*e-4*B*a*d*e^2+B*c*
d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(
1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+
a^(1/2)*e/c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$$

input
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4],x]
```

output
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}\,dx$$

$$\Big\downarrow \text{ 2261}$$

$$\int \sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}dx$$

input
```
Int[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right)\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx = \int (A+Bx^2)\sqrt{a-cx^4}\sqrt{d+ex^2}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx = \int \sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx = \int \sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx=\int \left(B\,x^2+A\right)\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2),x)
```

output
```
int((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a-cx^4}\,dx$$
$$=\frac{6\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,aex+\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,bdx+4\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,be\,x^3+8\left(\int\frac{\sqrt{e\,x^2+d}}{-ce\,x^6-cd}\right.}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2),x)
```

output
```
(6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*e*x + sqrt(d + e*x**2)*sqrt(a - c*x
**4)*b*d*x + 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*e*x**3 + 8*int((sqrt(d
+ e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)
*a*b*e**2 - 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2
 - c*d*x**4 - c*e*x**6),x)*a*c*d*e + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c*d**2 + 12*int((sqr
t(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6
),x)*a**2*e**2 + 10*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*
e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*d*e + 18*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*d*e - int((sqrt
(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a
*b*d**2)/(24*e)
```

3.37 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 530

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^2}\,dx = \frac{(Bd+4Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{8ex}+\frac{1}{4}Bx\sqrt{d+ex^2}\sqrt{a-cx^4} + \frac{c(Bd+12Ae)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{\sqrt{a}\sqrt{c}(5Bd-4Ae)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{(Bcd^2-4Acde+4aBe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/8*(4*A*e+B*d)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e/x+1/4*B*x*(e*x^2+d)^(1/
2)*(-c*x^4+a)^(1/2)+1/8*c*(12*A*e+B*d)*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(
1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2
*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(
1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/8*a^(1/2)*c^(1/2)*(-4*A*e+5*B*d
)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2
)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2
)*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/8*(-4*A*c*d*e+4*B*
a*e^2+B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)
*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1
/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^2}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^2}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^2,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}}{x^2}\,dx$$
$$\Big\downarrow \; 2251$$
$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}}{x^2}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2 + A)\,\sqrt{e\,x^2 + d}\,\sqrt{-c\,x^4 + a}}{x^2}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^2,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^2,x)
```

Fricas [F]

$$\int \frac{(A + Bx^2)\,\sqrt{d + ex^2}\sqrt{a - cx^4}}{x^2}\,dx = \int \frac{\sqrt{-cx^4 + a}(Bx^2 + A)\sqrt{ex^2 + d}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^2,x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^2}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}\sqrt{d+ex^2}}{x^2}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(-c*x**4+a)**(1/2)/x**2,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*sqrt(d + e*x**2)/x**2, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^2}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^2,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^2, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^2}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^2,x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^2}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}{x^2}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^2,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^2}\,dx$$
$$= \frac{-2\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,abe+4\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,acd+\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,bcd\,x^2-4\left(\int \frac{\sqrt{e\,x^2}}{-ce\,x^6}\right.}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*e + 4*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*a*c*d + sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d*x**2 - 4*int((sq
rt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**
6),x)*a*b*c*e**2*x + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d +
 a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c**2*d*e*x - int((sqrt(d + e*x**2)*s
qrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c**2*d**
2*x - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**2 + a*e*x**4 - c*d
*x**6 - c*e*x**8),x)*a**2*b*d*e*x + 8*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4))/(a*d*x**2 + a*e*x**4 - c*d*x**6 - c*e*x**8),x)*a**2*c*d**2*x + 4*int((
sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),
x)*a**2*c*d*e*x + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**
2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d**2*x)/(4*c*d*x)
```

3.38 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 524

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^4}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{3x^3} + \frac{B\sqrt{d+ex^2}\sqrt{a-cx^4}}{2x}$$
$$+ \frac{c(9Bd+2Ae)\left(d+\frac{\sqrt{ae}}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{6d\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{\sqrt{c}(4Acd^2+3aBde+2aAe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{6\sqrt{a}d\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{c(Bd+2Ae)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/3*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^3+1/2*B*(e*x^2+d)^(1/2)*(-c*x^4+
a)^(1/2)/x+1/6*c*(2*A*e+9*B*d)*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3
*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1
/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d/
(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/6*c^(1/2)*(2*A*a*e^2+4*A*c*d^2+3*B*a*d*
e)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/
2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/
2)*e/c^(1/2)))^(1/2))/a^(1/2)/d/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/2*c*(2*
A*e+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^
2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d
/(d+a^(1/2)*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^4}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^4}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^4,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}}{x^4}\,dx$$

$$\Big\downarrow \; 2251$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}}{x^4}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}{x^{4}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^4,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^4,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^4}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^4,x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^4}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}\sqrt{d+ex^2}}{x^4}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(-c*x**4+a)**(1/2)/x**4,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*sqrt(d + e*x**2)/x**4, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^4}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^4,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^4, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^4}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^4,x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^4}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}{x^4}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^4,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^4}\,dx = \frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,ade+\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,a\,e^2x^2-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,b\,d^2+\sqrt{e\,x^2+d}\,\sqrt{}}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^4,x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*d*e + sqrt(d + e*x**2)*sqrt(a - c*
x**4)*a*e**2*x**2 - sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*d**2 + sqrt(d + e*
x**2)*sqrt(a - c*x**4)*b*d*e*x**2 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*e**3*x**3 + int((sq
rt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**
6),x)*b*c*d*e**2*x**3 - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**
4 + a*e*x**6 - c*d*x**8 - c*e*x**10),x)*a**2*d**2*e*x**3 - 3*int((sqrt(d +
 e*x**2)*sqrt(a - c*x**4))/(a*d*x**4 + a*e*x**6 - c*d*x**8 - c*e*x**10),x)
*a*b*d**3*x**3 + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 -
 c*d*x**4 - c*e*x**6),x)*a*b*d*e**2*x**3 + int((sqrt(d + e*x**2)*sqrt(a -
c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c*d**3*x**3)/(d*e*x**
3)
```

3.39 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^6}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 551

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^6}\,dx$$
$$= -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{5x^5} - \frac{(5Bd+Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{15dx^3}$$
$$- \frac{c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(6Acd^2-5aBde+2aAe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15ad^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{\sqrt{c}(10Bcd^3+2Acd^2e+5aBde^2-2aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\ldots\right.}{15\sqrt{a}d^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{Bce\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/5*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^5-1/15*(A*e+5*B*d)*(e*x^2+d)^(1/
2)*(-c*x^4+a)^(1/2)/d/x^3-1/15*c*(d+a^(1/2)*e/c^(1/2))*(2*A*a*e^2+6*A*c*d^
2-5*B*a*d*e)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e
)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(
d/(d+a^(1/2)*e/c^(1/2))))^(1/2))/a/d^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/1
5*c^(1/2)*(-2*A*a*e^3+2*A*c*d^2*e+5*B*a*d*e^2+10*B*c*d^3)*(1-a/c/x^4)^(1/2
)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1
-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2))))^(1/2
))/a^(1/2)/d^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-B*c*e*(1-a/c/x^4)^(1/2)*x^
3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^
(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2))))^(1/2)
)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^6}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^6}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^6,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^6, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}}{x^6}\,dx$$
$$\Big\downarrow \text{ 2251}$$
$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}}{x^6}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^6,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{x^6}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^6,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^6,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^6}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^6,x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^6, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^6}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}\sqrt{d+ex^2}}{x^6}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(-c*x**4+a)**(1/2)/x**6,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*sqrt(d + e*x**2)/x**6, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^6}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^6,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^6, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^6}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^6,x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^6}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}{x^6}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^6,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^6}\,dx = \text{Too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^6,x)
```

output

```
( - 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*e - 5*sqrt(d + e*x**2)*sqrt(a -
c*x**4)*b*e*x**2 + 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d*x**2 - 40*int((
sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(4*a**2*d*e**2 + 4*a**2*e**3*x**2
+ 3*a*c*d**3 + 3*a*c*d**2*e*x**2 - 4*a*c*d*e**2*x**4 - 4*a*c*e**3*x**6 - 3
*c**2*d**3*x**4 - 3*c**2*d**2*e*x**6),x)*a*b*c*e**4*x**5 - 30*int((sqrt(d
+ e*x**2)*sqrt(a - c*x**4)*x**2)/(4*a**2*d*e**2 + 4*a**2*e**3*x**2 + 3*a*c
*d**3 + 3*a*c*d**2*e*x**2 - 4*a*c*d*e**2*x**4 - 4*a*c*e**3*x**6 - 3*c**2*d
**3*x**4 - 3*c**2*d**2*e*x**6),x)*b*c**2*d**2*e**2*x**5 + 8*int((sqrt(d +
e*x**2)*sqrt(a - c*x**4))/(4*a**2*d*e**2*x**4 + 4*a**2*e**3*x**6 + 3*a*c*d
**3*x**4 + 3*a*c*d**2*e*x**6 - 4*a*c*d*e**2*x**8 - 4*a*c*e**3*x**10 - 3*c*
*2*d**3*x**8 - 3*c**2*d**2*e*x**10),x)*a**3*e**4*x**5 - 20*int((sqrt(d + e
*x**2)*sqrt(a - c*x**4))/(4*a**2*d*e**2*x**4 + 4*a**2*e**3*x**6 + 3*a*c*d*
*3*x**4 + 3*a*c*d**2*e*x**6 - 4*a*c*d*e**2*x**8 - 4*a*c*e**3*x**10 - 3*c**
2*d**3*x**8 - 3*c**2*d**2*e*x**10),x)*a**2*b*d*e**3*x**5 + 30*int((sqrt(d
+ e*x**2)*sqrt(a - c*x**4))/(4*a**2*d*e**2*x**4 + 4*a**2*e**3*x**6 + 3*a*c
*d**3*x**4 + 3*a*c*d**2*e*x**6 - 4*a*c*d*e**2*x**8 - 4*a*c*e**3*x**10 - 3*
c**2*d**3*x**8 - 3*c**2*d**2*e*x**10),x)*a**2*c*d**2*e**2*x**5 - 15*int((s
qrt(d + e*x**2)*sqrt(a - c*x**4))/(4*a**2*d*e**2*x**4 + 4*a**2*e**3*x**6 +
 3*a*c*d**3*x**4 + 3*a*c*d**2*e*x**6 - 4*a*c*d*e**2*x**8 - 4*a*c*e**3*x**1
0 - 3*c**2*d**3*x**8 - 3*c**2*d**2*e*x**10),x)*a*b*c*d**3*e*x**5 + 18*i...
```

3.40 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^8}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 488

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^8}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{7x^7} - \frac{(7Bd+Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{35dx^5} + \frac{(10Acd^2-7aBde+4aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{105ad^2x^3} - \frac{2c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(21Bcd^3+8Acd^2e+7aBde^2-4aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\right.}{105ad^3\sqrt{d+ex^2}\sqrt{a-cx^4}} - \frac{2\sqrt{c}(cd^2-ae^2)(5Acd^2+7aBde-4aAe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\right.}{105a^{3/2}d^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/7*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^7-1/35*(A*e+7*B*d)*(e*x^2+d)^(1/
2)*(-c*x^4+a)^(1/2)/d/x^5+1/105*(4*A*a*e^2+10*A*c*d^2-7*B*a*d*e)*(e*x^2+d)
^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^3-2/105*c*(d+a^(1/2)*e/c^(1/2))*(-4*A*a*e^
3+8*A*c*d^2*e+7*B*a*d*e^2+21*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^
2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2
)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d^3/(e*x^2+d)^(
1/2)/(-c*x^4+a)^(1/2)-2/105*c^(1/2)*(-a*e^2+c*d^2)*(-4*A*a*e^2+5*A*c*d^2+7
*B*a*d*e)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x
^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(
d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/d^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^8}\,dx=\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^8}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^8,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^8, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}}{x^8}\,dx$$
$$\Big\downarrow \text{ 2251}$$
$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}}{x^8}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^8,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2 + A)\,\sqrt{e\,x^2 + d}\,\sqrt{-c\,x^4 + a}}{x^8}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^8,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^8,x)
```

Fricas [F]

$$\int \frac{(A + Bx^2)\,\sqrt{d + ex^2}\sqrt{a - cx^4}}{x^8}\,dx = \int \frac{\sqrt{-cx^4 + a}(Bx^2 + A)\sqrt{ex^2 + d}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^8,x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^8, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^8}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}\sqrt{d+ex^2}}{x^8}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(-c*x**4+a)**(1/2)/x**8,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*sqrt(d + e*x**2)/x**8, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^8}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^8,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^8, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^8}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^8,x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^8}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}{x^8}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^8,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^8, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^8}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^8,x)
```

output

```
( - 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*d*e**4 - 84*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*a**3*b*d*e**4*x**2 + 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)
*a**3*c*d**3*e**2 - 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c*d**2*e**3*
x**2 + 48*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c*d*e**4*x**4 - 24*sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*a**3*c*e**5*x**6 + 91*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*a**2*b*c*d**3*e**2*x**2 - 168*sqrt(d + e*x**2)*sqrt(a - c*x**4)*
a**2*b*c*d**2*e**3*x**4 + 252*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c*d
*e**4*x**6 + 30*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**4*e*x**2 -
60*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**3*e**2*x**4 + 42*sqrt(d
+ e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**2*e**3*x**6 + 35*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a*b*c**2*d**5*x**2 - 70*sqrt(d + e*x**2)*sqrt(a - c*x**4)
*a*b*c**2*d**4*e*x**4 + 399*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c**2*d**
3*e**2*x**6 + 90*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**3*d**4*e*x**6 + 10
5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c**3*d**5*x**6 - 576*int((sqrt(d + e
*x**2)*sqrt(a - c*x**4)*x**4)/(12*a**2*d*e**2 + 12*a**2*e**3*x**2 - a*c*d*
*3 - a*c*d**2*e*x**2 - 12*a*c*d*e**2*x**4 - 12*a*c*e**3*x**6 + c**2*d**3*x
**4 + c**2*d**2*e*x**6),x)*a**4*c**2*e**8*x**7 + 6048*int((sqrt(d + e*x**2
)*sqrt(a - c*x**4)*x**4)/(12*a**2*d*e**2 + 12*a**2*e**3*x**2 - a*c*d**3 -
a*c*d**2*e*x**2 - 12*a*c*d*e**2*x**4 - 12*a*c*e**3*x**6 + c**2*d**3*x**4 +
 c**2*d**2*e*x**6),x)*a**3*b*c**2*d*e**7*x**7 + 1056*int((sqrt(d + e*x*...
```

3.41 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{10}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 583

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{10}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{9x^9} - \frac{(9Bd+Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{63dx^7} + \frac{(14Acd^2-9aBde+6aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{315ad^2x^5} + \frac{2(15Bcd^3+4Acd^2e+6aBde^2-4aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{315ad^3x^3} - \frac{2c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(12aBde(2cd^2-ae^2)+A(21c^2d^4-9acd^2e^2+8a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\text{arcs}\right.}{315a^2d^4\sqrt{d+ex^2}\sqrt{a-cx^4}} - \frac{2\sqrt{c}(cd^2-ae^2)(15Bcd^3-3Acd^2e-12aBde^2+8aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\text{arcsin}\right.}{315a^{3/2}d^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/9*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^9-1/63*(A*e+9*B*d)*(e*x^2+d)^(1/
2)*(-c*x^4+a)^(1/2)/d/x^7+1/315*(6*A*a*e^2+14*A*c*d^2-9*B*a*d*e)*(e*x^2+d)
^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^5+2/315*(-4*A*a*e^3+4*A*c*d^2*e+6*B*a*d*e^
2+15*B*c*d^3)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^3/x^3-2/315*c*(d+a^(1/2
)*e/c^(1/2))*(12*a*B*d*e*(-a*e^2+2*c*d^2)+A*(8*a^2*e^4-9*a*c*d^2*e^2+21*c^
2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2
)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+
a^(1/2)*e/c^(1/2)))^(1/2))/a^2/d^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-2/315*
c^(1/2)*(-a*e^2+c*d^2)*(8*A*a*e^3-3*A*c*d^2*e-12*B*a*d*e^2+15*B*c*d^3)*(1-
a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ell
ipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c
^(1/2)))^(1/2))/a^(3/2)/d^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{10}}\,dx=\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{10}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^10,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^10, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}}{x^{10}}\,dx$$

$$\Big\downarrow \; 2251$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}}{x^{10}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^10,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{x^{10}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^10,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^10,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{10}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^10,x, algorithm="fr
icas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^10, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{10}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}\sqrt{d+ex^2}}{x^{10}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(-c*x**4+a)**(1/2)/x**10,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*sqrt(d + e*x**2)/x**10, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{10}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^10,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^10, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{10}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^10,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{10}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}{x^{10}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^10,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^10, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{10}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^10,x)
```

output

```
( - 1440*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*d**2*e**6 + 1440*sqrt(d +
e*x**2)*sqrt(a - c*x**4)*a**5*d*e**7*x**2 - 1728*sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*a**5*e**8*x**4 - 1440*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*b*d*
*2*e**6*x**2 - 864*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*b*d*e**7*x**4 +
2640*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c*d**3*e**5*x**2 - 2592*sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*a**4*c*d**2*e**6*x**4 + 288*sqrt(d + e*x**2)*s
qrt(a - c*x**4)*a**4*c*d*e**7*x**6 + 1620*sqrt(d + e*x**2)*sqrt(a - c*x**4
)*a**3*b*c*d**4*e**4*x**2 - 1944*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*
c*d**3*e**5*x**4 + 2304*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*c*d**2*e*
*6*x**6 + 10*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2*d**6*e**2 + 2290*
sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2*d**5*e**3*x**2 - 2748*sqrt(d +
 e*x**2)*sqrt(a - c*x**4)*a**3*c**2*d**4*e**4*x**4 + 1392*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*a**3*c**2*d**3*e**5*x**6 + 1485*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*a**2*b*c**2*d**6*e**2*x**2 - 1764*sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*a**2*b*c**2*d**5*e**3*x**4 + 324*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**
2*b*c**2*d**4*e**4*x**6 + 840*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**3*
d**7*e*x**2 - 1012*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**3*d**6*e**2*x
**4 + 458*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**3*d**5*e**3*x**6 + 945
*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c**3*d**8*x**2 - 1134*sqrt(d + e*x*
*2)*sqrt(a - c*x**4)*a*b*c**3*d**7*e*x**4 + 279*sqrt(d + e*x**2)*sqrt(a...
```

3.42 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{12}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 700

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{12}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{11x^{11}}$$

$$-\frac{(11Bd+Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{99dx^9}+\frac{(18Acd^2-11aBde+8aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{693ad^2x^7}$$

$$+\frac{2(77Bcd^3+16Acd^2e+33aBde^2-24aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{3465ad^3x^5}$$

$$+\frac{2(44aBde(cd^2-ae^2)+A(75c^2d^4-23acd^2e^2+32a^2e^4))\sqrt{d+ex^2}\sqrt{a-cx^4}}{3465a^2d^4x^3}$$

$$-\frac{2c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(2Ae(39c^2d^4+27acd^2e^2-32a^2e^4)+11B(21c^2d^5-9acd^3e^2+8a^2de^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{}}{(\sqrt{}}}}{3465a^2d^5\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{2\sqrt{c}(cd^2-ae^2)(11aBde(3cd^2-8ae^2)-A(75c^2d^4+6acd^2e^2-64a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{El}}{3465a^{5/2}d^5\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/11*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^11-1/99*(A*e+11*B*d)*(e*x^2+d)^
(1/2)*(-c*x^4+a)^(1/2)/d/x^9+1/693*(8*A*a*e^2+18*A*c*d^2-11*B*a*d*e)*(e*x^
2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^7+2/3465*(-24*A*a*e^3+16*A*c*d^2*e+33*
B*a*d*e^2+77*B*c*d^3)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^3/x^5+2/3465*(4
4*a*B*d*e*(-a*e^2+c*d^2)+A*(32*a^2*e^4-23*a*c*d^2*e^2+75*c^2*d^4))*(e*x^2+
d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/d^4/x^3-2/3465*c*(d+a^(1/2)*e/c^(1/2))*(2*A*
e*(-32*a^2*e^4+27*a*c*d^2*e^2+39*c^2*d^4)+11*B*(8*a^2*d*e^4-9*a*c*d^3*e^2+
21*c^2*d^5))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e
)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(
d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/d^5/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+2
/3465*c^(1/2)*(-a*e^2+c*d^2)*(11*a*B*d*e*(-8*a*e^2+3*c*d^2)-A*(-64*a^2*e^4
+6*a*c*d^2*e^2+75*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1
/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^
(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(5/2)/d^5/(e*x^2+d)^(1/2)
/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{12}}\,dx=\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{12}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^12,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^12, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\sqrt{d+ex^2}}{x^{12}}\,dx$$

$$\Big\downarrow \; 2251$$

$$\int \frac{\sqrt{a-cx^4}\,(A+Bx^2)\,\sqrt{d+ex^2}}{x^{12}}\,dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a - c*x^4])/x^12,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{x^{12}}\,dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^12,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^12,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{12}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)\sqrt{ex^2+d}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^12,x, algorithm="fr
icas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^12, x)
```

Sympy [F]

$$\int \frac{(A + Bx^2)\sqrt{d + ex^2}\sqrt{a - cx^4}}{x^{12}}\,dx = \int \frac{(A + Bx^2)\sqrt{a - cx^4}\sqrt{d + ex^2}}{x^{12}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(-c*x**4+a)**(1/2)/x**12,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*sqrt(d + e*x**2)/x**12, x)
```

Maxima [F]

$$\int \frac{(A + Bx^2)\sqrt{d + ex^2}\sqrt{a - cx^4}}{x^{12}}\,dx = \int \frac{\sqrt{-cx^4 + a}(Bx^2 + A)\sqrt{ex^2 + d}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^12,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^12, x)
```

Giac [F]

$$\int \frac{(A + Bx^2)\sqrt{d + ex^2}\sqrt{a - cx^4}}{x^{12}}\,dx = \int \frac{\sqrt{-cx^4 + a}(Bx^2 + A)\sqrt{ex^2 + d}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^12,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{12}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}{x^{12}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^12,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))/x^12, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{x^{12}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^12,x)
```

output

```
( - 120960*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**7*d**2*e**8 - 221760*sqrt(
d + e*x**2)*sqrt(a - c*x**4)*a**6*b*d**2*e**8*x**2 + 63360*sqrt(d + e*x**2
)*sqrt(a - c*x**4)*a**6*b*d*e**9*x**4 + 2016*sqrt(d + e*x**2)*sqrt(a - c*x
**4)*a**6*c*d**4*e**6 - 40320*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**6*c*d**
3*e**7*x**2 + 80640*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**6*c*d**2*e**8*x**
4 + 77616*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*b*c*d**4*e**6*x**2 - 8553
6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*b*c*d**3*e**7*x**4 - 158400*sqrt(
d + e*x**2)*sqrt(a - c*x**4)*a**5*b*c*d**2*e**8*x**6 + 316800*sqrt(d + e*x
**2)*sqrt(a - c*x**4)*a**5*b*c*d*e**9*x**8 - 380160*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*a**5*b*c*e**10*x**10 + 3528*sqrt(d + e*x**2)*sqrt(a - c*x**4)*
a**5*c**2*d**6*e**4 - 20496*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*c**2*d*
*5*e**5*x**2 + 22848*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*c**2*d**4*e**6
*x**4 - 80640*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*c**2*d**3*e**7*x**6 +
 161280*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*c**2*d**2*e**8*x**8 + 12166
0*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*b*c**2*d**6*e**4*x**2 - 133496*sq
rt(d + e*x**2)*sqrt(a - c*x**4)*a**4*b*c**2*d**5*e**5*x**4 - 7920*sqrt(d +
 e*x**2)*sqrt(a - c*x**4)*a**4*b*c**2*d**4*e**6*x**6 + 15840*sqrt(d + e*x*
*2)*sqrt(a - c*x**4)*a**4*b*c**2*d**3*e**7*x**8 - 849024*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a**4*b*c**2*d**2*e**8*x**10 + 210*sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*a**4*c**3*d**8*e**2 + 75180*sqrt(d + e*x**2)*sqrt(a - c*x**4...
```

3.43 $\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 874

$$\int x^2\left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx =$$

$$-\frac{(30Acde(3cd^2+28ae^2)-B(45c^2d^4-108acd^2e^2-256a^2e^4))\sqrt{d+ex^2}\sqrt{a-cx^4}}{3840c^2e^3x}$$

$$-\frac{(15Bcd^3-30Acd^2e+148aBde^2+120aAe^3)\,x\sqrt{d+ex^2}\sqrt{a-cx^4}}{1920ce^2}$$

$$+\frac{(3Bcd^2+90Acde-16aBe^2)\,x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{480ce}$$

$$+\frac{1}{80}(11Bd+10Ae)x^5\sqrt{d+ex^2}\sqrt{a-cx^4}+\frac{1}{10}Bex^7\sqrt{d+ex^2}\sqrt{a-cx^4}$$

$$-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)(30Acde(3cd^2+28ae^2)-B(45c^2d^4-108acd^2e^2-256a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\right.}{3840c^{3/2}e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{\sqrt{a}(30Acde(cd^2+36ae^2)-B(15c^2d^4-404acd^2e^2-256a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\text{arc}\right.}{3840c^{3/2}e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{(B(3c^2d^5-8acd^3e^2+48a^2de^4)-A(6c^2d^4e-48acd^2e^3-32a^2e^5))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(\right.}{256ce^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/3840*(30*A*c*d*e*(28*a*e^2+3*c*d^2)-B*(-256*a^2*e^4-108*a*c*d^2*e^2+45*
c^2*d^4))*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c^2/e^3/x-1/1920*(120*A*a*e^3-3
0*A*c*d^2*e+148*B*a*d*e^2+15*B*c*d^3)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c
/e^2+1/480*(90*A*c*d*e-16*B*a*e^2+3*B*c*d^2)*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a
)^(1/2)/c/e+1/80*(10*A*e+11*B*d)*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)+1/10
*B*e*x^7*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)-1/3840*(c^(1/2)*d+a^(1/2)*e)*(30
*A*c*d*e*(28*a*e^2+3*c*d^2)-B*(-256*a^2*e^4-108*a*c*d^2*e^2+45*c^2*d^4))*(
1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*E
llipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e
/c^(1/2)))^(1/2))/c^(3/2)/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/3840*a^(1
/2)*(30*A*c*d*e*(36*a*e^2+c*d^2)-B*(-256*a^2*e^4-404*a*c*d^2*e^2+15*c^2*d^
4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1
/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1
/2)*e/c^(1/2)))^(1/2))/c^(3/2)/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/256*
(B*(48*a^2*d*e^4-8*a*c*d^3*e^2+3*c^2*d^5)-A*(-32*a^2*e^5-48*a*c*d^2*e^3+6*
c^2*d^4*e))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)
/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int x^2 \left(A+Bx^2\right) \left(d+ex^2\right)^{3/2} \sqrt{a-cx^4}\,dx=\int x^2 \left(A+Bx^2\right) \left(d+ex^2\right)^{3/2} \sqrt{a-cx^4}\,dx$$

input
```
Integrate[x^2*(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4],x]
```

output
```
Integrate[x^2*(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a-cx^4}\left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 2251}$$

$$\int x^2\sqrt{a-cx^4}\left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}dx$$

input
```
Int[x^2*(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int x^{2}\left(B\,x^{2}+A\right)\left(e\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{-c\,x^{4}+a}dx$$

input
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

output
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}(Bx^2+A)\left(ex^2+d\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="fri
cas")
```

output
```
integral((B*e*x^6 + (B*d + A*e)*x^4 + A*d*x^2)*sqrt(-c*x^4 + a)*sqrt(e*x^2
 + d), x)
```

Sympy [F]

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int x^2(A+Bx^2)\sqrt{a-cx^4}\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(B*x**2+A)*(e*x**2+d)**(3/2)*(-c*x**4+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2)*sqrt(a - c*x**4)*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}(Bx^2+A)\left(ex^2+d\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int\sqrt{-cx^4+a}\left(Bx^2+A\right)\left(ex^2+d\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int x^2\,\left(B\,x^2+A\right)\,\sqrt{a-c\,x^4}\,{\left(e\,x^2+d\right)}^{3/2}\,dx$$

input
```
int(x^2*(A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2),x)
```

output
```
int(x^2*(A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\text{Too large to display}$$

input
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

output
```
( - 120*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*e**3*x - 148*sqrt(d + e*x**
2)*sqrt(a - c*x**4)*a*b*d*e**2*x - 64*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*
b*e**3*x**3 + 30*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**2*e*x + 360*sqrt
(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*e**2*x**3 + 240*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*a*c*e**3*x**5 - 15*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**3*
x + 12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*e*x**3 + 264*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*b*c*d*e**2*x**5 + 192*sqrt(d + e*x**2)*sqrt(a - c*x
**4)*b*c*e**3*x**7 + 256*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d
 + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*b*e**4 + 840*int((sqrt(d + e*x*
*2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*
c*d*e**3 + 108*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**
2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d**2*e**2 + 90*int((sqrt(d + e*x**2)*sqr
t(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c**2*d**3*
e - 45*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*
x**4 - c*e*x**6),x)*b*c**2*d**4 + 240*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**3*e**4 + 488*int((sq
rt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**
6),x)*a**2*b*d*e**3 + 780*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*
d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d**2*e**2 - 6*int((sqrt(d +
e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x...
```

3.44 $\int (A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 732

$$\int (A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}\,dx =$$

$$-\frac{(9Bcd^3-24Acd^2e+84aBde^2+64aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{384ce^2x}$$

$$+\frac{(3Bcd^2+56Acde-12aBe^2)\,x\sqrt{d+ex^2}\sqrt{a-cx^4}}{192ce}$$

$$+\frac{1}{48}(9Bd+8Ae)x^3\sqrt{d+ex^2}\sqrt{a-cx^4}+\frac{1}{8}Bex^5\sqrt{d+ex^2}\sqrt{a-cx^4}$$

$$-\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(9Bcd^3-24Acd^2e+84aBde^2+64aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{}{d}\right.}{384e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{\sqrt{a}(3Bcd^3+248Acd^2e+108aBde^2+64aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\right.}{384\sqrt{c}e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{(8Acde(cd^2+12ae^2)-B(3c^2d^4-24acd^2e^2-16a^2e^4))\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\right.\right.}{128ce^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/384*(64*A*a*e^3-24*A*c*d^2*e+84*B*a*d*e^2+9*B*c*d^3)*(e*x^2+d)^(1/2)*(-
c*x^4+a)^(1/2)/c/e^2/x+1/192*(56*A*c*d*e-12*B*a*e^2+3*B*c*d^2)*x*(e*x^2+d)
^(1/2)*(-c*x^4+a)^(1/2)/c/e+1/48*(8*A*e+9*B*d)*x^3*(e*x^2+d)^(1/2)*(-c*x^4
+a)^(1/2)+1/8*B*e*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)-1/384*(d+a^(1/2)*e/
c^(1/2))*(64*A*a*e^3-24*A*c*d^2*e+84*B*a*d*e^2+9*B*c*d^3)*(1-a/c/x^4)^(1/2
)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1
-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2
))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/384*a^(1/2)*(64*A*a*e^3+248*A*c*
d^2*e+108*B*a*d*e^2+3*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c
^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)
*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e/(e*x^2+d)^(1/2
)/(-c*x^4+a)^(1/2)+1/128*(8*A*c*d*e*(12*a*e^2+c*d^2)-B*(-16*a^2*e^4-24*a*c
*d^2*e^2+3*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a
^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),
2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a
)^(1/2)
```

Mathematica [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx$$

input
```
Integrate[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4],x]
```

output
```
Integrate[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-cx^4}(A+Bx^2)\left(d+ex^2\right)^{3/2}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \sqrt{a-cx^4}(A+Bx^2)\left(d+ex^2\right)^{3/2}dx$$

input
```
Int[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right)\left(e\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{-c\,x^{4}+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="fricas"
)
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
), x)
```

Sympy [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \left(A+Bx^2\right)\sqrt{a-cx^4}\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}\left(Bx^2+A\right)\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}\,dx=\int \sqrt{-cx^4+a}\left(Bx^2+A\right)\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2) (d+ex^2)^{3/2} \sqrt{a-cx^4}\, dx = \int (B\,x^2+A)\, \sqrt{a-c\,x^4}\, (e\,x^2+d)^{3/2}\, dx$$

input
```
int((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2),x)
```

output
```
int((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int (A+Bx^2) (d+ex^2)^{3/2} \sqrt{a-cx^4}\, dx = \frac{-12\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,ab\,e^2x+56\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,acdex+32\sqrt{e\,x^2+d}}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2),x)
```

output
```
( - 12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*e**2*x + 56*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a*c*d*e*x + 32*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*e**2
*x**3 + 3*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*x + 36*sqrt(d + e*x**
2)*sqrt(a - c*x**4)*b*c*d*e*x**3 + 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*
c*e**2*x**5 + 64*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x
**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*e**3 + 84*int((sqrt(d + e*x**2)*sqrt(
a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d*e**2 -
 24*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**
4 - c*e*x**6),x)*a*c**2*d**2*e + 9*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*
x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c**2*d**3 + 24*int((sqrt
(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6)
,x)*a**2*b*e**3 + 176*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d +
a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d*e**2 + 78*int((sqrt(d + e*x**2
)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d
**2*e + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x
**4 - c*e*x**6),x)*a**2*b*d*e**2 + 136*int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d**2*e - 3*int((sqrt
(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a
*b*c*d**3)/(192*c*e)
```

3.45 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 620

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^2}\,dx = \frac{(3Bcd^2+30Acde-8aBe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{48cex}$$

$$+\frac{1}{24}(7Bd+6Ae)x\sqrt{d+ex^2}\sqrt{a-cx^4}+\frac{1}{6}Bex^3\sqrt{d+ex^2}\sqrt{a-cx^4}$$

$$+\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(3Bcd^2+78Acde-8aBe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{\sqrt{a}(31Bcd^2+6Acde+8aBe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{(Bcd^3-6Acd^2e+12aBde^2+8aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{16e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/48*(30*A*c*d*e-8*B*a*e^2+3*B*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e
/x+1/24*(6*A*e+7*B*d)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)+1/6*B*e*x^3*(e*x^
2+d)^(1/2)*(-c*x^4+a)^(1/2)+1/48*(d+a^(1/2)*e/c^(1/2))*(78*A*c*d*e-8*B*a*e
^2+3*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*
e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*
(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/48*a
^(1/2)*(6*A*c*d*e+8*B*a*e^2+31*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*
x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x
^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/(e*x^2+
d)^(1/2)/(-c*x^4+a)^(1/2)+1/16*(8*A*a*e^3-6*A*c*d^2*e+12*B*a*d*e^2+B*c*d^3
)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2
)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(
1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^2}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^2}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^2,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)(d+ex^2)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \ 2251$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)(d+ex^2)^{3/2}}{x^2}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,(e\,x^{2}+d)^{\frac{3}{2}}\,\sqrt{-c\,x^{4}+a}}{x^{2}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^2,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^2,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^2}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^2,x, algorithm="fri
cas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)/x^2, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^2}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}(d+ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(-c*x**4+a)**(1/2)/x**2,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*(d + e*x**2)**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^2}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^2,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^2}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^2,x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^2}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}\,(e\,x^2+d)^{3/2}}{x^2}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^2,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^2}\,dx=\frac{-12\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,a^2e^2-22\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,abde}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^2,x)
```

output
```
( - 12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*e**2 - 22*sqrt(d + e*x**2)*s
qrt(a - c*x**4)*a*b*d*e + 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**2 +
6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*e*x**2 + 7*sqrt(d + e*x**2)*sqrt
(a - c*x**4)*b*c*d**2*x**2 + 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d*e*x
**4 - 24*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*
d*x**4 - c*e*x**6),x)*a**2*c*e**3*x - 36*int((sqrt(d + e*x**2)*sqrt(a - c*
x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d*e**2*x + 18*
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 -
c*e*x**6),x)*a*c**2*d**2*e*x - 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x*
*4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c**2*d**3*x - 12*int((sqrt
(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**2 + a*e*x**4 - c*d*x**6 - c*e*x**8)
,x)*a**3*d*e**2*x - 22*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**2 +
 a*e*x**4 - c*d*x**6 - c*e*x**8),x)*a**2*b*d**2*e*x + 48*int((sqrt(d + e*x
**2)*sqrt(a - c*x**4))/(a*d*x**2 + a*e*x**4 - c*d*x**6 - c*e*x**8),x)*a**2
*c*d**3*x + 42*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c
*d*x**4 - c*e*x**6),x)*a**2*c*d**2*e*x + 17*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d**3*x)/(24*c*d*
x)
```

3.46 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 569

$$\begin{aligned}\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^4}\,dx &= -\frac{Ad\sqrt{d+ex^2}\sqrt{a-cx^4}}{3x^3}\\
&+\frac{(5Bd+4Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{8x}+\frac{1}{4}Bex\sqrt{d+ex^2}\sqrt{a-cx^4}\\
&+\frac{c(39Bd+44Ae)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{24\sqrt{d+ex^2}\sqrt{a-cx^4}}\\
&-\frac{\sqrt{c}(16Acd^2-3aBde+20aAe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{24\sqrt{a}\sqrt{d+ex^2}\sqrt{a-cx^4}}\\
&-\frac{(3Bcd^2+12Acde-4aBe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output
```
-1/3*A*d*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^3+1/8*(4*A*e+5*B*d)*(e*x^2+d)^
(1/2)*(-c*x^4+a)^(1/2)/x+1/4*B*e*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)+1/24*c
*(44*A*e+39*B*d)*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x
^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^
2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/
(-c*x^4+a)^(1/2)-1/24*c^(1/2)*(20*A*a*e^2+16*A*c*d^2-3*B*a*d*e)*(1-a/c/x^4
)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(
1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2))
)^(1/2))/a^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/8*(12*A*c*d*e-4*B*a*e^
2+3*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e
)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2
)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^4}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^4}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^4,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)(d+ex^2)^{3/2}}{x^4}\,dx$$
$$\Big\downarrow \text{2251}$$
$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)(d+ex^2)^{3/2}}{x^4}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{\left(B\,x^{2}+A\right)\left(e\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{-c\,x^{4}+a}}{x^{4}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^4,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^4,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}{x^4}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^4,x, algorithm="fri
cas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)/x^4, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^4}\,dx=\int \frac{(A+Bx^2)\sqrt{a-cx^4}(d+ex^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(-c*x**4+a)**(1/2)/x**4,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*(d + e*x**2)**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^4}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^4,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^4}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^4,x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}{x^4}\,dx=\int \frac{\left(B\,x^2+A\right)\,\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^{3/2}}{x^4}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^4,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}{x^4}\,dx=\frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,abd\,e^2-2\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,ab\,e^3x^2-}{}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^4,x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*d*e**2 - 2*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*a*b*e**3*x**2 - 8*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**2*e +
 8*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*e**2*x**2 - 4*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*b*c*d**3 + 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*e
*x**2 + sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d*e**2*x**4 - 4*int((sqrt(d
+ e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)
*a*b*c*e**4*x**3 + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d +
a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c**2*d*e**3*x**3 + 3*int((sqrt(d + e*
x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c
**2*d**2*e**2*x**3 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**4 +
 a*e*x**6 - c*d*x**8 - c*e*x**10),x)*a**2*b*d**2*e**2*x**3 - 20*int((sqrt(
d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**4 + a*e*x**6 - c*d*x**8 - c*e*x**10)
,x)*a**2*c*d**3*e*x**3 - 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x
**4 + a*e*x**6 - c*d*x**8 - c*e*x**10),x)*a*b*c*d**4*x**3 + 4*int((sqrt(d
+ e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2
*c*d*e**3*x**3 + 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2
 - c*d*x**4 - c*e*x**6),x)*a*b*c*d**2*e**2*x**3 + 4*int((sqrt(d + e*x**2)*
sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c**2*d**3*e*
x**3 + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**
4 - c*e*x**6),x)*b*c**2*d**4*x**3)/(4*c*d*e*x**3)
```

3.47 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^6}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 592

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^6}\,dx = -\frac{Ad\sqrt{d+ex^2}\sqrt{a-cx^4}}{5x^5}$$

$$-\frac{(5Bd+6Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{15x^3}+\frac{Be\sqrt{d+ex^2}\sqrt{a-cx^4}}{2x}$$

$$-\frac{c\left(d+\frac{\sqrt{ae}}{\sqrt{c}}\right)(12Acd^2-55aBde-6aAe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{30ad\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$-\frac{\sqrt{c}(20Bcd^3+24Acd^2e+25aBde^2+6aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\right.}{30\sqrt{a}d\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$-\frac{ce(3Bd+2Ae)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/5*A*d*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^5-1/15*(6*A*e+5*B*d)*(e*x^2+d)
^(1/2)*(-c*x^4+a)^(1/2)/x^3+1/2*B*e*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x-1/3
0*c*(d+a^(1/2)*e/c^(1/2))*(-6*A*a*e^2+12*A*c*d^2-55*B*a*d*e)*(1-a/c/x^4)^(
1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2
*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(
1/2))/a/d/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/30*c^(1/2)*(6*A*a*e^3+24*A*c*
d^2*e+25*B*a*d*e^2+20*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c
^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)
*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/d/(e*x^2+d)^(1/2
)/(-c*x^4+a)^(1/2)-1/2*c*e*(2*A*e+3*B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e
*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)
/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/(e*x^2+d)^(
1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^6}\,dx=\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^6}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^6,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^6, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}\,(A+Bx^2)\,(d+ex^2)^{3/2}}{x^6}\,dx$$

$$\Big\downarrow \; 2251$$

$$\int \frac{\sqrt{a-cx^4}\,(A+Bx^2)\,(d+ex^2)^{3/2}}{x^6}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^6,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,(e\,x^{2}+d)^{\frac{3}{2}}\,\sqrt{-c\,x^{4}+a}}{x^{6}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^6,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^6,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^6}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^6,x, algorithm="fri
cas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)/x^6, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^6}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}(d+ex^2)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(-c*x**4+a)**(1/2)/x**6,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*(d + e*x**2)**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^6}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^6,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^6}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^6,x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^6}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}\,(e\,x^2+d)^{3/2}}{x^6}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^6,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^6}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^6,x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*d**2*e - 5*sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*a*d*e**2*x**2 + 5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*e**3*x**4
 - 10*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*d**2*e*x**2 + 10*sqrt(d + e*x**2
)*sqrt(a - c*x**4)*b*d*e**2*x**4 + sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d**
3*x**2 + 40*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d*e**2 +
4*a**2*e**3*x**2 + 3*a*c*d**3 + 3*a*c*d**2*e*x**2 - 4*a*c*d*e**2*x**4 - 4*
a*c*e**3*x**6 - 3*c**2*d**3*x**4 - 3*c**2*d**2*e*x**6),x)*a**2*c*e**6*x**5
 + 60*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d*e**2 + 4*a**2
*e**3*x**2 + 3*a*c*d**3 + 3*a*c*d**2*e*x**2 - 4*a*c*d*e**2*x**4 - 4*a*c*e*
*3*x**6 - 3*c**2*d**3*x**4 - 3*c**2*d**2*e*x**6),x)*a*b*c*d*e**5*x**5 + 30
*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d*e**2 + 4*a**2*e**3
*x**2 + 3*a*c*d**3 + 3*a*c*d**2*e*x**2 - 4*a*c*d*e**2*x**4 - 4*a*c*e**3*x*
*6 - 3*c**2*d**3*x**4 - 3*c**2*d**2*e*x**6),x)*a*c**2*d**2*e**4*x**5 + 45*
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d*e**2 + 4*a**2*e**3*
x**2 + 3*a*c*d**3 + 3*a*c*d**2*e*x**2 - 4*a*c*d*e**2*x**4 - 4*a*c*e**3*x**
6 - 3*c**2*d**3*x**4 - 3*c**2*d**2*e*x**6),x)*b*c**2*d**3*e**3*x**5 - 36*i
nt((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(4*a**2*d*e**2*x**4 + 4*a**2*e**3*x
**6 + 3*a*c*d**3*x**4 + 3*a*c*d**2*e*x**6 - 4*a*c*d*e**2*x**8 - 4*a*c*e**3
*x**10 - 3*c**2*d**3*x**8 - 3*c**2*d**2*e*x**10),x)*a**3*d**2*e**4*x**5 -
100*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(4*a**2*d*e**2*x**4 + 4*a**...
```

3.48 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^8}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 637

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^8}\,dx = -\frac{Ad\sqrt{d+ex^2}\sqrt{a-cx^4}}{7x^7} - \frac{(7Bd+8Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{35x^5} + \frac{(10Acd^2-42aBde-3aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{105adx^3} - \frac{c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(42Bcd^3+58Acd^2e-21aBde^2+6aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\right.}{105ad^2\sqrt{d+ex^2}\sqrt{a-cx^4}} - \frac{\sqrt{c}(21aBde(4cd^2+ae^2)+2A(5c^2d^4-2acd^2e^2-3a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\right.\right.}{105a^{3/2}d^2\sqrt{d+ex^2}\sqrt{a-cx^4}} - \frac{Bce^2\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/7*A*d*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^7-1/35*(8*A*e+7*B*d)*(e*x^2+d)
^(1/2)*(-c*x^4+a)^(1/2)/x^5+1/105*(-3*A*a*e^2+10*A*c*d^2-42*B*a*d*e)*(e*x^
2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^3-1/105*c*(d+a^(1/2)*e/c^(1/2))*(6*A*a*e
^3+58*A*c*d^2*e-21*B*a*d*e^2+42*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e
*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/
x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d^2/(e*x^2+d
)^(1/2)/(-c*x^4+a)^(1/2)-1/105*c^(1/2)*(21*a*B*d*e*(a*e^2+4*c*d^2)+2*A*(-3
*a^2*e^4-2*a*c*d^2*e^2+5*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d
)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(
1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/d^2/(e*x^2+d
)^(1/2)/(-c*x^4+a)^(1/2)-B*c*e^2*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/
(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1
/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c
*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^8}\,dx=\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^8}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^8,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^8, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\,(d+ex^2)^{3/2}}{x^8}\,dx$$
$$\Big\downarrow \text{ 2251}$$
$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)\,(d+ex^2)^{3/2}}{x^8}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^8,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{\left(B\,x^{2}+A\right)\left(e\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{-c\,x^{4}+a}}{x^{8}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^8,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^8,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}{x^8}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^8,x, algorithm="fri
cas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)/x^8, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^8}\,dx = \int \frac{(A+Bx^2)\,\sqrt{a-cx^4}(d+ex^2)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(-c*x**4+a)**(1/2)/x**8,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*(d + e*x**2)**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^8}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^8,x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^8}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^8,x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^8}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}\,(e\,x^2+d)^{3/2}}{x^8}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^8,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^8, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^8}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^8,x)
```

output
```
(336*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*e**6*x**2 + 840*sqrt(d + e*x**
2)*sqrt(a - c*x**4)*a**3*b*d*e**5*x**2 - 120*sqrt(d + e*x**2)*sqrt(a - c*x
**4)*a**3*c*d**3*e**3 - 532*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c*d**2*
e**4*x**2 + 1344*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c*d*e**5*x**4 - 16
80*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c*e**6*x**6 - 1078*sqrt(d + e*x*
*2)*sqrt(a - c*x**4)*a**2*b*c*d**3*e**3*x**2 + 2940*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*a**2*b*c*d**2*e**4*x**4 - 4200*sqrt(d + e*x**2)*sqrt(a - c*x**
4)*a**2*b*c*d*e**5*x**6 - 150*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**2*
d**5*e - 918*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**4*e**2*x**2 +
1752*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**3*e**3*x**4 - 3900*sqr
t(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**2*e**4*x**6 - 1960*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*a*b*c**2*d**5*e*x**2 + 3003*sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*a*b*c**2*d**4*e**2*x**4 - 9450*sqrt(d + e*x**2)*sqrt(a - c*x**4
)*a*b*c**2*d**3*e**3*x**6 - 150*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**3*d
**6*x**2 + 678*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**3*d**5*e*x**4 - 2610
*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**3*d**4*e**2*x**6 + 210*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*b*c**3*d**6*x**4 - 5250*sqrt(d + e*x**2)*sqrt(a - c
*x**4)*b*c**3*d**5*e*x**6 - 450*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c**4*d**
6*x**6 - 13440*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(4*a**2*d*e**2
 + 4*a**2*e**3*x**2 + 5*a*c*d**3 + 5*a*c*d**2*e*x**2 - 4*a*c*d*e**2*x**...
```

3.49 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{10}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 582

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{10}}\,dx = -\frac{Ad\sqrt{d+ex^2}\sqrt{a-cx^4}}{9x^9} - \frac{(9Bd+10Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{63x^7} + \frac{(14Acd^2-72aBde-3aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{315adx^5} + \frac{(30Bcd^3+38Acd^2e-9aBde^2+4aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{315ad^2x^3} - \frac{2c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(3aBde(29cd^2+3ae^2)+A(21c^2d^4+15acd^2e^2-4a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\text{arcs}\right.}{315a^2d^3\sqrt{d+ex^2}\sqrt{a-cx^4}} - \frac{2\sqrt{c}(cd^2-ae^2)(15Bcd^3+12Acd^2e+9aBde^2-4aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{315a^{3/2}d^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/9*A*d*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^9-1/63*(10*A*e+9*B*d)*(e*x^2+d
)^(1/2)*(-c*x^4+a)^(1/2)/x^7+1/315*(-3*A*a*e^2+14*A*c*d^2-72*B*a*d*e)*(e*x
^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^5+1/315*(4*A*a*e^3+38*A*c*d^2*e-9*B*a*d
*e^2+30*B*c*d^3)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^3-2/315*c*(d+a^(
1/2)*e/c^(1/2))*(3*a*B*d*e*(3*a*e^2+29*c*d^2)+A*(-4*a^2*e^4+15*a*c*d^2*e^2
+21*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*
e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*
(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/d^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-
2/315*c^(1/2)*(-a*e^2+c*d^2)*(-4*A*a*e^3+12*A*c*d^2*e+9*B*a*d*e^2+15*B*c*d
^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1
/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1
/2)*e/c^(1/2)))^(1/2))/a^(3/2)/d^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{10}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{10}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^10,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^10, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)(d+ex^2)^{3/2}}{x^{10}}\,dx$$
$$\Big\downarrow \; 2251$$
$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)(d+ex^2)^{3/2}}{x^{10}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^10,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{\left(B\,x^{2}+A\right)\left(e\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{-c\,x^{4}+a}}{x^{10}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^10,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^10,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^{10}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^10,x, algorithm="fr
icas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)/x^10, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{10}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}(d+ex^2)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(-c*x**4+a)**(1/2)/x**10,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*(d + e*x**2)**(3/2)/x**10, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{10}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^10,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^10, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{10}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^10,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}{x^{10}}\,dx=\int \frac{\left(B\,x^2+A\right)\,\sqrt{a-c\,x^4}\,{\left(e\,x^2+d\right)}^{3/2}}{x^{10}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^10,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^10, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}{x^{10}}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^10,x)
```

output

```
(5760*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*b*d*e**8*x**2 - 6912*sqrt(d +
 e*x**2)*sqrt(a - c*x**4)*a**5*b*e**9*x**4 - 1440*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*a**5*c*d**3*e**6 - 2592*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*c
*d*e**8*x**4 + 9720*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*b*c*d**3*e**6*x
**2 - 16848*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*b*c*d**2*e**7*x**4 + 11
52*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*b*c*d*e**8*x**6 + 4260*sqrt(d +
e*x**2)*sqrt(a - c*x**4)*a**4*c**2*d**4*e**5*x**2 - 4536*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a**4*c**2*d**3*e**6*x**4 + 2592*sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*a**4*c**2*d**2*e**7*x**6 + 11430*sqrt(d + e*x**2)*sqrt(a - c*x**4
)*a**3*b*c**2*d**5*e**4*x**2 - 13716*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**
3*b*c**2*d**4*e**5*x**4 + 7128*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*c*
*2*d**3*e**6*x**6 + 10*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**3*d**7*e*
*2 + 3775*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**3*d**6*e**3*x**2 - 451
2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**3*d**5*e**4*x**4 + 1716*sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*a**3*c**3*d**4*e**5*x**6 + 5895*sqrt(d + e*x**
2)*sqrt(a - c*x**4)*a**2*b*c**3*d**7*e**2*x**2 - 7038*sqrt(d + e*x**2)*sqr
t(a - c*x**4)*a**2*b*c**3*d**6*e**3*x**4 + 2286*sqrt(d + e*x**2)*sqrt(a -
c*x**4)*a**2*b*c**3*d**5*e**4*x**6 + 1785*sqrt(d + e*x**2)*sqrt(a - c*x**4
)*a**2*c**4*d**8*e*x**2 - 2146*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**4
*d**7*e**2*x**4 + 737*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**4*d**6*...
```

3.50 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{12}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 700

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{12}}\,dx =$$

$$-\frac{Ad\sqrt{d+ex^2}\sqrt{a-cx^4}}{11x^{11}}-\frac{(11Bd+12Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{99x^9}$$

$$+\frac{(18Acd^2-110aBde-3aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{693adx^7}$$

$$+\frac{(154Bcd^3+186Acd^2e-33aBde^2+18aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{3465ad^2x^5}$$

$$+\frac{2(75Ac^2d^4+209aBcd^3e+21aAcd^2e^2+22a^2Bde^3-12a^2Ae^4)\sqrt{d+ex^2}\sqrt{a-cx^4}}{3465a^2d^3x^3}$$

$$-\frac{2c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(3Ae(103c^2d^4-15acd^2e^2+8a^2e^4)+11B(21c^2d^5+15acd^3e^2-4a^2de^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}}{(\sqrt{c}}}}{3465a^2d^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$-\frac{2\sqrt{c}(cd^2-ae^2)(44aBde(3cd^2-ae^2)+3A(25c^2d^4-9acd^2e^2+8a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\text{Ellipti}}{3465a^{5/2}d^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/11*A*d*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^11-1/99*(12*A*e+11*B*d)*(e*x^
2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^9+1/693*(-3*A*a*e^2+18*A*c*d^2-110*B*a*d*e)*
(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^7+1/3465*(18*A*a*e^3+186*A*c*d^2*e-
33*B*a*d*e^2+154*B*c*d^3)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^5+2/346
5*(-12*A*a^2*e^4+21*A*a*c*d^2*e^2+75*A*c^2*d^4+22*B*a^2*d*e^3+209*B*a*c*d^
3*e)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/d^3/x^3-2/3465*c*(d+a^(1/2)*e/c^
(1/2))*(3*A*e*(8*a^2*e^4-15*a*c*d^2*e^2+103*c^2*d^4)+11*B*(-4*a^2*d*e^4+15
*a*c*d^3*e^2+21*c^2*d^5))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2
)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1
/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/d^4/(e*x^2+d)^(1/2)/(-c*x
^4+a)^(1/2)-2/3465*c^(1/2)*(-a*e^2+c*d^2)*(44*a*B*d*e*(-a*e^2+3*c*d^2)+3*A
*(8*a^2*e^4-9*a*c*d^2*e^2+25*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x
^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^
2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(5/2)/d^4/(e*x
^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{12}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{12}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^12,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^12, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)(d+ex^2)^{3/2}}{x^{12}}\,dx$$

$$\Big\downarrow \text{ 2251}$$

$$\int \frac{\sqrt{a-cx^4}\,(A+Bx^2)\,(d+ex^2)^{3/2}}{x^{12}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^12,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\,\sqrt{-c\,x^4+a}}{x^{12}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^12,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^12,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^{12}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^12,x, algorithm="fr
icas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)/x^12, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^{12}}\,dx = \int \frac{(A+Bx^2)\,\sqrt{a-cx^4}(d+ex^2)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(-c*x**4+a)**(1/2)/x**12,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*(d + e*x**2)**(3/2)/x**12, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^{12}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^12,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^12, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^{12}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^12,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^{12}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}\,(e\,x^2+d)^{3/2}}{x^{12}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^12,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^12, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}{x^{12}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^12,x)
```

output
```
( - 120960*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**7*d**3*e**8 - 221760*sqrt(
d + e*x**2)*sqrt(a - c*x**4)*a**7*d**2*e**9*x**2 + 63360*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a**7*d*e**10*x**4 - 221760*sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*a**6*b*d**3*e**8*x**2 - 126720*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**6*
b*d**2*e**9*x**4 + 2016*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**6*c*d**5*e**6
 + 37296*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**6*c*d**4*e**7*x**2 - 4896*sq
rt(d + e*x**2)*sqrt(a - c*x**4)*a**6*c*d**3*e**8*x**4 - 158400*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*a**6*c*d**2*e**9*x**6 + 316800*sqrt(d + e*x**2)*sqr
t(a - c*x**4)*a**6*c*d*e**10*x**8 - 380160*sqrt(d + e*x**2)*sqrt(a - c*x**
4)*a**6*c*e**11*x**10 + 3696*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*b*c*d*
*5*e**6*x**2 + 2112*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*b*c*d**4*e**7*x
**4 - 348480*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*b*c*d**3*e**8*x**6 + 6
96960*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*b*c*d**2*e**9*x**8 - 570240*s
qrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*b*c*d*e**10*x**10 + 3528*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*a**5*c**2*d**7*e**4 + 101164*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*a**5*c**2*d**6*e**5*x**2 - 110648*sqrt(d + e*x**2)*sqrt(a - c*
x**4)*a**5*c**2*d**5*e**6*x**4 - 88560*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a
**5*c**2*d**4*e**7*x**6 + 177120*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*c*
*2*d**3*e**8*x**8 - 849024*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*c**2*d**
2*e**9*x**10 + 215292*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*b*c**2*d**...
```

3.51 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{14}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 825

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{14}}\,dx =$$

$$-\frac{Ad\sqrt{d+ex^2}\sqrt{a-cx^4}}{13x^{13}}-\frac{(13Bd+14Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{143x^{11}}$$

$$+\frac{(22Acd^2-156aBde-3aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{1287adx^9}$$

$$+\frac{(234Bcd^3+274Acd^2e-39aBde^2+24aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{9009ad^2x^7}$$

$$+\frac{2(39aBde(31cd^2+3ae^2)+A(539c^2d^4+81acd^2e^2-72a^2e^4))\sqrt{d+ex^2}\sqrt{a-cx^4}}{45045a^2d^3x^5}$$

$$+\frac{2(Ae(1193c^2d^4-113acd^2e^2+96a^2e^4)+39B(25c^2d^5+7acd^3e^2-4a^2de^4))\sqrt{d+ex^2}\sqrt{a-cx^4}}{45045a^2d^4x^3}$$

$$-\frac{2c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(39aBde(103c^2d^4-15acd^2e^2+8a^2e^4)+A(1617c^3d^6+597ac^2d^4e^2+250a^2cd^2e^4-192a^3e}{45045a^3d^5\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$-\frac{2\sqrt{c}(cd^2-ae^2)(2Ae(327c^2d^4+53acd^2e^2-96a^2e^4)+39B(25c^2d^5-9acd^3e^2+8a^2de^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{}}{45045a^{5/2}d^5\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/13*A*d*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^13-1/143*(14*A*e+13*B*d)*(e*x
^2+d)^(1/2)*(-c*x^4+a)^(1/2)/x^11+1/1287*(-3*A*a*e^2+22*A*c*d^2-156*B*a*d*
e)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^9+1/9009*(24*A*a*e^3+274*A*c*d^2
*e-39*B*a*d*e^2+234*B*c*d^3)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^7+2/
45045*(39*a*B*d*e*(3*a*e^2+31*c*d^2)+A*(-72*a^2*e^4+81*a*c*d^2*e^2+539*c^2
*d^4))*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/d^3/x^5+2/45045*(A*e*(96*a^2*e
^4-113*a*c*d^2*e^2+1193*c^2*d^4)+39*B*(-4*a^2*d*e^4+7*a*c*d^3*e^2+25*c^2*d
^5))*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/d^4/x^3-2/45045*c*(d+a^(1/2)*e/c
^(1/2))*(39*a*B*d*e*(8*a^2*e^4-15*a*c*d^2*e^2+103*c^2*d^4)+A*(-192*a^3*e^6
+250*a^2*c*d^2*e^4+597*a*c^2*d^4*e^2+1617*c^3*d^6))*(1-a/c/x^4)^(1/2)*x^3*
(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/
2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^3
/d^5/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-2/45045*c^(1/2)*(-a*e^2+c*d^2)*(2*A*
e*(-96*a^2*e^4+53*a*c*d^2*e^2+327*c^2*d^4)+39*B*(8*a^2*d*e^4-9*a*c*d^3*e^2
+25*c^2*d^5))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*
e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*
(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(5/2)/d^5/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1
/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{14}}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{14}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^14,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^14, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}\left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}}{x^{14}}dx$$
$$\Big\downarrow \text{ 2251}$$
$$\int \frac{\sqrt{a-cx^4}\left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}}{x^{14}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4])/x^14,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{\left(B\,x^{2}+A\right)\left(e\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{-c\,x^{4}+a}}{x^{14}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^14,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^14,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}{x^{14}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{14}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^14,x, algorithm="fr
icas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)/x^14, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}{x^{14}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}(d+ex^2)^{\frac{3}{2}}}{x^{14}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(-c*x**4+a)**(1/2)/x**14,x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)*(d + e*x**2)**(3/2)/x**14, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}{x^{14}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{14}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^14,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^14, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{14}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{14}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^14,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^14, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{14}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}\,(e\,x^2+d)^{3/2}}{x^{14}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^14,x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2))/x^14, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a-cx^4}}{x^{14}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\,\sqrt{-c\,x^4+a}}{x^{14}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^14,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(-c*x^4+a)^(1/2)/x^14,x)
```

3.52 $\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 738

$$\begin{aligned}
&\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx \\
&= -\frac{(105Bcd^3-120Acd^2e-44aBde^2+64aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{384ce^4x} \\
&\quad+\frac{(35Bcd^2-40Acde-12aBe^2)\,x\sqrt{d+ex^2}\sqrt{a-cx^4}}{192ce^3} \\
&\quad-\frac{(7Bd-8Ae)x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{48e^2}+\frac{Bx^5\sqrt{d+ex^2}\sqrt{a-cx^4}}{8e} \\
&\quad-\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(105Bcd^3-120Acd^2e-44aBde^2+64aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{}}{\sqrt{c}}}}{\sqrt{2}}\right.\right.}{384e^4\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&\quad+\frac{\sqrt{a}(35Bcd^3-40Acd^2e-20aBde^2+64aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right.\right.}{384\sqrt{c}e^3\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&\quad+\frac{(8Acde(5cd^2-4ae^2)-B(35c^2d^4-24acd^2e^2-16a^2e^4))\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\mathrm{arcs}\right.}{128ce^4\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output
```
-1/384*(64*A*a*e^3-120*A*c*d^2*e-44*B*a*d*e^2+105*B*c*d^3)*(e*x^2+d)^(1/2)
*(-c*x^4+a)^(1/2)/c/e^4/x+1/192*(-40*A*c*d*e-12*B*a*e^2+35*B*c*d^2)*x*(e*x
^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e^3-1/48*(-8*A*e+7*B*d)*x^3*(e*x^2+d)^(1/2)
*(-c*x^4+a)^(1/2)/e^2+1/8*B*x^5*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e-1/384*(
d+a^(1/2)*e/c^(1/2))*(64*A*a*e^3-120*A*c*d^2*e-44*B*a*d*e^2+105*B*c*d^3)*(
1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*E
llipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e
/c^(1/2)))^(1/2))/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/384*a^(1/2)*(64*A
*a*e^3-40*A*c*d^2*e-20*B*a*d*e^2+35*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2
)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1
/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e^
3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/128*(8*A*c*d*e*(-4*a*e^2+5*c*d^2)-B*(
-16*a^2*e^4-24*a*c*d^2*e^2+35*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*
x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/
x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/e^4/(e*x^2
+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(x^4*(A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(x^4*(A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a-cx^4}(A+Bx^2)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \; 2251$$

$$\int \frac{x^4\sqrt{a-cx^4}(A+Bx^2)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(x^4*(A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{x^4(B\,x^2+A)\,\sqrt{-c\,x^4+a}}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
int(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^4(A+Bx^2)\,\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^4}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral((B*x^6 + A*x^4)*sqrt(-c*x^4 + a)/sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**4*(B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**4*(A + B*x**2)*sqrt(a - c*x**4)/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^4}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^4/sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^4}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^4/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{x^4\,(B\,x^2+A)\,\sqrt{a-c\,x^4}}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^4*(A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^4*(A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$
$$=\frac{-12\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,ab\,e^2x-40\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,acdex+32\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,ac\,e^2x^3+}{}$$

input
```
int(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
( - 12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*e**2*x - 40*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a*c*d*e*x + 32*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*e**2
*x**3 + 35*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*x - 28*sqrt(d + e*x*
*2)*sqrt(a - c*x**4)*b*c*d*e*x**3 + 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b
*c*e**2*x**5 + 64*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*
x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*e**3 - 44*int((sqrt(d + e*x**2)*sqrt
(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d*e**2
- 120*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x
**4 - c*e*x**6),x)*a*c**2*d**2*e + 105*int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c**2*d**3 + 24*int((
sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x
**6),x)*a**2*b*e**3 - 16*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d
 + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d*e**2 + 14*int((sqrt(d + e*x
**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*
c*d**2*e + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*
d*x**4 - c*e*x**6),x)*a**2*b*d*e**2 + 40*int((sqrt(d + e*x**2)*sqrt(a - c*
x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d**2*e - 35*int((s
qrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x
)*a*b*c*d**3)/(192*c*e**3)
```

3.53 $\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 629

$$\begin{aligned}
\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx &= \frac{(15Bcd^2-18Acde-8aBe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{48ce^3x} \\
&- \frac{(5Bd-6Ae)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{24e^2} + \frac{Bx^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{6e} \\
&+ \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(15Bcd^2-18Acde-8aBe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48e^3\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&- \frac{\sqrt{a}(5Bcd^2-6Acde-8aBe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48\sqrt{c}e^2\sqrt{d+ex^2}\sqrt{a-cx^4}} \\
&+ \frac{(5Bcd^3-6Acd^2e-4aBde^2+8aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{\ }{d+\cdots}\right.}{16e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}
\end{aligned}$$

output
```
1/48*(-18*A*c*d*e-8*B*a*e^2+15*B*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c
/e^3/x-1/24*(-6*A*e+5*B*d)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e^2+1/6*B*x^
3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e+1/48*(d+a^(1/2)*e/c^(1/2))*(-18*A*c*d
*e-8*B*a*e^2+15*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)
*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/
2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)
^(1/2)-1/48*a^(1/2)*(-6*A*c*d*e-8*B*a*e^2+5*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3
*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1
/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^
(1/2)/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/16*(8*A*a*e^3-6*A*c*d^2*e-4*B
*a*d*e^2+5*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^
(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2
,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(
1/2)
```

Mathematica [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(x^2*(A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(x^2*(A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a-cx^4}(A+Bx^2)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \; 2251$$

$$\int \frac{x^2\sqrt{a-cx^4}(A+Bx^2)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(x^2*(A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{x^2(B\,x^2+A)\,\sqrt{-c\,x^4+a}}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2(A+Bx^2)\,\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral((B*x^4 + A*x^2)*sqrt(-c*x^4 + a)/sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**2*(B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2)*sqrt(a - c*x**4)/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^2/sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^2/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2\,(B\,x^2+A)\,\sqrt{a-c\,x^4}}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^2*(A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^2*(A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{6\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,aex - 5\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,bdx + 4\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,be\,x^3 + 8\Big(\int \frac{\sqrt{e\,x^2+d}}{-ce\,x^6-c}}{}$$

input
```
int(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
(6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*e*x - 5*sqrt(d + e*x**2)*sqrt(a - c
*x**4)*b*d*x + 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*e*x**3 + 8*int((sqrt(
d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),
x)*a*b*e**2 + 18*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x
**2 - c*d*x**4 - c*e*x**6),x)*a*c*d*e - 15*int((sqrt(d + e*x**2)*sqrt(a -
c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c*d**2 + 12*int(
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*
x**6),x)*a**2*e**2 - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d +
 a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*d*e - 6*int((sqrt(d + e*x**2)*sqrt
(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*d*e + 5*int((
sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),
x)*a*b*d**2)/(24*e**2)
```

3.54 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 538

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = -\frac{(3Bd-4Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{8e^2x}+\frac{Bx\sqrt{d+ex^2}\sqrt{a-cx^4}}{4e}$$
$$-\frac{c(3Bd-4Ae)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{a}\sqrt{c}(Bd+4Ae)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{(3Bcd^2-4Acde-4aBe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/8*(-4*A*e+3*B*d)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e^2/x+1/4*B*x*(e*x^2+
d)^(1/2)*(-c*x^4+a)^(1/2)/e-1/8*c*(-4*A*e+3*B*d)*(d+a^(1/2)*e/c^(1/2))*(1-
a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ell
ipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c
^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/8*a^(1/2)*c^(1/2)*(
4*A*e+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/
x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/
(d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/8*(-4*A
*c*d*e-4*B*a*e^2+3*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1
/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2
^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*
x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{2261}$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{\sqrt{d+ex^2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2 + A)\,\sqrt{-c\,x^4 + a}}{\sqrt{e\,x^2 + d}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A + Bx^2)\,\sqrt{a - cx^4}}{\sqrt{d + ex^2}}\,dx = \int \frac{\sqrt{-cx^4 + a}(Bx^2 + A)}{\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/sqrt(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(1/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,bx - 4\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)ace + 3\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)bcd + 2\left(\int \frac{\sqrt{e}}{-ce}\right.}{4e}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*x - 4*int((sqrt(d + e*x**2)*sqrt(a -
c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*e + 3*int((sqr
t(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6
),x)*b*c*d + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**
2 - c*d*x**4 - c*e*x**6),x)*a*b*e + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*e - int((sqrt(d + e*x**
2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*d)/(4*e
)
```

3.55 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 488

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2\sqrt{d+ex^2}}\,dx = \frac{B\sqrt{d+ex^2}\sqrt{a-cx^4}}{2ex}$$
$$+ \frac{c(Bd+2Ae)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2de\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{\sqrt{a}\sqrt{c}(Bd-2Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2d\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{c(Bd-2Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/2*B*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e/x+1/2*c*(2*A*e+B*d)*(d+a^(1/2)*e/
c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^
2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d
+a^(1/2)*e/c^(1/2)))^(1/2))/d/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*a^(1/
2)*c^(1/2)*(-2*A*e+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*
d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2
),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1
/2)+1/2*c*(-2*A*e+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d
+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2
),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^
(1/2)
```

Mathematica [F]

$$\int \frac{(A + Bx^2)\sqrt{a - cx^4}}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{(A + Bx^2)\sqrt{a - cx^4}}{x^2\sqrt{d + ex^2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^2*Sqrt[d + e*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a - cx^4}(A + Bx^2)}{x^2\sqrt{d + ex^2}}\,dx$$
$$\Big\downarrow \; 2251$$
$$\int \frac{\sqrt{a - cx^4}(A + Bx^2)}{x^2\sqrt{d + ex^2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,\sqrt{-c\,x^{4}+a}}{x^{2}\sqrt{e\,x^{2}+d}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(1/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{a-cx^4}}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e*x^4 + d*x^2), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**2*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(sqrt(e*x^2 + d)*x^2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(sqrt(e*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{x^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^2*(d + e*x^2)^(1/2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^2*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2\sqrt{d+ex^2}}\,dx$$
$$= \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,a+2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)acex-\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)bcdx+2\left(\int \frac{}{-ce}\right.}{dx}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*a + 2*int((sqrt(d + e*x**2)*sqrt(a - c*
x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*e*x - int((sqrt(
d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),
x)*b*c*d*x + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**2 + a*e*x**
4 - c*d*x**6 - c*e*x**8),x)*a**2*d*x + int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*d*x)/(d*x)
```

3.56 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 486

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4\sqrt{d+ex^2}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{3dx^3}$$
$$+\frac{c(3Bd-2Ae)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3d^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{\sqrt{c}(2Acd^2+ae(3Bd-2Ae))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3\sqrt{a}d^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{Bc\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/3*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/d/x^3+1/3*c*(-2*A*e+3*B*d)*(d+a^(1
/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)
*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/3
*c^(1/2)*(2*A*c*d^2+a*e*(-2*A*e+3*B*d))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*
x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x
^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/d^2/(e*
x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-B*c*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)
/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(
1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-
c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4\sqrt{d+ex^2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^4*Sqrt[d + e*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^4\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \; 2251$$
$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^4\sqrt{d+ex^2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,\sqrt{-c\,x^4+a}}{x^4\sqrt{e\,x^2+d}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(1/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{a-cx^4}}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e*x^6 + d*x^4), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**4/(e*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**4*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(sqrt(e*x^2 + d)*x^4), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(sqrt(e*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{x^4\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^4*(d + e*x^2)^(1/2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^4*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4\sqrt{d+ex^2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,b-2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)bce\,x^3+2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^{10}-cd\,x^8+ae\,x^6+ad\,x^4}dx\right)a^2e\,x^3-}{}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*b - 2*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c*e*x**3 + 2*in
t((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**4 + a*e*x**6 - c*d*x**8 - c*
e*x**10),x)*a**2*e*x**3 - 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x
**4 + a*e*x**6 - c*d*x**8 - c*e*x**10),x)*a*b*d*x**3 - 2*int((sqrt(d + e*x
**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*e*x**
3 + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c
*e*x**6),x)*b*c*d*x**3)/(2*e*x**3)
```

3.57 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 413

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\sqrt{d+ex^2}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{5dx^5} - \frac{(5Bd-4Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{15d^2x^3}$$
$$-\frac{2c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(3Acd^2+5aBde-4aAe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\middle|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15ad^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{2\sqrt{c}(5Bd-4Ae)(cd^2-ae^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15\sqrt{a}d^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/5*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/d/x^5-1/15*(-4*A*e+5*B*d)*(e*x^2+d
)^(1/2)*(-c*x^4+a)^(1/2)/d^2/x^3-2/15*c*(d+a^(1/2)*e/c^(1/2))*(-4*A*a*e^2+
3*A*c*d^2+5*B*a*d*e)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a
^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2
^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(
1/2)-2/15*c^(1/2)*(-4*A*e+5*B*d)*(-a*e^2+c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(
1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c
^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)
/d^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\sqrt{d+ex^2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^6*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^6*Sqrt[d + e*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^6\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \ 2251$$
$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^6\sqrt{d+ex^2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^6*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{-c\,x^4+a}}{x^6\sqrt{e\,x^2+d}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(1/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^6}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e*x^8 + d*x^6), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**6/(e*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**6*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+dx^6}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(sqrt(e*x^2 + d)*x^6), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+dx^6}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(sqrt(e*x^2 + d)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\sqrt{d+ex^2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{x^6\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^6*(d + e*x^2)^(1/2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^6*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(1/2),x)
```

output
```
( - 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*d*e**2 + 10*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a*b*d*e**2*x**2 - 20*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*
b*e**3*x**4 - 3*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**2*e*x**2 + 10*sqr
t(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*e**2*x**4 + 15*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*b*c*d**3*x**2 - 30*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*
e*x**4 - 40*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 -
 c*d*x**4 - c*e*x**6),x)*a*b*c*e**4*x**5 + 20*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c**2*d*e**3*x
**5 - 60*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*
d*x**4 - c*e*x**6),x)*b*c**2*d**2*e**2*x**5 - 20*int((sqrt(d + e*x**2)*sqr
t(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d*e**3
*x**5 + 10*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 -
c*d*x**4 - c*e*x**6),x)*a*c**2*d**2*e**2*x**5 - 30*int((sqrt(d + e*x**2)*s
qrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c**2*d**
3*e*x**5 - 8*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**4 + a*e*x**6
- c*d*x**8 - c*e*x**10),x)*a**3*d*e**3*x**5 + 40*int((sqrt(d + e*x**2)*sqr
t(a - c*x**4))/(a*d*x**4 + a*e*x**6 - c*d*x**8 - c*e*x**10),x)*a**2*b*d**2
*e**2*x**5 - 9*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**4 + a*e*x**
6 - c*d*x**8 - c*e*x**10),x)*a**2*c*d**3*e*x**5 + 45*int((sqrt(d + e*x**2)
*sqrt(a - c*x**4))/(a*d*x**4 + a*e*x**6 - c*d*x**8 - c*e*x**10),x)*a*b*...
```

3.58 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 492

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\sqrt{d+ex^2}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{7dx^7} - \frac{(7Bd-6Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{35d^2x^5}$$
$$+ \frac{2(5Acd^2+14aBde-12aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{105ad^3x^3}$$
$$- \frac{2c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(21Bcd^3-13Acd^2e-28aBde^2+24aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{c}}{\sqrt{cx}}}}{\sqrt{2}}\right.\right.}{105ad^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{2\sqrt{c}(cd^2-ae^2)(5Acd^2-28aBde+24aAe^2)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\right.}{105a^{3/2}d^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/7*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/d/x^7-1/35*(-6*A*e+7*B*d)*(e*x^2+d
)^(1/2)*(-c*x^4+a)^(1/2)/d^2/x^5+2/105*(-12*A*a*e^2+5*A*c*d^2+14*B*a*d*e)*
(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^3/x^3-2/105*c*(d+a^(1/2)*e/c^(1/2))*(
24*A*a*e^3-13*A*c*d^2*e-28*B*a*d*e^2+21*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^
(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/
c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d^4/
(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-2/105*c^(1/2)*(-a*e^2+c*d^2)*(24*A*a*e^2+
5*A*c*d^2-28*B*a*d*e)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+
a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),
2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/d^4/(e*x^2+d)^(1/2)/(-c*x
^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\sqrt{d+ex^2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^8*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^8*Sqrt[d + e*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^8\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{2251}$$
$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^8\sqrt{d+ex^2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^8*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,\sqrt{-c\,x^4+a}}{x^8\sqrt{e\,x^2+d}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(1/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{a-cx^4}}{x^8\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e*x^10 + d*x^8), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\sqrt{d+ex^2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**8/(e*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**8*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(sqrt(e*x^2 + d)*x^8), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(sqrt(e*x^2 + d)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\sqrt{d+ex^2}}\,dx=\int \frac{\left(B\,x^2+A\right)\,\sqrt{a-c\,x^4}}{x^8\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^8*(d + e*x^2)^(1/2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^8*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\sqrt{d+ex^2}}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(1/2),x)
```

output

```
( - 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*d*e**4 + 2*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a**3*c*d**3*e**2 + 60*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a
**3*c*d**2*e**3*x**2 - 120*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c*d*e**4
*x**4 + 144*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c*e**5*x**6 - 84*sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*a**2*b*c*d**3*e**2*x**2 + 168*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*a**2*b*c*d**2*e**3*x**4 - 168*sqrt(d + e*x**2)*sqrt(a -
c*x**4)*a**2*b*c*d*e**4*x**6 - 5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c*
*2*d**4*e*x**2 + 10*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**3*e**2*
x**4 + 168*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**2*e**3*x**6 + 35
*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c**2*d**5*x**2 - 70*sqrt(d + e*x**2
)*sqrt(a - c*x**4)*a*b*c**2*d**4*e*x**4 - 126*sqrt(d + e*x**2)*sqrt(a - c*
x**4)*a*b*c**2*d**3*e**2*x**6 - 15*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**
3*d**4*e*x**6 + 105*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c**3*d**5*x**6 + 3
456*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(12*a**2*d*e**2 + 12*a**2
*e**3*x**2 - a*c*d**3 - a*c*d**2*e*x**2 - 12*a*c*d*e**2*x**4 - 12*a*c*e**3
*x**6 + c**2*d**3*x**4 + c**2*d**2*e*x**6),x)*a**4*c**2*e**8*x**7 - 4032*i
nt((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(12*a**2*d*e**2 + 12*a**2*e**3
*x**2 - a*c*d**3 - a*c*d**2*e*x**2 - 12*a*c*d*e**2*x**4 - 12*a*c*e**3*x**6
 + c**2*d**3*x**4 + c**2*d**2*e*x**6),x)*a**3*b*c**2*d*e**7*x**7 + 3744*in
t((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(12*a**2*d*e**2 + 12*a**2*e*...
```

3.59 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^{10}\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 588

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^{10}\sqrt{d+ex^2}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{9dx^9} - \frac{(9Bd-8Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{63d^2x^7}$$
$$+ \frac{2(7Acd^2+27aBde-24aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{315ad^3x^5}$$
$$+ \frac{2(15Bcd^3-11Acd^2e-36aBde^2+32aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{315ad^4x^3}$$
$$+ \frac{2c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(3aBde(13cd^2-24ae^2)-A(21c^2d^4+30acd^2e^2-64a^2e^4))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\Big(}{315a^2d^5\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{2\sqrt{c}(cd^2-ae^2)(15Bcd^3-18Acd^2e+72aBde^2-64aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\Big(\mathrm{arcsi}}{315a^{3/2}d^5\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/9*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/d/x^9-1/63*(-8*A*e+9*B*d)*(e*x^2+d
)^(1/2)*(-c*x^4+a)^(1/2)/d^2/x^7+2/315*(-24*A*a*e^2+7*A*c*d^2+27*B*a*d*e)*
(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^3/x^5+2/315*(32*A*a*e^3-11*A*c*d^2*e-
36*B*a*d*e^2+15*B*c*d^3)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^4/x^3+2/315*
c*(d+a^(1/2)*e/c^(1/2))*(3*a*B*d*e*(-24*a*e^2+13*c*d^2)-A*(-64*a^2*e^4+30*
a*c*d^2*e^2+21*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)
*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/
2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/d^5/(e*x^2+d)^(1/2)/(-c*x^
4+a)^(1/2)-2/315*c^(1/2)*(-a*e^2+c*d^2)*(-64*A*a*e^3-18*A*c*d^2*e+72*B*a*d
*e^2+15*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/
2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/
2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/d^5/(e*x^2+d)^(1/2)/(-c*x^4+a)
^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^{10}\sqrt{d+ex^2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^{10}\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^10*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^10*Sqrt[d + e*x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^{10}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{2251}$$
$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^{10}\sqrt{d+ex^2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^10*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,\sqrt{-c\,x^{4}+a}}{x^{10}\sqrt{e\,x^{2}+d}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^10/(e*x^2+d)^(1/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^10/(e*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{a-cx^4}}{x^{10}\sqrt{d+ex^2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^10/(e*x^2+d)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e*x^12 + d*x^10), x
)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^{10}\sqrt{d+ex^2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^{10}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**10/(e*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**10*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^{10}\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^10/(e*x^2+d)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(sqrt(e*x^2 + d)*x^10), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^{10}\sqrt{d+ex^2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^10/(e*x^2+d)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(sqrt(e*x^2 + d)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^{10}\sqrt{d+ex^2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{x^{10}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^10*(d + e*x^2)^(1/2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^10*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^{10}\sqrt{d+ex^2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^10/(e*x^2+d)^(1/2),x)
```

output

```
( - 1440*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*d*e**6 + 1440*sqrt(d + e*x
**2)*sqrt(a - c*x**4)*a**4*b*d*e**6*x**2 - 1728*sqrt(d + e*x**2)*sqrt(a -
c*x**4)*a**4*b*e**7*x**4 - 1680*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c*d
**2*e**5*x**2 + 2592*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c*d*e**6*x**4
+ 4320*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*c*d**3*e**4*x**2 - 5184*sq
rt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*c*d**2*e**5*x**4 + 288*sqrt(d + e*x
**2)*sqrt(a - c*x**4)*a**3*b*c*d*e**6*x**6 + 10*sqrt(d + e*x**2)*sqrt(a -
c*x**4)*a**3*c**2*d**5*e**2 - 1400*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*
c**2*d**4*e**3*x**2 + 1680*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2*d**
3*e**4*x**4 + 528*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2*d**2*e**5*x*
*6 + 3690*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c**2*d**5*e**2*x**2 - 4
428*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c**2*d**4*e**3*x**4 + 864*sqr
t(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c**2*d**3*e**4*x**6 - 105*sqrt(d + e
*x**2)*sqrt(a - c*x**4)*a**2*c**3*d**6*e*x**2 + 122*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*a**2*c**3*d**5*e**2*x**4 - 280*sqrt(d + e*x**2)*sqrt(a - c*x**
4)*a**2*c**3*d**4*e**3*x**6 + 945*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*
*3*d**7*x**2 - 1134*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c**3*d**6*e*x**4
 + 738*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c**3*d**5*e**2*x**6 - 27*sqrt
(d + e*x**2)*sqrt(a - c*x**4)*a*c**4*d**6*e*x**6 + 189*sqrt(d + e*x**2)*sq
rt(a - c*x**4)*b*c**4*d**7*x**6 - 1658880*int((sqrt(d + e*x**2)*sqrt(a ...
```

3.60 $\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 673

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = -\frac{d(Bd-Ae)x\sqrt{a-cx^4}}{e^3\sqrt{d+ex^2}}$$
$$+\frac{(105Bcd^2-90Acde-8aBe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{48ce^4x}$$
$$-\frac{(11Bd-6Ae)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{24e^3}+\frac{Bx^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{6e^2}$$
$$+\frac{(\sqrt{c}d+\sqrt{a}e)(105Bcd^2-90Acde-8aBe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48\sqrt{c}e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{\sqrt{a}(35Bcd^2-30Acde-8aBe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{48\sqrt{c}e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{(35Bcd^3-30Acd^2e-12aBde^2+8aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{}{d}\right.}{16e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-d*(-A*e+B*d)*x*(-c*x^4+a)^(1/2)/e^3/(e*x^2+d)^(1/2)+1/48*(-90*A*c*d*e-8*B
*a*e^2+105*B*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e^4/x-1/24*(-6*A*e+
11*B*d)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/e^3+1/6*B*x^3*(e*x^2+d)^(1/2)*(
-c*x^4+a)^(1/2)/e^2+1/48*(c^(1/2)*d+a^(1/2)*e)*(-90*A*c*d*e-8*B*a*e^2+105*
B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^
2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d
+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1
/48*a^(1/2)*(-30*A*c*d*e-8*B*a*e^2+35*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1
/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^
(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/
e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/16*(8*A*a*e^3-30*A*c*d^2*e-12*B*a*d
*e^2+35*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/
2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^
(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2
)
```

Mathematica [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^4*(A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^4*(A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2251}$$

$$\int \frac{x^4\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{3/2}}\,dx$$

input
```
Int[(x^4*(A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{x^4(B\,x^2+A)\sqrt{-c\,x^4+a}}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output
```
int(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="fri
cas")
```

output
```
integral((B*x^6 + A*x^4)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e
*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**4*(A + B*x**2)*sqrt(a - c*x**4)/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^4/(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^4/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^4\,(B\,x^2+A)\,\sqrt{a-c\,x^4}}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^4*(A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(x^4*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output
```
( - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*e**2*x + 2*sqrt(d + e*x**2)*s
qrt(a - c*x**4)*a*b*d*e*x + 6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*e*x*
*3 - 7*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*x**3 + 4*sqrt(d + e*x**2
)*sqrt(a - c*x**4)*b*c*d*e*x**5 - 8*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)
*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 -
 c*e**2*x**8),x)*a**2*c*d*e**3 - 8*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*
x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 -
c*e**2*x**8),x)*a**2*c*e**4*x**2 + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6
 - c*e**2*x**8),x)*a*b*c*d**2*e**2 + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x
**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x*
*6 - c*e**2*x**8),x)*a*b*c*d*e**3*x**2 + 30*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*
e*x**6 - c*e**2*x**8),x)*a*c**2*d**3*e + 30*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*
e*x**6 - c*e**2*x**8),x)*a*c**2*d**2*e**2*x**2 - 35*int((sqrt(d + e*x**2)*
sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4
- 2*c*d*e*x**6 - c*e**2*x**8),x)*b*c**2*d**4 - 35*int((sqrt(d + e*x**2)*sq
rt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 -
2*c*d*e*x**6 - c*e**2*x**8),x)*b*c**2*d**3*e*x**2 - 18*int((sqrt(d + e*...
```

3.61 $\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 580

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \frac{(Bd-Ae)x\sqrt{a-cx^4}}{e^2\sqrt{d+ex^2}} - \frac{3(5Bd-4Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{8e^3x} + \frac{Bx\sqrt{d+ex^2}\sqrt{a-cx^4}}{4e^2}$$
$$- \frac{3\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)(5Bd-4Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{\sqrt{a}\sqrt{c}(5Bd-4Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{(15Bcd^2-12Acde-4aBe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
(-A*e+B*d)*x*(-c*x^4+a)^(1/2)/e^2/(e*x^2+d)^(1/2)-3/8*(-4*A*e+5*B*d)*(e*x^
2+d)^(1/2)*(-c*x^4+a)^(1/2)/e^3/x+1/4*B*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)
/e^2-3/8*c^(1/2)*(c^(1/2)*d+a^(1/2)*e)*(-4*A*e+5*B*d)*(1-a/c/x^4)^(1/2)*x^
3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(
1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e
^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/8*a^(1/2)*c^(1/2)*(-4*A*e+5*B*d)*(1-
a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ell
ipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c
^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/8*(-12*A*c*d*e-4*B*
a*e^2+15*B*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1
/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2
^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/
2)
```

Mathematica [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^2*(A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*(A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{3/2}}\,dx$$
$$\Big\downarrow \ 2251$$
$$\int \frac{x^2\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{3/2}}dx$$

input
```
Int[(x^2*(A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{x^{2}\left(B\,x^{2}+A\right)\sqrt{-c\,x^{4}+a}}{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output
```
int(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="fri
cas")
```

output
```
integral((B*x^4 + A*x^2)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e
*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(A + B*x**2)*sqrt(a - c*x**4)/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^2/(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)*x^2/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2\,(B\,x^2+A)\,\sqrt{a-c\,x^4}}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^2*(A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2*(B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output

```
( - 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*e*x + 3*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*b*c*d*x**3 - 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a
*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x
**8),x)*a*b*c*d*e**2 - 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d
**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**
8),x)*a*b*c*e**3*x**2 - 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a
*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x
**8),x)*a*c**2*d**2*e - 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a
*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x
**8),x)*a*c**2*d*e**2*x**2 + 15*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**
6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e
**2*x**8),x)*b*c**2*d**3 + 15*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)
/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**
2*x**8),x)*b*c**2*d**2*e*x**2 + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*
x**2)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 -
c*e**2*x**8),x)*a**2*c*d**2*e + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*
x**2)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 -
c*e**2*x**8),x)*a**2*c*d*e**2*x**2 - 9*int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*x**2)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**
6 - c*e**2*x**8),x)*a*b*c*d**3 - 9*int((sqrt(d + e*x**2)*sqrt(a - c*x**...
```

3.62 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 546

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \frac{\left(\frac{A}{d}-\frac{B}{e}\right)x\sqrt{a-cx^4}}{\sqrt{d+ex^2}} + \frac{(3Bd-2Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{2de^2x}$$
$$+ \frac{\sqrt{c}\left(\sqrt{c}d+\sqrt{a}e\right)(3Bd-2Ae)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2de^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{\sqrt{a}\sqrt{c}(Bd-2Ae)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2de\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{c(3Bd-2Ae)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
(A/d-B/e)*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^(1/2)+1/2*(-2*A*e+3*B*d)*(e*x^2+d)^
(1/2)*(-c*x^4+a)^(1/2)/d/e^2/x+1/2*c^(1/2)*(c^(1/2)*d+a^(1/2)*e)*(-2*A*e+3
*B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^
(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^
(1/2)*e/c^(1/2)))^(1/2))/d/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/2*a^(1/2
)*c^(1/2)*(-2*A*e+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d
+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2)
,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(
1/2)+1/2*c*(-2*A*e+3*B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2
)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(
1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^
4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2261$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{(d+ex^2)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,\sqrt{-c\,x^{4}+a}}{(e\,x^{2}+d)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
 + d^2), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/(e*x**2+d)**(3/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(d + e*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{{(e\,x^2+d)}^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(3/2),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{(d+ex^2)^{3/2}}\,dx=\frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,ax+2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^6}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx\right)acd$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*x + 2*int((sqrt(d + e*x**2)*sqrt(a -
c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e
*x**6 - c*e**2*x**8),x)*a*c*d*e + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)
*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 -
 c*e**2*x**8),x)*a*c*e**2*x**2 - 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*
x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 -
c*e**2*x**8),x)*b*c*d**2 - 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/
(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2
*x**8),x)*b*c*d*e*x**2 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a
*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x
**8),x)*a*b*d**2 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d**2
+ 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x
)*a*b*d*e*x**2 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d
*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*
d**2 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d*e*x**2 +
a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*d*e*x**2)/
(3*d*(d + e*x**2))
```

3.63 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 518

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(d+ex^2)^{3/2}}\,dx = \frac{(Bd-Ae)x\sqrt{a-cx^4}}{d^2\sqrt{d+ex^2}} - \frac{(Bd-Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{d^2ex}$$
$$- \frac{c(Bd-2Ae)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{d^2e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{\sqrt{a}\sqrt{c}(Bd-2Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{d^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{Bc\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
(-A*e+B*d)*x*(-c*x^4+a)^(1/2)/d^2/(e*x^2+d)^(1/2)-(-A*e+B*d)*(e*x^2+d)^(1/
2)*(-c*x^4+a)^(1/2)/d^2/e/x-c*(-2*A*e+B*d)*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^
4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE
(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)
))^(1/2))/d^2/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+a^(1/2)*c^(1/2)*(-2*A*e+B
*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1
/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1
/2)*e/c^(1/2)))^(1/2))/d^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-B*c*(1-a/c/x^4
)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi
(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/
2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(d+ex^2)^{3/2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^2*(d + e*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^2(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2251}$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^2(d+ex^2)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{-c\,x^4+a}}{x^2\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(3/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2\,(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(3/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^2*x^6 + 2*d*e*x^4
 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**2*(d + e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((e*x^2 + d)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(3/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((e*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(d+ex^2)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{x^2\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^2*(d + e*x^2)^(3/2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^2*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^2(d+ex^2)^{3/2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,b-2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx\right)bcd}{}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^2/(e*x^2+d)^(3/2),x)
```

output

```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*b - 2*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*
e*x**6 - c*e**2*x**8),x)*b*c*d*e*x - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*x**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**
6 - c*e**2*x**8),x)*b*c*e**2*x**3 - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4)*x**2)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6
 - c*e**2*x**8),x)*a*c*d*e*x - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x*
*2)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*
e**2*x**8),x)*a*c*e**2*x**3 - int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)
/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**
2*x**8),x)*b*c*d**2*x - int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d*
*2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8
),x)*b*c*d*e*x**3 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2*x**2
 + 2*a*d*e*x**4 + a*e**2*x**6 - c*d**2*x**6 - 2*c*d*e*x**8 - c*e**2*x**10)
,x)*a**2*d*e*x + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2*x**2 +
2*a*d*e*x**4 + a*e**2*x**6 - c*d**2*x**6 - 2*c*d*e*x**8 - c*e**2*x**10),x)
*a**2*e**2*x**3 - int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2*x**2 + 2
*a*d*e*x**4 + a*e**2*x**6 - c*d**2*x**6 - 2*c*d*e*x**8 - c*e**2*x**10),x)*
a*b*d**2*x - int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2*x**2 + 2*a*d*
e*x**4 + a*e**2*x**6 - c*d**2*x**6 - 2*c*d*e*x**8 - c*e**2*x**10),x)*a*...
```

3.64 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 431

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(d+ex^2)^{3/2}}\,dx = -\frac{e(Bd-Ae)x\sqrt{a-cx^4}}{d^3\sqrt{d+ex^2}} - \frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{3d^2x^3} + \frac{(Bd-Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{d^3x} + \frac{2c(3Bd-4Ae)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3d^3\sqrt{d+ex^2}\sqrt{a-cx^4}} - \frac{2\sqrt{c}(Acd^2+3aBde-4aAe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3\sqrt{a}d^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-e*(-A*e+B*d)*x*(-c*x^4+a)^(1/2)/d^3/(e*x^2+d)^(1/2)-1/3*A*(e*x^2+d)^(1/2)
*(-c*x^4+a)^(1/2)/d^2/x^3+(-A*e+B*d)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/d^3/
x+2/3*c*(-4*A*e+3*B*d)*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2
)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1
/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/d^3/(e*x^2
+d)^(1/2)/(-c*x^4+a)^(1/2)-2/3*c^(1/2)*(-4*A*a*e^2+A*c*d^2+3*B*a*d*e)*(1-a
/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Elli
pticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^
(1/2)))^(1/2))/a^(1/2)/d^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^4*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^4*(d + e*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^4(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2251$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^4(d+ex^2)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^4*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^{2}+A)\,\sqrt{-c\,x^{4}+a}}{x^{4}\,(e\,x^{2}+d)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(3/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{a-cx^4}}{x^4\,(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(3/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^2*x^8 + 2*d*e*x^6
 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**4/(e*x**2+d)**(3/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**4*(d + e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((e*x^2 + d)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(3/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((e*x^2 + d)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{x^4\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^4*(d + e*x^2)^(3/2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^4*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^4(d+ex^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^4/(e*x^2+d)^(3/2),x)
```

output
```
( - 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*d*e**2 + 2*sqrt(d + e*x**2)*s
qrt(a - c*x**4)*a*b*d*e**2*x**2 + 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*
e**3*x**4 - sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**2*e*x**2 - 6*sqrt(d +
 e*x**2)*sqrt(a - c*x**4)*a*c*d*e**2*x**4 + sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*b*c*d**3*x**2 + 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*e*x**4 +
8*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*
e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*b*c*d*e**4*x**3
 + 8*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 +
 a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*b*c*e**5*x**
5 - 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2
 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*c**2*d**2*
e**3*x**3 - 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*
d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*c*
*2*d*e**4*x**5 + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 +
2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*
b*c**2*d**3*e**2*x**3 + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*
d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x*
*8),x)*b*c**2*d**2*e**3*x**5 + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x
**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c
*e**2*x**8),x)*a*b*c*d**2*e**3*x**3 + 12*int((sqrt(d + e*x**2)*sqrt(a -...
```

3.65 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 503

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(d+ex^2)^{3/2}}\,dx = \frac{e^2(Bd-Ae)x\sqrt{a-cx^4}}{d^4\sqrt{d+ex^2}} - \frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{5d^2x^5}$$
$$- \frac{(5Bd-9Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{15d^3x^3} - \frac{e(Bd-Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{d^4x}$$
$$- \frac{2c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(20aBde+3A(cd^2-8ae^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{15ad^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{2\sqrt{c}(5Bcd^3-9Acd^2e-20aBde^2+24aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right), \ldots\right.}{15\sqrt{a}d^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
e^2*(-A*e+B*d)*x*(-c*x^4+a)^(1/2)/d^4/(e*x^2+d)^(1/2)-1/5*A*(e*x^2+d)^(1/2
)*(-c*x^4+a)^(1/2)/d^2/x^5-1/15*(-9*A*e+5*B*d)*(e*x^2+d)^(1/2)*(-c*x^4+a)^
(1/2)/d^3/x^3-e*(-A*e+B*d)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/d^4/x-2/15*c*(
d+a^(1/2)*e/c^(1/2))*(20*a*B*d*e+3*A*(-8*a*e^2+c*d^2))*(1-a/c/x^4)^(1/2)*x
^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^
(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/
a/d^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-2/15*c^(1/2)*(24*A*a*e^3-9*A*c*d^2*
e-20*B*a*d*e^2+5*B*c*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2
)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1
/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/d^4/(e*x^2+d)^(1/2)/(
-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(d+ex^2)^{3/2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^6*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^6*(d + e*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^6(d+ex^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 2251}$$
$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^6(d+ex^2)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^6*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,\sqrt{-c\,x^4+a}}{x^6\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(3/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,\sqrt{a-cx^4}}{x^6\,(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(3/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^2*x^10 + 2*d*e*x^
8 + d^2*x^6), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**6/(e*x**2+d)**(3/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**6*(d + e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((e*x^2 + d)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(3/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((e*x^2 + d)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(d+ex^2)^{3/2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{a-c\,x^4}}{x^6\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^6*(d + e*x^2)^(3/2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^6*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^6(d+ex^2)^{3/2}}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^6/(e*x^2+d)^(3/2),x)
```

output

```
( - 6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*d*e**2 - 3*sqrt(d + e*x**2)*s
qrt(a - c*x**4)*a**2*c*d**2*e*x**2 - 6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a
**2*c*d*e**2*x**4 + 12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c*e**3*x**6
+ 5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*d**3*x**2 + 10*sqrt(d + e*x**2
)*sqrt(a - c*x**4)*a*b*c*d**2*e*x**4 - 10*sqrt(d + e*x**2)*sqrt(a - c*x**4
)*a*b*c*d*e**2*x**6 + 3*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**2*d**2*e*x*
*6 - 5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c**2*d**3*x**6 + 24*int((sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c
*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*c**2*d*e**4*x**5 + 24*int
((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*
x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*c**2*e**5*x**7 -
20*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a
*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*b*c**2*d**2*e*
*3*x**5 - 20*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*
e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*b*c*
*2*d*e**4*x**7 + 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 +
2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*
a*c**3*d**3*e**2*x**5 + 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*
d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x*
*8),x)*a*c**3*d**2*e**3*x**7 - 10*int((sqrt(d + e*x**2)*sqrt(a - c*x**4...
```

3.66 $\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 593

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8(d+ex^2)^{3/2}}\,dx = -\frac{e^3(Bd-Ae)x\sqrt{a-cx^4}}{d^5\sqrt{d+ex^2}} - \frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{7d^2x^7} - \frac{(7Bd-13Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{35d^3x^5} + \frac{(10Acd^2+63aBde-87aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{105ad^4x^3} + \frac{e^2(Bd-Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{d^5x} - \frac{2c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(21B(cd^3-8ade^2)-A(34cd^2e-192ae^3)\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\right.}{105ad^5\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{2\sqrt{c}(21aBde(3cd^2-8ae^2)-A(5c^2d^4+82acd^2e^2-192a^2e^4))\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\text{arcsi}\right.}{105a^{3/2}d^5\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-e^3*(-A*e+B*d)*x*(-c*x^4+a)^(1/2)/d^5/(e*x^2+d)^(1/2)-1/7*A*(e*x^2+d)^(1/
2)*(-c*x^4+a)^(1/2)/d^2/x^7-1/35*(-13*A*e+7*B*d)*(e*x^2+d)^(1/2)*(-c*x^4+a
)^(1/2)/d^3/x^5+1/105*(-87*A*a*e^2+10*A*c*d^2+63*B*a*d*e)*(e*x^2+d)^(1/2)*
(-c*x^4+a)^(1/2)/a/d^4/x^3+e^2*(-A*e+B*d)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)
/d^5/x-2/105*c*(d+a^(1/2)*e/c^(1/2))*(21*B*(-8*a*d*e^2+c*d^3)-A*(-192*a*e^
3+34*c*d^2*e))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)
*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d^5/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+2
/105*c^(1/2)*(21*a*B*d*e*(-8*a*e^2+3*c*d^2)-A*(-192*a^2*e^4+82*a*c*d^2*e^2
+5*c^2*d^4))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e
)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(
d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/d^5/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/
2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^8*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[a - c*x^4])/(x^8*(d + e*x^2)^(3/2)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^8(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2251$$

$$\int \frac{\sqrt{a-cx^4}(A+Bx^2)}{x^8(d+ex^2)^{3/2}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[a - c*x^4])/(x^8*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{\left(B\,x^{2}+A\right)\sqrt{-c\,x^{4}+a}}{x^{8}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(3/2),x)
```

output
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(3/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c*x^4 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/(e^2*x^12 + 2*d*e*x^
10 + d^2*x^8), x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8(d+ex^2)^{3/2}}\,dx = \int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x**2+A)*(-c*x**4+a)**(1/2)/x**8/(e*x**2+d)**(3/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(a - c*x**4)/(x**8*(d + e*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(3/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((e*x^2 + d)^(3/2)*x^8), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8(d+ex^2)^{3/2}}\,dx = \int \frac{\sqrt{-cx^4+a}(Bx^2+A)}{(ex^2+d)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(3/2),x, algorithm="gia
c")
```

output
```
integrate(sqrt(-c*x^4 + a)*(B*x^2 + A)/((e*x^2 + d)^(3/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{\left(B\,x^2+A\right)\,\sqrt{a-c\,x^4}}{x^8\,{\left(e\,x^2+d\right)}^{3/2}} \,dx$$

input
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^8*(d + e*x^2)^(3/2)),x)
```

output
```
int(((A + B*x^2)*(a - c*x^4)^(1/2))/(x^8*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{a-cx^4}}{x^8\left(d+ex^2\right)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(-c*x^4+a)^(1/2)/x^8/(e*x^2+d)^(3/2),x)
```

output

```
( - 2592*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*d*e**6 + 672*sqrt(d + e*x*
*2)*sqrt(a - c*x**4)*a**4*b*d*e**6*x**2 - 1344*sqrt(d + e*x**2)*sqrt(a - c
*x**4)*a**4*b*e**7*x**4 - 576*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c*d**
3*e**4 - 2160*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c*d**2*e**5*x**2 + 60
48*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c*d*e**6*x**4 + 2240*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*a**3*b*c*d**3*e**4*x**2 - 4480*sqrt(d + e*x**2)*sqr
t(a - c*x**4)*a**3*b*c*d**2*e**5*x**4 + 672*sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*a**3*b*c*d*e**6*x**6 + 18*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2*
d**5*e**2 + 600*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2*d**4*e**3*x**2
 - 816*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2*d**3*e**4*x**4 - 1296*s
qrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2*d**2*e**5*x**6 - 742*sqrt(d + e
*x**2)*sqrt(a - c*x**4)*a**2*b*c**2*d**5*e**2*x**2 + 1484*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*a**2*b*c**2*d**4*e**3*x**4 + 2240*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*a**2*b*c**2*d**3*e**4*x**6 - 15*sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*a**2*c**3*d**6*e*x**2 + 18*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**3
*d**5*e**2*x**4 + 792*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**3*d**4*e**
3*x**6 + 35*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c**3*d**7*x**2 - 70*sqrt
(d + e*x**2)*sqrt(a - c*x**4)*a*b*c**3*d**6*e*x**4 - 742*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a*b*c**3*d**5*e**2*x**6 - 21*sqrt(d + e*x**2)*sqrt(a - c*
x**4)*a*c**4*d**6*e*x**6 + 35*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c**4*...
```

3.67 $\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 668

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = -\frac{(15cCd^2-18Bcde+24Ace^2+16aCe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{48c^2e^3x} + \frac{(5Cd-6Be)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{24ce^2} - \frac{Cx^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{6ce} - \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(16aCe^2+3c(5Cd^2-6Bde+8Ae^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2}{d+\dots}\right)}{48ce^3\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{\sqrt{a}(5cCd^2-6Bcde+24Ace^2+16aCe^2)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{\dots}{d\dots}\right)}{48c^{3/2}e^2\sqrt{d+ex^2}\sqrt{a-cx^4}} - \frac{(4ae^2(Cd-2Be)+cd(5Cd^2-6Bde+8Ae^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\dots}}{\sqrt{2}}\right)\dots\right)}{16ce^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/48*(24*A*c*e^2-18*B*c*d*e+16*C*a*e^2+15*C*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^
4+a)^(1/2)/c^2/e^3/x+1/24*(-6*B*e+5*C*d)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2
)/c/e^2-1/6*C*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e-1/48*(d+a^(1/2)*e/c
^(1/2))*(16*C*a*e^2+3*c*(8*A*e^2-6*B*d*e+5*C*d^2))*(1-a/c/x^4)^(1/2)*x^3*(
a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2
)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/e^
3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/48*a^(1/2)*(24*A*c*e^2-6*B*c*d*e+16*C
*a*e^2+5*C*c*d^2)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1
/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1
/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(3/2)/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a
)^(1/2)-1/16*(4*a*e^2*(-2*B*e+C*d)+c*d*(8*A*e^2-6*B*d*e+5*C*d^2))*(1-a/c/x
^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Elliptic
Pi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(
1/2)))^(1/2))/c/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{ 2251}$$
$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{x^4(C\,x^4+B\,x^2+A)}{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}dx$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int(x^4*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^4}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(C*x^8 + B*x^6 + A*x^4)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*e*x^
6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{x^4(A + Bx^2 + Cx^4)}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{x^4(A + Bx^2 + Cx^4)}{\sqrt{a - cx^4}\sqrt{d + ex^2}}\,dx$$

input
```
integrate(x**4*(C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(x**4*(A + B*x**2 + C*x**4)/(sqrt(a - c*x**4)*sqrt(d + e*x**2)), x
)
```

Maxima [F]

$$\int \frac{x^4(A + Bx^2 + Cx^4)}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{(Cx^4 + Bx^2 + A)x^4}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{x^4(A + Bx^2 + Cx^4)}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{(Cx^4 + Bx^2 + A)x^4}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A + Bx^2 + Cx^4)}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{x^4\,(C\,x^4 + B\,x^2 + A)}{\sqrt{a - c\,x^4}\,\sqrt{e\,x^2 + d}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4))/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4))/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(A + Bx^2 + Cx^4)}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx$$
$$= \frac{-6\sqrt{e\,x^2 + d}\,\sqrt{-c\,x^4 + a}\,bex + 5\sqrt{e\,x^2 + d}\,\sqrt{-c\,x^4 + a}\,cdx - 4\sqrt{e\,x^2 + d}\,\sqrt{-c\,x^4 + a}\,ce\,x^3 + 40\Big(\int \frac{\sqrt{e\,x}}{-ce\,x}}{}$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*e*x + 5*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*c*d*x - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*e*x**3 + 40*int((s
qrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x*
*6),x)*a*c*e**2 - 18*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a
*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c*d*e + 15*int((sqrt(d + e*x**2)*sqrt(
a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c**2*d**2 + 12
*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 -
 c*e*x**6),x)*a*b*e**2 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a
*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*d*e + 6*int((sqrt(d + e*x**2)*
sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*d*e - 5*in
t((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**
6),x)*a*c*d**2)/(24*c*e**2)
```

3.68 $\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 551

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \frac{(3Cd-4Be)\sqrt{d+ex^2}\sqrt{a-cx^4}}{8ce^2x} - \frac{Cx\sqrt{d+ex^2}\sqrt{a-cx^4}}{4ce}$$
$$+ \frac{(3Cd-4Be)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{\sqrt{a}(Cd-4Be)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8\sqrt{c}e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{(4(2Ac+aC)e^2+cd(3Cd-4Be))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}\cdots}{\sqrt{\cdots}}}\cdots\right.}{8ce^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/8*(-4*B*e+3*C*d)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e^2/x-1/4*C*x*(e*x^2
+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e+1/8*(-4*B*e+3*C*d)*(d+a^(1/2)*e/c^(1/2))*(1
-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*El
lipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/
c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/8*a^(1/2)*(-4*B*e+
C*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(
1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(
1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/8*(4*
(2*A*c+C*a)*e^2+c*d*(-4*B*e+3*C*d))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+
d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)
^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/e^2/(e*x^2+d)^
(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{2251}$$

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{x^2(C\,x^4+B\,x^2+A)}{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}dx$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A + Bx^2 + Cx^4)}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\, dx = \int \frac{x^2(A + Bx^2 + Cx^4)}{\sqrt{a - cx^4}\sqrt{d + ex^2}}\, dx$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4)/(sqrt(a - c*x**4)*sqrt(d + e*x**2)), x
)
```

Maxima [F]

$$\int \frac{x^2(A + Bx^2 + Cx^4)}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\, dx = \int \frac{(Cx^4 + Bx^2 + A)x^2}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}}\, dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{x^2(A + Bx^2 + Cx^4)}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\, dx = \int \frac{(Cx^4 + Bx^2 + A)x^2}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}}\, dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{x^2\,(C\,x^4+B\,x^2+A)}{\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x+4\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)be-3\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)cd+6\left(\int \frac{\sqrt{e\,x^2}}{-ce\,x^6}\right.}{4e}$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*x + 4*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*e - 3*int((sqrt
(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6)
,x)*c*d + 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 -
 c*d*x**4 - c*e*x**6),x)*a*e + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*
d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*d)/(4*e)
```

3.69 $\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 476

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx &= -\frac{C\sqrt{d+ex^2}\sqrt{a-cx^4}}{2cex} \\ &- \frac{C\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &+ \frac{(2Ac+aC)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{d+ex^2}\sqrt{a-cx^4}} \\ &- \frac{(Cd-2Be)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e\sqrt{d+ex^2}\sqrt{a-cx^4}}\end{aligned}$$

output
```
-1/2*C*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e/x-1/2*C*(d+a^(1/2)*e/c^(1/2))*
(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*
EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*
e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*(2*A*c+C*a)*(1-a
/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Elli
pticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^
(1/2)))^(1/2))/a^(1/2)/c^(1/2)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/2*(-2*B*
e+C*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)
^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(
d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{2261}$$
$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{A + Bx^2 + Cx^4}{\sqrt{a - cx^4}\sqrt{d + ex^2}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt(a - c*x**4)*sqrt(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)c+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)b+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)a$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 -
c*e*x**6),x)*c + int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x
**2 - c*d*x**4 - c*e*x**6),x)*b + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/
(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a
```

3.70 $\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 427

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \frac{Ac\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{ad\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{\sqrt{c}(Bd-Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{a}d\sqrt{d+ex^2}\sqrt{a-cx^4}} + \frac{C\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
A*c*(d+a^(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2
)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1
/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d/(e*x^2+d)^(1/2)/(-c*x^4+a
)^(1/2)+c^(1/2)*(-A*e+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/
2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(
1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/d/(e*x^2+d)^(1/2)/(-
c*x^4+a)^(1/2)+C*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/
2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^
(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^2\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{A + Bx^2 + Cx^4}{x^2\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + Bx^2 + Cx^4}{x^2\sqrt{a - cx^4}\sqrt{d + ex^2}}\,dx$$
$$\Big\downarrow \; 2251$$
$$\int \frac{A + Bx^2 + Cx^4}{x^2\sqrt{a - cx^4}\sqrt{d + ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{x^{2}\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*e*x^8 +
c*d*x^6 - a*e*x^4 - a*d*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**2*sqrt(a - c*x**4)*sqrt(d + e*x**2)), x
)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^2\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$
$$=\frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}-2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)cex+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)bdx}{dx}$$

input
```
int((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4) - 2*int((sqrt(d + e*x**2)*sqrt(a - c
*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c*e*x + int((sqrt(d
 + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*d
*x)/(d*x)
```

3.71 $\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 368

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{3adx^3}$$
$$+\frac{c(3Bd-2Ae)\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3ad^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{c}(3ad(Cd-Be)+A(cd^2+2ae^2))\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right.}{3a^{3/2}d^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/3*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^3+1/3*c*(-2*A*e+3*B*d)*(d+a^
(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/
2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/
2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
+1/3*c^(1/2)*(3*a*d*(-B*e+C*d)+A*(2*a*e^2+c*d^2))*(1-a/c/x^4)^(1/2)*x^3*(a
^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)
/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/
2)/d^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{ 2251}$$

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^4*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{x^{4}\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*e*x^10 +
 c*d*x^8 - a*e*x^6 - a*d*x^4), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**4*sqrt(a - c*x**4)*sqrt(d + e*x**2)), x
)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^4\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$
$$=\frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,b+2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^{10}-cd\,x^8+ae\,x^6+ad\,x^4}dx\right)a^2e\,x^3-3\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^{10}-cd\,x^8+ae\,x^6+ad\,x^4}dx\right)abd\,x}{2ae\,x^3}$$

input
```
int((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*b + 2*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4))/(a*d*x**4 + a*e*x**6 - c*d*x**8 - c*e*x**10),x)*a**2*e*x**3 - 3*
int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**4 + a*e*x**6 - c*d*x**8 -
c*e*x**10),x)*a*b*d*x**3 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d
+ a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*e*x**3 + int((sqrt(d + e*x**2)*sq
rt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c*d*x**3)/(2*a
*e*x**3)
```

3.72 $\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 445

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{5adx^5}-\frac{(5Bd-4Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{15ad^2x^3}$$

$$+\frac{c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(5ad(3Cd-2Be)+A(9cd^2+8ae^2)\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\Big|\frac{\cdots}{d+\cdots}\right.}{15a^2d^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{\sqrt{c}(5Bcd^3-7Acd^2e-15aCd^2e+10aBde^2-8aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{15a^{3/2}d^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output

```
-1/5*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^5-1/15*(-4*A*e+5*B*d)*(e*x^2
+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^3+1/15*c*(d+a^(1/2)*e/c^(1/2))*(5*a*d*(
-2*B*e+3*C*d)+A*(8*a*e^2+9*c*d^2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d
)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(
1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/d^3/(e*x^2+d)^(1
/2)/(-c*x^4+a)^(1/2)+1/15*c^(1/2)*(-8*A*a*e^3-7*A*c*d^2*e+10*B*a*d*e^2+5*B
*c*d^3-15*C*a*d^2*e)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a
^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2
^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/d^3/(e*x^2+d)^(1/2)/(-c*x^
4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{ 2251}$$

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^6*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{x^{6}\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*e*x^12 +
 c*d*x^10 - a*e*x^8 - a*d*x^6), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**6/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**6*sqrt(a - c*x**4)*sqrt(d + e*x**2)), x
)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{x^6\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^6*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^6*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*d*e**2 - 20*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a*b*d*e**2*x**2 + 40*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*
b*e**3*x**4 - 6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**2*e*x**2 - 20*sqr
t(d + e*x**2)*sqrt(a - c*x**4)*a*c*d*e**2*x**4 - 15*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*b*c*d**3*x**2 + 30*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*d**2*
e*x**4 + 80*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 -
 c*d*x**4 - c*e*x**6),x)*a*b*c*e**4*x**5 - 40*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c**2*d*e**3*x
**5 + 60*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*
d*x**4 - c*e*x**6),x)*b*c**2*d**2*e**2*x**5 + 40*int((sqrt(d + e*x**2)*sqr
t(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*d*e**3
*x**5 - 20*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 -
c*d*x**4 - c*e*x**6),x)*a*c**2*d**2*e**2*x**5 + 30*int((sqrt(d + e*x**2)*s
qrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c**2*d**
3*e*x**5 - 16*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**4 + a*e*x**6
 - c*d*x**8 - c*e*x**10),x)*a**3*d*e**3*x**5 - 40*int((sqrt(d + e*x**2)*sq
rt(a - c*x**4))/(a*d*x**4 + a*e*x**6 - c*d*x**8 - c*e*x**10),x)*a**2*b*d**
2*e**2*x**5 - 18*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**4 + a*e*x
**6 - c*d*x**8 - c*e*x**10),x)*a**2*c*d**3*e*x**5 - 45*int((sqrt(d + e*x**
2)*sqrt(a - c*x**4))/(a*d*x**4 + a*e*x**6 - c*d*x**8 - c*e*x**10),x)*a*...
```

3.73 $\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 546

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{7adx^7} - \frac{(7Bd-6Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{35ad^2x^5}$$
$$- \frac{(25Acd^2+35aCd^2-28aBde+24aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{105a^2d^3x^3}$$
$$- \frac{c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(44Acd^2e+70aCd^2e+48aAe^3-7B(9cd^3+8ade^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\right.}{105a^2d^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{\sqrt{c}(A(25c^2d^4+32acd^2e^2+48a^2e^4)+7ad(cd^2(5Cd-7Be)+2ae^2(5Cd-4Be)))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(}{(\sqrt{c}d+}}}{105a^{5/2}d^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/7*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^7-1/35*(-6*A*e+7*B*d)*(e*x^2
+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^5-1/105*(24*A*a*e^2+25*A*c*d^2-28*B*a*d
*e+35*C*a*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/d^3/x^3-1/105*c*(d+a^(
1/2)*e/c^(1/2))*(44*A*c*d^2*e+70*C*a*d^2*e+48*A*a*e^3-7*B*(8*a*d*e^2+9*c*d
^3))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(
1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(
1/2)*e/c^(1/2)))^(1/2))/a^2/d^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/105*c^(
1/2)*(A*(48*a^2*e^4+32*a*c*d^2*e^2+25*c^2*d^4)+7*a*d*(c*d^2*(-7*B*e+5*C*d)
+2*a*e^2*(-4*B*e+5*C*d)))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2
)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1
/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(5/2)/d^4/(e*x^2+d)^(1/2)/(
-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^8*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(x^8*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \ 2251$$
$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^8*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{x^{8}\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*e*x^14 +
 c*d*x^12 - a*e*x^10 - a*d*x^8), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**8/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**8*sqrt(a - c*x**4)*sqrt(d + e*x**2)), x
)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^8), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^8\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^8*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^8*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\text{too large to display}$$

input
```
int((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output

```
( - 48*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*d*e**4 + 4*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a**3*c*d**3*e**2 + 120*sqrt(d + e*x**2)*sqrt(a - c*x**4)*
a**3*c*d**2*e**3*x**2 - 240*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c*d*e**
4*x**4 + 288*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c*e**5*x**6 - 84*sqrt(
d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c*d**3*e**2*x**2 + 168*sqrt(d + e*x**2
)*sqrt(a - c*x**4)*a**2*b*c*d**2*e**3*x**4 - 336*sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*a**2*b*c*d*e**4*x**6 - 10*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*
c**2*d**4*e*x**2 + 20*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**3*e**
2*x**4 + 336*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**2*e**3*x**6 -
105*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c**2*d**5*x**2 + 210*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*a*b*c**2*d**4*e*x**4 - 672*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*a*b*c**2*d**3*e**2*x**6 - 30*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a
*c**3*d**4*e*x**6 - 315*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c**3*d**5*x**6
 + 6912*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(12*a**2*d*e**2 + 12*
a**2*e**3*x**2 - a*c*d**3 - a*c*d**2*e*x**2 - 12*a*c*d*e**2*x**4 - 12*a*c*
e**3*x**6 + c**2*d**3*x**4 + c**2*d**2*e*x**6),x)*a**4*c**2*e**8*x**7 - 80
64*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(12*a**2*d*e**2 + 12*a**2*
e**3*x**2 - a*c*d**3 - a*c*d**2*e*x**2 - 12*a*c*d*e**2*x**4 - 12*a*c*e**3*
x**6 + c**2*d**3*x**4 + c**2*d**2*e*x**6),x)*a**3*b*c**2*d*e**7*x**7 + 748
8*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(12*a**2*d*e**2 + 12*a**...
```

3.74 $\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 662

$$\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{9adx^9}$$
$$-\frac{(9Bd-8Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{63ad^2x^7}$$
$$-\frac{(49Acd^2+63aCd^2-54aBde+48aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{315a^2d^3x^5}$$
$$-\frac{(75Bcd^3-62Acd^2e-84aCd^2e+72aBde^2-64aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{315a^2d^4x^3}$$
$$+\frac{c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(A(147c^2d^4+108acd^2e^2+128a^2e^4)+3ad(cd^2(63Cd-44Be)+8ae^2(7Cd-6Be)))\sqrt{1}}{315a^3d^5\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{c}(75Bc^2d^5-111Ac^2d^4e-147acCd^4e+96aBcd^3e^2-76aAcd^2e^3-168a^2Cd^2e^3+144a^2Bde^4-128}{315a^{5/2}d^5\sqrt{d+ex^2}\sqrt{a-cx}}$$

output
```
-1/9*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^9-1/63*(-8*A*e+9*B*d)*(e*x^2
+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^7-1/315*(48*A*a*e^2+49*A*c*d^2-54*B*a*d
*e+63*C*a*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/d^3/x^5-1/315*(-64*A*a
*e^3-62*A*c*d^2*e+72*B*a*d*e^2+75*B*c*d^3-84*C*a*d^2*e)*(e*x^2+d)^(1/2)*(-
c*x^4+a)^(1/2)/a^2/d^4/x^3+1/315*c*(d+a^(1/2)*e/c^(1/2))*(A*(128*a^2*e^4+1
08*a*c*d^2*e^2+147*c^2*d^4)+3*a*d*(c*d^2*(-44*B*e+63*C*d)+8*a*e^2*(-6*B*e+
7*C*d)))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^
2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d
+a^(1/2)*e/c^(1/2)))^(1/2))/a^3/d^5/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/315
*c^(1/2)*(-128*A*a^2*e^5-76*A*a*c*d^2*e^3-111*A*c^2*d^4*e+144*B*a^2*d*e^4+
96*B*a*c*d^3*e^2+75*B*c^2*d^5-168*C*a^2*d^2*e^3-147*C*a*c*d^4*e)*(1-a/c/x^
4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF
(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)
))^(1/2))/a^(5/2)/d^5/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^10*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(x^10*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{ 2251}$$
$$\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^10*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{x^{10}\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(-(C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*e*x^16 +
 c*d*x^14 - a*e*x^12 - a*d*x^10), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**10/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**10*sqrt(a - c*x**4)*sqrt(d + e*x**2)),
x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^10), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^{10}\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^10*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^10*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\text{too large to display}$$

input
```
int((C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output

```
( - 2880*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*d*e**6 - 8640*sqrt(d + e*x
**2)*sqrt(a - c*x**4)*a**4*b*d*e**6*x**2 + 10368*sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*a**4*b*e**7*x**4 - 3360*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c*
d**2*e**5*x**2 - 5184*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c*d*e**6*x**4
 - 10800*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*c*d**3*e**4*x**2 + 12960
*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*c*d**2*e**5*x**4 - 1728*sqrt(d +
 e*x**2)*sqrt(a - c*x**4)*a**3*b*c*d*e**6*x**6 + 20*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*a**3*c**2*d**5*e**2 - 2800*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a
**3*c**2*d**4*e**3*x**2 + 3360*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2
*d**3*e**4*x**4 - 14496*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2*d**2*e
**5*x**6 - 9540*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c**2*d**5*e**2*x*
*2 + 11448*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c**2*d**4*e**3*x**4 -
2160*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c**2*d**3*e**4*x**6 - 210*sq
rt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**3*d**6*e*x**2 + 316*sqrt(d + e*x**
2)*sqrt(a - c*x**4)*a**2*c**3*d**5*e**2*x**4 - 560*sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*a**2*c**3*d**4*e**3*x**6 - 4725*sqrt(d + e*x**2)*sqrt(a - c*x**
4)*a*b*c**3*d**7*x**2 + 5670*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c**3*d*
*6*e*x**4 - 1908*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c**3*d**5*e**2*x**6
 + 54*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**4*d**6*e*x**6 - 945*sqrt(d +
e*x**2)*sqrt(a - c*x**4)*b*c**4*d**7*x**6 - 3317760*int((sqrt(d + e*x**...
```

3.75 $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 815

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

$$= \frac{(4ae^2(25dD-32Ce)+3c(35d^3D-40Cd^2e+48Bde^2-64Ae^3))\sqrt{d+ex^2}\sqrt{a-cx^4}}{384c^2e^4x}$$

$$- \frac{(36aDe^2+c(35d^2D-40Cde+48Be^2))\,x\sqrt{d+ex^2}\sqrt{a-cx^4}}{192c^2e^3}$$

$$+ \frac{(7dD-8Ce)x^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{48ce^2} - \frac{Dx^5\sqrt{d+ex^2}\sqrt{a-cx^4}}{8ce}$$

$$+ \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(4ae^2(25dD-32Ce)+3c(35d^3D-40Cd^2e+48Bde^2-64Ae^3))\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x}}}{384ce^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$- \frac{\sqrt{a}(4ae^2(7dD-32Ce)+c(35d^3D-40Cd^2e+48Bde^2-192Ae^3))\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{Elliptic}}{384c^{3/2}e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+ \frac{(48a^2De^4+8ace^2(3d^2D-4Cde+8Be^2)+c^2d(35d^3D-40Cd^2e+48Bde^2-64Ae^3))\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{}}{128c^2e^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/384*(4*a*e^2*(-32*C*e+25*D*d)+3*c*(-64*A*e^3+48*B*d*e^2-40*C*d^2*e+35*D*
d^3))*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c^2/e^4/x-1/192*(36*a*D*e^2+c*(48*B
*e^2-40*C*d*e+35*D*d^2))*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c^2/e^3+1/48*(
-8*C*e+7*D*d)*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e^2-1/8*D*x^5*(e*x^2+
d)^(1/2)*(-c*x^4+a)^(1/2)/c/e+1/384*(d+a^(1/2)*e/c^(1/2))*(4*a*e^2*(-32*C*
e+25*D*d)+3*c*(-64*A*e^3+48*B*d*e^2-40*C*d^2*e+35*D*d^3))*(1-a/c/x^4)^(1/2
)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1
-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2
))/c/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/384*a^(1/2)*(4*a*e^2*(-32*C*e+
7*D*d)+c*(-192*A*e^3+48*B*d*e^2-40*C*d^2*e+35*D*d^3))*(1-a/c/x^4)^(1/2)*x^
3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(
1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c
^(3/2)/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/128*(48*a^2*D*e^4+8*a*c*e^2*
(8*B*e^2-4*C*d*e+3*D*d^2)+c^2*d*(-64*A*e^3+48*B*d*e^2-40*C*d^2*e+35*D*d^3)
)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2
)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(
1/2)*e/c^(1/2)))^(1/2))/c^2/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^
4]),x]
```

output
```
Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^
4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A + Bx^2 + Cx^4 + Dx^6)}{\sqrt{a - cx^4}\sqrt{d + ex^2}} dx$$
$$\Big\downarrow \ 2251$$
$$\int \frac{x^4 (A + Bx^2 + Cx^4 + Dx^6)}{\sqrt{a - cx^4}\sqrt{d + ex^2}} dx$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{x^4 (D x^6 + C\,x^4 + B\,x^2 + A)}{\sqrt{e\,x^2 + d}\,\sqrt{-c\,x^4 + a}} dx$$

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)x^4}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(D*x^10 + C*x^8 + B*x^6 + A*x^4)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)/(c*e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral(x**4*(A + B*x**2 + C*x**4 + D*x**6)/(sqrt(a - c*x**4)*sqrt(d + e*
x**2)), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)x^4}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^4/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)x^4}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^4/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{x^4\,(A+B\,x^2+C\,x^4+x^6\,D)}{\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)
),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output

```
( - 36*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*d*e**2*x - 48*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*b*c*e**2*x + 40*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c**2*d*
e*x - 32*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c**2*e**2*x**3 - 35*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*c*d**3*x + 28*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d
**2*e*x**3 - 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d*e**2*x**5 + 320*int(
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*
x**6),x)*a*c**2*e**3 - 100*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a
*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*d**2*e**2 - 144*int((sqrt(d +
e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b
*c**2*d*e**2 + 120*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e
*x**2 - c*d*x**4 - c*e*x**6),x)*c**3*d**2*e - 105*int((sqrt(d + e*x**2)*sq
rt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c**2*d**4 +
 72*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**
4 - c*e*x**6),x)*a**2*d*e**3 + 96*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x
**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c*e**3 + 16*int((sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x
)*a*c**2*d*e**2 - 14*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a
*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*d**3*e + 36*int((sqrt(d + e*x**2)*sq
rt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*d**2*e**2 +
 48*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 ...
```

3.76 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 676

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

$$= -\frac{(15cd^2D-18cCde+24Bce^2+16aDe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{48c^2e^3x}$$

$$+\frac{(5dD-6Ce)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{24ce^2}-\frac{Dx^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{6ce}$$

$$-\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(16aDe^2+3c(5d^2D-6Cde+8Be^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{\ }{d+}\right.}{48ce^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{\sqrt{a}(16aDe^2+c(5d^2D-6Cde+24Be^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{\ }{d}\right.}{48c^{3/2}e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$-\frac{(4ae^2(dD-2Ce)+c(5d^3D-6Cd^2e+8Bde^2-16Ae^3))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\text{arc}\right.}{16ce^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/48*(24*B*c*e^2-18*C*c*d*e+16*D*a*e^2+15*D*c*d^2)*(e*x^2+d)^(1/2)*(-c*x^
4+a)^(1/2)/c^2/e^3/x+1/24*(-6*C*e+5*D*d)*x*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2
)/c/e^2-1/6*D*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e-1/48*(d+a^(1/2)*e/c
^(1/2))*(16*a*D*e^2+3*c*(8*B*e^2-6*C*d*e+5*D*d^2))*(1-a/c/x^4)^(1/2)*x^3*(
a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2
)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/e^
3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/48*a^(1/2)*(16*a*D*e^2+c*(24*B*e^2-6*
C*d*e+5*D*d^2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2
)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2
)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(3/2)/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^
(1/2)-1/16*(4*a*e^2*(-2*C*e+D*d)+c*(-16*A*e^3+8*B*d*e^2-6*C*d^2*e+5*D*d^3)
)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2
)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(
1/2)*e/c^(1/2)))^(1/2))/c/e^3/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^
4]),x]
```

output
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^
4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \ 2251$$

$$\int \frac{x^2 (A + Bx^2 + Cx^4 + Dx^6)}{\sqrt{a - cx^4}\sqrt{d + ex^2}} dx$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{x^2 (D x^6 + C x^4 + B x^2 + A)}{\sqrt{e x^2 + d}\sqrt{-c x^4 + a}} dx$$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2 (A + Bx^2 + Cx^4 + Dx^6)}{\sqrt{d + ex^2}\sqrt{a - cx^4}} dx = \text{Timed out}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4 + D*x**6)/(sqrt(a - c*x**4)*sqrt(d + e*
x**2)), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)x^2}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^2/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)x^2}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^2/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{x^2\,(A+B\,x^2+C\,x^4+x^6\,D)}{\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)
),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$
$$=\frac{-6\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,cex+5\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,d^2x-4\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,de\,x^3+16\left(\int \frac{\sqrt{e\,x}}{-ce\,x}\right.}{}$$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*e*x + 5*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*d**2*x - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*d*e*x**3 + 16*int((
sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x
**6),x)*a*d*e**2 + 24*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d +
a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c*e**2 - 18*int((sqrt(d + e*x**2)*sqr
t(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c**2*d*e + 1
5*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4
- c*e*x**6),x)*c*d**3 + 36*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a
*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*e**2 + 2*int((sqrt(d + e*x**2)
*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*d**2*e
 + 6*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 -
c*e*x**6),x)*a*c*d*e - 5*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*
e*x**2 - c*d*x**4 - c*e*x**6),x)*a*d**3)/(24*c*e**2)
```

3.77 $\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 558

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \frac{(3dD-4Ce)\sqrt{d+ex^2}\sqrt{a-cx^4}}{8ce^2x} - \frac{Dx\sqrt{d+ex^2}\sqrt{a-cx^4}}{4ce}$$
$$+ \frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(3dD-4Ce)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$- \frac{(adD-8Ace-4aCe)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8\sqrt{a}\sqrt{c}e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{(4(2Bc+aD)e^2+cd(3dD-4Ce))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{\ }}{\sqrt{\ }}}\right.}{8ce^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
1/8*(-4*C*e+3*D*d)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e^2/x-1/4*D*x*(e*x^2
+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e+1/8*(d+a^(1/2)*e/c^(1/2))*(-4*C*e+3*D*d)*(1
-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*El
lipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/
c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/8*(-8*A*c*e-4*C*a*
e+D*a*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^
2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d
+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/c^(1/2)/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(
1/2)+1/8*(4*(2*B*c+D*a)*e^2+c*d*(-4*C*e+3*D*d))*(1-a/c/x^4)^(1/2)*x^3*(a^(
1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/
c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/e^
2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]), x
]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \; 2261$$
$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^6+C\,x^4+B\,x^2+A}{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algori
thm="fricas")
```

output
```
integral(-(D*x^6 + C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e*x^6 + c*d*x^4 - a*e*x^2 - a*d), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(sqrt(a - c*x**4)*sqrt(d + e*x**2)
), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algori
thm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)),
x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, algori
thm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/((a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,dx+4\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)c^2e-3\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)c\,d^2+2\left(\int \frac{}{}\right.}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*d*x + 4*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c**2*e - 3*int(
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*
x**6),x)*c*d**2 + 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*
e*x**2 - c*d*x**4 - c*e*x**6),x)*a*d*e + 4*int((sqrt(d + e*x**2)*sqrt(a -
c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c*e + 4*int((sqr
t(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*
a*c*e + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4
 - c*e*x**6),x)*a*d**2)/(4*c*e)
```

3.78 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 501

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = -\frac{D\sqrt{d+ex^2}\sqrt{a-cx^4}}{2cex}$$
$$-\frac{\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(adD-2Ace)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2ade\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{(2Bcd+adD-2Ace)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2\sqrt{a}\sqrt{c}d\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{(dD-2Ce)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/2*D*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e/x-1/2*(d+a^(1/2)*e/c^(1/2))*(-
2*A*c*e+D*a*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)
*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1
/2*(-2*A*c*e+2*B*c*d+D*a*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1
/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^
(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/c^(1/2)/d/(e*x^2+d)
^(1/2)/(-c*x^4+a)^(1/2)-1/2*(-2*C*e+D*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e
*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)
/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)
^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]
),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]
), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{ 2251}$$
$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^6 + C\,x^4 + B\,x^2 + A}{x^2\sqrt{e\,x^2 + d}\,\sqrt{-c\,x^4 + a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^2\sqrt{d + ex^2}\sqrt{a - cx^4}}\,dx = \int \frac{Dx^6 + Cx^4 + Bx^2 + A}{\sqrt{-cx^4 + a}\sqrt{ex^2 + d}x^2}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(D*x^6 + C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e*x^8 + c*d*x^6 - a*e*x^4 - a*d*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**2*sqrt(a - c*x**4)*sqrt(d + e*
x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^
2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^2\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$
$$=\frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}-2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)cex+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)d^2x+\left(\int \frac{\sqrt{e\,x^2}}{-ce\,x^6-}\right.}{dx}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4) - 2*int((sqrt(d + e*x**2)*sqrt(a - c
*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*c*e*x + int((sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x
)*d**2*x + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x
**4 - c*e*x**6),x)*b*d*x)/(d*x)
```

3.79 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 497

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{3adx^3}$$

$$+\frac{c(3Bd-2Ae)\left(d+\frac{\sqrt{ae}}{\sqrt{c}}\right)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{3ad^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{\sqrt{c}(3ad(Cd-Be)+A(cd^2+2ae^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{3a^{3/2}d^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{D\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{ae}}{\sqrt{c}}}\right)}{\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/3*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^3+1/3*c*(-2*A*e+3*B*d)*(d+a^
(1/2)*e/c^(1/2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/
2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/
2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
+1/3*c^(1/2)*(3*a*d*(-B*e+C*d)+A*(2*a*e^2+c*d^2))*(1-a/c/x^4)^(1/2)*x^3*(a
^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)
/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/
2)/d^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+D*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(
e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2
)/x^2)^(1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/(e*x^2+d)^
(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]
),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]
), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \; 2251$$
$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^{6}+C\,x^{4}+B\,x^{2}+A}{x^{4}\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(D*x^6 + C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e*x^10 + c*d*x^8 - a*e*x^6 - a*d*x^4), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**4*sqrt(a - c*x**4)*sqrt(d + e*
x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^
4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^
4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^4\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$
$$=\frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,b+2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^2}{-ce\,x^6-cd\,x^4+ae\,x^2+ad}dx\right)ade\,x^3+2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}{-ce\,x^{10}-cd\,x^8+ae\,x^6+ad\,x^4}dx\right)a^2e\,x^3-}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*b + 2*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*d*e*x**3 + 2*in
t((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x**4 + a*e*x**6 - c*d*x**8 - c*
e*x**10),x)*a**2*e*x**3 - 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d*x
**4 + a*e*x**6 - c*d*x**8 - c*e*x**10),x)*a*b*d*x**3 + 2*int((sqrt(d + e*x
**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c*e*x**
3 + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d + a*e*x**2 - c*d*x**4 - c
*e*x**6),x)*b*c*d*x**3)/(2*a*e*x**3)
```

3.80 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 452

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

$$= -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{5adx^5} - \frac{(5Bd-4Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{15ad^2x^3}$$

$$+ \frac{c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\left(5ad(3Cd-2Be)+A(9cd^2+8ae^2)\right)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{\ldots}{d+\ldots}\right)}{15a^2d^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+ \frac{\sqrt{c}(5Bcd^3+15ad^3D-7Acd^2e-15aCd^2e+10aBde^2-8aAe^3)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\ldots\right)}{15a^{3/2}d^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/5*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^5-1/15*(-4*A*e+5*B*d)*(e*x^2
+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^3+1/15*c*(d+a^(1/2)*e/c^(1/2))*(5*a*d*(
-2*B*e+3*C*d)+A*(8*a*e^2+9*c*d^2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d
)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(
1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/d^3/(e*x^2+d)^(1
/2)/(-c*x^4+a)^(1/2)+1/15*c^(1/2)*(-8*A*a*e^3-7*A*c*d^2*e+10*B*a*d*e^2+5*B
*c*d^3-15*C*a*d^2*e+15*D*a*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(
c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2
)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/d^3/(e*x^2+d)^(
1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]
),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]
), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{ 2251}$$
$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{D x^{6}+C\,x^{4}+B\,x^{2}+A}{x^{6}\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^6}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(D*x^6 + C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e*x^12 + c*d*x^10 - a*e*x^8 - a*d*x^6), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**6*sqrt(a - c*x**4)*sqrt(d + e*
x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^6}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^
6), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^6}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^
6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^6\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\text{too large to display}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c*d*e**2 - 40*sqrt(d + e*x**2
)*sqrt(a - c*x**4)*a**2*d**2*e**2*x**2 + 80*sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*a**2*d*e**3*x**4 - 20*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*d*e**2*x
**2 + 40*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*e**3*x**4 - 6*sqrt(d + e*
x**2)*sqrt(a - c*x**4)*a*c**2*d**2*e*x**2 - 20*sqrt(d + e*x**2)*sqrt(a - c
*x**4)*a*c**2*d*e**2*x**4 - 45*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**4*
x**2 + 90*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**3*e*x**4 - 15*sqrt(d +
e*x**2)*sqrt(a - c*x**4)*b*c**2*d**3*x**2 + 30*sqrt(d + e*x**2)*sqrt(a - c
*x**4)*b*c**2*d**2*e*x**4 + 160*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**
4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a**2*c*d*e**4*x**5 + 80*int((
sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x
**6),x)*a*b*c**2*e**4*x**5 - 40*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**
4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*c**3*d*e**3*x**5 + 180*int(
(sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*
x**6),x)*a*c**2*d**3*e**2*x**5 + 60*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)
*x**4)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*b*c**3*d**2*e**2*x**5 + 8
0*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4
- c*e*x**6),x)*a**2*c*d**2*e**3*x**5 + 40*int((sqrt(d + e*x**2)*sqrt(a - c
*x**4)*x**2)/(a*d + a*e*x**2 - c*d*x**4 - c*e*x**6),x)*a*b*c**2*d*e**3*x**
5 - 20*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d + a*e*x**2 - c...
```

3.81 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 561

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$
$$= -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{7adx^7} - \frac{(7Bd-6Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{35ad^2x^5}$$
$$- \frac{(25Acd^2+35aCd^2-28aBde+24aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{105a^2d^3x^3}$$
$$- \frac{c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(44Acd^2e-7B(9cd^3+8ade^2)-a(105d^3D-70Cd^2e-48Ae^3))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x}}}{105a^2d^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+ \frac{\sqrt{c}(A(25c^2d^4+32acd^2e^2+48a^2e^4)+7ad(cd^2(5Cd-7Be)-ae(15d^2D-10Cde+8Be^2)))\sqrt{1-\frac{a}{cx}}}{105a^{5/2}d^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-1/7*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^7-1/35*(-6*A*e+7*B*d)*(e*x^2
+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^5-1/105*(24*A*a*e^2+25*A*c*d^2-28*B*a*d
*e+35*C*a*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/d^3/x^3-1/105*c*(d+a^(
1/2)*e/c^(1/2))*(44*A*c*d^2*e-7*B*(8*a*d*e^2+9*c*d^3)-a*(-48*A*e^3-70*C*d^
2*e+105*D*d^3))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2
)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2
)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^2/d^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2
)+1/105*c^(1/2)*(A*(48*a^2*e^4+32*a*c*d^2*e^2+25*c^2*d^4)+7*a*d*(c*d^2*(-7
*B*e+5*C*d)-a*e*(8*B*e^2-10*C*d*e+15*D*d^2)))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/
2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(
1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(5/2)/d
^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]
),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]
), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{ 2251}$$
$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^{6}+C\,x^{4}+B\,x^{2}+A}{x^{8}\sqrt{e\,x^{2}+d}\,\sqrt{-c\,x^{4}+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(D*x^6 + C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e*x^14 + c*d*x^12 - a*e*x^10 - a*d*x^8), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**8*sqrt(a - c*x**4)*sqrt(d + e*
x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^
8), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^
8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^8\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx = \text{too large to display}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 48*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*d*e**4 + 4*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a**3*c*d**3*e**2 + 120*sqrt(d + e*x**2)*sqrt(a - c*x**4)*
a**3*c*d**2*e**3*x**2 - 240*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c*d*e**
4*x**4 + 288*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c*e**5*x**6 + 84*sqrt(
d + e*x**2)*sqrt(a - c*x**4)*a**3*d**4*e**2*x**2 - 168*sqrt(d + e*x**2)*sq
rt(a - c*x**4)*a**3*d**3*e**3*x**4 - 84*sqrt(d + e*x**2)*sqrt(a - c*x**4)*
a**2*b*c*d**3*e**2*x**2 + 168*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c*d
**2*e**3*x**4 - 336*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c*d*e**4*x**6
 - 10*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**4*e*x**2 + 20*sqrt(d
+ e*x**2)*sqrt(a - c*x**4)*a**2*c**2*d**3*e**2*x**4 + 336*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*a**2*c**2*d**2*e**3*x**6 - 175*sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*a**2*c*d**6*x**2 + 350*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c*d
**5*e*x**4 - 420*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c*d**4*e**2*x**6 -
 105*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c**2*d**5*x**2 + 210*sqrt(d + e
*x**2)*sqrt(a - c*x**4)*a*b*c**2*d**4*e*x**4 - 672*sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*a*b*c**2*d**3*e**2*x**6 - 30*sqrt(d + e*x**2)*sqrt(a - c*x**4)*
a*c**3*d**4*e*x**6 - 525*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c**2*d**6*x**
6 - 315*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c**3*d**5*x**6 + 6912*int((sqr
t(d + e*x**2)*sqrt(a - c*x**4)*x**4)/(12*a**2*d*e**2 + 12*a**2*e**3*x**2 -
 a*c*d**3 - a*c*d**2*e*x**2 - 12*a*c*d*e**2*x**4 - 12*a*c*e**3*x**6 + c...
```

3.82 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 691

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

$$= -\frac{A\sqrt{d+ex^2}\sqrt{a-cx^4}}{9adx^9} - \frac{(9Bd-8Ae)\sqrt{d+ex^2}\sqrt{a-cx^4}}{63ad^2x^7}$$

$$- \frac{(49Acd^2+63aCd^2-54aBde+48aAe^2)\sqrt{d+ex^2}\sqrt{a-cx^4}}{315a^2d^3x^5}$$

$$- \frac{(75Bcd^3+105ad^3D-62Acd^2e-84aCd^2e+72aBde^2-64aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{315a^2d^4x^3}$$

$$+ \frac{c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)(A(147c^2d^4+108acd^2e^2+128a^2e^4)+3ad(cd^2(63Cd-44Be)-2ae(35d^2D-28Cde+2}{315a^3d^5\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$- \frac{\sqrt{c}(111Ac^2d^4e-acd^2(105d^3D-147Cd^2e-76Ae^3)-2a^2e^2(105d^3D-84Cd^2e-64Ae^3)-3B(25c^2}{315a^{5/2}d^5\sqrt{d+ex^2}}$$

output
```
-1/9*A*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d/x^9-1/63*(-8*A*e+9*B*d)*(e*x^2
+d)^(1/2)*(-c*x^4+a)^(1/2)/a/d^2/x^7-1/315*(48*A*a*e^2+49*A*c*d^2-54*B*a*d
*e+63*C*a*d^2)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/d^3/x^5-1/315*(-64*A*a
*e^3-62*A*c*d^2*e+72*B*a*d*e^2+75*B*c*d^3-84*C*a*d^2*e+105*D*a*d^3)*(e*x^2
+d)^(1/2)*(-c*x^4+a)^(1/2)/a^2/d^4/x^3+1/315*c*(d+a^(1/2)*e/c^(1/2))*(A*(1
28*a^2*e^4+108*a*c*d^2*e^2+147*c^2*d^4)+3*a*d*(c*d^2*(-44*B*e+63*C*d)-2*a*
e*(24*B*e^2-28*C*d*e+35*D*d^2)))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/
(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/
2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^3/d^5/(e*x^2+d)^(1/2
)/(-c*x^4+a)^(1/2)-1/315*c^(1/2)*(111*A*c^2*d^4*e-a*c*d^2*(-76*A*e^3-147*C
*d^2*e+105*D*d^3)-2*a^2*e^2*(-64*A*e^3-84*C*d^2*e+105*D*d^3)-3*B*(48*a^2*d
*e^4+32*a*c*d^3*e^2+25*c^2*d^5))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/
(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/
2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(5/2)/d^5/(e*x^2+d)^
(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*Sqrt[d + e*x^2]*Sqrt[a - c*x^4
]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*Sqrt[d + e*x^2]*Sqrt[a - c*x^4
]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

$$\downarrow \text{2251}$$

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a-cx^4}\sqrt{d+ex^2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*Sqrt[d + e*x^2]*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^6+C\,x^4+B\,x^2+A}{x^{10}\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, a
lgorithm="fricas")
```

output
```
integral(-(D*x^6 + C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e*x^16 + c*d*x^14 - a*e*x^12 - a*d*x^10), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{a-cx^4}\sqrt{d+ex^2}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(e*x**2+d)**(1/2)/(-c*x**4+a)**(1
/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**10*sqrt(a - c*x**4)*sqrt(d + e
*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^
10), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)*x^
10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^{10}\,\sqrt{a-c\,x^4}\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))
,x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a - c*x^4)^(1/2)*(d + e*x^2)^(1/2))
, x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a-cx^4}}\,dx=\text{too large to display}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 2880*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*c*d*e**6 - 11520*sqrt(d +
e*x**2)*sqrt(a - c*x**4)*a**5*d**2*e**6*x**2 + 13824*sqrt(d + e*x**2)*sqrt
(a - c*x**4)*a**5*d*e**7*x**4 - 8640*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**
4*b*c*d*e**6*x**2 + 10368*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*b*c*e**7*
x**4 - 3360*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c**2*d**2*e**5*x**2 - 5
184*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c**2*d*e**6*x**4 - 19440*sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*a**4*c*d**4*e**4*x**2 + 23328*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*a**4*c*d**3*e**5*x**4 - 2304*sqrt(d + e*x**2)*sqrt(a - c
*x**4)*a**4*c*d**2*e**6*x**6 - 10800*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**
3*b*c**2*d**3*e**4*x**2 + 12960*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*c
**2*d**2*e**5*x**4 - 1728*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*c**2*d*
e**6*x**6 + 20*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**3*d**5*e**2 - 280
0*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**3*d**4*e**3*x**2 + 3360*sqrt(d
 + e*x**2)*sqrt(a - c*x**4)*a**3*c**3*d**3*e**4*x**4 - 14496*sqrt(d + e*x*
*2)*sqrt(a - c*x**4)*a**3*c**3*d**2*e**5*x**6 - 16920*sqrt(d + e*x**2)*sqr
t(a - c*x**4)*a**3*c**2*d**6*e**2*x**2 + 20304*sqrt(d + e*x**2)*sqrt(a - c
*x**4)*a**3*c**2*d**5*e**3*x**4 - 3888*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a
**3*c**2*d**4*e**4*x**6 - 9540*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c*
*3*d**5*e**2*x**2 + 11448*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c**3*d*
*4*e**3*x**4 - 2160*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c**3*d**3*...
```

3.83 $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 842

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \frac{d(d^3D-Cd^2e+Bde^2-Ae^3)\,x\sqrt{a-cx^4}}{e^3\left(cd^2-ae^2\right)\sqrt{d+ex^2}}$$
$$+\frac{(16a^2De^4+ace^2(41d^2D-42Cde+24Be^2)-3c^2d(35d^3D-30Cd^2e+24Bde^2-16Ae^3))\sqrt{d+ex^2}\sqrt{a}}{48c^2e^4\left(cd^2-ae^2\right)x}$$
$$+\frac{(11dD-6Ce)x\sqrt{d+ex^2}\sqrt{a-cx^4}}{24ce^3}-\frac{Dx^3\sqrt{d+ex^2}\sqrt{a-cx^4}}{6ce^2}$$
$$+\frac{(16a^2De^4+ace^2(41d^2D-42Cde+24Be^2)-3c^2d(35d^3D-30Cd^2e+24Bde^2-16Ae^3))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{}}{48c^{3/2}e^4\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{a}(16aDe^2+c(35d^2D-30Cde+24Be^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{}{d+}\right)}{48c^{3/2}e^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{(4ae^2(3dD-2Ce)+c(35d^3D-30Cd^2e+24Bde^2-16Ae^3))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,a\right.}{16ce^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
d*(-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*x*(-c*x^4+a)^(1/2)/e^3/(-a*e^2+c*d^2)/(e*
x^2+d)^(1/2)+1/48*(16*a^2*D*e^4+a*c*e^2*(24*B*e^2-42*C*d*e+41*D*d^2)-3*c^2
*d*(-16*A*e^3+24*B*d*e^2-30*C*d^2*e+35*D*d^3))*(e*x^2+d)^(1/2)*(-c*x^4+a)^
(1/2)/c^2/e^4/(-a*e^2+c*d^2)/x+1/24*(-6*C*e+11*D*d)*x*(e*x^2+d)^(1/2)*(-c*
x^4+a)^(1/2)/c/e^3-1/6*D*x^3*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e^2+1/48*(
16*a^2*D*e^4+a*c*e^2*(24*B*e^2-42*C*d*e+41*D*d^2)-3*c^2*d*(-16*A*e^3+24*B*
d*e^2-30*C*d^2*e+35*D*d^3))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1
/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^
(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(3/2)/e^4/(c^(1/2)*d-a^(1
/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/48*a^(1/2)*(16*a*D*e^2+c*(24*B*e
^2-30*C*d*e+35*D*d^2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d
+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2)
,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(3/2)/e^3/(e*x^2+d)^(1/2)/(-c*
x^4+a)^(1/2)-1/16*(4*a*e^2*(-2*C*e+3*D*d)+c*(-16*A*e^3+24*B*d*e^2-30*C*d^2
*e+35*D*d^3))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*
e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/
2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/e^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/((d + e*x^2)^(3/2)*Sqrt[a - c*
x^4]),x]
```

output
```
Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/((d + e*x^2)^(3/2)*Sqrt[a - c*
x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a-cx^4} \left(d+ex^2\right)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 2251}$$
$$\int \frac{x^4 \left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a-cx^4} \left(d+ex^2\right)^{3/2}}dx$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/((d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),
x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{x^4 (D x^6+C\,x^4+B\,x^2+A)}{(e\,x^2+d)^{\frac{3}{2}} \sqrt{-c\,x^4+a}}dx$$

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a-cx^4}\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(e*x**2+d)**(3/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral(x**4*(A + B*x**2 + C*x**4 + D*x**6)/(sqrt(a - c*x**4)*(d + e*x**2
)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)x^4}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^4/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3
/2)), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)x^4}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^4/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{x^4\,(A+B\,x^2+C\,x^4+x^6\,D)}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)
),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)
), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output

```
( - 12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*e**2*x - 2*sqrt(d + e*x**2)*s
qrt(a - c*x**4)*a*d**2*e*x - 6*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c**2*d*e*
x**3 + 7*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d**3*x**3 - 4*sqrt(d + e*x**2
)*sqrt(a - c*x**4)*c*d**2*e*x**5 - 24*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6
 - c*e**2*x**8),x)*a*c**2*d*e**3 - 24*int((sqrt(d + e*x**2)*sqrt(a - c*x**
4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6
 - c*e**2*x**8),x)*a*c**2*e**4*x**2 + 12*int((sqrt(d + e*x**2)*sqrt(a - c*
x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x
**6 - c*e**2*x**8),x)*a*c*d**3*e**2 + 12*int((sqrt(d + e*x**2)*sqrt(a - c*
x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x
**6 - c*e**2*x**8),x)*a*c*d**2*e**3*x**2 + 24*int((sqrt(d + e*x**2)*sqrt(a
 - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*
d*e*x**6 - c*e**2*x**8),x)*b*c**2*d**2*e**2 + 24*int((sqrt(d + e*x**2)*sqr
t(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2
*c*d*e*x**6 - c*e**2*x**8),x)*b*c**2*d*e**3*x**2 - 30*int((sqrt(d + e*x**2
)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**
4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*c**3*d**3*e - 30*int((sqrt(d + e*x**2)*
sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4
- 2*c*d*e*x**6 - c*e**2*x**8),x)*c**3*d**2*e**2*x**2 + 35*int((sqrt(d +...
```

3.84 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 717

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = -\frac{(d^3D-Cd^2e+Bde^2-Ae^3)\,x\sqrt{a-cx^4}}{e^2(cd^2-ae^2)\sqrt{d+ex^2}}$$

$$-\frac{(ae^2(7dD-4Ce)-c(15d^3D-12Cd^2e+8Bde^2-8Ae^3))\sqrt{d+ex^2}\sqrt{a-cx^4}}{8ce^3(cd^2-ae^2)\,x}$$

$$-\frac{Dx\sqrt{d+ex^2}\sqrt{a-cx^4}}{4ce^2}$$

$$-\frac{(ae^2(7dD-4Ce)-c(15d^3D-12Cd^2e+8Bde^2-8Ae^3))\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)\ldots\right)}{8\sqrt{c}e^3(\sqrt{c}d-\sqrt{a}e)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$-\frac{\sqrt{a}(5dD-4Ce)\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{8\sqrt{c}e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{(4aDe^2+c(15d^2D-12Cde+8Be^2))\sqrt{1-\frac{a}{cx^4}}\,x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\ldots}\right)}{8ce^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
-(-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*x*(-c*x^4+a)^(1/2)/e^2/(-a*e^2+c*d^2)/(e*x
^2+d)^(1/2)-1/8*(a*e^2*(-4*C*e+7*D*d)-c*(-8*A*e^3+8*B*d*e^2-12*C*d^2*e+15*
D*d^3))*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e^3/(-a*e^2+c*d^2)/x-1/4*D*x*(e
*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)/c/e^2-1/8*(a*e^2*(-4*C*e+7*D*d)-c*(-8*A*e^3
+8*B*d*e^2-12*C*d^2*e+15*D*d^3))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/
(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/
2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/e^3/(c^(1/2)*d
-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/8*a^(1/2)*(-4*C*e+5*D*d)*(1
-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*El
lipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/
c^(1/2)))^(1/2))/c^(1/2)/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/8*(4*a*D*e
^2+c*(8*B*e^2-12*C*d*e+15*D*d^2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)
/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(
1/2)*2^(1/2),2,2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c/e^3/(e*x^2+d)^(1
/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/((d + e*x^2)^(3/2)*Sqrt[a - c*
x^4]),x]
```

output
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/((d + e*x^2)^(3/2)*Sqrt[a - c*
x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a-cx^4}\,(d+ex^2)^{3/2}}\,dx$$

$$\downarrow \text{2251}$$

$$\int \frac{x^2 \left(A+Bx^2+Cx^4+Dx^6\right)}{\sqrt{a-cx^4} \left(d+ex^2\right)^{3/2}}dx$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/((d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),
x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{x^{2} \left(D x^{6}+C \,x^{4}+B \,x^{2}+A \right)}{\left(e \,x^{2}+d \right)^{\frac{3}{2}} \sqrt{-c \,x^{4}+a}}d x$$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2 (A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{(Dx^6+Cx^4+Bx^2+A)x^2}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(D*x^8 + C*x^6 + B*x^4 + A*x^2)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)
/(c*e^2*x^8 + 2*c*d*e*x^6 - 2*a*d*e*x^2 + (c*d^2 - a*e^2)*x^4 - a*d^2), x)
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{a-cx^4}\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(e*x**2+d)**(3/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4 + D*x**6)/(sqrt(a - c*x**4)*(d + e*x**2
)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)x^2}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^2/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3
/2)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)x^2}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^2/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{x^2\,(A+B\,x^2+C\,x^4+x^6\,D)}{\sqrt{a-c\,x^4}\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)
),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)
), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output

```
( - 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*d*e*x - 4*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*b*c*e*x - 3*sqrt(d + e*x**2)*sqrt(a - c*x**4)*c*d**2*x**3 - 4*
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e*
*2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*c*d**2*e**2 - 4*i
nt((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**
2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*c*d*e**3*x**2 - 8*
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e*
*2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*b*c**2*d*e**2 - 8*i
nt((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**
2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*b*c**2*e**3*x**2 + 1
2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*
e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*c**3*d**2*e + 12*
int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e*
*2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*c**3*d*e**2*x**2 -
15*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a
*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*c**2*d**4 - 15*i
nt((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**
2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*c**2*d**3*e*x**2 + 1
2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d**2 + 2*a*d*e*x**2 + a*
e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*c**2*d**2*e ...
```

3.85 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 658

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \frac{(d^3D-Cd^2e+Bde^2-Ae^3)\,x\sqrt{a-cx^4}}{de\,(cd^2-ae^2)\sqrt{d+ex^2}}$$
$$+\frac{(adDe^2-c(3d^3D-2Cd^2e+2Bde^2-2Ae^3))\sqrt{d+ex^2}\sqrt{a-cx^4}}{2cde^2\,(cd^2-ae^2)\,x}$$
$$+\frac{(adDe^2-c(3d^3D-2Cd^2e+2Bde^2-2Ae^3))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2\sqrt{c}de^2\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{(adD+2Ace)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2\sqrt{a}\sqrt{c}de\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$-\frac{(3dD-2Ce)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{2e^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
(-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*x*(-c*x^4+a)^(1/2)/d/e/(-a*e^2+c*d^2)/(e*x^
2+d)^(1/2)+1/2*(a*d*D*e^2-c*(-2*A*e^3+2*B*d*e^2-2*C*d^2*e+3*D*d^3))*(e*x^2
+d)^(1/2)*(-c*x^4+a)^(1/2)/c/d/e^2/(-a*e^2+c*d^2)/x+1/2*(a*d*D*e^2-c*(-2*A
*e^3+2*B*d*e^2-2*C*d^2*e+3*D*d^3))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d
)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(
1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/c^(1/2)/d/e^2/(c^(1/
2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/2*(2*A*c*e+D*a*d)*(1-a/
c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellip
ticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(
1/2)))^(1/2))/a^(1/2)/c^(1/2)/d/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)-1/2*(-2
*C*e+3*D*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)
/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e^2/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/((d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),
x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/((d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),
 x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a-cx^4}\,(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2261}$$

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a-cx^4}\,(d+ex^2)^{3/2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/((d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^6+C\,x^4+B\,x^2+A}{(e\,x^2+d)^{\frac{3}{2}}\sqrt{-c\,x^4+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algori
thm="fricas")
```

output
```
integral(-(D*x^6 + C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e^2*x^8 + 2*c*d*e*x^6 - 2*a*d*e*x^2 + (c*d^2 - a*e^2)*x^4 - a*d^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{a-cx^4}\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(e*x**2+d)**(3/2)/(-c*x**4+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(sqrt(a - c*x**4)*(d + e*x**2)**(3
/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algori
thm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2))
, x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, algori
thm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{\sqrt{a-c\,x^4}\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/((a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x-2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^6}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx\right)}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4)*x - 2*int((sqrt(d + e*x**2)*sqrt(a -
 c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*
e*x**6 - c*e**2*x**8),x)*c*d*e - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*
x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 -
c*e**2*x**8),x)*c*e**2*x**2 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**
6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e
**2*x**8),x)*d**3 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2
 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),
x)*d**2*e*x**2 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d**2 +
2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*
b*d**2 + 3*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**2)/(a*d**2 + 2*a*d*e*
x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*b*d*e*x*
*2 + 4*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d*e*x**2 + a*
e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*d**2 + 4*int((s
qrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c
*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*d*e*x**2)/(3*d*(d + e*x**2))
```

3.86 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 547

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\frac{(d^3D-Cd^2e+Bde^2-Ae^3)\sqrt{a-cx^4}}{de(cd^2-ae^2)x\sqrt{d+ex^2}}$$
$$+\frac{\sqrt{c}(Acd^2e+a(d^3D-Cd^2e+Bde^2-2Ae^3))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right)|\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{ad^2e\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{\sqrt{c}(Bd-2Ae)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{\sqrt{a}d^2\sqrt{d+ex^2}\sqrt{a-cx^4}}$$
$$+\frac{D\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{e\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
(-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*(-c*x^4+a)^(1/2)/d/e/(-a*e^2+c*d^2)/x/(e*x^
2+d)^(1/2)+c^(1/2)*(A*c*d^2*e+a*(-2*A*e^3+B*d*e^2-C*d^2*e+D*d^3))*(1-a/c/x
^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Elliptic
E(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2
)))^(1/2))/a/d^2/e/(c^(1/2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+
c^(1/2)*(-2*A*e+B*d)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a
^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2
^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(1/2)/d^2/(e*x^2+d)^(1/2)/(-c*x^
4+a)^(1/2)+D*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e
)/x^2)^(1/2)*EllipticPi(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2,2^(1/2
)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/e/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(d + e*x^2)^(3/2)*Sqrt[a - c*x^
4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(d + e*x^2)^(3/2)*Sqrt[a - c*x^
4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a-cx^4}(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2251$$

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a-cx^4}(d+ex^2)^{3/2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^{6}+C\,x^{4}+B\,x^{2}+A}{x^{2} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \sqrt{-c\,x^{4}+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^{6}+Cx^{4}+Bx^{2}+A}{\sqrt{-cx^{4}+a}(ex^{2}+d)^{\frac{3}{2}}x^{2}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(D*x^6 + C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e^2*x^10 + 2*c*d*e*x^8 - 2*a*d*e*x^4 + (c*d^2 - a*e^2)*x^6 - a*d^2*x^2), x
)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{a-cx^4}\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(e*x**2+d)**(3/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**2*sqrt(a - c*x**4)*(d + e*x**2
)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)*
x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)*
x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^2\,\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}\,dx=\frac{-\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-c\,x^4+a}\,x^4}{-c\,e^2x^8-2cde\,x^6+a\,e^2x^4-c\,d^2x^4+2ade\,x^2+a\,d^2}dx\right)d^3}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a - c*x**4) + int((sqrt(d + e*x**2)*sqrt(a - c*x
**4)*x**4)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x*
*6 - c*e**2*x**8),x)*d**3*x + int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**4)
/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**
2*x**8),x)*d**2*e*x**3 - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2
 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),
x)*a*d*e*x - 2*int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d*e*x
**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a*e**2*x*
*3 + int((sqrt(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d*e*x**2 + a*e*
*2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*b*d**2*x + int((sqr
t(d + e*x**2)*sqrt(a - c*x**4))/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d
**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*b*d*e*x**3)/(d*x*(d + e*x**2))
```

3.87 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 529

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \frac{(d^3D-Cd^2e+Bde^2-Ae^3)\sqrt{a-cx^4}}{de(cd^2-ae^2)x^3\sqrt{d+ex^2}}$$

$$-\frac{\left(\frac{Acd}{a}-3Cd+\frac{3d^2D}{e}+3Be-\frac{4Ae^2}{d}\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}{3d(cd^2-ae^2)x^3}$$

$$+\frac{\sqrt{c}(3Bcd^3-3ad^3D-5Acd^2e+3aCd^2e-6aBde^2+8aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}E\left(\arcsin\left(\frac{\sqrt{1-\cdots}}{\sqrt{\cdots}}\right.\right.}{3ad^3(\sqrt{c}d-\sqrt{a}e)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{\sqrt{c}(3ad(Cd-2Be)+A(cd^2+8ae^2))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{a}}{\sqrt{c}x^2}}}{\sqrt{2}}\right),\frac{2d}{d+\frac{\sqrt{a}e}{\sqrt{c}}}\right)}{3a^{3/2}d^3\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
(-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*(-c*x^4+a)^(1/2)/d/e/(-a*e^2+c*d^2)/x^3/(e*
x^2+d)^(1/2)-1/3*(A*c*d/a-3*C*d+3*d^2*D/e+3*B*e-4*A*e^2/d)*(e*x^2+d)^(1/2)
*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/x^3+1/3*c^(1/2)*(8*A*a*e^3-5*A*c*d^2*e-
6*B*a*d*e^2+3*B*c*d^3+3*C*a*d^2*e-3*D*a*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2
)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticE(1/2*(1-a^(1/2)/c^(1
/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a/d^3/(c^(
1/2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)+1/3*c^(1/2)*(3*a*d*(-2*
B*e+C*d)+A*(8*a*e^2+c*d^2))*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1
/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^
(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/d^3/(e*x^2+d)^(1/2)
/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(d + e*x^2)^(3/2)*Sqrt[a - c*x^
4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(d + e*x^2)^(3/2)*Sqrt[a - c*x^
4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a-cx^4}\left(d+ex^2\right)^{3/2}}\,dx$$
$$\Big\downarrow \ 2251$$
$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{a-cx^4}\left(d+ex^2\right)^{3/2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{D x^{6}+C\,x^{4}+B\,x^{2}+A}{x^{4} \left(e\,x^{2}+d \right)^{\frac{3}{2}} \sqrt{-c\,x^{4}+a}}d x$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4 \left(d+ex^2\right)^{3/2} \sqrt{a-cx^4}}\,dx=\int \frac{Dx^{6}+Cx^{4}+Bx^{2}+A}{\sqrt{-cx^{4}+a}{\left(ex^{2}+d\right)}^{\frac{3}{2}}x^{4}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(D*x^6 + C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e^2*x^12 + 2*c*d*e*x^10 - 2*a*d*e*x^6 + (c*d^2 - a*e^2)*x^8 - a*d^2*x^4),
x)
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^4 (d + ex^2)^{3/2} \sqrt{a - cx^4}} \, dx = \int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^4 \sqrt{a - cx^4} (d + ex^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(e*x**2+d)**(3/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**4*sqrt(a - c*x**4)*(d + e*x**2
)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^4 (d + ex^2)^{3/2} \sqrt{a - cx^4}} \, dx = \int \frac{Dx^6 + Cx^4 + Bx^2 + A}{\sqrt{-cx^4 + a} (ex^2 + d)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)*
x^4), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^4 (d + ex^2)^{3/2} \sqrt{a - cx^4}} \, dx = \int \frac{Dx^6 + Cx^4 + Bx^2 + A}{\sqrt{-cx^4 + a} (ex^2 + d)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)*
x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^4\,\sqrt{a-c\,x^4}\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}\,dx=\text{too large to display}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output

```
( - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c*d*e**2 - 8*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*a**2*d**2*e**2*x**2 - 16*sqrt(d + e*x**2)*sqrt(a - c*x**
4)*a**2*d*e**3*x**4 - 4*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*d*e**2*x**
2 - 8*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*e**3*x**4 - 2*sqrt(d + e*x**
2)*sqrt(a - c*x**4)*a*c**2*d**2*e*x**2 + 12*sqrt(d + e*x**2)*sqrt(a - c*x*
*4)*a*c**2*d*e**2*x**4 - sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**4*x**2 -
 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*c*d**3*e*x**4 + sqrt(d + e*x**2)*sq
rt(a - c*x**4)*b*c**2*d**3*x**2 + 2*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c*
*2*d**2*e*x**4 - 32*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 +
 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)
*a**2*c*d**2*e**4*x**3 - 32*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(
a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*
x**8),x)*a**2*c*d*e**5*x**5 - 16*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x*
*6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*
e**2*x**8),x)*a*b*c**2*d*e**4*x**3 - 16*int((sqrt(d + e*x**2)*sqrt(a - c*x
**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x*
*6 - c*e**2*x**8),x)*a*b*c**2*e**5*x**5 + 24*int((sqrt(d + e*x**2)*sqrt(a
- c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d
*e*x**6 - c*e**2*x**8),x)*a*c**3*d**2*e**3*x**3 + 24*int((sqrt(d + e*x**2)
*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x...
```

3.88 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 669

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx = \frac{(d^3D-Cd^2e+Bde^2-Ae^3)\sqrt{a-cx^4}}{de(cd^2-ae^2)x^5\sqrt{d+ex^2}}$$

$$-\frac{\left(\frac{Acd}{a}-5Cd+\frac{5d^2D}{e}+5Be-\frac{6Ae^2}{d}\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}{5d(cd^2-ae^2)x^5}$$

$$-\frac{(5Bcd^3-15ad^3D-9Acd^2e+15aCd^2e-20aBde^2+24aAe^3)\sqrt{d+ex^2}\sqrt{a-cx^4}}{15ad^3(cd^2-ae^2)x^3}$$

$$+\frac{\sqrt{c}(3A(3c^2d^4+8acd^2e^2-16a^2e^4)+5ad(cd^2(3Cd-5Be)+ae(3d^2D-6Cde+8Be^2)))\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{}}{15a^2d^4(\sqrt{c}d-\sqrt{a}e)\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

$$+\frac{\sqrt{c}(5Bcd^3+15ad^3D-12Acd^2e-30aCd^2e+40aBde^2-48aAe^3)\sqrt{1-\frac{a}{cx^4}}x^3\sqrt{\frac{\sqrt{a}(d+ex^2)}{(\sqrt{c}d+\sqrt{a}e)x^2}}\operatorname{EllipticF}(}{15a^{3/2}d^4\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
(-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*(-c*x^4+a)^(1/2)/d/e/(-a*e^2+c*d^2)/x^5/(e*
x^2+d)^(1/2)-1/5*(A*c*d/a-5*C*d+5*d^2*D/e+5*B*e-6*A*e^2/d)*(e*x^2+d)^(1/2)
*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/x^5-1/15*(24*A*a*e^3-9*A*c*d^2*e-20*B*a
*d*e^2+5*B*c*d^3+15*C*a*d^2*e-15*D*a*d^3)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2)
/a/d^3/(-a*e^2+c*d^2)/x^3+1/15*c^(1/2)*(3*A*(-16*a^2*e^4+8*a*c*d^2*e^2+3*c
^2*d^4)+5*a*d*(c*d^2*(-5*B*e+3*C*d)+a*e*(8*B*e^2-6*C*d*e+3*D*d^2)))*(1-a/c
/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellipt
icE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1
/2)))^(1/2))/a^2/d^4/(c^(1/2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2
)+1/15*c^(1/2)*(-48*A*a*e^3-12*A*c*d^2*e+40*B*a*d*e^2+5*B*c*d^3-30*C*a*d^2
*e+15*D*a*d^3)*(1-a/c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)
*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)
*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^(3/2)/d^4/(e*x^2+d)^(1/2)/(-c*x^4+a)^(
1/2)
```

Mathematica [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^6 (d + ex^2)^{3/2} \sqrt{a - cx^4}} \, dx = \int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^6 (d + ex^2)^{3/2} \sqrt{a - cx^4}} \, dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(d + e*x^2)^(3/2)*Sqrt[a - c*x^
4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(d + e*x^2)^(3/2)*Sqrt[a - c*x^
4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^6 \sqrt{a - cx^4} (d + ex^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{2251}$$

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a-cx^4}\,(d+ex^2)^{3/2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^6+C\,x^4+B\,x^2+A}{x^6\,(e\,x^2+d)^{\frac{3}{2}}\,\sqrt{-c\,x^4+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\,(d+ex^2)^{3/2}\,\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(D*x^6 + C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e^2*x^14 + 2*c*d*e*x^12 - 2*a*d*e*x^8 + (c*d^2 - a*e^2)*x^10 - a*d^2*x^6),
 x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{a-cx^4}\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(e*x**2+d)**(3/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**6*sqrt(a - c*x**4)*(d + e*x**2
)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)*
x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)*
x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^6\,\sqrt{a-c\,x^4}\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\text{too large to display}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output

```
( - 12*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*d*e**2 - 6*sqrt(d + e*x**2)*
sqrt(a - c*x**4)*a**2*c*d**2*e*x**2 - 12*sqrt(d + e*x**2)*sqrt(a - c*x**4)
*a**2*c*d*e**2*x**4 + 24*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*c*e**3*x**
6 - 15*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*d**4*x**2 - 30*sqrt(d + e*x*
*2)*sqrt(a - c*x**4)*a**2*d**3*e*x**4 - 5*sqrt(d + e*x**2)*sqrt(a - c*x**4
)*a*b*c*d**3*x**2 - 10*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*d**2*e*x**4
 - 20*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a*b*c*d*e**2*x**6 + 6*sqrt(d + e*x
**2)*sqrt(a - c*x**4)*a*c**2*d**2*e*x**6 + 15*sqrt(d + e*x**2)*sqrt(a - c*
x**4)*a*c*d**4*x**6 + 5*sqrt(d + e*x**2)*sqrt(a - c*x**4)*b*c**2*d**3*x**6
 + 48*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d*e*x**2
+ a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*c**2*d*e
**4*x**5 + 48*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 + 2*a*d
*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)*a**2*
c**2*e**5*x**7 - 40*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)/(a*d**2 +
 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**2*x**8),x)
*a*b*c**2*d**2*e**3*x**5 - 40*int((sqrt(d + e*x**2)*sqrt(a - c*x**4)*x**6)
/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6 - c*e**
2*x**8),x)*a*b*c**2*d*e**4*x**7 + 12*int((sqrt(d + e*x**2)*sqrt(a - c*x**4
)*x**6)/(a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4 - c*d**2*x**4 - 2*c*d*e*x**6
- c*e**2*x**8),x)*a*c**3*d**3*e**2*x**5 + 12*int((sqrt(d + e*x**2)*sqrt...
```

3.89 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 840

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}\,dx = \frac{\left(d^3D-Cd^2e+Bde^2-Ae^3\right)\sqrt{a-cx^4}}{de\left(cd^2-ae^2\right)x^7\sqrt{d+ex^2}}$$

$$-\frac{\left(\frac{Acd}{a}-7Cd+\frac{7d^2D}{e}+7Be-\frac{8Ae^2}{d}\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}{7d\left(cd^2-ae^2\right)x^7}$$

$$-\frac{\left(7Bcd^3-35ad^3D-13Acd^2e+35aCd^2e-42aBde^2+48aAe^3\right)\sqrt{d+ex^2}\sqrt{a-cx^4}}{35ad^3\left(cd^2-ae^2\right)x^5}$$

$$-\frac{\left(A(25c^2d^4+62acd^2e^2-192a^2e^4)+7ad(cd^2(5Cd-9Be)+ae(15d^2D-20Cde+24Be^2))\right)\sqrt{d+ex^2}\sqrt{\cdots}}{105a^2d^4\left(cd^2-ae^2\right)x^3}$$

$$-\frac{\sqrt{c}(107Ac^2d^4e+2a^2e^2(105d^3D-140Cd^2e-192Ae^3)-acd^2(105d^3D-175Cd^2e-172Ae^3)-21B(3c\cdots}{105a^2d^5\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{d+ex\cdots}}$$

$$+\frac{\sqrt{c}(A(25c^2d^4+116acd^2e^2+384a^2e^4)+7ad(cd^2(5Cd-12Be)-2ae(15d^2D-20Cde+24Be^2)))\sqrt{1-\cdots}}{105a^{5/2}d^5\sqrt{d+ex^2}\sqrt{a-cx^4}}$$

output
```
(-A*e^3+B*d*e^2-C*d^2*e+D*d^3)*(-c*x^4+a)^(1/2)/d/e/(-a*e^2+c*d^2)/x^7/(e*
x^2+d)^(1/2)-1/7*(A*c*d/a-7*C*d+7*d^2*D/e+7*B*e-8*A*e^2/d)*(e*x^2+d)^(1/2)
*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/x^7-1/35*(48*A*a*e^3-13*A*c*d^2*e-42*B*
a*d*e^2+7*B*c*d^3+35*C*a*d^2*e-35*D*a*d^3)*(e*x^2+d)^(1/2)*(-c*x^4+a)^(1/2
)/a/d^3/(-a*e^2+c*d^2)/x^5-1/105*(A*(-192*a^2*e^4+62*a*c*d^2*e^2+25*c^2*d^
4)+7*a*d*(c*d^2*(-9*B*e+5*C*d)+a*e*(24*B*e^2-20*C*d*e+15*D*d^2)))*(e*x^2+d
)^(1/2)*(-c*x^4+a)^(1/2)/a^2/d^4/(-a*e^2+c*d^2)/x^3-1/105*c^(1/2)*(107*A*c
^2*d^4*e+2*a^2*e^2*(-192*A*e^3-140*C*d^2*e+105*D*d^3)-a*c*d^2*(-172*A*e^3-
175*C*d^2*e+105*D*d^3)-21*B*(-16*a^2*d*e^4+8*a*c*d^3*e^2+3*c^2*d^5))*(1-a/
c/x^4)^(1/2)*x^3*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*Ellip
ticE(1/2*(1-a^(1/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(
1/2)))^(1/2))/a^2/d^5/(c^(1/2)*d-a^(1/2)*e)/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/
2)+1/105*c^(1/2)*(A*(384*a^2*e^4+116*a*c*d^2*e^2+25*c^2*d^4)+7*a*d*(c*d^2*
(-12*B*e+5*C*d)-2*a*e*(24*B*e^2-20*C*d*e+15*D*d^2)))*(1-a/c/x^4)^(1/2)*x^3
*(a^(1/2)*(e*x^2+d)/(c^(1/2)*d+a^(1/2)*e)/x^2)^(1/2)*EllipticF(1/2*(1-a^(1
/2)/c^(1/2)/x^2)^(1/2)*2^(1/2),2^(1/2)*(d/(d+a^(1/2)*e/c^(1/2)))^(1/2))/a^
(5/2)/d^5/(e*x^2+d)^(1/2)/(-c*x^4+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(d + e*x^2)^(3/2)*Sqrt[a - c*x^
4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(d + e*x^2)^(3/2)*Sqrt[a - c*x^
4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a-cx^4}\,(d+ex^2)^{3/2}}dx$$
$$\Big\downarrow \; 2251$$
$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a-cx^4}\,(d+ex^2)^{3/2}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(d + e*x^2)^(3/2)*Sqrt[a - c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2251
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p
, x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^6+C\,x^4+B\,x^2+A}{x^8\,(e\,x^2+d)^{\frac{3}{2}}\,\sqrt{-c\,x^4+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(D*x^6 + C*x^4 + B*x^2 + A)*sqrt(-c*x^4 + a)*sqrt(e*x^2 + d)/(c*
e^2*x^16 + 2*c*d*e*x^14 - 2*a*d*e*x^10 + (c*d^2 - a*e^2)*x^12 - a*d^2*x^8)
, x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{a-cx^4}\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(e*x**2+d)**(3/2)/(-c*x**4+a)**(1/
2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**8*sqrt(a - c*x**4)*(d + e*x**2
)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(d+ex^2)^{3/2}\sqrt{a-cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)*
x^8), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8 (d+ex^2)^{3/2} \sqrt{a-cx^4}} \, dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{-cx^4+a}(ex^2+d)^{\frac{3}{2}}x^8} \, dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(-c*x^4 + a)*(e*x^2 + d)^(3/2)*
x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8 (d+ex^2)^{3/2} \sqrt{a-cx^4}} \, dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^8\,\sqrt{a-c\,x^4}\,{\left(e\,x^2+d\right)}^{3/2}} \, dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a - c*x^4)^(1/2)*(d + e*x^2)^(3/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(d+ex^2\right)^{3/2}\sqrt{a-cx^4}}\,dx=\text{too large to display}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(3/2)/(-c*x^4+a)^(1/2),x)
```

output
```
( - 5184*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**5*c*d*e**6 - 5376*sqrt(d + e
*x**2)*sqrt(a - c*x**4)*a**5*d**2*e**6*x**2 + 10752*sqrt(d + e*x**2)*sqrt(
a - c*x**4)*a**5*d*e**7*x**4 - 4032*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4
*b*c*d*e**6*x**2 + 8064*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*b*c*e**7*x*
*4 - 1152*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c**2*d**3*e**4 - 4320*sqr
t(d + e*x**2)*sqrt(a - c*x**4)*a**4*c**2*d**2*e**5*x**2 - 12096*sqrt(d + e
*x**2)*sqrt(a - c*x**4)*a**4*c**2*d*e**6*x**4 - 2800*sqrt(d + e*x**2)*sqrt
(a - c*x**4)*a**4*c*d**4*e**4*x**2 + 5600*sqrt(d + e*x**2)*sqrt(a - c*x**4
)*a**4*c*d**3*e**5*x**4 - 5376*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**4*c*d*
*2*e**6*x**6 + 1680*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*c**2*d**3*e**
4*x**2 - 3360*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*c**2*d**2*e**5*x**4
 - 4032*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*b*c**2*d*e**6*x**6 + 36*sqr
t(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**3*d**5*e**2 + 1200*sqrt(d + e*x**2)
*sqrt(a - c*x**4)*a**3*c**3*d**4*e**3*x**2 - 7008*sqrt(d + e*x**2)*sqrt(a
- c*x**4)*a**3*c**3*d**3*e**4*x**4 - 14688*sqrt(d + e*x**2)*sqrt(a - c*x**
4)*a**3*c**3*d**2*e**5*x**6 + 1736*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*
c**2*d**6*e**2*x**2 - 3472*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2*d**
5*e**3*x**4 - 2800*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**3*c**2*d**4*e**4*x
**6 + 252*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c**3*d**5*e**2*x**2 - 5
04*sqrt(d + e*x**2)*sqrt(a - c*x**4)*a**2*b*c**3*d**4*e**3*x**4 + 1680*...
```

3.90 $\int \frac{x(1+2x^2)}{\sqrt{1+x^2}(1+x^2+x^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 77

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = -\frac{1}{2}\arctan\left(\sqrt{3}-2\sqrt{1+x^2}\right) + \frac{1}{2}\arctan\left(\sqrt{3}+2\sqrt{1+x^2}\right) - \frac{1}{2}\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt{1+x^2}}{2+x^2}\right)$$

output
```
1/2*arctan(-3^(1/2)+2*(x^2+1)^(1/2))+1/2*arctan(3^(1/2)+2*(x^2+1)^(1/2))-1
/2*3^(1/2)*arctanh(3^(1/2)*(x^2+1)^(1/2)/(x^2+2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = \frac{1}{2}\left(1-i\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{1+x^2}\right) + \frac{1}{2}\left(1+i\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\sqrt{1+x^2}\right)$$

input
```
Integrate[(x*(1 + 2*x^2))/(Sqrt[1 + x^2]*(1 + x^2 + x^4)),x]
```

output
```
((1 - I*Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*Sqrt[1 + x^2])/2])/2 + ((1 + I*Sq
rt[3])*ArcTan[((1 + I*Sqrt[3])*Sqrt[1 + x^2])/2])/2
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2238, 1197, 25, 1483, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2x^2+1)}{\sqrt{x^2+1}\,(x^4+x^2+1)}dx$$

$$\Big\downarrow \text{2238}$$

$$\frac{1}{2}\int \frac{2x^2+1}{\sqrt{x^2+1}\,(x^4+x^2+1)}dx^2$$

$$\Big\downarrow \text{1197}$$

$$\int -\frac{1-2x^4}{x^8-x^4+1}d\sqrt{x^2+1}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1-2x^4}{x^8-x^4+1}d\sqrt{x^2+1}$$

$$\Big\downarrow \text{1483}$$

$$-\frac{\int \frac{\sqrt{3}-3\sqrt{x^2+1}}{x^4-\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}}{2\sqrt{3}}-\frac{\int \frac{\sqrt{3}\left(\sqrt{3}\sqrt{x^2+1}+1\right)}{x^4+\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}}{2\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{3}-3\sqrt{x^2+1}}{x^4-\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}}{2\sqrt{3}}-\frac{1}{2}\int \frac{\sqrt{3}\sqrt{x^2+1}+1}{x^4+\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4+\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}-\frac{1}{2}\sqrt{3}\int \frac{2\sqrt{x^2+1}+\sqrt{3}}{x^4+\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}\right)-\frac{-\frac{1}{2}\sqrt{3}\int \frac{1}{x^4-\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}-\frac{3}{2}\int -\frac{\sqrt{3}-2\sqrt{x^2+1}}{x^4-\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4+\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}-\frac{1}{2}\sqrt{3}\int \frac{2\sqrt{x^2+1}+\sqrt{3}}{x^4+\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}\right)-\frac{\frac{3}{2}\int \frac{\sqrt{3}-2\sqrt{x^2+1}}{x^4-\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}-\frac{1}{2}\sqrt{3}\int \frac{1}{x^4-\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(-\int \frac{1}{-x^4-1}d\left(2\sqrt{x^2+1}+\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{2\sqrt{x^2+1}+\sqrt{3}}{x^4+\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}\right)-\frac{\sqrt{3}\int \frac{1}{-x^4-1}d\left(2\sqrt{x^2+1}-\sqrt{3}\right)+\frac{3}{2}\int \frac{\sqrt{3}-2\sqrt{x^2+1}}{x^4-\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\arctan\left(2\sqrt{x^2+1}+\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{2\sqrt{x^2+1}+\sqrt{3}}{x^4+\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}\right)-\frac{\frac{3}{2}\int \frac{\sqrt{3}-2\sqrt{x^2+1}}{x^4-\sqrt{3}\sqrt{x^2+1}+1}d\sqrt{x^2+1}+\sqrt{3}\arctan\left(\sqrt{3}-2\sqrt{x^2+1}\right)}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\arctan\left(2\sqrt{x^2+1}+\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\log\left(x^4+\sqrt{3}\sqrt{x^2+1}+1\right)\right)-\frac{\sqrt{3}\arctan\left(\sqrt{3}-2\sqrt{x^2+1}\right)-\frac{3}{2}\log\left(x^4-\sqrt{3}\sqrt{x^2+1}+1\right)}{2\sqrt{3}}$$

input
```
Int[(x*(1 + 2*x^2))/(Sqrt[1 + x^2]*(1 + x^2 + x^4)),x]
```

output
```
-1/2*(Sqrt[3]*ArcTan[Sqrt[3] - 2*Sqrt[1 + x^2]] - (3*Log[1 + x^4 - Sqrt[3]
*Sqrt[1 + x^2]])/2)/Sqrt[3] + (ArcTan[Sqrt[3] + 2*Sqrt[1 + x^2]] - (Sqrt[3
]*Log[1 + x^4 + Sqrt[3]*Sqrt[1 + x^2]])/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2238
```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x
)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
 && PolyQ[Px, x^2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,\ln\left(x^2+2+\sqrt{x^2+1}\,\sqrt{3}\right)}{4}+\frac{\arctan\left(\sqrt{3}+2\sqrt{x^2+1}\right)}{2}+\frac{\sqrt{3}\,\ln\left(x^2+2-\sqrt{x^2+1}\,\sqrt{3}\right)}{4}+\frac{\arctan\left(-\sqrt{3}+2\sqrt{x^2+1}\right)}{2}$
default	$-\frac{\sqrt{2}\,\sqrt{\frac{2(x-1)^2}{(-1-x)^2}+2}\left(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{2(x-1)^2}{(-1-x)^2}+2}\,\sqrt{3}}{2}\right)+\arctan\left(\frac{\sqrt{\frac{2(x-1)^2}{(-1-x)^2}+2}\,(x-1)}{\left(\frac{(x-1)^2}{(-1-x)^2}+1\right)(-1-x)}\right)\right)}{4\sqrt{\frac{\frac{(x-1)^2}{(-1-x)^2}+1}{\left(\frac{x-1}{-1-x}+1\right)^2}}\left(\frac{x-1}{-1-x}+1\right)}-\frac{\sqrt{2}\,\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\left(\sqrt{3}\,\mathrm{a}\right.}{}$
trager	$-4\ln\left(\frac{-16\,\mathrm{RootOf}\left(16\_Z^4-4\_Z^2+1\right)^5x^2+16\,\mathrm{RootOf}\left(16\_Z^4-4\_Z^2+1\right)^3x^2+12\,\mathrm{RootOf}\left(16\_Z^4-4\_Z^2+1\right)}{\left(4\,\mathrm{RootOf}\left(16\_Z^4-4\_Z^2+1\right)^2x-x-1\right)\left(4\,\mathrm{RootOf}\left(16\right.\right.}\right.$

input
```
int(x*(2*x^2+1)/(x^2+1)^(1/2)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*3^(1/2)*ln(x^2+2+(x^2+1)^(1/2)*3^(1/2))+1/2*arctan(3^(1/2)+2*(x^2+1)^
(1/2))+1/4*3^(1/2)*ln(x^2+2-(x^2+1)^(1/2)*3^(1/2))+1/2*arctan(-3^(1/2)+2*(
x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(59) = 118$.

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.95

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = \frac{1}{4}\sqrt{3}\log\left(2\,x^4+5\,x^2+2\,\sqrt{3}(x^3+x) - \left(2\,x^3+\sqrt{3}(2\,x^2+1)+4\,x\right)\sqrt{x^2+1}+2\right) - \frac{1}{4}\sqrt{3}\log\left(2\,x^4+5\,x^2-2\,\sqrt{3}(x^3+x) - \left(2\,x^3-\sqrt{3}(2\,x^2+1)+4\,x\right)\sqrt{x^2+1}+2\right) + \frac{1}{2}\arctan\left(\sqrt{3}+2\,\sqrt{x^2+1}\right) + \frac{1}{2}\arctan\left(-\sqrt{3}+2\,\sqrt{x^2+1}\right)$$

input

```
integrate(x*(2*x^2+1)/(x^2+1)^(1/2)/(x^4+x^2+1),x, algorithm="fricas")
```

output

```
1/4*sqrt(3)*log(2*x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) - (2*x^3 + sqrt(3)*(2*
x^2 + 1) + 4*x)*sqrt(x^2 + 1) + 2) - 1/4*sqrt(3)*log(2*x^4 + 5*x^2 - 2*sqr
t(3)*(x^3 + x) - (2*x^3 - sqrt(3)*(2*x^2 + 1) + 4*x)*sqrt(x^2 + 1) + 2) +
1/2*arctan(sqrt(3) + 2*sqrt(x^2 + 1)) + 1/2*arctan(-sqrt(3) + 2*sqrt(x^2 +
 1))
```

Sympy [F]

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = \int \frac{x(2x^2+1)}{\sqrt{x^2+1}\,(x^2-x+1)\,(x^2+x+1)}\,dx$$

input

```
integrate(x*(2*x**2+1)/(x**2+1)**(1/2)/(x**4+x**2+1),x)
```

output

```
Integral(x*(2*x**2 + 1)/(sqrt(x**2 + 1)*(x**2 - x + 1)*(x**2 + x + 1)), x)
```

Maxima [F]

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = \int \frac{(2\,x^2+1)x}{(x^4+x^2+1)\sqrt{x^2+1}}\,dx$$

input
```
integrate(x*(2*x^2+1)/(x^2+1)^(1/2)/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
integrate((2*x^2 + 1)*x/((x^4 + x^2 + 1)*sqrt(x^2 + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = -\frac{1}{4}\sqrt{3}\log\left(x^2+\sqrt{3}\sqrt{x^2+1}+2\right) + \frac{1}{4}\sqrt{3}\log\left(x^2-\sqrt{3}\sqrt{x^2+1}+2\right) + \frac{1}{2}\arctan\left(\sqrt{3}+2\sqrt{x^2+1}\right) + \frac{1}{2}\arctan\left(-\sqrt{3}+2\sqrt{x^2+1}\right)$$

input
```
integrate(x*(2*x^2+1)/(x^2+1)^(1/2)/(x^4+x^2+1),x, algorithm="giac")
```

output
```
-1/4*sqrt(3)*log(x^2 + sqrt(3)*sqrt(x^2 + 1) + 2) + 1/4*sqrt(3)*log(x^2 -
sqrt(3)*sqrt(x^2 + 1) + 2) + 1/2*arctan(sqrt(3) + 2*sqrt(x^2 + 1)) + 1/2*a
rctan(-sqrt(3) + 2*sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 397, normalized size of antiderivative = 5.16

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = \text{Too large to display}$$

input

```
int((x*(2*x^2 + 1))/((x^2 + 1)^(1/2)*(x^2 + x^4 + 1)),x)
```

output

```
((log(x - (3^(1/2)*1i)/2 - 1/2) - log(x/2 + (3^(1/2)/2 + 1i/2)*(x^2 + 1)^(
1/2) + (3^(1/2)*x*1i)/2 + 1))*((3^(1/2)*1i)/2 + 2*((3^(1/2)*1i)/2 + 1/2)^3
 + 1/2))/((((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2)*(3^(1/2)*1i + 4*((3^(1/2)*1
i)/2 + 1/2)^3 + 1)) + ((log(x - (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 - 1
i/2)*(x^2 + 1)^(1/2) - x/2 + (3^(1/2)*x*1i)/2 + 1))*((3^(1/2)*1i)/2 + 2*((
3^(1/2)*1i)/2 - 1/2)^3 - 1/2))/((((3^(1/2)*1i)/2 - 1/2)^2 + 1)^(1/2)*(3^(1
/2)*1i + 4*((3^(1/2)*1i)/2 - 1/2)^3 - 1)) + ((log(x + (3^(1/2)*1i)/2 - 1/2
) - log(x/2 + (3^(1/2)/2 - 1i/2)*(x^2 + 1)^(1/2) - (3^(1/2)*x*1i)/2 + 1))*
((3^(1/2)*1i)/2 + 2*((3^(1/2)*1i)/2 - 1/2)^3 - 1/2))/((((3^(1/2)*1i)/2 - 1
/2)^2 + 1)^(1/2)*(3^(1/2)*1i + 4*((3^(1/2)*1i)/2 - 1/2)^3 - 1)) + ((log(x
+ (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 + 1i/2)*(x^2 + 1)^(1/2) - x/2 - (
3^(1/2)*x*1i)/2 + 1))*((3^(1/2)*1i)/2 + 2*((3^(1/2)*1i)/2 + 1/2)^3 + 1/2))
/((((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2)*(3^(1/2)*1i + 4*((3^(1/2)*1i)/2 + 1
/2)^3 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.48

$$\begin{aligned}
&\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx \\
&= -\frac{atan\left(\frac{\sqrt{x^2+1}\,\sqrt{3}-2\sqrt{x^2+1}\,x+\sqrt{3}\,x-2x^2-2}{\sqrt{x^2+1}+x}\right)}{2} + \frac{atan\left(\frac{\sqrt{x^2+1}\,\sqrt{3}+2\sqrt{x^2+1}\,x+\sqrt{3}\,x+2x^2+2}{\sqrt{x^2+1}+x}\right)}{2} \\
&\quad + \frac{\sqrt{3}\log\left(\frac{-2\sqrt{x^2+1}\,\sqrt{3}\,x^2-\sqrt{x^2+1}\,\sqrt{3}+2\sqrt{x^2+1}\,x^3+4\sqrt{x^2+1}\,x-2\sqrt{3}\,x^3-2\sqrt{3}\,x+2x^4+5x^2+2}{2\sqrt{x^2+1}\,x+2x^2+1}\right)}{4} \\
&\quad - \frac{\sqrt{3}\log\left(\frac{2\sqrt{x^2+1}\,\sqrt{3}\,x^2+\sqrt{x^2+1}\,\sqrt{3}+2\sqrt{x^2+1}\,x^3+4\sqrt{x^2+1}\,x+2\sqrt{3}\,x^3+2\sqrt{3}\,x+2x^4+5x^2+2}{2\sqrt{x^2+1}\,x+2x^2+1}\right)}{4}
\end{aligned}$$

input
```
int(x*(2*x^2+1)/(x^2+1)^(1/2)/(x^4+x^2+1),x)
```

output
```
( - 2*atan((sqrt(x**2 + 1)*sqrt(3) - 2*sqrt(x**2 + 1)*x + sqrt(3)*x - 2*x*
*2 - 2)/(sqrt(x**2 + 1) + x)) + 2*atan((sqrt(x**2 + 1)*sqrt(3) + 2*sqrt(x*
*2 + 1)*x + sqrt(3)*x + 2*x**2 + 2)/(sqrt(x**2 + 1) + x)) + sqrt(3)*log((
- 2*sqrt(x**2 + 1)*sqrt(3)*x**2 - sqrt(x**2 + 1)*sqrt(3) + 2*sqrt(x**2 + 1
)*x**3 + 4*sqrt(x**2 + 1)*x - 2*sqrt(3)*x**3 - 2*sqrt(3)*x + 2*x**4 + 5*x*
*2 + 2)/(2*sqrt(x**2 + 1)*x + 2*x**2 + 1)) - sqrt(3)*log((2*sqrt(x**2 + 1)
*sqrt(3)*x**2 + sqrt(x**2 + 1)*sqrt(3) + 2*sqrt(x**2 + 1)*x**3 + 4*sqrt(x*
*2 + 1)*x + 2*sqrt(3)*x**3 + 2*sqrt(3)*x + 2*x**4 + 5*x**2 + 2)/(2*sqrt(x*
*2 + 1)*x + 2*x**2 + 1)))/4
```

3.91 $\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 273

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}$$

output
```
-1/2*(c^(1/2)*d-a^(1/2)*e)*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/2)/e^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+1/4*(
c^(1/2)*d+a^(1/2)*e)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/
2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*a^(1/2)*
(c^(1/2)*d/a^(1/2)-e)^2/c^(1/2)/d/e,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/
4)/c^(1/4)/d/e/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\sqrt{c}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(-\sqrt{c}\right.}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2+}}$$

input
```
Integrate[(Sqrt[a] + Sqrt[c]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Sqrt[c]*d*EllipticF[I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt
[b^2 - 4*a*c])] + (-(Sqrt[c]*d) + Sqrt[a]*e)*EllipticPi[((b + Sqrt[b^2 - 4
*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

$$\downarrow \text{ 2220}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

input
```
Int[(Sqrt[a] + Sqrt[c]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt
[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e
 + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[c]
*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.34

method	result
default	$\frac{\sqrt{c}\,\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-\frac{(\sqrt{c}\,d-\cdots}{\cdots}$
elliptic	$\frac{\sqrt{a(c\,x^4+b\,x^2+a)}\,\sqrt{c(c\,x^4+b\,x^2+a)}\,(\sqrt{a}+\sqrt{c}\,x^2)\left(\frac{c\sqrt{2}\,\sqrt{4+\frac{2b\,x^2}{a}-\frac{2x^2\sqrt{-4ac+b^2}}{a}}\,\sqrt{4+\frac{2b\,x^2}{a}+\frac{2x^2\sqrt{-4ac+b^2}}{a}}\ \operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{-b+\cdots}}{2}\cdots\right.}{4e\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{c^2x^4+bc\,x^2+ac}}\cdots\right.}{\cdots}$

input
```
int((a^(1/2)+c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
1/4*c^(1/2)/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*
x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-(c^(1/2)*d-a^(1/2)*e)/e/d*2^(1
/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2*(-4*a*c+b
^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4
+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((a^(1/2)+c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((a**(1/2)+c**(1/2)*x**2)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((sqrt(a) + sqrt(c)*x**2)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4))
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^(1/2)+c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((a^(1/2)+c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{a}+\sqrt{c}\,x^2}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((a^(1/2) + c^(1/2)*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((a^(1/2) + c^(1/2)*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx\\&=\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)\\&\quad+\sqrt{c}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)\end{aligned}$$

input
```
int((a^(1/2)+c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(a + b*x**2 + c*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**
4 + c*d*x**4 + c*e*x**6),x) + sqrt(c)*int((sqrt(a + b*x**2 + c*x**4)*x**2)
/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)
```

3.92 $\int \frac{\sqrt{a}-\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 421

$$\int \frac{\sqrt{a}-\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} + \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}}$$

output

```
1/2*(c^(1/2)*d+a^(1/2)*e)*arctan((a*e^2-b*d*e+c*d^2)^(1/2)*x/d^(1/2)/e^(1/
2)/(c*x^4+b*x^2+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+a^(1/4
)*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^
(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2)
)^(1/2))/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+b*x^2+a)^(1/2)-1/4*(c^(1/2)*d+a^(1/2
)*e)^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/
2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(c^(1/2)*d-a^(1/2)*e)^
2/a^(1/2)/c^(1/2)/d/e,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/c^(1/4)/d/e
/(c^(1/2)*d-a^(1/2)*e)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.69 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a}-\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx =$$
$$-\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(-\sqrt{c}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(\sqrt{c}d\right.}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2+}}$$

input
```
Integrate[(Sqrt[a] - Sqrt[c]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(-(Sqrt[c]*d*EllipticF[I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])]) + (Sqrt[c]*d + Sqrt[a]*e)*EllipticPi[((b + Sqrt[b^2 - 4
*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2224, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a}-\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \text{2224}$$
$$\frac{2\sqrt{a}\sqrt{c}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\sqrt[4]{a}\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{(ex^2+d)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}d-\sqrt{a}e}$$

$$\Big\downarrow \text{2220}$$

$$\frac{\sqrt[4]{a}\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{a}e+\sqrt{c}d)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}-\frac{(\sqrt{c}d-\sqrt{a}e)\text{ ar}}{2\sqrt{d}\sqrt{}}\right.}{\sqrt{c}d-\sqrt{a}e}$$

input
```
Int[(Sqrt[a] - Sqrt[c]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a
]*Sqrt[c]))/4])/((Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt
[c]*d + Sqrt[a]*e)*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 - b*d
*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e
]*Sqrt[c*d^2 - b*d*e + a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/
4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a +
b*x^2 + c*x^4])))/(Sqrt[c]*d - Sqrt[a]*e)
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2224
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[2*A*(B/(B*d + A*e))   Int[1/Sqrt[a + b*x
^2 + c*x^4], x], x] - Simp[(B*d - A*e)/(B*d + A*e)   Int[(A - B*x^2)/((d +
e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x]
&& NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && NegQ[B/A]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.86

method	result
default	$-\frac{\sqrt{c}\,\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+\frac{(\sqrt{c}\ldots}{\ldots}$
elliptic	$\frac{\left(\sqrt{a}-\sqrt{c}\,x^2\right)\sqrt{a\left(c\,x^4+b\,x^2+a\right)}\,\sqrt{c\left(c\,x^4+b\,x^2+a\right)}\left(-\frac{c\sqrt{2}\,\sqrt{4+\frac{2b\,x^2}{a}-\frac{2x^2\sqrt{-4ac+b^2}}{a}}\,\sqrt{4+\frac{2b\,x^2}{a}+\frac{2x^2\sqrt{-4ac+b^2}}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{-b+\ldots}}{\ldots}\right.}{4e\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{c^2x^4+bc\,x^2+ac}}\ldots\right.}{\ldots}$

input
```
int((a^(1/2)-c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
-1/4*c^(1/2)/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b
*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(c^(1/2)*d+a^(1/2)*e)/e/d*2^(
1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2-1/2/a*x^2*(-4*a*c+
b^2)^(1/2))^(1/2)*(1+1/2*b/a*x^2+1/2/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^
4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),-2/(-b+(-4*a*c+b^2)^(1/2))*a/d*e,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}-\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((a^(1/2)-c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a}-\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\int \left(-\frac{\sqrt{a}}{d\sqrt{a+bx^2+cx^4}+ex^2\sqrt{a+bx^2+cx^4}}\right)dx - \int \frac{\sqrt{c}x^2}{d\sqrt{a+bx^2+cx^4}+ex^2\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((a**(1/2)-c**(1/2)*x**2)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
-Integral(-sqrt(a)/(d*sqrt(a + b*x**2 + c*x**4) + e*x**2*sqrt(a + b*x**2 +
 c*x**4)), x) - Integral(sqrt(c)*x**2/(d*sqrt(a + b*x**2 + c*x**4) + e*x**
2*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a}-\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^(1/2)-c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}-\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((a^(1/2)-c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}-\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((a^(1/2) - c^(1/2)*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((a^(1/2) - c^(1/2)*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{\sqrt{a}-\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx\\&=\sqrt{a}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)\\&\quad-\sqrt{c}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)\end{aligned}$$

input
```
int((a^(1/2)-c^(1/2)*x^2)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(a + b*x**2 + c*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**
4 + c*d*x**4 + c*e*x**6),x) - sqrt(c)*int((sqrt(a + b*x**2 + c*x**4)*x**2)
/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)
```

3.93 $\int \frac{x^5(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 82

$$\int \frac{x^5(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\frac{13}{8}\sqrt{1+x^2+x^4}+\frac{3}{4}x^2\sqrt{1+x^2+x^4} +\frac{9}{16}\text{arcsinh}\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{2}\text{arctanh}\left(\frac{1-x^2}{2\sqrt{1+x^2+x^4}}\right)$$

output
```
-13/8*(x^4+x^2+1)^(1/2)+3/4*x^2*(x^4+x^2+1)^(1/2)+9/16*arcsinh(1/3*(2*x^2+
1)*3^(1/2))+1/2*arctanh(1/2*(-x^2+1)/(x^4+x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^5(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{1}{8}(-13+6x^2)\sqrt{1+x^2+x^4} - \text{arctanh}\left(1+x^2-\sqrt{1+x^2+x^4}\right) - \frac{9}{16}\log\left(-1-2x^2+2\sqrt{1+x^2+x^4}\right)$$

input
```
Integrate[(x^5*(2 + 3*x^2))/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
((-13 + 6*x^2)*Sqrt[1 + x^2 + x^4])/8 - ArcTanh[1 + x^2 - Sqrt[1 + x^2 + x
^4]] - (9*Log[-1 - 2*x^2 + 2*Sqrt[1 + x^2 + x^4]])/16
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2236, 2236, 2252, 2238, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (3x^2+2)}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{2236}$$

$$\frac{1}{4}\int \frac{4x^5(3x^2+2)-3x(x^2+1)(4x^4+3x^2+2)}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{3}{4}\sqrt{x^4+x^2+1}x^2$$

$$\Big\downarrow \text{2236}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\frac{13(x^2+1)(2x^4+x^2)}{x}+2(4x^5(3x^2+2)-3x(x^2+1)(4x^4+3x^2+2))}{(x^2+1)\sqrt{x^4+x^2+1}}dx-\frac{13}{2}\sqrt{x^4+x^2+1}\right)+\frac{3}{4}\sqrt{x^4+x^2+1}x^2$$

$$\Big\downarrow \text{2252}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{x(9x^2+1)}{(x^2+1)\sqrt{x^4+x^2+1}}dx-\frac{13}{2}\sqrt{x^4+x^2+1}\right)+\frac{3}{4}\sqrt{x^4+x^2+1}x^2$$

$$\Big\downarrow \text{2238}$$

$$\frac{1}{4}\left(\frac{1}{4}\int \frac{9x^2+1}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2-\frac{13}{2}\sqrt{x^4+x^2+1}\right)+\frac{3}{4}\sqrt{x^4+x^2+1}x^2$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(9\int \frac{1}{\sqrt{x^4+x^2+1}}dx^2-8\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2\right)-\frac{13}{2}\sqrt{x^4+x^2+1}\right)+\frac{3}{4}\sqrt{x^4+x^2+1}x^2$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(3\sqrt{3}\int \frac{1}{\sqrt{\frac{x^4}{3}+1}}d(2x^2+1)-8\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2\right)-\frac{13}{2}\sqrt{x^4+x^2+1}\right)+\frac{3}{4}\sqrt{x^4+x^2+1}x^2$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(9\text{arcsinh}\left(\frac{2x^2+1}{\sqrt{3}}\right)-8\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2\right)-\frac{13}{2}\sqrt{x^4+x^2+1}\right)+\frac{3}{4}\sqrt{x^4+x^2+1}x^2$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(16\int \frac{1}{4-x^4}d\frac{1-x^2}{\sqrt{x^4+x^2+1}}+9\text{arcsinh}\left(\frac{2x^2+1}{\sqrt{3}}\right)\right)-\frac{13}{2}\sqrt{x^4+x^2+1}\right)+\frac{3}{4}\sqrt{x^4+x^2+1}x^2$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{1}{4}\left(9\text{arcsinh}\left(\frac{2x^2+1}{\sqrt{3}}\right)+8\text{arctanh}\left(\frac{1-x^2}{2\sqrt{x^4+x^2+1}}\right)\right)-\frac{13}{2}\sqrt{x^4+x^2+1}\right)+\frac{3}{4}\sqrt{x^4+x^2+1}x^2$$

input
```
Int[(x^5*(2 + 3*x^2))/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
(3*x^2*Sqrt[1 + x^2 + x^4])/4 + ((-13*Sqrt[1 + x^2 + x^4])/2 + (9*ArcSinh[
(1 + 2*x^2)/Sqrt[3]] + 8*ArcTanh[(1 - x^2)/(2*Sqrt[1 + x^2 + x^4])])/4)/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2236
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]),
 x_Symbol] :> With[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt
[a + b*x^2 + c*x^4]/(c*e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q
 - 3)*Px - Coeff[Px, x, q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + b*(q - 4)*x^2
 + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; GtQ[q,
4]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Px, x]
```

rule 2238
```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x
)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
 && PolyQ[Px, x^2]
```

rule 2252
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> With[{m = Expon[Px, x, Min]}, Int[x^m*ExpandToSum[Px/x^m
, x]*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x] /; GtQ[m, 0] &&  !MatchQ[Px, x
^m*(u_.)]] /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\frac{9 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\,(x^2+\frac{1}{2})}{3}\right)}{16}-\frac{13\sqrt{x^4+x^2+1}}{8}+\frac{3x^2\sqrt{x^4+x^2+1}}{4}+\frac{\operatorname{arctanh}\left(\frac{-x^2+1}{2\sqrt{(x^2+1)^2-x^2}}\right)}{2}$$

input
```
int(x^5*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
9/16*arcsinh(2/3*3^(1/2)*(x^2+1/2))-13/8*(x^4+x^2+1)^(1/2)+3/4*x^2*(x^4+x^
2+1)^(1/2)+1/2*arctanh(1/2*(-x^2+1)/((x^2+1)^2-x^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

$$\int \frac{x^5(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{1}{8}\sqrt{x^4+x^2+1}(6\,x^2-13) + \frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}-2\right) - \frac{9}{16}\log\left(-2\,x^2+2\sqrt{x^4+x^2+1}-1\right)$$

input
```
integrate(x^5*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(x^4 + x^2 + 1)*(6*x^2 - 13) + 1/2*log(-x^2 + sqrt(x^4 + x^2 + 1))
 - 1/2*log(-x^2 + sqrt(x^4 + x^2 + 1) - 2) - 9/16*log(-2*x^2 + 2*sqrt(x^4
+ x^2 + 1) - 1)
```

Sympy [F]

$$\int \frac{x^5(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^5\cdot(3x^2+2)}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)}\,dx$$

input
```
integrate(x**5*(3*x**2+2)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(x**5*(3*x**2 + 2)/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)
), x)
```

Maxima [F]

$$\int \frac{x^5(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{(3x^2+2)x^5}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate(x^5*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^5/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^5(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx &= \frac{1}{8}\sqrt{x^4+x^2+1}(6x^2-13) \\ &\quad - \frac{1}{2}\log\left(x^2-\sqrt{x^4+x^2+1}+2\right) \\ &\quad + \frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}\right) \\ &\quad - \frac{9}{16}\log\left(-2x^2+2\sqrt{x^4+x^2+1}-1\right)\end{aligned}$$

input
```
integrate(x^5*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(x^4 + x^2 + 1)*(6*x^2 - 13) - 1/2*log(x^2 - sqrt(x^4 + x^2 + 1) +
 2) + 1/2*log(-x^2 + sqrt(x^4 + x^2 + 1)) - 9/16*log(-2*x^2 + 2*sqrt(x^4 +
 x^2 + 1) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^5\,(3\,x^2+2)}{(x^2+1)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((x^5*(3*x^2 + 2))/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((x^5*(3*x^2 + 2))/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^5(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = 3\left(\int \frac{x^7}{\sqrt{x^4+x^2+1}\,x^2+\sqrt{x^4+x^2+1}}dx\right) + 2\left(\int \frac{x^5}{\sqrt{x^4+x^2+1}\,x^2+\sqrt{x^4+x^2+1}}dx\right)$$

input
```
int(x^5*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
3*int(x**7/(sqrt(x**4 + x**2 + 1)*x**2 + sqrt(x**4 + x**2 + 1)),x) + 2*int
(x**5/(sqrt(x**4 + x**2 + 1)*x**2 + sqrt(x**4 + x**2 + 1)),x)
```

3.94 $\int \frac{x^3(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{x^3(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{3}{2}\sqrt{1+x^2+x^4}-\frac{5}{4}\operatorname{arcsinh}\left(\frac{1+2x^2}{\sqrt{3}}\right)-\frac{1}{2}\operatorname{arctanh}\left(\frac{1-x^2}{2\sqrt{1+x^2+x^4}}\right)$$

output
```
3/2*(x^4+x^2+1)^(1/2)-5/4*arcsinh(1/3*(2*x^2+1)*3^(1/2))-1/2*arctanh(1/2*(
-x^2+1)/(x^4+x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{x^3(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{3}{2}\sqrt{1+x^2+x^4}+\operatorname{arctanh}\left(1+x^2-\sqrt{1+x^2+x^4}\right)+\frac{5}{4}\log\left(-1-2x^2+2\sqrt{1+x^2+x^4}\right)$$

input
```
Integrate[(x^3*(2 + 3*x^2))/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
(3*Sqrt[1 + x^2 + x^4])/2 + ArcTanh[1 + x^2 - Sqrt[1 + x^2 + x^4]] + (5*Lo
g[-1 - 2*x^2 + 2*Sqrt[1 + x^2 + x^4]])/4
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2236, 2252, 2238, 25, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(3x^2+2)}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{ 2236}$$

$$\frac{1}{2}\int \frac{2x^3(3x^2+2)-\frac{3(x^2+1)(2x^4+x^2)}{x}}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{3}{2}\sqrt{x^4+x^2+1}$$

$$\Big\downarrow \text{ 2252}$$

$$\frac{1}{2}\int \frac{x(-5x^2-3)}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{3}{2}\sqrt{x^4+x^2+1}$$

$$\Big\downarrow \text{ 2238}$$

$$\frac{1}{4}\int -\frac{5x^2+3}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2+\frac{3}{2}\sqrt{x^4+x^2+1}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3}{2}\sqrt{x^4+x^2+1}-\frac{1}{4}\int \frac{5x^2+3}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{4}\left(2\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2-5\int \frac{1}{\sqrt{x^4+x^2+1}}dx^2\right)+\frac{3}{2}\sqrt{x^4+x^2+1}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{4}\left(2\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2-\frac{5\int \frac{1}{\sqrt{\frac{x^4}{3}+1}}d(2x^2+1)}{\sqrt{3}}\right)+\frac{3}{2}\sqrt{x^4+x^2+1}$$

$\Big\downarrow$ 222

$$\frac{1}{4}\left(2\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2-5\text{arcsinh}\left(\frac{2x^2+1}{\sqrt{3}}\right)\right)+\frac{3}{2}\sqrt{x^4+x^2+1}$$

$\Big\downarrow$ 1154

$$\frac{1}{4}\left(-4\int \frac{1}{4-x^4}d\frac{1-x^2}{\sqrt{x^4+x^2+1}}-5\text{arcsinh}\left(\frac{2x^2+1}{\sqrt{3}}\right)\right)+\frac{3}{2}\sqrt{x^4+x^2+1}$$

$\Big\downarrow$ 219

$$\frac{1}{4}\left(-5\text{arcsinh}\left(\frac{2x^2+1}{\sqrt{3}}\right)-2\text{arctanh}\left(\frac{1-x^2}{2\sqrt{x^4+x^2+1}}\right)\right)+\frac{3}{2}\sqrt{x^4+x^2+1}$$

input
```
Int[(x^3*(2 + 3*x^2))/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
(3*Sqrt[1 + x^2 + x^4])/2 + (-5*ArcSinh[(1 + 2*x^2)/Sqrt[3]] - 2*ArcTanh[(
1 - x^2)/(2*Sqrt[1 + x^2 + x^4])])/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2236
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]),
 x_Symbol] :> With[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt
[a + b*x^2 + c*x^4]/(c*e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q
 - 3)*Px - Coeff[Px, x, q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + b*(q - 4)*x^2
 + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; GtQ[q,
4]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Px, x]
```

rule 2238
```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x
)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
 && PolyQ[Px, x^2]
```

rule 2252
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> With[{m = Expon[Px, x, Min]}, Int[x^m*ExpandToSum[Px/x^m
, x]*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x] /; GtQ[m, 0] &&  !MatchQ[Px, x
^m*(u_.)]] /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$-\frac{5\,\operatorname{arcsinh}\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)}{4}+\frac{3\sqrt{x^4+x^2+1}}{2}+\frac{\operatorname{arctanh}\left(\frac{x^2-1}{2\sqrt{x^4+x^2+1}}\right)}{2}$
default	$-\frac{\operatorname{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)}{2}-\frac{5\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x^2+\frac{1}{2}\right)}{3}\right)}{4}+\frac{3\sqrt{x^4+x^2+1}}{2}$
risch	$-\frac{\operatorname{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)}{2}-\frac{5\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x^2+\frac{1}{2}\right)}{3}\right)}{4}+\frac{3\sqrt{x^4+x^2+1}}{2}$
elliptic	$-\frac{\operatorname{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)}{2}-\frac{5\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(x^2+\frac{1}{2}\right)}{3}\right)}{4}+\frac{3\sqrt{x^4+x^2+1}}{2}$
trager	$\frac{3\sqrt{x^4+x^2+1}}{2}-\frac{\ln\left(\frac{512x^{14}+512\sqrt{x^4+x^2+1}\,x^{12}+3328x^{12}+3072\sqrt{x^4+x^2+1}\,x^{10}+9696x^{10}+7968\sqrt{x^4+x^2+1}\,x^8+16592x^8+11552\sqrt{\ldots}}{\left(x^2+1\ldots\right.}\right.}{4}$

input
```
int(x^3*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/4*arcsinh(1/3*(2*x^2+1)*3^(1/2))+3/2*(x^4+x^2+1)^(1/2)+1/2*arctanh(1/2*
(x^2-1)/(x^4+x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

$$\int \frac{x^3(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx=\frac{3}{2}\sqrt{x^4+x^2+1}-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}-2\right)+\frac{5}{4}\log\left(-2\,x^2+2\sqrt{x^4+x^2+1}-1\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
3/2*sqrt(x^4 + x^2 + 1) - 1/2*log(-x^2 + sqrt(x^4 + x^2 + 1)) + 1/2*log(-x
^2 + sqrt(x^4 + x^2 + 1) - 2) + 5/4*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1
)
```

Sympy [F]

$$\int \frac{x^3(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^3\cdot(3x^2+2)}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)}\,dx$$

input
```
integrate(x**3*(3*x**2+2)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(x**3*(3*x**2 + 2)/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)
), x)
```

Maxima [F]

$$\int \frac{x^3(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{(3x^2+2)x^3}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate(x^3*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^3/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

$$\int \frac{x^3(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{3}{2}\sqrt{x^4+x^2+1} + \frac{1}{2}\log\left(x^2-\sqrt{x^4+x^2+1}+2\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}\right) + \frac{5}{4}\log\left(-2\,x^2+2\sqrt{x^4+x^2+1}-1\right)$$

input
```
integrate(x^3*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
3/2*sqrt(x^4 + x^2 + 1) + 1/2*log(x^2 - sqrt(x^4 + x^2 + 1) + 2) - 1/2*log
(-x^2 + sqrt(x^4 + x^2 + 1)) + 5/4*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^3\,(3\,x^2+2)}{(x^2+1)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((x^3*(3*x^2 + 2))/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((x^3*(3*x^2 + 2))/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = 3\left(\int \frac{x^5}{\sqrt{x^4+x^2+1}\,x^2+\sqrt{x^4+x^2+1}}dx\right) + 2\left(\int \frac{x^3}{\sqrt{x^4+x^2+1}\,x^2+\sqrt{x^4+x^2+1}}dx\right)$$

input
```
int(x^3*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
3*int(x**5/(sqrt(x**4 + x**2 + 1)*x**2 + sqrt(x**4 + x**2 + 1)),x) + 2*int
(x**3/(sqrt(x**4 + x**2 + 1)*x**2 + sqrt(x**4 + x**2 + 1)),x)
```

3.95 $\int \frac{x(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 47

$$\int \frac{x(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{3}{2}\text{arcsinh}\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{2}\text{arctanh}\left(\frac{1-x^2}{2\sqrt{1+x^2+x^4}}\right)$$

output
```
3/2*arcsinh(1/3*(2*x^2+1)*3^(1/2))+1/2*arctanh(1/2*(-x^2+1)/(x^4+x^2+1)^(1
/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{x(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\text{arctanh}\left(1+x^2-\sqrt{1+x^2+x^4}\right) - \frac{3}{2}\log\left(-1-2x^2+2\sqrt{1+x^2+x^4}\right)$$

input
```
Integrate[(x*(2 + 3*x^2))/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
-ArcTanh[1 + x^2 - Sqrt[1 + x^2 + x^4]] - (3*Log[-1 - 2*x^2 + 2*Sqrt[1 + x
^2 + x^4]])/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2238, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(3x^2+2)}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{2238}$$

$$\frac{1}{2}\int \frac{3x^2+2}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{x^4+x^2+1}}dx^2-\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2\right)$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}\left(\sqrt{3}\int \frac{1}{\sqrt{\frac{x^4}{3}+1}}d(2x^2+1)-\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(3\text{arcsinh}\left(\frac{2x^2+1}{\sqrt{3}}\right)-\int \frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}dx^2\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(2\int \frac{1}{4-x^4}d\frac{1-x^2}{\sqrt{x^4+x^2+1}}+3\text{arcsinh}\left(\frac{2x^2+1}{\sqrt{3}}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(3\text{arcsinh}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\text{arctanh}\left(\frac{1-x^2}{2\sqrt{x^4+x^2+1}}\right)\right)$$

input
```
Int[(x*(2 + 3*x^2))/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
(3*ArcSinh[(1 + 2*x^2)/Sqrt[3]] + ArcTanh[(1 - x^2)/(2*Sqrt[1 + x^2 + x^4]
)])/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2238
```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x
)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
 && PolyQ[Px, x^2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{3\,\mathrm{arcsinh}\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)}{2}-\frac{\mathrm{arctanh}\left(\frac{x^2-1}{2\sqrt{x^4+x^2+1}}\right)}{2}$	37
default	$\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(x^2+\frac{1}{2}\right)}{3}\right)}{2}+\frac{\mathrm{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)}{2}$	42
elliptic	$\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\left(x^2+\frac{1}{2}\right)}{3}\right)}{2}+\frac{\mathrm{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)}{2}$	42
trager	$-\frac{\ln\left(-\frac{-32x^8+32\sqrt{x^4+x^2+1}\,x^6-112x^6+96\sqrt{x^4+x^2+1}\,x^4-162x^4+102x^2\sqrt{x^4+x^2+1}-121x^2+40\sqrt{x^4+x^2+1}-41}{x^2+1}\right)}{2}$	92

input
```
int(x*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*arcsinh(1/3*(2*x^2+1)*3^(1/2))-1/2*arctanh(1/2*(x^2-1)/(x^4+x^2+1)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \frac{x(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}-2\right) - \frac{3}{2}\log\left(-2\,x^2+2\sqrt{x^4+x^2+1}-1\right)$$

input
```
integrate(x*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log(-x^2 + sqrt(x^4 + x^2 + 1)) - 1/2*log(-x^2 + sqrt(x^4 + x^2 + 1) -
 2) - 3/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

Sympy [F]

$$\int \frac{x(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{x(3x^2+2)}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)}\,dx$$

input
```
integrate(x*(3*x**2+2)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(x*(3*x**2 + 2)/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)),
x)
```

Maxima [F]

$$\int \frac{x(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate(x*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \frac{x(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\frac{1}{2}\log\left(x^2-\sqrt{x^4+x^2+1}+2\right) + \frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}\right) - \frac{3}{2}\log\left(-2\,x^2+2\,\sqrt{x^4+x^2+1}-1\right)$$

input
```
integrate(x*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(x^2 - sqrt(x^4 + x^2 + 1) + 2) + 1/2*log(-x^2 + sqrt(x^4 + x^2 +
1)) - 3/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{x\,(3\,x^2+2)}{(x^2+1)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((x*(3*x^2 + 2))/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((x*(3*x^2 + 2))/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = 3\left(\int \frac{x^3}{\sqrt{x^4+x^2+1}\,x^2+\sqrt{x^4+x^2+1}}dx\right) + 2\left(\int \frac{x}{\sqrt{x^4+x^2+1}\,x^2+\sqrt{x^4+x^2+1}}dx\right)$$

input
```
int(x*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
3*int(x**3/(sqrt(x**4 + x**2 + 1)*x**2 + sqrt(x**4 + x**2 + 1)),x) + 2*int
(x/(sqrt(x**4 + x**2 + 1)*x**2 + sqrt(x**4 + x**2 + 1)),x)
```

3.96 $\int \frac{2+3x^2}{x(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 53

$$\int \frac{2+3x^2}{x(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\frac{1}{2}\text{arctanh}\left(\frac{1-x^2}{2\sqrt{1+x^2+x^4}}\right) - \text{arctanh}\left(\frac{2+x^2}{2\sqrt{1+x^2+x^4}}\right)$$

output
```
-1/2*arctanh(1/2*(-x^2+1)/(x^4+x^2+1)^(1/2))-arctanh(1/2*(x^2+2)/(x^4+x^2+
1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{2+3x^2}{x(1+x^2)\sqrt{1+x^2+x^4}}\,dx = 2\text{arctanh}\left(x^2-\sqrt{1+x^2+x^4}\right) + \text{arctanh}\left(1+x^2-\sqrt{1+x^2+x^4}\right)$$

input
```
Integrate[(2 + 3*x^2)/(x*(1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
2*ArcTanh[x^2 - Sqrt[1 + x^2 + x^4]] + ArcTanh[1 + x^2 - Sqrt[1 + x^2 + x^
4]]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2248, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{2248}$$

$$\int \left(\frac{x}{(x^2+1)\sqrt{x^4+x^2+1}}+\frac{2}{\sqrt{x^4+x^2+1}x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\text{arctanh}\left(\frac{1-x^2}{2\sqrt{x^4+x^2+1}}\right)-\text{arctanh}\left(\frac{x^2+2}{2\sqrt{x^4+x^2+1}}\right)$$

input
```
Int[(2 + 3*x^2)/(x*(1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
-1/2*ArcTanh[(1 - x^2)/(2*Sqrt[1 + x^2 + x^4])] - ArcTanh[(2 + x^2)/(2*Sqr
t[1 + x^2 + x^4])]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{x^{2}-1}{2 \sqrt{x^{4}+x^{2}+1}}\right)}{2}-\operatorname{arctanh}\left(\frac{x^{2}+2}{2 \sqrt{x^{4}+x^{2}+1}}\right)$	42
default	$-\frac{\operatorname{arctanh}\left(\frac{-x^{2}+1}{2 \sqrt{\left(x^{2}+1\right)^{2}-x^{2}}}\right)}{2}-\operatorname{arctanh}\left(\frac{x^{2}+2}{2 \sqrt{x^{4}+x^{2}+1}}\right)$	49
elliptic	$-\frac{\operatorname{arctanh}\left(\frac{-x^{2}+1}{2 \sqrt{\left(x^{2}+1\right)^{2}-x^{2}}}\right)}{2}-\operatorname{arctanh}\left(\frac{x^{2}+2}{2 \sqrt{x^{4}+x^{2}+1}}\right)$	49
trager	$-\frac{\ln\left(\frac{x^{6}+2 \sqrt{x^{4}+x^{2}+1}\, x^{4}+7 x^{4}+4 x^{2} \sqrt{x^{4}+x^{2}+1}+8 x^{2}+8 \sqrt{x^{4}+x^{2}+1}+8}{x^{4} \left(x^{2}+1\right)}\right)}{2}$	72

input
```
int((3*x^2+2)/x/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(1/2*(x^2-1)/(x^4+x^2+1)^(1/2))-arctanh(1/2*(x^2+2)/(x^4+x^2+1)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{2+3x^2}{x\,(1+x^2)\,\sqrt{1+x^2+x^4}}\,dx = -\log\left(-x^2+\sqrt{x^4+x^2+1}+1\right) - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}\right) + \log\left(-x^2+\sqrt{x^4+x^2+1}-1\right) + \frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}-2\right)$$

input
```
integrate((3*x^2+2)/x/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x^2 + sqrt(x^4 + x^2 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + x^2 + 1))
 + log(-x^2 + sqrt(x^4 + x^2 + 1) - 1) + 1/2*log(-x^2 + sqrt(x^4 + x^2 + 1
) - 2)
```

Sympy [F]

$$\int \frac{2+3x^2}{x(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3x^2+2}{x\sqrt{(x^2-x+1)(x^2+x+1)}(x^2+1)}\,dx$$

input
```
integrate((3*x**2+2)/x/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(x*sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)),
x)
```

Maxima [F]

$$\int \frac{2+3x^2}{x(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3x^2+2}{\sqrt{x^4+x^2+1}(x^2+1)x}\,dx$$

input
```
integrate((3*x^2+2)/x/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\begin{aligned}\int \frac{2+3x^2}{x(1+x^2)\sqrt{1+x^2+x^4}}\,dx = {} & \frac{1}{2}\log\left(x^2-\sqrt{x^4+x^2+1}+2\right) \\ & +\log\left(x^2-\sqrt{x^4+x^2+1}+1\right) \\ & -\log\left(-x^2+\sqrt{x^4+x^2+1}+1\right) \\ & -\frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}\right)\end{aligned}$$

input
```
integrate((3*x^2+2)/x/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(x^2 - sqrt(x^4 + x^2 + 1) + 2) + log(x^2 - sqrt(x^4 + x^2 + 1) + 1
) - log(-x^2 + sqrt(x^4 + x^2 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4 + x^2 +
1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{x\,\left(x^2+1\right)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((3*x^2 + 2)/(x*(x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((3*x^2 + 2)/(x*(x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \frac{2+3x^2}{x(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\log\left(\frac{6\sqrt{x^4+x^2+1}+6x^2+6}{\sqrt{3}}\right) + \frac{\log\left(\frac{6\sqrt{x^4+x^2+1}+6x^2}{\sqrt{3}}\right)}{2} + \log\left(\frac{2\sqrt{x^4+x^2+1}+2x^2-2}{\sqrt{3}}\right) - \frac{\log\left(\frac{2\sqrt{x^4+x^2+1}+2x^2+4}{\sqrt{3}}\right)}{2}$$

input
```
int((3*x^2+2)/x/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
( - 2*log((6*sqrt(x**4 + x**2 + 1) + 6*x**2 + 6)/sqrt(3)) + log((6*sqrt(x*
*4 + x**2 + 1) + 6*x**2)/sqrt(3)) + 2*log((2*sqrt(x**4 + x**2 + 1) + 2*x**
2 - 2)/sqrt(3)) - log((2*sqrt(x**4 + x**2 + 1) + 2*x**2 + 4)/sqrt(3)))/2
```

3.97 $\int \frac{2+3x^2}{x^3(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 46

$$\int \frac{2+3x^2}{x^3\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx=-\frac{\sqrt{1+x^2+x^4}}{x^2}+\frac{1}{2}\text{arctanh}\left(\frac{1-x^2}{2\sqrt{1+x^2+x^4}}\right)$$

output
```
-(x^4+x^2+1)^(1/2)/x^2+1/2*arctanh(1/2*(-x^2+1)/(x^4+x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{2+3x^2}{x^3\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx=-\frac{\sqrt{1+x^2+x^4}}{x^2}-\text{arctanh}\left(1+x^2-\sqrt{1+x^2+x^4}\right)$$

input
```
Integrate[(2 + 3*x^2)/(x^3*(1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
-(Sqrt[1 + x^2 + x^4]/x^2) - ArcTanh[1 + x^2 - Sqrt[1 + x^2 + x^4]]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2248, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^3\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{2248}$$

$$\int \left(-\frac{x}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}+\frac{1}{\sqrt{x^4+x^2+1}x}+\frac{2}{\sqrt{x^4+x^2+1}x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\text{arctanh}\left(\frac{1-x^2}{2\sqrt{x^4+x^2+1}}\right)-\frac{\sqrt{x^4+x^2+1}}{x^2}$$

input
```
Int[(2 + 3*x^2)/(x^3*(1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
-(Sqrt[1 + x^2 + x^4]/x^2) + ArcTanh[(1 - x^2)/(2*Sqrt[1 + x^2 + x^4])]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$\frac{-\operatorname{arctanh}\left(\frac{x^2-1}{2\sqrt{x^4+x^2+1}}\right)x^2-2\sqrt{x^4+x^2+1}}{2x^2}$	42
default	$-\frac{\sqrt{x^4+x^2+1}}{x^2}+\frac{\operatorname{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)}{2}$	44
risch	$-\frac{\sqrt{x^4+x^2+1}}{x^2}+\frac{\operatorname{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)}{2}$	44
elliptic	$-\frac{\sqrt{x^4+x^2+1}}{x^2}+\frac{\operatorname{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)}{2}$	44
trager	$-\frac{\sqrt{x^4+x^2+1}}{x^2}+\frac{\ln\left(\frac{-x^2+2\sqrt{x^4+x^2+1}+1}{x^2+1}\right)}{2}$	47

input
```
int((3*x^2+2)/x^3/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-arctanh(1/2*(x^2-1)/(x^4+x^2+1)^(1/2))*x^2-2*(x^4+x^2+1)^(1/2))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{2+3x^2}{x^3\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx$$
$$=\frac{x^2\log\left(-x^2+\sqrt{x^4+x^2+1}\right)-x^2\log\left(-x^2+\sqrt{x^4+x^2+1}-2\right)-2\,x^2-2\,\sqrt{x^4+x^2+1}}{2\,x^2}$$

input
```
integrate((3*x^2+2)/x^3/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(x^2*log(-x^2 + sqrt(x^4 + x^2 + 1)) - x^2*log(-x^2 + sqrt(x^4 + x^2 +
 1) - 2) - 2*x^2 - 2*sqrt(x^4 + x^2 + 1))/x^2
```

Sympy [F]

$$\int \frac{2+3x^2}{x^3\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx=\int \frac{3x^2+2}{x^3\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\left(x^2+1\right)}\,dx$$

input
```
integrate((3*x**2+2)/x**3/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**3*sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)
), x)
```

Maxima [F]

$$\int \frac{2+3x^2}{x^3\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx=\int \frac{3\,x^2+2}{\sqrt{x^4+x^2+1}(x^2+1)x^3}\,dx$$

input
```
integrate((3*x^2+2)/x^3/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(36) = 72.

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \frac{2+3x^2}{x^3\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx=\frac{x^2-\sqrt{x^4+x^2+1}+2}{\left(x^2-\sqrt{x^4+x^2+1}\right)^2-1}-\frac{1}{2}\log\left(x^2-\sqrt{x^4+x^2+1}+2\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}\right)$$

input
```
integrate((3*x^2+2)/x^3/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
(x^2 - sqrt(x^4 + x^2 + 1) + 2)/((x^2 - sqrt(x^4 + x^2 + 1))^2 - 1) - 1/2*
log(x^2 - sqrt(x^4 + x^2 + 1) + 2) + 1/2*log(-x^2 + sqrt(x^4 + x^2 + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^3(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{x^3\,\left(x^2+1\right)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((3*x^2 + 2)/(x^3*(x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((3*x^2 + 2)/(x^3*(x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{2+3x^2}{x^3(1+x^2)\sqrt{1+x^2+x^4}}\,dx = 2\left(\int \frac{1}{\sqrt{x^4+x^2+1}\,x^5+\sqrt{x^4+x^2+1}\,x^3}dx\right) - \frac{3\log\left(\frac{6\sqrt{x^4+x^2+1}+6x^2+6}{\sqrt{3}}\right)}{2} - \frac{3\log\left(\frac{6\sqrt{x^4+x^2+1}+6x^2}{\sqrt{3}}\right)}{2} + \frac{3\log\left(\frac{2\sqrt{x^4+x^2+1}+2x^2-2}{\sqrt{3}}\right)}{2} + \frac{3\log\left(\frac{2\sqrt{x^4+x^2+1}+2x^2+4}{\sqrt{3}}\right)}{2}$$

input
```
int((3*x^2+2)/x^3/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
(4*int(1/(sqrt(x**4 + x**2 + 1)*x**5 + sqrt(x**4 + x**2 + 1)*x**3),x) - 3*
log((6*sqrt(x**4 + x**2 + 1) + 6*x**2 + 6)/sqrt(3)) - 3*log((6*sqrt(x**4 +
 x**2 + 1) + 6*x**2)/sqrt(3)) + 3*log((2*sqrt(x**4 + x**2 + 1) + 2*x**2 -
2)/sqrt(3)) + 3*log((2*sqrt(x**4 + x**2 + 1) + 2*x**2 + 4)/sqrt(3)))/2
```

3.98 $\int \frac{2+3x^2}{x^5(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 93

$$\int \frac{2+3x^2}{x^5(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\frac{\sqrt{1+x^2+x^4}}{2x^4}+\frac{\sqrt{1+x^2+x^4}}{4x^2} - \frac{1}{2}\operatorname{arctanh}\left(\frac{1-x^2}{2\sqrt{1+x^2+x^4}}\right) + \frac{7}{8}\operatorname{arctanh}\left(\frac{2+x^2}{2\sqrt{1+x^2+x^4}}\right)$$

output
```
-1/2*(x^4+x^2+1)^(1/2)/x^4+1/4*(x^4+x^2+1)^(1/2)/x^2-1/2*arctanh(1/2*(-x^2
+1)/(x^4+x^2+1)^(1/2))+7/8*arctanh(1/2*(x^2+2)/(x^4+x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int \frac{2+3x^2}{x^5(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{(-2+x^2)\sqrt{1+x^2+x^4}}{4x^4} - \frac{7}{4}\operatorname{arctanh}\left(x^2-\sqrt{1+x^2+x^4}\right) + \operatorname{arctanh}\left(1+x^2-\sqrt{1+x^2+x^4}\right)$$

input
```
Integrate[(2 + 3*x^2)/(x^5*(1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
((-2 + x^2)*Sqrt[1 + x^2 + x^4])/(4*x^4) - (7*ArcTanh[x^2 - Sqrt[1 + x^2 +
 x^4]])/4 + ArcTanh[1 + x^2 - Sqrt[1 + x^2 + x^4]]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2248, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^5 (x^2+1) \sqrt{x^4+x^2+1}} dx$$

$$\Big\downarrow \text{2248}$$

$$\int \left(\frac{x}{(x^2+1)\sqrt{x^4+x^2+1}} - \frac{1}{\sqrt{x^4+x^2+1}x} + \frac{2}{\sqrt{x^4+x^2+1}x^5} + \frac{1}{\sqrt{x^4+x^2+1}x^3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\text{arctanh}\left(\frac{1-x^2}{2\sqrt{x^4+x^2+1}}\right) + \frac{7}{8}\text{arctanh}\left(\frac{x^2+2}{2\sqrt{x^4+x^2+1}}\right) + \frac{\sqrt{x^4+x^2+1}}{4x^2} - \frac{\sqrt{x^4+x^2+1}}{2x^4}$$

input
```
Int[(2 + 3*x^2)/(x^5*(1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
-1/2*Sqrt[1 + x^2 + x^4]/x^4 + Sqrt[1 + x^2 + x^4]/(4*x^2) - ArcTanh[(1 -
x^2)/(2*Sqrt[1 + x^2 + x^4])]/2 + (7*ArcTanh[(2 + x^2)/(2*Sqrt[1 + x^2 + x
^4])])/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

method	result
default	$\frac{\sqrt{x^4+x^2+1}}{4x^2}+\frac{7\,\mathrm{arctanh}\left(\frac{x^2+2}{2\sqrt{x^4+x^2+1}}\right)}{8}-\frac{\mathrm{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)}{2}-\frac{\sqrt{x^4+x^2+1}}{2x^4}$
risch	$\frac{x^6-x^4-x^2-2}{4x^4\sqrt{x^4+x^2+1}}+\frac{7\,\mathrm{arctanh}\left(\frac{x^2+2}{2\sqrt{x^4+x^2+1}}\right)}{8}-\frac{\mathrm{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)}{2}$
elliptic	$\frac{\sqrt{x^4+x^2+1}}{4x^2}+\frac{7\,\mathrm{arctanh}\left(\frac{x^2+2}{2\sqrt{x^4+x^2+1}}\right)}{8}-\frac{\mathrm{arctanh}\left(\frac{-x^2+1}{2\sqrt{\left(x^2+1\right)^2-x^2}}\right)}{2}-\frac{\sqrt{x^4+x^2+1}}{2x^4}$
pseudoelliptic	$\frac{7\,\mathrm{arctanh}\left(\frac{x^2+2}{2\sqrt{x^4+x^2+1}}\right)x^4+4\,\mathrm{arctanh}\left(\frac{x^2-1}{2\sqrt{x^4+x^2+1}}\right)x^4+2x^2\sqrt{x^4+x^2+1}-4\sqrt{x^4+x^2+1}}{8x^4}$
trager	$\frac{\left(x^2-2\right)\sqrt{x^4+x^2+1}}{4x^4}-\frac{\ln\left(-\frac{-88573x^{22}+88574\sqrt{x^4+x^2+1}\,x^{20}-620014x^{20}+575728\sqrt{x^4+x^2+1}\,x^{18}-2103622x^{18}+1782540\sqrt{x^4+x}}{}\right.}{}$

input
```
int((3*x^2+2)/x^5/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^4+x^2+1)^(1/2)/x^2+7/8*arctanh(1/2*(x^2+2)/(x^4+x^2+1)^(1/2))-1/2*a
rctanh(1/2*(-x^2+1)/((x^2+1)^2-x^2)^(1/2))-1/2*(x^4+x^2+1)^(1/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.28

$$\int \frac{2+3x^2}{x^5\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx = \frac{7\,x^4\log\left(-x^2+\sqrt{x^4+x^2+1}+1\right)-4\,x^4\log\left(-x^2+\sqrt{x^4+x^2+1}\right)-7\,x^4\log\left(-x^2+\sqrt{x^4+x^2+1}-\right.}{8\,x^4}$$

input
```
integrate((3*x^2+2)/x^5/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(7*x^4*log(-x^2 + sqrt(x^4 + x^2 + 1) + 1) - 4*x^4*log(-x^2 + sqrt(x^4
 + x^2 + 1)) - 7*x^4*log(-x^2 + sqrt(x^4 + x^2 + 1) - 1) + 4*x^4*log(-x^2
+ sqrt(x^4 + x^2 + 1) - 2) + 2*x^4 + 2*sqrt(x^4 + x^2 + 1)*(x^2 - 2))/x^4
```

Sympy [F]

$$\int \frac{2+3x^2}{x^5\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^5\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}\left(x^2+1\right)}\,dx$$

input
```
integrate((3*x**2+2)/x**5/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**5*sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)
), x)
```

Maxima [F]

$$\int \frac{2+3x^2}{x^5\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+x^2+1}(x^2+1)x^5}\,dx$$

input
```
integrate((3*x^2+2)/x^5/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(71) = 142$.

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.75

$$\int \frac{2+3x^2}{x^5(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{3\left(x^2-\sqrt{x^4+x^2+1}\right)^3+4\left(x^2-\sqrt{x^4+x^2+1}\right)^2+7\,x^2-7\sqrt{x^4+x^2+1}+4}{4\left(\left(x^2-\sqrt{x^4+x^2+1}\right)^2-1\right)^2} + \frac{1}{2}\log\left(x^2-\sqrt{x^4+x^2+1}+2\right)-\frac{7}{8}\log\left(x^2-\sqrt{x^4+x^2+1}+1\right) + \frac{7}{8}\log\left(-x^2+\sqrt{x^4+x^2+1}+1\right)-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+x^2+1}\right)$$

input
```
integrate((3*x^2+2)/x^5/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*(x^2 - sqrt(x^4 + x^2 + 1))^3 + 4*(x^2 - sqrt(x^4 + x^2 + 1))^2 + 7
*x^2 - 7*sqrt(x^4 + x^2 + 1) + 4)/((x^2 - sqrt(x^4 + x^2 + 1))^2 - 1)^2 +
1/2*log(x^2 - sqrt(x^4 + x^2 + 1) + 2) - 7/8*log(x^2 - sqrt(x^4 + x^2 + 1)
 + 1) + 7/8*log(-x^2 + sqrt(x^4 + x^2 + 1) + 1) - 1/2*log(-x^2 + sqrt(x^4
+ x^2 + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^5(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{x^5\,\left(x^2+1\right)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((3*x^2 + 2)/(x^5*(x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((3*x^2 + 2)/(x^5*(x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{2+3x^2}{x^5 (1+x^2) \sqrt{1+x^2+x^4}} \, dx = 2\left(\int \frac{1}{\sqrt{x^4+x^2+1}\, x^7 + \sqrt{x^4+x^2+1}\, x^5} dx\right) + 3\left(\int \frac{1}{\sqrt{x^4+x^2+1}\, x^5 + \sqrt{x^4+x^2+1}\, x^3} dx\right)$$

input
```
int((3*x^2+2)/x^5/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
2*int(1/(sqrt(x**4 + x**2 + 1)*x**7 + sqrt(x**4 + x**2 + 1)*x**5),x) + 3*i
nt(1/(sqrt(x**4 + x**2 + 1)*x**5 + sqrt(x**4 + x**2 + 1)*x**3),x)
```

3.99 $\int \frac{x^4(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 152

$$\begin{aligned}\int \frac{x^4(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx &= x\sqrt{1+x^2+x^4}-\frac{3x\sqrt{1+x^2+x^4}}{1+x^2}\\ &\quad-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\\ &\quad+\frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\middle|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}\\ &\quad-\frac{7(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}\end{aligned}$$

output
```
x*(x^4+x^2+1)^(1/2)-3*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/2*arctan(x/(x^4+x^2+1)
^(1/2))+3*(x^2+1)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(x))
,1/2)/(x^4+x^2+1)^(1/2)-7/4*(x^2+1)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)*InverseJ
acobiAM(2*arctan(x),1/2)/(x^4+x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

$$\int \frac{x^4(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x+x^3+x^5+3\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(-E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\text{EllipticF}\left(i\text{arc}\right.\right.}{\sqrt{}}$$

input
```
Integrate[(x^4*(2 + 3*x^2))/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
(x + x^3 + x^5 + 3*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)
*x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSi
nh[(-1)^(5/6)*x], (-1)^(2/3)]) - (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[
1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(
2/3)])/Sqrt[1 + x^2 + x^4]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2236, 27, 2230, 25, 1509, 2214, 1416, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(3x^2+2)}{(x^2+1)\sqrt{x^4+x^2+1}}\,dx$$

$$\Big\downarrow \text{2236}$$

$$\frac{1}{3}\int \frac{3\left(x^4(3x^2+2)-(x^2+1)\left(3x^4+2x^2+1\right)\right)}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^4\left(3x^2+2\right)-\left(x^2+1\right)\left(3x^4+2x^2+1\right)}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx+\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \ 2230$$

$$3\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx+\int -\frac{3x^2+4}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx+\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \ 25$$

$$3\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx-\int \frac{3x^2+4}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx+\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \ 1509$$

$$-\int \frac{3x^2+4}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx+3\left(\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)+\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \ 2214$$

$$-\frac{7}{2}\int \frac{1}{\sqrt{x^4+x^2+1}}dx-\frac{1}{2}\int \frac{1-x^2}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx+3\left(\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)+\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \ 1416$$

$$-\frac{1}{2}\int \frac{1-x^2}{\left(x^2+1\right)\sqrt{x^4+x^2+1}}dx-\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+3\left(\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)+\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \ 2212$$

$$-\frac{1}{2}\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}-\frac{7\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+3\left(\frac{\left(x^2+1\right)\sqrt{\frac{x^4+x^2+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)+\sqrt{x^4+x^2+1}x$$

$$\Big\downarrow \ 216$$

$$-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{7(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\mathrm{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+3\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,|\,\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)+\sqrt{x^4+x^2+1}x$$

input
```
Int[(x^4*(2 + 3*x^2))/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
x*Sqrt[1 + x^2 + x^4] - ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + 3*(-((x*Sqrt[1 +
 x^2 + x^4])/(1 + x^2)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Ell
ipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]) - (7*(1 + x^2)*Sqrt[(1 + x^
2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2214
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[(B*d + A*e)/(2*d*e)   Int[1/Sqrt[a + b*x
^2 + c*x^4], x], x] - Simp[(B*d - A*e)/(2*d*e)   Int[(d - e*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && E
qQ[c*d^2 - a*e^2, 0] && NeQ[B*d + A*e, 0]
```

rule 2230
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-C/e^2   Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]
+ Simp[1/e^2   Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2
+ c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && Eq
Q[c*d^2 - a*e^2, 0]
```

rule 2236
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]),
 x_Symbol] :> With[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt
[a + b*x^2 + c*x^4]/(c*e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q
 - 3)*Px - Coeff[Px, x, q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + b*(q - 4)*x^2
 + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; GtQ[q,
4]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Px, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.57

method	result
default	$x\sqrt{x^4+x^2+1}+\frac{12\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{-2+2i\sqrt{}}}{2}\cdots\right.\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}$
risch	$x\sqrt{x^4+x^2+1}+\frac{12\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{-2+2i\sqrt{}}}{2}\cdots\right.\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}$
elliptic	$x\sqrt{x^4+x^2+1}+\frac{12\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}-\frac{12\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}\cdots}}{\sqrt{-2+2\cdots}}$

input
```
int(x^4*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(x^4+x^2+1)^(1/2)+12/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)
^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*
(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-Ellipt
icE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))-1/(-1/2+1/2*
I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*
3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-
1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.82

$$\int \frac{x^4(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx =$$

$$-\frac{6\left(\sqrt{-3}x-x\right)\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}}{x}\right) \mid \frac{1}{2}\sqrt{-3}-\frac{1}{2})-\left(5\sqrt{-3}x-7x\right)\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}F(\mathrm{a}\cdots}{4x}$$

input
```
integrate(x^4*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(6*(sqrt(-3)*x - x)*sqrt(1/2*sqrt(-3) - 1/2)*elliptic_e(arcsin(sqrt(1
/2*sqrt(-3) - 1/2)/x), 1/2*sqrt(-3) - 1/2) - (5*sqrt(-3)*x - 7*x)*sqrt(1/2
*sqrt(-3) - 1/2)*elliptic_f(arcsin(sqrt(1/2*sqrt(-3) - 1/2)/x), 1/2*sqrt(-
3) - 1/2) + 2*x*arctan(x/sqrt(x^4 + x^2 + 1)) - 4*sqrt(x^4 + x^2 + 1)*(x^2
 - 3))/x
```

Sympy [F]

$$\int \frac{x^4(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^4\cdot(3x^2+2)}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)}\,dx$$

input
```
integrate(x**4*(3*x**2+2)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(x**4*(3*x**2 + 2)/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)
), x)
```

Maxima [F]

$$\int \frac{x^4(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^4}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate(x^4*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^4/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x^4(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^4}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate(x^4*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)*x^4/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^4\,(3\,x^2+2)}{(x^2+1)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((x^4*(3*x^2 + 2))/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((x^4*(3*x^2 + 2))/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \sqrt{x^4+x^2+1}\,x - \left(\int \frac{\sqrt{x^4+x^2+1}}{x^6+2x^4+2x^2+1}dx\right) - 3\left(\int \frac{\sqrt{x^4+x^2+1}\,x^4}{x^6+2x^4+2x^2+1}dx\right) - 3\left(\int \frac{\sqrt{x^4+x^2+1}\,x^2}{x^6+2x^4+2x^2+1}dx\right)$$

input
```
int(x^4*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
sqrt(x**4 + x**2 + 1)*x - int(sqrt(x**4 + x**2 + 1)/(x**6 + 2*x**4 + 2*x**
2 + 1),x) - 3*int((sqrt(x**4 + x**2 + 1)*x**4)/(x**6 + 2*x**4 + 2*x**2 + 1
),x) - 3*int((sqrt(x**4 + x**2 + 1)*x**2)/(x**6 + 2*x**4 + 2*x**2 + 1),x)
```

3.100 $\int \frac{x^2(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 138

$$\int \frac{x^2(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{3x\sqrt{1+x^2+x^4}}{1+x^2}+\frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

output
```
3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/2*arctan(x/(x^4+x^2+1)^(1/2))-3*(x^2+1)*((
x^4+x^2+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(x)),1/2)/(x^4+x^2+1)^(1
/2)+5/4*(x^2+1)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(x),
1/2)/(x^4+x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{x^2(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(3E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-\left(3+\sqrt[3]{-1}\right)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\right.\right.}{\sqrt{1+x^2+x^4}}$$

input
```
Integrate[(x^2*(2 + 3*x^2))/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(3*EllipticE
[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - (3 + (-1)^(1/3))*EllipticF[I*ArcSi
nh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*EllipticPi[(-1)^(1/3), I*ArcSin
h[(-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2230, 1509, 2214, 1416, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(3x^2+2)}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$
$$\Big\downarrow \text{ 2230}$$
$$\int \frac{2x^2+3}{(x^2+1)\sqrt{x^4+x^2+1}}dx-3\int \frac{1-x^2}{\sqrt{x^4+x^2+1}}dx$$
$$\Big\downarrow \text{ 1509}$$
$$\int \frac{2x^2+3}{(x^2+1)\sqrt{x^4+x^2+1}}dx-3\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)$$

$$\Big\downarrow \text{2214}$$

$$\frac{5}{2}\int \frac{1}{\sqrt{x^4+x^2+1}}dx+\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx-3\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx+\frac{5(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-3\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)$$

$$\Big\downarrow \text{2212}$$

$$\frac{1}{2}\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}+\frac{5(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-3\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{5(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-3\left(\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}-\frac{x\sqrt{x^4+x^2+1}}{x^2+1}\right)$$

input
```
Int[(x^2*(2 + 3*x^2))/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - 3*(-((x*Sqrt[1 + x^2 + x^4])/(1 + x^2))
+ ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4]
)/Sqrt[1 + x^2 + x^4]) + (5*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2214
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[(B*d + A*e)/(2*d*e)   Int[1/Sqrt[a + b*x
^2 + c*x^4], x], x] - Simp[(B*d - A*e)/(2*d*e)   Int[(d - e*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && E
qQ[c*d^2 - a*e^2, 0] && NeQ[B*d + A*e, 0]
```

rule 2230
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Simp[-C/e^2   Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]
+ Simp[1/e^2   Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2
+ c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && Eq
Q[c*d^2 - a*e^2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.23

method	result
default	$\frac{\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}-\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{\sqrt{-2+2i\sqrt{3}}}$
elliptic	$-\frac{2\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{12\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{}}{}\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{}\right.}$

input
```
int(x^2*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^
2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2
))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*
3^(1/2))^(1/2))-2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2
)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2
+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-12/(-2+2*I*3^(1/2))^(1/2)*
(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4
+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-
2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3
^(1/2))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int \frac{x^2(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{6\left(\sqrt{-3}x-x\right)\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}}{x}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)-\left(7\sqrt{-3}x-5\,x\right)\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}F(\mathrm{arcs}}{4\,x}$$

input
```
integrate(x^2*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(6*(sqrt(-3)*x - x)*sqrt(1/2*sqrt(-3) - 1/2)*elliptic_e(arcsin(sqrt(1/
2*sqrt(-3) - 1/2)/x), 1/2*sqrt(-3) - 1/2) - (7*sqrt(-3)*x - 5*x)*sqrt(1/2*
sqrt(-3) - 1/2)*elliptic_f(arcsin(sqrt(1/2*sqrt(-3) - 1/2)/x), 1/2*sqrt(-3
) - 1/2) + 2*x*arctan(x/sqrt(x^4 + x^2 + 1)) + 12*sqrt(x^4 + x^2 + 1))/x
```

Sympy [F]

$$\int \frac{x^2(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^2\cdot(3x^2+2)}{\sqrt{(x^2-x+1)\,(x^2+x+1)}\,(x^2+1)}\,dx$$

input
```
integrate(x**2*(3*x**2+2)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral(x**2*(3*x**2 + 2)/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)
), x)
```

Maxima [F]

$$\int \frac{x^2(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate(x^2*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)*x^2/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{x^2(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate(x^2*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)*x^2/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{x^2\,(3\,x^2+2)}{(x^2+1)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((x^2*(3*x^2 + 2))/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((x^2*(3*x^2 + 2))/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(2+3x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = 3\left(\int \frac{\sqrt{x^4+x^2+1}\,x^4}{x^6+2x^4+2x^2+1}dx\right) + 2\left(\int \frac{\sqrt{x^4+x^2+1}\,x^2}{x^6+2x^4+2x^2+1}dx\right)$$

input
```
int(x^2*(3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
3*int((sqrt(x**4 + x**2 + 1)*x**4)/(x**6 + 2*x**4 + 2*x**2 + 1),x) + 2*int
((sqrt(x**4 + x**2 + 1)*x**2)/(x**6 + 2*x**4 + 2*x**2 + 1),x)
```

3.101 $\int \frac{2+3x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{2+3x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

output
```
-1/2*arctan(x/(x^4+x^2+1)^(1/2))+5/4*(x^2+1)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)
*InverseJacobiAM(2*arctan(x),1/2)/(x^4+x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{2+3x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(3\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)-\operatorname{EllipticPi}\left(\sqrt[3]{-1},i\mathrm{ar}\right.\right.}{\sqrt{1+x^2+x^4}}$$

input
```
Integrate[(2 + 3*x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(3*EllipticF
[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticPi[(-1)^(1/3), I*ArcSinh[(
-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2214, 1416, 2212, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{ 2214}$$

$$\frac{5}{2}\int \frac{1}{\sqrt{x^4+x^2+1}}dx-\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{5(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{1}{2}\int \frac{1-x^2}{(x^2+1)\sqrt{x^4+x^2+1}}dx$$

$$\Big\downarrow \text{ 2212}$$

$$\frac{5(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{1}{2}\int \frac{1}{\frac{x^2}{x^4+x^2+1}+1}d\frac{x}{\sqrt{x^4+x^2+1}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{5(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

input
```
Int[(2 + 3*x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
-1/2*ArcTan[x/Sqrt[1 + x^2 + x^4]] + (5*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1
+ x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2212
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[A   Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
 x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &
& EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

rule 2214
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Simp[(B*d + A*e)/(2*d*e)   Int[1/Sqrt[a + b*x
^2 + c*x^4], x], x] - Simp[(B*d - A*e)/(2*d*e)   Int[(d - e*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && E
qQ[c*d^2 - a*e^2, 0] && NeQ[B*d + A*e, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.72

method	result
default	$\frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticPi}\left(\cdots\right.}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x\cdots}}$
elliptic	$\frac{6\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{1}{2}+\cdots}\right.}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+\cdots}}$

input
```
int((3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I
*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2))-1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/
2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/
2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1
/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \frac{2+3x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\frac{5}{4}\left(\sqrt{-3}+1\right)\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}F(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}) - \frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

input
```
integrate((3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-5/4*(sqrt(-3) + 1)*sqrt(1/2*sqrt(-3) - 1/2)*elliptic_f(arcsin(x*sqrt(1/2*
sqrt(-3) - 1/2)), 1/2*sqrt(-3) - 1/2) - 1/2*arctan(x/sqrt(x^4 + x^2 + 1))
```

Sympy [F]

$$\int \frac{2+3x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3x^2+2}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)}\,dx$$

input
```
integrate((3*x**2+2)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{2+3x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate((3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{2+3x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+x^2+1}(x^2+1)}\,dx$$

input
```
integrate((3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx=\int \frac{3\,x^2+2}{(x^2+1)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((3*x^2 + 2)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((3*x^2 + 2)/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{2+3x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx=2\left(\int \frac{\sqrt{x^4+x^2+1}}{x^6+2x^4+2x^2+1}dx\right)+3\left(\int \frac{\sqrt{x^4+x^2+1}\,x^2}{x^6+2x^4+2x^2+1}dx\right)$$

input
```
int((3*x^2+2)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
2*int(sqrt(x**4 + x**2 + 1)/(x**6 + 2*x**4 + 2*x**2 + 1),x) + 3*int((sqrt(
x**4 + x**2 + 1)*x**2)/(x**6 + 2*x**4 + 2*x**2 + 1),x)
```

3.102 $\int \frac{2+3x^2}{x^2(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 155

$$\begin{aligned}\int \frac{2+3x^2}{x^2(1+x^2)\sqrt{1+x^2+x^4}}\,dx = &-\frac{2\sqrt{1+x^2+x^4}}{x}+\frac{2x\sqrt{1+x^2+x^4}}{1+x^2}\\ &+\frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\\ &-\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}\\ &+\frac{5(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}\end{aligned}$$

output
```
-2*(x^4+x^2+1)^(1/2)/x+2*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/2*arctan(x/(x^4+x^2
+1)^(1/2))-2*(x^2+1)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(
x)),1/2)/(x^4+x^2+1)^(1/2)+5/4*(x^2+1)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)*Inver
seJacobiAM(2*arctan(x),1/2)/(x^4+x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.06

$$\int \frac{2+3x^2}{x^2(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{-\frac{2(1+x^2+x^4)}{x}+2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\text{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-\text{EllipticF}\left(i\text{arcsi}\right.\right.}{\sqrt{}}$$

input
```
Integrate[(2 + 3*x^2)/(x^2*(1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
((-2*(1 + x^2 + x^4))/x + 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticF
[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) + (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^
2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x]
, (-1)^(2/3)])/Sqrt[1 + x^2 + x^4]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2248, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^2(x^2+1)\sqrt{x^4+x^2+1}}\,dx$$
$$\Big\downarrow \text{2248}$$
$$\int \left(\frac{1}{(x^2+1)\sqrt{x^4+x^2+1}}+\frac{2}{x^2\sqrt{x^4+x^2+1}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{5(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{4}\right.\right)}{\sqrt{x^4+x^2+1}}+\frac{2\sqrt{x^4+x^2+1}x}{x^2+1}-\frac{2\sqrt{x^4+x^2+1}}{x}$$

input
```
Int[(2 + 3*x^2)/(x^2*(1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
(-2*Sqrt[1 + x^2 + x^4])/x + (2*x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + ArcTan[
x/Sqrt[1 + x^2 + x^4]]/2 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*
EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] + (5*(1 + x^2)*Sqrt[(1 +
x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4
])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.55

method	result
default	$\frac{\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}-\frac{2\sqrt{x^4+x^2+1}}{x}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}}{}$
risch	$\frac{\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,\operatorname{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}-\frac{2\sqrt{x^4+x^2+1}}{x}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}}{}$
elliptic	$-\frac{2\sqrt{x^4+x^2+1}}{x}-\frac{8\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{3}}{2}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}+\frac{8\sqrt{1+\frac{x^2}{2}-\frac{i x^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{i x^2\sqrt{}}{2}}}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{}}$

input
```
int((3*x^2+2)/x^2/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^
2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2
))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*
3^(1/2))^(1/2))-2/x*(x^4+x^2+1)^(1/2)-8/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/
2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/
2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/
2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

$$\int \frac{2+3x^2}{x^2\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx = -\frac{4\left(\sqrt{-3}x-x\right)\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}E\left(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)-\left(3\sqrt{-3}x-5\,x\right)\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}}{4\,x}$$

input
```
integrate((3*x^2+2)/x^2/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(4*(sqrt(-3)*x - x)*sqrt(1/2*sqrt(-3) - 1/2)*elliptic_e(arcsin(x*sqrt
(1/2*sqrt(-3) - 1/2)), 1/2*sqrt(-3) - 1/2) - (3*sqrt(-3)*x - 5*x)*sqrt(1/2
*sqrt(-3) - 1/2)*elliptic_f(arcsin(x*sqrt(1/2*sqrt(-3) - 1/2)), 1/2*sqrt(-
3) - 1/2) - 2*x*arctan(x/sqrt(x^4 + x^2 + 1)) + 8*sqrt(x^4 + x^2 + 1))/x
```

Sympy [F]

$$\int \frac{2+3x^2}{x^2(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^2\sqrt{(x^2-x+1)(x^2+x+1)}(x^2+1)}\,dx$$

input
```
integrate((3*x**2+2)/x**2/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**2*sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)
), x)
```

Maxima [F]

$$\int \frac{2+3x^2}{x^2(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+x^2+1}(x^2+1)x^2}\,dx$$

input
```
integrate((3*x^2+2)/x^2/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)*x^2), x)
```

Giac [F]

$$\int \frac{2+3x^2}{x^2(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{\sqrt{x^4+x^2+1}(x^2+1)x^2}\,dx$$

input
```
integrate((3*x^2+2)/x^2/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^2(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3\,x^2+2}{x^2\,(x^2+1)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((3*x^2 + 2)/(x^2*(x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((3*x^2 + 2)/(x^2*(x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{2+3x^2}{x^2(1+x^2)\sqrt{1+x^2+x^4}}\,dx = 2\left(\int \frac{\sqrt{x^4+x^2+1}}{x^8+2x^6+2x^4+x^2}dx\right) + 3\left(\int \frac{\sqrt{x^4+x^2+1}}{x^6+2x^4+2x^2+1}dx\right)$$

input
```
int((3*x^2+2)/x^2/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output
```
2*int(sqrt(x**4 + x**2 + 1)/(x**8 + 2*x**6 + 2*x**4 + x**2),x) + 3*int(sqr
t(x**4 + x**2 + 1)/(x**6 + 2*x**4 + 2*x**2 + 1),x)
```

3.103 $\int \frac{2+3x^2}{x^4(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 180

$$\begin{aligned}\int \frac{2+3x^2}{x^4(1+x^2)\sqrt{1+x^2+x^4}}\,dx =& -\frac{2\sqrt{1+x^2+x^4}}{3x^3}+\frac{\sqrt{1+x^2+x^4}}{3x} \\ &-\frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)}-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) \\ &+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}} \\ &-\frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}\end{aligned}$$

output
```
-2/3*(x^4+x^2+1)^(1/2)/x^3+1/3*(x^4+x^2+1)^(1/2)/x-x*(x^4+x^2+1)^(1/2)/(3*
x^2+3)-1/2*arctan(x/(x^4+x^2+1)^(1/2))+1/3*(x^2+1)*((x^4+x^2+1)/(x^2+1)^2)
^(1/2)*EllipticE(sin(2*arctan(x)),1/2)/(x^4+x^2+1)^(1/2)-3/4*(x^2+1)*((x^4
+x^2+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2)/(x^4+x^2+1)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.44 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.21

$$\int \frac{2+3x^2}{x^4(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$
$$= \frac{-2-x^2-x^4+x^6-\sqrt[3]{-1}x^3\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}E\left(i\operatorname{arcsinh}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-(-1)^{5/6}x^3}{}$$

input
```
Integrate[(2 + 3*x^2)/(x^4*(1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
(-2 - x^2 - x^4 + x^6 - (-1)^(1/3)*x^3*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - (-1)^(5/6)
*x^3*Sqrt[3 + 3*(-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSi
nh[(-1)^(5/6)*x], (-1)^(2/3)] - 3*(-1)^(2/3)*x^3*Sqrt[1 + (-1)^(1/3)*x^2]*
Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (
-1)^(2/3)])/(3*x^3*Sqrt[1 + x^2 + x^4])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2248, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2}{x^4(x^2+1)\sqrt{x^4+x^2+1}}\,dx$$
$$\Big\downarrow \text{2248}$$
$$\int \left(\frac{1}{(-x^2-1)\sqrt{x^4+x^2+1}}+\frac{1}{x^2\sqrt{x^4+x^2+1}}+\frac{2}{x^4\sqrt{x^4+x^2+1}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\arctan(x)\,|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}-\frac{\sqrt{x^4+x^2+1}x}{3(x^2+1)}+\frac{\sqrt{x^4+x^2+1}}{3x}-\frac{2\sqrt{x^4+x^2+1}}{3x^3}$$

input
```
Int[(2 + 3*x^2)/(x^4*(1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

output
```
(-2*Sqrt[1 + x^2 + x^4])/(3*x^3) + Sqrt[1 + x^2 + x^4]/(3*x) - (x*Sqrt[1 +
 x^2 + x^4])/(3*(1 + x^2)) - ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*
Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 +
 x^2 + x^4]) - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*
ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.88

method	result
default	$\frac{\sqrt{x^4+x^2+1}}{3x}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2}}{2}\right.\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}$
risch	$\frac{x^6-x^4-x^2-2}{3x^3\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-}}{}\right.\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}$
elliptic	$-\frac{2\sqrt{x^4+x^2+1}}{3x^3}+\frac{\sqrt{x^4+x^2+1}}{3x}-\frac{4\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{}}{2}}}{}$

input
```
int((3*x^2+2)/x^4/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/x*(x^4+x^2+1)^(1/2)+4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))
*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1
/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-E
llipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))-2/3*(x^
4+x^2+1)^(1/2)/x^3-4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)
^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*
x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-1/(-1/2+1/2*I*3^(1/2)
)^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^
(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1
/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.72

$$\int \frac{2+3x^2}{x^4(1+x^2)\sqrt{1+x^2+x^4}}\,dx = -\frac{6\,x^3\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right) - 2\left(\sqrt{-3}x^3 - x^3\right)\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}E\left(\arcsin\left(x\sqrt{\frac{1}{2}\sqrt{-3}-\frac{1}{2}}\right)\mid\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right) - (}{12\,x^3}$$

input
```
integrate((3*x^2+2)/x^4/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(6*x^3*arctan(x/sqrt(x^4 + x^2 + 1)) - 2*(sqrt(-3)*x^3 - x^3)*sqrt(1
/2*sqrt(-3) - 1/2)*elliptic_e(arcsin(x*sqrt(1/2*sqrt(-3) - 1/2)), 1/2*sqrt
(-3) - 1/2) - (5*sqrt(-3)*x^3 + 9*x^3)*sqrt(1/2*sqrt(-3) - 1/2)*elliptic_f
(arcsin(x*sqrt(1/2*sqrt(-3) - 1/2)), 1/2*sqrt(-3) - 1/2) - 4*sqrt(x^4 + x^
2 + 1)*(x^2 - 2))/x^3
```

Sympy [F]

$$\int \frac{2+3x^2}{x^4(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \int \frac{3x^2+2}{x^4\sqrt{(x^2-x+1)(x^2+x+1)}(x^2+1)}\,dx$$

input
```
integrate((3*x**2+2)/x**4/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

output
```
Integral((3*x**2 + 2)/(x**4*sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)
), x)
```

Maxima [F]

$$\int \frac{2+3x^2}{x^4\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx=\int \frac{3\,x^2+2}{\sqrt{x^4+x^2+1}(x^2+1)x^4}\,dx$$

input
```
integrate((3*x^2+2)/x^4/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)*x^4), x)
```

Giac [F]

$$\int \frac{2+3x^2}{x^4\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx=\int \frac{3\,x^2+2}{\sqrt{x^4+x^2+1}(x^2+1)x^4}\,dx$$

input
```
integrate((3*x^2+2)/x^4/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2}{x^4\left(1+x^2\right)\sqrt{1+x^2+x^4}}\,dx=\int \frac{3\,x^2+2}{x^4\,\left(x^2+1\right)\,\sqrt{x^4+x^2+1}}\,dx$$

input
```
int((3*x^2 + 2)/(x^4*(x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

output
```
int((3*x^2 + 2)/(x^4*(x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{2+3x^2}{x^4(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{-2\sqrt{x^4+x^2+1}-\left(\int \frac{\sqrt{x^4+x^2+1}}{x^8+2x^6+2x^4+x^2}dx\right)x^3-6\left(\int \frac{\sqrt{x^4+x^2+1}}{x^6+2x^4+2x^2+1}dx\right)x^3-2\left(\int \frac{\sqrt{x^4+x^2+1}\,x^2}{x^6+2x^4+2x^2+1}dx\right)x^3}{3x^3}$$

input

```
int((3*x^2+2)/x^4/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

output

```
( - 2*sqrt(x**4 + x**2 + 1) - int(sqrt(x**4 + x**2 + 1)/(x**8 + 2*x**6 + 2
*x**4 + x**2),x)*x**3 - 6*int(sqrt(x**4 + x**2 + 1)/(x**6 + 2*x**4 + 2*x**
2 + 1),x)*x**3 - 2*int((sqrt(x**4 + x**2 + 1)*x**2)/(x**6 + 2*x**4 + 2*x**
2 + 1),x)*x**3)/(3*x**3)
```

3.104 $\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1604

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

output
```
1/3840*(10*A*c*e*(15*c^3*d^3+15*b^3*e^3-c^2*d*e*(-20*a*e+7*b*d)-b*c*e^2*(5
2*a*e+7*b*d))-B*(105*c^4*d^4+105*b^4*e^4-4*c^3*d^2*e*(-23*a*e+10*b*d)-20*b
^2*c*e^3*(23*a*e+2*b*d)-2*c^2*e^2*(-128*a^2*e^2-72*a*b*d*e+17*b^2*d^2)))*(
e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^4/e^4/x-1/1920*(10*A*c*e*(5*c^2*d^2
+5*b^2*e^2-2*c*e*(6*a*e+b*d))-B*(35*c^3*d^3+35*b^3*e^3-c^2*d*e*(-28*a*e+11
*b*d)-b*c*e^2*(116*a*e+11*b*d)))*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c
^3/e^3+1/480*(10*A*c*e*(b*e+c*d)-B*(7*c^2*d^2+7*b^2*e^2-2*c*e*(8*a*e+b*d))
)*x^3*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e^2+1/80*(10*A*c*e+B*b*e+B
*c*d)*x^5*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e+1/10*B*x^7*(e*x^2+d)^(
1/2)*(c*x^4+b*x^2+a)^(1/2)-1/7680*(-4*a*c+b^2)^(1/2)*(10*A*c*e*(15*c^3*d^3
+15*b^3*e^3-c^2*d*e*(-20*a*e+7*b*d)-b*c*e^2*(52*a*e+7*b*d))-B*(105*c^4*d^4
+105*b^4*e^4-4*c^3*d^2*e*(-23*a*e+10*b*d)-20*b^2*c*e^3*(23*a*e+2*b*d)-2*c^
2*e^2*(-128*a^2*e^2-72*a*b*d*e+17*b^2*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b
^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-
2*a*e))^(1/2))*2^(1/2)/c^4/e^4/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a
*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/3840*(-4*a*c+b^2)^(1/2)*(10*A*c*e*(5*c^
3*d^3+15*b^3*e^3-c^2*d*e*(-44*a*e+3*b*d)-b*c*e^2*(52*a*e+17*b*d))-B*(35*c^
4*d^4+105*b^4*e^4-18*c^3*d^2*e*(-2*a*e+b*d)-10*b^2*c*e^3*(46*a*e+11*b*d)-4
*c^2*e^2*(-64*a^2*e^2-94*a*b*d*e+3*b^2*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a...
```

Mathematica [F]

$$\int x^4 (A + Bx^2) \sqrt{d + ex^2} \sqrt{a + bx^2 + cx^4} \, dx = \int x^4 (A + Bx^2) \sqrt{d + ex^2} \sqrt{a + bx^2 + cx^4} \, dx$$

input
```
Integrate[x^4*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
Integrate[x^4*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (A+Bx^2) \sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2250}$$

$$\int x^4 (A+Bx^2) \sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}dx$$

input
```
Int[x^4*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int x^4 (B\,x^2+A)\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$
$$=\int \sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}x^4\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm
="fricas")
```

output
```
integral((B*x^6 + A*x^4)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$
$$=\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**4*(B*x**2+A)*(e*x**2+d)**(1/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**4*(A + B*x**2)*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$
$$=\int \sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}x^4\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)*x^4, x)
```

Giac [F]

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}x^4\,dx$$

input
```
integrate(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx = \int x^4\,\left(B\,x^2+A\right)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(x^4*(A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^4*(A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^4(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$
$$=\int x^4(B\,x^2+A)\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(x^4*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

3.105 $\int x^2(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1229

$$\int x^2(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

output
```
-1/384*(8*A*c*e*(3*c^2*d^2+3*b^2*e^2-2*c*e*(4*a*e+b*d))-B*(15*c^3*d^3+15*b
^3*e^3-c^2*d*e*(-20*a*e+7*b*d)-b*c*e^2*(52*a*e+7*b*d)))*(e*x^2+d)^(1/2)*(c
*x^4+b*x^2+a)^(1/2)/c^3/e^3/x+1/192*(8*A*c*e*(b*e+c*d)-B*(5*c^2*d^2+5*b^2*
e^2-2*c*e*(6*a*e+b*d)))*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e^2+1/
48*(8*A*c*e+B*b*e+B*c*d)*x^3*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e+1/8
*B*x^5*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)+1/768*(-4*a*c+b^2)^(1/2)*(8*A
*c*e*(3*c^2*d^2+3*b^2*e^2-2*c*e*(4*a*e+b*d))-B*(15*c^3*d^3+15*b^3*e^3-c^2*
d*e*(-20*a*e+7*b*d)-b*c*e^2*(52*a*e+7*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b
^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-
2*a*e))^(1/2))*2^(1/2)/c^3/e^3/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a
*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/384*(-4*a*c+b^2)^(1/2)*(8*A*c*e*(c^2*d^
2+3*b^2*e^2-4*c*e*(2*a*e+b*d))-B*(5*c^3*d^3+15*b^3*e^3-c^2*d*e*(-44*a*e+3*
b*d)-b*c*e^2*(52*a*e+17*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a
*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b
+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/
(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^3/e^2/(e*x^2+d)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)+1/64*(-4*a*c+b^2)^(1/2)*(8*A*c*e*(-b*e+c*d)*(4*a*c*e^
2-b^2*e^2+c^2*d^2)-B*(5*c^4*d^4+5*b^4*e^4-4*c^3*d^2*e*(-2*a*e+b*d)-4*b^2*c
*e^3*(6*a*e+b*d)-2*c^2*e^2*(-8*a^2*e^2-8*a*b*d*e+b^2*d^2)))*(-a*(c+a/x^...
```

Mathematica [F]

$$\int x^2(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$
$$=\int x^2(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
Integrate[x^2*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
Integrate[x^2*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (A + Bx^2) \sqrt{d + ex^2} \sqrt{a + bx^2 + cx^4} \, dx$$

$$\Big\downarrow \; 2250$$

$$\int x^2 (A + Bx^2) \sqrt{d + ex^2} \sqrt{a + bx^2 + cx^4} dx$$

input
```
Int[x^2*(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int x^2 \left(B\,x^2 + A\right) \sqrt{e\,x^2 + d}\,\sqrt{c\,x^4 + b\,x^2 + a}dx$$

input
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int x^2 \left(A + Bx^2\right) \sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}\, dx = \text{Timed out}$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int x^2 \left(A + Bx^2\right) \sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}\, dx = \int x^2 \left(A + Bx^2\right) \sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}\, dx$$

input
```
integrate(x**2*(B*x**2+A)*(e*x**2+d)**(1/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2)*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int x^2 \left(A + Bx^2\right) \sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}\, dx = \int \sqrt{cx^4 + bx^2 + a}\left(Bx^2 + A\right)\sqrt{ex^2 + d}x^2\, dx$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)*x^2, x)
```

Giac [F]

$$\int x^2(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$
$$=\int \sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}x^2\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$
$$=\int x^2\,(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(x^2*(A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^2*(A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^2(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx=\text{too large to display}$$

input
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
(20*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*c*e**2*x + 8*sqrt(d + e
*x**2)*sqrt(a + b*x**2 + c*x**4)*a*c**2*d*e*x + 32*sqrt(d + e*x**2)*sqrt(a
 + b*x**2 + c*x**4)*a*c**2*e**2*x**3 - 5*sqrt(d + e*x**2)*sqrt(a + b*x**2
+ c*x**4)*b**3*e**2*x + 2*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*
c*d*e*x + 4*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*c*e**2*x**3 -
5*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c**2*d**2*x + 4*sqrt(d + e*
x**2)*sqrt(a + b*x**2 + c*x**4)*b*c**2*d*e*x**3 + 24*sqrt(d + e*x**2)*sqrt
(a + b*x**2 + c*x**4)*b*c**2*e**2*x**5 + 64*int((sqrt(d + e*x**2)*sqrt(a +
 b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 +
 c*e*x**6),x)*a**2*c**2*e**3 - 76*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 +
c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6)
,x)*a*b**2*c*e**3 + 36*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**
4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*b*c**
2*d*e**2 - 24*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d +
 a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*c**3*d**2*e +
15*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 +
 b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b**4*e**3 - 7*int((sqrt(d +
 e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*
x**4 + c*d*x**4 + c*e*x**6),x)*b**3*c*d*e**2 - 7*int((sqrt(d + e*x**2)*sqr
t(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*...
```

3.106 $\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 964

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{(6Ace(cd+be)-B(3c^2d^2+3b^2e^2-2ce(bd+4ae)))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{48c^2e^2x}$$

$$+ \frac{(Bcd+bBe+6Ace)x\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{24ce} + \frac{1}{6}Bx^3\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}$$

$$- \frac{\sqrt{b^2-4ac}(6Ace(cd+be)-B(3c^2d^2+3b^2e^2-2ce(bd+4ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}x\sqrt{d+ex^2}E\left(\arcsin\left(\right.\right.}{48\sqrt{2}c^2e^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{\sqrt{b^2-4ac}(6Ace(5cd-be)+B(c^2d^2+3b^2e^2-4ce(bd+2ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3}{24\sqrt{2}c^2e\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$+ \frac{\sqrt{b^2-4ac}(B(cd-be)\left(c^2d^2-b^2e^2+4ace^2\right)-2Ace(c^2d^2+b^2e^2-2ce(bd+2ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{}}{4\sqrt{2}c^2\left(b+\sqrt{b^2-4ac}\right)e^2\sqrt{d+ex^2}}$$

output
```
1/48*(6*A*c*e*(b*e+c*d)-B*(3*c^2*d^2+3*b^2*e^2-2*c*e*(4*a*e+b*d)))*(e*x^2+
d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e^2/x+1/24*(6*A*c*e+B*b*e+B*c*d)*x*(e*x
^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e+1/6*B*x^3*(e*x^2+d)^(1/2)*(c*x^4+b*x
^2+a)^(1/2)-1/96*(-4*a*c+b^2)^(1/2)*(6*A*c*e*(b*e+c*d)-B*(3*c^2*d^2+3*b^2*
e^2-2*c*e*(4*a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d
)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(
1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2
)/c^2/e^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x
^2+a)^(1/2)-1/48*(-4*a*c+b^2)^(1/2)*(6*A*c*e*(-b*e+5*c*d)+B*(c^2*d^2+3*b^2
*e^2-4*c*e*(2*a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/
x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x
^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(
-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^2/e/(e*x^2+d)^(1/2)/(c*x^4+b*
x^2+a)^(1/2)+1/8*(-4*a*c+b^2)^(1/2)*(B*(-b*e+c*d)*(4*a*c*e^2-b^2*e^2+c^2*d
^2)-2*A*c*e*(c^2*d^2+b^2*e^2-2*c*e*(2*a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a
*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*E
llipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b
^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*
c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^2/(b+(-4*a*c+b^2)^(1/2))/e^2/(e*x^
2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx=\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
Integrate[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 2260$$

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}dx$$

input
```
Int[(A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int (B\,x^2+A)\sqrt{e\,x^2+d}\sqrt{c\,x^4+b\,x^2+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Sympy [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx = \int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int (A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Giac [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}\left(Bx^2+A\right)\sqrt{ex^2+d}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx=\int \left(B\,x^2+A\right)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \left(A+Bx^2\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(6*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*c*e*x + sqrt(d + e*x**2)*s
qrt(a + b*x**2 + c*x**4)*b**2*e*x + sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x
**4)*b*c*d*x + 4*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c*e*x**3 + 1
4*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 +
b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*b*c*e**2 + 6*int((sqrt(d +
 e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*
x**4 + c*d*x**4 + c*e*x**6),x)*a*c**2*d*e - 3*int((sqrt(d + e*x**2)*sqrt(a
 + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4
 + c*e*x**6),x)*b**3*e**2 + 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x*
*4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*
b**2*c*d*e - 3*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d
+ a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b*c**2*d**2 + 1
2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 +
b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a**2*c*e**2 - 2*int((sqrt(d
+ e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e
*x**4 + c*d*x**4 + c*e*x**6),x)*a*b**2*e**2 + 22*int((sqrt(d + e*x**2)*sqr
t(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x
**4 + c*e*x**6),x)*a*b*c*d*e - 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c
*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),
x)*b**3*d*e - 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(...
```

3.107 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 805

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$

$$= \frac{(Bcd+bBe+4Ace)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{8cex}+\frac{1}{4}Bx\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}$$

$$- \frac{\sqrt{b^2-4ac}(Bcd+bBe+12Ace)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right)}{8\sqrt{2}ce\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{\sqrt{b^2-4ac}(8Abcd+5aBcd-abBe-4aAce)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticF}\left(\text{arcsin}\right.}{4\sqrt{2}ac\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$+ \frac{\sqrt{b^2-4ac}(4Ace(cd+be)-B(c^2d^2+b^2e^2-2ce(bd+2ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{E}}{2\sqrt{2}c\left(b+\sqrt{b^2-4ac}\right)e\sqrt{d+ex^2}\sqrt{a+bx^2+}}$$

output
```
1/8*(4*A*c*e+B*b*e+B*c*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e/x+1/4*
B*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)-1/16*(-4*a*c+b^2)^(1/2)*(12*A*c*
e+B*b*e+B*c*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*E
llipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4
*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c/e/(-a
*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1
/8*(-4*a*c+b^2)^(1/2)*(-4*A*a*c*e+8*A*b*c*d-B*a*b*e+5*B*a*c*d)*(-a*(c+a/x^
4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e
))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2
^(1/2)/a/c/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/4*(-4*a*c+b^2)^(1/2)*(4
*A*c*e*(b*e+c*d)-B*(c^2*d^2+b^2*e^2-2*c*e*(2*a*e+b*d)))*(-a*(c+a/x^4+b/x^2
)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2
)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-
4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d
+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c/(b+(-4*a*c+b^2)^(1/2))/e/(e
*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^2,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$

$$\Big\downarrow 2250$$

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm
="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx=\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(c*x**4+b*x**2+a)**(1/2)/x**2,x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/x**2, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx=\int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^2, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx=\int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm
="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^2,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x)
```

output

```
(6*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*e + 4*sqrt(d + e*x**2)*s
qrt(a + b*x**2 + c*x**4)*a*c*d + 2*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x*
*4)*b**2*d + sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*e*x**2 + sqrt
(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c*d*x**2 - 8*int((sqrt(d + e*x**2
)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*b*d*e + a*b*e**2*x**2 + a*c*d**2 + a*
c*d*e*x**2 + b**2*d*e*x**2 + b**2*e**2*x**4 + b*c*d**2*x**2 + 2*b*c*d*e*x*
*4 + b*c*e**2*x**6 + c**2*d**2*x**4 + c**2*d*e*x**6),x)*a*b**2*c*e**3*x -
12*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*b*d*e + a*b*e*
*2*x**2 + a*c*d**2 + a*c*d*e*x**2 + b**2*d*e*x**2 + b**2*e**2*x**4 + b*c*d
**2*x**2 + 2*b*c*d*e*x**4 + b*c*e**2*x**6 + c**2*d**2*x**4 + c**2*d*e*x**6
),x)*a*b*c**2*d*e**2*x - 4*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)
*x**4)/(a*b*d*e + a*b*e**2*x**2 + a*c*d**2 + a*c*d*e*x**2 + b**2*d*e*x**2
+ b**2*e**2*x**4 + b*c*d**2*x**2 + 2*b*c*d*e*x**4 + b*c*e**2*x**6 + c**2*d
**2*x**4 + c**2*d*e*x**6),x)*a*c**3*d**2*e*x + int((sqrt(d + e*x**2)*sqrt(
a + b*x**2 + c*x**4)*x**4)/(a*b*d*e + a*b*e**2*x**2 + a*c*d**2 + a*c*d*e*x
**2 + b**2*d*e*x**2 + b**2*e**2*x**4 + b*c*d**2*x**2 + 2*b*c*d*e*x**4 + b*
c*e**2*x**6 + c**2*d**2*x**4 + c**2*d*e*x**6),x)*b**4*e**3*x - int((sqrt(d
 + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*b*d*e + a*b*e**2*x**2 + a*c*
d**2 + a*c*d*e*x**2 + b**2*d*e*x**2 + b**2*e**2*x**4 + b*c*d**2*x**2 + 2*b
*c*d*e*x**4 + b*c*e**2*x**6 + c**2*d**2*x**4 + c**2*d*e*x**6),x)*b**3*c...
```

3.108 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 760

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

$$= -\frac{A\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{3x^3} + \frac{B\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{2x}$$

$$- \frac{\sqrt{b^2-4ac}(9aBd+2A(bd+ae))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right.}{6\sqrt{2}ad\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{\sqrt{b^2-4ac}(4Acd^2-3aBde-2aAe^2+2bd(3Bd+Ae))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{Elliptic}}{3\sqrt{2}ad\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}(Bcd+bBe+2Ace)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\mathrm{arcsin}\right.}{\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/3*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^3+1/2*B*(e*x^2+d)^(1/2)*(c*
x^4+b*x^2+a)^(1/2)/x-1/12*(-4*a*c+b^2)^(1/2)*(9*B*a*d+2*A*(a*e+b*d))*(-a*(
c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2
*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b
*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/d/(-a*(e+d/x^2)/((b+(-4*a
*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/6*(-4*a*c+b^2)^(1/2
)*(4*A*c*d^2-3*a*B*d*e-2*A*a*e^2+2*b*d*(A*e+3*B*d))*(-a*(c+a/x^4+b/x^2)/(-
4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^
3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*(
(-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/d/
(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+2^(1/2)*(-4*a*c+b^2)^(1/2)*(2*A*c*e+
B*b*e+B*c*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4
*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(
1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/(b+(-4
*a*c+b^2)^(1/2))/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^4,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

$$\Big\downarrow \text{ 2250}$$

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^4}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm
="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(c*x**4+b*x**2+a)**(1/2)/x**4,x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/x**4, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^4, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm
="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^4}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^4,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x)
```

output

```
( - sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a**2*b*e**2 - 2*sqrt(d + e*
x**2)*sqrt(a + b*x**2 + c*x**4)*a**2*c*d*e + 2*sqrt(d + e*x**2)*sqrt(a + b
*x**2 + c*x**4)*a**2*c*e**2*x**2 - 3*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4)*a*b**2*d*e + 2*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b**2*e**
2*x**2 - 3*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*c*d**2 + 4*sqrt(
d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*c*d*e*x**2 + 2*sqrt(d + e*x**2)*
sqrt(a + b*x**2 + c*x**4)*b**3*d*e*x**2 + 2*sqrt(d + e*x**2)*sqrt(a + b*x*
*2 + c*x**4)*b**2*c*d**2*x**2 - 4*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 +
c*x**4)*x**4)/(a**2*b*d*e**2 + a**2*b*e**3*x**2 + a**2*c*d**2*e + a**2*c*d
*e**2*x**2 + a*b**2*d**2*e + 2*a*b**2*d*e**2*x**2 + a*b**2*e**3*x**4 + a*b
*c*d**3 + 2*a*b*c*d**2*e*x**2 + 2*a*b*c*d*e**2*x**4 + a*b*c*e**3*x**6 + a*
c**2*d**2*e*x**4 + a*c**2*d*e**2*x**6 + b**3*d**2*e*x**2 + b**3*d*e**2*x**
4 + b**2*c*d**3*x**2 + 2*b**2*c*d**2*e*x**4 + b**2*c*d*e**2*x**6 + b*c**2*
d**3*x**4 + b*c**2*d**2*e*x**6),x)*a**3*b*c**2*e**5*x**3 - 4*int((sqrt(d +
 e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2*b*d*e**2 + a**2*b*e**3*x**2
 + a**2*c*d**2*e + a**2*c*d*e**2*x**2 + a*b**2*d**2*e + 2*a*b**2*d*e**2*x*
*2 + a*b**2*e**3*x**4 + a*b*c*d**3 + 2*a*b*c*d**2*e*x**2 + 2*a*b*c*d*e**2*
x**4 + a*b*c*e**3*x**6 + a*c**2*d**2*e*x**4 + a*c**2*d*e**2*x**6 + b**3*d*
*2*e*x**2 + b**3*d*e**2*x**4 + b**2*c*d**3*x**2 + 2*b**2*c*d**2*e*x**4 + b
**2*c*d*e**2*x**6 + b*c**2*d**3*x**4 + b*c**2*d**2*e*x**6),x)*a**3*c**3...
```

3.109 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 816

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

$$= -\frac{A\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{(Abd+5aBd+aAe)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{15adx^3}$$

$$- \frac{\sqrt{b^2-4ac}(5aBd(bd+ae)-2A(b^2d^2-abde-a(3cd^2-ae^2)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}x\sqrt{d+ex^2}E\left(\arcsin\left(\ldots\right.\right.}{15\sqrt{2}a^2d^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}(A(bd-2ae)\left(cd^2-e(bd-ae)\right)-5aBd(2cd^2+e(bd-ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a(\ldots}{(b+\sqrt{b^2\ldots}}}\ldots}{15a^2d^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$+ \frac{2\sqrt{2}Bc\sqrt{b^2-4ac}e\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3\,\mathrm{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\ldots\right.}{\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/5*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^5-1/15*(A*a*e+A*b*d+5*B*a*d
)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/d/x^3-1/30*(-4*a*c+b^2)^(1/2)*(5
*a*B*d*(a*e+b*d)-2*A*(b^2*d^2-a*b*d*e-a*(-a*e^2+3*c*d^2)))*(-a*(c+a/x^4+b/
x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c
+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a^2/d^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2
)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/15*2^(1/2)*(-4*a*c+b^2)^(
1/2)*(A*(-2*a*e+b*d)*(c*d^2-e*(-a*e+b*d))-5*a*B*d*(2*c*d^2+e*(-a*e+b*d)))*
(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/
2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e
))^(1/2))/a^2/d^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+2*2^(1/2)*B*c*(-4*
a*c+b^2)^(1/2)*e*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b
+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4
*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
,2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/(b
+(-4*a*c+b^2)^(1/2))/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^6,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^6, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

$$\downarrow 2250$$

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^6,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^6}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm
="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^6, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(c*x**4+b*x**2+a)**(1/2)/x**6,x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/x**6, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^6, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm
="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^6}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^6,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^6}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

3.110 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 726

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx$$

$$= -\frac{A\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{7x^7} - \frac{(Abd+7aBd+aAe)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{35adx^5}$$

$$- \frac{(7aBd(bd+ae)-A(4b^2d^2-2abde-2a(5cd^2-2ae^2)))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{105a^2d^2x^3}$$

$$+ \frac{\sqrt{b^2-4ac}(14aBd(b^2d^2-abde-a(3cd^2-ae^2))-A(8b^3d^3-5ab^2d^2e+8a^2e(2cd^2+ae^2)-abd(29cd}{105\sqrt{2}a^3d^3\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{}}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)(7aBd(bd-2ae)-A(4b^2d^2+abde-2a(5cd^2+4ae^2)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x}\right.}{b^2-4ac}}}{105a^3d^3\sqrt{d+ex^2}\sqrt{a+bx^2+}}$$

output
```
-1/7*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^7-1/35*(A*a*e+A*b*d+7*B*a*d
)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/d/x^5-1/105*(7*a*B*d*(a*e+b*d)-A
*(4*b^2*d^2-2*a*b*d*e-2*a*(-2*a*e^2+5*c*d^2)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^
2+a)^(1/2)/a^2/d^2/x^3+1/210*(-4*a*c+b^2)^(1/2)*(14*a*B*d*(b^2*d^2-a*b*d*e
-a*(-a*e^2+3*c*d^2))-A*(8*b^3*d^3-5*a*b^2*d^2*e+8*a^2*e*(a*e^2+2*c*d^2)-a*
b*d*(5*a*e^2+29*c*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+
d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^
(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/
2)/a^3/d^3/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*
x^2+a)^(1/2)+1/105*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a*e^2-b*d*e+c*d^2)*(7*a*B*d
*(-2*a*e+b*d)-A*(4*b^2*d^2+a*b*d*e-2*a*(4*a*e^2+5*c*d^2)))*(-a*(c+a/x^4+b/
x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(
1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^
(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/a^3/d
^3/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^8,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^8, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx$$

$$\Big\downarrow \text{2250}$$

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^8,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^8}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x, algorithm
="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^8, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(c*x**4+b*x**2+a)**(1/2)/x**8,x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/x**8, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^8, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x, algorithm
="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^8}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^8,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^8, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^8}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x)
```

3.111 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 958

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx$$

$$= -\frac{A\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{9x^9} - \frac{(Abd+9aBd+aAe)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{63adx^7}$$

$$- \frac{(9aBd(bd+ae)-A(6b^2d^2-2abde-2a(7cd^2-3ae^2)))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{315a^2d^2x^5}$$

$$+ \frac{(6aBd(2b^2d^2-abde-a(5cd^2-2ae^2))-A(8b^3d^3-3ab^2d^2e+8a^2e(cd^2+ae^2)-3abd(9cd^2+ae^2)))}{315a^3d^3x^3}$$

$$- \frac{\sqrt{b^2-4ac}(3aBd(8b^3d^3-5ab^2d^2e+8a^2e(2cd^2+ae^2)-abd(29cd^2+5ae^2))-2A(8b^4d^4-4ab^3d^3e+}{}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)(3aBd(4b^2d^2+abde-2a(5cd^2+4ae^2))-A(8b^3d^3-27abcd^3+3ab^2}{315a^4d^4\sqrt{}}$$

output
```
-1/9*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^9-1/63*(A*a*e+A*b*d+9*B*a*d
)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/d/x^7-1/315*(9*a*B*d*(a*e+b*d)-A
*(6*b^2*d^2-2*a*b*d*e-2*a*(-3*a*e^2+7*c*d^2)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^
2+a)^(1/2)/a^2/d^2/x^5+1/315*(6*a*B*d*(2*b^2*d^2-a*b*d*e-a*(-2*a*e^2+5*c*d
^2))-A*(8*b^3*d^3-3*a*b^2*d^2*e+8*a^2*e*(a*e^2+c*d^2)-3*a*b*d*(a*e^2+9*c*d
^2)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^3/d^3/x^3-1/630*(-4*a*c+b^2)
^(1/2)*(3*a*B*d*(8*b^3*d^3-5*a*b^2*d^2*e+8*a^2*e*(a*e^2+2*c*d^2)-a*b*d*(5*
a*e^2+29*c*d^2))-2*A*(8*b^4*d^4-4*a*b^3*d^3*e+a^2*b*d*e*(-4*a*e^2+15*c*d^2
)-3*a*b^2*d^2*(a*e^2+12*c*d^2)+a^2*(8*a^2*e^4+9*a*c*d^2*e^2+21*c^2*d^4)))*
(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1
+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)
*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a^4/d^4/(-a*(e+d/x^2)/
((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/315*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(a*e^2-b*d*e+c*d^2)*(3*a*B*d*(4*b^2*d^2+a*b*d*e-2*a*(
4*a*e^2+5*c*d^2))-A*(8*b^3*d^3-27*a*b*c*d^3+3*a*b^2*d^2*e-2*a^2*e*(8*a*e^2
+3*c*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*
a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(
1/2)*d-2*a*e))^(1/2))/a^4/d^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx=\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^10,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^10, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx$$
$$\Big\downarrow \ 2250$$
$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^10,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^{10}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x, algorith
m="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^10, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(c*x**4+b*x**2+a)**(1/2)/x**10,x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/x**10, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x, algorith
m="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^10, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x, algorith
m="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^{10}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^10,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^10, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^{10}}\,dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x)
```

3.112 $\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}} \, dx$

Optimal result

Integrand size = 38, antiderivative size = 1295

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}} \, dx = \text{Too large to display}$$

output
```
-1/11*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^11-1/99*(A*a*e+A*b*d+11*B*
a*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/d/x^9-1/693*(11*a*B*d*(a*e+b*
d)-A*(8*b^2*d^2-2*a*b*d*e-2*a*(-4*a*e^2+9*c*d^2)))*(e*x^2+d)^(1/2)*(c*x^4+
b*x^2+a)^(1/2)/a^2/d^2/x^7+1/3465*(22*a*B*d*(3*b^2*d^2-a*b*d*e-a*(-3*a*e^2
+7*c*d^2))-A*(48*b^3*d^3-13*a*b^2*d^2*e+16*a^2*e*(3*a*e^2+2*c*d^2)-a*b*d*(
13*a*e^2+157*c*d^2)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^3/d^3/x^5-1/
3465*(11*a*B*d*(8*b^3*d^3-3*a*b^2*d^2*e+8*a^2*e*(a*e^2+c*d^2)-3*a*b*d*(a*e
^2+9*c*d^2))-2*A*(32*b^4*d^4-10*a*b^3*d^3*e+5*a^2*b*d*e*(-2*a*e^2+7*c*d^2)
-3*a*b^2*d^2*(3*a*e^2+46*c*d^2)+a^2*(32*a^2*e^4+23*a*c*d^2*e^2+75*c^2*d^4)
))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^4/d^4/x^3-1/6930*(-4*a*c+b^2)^(
1/2)*(A*(128*b^5*d^5-56*a*b^4*d^4*e+a^2*b^2*d^2*e*(-37*a*e^2+258*c*d^2)-a*
b^3*d^3*(37*a*e^2+696*c*d^2)+a^2*b*d*(-56*a^2*e^4+135*a*c*d^2*e^2+771*c^2*
d^4)-4*a^3*e*(-32*a^2*e^4-27*a*c*d^2*e^2+39*c^2*d^4))-22*a*B*d*(8*b^4*d^4-
4*a*b^3*d^3*e+a^2*b*d*e*(-4*a*e^2+15*c*d^2)-3*a*b^2*d^2*(a*e^2+12*c*d^2)+a
^2*(8*a^2*e^4+9*a*c*d^2*e^2+21*c^2*d^4)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2)
)^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))
^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a
*e))^(1/2))*2^(1/2)/a^5/d^5/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e)
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/3465*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a*e^2-b*d
*e+c*d^2)*(11*a*B*d*(8*b^3*d^3-27*a*b*c*d^3+3*a*b^2*d^2*e-2*a^2*e*(8*a*...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx=\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx$$

input
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^12,x]
```

output
```
Integrate[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^12, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx$$
$$\Big\downarrow \text{ 2250}$$
$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}}dx$$

input
```
Int[((A + B*x^2)*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4])/x^12,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^{12}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x, algorith
m="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^12, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx = \int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(1/2)*(c*x**4+b*x**2+a)**(1/2)/x**12,x)
```

output
```
Integral((A + B*x**2)*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/x**12, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x, algorith
m="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^12, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)\sqrt{ex^2+d}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x, algorith
m="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*sqrt(e*x^2 + d)/x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^{12}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^12,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2))/x^12, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{x^{12}}\,dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x)
```

3.113 $\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1576

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

output
```
-1/3840*(10*A*c*e*(9*c^3*d^3-15*b^3*e^3-3*c^2*d*e*(28*a*e+3*b*d)+b*c*e^2*(
52*a*e+31*b*d))-B*(45*c^4*d^4-105*b^4*e^4-6*c^3*d^2*e*(-18*a*e+5*b*d)+10*b
^2*c*e^3*(46*a*e+19*b*d)-4*c^2*e^2*(64*a^2*e^2+166*a*b*d*e+9*b^2*d^2)))*(e
*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^4/e^3/x+1/1920*(10*A*c*e*(3*c^2*d^2-
5*b^2*e^2+2*c*e*(6*a*e+5*b*d))-B*(15*c^3*d^3-35*b^3*e^3+b*c*e^2*(116*a*e+6
1*b*d)-c^2*d*e*(148*a*e+9*b*d)))*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c
^3/e^2+1/480*(10*A*c*e*(b*e+9*c*d)+B*(3*c^2*d^2-7*b^2*e^2+4*c*e*(4*a*e+3*b
*d)))*x^3*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e+1/80*(10*A*c*e+B*b*e
+11*B*c*d)*x^5*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c+1/10*B*e*x^7*(e*x^2
+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)+1/7680*(-4*a*c+b^2)^(1/2)*(10*A*c*e*(9*c^3
*d^3-15*b^3*e^3-3*c^2*d*e*(28*a*e+3*b*d)+b*c*e^2*(52*a*e+31*b*d))-B*(45*c^
4*d^4-105*b^4*e^4-6*c^3*d^2*e*(-18*a*e+5*b*d)+10*b^2*c*e^3*(46*a*e+19*b*d)
-4*c^2*e^2*(64*a^2*e^2+166*a*b*d*e+9*b^2*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*
c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)
*d-2*a*e))^(1/2))*2^(1/2)/c^4/e^3/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-
2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/3840*(-4*a*c+b^2)^(1/2)*(10*A*c*e*(3
*c^3*d^3-15*b^3*e^3+b*c*e^2*(52*a*e+41*b*d)-c^2*d*e*(108*a*e+29*b*d))-B*(1
5*c^4*d^4-105*b^4*e^4-4*c^3*d^2*e*(-101*a*e+3*b*d)+20*b^2*c*e^3*(23*a*e+13
*b*d)-2*c^2*e^2*(128*a^2*e^2+448*a*b*d*e+79*b^2*d^2)))*(-a*(c+a/x^4+b/x...
```

Mathematica [F]

$$\int x^2(A+Bx^2)\,(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int x^2(A+Bx^2)\,(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
Integrate[x^2*(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
Integrate[x^2*(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (A+Bx^2) (d+ex^2)^{3/2} \sqrt{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{ 2250}$$

$$\int x^2 (A+Bx^2) (d+ex^2)^{3/2} \sqrt{a+bx^2+cx^4} dx$$

input
```
Int[x^2*(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int x^2 (B\,x^2+A) \left(e\,x^2+d\right)^{\frac{3}{2}} \sqrt{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm
="fricas")
```

output
```
integral((B*e*x^6 + (B*d + A*e)*x^4 + A*d*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqr
t(e*x^2 + d), x)
```

Sympy [F]

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int x^2(A+Bx^2)\left(d+ex^2\right)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**2*(B*x**2+A)*(e*x**2+d)**(3/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2)*(d + e*x**2)**(3/2)*sqrt(a + b*x**2 + c*x**4),
x)
```

Maxima [F]

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int\sqrt{cx^4+bx^2+a}(Bx^2+A)\left(ex^2+d\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int x^2\left(B\,x^2+A\right)\left(e\,x^2+d\right)^{3/2}\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(x^2*(A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^2*(A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^2 (A + Bx^2) (d + ex^2)^{3/2} \sqrt{a + bx^2 + cx^4} \, dx = \int x^2 (B x^2 + A) (e x^2 + d)^{\frac{3}{2}} \sqrt{c x^4 + b x^2 + a} dx$$

input
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(x^2*(B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

3.114 $\int (A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1241

$$\int (A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

output
```
1/384*(8*A*c*e*(3*c^2*d^2-3*b^2*e^2+8*c*e*(a*e+b*d))-B*(9*c^3*d^3-15*b^3*e
^3-3*c^2*d*e*(28*a*e+3*b*d)+b*c*e^2*(52*a*e+31*b*d)))*(e*x^2+d)^(1/2)*(c*x
^4+b*x^2+a)^(1/2)/c^3/e^2/x+1/192*(8*A*c*e*(b*e+7*c*d)+B*(3*c^2*d^2-5*b^2*
e^2+2*c*e*(6*a*e+5*b*d)))*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e+1/
48*(8*A*c*e+B*b*e+9*B*c*d)*x^3*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c+1/8
*B*e*x^5*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)-1/768*(-4*a*c+b^2)^(1/2)*(8
*A*c*e*(3*c^2*d^2-3*b^2*e^2+8*c*e*(a*e+b*d))-B*(9*c^3*d^3-15*b^3*e^3-3*c^2
*d*e*(28*a*e+3*b*d)+b*c*e^2*(52*a*e+31*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+
b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d
-2*a*e))^(1/2))*2^(1/2)/c^3/e^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*
a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/384*(-4*a*c+b^2)^(1/2)*(8*A*c*e*(31*c^
2*d^2+3*b^2*e^2-2*c*e*(4*a*e+5*b*d))+B*(3*c^3*d^3-15*b^3*e^3+b*c*e^2*(52*a
*e+41*b*d)-c^2*d*e*(108*a*e+29*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1
/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/
2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(
1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^3/e/(e*x^2+d)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)-1/64*(-4*a*c+b^2)^(1/2)*(8*A*c*e*(c^3*d^3-b^3*e^
3-3*c^2*d*e*(4*a*e+b*d)+b*c*e^2*(4*a*e+3*b*d))-B*(3*c^4*d^4-5*b^4*e^4-4*c^
3*d^2*e*(-6*a*e+b*d)+12*b^2*c*e^3*(2*a*e+b*d)-2*c^2*e^2*(8*a^2*e^2+24*a...
```

Mathematica [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
Integrate[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
Integrate[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2260}$$

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}dx$$

input
```
Int[(A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \left(B\,x^{2}+A\right)\left(e\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{c\,x^{4}+b\,x^{2}+a}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\text{Timed out}$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \left(A+Bx^2\right)\left(d+ex^2\right)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \left(A+Bx^2\right)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}\left(Bx^2+A\right)\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2), x)
```

Giac [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}\left(Bx^2+A\right)\left(ex^2+d\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \left(B\,x^2+A\right)\,\left(e\,x^2+d\right)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int (A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
(20*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*c*e**2*x + 56*sqrt(d +
e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*c**2*d*e*x + 32*sqrt(d + e*x**2)*sqrt(
a + b*x**2 + c*x**4)*a*c**2*e**2*x**3 - 5*sqrt(d + e*x**2)*sqrt(a + b*x**2
 + c*x**4)*b**3*e**2*x + 10*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**
2*c*d*e*x + 4*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*c*e**2*x**3
+ 3*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c**2*d**2*x + 36*sqrt(d +
 e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c**2*d*e*x**3 + 24*sqrt(d + e*x**2)*s
qrt(a + b*x**2 + c*x**4)*b*c**2*e**2*x**5 + 64*int((sqrt(d + e*x**2)*sqrt(
a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**
4 + c*e*x**6),x)*a**2*c**2*e**3 - 76*int((sqrt(d + e*x**2)*sqrt(a + b*x**2
 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x*
*6),x)*a*b**2*c*e**3 + 148*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)
*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*b
*c**2*d*e**2 + 24*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a
*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*c**3*d**2*
e + 15*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x*
*2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b**4*e**3 - 31*int((sqr
t(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 +
 b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b**3*c*d*e**2 + 9*int((sqrt(d + e*x**2
)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4...
```

3.115 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 993

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \frac{(6Ace(5cd+be)+B(3c^2d^2-3b^2e^2+8ce(bd+ae)))\sqrt{d-}}{48c^2ex}$$

$$+\frac{(7Bcd+bBe+6Ace)x\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{24c}+\frac{1}{6}Bex^3\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}$$

$$-\frac{\sqrt{b^2-4ac}(6Ace(13cd+be)+B(3c^2d^2-3b^2e^2+8ce(bd+ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\right.\right.}{48\sqrt{2}c^2e\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{b^2-4ac}(6Ac(8bcd^2+acde-abe^2)+aB(31c^2d^2+3b^2e^2-2ce(5bd+4ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a}{(b+\sqrt{b}}}}{24\sqrt{2}ac^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt{b^2-4ac}(2Ace(3c^2d^2-b^2e^2+2ce(3bd+2ae))-B(c^3d^3-b^3e^3-3c^2de(bd+4ae)+bce^2(3bd+4ae)))}{4\sqrt{2}c^2\left(b+\sqrt{b^2-4ac}\right)e\sqrt{}}$$

output
```
1/48*(6*A*c*e*(b*e+5*c*d)+B*(3*c^2*d^2-3*b^2*e^2+8*c*e*(a*e+b*d)))*(e*x^2+
d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e/x+1/24*(6*A*c*e+B*b*e+7*B*c*d)*x*(e*x
^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c+1/6*B*e*x^3*(e*x^2+d)^(1/2)*(c*x^4+b*x
^2+a)^(1/2)-1/96*(-4*a*c+b^2)^(1/2)*(6*A*c*e*(b*e+13*c*d)+B*(3*c^2*d^2-3*b
^2*e^2+8*c*e*(a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+
d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^
(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/
2)/c^2/e/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^
2+a)^(1/2)-1/48*(-4*a*c+b^2)^(1/2)*(6*A*c*(-a*b*e^2+a*c*d*e+8*b*c*d^2)+a*B
*(31*c^2*d^2+3*b^2*e^2-2*c*e*(4*a*e+5*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b
^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*Ellip
ticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c
+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/c^2/(e*x^
2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/8*(-4*a*c+b^2)^(1/2)*(2*A*c*e*(3*c^2*d^
2-b^2*e^2+2*c*e*(2*a*e+3*b*d))-B*(c^3*d^3-b^3*e^3-3*c^2*d*e*(4*a*e+b*d)+b*
c*e^2*(4*a*e+3*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2
)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2
)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(
1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2
))*2^(1/2)/c^2/(b+(-4*a*c+b^2)^(1/2))/e/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^2,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$
$$\Big\downarrow \; 2250$$
$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a+bx^2+cx^4}}{x^2}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm
="fricas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*
x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(c*x**4+b*x**2+a)**(1/2)/x**2,x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)*sqrt(a + b*x**2 + c*x**4)/x**2,
x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm
="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^2,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^2,x)
```

output
```
(12*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a**2*c*e**2 - 2*sqrt(d + e*
x**2)*sqrt(a + b*x**2 + c*x**4)*a*b**2*e**2 + 58*sqrt(d + e*x**2)*sqrt(a +
 b*x**2 + c*x**4)*a*b*c*d*e + 6*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)
*a*b*c*e**2*x**2 + 24*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*c**2*d*
*2 + 6*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*c**2*d*e*x**2 - 2*sqrt
(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**3*d*e + sqrt(d + e*x**2)*sqrt(a
+ b*x**2 + c*x**4)*b**3*e**2*x**2 + 10*sqrt(d + e*x**2)*sqrt(a + b*x**2 +
c*x**4)*b**2*c*d**2 + 8*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*c*
d*e*x**2 + 4*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*c*e**2*x**4 +
 7*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c**2*d**2*x**2 + 4*sqrt(d
+ e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*c**2*d*e*x**4 - 24*int((sqrt(d + e*x
**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*b*d*e + a*b*e**2*x**2 + a*c*d**2 +
 a*c*d*e*x**2 + b**2*d*e*x**2 + b**2*e**2*x**4 + b*c*d**2*x**2 + 2*b*c*d*e
*x**4 + b*c*e**2*x**6 + c**2*d**2*x**4 + c**2*d*e*x**6),x)*a**2*b*c**2*e**
4*x - 24*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*b*d*e +
a*b*e**2*x**2 + a*c*d**2 + a*c*d*e*x**2 + b**2*d*e*x**2 + b**2*e**2*x**4 +
 b*c*d**2*x**2 + 2*b*c*d*e*x**4 + b*c*e**2*x**6 + c**2*d**2*x**4 + c**2*d*
e*x**6),x)*a**2*c**3*d*e**3*x + 18*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 +
 c*x**4)*x**4)/(a*b*d*e + a*b*e**2*x**2 + a*c*d**2 + a*c*d*e*x**2 + b**2*d
*e*x**2 + b**2*e**2*x**4 + b*c*d**2*x**2 + 2*b*c*d*e*x**4 + b*c*e**2*x*...
```

3.116 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 874

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx=-\frac{Ad\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{3x^3}$$

$$+\frac{(5Bcd+bBe+4Ace)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{8cx}+\frac{1}{4}Bex\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}$$

$$-\frac{\sqrt{b^2-4ac}(8Abcd+39aBcd+3abBe+44aAce)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}x\sqrt{d+ex^2}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{\ldots}{bd\ldots}\right.}{24\sqrt{2}ac\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{b^2-4ac}(24bBcd^2+16Ac^2d^2+32Abcde+3aBcde-3abBe^2-20aAce^2)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\ldots\right)}{\left(b+\sqrt{b^2-\ldots}\right.}}}{12\sqrt{2}ac\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt{b^2-4ac}(4Ace(3cd+be)+B(3c^2d^2-b^2e^2+2ce(3bd+2ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3\,\mathrm{E}\ldots}{2\sqrt{2}c\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex^2}\sqrt{a+bx^2+c\ldots}}$$

output
```
-1/3*A*d*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^3+1/8*(4*A*c*e+B*b*e+5*B*
c*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/x+1/4*B*e*x*(e*x^2+d)^(1/2)*(
c*x^4+b*x^2+a)^(1/2)-1/48*(-4*a*c+b^2)^(1/2)*(44*A*a*c*e+8*A*b*c*d+3*B*a*b
*e+39*B*a*c*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*E
llipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4
*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/c/(-a
*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1
/24*(-4*a*c+b^2)^(1/2)*(-20*A*a*c*e^2+32*A*b*c*d*e+16*A*c^2*d^2-3*B*a*b*e^
2+3*B*a*c*d*e+24*B*b*c*d^2)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e
+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*
a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*
d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/c/(e*x^2+d)^(1/2)/(c*x^4+b
*x^2+a)^(1/2)+1/4*(-4*a*c+b^2)^(1/2)*(4*A*c*e*(b*e+3*c*d)+B*(3*c^2*d^2-b^2
*e^2+2*c*e*(2*a*e+3*b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+
d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*
a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b
^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))
^(1/2))*2^(1/2)/c/(b+(-4*a*c+b^2)^(1/2))/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^4,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^4, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

$$\Big\downarrow \; 2250$$

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^4}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}}{x^4}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm
="fricas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*
x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(c*x**4+b*x**2+a)**(1/2)/x**4,x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)*sqrt(a + b*x**2 + c*x**4)/x**4,
x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a+bx^2+cx^4}}{x^4}\,dx=\int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm
="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a+bx^2+cx^4}}{x^4}\,dx=\int \frac{(B\,x^2+A)\,\left(e\,x^2+d\right)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}}{x^4}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^4,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\,\sqrt{a+bx^2+cx^4}}{x^4}\,dx=\text{too large to display}$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^4,x)
```

output

```
( - 7*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a**2*b*d*e**2 + 6*sqrt(d
+ e*x**2)*sqrt(a + b*x**2 + c*x**4)*a**2*b*e**3*x**2 - 8*sqrt(d + e*x**2)*
sqrt(a + b*x**2 + c*x**4)*a**2*c*d**2*e + 8*sqrt(d + e*x**2)*sqrt(a + b*x*
*2 + c*x**4)*a**2*c*d*e**2*x**2 - 7*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x
**4)*a*b**2*d**2*e + 12*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b**2*
d*e**2*x**2 + sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b**2*e**3*x**4
- 6*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*c*d**3 + 12*sqrt(d + e*
x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*c*d**2*e*x**2 + sqrt(d + e*x**2)*sqrt(
a + b*x**2 + c*x**4)*a*b*c*d*e**2*x**4 + 6*sqrt(d + e*x**2)*sqrt(a + b*x**
2 + c*x**4)*b**3*d**2*e*x**2 + sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*
b**3*d*e**2*x**4 + 4*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*c*d**
3*x**2 + sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*c*d**2*e*x**4 - 8
*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2*b*d*e**2 + a*
*2*b*e**3*x**2 + a**2*c*d**2*e + a**2*c*d*e**2*x**2 + a*b**2*d**2*e + 2*a*
b**2*d*e**2*x**2 + a*b**2*e**3*x**4 + a*b*c*d**3 + 2*a*b*c*d**2*e*x**2 + 2
*a*b*c*d*e**2*x**4 + a*b*c*e**3*x**6 + a*c**2*d**2*e*x**4 + a*c**2*d*e**2*
x**6 + b**3*d**2*e*x**2 + b**3*d*e**2*x**4 + b**2*c*d**3*x**2 + 2*b**2*c*d
**2*e*x**4 + b**2*c*d*e**2*x**6 + b*c**2*d**3*x**4 + b*c**2*d**2*e*x**6),x
)*a**3*b**2*c*e**6*x**3 - 20*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**
4)*x**4)/(a**2*b*d*e**2 + a**2*b*e**3*x**2 + a**2*c*d**2*e + a**2*c*d*e...
```

3.117 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 882

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = -\frac{Ad\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{5x^5}$$

$$-\frac{(Abd+5aBd+6aAe)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{15ax^3}+\frac{Be\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{2x}$$

$$-\frac{\sqrt{b^2-4ac}(5aBd(2bd+11ae)-2A(2b^2d^2-7abde-3a(2cd^2+ae^2)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}x\sqrt{d+ex^2}E\left(\text{arc}\right.}{30\sqrt{2}a^2d\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{\sqrt{b^2-4ac}(2A(bcd^3-b^2d^2e-12acd^2e-2abde^2+3a^2e^3)-5aBd(4cd^2+e(8bd-5ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}}{15\sqrt{2}a^2d\sqrt{d+ex^2}\sqrt{a+bx^2+c}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}e(3Bcd+bBe+2Ace)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3\,\text{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\right.}{\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/5*A*d*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^5-1/15*(6*A*a*e+A*b*d+5*B
*a*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/x^3+1/2*B*e*(e*x^2+d)^(1/2)*
(c*x^4+b*x^2+a)^(1/2)/x-1/60*(-4*a*c+b^2)^(1/2)*(5*a*B*d*(11*a*e+2*b*d)-2*
A*(2*b^2*d^2-7*a*b*d*e-3*a*(a*e^2+2*c*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b
^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-
2*a*e))^(1/2))*2^(1/2)/a^2/d/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e
))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/30*(-4*a*c+b^2)^(1/2)*(2*A*(3*a^2*e^3-2*a
*b*d*e^2-12*a*c*d^2*e-b^2*d^2*e+b*c*d^3)-5*a*B*d*(4*c*d^2+e*(-5*a*e+8*b*d)
))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^
(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2)
)^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*
a*e))^(1/2))*2^(1/2)/a^2/d/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+2^(1/2)*(
-4*a*c+b^2)^(1/2)*e*(2*A*c*e+B*b*e+3*B*c*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^
2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*Ellipt
icPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(
1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2
)^(1/2)*d-2*a*e))^(1/2))/(b+(-4*a*c+b^2)^(1/2))/(e*x^2+d)^(1/2)/(c*x^4+b*x
^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx=\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^6,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^6, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$
$$\Big\downarrow \text{2250}$$
$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^6,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\,\sqrt{c\,x^4+b\,x^2+a}}{x^6}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm
="fricas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*
x^2 + d)/x^6, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(c*x**4+b*x**2+a)**(1/2)/x**6,x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)*sqrt(a + b*x**2 + c*x**4)/x**6,
x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm
="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{3/2}\sqrt{c\,x^4+b\,x^2+a}}{x^6}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^6,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}}{x^6}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

3.118 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1013

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \text{Too large to display}$$

output
```
-1/7*A*d*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^7-1/35*(8*A*a*e+A*b*d+7*B
*a*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/x^5-1/105*(7*a*B*d*(6*a*e+b*
d)-A*(4*b^2*d^2-9*a*b*d*e-a*(3*a*e^2+10*c*d^2)))*(e*x^2+d)^(1/2)*(c*x^4+b*
x^2+a)^(1/2)/a^2/d/x^3-1/210*(-4*a*c+b^2)^(1/2)*(A*(-2*a*e+b*d)*(8*b^2*d^2
-3*a*b*d*e-a*(-3*a*e^2+29*c*d^2))-7*a*B*d*(2*b^2*d^2-7*a*b*d*e-3*a*(a*e^2+
2*c*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*Ellip
ticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c
+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a^3/d^2/(-a
*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1
/105*2^(1/2)*(-4*a*c+b^2)^(1/2)*(7*a*B*d*(3*a^2*e^3-2*a*b*d*e^2-12*a*c*d^2
*e-b^2*d^2*e+b*c*d^3)+2*A*(2*b^3*d^3*e-2*a*b*d*e*(-3*a*e^2+c*d^2)-b^2*(5*a
*d^2*e^2+2*c*d^4)+a*(-3*a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)))*(-a*(c+a/x^4+b/
x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(
1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^
(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/a^3/d
^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+2*2^(1/2)*B*c*(-4*a*c+b^2)^(1/2)*
e^2*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)
^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4
*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/(b+(-4*a*c+b...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^8,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^8, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx$$
$$\Big\downarrow \text{ 2250}$$
$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^8,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}}{x^8}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x, algorithm
="fricas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*
x^2 + d)/x^8, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}{x^8}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(c*x**4+b*x**2+a)**(1/2)/x**8,x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)*sqrt(a + b*x**2 + c*x**4)/x**8,
x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x, algorithm
="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x, algorithm
="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}}{x^8}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^8,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^8, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^8}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\,\sqrt{c\,x^4+b\,x^2+a}}{x^8}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^8,x)
```

3.119 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 954

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx = -\frac{Ad\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{9x^9}$$

$$-\frac{(Abd+9aBd+10aAe)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{63ax^7}$$

$$-\frac{(9aBd(bd+8ae)-A(6b^2d^2-11abde-a(14cd^2+3ae^2)))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{315a^2dx^5}$$

$$-\frac{(A(8b^3d^3-15ab^2d^2e+38a^2cd^2e-4a^3e^3-3abd(9cd^2-ae^2))-3aBd(4b^2d^2-9abde-a(10cd^2+3ae^2)}{315a^3d^2x^3}$$

$$-\frac{\sqrt{b^2-4ac}(3aBd(bd-2ae)(8b^2d^2-3abde-a(29cd^2-3ae^2))-A(16b^4d^4-32ab^3d^3e-9ab^2d^2(8cd^2-}{315\sqrt{2}a}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)(6aBd(2b^2d^2-3abde-a(5cd^2-3ae^2))-A(8b^3d^3-9ab^2d^2e+8a^2e(3c}{315a^4d^3}$$

output
```
-1/9*A*d*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^9-1/63*(10*A*a*e+A*b*d+9*
B*a*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/x^7-1/315*(9*a*B*d*(8*a*e+b
*d)-A*(6*b^2*d^2-11*a*b*d*e-a*(3*a*e^2+14*c*d^2)))*(e*x^2+d)^(1/2)*(c*x^4+
b*x^2+a)^(1/2)/a^2/d/x^5-1/315*(A*(8*b^3*d^3-15*a*b^2*d^2*e+38*a^2*c*d^2*e
-4*a^3*e^3-3*a*b*d*(-a*e^2+9*c*d^2))-3*a*B*d*(4*b^2*d^2-9*a*b*d*e-a*(3*a*e
^2+10*c*d^2)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^3/d^2/x^3-1/630*(-4
*a*c+b^2)^(1/2)*(3*a*B*d*(-2*a*e+b*d)*(8*b^2*d^2-3*a*b*d*e-a*(-3*a*e^2+29*
c*d^2))-A*(16*b^4*d^4-32*a*b^3*d^3*e-9*a*b^2*d^2*(-a*e^2+8*c*d^2)+a^2*b*d*
e*(7*a*e^2+117*c*d^2)+2*a^2*(-4*a^2*e^4-15*a*c*d^2*e^2+21*c^2*d^4)))*(-a*(
c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2
*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b
*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a^4/d^3/(-a*(e+d/x^2)/((b+(
-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/315*2^(1/2)*(-4
*a*c+b^2)^(1/2)*(a*e^2-b*d*e+c*d^2)*(6*a*B*d*(2*b^2*d^2-3*a*b*d*e-a*(-3*a*
e^2+5*c*d^2))-A*(8*b^3*d^3-9*a*b^2*d^2*e+8*a^2*e*(a*e^2+3*c*d^2)-3*a*b*d*(
a*e^2+9*c*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b
+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b
^2)^(1/2)*d-2*a*e))^(1/2))/a^4/d^3/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^10,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^10, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx$$
$$\Big\downarrow \text{ 2250}$$
$$\int \frac{(A+Bx^2)\,(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^10,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}}{x^{10}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x, algorith
m="fricas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*
x^2 + d)/x^10, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(c*x**4+b*x**2+a)**(1/2)/x**10,x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)*sqrt(a + b*x**2 + c*x**4)/x**10,
 x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x, algorith
m="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^10, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx=\int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x, algorith
m="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx=\int \frac{\left(B\,x^2+A\right)\,\left(e\,x^2+d\right)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}}{x^{10}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^10,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^10, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{10}}\,dx=\int \frac{\left(B\,x^2+A\right)\left(e\,x^2+d\right)^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}}{x^{10}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^10,x)
```

3.120 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1306

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx = \text{Too large to display}$$

output
```
-1/11*A*d*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^11-1/99*(12*A*a*e+A*b*d+
11*B*a*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/x^9-1/693*(11*a*B*d*(10*
a*e+b*d)-A*(8*b^2*d^2-13*a*b*d*e-3*a*(a*e^2+6*c*d^2)))*(e*x^2+d)^(1/2)*(c*
x^4+b*x^2+a)^(1/2)/a^2/d/x^7-1/3465*(A*(48*b^3*d^3-79*a*b^2*d^2*e-a*b*d*(-
9*a*e^2+157*c*d^2)+6*a^2*e*(-3*a*e^2+31*c*d^2))-11*a*B*d*(6*b^2*d^2-11*a*b
*d*e-a*(3*a*e^2+14*c*d^2)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^3/d^2/
x^5-1/3465*(11*a*B*d*(8*b^3*d^3-15*a*b^2*d^2*e+2*a^2*e*(-2*a*e^2+19*c*d^2)
-3*a*b*d*(-a*e^2+9*c*d^2))-A*(64*b^4*d^4-108*a*b^3*d^3*e-3*a*b^2*d^2*(-5*a
*e^2+92*c*d^2)+a^2*b*d*e*(13*a*e^2+367*c*d^2)+6*a^2*(-4*a^2*e^4-7*a*c*d^2*
e^2+25*c^2*d^4)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^4/d^3/x^3+1/6930
*(-4*a*c+b^2)^(1/2)*(11*a*B*d*(16*b^4*d^4-32*a*b^3*d^3*e-9*a*b^2*d^2*(-a*e
^2+8*c*d^2)+a^2*b*d*e*(7*a*e^2+117*c*d^2)+2*a^2*(-4*a^2*e^4-15*a*c*d^2*e^2
+21*c^2*d^4))-A*(128*b^5*d^5-232*a*b^4*d^4*e-3*a*b^3*d^3*(-17*a*e^2+232*c*
d^2)+a^2*b^2*d^2*e*(29*a*e^2+1050*c*d^2)-6*a^3*e*(8*a^2*e^4+15*a*c*d^2*e^2
+103*c^2*d^4)+a^2*b*d*(32*a^2*e^4-195*a*c*d^2*e^2+771*c^2*d^4)))*(-a*(c+a/
x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x
^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(
-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a^5/d^4/(-a*(e+d/x^2)/((b+(-4*a
*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/3465*2^(1/2)*(-4*a*
c+b^2)^(1/2)*(a*e^2-b*d*e+c*d^2)*(11*a*B*d*(8*b^3*d^3-9*a*b^2*d^2*e+8*a...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^12,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^12, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx$$
$$\Big\downarrow \; 2250$$
$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^12,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}}{x^{12}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx=\int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x, algorith
m="fricas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*
x^2 + d)/x^12, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(c*x**4+b*x**2+a)**(1/2)/x**12,x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)*sqrt(a + b*x**2 + c*x**4)/x**12,
 x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx=\int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x, algorith
m="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^12, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x, algorith
m="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}}{x^{12}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^12,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^12, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{12}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\,\sqrt{c\,x^4+b\,x^2+a}}{x^{12}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^12,x)
```

3.121 $\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1746

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx = \text{Too large to display}$$

output
```
-1/13*A*d*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/x^13-1/143*(14*A*a*e+A*b*d
+13*B*a*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/x^11-1/1287*(13*a*B*d*(
12*a*e+b*d)-A*(10*b^2*d^2-15*a*b*d*e-a*(3*a*e^2+22*c*d^2)))*(e*x^2+d)^(1/2
)*(c*x^4+b*x^2+a)^(1/2)/a^2/d/x^9-1/9009*(A*(80*b^3*d^3-121*a*b^2*d^2*e+27
4*a^2*c*d^2*e-24*a^3*e^3-a*b*d*(-9*a*e^2+257*c*d^2))-13*a*B*d*(8*b^2*d^2-1
3*a*b*d*e-3*a*(a*e^2+6*c*d^2)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^3/
d^2/x^7-1/45045*(13*a*B*d*(48*b^3*d^3-79*a*b^2*d^2*e-a*b*d*(-9*a*e^2+157*c
*d^2)+6*a^2*e*(-3*a*e^2+31*c*d^2))-A*(480*b^4*d^4-736*a*b^3*d^3*e-a*b^2*d^
2*(-63*a*e^2+2032*c*d^2)+a^2*b*d*e*(57*a*e^2+2419*c*d^2)+2*a^2*(-72*a^2*e^
4-81*a*c*d^2*e^2+539*c^2*d^4)))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^4/
d^3/x^5+1/45045*(13*a*B*d*(64*b^4*d^4-108*a*b^3*d^3*e-3*a*b^2*d^2*(-5*a*e^
2+92*c*d^2)+a^2*b*d*e*(13*a*e^2+367*c*d^2)+6*a^2*(-4*a^2*e^4-7*a*c*d^2*e^2
+25*c^2*d^4))-A*(640*b^5*d^5-1008*a*b^4*d^4*e-a*b^3*d^3*(-111*a*e^2+3376*c
*d^2)+a^2*b^2*d^2*e*(85*a*e^2+4366*c*d^2)+3*a^2*b*d*(28*a^2*e^4-133*a*c*d^
2*e^2+1193*c^2*d^4)-2*a^3*e*(96*a^2*e^4+113*a*c*d^2*e^2+1193*c^2*d^4)))*(e
*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^5/d^4/x^3-1/90090*(-4*a*c+b^2)^(1/2)
*(13*a*B*d*(128*b^5*d^5-232*a*b^4*d^4*e-3*a*b^3*d^3*(-17*a*e^2+232*c*d^2)+
a^2*b^2*d^2*e*(29*a*e^2+1050*c*d^2)-6*a^3*e*(8*a^2*e^4+15*a*c*d^2*e^2+103*
c^2*d^4)+a^2*b*d*(32*a^2*e^4-195*a*c*d^2*e^2+771*c^2*d^4))-A*(1280*b^6*d^6
-2176*a*b^5*d^5*e-2*a*b^4*d^4*(-207*a*e^2+4096*c*d^2)+a^2*b^3*d^3*e*(20...
```

Mathematica [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx=\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx$$

input
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^14,x]
```

output
```
Integrate[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^14, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx$$

$$\Big\downarrow \text{ 2250}$$

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}dx$$

input
```
Int[((A + B*x^2)*(d + e*x^2)^(3/2)*Sqrt[a + b*x^2 + c*x^4])/x^14,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}}{x^{14}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^14,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^14,x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{14}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^14,x, algorith
m="fricas")
```

output
```
integral((B*e*x^4 + (B*d + A*e)*x^2 + A*d)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*
x^2 + d)/x^14, x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx$$

input
```
integrate((B*x**2+A)*(e*x**2+d)**(3/2)*(c*x**4+b*x**2+a)**(1/2)/x**14,x)
```

output
```
Integral((A + B*x**2)*(d + e*x**2)**(3/2)*sqrt(a + b*x**2 + c*x**4)/x**14,
 x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{14}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^14,x, algorith
m="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^14, x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}(Bx^2+A)(ex^2+d)^{\frac{3}{2}}}{x^{14}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^14,x, algorith
m="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)*(e*x^2 + d)^(3/2)/x^14, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{3/2}\sqrt{c\,x^4+b\,x^2+a}}{x^{14}}\,dx$$

input
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^14,x)
```

output
```
int(((A + B*x^2)*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)^(1/2))/x^14, x)
```

Reduce [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^{3/2}\sqrt{a+bx^2+cx^4}}{x^{14}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}}{x^{14}}dx$$

input
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^14,x)
```

output
```
int((B*x^2+A)*(e*x^2+d)^(3/2)*(c*x^4+b*x^2+a)^(1/2)/x^14,x)
```

3.122 $\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 806

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{(3cCd-4Bce+3bCe)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{8c^2e^2x} + \frac{Cx\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{4ce}$$

$$+ \frac{\sqrt{b^2-4ac}(3cCd-4Bce+3bCe)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right.}{8\sqrt{2}c^2e^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{\sqrt{b^2-4ac}(cCd-4Bce+3bCe)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4a}}}}{\sqrt{2}}\right.\right.}{4\sqrt{2}c^2e\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{\sqrt{b^2-4ac}(4ce(bCd-2Ace+aCe)-(cd+be)(3cCd-4Bce+3bCe))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x}\right.}{\left(b+\sqrt{b^2-4ac}\right.}}}{2\sqrt{2}c^2\left(b+\sqrt{b^2-4ac}\right)e^2\sqrt{d+ex^2}\sqrt{a+}}$$

output
```
-1/8*(-4*B*c*e+3*C*b*e+3*C*c*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/
e^2/x+1/4*C*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e+1/16*(-4*a*c+b^2)^
(1/2)*(-4*B*c*e+3*C*b*e+3*C*c*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x
*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2
))*2^(1/2)/c^2/e^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(
c*x^4+b*x^2+a)^(1/2)-1/8*(-4*a*c+b^2)^(1/2)*(-4*B*c*e+3*C*b*e+C*c*d)*(-a*(
c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d
-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*
2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1
/2))*2^(1/2)/c^2/e/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/4*(-4*a*c+b^2)^
(1/2)*(4*c*e*(-2*A*c*e+C*a*e+C*b*d)-(b*e+c*d)*(-4*B*c*e+3*C*b*e+3*C*c*d))*
(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/
2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^
(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c
+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^2/(b+(-4*
a*c+b^2)^(1/2))/e^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^
4]),x]
```

output
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^
4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$
$$\big\downarrow \text{2250}$$
$$\int \frac{x^2 (A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{x^2 (C\,x^4+B\,x^2+A)}{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(1/2
),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4)/(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4)), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)
), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^2}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^2\,(C\,x^4+B\,x^2+A)}{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)
),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,x+\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,x^4}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)be-3\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,x^4}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}\right.}{}$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x + int((sqrt(d + e*x**2)*sqrt
(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x*
*4 + c*e*x**6),x)*b*e - 3*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*
x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*c*d
+ 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2
+ b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*e - 2*int((sqrt(d + e*x*
*2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4
+ c*d*x**4 + c*e*x**6),x)*b*d - int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4))/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*d
)/(4*e)
```

3.123 $\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 697

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{C\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{2cex}$$

$$-\frac{\sqrt{b^2-4ac}\,C\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}\,d}{bd+\sqrt{b^2-4ac}\,d-2ae}\right)}{2\sqrt{2}ce\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{b^2-4ac}(2Ac-aC)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{}}{bd+\sqrt{b^2}}\right.}{\sqrt{2}ac\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(cCd-2Bce+bCe)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\right.}{c\left(b+\sqrt{b^2-4ac}\right)e\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
1/2*C*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e/x-1/4*(-4*a*c+b^2)^(1/2)*C
*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(
1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2
)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c/e/(-a*(e+d/x^2)/((b
+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1/2*(-4*a*c+b^2
)^(1/2)*(2*A*c-C*a)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/
((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*
c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/c/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)-2^(1/2)*(-4*a*c+b^2)^(1/2)*(-2*B*c*e+C*b*e+C*c*d)*(-a*(c+a/x^4+b/x^2
)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2
)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-
4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d
+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))/e/(e*x^2+d)^
(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]), x
]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 2260$$

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}} dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^4 + B\,x^2 + A}{\sqrt{e\,x^2 + d}\,\sqrt{c\,x^4 + b\,x^2 + a}} dx$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)
), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x
)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}dx$$

input
```
int((C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(d + e*x**2),x)
```

3.124 $\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 634

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{A\sqrt{b^2-4ac}\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right)}{\sqrt{2}ad\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(Bd-Ae)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\ldots}{bd+\ldots}\right.}{ad\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\ldots\right.}{\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/2*A*(-4*a*c+b^2)^(1/2)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2
+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1
/2)/a/d/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2
+a)^(1/2)-2^(1/2)*(-4*a*c+b^2)^(1/2)*(-A*e+B*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*
c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*El
lipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*
a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/a/d/(e*x^2+d)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)+2*2^(1/2)*(-4*a*c+b^2)^(1/2)*C*(-a*(c+a/x^4+b/x^
2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/
2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(
-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*
d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/(b+(-4*a*c+b^2)^(1/2))/(e*x^2+d)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]
),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]
), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 2250$$

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^4+B\,x^2+A}{x^2\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)/(c*e*
x^8 + (c*d + b*e)*x^6 + (b*d + a*e)*x^4 + a*d*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(1/2
),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**2*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2
), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^2\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\sqrt{ex^2 + dx^2}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{x^2\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{C\,x^4 + B\,x^2 + A}{x^2\,\sqrt{e\,x^2 + d}\,\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^2\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{\sqrt{e\,x^2 + d}\,\sqrt{c\,x^4 + b\,x^2 + a}}{e\,x^4 + d\,x^2}dx$$

input
```
int((C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(d*x**2 + e*x**4),x)
```

3.125 $\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 483

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{3adx^3}$$

$$-\frac{\sqrt{b^2-4ac}(3aBd-2A(bd+ae))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}x\sqrt{d+ex^2}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right.}{3\sqrt{2}a^2d^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(3ad(Cd-Be)-A(cd^2-e(bd+2ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}x^3\ \mathrm{Elliptic}}{3a^2d^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/3*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/d/x^3-1/6*(-4*a*c+b^2)^(1/2
)*(3*B*a*d-2*A*(a*e+b*d))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2
+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1
/2)/a^2/d^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b
*x^2+a)^(1/2)-1/3*2^(1/2)*(-4*a*c+b^2)^(1/2)*(3*a*d*(-B*e+C*d)-A*(c*d^2-e*
(2*a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-
4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)
^(1/2)*d-2*a*e))^(1/2))/a^2/d^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]
),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]
), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \ 2250$$
$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^4*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{x^{4}\sqrt{e\,x^{2}+d}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)/(c*e*
x^10 + (c*d + b*e)*x^8 + (b*d + a*e)*x^6 + a*d*x^4), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(1/2
),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**4*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4
), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{x^4\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b + 2*int((sqrt(d + e*x**2)
*sqrt(a + b*x**2 + c*x**4))/(a**2*d*e*x**4 + a**2*e**2*x**6 + a*b*d**2*x**
4 + 2*a*b*d*e*x**6 + a*b*e**2*x**8 + a*c*d*e*x**8 + a*c*e**2*x**10 + b**2*
d**2*x**6 + b**2*d*e*x**8 + b*c*d**2*x**8 + b*c*d*e*x**10),x)*a**3*e**2*x*
*3 + int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a**2*d*e*x**4 + a**
2*e**2*x**6 + a*b*d**2*x**4 + 2*a*b*d*e*x**6 + a*b*e**2*x**8 + a*c*d*e*x**
8 + a*c*e**2*x**10 + b**2*d**2*x**6 + b**2*d*e*x**8 + b*c*d**2*x**8 + b*c*
d*e*x**10),x)*a**2*b*d*e*x**3 - int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4))/(a**2*d*e*x**4 + a**2*e**2*x**6 + a*b*d**2*x**4 + 2*a*b*d*e*x**6 +
a*b*e**2*x**8 + a*c*d*e*x**8 + a*c*e**2*x**10 + b**2*d**2*x**6 + b**2*d*e*
x**8 + b*c*d**2*x**8 + b*c*d*e*x**10),x)*a*b**2*d**2*x**3 + 2*int((sqrt(d
+ e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a**2*d*e + a**2*e**2*x**2 + a*b*d**2
 + 2*a*b*d*e*x**2 + a*b*e**2*x**4 + a*c*d*e*x**4 + a*c*e**2*x**6 + b**2*d*
*2*x**2 + b**2*d*e*x**4 + b*c*d**2*x**4 + b*c*d*e*x**6),x)*a**2*c*e**2*x**
3 - int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a**2*d*e + a**2*e**2
*x**2 + a*b*d**2 + 2*a*b*d*e*x**2 + a*b*e**2*x**4 + a*c*d*e*x**4 + a*c*e**
2*x**6 + b**2*d**2*x**2 + b**2*d*e*x**4 + b*c*d**2*x**4 + b*c*d*e*x**6),x)
*a*b**2*e**2*x**3 + 3*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a*
*2*d*e + a**2*e**2*x**2 + a*b*d**2 + 2*a*b*d*e*x**2 + a*b*e**2*x**4 + a*c*
d*e*x**4 + a*c*e**2*x**6 + b**2*d**2*x**2 + b**2*d*e*x**4 + b*c*d**2*x*...
```

3.126 $\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 619

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{A\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{5adx^5} - \frac{(5aBd-4A(bd+ae))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{15a^2d^2x^3}$$

$$+ \frac{\sqrt{b^2-4ac}(5ad(2bBd-3aCd+2aBe)-A(8b^2d^2+7abde-a(9cd^2-8ae^2)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+}}{15\sqrt{2}a^3d^3\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}(A(4bcd^3-4b^2d^2e+7acd^2e-3abde^2-8a^2e^3)-5ad(3aCde+B(cd^2-e(bd+2ae))))}{15a^3d^3\sqrt{d+ex^2}\sqrt{a+}}$$

output
```
-1/5*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/d/x^5-1/15*(5*B*a*d-4*A*(a*
e+b*d))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/d^2/x^3+1/30*(-4*a*c+b^2
)^(1/2)*(5*a*d*(2*B*a*e+2*B*b*d-3*C*a*d)-A*(8*b^2*d^2+7*a*b*d*e-a*(-8*a*e^
2+9*c*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*Ell
ipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a
*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a^3/d^3/(
-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)
-1/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(A*(-8*a^2*e^3-3*a*b*d*e^2+7*a*c*d^2*e-4*
b^2*d^2*e+4*b*c*d^3)-5*a*d*(3*C*a*d*e+B*(c*d^2-e*(2*a*e+b*d))))*(-a*(c+a/x
^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*
e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/
a^3/d^3/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]
),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]
), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 2250$$

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^6*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^4 + B\,x^2 + A}{x^6\sqrt{e\,x^2 + d}\,\sqrt{c\,x^4 + b\,x^2 + a}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\sqrt{ex^2 + d}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)/(c*e*
x^12 + (c*d + b*e)*x^10 + (b*d + a*e)*x^8 + a*d*x^6), x)
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**6/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(1/2
),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**6*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\sqrt{ex^2 + d}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6
), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^6\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\text{too large to display}$$

input
```
int((C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - 8*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a**3*b*e**3 + 4*sqrt(d +
e*x**2)*sqrt(a + b*x**2 + c*x**4)*a**3*c*d*e**2 - 16*sqrt(d + e*x**2)*sqrt
(a + b*x**2 + c*x**4)*a**2*b**2*d*e**2 + 12*sqrt(d + e*x**2)*sqrt(a + b*x*
*2 + c*x**4)*a**2*b**2*e**3*x**2 - 4*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4)*a**2*b*c*d**2*e + 26*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a**2
*b*c*d*e**2*x**2 - 80*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a**2*b*c*
e**3*x**4 - 6*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a**2*c**2*d**2*e*
x**2 + 20*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a**2*c**2*d*e**2*x**4
 - 8*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b**3*d**2*e - 3*sqrt(d +
 e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b**3*d*e**2*x**2 + 30*sqrt(d + e*x**2
)*sqrt(a + b*x**2 + c*x**4)*a*b**3*e**3*x**4 - 8*sqrt(d + e*x**2)*sqrt(a +
 b*x**2 + c*x**4)*a*b**2*c*d**3 + 14*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4)*a*b**2*c*d**2*e*x**2 - 100*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4
)*a*b**2*c*d*e**2*x**4 - 3*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*
c**2*d**3*x**2 + 10*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*d*
*2*e*x**4 + 30*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**4*d*e**2*x**4
 - 20*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**3*c*d**2*e*x**4 - 10*s
qrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b**2*c**2*d**3*x**4 + 320*int((s
qrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(2*a**2*b*d*e**2 + 2*a**2*
b*e**3*x**2 - a**2*c*d**2*e - a**2*c*d*e**2*x**2 + 2*a*b**2*d**2*e + 4*...
```

3.127 $\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 851

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$
$$= -\frac{A\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{7adx^7} - \frac{(7aBd-6A(bd+ae))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{35a^2d^2x^5}$$
$$+\frac{(7ad(4bBd-5aCd+4aBe)-A(24b^2d^2+23abde-a(25cd^2-24ae^2)))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{105a^3d^3x^3}$$
$$+\frac{\sqrt{b^2-4ac}(4A(12b^3d^3+10ab^2d^2e-a^2e(11cd^2-12ae^2)-2abd(13cd^2-5ae^2))-7ad(8b^2Bd^2-abd(}{105\sqrt{2}a^4d^4\sqrt{-}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(A(24b^3d^3e-2abde(33cd^2-8ae^2)-b^2(24cd^4-17ad^2e^2)+a(25c^2d^4-32acd^2e^2+48a}{}$$

output
```
-1/7*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/d/x^7-1/35*(7*B*a*d-6*A*(a*
e+b*d))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/d^2/x^5+1/105*(7*a*d*(4*
B*a*e+4*B*b*d-5*C*a*d)-A*(24*b^2*d^2+23*a*b*d*e-a*(-24*a*e^2+25*c*d^2)))*(
e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^3/d^3/x^3+1/210*(-4*a*c+b^2)^(1/2)*
(4*A*(12*b^3*d^3+10*a*b^2*d^2*e-a^2*e*(-12*a*e^2+11*c*d^2)-2*a*b*d*(-5*a*e
^2+13*c*d^2))-7*a*d*(8*b^2*B*d^2-a*b*d*(-7*B*e+10*C*d)-a*(-8*B*a*e^2+9*B*c
*d^2+10*C*a*d*e)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/
2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*
((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a^4
/d^4/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)
^(1/2)-1/105*2^(1/2)*(-4*a*c+b^2)^(1/2)*(A*(24*b^3*d^3*e-2*a*b*d*e*(-8*a*e
^2+33*c*d^2)-b^2*(-17*a*d^2*e^2+24*c*d^4)+a*(48*a^2*e^4-32*a*c*d^2*e^2+25*
c^2*d^4))-7*a*d*(4*b^2*B*d^2*e-b*d*(-3*B*a*e^2+4*B*c*d^2+5*C*a*d*e)+a*(c*d
^2*(-7*B*e+5*C*d)-2*a*e^2*(-4*B*e+5*C*d))))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^
2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*Ellipt
icF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+
b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/a^4/d^4/(e*x^2+d)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^8*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]
),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4)/(x^8*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]
), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \; 2250$$
$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^8*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{C\,x^4+B\,x^2+A}{x^8\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)/(c*e*
x^14 + (c*d + b*e)*x^12 + (b*d + a*e)*x^10 + a*d*x^8), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((C*x**4+B*x**2+A)/x**8/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(1/2
),x)
```

output
```
Integral((A + B*x**2 + C*x**4)/(x**8*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^8
), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^8
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^8\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^8*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),
x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^8*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^8\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

3.128 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 1032

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

output
```
1/48*(15*b^2*D*e^2+2*c*e*(-9*C*b*e-8*D*a*e+7*D*b*d)+3*c^2*(8*B*e^2-6*C*d*e
+5*D*d^2))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^3/e^3/x-1/24*(-6*C*c*e+
5*D*b*e+5*D*c*d)*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e^2+1/6*D*x^3
*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e-1/96*(-4*a*c+b^2)^(1/2)*(15*b^2
*D*e^2+2*c*e*(-9*C*b*e-8*D*a*e+7*D*b*d)+3*c^2*(8*B*e^2-6*C*d*e+5*D*d^2))*(
-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+
(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*
d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^3/e^3/(-a*(e+d/x^2)/(
(b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)+1/48*(-4*a*c+
b^2)^(1/2)*(15*b^2*D*e^2+2*c*e*(-9*C*b*e-8*D*a*e+2*D*b*d)+c^2*(24*B*e^2-6*
C*d*e+5*D*d^2))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+
(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^
2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/c^3/e^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)-1/8*(-4*a*c+b^2)^(1/2)*(5*b^3*D*e^3+3*b*c*e^2*(-2*C*b*e-4*D*a*e+D*b*
d)+c^3*(-16*A*e^3+8*B*d*e^2-6*C*d^2*e+5*D*d^3)-c^2*e*(4*a*e*(-2*C*e+D*d)-b
*(8*B*e^2-4*C*d*e+3*D*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(
e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+
2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c
+b^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*...
```

Mathematica [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(Sqrt[d + e*x^2]*Sqrt[a + b*x^
2 + c*x^4]),x]
```

output
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(Sqrt[d + e*x^2]*Sqrt[a + b*x^
2 + c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$
$$\Big\downarrow \text{ 2250}$$
$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*
x^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{x^2(Dx^6+C\,x^4+B\,x^2+A)}{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a
)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x**2 + C*x**4 + D*x**6)/(sqrt(d + e*x**2)*sqrt(a + b*
x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Dx^6+Cx^4+Bx^2+A)x^2}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^2/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*
x^2 + d)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(Dx^6+Cx^4+Bx^2+A)x^2}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*x^2/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*
x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^2\,(A+B\,x^2+C\,x^4+x^6\,D)}{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^
4)^(1/2)),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^
4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\text{Too large to display}$$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - 5*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b*d*e*x + 6*sqrt(d + e*x*
*2)*sqrt(a + b*x**2 + c*x**4)*c**2*e*x - 5*sqrt(d + e*x**2)*sqrt(a + b*x**
2 + c*x**4)*c*d**2*x + 4*sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*c*d*e*
x**3 - 16*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e
*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*c*d*e**2 + 15*int(
(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x*
*2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b**2*d*e**2 + 6*int((sqrt(d + e*x*
*2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4
 + c*d*x**4 + c*e*x**6),x)*b*c**2*e**2 + 14*int((sqrt(d + e*x**2)*sqrt(a +
b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 +
c*e*x**6),x)*b*c*d**2*e - 18*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**
4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*c
**3*d*e + 15*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d +
a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*c**2*d**3 + 10*in
t((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*
x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*b*d*e**2 + 12*int((sqrt(d + e*
x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**
4 + c*d*x**4 + c*e*x**6),x)*a*c**2*e**2 - 2*int((sqrt(d + e*x**2)*sqrt(a +
 b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 +
 c*e*x**6),x)*a*c*d**2*e + 10*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c...
```

3.129 $\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 819

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{(3cdD-4cCe+3bDe)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{8c^2e^2x} + \frac{Dx\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{4ce}$$

$$+ \frac{\sqrt{b^2-4ac}(3cdD-4cCe+3bDe)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2a\cdots}\right.}{8\sqrt{2}c^2e^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{\sqrt{b^2-4ac}(8Ac^2e+a(cdD-4cCe+3bDe))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{EllipticF}\left(\arcsin\cdots\right.}{4\sqrt{2}ac^2e\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$- \frac{\sqrt{b^2-4ac}(4ce(bdD-2Bce+aDe)-(cd+be)(3cdD-4cCe+3bDe))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\cdots\right)}{\left(b+\sqrt{b^2-4a\cdots}\right.}}}{2\sqrt{2}c^2\left(b+\sqrt{b^2-4ac}\right)e^2\sqrt{d+ex^2}\sqrt{a+\cdots}}$$

output
```
-1/8*(-4*C*c*e+3*D*b*e+3*D*c*d)*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^2/
e^2/x+1/4*D*x*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e+1/16*(-4*a*c+b^2)^
(1/2)*(-4*C*c*e+3*D*b*e+3*D*c*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x
*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2
))*2^(1/2)/c^2/e^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(
c*x^4+b*x^2+a)^(1/2)-1/8*(-4*a*c+b^2)^(1/2)*(8*A*c^2*e+a*(-4*C*c*e+3*D*b*e
+D*c*d))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c
+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2
)*d-2*a*e))^(1/2))*2^(1/2)/a/c^2/e/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)-1
/4*(-4*a*c+b^2)^(1/2)*(4*c*e*(-2*B*c*e+D*a*e+D*b*d)-(b*e+c*d)*(-4*C*c*e+3*
D*b*e+3*D*c*d))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+
(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),
2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(
1/2)/c^2/(b+(-4*a*c+b^2)^(1/2))/e^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*
x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*
x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 2260}$$

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^4]),
x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2260
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Unintegrable[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^6+C\,x^4+B\,x^2+A}{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, a
lgorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 +
d)/(c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a)**(1
/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(sqrt(d + e*x**2)*sqrt(a + b*x**2
+ c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/
2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,dx-3\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,x^4}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2+ad}dx\right)bde+4\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}\,x^4}{ce\,x^6+be\,x^4+cd\,x^4+ae\,x^2+bd\,x^2}\right.}{}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*d*x - 3*int((sqrt(d + e*x**2)*
sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*
d*x**4 + c*e*x**6),x)*b*d*e + 4*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*
x**4)*x**4)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x
)*c**2*e - 3*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*d +
a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*c*d**2 - 2*int((s
qrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2
 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*d*e + 4*int((sqrt(d + e*x**2)*sqrt
(a + b*x**2 + c*x**4)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x*
*4 + c*e*x**6),x)*b*c*e - 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4
)*x**2)/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*b*
d**2 + 4*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a*d + a*e*x**2
+ b*d*x**2 + b*e*x**4 + c*d*x**4 + c*e*x**6),x)*a*c*e - int((sqrt(d + e*x*
*2)*sqrt(a + b*x**2 + c*x**4))/(a*d + a*e*x**2 + b*d*x**2 + b*e*x**4 + c*d
*x**4 + c*e*x**6),x)*a*d**2)/(4*c*e)
```

3.130 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 722

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{D\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{2cex}$$

$$-\frac{\sqrt{b^2-4ac}(adD+2Ace)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right)}{2\sqrt{2}acde\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{b^2-4ac}(2Bcd-adD-2Ace)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{\sqrt{2}acd\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(cdD-2cCe+bDe)\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\text{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\right.}{c\left(b+\sqrt{b^2-4ac}\right)e\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
1/2*D*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c/e/x-1/4*(-4*a*c+b^2)^(1/2)*(
2*A*c*e+D*a*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*E
llipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4
*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/c/d/e
/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/
2)-1/2*(-4*a*c+b^2)^(1/2)*(-2*A*c*e+2*B*c*d-D*a*d)*(-a*(c+a/x^4+b/x^2)/(-4
*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3
*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((
-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a/c/d
/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)-2^(1/2)*(-4*a*c+b^2)^(1/2)*(-2*C*c*
e+D*b*e+D*c*d)*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(
-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*a/x^2)/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),2
^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/c/(b
+(-4*a*c+b^2)^(1/2))/e/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*Sqrt[d + e*x^2]*Sqrt[a + b*x^2
+ c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*Sqrt[d + e*x^2]*Sqrt[a + b*x^2
+ c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$\downarrow 2250$$

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^
4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^6+C\,x^4+B\,x^2+A}{x^2\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 +
d)/(c*e*x^8 + (c*d + b*e)*x^6 + (b*d + a*e)*x^4 + a*d*x^2), x)
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^2\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^2\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a
)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**2*sqrt(d + e*x**2)*sqrt(a + b*
x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^2\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{Dx^6 + Cx^4 + Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\sqrt{ex^2 + d}x^2}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^2\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)
^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\text{Too large to display}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*c + int((sqrt(d + e*x**2)*sqrt
(a + b*x**2 + c*x**4)*x**4)/(a*b*d*e + a*b*e**2*x**2 + a*c*d**2 + a*c*d*e*
x**2 + b**2*d*e*x**2 + b**2*e**2*x**4 + b*c*d**2*x**2 + 2*b*c*d*e*x**4 + b
*c*e**2*x**6 + c**2*d**2*x**4 + c**2*d*e*x**6),x)*b**2*d*e**2*x - 2*int((s
qrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*b*d*e + a*b*e**2*x**2 +
 a*c*d**2 + a*c*d*e*x**2 + b**2*d*e*x**2 + b**2*e**2*x**4 + b*c*d**2*x**2
+ 2*b*c*d*e*x**4 + b*c*e**2*x**6 + c**2*d**2*x**4 + c**2*d*e*x**6),x)*b*c*
*2*e**2*x + 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**4)/(a*b*d
*e + a*b*e**2*x**2 + a*c*d**2 + a*c*d*e*x**2 + b**2*d*e*x**2 + b**2*e**2*x
**4 + b*c*d**2*x**2 + 2*b*c*d*e*x**4 + b*c*e**2*x**6 + c**2*d**2*x**4 + c*
*2*d*e*x**6),x)*b*c*d**2*e*x - 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c
*x**4)*x**4)/(a*b*d*e + a*b*e**2*x**2 + a*c*d**2 + a*c*d*e*x**2 + b**2*d*e
*x**2 + b**2*e**2*x**4 + b*c*d**2*x**2 + 2*b*c*d*e*x**4 + b*c*e**2*x**6 +
c**2*d**2*x**4 + c**2*d*e*x**6),x)*c**3*d*e*x + int((sqrt(d + e*x**2)*sqrt
(a + b*x**2 + c*x**4)*x**4)/(a*b*d*e + a*b*e**2*x**2 + a*c*d**2 + a*c*d*e*
x**2 + b**2*d*e*x**2 + b**2*e**2*x**4 + b*c*d**2*x**2 + 2*b*c*d*e*x**4 + b
*c*e**2*x**6 + c**2*d**2*x**4 + c**2*d*e*x**6),x)*c**2*d**3*x + int((sqrt(
d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a*b*d*e*x**2 + a*b*e**2*x**4 + a*c
*d**2*x**2 + a*c*d*e*x**4 + b**2*d*e*x**4 + b**2*e**2*x**6 + b*c*d**2*x**4
 + 2*b*c*d*e*x**6 + b*c*e**2*x**8 + c**2*d**2*x**6 + c**2*d*e*x**8),x)*...
```

3.131 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 719

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{3adx^3}$$

$$-\frac{\sqrt{b^2-4ac}(3aBd-2A(bd+ae))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\,x\sqrt{d+ex^2}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\frac{2\sqrt{b^2-4ac}d}{bd+\sqrt{b^2-4ac}d-2ae}\right.}{3\sqrt{2}a^2d^2\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(3ad(Cd-Be)-A(cd^2-e(bd+2ae)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{Elliptic}}{3a^2d^2\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}D\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\,x^3\,\mathrm{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+\frac{2a}{x^2}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\right.}{\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/3*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/d/x^3-1/6*(-4*a*c+b^2)^(1/2
)*(3*B*a*d-2*A*(a*e+b*d))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2
+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2
^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1
/2)/a^2/d^2/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b
*x^2+a)^(1/2)-1/3*2^(1/2)*(-4*a*c+b^2)^(1/2)*(3*a*d*(-B*e+C*d)-A*(c*d^2-e*
(2*a*e+b*d)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-
4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)
^(1/2)*d-2*a*e))^(1/2))/a^2/d^2/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)+2*2^
(1/2)*(-4*a*c+b^2)^(1/2)*D*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+
d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)*x^3*EllipticPi(1/2*(1+(b+2*
a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b
^2)^(1/2)),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))
^(1/2))/(b+(-4*a*c+b^2)^(1/2))/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*Sqrt[d + e*x^2]*Sqrt[a + b*x^2
+ c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*Sqrt[d + e*x^2]*Sqrt[a + b*x^2
+ c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$\downarrow 2250$$

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^
4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^6+C\,x^4+B\,x^2+A}{x^4\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^4}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 +
d)/(c*e*x^10 + (c*d + b*e)*x^8 + (b*d + a*e)*x^6 + a*d*x^4), x)
```

Sympy [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^4\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^4\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a
)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**4*sqrt(d + e*x**2)*sqrt(a + b*
x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^4\sqrt{d + ex^2}\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{Dx^6 + Cx^4 + Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}\sqrt{ex^2 + d}x^4}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+dx^4}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^4\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)
^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
( - sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*b + 2*int((sqrt(d + e*x**2)
*sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2*d*e + a**2*e**2*x**2 + a*b*d**2 + 2
*a*b*d*e*x**2 + a*b*e**2*x**4 + a*c*d*e*x**4 + a*c*e**2*x**6 + b**2*d**2*x
**2 + b**2*d*e*x**4 + b*c*d**2*x**4 + b*c*d*e*x**6),x)*a**2*d*e**2*x**3 +
4*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2*d*e + a**2*e
**2*x**2 + a*b*d**2 + 2*a*b*d*e*x**2 + a*b*e**2*x**4 + a*c*d*e*x**4 + a*c*
e**2*x**6 + b**2*d**2*x**2 + b**2*d*e*x**4 + b*c*d**2*x**4 + b*c*d*e*x**6)
,x)*a*b*d**2*e*x**3 + 2*int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)*x*
*2)/(a**2*d*e + a**2*e**2*x**2 + a*b*d**2 + 2*a*b*d*e*x**2 + a*b*e**2*x**4
 + a*c*d*e*x**4 + a*c*e**2*x**6 + b**2*d**2*x**2 + b**2*d*e*x**4 + b*c*d**
2*x**4 + b*c*d*e*x**6),x)*b**2*d**3*x**3 + 2*int((sqrt(d + e*x**2)*sqrt(a
+ b*x**2 + c*x**4))/(a**2*d*e*x**4 + a**2*e**2*x**6 + a*b*d**2*x**4 + 2*a*
b*d*e*x**6 + a*b*e**2*x**8 + a*c*d*e*x**8 + a*c*e**2*x**10 + b**2*d**2*x**
6 + b**2*d*e*x**8 + b*c*d**2*x**8 + b*c*d*e*x**10),x)*a**3*e**2*x**3 + int
((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(a**2*d*e*x**4 + a**2*e**2*x
**6 + a*b*d**2*x**4 + 2*a*b*d*e*x**6 + a*b*e**2*x**8 + a*c*d*e*x**8 + a*c*
e**2*x**10 + b**2*d**2*x**6 + b**2*d*e*x**8 + b*c*d**2*x**8 + b*c*d*e*x**1
0),x)*a**2*b*d*e*x**3 - int((sqrt(d + e*x**2)*sqrt(a + b*x**2 + c*x**4))/(
a**2*d*e*x**4 + a**2*e**2*x**6 + a*b*d**2*x**4 + 2*a*b*d*e*x**6 + a*b*e**2
*x**8 + a*c*d*e*x**8 + a*c*e**2*x**10 + b**2*d**2*x**6 + b**2*d*e*x**8 ...
```

3.132 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 626

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{5adx^5} - \frac{(5aBd-4A(bd+ae))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{15a^2d^2x^3}$$

$$+\frac{\sqrt{b^2-4ac}(5ad(2bBd-3aCd+2aBe)-A(8b^2d^2+7abde-a(9cd^2-8ae^2)))\sqrt{-\frac{a\left(c+\frac{a}{x^4}+\frac{b}{x^2}\right)}{b^2-4ac}}x\sqrt{d+}}{15\sqrt{2}a^3d^3\sqrt{-\frac{a\left(e+\frac{d}{x^2}\right)}{\left(b+\sqrt{b^2-4ac}\right)d-2ae}}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(A(4bcd^3-4b^2d^2e+7acd^2e-3abde^2-8a^2e^3)+5ad(3ad(dD-Ce)-B(cd^2-bde-}{15a^3d^3\sqrt{d+ex^2}\sqrt{}}$$

output
```
-1/5*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/d/x^5-1/15*(5*B*a*d-4*A*(a*
e+b*d))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/d^2/x^3+1/30*(-4*a*c+b^2
)^(1/2)*(5*a*d*(2*B*a*e+2*B*b*d-3*C*a*d)-A*(8*b^2*d^2+7*a*b*d*e-a*(-8*a*e^
2+9*c*d^2)))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*x^2+d)^(1/2)*Ell
ipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*((-4*a
*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2^(1/2)/a^3/d^3/(
-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^4+b*x^2+a)^(1/2)
-1/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(A*(-8*a^2*e^3-3*a*b*d*e^2+7*a*c*d^2*e-4*
b^2*d^2*e+4*b*c*d^3)+5*a*d*(3*a*d*(-C*e+D*d)-B*(-2*a*e^2-b*d*e+c*d^2)))*(-
a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2)
)*d-2*a*e))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/
2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))
^(1/2))/a^3/d^3/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*Sqrt[d + e*x^2]*Sqrt[a + b*x^2
+ c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*Sqrt[d + e*x^2]*Sqrt[a + b*x^2
+ c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 2250$$

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^
4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{D x^6+C\,x^4+B\,x^2+A}{x^6\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^6}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 +
d)/(c*e*x^12 + (c*d + b*e)*x^10 + (b*d + a*e)*x^8 + a*d*x^6), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a
)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**6*sqrt(d + e*x**2)*sqrt(a + b*
x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^6}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+dx^6}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^6\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)
^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Dx^6+C\,x^4+B\,x^2+A}{x^6\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

3.133 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 865

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{A\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{7adx^7} - \frac{(7aBd-6A(bd+ae))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{35a^2d^2x^5}$$

$$+ \frac{(7ad(4bBd-5aCd+4aBe)-A(24b^2d^2+23abde-a(25cd^2-24ae^2)))\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}{105a^3d^3x^3}$$

$$+ \frac{\sqrt{b^2-4ac}(4A(12b^3d^3+10ab^2d^2e-a^2e(11cd^2-12ae^2)-2abd(13cd^2-5ae^2))-7ad(8b^2Bd^2-abd(}{105\sqrt{2}a^4d^4}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}(A(24b^3d^3e-2abde(33cd^2-8ae^2)-b^2(24cd^4-17ad^2e^2)+a(25c^2d^4-32acd^2e^2+48a}{}$$

output
```
-1/7*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/d/x^7-1/35*(7*B*a*d-6*A*(a*
e+b*d))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/d^2/x^5+1/105*(7*a*d*(4*
B*a*e+4*B*b*d-5*C*a*d)-A*(24*b^2*d^2+23*a*b*d*e-a*(-24*a*e^2+25*c*d^2)))*(
e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^3/d^3/x^3+1/210*(-4*a*c+b^2)^(1/2)*
(4*A*(12*b^3*d^3+10*a*b^2*d^2*e-a^2*e*(-12*a*e^2+11*c*d^2)-2*a*b*d*(-5*a*e
^2+13*c*d^2))-7*a*d*(8*b^2*B*d^2-a*b*d*(-7*B*e+10*C*d)+a*(5*a*d*(-2*C*e+3*
D*d)-B*(-8*a*e^2+9*c*d^2))))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x*(e*
x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))*2
^(1/2)/a^4/d^4/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(c*x^
4+b*x^2+a)^(1/2)-1/105*2^(1/2)*(-4*a*c+b^2)^(1/2)*(A*(24*b^3*d^3*e-2*a*b*d
*e*(-8*a*e^2+33*c*d^2)-b^2*(-17*a*d^2*e^2+24*c*d^4)+a*(48*a^2*e^4-32*a*c*d
^2*e^2+25*c^2*d^4))-7*a*d*(4*b^2*B*d^2*e-b*d*(-3*B*a*e^2+4*B*c*d^2+5*C*a*d
*e)+a*(c*d^2*(-7*B*e+5*C*d)+a*e*(8*B*e^2-10*C*d*e+15*D*d^2))))*(-a*(c+a/x^
4+b/x^2)/(-4*a*c+b^2))^(1/2)*(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e
))^(1/2)*x^3*EllipticF(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2))/a
^4/d^4/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*Sqrt[d + e*x^2]*Sqrt[a + b*x^2
+ c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*Sqrt[d + e*x^2]*Sqrt[a + b*x^2
+ c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$
$$\Big\downarrow \text{ 2250}$$
$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x^
4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^6+C\,x^4+B\,x^2+A}{x^8\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+dx^8}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 +
d)/(c*e*x^14 + (c*d + b*e)*x^12 + (b*d + a*e)*x^10 + a*d*x^8), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a
)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**8*sqrt(d + e*x**2)*sqrt(a + b*
x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+dx^8}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)*x^8), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^8}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),
x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^8\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)
^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+C\,x^4+B\,x^2+A}{x^8\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

3.134 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 1220

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

output
```
-1/9*A*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/d/x^9-1/63*(9*B*a*d-8*A*(a*
e+b*d))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/d^2/x^7+1/315*(9*a*d*(6*
B*a*e+6*B*b*d-7*C*a*d)-A*(48*b^2*d^2+47*a*b*d*e-a*(-48*a*e^2+49*c*d^2)))*(
e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^3/d^3/x^5+1/315*(2*A*(32*b^3*d^3+30
*a*b^2*d^2*e-a^2*e*(-32*a*e^2+31*c*d^2)-6*a*b*d*(-5*a*e^2+11*c*d^2))-3*a*d
*(24*b^2*B*d^2-a*b*d*(-23*B*e+28*C*d)+a*(7*a*d*(-4*C*e+5*D*d)-B*(-24*a*e^2
+25*c*d^2))))*(e*x^2+d)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^4/d^4/x^3-1/630*(-4*
a*c+b^2)^(1/2)*(A*(128*b^4*d^4+104*a*b^3*d^3*e-2*a^2*b*d*e*(-52*a*e^2+111*
c*d^2)-3*a*b^2*d^2*(-33*a*e^2+136*c*d^2)+a^2*(128*a^2*e^4-108*a*c*d^2*e^2+
147*c^2*d^4))-3*a*d*(48*b^3*B*d^3-8*a*b^2*d^2*(-5*B*e+7*C*d)+a*b*d*(7*a*d*
(-7*C*e+10*D*d)-8*B*(-5*a*e^2+13*c*d^2))+a^2*(c*d^2*(-44*B*e+63*C*d)+2*a*e
*(24*B*e^2-28*C*d*e+35*D*d^2))))*(-a*(c+a/x^4+b/x^2)/(-4*a*c+b^2))^(1/2)*x
*(e*x^2+d)^(1/2)*EllipticE(1/2*(1+(b+2*a/x^2)/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)*d/(b*d+(-4*a*c+b^2)^(1/2)*d-2*a*e))^(1/2
))*2^(1/2)/a^5/d^5/(-a*(e+d/x^2)/((b+(-4*a*c+b^2)^(1/2))*d-2*a*e))^(1/2)/(
c*x^4+b*x^2+a)^(1/2)+1/315*2^(1/2)*(-4*a*c+b^2)^(1/2)*(A*(64*b^4*d^4*e-3*a
*b^2*d^2*e*(-13*a*e^2+80*c*d^2)-b^3*(-44*a*d^3*e^2+64*c*d^5)+2*a*b*d*(20*a
^2*e^4-63*a*c*d^2*e^2+66*c^2*d^4)+a^2*e*(128*a^2*e^4-76*a*c*d^2*e^2+111*c^
2*d^4))-3*a*d*(24*b^3*B*d^3*e-b^2*d^2*(-17*B*a*e^2+24*B*c*d^2+28*C*a*d*e)+
a*b*d*(2*c*d^2*(-33*B*e+14*C*d)+a*e*(16*B*e^2-21*C*d*e+35*D*d^2))+a*(B*...
```

Mathematica [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*Sqrt[d + e*x^2]*Sqrt[a + b*x^2
 + c*x^4]),x]
```

output
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*Sqrt[d + e*x^2]*Sqrt[a + b*x^2
 + c*x^4]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \text{ 2250}$$
$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}dx$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*Sqrt[d + e*x^2]*Sqrt[a + b*x^2 + c*x
^4]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2250
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Unintegrable[Px*(f*x)^m*(d + e*x^2)^
q*(a + b*x^2 + c*x^4)^p, x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && Pol
yQ[Px, x]
```

Maple [F]

$$\int \frac{Dx^6+C\,x^4+B\,x^2+A}{x^{10}\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
,x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2 +
d)/(c*e*x^16 + (c*d + b*e)*x^14 + (b*d + a*e)*x^12 + a*d*x^10), x)
```

Sympy [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(e*x**2+d)**(1/2)/(c*x**4+b*x**2+
a)**(1/2),x)
```

output
```
Integral((A + B*x**2 + C*x**4 + D*x**6)/(x**10*sqrt(d + e*x**2)*sqrt(a + b
*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
,x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)*x^10), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+Cx^4+Bx^2+A}{\sqrt{cx^4+bx^2+a}\sqrt{ex^2+d}x^{10}}\,dx$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
,x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(e*x^2
+ d)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^{10}\,\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4
)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4
)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\sqrt{d+ex^2}\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Dx^6+C\,x^4+B\,x^2+A}{x^{10}\sqrt{e\,x^2+d}\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                      Small rewrite of logic in main function to make it*)
(*                      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file