Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.3-General-trinomial/123-1.2.3.1

Nasser M. Abbasi

May 18, 2024 Compiled on May 18, 2024 at 8:43pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**117**]. This is test number [123].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (117)	0.00 (0)
Mathematica	100.00 (117)	0.00 (0)
Maple	84.62 (99)	15.38 (18)
Reduce	74.36 (87)	25.64 (30)
Giac	73.50 (86)	26.50 (31)
Fricas	64.10 (75)	35.90 (42)
Maxima	54.70 (64)	45.30 (53)
Mupad	52.14 (61)	47.86 (56)
Sympy	42.74 (50)	57.26 (67)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

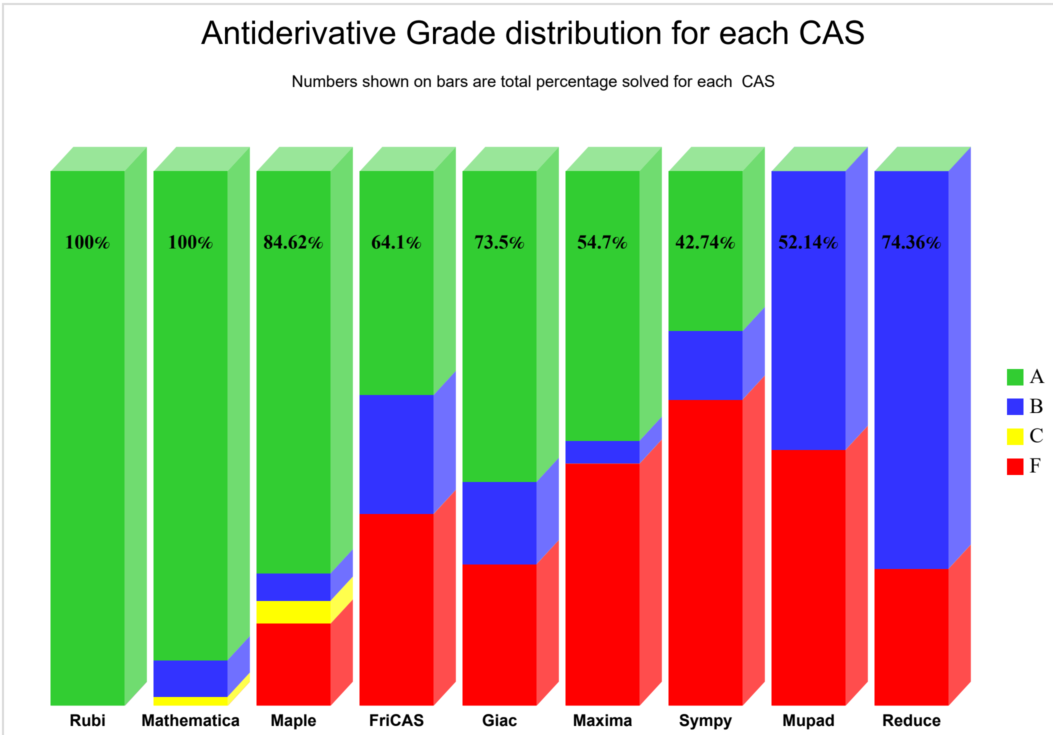
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

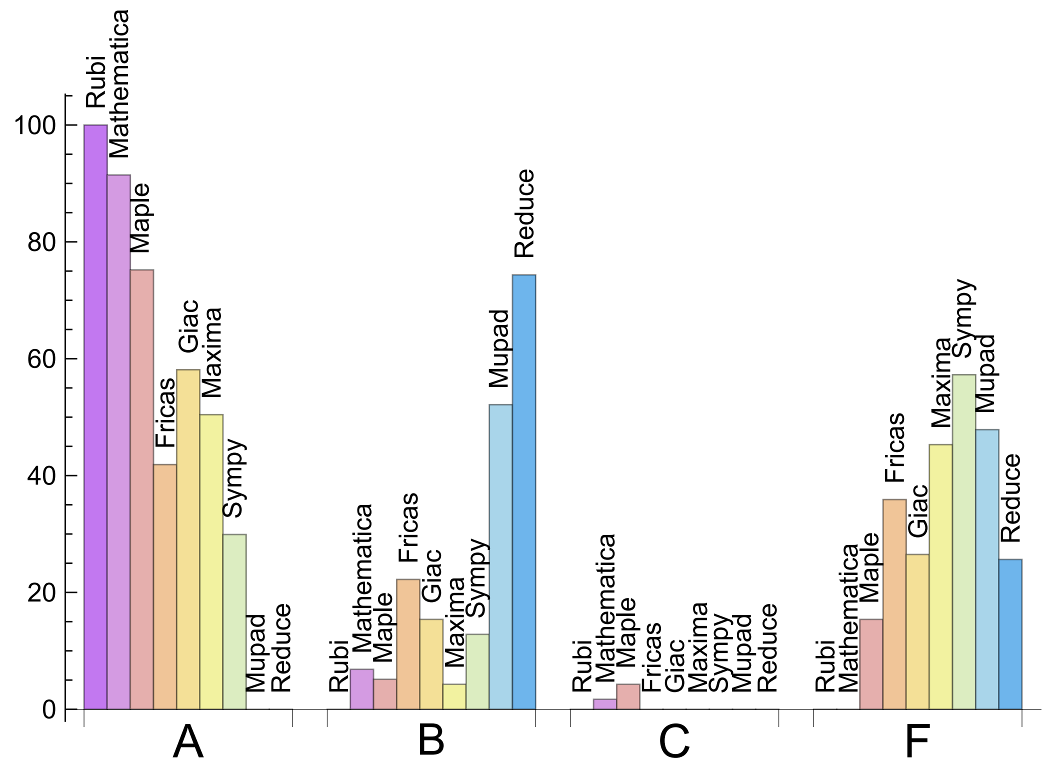
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	91.453	6.838	1.709	0.000
Maple	75.214	5.128	4.274	15.385
Giac	58.120	15.385	0.000	26.496
Maxima	50.427	4.274	0.000	45.299
Fricas	41.880	22.222	0.000	35.897
Sympy	29.915	12.821	0.000	57.265
Mupad	0.000	52.137	0.000	47.863
Reduce	0.000	74.359	0.000	25.641

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	18	100.00	0.00	0.00
Reduce	30	100.00	0.00	0.00
Giac	31	74.19	6.45	19.35
Fricas	42	21.43	64.29	14.29
Maxima	53	71.70	0.00	28.30
Mupad	56	0.00	100.00	0.00
Sympy	67	86.57	13.43	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Fricas	0.13
Reduce	0.19
Maple	0.26
Rubi	0.28
Giac	0.31
Mathematica	0.53
Sympy	1.93
Mupad	14.61

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	68.72	0.81	56.50	0.77
Maple	104.37	0.88	75.00	0.83
Rubi	115.52	0.94	97.00	1.01
Mathematica	162.51	1.08	83.00	0.95
Giac	162.83	1.27	81.50	0.81
Reduce	194.45	1.60	68.00	0.90
Fricas	345.96	2.71	130.00	1.48
Sympy	370.82	3.77	147.00	1.71
Mupad	381.66	2.02	65.00	0.95

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

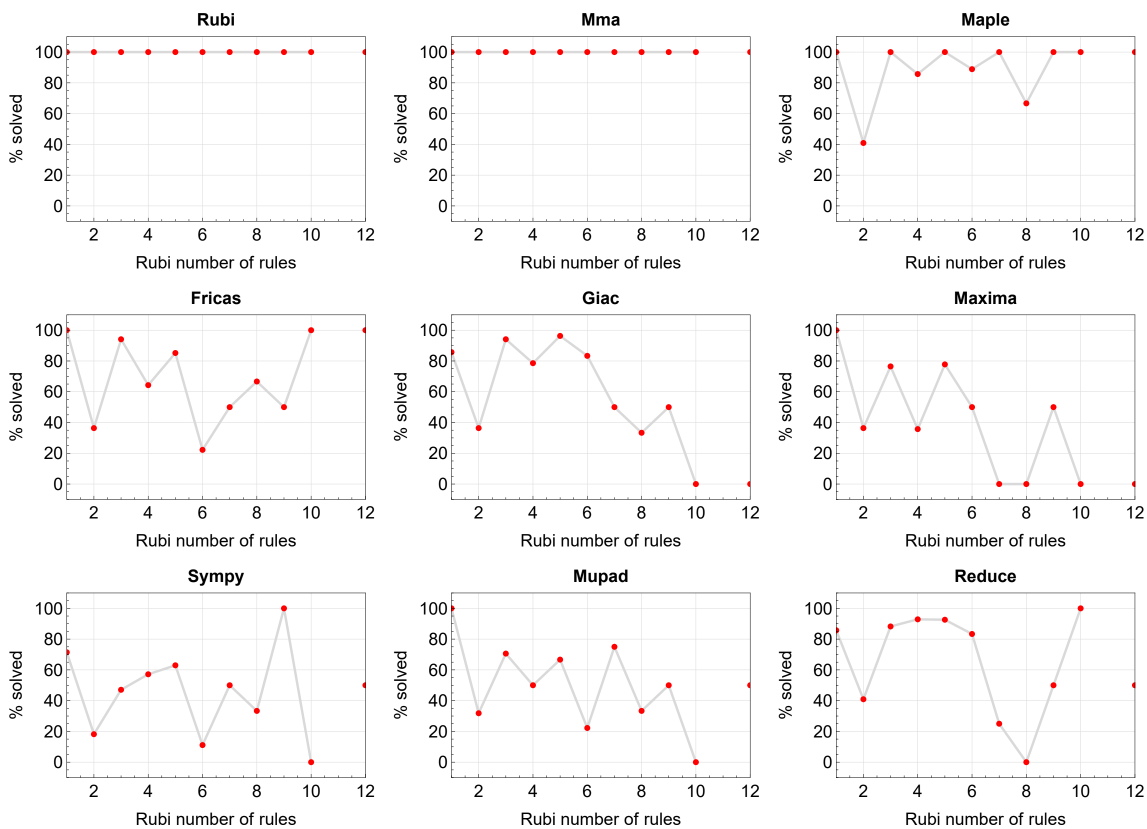


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

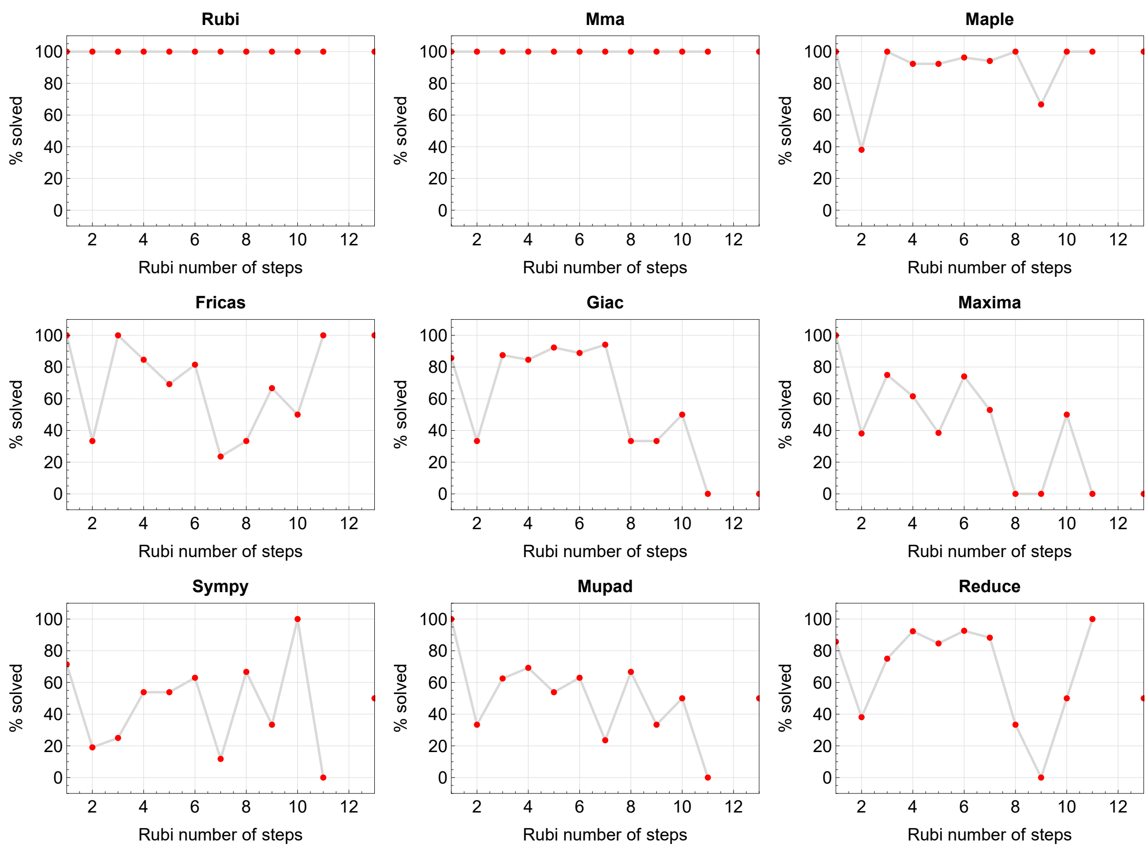


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

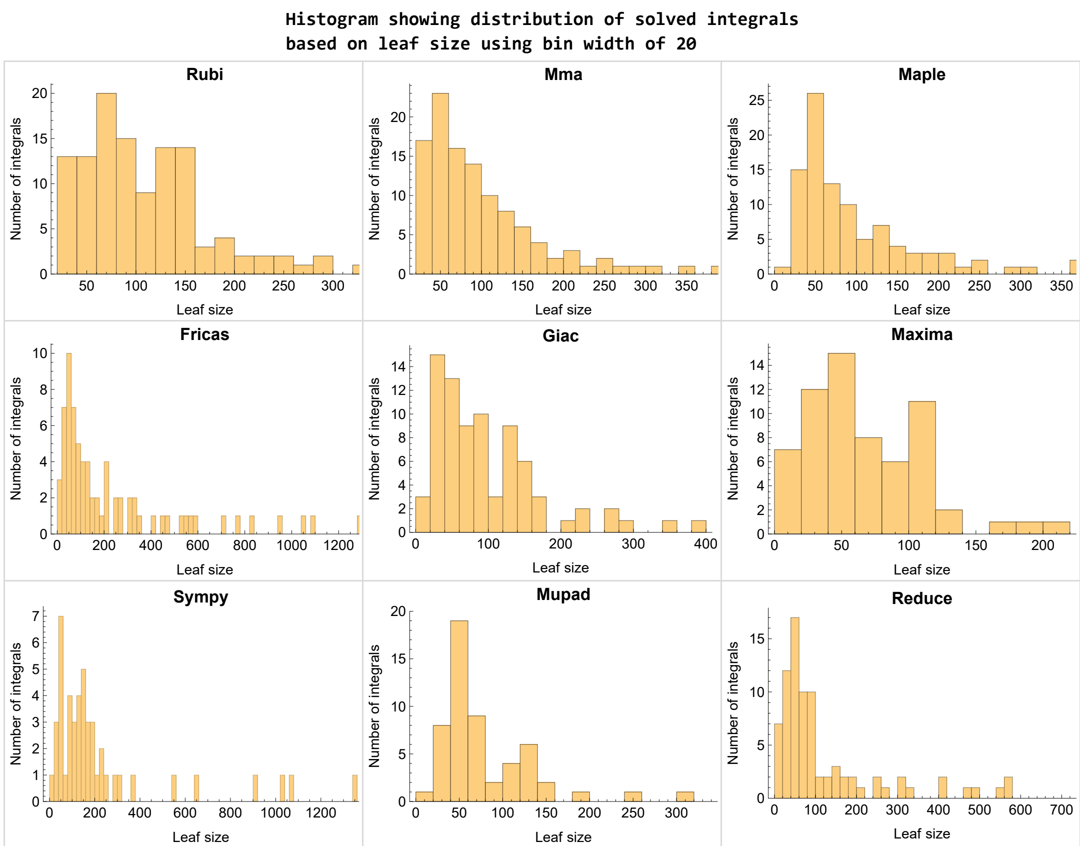


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

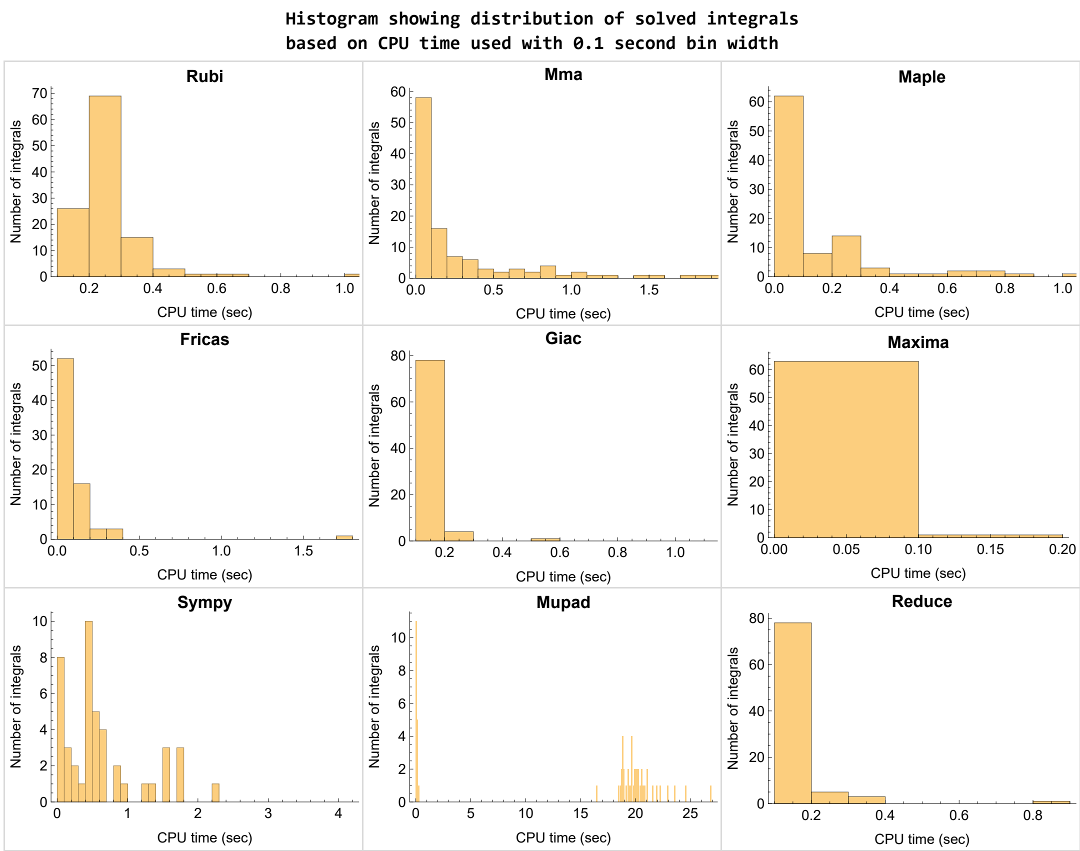


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

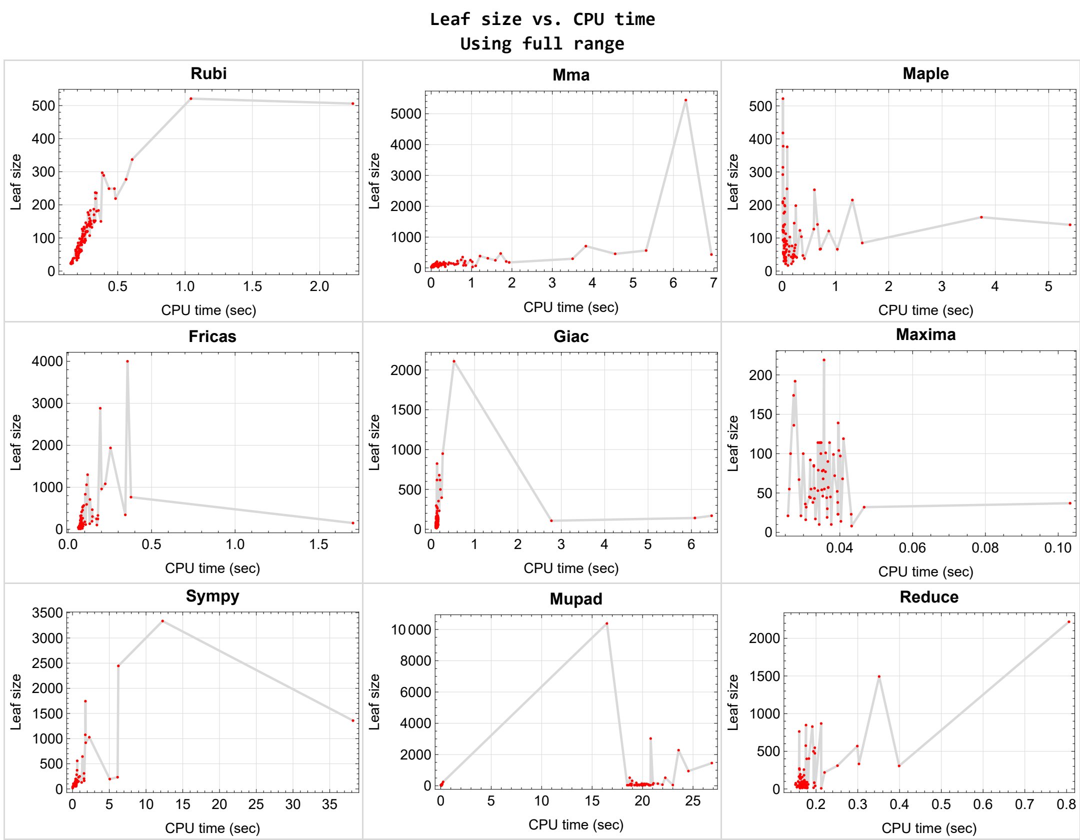


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{98, 99, 100, 101, 102, 103, 104, 105\}$

Maple $\{45, 75\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

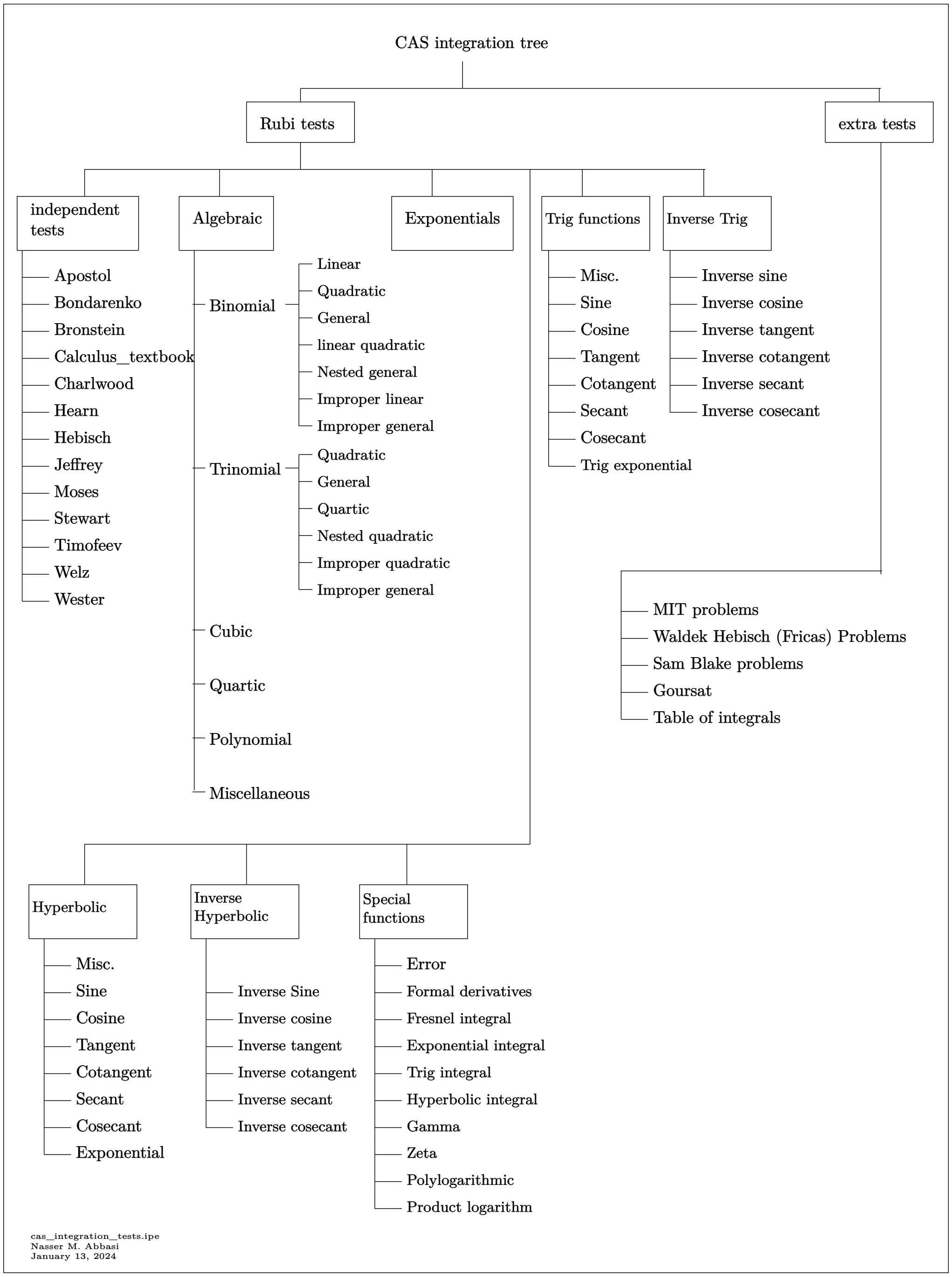
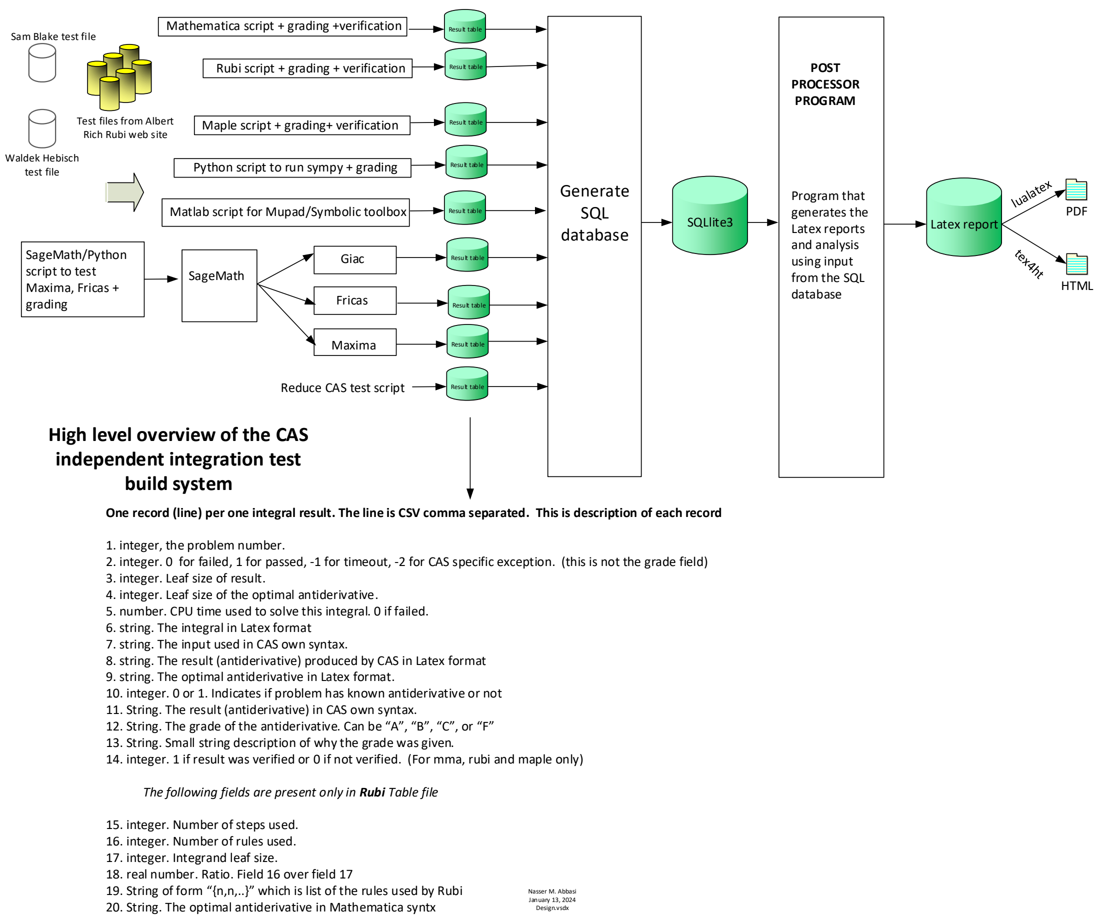


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117 }

B grade { 69, 97, 99, 100, 101, 102, 104, 105 }

C grade { 22, 23 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 66, 67, 68, 70, 71, 72, 73, 74, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 108, 112, 113, 114 }

B grade { 36, 61, 62, 63, 64, 69 }

C grade { 21, 22, 23, 45, 75 }

F normal fail { 6, 7, 79, 97, 98, 99, 100, 101, 102, 103, 104, 105, 109, 110, 111, 115, 116, 117 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 24, 25, 26, 27, 37, 38, 39, 42, 50, 51, 52, 53, 65, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 92, 96, 108, 112, 113, 114 }

B grade { 18, 19, 21, 22, 23, 28, 29, 34, 35, 36, 40, 41, 43, 44, 45, 47, 48, 54, 55, 69, 70, 71, 94, 95, 106, 107 }

C grade { }

F normal fail { 97, 98, 99, 109, 110, 111, 115, 116, 117 }

F(-1) timedout fail { 30, 31, 32, 33, 46, 49, 56, 57, 58, 59, 60, 61, 62, 63, 64, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93 }

F(-2) exception fail { 100, 101, 102, 103, 104, 105 }

Maxima

A grade { 1, 2, 3, 4, 5, 11, 12, 13, 20, 24, 25, 26, 37, 38, 39, 40, 42, 45, 46, 47, 48, 49, 50, 51, 52, 66, 67, 68, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 108, 112, 113, 114 }

B grade { 36, 41, 65, 69, 71 }

C grade { }

F normal fail { 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 21, 22, 23, 30, 31, 32, 33, 34, 35, 43, 44, 82, 97, 98, 99, 100, 101, 102, 103, 104, 105, 109, 110, 111, 115, 116, 117 }

F(-1) timedout fail { }

F(-2) exception fail { 27, 28, 29, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64 }

Giac

A grade { 1, 8, 9, 10, 11, 12, 13, 20, 24, 25, 26, 27, 28, 29, 33, 34, 37, 38, 39, 40, 41, 42, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 108, 113, 114 }

B grade { 18, 19, 21, 31, 35, 36, 43, 47, 48, 62, 63, 64, 69, 94, 95, 106, 107, 112 }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 22, 23, 97, 98, 99, 100, 101, 102, 103, 104, 105, 109, 110, 111, 115, 116, 117 }

F(-1) timedout fail { 14, 15 }

F(-2) exception fail { 16, 17, 30, 32, 56, 57 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 36, 37, 38, 39, 40, 41, 42, 50, 51, 52, 53, 54, 55, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 75, 78, 79, 86, 94, 95, 96, 106, 107, 108 }

C grade { }

F normal fail { }

F(-1) timedout fail { 14, 15, 18, 19, 30, 31, 32, 33, 34, 35, 43, 44, 45, 46, 47, 48, 49, 56, 57, 59, 72, 73, 74, 76, 77, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 104, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117 }

F(-2) exception fail { }

Sympy

A grade { 1, 21, 22, 24, 25, 26, 30, 31, 32, 33, 37, 38, 42, 43, 44, 45, 46, 49, 50, 51, 52, 56, 57, 58, 59, 65, 66, 67, 72, 73, 74, 75, 76, 96, 108 }

B grade { 27, 28, 36, 39, 40, 41, 53, 68, 69, 70, 71, 94, 95, 106, 107 }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 34, 35, 47, 48, 60, 61, 62, 63, 64, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 97, 100, 101, 102, 103, 104, 105, 109, 110, 111, 112, 113, 114, 115, 116, 117 }

F(-1) timedout fail { 23, 29, 54, 55, 81, 90, 93, 98, 99 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 14, 15, 16, 17, 18, 20, 21, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 108, 112, 113, 114 }

C grade { }

F normal fail { 6, 7, 8, 9, 10, 11, 12, 13, 19, 22, 23, 30, 56, 57, 58, 97, 98, 99, 100, 101, 102, 103, 104, 105, 109, 110, 111, 115, 116, 117 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	53	48	36	37	46	48	38	46	51
N.S.	1	1.10	1.00	0.75	0.77	0.96	1.00	0.79	0.96	1.06
time (sec)	N/A	0.211	0.023	0.053	0.103	0.071	0.082	0.129	0.178	0.110

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	64	61	90	97	0	0	90	62
N.S.	1	1.11	0.59	0.56	0.83	0.90	0.00	0.00	0.83	0.57
time (sec)	N/A	0.275	0.081	0.227	0.037	0.073	0.000	0.000	0.178	20.111

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	46	50	68	75	0	0	68	51
N.S.	1	1.08	0.58	0.62	0.85	0.94	0.00	0.00	0.85	0.64
time (sec)	N/A	0.229	0.055	0.207	0.035	0.067	0.000	0.000	0.164	19.697

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	46	53	0	0	46	40
N.S.	1	1.00	0.67	0.75	0.88	1.02	0.00	0.00	0.88	0.77
time (sec)	N/A	0.191	0.050	0.200	0.035	0.069	0.000	0.000	0.167	19.681

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	0	14	29
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	0.00	0.56	1.16
time (sec)	N/A	0.153	0.023	0.205	0.040	0.064	0.000	0.000	0.176	19.476

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	112	130	0	0	319	0	0	15	42
N.S.	1	0.68	0.79	0.00	0.00	1.95	0.00	0.00	0.09	0.26
time (sec)	N/A	0.239	0.166	0.000	0.000	0.074	0.000	0.000	0.169	18.480

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	170	184	0	0	543	0	0	33	42
N.S.	1	0.78	0.85	0.00	0.00	2.50	0.00	0.00	0.15	0.19
time (sec)	N/A	0.324	0.312	0.000	0.000	0.089	0.000	0.000	0.176	18.900

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	249	251	246	0	266	0	169	114	42
N.S.	1	0.96	0.97	0.95	0.00	1.02	0.00	0.65	0.44	0.16
time (sec)	N/A	0.435	0.973	0.606	0.000	0.077	0.000	6.466	0.205	18.890

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	187	223	198	0	214	0	141	74	42
N.S.	1	0.90	1.08	0.96	0.00	1.03	0.00	0.68	0.36	0.20
time (sec)	N/A	0.322	0.651	0.257	0.000	0.074	0.000	6.080	0.187	19.192

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	127	170	145	0	214	0	106	31	42
N.S.	1	0.82	1.10	0.94	0.00	1.39	0.00	0.69	0.20	0.27
time (sec)	N/A	0.235	0.442	0.227	0.000	0.074	0.000	2.773	0.181	18.640

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	17	21	0	14	15	21
N.S.	1	1.00	1.00	0.96	0.74	0.91	0.00	0.61	0.65	0.91
time (sec)	N/A	0.151	0.174	0.176	0.033	0.069	0.000	0.129	0.169	18.895

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	83	46	41	38	54	0	57	33	51
N.S.	1	1.06	0.59	0.53	0.49	0.69	0.00	0.73	0.42	0.65
time (sec)	N/A	0.226	0.349	0.210	0.039	0.074	0.000	0.134	0.171	19.354

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	152	68	70	72	87	0	89	53	124
N.S.	1	1.13	0.51	0.52	0.54	0.65	0.00	0.66	0.40	0.93
time (sec)	N/A	0.322	1.104	0.237	0.039	0.077	0.000	0.126	0.183	20.414

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	219	208	249	0	959	0	0	332	0
N.S.	1	1.07	1.02	1.22	0.00	4.70	0.00	0.00	1.63	0.00
time (sec)	N/A	0.484	1.859	0.091	0.000	0.201	0.000	0.000	0.303	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	150	158	179	0	709	0	0	182	0
N.S.	1	1.03	1.09	1.23	0.00	4.89	0.00	0.00	1.26	0.00
time (sec)	N/A	0.375	0.825	0.049	0.000	0.131	0.000	0.000	0.171	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	131	121	0	590	0	0	828	100
N.S.	1	1.02	1.25	1.15	0.00	5.62	0.00	0.00	7.89	0.95
time (sec)	N/A	0.291	0.260	0.030	0.000	0.111	0.000	0.000	0.191	0.137

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	89	88	0	171	0	0	61	53
N.S.	1	1.00	1.33	1.31	0.00	2.55	0.00	0.00	0.91	0.79
time (sec)	N/A	0.215	0.217	0.038	0.000	0.081	0.000	0.000	0.181	18.838

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	148	140	197	0	465	0	231	499	0
N.S.	1	1.18	1.12	1.58	0.00	3.72	0.00	1.85	3.99	0.00
time (sec)	N/A	0.326	0.663	0.053	0.000	0.145	0.000	0.187	0.194	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	249	246	376	0	1081	0	499	16	0
N.S.	1	1.18	1.17	1.78	0.00	5.12	0.00	2.36	0.08	0.00
time (sec)	N/A	0.475	1.590	0.095	0.000	0.223	0.000	0.213	200.024	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	46	32	40	8	8	0	29	8	134
N.S.	1	0.66	0.46	0.57	0.11	0.11	0.00	0.41	0.11	1.91
time (sec)	N/A	0.195	1.024	0.046	0.043	0.075	0.000	0.151	0.212	19.920

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	183	202	57	0	1059	129	2109	547	3026
N.S.	1	1.02	1.13	0.32	0.00	5.92	0.72	11.78	3.06	16.91
time (sec)	N/A	0.356	0.146	0.058	0.000	0.111	1.240	0.527	0.197	20.805

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	521	70	59	0	2882	196	0	48	2280
N.S.	1	0.83	0.11	0.09	0.00	4.57	0.31	0.00	0.08	3.61
time (sec)	N/A	1.044	0.044	0.132	0.000	0.193	5.064	0.000	0.200	23.569

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	337	70	59	0	4001	0	0	16	10382
N.S.	1	0.90	0.19	0.16	0.00	10.64	0.00	0.00	0.04	27.61
time (sec)	N/A	0.607	0.050	0.073	0.000	0.356	0.000	0.000	200.022	16.467

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	92	97	67	84	81	104	82	84	72
N.S.	1	1.08	1.14	0.79	0.99	0.95	1.22	0.96	0.99	0.85
time (sec)	N/A	0.261	0.072	0.722	0.033	0.068	0.117	0.106	0.196	19.604

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	58	52	38	45	45	51	42	41	41
N.S.	1	1.09	0.98	0.72	0.85	0.85	0.96	0.79	0.77	0.77
time (sec)	N/A	0.222	0.031	0.418	0.037	0.065	0.092	0.106	0.198	0.026

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	19	16	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.86	0.73	0.73	0.73
time (sec)	N/A	0.153	0.004	0.110	0.031	0.068	0.022	0.115	0.194	0.035

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	61	57	0	262	250	56	82	114
N.S.	1	1.07	1.02	0.95	0.00	4.37	4.17	0.93	1.37	1.90
time (sec)	N/A	0.228	0.122	0.067	0.000	0.088	0.970	0.124	0.196	0.138

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	81	75	0	565	1358	81	219	117
N.S.	1	1.01	1.05	0.97	0.00	7.34	17.64	1.05	2.84	1.52
time (sec)	N/A	0.219	0.192	0.133	0.000	0.094	38.165	0.115	0.220	18.798

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	133	119	123	0	1299	0	139	569	252
N.S.	1	1.17	1.04	1.08	0.00	11.39	0.00	1.22	4.99	2.21
time (sec)	N/A	0.270	0.275	0.334	0.000	0.117	0.000	0.113	0.299	0.240

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	219	197	175	0	0	1076	0	14	0
N.S.	1	1.10	0.98	0.88	0.00	0.00	5.38	0.00	0.07	0.00
time (sec)	N/A	0.335	1.018	0.032	0.000	0.000	1.737	0.000	200.033	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	170	140	134	0	0	371	396	306	0
N.S.	1	1.09	0.90	0.86	0.00	0.00	2.38	2.54	1.96	0.00
time (sec)	N/A	0.285	0.550	0.023	0.000	0.000	0.616	0.242	0.399	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	121	103	93	0	0	192	0	158	0
N.S.	1	1.08	0.92	0.83	0.00	0.00	1.71	0.00	1.41	0.00
time (sec)	N/A	0.241	0.334	0.018	0.000	0.000	0.478	0.000	0.167	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	69	66	52	0	0	148	54	59	0
N.S.	1	1.05	1.00	0.79	0.00	0.00	2.24	0.82	0.89	0.00
time (sec)	N/A	0.203	0.199	0.023	0.000	0.000	0.350	0.164	0.161	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	57	0	100	0	43	101	0
N.S.	1	1.00	1.00	1.46	0.00	2.56	0.00	1.10	2.59	0.00
time (sec)	N/A	0.176	0.350	0.020	0.000	0.173	0.000	0.129	0.169	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	85	81	95	0	342	0	150	309	0
N.S.	1	1.02	0.98	1.14	0.00	4.12	0.00	1.81	3.72	0.00
time (sec)	N/A	0.205	0.800	0.021	0.000	0.343	0.000	0.139	0.251	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	76	66	100	69	76	65	67	65
N.S.	1	1.05	2.00	1.74	2.63	1.82	2.00	1.71	1.76	1.71
time (sec)	N/A	0.207	0.029	1.036	0.030	0.090	0.135	0.108	0.175	0.042

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	52	44	55	47	51	43	43	43
N.S.	1	1.05	1.37	1.16	1.45	1.24	1.34	1.13	1.13	1.13
time (sec)	N/A	0.193	0.022	0.210	0.032	0.071	0.097	0.110	0.159	0.027

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	22	21	21	24	21	21	21
N.S.	1	1.00	0.96	0.81	0.78	0.78	0.89	0.78	0.78	0.78
time (sec)	N/A	0.156	0.018	0.065	0.026	0.068	0.027	0.112	0.162	0.040

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	30	32	50	80	30	39	29
N.S.	1	1.00	0.88	0.91	0.97	1.52	2.42	0.91	1.18	0.88
time (sec)	N/A	0.196	0.037	0.033	0.047	0.074	0.215	0.111	0.162	0.084

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	30	31	45	72	97	22	42	45
N.S.	1	1.11	0.83	0.86	1.25	2.00	2.69	0.61	1.17	1.25
time (sec)	N/A	0.195	0.038	0.051	0.032	0.070	0.464	0.108	0.156	20.257

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	30	31	67	120	148	22	66	68
N.S.	1	1.05	0.79	0.82	1.76	3.16	3.89	0.58	1.74	1.79
time (sec)	N/A	0.196	0.038	0.071	0.029	0.078	0.815	0.108	0.167	19.515

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	47	55	50	54	53	51	49	46	44
N.S.	1	1.18	1.38	1.25	1.35	1.32	1.28	1.22	1.15	1.10
time (sec)	N/A	0.192	0.026	0.219	0.035	0.076	0.174	0.117	0.163	0.042

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	91	76	0	246	162	128	56	0
N.S.	1	1.03	1.28	1.07	0.00	3.46	2.28	1.80	0.79	0.00
time (sec)	N/A	0.205	0.050	0.054	0.000	0.170	1.591	0.115	0.161	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	67	54	0	188	133	67	32	0
N.S.	1	1.03	0.94	0.76	0.00	2.65	1.87	0.94	0.45	0.00
time (sec)	N/A	0.202	0.034	0.042	0.000	0.143	0.651	0.111	0.176	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	43	31	10	130	105	19	12	0
N.S.	1	1.06	0.64	0.46	0.15	1.94	1.57	0.28	0.18	0.00
time (sec)	N/A	0.198	0.011	0.069	0.034	0.130	0.517	0.112	0.179	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	50	41	23	0	124	45	20	0
N.S.	1	1.01	0.67	0.55	0.31	0.00	1.65	0.60	0.27	0.00
time (sec)	N/A	0.205	0.034	0.043	0.039	0.000	0.408	0.117	0.162	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	71	40	32	23	328	0	137	22	0
N.S.	1	1.08	0.61	0.48	0.35	4.97	0.00	2.08	0.33	0.00
time (sec)	N/A	0.200	0.046	0.059	0.043	0.179	0.000	0.131	0.179	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	43	32	68	764	0	202	53	0
N.S.	1	1.03	0.61	0.45	0.96	10.76	0.00	2.85	0.75	0.00
time (sec)	N/A	0.201	0.047	0.069	0.041	0.377	0.000	0.131	0.167	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	50	41	23	0	124	45	20	0
N.S.	1	1.01	0.67	0.55	0.31	0.00	1.65	0.60	0.27	0.00
time (sec)	N/A	0.205	0.045	0.052	0.039	0.000	0.412	0.112	0.170	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	152	164	140	136	140	158	139	138	125
N.S.	1	1.04	1.12	0.96	0.93	0.96	1.08	0.95	0.95	0.86
time (sec)	N/A	0.333	0.116	5.401	0.027	0.072	0.584	0.130	0.162	18.910

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	97	101	85	85	86	99	84	83	73
N.S.	1	1.05	1.10	0.92	0.92	0.93	1.08	0.91	0.90	0.79
time (sec)	N/A	0.257	0.063	1.503	0.033	0.068	0.465	0.114	0.160	0.042

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	60	56	41	44	44	51	42	44	41
N.S.	1	1.15	1.08	0.79	0.85	0.85	0.98	0.81	0.85	0.79
time (sec)	N/A	0.214	0.034	0.274	0.032	0.061	0.423	0.105	0.166	0.031

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	86	86	83	0	413	644	75	148	517
N.S.	1	1.01	1.01	0.98	0.00	4.86	7.58	0.88	1.74	6.08
time (sec)	N/A	0.254	0.246	0.071	0.000	0.081	1.352	0.125	0.163	18.725

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	87	101	104	0	833	0	96	258	142
N.S.	1	0.96	1.11	1.14	0.00	9.15	0.00	1.05	2.84	1.56
time (sec)	N/A	0.224	0.267	0.362	0.000	0.103	0.000	0.114	0.160	0.159

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	159	146	215	0	1937	0	158	848	319
N.S.	1	1.04	0.95	1.41	0.00	12.66	0.00	1.03	5.54	2.08
time (sec)	N/A	0.290	0.493	1.318	0.000	0.254	0.000	0.114	0.176	18.968

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	289	312	378	0	0	1744	0	16	0
N.S.	1	1.08	1.16	1.41	0.00	0.00	6.51	0.00	0.06	0.00
time (sec)	N/A	0.396	1.403	0.020	0.000	0.000	1.758	0.000	200.017	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	236	219	292	0	0	559	0	16	0
N.S.	1	1.12	1.04	1.38	0.00	0.00	2.65	0.00	0.08	0.00
time (sec)	N/A	0.341	0.842	0.013	0.000	0.000	0.638	0.000	200.024	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	183	146	206	0	0	282	132	16	193
N.S.	1	1.19	0.95	1.34	0.00	0.00	1.83	0.86	0.10	1.25
time (sec)	N/A	0.297	0.359	0.014	0.000	0.000	0.559	0.155	200.024	19.384

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	128	103	119	0	0	199	80	137	0
N.S.	1	1.29	1.04	1.20	0.00	0.00	2.01	0.81	1.38	0.00
time (sec)	N/A	0.253	0.320	0.018	0.000	0.000	0.413	0.152	0.163	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	146	107	124	0	0	0	99	473	84
N.S.	1	1.45	1.06	1.23	0.00	0.00	0.00	0.98	4.68	0.83
time (sec)	N/A	0.262	0.580	0.013	0.000	0.000	0.000	0.160	0.197	20.596

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	98	84	210	0	0	0	151	403	159
N.S.	1	0.92	0.79	1.96	0.00	0.00	0.00	1.41	3.77	1.49
time (sec)	N/A	0.218	0.859	0.013	0.000	0.000	0.000	0.161	0.183	21.084

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	177	178	314	0	0	0	353	869	515
N.S.	1	1.06	1.07	1.88	0.00	0.00	0.00	2.11	5.20	3.08
time (sec)	N/A	0.278	1.932	0.015	0.000	0.000	0.000	0.168	0.212	22.253

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	237	296	418	0	0	0	618	1493	942
N.S.	1	1.07	1.33	1.88	0.00	0.00	0.00	2.78	6.73	4.24
time (sec)	N/A	0.333	3.503	0.015	0.000	0.000	0.000	0.205	0.351	24.545

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	297	434	522	0	0	0	949	2219	1455
N.S.	1	1.06	1.56	1.87	0.00	0.00	0.00	3.40	7.95	5.22
time (sec)	N/A	0.385	6.941	0.016	0.000	0.000	0.000	0.266	0.806	26.864

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	78	66	100	72	80	65	66	65
N.S.	1	1.03	1.32	1.12	1.69	1.22	1.36	1.10	1.12	1.10
time (sec)	N/A	0.231	0.028	0.711	0.026	0.070	0.532	0.118	0.152	19.642

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	54	44	55	46	53	43	44	43
N.S.	1	1.07	0.95	0.77	0.96	0.81	0.93	0.75	0.77	0.75
time (sec)	N/A	0.226	0.023	0.251	0.026	0.069	0.494	0.119	0.162	0.027

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	22	21	21	27	21	24	21
N.S.	1	1.00	1.03	0.76	0.72	0.72	0.93	0.72	0.83	0.72
time (sec)	N/A	0.164	0.017	0.069	0.029	0.068	0.022	0.108	0.154	0.041

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	47	56	41	43	68	109	41	57	43
N.S.	1	1.02	1.22	0.89	0.93	1.48	2.37	0.89	1.24	0.93
time (sec)	N/A	0.207	0.058	0.064	0.033	0.081	0.213	0.113	0.151	20.029

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	23	40	48	56	109	141	33	34	56
N.S.	1	1.44	2.50	3.00	3.50	6.81	8.81	2.06	2.12	3.50
time (sec)	N/A	0.159	0.044	0.125	0.033	0.075	0.444	0.131	0.162	20.128

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	43	48	78	176	233	33	77	80
N.S.	1	1.07	0.75	0.84	1.37	3.09	4.09	0.58	1.35	1.40
time (sec)	N/A	0.214	0.051	0.227	0.035	0.100	0.821	0.113	0.151	21.021

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	43	47	100	244	309	33	99	101
N.S.	1	1.09	0.77	0.84	1.79	4.36	5.52	0.59	1.77	1.80
time (sec)	N/A	0.212	0.046	0.390	0.035	0.179	1.563	0.116	0.157	0.115

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	107	117	98	114	84	233	140	77	0
N.S.	1	0.78	0.85	0.72	0.83	0.61	1.70	1.02	0.56	0.00
time (sec)	N/A	0.251	0.057	0.048	0.034	0.071	6.131	0.153	0.170	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	107	93	76	114	61	204	102	55	0
N.S.	1	0.78	0.68	0.55	0.83	0.45	1.49	0.74	0.40	0.00
time (sec)	N/A	0.243	0.042	0.039	0.035	0.073	1.563	0.130	0.164	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	93	67	54	114	32	175	64	33	0
N.S.	1	0.68	0.49	0.39	0.83	0.23	1.28	0.47	0.24	0.00
time (sec)	N/A	0.231	0.034	0.037	0.034	0.064	0.625	0.110	0.164	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	88	59	43	42	114	10	146	26	13	71
N.S.	1	0.67	0.49	0.48	1.30	0.11	1.66	0.30	0.15	0.81
time (sec)	N/A	0.189	0.010	0.070	0.035	0.064	0.511	0.114	0.167	21.972

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	85	65	52	36	33	170	61	33	0
N.S.	1	0.58	0.44	0.35	0.24	0.22	1.16	0.41	0.22	0.00
time (sec)	N/A	0.228	0.046	0.046	0.031	0.073	0.470	0.144	0.157	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	99	72	81	55	113	0	64	81	0
N.S.	1	0.76	0.55	0.62	0.42	0.87	0.00	0.49	0.62	0.00
time (sec)	N/A	0.236	0.072	0.092	0.036	0.090	0.000	0.127	0.175	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	105	56	43	53	136	0	43	66	53
N.S.	1	0.78	0.41	0.32	0.39	1.01	0.00	0.32	0.49	0.39
time (sec)	N/A	0.236	0.060	0.171	0.034	0.090	0.000	0.120	0.170	22.999

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	138	83	0	77	110	0	229	98	138
N.S.	1	0.97	0.58	0.00	0.54	0.77	0.00	1.61	0.69	0.97
time (sec)	N/A	0.278	0.147	0.000	0.036	0.105	0.000	0.137	0.165	21.514

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	112	93	103	114	147	0	83	92	0
N.S.	1	0.64	0.53	0.59	0.65	0.84	0.00	0.47	0.52	0.00
time (sec)	N/A	0.256	0.117	0.145	0.037	1.706	0.000	0.137	0.175	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	152	121	163	119	0	0	105	173	0
N.S.	1	0.57	0.45	0.61	0.44	0.00	0.00	0.39	0.65	0.00
time (sec)	N/A	0.298	0.169	3.739	0.041	0.000	0.000	0.160	0.161	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	88	66	65	0	0	0	80	36	0
N.S.	1	0.50	0.38	0.37	0.00	0.00	0.00	0.46	0.21	0.00
time (sec)	N/A	0.246	0.058	0.083	0.000	0.000	0.000	0.142	0.158	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	148	125	113	79	0	0	173	83	0
N.S.	1	0.38	0.32	0.29	0.20	0.00	0.00	0.45	0.21	0.00
time (sec)	N/A	0.295	0.094	0.068	0.035	0.000	0.000	0.137	0.169	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	120	99	91	57	0	0	128	59	0
N.S.	1	0.42	0.34	0.32	0.20	0.00	0.00	0.45	0.21	0.00
time (sec)	N/A	0.267	0.082	0.045	0.037	0.000	0.000	0.134	0.169	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	91	75	69	30	0	0	79	30	0
N.S.	1	0.49	0.41	0.38	0.16	0.00	0.00	0.43	0.16	0.00
time (sec)	N/A	0.234	0.056	0.040	0.037	0.000	0.000	0.141	0.168	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	59	49	47	10	0	0	34	10	39
N.S.	1	0.67	0.56	0.53	0.11	0.00	0.00	0.39	0.11	0.44
time (sec)	N/A	0.191	0.024	0.038	0.038	0.000	0.000	0.119	0.161	20.791

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	96	86	78	44	0	0	77	43	0
N.S.	1	0.51	0.45	0.41	0.23	0.00	0.00	0.41	0.23	0.00
time (sec)	N/A	0.257	0.048	0.039	0.036	0.000	0.000	0.157	0.163	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	139	126	141	97	0	0	127	117	0
N.S.	1	0.46	0.42	0.47	0.32	0.00	0.00	0.42	0.39	0.00
time (sec)	N/A	0.292	0.128	0.049	0.040	0.000	0.000	0.166	0.172	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	181	152	199	139	0	0	147	195	0
N.S.	1	0.44	0.37	0.49	0.34	0.00	0.00	0.36	0.48	0.00
time (sec)	N/A	0.342	0.125	0.047	0.040	0.000	0.000	0.154	0.162	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	118	98	91	57	0	0	126	60	0
N.S.	1	0.41	0.34	0.32	0.20	0.00	0.00	0.44	0.21	0.00
time (sec)	N/A	0.270	0.073	0.082	0.037	0.000	0.000	0.132	0.170	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	119	101	91	52	0	0	125	52	0
N.S.	1	0.42	0.35	0.32	0.18	0.00	0.00	0.44	0.18	0.00
time (sec)	N/A	0.262	0.075	0.064	0.039	0.000	0.000	0.155	0.172	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	139	98	141	99	302	0	84	163	0
N.S.	1	0.63	0.44	0.64	0.45	1.36	0.00	0.38	0.73	0.00
time (sec)	N/A	0.284	0.117	0.664	0.038	0.146	0.000	0.137	0.159	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	150	124	117	79	0	0	172	82	0
N.S.	1	0.39	0.32	0.30	0.20	0.00	0.00	0.44	0.21	0.00
time (sec)	N/A	0.296	0.085	0.026	0.034	0.000	0.000	0.150	0.170	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	120	127	174	525	3337	824	763	126
N.S.	1	1.00	0.91	0.96	1.32	3.98	25.28	6.24	5.78	0.95
time (sec)	N/A	0.308	0.621	0.595	0.027	0.087	12.257	0.136	0.160	20.216

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	76	92	205	1027	296	271	75
N.S.	1	1.00	0.90	0.96	1.16	2.59	13.00	3.75	3.43	0.95
time (sec)	N/A	0.233	0.146	0.186	0.032	0.073	2.283	0.116	0.160	20.557

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	32	51	37	32	53	30
N.S.	1	1.00	1.00	0.97	1.00	1.59	1.16	1.00	1.66	0.94
time (sec)	N/A	0.162	0.064	0.056	0.031	0.074	0.030	0.128	0.174	20.664

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	133	261	0	0	0	0	0	18	0
N.S.	1	1.07	2.10	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.249	0.743	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	277	456	0	0	0	0	0	51	0
N.S.	1	0.98	1.61	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.562	4.557	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	489	506	5445	0	0	0	0	0	100	0
N.S.	1	1.03	11.13	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.246	6.306	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	706	0	0	0	0	0	0	0
N.S.	1	1.00	4.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	3.830	0.000	0.000	0.000	0.000	0.000	0.417	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	466	0	0	0	0	0	0	0
N.S.	1	1.00	3.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	1.722	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	351	0	0	0	0	0	274	0
N.S.	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	1.97	0.00
time (sec)	N/A	0.283	0.780	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	166	0	0	0	0	0	34	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.273	0.219	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	384	0	0	0	0	0	67	0
N.S.	1	1.00	2.70	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.284	1.210	0.000	0.000	0.000	0.000	0.000	0.344	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	564	0	0	0	0	0	116	0
N.S.	1	1.00	3.97	0.00	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.286	5.323	0.000	0.000	0.000	0.000	0.000	0.697	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	135	114	121	192	454	2445	618	575	120
N.S.	1	1.07	0.90	0.96	1.52	3.60	19.40	4.90	4.56	0.95
time (sec)	N/A	0.294	0.421	0.875	0.028	0.087	6.236	0.134	0.175	20.057

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	91	74	79	104	216	918	278	257	78
N.S.	1	1.11	0.90	0.96	1.27	2.63	11.20	3.39	3.13	0.95
time (sec)	N/A	0.243	0.162	0.251	0.040	0.080	1.792	0.133	0.171	19.805

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	37	38	65	44	38	66	36
N.S.	1	1.00	0.89	0.97	1.00	1.71	1.16	1.00	1.74	0.95
time (sec)	N/A	0.168	0.066	0.063	0.033	0.076	0.024	0.110	0.169	19.902

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	0	0	24	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.164	0.023	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	0	0	50	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.08	0.00
time (sec)	N/A	0.163	0.022	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	0	0	76	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	3.17	0.00
time (sec)	N/A	0.161	0.022	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	149	116	220	219	324	0	679	400	0
N.S.	1	0.46	0.36	0.68	0.68	1.00	0.00	2.10	1.23	0.00
time (sec)	N/A	0.290	0.253	0.043	0.036	0.079	0.000	0.186	0.178	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	105	122	138	101	130	0	263	146	0
N.S.	1	0.51	0.59	0.67	0.49	0.63	0.00	1.28	0.71	0.00
time (sec)	N/A	0.244	0.116	0.025	0.036	0.075	0.000	0.145	0.163	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	57	39	56	19	20	0	25	17	0
N.S.	1	0.65	0.44	0.64	0.22	0.23	0.00	0.28	0.19	0.00
time (sec)	N/A	0.196	0.016	0.024	0.036	0.085	0.000	0.127	0.173	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	62	44	0	0	0	0	0	11	0
N.S.	1	1.13	0.80	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.189	0.035	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	64	46	0	0	0	0	0	37	0
N.S.	1	1.16	0.84	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.187	0.033	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	64	46	0	0	0	0	0	63	0
N.S.	1	1.16	0.84	0.00	0.00	0.00	0.00	0.00	1.15	0.00
time (sec)	N/A	0.190	0.034	0.000	0.000	0.000	0.000	0.000	0.181	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [22] had the largest ratio of [.857142999999999988]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	9	1.10	11	0.818
2	A	4	4	1.11	15	0.267
3	A	3	3	1.08	15	0.200
4	A	2	2	1.00	15	0.133
5	A	1	1	1.00	15	0.067
6	A	7	6	0.68	15	0.400
7	A	9	8	0.78	15	0.533
8	A	7	7	0.96	15	0.467
9	A	5	5	0.90	15	0.333
10	A	3	3	0.82	15	0.200
11	A	1	1	1.00	15	0.067
12	A	3	3	1.06	15	0.200
13	A	5	5	1.13	15	0.333
14	A	13	12	1.07	16	0.750
15	A	11	10	1.03	16	0.625
16	A	8	7	1.02	16	0.438
17	A	5	4	1.00	16	0.250
18	A	7	6	1.18	16	0.375
19	A	9	8	1.18	16	0.500
20	A	2	2	0.66	22	0.091
21	A	4	4	1.02	14	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	13	12	0.83	14	0.857
23	A	6	6	0.90	14	0.429
24	A	4	3	1.08	14	0.214
25	A	4	3	1.09	14	0.214
26	A	1	1	1.00	12	0.083
27	A	6	5	1.07	14	0.357
28	A	5	4	1.01	14	0.286
29	A	6	5	1.17	14	0.357
30	A	8	7	1.10	16	0.438
31	A	7	6	1.09	16	0.375
32	A	6	5	1.08	16	0.312
33	A	5	4	1.05	16	0.250
34	A	3	2	1.00	16	0.125
35	A	4	3	1.02	16	0.188
36	A	6	5	1.05	20	0.250
37	A	6	5	1.05	20	0.250
38	A	1	1	1.00	18	0.056
39	A	6	5	1.00	20	0.250
40	A	6	5	1.11	20	0.250
41	A	6	5	1.05	20	0.250
42	A	6	5	1.18	23	0.217
43	A	5	4	1.03	22	0.182
44	A	5	4	1.03	22	0.182
45	A	5	4	1.06	22	0.182
46	A	5	4	1.01	22	0.182
47	A	4	3	1.08	22	0.136
48	A	4	3	1.03	22	0.136
49	A	5	4	1.01	22	0.182
50	A	4	3	1.04	18	0.167
51	A	4	3	1.05	18	0.167
52	A	4	3	1.15	18	0.167
53	A	4	3	1.01	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	5	4	0.96	18	0.222
55	A	7	6	1.04	18	0.333
56	A	10	9	1.08	20	0.450
57	A	9	8	1.12	20	0.400
58	A	8	7	1.19	20	0.350
59	A	7	6	1.29	20	0.300
60	A	6	5	1.45	20	0.250
61	A	4	3	0.92	20	0.150
62	A	5	4	1.06	20	0.200
63	A	6	5	1.07	20	0.250
64	A	7	6	1.06	20	0.300
65	A	6	5	1.03	24	0.208
66	A	6	5	1.07	24	0.208
67	A	1	1	1.00	22	0.045
68	A	6	5	1.02	24	0.208
69	A	2	2	1.44	24	0.083
70	A	6	5	1.07	24	0.208
71	A	6	5	1.09	24	0.208
72	A	6	5	0.78	26	0.192
73	A	6	5	0.78	26	0.192
74	A	6	5	0.68	26	0.192
75	A	2	2	0.67	26	0.077
76	A	6	5	0.58	26	0.192
77	A	6	5	0.76	26	0.192
78	A	6	5	0.78	26	0.192
79	A	5	4	0.97	24	0.167
80	A	6	5	0.64	26	0.192
81	A	6	5	0.57	26	0.192
82	A	7	6	0.50	24	0.250
83	A	7	6	0.38	26	0.231
84	A	7	6	0.42	26	0.231
85	A	7	6	0.49	26	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	0.67	26	0.077
87	A	7	6	0.51	26	0.231
88	A	7	6	0.46	26	0.231
89	A	7	6	0.44	26	0.231
90	A	7	6	0.41	26	0.231
91	A	7	6	0.42	26	0.231
92	A	6	5	0.63	26	0.192
93	A	7	6	0.39	26	0.231
94	A	2	2	1.00	16	0.125
95	A	2	2	1.00	16	0.125
96	A	1	1	1.00	14	0.071
97	A	2	2	1.07	16	0.125
98	A	4	4	0.98	16	0.250
99	A	6	6	1.03	16	0.375
100	A	2	2	1.00	18	0.111
101	A	2	2	1.00	18	0.111
102	A	2	2	1.00	18	0.111
103	A	2	2	1.00	18	0.111
104	A	2	2	1.00	18	0.111
105	A	2	2	1.00	18	0.111
106	A	3	3	1.07	22	0.136
107	A	3	3	1.11	22	0.136
108	A	1	1	1.00	20	0.050
109	A	2	2	1.00	22	0.091
110	A	2	2	1.00	22	0.091
111	A	2	2	1.00	22	0.091
112	A	3	3	0.46	24	0.125
113	A	3	3	0.51	24	0.125
114	A	2	2	0.65	24	0.083
115	A	2	2	1.13	24	0.083
116	A	2	2	1.16	24	0.083
117	A	2	2	1.16	24	0.083

CHAPTER 3

Listing of integrals

3.1 $\int \frac{1}{-x^3+x^6} dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{1}{-x^3+x^6} dx = \frac{1}{2x^2} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)$$

output
```
1/2/x^2-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*ln(1-x)-1/6*ln(x^2+x+1
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+x^6} dx = \frac{1}{2x^2} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)$$

input
```
Integrate[(-x^3 + x^6)^(-1),x]
```

output
```
1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
 x^2]/6
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {2026, 847, 750, 16, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6-x^3}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^3\left(x^3-1\right)}dx$$

$$\Big\downarrow \text{847}$$

$$\int \frac{1}{x^3-1}dx+\frac{1}{2x^2}$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{3}\int -\frac{x+2}{x^2+x+1}dx+\frac{1}{3}\int \frac{1}{x-1}dx+\frac{1}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int -\frac{x+2}{x^2+x+1}dx+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\int \frac{x+2}{x^2+x+1}dx+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2+x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(3\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

$$\downarrow \text{1103}$$

$$\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)+\frac{1}{2x^2}+\frac{1}{3}\log(1-x)$$

input
```
Int[(-x^3 + x^6)^(-1),x]
```

output
```
1/(2*x^2) + Log[1 - x]/3 + (-(Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]) - Log[1 +
 x + x^2]/2)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{1}{2x^2}+\frac{\ln(x-1)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt{3}\left(x+\frac{1}{2}\right)}{3}\right)}{3}$	36
default	$\frac{\ln(x-1)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{1}{2x^2}$	38
meijerg	$-\frac{(-1)^{\frac{2}{3}}\left(\frac{3(-1)^{\frac{1}{3}}}{2x^2}+\frac{x(-1)^{\frac{1}{3}}\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{\left(x^3\right)^{\frac{1}{3}}}\right)}{3}$	78

input
```
int(1/(x^6-x^3),x,method=_RETURNVERBOSE)
```

output
```
1/2/x^2+1/3*ln(x-1)-1/6*ln(x^2+x+1)-1/3*3^(1/2)*arctan(2/3*3^(1/2)*(x+1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{2\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x^2\log\left(x^2+x+1\right)-2x^2\log\left(x-1\right)-3}{6x^2}$$

input
```
integrate(1/(x^6-x^3),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) + x^2*log(x^2 + x + 1) -
 2*x^2*log(x - 1) - 3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+x^6}\,dx = \frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{2x^2}$$

input
```
integrate(1/(x**6-x**3),x)
```

output
```
log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/
3)/3 + 1/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

input
```
integrate(1/(x^6-x^3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1
) + 1/3*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{1}{-x^3+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(|x-1|\right)$$

input
```
integrate(1/(x^6-x^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1
) + 1/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{-x^3+x^6}\,dx = \frac{\ln(x-1)}{3} + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \frac{1}{2\,x^2}$$

input
```
int(-1/(x^3 - x^6),x)
```

output
```
log(x - 1)/3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(
x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1}{-x^3+x^6}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^2 - \log(x^2+x+1)\,x^2 + 2\log(x-1)\,x^2 + 3}{6x^2}$$

input
```
int(1/(x^6-x^3),x)
```

output
```
( - 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*x**2 - log(x**2 + x + 1)*x**2 + 2*lo
g(x - 1)*x**2 + 3)/(6*x**2)
```

3.2 $\int (ax^3+bx^6)^{11/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int (ax^3+bx^6)^{11/3}\,dx=-\frac{81a^3(ax^3+bx^6)^{14/3}}{54740b^4x^{14}}+\frac{27a^2(ax^3+bx^6)^{14/3}}{3910b^3x^{11}}-\frac{9a(ax^3+bx^6)^{14/3}}{460b^2x^8}+\frac{(ax^3+bx^6)^{14/3}}{23bx^5}$$

output
```
-81/54740*a^3*(b*x^6+a*x^3)^(14/3)/b^4/x^14+27/3910*a^2*(b*x^6+a*x^3)^(14/
3)/b^3/x^11-9/460*a*(b*x^6+a*x^3)^(14/3)/b^2/x^8+1/23*(b*x^6+a*x^3)^(14/3)
/b/x^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int (ax^3+bx^6)^{11/3}\,dx=\frac{(a+bx^3)\,(x^3(a+bx^3))^{11/3}\,(-81a^3+378a^2bx^3-1071ab^2x^6+2380b^3x^9)}{54740b^4x^{11}}$$

input
```
Integrate[(a*x^3 + b*x^6)^(11/3),x]
```

output
```
((a + b*x^3)*(x^3*(a + b*x^3))^(11/3)*(-81*a^3 + 378*a^2*b*x^3 - 1071*a*b^
2*x^6 + 2380*b^3*x^9))/(54740*b^4*x^11)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^3+bx^6\right)^{11/3}dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{\left(ax^3+bx^6\right)^{14/3}}{23bx^5}-\frac{9a\int \frac{\left(bx^6+ax^3\right)^{11/3}}{x^3}dx}{23b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{\left(ax^3+bx^6\right)^{14/3}}{23bx^5}-\frac{9a\left(\frac{\left(ax^3+bx^6\right)^{14/3}}{20bx^8}-\frac{3a\int \frac{\left(bx^6+ax^3\right)^{11/3}}{x^6}dx}{10b}\right)}{23b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{\left(ax^3+bx^6\right)^{14/3}}{23bx^5}-\frac{9a\left(\frac{\left(ax^3+bx^6\right)^{14/3}}{20bx^8}-\frac{3a\left(\frac{\left(ax^3+bx^6\right)^{14/3}}{17bx^{11}}-\frac{3a\int \frac{\left(bx^6+ax^3\right)^{11/3}}{x^9}dx}{17b}\right)}{10b}\right)}{23b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{\left(ax^3+bx^6\right)^{14/3}}{23bx^5}-\frac{9a\left(\frac{\left(ax^3+bx^6\right)^{14/3}}{20bx^8}-\frac{3a\left(\frac{\left(ax^3+bx^6\right)^{14/3}}{17bx^{11}}-\frac{3a\left(ax^3+bx^6\right)^{14/3}}{238b^2x^{14}}\right)}{10b}\right)}{23b}$$

input
```
Int[(a*x^3 + b*x^6)^(11/3),x]
```

output
```
(a*x^3 + b*x^6)^(14/3)/(23*b*x^5) - (9*a*((a*x^3 + b*x^6)^(14/3)/(20*b*x^8
) - (3*a*((-3*a*(a*x^3 + b*x^6)^(14/3))/(238*b^2*x^14) + (a*x^3 + b*x^6)^(
14/3)/(17*b*x^11)))/(10*b)))/(23*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{(b\,x^3+a)\left(-2380x^9b^3+1071a\,x^6b^2-378a^2x^3b+81a^3\right)\left(b\,x^6+a\,x^3\right)^{\frac{11}{3}}}{54740b^4x^{11}}$	61
orering	$-\frac{(b\,x^3+a)\left(-2380x^9b^3+1071a\,x^6b^2-378a^2x^3b+81a^3\right)\left(b\,x^6+a\,x^3\right)^{\frac{11}{3}}}{54740b^4x^{11}}$	61
trager	$-\frac{\left(-2380b^7x^{21}-8449a\,b^6x^{18}-10374a^2b^5x^{15}-4525a^3b^4x^{12}-40a^4x^9b^3+45a^5x^6b^2-54a^6b\,x^3+81a^7\right)\left(b\,x^6+a\,x^3\right)^{\frac{2}{3}}}{54740b^4x^2}$	98
risch	$-\frac{\left(x^3\left(b\,x^3+a\right)\right)^{\frac{2}{3}}\left(-2380b^7x^{21}-8449a\,b^6x^{18}-10374a^2b^5x^{15}-4525a^3b^4x^{12}-40a^4x^9b^3+45a^5x^6b^2-54a^6b\,x^3+81a^7\right)}{54740x^2b^4}$	98

input
```
int((b*x^6+a*x^3)^(11/3),x,method=_RETURNVERBOSE)
```

output
```
-1/54740*(b*x^3+a)*(-2380*b^3*x^9+1071*a*b^2*x^6-378*a^2*b*x^3+81*a^3)*(b*
x^6+a*x^3)^(11/3)/b^4/x^11
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int (ax^3 +bx^6)^{11/3}\,dx = \frac{(2380\,b^7x^{21}+8449\,ab^6x^{18}+10374\,a^2b^5x^{15}+4525\,a^3b^4x^{12}+40\,a^4b^3x^9-45\,a^5b^2x^6+54\,a^6b}{54740\,b^4x^2}$$

input
```
integrate((b*x^6+a*x^3)^(11/3),x, algorithm="fricas")
```

output
```
1/54740*(2380*b^7*x^21 + 8449*a*b^6*x^18 + 10374*a^2*b^5*x^15 + 4525*a^3*b
^4*x^12 + 40*a^4*b^3*x^9 - 45*a^5*b^2*x^6 + 54*a^6*b*x^3 - 81*a^7)*(b*x^6
+ a*x^3)^(2/3)/(b^4*x^2)
```

Sympy [F]

$$\int \left(ax^3+bx^6\right)^{11/3}\,dx=\int \left(ax^3+bx^6\right)^{\frac{11}{3}}\,dx$$

input
```
integrate((b*x**6+a*x**3)**(11/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(11/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \left(ax^3+bx^6\right)^{11/3}\,dx=\frac{(2380\,b^7x^{21}+8449\,ab^6x^{18}+10374\,a^2b^5x^{15}+4525\,a^3b^4x^{12}+40\,a^4b^3x^9-45\,a^5b^2x^6+54\,a^6b}{54740\,b^4}$$

input
```
integrate((b*x^6+a*x^3)^(11/3),x, algorithm="maxima")
```

output
```
1/54740*(2380*b^7*x^21 + 8449*a*b^6*x^18 + 10374*a^2*b^5*x^15 + 4525*a^3*b
^4*x^12 + 40*a^4*b^3*x^9 - 45*a^5*b^2*x^6 + 54*a^6*b*x^3 - 81*a^7)*(b*x^3
+ a)^(2/3)/b^4
```

Giac [F]

$$\int \left(ax^3+bx^6\right)^{11/3}\,dx=\int \left(bx^6+ax^3\right)^{\frac{11}{3}}\,dx$$

input
```
integrate((b*x^6+a*x^3)^(11/3),x, algorithm="giac")
```

output
```
integrate((b*x^6 + a*x^3)^(11/3), x)
```

Mupad [B] (verification not implemented)

Time = 20.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

$$\int \left(ax^3+bx^6\right)^{11/3}\,dx = -\frac{\left(b\,x^3+a\right)^4\,\left(b\,x^6+a\,x^3\right)^{2/3}\,\left(81\,a^3-378\,a^2\,b\,x^3+1071\,a\,b^2\,x^6-2380\,b^3\,x^9\right)}{54740\,b^4\,x^2}$$

input
```
int((a*x^3 + b*x^6)^(11/3),x)
```

output
```
-((a + b*x^3)^4*(a*x^3 + b*x^6)^(2/3)*(81*a^3 - 2380*b^3*x^9 - 378*a^2*b*x
^3 + 1071*a*b^2*x^6))/(54740*b^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \left(ax^3+bx^6\right)^{11/3}\,dx = \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(2380b^7x^{21}+8449a\,b^6x^{18}+10374a^2b^5x^{15}+4525a^3b^4x^{12}+40a^4b^3x^9-45a^5b^2x^6\right.}{54740b^4}$$

input
```
int((b*x^6+a*x^3)^(11/3),x)
```

output
```
((a + b*x**3)**(2/3)*( - 81*a**7 + 54*a**6*b*x**3 - 45*a**5*b**2*x**6 + 40
*a**4*b**3*x**9 + 4525*a**3*b**4*x**12 + 10374*a**2*b**5*x**15 + 8449*a*b*
*6*x**18 + 2380*b**7*x**21))/(54740*b**4)
```

3.3 $\int (ax^3 + bx^6)^{8/3} dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int (ax^3 + bx^6)^{8/3} dx = \frac{9a^2(ax^3+bx^6)^{11/3}}{1309b^3x^{11}} - \frac{3a(ax^3+bx^6)^{11/3}}{119b^2x^8} + \frac{(ax^3+bx^6)^{11/3}}{17bx^5}$$

output
```
9/1309*a^2*(b*x^6+a*x^3)^(11/3)/b^3/x^11-3/119*a*(b*x^6+a*x^3)^(11/3)/b^2/
x^8+1/17*(b*x^6+a*x^3)^(11/3)/b/x^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int (ax^3 + bx^6)^{8/3} dx = \frac{(x^3(a+bx^3))^{11/3}(9a^2 - 33abx^3 + 77b^2x^6)}{1309b^3x^{11}}$$

input
```
Integrate[(a*x^3 + b*x^6)^(8/3),x]
```

output
```
((x^3*(a + b*x^3))^(11/3)*(9*a^2 - 33*a*b*x^3 + 77*b^2*x^6))/(1309*b^3*x^1
1)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1908, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^3+bx^6\right)^{8/3}dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{\left(ax^3+bx^6\right)^{11/3}}{17bx^5}-\frac{6a\int \frac{\left(bx^6+ax^3\right)^{8/3}}{x^3}dx}{17b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{\left(ax^3+bx^6\right)^{11/3}}{17bx^5}-\frac{6a\left(\frac{\left(ax^3+bx^6\right)^{11/3}}{14bx^8}-\frac{3a\int \frac{\left(bx^6+ax^3\right)^{8/3}}{x^6}dx}{14b}\right)}{17b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{\left(ax^3+bx^6\right)^{11/3}}{17bx^5}-\frac{6a\left(\frac{\left(ax^3+bx^6\right)^{11/3}}{14bx^8}-\frac{3a\left(ax^3+bx^6\right)^{11/3}}{154b^2x^{11}}\right)}{17b}$$

input
```
Int[(a*x^3 + b*x^6)^(8/3),x]
```

output
```
(a*x^3 + b*x^6)^(11/3)/(17*b*x^5) - (6*a*((-3*a*(a*x^3 + b*x^6)^(11/3))/(1
54*b^2*x^11) + (a*x^3 + b*x^6)^(11/3)/(14*b*x^8)))/(17*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{(b\,x^{3}+a)\,(77b^{2}x^{6}-33a\,x^{3}b+9a^{2})\,(b\,x^{6}+a\,x^{3})^{\frac{8}{3}}}{1309b^{3}x^{8}}$	50
orering	$\frac{(b\,x^{3}+a)\,(77b^{2}x^{6}-33a\,x^{3}b+9a^{2})\,(b\,x^{6}+a\,x^{3})^{\frac{8}{3}}}{1309b^{3}x^{8}}$	50
trager	$\frac{(77b^{5}x^{15}+198a\,b^{4}x^{12}+141a^{2}b^{3}x^{9}+5a^{3}x^{6}b^{2}-6a^{4}b\,x^{3}+9a^{5})\,(b\,x^{6}+a\,x^{3})^{\frac{2}{3}}}{1309b^{3}x^{2}}$	76
risch	$\frac{(x^{3}\,(b\,x^{3}+a))^{\frac{2}{3}}\,(77b^{5}x^{15}+198a\,b^{4}x^{12}+141a^{2}b^{3}x^{9}+5a^{3}x^{6}b^{2}-6a^{4}b\,x^{3}+9a^{5})}{1309x^{2}b^{3}}$	76

input
```
int((b*x^6+a*x^3)^(8/3),x,method=_RETURNVERBOSE)
```

output
```
1/1309*(b*x^3+a)*(77*b^2*x^6-33*a*b*x^3+9*a^2)*(b*x^6+a*x^3)^(8/3)/b^3/x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \left(ax^3+bx^6\right)^{8/3}\,dx = \frac{\left(77\,b^5x^{15}+198\,ab^4x^{12}+141\,a^2b^3x^9+5\,a^3b^2x^6-6\,a^4bx^3+9\,a^5\right)\left(bx^6+ax^3\right)^{\frac{2}{3}}}{1309\,b^3x^2}$$

input
```
integrate((b*x^6+a*x^3)^(8/3),x, algorithm="fricas")
```

output
```
1/1309*(77*b^5*x^15 + 198*a*b^4*x^12 + 141*a^2*b^3*x^9 + 5*a^3*b^2*x^6 - 6
*a^4*b*x^3 + 9*a^5)*(b*x^6 + a*x^3)^(2/3)/(b^3*x^2)
```

Sympy [F]

$$\int \left(ax^3+bx^6\right)^{8/3}\,dx = \int \left(ax^3+bx^6\right)^{\frac{8}{3}}\,dx$$

input
```
integrate((b*x**6+a*x**3)**(8/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(8/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int (ax^3 + bx^6)^{8/3} \, dx = \frac{(77 b^5 x^{15} + 198 ab^4 x^{12} + 141 a^2 b^3 x^9 + 5 a^3 b^2 x^6 - 6 a^4 b x^3 + 9 a^5)(bx^3 + a)^{\frac{2}{3}}}{1309 b^3}$$

input
```
integrate((b*x^6+a*x^3)^(8/3),x, algorithm="maxima")
```

output
```
1/1309*(77*b^5*x^15 + 198*a*b^4*x^12 + 141*a^2*b^3*x^9 + 5*a^3*b^2*x^6 - 6
*a^4*b*x^3 + 9*a^5)*(b*x^3 + a)^(2/3)/b^3
```

Giac [F]

$$\int (ax^3 + bx^6)^{8/3} \, dx = \int (bx^6 + ax^3)^{\frac{8}{3}} \, dx$$

input
```
integrate((b*x^6+a*x^3)^(8/3),x, algorithm="giac")
```

output
```
integrate((b*x^6 + a*x^3)^(8/3), x)
```

Mupad [B] (verification not implemented)

Time = 19.70 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int (ax^3 + bx^6)^{8/3} \, dx = \frac{(b x^3 + a)^3 (b x^6 + a x^3)^{2/3} (9 a^2 - 33 a b x^3 + 77 b^2 x^6)}{1309 b^3 x^2}$$

input
```
int((a*x^3 + b*x^6)^(8/3),x)
```

output
```
((a + b*x^3)^3*(a*x^3 + b*x^6)^(2/3)*(9*a^2 + 77*b^2*x^6 - 33*a*b*x^3))/(1
309*b^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \left(ax^3+bx^6\right)^{8/3} dx = \frac{(b\,x^3+a)^{\frac{2}{3}}\,(77b^5x^{15}+198a\,b^4x^{12}+141a^2b^3x^9+5a^3b^2x^6-6a^4b\,x^3+9a^5)}{1309b^3}$$

input
```
int((b*x^6+a*x^3)^(8/3),x)
```

output
```
((a + b*x**3)**(2/3)*(9*a**5 - 6*a**4*b*x**3 + 5*a**3*b**2*x**6 + 141*a**2
*b**3*x**9 + 198*a*b**4*x**12 + 77*b**5*x**15))/(1309*b**3)
```

3.4 $\int (ax^3+bx^6)^{5/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int (ax^3+bx^6)^{5/3}\,dx = -\frac{3a(ax^3+bx^6)^{8/3}}{88b^2x^8}+\frac{(ax^3+bx^6)^{8/3}}{11bx^5}$$

output
```
-3/88*a*(b*x^6+a*x^3)^(8/3)/b^2/x^8+1/11*(b*x^6+a*x^3)^(8/3)/b/x^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int (ax^3+bx^6)^{5/3}\,dx = \frac{(x^3(a+bx^3))^{8/3}\,(-3a+8bx^3)}{88b^2x^8}$$

input
```
Integrate[(a*x^3 + b*x^6)^(5/3),x]
```

output
```
((x^3*(a + b*x^3))^(8/3)*(-3*a + 8*b*x^3))/(88*b^2*x^8)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^3+bx^6\right)^{5/3} dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\int \frac{\left(bx^6+ax^3\right)^{5/3}}{x^3}dx}{11b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

input
```
Int[(a*x^3 + b*x^6)^(5/3),x]
```

output
```
(-3*a*(a*x^3 + b*x^6)^(8/3))/(88*b^2*x^8) + (a*x^3 + b*x^6)^(8/3)/(11*b*x^
5)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)\left(-8 b\,x^{3}+3 a\right)\left(b\,x^{6}+a\,x^{3}\right)^{\frac{5}{3}}}{88 b^{2} x^{5}}$	39
orering	$-\frac{\left(b\,x^{3}+a\right)\left(-8 b\,x^{3}+3 a\right)\left(b\,x^{6}+a\,x^{3}\right)^{\frac{5}{3}}}{88 b^{2} x^{5}}$	39
trager	$-\frac{\left(-8 x^{9} b^{3}-13 a\,x^{6} b^{2}-2 a^{2} x^{3} b+3 a^{3}\right)\left(b\,x^{6}+a\,x^{3}\right)^{\frac{2}{3}}}{88 b^{2} x^{2}}$	54
risch	$-\frac{\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{2}{3}}\left(-8 x^{9} b^{3}-13 a\,x^{6} b^{2}-2 a^{2} x^{3} b+3 a^{3}\right)}{88 x^{2} b^{2}}$	54

input
```
int((b*x^6+a*x^3)^(5/3),x,method=_RETURNVERBOSE)
```

output
```
-1/88*(b*x^3+a)*(-8*b*x^3+3*a)*(b*x^6+a*x^3)^(5/3)/b^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx=\frac{(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3)(bx^6+ax^3)^{\frac{2}{3}}}{88\,b^2x^2}$$

input
```
integrate((b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
```

output
```
1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^6 + a*x^3)^(2/3
)/(b^2*x^2)
```

Sympy [F]

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx=\int \left(ax^3+bx^6\right)^{\frac{5}{3}}\,dx$$

input
```
integrate((b*x**6+a*x**3)**(5/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(5/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx=\frac{(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3)(bx^3+a)^{\frac{2}{3}}}{88\,b^2}$$

input
```
integrate((b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
```

output
```
1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^3 + a)^(2/3)/b^
2
```

Giac [F]

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx=\int \left(bx^6+ax^3\right)^{\frac{5}{3}}\,dx$$

input
```
integrate((b*x^6+a*x^3)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x^6 + a*x^3)^(5/3), x)
```

Mupad [B] (verification not implemented)

Time = 19.68 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx=-\frac{{\left(b\,x^3+a\right)}^2\,{\left(b\,x^6+a\,x^3\right)}^{2/3}\,\left(3\,a-8\,b\,x^3\right)}{88\,b^2\,x^2}$$

input
```
int((a*x^3 + b*x^6)^(5/3),x)
```

output
```
-((a + b*x^3)^2*(a*x^3 + b*x^6)^(2/3)*(3*a - 8*b*x^3))/(88*b^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \left(ax^3+bx^6\right)^{5/3}\,dx=\frac{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(8b^3x^9+13a\,b^2x^6+2a^2b\,x^3-3a^3\right)}{88b^2}$$

input
```
int((b*x^6+a*x^3)^(5/3),x)
```

output
```
((a + b*x**3)**(2/3)*( - 3*a**3 + 2*a**2*b*x**3 + 13*a*b**2*x**6 + 8*b**3*
x**9))/(88*b**2)
```

3.5 $\int (ax^3+bx^6)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int (ax^3+bx^6)^{2/3}\,dx=\frac{(ax^3+bx^6)^{5/3}}{5bx^5}$$

output
```
1/5*(b*x^6+a*x^3)^(5/3)/b/x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ax^3+bx^6)^{2/3}\,dx=\frac{(x^3(a+bx^3))^{5/3}}{5bx^5}$$

input
```
Integrate[(a*x^3 + b*x^6)^(2/3),x]
```

output
```
(x^3*(a + b*x^3))^(5/3)/(5*b*x^5)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^3+bx^6\right)^{2/3}\,dx$$

$$\Big\downarrow \text{ 1906}$$

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

input
```
Int[(a*x^3 + b*x^6)^(2/3),x]
```

output
```
(a*x^3 + b*x^6)^(5/3)/(5*b*x^5)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{(b\,x^3+a)\,(b\,x^6+a\,x^3)^{\frac{2}{3}}}{5b\,x^2}$	29
trager	$\frac{(b\,x^3+a)\,(b\,x^6+a\,x^3)^{\frac{2}{3}}}{5b\,x^2}$	29
risch	$\frac{(x^3\,(b\,x^3+a))^{\frac{2}{3}}\,(b\,x^3+a)}{5x^2b}$	29
orering	$\frac{(b\,x^3+a)\,(b\,x^6+a\,x^3)^{\frac{2}{3}}}{5b\,x^2}$	29

input
```
int((b*x^6+a*x^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/5*(b*x^3+a)/b/x^2*(b*x^6+a*x^3)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int (ax^3+bx^6)^{2/3}\,dx=\frac{(bx^6+ax^3)^{\frac{2}{3}}(bx^3+a)}{5\,bx^2}$$

input
```
integrate((b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
```

output
```
1/5*(b*x^6 + a*x^3)^(2/3)*(b*x^3 + a)/(b*x^2)
```

Sympy [F]

$$\int \left(ax^3+bx^6\right)^{2/3}\,dx=\int \left(ax^3+bx^6\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x**6+a*x**3)**(2/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \left(ax^3+bx^6\right)^{2/3}\,dx=\frac{(bx^3+a)^{\frac{5}{3}}}{5\,b}$$

input
```
integrate((b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
```

output
```
1/5*(b*x^3 + a)^(5/3)/b
```

Giac [F]

$$\int \left(ax^3+bx^6\right)^{2/3}\,dx=\int \left(bx^6+ax^3\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x^6+a*x^3)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^6 + a*x^3)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 19.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \left(ax^3+bx^6\right)^{2/3} dx = \frac{\left(\frac{a}{5\,b}+\frac{x^3}{5}\right)\,\left(b\,x^6+a\,x^3\right)^{2/3}}{x^2}$$

input
```
int((a*x^3 + b*x^6)^(2/3),x)
```

output
```
((a/(5*b) + x^3/5)*(a*x^3 + b*x^6)^(2/3))/x^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \left(ax^3+bx^6\right)^{2/3} dx = \frac{\left(b\,x^3+a\right)^{\frac{5}{3}}}{5b}$$

input
```
int((b*x^6+a*x^3)^(2/3),x)
```

output
```
((a + b*x**3)**(2/3)*(a + b*x**3))/(5*b)
```

3.6 $\int \frac{1}{\sqrt[3]{ax^3+bx^6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 164

$$\int \frac{1}{\sqrt[3]{ax^3+bx^6}}\,dx=\frac{x\sqrt[3]{a+bx^3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{ax^3+bx^6}}-\frac{x\sqrt[3]{a+bx^3}\log(x)}{2\sqrt[3]{a}\sqrt[3]{ax^3+bx^6}}+\frac{x\sqrt[3]{a+bx^3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\sqrt[3]{ax^3+bx^6}}$$

output
```
1/3*x*(b*x^3+a)^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/
3))*3^(1/2)/a^(1/3)/(b*x^6+a*x^3)^(1/3)-1/2*x*(b*x^3+a)^(1/3)*ln(x)/a^(1/3
)/(b*x^6+a*x^3)^(1/3)+1/2*x*(b*x^3+a)^(1/3)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^
(1/3)/(b*x^6+a*x^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt[3]{ax^3+bx^6}}\,dx$$
$$= \frac{x\sqrt[3]{a+bx^3}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)-\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a\right.\right.}{6\sqrt[3]{a}\sqrt[3]{x^3(a+bx^3)}}$$

input
```
Integrate[(a*x^3 + b*x^6)^(-1/3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))
/Sqrt[3]] + 2*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a
 + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/(6*a^(1/3)*(x^3*(a + b*x^3))^(1/3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1917, 798, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{ax^3+bx^6}}\,dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x\sqrt[3]{a+bx^3}\int \frac{1}{x\sqrt[3]{bx^3+a}}dx}{\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \text{798}$$

$$\frac{x\sqrt[3]{a+bx^3}\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{3\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \text{67}$$

$$\frac{x\sqrt[3]{a+bx^3}\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{3\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \text{16}$$

$$\frac{x\sqrt[3]{a+bx^3}\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{3\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{x\sqrt[3]{a+bx^3}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{3\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \text{217}$$

$$\frac{x\sqrt[3]{a+bx^3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{3\sqrt[3]{ax^3+bx^6}}$$

input
```
Int[(a*x^3 + b*x^6)^(-1/3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/
Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1
/3)])/(2*a^(1/3))))/(3*(a*x^3 + b*x^6)^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{6}+a\,x^{3}\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(b*x^6+a*x^3)^(1/3),x)
```

output
```
int(1/(b*x^6+a*x^3)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.95

$$\int \frac{1}{\sqrt[3]{ax^3+bx^6}}\,dx = \left[\frac{3\,\sqrt{\frac{1}{3}}a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^5+3\,ax^2-3\,\left(bx^6+ax^3\right)^{\frac{1}{3}}a^{\frac{2}{3}}x-3\,\sqrt{\frac{1}{3}}\left(a^{\frac{4}{3}}x^2+\left(bx^6+ax^3\right)^{\frac{1}{3}}ax-2\,\left(bx^6+ax^3\right)^{\frac{2}{3}}a^{\frac{2}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}}{x^5}\right)+2\,a^{\frac{2}{3}}\log\left(\right.}{6\,a}\right.$$

input
```
integrate(1/(b*x^6+a*x^3)^(1/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^5 + 3*a*x^2 - 3*(b*x^6 + a
*x^3)^(1/3)*a^(2/3)*x - 3*sqrt(1/3)*(a^(4/3)*x^2 + (b*x^6 + a*x^3)^(1/3)*a
*x - 2*(b*x^6 + a*x^3)^(2/3)*a^(2/3))*sqrt(-1/a^(2/3)))/x^5) + 2*a^(2/3)*l
og(-(a^(1/3)*x - (b*x^6 + a*x^3)^(1/3))/x) - a^(2/3)*log((a^(2/3)*x^2 + (b
*x^6 + a*x^3)^(1/3)*a^(1/3)*x + (b*x^6 + a*x^3)^(2/3))/x^2))/a, 1/6*(6*sqr
t(1/3)*a^(2/3)*arctan(sqrt(1/3)*(a^(1/3)*x + 2*(b*x^6 + a*x^3)^(1/3))/(a^(
1/3)*x)) + 2*a^(2/3)*log(-(a^(1/3)*x - (b*x^6 + a*x^3)^(1/3))/x) - a^(2/3)
*log((a^(2/3)*x^2 + (b*x^6 + a*x^3)^(1/3)*a^(1/3)*x + (b*x^6 + a*x^3)^(2/3
))/x^2))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{ax^3+bx^6}}\,dx = \int \frac{1}{\sqrt[3]{ax^3+bx^6}}\,dx$$

input
```
integrate(1/(b*x**6+a*x**3)**(1/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{ax^3+bx^6}}\,dx = \int \frac{1}{(bx^6+ax^3)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^6+a*x^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^6 + a*x^3)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{ax^3+bx^6}}\,dx = \int \frac{1}{(bx^6+ax^3)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^6+a*x^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^6 + a*x^3)^(-1/3), x)
```

Mupad [B] (verification not implemented)

Time = 18.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.26

$$\int \frac{1}{\sqrt[3]{ax^3+bx^6}}\,dx = -\frac{x\left(\frac{a}{b\,x^3}+1\right)^{1/3}{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{a}{b\,x^3}\right)}{\left(b\,x^6+a\,x^3\right)^{1/3}}$$

input
```
int(1/(a*x^3 + b*x^6)^(1/3),x)
```

output
```
-(x*(a/(b*x^3) + 1)^(1/3)*hypergeom([1/3, 1/3], 4/3, -a/(b*x^3)))/(a*x^3 +
 b*x^6)^(1/3)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{ax^3+bx^6}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}x}dx$$

input
```
int(1/(b*x^6+a*x^3)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x),x)
```

3.7 $\int \frac{1}{(ax^3+bx^6)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 217

$$\int \frac{1}{(ax^3+bx^6)^{4/3}}\,dx = \frac{1}{ax^2\sqrt[3]{ax^3+bx^6}} - \frac{4(ax^3+bx^6)^{2/3}}{3a^2x^5} - \frac{4bx\sqrt[3]{a+bx^3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}\sqrt[3]{ax^3+bx^6}} + \frac{2bx\sqrt[3]{a+bx^3}\log(x)}{3a^{7/3}\sqrt[3]{ax^3+bx^6}} - \frac{2bx\sqrt[3]{a+bx^3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{7/3}\sqrt[3]{ax^3+bx^6}}$$

output
```
1/a/x^2/(b*x^6+a*x^3)^(1/3)-4/3*(b*x^6+a*x^3)^(2/3)/a^2/x^5-4/9*b*x*(b*x^3
+a)^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/
a^(7/3)/(b*x^6+a*x^3)^(1/3)+2/3*b*x*(b*x^3+a)^(1/3)*ln(x)/a^(7/3)/(b*x^6+a
*x^3)^(1/3)-2/3*b*x*(b*x^3+a)^(1/3)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(7/3)/(b
*x^6+a*x^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.85

$$\int \frac{1}{(ax^3+bx^6)^{4/3}}\,dx =$$

$$-\frac{3a^{4/3}+12\sqrt[3]{a}bx^3+4\sqrt{3}bx^3\sqrt[3]{a+bx^3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4bx^3\sqrt[3]{a+bx^3}\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{9a^{7/3}x^2\sqrt[3]{x^3(a+bx^3)}}$$

input
```
Integrate[(a*x^3 + b*x^6)^(-4/3),x]
```

output
```
-1/9*(3*a^(4/3) + 12*a^(1/3)*b*x^3 + 4*Sqrt[3]*b*x^3*(a + b*x^3)^(1/3)*Arc
Tan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 4*b*x^3*(a + b*x^3)^(1/
3)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 2*b*x^3*(a + b*x^3)^(1/3)*Log[a^(2/
3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(a^(7/3)*x^2*(x^3*(a
+ b*x^3))^(1/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.78, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1912, 1931, 1917, 798, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^3+bx^6)^{4/3}}dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{4\int \frac{1}{x^3\sqrt[3]{bx^6+ax^3}}dx}{a}+\frac{1}{ax^2\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{4\left(-\frac{b\int \frac{1}{\sqrt[3]{bx^6+ax^3}}dx}{3a}-\frac{(ax^3+bx^6)^{2/3}}{3ax^5}\right)}{a}+\frac{1}{ax^2\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{4\left(-\frac{bx\sqrt[3]{a+bx^3}\int \frac{1}{x\sqrt[3]{bx^3+a}}dx}{3a\sqrt[3]{ax^3+bx^6}}-\frac{(ax^3+bx^6)^{2/3}}{3ax^5}\right)}{a}+\frac{1}{ax^2\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{4\left(-\frac{bx\sqrt[3]{a+bx^3}\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3}{9a\sqrt[3]{ax^3+bx^6}}-\frac{(ax^3+bx^6)^{2/3}}{3ax^5}\right)}{a}+\frac{1}{ax^2\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \text{ 67}$$

$$\frac{4\left(-\frac{bx\sqrt[3]{a+bx^3}\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{9a\sqrt[3]{ax^3+bx^6}}-\frac{(ax^3+bx^6)^{2/3}}{3ax^5}\right)}{a}$$

$$\frac{1}{ax^2\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{4\left(-\frac{bx\sqrt[3]{a+bx^3}\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{9a\sqrt[3]{ax^3+bx^6}}-\frac{(ax^3+bx^6)^{2/3}}{3ax^5}\right)}{a}+$$

$$\frac{1}{ax^2\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \; 1082$$

$$\frac{4\left(-\frac{bx\sqrt[3]{a+bx^3}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{9a\sqrt[3]{ax^3+bx^6}}-\frac{\left(ax^3+bx^6\right)^{2/3}}{3ax^5}\right)}{a}+\frac{1}{ax^2\sqrt[3]{ax^3+bx^6}}$$

$$\Big\downarrow \; 217$$

$$\frac{4\left(-\frac{bx\sqrt[3]{a+bx^3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)}{9a\sqrt[3]{ax^3+bx^6}}-\frac{\left(ax^3+bx^6\right)^{2/3}}{3ax^5}\right)}{a}+\frac{1}{ax^2\sqrt[3]{ax^3+bx^6}}$$

input

```
Int[(a*x^3 + b*x^6)^(-4/3),x]
```

output

```
1/(a*x^2*(a*x^3 + b*x^6)^(1/3)) + (4*(-1/3*(a*x^3 + b*x^6)^(2/3)/(a*x^5) -
 (b*x*(a + b*x^3)^(1/3)*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3
))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)
^(1/3)])/(2*a^(1/3)))))/(9*a*(a*x^3 + b*x^6)^(1/3))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{6}+a\,x^{3}\right)^{\frac{4}{3}}}dx$$

input
```
int(1/(b*x^6+a*x^3)^(4/3),x)
```

output
```
int(1/(b*x^6+a*x^3)^(4/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.50

$$\int \frac{1}{\left(ax^3+bx^6\right)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^6+a*x^3)^(4/3),x, algorithm="fricas")
```

output
```
[1/9*(6*sqrt(1/3)*(a*b^2*x^8 + a^2*b*x^5)*sqrt((-a)^(1/3)/a)*log((2*b*x^5
+ 3*a*x^2 - 3*(b*x^6 + a*x^3)^(1/3)*(-a)^(2/3)*x - 3*sqrt(1/3)*((-a)^(1/3)
*a*x^2 - (b*x^6 + a*x^3)^(1/3)*a*x + 2*(b*x^6 + a*x^3)^(2/3)*(-a)^(2/3))*s
qrt((-a)^(1/3)/a))/x^5) - 4*(b^2*x^8 + a*b*x^5)*(-a)^(2/3)*log(((-a)^(1/3)
*x + (b*x^6 + a*x^3)^(1/3))/x) + 2*(b^2*x^8 + a*b*x^5)*(-a)^(2/3)*log(((-a
)^(2/3)*x^2 - (b*x^6 + a*x^3)^(1/3)*(-a)^(1/3)*x + (b*x^6 + a*x^3)^(2/3))/
x^2) - 3*(b*x^6 + a*x^3)^(2/3)*(4*a*b*x^3 + a^2))/(a^3*b*x^8 + a^4*x^5), -
1/9*(12*sqrt(1/3)*(a*b^2*x^8 + a^2*b*x^5)*sqrt(-(-a)^(1/3)/a)*arctan(-sqrt
(1/3)*((-a)^(1/3)*x - 2*(b*x^6 + a*x^3)^(1/3))*sqrt(-(-a)^(1/3)/a)/x) + 4*
(b^2*x^8 + a*b*x^5)*(-a)^(2/3)*log(((-a)^(1/3)*x + (b*x^6 + a*x^3)^(1/3))/
x) - 2*(b^2*x^8 + a*b*x^5)*(-a)^(2/3)*log(((-a)^(2/3)*x^2 - (b*x^6 + a*x^3
)^(1/3)*(-a)^(1/3)*x + (b*x^6 + a*x^3)^(2/3))/x^2) + 3*(b*x^6 + a*x^3)^(2/
3)*(4*a*b*x^3 + a^2))/(a^3*b*x^8 + a^4*x^5)]
```

Sympy [F]

$$\int \frac{1}{(ax^3+bx^6)^{4/3}}\,dx=\int \frac{1}{(ax^3+bx^6)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x**6+a*x**3)**(4/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(ax^3+bx^6)^{4/3}}\,dx=\int \frac{1}{(bx^6+ax^3)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^6+a*x^3)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^6 + a*x^3)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(ax^3+bx^6)^{4/3}}\,dx=\int \frac{1}{(bx^6+ax^3)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^6+a*x^3)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^6 + a*x^3)^(-4/3), x)
```

Mupad [B] (verification not implemented)

Time = 18.90 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.19

$$\int \frac{1}{(ax^3+bx^6)^{4/3}}\,dx=-\frac{x\,\left(\frac{a}{b\,x^3}+1\right)^{4/3}\,{}_2\mathrm{F}_1\left(\frac{4}{3},\frac{7}{3};\ \frac{10}{3};\ -\frac{a}{b\,x^3}\right)}{7\,(b\,x^6+a\,x^3)^{4/3}}$$

input
```
int(1/(a*x^3 + b*x^6)^(4/3),x)
```

output
```
-(x*(a/(b*x^3) + 1)^(4/3)*hypergeom([4/3, 7/3], 10/3, -a/(b*x^3)))/(7*(a*x
^3 + b*x^6)^(4/3))
```

Reduce [F]

$$\int \frac{1}{(ax^3+bx^6)^{4/3}}\,dx=\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a\,x^4+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^7}dx$$

input
```
int(1/(b*x^6+a*x^3)^(4/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*x**4 + (a + b*x**3)**(1/3)*b*x**7),x)
```

3.8 $\int (ax^3+bx^6)^{7/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 260

$$\int (ax^3+bx^6)^{7/3}\,dx = -\frac{7a^4x\sqrt[3]{ax^3+bx^6}}{1458b^2}+\frac{7a^3x^4\sqrt[3]{ax^3+bx^6}}{2430b}+\frac{7}{405}a^2x^7\sqrt[3]{ax^3+bx^6}+\frac{7}{180}ax^4(ax^3+bx^6)^{4/3}+\frac{1}{15}x(ax^3+bx^6)^{7/3}-\frac{7a^5x^2(a+bx^3)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{729\sqrt{3}b^{8/3}(ax^3+bx^6)^{2/3}}-\frac{7a^5x^2(a+bx^3)^{2/3}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{1458b^{8/3}(ax^3+bx^6)^{2/3}}$$

output
```
-7/1458*a^4*x*(b*x^6+a*x^3)^(1/3)/b^2+7/2430*a^3*x^4*(b*x^6+a*x^3)^(1/3)/b
+7/405*a^2*x^7*(b*x^6+a*x^3)^(1/3)+7/180*a*x^4*(b*x^6+a*x^3)^(4/3)+1/15*x*
(b*x^6+a*x^3)^(7/3)-7/2187*a^5*x^2*(b*x^3+a)^(2/3)*arctan(1/3*(1+2*b^(1/3)
*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(8/3)/(b*x^6+a*x^3)^(2/3)-7/1458*a^
5*x^2*(b*x^3+a)^(2/3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(8/3)/(b*x^6+a*x^3)^
(2/3)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.97

$$\int (ax^3+bx^6)^{7/3}\,dx = \frac{x^2\left(-210a^5b^{2/3}x^2-84a^4b^{5/3}x^5+5499a^3b^{8/3}x^8+12906a^2b^{11/3}x^{11}+10449ab^{14/3}x^{14}+2916b\right.}{}$$

input
```
Integrate[(a*x^3 + b*x^6)^(7/3),x]
```

output
```
(x^2*(-210*a^5*b^(2/3)*x^2 - 84*a^4*b^(5/3)*x^5 + 5499*a^3*b^(8/3)*x^8 + 1
2906*a^2*b^(11/3)*x^11 + 10449*a*b^(14/3)*x^14 + 2916*b^(17/3)*x^17 - 140*
Sqrt[3]*a^5*(a + b*x^3)^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a
 + b*x^3)^(1/3))] - 140*a^5*(a + b*x^3)^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^
3)^(1/3)] + 70*a^5*(a + b*x^3)^(2/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^
3)^(1/3) + (a + b*x^3)^(2/3)]))/(43740*b^(8/3)*(x^3*(a + b*x^3))^(2/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1910, 1927, 1927, 1930, 1930, 1938, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^3+bx^6)^{7/3}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{7}{15}a\int x^3(bx^6+ax^3)^{4/3}dx+\frac{1}{15}x(ax^3+bx^6)^{7/3}$$

$$\Big\downarrow \text{1927}$$

$$\frac{7}{15}a\left(\frac{1}{3}a\int x^6\sqrt[3]{bx^6+ax^3}dx+\frac{1}{12}x^4(ax^3+bx^6)^{4/3}\right)+\frac{1}{15}x(ax^3+bx^6)^{7/3}$$

$\downarrow$ 1927

$$\frac{7}{15}a\left(\frac{1}{3}a\left(\frac{1}{9}a\int\frac{x^9}{(bx^6+ax^3)^{2/3}}dx+\frac{1}{9}x^7\sqrt[3]{ax^3+bx^6}\right)+\frac{1}{12}x^4\left(ax^3+bx^6\right)^{4/3}\right)+\frac{1}{15}x\left(ax^3+bx^6\right)^{7/3}$$

$\downarrow$ 1930

$$\frac{7}{15}a\left(\frac{1}{3}a\left(\frac{1}{9}a\left(\frac{x^4\sqrt[3]{ax^3+bx^6}}{6b}-\frac{5a\int\frac{x^6}{(bx^6+ax^3)^{2/3}}dx}{6b}\right)+\frac{1}{9}x^7\sqrt[3]{ax^3+bx^6}\right)+\frac{1}{12}x^4\left(ax^3+bx^6\right)^{4/3}\right)+\frac{1}{15}x\left(ax^3+bx^6\right)^{7/3}$$

$\downarrow$ 1930

$$\frac{7}{15}a\left(\frac{1}{3}a\left(\frac{1}{9}a\left(\frac{x^4\sqrt[3]{ax^3+bx^6}}{6b}-\frac{5a\left(\frac{x\sqrt[3]{ax^3+bx^6}}{3b}-\frac{2a\int\frac{x^3}{\left(bx^6+ax^3\right)^{2/3}}dx}{3b}\right)}{6b}\right)+\frac{1}{9}x^7\sqrt[3]{ax^3+bx^6}\right)+\frac{1}{12}x^4(ax^3\right.$$

$$\frac{1}{15}x\left(ax^3+bx^6\right)^{7/3}$$

$\downarrow$ 1938

$$\frac{7}{15}a\left(\frac{1}{3}a\left(\frac{1}{9}a\left(\frac{x^4\sqrt[3]{ax^3+bx^6}}{6b}-\frac{5a\left(\frac{x\sqrt[3]{ax^3+bx^6}}{3b}-\frac{2ax^2\left(a+bx^3\right)^{2/3}\int\frac{x}{\left(bx^3+a\right)^{2/3}}dx}{3b(ax^3+bx^6)^{2/3}}\right)}{6b}\right)+\frac{1}{9}x^7\sqrt[3]{ax^3+bx^6}\right)+\right.$$

$$\frac{1}{15}x\left(ax^3+bx^6\right)^{7/3}$$

$\downarrow$ 853

$$\frac{7}{15}a\Bigg(\frac{1}{3}a\Bigg(\frac{1}{9}a\Bigg(\frac{x^4\sqrt[3]{ax^3+bx^6}}{6b}-\frac{5a\Bigg(\frac{x\sqrt[3]{ax^3+bx^6}}{3b}-\frac{2ax^2\left(a+bx^3\right)^{2/3}\Bigg(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\Big(\sqrt[3]{b}x-\sqrt[3]{a+bx}}{2b^{2/3}}}{3b\left(ax^3+bx^6\right)^{2/3}}}{6b}$$

$$\frac{1}{15}x\left(ax^3+bx^6\right)^{7/3}$$

input
```
Int[(a*x^3 + b*x^6)^(7/3),x]
```

output
```
(x*(a*x^3 + b*x^6)^(7/3))/15 + (7*a*((x^4*(a*x^3 + b*x^6)^(4/3))/12 + (a*(
(x^7*(a*x^3 + b*x^6)^(1/3))/9 + (a*((x^4*(a*x^3 + b*x^6)^(1/3))/(6*b) - (5
*a*((x*(a*x^3 + b*x^6)^(1/3))/(3*b) - (2*a*x^2*(a + b*x^3)^(2/3)*(-(ArcTan
[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))) - Log[b
^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3))))/(3*b*(a*x^3 + b*x^6)^(2/3))))/
(6*b)))/9))/3))/15
```

Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$\frac{2916\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}} b^{\frac{14}{3}} x^{13}+7533 a\,b^{\frac{11}{3}} x^{10} \left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}}+5373 a^{2} b^{\frac{8}{3}} x^{7} \left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}}+126 a^{3} b^{\frac{5}{3}} x^{4} \left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}}-21}{}$

input
```
int((b*x^6+a*x^3)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
1/43740/b^(8/3)*(2916*(x^3*(b*x^3+a))^(1/3)*b^(14/3)*x^13+7533*a*b^(11/3)*
x^10*(x^3*(b*x^3+a))^(1/3)+5373*a^2*b^(8/3)*x^7*(x^3*(b*x^3+a))^(1/3)+126*
a^3*b^(5/3)*x^4*(x^3*(b*x^3+a))^(1/3)-210*a^4*x*(x^3*(b*x^3+a))^(1/3)*b^(2
/3)+140*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x^2+2*(x^3*(b*x^3+a))^(1/3))/b
^(1/3)/x^2)*a^5-140*ln((-b^(1/3)*x^2+(x^3*(b*x^3+a))^(1/3))/x^2)*a^5+70*ln
((b^(2/3)*x^4+b^(1/3)*(x^3*(b*x^3+a))^(1/3)*x^2+(x^3*(b*x^3+a))^(2/3))/x^4
)*a^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.02

$$\int \left(ax^3+bx^6\right)^{7/3}\,dx = \frac{420\sqrt{\frac{1}{3}}a^5b\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(-b^2\right)^{\frac{1}{3}}bx^2-2\left(bx^6+ax^3\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{b^2x^2}\right)-140\left(-b^2\right)^{\frac{2}{3}}}{}$$

input
```
integrate((b*x^6+a*x^3)^(7/3),x, algorithm="fricas")
```

output
```
1/43740*(420*sqrt(1/3)*a^5*b*sqrt(-(-b^2)^(1/3))*arctan(-sqrt(1/3)*((-b^2)
^(1/3)*b*x^2 - 2*(b*x^6 + a*x^3)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(
b^2*x^2)) - 140*(-b^2)^(2/3)*a^5*log(-((-b^2)^(2/3)*x^2 - (b*x^6 + a*x^3)^
(1/3)*b)/x^2) + 70*(-b^2)^(2/3)*a^5*log(-((-b^2)^(1/3)*b*x^4 - (b*x^6 + a*
x^3)^(1/3)*(-b^2)^(2/3)*x^2 - (b*x^6 + a*x^3)^(2/3)*b)/x^4) + 3*(972*b^6*x
^13 + 2511*a*b^5*x^10 + 1791*a^2*b^4*x^7 + 42*a^3*b^3*x^4 - 70*a^4*b^2*x)*
(b*x^6 + a*x^3)^(1/3))/b^4
```

Sympy [F]

$$\int \left(ax^3+bx^6\right)^{7/3}\,dx=\int \left(ax^3+bx^6\right)^{\frac{7}{3}}\,dx$$

input
```
integrate((b*x**6+a*x**3)**(7/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(7/3), x)
```

Maxima [F]

$$\int \left(ax^3+bx^6\right)^{7/3}\,dx=\int \left(bx^6+ax^3\right)^{\frac{7}{3}}\,dx$$

input
```
integrate((b*x^6+a*x^3)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*x^6 + a*x^3)^(7/3), x)
```

Giac [A] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.65

$$\int \left(ax^3+bx^6\right)^{7/3}\,dx =$$

$$-\frac{1}{43740}\left(\frac{3\left(70\left(b+\frac{a}{x^3}\right)^{\frac{13}{3}}-322\left(b+\frac{a}{x^3}\right)^{\frac{10}{3}}b-1245\left(b+\frac{a}{x^3}\right)^{\frac{7}{3}}b^2+665\left(b+\frac{a}{x^3}\right)^{\frac{4}{3}}b^3-140\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}b^4\right)x^1}{a^5b^2}\right.$$

input
```
integrate((b*x^6+a*x^3)^(7/3),x, algorithm="giac")
```

output
```
-1/43740*(3*(70*(b + a/x^3)^(13/3) - 322*(b + a/x^3)^(10/3)*b - 1245*(b +
a/x^3)^(7/3)*b^2 + 665*(b + a/x^3)^(4/3)*b^3 - 140*(b + a/x^3)^(1/3)*b^4)*
x^15/(a^5*b^2) - 140*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b + a/x^3)^(1/3) + b^(
1/3))/b^(1/3))/b^(8/3) - 70*log((b + a/x^3)^(2/3) + (b + a/x^3)^(1/3)*b^(1
/3) + b^(2/3))/b^(8/3) + 140*log(abs((b + a/x^3)^(1/3) - b^(1/3)))/b^(8/3)
)*a^5
```

Mupad [B] (verification not implemented)

Time = 18.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.16

$$\int \left(ax^3+bx^6\right)^{7/3}\,dx = \frac{x\left(b\,x^6+a\,x^3\right)^{7/3}{}_2\mathrm{F}_1\left(-\frac{7}{3},\frac{8}{3};\ \frac{11}{3};\ -\frac{b\,x^3}{a}\right)}{8\left(\frac{b\,x^3}{a}+1\right)^{7/3}}$$

input
```
int((a*x^3 + b*x^6)^(7/3),x)
```

output
```
(x*(a*x^3 + b*x^6)^(7/3)*hypergeom([-7/3, 8/3], 11/3, -(b*x^3)/a))/(8*((b*
x^3)/a + 1)^(7/3))
```

Reduce [F]

$$\int \left(ax^3+bx^6\right)^{7/3} dx = \frac{-70(b\,x^3+a)^{\frac{1}{3}}\,a^4x^2+42(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,x^5+1791(b\,x^3+a)^{\frac{1}{3}}\,a^2b^2x^8+2511(b\,x^3+a)^{\frac{1}{3}}\,a\,b^3x}{14580b^2}$$

input
```
int((b*x^6+a*x^3)^(7/3),x)
```

output
```
( - 70*(a + b*x**3)**(1/3)*a**4*x**2 + 42*(a + b*x**3)**(1/3)*a**3*b*x**5
+ 1791*(a + b*x**3)**(1/3)*a**2*b**2*x**8 + 2511*(a + b*x**3)**(1/3)*a*b**
3*x**11 + 972*(a + b*x**3)**(1/3)*b**4*x**14 + 140*int(((a + b*x**3)**(1/3
)*x)/(a + b*x**3),x)*a**5)/(14580*b**2)
```

3.9 $\int (ax^3+bx^6)^{4/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 207

$$\int (ax^3+bx^6)^{4/3}\,dx = \frac{2a^2x\sqrt[3]{ax^3+bx^6}}{81b}+\frac{2}{27}ax^4\sqrt[3]{ax^3+bx^6}+\frac{1}{9}x(ax^3+bx^6)^{4/3}+\frac{4a^3x^2(a+bx^3)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}b^{5/3}(ax^3+bx^6)^{2/3}}+\frac{2a^3x^2(a+bx^3)^{2/3}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{81b^{5/3}(ax^3+bx^6)^{2/3}}$$

output
```
2/81*a^2*x*(b*x^6+a*x^3)^(1/3)/b+2/27*a*x^4*(b*x^6+a*x^3)^(1/3)+1/9*x*(b*x
^6+a*x^3)^(4/3)+4/243*a^3*x^2*(b*x^3+a)^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b
*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(5/3)/(b*x^6+a*x^3)^(2/3)+2/81*a^3*x^2*(
b*x^3+a)^(2/3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/(b*x^6+a*x^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.08

$$\int (ax^3+bx^6)^{4/3}\,dx=\frac{x^2(a+bx^3)^{2/3}\left(6a^2b^{2/3}x^2\sqrt[3]{a+bx^3}+45ab^{5/3}x^5\sqrt[3]{a+bx^3}+27b^{8/3}x^8\sqrt[3]{a+bx^3}+4\sqrt{3}a^3\,\mathrm{arc}\right.}{243}$$

input
```
Integrate[(a*x^3 + b*x^6)^(4/3),x]
```

output
```
(x^2*(a + b*x^3)^(2/3)*(6*a^2*b^(2/3)*x^2*(a + b*x^3)^(1/3) + 45*a*b^(5/3)
*x^5*(a + b*x^3)^(1/3) + 27*b^(8/3)*x^8*(a + b*x^3)^(1/3) + 4*Sqrt[3]*a^3*
ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 4*a^3*Log[
-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - 2*a^3*Log[b^(2/3)*x^2 + b^(1/3)*x*(a +
 b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/(243*b^(5/3)*(x^3*(a + b*x^3))^(2/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1910, 1927, 1930, 1938, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^3+bx^6)^{4/3}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{4}{9}a\int x^3\sqrt[3]{bx^6+ax^3}dx+\frac{1}{9}x(ax^3+bx^6)^{4/3}$$

$$\Big\downarrow \text{1927}$$

$$\frac{4}{9}a\left(\frac{1}{6}a\int \frac{x^6}{(bx^6+ax^3)^{2/3}}dx+\frac{1}{6}x^4\sqrt[3]{ax^3+bx^6}\right)+\frac{1}{9}x(ax^3+bx^6)^{4/3}$$

$$\Big\downarrow \ 1930$$

$$\frac{4}{9}a\left(\frac{1}{6}a\left(\frac{x\sqrt[3]{ax^3+bx^6}}{3b}-\frac{2a\int \frac{x^3}{(bx^6+ax^3)^{2/3}}dx}{3b}\right)+\frac{1}{6}x^4\sqrt[3]{ax^3+bx^6}\right)+\frac{1}{9}x\left(ax^3+bx^6\right)^{4/3}$$

$$\Big\downarrow \ 1938$$

$$\frac{4}{9}a\left(\frac{1}{6}a\left(\frac{x\sqrt[3]{ax^3+bx^6}}{3b}-\frac{2ax^2\left(a+bx^3\right)^{2/3}\int \frac{x}{(bx^3+a)^{2/3}}dx}{3b\left(ax^3+bx^6\right)^{2/3}}\right)+\frac{1}{6}x^4\sqrt[3]{ax^3+bx^6}\right)+\frac{1}{9}x\left(ax^3+bx^6\right)^{4/3}$$

$$\Big\downarrow \ 853$$

$$\frac{4}{9}a\left(\frac{1}{6}a\left(\frac{x\sqrt[3]{ax^3+bx^6}}{3b}-\frac{2ax^2\left(a+bx^3\right)^{2/3}\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{3b\left(ax^3+bx^6\right)^{2/3}}\right)+\frac{1}{6}x^4\sqrt[3]{ax^3}\right)$$

$$\frac{1}{9}x\left(ax^3+bx^6\right)^{4/3}$$

input
```
Int[(a*x^3 + b*x^6)^(4/3),x]
```

output
```
(x*(a*x^3 + b*x^6)^(4/3))/9 + (4*a*((x^4*(a*x^3 + b*x^6)^(1/3))/6 + (a*((x
*(a*x^3 + b*x^6)^(1/3))/(3*b) - (2*a*x^2*(a + b*x^3)^(2/3)*(-(ArcTan[(1 +
(2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))) - Log[b^(1/3)
*x - (a + b*x^3)^(1/3)]/(2*b^(2/3))))/(3*b*(a*x^3 + b*x^6)^(2/3))))/6))/9
```

Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{27\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}} b^{\frac{8}{3}} x^{7}+45 a\,b^{\frac{5}{3}} x^{4}\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}}+6 a^{2} x\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}} b^{\frac{2}{3}}-4 \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x^{2}+2\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x^{2}}\right)}{243 b^{\frac{5}{3}}}$

input

```
int((b*x^6+a*x^3)^(4/3),x,method=_RETURNVERBOSE)
```

output

```
1/243*(27*(x^3*(b*x^3+a))^(1/3)*b^(8/3)*x^7+45*a*b^(5/3)*x^4*(x^3*(b*x^3+a
))^(1/3)+6*a^2*x*(x^3*(b*x^3+a))^(1/3)*b^(2/3)-4*3^(1/2)*arctan(1/3*3^(1/2
)*(b^(1/3)*x^2+2*(x^3*(b*x^3+a))^(1/3))/b^(1/3)/x^2)*a^3+4*ln((-b^(1/3)*x^
2+(x^3*(b*x^3+a))^(1/3))/x^2)*a^3-2*ln((b^(2/3)*x^4+b^(1/3)*(x^3*(b*x^3+a)
)^(1/3)*x^2+(x^3*(b*x^3+a))^(2/3))/x^4)*a^3)/b^(5/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03

$$\int \left(ax^3+bx^6\right)^{4/3}\,dx =$$

$$-\frac{12\sqrt{\frac{1}{3}}a^3(b^2)^{\frac{1}{6}}b\arctan\left(\frac{\sqrt{\frac{1}{3}}\left((b^2)^{\frac{1}{3}}bx^2+2\,(bx^6+ax^3)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{b^2x^2}\right)-4\,a^3(b^2)^{\frac{2}{3}}\log\left(-\frac{(b^2)^{\frac{2}{3}}x^2-(bx^6+ax^3)^{\frac{1}{3}}b}{x^2}\right)+2\,a}{243\,b^3}$$

input

```
integrate((b*x^6+a*x^3)^(4/3),x, algorithm="fricas")
```

output

```
-1/243*(12*sqrt(1/3)*a^3*(b^2)^(1/6)*b*arctan(sqrt(1/3)*((b^2)^(1/3)*b*x^2
 + 2*(b*x^6 + a*x^3)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x^2)) - 4*a^3*(b^
2)^(2/3)*log(-((b^2)^(2/3)*x^2 - (b*x^6 + a*x^3)^(1/3)*b)/x^2) + 2*a^3*(b^
2)^(2/3)*log(((b^2)^(1/3)*b*x^4 + (b*x^6 + a*x^3)^(1/3)*(b^2)^(2/3)*x^2 +
(b*x^6 + a*x^3)^(2/3)*b)/x^4) - 3*(9*b^4*x^7 + 15*a*b^3*x^4 + 2*a^2*b^2*x)
*(b*x^6 + a*x^3)^(1/3))/b^3
```

Sympy [F]

$$\int \left(ax^3+bx^6\right)^{4/3}\,dx=\int \left(ax^3+bx^6\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*x**6+a*x**3)**(4/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(4/3), x)
```

Maxima [F]

$$\int \left(ax^3+bx^6\right)^{4/3}\,dx=\int \left(bx^6+ax^3\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*x^6+a*x^3)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^6 + a*x^3)^(4/3), x)
```

Giac [A] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.68

$$\int \left(ax^3+bx^6\right)^{4/3}\,dx=\frac{1}{243}\left(\frac{3\left(2\left(b+\frac{a}{x^3}\right)^{\frac{7}{3}}+11\left(b+\frac{a}{x^3}\right)^{\frac{4}{3}}b-4\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}b^2\right)x^9}{a^3b}-\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{5}{3}}}\right.$$

input
```
integrate((b*x^6+a*x^3)^(4/3),x, algorithm="giac")
```

output
```
1/243*(3*(2*(b + a/x^3)^(7/3) + 11*(b + a/x^3)^(4/3)*b - 4*(b + a/x^3)^(1/
3)*b^2)*x^9/(a^3*b) - 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b + a/x^3)^(1/3) +
b^(1/3))/b^(1/3))/b^(5/3) - 2*log((b + a/x^3)^(2/3) + (b + a/x^3)^(1/3)*b^
(1/3) + b^(2/3))/b^(5/3) + 4*log(abs((b + a/x^3)^(1/3) - b^(1/3)))/b^(5/3)
)*a^3
```

Mupad [B] (verification not implemented)

Time = 19.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.20

$$\int \left(ax^3+bx^6\right)^{4/3}\,dx=\frac{x\,\left(b\,x^6+a\,x^3\right)^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{4}{3},\frac{5}{3};\ \frac{8}{3};\ -\frac{b\,x^3}{a}\right)}{5\,\left(\frac{b\,x^3}{a}+1\right)^{4/3}}$$

input
```
int((a*x^3 + b*x^6)^(4/3),x)
```

output
```
(x*(a*x^3 + b*x^6)^(4/3)*hypergeom([-4/3, 5/3], 8/3, -(b*x^3)/a))/(5*((b*x
^3)/a + 1)^(4/3))
```

Reduce [F]

$$\int \left(ax^3+bx^6\right)^{4/3}\,dx=\frac{2(b\,x^3+a)^{\frac{1}{3}}\,a^2x^2+15(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^5+9(b\,x^3+a)^{\frac{1}{3}}\,b^2x^8-4\left(\int\frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^3}{81b}$$

input
```
int((b*x^6+a*x^3)^(4/3),x)
```

output
```
(2*(a + b*x**3)**(1/3)*a**2*x**2 + 15*(a + b*x**3)**(1/3)*a*b*x**5 + 9*(a
+ b*x**3)**(1/3)*b**2*x**8 - 4*int(((a + b*x**3)**(1/3)*x)/(a + b*x**3),x)
*a**3)/(81*b)
```

3.10 $\int \sqrt[3]{ax^3+bx^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 154

$$\int \sqrt[3]{ax^3+bx^6}\,dx=\frac{1}{3}x\sqrt[3]{ax^3+bx^6}-\frac{ax^2(a+bx^3)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}\left(ax^3+bx^6\right)^{2/3}}-\frac{ax^2(a+bx^3)^{2/3}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}\left(ax^3+bx^6\right)^{2/3}}$$

output
```
1/3*x*(b*x^6+a*x^3)^(1/3)-1/9*a*x^2*(b*x^3+a)^(2/3)*arctan(1/3*(1+2*b^(1/3
)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/(b*x^6+a*x^3)^(2/3)-1/6*a*x^
2*(b*x^3+a)^(2/3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)/(b*x^6+a*x^3)^(2/3
)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10

$$\int \sqrt[3]{ax^3+bx^6}\,dx = \frac{x^2(a+bx^3)^{2/3}\left(6b^{2/3}x^2\sqrt[3]{a+bx^3}-2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)+a\right.}{18b^{2/3}\left(x^3\left(a+bx^3\right)\right)^{2/3}}$$

input
```
Integrate[(a*x^3 + b*x^6)^(1/3),x]
```

output
```
(x^2*(a + b*x^3)^(2/3)*(6*b^(2/3)*x^2*(a + b*x^3)^(1/3) - 2*Sqrt[3]*a*ArcT
an[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*a*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)] + a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1
/3) + (a + b*x^3)^(2/3)]))/(18*b^(2/3)*(x^3*(a + b*x^3))^(2/3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.82, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1910, 1938, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{ax^3+bx^6}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{1}{3}a\int \frac{x^3}{(bx^6+ax^3)^{2/3}}dx+\frac{1}{3}x\sqrt[3]{ax^3+bx^6}$$

$$\Big\downarrow \text{1938}$$

$$\frac{ax^2\left(a+bx^3\right)^{2/3}\int \frac{x}{(bx^3+a)^{2/3}}dx}{3\left(ax^3+bx^6\right)^{2/3}}+\frac{1}{3}x\sqrt[3]{ax^3+bx^6}$$

$$\downarrow 853$$

$$\frac{ax^2(a+bx^3)^{2/3}\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{3(ax^3+bx^6)^{2/3}}+\frac{1}{3}x\sqrt[3]{ax^3+bx^6}$$

input
```
Int[(a*x^3 + b*x^6)^(1/3),x]
```

output
```
(x*(a*x^3 + b*x^6)^(1/3))/3 + (a*x^2*(a + b*x^3)^(2/3)*(-(ArcTan[(1 + (2*b
^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))) - Log[b^(1/3)*x -
 (a + b*x^3)^(1/3)]/(2*b^(2/3))))/(3*(a*x^3 + b*x^6)^(2/3))
```

Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{6\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}} x\,b^{\frac{2}{3}}+2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x^{2}+2\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x^{2}}\right) a-2\ln\left(\frac{-b^{\frac{1}{3}} x^{2}+\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}}}{x^{2}}\right) a+\ln\left(\frac{b^{\frac{2}{3}} x^{4}+b^{\frac{1}{3}}\left(\right.}{}\right.}{18 b^{\frac{2}{3}}}$

input
```
int((b*x^6+a*x^3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/18*(6*(x^3*(b*x^3+a))^(1/3)*x*b^(2/3)+2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1
/3)*x^2+2*(x^3*(b*x^3+a))^(1/3))/b^(1/3)/x^2)*a-2*ln((-b^(1/3)*x^2+(x^3*(b
*x^3+a))^(1/3))/x^2)*a+ln((b^(2/3)*x^4+b^(1/3)*(x^3*(b*x^3+a))^(1/3)*x^2+(
x^3*(b*x^3+a))^(2/3))/x^4)*a)/b^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.39

$$\int \sqrt[3]{ax^3+bx^6}\,dx$$
$$= \frac{6\sqrt{\frac{1}{3}}ab\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(-b^2\right)^{\frac{1}{3}}bx^2-2\left(bx^6+ax^3\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{b^2x^2}\right)+6\left(bx^6+ax^3\right)^{\frac{1}{3}}b^2x-2\left(-b^2\right.}{18\,b^2}$$

input
```
integrate((b*x^6+a*x^3)^(1/3),x, algorithm="fricas")
```

output
```
1/18*(6*sqrt(1/3)*a*b*sqrt(-(-b^2)^(1/3))*arctan(-sqrt(1/3)*((-b^2)^(1/3)*
b*x^2 - 2*(b*x^6 + a*x^3)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x^2
)) + 6*(b*x^6 + a*x^3)^(1/3)*b^2*x - 2*(-b^2)^(2/3)*a*log(-((-b^2)^(2/3)*x
^2 - (b*x^6 + a*x^3)^(1/3)*b)/x^2) + (-b^2)^(2/3)*a*log(-((-b^2)^(1/3)*b*x
^4 - (b*x^6 + a*x^3)^(1/3)*(-b^2)^(2/3)*x^2 - (b*x^6 + a*x^3)^(2/3)*b)/x^4
))/b^2
```

Sympy [F]

$$\int \sqrt[3]{ax^3+bx^6}\,dx=\int \sqrt[3]{ax^3+bx^6}\,dx$$

input
```
integrate((b*x**6+a*x**3)**(1/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{ax^3+bx^6}\,dx=\int \left(bx^6+ax^3\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^6+a*x^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^6 + a*x^3)^(1/3), x)
```

Giac [A] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.69

$$\int \sqrt[3]{ax^3+bx^6}\,dx$$
$$=\frac{1}{18}\left(\frac{6\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}x^3}{a}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{2}{3}}}+\frac{\log\left(\left(b+\frac{a}{x^3}\right)^{\frac{2}{3}}+\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{2}{3}}}-\frac{2\log\right.$$

input
```
integrate((b*x^6+a*x^3)^(1/3),x, algorithm="giac")
```

output
```
1/18*(6*(b + a/x^3)^(1/3)*x^3/a + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b + a/x
^3)^(1/3) + b^(1/3))/b^(1/3))/b^(2/3) + log((b + a/x^3)^(2/3) + (b + a/x^3
)^(1/3)*b^(1/3) + b^(2/3))/b^(2/3) - 2*log(abs((b + a/x^3)^(1/3) - b^(1/3)
))/b^(2/3))*a
```

Mupad [B] (verification not implemented)

Time = 18.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.27

$$\int \sqrt[3]{ax^3+bx^6}\,dx = \frac{x\,(b\,x^6+a\,x^3)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{2}{3};\;\frac{5}{3};\;-\frac{b\,x^3}{a}\right)}{2\,\left(\frac{b\,x^3}{a}+1\right)^{1/3}}$$

input
```
int((a*x^3 + b*x^6)^(1/3),x)
```

output
```
(x*(a*x^3 + b*x^6)^(1/3)*hypergeom([-1/3, 2/3], 5/3, -(b*x^3)/a))/(2*((b*x
^3)/a + 1)^(1/3))
```

Reduce [F]

$$\int \sqrt[3]{ax^3+bx^6}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,x^2}{3} + \frac{\left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a}{3}$$

input
```
int((b*x^6+a*x^3)^(1/3),x)
```

output
```
((a + b*x**3)**(1/3)*x**2 + int(((a + b*x**3)**(1/3)*x)/(a + b*x**3),x)*a)
/3
```

3.11 $\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx=-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

output
```
-(b*x^6+a*x^3)^(1/3)/a/x^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx=-\frac{\sqrt[3]{x^3(a+bx^3)}}{ax^2}$$

input
```
Integrate[(a*x^3 + b*x^6)^(-2/3),x]
```

output
```
-((x^3*(a + b*x^3))^(1/3)/(a*x^2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx$$

$$\Big\downarrow \; 1906$$

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

input
```
Int[(a*x^3 + b*x^6)^(-2/3),x]
```

output
```
-((a*x^3 + b*x^6)^(1/3)/(a*x^2))
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
trager	$-\frac{\left(b\,x^{6}+a\,x^{3}\right)^{\frac{1}{3}}}{a\,x^{2}}$	22
pseudoelliptic	$-\frac{\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{1}{3}}}{a\,x^{2}}$	22
gosper	$-\frac{x\left(b\,x^{3}+a\right)}{a\left(b\,x^{6}+a\,x^{3}\right)^{\frac{2}{3}}}$	27
risch	$-\frac{x\left(b\,x^{3}+a\right)}{\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{2}{3}}a}$	27
orering	$-\frac{x\left(b\,x^{3}+a\right)}{a\left(b\,x^{6}+a\,x^{3}\right)^{\frac{2}{3}}}$	27

input
```
int(1/(b*x^6+a*x^3)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^6+a*x^3)^(1/3)/a/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{(bx^6+ax^3)^{\frac{1}{3}}}{ax^2}$$

input
```
integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
```

output
```
-(b*x^6 + a*x^3)^(1/3)/(a*x^2)
```

Sympy [F]

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = \int \frac{1}{(ax^3+bx^6)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x**6+a*x**3)**(2/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(-2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{(bx^3+a)^{\frac{1}{3}}}{ax}$$

input
```
integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
```

output
```
-(b*x^3 + a)^(1/3)/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx = -\frac{\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}}{a}$$

input
```
integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="giac")
```

output
```
-(b + a/x^3)^(1/3)/a
```

Mupad [B] (verification not implemented)

Time = 18.89 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx=-\frac{\left(b\,x^6+a\,x^3\right)^{1/3}}{a\,x^2}$$

input
```
int(1/(a*x^3 + b*x^6)^(2/3),x)
```

output
```
-(a*x^3 + b*x^6)^(1/3)/(a*x^2)
```

Reduce [F]

$$\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}x^2}dx$$

input
```
int(1/(b*x^6+a*x^3)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**2),x)
```

3.12 $\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = -\frac{1}{4ax^2(ax^3+bx^6)^{2/3}} + \frac{3bx}{2a^2(ax^3+bx^6)^{2/3}} + \frac{9b^2x^4}{4a^3(ax^3+bx^6)^{2/3}}$$

output
```
-1/4/a/x^2/(b*x^6+a*x^3)^(2/3)+3/2*b*x/a^2/(b*x^6+a*x^3)^(2/3)+9/4*b^2*x^4
/a^3/(b*x^6+a*x^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{-a^2+6abx^3+9b^2x^6}{4a^3x^2(x^3(a+bx^3))^{2/3}}$$

input
```
Integrate[(a*x^3 + b*x^6)^(-5/3),x]
```

output
```
(-a^2 + 6*a*b*x^3 + 9*b^2*x^6)/(4*a^3*x^2*(x^3*(a + b*x^3))^(2/3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1907, 1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}dx$$

$$\Big\downarrow \text{1907}$$

$$\frac{3\int \frac{1}{x^3(bx^6+ax^3)^{2/3}}dx}{a}+\frac{1}{2ax^2(ax^3+bx^6)^{2/3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{3\left(-\frac{3b\int \frac{1}{(bx^6+ax^3)^{2/3}}dx}{4a}-\frac{\sqrt[3]{ax^3+bx^6}}{4ax^5}\right)}{a}+\frac{1}{2ax^2(ax^3+bx^6)^{2/3}}$$

$$\Big\downarrow \text{1906}$$

$$\frac{3\left(\frac{3b\sqrt[3]{ax^3+bx^6}}{4a^2x^2}-\frac{\sqrt[3]{ax^3+bx^6}}{4ax^5}\right)}{a}+\frac{1}{2ax^2(ax^3+bx^6)^{2/3}}$$

input
```
Int[(a*x^3 + b*x^6)^(-5/3),x]
```

output
```
1/(2*a*x^2*(a*x^3 + b*x^6)^(2/3)) + (3*(-1/4*(a*x^3 + b*x^6)^(1/3)/(a*x^5)
 + (3*b*(a*x^3 + b*x^6)^(1/3))/(4*a^2*x^2)))/a
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 1907
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b, j, n}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j +
1)/(n - j)], 0] && LtQ[p, -1]
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.53

method	result	size
pseudoelliptic	$-\frac{-9 b^{2} x^{6}-6 a \,x^{3} b +a^{2}}{4 x^{2} \left(x^{3} \left(b \,x^{3}+a \right)\right)^{\frac{2}{3}} a^{3}}$	41
gosper	$-\frac{x \left(b \,x^{3}+a \right) \left(-9 b^{2} x^{6}-6 a \,x^{3} b +a^{2}\right)}{4 a^{3} \left(b \,x^{6}+a \,x^{3}\right)^{\frac{5}{3}}}$	46
orering	$-\frac{x \left(b \,x^{3}+a \right) \left(-9 b^{2} x^{6}-6 a \,x^{3} b +a^{2}\right)}{4 a^{3} \left(b \,x^{6}+a \,x^{3}\right)^{\frac{5}{3}}}$	46
trager	$-\frac{\left(-9 b^{2} x^{6}-6 a \,x^{3} b +a^{2}\right) \left(b \,x^{6}+a \,x^{3}\right)^{\frac{1}{3}}}{4 \left(b \,x^{3}+a \right) a^{3} x^{5}}$	50
risch	$-\frac{\left(b \,x^{3}+a \right) \left(-7 b \,x^{3}+a \right)}{4 a^{3} x^{2} \left(x^{3} \left(b \,x^{3}+a \right)\right)^{\frac{2}{3}}}+\frac{b^{2} x^{4}}{2 a^{3} \left(x^{3} \left(b \,x^{3}+a \right)\right)^{\frac{2}{3}}}$	62

input
```
int(1/(b*x^6+a*x^3)^(5/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^2*(-9*b^2*x^6-6*a*b*x^3+a^2)/(x^3*(b*x^3+a))^(2/3)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{(9\,b^2x^6+6\,abx^3-a^2)(bx^6+ax^3)^{\frac{1}{3}}}{4\,(a^3bx^8+a^4x^5)}$$

input
```
integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
```

output
```
1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)*(b*x^6 + a*x^3)^(1/3)/(a^3*b*x^8 + a^4*x
^5)
```

Sympy [F]

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \int \frac{1}{(ax^3+bx^6)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*x**6+a*x**3)**(5/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(-5/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{9\,b^2x^6+6\,abx^3-a^2}{4\,(bx^3+a)^{\frac{2}{3}}a^3x^4}$$

input
```
integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
```

output
```
1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)/((b*x^3 + a)^(2/3)*a^3*x^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{\frac{2\,b^2}{a\left(b+\frac{a}{x^3}\right)^{\frac{2}{3}}} - \frac{a^3\left(b+\frac{a}{x^3}\right)^{\frac{4}{3}} - 8\,a^3\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}b}{a^4}}{4\,a^2}$$

input
```
integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="giac")
```

output
```
1/4*(2*b^2/(a*(b + a/x^3)^(2/3)) - (a^3*(b + a/x^3)^(4/3) - 8*a^3*(b + a/x
^3)^(1/3)*b)/a^4)/a^2
```

Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx = \frac{(b\,x^6+a\,x^3)^{1/3}\,(-a^2+6\,a\,b\,x^3+9\,b^2\,x^6)}{4\,a^3\,x^5\,\left(b\,x^3+a\right)}$$

input
```
int(1/(a*x^3 + b*x^6)^(5/3),x)
```

output
```
((a*x^3 + b*x^6)^(1/3)*(9*b^2*x^6 - a^2 + 6*a*b*x^3))/(4*a^3*x^5*(a + b*x^
3))
```

Reduce [F]

$$\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx=\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a\,x^5+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^8}dx$$

input
```
int(1/(b*x^6+a*x^3)^(5/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*a*x**5 + (a + b*x**3)**(2/3)*b*x**8),x)
```

3.13 $\int \frac{1}{(ax^3+bx^6)^{8/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 134

$$\int \frac{1}{(ax^3+bx^6)^{8/3}}\,dx = -\frac{1}{7ax^2(ax^3+bx^6)^{5/3}}+\frac{3bx}{7a^2(ax^3+bx^6)^{5/3}} - \frac{27b^2x^4}{7a^3(ax^3+bx^6)^{5/3}}-\frac{81b^3x^7}{7a^4(ax^3+bx^6)^{5/3}}-\frac{243b^4x^{10}}{35a^5(ax^3+bx^6)^{5/3}}$$

output
```
-1/7/a/x^2/(b*x^6+a*x^3)^(5/3)+3/7*b*x/a^2/(b*x^6+a*x^3)^(5/3)-27/7*b^2*x^
4/a^3/(b*x^6+a*x^3)^(5/3)-81/7*b^3*x^7/a^4/(b*x^6+a*x^3)^(5/3)-243/35*b^4*
x^10/a^5/(b*x^6+a*x^3)^(5/3)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.51

$$\int \frac{1}{(ax^3+bx^6)^{8/3}}\,dx = \frac{-5a^4+15a^3bx^3-135a^2b^2x^6-405ab^3x^9-243b^4x^{12}}{35a^5x^2(x^3(a+bx^3))^{5/3}}$$

input
```
Integrate[(a*x^3 + b*x^6)^(-8/3),x]
```

output
```
(-5*a^4 + 15*a^3*b*x^3 - 135*a^2*b^2*x^6 - 405*a*b^3*x^9 - 243*b^4*x^12)/(
35*a^5*x^2*(x^3*(a + b*x^3))^(5/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1907, 1921, 1922, 1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^3+bx^6)^{8/3}}\,dx$$

$$\Big\downarrow \text{1907}$$

$$\frac{12\int \frac{1}{x^3(bx^6+ax^3)^{5/3}}dx}{5a}+\frac{1}{5ax^2(ax^3+bx^6)^{5/3}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{12\left(\frac{9\int \frac{1}{x^6\left(bx^6+ax^3\right)^{2/3}}dx}{2a}+\frac{1}{2ax^5(ax^3+bx^6)^{2/3}}\right)}{5a}+\frac{1}{5ax^2(ax^3+bx^6)^{5/3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{12\left(\frac{9\left(-\frac{6b\int \frac{1}{x^3\left(bx^6+ax^3\right)^{2/3}}dx}{7a}-\frac{\sqrt[3]{ax^3+bx^6}}{7ax^8}\right)}{2a}+\frac{1}{2ax^5(ax^3+bx^6)^{2/3}}\right)}{5a}+\frac{1}{5ax^2(ax^3+bx^6)^{5/3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{12\left(\frac{9\left(-\frac{6b\left(-\frac{3b\int \frac{1}{\left(bx^6+ax^3\right)^{2/3}}dx}{4a}-\frac{\sqrt[3]{ax^3+bx^6}}{4ax^5}\right)}{7a}-\frac{\sqrt[3]{ax^3+bx^6}}{7ax^8}\right)}{2a}+\frac{1}{2ax^5(ax^3+bx^6)^{2/3}}\right)}{5a}+\frac{1}{5ax^2\left(ax^3+bx^6\right)^{5/3}}$$

$$\Big\downarrow \; 1906$$

$$\frac{12\left(\frac{9\left(-\frac{6b\left(\frac{3b\sqrt[3]{ax^3+bx^6}}{4a^2x^2}-\frac{\sqrt[3]{ax^3+bx^6}}{4ax^5}\right)}{7a}-\frac{\sqrt[3]{ax^3+bx^6}}{7ax^8}\right)}{2a}+\frac{1}{2ax^5(ax^3+bx^6)^{2/3}}\right)}{5a}+\frac{1}{5ax^2\left(ax^3+bx^6\right)^{5/3}}$$

input
```
Int[(a*x^3 + b*x^6)^(-8/3),x]
```

output
```
1/(5*a*x^2*(a*x^3 + b*x^6)^(5/3)) + (12*(1/(2*a*x^5*(a*x^3 + b*x^6)^(2/3))
 + (9*(-1/7*(a*x^3 + b*x^6)^(1/3)/(a*x^8) - (6*b*(-1/4*(a*x^3 + b*x^6)^(1/
3)/(a*x^5) + (3*b*(a*x^3 + b*x^6)^(1/3))/(4*a^2*x^2)))/(7*a)))/(2*a)))/(5*
a)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 1907
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b, j, n}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j +
1)/(n - j)], 0] && LtQ[p, -1]
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{x(b\,x^{3}+a)\left(243b^{4}x^{12}+405a\,b^{3}x^{9}+135b^{2}x^{6}a^{2}-15b\,x^{3}a^{3}+5a^{4}\right)}{35a^{5}\left(b\,x^{6}+a\,x^{3}\right)^{\frac{8}{3}}}$	70
orering	$-\frac{x(b\,x^{3}+a)\left(243b^{4}x^{12}+405a\,b^{3}x^{9}+135b^{2}x^{6}a^{2}-15b\,x^{3}a^{3}+5a^{4}\right)}{35a^{5}\left(b\,x^{6}+a\,x^{3}\right)^{\frac{8}{3}}}$	70
pseudoelliptic	$-\frac{\frac{243}{5}b^{4}x^{12}+81a\,b^{3}x^{9}+27b^{2}x^{6}a^{2}-3b\,x^{3}a^{3}+a^{4}}{7\left(x^{3}(b\,x^{3}+a)\right)^{\frac{2}{3}}x^{5}(b\,x^{3}+a)a^{5}}$	72
trager	$-\frac{\left(243b^{4}x^{12}+405a\,b^{3}x^{9}+135b^{2}x^{6}a^{2}-15b\,x^{3}a^{3}+5a^{4}\right)\left(b\,x^{6}+a\,x^{3}\right)^{\frac{1}{3}}}{35x^{8}a^{5}\left(b\,x^{3}+a\right)^{2}}$	74
risch	$-\frac{(b\,x^{3}+a)\left(36b^{2}x^{6}-5a\,x^{3}b+a^{2}\right)}{7a^{5}x^{5}\left(x^{3}(b\,x^{3}+a)\right)^{\frac{2}{3}}}-\frac{b^{3}x^{4}\left(9b\,x^{3}+10a\right)}{5(b\,x^{3}+a)a^{5}\left(x^{3}(b\,x^{3}+a)\right)^{\frac{2}{3}}}$	92

input
```
int(1/(b*x^6+a*x^3)^(8/3),x,method=_RETURNVERBOSE)
```

output
```
-1/35*x*(b*x^3+a)*(243*b^4*x^12+405*a*b^3*x^9+135*a^2*b^2*x^6-15*a^3*b*x^3
+5*a^4)/a^5/(b*x^6+a*x^3)^(8/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.65

$$\int \frac{1}{(ax^3+bx^6)^{8/3}}\,dx = -\frac{(243\,b^4x^{12}+405\,ab^3x^9+135\,a^2b^2x^6-15\,a^3bx^3+5\,a^4)(bx^6+ax^3)^{\frac{1}{3}}}{35\,(a^5b^2x^{14}+2\,a^6bx^{11}+a^7x^8)}$$

input
```
integrate(1/(b*x^6+a*x^3)^(8/3),x, algorithm="fricas")
```

output
```
-1/35*(243*b^4*x^12 + 405*a*b^3*x^9 + 135*a^2*b^2*x^6 - 15*a^3*b*x^3 + 5*a
^4)*(b*x^6 + a*x^3)^(1/3)/(a^5*b^2*x^14 + 2*a^6*b*x^11 + a^7*x^8)
```

Sympy [F]

$$\int \frac{1}{(ax^3+bx^6)^{8/3}}\,dx = \int \frac{1}{(ax^3+bx^6)^{\frac{8}{3}}}\,dx$$

input
```
integrate(1/(b*x**6+a*x**3)**(8/3),x)
```

output
```
Integral((a*x**3 + b*x**6)**(-8/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.54

$$\int \frac{1}{(ax^3+bx^6)^{8/3}}\,dx = -\frac{243\,b^4x^{12}+405\,ab^3x^9+135\,a^2b^2x^6-15\,a^3bx^3+5\,a^4}{35\,(a^5bx^{10}+a^6x^7)(bx^3+a)^{\frac{2}{3}}}$$

input
```
integrate(1/(b*x^6+a*x^3)^(8/3),x, algorithm="maxima")
```

output
```
-1/35*(243*b^4*x^12 + 405*a*b^3*x^9 + 135*a^2*b^2*x^6 - 15*a^3*b*x^3 + 5*a
^4)/((a^5*b*x^10 + a^6*x^7)*(b*x^3 + a)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.66

$$\int \frac{1}{(ax^3+bx^6)^{8/3}}\,dx = -\frac{\frac{7\left(10\left(b+\frac{a}{x^3}\right)b^3-b^4\right)}{a\left(b+\frac{a}{x^3}\right)^{\frac{5}{3}}}+\frac{5\left(a^6\left(b+\frac{a}{x^3}\right)^{\frac{7}{3}}-7\,a^6\left(b+\frac{a}{x^3}\right)^{\frac{4}{3}}b+42\,a^6\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}b^2\right)}{a^7}}{35\,a^4}$$

input
```
integrate(1/(b*x^6+a*x^3)^(8/3),x, algorithm="giac")
```

output
```
-1/35*(7*(10*(b + a/x^3)*b^3 - b^4)/(a*(b + a/x^3)^(5/3)) + 5*(a^6*(b + a/
x^3)^(7/3) - 7*a^6*(b + a/x^3)^(4/3)*b + 42*a^6*(b + a/x^3)^(1/3)*b^2)/a^7
)/a^4
```

Mupad [B] (verification not implemented)

Time = 20.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \frac{1}{(ax^3+bx^6)^{8/3}}\,dx = \frac{5\,b\,{\left(b\,x^6+a\,x^3\right)}^{1/3}}{7\,a^4\,x^5}-\frac{{\left(b\,x^6+a\,x^3\right)}^{1/3}}{7\,a^3\,x^8}-\frac{{\left(b\,x^6+a\,x^3\right)}^{1/3}\,\left(\frac{187\,b^2}{35\,a^4}+\frac{243\,b^3\,x^3}{35\,a^5}\right)}{x^2\,\left(b\,x^3+a\right)}+\frac{b^2\,{\left(b\,x^6+a\,x^3\right)}^{1/3}}{5\,a^3\,x^2\,{\left(b\,x^3+a\right)}^2}$$

input
```
int(1/(a*x^3 + b*x^6)^(8/3),x)
```

output
```
(5*b*(a*x^3 + b*x^6)^(1/3))/(7*a^4*x^5) - (a*x^3 + b*x^6)^(1/3)/(7*a^3*x^8
) - ((a*x^3 + b*x^6)^(1/3)*((187*b^2)/(35*a^4) + (243*b^3*x^3)/(35*a^5)))/
(x^2*(a + b*x^3)) + (b^2*(a*x^3 + b*x^6)^(1/3))/(5*a^3*x^2*(a + b*x^3)^2)
```

Reduce [F]

$$\int \frac{1}{(ax^3+bx^6)^{8/3}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}} a^2x^8+2\left(b\,x^3+a\right)^{\frac{2}{3}} ab\,x^{11}+\left(b\,x^3+a\right)^{\frac{2}{3}} b^2x^{14}}dx$$

input
```
int(1/(b*x^6+a*x^3)^(8/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*a**2*x**8 + 2*(a + b*x**3)**(2/3)*a*b*x**11 + (
a + b*x**3)**(2/3)*b**2*x**14),x)
```

3.14 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx$

Optimal result

Integrand size = 16, antiderivative size = 204

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c} + \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} x + \frac{5}{2}a^{3/2}b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) + \frac{5(b^4-24ab^2c-48a^2c^2)\operatorname{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{128c^{3/2}}$$

output

```
-5/24*(a+c/x^2+b/x)^(3/2)*(7*b+6*c/x)-5/64*(a+c/x^2+b/x)^(1/2)*(b*(44*a*c+
b^2)+2*c*(12*a*c+b^2)/x)/c+(a+c/x^2+b/x)^(5/2)*x+5/2*a^(3/2)*b*arctanh(1/2
*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))+5/128*(-48*a^2*c^2-24*a*b^2*c+b^4)
*arctanh(1/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx =$$
$$-\frac{\sqrt{a+\frac{c+bx}{x^2}}\left(15\left(b^4-24ab^2c-48a^2c^2\right)x^4\text{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)+\sqrt{c}\left(\sqrt{c+x(b+ax)}(48c^3+15b^3x^3\right.\right.}{192c^{3/2}x^3}$$

input
```
Integrate[(a + c/x^2 + b/x)^(5/2),x]
```

output
```
-1/192*(Sqrt[a + (c + b*x)/x^2]*(15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*x^4*Ar
cTanh[(Sqrt[a]*x - Sqrt[c + x*(b + a*x)])/Sqrt[c]] + Sqrt[c]*(Sqrt[c + x*(
b + a*x)]*(48*c^3 + 15*b^3*x^3 + 8*c^2*x*(17*b + 27*a*x) + 2*c*x^2*(59*b^2
 + 278*a*b*x - 96*a^2*x^2)) + 480*a^(3/2)*b*c*x^4*Log[b + 2*a*x - 2*Sqrt[a
]*Sqrt[c + x*(b + a*x)]])))/(c^(3/2)*x^3*Sqrt[c + x*(b + a*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {1681, 1161, 1231, 25, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2} dx$$
$$\Big\downarrow \text{1681}$$
$$-\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2} x^2 d\frac{1}{x}$$
$$\Big\downarrow \text{1161}$$
$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{2}\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}\left(b+\frac{2c}{x}\right) x d\frac{1}{x}$$

$$\Big\downarrow \text{1231}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{2}\left(\frac{1}{12}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{\int -c\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(8ab+\frac{b^2+12ac}{x}\right)xd\frac{1}{x}}{8c}\right)$$

$$\Big\downarrow \text{25}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{2}\left(\frac{\int c\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(8ab+\frac{b^2+12ac}{x}\right)xd\frac{1}{x}}{8c}+\frac{1}{12}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{27}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{2}\left(\frac{1}{8}\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(8ab+\frac{b^2+12ac}{x}\right)xd\frac{1}{x}+\frac{1}{12}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{1231}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{2}\left(\frac{1}{8}\left(\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+b(44ac+b^2)\right)}{4c}-\frac{\int -\frac{\left(64a^2bc-\frac{b^4-24acb^2-48a^2c^2}{x}\right)x}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{4c}\right)+\frac{1}{12}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}\right.$$

$$\Big\downarrow \text{27}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{2}\left(\frac{1}{8}\left(\frac{\int \frac{\left(64a^2bc-\frac{b^4-24acb^2-48a^2c^2}{x}\right)x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+b(44ac+b^2)\right)}{4c}\right)+\frac{1}{12}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\right.$$

$$\Big\downarrow \text{1269}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{2}\left(\frac{1}{8}\left(\frac{64a^2bc\int\frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}-\left(-48a^2c^2-24ab^2c+b^4\right)\int\frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+b(44ac+\right.}{4c}\right.\right.$$

$\Big\downarrow$ 1092

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{2}\left(\frac{1}{8}\left(\frac{64a^2bc\int\frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}-2\left(-48a^2c^2-24ab^2c+b^4\right)\int\frac{1}{4c-\frac{1}{x^2}}d\frac{b+\frac{2c}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+b(4\right.}{4c}\right.\right.$$

$\Big\downarrow$ 219

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{2}\left(\frac{1}{8}\left(\frac{64a^2bc\int\frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}-\frac{(-48a^2c^2-24ab^2c+b^4)\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+b(44ac\right.}{4c}\right.\right.$$

$\Big\downarrow$ 1154

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{2}\left(\frac{1}{8}\left(\frac{-128a^2bc\int\frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}-\frac{(-48a^2c^2-24ab^2c+b^4)\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac+b^2)}{x}+\right.}{4c}\right.\right.$$

$\Big\downarrow$ 219

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-$$

$$\frac{5}{2}\left(\frac{1}{8}\left(\frac{-64a^{3/2}bc\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)-\frac{(-48a^2c^2-24ab^2c+b^4)\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{2c(12ac}{x}\right.}{4}\right.\right.$$

input
```
Int[(a + c/x^2 + b/x)^(5/2),x]
```

output
```
(a + c/x^2 + b/x)^(5/2)*x - (5*(((a + c/x^2 + b/x)^(3/2)*(7*b + (6*c)/x))/
12 + ((Sqrt[a + c/x^2 + b/x]*(b*(b^2 + 44*a*c) + (2*c*(b^2 + 12*a*c))/x))/
(4*c) + (-64*a^(3/2)*b*c*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b
/x])] - ((b^4 - 24*a*b^2*c - 48*a^2*c^2)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*
Sqrt[a + c/x^2 + b/x])])/Sqrt[c])/(8*c))/8))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(556abc\,x^3+15b^3x^3+216a\,c^2x^2+118b^2c\,x^2+136b\,c^2x+48c^3)\sqrt{\frac{a\,x^2+bx+c}{x^2}}}{192x^3c}+\frac{\left(-\frac{(240a^2c^2+120a\,b^2c-5b^4)\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+\cdots}}{x}\right)}{\sqrt{c}}\cdots\right.}{\cdots}$
default	$\frac{\left(\frac{a\,x^2+bx+c}{x^2}\right)^{\frac{5}{2}}x\left(-360\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)c^{\frac{7}{2}}a^{\frac{5}{2}}b^2x^4+660a^{\frac{5}{2}}\sqrt{a\,x^2+bx+c}\,b^2c^3x^4+152a^{\frac{7}{2}}(a\,x^2+bx+c)^{\frac{5}{2}}bc\,x^5-152a^{\frac{5}{2}}\cdots\right.}{\cdots}$

input
```
int((a+c/x^2+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(556*a*b*c*x^3+15*b^3*x^3+216*a*c^2*x^2+118*b^2*c*x^2+136*b*c^2*x+4
8*c^3)/x^3/c*((a*x^2+b*x+c)/x^2)^(1/2)+1/128/c*(-(240*a^2*c^2+120*a*b^2*c-
5*b^4)/c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)+384*a^(3/2)*b
*c*ln((1/2*b+x*a)/a^(1/2)+(a*x^2+b*x+c)^(1/2))+128*a^3*c*(1/a*(a*x^2+b*x+c
)^(1/2)-1/2*b/a^(3/2)*ln((1/2*b+x*a)/a^(1/2)+(a*x^2+b*x+c)^(1/2))))*((a*x^
2+b*x+c)/x^2)^(1/2)*x/(a*x^2+b*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 959, normalized size of antiderivative = 4.70

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(960*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(
2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 15*(b^4 - 24*a*b^2*c
 - 48*a^2*c^2)*sqrt(c)*x^3*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(
b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 4*(192*a^2*c^2*
x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2
+ 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/768*(1920*sqr
t(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x +
c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt
(c)*x^3*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt
(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 4*(192*a^2*c^2*x^4 - 136*b*c^3*x -
 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*s
qrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), 1/384*(480*a^(3/2)*b*c^2*x^3*log(-8
*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 +
 b*x + c)/x^2)) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1
/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x +
 c^2)) + 2*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c
^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^
2*x^3), -1/384*(960*sqrt(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-
a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b
^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqr...
```

Sympy [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx=\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+c/x**2+b/x)**(5/2),x)
```

output
```
Integral((a + b/x + c/x**2)**(5/2), x)
```

Maxima [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x + c/x^2)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2} dx$$

input
```
int((a + b/x + c/x^2)^(5/2),x)
```

output
```
int((a + b/x + c/x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.63

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = \frac{384\sqrt{a\,x^2+bx+c}\,a^2c^2x^4 - 1112\sqrt{a\,x^2+bx+c}\,ab\,c^2x^3 - 432\sqrt{a\,x^2+bx+c}\,a\,c^3x^2 - 30\sqrt{a}}{}$$

input
```
int((a+c/x^2+b/x)^(5/2),x)
```

output
```
(384*sqrt(a*x**2 + b*x + c)*a**2*c**2*x**4 - 1112*sqrt(a*x**2 + b*x + c)*a
*b*c**2*x**3 - 432*sqrt(a*x**2 + b*x + c)*a*c**3*x**2 - 30*sqrt(a*x**2 + b
*x + c)*b**3*c*x**3 - 236*sqrt(a*x**2 + b*x + c)*b**2*c**2*x**2 - 272*sqrt
(a*x**2 + b*x + c)*b*c**3*x - 96*sqrt(a*x**2 + b*x + c)*c**4 + 960*sqrt(a)
*log( - 2*sqrt(a)*sqrt(a*x**2 + b*x + c) - 2*a*x - b)*a*b*c**2*x**4 + 720*
sqrt(c)*log(2*sqrt(c)*sqrt(a*x**2 + b*x + c) - b*x - 2*c)*a**2*c**2*x**4 +
 360*sqrt(c)*log(2*sqrt(c)*sqrt(a*x**2 + b*x + c) - b*x - 2*c)*a*b**2*c*x*
*4 - 15*sqrt(c)*log(2*sqrt(c)*sqrt(a*x**2 + b*x + c) - b*x - 2*c)*b**4*x**
4 - 720*sqrt(c)*log(x)*a**2*c**2*x**4 - 360*sqrt(c)*log(x)*a*b**2*c*x**4 +
 15*sqrt(c)*log(x)*b**4*x**4)/(384*c**2*x**4)
```

3.15 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx$

Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}x + \frac{3}{2}\sqrt{a}b\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) - \frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{8\sqrt{c}}$$

output
```
-3/4*(a+c/x^2+b/x)^(1/2)*(3*b+2*c/x)+(a+c/x^2+b/x)^(3/2)*x+3/2*a^(1/2)*b*a
rctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))-3/8*(4*a*c+b^2)*arctanh(
1/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.09

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \frac{\sqrt{a+\frac{c+bx}{x^2}}\left(3(b^2+4ac)\,x^2\text{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)-\sqrt{c}\left((2c+x(5b-4ax))\sqrt{c+x(b+a\right.\right.}{4\sqrt{c}x\sqrt{c+x(b+ax)}}$$

input
```
Integrate[(a + c/x^2 + b/x)^(3/2),x]
```

output
```
(Sqrt[a + (c + b*x)/x^2]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(Sqrt[a]*x - Sqrt[c
+ x*(b + a*x)])/Sqrt[c]] - Sqrt[c]*((2*c + x*(5*b - 4*a*x))*Sqrt[c + x*(b
+ a*x)] + 6*Sqrt[a]*b*x^2*Log[b + 2*a*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]]
)))/(4*Sqrt[c]*x*Sqrt[c + x*(b + a*x)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1681, 1161, 1231, 25, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{1681}$$

$$-\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{1161}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{2}\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(b+\frac{2c}{x}\right)xd\frac{1}{x}$$

$$\Big\downarrow \text{1231}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{2}\left(\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}-\frac{\int -\frac{c\left(4ab+\frac{b^2+4ac}{x}\right)x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{4c}\right)$$

$$\Big\downarrow \text{25}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{2}\left(\frac{\int \frac{c\left(4ab+\frac{b^2+4ac}{x}\right)x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{4c}+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{2}\left(\frac{1}{4}\int \frac{\left(4ab+\frac{b^2+4ac}{x}\right)x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$$\Big\downarrow \text{1269}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-$$
$$\frac{3}{2}\left(\frac{1}{4}\left(\left(4ac+b^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}+4ab\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}\right)+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$$\Big\downarrow \text{1092}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-$$
$$\frac{3}{2}\left(\frac{1}{4}\left(2\left(4ac+b^2\right)\int \frac{1}{4c-\frac{1}{x^2}}d\frac{b+\frac{2c}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+4ab\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}\right)+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$$\Big\downarrow \text{219}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-$$
$$\frac{3}{2}\left(\frac{1}{4}\left(4ab\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}+\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}\right)+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$$\Big\downarrow \text{1154}$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-$$
$$\frac{3}{2}\left(\frac{1}{4}\left(\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}-8ab\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

$$\Big\downarrow 219$$

$$x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-$$

$$\frac{3}{2}\left(\frac{1}{4}\left(\frac{(4ac+b^2)\,\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{c}}-4\sqrt{a}b\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)\right)+\frac{1}{2}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\right)$$

input
```
Int[(a + c/x^2 + b/x)^(3/2),x]
```

output
```
(a + c/x^2 + b/x)^(3/2)*x - (3*((Sqrt[a + c/x^2 + b/x]*(3*b + (2*c)/x))/2
+ (-4*Sqrt[a]*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])] + (
(b^2 + 4*a*c)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/Sq
rt[c])/4))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{(5bx+2c)\sqrt{\frac{a\,x^2+bx+c}{x^2}}}{4x}+\frac{\left(-\frac{3\sqrt{c}\,\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)a}{2}-\frac{3\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)b^2}{8\sqrt{c}}+a\sqrt{a\,x^2+bx+c}+\frac{3b\sqrt{a}\,\ln\left(\cdots\right.}{\cdots}\right.}{\sqrt{a\,x^2+bx+c}}$
default	$-\frac{\left(\frac{a\,x^2+bx+c}{x^2}\right)^{\frac{3}{2}}x\left(12a^{\frac{5}{2}}c^{\frac{5}{2}}\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)x^2-2a^{\frac{5}{2}}\left(a\,x^2+bx+c\right)^{\frac{3}{2}}b\,x^3-4a^{\frac{5}{2}}\left(a\,x^2+bx+c\right)^{\frac{3}{2}}c\,x^2-6a^{\frac{5}{2}}\sqrt{a\,x^2+bx+c}\cdots\right.}{\cdots}$

input
```
int((a+c/x^2+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(5*b*x+2*c)/x*((a*x^2+b*x+c)/x^2)^(1/2)+(-3/2*c^(1/2)*ln((2*c+b*x+2*c
^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*a-3/8/c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+
b*x+c)^(1/2))/x)*b^2+a*(a*x^2+b*x+c)^(1/2)+3/2*b*a^(1/2)*ln((1/2*b+x*a)/a^
(1/2)+(a*x^2+b*x+c)^(1/2)))*((a*x^2+b*x+c)/x^2)^(1/2)*x/(a*x^2+b*x+c)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 709, normalized size of antiderivative = 4.89

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(12*sqrt(a)*b*c*x*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^
2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*sqrt(c)*x*
log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt
((a*x^2 + b*x + c)/x^2))/x^2) + 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*sqrt((a*x^
2 + b*x + c)/x^2))/(c*x), -1/16*(24*sqrt(-a)*b*c*x*arctan(1/2*(2*a*x^2 + b
*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 3*(b^2
 + 4*a*c)*sqrt(c)*x*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 +
 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 4*(4*a*c*x^2 - 5*b*c*x
 - 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), 1/8*(6*sqrt(a)*b*c*x*log(-8*
a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 +
b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqr
t(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(4*a*c*x^2
- 5*b*c*x - 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), -1/8*(12*sqrt(-a)*b
*c*x*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*
x^2 + a*b*x + a*c)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x
)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - 2*(4*a*c
*x^2 - 5*b*c*x - 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x)]
```

Sympy [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+c/x**2+b/x)**(3/2),x)
```

output
```
Integral((a + b/x + c/x**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x + c/x^2)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2} dx$$

input
```
int((a + b/x + c/x^2)^(3/2),x)
```

output
```
int((a + b/x + c/x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = \frac{8\sqrt{a\,x^2+bx+c}\,ac\,x^2-10\sqrt{a\,x^2+bx+c}\,bcx-4\sqrt{a\,x^2+bx+c}\,c^2+12\sqrt{a}\log\left(-2\sqrt{a}\,\sqrt{a\,x$$

input
```
int((a+c/x^2+b/x)^(3/2),x)
```

output

```
(8*sqrt(a*x**2 + b*x + c)*a*c*x**2 - 10*sqrt(a*x**2 + b*x + c)*b*c*x - 4*s
qrt(a*x**2 + b*x + c)*c**2 + 12*sqrt(a)*log( - 2*sqrt(a)*sqrt(a*x**2 + b*x
 + c) - 2*a*x - b)*b*c*x**2 + 12*sqrt(c)*log(2*sqrt(c)*sqrt(a*x**2 + b*x +
 c) - b*x - 2*c)*a*c*x**2 + 3*sqrt(c)*log(2*sqrt(c)*sqrt(a*x**2 + b*x + c)
 - b*x - 2*c)*b**2*x**2 - 12*sqrt(c)*log(x)*a*c*x**2 - 3*sqrt(c)*log(x)*b*
*2*x**2)/(8*c*x**2)
```

3.16 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x+\frac{b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2\sqrt{a}}-\sqrt{c}\operatorname{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)$$

output
```
(a+c/x^2+b/x)^(1/2)*x+1/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1
/2))/a^(1/2)-c^(1/2)*arctanh(1/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$$
$$=\frac{x\sqrt{a+\frac{c+bx}{x^2}}\left(2\sqrt{a}\sqrt{c+x(b+ax)}+4\sqrt{a}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)-b\log\left(b+2ax-2\sqrt{a}\sqrt{c+x(b\right.\right.}{2\sqrt{a}\sqrt{c+x(b+ax)}}$$

input
```
Integrate[Sqrt[a + c/x^2 + b/x],x]
```

output
```
(x*Sqrt[a + (c + b*x)/x^2]*(2*Sqrt[a]*Sqrt[c + x*(b + a*x)] + 4*Sqrt[a]*Sq
rt[c]*ArcTanh[(Sqrt[a]*x - Sqrt[c + x*(b + a*x)])/Sqrt[c]] - b*Log[b + 2*a
*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]]))/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1681, 1161, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}dx$$

$$\Big\downarrow \text{ 1681}$$

$$-\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 1161}$$

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}-\frac{1}{2}\int \frac{\left(b+\frac{2c}{x}\right)x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{2}\left(-2c\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}-b\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{2}\left(-4c\int \frac{1}{4c-\frac{1}{x^2}}d\frac{b+\frac{2c}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}-b\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(-b\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}-2\sqrt{c}\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{2}\left(2b\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}-2\sqrt{c}\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{\sqrt{a}}-2\sqrt{c}\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)\right)+x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

input
```
Int[Sqrt[a + c/x^2 + b/x],x]
```

output
```
Sqrt[a + c/x^2 + b/x]*x + ((b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^
2 + b/x])])/Sqrt[a] - 2*Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a +
c/x^2 + b/x])])/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\,x\left(-2\sqrt{c}\,\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^{2}+bx+c}}{x}\right)\sqrt{a}+b\ln\left(\frac{2\sqrt{a\,x^{2}+bx+c}\,\sqrt{a}+2xa+b}{2\sqrt{a}}\right)+2\sqrt{a\,x^{2}+bx+c}\,\sqrt{a}\right)}{2\sqrt{a\,x^{2}+bx+c}\,\sqrt{a}}$	121

input
```
int((a+c/x^2+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x^2+b*x+c)/x^2)^(1/2)*x*(-2*c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b
*x+c)^(1/2))/x)*a^(1/2)+b*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*x*a+b)/a
^(1/2))+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/(a*x^2+b*x+c)^(1/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 590, normalized size of antiderivative = 5.62

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$$
$$= \left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{a}b\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac-4\,(2\,ax^2+bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+2\,a\sqrt{c}\log\left(\right.}{4\,a}\right.$$

input
```
integrate((a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b
*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2))
+ 2*a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x
)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2))/a, 1/2*(2*a*x*sqrt((a*x^2 + b
*x + c)/x^2) - sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2
+ b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + a*sqrt(c)*log(-(8*b*c*x + (b^2
+ 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^
2))/x^2))/a, 1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + 4*a*sqrt(-c)*arctan(
1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x
+ c^2)) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 +
b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x
 + c)/x^2) - sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 +
b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 2*a*sqrt(-c)*arctan(1/2*(b*x^2 +
2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)))/a]
```

Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx = \int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
integrate((a+c/x**2+b/x)**(1/2),x)
```

output
```
Integral(sqrt(a + b/x + c/x**2), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\int \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

input
```
integrate((a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x + c/x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx=x\sqrt{\frac{1}{x^2}}\,\sqrt{a\,x^2+b\,x+c}-\sqrt{c}\,x\,\ln\left(\frac{2\,c+2\,\sqrt{c}\,\sqrt{a\,x^2+b\,x+c}+b\,x}{x}\right)\sqrt{\frac{1}{x^2}}+\frac{b\,x\,\ln\left(\frac{\frac{b}{2}+\sqrt{a}\,\sqrt{a\,x^2+b\,x+c}+a\,x}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\,\sqrt{a}}$$

input
```
int((a + b/x + c/x^2)^(1/2),x)
```

output
```
x*(1/x^2)^(1/2)*(c + b*x + a*x^2)^(1/2) - c^(1/2)*x*log((2*c + 2*c^(1/2)*(
c + b*x + a*x^2)^(1/2) + b*x)/x)*(1/x^2)^(1/2) + (b*x*log((b/2 + a^(1/2)*(
c + b*x + a*x^2)^(1/2) + a*x)/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 828, normalized size of antiderivative = 7.89

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx = \text{Too large to display}$$

input
```
int((a+c/x^2+b/x)^(1/2),x)
```

output
```
( - 4*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(a)*sqr
t(a*x**2 + b*x + c) + 2*a*x + b)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*a*c - 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(a)*
sqrt(a*x**2 + b*x + c) + 2*a*x + b)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**
2))*a*b + 8*sqrt(a*x**2 + b*x + c)*a**2*c - 2*sqrt(a*x**2 + b*x + c)*a*b**
2 + 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(2*sqrt(a)*sqrt(
a*x**2 + b*x + c) - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*a*x + b)*
a*c - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(2*sqrt(a)*sqr
t(a*x**2 + b*x + c) + sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*a*x + b
)*a*c - sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(2*sqrt(a)*sqr
t(a*x**2 + b*x + c) - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*a*x + b
)*a*b + sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(2*sqrt(a)*sqr
t(a*x**2 + b*x + c) + sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*a*x + b
)*a*b + 4*sqrt(a)*log((2*sqrt(a)*sqrt(a*x**2 + b*x + c) + 2*a*x + b)/sqrt(
4*a*c - b**2))*a*b*c - sqrt(a)*log((2*sqrt(a)*sqrt(a*x**2 + b*x + c) + 2*a
*x + b)/sqrt(4*a*c - b**2))*b**3 - 4*sqrt(c)*log(8*sqrt(a)*sqrt(a*x**2 + b
*x + c)*a*x + 4*sqrt(a)*sqrt(a*x**2 + b*x + c)*b + 4*sqrt(c)*sqrt(a)*b + 8
*a**2*x**2 + 8*a*b*x)*a**2*c + sqrt(c)*log(8*sqrt(a)*sqrt(a*x**2 + b*x + c
)*a*x + 4*sqrt(a)*sqrt(a*x**2 + b*x + c)*b + 4*sqrt(c)*sqrt(a)*b + 8*a**2*
x**2 + 8*a*b*x)*a*b**2 + 4*sqrt(c)*log(2*sqrt(a)*sqrt(a*x**2 + b*x + c)...
```

3.17 $\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} dx = \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x}{a} - \frac{b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{3/2}}$$

output
```
(a+c/x^2+b/x)^(1/2)*x/a-1/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^
(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} dx = \frac{2\sqrt{a}(c+x(b+ax)) - b\sqrt{c+x(b+ax)}\operatorname{arctanh}\left(\frac{b+2ax}{2\sqrt{a}\sqrt{c+x(b+ax)}}\right)}{2a^{3/2}x\sqrt{a+\frac{c+bx}{x^2}}}$$

input
```
Integrate[1/Sqrt[a + c/x^2 + b/x],x]
```

output
```
(2*Sqrt[a]*(c + x*(b + a*x)) - b*Sqrt[c + x*(b + a*x)]*ArcTanh[(b + 2*a*x)
/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])])/(2*a^(3/2)*x*Sqrt[a + (c + b*x)/x^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1681, 1157, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}dx$$
$$\Big\downarrow \text{ 1681}$$
$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}$$
$$\Big\downarrow \text{ 1157}$$
$$\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{2a}+\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}$$
$$\Big\downarrow \text{ 1154}$$
$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{a}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\,\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

input
```
Int[1/Sqrt[a + c/x^2 + b/x],x]
```

output
```
(Sqrt[a + c/x^2 + b/x]*x)/a - (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c
/x^2 + b/x])])/(2*a^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{\sqrt{a\,x^{2}+b x +c}\,\left(2 \sqrt{a\,x^{2}+b x +c}\, a^{\frac{3}{2}}-b \ln \left(\frac{2 \sqrt{a\,x^{2}+b x +c}\,\sqrt{a}+2 x a +b}{2 \sqrt{a}}\right) a \right)}{2 \sqrt{\frac{a\,x^{2}+b x +c}{x^{2}}}\, x\, a^{\frac{5}{2}}}$	88
risch	$\frac{a\,x^{2}+b x +c}{a \sqrt{\frac{a\,x^{2}+b x +c}{x^{2}}}\, x}-\frac{b \ln \left(\frac{\frac{b}{2}+x a}{\sqrt{a}}+\sqrt{a\,x^{2}+b x +c}\right) \sqrt{a\,x^{2}+b x +c}}{2 a^{\frac{3}{2}} \sqrt{\frac{a\,x^{2}+b x +c}{x^{2}}}\, x}$	97

input
```
int(1/(a+c/x^2+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^2+b*x+c)^(1/2)*(2*(a*x^2+b*x+c)^(1/2)*a^(3/2)-b*ln(1/2*(2*(a*x^2+
b*x+c)^(1/2)*a^(1/2)+2*x*a+b)/a^(1/2))*a)/((a*x^2+b*x+c)/x^2)^(1/2)/x/a^(5
/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.55

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = \left[\frac{4\,ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{a}b\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac+4\,(2\,ax^2+bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)}{4\,a^2}, \frac{2\,ax\sqrt{\frac{ax^2+bx}{x^2}}}{}\right.$$

input
```
integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b
*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)))
/a^2, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(-a)*b*arctan(1/2*(2*a*
x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)))/
a^2]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\,dx$$

input
```
integrate(1/(a+c/x**2+b/x)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b/x + c/x**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\,dx$$

input
```
integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a + b/x + c/x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [B] (verification not implemented)

Time = 18.84 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = \frac{x\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\,\mathrm{atanh}\left(\frac{a+\frac{b}{2\,x}}{\sqrt{a}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\,a^{3/2}}$$

input

```
int(1/(a + b/x + c/x^2)^(1/2),x)
```

output

```
(x*(a + b/x + c/x^2)^(1/2))/a - (b*atanh((a + b/(2*x))/(a^(1/2)*(a + b/x +
 c/x^2)^(1/2))))/(2*a^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx = \frac{2\sqrt{a\,x^2+bx+c}\,a-\sqrt{a}\log\left(\frac{2\sqrt{a}\,\sqrt{a\,x^2+bx+c}+2ax+b}{\sqrt{4ac-b^2}}\right)b}{2a^2}$$

input

```
int(1/(a+c/x^2+b/x)^(1/2),x)
```

output

```
(2*sqrt(a*x**2 + b*x + c)*a - sqrt(a)*log((2*sqrt(a)*sqrt(a*x**2 + b*x + c
) + 2*a*x + b)/sqrt(4*a*c - b**2))*b)/(2*a**2)
```

3.18 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx = \frac{b(3b^2-10ac)+\frac{c(3b^2-8ac)}{x}}{a^2(b^2-4ac)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} + \frac{x}{a\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{3b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{5/2}}$$

output

```
(b*(-10*a*c+3*b^2)+c*(-8*a*c+3*b^2)/x)/a^2/(-4*a*c+b^2)/(a+c/x^2+b/x)^(1/2
)+x/a/(a+c/x^2+b/x)^(1/2)-3/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x
)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{2\sqrt{a}(-3b^3x+10abcx+4ac(2c+ax^2)-b^2(3c+ax^2))-3b(b^2-4ac)\sqrt{c+x(b+ax)}\log\left(a^2\left(b+2ax\right.\right.}{2a^{5/2}\left(b^2-4ac\right)x\sqrt{a+\frac{c+bx}{x^2}}}$$

input
```
Integrate[(a + c/x^2 + b/x)^(-3/2),x]
```

output
```
-1/2*(2*Sqrt[a]*(-3*b^3*x + 10*a*b*c*x + 4*a*c*(2*c + a*x^2) - b^2*(3*c +
a*x^2)) - 3*b*(b^2 - 4*a*c)*Sqrt[c + x*(b + a*x)]*Log[a^2*(b + 2*a*x - 2*S
qrt[a]*Sqrt[c + x*(b + a*x)])])/(a^(5/2)*(b^2 - 4*a*c)*x*Sqrt[a + (c + b*x
)/x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1681, 1165, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1681}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{2\int -\frac{\left(3b^2+\frac{2cb}{x}-8ac\right)x^2}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{\left(3b^2+\frac{2cb}{x}-8ac\right)x^2}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

$$\Big\downarrow \ 1228$$

$$-\frac{-\frac{3b(b^2-4ac)\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{2a}-\frac{x(3b^2-8ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

$$\Big\downarrow \ 1154$$

$$-\frac{\frac{3b(b^2-4ac)\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{a}-\frac{x(3b^2-8ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{3b(b^2-4ac)\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}-\frac{x(3b^2-8ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a\left(b^2-4ac\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

input
```
Int[(a + c/x^2 + b/x)^(-3/2),x]
```

output
```
(-2*(b^2 - 2*a*c + (b*c)/x)*x)/(a*(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/x]) - (
-(((3*b^2 - 8*a*c)*Sqrt[a + c/x^2 + b/x]*x)/a) + (3*b*(b^2 - 4*a*c)*ArcTan
h[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(3/2)))/(a*(b^2 - 4
*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.58

method	result
default	$\frac{(a\,x^2+bx+c)\left(8a^{\frac{7}{2}}c\,x^2-2a^{\frac{5}{2}}b^2x^2+20a^{\frac{5}{2}}bcx-6a^{\frac{3}{2}}b^3x+16a^{\frac{5}{2}}c^2-6a^{\frac{3}{2}}b^2c-12\ln\left(\frac{2\sqrt{a\,x^2+bx+c}\,\sqrt{a}+2xa+b}{2\sqrt{a}}\right)\sqrt{a\,x^2+bx+c}\,a^2bc+3\ln}{2a^{\frac{7}{2}}\left(\frac{a\,x^2+bx+c}{x^2}\right)^{\frac{3}{2}}x^3(4ac-b^2)}$
risch	$\frac{a\,x^2+bx+c}{a^2\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,x}+\frac{\left(\frac{c}{a^2\sqrt{a\,x^2+bx+c}}-\frac{b^2}{4a^3\sqrt{a\,x^2+bx+c}}-\frac{b^3x}{2a^2\left(4ac-b^2\right)\sqrt{a\,x^2+bx+c}}-\frac{b^4}{4a^3\left(4ac-b^2\right)\sqrt{a\,x^2+bx+c}}+\frac{3bx}{2a^2\sqrt{a\,x^2+bx+c}}-\frac{3}{}\right.}{\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,x}$

input
```
int(1/(a+c/x^2+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^2+b*x+c)/a^(7/2)*(8*a^(7/2)*c*x^2-2*a^(5/2)*b^2*x^2+20*a^(5/2)*b*
c*x-6*a^(3/2)*b^3*x+16*a^(5/2)*c^2-6*a^(3/2)*b^2*c-12*ln(1/2*(2*(a*x^2+b*x
+c)^(1/2)*a^(1/2)+2*x*a+b)/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a^2*b*c+3*ln(1/2*(
2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*x*a+b)/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a*b^3)
/((a*x^2+b*x+c)/x^2)^(3/2)/x^3/(4*a*c-b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(111) = 222$.

Time = 0.14 (sec) , antiderivative size = 465, normalized size of antiderivative = 3.72

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx = \left[\frac{3\left(b^3c-4\,abc^2+(ab^3-4\,a^2bc)x^2+(b^4-4\,ab^2c)x\right)\sqrt{a}\log\left(-8\,a^2x^2-8\,abx-b\right.}{4\left(a^3b^2c-\right.}\right.$$

input
```
integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x
)*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(
a)*sqrt((a*x^2 + b*x + c)/x^2)) + 4*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 -
10*a^2*b*c)*x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/
(a^3*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x
), 1/2*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)
*x)*sqrt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^
2)/(a^2*x^2 + a*b*x + a*c)) + 2*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a
^2*b*c)*x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3
*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x)]
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+c/x**2+b/x)**(3/2),x)
```

output
```
Integral((a + b/x + c/x**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x + c/x^2)^(-3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(111) = 222$.

Time = 0.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.85

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{\left(3\,b^3\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-12\,abc\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{2\left(a^{\frac{5}{2}}b^2-4\,a^{\frac{7}{2}}c\right)} + \frac{\left(\frac{(ab^2-4\,a^2c)x}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}+\frac{3\,b^3-10\,abc}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}\right)x+\frac{3\,b^2c-8\,ac^2}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}}{\sqrt{ax^2+bx+c}} + \frac{3\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx+c}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*b^3*log(abs(b - 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(b - 2*sqrt(
a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/(a^(5/2)
*b^2 - 4*a^(7/2)*c) + (((a*b^2 - 4*a^2*c)*x/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(
x)) + (3*b^3 - 10*a*b*c)/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(x)))*x + (3*b^2*c -
 8*a*c^2)/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(x)))/sqrt(a*x^2 + b*x + c) + 3/2*b
*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x + c))*sqrt(a) + b))/(a^(5/2)*sgn(
x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/x + c/x^2)^(3/2),x)
```

output
```
int(1/(a + b/x + c/x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.99

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx = \frac{8\sqrt{a\,x^2+bx+c}\,a^3c\,x^2-2\sqrt{a\,x^2+bx+c}\,a^2b^2x^2+20\sqrt{a\,x^2+bx+c}\,a^2bcx+16\sqrt{}}{}$$

input
```
int(1/(a+c/x^2+b/x)^(3/2),x)
```

output
```
(8*sqrt(a*x**2 + b*x + c)*a**3*c*x**2 - 2*sqrt(a*x**2 + b*x + c)*a**2*b**2
*x**2 + 20*sqrt(a*x**2 + b*x + c)*a**2*b*c*x + 16*sqrt(a*x**2 + b*x + c)*a
**2*c**2 - 6*sqrt(a*x**2 + b*x + c)*a*b**3*x - 6*sqrt(a*x**2 + b*x + c)*a*
b**2*c - 12*sqrt(a)*log((2*sqrt(a)*sqrt(a*x**2 + b*x + c) + 2*a*x + b)/sqr
t(4*a*c - b**2))*a**2*b*c*x**2 + 3*sqrt(a)*log((2*sqrt(a)*sqrt(a*x**2 + b*
x + c) + 2*a*x + b)/sqrt(4*a*c - b**2))*a*b**3*x**2 - 12*sqrt(a)*log((2*sq
rt(a)*sqrt(a*x**2 + b*x + c) + 2*a*x + b)/sqrt(4*a*c - b**2))*a*b**2*c*x -
 12*sqrt(a)*log((2*sqrt(a)*sqrt(a*x**2 + b*x + c) + 2*a*x + b)/sqrt(4*a*c
- b**2))*a*b*c**2 + 3*sqrt(a)*log((2*sqrt(a)*sqrt(a*x**2 + b*x + c) + 2*a*
x + b)/sqrt(4*a*c - b**2))*b**4*x + 3*sqrt(a)*log((2*sqrt(a)*sqrt(a*x**2 +
 b*x + c) + 2*a*x + b)/sqrt(4*a*c - b**2))*b**3*c + 12*sqrt(a)*a**2*b*c*x*
*2 - 4*sqrt(a)*a*b**3*x**2 + 12*sqrt(a)*a*b**2*c*x + 12*sqrt(a)*a*b*c**2 -
 4*sqrt(a)*b**4*x - 4*sqrt(a)*b**3*c)/(2*a**3*(4*a**2*c*x**2 - a*b**2*x**2
 + 4*a*b*c*x + 4*a*c**2 - b**3*x - b**2*c))
```

3.19 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 211

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}} dx = \frac{b(5b^2-18ac)+\frac{c(5b^2-16ac)}{x}}{3a^2(b^2-4ac)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} + \frac{b(15b^4-110ab^2c+184a^2c^2)+\frac{c(15b^4-100ab^2c+128a^2c^2)}{x}}{3a^3(b^2-4ac)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} + \frac{x}{a\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{5b\operatorname{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{7/2}}$$

output

```
1/3*(b*(-18*a*c+5*b^2)+c*(-16*a*c+5*b^2)/x)/a^2/(-4*a*c+b^2)/(a+c/x^2+b/x)
^(3/2)+1/3*(b*(184*a^2*c^2-110*a*b^2*c+15*b^4)+c*(128*a^2*c^2-100*a*b^2*c+
15*b^4)/x)/a^3/(-4*a*c+b^2)^2/(a+c/x^2+b/x)^(1/2)+x/a/(a+c/x^2+b/x)^(3/2)-
5/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\frac{2\sqrt{a}(c+x(b+ax))\left(15b^6x^2+8a^2bc^2x(39c+32ax^2)-2ab^3cx(105c+74ax^2)+10b^5(3cx+2ax^3)+3b^4(5c^2-30acx^2+\right.}{(b^2-4ac)^2}}{}$$

input
```
Integrate[(a + c/x^2 + b/x)^(-5/2),x]
```

output
```
((2*Sqrt[a]*(c + x*(b + a*x))*(15*b^6*x^2 + 8*a^2*b*c^2*x*(39*c + 32*a*x^2
) - 2*a*b^3*c*x*(105*c + 74*a*x^2) + 10*b^5*(3*c*x + 2*a*x^3) + 3*b^4*(5*c
^2 - 30*a*c*x^2 + a^2*x^4) + 16*a^2*c^2*(8*c^2 + 12*a*c*x^2 + 3*a^2*x^4) -
 4*a*b^2*c*(25*c^2 - 12*a*c*x^2 + 6*a^2*x^4)))/(b^2 - 4*a*c)^2 + 15*b*(c +
 x*(b + a*x))^(5/2)*Log[a^3*(b + 2*a*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)])]
)/(6*a^(7/2)*x^5*(a + (c + b*x)/x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1681, 1165, 27, 1235, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1681}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{2\int -\frac{\left(5b^2+\frac{6cb}{x}-16ac\right)x^2}{2\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}d\frac{1}{x}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(5b^2+\frac{6cb}{x}-16ac\right)x^2}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}d\frac{1}{x}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{1235}$$

$$-\frac{\frac{2x\left(32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}-32ab^2c+5b^4\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}-\frac{2\int -\frac{\left(15b^4-100acb^2+\frac{2c\left(5b^2-28ac\right)b}{x}+128a^2c^2\right)x^2}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{a(b^2-4ac)}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\left(15b^4-100acb^2+\frac{2c\left(5b^2-28ac\right)b}{x}+128a^2c^2\right)x^2}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{a(b^2-4ac)}+\frac{2x\left(32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}-32ab^2c+5b^4\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{1228}$$

$$-\frac{\frac{-\frac{15b\left(b^2-4ac\right)^2\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{2a}-\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a(b^2-4ac)}+\frac{2x\left(32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}-32ab^2c+5b^4\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{\frac{\frac{15b\left(b^2-4ac\right)^2\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{a}-\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a(b^2-4ac)}+\frac{2x\left(32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}-32ab^2c+5b^4\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

$$\downarrow \text{219}$$

$$-\frac{\frac{2x\left(32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}-32ab^2c+5b^4\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{\frac{15b\left(b^2-4ac\right)^2\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}-\frac{x\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}}{a(b^2-4ac)}}{3a\left(b^2-4ac\right)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

input
```
Int[(a + c/x^2 + b/x)^(-5/2),x]
```

output
```
(-2*(b^2 - 2*a*c + (b*c)/x)*x)/(3*a*(b^2 - 4*a*c)*(a + c/x^2 + b/x)^(3/2))
 - ((2*(5*b^4 - 32*a*b^2*c + 32*a^2*c^2 + (b*c*(5*b^2 - 28*a*c))/x)*x)/(a*
(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/x]) + (-(((15*b^4 - 100*a*b^2*c + 128*a^2
*c^2)*Sqrt[a + c/x^2 + b/x]*x)/a) + (15*b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b
/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(3/2)))/(a*(b^2 - 4*a*c)))/(3
*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1681
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.78

method	result
default	$\frac{(a\,x^2+bx+c)\left(96a^{\frac{13}{2}}c^2x^4-48a^{\frac{11}{2}}b^2c\,x^4+512a^{\frac{11}{2}}b\,c^2x^3+6a^{\frac{9}{2}}b^4x^4+384a^{\frac{11}{2}}c^3x^2-296a^{\frac{9}{2}}b^3c\,x^3+96a^{\frac{9}{2}}b^2c^2x^2+40a^{\frac{7}{2}}b^5x^3+624a^{\frac{9}{2}}\right.}{}$
risch	Expression too large to display

input
```
int(1/(a+c/x^2+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x^2+b*x+c)*(96*a^(13/2)*c^2*x^4-48*a^(11/2)*b^2*c*x^4+512*a^(11/2)*
b*c^2*x^3+6*a^(9/2)*b^4*x^4+384*a^(11/2)*c^3*x^2-296*a^(9/2)*b^3*c*x^3+96*
a^(9/2)*b^2*c^2*x^2+40*a^(7/2)*b^5*x^3+624*a^(9/2)*b*c^3*x-180*a^(7/2)*b^4
*c*x^2+256*a^(9/2)*c^4-420*a^(7/2)*b^3*c^2*x+30*a^(5/2)*b^6*x^2-200*a^(7/2
)*b^2*c^3+60*a^(5/2)*b^5*c*x+30*a^(5/2)*b^4*c^2-240*ln(1/2*(2*(a*x^2+b*x+c
)^(1/2)*a^(1/2)+2*x*a+b)/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^4*b*c^2+120*ln(1/2
*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*x*a+b)/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^3*
b^3*c-15*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*x*a+b)/a^(1/2))*(a*x^2+b*
x+c)^(3/2)*a^2*b^5)/a^(11/2)/((a*x^2+b*x+c)/x^2)^(5/2)/x^5/(4*a*c-b^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(191) = 382$.

Time = 0.22 (sec) , antiderivative size = 1081, normalized size of antiderivative = 5.12

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^2*b^5 - 8*a^3*b^3*c +
 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^3 + (b^7 -
 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*
x)*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt
(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 4*(3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*a*b^6 - 30
*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35*a^2*b^3*
c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3*c^4)*x
)*sqrt((a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4 +
 (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*
a^7*b*c^2)*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^5*c -
 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x), 1/6*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4
*c + 16*a^3*b^2*c^2)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c
 - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x)*sqrt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sq
rt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 2*(3*(a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b
*c^2)*x^4 + 3*(5*a*b^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 +
 6*(5*a*b^5*c - 35*a^2*b^3*c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a
^2*b^2*c^3 + 128*a^3*c^4)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 ...
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+c/x**2+b/x)**(5/2),x)
```

output
```
Integral((a + b/x + c/x**2)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x + c/x^2)^(-5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. 2(191) = 382.

Time = 0.21 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.36

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx =$$
$$-\frac{\left(15\,b^5\sqrt{c}\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-120\,ab^3c^{\frac{3}{2}}\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+240\,a^2bc^{\frac{5}{2}}\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+30\,\sqrt{a}b^4c\right.}{6\left(a^{\frac{7}{2}}b^4\sqrt{c}-8\,a^{\frac{9}{2}}b^2c^{\frac{3}{2}}+16\,a^{\frac{11}{2}}c^{\frac{5}{2}}\right)}$$
$$+\frac{\left(\left(\left(\frac{3\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2\right)x}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}+\frac{4\left(5\,ab^5-37\,a^2b^3c+64\,a^3bc^2\right)}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}\right)x+\frac{3\left(5\,b^6-30\,ab^4c+16\,a^2b^2c^2+64\,a^3c^3\right)}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}\right.\right.}{3\left(ax^2+bx+c\right)^{\frac{3}{2}}}$$
$$+\frac{5\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx+c}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(15*b^5*sqrt(c)*log(abs(b - 2*sqrt(a)*sqrt(c))) - 120*a*b^3*c^(3/2)*l
og(abs(b - 2*sqrt(a)*sqrt(c))) + 240*a^2*b*c^(5/2)*log(abs(b - 2*sqrt(a)*s
qrt(c))) + 30*sqrt(a)*b^4*c - 200*a^(3/2)*b^2*c^2 + 256*a^(5/2)*c^3)*sgn(x
)/(a^(7/2)*b^4*sqrt(c) - 8*a^(9/2)*b^2*c^(3/2) + 16*a^(11/2)*c^(5/2)) + 1/
3*((((3*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*x/(a^3*b^4*sgn(x) - 8*a^4*b^2
*c*sgn(x) + 16*a^5*c^2*sgn(x)) + 4*(5*a*b^5 - 37*a^2*b^3*c + 64*a^3*b*c^2)
/(a^3*b^4*sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*a^5*c^2*sgn(x)))*x + 3*(5*b^6 -
 30*a*b^4*c + 16*a^2*b^2*c^2 + 64*a^3*c^3)/(a^3*b^4*sgn(x) - 8*a^4*b^2*c*s
gn(x) + 16*a^5*c^2*sgn(x)))*x + 6*(5*b^5*c - 35*a*b^3*c^2 + 52*a^2*b*c^3)/
(a^3*b^4*sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*a^5*c^2*sgn(x)))*x + (15*b^4*c^2
 - 100*a*b^2*c^3 + 128*a^2*c^4)/(a^3*b^4*sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*
a^5*c^2*sgn(x)))/(a*x^2 + b*x + c)^(3/2) + 5/2*b*log(abs(2*(sqrt(a)*x - sq
rt(a*x^2 + b*x + c))*sqrt(a) + b))/(a^(7/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}}\,dx$$

input
```
int(1/(a + b/x + c/x^2)^(5/2),x)
```

output
```
int(1/(a + b/x + c/x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a+c/x^2+b/x)^(5/2),x)
```

output
```
int(1/(a+c/x^2+b/x)^(5/2),x)
```

3.20 $\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx=\frac{a\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,x}{a+\frac{b}{x}}+\frac{b\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\log(x)}{a+\frac{b}{x}}$$

output
```
a*(a^2+b^2/x^2+2*a*b/x)^(1/2)*x/(a+b/x)+b*(a^2+b^2/x^2+2*a*b/x)^(1/2)*ln(x
)/(a+b/x)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx=\frac{x\sqrt{\frac{(b+ax)^2}{x^2}}(ax+b\log(x))}{b+ax}$$

input
```
Integrate[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

output
```
(x*Sqrt[(b + a*x)^2/x^2]*(a*x + b*Log[x]))/(b + a*x)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.66, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}\int \left(\frac{b^2}{x}+ab\right)dx}{b\left(a+\frac{b}{x}\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}\left(abx+b^2\log(x)\right)}{b\left(a+\frac{b}{x}\right)}$$

input
```
Int[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

output
```
(Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*(a*b*x + b^2*Log[x]))/(b*(a + b/x))
```

Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{\sqrt{\frac{a^{2}x^{2}+2abx+b^{2}}{x^{2}}}\, x(xa+b\ln(x))}{xa+b}$	40
risch	$\frac{\sqrt{\frac{(xa+b)^{2}}{x^{2}}}\, x^{2}a}{xa+b}+\frac{\sqrt{\frac{(xa+b)^{2}}{x^{2}}}\, xb\ln(x)}{xa+b}$	52

input
```
int((a^2+b^2/x^2+2*a*b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a^2*x^2+2*a*b*x+b^2)/x^2)^(1/2)/(a*x+b)*x*(x*a+b*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx=ax+b\log\left(x\right)$$

input
```
integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="fricas")
```

output
```
a*x + b*log(x)
```

Sympy [F]

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx=\int \sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}\,dx$$

input
```
integrate((a**2+b**2/x**2+2*a*b/x)**(1/2),x)
```

output
```
Integral(sqrt(a**2 + 2*a*b/x + b**2/x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx = ax + b\log(x)$$

input
```
integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="maxima")
```

output
```
a*x + b*log(x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx = ax\mathrm{sgn}\left(ax^2+bx\right) + b\log\left(|x|\right)\mathrm{sgn}\left(ax^2+bx\right)$$

input
```
integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="giac")
```

output
```
a*x*sgn(a*x^2 + b*x) + b*log(abs(x))*sgn(a*x^2 + b*x)
```

Mupad [B] (verification not implemented)

Time = 19.92 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.91

$$\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx = x\sqrt{\frac{1}{x^2}}\sqrt{a^2\,x^2+2\,a\,b\,x+b^2} - x\,\ln\left(\frac{2\sqrt{b^2}\sqrt{a^2\,x^2+2\,a\,b\,x+b^2}+2\,b^2+2\,a\,b\,x}{x}\right)\sqrt{b^2}\sqrt{\frac{1}{x^2}} + \frac{a\,b\,x\,\ln\left(\frac{a\,b+\sqrt{a^2}\sqrt{a^2\,x^2+2\,a\,b\,x+b^2}+a^2\,x}{\sqrt{a^2}}\right)\sqrt{\frac{1}{x^2}}}{\sqrt{a^2}}$$

input
```
int((a^2 + b^2/x^2 + (2*a*b)/x)^(1/2),x)
```

output
```
x*(1/x^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) - x*log((2*(b^2)^(1/2)*(b^
2 + a^2*x^2 + 2*a*b*x)^(1/2) + 2*b^2 + 2*a*b*x)/x)*(b^2)^(1/2)*(1/x^2)^(1/
2) + (a*b*x*log((a*b + (a^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) + a^2*x
)/(a^2)^(1/2))*(1/x^2)^(1/2))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2 + \frac{b^2}{x^2} + \frac{2ab}{x}}\, dx = \log(x)\, b + ax$$

input
```
int((a^2+b^2/x^2+2*a*b/x)^(1/2),x)
```

output
```
log(x)*b + a*x
```

3.21 $\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
x/c-1/2*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*
(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^
2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.13

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx=\frac{x}{c}-\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[(c + a/x^4 + b/x^2)^(-1),x]
```

output
```
x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1679, 1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{a}{x^4}+\frac{b}{x^2}+c}dx$$

$$\Big\downarrow \text{1679}$$

$$\int \frac{x^4}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x}{c}-\frac{\int \frac{bx^2+a}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \text{1480}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}$$

input
```
Int[(c + a/x^4 + b/x^2)^(-1),x]
```

output
```
x/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.32

method	result
risch	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+b\_Z^{2}+a\right)}\frac{\left(-\_R^{2}b-a\right)\ln\left(x-\_R\right)}{2\_R^{3}c+\_Rb}}{2c}$
default	$\frac{x}{c}-\frac{\left(-b\sqrt{-4ac+b^{2}}-2ac+b^{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2c\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\left(-b\sqrt{-4ac+b^{2}}+2ac-b^{2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right.}{2c\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}$

input
```
int(1/(c+a/x^4+b/x^2),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/2/c*sum((-_R^2*b-a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b
+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(143) = 286.

Time = 0.11 (sec) , antiderivative size = 1059, normalized size of antiderivative = 5.92

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^4+b/x^2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b
^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*
c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3
*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*
c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2
*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4
*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*
sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sq
rt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3
- 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l
og(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^
3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*s...
```

Sympy [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.72

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$$
$$=\operatorname{RootSum}\left(t^4\cdot\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\cdot\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32\right.\right.\right.$$
$$+\frac{x}{c}$$

input
```
integrate(1/(c+a/x**4+b/x**2),x)
```

output
```
RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48
*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t*
*3*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b*
*4)/(a**2*c - a*b**2)))) + x/c
```

Maxima [F]

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \int \frac{1}{c+\frac{b}{x^2}+\frac{a}{x^4}}\,dx$$

input
```
integrate(1/(c+a/x^4+b/x^2),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2109 vs. 2(143) = 286.

Time = 0.53 (sec) , antiderivative size = 2109, normalized size of antiderivative = 11.78

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^4+b/x^2),x, algorithm="giac")
```

output
```
x/c - 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c
^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 + 2*
(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*s...
```

Mupad [B] (verification not implemented)

Time = 20.80 (sec) , antiderivative size = 3026, normalized size of antiderivative = 16.91

$$\int \frac{1}{c + \frac{a}{x^4} + \frac{b}{x^2}} \, dx = \text{Too large to display}$$

input
```
int(1/(c + a/x^4 + b/x^2),x)
```

output
```
x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)
*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c
)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) -
 (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c +
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4
*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^
4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b
^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/(((
(16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 ...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.06

$$\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$$
$$= \frac{2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)bc+4\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)ac-2\sqrt{c}\,\sqrt{2}}{}$$

input
```
int(1/(c+a/x^4+b/x^2),x)
```

output
```
(2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c + 4*sqrt(c)*sqrt(2*sqrt(c)*
sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2 - 2*
sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c - 4*sqrt(c)*sqrt(2*sqrt(c)*sqr
t(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2 - sqrt(
a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt
(a) + sqrt(c)*x**2)*b*c + sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*s
qrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b*c + 2*sqrt(c)*sqrt(2*sqr
t(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)
*x**2)*a*c - sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqr
t(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**2 - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt
(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c +
 sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + s
qrt(a) + sqrt(c)*x**2)*b**2 + 16*a*c**2*x - 4*b**2*c*x)/(4*c**2*(4*a*c - b
**2))
```

3.22 $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 631

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

output
```
x/c+1/6*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(1-2*2^(1/3)*c^(1/3
)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)/c^(4/3)/(b-(-4*
a*c+b^2)^(1/2))^(2/3)+1/6*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(1/3*(
1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*2^(2/3)*3^(1/
2)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/6*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(
1/2))*ln((b-(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(2/3)/c^(4/3)/(
b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*ln((b+
(-4*a*c+b^2)^(1/2))^(1/3)+2^(1/3)*c^(1/3)*x)*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^
2)^(1/2))^(2/3)+1/12*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*ln((b-(-4*a*c+b^2
)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b-(-4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2
/3)*x^2)*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*(b+(-2*a*c+b^2)
/(-4*a*c+b^2)^(1/2))*ln((b+(-4*a*c+b^2)^(1/2))^(2/3)-2^(1/3)*c^(1/3)*(b+(-
4*a*c+b^2)^(1/2))^(1/3)*x+2^(2/3)*c^(2/3)*x^2)*2^(2/3)/c^(4/3)/(b+(-4*a*c+
b^2)^(1/2))^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.11

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \frac{x}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

input
```
Integrate[(c + a/x^6 + b/x^3)^(-1),x]
```

output
```
x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)
/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 521, normalized size of antiderivative = 0.83, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {1679, 1703, 1752, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{a}{x^6}+\frac{b}{x^3}+c}dx$$

$$\Big\downarrow \text{1679}$$

$$\int \frac{x^6}{a+bx^3+cx^6}dx$$

$$\Big\downarrow \text{1703}$$

$$\frac{x}{c}-\frac{\int \frac{bx^3+a}{cx^6+bx^3+a}dx}{c}$$

$$\Big\downarrow \text{1752}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^3+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{750}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\int \frac{1}{\sqrt[3]{c}x+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right)}{}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2^{2/3}\int \frac{2\left(2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x\right)}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\ldots}$$

$$\frac{x}{c}-$$

$\downarrow$ 27

$$\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{\ldots}\right.}{3\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\ldots}$$

$$\frac{x}{c}-$$

$\downarrow$ 1142

$$\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}-\frac{\int -\frac{\sqrt[3]{\ldots}}{2c^{2/3}x^2-2^{2/3}\ldots}}{\ldots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{\ldots}$$

$$\frac{x}{c}-$$

$\downarrow$ 25

$$\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{\int\frac{\sqrt[3]{c}\ \cdots}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{\cdots}}\cdots}{\cdots}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\right.\frac{x}{c}-$$

$\downarrow$ 27

$$\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\ 2^{2/3}\left(\frac{3\sqrt[3]{b-\sqrt{b^2-4ac}}\int\frac{1}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx}{2\sqrt[3]{2}}+\frac{1}{4}\int\frac{\cdots}{2c^{2/3}x^2-2^{2/3}\cdots}\cdots\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}\cdots}\right.\frac{x}{c}-$$

$\downarrow$ 1082

$$\frac{x}{c}-\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\;2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)^2-3}\cdots}{2\sqrt[3]{c}}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$\downarrow$ 217

$$\frac{x}{c}-\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\;2^{2/3}\left(\frac{1}{4}\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-4\sqrt[3]{c}x}{2c^{2/3}x^2-2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\cdots\right)}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\cdots\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{x}{c} - \frac{\frac{1}{2}\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{2\,2^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2\sqrt[3]{c}} - \frac{\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right.}{4\sqrt[3]{c}}\right.}{3\left(b-\sqrt{b^2-4ac}\right)^{2/3}}\right.}{}$$

input
```
Int[(c + a/x^6 + b/x^3)^(-1),x]
```

output
```
x/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b - Sqrt[b^2
- 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(
2/3)) + (2*2^(2/3)*(-1/2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - S
qrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/c^(1/3) - Log[(b - Sqrt[b^2 - 4*a*c])^(
2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x
^2]/(4*c^(1/3))))/(3*(b - Sqrt[b^2 - 4*a*c])^(2/3))))/2 + ((b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*((2^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/
3)*c^(1/3)*x])/(3*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (2*2^(2/3)*(-1/
2*(Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3)
)/Sqrt[3]])/c^(1/3) - Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*
(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2]/(4*c^(1/3))))/(3*(b
 + Sqrt[b^2 - 4*a*c])^(2/3))))/2)/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{6}+b\_Z^{3}+a\right)}\frac{\left(-\_R^{3} b-a\right)\ln\left(x-\_R\right)}{2\_R^{5} c+\_R^{2} b}}{3c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{6}+b\_Z^{3}+a\right)}\frac{\left(-\_R^{3} b-a\right)\ln\left(x-\_R\right)}{2\_R^{5} c+\_R^{2} b}}{3c}$	59

input
```
int(1/(c+a/x^6+b/x^3),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3
*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2882 vs. $2(495) = 990$.

Time = 0.19 (sec) , antiderivative size = 2882, normalized size of antiderivative = 4.57

$$\int \frac{1}{c + \frac{a}{x^6} + \frac{b}{x^3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^6+b/x^3),x, algorithm="fricas")
```

output
```
-1/6*((1/2)^(1/3)*(sqrt(-3)*c + c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*
sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*
c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))
^(1/3)*log(4*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*
b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3 + sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^
2*c^2 - 8*a^3*c^3) - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 + sqrt(-3)*(b^5
*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2
 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 -
 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6
*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9
+ 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)) - (1/2)^(1/
3)*(sqrt(-3)*c - c)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*
c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*log(4*(a
*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*x - (1/2)^(1/3)*(b^6 - 8*a*b^4*c + 18*a^2*
b^2*c^2 - 8*a^3*c^3 - sqrt(-3)*(b^6 - 8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c
^3) - (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(-3)*(b^5*c^4 - 8*a*b^3*
c^5 + 16*a^2*b*c^6))*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c
^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))
*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*...
```

Sympy [A] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.31

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$$
$$=\mathrm{RootSum}\left(t^6\cdot\left(46656a^3c^7-34992a^2b^2c^6+8748ab^4c^5-729b^6c^4\right)+t^3\cdot\left(864a^3bc^3-864a^2b^3c^2+270ab^5\right.\right.$$
$$+\frac{x}{c}$$

input
```
integrate(1/(c+a/x**6+b/x**3),x)
```

output
```
RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
 729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5
*c - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*
_t**4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2
*c**2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)
))) + x/c
```

Maxima [F]

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\int \frac{1}{c+\frac{b}{x^3}+\frac{a}{x^6}}\,dx$$

input
```
integrate(1/(c+a/x^6+b/x^3),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

Giac [F]

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\int \frac{1}{c+\frac{b}{x^3}+\frac{a}{x^6}}\,dx$$

input
```
integrate(1/(c+a/x^6+b/x^3),x, algorithm="giac")
```

output
```
integrate(1/(c + b/x^3 + a/x^6), x)
```

Mupad [B] (verification not implemented)

Time = 23.57 (sec) , antiderivative size = 2280, normalized size of antiderivative = 3.61

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx=\text{Too large to display}$$

input
```
int(1/(c + a/x^6 + b/x^3),x)
```

output
```
log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a
*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^
4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)
^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/
(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c +
 log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*
(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^
4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)
^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^
4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(
54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3
*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((
b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2
*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^
(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-...
```

Reduce [F]

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \frac{-\left(\int \frac{x^3}{c\,x^6+b\,x^3+a}dx\right)b-\left(\int \frac{1}{c\,x^6+b\,x^3+a}dx\right)a+x}{c}$$

input
```
int(1/(c+a/x^6+b/x^3),x)
```

output
```
( - int(x**3/(a + b*x**3 + c*x**6),x)*b - int(1/(a + b*x**3 + c*x**6),x)*a
 + x)/c
```

3.23 $\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}} dx$

Optimal result

Integrand size = 14, antiderivative size = 376

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \frac{x}{c} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
x/c+1/4*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b-(
-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/
4*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c
+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*(b+(
-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)
^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*(b-(-2*a*
c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2
))^(1/4))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.19

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx=\frac{x}{c}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

input
```
Integrate[(c + a/x^8 + b/x^4)^(-1),x]
```

output
```
x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)
/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1679, 1703, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{a}{x^8}+\frac{b}{x^4}+c}dx$$

$$\Big\downarrow \text{1679}$$

$$\int \frac{x^8}{a+bx^4+cx^8}dx$$

$\downarrow$ 1703

$$\frac{x}{c}-\frac{\int \frac{bx^4+a}{cx^8+bx^4+a}dx}{c}$$

$\downarrow$ 1752

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^4+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$\downarrow$ 756

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x^2+\sqrt{-b-\sqrt{b^2-4ac}}}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x}}{\sqrt{\sqrt{b^2-4ac}-b}}\right.}{c}$$

$\downarrow$ 218

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}dx}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}}}{\sqrt{\sqrt{b^2-4}}}\right.}{c}$$

$\downarrow$ 221

$$\frac{x}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\text{arcta}}{\sqrt[4]{}}\right.}{c}$$

input
```
Int[(c + a/x^8 + b/x^4)^(-1),x]
```

output
```
x/c - (((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c]
)^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2
^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + ((b - (b^2 - 2*a*c)/S
qrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)
*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^
2 - 4*a*c])^(3/4))))/2)/c
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1679
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(
2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n]
 && LtQ[n, 0] && IntegerQ[p]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)}\frac{\left(-\_R^{4}b-a\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+b\_Z^{4}+a\right)}\frac{\left(-\_R^{4}b-a\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	59

input
```
int(1/(c+a/x^8+b/x^4),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4
*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4001 vs. $2(296) = 592$.

Time = 0.36 (sec) , antiderivative size = 4001, normalized size of antiderivative = 10.64

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c+a/x^8+b/x^4),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx=\text{Timed out}$$

input
```
integrate(1/(c+a/x**8+b/x**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx=\int \frac{1}{c+\frac{b}{x^4}+\frac{a}{x^8}}\,dx$$

input
```
integrate(1/(c+a/x^8+b/x^4),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

Giac [F]

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx=\int \frac{1}{c+\frac{b}{x^4}+\frac{a}{x^8}}\,dx$$

input
```
integrate(1/(c+a/x^8+b/x^4),x, algorithm="giac")
```

output
```
integrate(1/(c + b/x^4 + a/x^8), x)
```

Mupad [B] (verification not implemented)

Time = 16.47 (sec) , antiderivative size = 10382, normalized size of antiderivative = 27.61

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \text{Too large to display}$$

input
```
int(1/(c + a/x^8 + b/x^4),x)
```

output
```
atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x
*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4
- 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*
c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4
*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2)
+ 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2
)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^
4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)
*1i - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x
*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 1
20*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4
- 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2...
```

Reduce [F]

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx = \int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}dx$$

input
```
int(1/(c+a/x^8+b/x^4),x)
```

output
```
int(1/(c+a/x^8+b/x^4),x)
```

3.24 $\int \left(a+b\sqrt{x}+cx\right)^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \left(a+b\sqrt{x}+cx\right)^3\,dx=a^3x+2a^2bx^{3/2}+\frac{3}{2}a\left(b^2+ac\right)x^2+\frac{2}{5}b\left(b^2+6ac\right)x^{5/2}+c\left(b^2+ac\right)x^3+\frac{6}{7}bc^2x^{7/2}+\frac{c^3x^4}{4}$$

output
```
a^3*x+2*a^2*b*x^(3/2)+3/2*a*(a*c+b^2)*x^2+2/5*b*(6*a*c+b^2)*x^(5/2)+c*(a*c
+b^2)*x^3+6/7*b*c^2*x^(7/2)+1/4*c^3*x^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int \left(a+b\sqrt{x}+cx\right)^3\,dx=\frac{1}{140}\left(140a^3x+280a^2bx^{3/2}+210ab^2x^2+210a^2cx^2+56b^3x^{5/2}+336abcx^{5/2}+140b^2cx^3+140ac^2x^3+120bc^2x^{7/2}+35c^3x^4\right)$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^3,x]
```

output
```
(140*a^3*x + 280*a^2*b*x^(3/2) + 210*a*b^2*x^2 + 210*a^2*c*x^2 + 56*b^3*x^
(5/2) + 336*a*b*c*x^(5/2) + 140*b^2*c*x^3 + 140*a*c^2*x^3 + 120*b*c^2*x^(7
/2) + 35*c^3*x^4)/140
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1680, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}+cx\right)^3 \, dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \sqrt{x}\left(a+cx+b\sqrt{x}\right)^3 d\sqrt{x}$$

$$\Big\downarrow \text{1140}$$

$$2\int \left(c^3x^{7/2}+3bc^2x^3+3c\left(b^2+ac\right)x^{5/2}+b\left(b^2+6ac\right)x^2+3a\left(b^2+ac\right)x^{3/2}+3a^2bx+a^3\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^3x}{2}+a^2bx^{3/2}+\frac{1}{5}bx^{5/2}\left(6ac+b^2\right)+\frac{1}{2}cx^3\left(ac+b^2\right)+\frac{3}{4}ax^2\left(ac+b^2\right)+\frac{3}{7}bc^2x^{7/2}+\frac{c^3x^4}{8}\right)$$

input
```
Int[(a + b*Sqrt[x] + c*x)^3,x]
```

output
```
2*((a^3*x)/2 + a^2*b*x^(3/2) + (3*a*(b^2 + a*c)*x^2)/4 + (b*(b^2 + 6*a*c)*
x^(5/2))/5 + (c*(b^2 + a*c)*x^3)/2 + (3*b*c^2*x^(7/2))/7 + (c^3*x^4)/8)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.79

method	result
default	$\frac{2b^3x^{\frac{5}{2}}}{5}+3b^2\left(\frac{1}{3}c\,x^3+\frac{1}{2}a\,x^2\right)+3b\left(\frac{2c^2x^{\frac{7}{2}}}{7}+\frac{4ac\,x^{\frac{5}{2}}}{5}+\frac{2a^2x^{\frac{3}{2}}}{3}\right)+\frac{(cx+a)^4}{4c}$
derivativedivides	$\frac{c^3x^4}{4}+\frac{6b\,c^2x^{\frac{7}{2}}}{7}+\frac{(a\,c^2+2b^2c+c(2ac+b^2))x^3}{3}+\frac{2(4abc+b(2ac+b^2))x^{\frac{5}{2}}}{5}+\frac{(a(2ac+b^2)+2b^2a+a^2c)x^2}{2}+2a$
trager	$\frac{(c^3x^3+4a\,c^2x^2+4b^2c\,x^2+c^3x^2+6a^2cx+6a\,b^2x+4a\,c^2x+4b^2cx+c^3x+4a^3+6a^2c+6b^2a+4a\,c^2+4b^2c+c^3)(x-1)}{4}+2b$
orering	$-\frac{(65x^6c^7+242x^5c^6a-143b^2c^5x^5+219x^4c^5a^2-222a\,b^2c^4x^4+84b^4c^3x^4-308x^3c^4a^3+294a^2b^2c^3x^3-98a\,b^4c^2x^3+448a}{140(-c^3x^3-a\,c^2x^2+b^2c\,x^2+a^2cx-a\,b^2x+a^3)c^2(c^2x^2+2acx-}$

input
```
int((a+b*x^(1/2)+c*x)^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*b^3*x^(5/2)+3*b^2*(1/3*c*x^3+1/2*a*x^2)+3*b*(2/7*c^2*x^(7/2)+4/5*a*c*x
^(5/2)+2/3*a^2*x^(3/2))+1/4*(c*x+a)^4/c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \left(a+b\sqrt{x}+cx\right)^3\,dx=\frac{1}{4}\,c^3x^4+a^3x+\left(b^2c+ac^2\right)x^3+\frac{3}{2}\left(ab^2+a^2c\right)x^2+\frac{2}{35}\left(15\,bc^2x^3+35\,a^2bx+7\left(b^3+6\,abc\right)x^2\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2)+c*x)^3,x, algorithm="fricas")
```

output
```
1/4*c^3*x^4 + a^3*x + (b^2*c + a*c^2)*x^3 + 3/2*(a*b^2 + a^2*c)*x^2 + 2/35
*(15*b*c^2*x^3 + 35*a^2*b*x + 7*(b^3 + 6*a*b*c)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \left(a+b\sqrt{x}+cx\right)^3\,dx=a^3x+2a^2bx^{\frac{3}{2}}+\frac{3a^2cx^2}{2}+\frac{3ab^2x^2}{2}+\frac{12abcx^{\frac{5}{2}}}{5}+ac^2x^3+\frac{2b^3x^{\frac{5}{2}}}{5}+b^2cx^3+\frac{6bc^2x^{\frac{7}{2}}}{7}+\frac{c^3x^4}{4}$$

input
```
integrate((a+b*x**(1/2)+c*x)**3,x)
```

output
```
a**3*x + 2*a**2*b*x**(3/2) + 3*a**2*c*x**2/2 + 3*a*b**2*x**2/2 + 12*a*b*c*
x**(5/2)/5 + a*c**2*x**3 + 2*b**3*x**(5/2)/5 + b**2*c*x**3 + 6*b*c**2*x**(
7/2)/7 + c**3*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \left(a+b\sqrt{x}+cx\right)^3\,dx=\frac{1}{4}\,c^3x^4+\frac{6}{7}\,bc^2x^{\frac{7}{2}}+b^2cx^3+\frac{2}{5}\,b^3x^{\frac{5}{2}}+a^3x+\frac{1}{2}\left(3\,cx^2+4\,bx^{\frac{3}{2}}\right)a^2+\frac{1}{10}\left(10\,c^2x^3+24\,bcx^{\frac{5}{2}}+15\,b^2x^2\right)a$$

input
```
integrate((a+b*x^(1/2)+c*x)^3,x, algorithm="maxima")
```

output
```
1/4*c^3*x^4 + 6/7*b*c^2*x^(7/2) + b^2*c*x^3 + 2/5*b^3*x^(5/2) + a^3*x + 1/
2*(3*c*x^2 + 4*b*x^(3/2))*a^2 + 1/10*(10*c^2*x^3 + 24*b*c*x^(5/2) + 15*b^2
*x^2)*a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \left(a+b\sqrt{x}+cx\right)^3\,dx=\frac{1}{4}\,c^3x^4+\frac{6}{7}\,bc^2x^{\frac{7}{2}}+b^2cx^3+ac^2x^3+\frac{2}{5}\,b^3x^{\frac{5}{2}}+\frac{12}{5}\,abcx^{\frac{5}{2}}+\frac{3}{2}\,ab^2x^2+\frac{3}{2}\,a^2cx^2+2\,a^2bx^{\frac{3}{2}}+a^3x$$

input
```
integrate((a+b*x^(1/2)+c*x)^3,x, algorithm="giac")
```

output
```
1/4*c^3*x^4 + 6/7*b*c^2*x^(7/2) + b^2*c*x^3 + a*c^2*x^3 + 2/5*b^3*x^(5/2)
+ 12/5*a*b*c*x^(5/2) + 3/2*a*b^2*x^2 + 3/2*a^2*c*x^2 + 2*a^2*b*x^(3/2) + a
^3*x
```

Mupad [B] (verification not implemented)

Time = 19.60 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt{x}+cx\right)^3\,dx=x^{5/2}\left(\frac{2\,b^3}{5}+\frac{12\,a\,c\,b}{5}\right)+a^3\,x+\frac{c^3\,x^4}{4}+2\,a^2\,b\,x^{3/2}+\frac{6\,b\,c^2\,x^{7/2}}{7}+\frac{3\,a\,x^2\,\left(b^2+a\,c\right)}{2}+c\,x^3\,\left(b^2+a\,c\right)$$

input
```
int((a + c*x + b*x^(1/2))^3,x)
```

output
```
x^(5/2)*((2*b^3)/5 + (12*a*b*c)/5) + a^3*x + (c^3*x^4)/4 + 2*a^2*b*x^(3/2)
 + (6*b*c^2*x^(7/2))/7 + (3*a*x^2*(a*c + b^2))/2 + c*x^3*(a*c + b^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \left(a+b\sqrt{x}+cx\right)^3\,dx$$

$$=\frac{x\left(280\sqrt{x}\,a^2b+336\sqrt{x}\,abcx+56\sqrt{x}\,b^3x+120\sqrt{x}\,b\,c^2x^2+140a^3+210a^2cx+210a\,b^2x+140a\,c^2x^2+1\right.}{140}$$

input
```
int((a+b*x^(1/2)+c*x)^3,x)
```

output
```
(x*(280*sqrt(x)*a**2*b + 336*sqrt(x)*a*b*c*x + 56*sqrt(x)*b**3*x + 120*sqr
t(x)*b*c**2*x**2 + 140*a**3 + 210*a**2*c*x + 210*a*b**2*x + 140*a*c**2*x**
2 + 140*b**2*c*x**2 + 35*c**3*x**3))/140
```

3.25 $\int \left(a+b\sqrt{x}+cx\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \left(a+b\sqrt{x}+cx\right)^2\,dx=a^2x+\frac{4}{3}abx^{3/2}+\frac{1}{2}\left(b^2+2ac\right)x^2+\frac{4}{5}bcx^{5/2}+\frac{c^2x^3}{3}$$

output
```
a^2*x+4/3*a*b*x^(3/2)+1/2*(2*a*c+b^2)*x^2+4/5*b*c*x^(5/2)+1/3*c^2*x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \left(a+b\sqrt{x}+cx\right)^2\,dx=\frac{1}{30}\left(30a^2x+40abx^{3/2}+15b^2x^2+30acx^2+24bcx^{5/2}+10c^2x^3\right)$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^2,x]
```

output
```
(30*a^2*x + 40*a*b*x^(3/2) + 15*b^2*x^2 + 30*a*c*x^2 + 24*b*c*x^(5/2) + 10
*c^2*x^3)/30
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1680, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}+cx\right)^2 \, dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \sqrt{x}\left(a+cx+b\sqrt{x}\right)^2 d\sqrt{x}$$

$$\Big\downarrow \text{1140}$$

$$2\int \left(c^2x^{5/2}+2bcx^2+\left(b^2+2ac\right)x^{3/2}+2abx+a^2\sqrt{x}\right) d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^2x}{2}+\frac{1}{4}x^2\left(2ac+b^2\right)+\frac{2}{3}abx^{3/2}+\frac{2}{5}bcx^{5/2}+\frac{c^2x^3}{6}\right)$$

input
```
Int[(a + b*Sqrt[x] + c*x)^2,x]
```

output
```
2*((a^2*x)/2 + (2*a*b*x^(3/2))/3 + ((b^2 + 2*a*c)*x^2)/4 + (2*b*c*x^(5/2))
/5 + (c^2*x^3)/6)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

method	result
default	$\frac{b^2x^2}{2}+2b\left(\frac{2c\,x^{\frac{5}{2}}}{5}+\frac{2a\,x^{\frac{3}{2}}}{3}\right)+\frac{(cx+a)^3}{3c}$
derivativedivides	$x\,a^2+\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{(2ac+b^2)x^2}{2}+\frac{4bc\,x^{\frac{5}{2}}}{5}+\frac{c^2x^3}{3}$
trager	$\frac{(2c^2x^2+6acx+3b^2x+2c^2x+6a^2+6ac+3b^2+2c^2)(x-1)}{6}+\frac{4b\,x^{\frac{3}{2}}(3cx+5a)}{15}$
orering	$-\frac{(18c^4x^4+28a\,c^3x^3-21b^2c^2x^3-40a^2c^2x^2+25a\,b^2c\,x^2+10a^4)(a+b\sqrt{x}+cx)^2}{30(-c^3x^3-a\,c^2x^2+b^2c\,x^2+a^2cx-a\,b^2x+a^3)c}+\frac{2(2c^4x^4+4a\,c^3x^3-3b^2c^2x^3-8a^2c^2\cdots}{15c(c^2x^2+2\cdots}$

input
```
int((a+b*x^(1/2)+c*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2*x^2+2*b*(2/5*c*x^(5/2)+2/3*a*x^(3/2))+1/3*(c*x+a)^3/c
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt{x}+cx\right)^2\,dx=\frac{1}{3}\,c^2x^3+a^2x+\frac{1}{2}\,(b^2+2\,ac)x^2+\frac{4}{15}\,(3\,bcx^2+5\,abx)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2)+c*x)^2,x, algorithm="fricas")
```

output
```
1/3*c^2*x^3 + a^2*x + 1/2*(b^2 + 2*a*c)*x^2 + 4/15*(3*b*c*x^2 + 5*a*b*x)*s
qrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \left(a+b\sqrt{x}+cx\right)^2\,dx=a^2x+\frac{4abx^{\frac{3}{2}}}{3}+acx^2+\frac{b^2x^2}{2}+\frac{4bcx^{\frac{5}{2}}}{5}+\frac{c^2x^3}{3}$$

input
```
integrate((a+b*x**(1/2)+c*x)**2,x)
```

output
```
a**2*x + 4*a*b*x**(3/2)/3 + a*c*x**2 + b**2*x**2/2 + 4*b*c*x**(5/2)/5 + c*
*2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt{x}+cx\right)^2\,dx=\frac{1}{3}\,c^2x^3+\frac{4}{5}\,bcx^{\frac{5}{2}}+\frac{1}{2}\,b^2x^2+a^2x+\frac{1}{3}\left(3\,cx^2+4\,bx^{\frac{3}{2}}\right)a$$

input
```
integrate((a+b*x^(1/2)+c*x)^2,x, algorithm="maxima")
```

output
```
1/3*c^2*x^3 + 4/5*b*c*x^(5/2) + 1/2*b^2*x^2 + a^2*x + 1/3*(3*c*x^2 + 4*b*x
^(3/2))*a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt{x}+cx\right)^2\,dx=\frac{1}{3}\,c^2x^3+\frac{4}{5}\,bcx^{\frac{5}{2}}+\frac{1}{2}\,b^2x^2+acx^2+\frac{4}{3}\,abx^{\frac{3}{2}}+a^2x$$

input
```
integrate((a+b*x^(1/2)+c*x)^2,x, algorithm="giac")
```

output
```
1/3*c^2*x^3 + 4/5*b*c*x^(5/2) + 1/2*b^2*x^2 + a*c*x^2 + 4/3*a*b*x^(3/2) +
a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt{x}+cx\right)^2\,dx=a^2\,x+x^2\,\left(\frac{b^2}{2}+a\,c\right)+\frac{c^2\,x^3}{3}+\frac{4\,a\,b\,x^{3/2}}{3}+\frac{4\,b\,c\,x^{5/2}}{5}$$

input
```
int((a + c*x + b*x^(1/2))^2,x)
```

output
```
a^2*x + x^2*(a*c + b^2/2) + (c^2*x^3)/3 + (4*a*b*x^(3/2))/3 + (4*b*c*x^(5/
2))/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt{x}+cx\right)^2\,dx=\frac{x\left(40\sqrt{x}\,ab+24\sqrt{x}\,bcx+30a^2+30acx+15b^2x+10c^2x^2\right)}{30}$$

input
```
int((a+b*x^(1/2)+c*x)^2,x)
```

output
```
(x*(40*sqrt(x)*a*b + 24*sqrt(x)*b*c*x + 30*a**2 + 30*a*c*x + 15*b**2*x + 1
0*c**2*x**2))/30
```

3.26 $\int (a+b\sqrt{x}+cx)\ dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int (a+b\sqrt{x}+cx)\ dx = ax+\frac{2}{3}bx^{3/2}+\frac{cx^2}{2}$$

output
```
a*x+2/3*b*x^(3/2)+1/2*c*x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (a+b\sqrt{x}+cx)\ dx = ax+\frac{2}{3}bx^{3/2}+\frac{cx^2}{2}$$

input
```
Integrate[a + b*Sqrt[x] + c*x,x]
```

output
```
a*x + (2*b*x^(3/2))/3 + (c*x^2)/2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\sqrt{x} + cx\right) dx$$

$$\Big\downarrow \text{2009}$$

$$ax + \frac{2}{3}bx^{3/2} + \frac{cx^2}{2}$$

input
```
Int[a + b*Sqrt[x] + c*x,x]
```

output
```
a*x + (2*b*x^(3/2))/3 + (c*x^2)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$xa+\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{c\,x^{2}}{2}$	17
default	$xa+\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{c\,x^{2}}{2}$	17
risch	$xa+\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{c\,x^{2}}{2}$	17
parts	$xa+\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{c\,x^{2}}{2}$	17
trager	$\frac{(x-1)(cx+2a+c)}{2}+\frac{2b\,x^{\frac{3}{2}}}{3}$	21
orering	$\frac{x(-5cx+6a)(a+b\sqrt{x}+cx)}{-6cx+6a}-\frac{x^{2}(-cx+2a)\left(\frac{b}{2\sqrt{x}}+c\right)}{3(-cx+a)}$	60

input
```
int(a+b*x^(1/2)+c*x,x,method=_RETURNVERBOSE)
```

output
```
x*a+2/3*b*x^(3/2)+1/2*c*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(a+b\sqrt{x}+cx\right)\,dx=\frac{1}{2}\,cx^{2}+\frac{2}{3}\,bx^{\frac{3}{2}}+ax$$

input
```
integrate(a+b*x^(1/2)+c*x,x, algorithm="fricas")
```

output
```
1/2*c*x^2 + 2/3*b*x^(3/2) + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \left(a+b\sqrt{x}+cx\right)\,dx=ax+\frac{2bx^{\frac{3}{2}}}{3}+\frac{cx^2}{2}$$

input
```
integrate(a+b*x**(1/2)+c*x,x)
```

output
```
a*x + 2*b*x**(3/2)/3 + c*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(a+b\sqrt{x}+cx\right)\,dx=\frac{1}{2}\,cx^2+\frac{2}{3}\,bx^{\frac{3}{2}}+ax$$

input
```
integrate(a+b*x^(1/2)+c*x,x, algorithm="maxima")
```

output
```
1/2*c*x^2 + 2/3*b*x^(3/2) + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(a+b\sqrt{x}+cx\right)\,dx=\frac{1}{2}\,cx^2+\frac{2}{3}\,bx^{\frac{3}{2}}+ax$$

input
```
integrate(a+b*x^(1/2)+c*x,x, algorithm="giac")
```

output
```
1/2*c*x^2 + 2/3*b*x^(3/2) + a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(a+b\sqrt{x}+cx\right)\,dx=a\,x+\frac{2\,b\,x^{3/2}}{3}+\frac{c\,x^2}{2}$$

input
```
int(a + c*x + b*x^(1/2),x)
```

output
```
a*x + (2*b*x^(3/2))/3 + (c*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(a+b\sqrt{x}+cx\right)\,dx=\frac{x\left(4\sqrt{x}\,b+6a+3cx\right)}{6}$$

input
```
int(a+b*x^(1/2)+c*x,x)
```

output
```
(x*(4*sqrt(x)*b + 6*a + 3*c*x))/6
```

3.27 $\int \frac{1}{a+b\sqrt{x}+cx} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{a+b\sqrt{x}+cx} \, dx = \frac{2b\text{arctanh}\left(\frac{b+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log\left(a+b\sqrt{x}+cx\right)}{c}$$

output
```
2*b*arctanh((b+2*c*x^(1/2))/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)+ln(a+
b*x^(1/2)+c*x)/c
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{1}{a+b\sqrt{x}+cx} \, dx = \frac{-\frac{2b\arctan\left(\frac{b+2c\sqrt{x}}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log\left(a+b\sqrt{x}+cx\right)}{c}$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^(-1),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*Sqrt[x])/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] +
Log[a + b*Sqrt[x] + c*x])/c
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1680, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sqrt{x}+cx}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \frac{\sqrt{x}}{a+cx+b\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{1142}$$

$$2\left(\frac{\int \frac{b+2c\sqrt{x}}{a+cx+b\sqrt{x}}d\sqrt{x}}{2c}-\frac{b\int \frac{1}{a+cx+b\sqrt{x}}d\sqrt{x}}{2c}\right)$$

$$\Big\downarrow \text{1083}$$

$$2\left(\frac{b\int \frac{1}{b^2-4ac-x}d\left(b+2c\sqrt{x}\right)}{c}+\frac{\int \frac{b+2c\sqrt{x}}{a+cx+b\sqrt{x}}d\sqrt{x}}{2c}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{\int \frac{b+2c\sqrt{x}}{a+cx+b\sqrt{x}}d\sqrt{x}}{2c}+\frac{b\text{arctanh}\left(\frac{b+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{1103}$$

$$2\left(\frac{b\text{arctanh}\left(\frac{b+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+b\sqrt{x}+cx\right)}{2c}\right)$$

input
```
Int[(a + b*Sqrt[x] + c*x)^(-1),x]
```

output
```
2*((b*ArcTanh[(b + 2*c*Sqrt[x])/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c])
+ Log[a + b*Sqrt[x] + c*x]/(2*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\ln(a+b\sqrt{x}+cx)}{c}-\frac{2b\arctan\left(\frac{b+2c\sqrt{x}}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}}$
default	$\frac{\ln(a+b\sqrt{x}+cx)}{2c}-\frac{b\arctan\left(\frac{b+2c\sqrt{x}}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}}-\frac{\ln(-b\sqrt{x}+cx+a)}{2c}-\frac{b\arctan\left(\frac{2c\sqrt{x}-b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}}+\frac{2a\arctan\left(\frac{2c^2x+2ac-b^2}{\sqrt{4a\,b^2c-b^4}}\right)}{\sqrt{4a\,b^2c-b^4}}$

input

```
int(1/(a+b*x^(1/2)+c*x),x,method=_RETURNVERBOSE)
```

output

```
ln(a+b*x^(1/2)+c*x)/c-2*b/c/(4*a*c-b^2)^(1/2)*arctan((b+2*c*x^(1/2))/(4*a*
c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 4.37

$$\int \frac{1}{a+b\sqrt{x}+cx}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^3x^2-b^2cx+ab^2-2\,a^2c-(bcx-ab)\sqrt{b^2-4\,ac}-\left(b^3-4\,abc-(2\,c^2x-b^2+2\,ac)\sqrt{b^2-4\,ac}\right)\sqrt{x}}{c^2x^2+a^2-(b^2-2\,ac)x}\right)+(b^2-4\,ac)\log}{b^2c-4\,ac^2}\right.$$

input

```
integrate(1/(a+b*x^(1/2)+c*x),x, algorithm="fricas")
```

output

```
[(sqrt(b^2 - 4*a*c)*b*log((2*c^3*x^2 - b^2*c*x + a*b^2 - 2*a^2*c - (b*c*x
- a*b)*sqrt(b^2 - 4*a*c) - (b^3 - 4*a*b*c - (2*c^2*x - b^2 + 2*a*c)*sqrt(b
^2 - 4*a*c))*sqrt(x))/(c^2*x^2 + a^2 - (b^2 - 2*a*c)*x)) + (b^2 - 4*a*c)*l
og(c*x + b*sqrt(x) + a))/(b^2*c - 4*a*c^2), (2*sqrt(-b^2 + 4*a*c)*b*arctan
(-(2*sqrt(-b^2 + 4*a*c)*c*sqrt(x) + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) +
 (b^2 - 4*a*c)*log(c*x + b*sqrt(x) + a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(54) = 108$.

Time = 0.97 (sec) , antiderivative size = 250, normalized size of antiderivative = 4.17

$$\int \frac{1}{a+b\sqrt{x}+cx}\,dx = \begin{cases} -\frac{2a\log\left(\frac{a}{b}+\sqrt{x}\right)}{b^2}+\frac{2\sqrt{x}}{b} & \text{for } c = \\ \frac{2b\log\left(\frac{b}{2c}+\sqrt{x}\right)}{bc+2c^2\sqrt{x}}+\frac{2b}{bc+2c^2\sqrt{x}}+\frac{4c\sqrt{x}\log\left(\frac{b}{2c}+\sqrt{x}\right)}{bc+2c^2\sqrt{x}} & \text{for } a = \\ -\frac{b\log\left(\frac{b}{2c}+\sqrt{x}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}}+\frac{b\log\left(\frac{b}{2c}+\sqrt{x}+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}}+\frac{\log\left(\frac{b}{2c}+\sqrt{x}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c}+\frac{\log\left(\frac{b}{2c}+\sqrt{x}+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c} & \text{other} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2)+c*x),x)
```

output
```
Piecewise((-2*a*log(a/b + sqrt(x))/b**2 + 2*sqrt(x)/b, Eq(c, 0)), (2*b*log
(b/(2*c) + sqrt(x))/(b*c + 2*c**2*sqrt(x)) + 2*b/(b*c + 2*c**2*sqrt(x)) +
4*c*sqrt(x)*log(b/(2*c) + sqrt(x))/(b*c + 2*c**2*sqrt(x)), Eq(a, b**2/(4*c
))), (-b*log(b/(2*c) + sqrt(x) - sqrt(-4*a*c + b**2)/(2*c))/(c*sqrt(-4*a*c
 + b**2)) + b*log(b/(2*c) + sqrt(x) + sqrt(-4*a*c + b**2)/(2*c))/(c*sqrt(-
4*a*c + b**2)) + log(b/(2*c) + sqrt(x) - sqrt(-4*a*c + b**2)/(2*c))/c + lo
g(b/(2*c) + sqrt(x) + sqrt(-4*a*c + b**2)/(2*c))/c, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\sqrt{x}+cx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{1}{a+b\sqrt{x}+cx}\,dx = -\frac{2\,b\arctan\left(\frac{2\,c\sqrt{x}+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c} + \frac{\log\left(cx+b\sqrt{x}+a\right)}{c}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x),x, algorithm="giac")
```

output
```
-2*b*arctan((2*c*sqrt(x) + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) +
 log(c*x + b*sqrt(x) + a)/c
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.90

$$\int \frac{1}{a+b\sqrt{x}+cx}\,dx = \frac{4\,a\,c\,\ln\left(a+c\,x+b\,\sqrt{x}\right)}{4\,a\,c^2-b^2\,c} - \frac{2\,b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,\sqrt{x}}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} - \frac{b^2\,\ln\left(a+c\,x+b\,\sqrt{x}\right)}{4\,a\,c^2-b^2\,c}$$

input
```
int(1/(a + c*x + b*x^(1/2)),x)
```

output
```
(4*a*c*log(a + c*x + b*x^(1/2)))/(4*a*c^2 - b^2*c) - (2*b*atan(b/(4*a*c -
b^2)^(1/2) + (2*c*x^(1/2))/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) -
 (b^2*log(a + c*x + b*x^(1/2)))/(4*a*c^2 - b^2*c)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

$$\int \frac{1}{a+b\sqrt{x}+cx}\,dx$$
$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2\sqrt{x}\,c+b}{\sqrt{4ac-b^2}}\right)b+4\log\left(\sqrt{x}\,b+a+cx\right)ac-\log\left(\sqrt{x}\,b+a+cx\right)b^2}{c\,(4ac-b^2)}$$

input
```
int(1/(a+b*x^(1/2)+c*x),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((2*sqrt(x)*c + b)/sqrt(4*a*c - b**2))*b + 4*
log(sqrt(x)*b + a + c*x)*a*c - log(sqrt(x)*b + a + c*x)*b**2)/(c*(4*a*c -
b**2))
```

3.28 $\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^2}\,dx=\frac{2\left(2a+b\sqrt{x}\right)}{\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)}-\frac{4b\text{arctanh}\left(\frac{b+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

output
```
2*(2*a+b*x^(1/2))/(-4*a*c+b^2)/(a+b*x^(1/2)+c*x)-4*b*arctanh((b+2*c*x^(1/2
))/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^2}\,dx=\frac{2\left(2a+b\sqrt{x}\right)}{\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)}-\frac{4b\arctan\left(\frac{b+2c\sqrt{x}}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}}$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^(-2),x]
```

output
```
(2*(2*a + b*Sqrt[x]))/((b^2 - 4*a*c)*(a + b*Sqrt[x] + c*x)) - (4*b*ArcTan[
(b + 2*c*Sqrt[x])/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1680, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^2}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \frac{\sqrt{x}}{\left(a+cx+b\sqrt{x}\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{1159}$$

$$2\left(\frac{b\int \frac{1}{a+cx+b\sqrt{x}}d\sqrt{x}}{b^2-4ac}+\frac{2a+b\sqrt{x}}{\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)}\right)$$

$$\Big\downarrow \text{1083}$$

$$2\left(\frac{2a+b\sqrt{x}}{\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)}-\frac{2b\int \frac{1}{b^2-4ac-x}d\left(b+2c\sqrt{x}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{2a+b\sqrt{x}}{\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)}-\frac{2b\text{arctanh}\left(\frac{b+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Int[(a + b*Sqrt[x] + c*x)^(-2),x]
```

output
```
2*((2*a + b*Sqrt[x])/((b^2 - 4*a*c)*(a + b*Sqrt[x] + c*x)) - (2*b*ArcTanh[
(b + 2*c*Sqrt[x])/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-2b\sqrt{x}-4a}{(4ac-b^2)(a+b\sqrt{x}+cx)}-\frac{4b\arctan\left(\frac{b+2c\sqrt{x}}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
default	$b^2\left(\frac{(-2ac+b^2)x-2a^2}{(4a\,b^2c-b^4)(c^2x^2+2acx-b^2x+a^2)}+\frac{2(-2ac+b^2)\arctan\left(\frac{2c^2x+2ac-b^2}{\sqrt{4a\,b^2c-b^4}}\right)}{(4a\,b^2c-b^4)^{\frac{3}{2}}}\right)-\frac{b\sqrt{x}}{(a+b\sqrt{x}+cx)(4ac-b^2)}-\ \cdots$

input
```
int(1/(a+b*x^(1/2)+c*x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-b*x^(1/2)-2*a)/(4*a*c-b^2)/(a+b*x^(1/2)+c*x)-4*b/(4*a*c-b^2)^(3/2)*arc
tan((b+2*c*x^(1/2))/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(67) = 134$.

Time = 0.09 (sec) , antiderivative size = 565, normalized size of antiderivative = 7.34

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^2}\,dx$$
$$= \left[\frac{2\left(2\,a^2b^2-8\,a^3c-(bc^2x^2+a^2b-(b^3-2\,abc)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^3x^2-b^2cx+ab^2-2\,a^2c-(bcx-ab)\sqrt{b^2-4\,ac}-\left(b^3\cdots\right.}{c^2x^2+a^2-(b^2-\cdots}\right.\right.}{a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^4c^2-8\,ab^2c^3+16\,a^2c\cdots}\right.$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^2,x, algorithm="fricas")
```

output
```
[2*(2*a^2*b^2 - 8*a^3*c - (b*c^2*x^2 + a^2*b - (b^3 - 2*a*b*c)*x)*sqrt(b^2
 - 4*a*c)*log((2*c^3*x^2 - b^2*c*x + a*b^2 - 2*a^2*c - (b*c*x - a*b)*sqrt(
b^2 - 4*a*c) - (b^3 - 4*a*b*c - (2*c^2*x - b^2 + 2*a*c)*sqrt(b^2 - 4*a*c))
*sqrt(x))/(c^2*x^2 + a^2 - (b^2 - 2*a*c)*x)) - (b^4 - 6*a*b^2*c + 8*a^2*c^
2)*x - (a*b^3 - 4*a^2*b*c - (b^3*c - 4*a*b*c^2)*x)*sqrt(x))/(a^2*b^4 - 8*a
^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 - (b^6 -
10*a*b^4*c + 32*a^2*b^2*c^2 - 32*a^3*c^3)*x), 2*(2*a^2*b^2 - 8*a^3*c - 2*(
b*c^2*x^2 + a^2*b - (b^3 - 2*a*b*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*sqrt(
-b^2 + 4*a*c)*c*sqrt(x) + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - (b^4 - 6*
a*b^2*c + 8*a^2*c^2)*x - (a*b^3 - 4*a^2*b*c - (b^3*c - 4*a*b*c^2)*x)*sqrt(
x))/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)*x^2 - (b^6 - 10*a*b^4*c + 32*a^2*b^2*c^2 - 32*a^3*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1358 vs. $2(70) = 140$.

Time = 38.16 (sec) , antiderivative size = 1358, normalized size of antiderivative = 17.64

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x**(1/2)+c*x)**2,x)
```

output
```
Piecewise((2*a*log(a/b + sqrt(x))/(a*b**2 + b**3*sqrt(x)) + 2*a/(a*b**2 +
b**3*sqrt(x)) + 2*b*sqrt(x)*log(a/b + sqrt(x))/(a*b**2 + b**3*sqrt(x)), Eq
(c, 0)), (-4*b/(3*b**3 + 18*b**2*c*sqrt(x) + 36*b*c**2*x + 24*c**3*x**(3/2
)) - 24*c*sqrt(x)/(3*b**3 + 18*b**2*c*sqrt(x) + 36*b*c**2*x + 24*c**3*x**(
3/2)), Eq(a, b**2/(4*c))), (-2*a*b*log(b/(2*c) + sqrt(x) - sqrt(-4*a*c + b
**2)/(2*c))/(4*a**2*c*sqrt(-4*a*c + b**2) - a*b**2*sqrt(-4*a*c + b**2) + 4
*a*b*c*sqrt(x)*sqrt(-4*a*c + b**2) + 4*a*c**2*x*sqrt(-4*a*c + b**2) - b**3
*sqrt(x)*sqrt(-4*a*c + b**2) - b**2*c*x*sqrt(-4*a*c + b**2)) + 2*a*b*log(b
/(2*c) + sqrt(x) + sqrt(-4*a*c + b**2)/(2*c))/(4*a**2*c*sqrt(-4*a*c + b**2
) - a*b**2*sqrt(-4*a*c + b**2) + 4*a*b*c*sqrt(x)*sqrt(-4*a*c + b**2) + 4*a
*c**2*x*sqrt(-4*a*c + b**2) - b**3*sqrt(x)*sqrt(-4*a*c + b**2) - b**2*c*x*
sqrt(-4*a*c + b**2)) - 4*a*sqrt(-4*a*c + b**2)/(4*a**2*c*sqrt(-4*a*c + b**
2) - a*b**2*sqrt(-4*a*c + b**2) + 4*a*b*c*sqrt(x)*sqrt(-4*a*c + b**2) + 4*
a*c**2*x*sqrt(-4*a*c + b**2) - b**3*sqrt(x)*sqrt(-4*a*c + b**2) - b**2*c*x
*sqrt(-4*a*c + b**2)) - 2*b**2*sqrt(x)*log(b/(2*c) + sqrt(x) - sqrt(-4*a*c
 + b**2)/(2*c))/(4*a**2*c*sqrt(-4*a*c + b**2) - a*b**2*sqrt(-4*a*c + b**2)
 + 4*a*b*c*sqrt(x)*sqrt(-4*a*c + b**2) + 4*a*c**2*x*sqrt(-4*a*c + b**2) -
b**3*sqrt(x)*sqrt(-4*a*c + b**2) - b**2*c*x*sqrt(-4*a*c + b**2)) + 2*b**2*
sqrt(x)*log(b/(2*c) + sqrt(x) + sqrt(-4*a*c + b**2)/(2*c))/(4*a**2*c*sqrt(
-4*a*c + b**2) - a*b**2*sqrt(-4*a*c + b**2) + 4*a*b*c*sqrt(x)*sqrt(-4*a...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^2}\,dx = \frac{4\,b\arctan\left(\frac{2\,c\sqrt{x}+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}} + \frac{2\,\left(b\sqrt{x}+2\,a\right)}{\left(b^2-4\,ac\right)\left(cx+b\sqrt{x}+a\right)}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^2,x, algorithm="giac")
```

output
```
4*b*arctan((2*c*sqrt(x) + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2
+ 4*a*c)) + 2*(b*sqrt(x) + 2*a)/((b^2 - 4*a*c)*(c*x + b*sqrt(x) + a))
```

Mupad [B] (verification not implemented)

Time = 18.80 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.52

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^2}\,dx$$
$$= -\frac{\frac{4a}{4ac-b^2}+\frac{2b\sqrt{x}}{4ac-b^2}}{a+cx+b\sqrt{x}} - \frac{4b\,\mathrm{atan}\left(\frac{\left(\frac{2b^2}{(4ac-b^2)^{3/2}}+\frac{4bc\sqrt{x}}{(4ac-b^2)^{3/2}}\right)(4ac-b^2)}{2b}\right)}{(4ac-b^2)^{3/2}}$$

input
```
int(1/(a + c*x + b*x^(1/2))^2,x)
```

output
```
- ((4*a)/(4*a*c - b^2) + (2*b*x^(1/2))/(4*a*c - b^2))/(a + c*x + b*x^(1/2)
) - (4*b*atan((((2*b^2)/(4*a*c - b^2)^(3/2) + (4*b*c*x^(1/2))/(4*a*c - b^2
)^(3/2))*(4*a*c - b^2))/(2*b)))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.84

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^2}\,dx$$
$$= \frac{-4\sqrt{x}\,\sqrt{4ac-b^2}\,atan\left(\frac{2\sqrt{x}\,c+b}{\sqrt{4ac-b^2}}\right)b^2-4\sqrt{4ac-b^2}\,atan\left(\frac{2\sqrt{x}\,c+b}{\sqrt{4ac-b^2}}\right)ab-4\sqrt{4ac-b^2}\,atan\left(\frac{2\sqrt{x}\,c+b}{\sqrt{4ac-b^2}}\right)bcx-}{16\sqrt{x}\,a^2bc^2-8\sqrt{x}\,ab^3c+\sqrt{x}\,b^5+16a^3c^2-8a^2b^2c+16a^2c^3x+ab^4-8ab^2c^2x}$$

input
```
int(1/(a+b*x^(1/2)+c*x)^2,x)
```

output
```
(2*( - 2*sqrt(x)*sqrt(4*a*c - b**2)*atan((2*sqrt(x)*c + b)/sqrt(4*a*c - b*
*2))*b**2 - 2*sqrt(4*a*c - b**2)*atan((2*sqrt(x)*c + b)/sqrt(4*a*c - b**2)
)*a*b - 2*sqrt(4*a*c - b**2)*atan((2*sqrt(x)*c + b)/sqrt(4*a*c - b**2))*b*
c*x - 4*a**2*c + a*b**2 + 4*a*c**2*x - b**2*c*x))/(16*sqrt(x)*a**2*b*c**2
- 8*sqrt(x)*a*b**3*c + sqrt(x)*b**5 + 16*a**3*c**2 - 8*a**2*b**2*c + 16*a*
*2*c**3*x + a*b**4 - 8*a*b**2*c**2*x + b**4*c*x)
```

3.29 $\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 114

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^3} \, dx=\frac{2a+b\sqrt{x}}{(b^2-4ac)\left(a+b\sqrt{x}+cx\right)^2}-\frac{3b\left(b+2c\sqrt{x}\right)}{(b^2-4ac)^2\left(a+b\sqrt{x}+cx\right)}+\frac{12bc\,\text{arctanh}\left(\frac{b+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
(2*a+b*x^(1/2))/(-4*a*c+b^2)/(a+b*x^(1/2)+c*x)^2-3*b*(b+2*c*x^(1/2))/(-4*a
*c+b^2)^2/(a+b*x^(1/2)+c*x)+12*b*c*arctanh((b+2*c*x^(1/2))/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^3} \, dx=-\frac{8a^2c+ab\left(b+10c\sqrt{x}\right)+b\sqrt{x}\left(2b^2+9bc\sqrt{x}+6c^2x\right)}{(b^2-4ac)^2\left(a+b\sqrt{x}+cx\right)^2}-\frac{12bc\arctan\left(\frac{b+2c\sqrt{x}}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^(-3),x]
```

output
```
-((8*a^2*c + a*b*(b + 10*c*Sqrt[x]) + b*Sqrt[x]*(2*b^2 + 9*b*c*Sqrt[x] + 6
*c^2*x))/((b^2 - 4*a*c)^2*(a + b*Sqrt[x] + c*x)^2)) - (12*b*c*ArcTan[(b +
2*c*Sqrt[x])/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1680, 1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^3}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \frac{\sqrt{x}}{\left(a+cx+b\sqrt{x}\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{1159}$$

$$2\left(\frac{3b\int \frac{1}{\left(a+cx+b\sqrt{x}\right)^2}d\sqrt{x}}{2\left(b^2-4ac\right)}+\frac{2a+b\sqrt{x}}{2\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)^2}\right)$$

$$\Big\downarrow \text{1086}$$

$$2\left(\frac{3b\left(-\frac{2c\int \frac{1}{a+cx+b\sqrt{x}}d\sqrt{x}}{b^2-4ac}-\frac{b+2c\sqrt{x}}{\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)}\right)}{2\left(b^2-4ac\right)}+\frac{2a+b\sqrt{x}}{2\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$2\left(\frac{3b\left(\frac{4c\int \frac{1}{b^2-4ac-x}d\left(b+2c\sqrt{x}\right)}{b^2-4ac}-\frac{b+2c\sqrt{x}}{\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)}\right)}{2\left(b^2-4ac\right)}+\frac{2a+b\sqrt{x}}{2\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)^2}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{3b\left(\frac{4c\,\text{arctanh}\left(\frac{b+2c\sqrt{x}}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2c\sqrt{x}}{(b^2-4ac)(a+b\sqrt{x}+cx)}\right)}{2\,(b^2-4ac)}+\frac{2a+b\sqrt{x}}{2\,(b^2-4ac)\,(a+b\sqrt{x}+cx)^2}\right)$$

input
```
Int[(a + b*Sqrt[x] + c*x)^(-3),x]
```

output
```
2*((2*a + b*Sqrt[x])/(2*(b^2 - 4*a*c)*(a + b*Sqrt[x] + c*x)^2) + (3*b*(-((
b + 2*c*Sqrt[x])/((b^2 - 4*a*c)*(a + b*Sqrt[x] + c*x))) + (4*c*ArcTanh[(b
+ 2*c*Sqrt[x])/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c))
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{-b\sqrt{x}-2a}{(4ac-b^2)(a+b\sqrt{x}+cx)^2}-\frac{3b\left(\frac{b+2c\sqrt{x}}{(4ac-b^2)(a+b\sqrt{x}+cx)}+\frac{4c\arctan\left(\frac{b+2c\sqrt{x}}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}\right)}{4ac-b^2}$	123
default	Expression too large to display	4753

input
```
int(1/(a+b*x^(1/2)+c*x)^3,x,method=_RETURNVERBOSE)
```

output
```
(-b*x^(1/2)-2*a)/(4*a*c-b^2)/(a+b*x^(1/2)+c*x)^2-3*b/(4*a*c-b^2)*((b+2*c*x
^(1/2))/(4*a*c-b^2)/(a+b*x^(1/2)+c*x)+4*c/(4*a*c-b^2)^(3/2)*arctan((b+2*c*
x^(1/2))/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(100) = 200.

Time = 0.12 (sec) , antiderivative size = 1299, normalized size of antiderivative = 11.39

$$\int \frac{1}{(a+b\sqrt{x}+cx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^3,x, algorithm="fricas")
```

output
```
[-(a^3*b^4 + 4*a^4*b^2*c - 32*a^5*c^2 - 3*(b^4*c^3 - 4*a*b^2*c^4)*x^3 + (5
*b^6*c - 33*a*b^4*c^2 + 60*a^2*b^2*c^3 - 32*a^3*c^4)*x^2 - 6*(b*c^5*x^4 +
a^4*b*c - 2*(b^3*c^3 - 2*a*b*c^4)*x^3 + (b^5*c - 4*a*b^3*c^2 + 6*a^2*b*c^3
)*x^2 - 2*(a^2*b^3*c - 2*a^3*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^3*x^2 -
b^2*c*x + a*b^2 - 2*a^2*c - (b*c*x - a*b)*sqrt(b^2 - 4*a*c) - (b^3 - 4*a*b
*c - (2*c^2*x - b^2 + 2*a*c)*sqrt(b^2 - 4*a*c))*sqrt(x))/(c^2*x^2 + a^2 -
(b^2 - 2*a*c)*x)) - (3*a*b^6 - 11*a^2*b^4*c - 20*a^3*b^2*c^2 + 64*a^4*c^3)
*x - 2*(3*a^3*b^3*c - 12*a^4*b*c^2 - 3*(b^3*c^4 - 4*a*b*c^5)*x^3 + (5*b^5*
c^2 - 31*a*b^3*c^3 + 44*a^2*b*c^4)*x^2 - (b^7 - 7*a*b^5*c + 17*a^2*b^3*c^2
 - 20*a^3*b*c^3)*x)*sqrt(x))/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64
*a^7*c^3 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^4 - 2*
(b^8*c^2 - 14*a*b^6*c^3 + 72*a^2*b^4*c^4 - 160*a^3*b^2*c^5 + 128*a^4*c^6)*
x^3 + (b^10 - 16*a*b^8*c + 102*a^2*b^6*c^2 - 328*a^3*b^4*c^3 + 544*a^4*b^2
*c^4 - 384*a^5*c^5)*x^2 - 2*(a^2*b^8 - 14*a^3*b^6*c + 72*a^4*b^4*c^2 - 160
*a^5*b^2*c^3 + 128*a^6*c^4)*x), -(a^3*b^4 + 4*a^4*b^2*c - 32*a^5*c^2 - 3*(
b^4*c^3 - 4*a*b^2*c^4)*x^3 + (5*b^6*c - 33*a*b^4*c^2 + 60*a^2*b^2*c^3 - 32
*a^3*c^4)*x^2 - 12*(b*c^5*x^4 + a^4*b*c - 2*(b^3*c^3 - 2*a*b*c^4)*x^3 + (b
^5*c - 4*a*b^3*c^2 + 6*a^2*b*c^3)*x^2 - 2*(a^2*b^3*c - 2*a^3*b*c^2)*x)*sqr
t(-b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*sqrt(x) + sqrt(-b^2 + 4*a*
c)*b)/(b^2 - 4*a*c)) - (3*a*b^6 - 11*a^2*b^4*c - 20*a^3*b^2*c^2 + 64*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a + b\sqrt{x} + cx\right)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/(a+b*x**(1/2)+c*x)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^3}\,dx = -\frac{12\,bc\arctan\left(\frac{2\,c\sqrt{x}+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,bc^2x^{\frac{3}{2}}+9\,b^2cx+2\,b^3\sqrt{x}+10\,abc\sqrt{x}+ab^2+8\,a^2c}{(b^4-8\,ab^2c+16\,a^2c^2)\left(cx+b\sqrt{x}+a\right)^2}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^3,x, algorithm="giac")
```

output
```
-12*b*c*arctan((2*c*sqrt(x) + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 1
6*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - (6*b*c^2*x^(3/2) + 9*b^2*c*x + 2*b^3*sqrt
(x) + 10*a*b*c*sqrt(x) + a*b^2 + 8*a^2*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*
(c*x + b*sqrt(x) + a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.21

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^3}\,dx = -\frac{\frac{8\,c\,a^2+a\,b^2}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{2\,b\,\sqrt{x}\,\left(b^2+5\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{9\,b^2\,c\,x}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{6\,b\,c^2\,x^{3/2}}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^2+2\,a\,b\,\sqrt{x}+2\,b\,c\,x^{3/2}} - \frac{12\,b\,c\,\mathrm{atan}\left(\frac{\left(\frac{6\,b^2\,c}{\left(4\,a\,c-b^2\right)^{5/2}}+\frac{12\,b\,c^2\,\sqrt{x}}{\left(4\,a\,c-b^2\right)^{5/2}}\right)\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,b\,c}\right)}{\left(4\,a\,c-b^2\right)^{5/2}}$$

input
```
int(1/(a + c*x + b*x^(1/2))^3,x)
```

output
```
- ((a*b^2 + 8*a^2*c)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (2*b*x^(1/2)*(5*a*c
+ b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b^2*c*x)/(b^4 + 16*a^2*c^2 - 8
*a*b^2*c) + (6*b*c^2*x^(3/2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x*(2*a*c +
b^2) + a^2 + c^2*x^2 + 2*a*b*x^(1/2) + 2*b*c*x^(3/2)) - (12*b*c*atan((((6*
b^2*c)/(4*a*c - b^2)^(5/2) + (12*b*c^2*x^(1/2))/(4*a*c - b^2)^(5/2))*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))/(6*b*c)))/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 569, normalized size of antiderivative = 4.99

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^3}\,dx = \frac{-24\sqrt{x}\,\sqrt{4ac-b^2}\,atan\left(\frac{2\sqrt{x}\,c+b}{\sqrt{4ac-b^2}}\right)a\,b^2c-24\sqrt{x}\,\sqrt{4ac-b^2}\,atan\left(\frac{2\sqrt{x}\,c+b}{\sqrt{4ac-b^2}}\right)b^2c^2x-12\sqrt{4ac-b^2}\,atan\left(\frac{2\sqrt{}}{\sqrt{4}}\right.}{128\sqrt{x}\,a^4b\,c^3-96\sqrt{x}\,a^3b^3c^2+128\sqrt{x}\,a^3b\,c^4x+24\sqrt{x}\,a^2b^5c-9}$$

input
```
int(1/(a+b*x^(1/2)+c*x)^3,x)
```

output
```
( - 24*sqrt(x)*sqrt(4*a*c - b**2)*atan((2*sqrt(x)*c + b)/sqrt(4*a*c - b**2
))*a*b**2*c - 24*sqrt(x)*sqrt(4*a*c - b**2)*atan((2*sqrt(x)*c + b)/sqrt(4*
a*c - b**2))*b**2*c**2*x - 12*sqrt(4*a*c - b**2)*atan((2*sqrt(x)*c + b)/sq
rt(4*a*c - b**2))*a**2*b*c - 24*sqrt(4*a*c - b**2)*atan((2*sqrt(x)*c + b)/
sqrt(4*a*c - b**2))*a*b*c**2*x - 12*sqrt(4*a*c - b**2)*atan((2*sqrt(x)*c +
 b)/sqrt(4*a*c - b**2))*b**3*c*x - 12*sqrt(4*a*c - b**2)*atan((2*sqrt(x)*c
 + b)/sqrt(4*a*c - b**2))*b*c**3*x**2 - 16*sqrt(x)*a**2*b*c**2 - 4*sqrt(x)
*a*b**3*c + 2*sqrt(x)*b**5 - 20*a**3*c**2 + a**2*b**2*c + 24*a**2*c**3*x +
 a*b**4 - 30*a*b**2*c**2*x + 12*a*c**4*x**2 + 6*b**4*c*x - 3*b**2*c**3*x**
2)/(128*sqrt(x)*a**4*b*c**3 - 96*sqrt(x)*a**3*b**3*c**2 + 128*sqrt(x)*a**3
*b*c**4*x + 24*sqrt(x)*a**2*b**5*c - 96*sqrt(x)*a**2*b**3*c**3*x - 2*sqrt(
x)*a*b**7 + 24*sqrt(x)*a*b**5*c**2*x - 2*sqrt(x)*b**7*c*x + 64*a**5*c**3 -
 48*a**4*b**2*c**2 + 128*a**4*c**4*x + 12*a**3*b**4*c - 32*a**3*b**2*c**3*
x + 64*a**3*c**5*x**2 - a**2*b**6 - 24*a**2*b**4*c**2*x - 48*a**2*b**2*c**
4*x**2 + 10*a*b**6*c*x + 12*a*b**4*c**3*x**2 - b**8*x - b**6*c**2*x**2)
```

3.30 $\int \left(a+b\sqrt{x}+cx\right)^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 200

$$\int \left(a+b\sqrt{x}+cx\right)^{5/2}\,dx = -\frac{5b(b^2-4ac)^2\left(b+2c\sqrt{x}\right)\sqrt{a+b\sqrt{x}+cx}}{512c^4} + \frac{5b(b^2-4ac)\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{3/2}}{192c^3} - \frac{b\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{5/2}}{12c^2} + \frac{2\left(a+b\sqrt{x}+cx\right)^{7/2}}{7c} + \frac{5b(b^2-4ac)^3\operatorname{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{1024c^{9/2}}$$

output
```
-5/512*b*(-4*a*c+b^2)^2*(b+2*c*x^(1/2))*(a+b*x^(1/2)+c*x)^(1/2)/c^4+5/192*
b*(-4*a*c+b^2)*(b+2*c*x^(1/2))*(a+b*x^(1/2)+c*x)^(3/2)/c^3-1/12*b*(b+2*c*x
^(1/2))*(a+b*x^(1/2)+c*x)^(5/2)/c^2+2/7*(a+b*x^(1/2)+c*x)^(7/2)/c+5/1024*b
*(-4*a*c+b^2)^3*arctanh(1/2*(b+2*c*x^(1/2))/c^(1/2)/(a+b*x^(1/2)+c*x)^(1/2
))/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98

$$\int \left(a+b\sqrt{x}+cx\right)^{5/2}\,dx=\frac{\sqrt{a+b\sqrt{x}+cx}\left(-105b^6+70b^5c\sqrt{x}-56b^4c(-20a+cx)+48b^3c^2\sqrt{x}(-14a+cx)+3072c^3(\cdots\right.}{10752c\cdots}-\frac{5b(b^2-4ac)^3\log\left(b+2c\sqrt{x}-2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}\right)}{1024c^{9/2}}$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^(5/2),x]
```

output
```
(Sqrt[a + b*Sqrt[x] + c*x]*(-105*b^6 + 70*b^5*c*Sqrt[x] - 56*b^4*c*(-20*a
+ c*x) + 48*b^3*c^2*Sqrt[x]*(-14*a + c*x) + 3072*c^3*(a + c*x)^3 + 32*b*c^
3*Sqrt[x]*(57*a^2 + 394*a*c*x + 232*c^2*x^2) + 16*b^2*c^2*(-231*a^2 + 30*a
*c*x + 296*c^2*x^2)))/(10752*c^4) - (5*b*(b^2 - 4*a*c)^3*Log[b + 2*c*Sqrt[
x] - 2*Sqrt[c]*Sqrt[a + b*Sqrt[x] + c*x]])/(1024*c^(9/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1680, 1160, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}+cx\right)^{5/2}\,dx$$

$$\Big\downarrow \text{ 1680}$$

$$2\int \sqrt{x}\left(a+cx+b\sqrt{x}\right)^{5/2}\,d\sqrt{x}$$

$$\Big\downarrow \text{ 1160}$$

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{7/2}}{7c}-\frac{b\int \left(a+cx+b\sqrt{x}\right)^{5/2}d\sqrt{x}}{2c}\right)$$

$\Big\downarrow$ 1087

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{7/2}}{7c}-\frac{b\left(\frac{\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\int \left(a+cx+b\sqrt{x}\right)^{3/2}d\sqrt{x}}{24c}\right)}{2c}\right)$$

$\Big\downarrow$ 1087

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{7/2}}{7c}-\frac{b\left(\frac{\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{a+cx+b\sqrt{x}}d\sqrt{x}}{16c}\right)}{24c}\right)}{2c}\right)$$

$\Big\downarrow$ 1087

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{7/2}}{7c}-\frac{b\left(\frac{\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt{x}\right)\sqrt{a+b\sqrt{x}+cx}}{4c}-\cdots\right.}{16c}\right.}{24c}\right.}{2c}\right.$$

$\Big\downarrow$ 1092

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{7/2}}{7c}-\frac{b\left(\frac{\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt{x}\right)\sqrt{a+b\sqrt{x}+cx}}{4c}-\cdots\right.}{16c}\right.}{24c}\right.}{2c}\right.$$

$$\downarrow 219$$

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{7/2}}{7c}-\frac{b\left(\frac{\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt{x}\right)\sqrt{a+b\sqrt{x}+cx}}{4c}-\cdots\right.}{\cdots}\right.}{24c}\right.}{2c}\right.$$

input
```
Int[(a + b*Sqrt[x] + c*x)^(5/2),x]
```

output
```
2*((a + b*Sqrt[x] + c*x)^(7/2)/(7*c) - (b*(((b + 2*c*Sqrt[x])*(a + b*Sqrt[
x] + c*x)^(5/2))/(12*c) - (5*(b^2 - 4*a*c)*(((b + 2*c*Sqrt[x])*(a + b*Sqrt
[x] + c*x)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*Sqrt[x])*Sqrt[a + b*
Sqrt[x] + c*x])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c
]*Sqrt[a + b*Sqrt[x] + c*x])])/(8*c^(3/2))))/(16*c)))/(24*c)))/(2*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{2(a+b\sqrt{x}+cx)^{\frac{7}{2}}}{7c} - \frac{b\left(\frac{(b+2c\sqrt{x})(a+b\sqrt{x}+cx)^{\frac{5}{2}}}{12c}+\frac{5(4ac-b^2)\left(\frac{(b+2c\sqrt{x})(a+b\sqrt{x}+cx)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(b+2c\sqrt{x})\sqrt{a+b\sqrt{x}-}}{4c}\right.}{}\right.}{24c}\right.}{c}$
default	$\frac{2(a+b\sqrt{x}+cx)^{\frac{7}{2}}}{7c} - \frac{b\left(\frac{(b+2c\sqrt{x})(a+b\sqrt{x}+cx)^{\frac{5}{2}}}{12c}+\frac{5(4ac-b^2)\left(\frac{(b+2c\sqrt{x})(a+b\sqrt{x}+cx)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(b+2c\sqrt{x})\sqrt{a+b\sqrt{x}-}}{4c}\right.}{}\right.}{24c}\right.}{c}$

input
```
int((a+b*x^(1/2)+c*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(a+b*x^(1/2)+c*x)^(7/2)/c-b/c*(1/12*(b+2*c*x^(1/2))/c*(a+b*x^(1/2)+c*x
)^(5/2)+5/24*(4*a*c-b^2)/c*(1/8*(b+2*c*x^(1/2))/c*(a+b*x^(1/2)+c*x)^(3/2)+
3/16*(4*a*c-b^2)/c*(1/4*(b+2*c*x^(1/2))/c*(a+b*x^(1/2)+c*x)^(1/2)+1/8*(4*a
*c-b^2)/c^(3/2)*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+b*x^(1/2)+c*x)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{x}+cx\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*x^(1/2)+c*x)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 1076, normalized size of antiderivative = 5.38

$$\int \left(a+b\sqrt{x}+cx\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*x**(1/2)+c*x)**(5/2),x)
```

output
```
2*Piecewise(((-a*(3*a**2*b - 3*a*(359*a*b*c/84 + b**3 - 9*b*(15*a*c**2/7 +
 185*b**2*c/168)/(10*c))/(4*c) - 5*b*(3*a**2*c + 3*a*b**2 - 4*a*(15*a*c**2
/7 + 185*b**2*c/168)/(5*c) - 7*b*(359*a*b*c/84 + b**3 - 9*b*(15*a*c**2/7 +
 185*b**2*c/168)/(10*c))/(8*c))/(6*c))/(2*c) - b*(a**3 - 2*a*(3*a**2*c + 3
*a*b**2 - 4*a*(15*a*c**2/7 + 185*b**2*c/168)/(5*c) - 7*b*(359*a*b*c/84 + b
**3 - 9*b*(15*a*c**2/7 + 185*b**2*c/168)/(10*c))/(8*c))/(3*c) - 3*b*(3*a**
2*b - 3*a*(359*a*b*c/84 + b**3 - 9*b*(15*a*c**2/7 + 185*b**2*c/168)/(10*c)
)/(4*c) - 5*b*(3*a**2*c + 3*a*b**2 - 4*a*(15*a*c**2/7 + 185*b**2*c/168)/(5
*c) - 7*b*(359*a*b*c/84 + b**3 - 9*b*(15*a*c**2/7 + 185*b**2*c/168)/(10*c)
)/(8*c))/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*sqrt
(x) + c*x) + 2*c*sqrt(x))/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + sqr
t(x))*log(b/(2*c) + sqrt(x))/sqrt(c*(b/(2*c) + sqrt(x))**2), True)) + sqrt
(a + b*sqrt(x) + c*x)*(29*b*c*x**(5/2)/84 + c**2*x**3/7 + x**(3/2)*(359*a*
b*c/84 + b**3 - 9*b*(15*a*c**2/7 + 185*b**2*c/168)/(10*c))/(4*c) + sqrt(x)
*(3*a**2*b - 3*a*(359*a*b*c/84 + b**3 - 9*b*(15*a*c**2/7 + 185*b**2*c/168)
/(10*c))/(4*c) - 5*b*(3*a**2*c + 3*a*b**2 - 4*a*(15*a*c**2/7 + 185*b**2*c/
168)/(5*c) - 7*b*(359*a*b*c/84 + b**3 - 9*b*(15*a*c**2/7 + 185*b**2*c/168)
/(10*c))/(8*c))/(6*c))/(2*c) + x**2*(15*a*c**2/7 + 185*b**2*c/168)/(5*c) +
 x*(3*a**2*c + 3*a*b**2 - 4*a*(15*a*c**2/7 + 185*b**2*c/168)/(5*c) - 7*b*(
359*a*b*c/84 + b**3 - 9*b*(15*a*c**2/7 + 185*b**2*c/168)/(10*c))/(8*c))...
```

Maxima [F]

$$\int \left(a + b\sqrt{x} + cx\right)^{5/2} dx = \int \left(cx + b\sqrt{x} + a\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\sqrt{x}+cx\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^(1/2)+c*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage3:=type(sage2):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m
& i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen &
 e,const
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{x}+cx\right)^{5/2} dx = \int \left(a+c\,x+b\,\sqrt{x}\right)^{5/2} dx$$

input
```
int((a + c*x + b*x^(1/2))^(5/2),x)
```

output
```
int((a + c*x + b*x^(1/2))^(5/2), x)
```

Reduce [F]

$$\int \left(a+b\sqrt{x}+cx\right)^{5/2} dx = \int \left(a+b\sqrt{x}+cx\right)^{\frac{5}{2}} dx$$

input
```
int((a+b*x^(1/2)+c*x)^(5/2),x)
```

output
```
int((a+b*x^(1/2)+c*x)^(5/2),x)
```

3.31 $\int \left(a+b\sqrt{x}+cx\right)^{3/2} dx$

Optimal result

Integrand size = 16, antiderivative size = 156

$$\int \left(a+b\sqrt{x}+cx\right)^{3/2} dx = \frac{3b(b^2-4ac)\left(b+2c\sqrt{x}\right)\sqrt{a+b\sqrt{x}+cx}}{64c^3} - \frac{b\left(b+2c\sqrt{x}\right)\left(a+b\sqrt{x}+cx\right)^{3/2}}{8c^2} + \frac{2\left(a+b\sqrt{x}+cx\right)^{5/2}}{5c} - \frac{3b(b^2-4ac)^2 \operatorname{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{128c^{7/2}}$$

output
```
3/64*b*(-4*a*c+b^2)*(b+2*c*x^(1/2))*(a+b*x^(1/2)+c*x)^(1/2)/c^3-1/8*b*(b+2
*c*x^(1/2))*(a+b*x^(1/2)+c*x)^(3/2)/c^2+2/5*(a+b*x^(1/2)+c*x)^(5/2)/c-3/12
8*b*(-4*a*c+b^2)^2*arctanh(1/2*(b+2*c*x^(1/2))/c^(1/2)/(a+b*x^(1/2)+c*x)^(
1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$\int (a+b\sqrt{x}+cx)^{3/2}\,dx = \frac{\sqrt{a+b\sqrt{x}+cx}\,(15b^4-10b^3c\sqrt{x}+128c^2(a+cx)^2+4b^2c(-25a+2cx)+8bc^2\sqrt{x}(7a+22c}{320c^3} + \frac{3b(b^2-4ac)^2\log\left(b+2c\sqrt{x}-2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}\right)}{128c^{7/2}}$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^(3/2),x]
```

output
```
(Sqrt[a + b*Sqrt[x] + c*x]*(15*b^4 - 10*b^3*c*Sqrt[x] + 128*c^2*(a + c*x)^
2 + 4*b^2*c*(-25*a + 2*c*x) + 8*b*c^2*Sqrt[x]*(7*a + 22*c*x)))/(320*c^3) +
 (3*b*(b^2 - 4*a*c)^2*Log[b + 2*c*Sqrt[x] - 2*Sqrt[c]*Sqrt[a + b*Sqrt[x] +
 c*x]])/(128*c^(7/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1680, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}+cx\right)^{3/2}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \sqrt{x}\left(a+cx+b\sqrt{x}\right)^{3/2}d\sqrt{x}$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{5/2}}{5c}-\frac{b\int \left(a+cx+b\sqrt{x}\right)^{3/2}d\sqrt{x}}{2c}\right)$$

$\downarrow$ 1087

$$2\left(\frac{(a+b\sqrt{x}+cx)^{5/2}}{5c}-\frac{b\left(\frac{(b+2c\sqrt{x})(a+b\sqrt{x}+cx)^{3/2}}{8c}-\frac{3(b^2-4ac)\int \sqrt{a+cx+b\sqrt{x}}d\sqrt{x}}{16c}\right)}{2c}\right)$$

$\downarrow$ 1087

$$2\left(\frac{(a+b\sqrt{x}+cx)^{5/2}}{5c}-\frac{b\left(\frac{(b+2c\sqrt{x})(a+b\sqrt{x}+cx)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2c\sqrt{x})\sqrt{a+b\sqrt{x}+cx}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}}{8c}\right)}{16c}\right)}{2c}\right)$$

$\downarrow$ 1092

$$2\left(\frac{(a+b\sqrt{x}+cx)^{5/2}}{5c}-\frac{b\left(\frac{(b+2c\sqrt{x})(a+b\sqrt{x}+cx)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2c\sqrt{x})\sqrt{a+b\sqrt{x}+cx}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-x}d\frac{b+2c\sqrt{x}}{\sqrt{a+cx+b\sqrt{x}}}}{4c}\right)}{16c}\right)}{2c}\right)$$

$\downarrow$ 219

$$2\left(\frac{(a+b\sqrt{x}+cx)^{5/2}}{5c}-\frac{b\left(\frac{(b+2c\sqrt{x})(a+b\sqrt{x}+cx)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2c\sqrt{x})\sqrt{a+b\sqrt{x}+cx}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{8c^{3/2}}\right)}{16c}\right)}{2c}\right)$$

input
```
Int[(a + b*Sqrt[x] + c*x)^(3/2),x]
```

output
```
2*((a + b*Sqrt[x] + c*x)^(5/2)/(5*c) - (b*(((b + 2*c*Sqrt[x])*(a + b*Sqrt[
x] + c*x)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*Sqrt[x])*Sqrt[a + b*S
qrt[x] + c*x])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]
*Sqrt[a + b*Sqrt[x] + c*x])])/(8*c^(3/2))))/(16*c)))/(2*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{2(a+b\sqrt{x}+cx)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(b+2c\sqrt{x})(a+b\sqrt{x}+cx)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(b+2c\sqrt{x})\sqrt{a+b\sqrt{x}+cx}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+b\sqrt{x}+}\right.}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{c}$
default	$\frac{2(a+b\sqrt{x}+cx)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(b+2c\sqrt{x})(a+b\sqrt{x}+cx)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(b+2c\sqrt{x})\sqrt{a+b\sqrt{x}+cx}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+b\sqrt{x}+}\right.}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{c}$

input
```
int((a+b*x^(1/2)+c*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a+b*x^(1/2)+c*x)^(5/2)/c-b/c*(1/8*(b+2*c*x^(1/2))/c*(a+b*x^(1/2)+c*x)
^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(b+2*c*x^(1/2))/c*(a+b*x^(1/2)+c*x)^(1/2)+1
/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+b*x^(1/2)+c*x)^(1/2
))))
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{x}+cx\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*x^(1/2)+c*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.38

$$\int \left(a+b\sqrt{x}+cx\right)^{3/2}\,dx=2\begin{cases}\left(-\frac{a\left(\frac{47ab}{40}-\frac{5b\left(\frac{6ac}{5}+\frac{3b^2}{80}\right)}{6c}\right)}{2c}-\frac{b\left(a^2-\frac{2a\left(\frac{6ac}{5}+\frac{3b^2}{80}\right)}{3c}-\frac{3b\left(\frac{47ab}{40}-\frac{5b\left(\frac{6ac}{5}+\frac{3b^2}{80}\right)}{6c}\right)}{4c}\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+b\sqrt{x}}\right.}{\sqrt{c}}\\ \frac{\left(\frac{b}{2c}+\sqrt{x}\right)\log\left(\frac{b}{2c}+\right.}{\sqrt{c\left(\frac{b}{2c}+\sqrt{x}\right)^2}}\end{cases}\right. \\ \frac{2\left(-\frac{a\left(a+b\sqrt{x}\right)^{\frac{5}{2}}}{5}+\frac{\left(a+b\sqrt{x}\right)^{\frac{7}{2}}}{7}\right)}{b^2} \\ \frac{a^{\frac{3}{2}}x}{2}\end{cases}$$

input
```
integrate((a+b*x**(1/2)+c*x)**(3/2),x)
```

output
```
2*Piecewise(((-a*(47*a*b/40 - 5*b*(6*a*c/5 + 3*b**2/80)/(6*c))/(2*c) - b*(
a**2 - 2*a*(6*a*c/5 + 3*b**2/80)/(3*c) - 3*b*(47*a*b/40 - 5*b*(6*a*c/5 + 3
*b**2/80)/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*sqr
t(x) + c*x) + 2*c*sqrt(x))/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + sq
rt(x))*log(b/(2*c) + sqrt(x))/sqrt(c*(b/(2*c) + sqrt(x))**2), True)) + sqr
t(a + b*sqrt(x) + c*x)*(11*b*x**(3/2)/40 + c*x**2/5 + sqrt(x)*(47*a*b/40 -
 5*b*(6*a*c/5 + 3*b**2/80)/(6*c))/(2*c) + x*(6*a*c/5 + 3*b**2/80)/(3*c) +
(a**2 - 2*a*(6*a*c/5 + 3*b**2/80)/(3*c) - 3*b*(47*a*b/40 - 5*b*(6*a*c/5 +
3*b**2/80)/(6*c))/(4*c))/c), Ne(c, 0)), (2*(-a*(a + b*sqrt(x))**(5/2)/5 +
(a + b*sqrt(x))**(7/2)/7)/b**2, Ne(b, 0)), (a**(3/2)*x/2, True))
```

Maxima [F]

$$\int \left(a+b\sqrt{x}+cx\right)^{3/2}\,dx=\int \left(cx+b\sqrt{x}+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(120) = 240$.

Time = 0.24 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.54

$$\int \left(a+b\sqrt{x}+cx\right)^{3/2}\,dx=\frac{1}{24}\left(2\sqrt{cx+b\sqrt{x}+a}\left(2\sqrt{x}\left(4\sqrt{x}+\frac{b}{c}\right)-\frac{3\,b^2-8\,ac}{c^2}\right)-\frac{3\,(b^3-4\,abc)\log\left(\left|2\sqrt{c}\left(\sqrt{c}\sqrt{x}\right.\right.\right.}{c^{\frac{5}{2}}}\right.$$
$$+\frac{1}{192}\left(2\sqrt{cx+b\sqrt{x}+a}\left(2\left(4\sqrt{x}\left(6\sqrt{x}+\frac{b}{c}\right)-\frac{5\,b^2c-12\,ac^2}{c^3}\right)\sqrt{x}+\frac{15\,b^3-52\,abc}{c^3}\right)+\frac{3\,(5\,b^4-24\,a}{}\right.$$
$$+\frac{1}{1920}\left(2\sqrt{cx+b\sqrt{x}+a}\left(2\left(4\left(6\sqrt{x}\left(8\sqrt{x}+\frac{b}{c}\right)-\frac{7\,b^2c^2-16\,ac^3}{c^4}\right)\sqrt{x}+\frac{35\,b^3c-116\,abc^2}{c^4}\right)\sqrt{x}-1\right.\right.$$

input
```
integrate((a+b*x^(1/2)+c*x)^(3/2),x, algorithm="giac")
```

output
```
1/24*(2*sqrt(c*x + b*sqrt(x) + a)*(2*sqrt(x)*(4*sqrt(x) + b/c) - (3*b^2 -
8*a*c)/c^2) - 3*(b^3 - 4*a*b*c)*log(abs(2*sqrt(c)*(sqrt(c)*sqrt(x) - sqrt(
c*x + b*sqrt(x) + a)) + b))/c^(5/2))*a + 1/192*(2*sqrt(c*x + b*sqrt(x) + a
)*(2*(4*sqrt(x)*(6*sqrt(x) + b/c) - (5*b^2*c - 12*a*c^2)/c^3)*sqrt(x) + (1
5*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*log(abs(2*sqr
t(c)*(sqrt(c)*sqrt(x) - sqrt(c*x + b*sqrt(x) + a)) + b))/c^(7/2))*b + 1/19
20*(2*sqrt(c*x + b*sqrt(x) + a)*(2*(4*(6*sqrt(x)*(8*sqrt(x) + b/c) - (7*b^
2*c^2 - 16*a*c^3)/c^4)*sqrt(x) + (35*b^3*c - 116*a*b*c^2)/c^4)*sqrt(x) - (
105*b^4 - 460*a*b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^
2*b*c^2)*log(abs(2*sqrt(c)*(sqrt(c)*sqrt(x) - sqrt(c*x + b*sqrt(x) + a)) +
 b))/c^(9/2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{x}+cx\right)^{3/2}\,dx=\int \left(a+c\,x+b\,\sqrt{x}\right)^{3/2}\,dx$$

input
```
int((a + c*x + b*x^(1/2))^(3/2),x)
```

output
```
int((a + c*x + b*x^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.96

$$\int \left(a+b\sqrt{x}+cx\right)^{3/2}\,dx=\frac{112\sqrt{x}\,\sqrt{\sqrt{x}\,b+a+cx}\,ab\,c^3-20\sqrt{x}\,\sqrt{\sqrt{x}\,b+a+cx}\,b^3c^2+352\sqrt{x}\,\sqrt{\sqrt{x}\,b+a+cx}\,b\,c^4x+\cdots}{\cdots}$$

input
```
int((a+b*x^(1/2)+c*x)^(3/2),x)
```

output

```
(112*sqrt(x)*sqrt(sqrt(x)*b + a + c*x)*a*b*c**3 - 20*sqrt(x)*sqrt(sqrt(x)*
b + a + c*x)*b**3*c**2 + 352*sqrt(x)*sqrt(sqrt(x)*b + a + c*x)*b*c**4*x +
256*sqrt(sqrt(x)*b + a + c*x)*a**2*c**3 - 200*sqrt(sqrt(x)*b + a + c*x)*a*
b**2*c**2 + 512*sqrt(sqrt(x)*b + a + c*x)*a*c**4*x + 30*sqrt(sqrt(x)*b + a
 + c*x)*b**4*c + 16*sqrt(sqrt(x)*b + a + c*x)*b**2*c**3*x + 256*sqrt(sqrt(
x)*b + a + c*x)*c**5*x**2 - 240*sqrt(c)*log((2*sqrt(c)*sqrt(sqrt(x)*b + a
+ c*x) + 2*sqrt(x)*c + b)/sqrt(4*a*c - b**2))*a**2*b*c**2 + 120*sqrt(c)*lo
g((2*sqrt(c)*sqrt(sqrt(x)*b + a + c*x) + 2*sqrt(x)*c + b)/sqrt(4*a*c - b**
2))*a*b**3*c - 15*sqrt(c)*log((2*sqrt(c)*sqrt(sqrt(x)*b + a + c*x) + 2*sqr
t(x)*c + b)/sqrt(4*a*c - b**2))*b**5)/(640*c**4)
```

3.32 $\int \sqrt{a+b\sqrt{x}+cx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \sqrt{a+b\sqrt{x}+cx}\,dx=-\frac{b\left(b+2c\sqrt{x}\right)\sqrt{a+b\sqrt{x}+cx}}{4c^2}+\frac{2\left(a+b\sqrt{x}+cx\right)^{3/2}}{3c}+\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{8c^{5/2}}$$

output
```
-1/4*b*(b+2*c*x^(1/2))*(a+b*x^(1/2)+c*x)^(1/2)/c^2+2/3*(a+b*x^(1/2)+c*x)^(
3/2)/c+1/8*b*(-4*a*c+b^2)*arctanh(1/2*(b+2*c*x^(1/2))/c^(1/2)/(a+b*x^(1/2)
+c*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.92

$$\int \sqrt{a+b\sqrt{x}+cx}\,dx=\frac{\sqrt{a+b\sqrt{x}+cx}\left(-3b^2+2bc\sqrt{x}+8c(a+cx)\right)}{12c^2}-\frac{(b^3-4abc)\log\left(c^2\left(b+2c\sqrt{x}-2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}\right)\right)}{8c^{5/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[x] + c*x],x]
```

output
```
(Sqrt[a + b*Sqrt[x] + c*x]*(-3*b^2 + 2*b*c*Sqrt[x] + 8*c*(a + c*x)))/(12*c
^2) - ((b^3 - 4*a*b*c)*Log[c^2*(b + 2*c*Sqrt[x] - 2*Sqrt[c]*Sqrt[a + b*Sqr
t[x] + c*x])])/(8*c^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1680, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sqrt{x}+cx}\,dx$$

$$\Big\downarrow \text{ 1680}$$

$$2\int \sqrt{x}\sqrt{a+cx+b\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1160}$$

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{3/2}}{3c}-\frac{b\int \sqrt{a+cx+b\sqrt{x}}d\sqrt{x}}{2c}\right)$$

$$\Big\downarrow \text{ 1087}$$

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{3/2}}{3c}-\frac{b\left(\frac{\left(b+2c\sqrt{x}\right)\sqrt{a+b\sqrt{x}+cx}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}}{8c}\right)}{2c}\right)$$

$$\Big\downarrow \text{ 1092}$$

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{3/2}}{3c}-\frac{b\left(\frac{\left(b+2c\sqrt{x}\right)\sqrt{a+b\sqrt{x}+cx}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-x}d\frac{b+2c\sqrt{x}}{\sqrt{a+cx+b\sqrt{x}}}}{4c}\right)}{2c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$2\left(\frac{\left(a+b\sqrt{x}+cx\right)^{3/2}}{3c}-\frac{b\left(\frac{\left(b+2c\sqrt{x}\right)\sqrt{a+b\sqrt{x}+cx}}{4c}-\frac{\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{8c^{3/2}}\right)}{2c}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[x] + c*x],x]
```

output
```
2*((a + b*Sqrt[x] + c*x)^(3/2)/(3*c) - (b*(((b + 2*c*Sqrt[x])*Sqrt[a + b*S
qrt[x] + c*x])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]
*Sqrt[a + b*Sqrt[x] + c*x])])/(8*c^(3/2))))/(2*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2\left(a+b\sqrt{x}+cx\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{\left(b+2c\sqrt{x}\right)\sqrt{a+b\sqrt{x}+cx}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+b\sqrt{x}+cx}\right)}{8c^{\frac{3}{2}}}\right)}{c}$	93
default	$\frac{2\left(a+b\sqrt{x}+cx\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{\left(b+2c\sqrt{x}\right)\sqrt{a+b\sqrt{x}+cx}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+b\sqrt{x}+cx}\right)}{8c^{\frac{3}{2}}}\right)}{c}$	93

input
```
int((a+b*x^(1/2)+c*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a+b*x^(1/2)+c*x)^(3/2)/c-b/c*(1/4*(b+2*c*x^(1/2))/c*(a+b*x^(1/2)+c*x)
^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+b*x^(1/2)+c
*x)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int\sqrt{a+b\sqrt{x}+cx}\,dx=\text{Timed out}$$

input
```
integrate((a+b*x^(1/2)+c*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.71

$$\int \sqrt{a+b\sqrt{x}+cx}\,dx$$
$$=2\left(\begin{cases}\left(-\frac{ab}{12c}-\frac{b\left(\frac{a}{3}-\frac{b^2}{8c}\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}+2c\sqrt{x}\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0\\ \frac{\left(\frac{b}{2c}+\sqrt{x}\right)\log\left(\frac{b}{2c}+\sqrt{x}\right)}{\sqrt{c\left(\frac{b}{2c}+\sqrt{x}\right)^2}} & \text{otherwise}\end{cases}\right)+\sqrt{a+b\sqrt{x}+cx}\left(\frac{b\sqrt{x}}{12c}+\frac{x}{3}+\right. \\ \frac{2\left(-\frac{a\left(a+b\sqrt{x}\right)^{\frac{3}{2}}}{3}+\frac{\left(a+b\sqrt{x}\right)^{\frac{5}{2}}}{5}\right)}{b^2} \\ \frac{\sqrt{a}x}{2}\end{cases}\right.$$

input
```
integrate((a+b*x**(1/2)+c*x)**(1/2),x)
```

output
```
2*Piecewise(((-a*b/(12*c) - b*(a/3 - b**2/(8*c))/(2*c))*Piecewise((log(b +
 2*sqrt(c)*sqrt(a + b*sqrt(x) + c*x) + 2*c*sqrt(x))/sqrt(c), Ne(a - b**2/(
4*c), 0)), ((b/(2*c) + sqrt(x))*log(b/(2*c) + sqrt(x))/sqrt(c*(b/(2*c) + s
qrt(x))**2), True)) + sqrt(a + b*sqrt(x) + c*x)*(b*sqrt(x)/(12*c) + x/3 +
(a/3 - b**2/(8*c))/c), Ne(c, 0)), (2*(-a*(a + b*sqrt(x))**(3/2)/3 + (a + b
*sqrt(x))**(5/2)/5)/b**2, Ne(b, 0)), (sqrt(a)*x/2, True))
```

Maxima [F]

$$\int \sqrt{a+b\sqrt{x}+cx}\,dx=\int \sqrt{cx+b\sqrt{x}+a}\,dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x + b*sqrt(x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\sqrt{x}+cx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^(1/2)+c*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{x}+cx}\,dx = \int \sqrt{a+c\,x+b\,\sqrt{x}}\,dx$$

input
```
int((a + c*x + b*x^(1/2))^(1/2),x)
```

output
```
int((a + c*x + b*x^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.41

$$\int \sqrt{a+b\sqrt{x}+cx}\,dx$$
$$= \frac{4\sqrt{x}\,\sqrt{\sqrt{x}\,b+a+cx}\,b\,c^2+16\sqrt{\sqrt{x}\,b+a+cx}\,a\,c^2-6\sqrt{\sqrt{x}\,b+a+cx}\,b^2c+16\sqrt{\sqrt{x}\,b+a+cx}\,c^3x-}{24c^3}$$

input
```
int((a+b*x^(1/2)+c*x)^(1/2),x)
```

output
```
(4*sqrt(x)*sqrt(sqrt(x)*b + a + c*x)*b*c**2 + 16*sqrt(sqrt(x)*b + a + c*x)
*a*c**2 - 6*sqrt(sqrt(x)*b + a + c*x)*b**2*c + 16*sqrt(sqrt(x)*b + a + c*x
)*c**3*x - 12*sqrt(c)*log((2*sqrt(c)*sqrt(sqrt(x)*b + a + c*x) + 2*sqrt(x)
*c + b)/sqrt(4*a*c - b**2))*a*b*c + 3*sqrt(c)*log((2*sqrt(c)*sqrt(sqrt(x)*
b + a + c*x) + 2*sqrt(x)*c + b)/sqrt(4*a*c - b**2))*b**3)/(24*c**3)
```

3.33 $\int \frac{1}{\sqrt{a+b\sqrt{x}+cx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{1}{\sqrt{a+b\sqrt{x}+cx}}\,dx = \frac{2\sqrt{a+b\sqrt{x}+cx}}{c} - \frac{b\operatorname{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{c^{3/2}}$$

output
```
2*(a+b*x^(1/2)+c*x)^(1/2)/c-b*arctanh(1/2*(b+2*c*x^(1/2))/c^(1/2)/(a+b*x^(
1/2)+c*x)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\sqrt{x}+cx}}\,dx = \frac{2\sqrt{a+b\sqrt{x}+cx}}{c} - \frac{b\operatorname{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{c^{3/2}}$$

input
```
Integrate[1/Sqrt[a + b*Sqrt[x] + c*x],x]
```

output
```
(2*Sqrt[a + b*Sqrt[x] + c*x])/c - (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*
Sqrt[a + b*Sqrt[x] + c*x])])/c^(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1680, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sqrt{x}+cx}}dx$$

$$\Big\downarrow \text{ 1680}$$

$$2\int \frac{\sqrt{x}}{\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1160}$$

$$2\left(\frac{\sqrt{a+b\sqrt{x}+cx}}{c}-\frac{b\int \frac{1}{\sqrt{a+cx+b\sqrt{x}}}d\sqrt{x}}{2c}\right)$$

$$\Big\downarrow \text{ 1092}$$

$$2\left(\frac{\sqrt{a+b\sqrt{x}+cx}}{c}-\frac{b\int \frac{1}{4c-x}d\frac{b+2c\sqrt{x}}{\sqrt{a+cx+b\sqrt{x}}}}{c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$2\left(\frac{\sqrt{a+b\sqrt{x}+cx}}{c}-\frac{b\,\text{arctanh}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{2c^{3/2}}\right)$$

input
```
Int[1/Sqrt[a + b*Sqrt[x] + c*x],x]
```

output
```
2*(Sqrt[a + b*Sqrt[x] + c*x]/c - (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*S
qrt[a + b*Sqrt[x] + c*x])])/(2*c^(3/2)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\sqrt{x}+cx}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+b\sqrt{x}+cx}\right)}{c^{\frac{3}{2}}}$	52
default	$\frac{2\sqrt{a+b\sqrt{x}+cx}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+b\sqrt{x}+cx}\right)}{c^{\frac{3}{2}}}$	52

input
```
int(1/(a+b*x^(1/2)+c*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a+b*x^(1/2)+c*x)^(1/2)/c-b/c^(3/2)*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+b*x^
(1/2)+c*x)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{x}+cx}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.24

$$\int \frac{1}{\sqrt{a+b\sqrt{x}+cx}}\,dx = 2\left(\begin{cases} -\frac{b\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}+2c\sqrt{x}\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0 \\ \frac{\left(\frac{b}{2c}+\sqrt{x}\right)\log\left(\frac{b}{2c}+\sqrt{x}\right)}{\sqrt{c\left(\frac{b}{2c}+\sqrt{x}\right)^2}} & \text{otherwise} \end{cases}\right)}{2c} + \frac{\sqrt{a+b\sqrt{x}+cx}}{c} & \text{for } c\neq 0 \\ \frac{2\left(-a\sqrt{a+b\sqrt{x}}+\frac{(a+b\sqrt{x})^{\frac{3}{2}}}{3}\right)}{b^2} & \text{for } b\neq 0 \\ \frac{x}{2\sqrt{a}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(1/(a+b*x**(1/2)+c*x)**(1/2),x)
```

output
```
2*Piecewise((-b*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*sqrt(x) + c*x) + 2
*c*sqrt(x))/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + sqrt(x))*log(b/(2
*c) + sqrt(x))/sqrt(c*(b/(2*c) + sqrt(x))**2), True))/(2*c) + sqrt(a + b*s
qrt(x) + c*x)/c, Ne(c, 0)), (2*(-a*sqrt(a + b*sqrt(x)) + (a + b*sqrt(x))**
(3/2)/3)/b**2, Ne(b, 0)), (x/(2*sqrt(a)), True))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sqrt{x}+cx}}\,dx=\int \frac{1}{\sqrt{cx+b\sqrt{x}+a}}\,dx$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x + b*sqrt(x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{a+b\sqrt{x}+cx}}\,dx=\frac{b\log\left(\left|2\sqrt{c}\left(\sqrt{c}\sqrt{x}-\sqrt{cx+b\sqrt{x}+a}\right)+b\right|\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{cx+b\sqrt{x}+a}}{c}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^(1/2),x, algorithm="giac")
```

output
```
b*log(abs(2*sqrt(c)*(sqrt(c)*sqrt(x) - sqrt(c*x + b*sqrt(x) + a)) + b))/c^
(3/2) + 2*sqrt(c*x + b*sqrt(x) + a)/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt{x}+cx}}\,dx=\int \frac{1}{\sqrt{a+c\,x+b\,\sqrt{x}}}\,dx$$

input
```
int(1/(a + c*x + b*x^(1/2))^(1/2),x)
```

output
```
int(1/(a + c*x + b*x^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{a+b\sqrt{x}+cx}}\,dx = \frac{2\sqrt{\sqrt{x}\,b+a+cx}\,c-\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{\sqrt{x}\,b+a+cx}+2\sqrt{x}\,c+b}{\sqrt{4ac-b^2}}\right)b}{c^2}$$

input
```
int(1/(a+b*x^(1/2)+c*x)^(1/2),x)
```

output
```
(2*sqrt(sqrt(x)*b + a + c*x)*c - sqrt(c)*log((2*sqrt(c)*sqrt(sqrt(x)*b + a
 + c*x) + 2*sqrt(x)*c + b)/sqrt(4*a*c - b**2))*b)/c**2
```

3.34 $\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{3/2}}\,dx=\frac{4\left(2a+b\sqrt{x}\right)}{\left(b^2-4ac\right)\sqrt{a+b\sqrt{x}+cx}}$$

output
```
4*(2*a+b*x^(1/2))/(-4*a*c+b^2)/(a+b*x^(1/2)+c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{3/2}}\,dx=\frac{4\left(2a+b\sqrt{x}\right)}{\left(b^2-4ac\right)\sqrt{a+b\sqrt{x}+cx}}$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^(-3/2),x]
```

output
```
(4*(2*a + b*Sqrt[x]))/((b^2 - 4*a*c)*Sqrt[a + b*Sqrt[x] + c*x])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1680, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1680}$$

$$2\int \frac{\sqrt{x}}{\left(a+cx+b\sqrt{x}\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1158}$$

$$\frac{4\left(2a+b\sqrt{x}\right)}{\left(b^2-4ac\right)\sqrt{a+b\sqrt{x}+cx}}$$

input
```
Int[(a + b*Sqrt[x] + c*x)^(-3/2),x]
```

output
```
(4*(2*a + b*Sqrt[x]))/((b^2 - 4*a*c)*Sqrt[a + b*Sqrt[x] + c*x])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

method	result	size
derivativedivides	$-\frac{2}{c\sqrt{a+b\sqrt{x}+cx}}-\frac{2b\left(b+2c\sqrt{x}\right)}{c\left(4ac-b^{2}\right)\sqrt{a+b\sqrt{x}+cx}}$	57
default	$-\frac{2}{c\sqrt{a+b\sqrt{x}+cx}}-\frac{2b\left(b+2c\sqrt{x}\right)}{c\left(4ac-b^{2}\right)\sqrt{a+b\sqrt{x}+cx}}$	57

input
```
int(1/(a+b*x^(1/2)+c*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/c/(a+b*x^(1/2)+c*x)^(1/2)-2*b/c*(b+2*c*x^(1/2))/(4*a*c-b^2)/(a+b*x^(1/2
)+c*x)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(33) = 66.

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.56

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{3/2}}\,dx = \frac{4\left(2\,a^{2}-(b^{2}-2\,ac)x+(bcx-ab)\sqrt{x}\right)\sqrt{cx+b\sqrt{x}+a}}{a^{2}b^{2}-4\,a^{3}c+(b^{2}c^{2}-4\,ac^{3})x^{2}-(b^{4}-6\,ab^{2}c+8\,a^{2}c^{2})x}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^(3/2),x, algorithm="fricas")
```

output
```
4*(2*a^2 - (b^2 - 2*a*c)*x + (b*c*x - a*b)*sqrt(x))*sqrt(c*x + b*sqrt(x) +
 a)/(a^2*b^2 - 4*a^3*c + (b^2*c^2 - 4*a*c^3)*x^2 - (b^4 - 6*a*b^2*c + 8*a^
2*c^2)*x)
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{3/2}}\,dx = \int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x**(1/2)+c*x)**(3/2),x)
```

output
```
Integral((a + b*sqrt(x) + c*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{3/2}}\,dx = \int \frac{1}{\left(cx+b\sqrt{x}+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^(-3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{3/2}}\,dx = \frac{4\left(\frac{b\sqrt{x}}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}\right)}{\sqrt{cx+b\sqrt{x}+a}}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^(3/2),x, algorithm="giac")
```

output
```
4*(b*sqrt(x)/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x + b*sqrt(x) + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{3/2}}\,dx=\int \frac{1}{\left(a+c\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(a + c*x + b*x^(1/2))^(3/2),x)
```

output
```
int(1/(a + c*x + b*x^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.59

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{3/2}}\,dx=\frac{-4\sqrt{x}\,\sqrt{\sqrt{x}\,b+a+cx}\,bc-8\sqrt{\sqrt{x}\,b+a+cx}\,ac-4\sqrt{x}\,\sqrt{c}\,b^2-4\sqrt{c}\,ab-4\sqrt{\ }}{c\left(4\sqrt{x}\,abc-\sqrt{x}\,b^3+4a^2c-a\,b^2+4a\,c^2x-b^2cx\right)}$$

input
```
int(1/(a+b*x^(1/2)+c*x)^(3/2),x)
```

output
```
(4*( - sqrt(x)*sqrt(sqrt(x)*b + a + c*x)*b*c - 2*sqrt(sqrt(x)*b + a + c*x)
*a*c - sqrt(x)*sqrt(c)*b**2 - sqrt(c)*a*b - sqrt(c)*b*c*x))/(c*(4*sqrt(x)*
a*b*c - sqrt(x)*b**3 + 4*a**2*c - a*b**2 + 4*a*c**2*x - b**2*c*x))
```

3.35 $\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{5/2}}\,dx=\frac{4\left(2a+b\sqrt{x}\right)}{3\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)^{3/2}}-\frac{16b\left(b+2c\sqrt{x}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+b\sqrt{x}+cx}}$$

output
```
4/3*(2*a+b*x^(1/2))/(-4*a*c+b^2)/(a+b*x^(1/2)+c*x)^(3/2)-16/3*b*(b+2*c*x^(
1/2))/(-4*a*c+b^2)^2/(a+b*x^(1/2)+c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{5/2}}\,dx=-\frac{4\left(2ab^2+8a^2c+3b^3\sqrt{x}+12abc\sqrt{x}+12b^2cx+8bc^2x^{3/2}\right)}{3\left(b^2-4ac\right)^2\left(a+b\sqrt{x}+cx\right)^{3/2}}$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^(-5/2),x]
```

output
```
(-4*(2*a*b^2 + 8*a^2*c + 3*b^3*Sqrt[x] + 12*a*b*c*Sqrt[x] + 12*b^2*c*x + 8
*b*c^2*x^(3/2)))/(3*(b^2 - 4*a*c)^2*(a + b*Sqrt[x] + c*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1680, 1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{5/2}}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \frac{\sqrt{x}}{\left(a+cx+b\sqrt{x}\right)^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1159}$$

$$2\left(\frac{4b\int \frac{1}{\left(a+cx+b\sqrt{x}\right)^{3/2}}d\sqrt{x}}{3\left(b^2-4ac\right)}+\frac{2\left(2a+b\sqrt{x}\right)}{3\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)^{3/2}}\right)$$

$$\Big\downarrow \text{1088}$$

$$2\left(\frac{2\left(2a+b\sqrt{x}\right)}{3\left(b^2-4ac\right)\left(a+b\sqrt{x}+cx\right)^{3/2}}-\frac{8b\left(b+2c\sqrt{x}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+b\sqrt{x}+cx}}\right)$$

input
```
Int[(a + b*Sqrt[x] + c*x)^(-5/2),x]
```

output
```
2*((2*(2*a + b*Sqrt[x]))/(3*(b^2 - 4*a*c)*(a + b*Sqrt[x] + c*x)^(3/2)) - (
8*b*(b + 2*c*Sqrt[x]))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*Sqrt[x] + c*x]))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\frac{2}{3c\left(a+b\sqrt{x}+cx\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{2b}{3}+\frac{4c\sqrt{x}}{3}}{\left(4ac-b^{2}\right)\left(a+b\sqrt{x}+cx\right)^{\frac{3}{2}}}+\frac{16c\left(b+2c\sqrt{x}\right)}{3\left(4ac-b^{2}\right)^{2}\sqrt{a+b\sqrt{x}+cx}}\right)}{c}$	95
default	$-\frac{2}{3c\left(a+b\sqrt{x}+cx\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{2b}{3}+\frac{4c\sqrt{x}}{3}}{\left(4ac-b^{2}\right)\left(a+b\sqrt{x}+cx\right)^{\frac{3}{2}}}+\frac{16c\left(b+2c\sqrt{x}\right)}{3\left(4ac-b^{2}\right)^{2}\sqrt{a+b\sqrt{x}+cx}}\right)}{c}$	95

input
```
int(1/(a+b*x^(1/2)+c*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/c/(a+b*x^(1/2)+c*x)^(3/2)-b/c*(2/3*(b+2*c*x^(1/2))/(4*a*c-b^2)/(a+b*x
^(1/2)+c*x)^(3/2)+16/3*c/(4*a*c-b^2)^2*(b+2*c*x^(1/2))/(a+b*x^(1/2)+c*x)^(
1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(67) = 134$.

Time = 0.34 (sec) , antiderivative size = 342, normalized size of antiderivative = 4.12

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{5/2}}\,dx = \frac{4\left(4\,b^2c^3x^3-2\,a^3b^2-8\,a^4c-2\left(3\,b^4c-7\,ab^2c^2+4\,a^2c^3\right)x^2+4\left(ab^4\right.\right.}{3\left(a^4b^4-8\,a^5b^2c+16\,a^6c^2+\left(b^4c^4-8\,ab^2c^5+16\,a^2c^6\right)x^4-2\left(b^6c^2-10\,ab^4c^3\right.\right.}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^(5/2),x, algorithm="fricas")
```

output
```
4/3*(4*b^2*c^3*x^3 - 2*a^3*b^2 - 8*a^4*c - 2*(3*b^4*c - 7*a*b^2*c^2 + 4*a^
2*c^3)*x^2 + 4*(a*b^4 - 4*a^3*c^2)*x - (8*b*c^4*x^3 - a^2*b^3 - 4*a^3*b*c
- (13*b^3*c^2 - 28*a*b*c^3)*x^2 + (3*b^5 - 10*a*b^3*c + 16*a^2*b*c^2)*x)*s
qrt(x))*sqrt(c*x + b*sqrt(x) + a)/(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + (b
^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 - 2*(b^6*c^2 - 10*a*b^4*c^3 + 32*a^
2*b^2*c^4 - 32*a^3*c^5)*x^3 + (b^8 - 12*a*b^6*c + 54*a^2*b^4*c^2 - 112*a^3
*b^2*c^3 + 96*a^4*c^4)*x^2 - 2*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 -
32*a^5*c^3)*x)
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*x**(1/2)+c*x)**(5/2),x)
```

output
```
Integral((a + b*sqrt(x) + c*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{5/2}}\,dx = \int \frac{1}{\left(cx+b\sqrt{x}+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x + b*sqrt(x) + a)^(-5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(67) = 134$.

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.81

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{5/2}}\,dx = -\frac{4\left(\left(4\left(\frac{2\,bc^2\sqrt{x}}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\,b^2c}{b^4-8\,ab^2c+16\,a^2c^2}\right)\sqrt{x}+\frac{3\left(b^3+4\,abc\right)}{b^4-8\,ab^2c+16\,a^2c^2}\right)\sqrt{x}+\frac{2\left(ab^2+4\,a^2c\right)}{b^4-8\,ab^2c+16\,a^2c^2}\right)}{3\left(cx+b\sqrt{x}+a\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b*x^(1/2)+c*x)^(5/2),x, algorithm="giac")
```

output
```
-4/3*((4*(2*b*c^2*sqrt(x)/(b^4 - 8*a*b^2*c + 16*a^2*c^2) + 3*b^2*c/(b^4 -
8*a*b^2*c + 16*a^2*c^2))*sqrt(x) + 3*(b^3 + 4*a*b*c)/(b^4 - 8*a*b^2*c + 16
*a^2*c^2))*sqrt(x) + 2*(a*b^2 + 4*a^2*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(
c*x + b*sqrt(x) + a)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{5/2}}\,dx=\int \frac{1}{\left(a+c\,x+b\,\sqrt{x}\right)^{5/2}}\,dx$$

input
```
int(1/(a + c*x + b*x^(1/2))^(5/2),x)
```

output
```
int(1/(a + c*x + b*x^(1/2))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.72

$$\int \frac{1}{\left(a+b\sqrt{x}+cx\right)^{5/2}}\,dx=\frac{-48\sqrt{x}\,\sqrt{\sqrt{x}\,b+a+cx}\,abc-12\sqrt{x}\,\sqrt{\sqrt{x}\,b+a+cx}\,b^3-32\sqrt{x}\,\sqrt{\sqrt{x}\,b+a+}}{96\sqrt{x}\,a^3b\,c^2-48\sqrt{x}\,a^2b^3c+96\sqrt{x}\,a^2b\,c^3x}$$

input
```
int(1/(a+b*x^(1/2)+c*x)^(5/2),x)
```

output
```
(4*( - 12*sqrt(x)*sqrt(sqrt(x)*b + a + c*x)*a*b*c - 3*sqrt(x)*sqrt(sqrt(x)
*b + a + c*x)*b**3 - 8*sqrt(x)*sqrt(sqrt(x)*b + a + c*x)*b*c**2*x - 8*sqrt
(sqrt(x)*b + a + c*x)*a**2*c - 2*sqrt(sqrt(x)*b + a + c*x)*a*b**2 - 12*sqr
t(sqrt(x)*b + a + c*x)*b**2*c*x + 16*sqrt(x)*sqrt(c)*a*b**2 + 16*sqrt(x)*s
qrt(c)*b**2*c*x + 8*sqrt(c)*a**2*b + 16*sqrt(c)*a*b*c*x + 8*sqrt(c)*b**3*x
 + 8*sqrt(c)*b*c**2*x**2))/(3*(32*sqrt(x)*a**3*b*c**2 - 16*sqrt(x)*a**2*b*
*3*c + 32*sqrt(x)*a**2*b*c**3*x + 2*sqrt(x)*a*b**5 - 16*sqrt(x)*a*b**3*c**
2*x + 2*sqrt(x)*b**5*c*x + 16*a**4*c**2 - 8*a**3*b**2*c + 32*a**3*c**3*x +
 a**2*b**4 + 16*a**2*c**4*x**2 - 6*a*b**4*c*x - 8*a*b**2*c**3*x**2 + b**6*
x + b**4*c**2*x**2))
```

3.36 $\int \left(a^2+2ac\sqrt{x}+c^2x\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^3\,dx=-\frac{2a\left(a+c\sqrt{x}\right)^7}{7c^2}+\frac{\left(a+c\sqrt{x}\right)^8}{4c^2}$$

output
```
-2/7*a*(a+c*x^(1/2))^7/c^2+1/4*(a+c*x^(1/2))^8/c^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^3\,dx=\frac{1}{28}\left(28a^6x+112a^5cx^{3/2}+210a^4c^2x^2+224a^3c^3x^{5/2}+140a^2c^4x^3+48ac^5x^{7/2}+7c^6x^4\right)$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^3,x]
```

output
```
(28*a^6*x + 112*a^5*c*x^(3/2) + 210*a^4*c^2*x^2 + 224*a^3*c^3*x^(5/2) + 14
0*a^2*c^4*x^3 + 48*a*c^5*x^(7/2) + 7*c^6*x^4)/28
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1379, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^3\,dx$$

$$\Big\downarrow \text{1379}$$

$$\frac{\int \left(\sqrt{x}c^2+ac\right)^6dx}{c^6}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int c^6\left(a+c\sqrt{x}\right)^6\sqrt{x}d\sqrt{x}}{c^6}$$

$$\Big\downarrow \text{27}$$

$$2\int \left(a+c\sqrt{x}\right)^6\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{\left(a+c\sqrt{x}\right)^7}{c}-\frac{a\left(a+c\sqrt{x}\right)^6}{c}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\left(a+c\sqrt{x}\right)^8}{8c^2}-\frac{a\left(a+c\sqrt{x}\right)^7}{7c^2}\right)$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^3,x]
```

output
```
2*(-1/7*(a*(a + c*Sqrt[x])^7)/c^2 + (a + c*Sqrt[x])^8/(8*c^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 1.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

method	result
derivativedivides	$\frac{c^6x^4}{4}+\frac{12a\,c^5x^{\frac{7}{2}}}{7}+5a^2c^4x^3+8a^3c^3x^{\frac{5}{2}}+\frac{15a^4c^2x^2}{2}+4a^5c\,x^{\frac{3}{2}}+a^6x$
default	$\frac{16a^3c^3x^{\frac{5}{2}}}{5}+12a^2c^2\left(\frac{1}{3}c^2x^3+\frac{1}{2}a^2x^2\right)+6ac\left(\frac{2c^4x^{\frac{7}{2}}}{7}+\frac{4a^2c^2x^{\frac{5}{2}}}{5}+\frac{2a^4x^{\frac{3}{2}}}{3}\right)+\frac{(c^2x+a^2)^4}{4c^2}$
trager	$\frac{(c^6x^3+20a^2c^4x^2+c^6x^2+30a^4c^2x+20a^2c^4x+c^6x+4a^6+30a^4c^2+20a^2c^4+c^6)(x-1)}{4}+\frac{4ac\,x^{\frac{3}{2}}(3c^4x^2+14a^2c^2x+7a^4}{7}$
orering	$\frac{x(-13c^{10}x^5+66a^2c^8x^4-135a^4c^6x^3+140a^6c^4x^2-70a^8c^2x+28a^{10})(a^2+2ac\sqrt{x}+c^2x)^3}{28(-c^2x+a^2)^5}-\frac{x^2(-3c^{10}x^5+18a^2c^8x^4-}{}$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*c^6*x^4+12/7*a*c^5*x^(7/2)+5*a^2*c^4*x^3+8*a^3*c^3*x^(5/2)+15/2*a^4*c^
2*x^2+4*a^5*c*x^(3/2)+a^6*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(30) = 60$.

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int \left(a^2 + 2ac\sqrt{x} + c^2x\right)^3 \, dx = \frac{1}{4}\, c^6x^4 + 5\, a^2c^4x^3 + \frac{15}{2}\, a^4c^2x^2 + a^6x + \frac{4}{7}\left(3\, ac^5x^3 + 14\, a^3c^3x^2 + 7\, a^5cx\right)\sqrt{x}$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^3,x, algorithm="fricas")
```

output
```
1/4*c^6*x^4 + 5*a^2*c^4*x^3 + 15/2*a^4*c^2*x^2 + a^6*x + 4/7*(3*a*c^5*x^3
+ 14*a^3*c^3*x^2 + 7*a^5*c*x)*sqrt(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(32) = 64$.

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \left(a^2 + 2ac\sqrt{x} + c^2x\right)^3 \, dx = a^6x + 4a^5cx^{\frac{3}{2}} + \frac{15a^4c^2x^2}{2} + 8a^3c^3x^{\frac{5}{2}} + 5a^2c^4x^3 + \frac{12ac^5x^{\frac{7}{2}}}{7} + \frac{c^6x^4}{4}$$

input
```
integrate((a**2+2*a*c*x**(1/2)+c**2*x)**3,x)
```

output
```
a**6*x + 4*a**5*c*x**(3/2) + 15*a**4*c**2*x**2/2 + 8*a**3*c**3*x**(5/2) +
5*a**2*c**4*x**3 + 12*a*c**5*x**(7/2)/7 + c**6*x**4/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(30) = 60$.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.63

$$\int \left(a^2 + 2ac\sqrt{x} + c^2x\right)^3 \, dx = \frac{1}{4}\,c^6x^4 + \frac{12}{7}\,ac^5x^{\frac{7}{2}} + 4\,a^2c^4x^3 + \frac{16}{5}\,a^3c^3x^{\frac{5}{2}} + a^6x + \frac{1}{2}\left(3\,c^2x^2 + 8\,acx^{\frac{3}{2}}\right)a^4 + \frac{1}{5}\left(5\,c^4x^3 + 24\,ac^3x^{\frac{5}{2}} + 30\,a^2c^2x^2\right)a^2$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^3,x, algorithm="maxima")
```

output
```
1/4*c^6*x^4 + 12/7*a*c^5*x^(7/2) + 4*a^2*c^4*x^3 + 16/5*a^3*c^3*x^(5/2) +
a^6*x + 1/2*(3*c^2*x^2 + 8*a*c*x^(3/2))*a^4 + 1/5*(5*c^4*x^3 + 24*a*c^3*x^
(5/2) + 30*a^2*c^2*x^2)*a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \left(a^2 + 2ac\sqrt{x} + c^2x\right)^3 \, dx = \frac{1}{4}\,c^6x^4 + \frac{12}{7}\,ac^5x^{\frac{7}{2}} + 5\,a^2c^4x^3 + 8\,a^3c^3x^{\frac{5}{2}} + \frac{15}{2}\,a^4c^2x^2 + 4\,a^5cx^{\frac{3}{2}} + a^6x$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^3,x, algorithm="giac")
```

output
```
1/4*c^6*x^4 + 12/7*a*c^5*x^(7/2) + 5*a^2*c^4*x^3 + 8*a^3*c^3*x^(5/2) + 15/
2*a^4*c^2*x^2 + 4*a^5*c*x^(3/2) + a^6*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^3\,dx = a^6\,x+\frac{c^6\,x^4}{4}+4\,a^5\,c\,x^{3/2}+\frac{12\,a\,c^5\,x^{7/2}}{7}+\frac{15\,a^4\,c^2\,x^2}{2}+5\,a^2\,c^4\,x^3+8\,a^3\,c^3\,x^{5/2}$$

input
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^3,x)
```

output
```
a^6*x + (c^6*x^4)/4 + 4*a^5*c*x^(3/2) + (12*a*c^5*x^(7/2))/7 + (15*a^4*c^2
*x^2)/2 + 5*a^2*c^4*x^3 + 8*a^3*c^3*x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^3\,dx = \frac{x\left(112\sqrt{x}\,a^5c+224\sqrt{x}\,a^3c^3x+48\sqrt{x}\,a\,c^5x^2+28a^6+210a^4c^2x+140a^2c^4x^2+7c^6x^3\right)}{28}$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^3,x)
```

output
```
(x*(112*sqrt(x)*a**5*c + 224*sqrt(x)*a**3*c**3*x + 48*sqrt(x)*a*c**5*x**2
+ 28*a**6 + 210*a**4*c**2*x + 140*a**2*c**4*x**2 + 7*c**6*x**3))/28
```

3.37 $\int \left(a^2+2ac\sqrt{x}+c^2x\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^2\,dx=-\frac{2a\left(a+c\sqrt{x}\right)^5}{5c^2}+\frac{\left(a+c\sqrt{x}\right)^6}{3c^2}$$

output
```
-2/5*a*(a+c*x^(1/2))^5/c^2+1/3*(a+c*x^(1/2))^6/c^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^2\,dx=\frac{1}{15}\left(15a^4x+40a^3cx^{3/2}+45a^2c^2x^2+24ac^3x^{5/2}+5c^4x^3\right)$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^2,x]
```

output
```
(15*a^4*x + 40*a^3*c*x^(3/2) + 45*a^2*c^2*x^2 + 24*a*c^3*x^(5/2) + 5*c^4*x
^3)/15
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1379, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^2 dx$$

$$\Big\downarrow \text{1379}$$

$$\frac{\int \left(\sqrt{x}c^2+ac\right)^4 dx}{c^4}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int c^4\left(a+c\sqrt{x}\right)^4\sqrt{x}d\sqrt{x}}{c^4}$$

$$\Big\downarrow \text{27}$$

$$2\int \left(a+c\sqrt{x}\right)^4\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{\left(a+c\sqrt{x}\right)^5}{c}-\frac{a\left(a+c\sqrt{x}\right)^4}{c}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\left(a+c\sqrt{x}\right)^6}{6c^2}-\frac{a\left(a+c\sqrt{x}\right)^5}{5c^2}\right)$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^2,x]
```

output
```
2*(-1/5*(a*(a + c*Sqrt[x])^5)/c^2 + (a + c*Sqrt[x])^6/(6*c^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{c^4x^3}{3}+\frac{8a\,c^3x^{\frac{5}{2}}}{5}+3a^2c^2x^2+\frac{8a^3c\,x^{\frac{3}{2}}}{3}+a^4x$
default	$2a^2c^2x^2+4ac\left(\frac{2c^2x^{\frac{5}{2}}}{5}+\frac{2a^2x^{\frac{3}{2}}}{3}\right)+\frac{\left(c^2x+a^2\right)^3}{3c^2}$
trager	$\frac{\left(c^4x^2+9a^2c^2x+c^4x+3a^4+9a^2c^2+c^4\right)(x-1)}{3}+\frac{8ac\,x^{\frac{3}{2}}\left(3c^2x+5a^2\right)}{15}$
orering	$-\frac{\left(9c^8x^4-28a^2c^6x^3+30a^4c^4x^2+5a^8\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^2}{15c^2\left(-c^2x+a^2\right)^3}+\frac{4\left(c^8x^4-4a^2c^6x^3+6a^4c^4x^2+5a^8\right)x\left(a^2+2ac\sqrt{x}+c^2x\right)\left(\right.}{15c^2\left(-c^2x+a^2\right)^3}$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*c^4*x^3+8/5*a*c^3*x^(5/2)+3*a^2*c^2*x^2+8/3*a^3*c*x^(3/2)+a^4*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^2\,dx=\frac{1}{3}\,c^4x^3+3\,a^2c^2x^2+a^4x+\frac{8}{15}\,\left(3\,ac^3x^2+5\,a^3cx\right)\sqrt{x}$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^2,x, algorithm="fricas")
```

output
```
1/3*c^4*x^3 + 3*a^2*c^2*x^2 + a^4*x + 8/15*(3*a*c^3*x^2 + 5*a^3*c*x)*sqrt(
x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^2\,dx=a^4x+\frac{8a^3cx^{\frac{3}{2}}}{3}+3a^2c^2x^2+\frac{8ac^3x^{\frac{5}{2}}}{5}+\frac{c^4x^3}{3}$$

input
```
integrate((a**2+2*a*c*x**(1/2)+c**2*x)**2,x)
```

output
```
a**4*x + 8*a**3*c*x**(3/2)/3 + 3*a**2*c**2*x**2 + 8*a*c**3*x**(5/2)/5 + c*
*4*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^2 \, dx=\frac{1}{3}\,c^4x^3+\frac{8}{5}\,ac^3x^{\frac{5}{2}}+2\,a^2c^2x^2+a^4x+\frac{1}{3}\left(3\,c^2x^2+8\,acx^{\frac{3}{2}}\right)a^2$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^2,x, algorithm="maxima")
```

output
```
1/3*c^4*x^3 + 8/5*a*c^3*x^(5/2) + 2*a^2*c^2*x^2 + a^4*x + 1/3*(3*c^2*x^2 +
 8*a*c*x^(3/2))*a^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^2 \, dx=\frac{1}{3}\,c^4x^3+\frac{8}{5}\,ac^3x^{\frac{5}{2}}+3\,a^2c^2x^2+\frac{8}{3}\,a^3cx^{\frac{3}{2}}+a^4x$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^2,x, algorithm="giac")
```

output
```
1/3*c^4*x^3 + 8/5*a*c^3*x^(5/2) + 3*a^2*c^2*x^2 + 8/3*a^3*c*x^(3/2) + a^4*
x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^2 \, dx=a^4\,x+\frac{c^4\,x^3}{3}+\frac{8\,a^3\,c\,x^{3/2}}{3}+\frac{8\,a\,c^3\,x^{5/2}}{5}+3\,a^2\,c^2\,x^2$$

input
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^2,x)
```

output
```
a^4*x + (c^4*x^3)/3 + (8*a^3*c*x^(3/2))/3 + (8*a*c^3*x^(5/2))/5 + 3*a^2*c^
2*x^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^2\,dx=\frac{x\left(40\sqrt{x}\,a^3c+24\sqrt{x}\,a\,c^3x+15a^4+45a^2c^2x+5c^4x^2\right)}{15}$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^2,x)
```

output
```
(x*(40*sqrt(x)*a**3*c + 24*sqrt(x)*a*c**3*x + 15*a**4 + 45*a**2*c**2*x + 5
*c**4*x**2))/15
```

3.38 $\int \left(a^2+2ac\sqrt{x}+c^2x\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)\,dx=a^2x+\frac{4}{3}acx^{3/2}+\frac{c^2x^2}{2}$$

output
```
a^2*x+4/3*a*c*x^(3/2)+1/2*c^2*x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)\,dx=\frac{1}{6}x\left(6a^2+8ac\sqrt{x}+3c^2x\right)$$

input
```
Integrate[a^2 + 2*a*c*Sqrt[x] + c^2*x,x]
```

output
```
(x*(6*a^2 + 8*a*c*Sqrt[x] + 3*c^2*x))/6
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2 + 2ac\sqrt{x} + c^2x\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2x + \frac{4}{3}acx^{3/2} + \frac{c^2x^2}{2}$$

input
```
Int[a^2 + 2*a*c*Sqrt[x] + c^2*x,x]
```

output
```
a^2*x + (4*a*c*x^(3/2))/3 + (c^2*x^2)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$x\,a^{2}+\frac{4 a c\,x^{\frac{3}{2}}}{3}+\frac{c^{2} x^{2}}{2}$	22
default	$x\,a^{2}+\frac{4 a c\,x^{\frac{3}{2}}}{3}+\frac{c^{2} x^{2}}{2}$	22
risch	$x\,a^{2}+\frac{4 a c\,x^{\frac{3}{2}}}{3}+\frac{c^{2} x^{2}}{2}$	22
parts	$x\,a^{2}+\frac{4 a c\,x^{\frac{3}{2}}}{3}+\frac{c^{2} x^{2}}{2}$	22
trager	$\frac{\left(x -1\right) \left(c^{2} x +2 a^{2}+c^{2}\right)}{2}+\frac{4 a c\,x^{\frac{3}{2}}}{3}$	28
orering	$\frac{x \left(-5 c^{2} x +6 a^{2}\right) \left(a^{2}+2 a c \sqrt{x}+c^{2} x \right)}{-6 c^{2} x +6 a^{2}}-\frac{x^{2} \left(-c^{2} x +2 a^{2}\right) \left(\frac{a c}{\sqrt{x}}+c^{2}\right)}{3 \left(-c^{2} x +a^{2}\right)}$	84

input
```
int(a^2+2*a*c*x^(1/2)+c^2*x,x,method=_RETURNVERBOSE)
```

output
```
x*a^2+4/3*a*c*x^(3/2)+1/2*c^2*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)\,dx=\frac{1}{2}\,c^{2}x^{2}+\frac{4}{3}\,acx^{\frac{3}{2}}+a^{2}x$$

input
```
integrate(a^2+2*a*c*x^(1/2)+c^2*x,x, algorithm="fricas")
```

output
```
1/2*c^2*x^2 + 4/3*a*c*x^(3/2) + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \left(a^2 + 2ac\sqrt{x} + c^2x\right) \, dx = a^2x + \frac{4acx^{\frac{3}{2}}}{3} + \frac{c^2x^2}{2}$$

input
```
integrate(a**2+2*a*c*x**(1/2)+c**2*x,x)
```

output
```
a**2*x + 4*a*c*x**(3/2)/3 + c**2*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \left(a^2 + 2ac\sqrt{x} + c^2x\right) \, dx = \frac{1}{2}\,c^2x^2 + \frac{4}{3}\,acx^{\frac{3}{2}} + a^2x$$

input
```
integrate(a^2+2*a*c*x^(1/2)+c^2*x,x, algorithm="maxima")
```

output
```
1/2*c^2*x^2 + 4/3*a*c*x^(3/2) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \left(a^2 + 2ac\sqrt{x} + c^2x\right) \, dx = \frac{1}{2}\,c^2x^2 + \frac{4}{3}\,acx^{\frac{3}{2}} + a^2x$$

input
```
integrate(a^2+2*a*c*x^(1/2)+c^2*x,x, algorithm="giac")
```

output
```
1/2*c^2*x^2 + 4/3*a*c*x^(3/2) + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)\,dx=a^2\,x+\frac{c^2\,x^2}{2}+\frac{4\,a\,c\,x^{3/2}}{3}$$

input
```
int(c^2*x + a^2 + 2*a*c*x^(1/2),x)
```

output
```
a^2*x + (c^2*x^2)/2 + (4*a*c*x^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)\,dx=\frac{x\left(8\sqrt{x}\,ac+6a^2+3c^2x\right)}{6}$$

input
```
int(a^2+2*a*c*x^(1/2)+c^2*x,x)
```

output
```
(x*(8*sqrt(x)*a*c + 6*a**2 + 3*c**2*x))/6
```

3.39 $\int \frac{1}{a^2+2ac\sqrt{x}+c^2x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 33

$$\int \frac{1}{a^2+2ac\sqrt{x}+c^2x}\,dx=\frac{2a}{c^2\left(a+c\sqrt{x}\right)}+\frac{2\log\left(a+c\sqrt{x}\right)}{c^2}$$

output
```
2*a/c^2/(a+c*x^(1/2))+2*ln(a+c*x^(1/2))/c^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{a^2+2ac\sqrt{x}+c^2x}\,dx=\frac{2\left(\frac{a}{a+c\sqrt{x}}+\log\left(a+c\sqrt{x}\right)\right)}{c^2}$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(-1),x]
```

output
```
(2*(a/(a + c*Sqrt[x]) + Log[a + c*Sqrt[x]]))/c^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1379, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+2ac\sqrt{x}+c^2x}dx$$

$$\Big\downarrow \text{ 1379}$$

$$c^2\int \frac{1}{\left(\sqrt{x}c^2+ac\right)^2}dx$$

$$\Big\downarrow \text{ 774}$$

$$2c^2\int \frac{\sqrt{x}}{c^2\left(a+c\sqrt{x}\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{\sqrt{x}}{\left(a+c\sqrt{x}\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{ 49}$$

$$2\int \left(\frac{1}{c\left(a+c\sqrt{x}\right)}-\frac{a}{c\left(a+c\sqrt{x}\right)^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{a}{c^2\left(a+c\sqrt{x}\right)}+\frac{\log\left(a+c\sqrt{x}\right)}{c^2}\right)$$

input

```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(-1),x]
```

output

```
2*(a/(c^2*(a + c*Sqrt[x])) + Log[a + c*Sqrt[x]]/c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{2a}{c^2(a+c\sqrt{x})}+\frac{2\ln(a+c\sqrt{x})}{c^2}$	30
default	$-\frac{2a^2}{c^2(c^2x-a^2)}+\frac{\ln(c^2x-a^2)}{c^2}+\frac{a}{c^2(c\sqrt{x}-a)}-\frac{\ln(c\sqrt{x}-a)}{c^2}+\frac{a}{c^2(a+c\sqrt{x})}+\frac{\ln(a+c\sqrt{x})}{c^2}$	96

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x),x,method=_RETURNVERBOSE)
```

output
```
2*a/c^2/(a+c*x^(1/2))+2*ln(a+c*x^(1/2))/c^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{1}{a^2 + 2ac\sqrt{x} + c^2x}\,dx = \frac{2\left(ac\sqrt{x} - a^2 + (c^2x - a^2)\log\left(c\sqrt{x} + a\right)\right)}{c^4x - a^2c^2}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x),x, algorithm="fricas")
```

output
```
2*(a*c*sqrt(x) - a^2 + (c^2*x - a^2)*log(c*sqrt(x) + a))/(c^4*x - a^2*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(29) = 58.

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.42

$$\int \frac{1}{a^2 + 2ac\sqrt{x} + c^2x}\,dx = \begin{cases} \frac{2a\log\left(\frac{a}{c}+\sqrt{x}\right)}{ac^2+c^3\sqrt{x}} + \frac{2a}{ac^2+c^3\sqrt{x}} + \frac{2c\sqrt{x}\log\left(\frac{a}{c}+\sqrt{x}\right)}{ac^2+c^3\sqrt{x}} & \text{for } c \neq 0 \\ \frac{x}{a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a**2+2*a*c*x**(1/2)+c**2*x),x)
```

output
```
Piecewise((2*a*log(a/c + sqrt(x))/(a*c**2 + c**3*sqrt(x)) + 2*a/(a*c**2 +
c**3*sqrt(x)) + 2*c*sqrt(x)*log(a/c + sqrt(x))/(a*c**2 + c**3*sqrt(x)), Ne
(c, 0)), (x/a**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{a^2 + 2ac\sqrt{x} + c^2x}\,dx = \frac{2\,a}{c^3\sqrt{x} + ac^2} + \frac{2\,\log\left(c\sqrt{x} + a\right)}{c^2}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x),x, algorithm="maxima")
```

output
```
2*a/(c^3*sqrt(x) + a*c^2) + 2*log(c*sqrt(x) + a)/c^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1}{a^2 + 2ac\sqrt{x} + c^2x}\,dx = \frac{2\log\left(\left|c\sqrt{x} + a\right|\right)}{c^2} + \frac{2a}{(c\sqrt{x} + a)c^2}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x),x, algorithm="giac")
```

output
```
2*log(abs(c*sqrt(x) + a))/c^2 + 2*a/((c*sqrt(x) + a)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{a^2 + 2ac\sqrt{x} + c^2x}\,dx = \frac{2\ln\left(a + c\sqrt{x}\right)}{c^2} + \frac{2a}{c^2\left(a + c\sqrt{x}\right)}$$

input
```
int(1/(c^2*x + a^2 + 2*a*c*x^(1/2)),x)
```

output
```
(2*log(a + c*x^(1/2)))/c^2 + (2*a)/(c^2*(a + c*x^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{1}{a^2 + 2ac\sqrt{x} + c^2x}\,dx = \frac{2\sqrt{x}\log\left(\sqrt{x}\,c + a\right)c - 2\sqrt{x}\,c + 2\log\left(\sqrt{x}\,c + a\right)a}{c^2\left(\sqrt{x}\,c + a\right)}$$

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x),x)
```

output
```
(2*(sqrt(x)*log(sqrt(x)*c + a)*c - sqrt(x)*c + log(sqrt(x)*c + a)*a))/(c**
2*(sqrt(x)*c + a))
```

3.40 $\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^2}\,dx=\frac{2a}{3c^2\left(a+c\sqrt{x}\right)^3}-\frac{1}{c^2\left(a+c\sqrt{x}\right)^2}$$

output
```
2/3*a/c^2/(a+c*x^(1/2))^3-1/c^2/(a+c*x^(1/2))^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^2}\,dx=\frac{-a-3c\sqrt{x}}{3c^2\left(a+c\sqrt{x}\right)^3}$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(-2),x]
```

output
```
(-a - 3*c*Sqrt[x])/(3*c^2*(a + c*Sqrt[x])^3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1379, 774, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^2}dx$$

$$\Big\downarrow \text{1379}$$

$$c^4\int \frac{1}{\left(\sqrt{x}c^2+ac\right)^4}dx$$

$$\Big\downarrow \text{774}$$

$$2c^4\int \frac{\sqrt{x}}{c^4\left(a+c\sqrt{x}\right)^4}d\sqrt{x}$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\sqrt{x}}{\left(a+c\sqrt{x}\right)^4}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{1}{c\left(a+c\sqrt{x}\right)^3}-\frac{a}{c\left(a+c\sqrt{x}\right)^4}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a}{3c^2\left(a+c\sqrt{x}\right)^3}-\frac{1}{2c^2\left(a+c\sqrt{x}\right)^2}\right)$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(-2),x]
```

output
```
2*(a/(3*c^2*(a + c*Sqrt[x])^3) - 1/(2*c^2*(a + c*Sqrt[x])^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{2a}{3c^2\left(a+c\sqrt{x}\right)^3}-\frac{1}{c^2\left(a+c\sqrt{x}\right)^2}$
trager	$\frac{(x-1)\left(-a^4c^4x^2+6a^2c^6x^2+3c^8x^2+6a^6c^2x-28a^4c^4x+6a^2c^6x+3a^8+6a^6c^2-a^4c^4\right)}{3(-c^2x+a^2)^3\left(a^6-3a^4c^2+3a^2c^4-c^6\right)}-\frac{8acx^{\frac{3}{2}}}{3(-c^2x+a^2)^3}$
default	$-\frac{a^4}{3c^2(c^2x-a^2)^3}+c^4\left(-\frac{1}{c^6(c^2x-a^2)}-\frac{a^2}{c^6(c^2x-a^2)^2}-\frac{a^4}{3c^6(c^2x-a^2)^3}\right)+6a^2c^2\left(-\frac{1}{2(c^2x-a^2)^2c^4}-\right.$

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*a/c^2/(a+c*x^(1/2))^3-1/c^2/(a+c*x^(1/2))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^2}\,dx = -\frac{3\,c^4x^2 - 8\,ac^3x^{\frac{3}{2}} + 6\,a^2c^2x - a^4}{3\,(c^8x^3 - 3\,a^2c^6x^2 + 3\,a^4c^4x - a^6c^2)}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*c^4*x^2 - 8*a*c^3*x^(3/2) + 6*a^2*c^2*x - a^4)/(c^8*x^3 - 3*a^2*c^
6*x^2 + 3*a^4*c^4*x - a^6*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(32) = 64$.

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.69

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^2}\,dx = \begin{cases} -\frac{a}{3a^3c^2+9a^2c^3\sqrt{x}+9ac^4x+3c^5x^{\frac{3}{2}}} - \frac{3c\sqrt{x}}{3a^3c^2+9a^2c^3\sqrt{x}+9ac^4x+3c^5x^{\frac{3}{2}}} & \text{for } c \neq 0 \\ \frac{x}{a^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a**2+2*a*c*x**(1/2)+c**2*x)**2,x)
```

output
```
Piecewise((-a/(3*a**3*c**2 + 9*a**2*c**3*sqrt(x) + 9*a*c**4*x + 3*c**5*x**
(3/2)) - 3*c*sqrt(x)/(3*a**3*c**2 + 9*a**2*c**3*sqrt(x) + 9*a*c**4*x + 3*c
**5*x**(3/2)), Ne(c, 0)), (x/a**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^2}\,dx = -\frac{3\,c\sqrt{x}+a}{3\,\left(c^5x^{\frac{3}{2}}+3\,ac^4x+3\,a^2c^3\sqrt{x}+a^3c^2\right)}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^2,x, algorithm="maxima")
```

output
```
-1/3*(3*c*sqrt(x) + a)/(c^5*x^(3/2) + 3*a*c^4*x + 3*a^2*c^3*sqrt(x) + a^3*
c^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^2}\,dx = -\frac{3\,c\sqrt{x}+a}{3\,\left(c\sqrt{x}+a\right)^3c^2}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^2,x, algorithm="giac")
```

output
```
-1/3*(3*c*sqrt(x) + a)/((c*sqrt(x) + a)^3*c^2)
```

Mupad [B] (verification not implemented)

Time = 20.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^2}\,dx = -\frac{\frac{a}{3\,c^2}+\frac{\sqrt{x}}{c}}{a^3+c^3\,x^{3/2}+3\,a^2\,c\,\sqrt{x}+3\,a\,c^2\,x}$$

input
```
int(1/(c^2*x + a^2 + 2*a*c*x^(1/2))^2,x)
```

output
```
-(a/(3*c^2) + x^(1/2)/c)/(a^3 + c^3*x^(3/2) + 3*a^2*c*x^(1/2) + 3*a*c^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^2}\,dx = \frac{-3\sqrt{x}\,c-a}{3c^2\left(3\sqrt{x}\,a^2c+\sqrt{x}\,c^3x+a^3+3a\,c^2x\right)}$$

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x)^2,x)
```

output
```
( - 3*sqrt(x)*c - a)/(3*c**2*(3*sqrt(x)*a**2*c + sqrt(x)*c**3*x + a**3 + 3
*a*c**2*x))
```

3.41 $\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^3}\,dx=\frac{2a}{5c^2\left(a+c\sqrt{x}\right)^5}-\frac{1}{2c^2\left(a+c\sqrt{x}\right)^4}$$

output
```
2/5*a/c^2/(a+c*x^(1/2))^5-1/2/c^2/(a+c*x^(1/2))^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^3}\,dx=\frac{-a-5c\sqrt{x}}{10c^2\left(a+c\sqrt{x}\right)^5}$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(-3),x]
```

output
```
(-a - 5*c*Sqrt[x])/(10*c^2*(a + c*Sqrt[x])^5)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1379, 774, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^3}dx$$

$$\Big\downarrow \; 1379$$

$$c^6\int \frac{1}{\left(\sqrt{x}c^2+ac\right)^6}dx$$

$$\Big\downarrow \; 774$$

$$2c^6\int \frac{\sqrt{x}}{c^6\left(a+c\sqrt{x}\right)^6}d\sqrt{x}$$

$$\Big\downarrow \; 27$$

$$2\int \frac{\sqrt{x}}{\left(a+c\sqrt{x}\right)^6}d\sqrt{x}$$

$$\Big\downarrow \; 53$$

$$2\int \left(\frac{1}{c\left(a+c\sqrt{x}\right)^5}-\frac{a}{c\left(a+c\sqrt{x}\right)^6}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{a}{5c^2\left(a+c\sqrt{x}\right)^5}-\frac{1}{4c^2\left(a+c\sqrt{x}\right)^4}\right)$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(-3),x]
```

output
```
2*(a/(5*c^2*(a + c*Sqrt[x])^5) - 1/(4*c^2*(a + c*Sqrt[x])^4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{2a}{5c^2\left(a+c\sqrt{x}\right)^5}-\frac{1}{2c^2\left(a+c\sqrt{x}\right)^4}$
default	$c^6\left(-\frac{1}{2c^8(c^2x-a^2)^2}-\frac{a^6}{5c^8(c^2x-a^2)^5}-\frac{3a^4}{4c^8(c^2x-a^2)^4}-\frac{a^2}{c^8(c^2x-a^2)^3}\right)-\frac{a^6}{5c^2(c^2x-a^2)^5}+\frac{a}{5c^2\left(c\sqrt{x}-a\right)^5}$
trager	$\frac{(x-1)\left(-a^6c^8x^4+15a^4c^{10}x^4+45a^2c^{12}x^4+5c^{14}x^4+5a^8c^6x^3-76a^6c^8x^3-210a^4c^{10}x^3+20a^2c^{12}x^3+5c^{14}x^3-5a^{10}c^4x^2-\right.}{10(-c^2x+a^2)}$

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*a/c^2/(a+c*x^(1/2))^5-1/2/c^2/(a+c*x^(1/2))^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.16

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^3}\,dx = -\frac{5\,c^6x^3+45\,a^2c^4x^2+15\,a^4c^2x-a^6-8\left(3\,ac^5x^2+5\,a^3c^3x\right)\sqrt{x}}{10\left(c^{12}x^5-5\,a^2c^{10}x^4+10\,a^4c^8x^3-10\,a^6c^6x^2+5\,a^8c^4x-a^{10}c^2\right)}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^3,x, algorithm="fricas")
```

output
```
-1/10*(5*c^6*x^3 + 45*a^2*c^4*x^2 + 15*a^4*c^2*x - a^6 - 8*(3*a*c^5*x^2 +
5*a^3*c^3*x)*sqrt(x))/(c^12*x^5 - 5*a^2*c^10*x^4 + 10*a^4*c^8*x^3 - 10*a^6
*c^6*x^2 + 5*a^8*c^4*x - a^10*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(34) = 68$.

Time = 0.81 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.89

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^3}\,dx = \begin{cases} -\frac{a}{10a^5c^2+50a^4c^3\sqrt{x}+100a^3c^4x+100a^2c^5x^{\frac{3}{2}}+50ac^6x^2+10c^7x^{\frac{5}{2}}} - \frac{5c\sqrt{x}}{10a^5c^2+50a^4c^3\sqrt{x}+100a^3c^4x+100a^2c^5x^{\frac{3}{2}}+50ac^6x^2+10c^7x^{\frac{5}{2}}} & \text{for} \\ \frac{x}{a^6} & \text{ot} \end{cases}$$

input
```
integrate(1/(a**2+2*a*c*x**(1/2)+c**2*x)**3,x)
```

output
```
Piecewise((-a/(10*a**5*c**2 + 50*a**4*c**3*sqrt(x) + 100*a**3*c**4*x + 100
*a**2*c**5*x**(3/2) + 50*a*c**6*x**2 + 10*c**7*x**(5/2)) - 5*c*sqrt(x)/(10
*a**5*c**2 + 50*a**4*c**3*sqrt(x) + 100*a**3*c**4*x + 100*a**2*c**5*x**(3/
2) + 50*a*c**6*x**2 + 10*c**7*x**(5/2)), Ne(c, 0)), (x/a**6, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^3}\,dx = -\frac{5\,c\sqrt{x}+a}{10\,\left(c^7x^{\frac{5}{2}}+5\,ac^6x^2+10\,a^2c^5x^{\frac{3}{2}}+10\,a^3c^4x+5\,a^4c^3\sqrt{x}+a^5c^2\right)}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^3,x, algorithm="maxima")
```

output
```
-1/10*(5*c*sqrt(x) + a)/(c^7*x^(5/2) + 5*a*c^6*x^2 + 10*a^2*c^5*x^(3/2) +
10*a^3*c^4*x + 5*a^4*c^3*sqrt(x) + a^5*c^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^3}\,dx = -\frac{5\,c\sqrt{x}+a}{10\,\left(c\sqrt{x}+a\right)^5c^2}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^3,x, algorithm="giac")
```

output
```
-1/10*(5*c*sqrt(x) + a)/((c*sqrt(x) + a)^5*c^2)
```

Mupad [B] (verification not implemented)

Time = 19.52 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^3}\,dx$$
$$= -\frac{\frac{a}{10\,c^2}+\frac{\sqrt{x}}{2\,c}}{a^5+c^5\,x^{5/2}+10\,a^3\,c^2\,x+5\,a\,c^4\,x^2+5\,a^4\,c\,\sqrt{x}+10\,a^2\,c^3\,x^{3/2}}$$

input
```
int(1/(c^2*x + a^2 + 2*a*c*x^(1/2))^3,x)
```

output
```
-(a/(10*c^2) + x^(1/2)/(2*c))/(a^5 + c^5*x^(5/2) + 10*a^3*c^2*x + 5*a*c^4*
x^2 + 5*a^4*c*x^(1/2) + 10*a^2*c^3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^3}\,dx$$
$$= \frac{-5\sqrt{x}\,c-a}{10c^2\left(5\sqrt{x}\,a^4c+10\sqrt{x}\,a^2c^3x+\sqrt{x}\,c^5x^2+a^5+10a^3c^2x+5a\,c^4x^2\right)}$$

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x)^3,x)
```

output
```
( - 5*sqrt(x)*c - a)/(10*c**2*(5*sqrt(x)*a**4*c + 10*sqrt(x)*a**2*c**3*x +
 sqrt(x)*c**5*x**2 + a**5 + 10*a**3*c**2*x + 5*a*c**4*x**2))
```

3.42 $\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx=-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}+\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}$$

output
```
-1/160*b*(b+2*c*x^(1/2))^5/c^4+1/192*(b+2*c*x^(1/2))^6/c^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx=\frac{15b^4x+80b^3cx^{3/2}+180b^2c^2x^2+192bc^3x^{5/2}+80c^4x^3}{240c^2}$$

input
```
Integrate[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

output
```
(15*b^4*x + 80*b^3*c*x^(3/2) + 180*b^2*c^2*x^2 + 192*b*c^3*x^(5/2) + 80*c^
4*x^3)/(240*c^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1379, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx$$

$$\Big\downarrow \text{1379}$$

$$\frac{\int \left(\frac{b}{2}+c\sqrt{x}\right)^4 dx}{c^2}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int \frac{1}{16}\left(b+2c\sqrt{x}\right)^4 \sqrt{x}d\sqrt{x}}{c^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(b+2c\sqrt{x}\right)^4 \sqrt{x}d\sqrt{x}}{8c^2}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{\left(b+2c\sqrt{x}\right)^5}{2c}-\frac{b\left(b+2c\sqrt{x}\right)^4}{2c}\right)d\sqrt{x}}{8c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\left(b+2c\sqrt{x}\right)^6}{24c^2}-\frac{b\left(b+2c\sqrt{x}\right)^5}{20c^2}}{8c^2}$$

input
```
Int[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

output
```
(-1/20*(b*(b + 2*c*Sqrt[x])^5)/c^2 + (b + 2*c*Sqrt[x])^6/(24*c^2))/(8*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{8c^4x^3}{3}+\frac{32b\,c^3x^{\frac{5}{2}}}{5}+6b^2c^2x^2+\frac{8b^3c\,x^{\frac{3}{2}}}{3}+\frac{b^4x}{2}}{8c^2}$
default	$\frac{b^2x^2}{2}+\frac{b\left(\frac{8c^2x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{3}{2}}}{3}\right)}{2c}+\frac{\left(\frac{b^2}{c}+4cx\right)^3}{192c}$
trager	$\frac{\frac{\left(16c^4x^2+36b^2x\,c^2+16c^4x+3b^4+36b^2c^2+16c^4\right)(x-1)}{3}+\frac{16bc\,x^{\frac{3}{2}}\left(12c^2x+5b^2\right)}{15}}{16c^2}$
orering	$\frac{x\left(-576c^6x^3+448x^2c^4b^2-120x\,c^2b^4+15b^6\right)\left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2}{15\left(-4c^2x+b^2\right)^3}-\frac{4x^2\left(-64c^6x^3+64x^2c^4b^2-24x\,c^2b^4+5b^6\right)\left(\frac{b^2}{4c}+b\sqrt{x}\right.}{15\left(-4c^2x+b^2\right)^3}$

input
```
int((1/4*b^2/c+b*x^(1/2)+c*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/c^2*(8/3*c^4*x^3+32/5*b*c^3*x^(5/2)+6*b^2*c^2*x^2+8/3*b^3*c*x^(3/2)+1/
2*b^4*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2\,dx=\frac{80\,c^4x^3+180\,b^2c^2x^2+15\,b^4x+16\,(12\,bc^3x^2+5\,b^3cx)\sqrt{x}}{240\,c^2}$$

input
```
integrate((1/4*b^2/c+b*x^(1/2)+c*x)^2,x, algorithm="fricas")
```

output
```
1/240*(80*c^4*x^3 + 180*b^2*c^2*x^2 + 15*b^4*x + 16*(12*b*c^3*x^2 + 5*b^3*
c*x)*sqrt(x))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2\,dx=\frac{b^4x}{16c^2}+\frac{b^3x^{\frac{3}{2}}}{3c}+\frac{3b^2x^2}{4}+\frac{4bcx^{\frac{5}{2}}}{5}+\frac{c^2x^3}{3}$$

input
```
integrate((1/4*b**2/c+b*x**(1/2)+c*x)**2,x)
```

output
```
b**4*x/(16*c**2) + b**3*x**(3/2)/(3*c) + 3*b**2*x**2/4 + 4*b*c*x**(5/2)/5
+ c**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{1}{3}c^2x^3+\frac{4}{5}bcx^{\frac{5}{2}}+\frac{1}{2}b^2x^2+\frac{b^4x}{16c^2}+\frac{\left(3cx^2+4bx^{\frac{3}{2}}\right)b^2}{12c}$$

input
```
integrate((1/4*b^2/c+b*x^(1/2)+c*x)^2,x, algorithm="maxima")
```

output
```
1/3*c^2*x^3 + 4/5*b*c*x^(5/2) + 1/2*b^2*x^2 + 1/16*b^4*x/c^2 + 1/12*(3*c*x
^2 + 4*b*x^(3/2))*b^2/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{80c^4x^3+192bc^3x^{\frac{5}{2}}+180b^2c^2x^2+80b^3cx^{\frac{3}{2}}+15b^4x}{240c^2}$$

input
```
integrate((1/4*b^2/c+b*x^(1/2)+c*x)^2,x, algorithm="giac")
```

output
```
1/240*(80*c^4*x^3 + 192*b*c^3*x^(5/2) + 180*b^2*c^2*x^2 + 80*b^3*c*x^(3/2)
 + 15*b^4*x)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{3b^2x^2}{4}+\frac{c^2x^3}{3}+\frac{b^4x}{16c^2}+\frac{b^3x^{3/2}}{3c}+\frac{4bcx^{5/2}}{5}$$

input
```
int((c*x + b*x^(1/2) + b^2/(4*c))^2,x)
```

output
```
(3*b^2*x^2)/4 + (c^2*x^3)/3 + (b^4*x)/(16*c^2) + (b^3*x^(3/2))/(3*c) + (4*
b*c*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{x\left(80\sqrt{x}\,b^3c+192\sqrt{x}\,b\,c^3x+15b^4+180b^2c^2x+80c^4x^2\right)}{240c^2}$$

input
```
int((1/4*b^2/c+b*x^(1/2)+c*x)^2,x)
```

output
```
(x*(80*sqrt(x)*b**3*c + 192*sqrt(x)*b*c**3*x + 15*b**4 + 180*b**2*c**2*x +
 80*c**4*x**2))/(240*c**2)
```

3.43 $\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}\,dx = -\frac{a\left(a+c\sqrt{x}\right)^5\sqrt{a^2+2ac\sqrt{x}+c^2x}}{3c^2}+\frac{2\left(a^2+2ac\sqrt{x}+c^2x\right)^{7/2}}{7c^2}$$

output
```
-1/3*a*(a+c*x^(1/2))^5*(a^2+2*a*c*x^(1/2)+c^2*x)^(1/2)/c^2+2/7*(a^2+2*a*c*
x^(1/2)+c^2*x)^(7/2)/c^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.28

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}\,dx = \frac{\left(\left(a+c\sqrt{x}\right)^2\right)^{5/2}\left(21a^5x+70a^4cx^{3/2}+105a^3c^2x^2+84a^2c^3x^{5/2}+35ac^4x^3+6c^5x^{7/2}\right)}{21\left(a+c\sqrt{x}\right)^5}$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(5/2),x]
```

output
```
(((a + c*Sqrt[x])^2)^(5/2)*(21*a^5*x + 70*a^4*c*x^(3/2) + 105*a^3*c^2*x^2
+ 84*a^2*c^3*x^(5/2) + 35*a*c^4*x^3 + 6*c^5*x^(7/2)))/(21*(a + c*Sqrt[x])^
5)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1680, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \sqrt{x}\left(a^2+2c\sqrt{x}a+c^2x\right)^{5/2}d\sqrt{x}$$

$$\Big\downarrow \text{1100}$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{7/2}}{7c^2}-\frac{a\int \left(a^2+2c\sqrt{x}a+c^2x\right)^{5/2}d\sqrt{x}}{c}\right)$$

$$\Big\downarrow \text{1079}$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{7/2}}{7c^2}-\frac{a\sqrt{a^2+2ac\sqrt{x}+c^2x}\int \left(\sqrt{x}c^2+ac\right)^5d\sqrt{x}}{c^6\left(a+c\sqrt{x}\right)}\right)$$

$$\Big\downarrow \text{17}$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{7/2}}{7c^2}-\frac{a\left(a+c\sqrt{x}\right)^5\sqrt{a^2+2ac\sqrt{x}+c^2x}}{6c^2}\right)$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(5/2),x]
```

output
```
2*(-1/6*(a*(a + c*Sqrt[x])^5*Sqrt[a^2 + 2*a*c*Sqrt[x] + c^2*x])/c^2 + (a^2
 + 2*a*c*Sqrt[x] + c^2*x)^(7/2)/(7*c^2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\left(\left(a+c\sqrt{x}\right)^{2}\right)^{\frac{5}{2}} x\left(6c^{5}x^{\frac{5}{2}}+35c^{4}x^{2}a+84a^{2}c^{3}x^{\frac{3}{2}}+105c^{2}a^{3}x+70a^{4}c\sqrt{x}+21a^{5}\right)}{21\left(a+c\sqrt{x}\right)^{5}}$	76
default	$\frac{\left(a^{2}+2ac\sqrt{x}+c^{2}x\right)^{\frac{5}{2}}\left(6c^{5}x^{\frac{7}{2}}+84a^{2}c^{3}x^{\frac{5}{2}}+70a^{4}c\,x^{\frac{3}{2}}+35c^{4}x^{3}a+105c^{2}a^{3}x^{2}+21x\,a^{5}\right)}{21\left(a+c\sqrt{x}\right)^{5}}$	85

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*((a+c*x^(1/2))^2)^(5/2)*x*(6*c^5*x^(5/2)+35*c^4*x^2*a+84*a^2*c^3*x^(3
/2)+105*c^2*a^3*x+70*a^4*c*x^(1/2)+21*a^5)/(a+c*x^(1/2))^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(57) = 114$.

Time = 0.17 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.46

$$\int \left(a^2 + 2ac\sqrt{x} + c^2x\right)^{5/2} dx = \frac{\left(7\,a^8 - 42\,a^6c^2 + 48\,a^4c^4 - 17\,a^2c^6 - 6\,(3\,a^2c^6 + c^8)x^4 - 49\,(3\,a^4c^4 + a^2c^6)x^3 + 35\,(3\,a^6c^2 + \right.}{}$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^(5/2),x, algorithm="fricas")
```

output
```
1/21*(7*a^8 - 42*a^6*c^2 + 48*a^4*c^4 - 17*a^2*c^6 - 6*(3*a^2*c^6 + c^8)*x
^4 - 49*(3*a^4*c^4 + a^2*c^6)*x^3 + 35*(3*a^6*c^2 + a^4*c^4)*x^2 + 21*(3*a
^8 + a^6*c^2)*x - (7*a^7*c - 42*a^5*c^3 + 48*a^3*c^5 - 17*a*c^7 + 29*(3*a^
3*c^5 + a*c^7)*x^3 + 21*(3*a^5*c^3 + a^3*c^5)*x^2 - 49*(3*a^7*c + a^5*c^3)
*x)*sqrt(x))*sqrt(c^2*x + 2*a*c*sqrt(x) + a^2)/(3*a^4 + a^2*c^2 - (3*a^2*c
^2 + c^4)*x)
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.28

$$\int \left(a^2 + 2ac\sqrt{x} + c^2x\right)^{5/2} dx = 2\left(\begin{cases} \sqrt{a^2 + 2ac\sqrt{x} + c^2x}\left(-\frac{a^6}{42c^2} + \frac{a^5\sqrt{x}}{42c} + \frac{10a^4x}{21} + \frac{25a^3cx^{\frac{3}{2}}}{21} + \frac{55a^2c^2x^2}{42} + \frac{29ac^3x^{\frac{5}{2}}}{42} + \frac{c^4x^3}{7}\right) & \text{for} \\ \frac{-\frac{a^2\left(a^2+2ac\sqrt{x}\right)^{\frac{7}{2}}}{7} + \frac{\left(a^2+2ac\sqrt{x}\right)^{\frac{9}{2}}}{9}}{2a^2c^2} & \text{for} \\ \frac{x\left(a^2\right)^{\frac{5}{2}}}{2} & \text{oth} \end{cases}\right.$$

input
```
integrate((a**2+2*a*c*x**(1/2)+c**2*x)**(5/2),x)
```

output
```
2*Piecewise((sqrt(a**2 + 2*a*c*sqrt(x) + c**2*x)*(-a**6/(42*c**2) + a**5*s
qrt(x)/(42*c) + 10*a**4*x/21 + 25*a**3*c*x**(3/2)/21 + 55*a**2*c**2*x**2/4
2 + 29*a*c**3*x**(5/2)/42 + c**4*x**3/7), Ne(c**2, 0)), ((-a**2*(a**2 + 2*
a*c*sqrt(x))**(7/2)/7 + (a**2 + 2*a*c*sqrt(x))**(9/2)/9)/(2*a**2*c**2), Ne
(a*c, 0)), (x*(a**2)**(5/2)/2, True))
```

Maxima [F]

$$\int \left(a^2 + 2ac\sqrt{x} + c^2x\right)^{5/2} dx = \int \left(c^2x + 2\,ac\sqrt{x} + a^2\right)^{\frac{5}{2}} dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^(5/2),x, algorithm="maxima")
```

output
```
2/7*c^5*x^(7/2) + 4*a^2*c^3*x^(5/2) + 10/3*a^4*c*x^(3/2) + a^5*x + 5*a*c^4
*integrate(x^2, x) + 10*a^3*c^2*integrate(x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(57) = 114$.

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.80

$$\begin{aligned}\int \left(a^2 + 2ac\sqrt{x} + c^2x\right)^{5/2} dx &= \frac{2}{7}\,c^4x^{\frac{7}{2}}|c|\mathrm{sgn}(a)\,\mathrm{sgn}(c) + \frac{4}{3}\,ac^3x^3|c|\mathrm{sgn}(a)\,\mathrm{sgn}(c) \\ &+ \frac{12}{5}\,a^2c^2x^{\frac{5}{2}}|c|\mathrm{sgn}(a)\,\mathrm{sgn}(c) + 2\,a^3cx^2|c|\mathrm{sgn}(a)\,\mathrm{sgn}(c) + \frac{2}{3}\,a^4x^{\frac{3}{2}}|c|\mathrm{sgn}(a)\,\mathrm{sgn}(c) \\ &+ \frac{1}{3}\,c^4x^3|a| + \frac{8}{5}\,ac^3x^{\frac{5}{2}}|a| + 3\,a^2c^2x^2|a| + \frac{8}{3}\,a^3cx^{\frac{3}{2}}|a| + a^4x|a|\end{aligned}$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^(5/2),x, algorithm="giac")
```

output
```
2/7*c^4*x^(7/2)*abs(c)*sgn(a)*sgn(c) + 4/3*a*c^3*x^3*abs(c)*sgn(a)*sgn(c)
+ 12/5*a^2*c^2*x^(5/2)*abs(c)*sgn(a)*sgn(c) + 2*a^3*c*x^2*abs(c)*sgn(a)*sg
n(c) + 2/3*a^4*x^(3/2)*abs(c)*sgn(a)*sgn(c) + 1/3*c^4*x^3*abs(a) + 8/5*a*c
^3*x^(5/2)*abs(a) + 3*a^2*c^2*x^2*abs(a) + 8/3*a^3*c*x^(3/2)*abs(a) + a^4*
x*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}\,dx=\int \left(c^2\,x+a^2+2\,a\,c\,\sqrt{x}\right)^{5/2}dx$$

input
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^(5/2),x)
```

output
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}\,dx=\frac{x\left(70\sqrt{x}\,a^4c+84\sqrt{x}\,a^2c^3x+6\sqrt{x}\,c^5x^2+21a^5+105a^3c^2x+35a\,c^4x^2\right)}{21}$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^(5/2),x)
```

output
```
(x*(70*sqrt(x)*a**4*c + 84*sqrt(x)*a**2*c**3*x + 6*sqrt(x)*c**5*x**2 + 21*
a**5 + 105*a**3*c**2*x + 35*a*c**4*x**2))/21
```

3.44 $\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}\,dx = -\frac{a\left(a+c\sqrt{x}\right)^3\sqrt{a^2+2ac\sqrt{x}+c^2x}}{2c^2}+\frac{2\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}{5c^2}$$

output
```
-1/2*a*(a+c*x^(1/2))^3*(a^2+2*a*c*x^(1/2)+c^2*x)^(1/2)/c^2+2/5*(a^2+2*a*c*
x^(1/2)+c^2*x)^(5/2)/c^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}\,dx = \frac{\left(\left(a+c\sqrt{x}\right)^2\right)^{3/2}\left(10a^3x+20a^2cx^{3/2}+15ac^2x^2+4c^3x^{5/2}\right)}{10\left(a+c\sqrt{x}\right)^3}$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(3/2),x]
```

output
```
(((a + c*Sqrt[x])^2)^(3/2)*(10*a^3*x + 20*a^2*c*x^(3/2) + 15*a*c^2*x^2 + 4
*c^3*x^(5/2)))/(10*(a + c*Sqrt[x])^3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1680, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2} dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \sqrt{x}\left(a^2+2c\sqrt{x}a+c^2x\right)^{3/2} d\sqrt{x}$$

$$\Big\downarrow \text{1100}$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}{5c^2}-\frac{a\int \left(a^2+2c\sqrt{x}a+c^2x\right)^{3/2} d\sqrt{x}}{c}\right)$$

$$\Big\downarrow \text{1079}$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}{5c^2}-\frac{a\sqrt{a^2+2ac\sqrt{x}+c^2x}\int \left(\sqrt{x}c^2+ac\right)^3 d\sqrt{x}}{c^4\left(a+c\sqrt{x}\right)}\right)$$

$$\Big\downarrow \text{17}$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}{5c^2}-\frac{a\left(a+c\sqrt{x}\right)^3\sqrt{a^2+2ac\sqrt{x}+c^2x}}{4c^2}\right)$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(3/2),x]
```

output
```
2*(-1/4*(a*(a + c*Sqrt[x])^3*Sqrt[a^2 + 2*a*c*Sqrt[x] + c^2*x])/c^2 + (a^2
 + 2*a*c*Sqrt[x] + c^2*x)^(5/2)/(5*c^2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\left(\left(a+c\sqrt{x}\right)^{2}\right)^{\frac{3}{2}} x \left(4c^{3}x^{\frac{3}{2}}+15a\,c^{2}x+20a^{2}c\sqrt{x}+10a^{3}\right)}{10\left(a+c\sqrt{x}\right)^{3}}$	54
default	$\frac{\left(a^{2}+2ac\sqrt{x}+c^{2}x\right)^{\frac{3}{2}} \left(4c^{3}x^{\frac{5}{2}}+20a^{2}c\,x^{\frac{3}{2}}+15a\,c^{2}x^{2}+10a^{3}x\right)}{10\left(a+c\sqrt{x}\right)^{3}}$	63

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*((a+c*x^(1/2))^2)^(3/2)*x*(4*c^3*x^(3/2)+15*a*c^2*x+20*a^2*c*x^(1/2)+
10*a^3)/(a+c*x^(1/2))^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(57) = 114$.

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.65

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}\,dx = -\frac{\left(10\,a^6-9\,a^4c^2+3\,a^2c^4+4\,(3\,a^2c^4+c^6)x^3+5\,(3\,a^4c^2+a^2c^4)x^2-10\,(3\,a^6+a^4c^2)x-(10\,a^5c-9\,a^3c^3+\right.}{10\,(3\,a^4+a^2c^2-(3\,a^2c^2+c^4)x}$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(10*a^6 - 9*a^4*c^2 + 3*a^2*c^4 + 4*(3*a^2*c^4 + c^6)*x^3 + 5*(3*a^4
*c^2 + a^2*c^4)*x^2 - 10*(3*a^6 + a^4*c^2)*x - (10*a^5*c - 9*a^3*c^3 + 3*a
*c^5 - 11*(3*a^3*c^3 + a*c^5)*x^2 + 10*(3*a^5*c + a^3*c^3)*x)*sqrt(x))*sqr
t(c^2*x + 2*a*c*sqrt(x) + a^2)/(3*a^4 + a^2*c^2 - (3*a^2*c^2 + c^4)*x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.87

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}\,dx = 2\left(\begin{cases} \sqrt{a^2+2ac\sqrt{x}+c^2x}\left(-\frac{a^4}{20c^2}+\frac{a^3\sqrt{x}}{20c}+\frac{9a^2x}{20}+\frac{11acx^{\frac{3}{2}}}{20}+\frac{c^2x^2}{5}\right) & \text{for } c^2\neq 0 \\ \frac{-\frac{a^2\left(a^2+2ac\sqrt{x}\right)^{\frac{5}{2}}}{5}+\frac{\left(a^2+2ac\sqrt{x}\right)^{\frac{7}{2}}}{7}}{2a^2c^2} & \text{for } ac\neq 0 \\ \frac{x\left(a^2\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a**2+2*a*c*x**(1/2)+c**2*x)**(3/2),x)
```

output
```
2*Piecewise((sqrt(a**2 + 2*a*c*sqrt(x) + c**2*x)*(-a**4/(20*c**2) + a**3*s
qrt(x)/(20*c) + 9*a**2*x/20 + 11*a*c*x**(3/2)/20 + c**2*x**2/5), Ne(c**2,
0)), ((-a**2*(a**2 + 2*a*c*sqrt(x))**(5/2)/5 + (a**2 + 2*a*c*sqrt(x))**(7/
2)/7)/(2*a**2*c**2), Ne(a*c, 0)), (x*(a**2)**(3/2)/2, True))
```

Maxima [F]

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}\,dx=\int \left(c^2x+2\,ac\sqrt{x}+a^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^(3/2),x, algorithm="maxima")
```

output
```
2/5*c^3*x^(5/2) + 2*a^2*c*x^(3/2) + a^3*x + 3*a*c^2*integrate(x, x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}\,dx=\frac{2}{5}\,c^2x^{\frac{5}{2}}|c|\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(c\right)+acx^2|c|\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(c\right)+\frac{2}{3}\,a^2x^{\frac{3}{2}}|c|\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(c\right)+\frac{1}{2}\,c^2x^2|a|+\frac{4}{3}\,acx^{\frac{3}{2}}|a|+a^2x|a|$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^(3/2),x, algorithm="giac")
```

output
```
2/5*c^2*x^(5/2)*abs(c)*sgn(a)*sgn(c) + a*c*x^2*abs(c)*sgn(a)*sgn(c) + 2/3*
a^2*x^(3/2)*abs(c)*sgn(a)*sgn(c) + 1/2*c^2*x^2*abs(a) + 4/3*a*c*x^(3/2)*ab
s(a) + a^2*x*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}\,dx=\int {\left(c^2\,x+a^2+2\,a\,c\,\sqrt{x}\right)}^{3/2}\,dx$$

input
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^(3/2),x)
```

output
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.45

$$\int \left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}\,dx=\frac{x\left(20\sqrt{x}\,a^2c+4\sqrt{x}\,c^3x+10a^3+15a\,c^2x\right)}{10}$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^(3/2),x)
```

output
```
(x*(20*sqrt(x)*a**2*c + 4*sqrt(x)*c**3*x + 10*a**3 + 15*a*c**2*x))/10
```

3.45 $\int \sqrt{a^2+2ac\sqrt{x}+c^2x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \sqrt{a^2+2ac\sqrt{x}+c^2x}\,dx = -\frac{a\left(a+c\sqrt{x}\right)\sqrt{a^2+2ac\sqrt{x}+c^2x}}{c^2}+\frac{2\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}{3c^2}$$

output
```
-a*(a+c*x^(1/2))*(a^2+2*a*c*x^(1/2)+c^2*x)^(1/2)/c^2+2/3*(a^2+2*a*c*x^(1/2
)+c^2*x)^(3/2)/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \sqrt{a^2+2ac\sqrt{x}+c^2x}\,dx = \frac{\sqrt{\left(a+c\sqrt{x}\right)^2}\left(3ax+2cx^{3/2}\right)}{3\left(a+c\sqrt{x}\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*c*Sqrt[x] + c^2*x],x]
```

output
```
(Sqrt[(a + c*Sqrt[x])^2]*(3*a*x + 2*c*x^(3/2)))/(3*(a + c*Sqrt[x]))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1680, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2ac\sqrt{x}+c^2x}\,dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \sqrt{x}\sqrt{a^2+2c\sqrt{x}a+c^2x}d\sqrt{x}$$

$$\Big\downarrow \text{1100}$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}{3c^2}-\frac{a\int \sqrt{a^2+2c\sqrt{x}a+c^2x}d\sqrt{x}}{c}\right)$$

$$\Big\downarrow \text{1079}$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}{3c^2}-\frac{a\sqrt{a^2+2ac\sqrt{x}+c^2x}\int \left(\sqrt{x}c^2+ac\right)d\sqrt{x}}{c^2\left(a+c\sqrt{x}\right)}\right)$$

$$\Big\downarrow \text{17}$$

$$2\left(\frac{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}{3c^2}-\frac{a\left(a+c\sqrt{x}\right)\sqrt{a^2+2ac\sqrt{x}+c^2x}}{2c^2}\right)$$

input
```
Int[Sqrt[a^2 + 2*a*c*Sqrt[x] + c^2*x],x]
```

output
```
2*(-1/2*(a*(a + c*Sqrt[x])*Sqrt[a^2 + 2*a*c*Sqrt[x] + c^2*x])/c^2 + (a^2 +
 2*a*c*Sqrt[x] + c^2*x)^(3/2)/(3*c^2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

method	result	size
derivativedivides	$-\frac{\operatorname{csgn}\left(a+c\sqrt{x}\right)\left(a+c\sqrt{x}\right)^{2}\left(-2c\sqrt{x}+a\right)}{3c^{2}}$	31
default	$\frac{\sqrt{a^{2}+2ac\sqrt{x}+c^{2}x}\,\left(2c\,x^{\frac{3}{2}}+3xa\right)}{3a+3c\sqrt{x}}$	41

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*csgn(a+c*x^(1/2))*(a+c*x^(1/2))^2*(-2*c*x^(1/2)+a)/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(55) = 110$.

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.94

$$\int \sqrt{a^2 + 2ac\sqrt{x} + c^2x}\,dx = -\frac{\left(7\,a^4 - 3\,a^2c^2 + 2\,(3\,a^2c^2 + c^4)x^2 - 3\,(3\,a^4 + a^2c^2)x - (7\,a^3c - 3\,ac^3 - (3\,a^3c + ac^3)x)\sqrt{x}\right)\sqrt{c^2x + 2\,a}}{3\,(3\,a^4 + a^2c^2 - (3\,a^2c^2 + c^4)x)}$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(7*a^4 - 3*a^2*c^2 + 2*(3*a^2*c^2 + c^4)*x^2 - 3*(3*a^4 + a^2*c^2)*x
- (7*a^3*c - 3*a*c^3 - (3*a^3*c + a*c^3)*x)*sqrt(x))*sqrt(c^2*x + 2*a*c*sq
rt(x) + a^2)/(3*a^4 + a^2*c^2 - (3*a^2*c^2 + c^4)*x)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.57

$$\int \sqrt{a^2 + 2ac\sqrt{x} + c^2x}\,dx = 2\left(\begin{cases} \sqrt{a^2 + 2ac\sqrt{x} + c^2x}\left(-\frac{a^2}{6c^2} + \frac{a\sqrt{x}}{6c} + \frac{x}{3}\right) & \text{for}\ c^2 \neq 0 \\ \frac{-\frac{a^2\left(a^2+2ac\sqrt{x}\right)^{\frac{3}{2}}}{3} + \frac{\left(a^2+2ac\sqrt{x}\right)^{\frac{5}{2}}}{5}}{2a^2c^2} & \text{for}\ ac \neq 0 \\ \frac{x\sqrt{a^2}}{2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a**2+2*a*c*x**(1/2)+c**2*x)**(1/2),x)
```

output
```
2*Piecewise((sqrt(a**2 + 2*a*c*sqrt(x) + c**2*x)*(-a**2/(6*c**2) + a*sqrt(
x)/(6*c) + x/3), Ne(c**2, 0)), ((-a**2*(a**2 + 2*a*c*sqrt(x))**(3/2)/3 + (
a**2 + 2*a*c*sqrt(x))**(5/2)/5)/(2*a**2*c**2), Ne(a*c, 0)), (x*sqrt(a**2)/
2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.15

$$\int \sqrt{a^2+2ac\sqrt{x}+c^2x}\,dx=\frac{2}{3}\,cx^{\frac{3}{2}}+ax$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*c*x^(3/2) + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.28

$$\int \sqrt{a^2+2ac\sqrt{x}+c^2x}\,dx=\frac{2}{3}\,\sqrt{c^2x}x\mathrm{sgn}(a)\,\mathrm{sgn}(c)+x|a|$$

input
```
integrate((a^2+2*a*c*x^(1/2)+c^2*x)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(c^2*x)*x*sgn(a)*sgn(c) + x*abs(a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+2ac\sqrt{x}+c^2x}\,dx=\int \sqrt{c^2\,x+a^2+2\,a\,c\,\sqrt{x}}\,dx$$

input
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^(1/2),x)
```

output
```
int((c^2*x + a^2 + 2*a*c*x^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.18

$$\int \sqrt{a^2 + 2ac\sqrt{x} + c^2x}\, dx = \frac{x(2\sqrt{x}\, c + 3a)}{3}$$

input
```
int((a^2+2*a*c*x^(1/2)+c^2*x)^(1/2),x)
```

output
```
(x*(2*sqrt(x)*c + 3*a))/3
```

3.46 $\int \frac{1}{\sqrt{a^2+2ac\sqrt{x}+c^2x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{1}{\sqrt{a^2+2ac\sqrt{x}+c^2x}}\,dx=\frac{2\sqrt{a^2+2ac\sqrt{x}+c^2x}}{c^2}-\frac{2a\left(a+c\sqrt{x}\right)\log\left(a+c\sqrt{x}\right)}{c^2\sqrt{a^2+2ac\sqrt{x}+c^2x}}$$

output
```
2*(a^2+2*a*c*x^(1/2)+c^2*x)^(1/2)/c^2-2*a*(a+c*x^(1/2))*ln(a+c*x^(1/2))/c^
2/(a^2+2*a*c*x^(1/2)+c^2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{a^2+2ac\sqrt{x}+c^2x}}\,dx=\frac{2\left(a+c\sqrt{x}\right)\left(c\sqrt{x}-a\log\left(a+c\sqrt{x}\right)\right)}{c^2\sqrt{\left(a+c\sqrt{x}\right)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*c*Sqrt[x] + c^2*x],x]
```

output
```
(2*(a + c*Sqrt[x])*(c*Sqrt[x] - a*Log[a + c*Sqrt[x]]))/(c^2*Sqrt[(a + c*Sq
rt[x])^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1680, 1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2ac\sqrt{x}+c^2x}}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \frac{\sqrt{x}}{\sqrt{a^2+2c\sqrt{x}a+c^2x}}d\sqrt{x}$$

$$\Big\downarrow \text{1100}$$

$$2\left(\frac{\sqrt{a^2+2ac\sqrt{x}+c^2x}}{c^2}-\frac{a\int \frac{1}{\sqrt{a^2+2c\sqrt{x}a+c^2x}}d\sqrt{x}}{c}\right)$$

$$\Big\downarrow \text{1079}$$

$$2\left(\frac{\sqrt{a^2+2ac\sqrt{x}+c^2x}}{c^2}-\frac{a\left(a+c\sqrt{x}\right)\int \frac{1}{\sqrt{x}c^2+ac}d\sqrt{x}}{\sqrt{a^2+2ac\sqrt{x}+c^2x}}\right)$$

$$\Big\downarrow \text{16}$$

$$2\left(\frac{\sqrt{a^2+2ac\sqrt{x}+c^2x}}{c^2}-\frac{a\left(a+c\sqrt{x}\right)\log\left(a+c\sqrt{x}\right)}{c^2\sqrt{a^2+2ac\sqrt{x}+c^2x}}\right)$$

input
```
Int[1/Sqrt[a^2 + 2*a*c*Sqrt[x] + c^2*x],x]
```

output
```
2*(Sqrt[a^2 + 2*a*c*Sqrt[x] + c^2*x]/c^2 - (a*(a + c*Sqrt[x])*Log[a + c*Sq
rt[x]])/(c^2*Sqrt[a^2 + 2*a*c*Sqrt[x] + c^2*x]))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$-\frac{2\left(a+c\sqrt{x}\right)\left(a\ln\left(a+c\sqrt{x}\right)-c\sqrt{x}\right)}{\sqrt{\left(a+c\sqrt{x}\right)^{2}}\,c^{2}}$	41
default	$\frac{\left(a+c\sqrt{x}\right)\left(2c\sqrt{x}+a\ln\left(c\sqrt{x}-a\right)-a\ln\left(a+c\sqrt{x}\right)-a\ln\left(c^{2}x-a^{2}\right)\right)}{\sqrt{a^{2}+2ac\sqrt{x}+c^{2}x}\,c^{2}}$	75

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a+c*x^(1/2))*(a*ln(a+c*x^(1/2))-c*x^(1/2))/((a+c*x^(1/2))^2)^(1/2)/c^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2ac\sqrt{x}+c^2x}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{a^2+2ac\sqrt{x}+c^2x}}\,dx = 2\left(\begin{cases} -\frac{a\left(\frac{a}{c}+\sqrt{x}\right)\log\left(\frac{a}{c}+\sqrt{x}\right)}{c\sqrt{c^2\left(\frac{a}{c}+\sqrt{x}\right)^2}}+\frac{\sqrt{a^2+2ac\sqrt{x}+c^2x}}{c^2} & \text{for } c^2\neq 0 \\ \frac{-a^2\sqrt{a^2+2ac\sqrt{x}}+\frac{\left(a^2+2ac\sqrt{x}\right)^{\frac{3}{2}}}{3}}{2a^2c^2} & \text{for } ac\neq 0 \\ \frac{x}{2\sqrt{a^2}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(1/(a**2+2*a*c*x**(1/2)+c**2*x)**(1/2),x)
```

output
```
2*Piecewise((-a*(a/c + sqrt(x))*log(a/c + sqrt(x))/(c*sqrt(c**2*(a/c + sqr
t(x))**2)) + sqrt(a**2 + 2*a*c*sqrt(x) + c**2*x)/c**2, Ne(c**2, 0)), ((-a*
*2*sqrt(a**2 + 2*a*c*sqrt(x)) + (a**2 + 2*a*c*sqrt(x))**(3/2)/3)/(2*a**2*c
**2), Ne(a*c, 0)), (x/(2*sqrt(a**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{a^2+2ac\sqrt{x}+c^2x}}\,dx = -\frac{2\,a\log\left(c\sqrt{x}+a\right)}{c^2}+\frac{2\,\sqrt{x}}{c}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(1/2),x, algorithm="maxima")
```

output
```
-2*a*log(c*sqrt(x) + a)/c^2 + 2*sqrt(x)/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{a^2+2ac\sqrt{x}+c^2x}}\,dx = -\frac{2\,|a|\log\left(\left|\sqrt{c^2x}\mathrm{sgn}(a)\,\mathrm{sgn}(c)+|a|\right|\right)}{c^2}+\frac{2\,\sqrt{c^2x}}{c^2\mathrm{sgn}\,(a)\,\mathrm{sgn}\,(c)}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(1/2),x, algorithm="giac")
```

output
```
-2*abs(a)*log(abs(sqrt(c^2*x)*sgn(a)*sgn(c) + abs(a)))/c^2 + 2*sqrt(c^2*x)
/(c^2*sgn(a)*sgn(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2ac\sqrt{x}+c^2x}}\,dx = \int \frac{1}{\sqrt{c^2\,x+a^2+2\,a\,c\,\sqrt{x}}}\,dx$$

input
```
int(1/(c^2*x + a^2 + 2*a*c*x^(1/2))^(1/2),x)
```

output
```
int(1/(c^2*x + a^2 + 2*a*c*x^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.27

$$\int \frac{1}{\sqrt{a^2+2ac\sqrt{x}+c^2x}}\,dx=\frac{2\sqrt{x}\,c-2\log\left(\sqrt{x}\,c+a\right)a}{c^2}$$

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(1/2),x)
```

output
```
(2*(sqrt(x)*c - log(sqrt(x)*c + a)*a))/c**2
```

3.47 $\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}\,dx=-\frac{2}{c^2\sqrt{a^2+2ac\sqrt{x}+c^2x}}+\frac{a}{c^2\left(a+c\sqrt{x}\right)\sqrt{a^2+2ac\sqrt{x}+c^2x}}$$

output
```
-2/c^2/(a^2+2*a*c*x^(1/2)+c^2*x)^(1/2)+a/c^2/(a+c*x^(1/2))/(a^2+2*a*c*x^(1
/2)+c^2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}\,dx=\frac{\left(-a-2c\sqrt{x}\right)\left(a+c\sqrt{x}\right)}{c^2\left(\left(a+c\sqrt{x}\right)^2\right)^{3/2}}$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(-3/2),x]
```

output
```
((-a - 2*c*Sqrt[x])*(a + c*Sqrt[x]))/(c^2*((a + c*Sqrt[x])^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1680, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \frac{\sqrt{x}}{\left(a^2+2c\sqrt{x}a+c^2x\right)^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1100}$$

$$2\left(-\frac{a\int \frac{1}{\left(a^2+2c\sqrt{x}a+c^2x\right)^{3/2}}d\sqrt{x}}{c}-\frac{1}{c^2\sqrt{a^2+2ac\sqrt{x}+c^2x}}\right)$$

$$\Big\downarrow \text{1078}$$

$$2\left(\frac{a}{2c^2\left(a+c\sqrt{x}\right)\sqrt{a^2+2ac\sqrt{x}+c^2x}}-\frac{1}{c^2\sqrt{a^2+2ac\sqrt{x}+c^2x}}\right)$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(-3/2),x]
```

output
```
2*(-(1/(c^2*Sqrt[a^2 + 2*a*c*Sqrt[x] + c^2*x])) + a/(2*c^2*(a + c*Sqrt[x])
*Sqrt[a^2 + 2*a*c*Sqrt[x] + c^2*x]))
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

method	result	size
derivativedivides	$-\frac{\left(2c\sqrt{x}+a\right)\left(a+c\sqrt{x}\right)}{c^{2}\left(\left(a+c\sqrt{x}\right)^{2}\right)^{\frac{3}{2}}}$	32
default	$-\frac{\left(-3x^{3}a\,c^{6}+2x^{\frac{7}{2}}c^{7}+7a^{3}c^{4}x^{2}-4x^{\frac{5}{2}}a^{2}c^{5}+2x^{\frac{3}{2}}a^{4}c^{3}-5x\,c^{2}a^{5}+a^{7}\right)\left(a+c\sqrt{x}\right)}{c^{2}\left(c\sqrt{x}-a\right)^{2}\left(-c^{2}x+a^{2}\right)^{2}\left(a^{2}+2ac\sqrt{x}+c^{2}x\right)^{\frac{3}{2}}}$	117

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(2*c*x^(1/2)+a)*(a+c*x^(1/2))/c^2/((a+c*x^(1/2))^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(56) = 112$.

Time = 0.18 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.97

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}\,dx =$$
$$-\frac{\left(5\,a^8+10\,a^6c^2+a^4c^4-(9\,a^6c^2-20\,a^4c^4-7\,a^2c^6+2\,c^8)x^2-(3\,a^8+5\,a^6c^2+21\,a^4c^4+3\,a^2c^6)x-(5\,a^7\right.}{3\,a^{12}-5\,a^{10}c^2+a^8c^4+a^6c^6-(3\,a^6c^6-5\,a^4c^8+a^2c^{10}+c^{12})x^3+}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(3/2),x, algorithm="fricas")
```

output
```
-(5*a^8 + 10*a^6*c^2 + a^4*c^4 - (9*a^6*c^2 - 20*a^4*c^4 - 7*a^2*c^6 + 2*c
^8)*x^2 - (3*a^8 + 5*a^6*c^2 + 21*a^4*c^4 + 3*a^2*c^6)*x - (5*a^7*c + 10*a
^5*c^3 + a^3*c^5 - (3*a^5*c^3 - 10*a^3*c^5 - 9*a*c^7)*x^2 - (9*a^7*c - 5*a
^5*c^3 + 23*a^3*c^5 + 5*a*c^7)*x)*sqrt(x))*sqrt(c^2*x + 2*a*c*sqrt(x) + a^
2)/(3*a^12 - 5*a^10*c^2 + a^8*c^4 + a^6*c^6 - (3*a^6*c^6 - 5*a^4*c^8 + a^2
*c^10 + c^12)*x^3 + 3*(3*a^8*c^4 - 5*a^6*c^6 + a^4*c^8 + a^2*c^10)*x^2 - 3
*(3*a^10*c^2 - 5*a^8*c^4 + a^6*c^6 + a^4*c^8)*x)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}\,dx = \int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*c*x**(1/2)+c**2*x)**(3/2),x)
```

output
```
Integral((a**2 + 2*a*c*sqrt(x) + c**2*x)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.35

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}\,dx=\frac{x}{ac^2x+2\,a^2c\sqrt{x}+a^3}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(3/2),x, algorithm="maxima")
```

output
```
x/(a*c^2*x + 2*a^2*c*sqrt(x) + a^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(56) = 112.

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.08

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}\,dx= -\frac{|a||c|\log\left(\left|\sqrt{x}|c|\mathrm{sgn}(a)\,\mathrm{sgn}(c)+|a|\right|\right)\mathrm{sgn}(a)\,\mathrm{sgn}(c)}{a^2c^2|c|\mathrm{sgn}\,(a)\,\mathrm{sgn}\,(c)-ac^3|a|} - \frac{c|a|\log\left(\left|c\sqrt{x}+a\right|\right)}{ac^2|a||c|\mathrm{sgn}\,(a)\,\mathrm{sgn}\,(c)-a^2c^3} + \frac{a^2|c|\mathrm{sgn}(a)\,\mathrm{sgn}(c)-ac|a|}{\left(|a||c|\mathrm{sgn}\,(a)\,\mathrm{sgn}\,(c)-ac\right)\left(c\sqrt{x}+a\right)ac^2}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(3/2),x, algorithm="giac")
```

output
```
-abs(a)*abs(c)*log(abs(sqrt(x)*abs(c)*sgn(a)*sgn(c) + abs(a)))*sgn(a)*sgn(
c)/(a^2*c^2*abs(c)*sgn(a)*sgn(c) - a*c^3*abs(a)) - c*abs(a)*log(abs(c*sqrt
(x) + a))/(a*c^2*abs(a)*abs(c)*sgn(a)*sgn(c) - a^2*c^3) + (a^2*abs(c)*sgn(
a)*sgn(c) - a*c*abs(a))/((abs(a)*abs(c)*sgn(a)*sgn(c) - a*c)*(c*sqrt(x) +
a)*a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}\,dx=\int \frac{1}{\left(c^2\,x+a^2+2\,a\,c\,\sqrt{x}\right)^{3/2}}\,dx$$

input
```
int(1/(c^2*x + a^2 + 2*a*c*x^(1/2))^(3/2),x)
```

output
```
int(1/(c^2*x + a^2 + 2*a*c*x^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.33

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}\,dx=\frac{x}{a\left(2\sqrt{x}\,ac+a^2+c^2x\right)}$$

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(3/2),x)
```

output
```
x/(a*(2*sqrt(x)*a*c + a**2 + c**2*x))
```

3.48 $\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}\,dx = -\frac{2}{3c^2\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}} + \frac{a}{2c^2\left(a+c\sqrt{x}\right)^3\sqrt{a^2+2ac\sqrt{x}+c^2x}}$$

output
```
-2/3/c^2/(a^2+2*a*c*x^(1/2)+c^2*x)^(3/2)+1/2*a/c^2/(a+c*x^(1/2))^3/(a^2+2*
a*c*x^(1/2)+c^2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}\,dx = \frac{\left(-a-4c\sqrt{x}\right)\left(a+c\sqrt{x}\right)}{6c^2\left(\left(a+c\sqrt{x}\right)^2\right)^{5/2}}$$

input
```
Integrate[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(-5/2),x]
```

output
```
((-a - 4*c*Sqrt[x])*(a + c*Sqrt[x]))/(6*c^2*((a + c*Sqrt[x])^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1680, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \frac{\sqrt{x}}{\left(a^2+2c\sqrt{x}a+c^2x\right)^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{1100}$$

$$2\left(-\frac{a\int \frac{1}{\left(a^2+2c\sqrt{x}a+c^2x\right)^{5/2}}d\sqrt{x}}{c}-\frac{1}{3c^2\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}\right)$$

$$\Big\downarrow \text{1078}$$

$$2\left(\frac{a}{4c^2\left(a+c\sqrt{x}\right)\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}-\frac{1}{3c^2\left(a^2+2ac\sqrt{x}+c^2x\right)^{3/2}}\right)$$

input
```
Int[(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(-5/2),x]
```

output
```
2*(-1/3*1/(c^2*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(3/2)) + a/(4*c^2*(a + c*Sqrt
[x])*(a^2 + 2*a*c*Sqrt[x] + c^2*x)^(3/2)))
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.45

method	result
derivativedivides	$-\frac{\left(4 c \sqrt{x}+a\right) \left(a+c \sqrt{x}\right)}{6 c^{2} \left(\left(a+c \sqrt{x}\right)^{2}\right)^{\frac{5}{2}}}$
default	$-\frac{\left(-15 x^{6} a\, c^{12}+50 x^{5} a^{3} c^{10}+4 x^{\frac{11}{2}} a^{2} c^{11}-56 x^{\frac{9}{2}} a^{4} c^{9}+104 x^{\frac{7}{2}} a^{6} c^{7}+a^{13}-14 a^{11} c^{2} x+4 x^{\frac{13}{2}} c^{13}-4 a^{7} c^{6} x^{3}+31 a^{9} c^{4} x^{2}-7}{6 c^{2} \left(c \sqrt{x}-a\right)^{4} \left(-c^{2} x+a^{2}\right)^{4} \left(a^{2}+2 a c \sqrt{x}+c^{2} x\right)^{\frac{5}{2}}}$

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(4*c*x^(1/2)+a)*(a+c*x^(1/2))/c^2/((a+c*x^(1/2))^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(57) = 114$.

Time = 0.38 (sec) , antiderivative size = 764, normalized size of antiderivative = 10.76

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(38*a^14 + 133*a^12*c^2 + 21*a^10*c^4 - a^8*c^6 + a^6*c^8 - (3*a^8*c^
6 - 49*a^6*c^8 - 119*a^4*c^10 - 27*a^2*c^12)*x^4 - 4*(38*a^8*c^6 + 133*a^6
*c^8 + 21*a^4*c^10 - a^2*c^12 + c^14)*x^3 - (123*a^12*c^2 - 679*a^10*c^4 -
 224*a^8*c^6 - 372*a^6*c^8 - 35*a^4*c^10 + 35*a^2*c^12)*x^2 - 2*(9*a^14 +
43*a^12*c^2 + 308*a^10*c^4 + 24*a^8*c^6 - 5*a^6*c^8 + 5*a^4*c^10)*x - (38*
a^13*c + 133*a^11*c^3 + 21*a^9*c^5 - a^7*c^7 + a^5*c^9 - (3*a^7*c^7 - 49*a
^5*c^9 - 119*a^3*c^11 - 27*a*c^13)*x^4 + 4*(3*a^9*c^5 - 49*a^7*c^7 - 119*a
^5*c^9 - 27*a^3*c^11)*x^3 - (75*a^11*c^3 - 503*a^9*c^5 - 448*a^7*c^7 - 276
*a^5*c^9 - 19*a^3*c^11 + 19*a*c^13)*x^2 - 2*(39*a^13*c - 67*a^11*c^3 + 448
*a^9*c^5 - 36*a^7*c^7 - 15*a^5*c^9 + 15*a^3*c^11)*x)*sqrt(x))*sqrt(c^2*x +
 2*a*c*sqrt(x) + a^2)/(3*a^20 - 11*a^18*c^2 + 14*a^16*c^4 - 6*a^14*c^6 - a
^12*c^8 + a^10*c^10 - (3*a^10*c^10 - 11*a^8*c^12 + 14*a^6*c^14 - 6*a^4*c^1
6 - a^2*c^18 + c^20)*x^5 + 5*(3*a^12*c^8 - 11*a^10*c^10 + 14*a^8*c^12 - 6*
a^6*c^14 - a^4*c^16 + a^2*c^18)*x^4 - 10*(3*a^14*c^6 - 11*a^12*c^8 + 14*a^
10*c^10 - 6*a^8*c^12 - a^6*c^14 + a^4*c^16)*x^3 + 10*(3*a^16*c^4 - 11*a^14
*c^6 + 14*a^12*c^8 - 6*a^10*c^10 - a^8*c^12 + a^6*c^14)*x^2 - 5*(3*a^18*c^
2 - 11*a^16*c^4 + 14*a^14*c^6 - 6*a^12*c^8 - a^10*c^10 + a^8*c^12)*x)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}\,dx = \int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*c*x**(1/2)+c**2*x)**(5/2),x)
```

output
```
Integral((a**2 + 2*a*c*sqrt(x) + c**2*x)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}\,dx = \frac{c^2x^2+4\,acx^{\frac{3}{2}}+6\,a^2x}{6\left(a^3c^4x^2+4\,a^4c^3x^{\frac{3}{2}}+6\,a^5c^2x+4\,a^6c\sqrt{x}+a^7\right)}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(c^2*x^2 + 4*a*c*x^(3/2) + 6*a^2*x)/(a^3*c^4*x^2 + 4*a^4*c^3*x^(3/2) +
 6*a^5*c^2*x + 4*a^6*c*sqrt(x) + a^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(57) = 114$.

Time = 0.13 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.85

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}\,dx = -\frac{c^2|a||c|\log\left(\left|\sqrt{x}|c|\mathrm{sgn}(a)\,\mathrm{sgn}(c)+|a|\right|\right)\mathrm{sgn}(a)\,\mathrm{sgn}(c)}{4\left(a^4c^4|c|\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(c\right)-a^3c^5|a|\right)} - \frac{c^3|a|\log\left(\left|c\sqrt{x}+a\right|\right)}{4\left(a^3c^4|a||c|\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(c\right)-a^4c^5\right)} - \frac{2\,a^4|c|\mathrm{sgn}(a)\,\mathrm{sgn}(c)-2\,a^3c|a|+3\left(a^2c^2|c|\mathrm{sgn}(a)\,\mathrm{sgn}(c)-ac^3|a|\right)x+9\left(a^3c|c|\mathrm{sgn}(a)\,\mathrm{sgn}(c)-a^2c^2|a|\right)\sqrt{x}}{12\left(|a||c|\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(c\right)-ac\right)\left(c\sqrt{x}+a\right)^3a^3c^2}$$

input
```
integrate(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(5/2),x, algorithm="giac")
```

output
```
-1/4*c^2*abs(a)*abs(c)*log(abs(sqrt(x)*abs(c)*sgn(a)*sgn(c) + abs(a)))*sgn
(a)*sgn(c)/(a^4*c^4*abs(c)*sgn(a)*sgn(c) - a^3*c^5*abs(a)) - 1/4*c^3*abs(a
)*log(abs(c*sqrt(x) + a))/(a^3*c^4*abs(a)*abs(c)*sgn(a)*sgn(c) - a^4*c^5)
- 1/12*(2*a^4*abs(c)*sgn(a)*sgn(c) - 2*a^3*c*abs(a) + 3*(a^2*c^2*abs(c)*sg
n(a)*sgn(c) - a*c^3*abs(a))*x + 9*(a^3*c*abs(c)*sgn(a)*sgn(c) - a^2*c^2*ab
s(a))*sqrt(x))/((abs(a)*abs(c)*sgn(a)*sgn(c) - a*c)*(c*sqrt(x) + a)^3*a^3*
c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}\,dx=\int \frac{1}{\left(c^2\,x+a^2+2\,a\,c\,\sqrt{x}\right)^{5/2}}\,dx$$

input
```
int(1/(c^2*x + a^2 + 2*a*c*x^(1/2))^(5/2),x)
```

output
```
int(1/(c^2*x + a^2 + 2*a*c*x^(1/2))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a^2+2ac\sqrt{x}+c^2x\right)^{5/2}}\,dx=\frac{-4\sqrt{x}\,c-a}{6c^2\left(4\sqrt{x}\,a^3c+4\sqrt{x}\,a\,c^3x+a^4+6a^2c^2x+c^4x^2\right)}$$

input
```
int(1/(a^2+2*a*c*x^(1/2)+c^2*x)^(5/2),x)
```

output
```
( - 4*sqrt(x)*c - a)/(6*c**2*(4*sqrt(x)*a**3*c + 4*sqrt(x)*a*c**3*x + a**4
 + 6*a**2*c**2*x + c**4*x**2))
```

3.49 $\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx=\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

output
```
2*(a^2+2*a*b*x^(1/2)+b^2*x)^(1/2)/b^2-2*a*(a+b*x^(1/2))*ln(a+b*x^(1/2))/b^
2/(a^2+2*a*b*x^(1/2)+b^2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx=\frac{2\left(a+b\sqrt{x}\right)\left(b\sqrt{x}-a\log\left(a+b\sqrt{x}\right)\right)}{b^2\sqrt{\left(a+b\sqrt{x}\right)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

output
```
(2*(a + b*Sqrt[x])*(b*Sqrt[x] - a*Log[a + b*Sqrt[x]]))/(b^2*Sqrt[(a + b*Sq
rt[x])^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1680, 1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \frac{\sqrt{x}}{\sqrt{a^2+2b\sqrt{x}a+b^2x}}d\sqrt{x}$$

$$\Big\downarrow \text{1100}$$

$$2\left(\frac{\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{a\int \frac{1}{\sqrt{a^2+2b\sqrt{x}a+b^2x}}d\sqrt{x}}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$2\left(\frac{\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{a\left(a+b\sqrt{x}\right)\int \frac{1}{\sqrt{x}b^2+ab}d\sqrt{x}}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\right)$$

$$\Big\downarrow \text{16}$$

$$2\left(\frac{\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}\right)$$

input
```
Int[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

output
```
2*(Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x]/b^2 - (a*(a + b*Sqrt[x])*Log[a + b*Sq
rt[x]])/(b^2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x]))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$-\frac{2\left(a+b\sqrt{x}\right)\left(a\ln\left(a+b\sqrt{x}\right)-b\sqrt{x}\right)}{\sqrt{\left(a+b\sqrt{x}\right)^{2}}\, b^{2}}$	41
default	$\frac{\left(a+b\sqrt{x}\right)\left(2b\sqrt{x}+a\ln\left(b\sqrt{x}-a\right)-a\ln\left(a+b\sqrt{x}\right)-a\ln\left(b^{2}x-a^{2}\right)\right)}{\sqrt{a^{2}+2ab\sqrt{x}+b^{2}x}\, b^{2}}$	75

input
```
int(1/(a^2+2*a*b*x^(1/2)+b^2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a+b*x^(1/2))*(a*ln(a+b*x^(1/2))-b*x^(1/2))/((a+b*x^(1/2))^2)^(1/2)/b^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/2)+b^2*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx=2\left(\begin{cases}-\frac{a\left(\frac{a}{b}+\sqrt{x}\right)\log\left(\frac{a}{b}+\sqrt{x}\right)}{b\sqrt{b^2\left(\frac{a}{b}+\sqrt{x}\right)^2}}+\frac{\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} & \text{for } b^2\neq 0\\ \frac{-a^2\sqrt{a^2+2ab\sqrt{x}}+\frac{\left(a^2+2ab\sqrt{x}\right)^{\frac{3}{2}}}{3}}{2a^2b^2} & \text{for } ab\neq 0\\ \frac{x}{2\sqrt{a^2}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(1/(a**2+2*a*b*x**(1/2)+b**2*x)**(1/2),x)
```

output
```
2*Piecewise((-a*(a/b + sqrt(x))*log(a/b + sqrt(x))/(b*sqrt(b**2*(a/b + sqr
t(x))**2)) + sqrt(a**2 + 2*a*b*sqrt(x) + b**2*x)/b**2, Ne(b**2, 0)), ((-a*
*2*sqrt(a**2 + 2*a*b*sqrt(x)) + (a**2 + 2*a*b*sqrt(x))**(3/2)/3)/(2*a**2*b
**2), Ne(a*b, 0)), (x/(2*sqrt(a**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = -\frac{2\,a\log\left(b\sqrt{x}+a\right)}{b^2}+\frac{2\,\sqrt{x}}{b}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/2)+b^2*x)^(1/2),x, algorithm="maxima")
```

output
```
-2*a*log(b*sqrt(x) + a)/b^2 + 2*sqrt(x)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = -\frac{2\,|a|\log\left(\left|\sqrt{b^2x}\mathrm{sgn}(a)\,\mathrm{sgn}(b)+|a|\right|\right)}{b^2}+\frac{2\,\sqrt{b^2x}}{b^2\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(b\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/2)+b^2*x)^(1/2),x, algorithm="giac")
```

output
```
-2*abs(a)*log(abs(sqrt(b^2*x)*sgn(a)*sgn(b) + abs(a)))/b^2 + 2*sqrt(b^2*x)
/(b^2*sgn(a)*sgn(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = \int \frac{1}{\sqrt{b^2\,x+a^2+2\,a\,b\,\sqrt{x}}}\,dx$$

input
```
int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2),x)
```

output
```
int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.27

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx=\frac{2\sqrt{x}\,b-2\log\left(\sqrt{x}\,b+a\right)a}{b^2}$$

input
```
int(1/(a^2+2*a*b*x^(1/2)+b^2*x)^(1/2),x)
```

output
```
(2*(sqrt(x)*b - log(sqrt(x)*b + a)*a))/b**2
```

3.50 $\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^4\,dx$

Optimal result

Integrand size = 18, antiderivative size = 146

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^4\,dx = a^4x+3a^3bx^{4/3}+\frac{6}{5}a^2\left(3b^2+2ac\right)x^{5/3}$$
$$+2ab\left(b^2+3ac\right)x^2+\frac{3}{7}\left(b^4+12ab^2c+6a^2c^2\right)x^{7/3}+\frac{3}{2}bc\left(b^2+3ac\right)x^{8/3}+\frac{2}{3}c^2\left(3b^2+2ac\right)x^3+\frac{6}{5}bc^3x^{10/3}+\frac{3}{11}c^4x^{11}$$

output
```
a^4*x+3*a^3*b*x^(4/3)+6/5*a^2*(2*a*c+3*b^2)*x^(5/3)+2*a*b*(3*a*c+b^2)*x^2+
3/7*(6*a^2*c^2+12*a*b^2*c+b^4)*x^(7/3)+3/2*b*c*(3*a*c+b^2)*x^(8/3)+2/3*c^2
*(2*a*c+3*b^2)*x^3+6/5*b*c^3*x^(10/3)+3/11*c^4*x^(11/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12

$$\int \left(a+b\sqrt[3]{x}\right.$$
$$\left.+cx^{2/3}\right)^4\,dx = \frac{2310a^4x+6930a^3bx^{4/3}+8316a^2b^2x^{5/3}+5544a^3cx^{5/3}+4620ab^3x^2+13860a^2bcx^2+990b^4x}{}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^4,x]
```

output
```
(2310*a^4*x + 6930*a^3*b*x^(4/3) + 8316*a^2*b^2*x^(5/3) + 5544*a^3*c*x^(5/
3) + 4620*a*b^3*x^2 + 13860*a^2*b*c*x^2 + 990*b^4*x^(7/3) + 11880*a*b^2*c*
x^(7/3) + 5940*a^2*c^2*x^(7/3) + 3465*b^3*c*x^(8/3) + 10395*a*b*c^2*x^(8/3
) + 4620*b^2*c^2*x^3 + 3080*a*c^3*x^3 + 2772*b*c^3*x^(10/3) + 630*c^4*x^(1
1/3))/2310
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1680, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^4dx$$

$$\Big\downarrow \text{1680}$$

$$3\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^4x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1140}$$

$$3\int \left(x^{2/3}a^4+4bxa^3+2\left(3b^2+2ac\right)x^{4/3}a^2+4b\left(b^2+3ac\right)x^{5/3}a+c^4x^{10/3}+4bc^3x^3+2c^2\left(3b^2+2ac\right)x^{8/3}+4bc$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^4x}{3}+a^3bx^{4/3}+\frac{2}{5}a^2x^{5/3}\left(2ac+3b^2\right)+\frac{1}{7}x^{7/3}\left(6a^2c^2+12ab^2c+b^4\right)+\frac{2}{9}c^2x^3\left(2ac+3b^2\right)+\frac{1}{2}bcx^{8/3}\left(3ac+b^2\right)$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^4,x]
```

output
```
3*((a^4*x)/3 + a^3*b*x^(4/3) + (2*a^2*(3*b^2 + 2*a*c)*x^(5/3))/5 + (2*a*b*
(b^2 + 3*a*c)*x^2)/3 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*x^(7/3))/7 + (b*c*(
b^2 + 3*a*c)*x^(8/3))/2 + (2*c^2*(3*b^2 + 2*a*c)*x^3)/9 + (2*b*c^3*x^(10/3
))/5 + (c^4*x^(11/3))/11)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.40 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.96

method	result
default	$a^4x+\frac{3b^4x^{\frac{7}{3}}}{7}+\frac{3c^4x^{\frac{11}{3}}}{11}+2b^3a\,x^2+\frac{4a\,c^3x^3}{3}+\frac{18a^2b^2x^{\frac{5}{3}}}{5}+\frac{18a^2c^2x^{\frac{7}{3}}}{7}+3a^3b\,x^{\frac{4}{3}}+\frac{12a^3c\,x^{\frac{5}{3}}}{5}+6$
derivativedivides	$\frac{3c^4x^{\frac{11}{3}}}{11}+\frac{6b\,c^3x^{\frac{10}{3}}}{5}+\frac{(2(2ac+b^2)c^2+4b^2c^2)x^3}{3}+\frac{3(4ba\,c^2+4(2ac+b^2)bc)x^{\frac{8}{3}}}{8}+\frac{3\left(2a^2c^2+8a\,b^2c+(2ac+b^2)^2\right)}{7}$
trager	$\frac{(4a\,c^3x^2+6b^2c^2x^2+18a^2bcx+6a\,b^3x+4a\,c^3x+6b^2x\,c^2+3a^4+18a^2bc+6b^3a+4a\,c^3+6b^2c^2)(x-1)}{3}+\frac{3x^{\frac{4}{3}}(14b\,x^2c^3+3}{}$
orering	Expression too large to display

input
```
int((a+b*x^(1/3)+c*x^(2/3))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x+3/7*b^4*x^(7/3)+3/11*c^4*x^(11/3)+2*b^3*a*x^2+4/3*a*c^3*x^3+18/5*a^2
*b^2*x^(5/3)+18/7*a^2*c^2*x^(7/3)+3*a^3*b*x^(4/3)+12/5*a^3*c*x^(5/3)+6/5*b
*c^3*x^(10/3)+2*b^2*c^2*x^3+3/2*b^3*c*x^(8/3)+9/2*b*a*c^2*x^(8/3)+36/7*a*b
^2*c*x^(7/3)+6*a^2*b*c*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.96

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^4\,dx = a^4x+\frac{2}{3}\left(3\,b^2c^2+2\,ac^3\right)x^3+2\left(ab^3+3\,a^2bc\right)x^2 + \frac{3}{110}\left(10\,c^4x^3+55\left(b^3c+3\,abc^2\right)x^2+44\left(3\,a^2b^2+2\,a^3c\right)x\right)x^{\frac{2}{3}} + \frac{3}{35}\left(14\,bc^3x^3+35\,a^3bx+5\left(b^4+12\,ab^2c+6\,a^2c^2\right)x^2\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^4,x, algorithm="fricas")
```

output
```
a^4*x + 2/3*(3*b^2*c^2 + 2*a*c^3)*x^3 + 2*(a*b^3 + 3*a^2*b*c)*x^2 + 3/110*
(10*c^4*x^3 + 55*(b^3*c + 3*a*b*c^2)*x^2 + 44*(3*a^2*b^2 + 2*a^3*c)*x)*x^(
2/3) + 3/35*(14*b*c^3*x^3 + 35*a^3*b*x + 5*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*
x^2)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.08

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^4\,dx = a^4x+3a^3bx^{\frac{4}{3}}+\frac{6bc^3x^{\frac{10}{3}}}{5}+\frac{3c^4x^{\frac{11}{3}}}{11} + \frac{3x^{\frac{8}{3}}\cdot\left(12abc^2+4b^3c\right)}{8}+\frac{3x^{\frac{7}{3}}\cdot\left(6a^2c^2+12ab^2c+b^4\right)}{7} + \frac{3x^{\frac{5}{3}}\cdot\left(4a^3c+6a^2b^2\right)}{5}+\frac{x^3\cdot\left(4ac^3+6b^2c^2\right)}{3}+\frac{x^2\cdot\left(12a^2bc+4ab^3\right)}{2}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**4,x)
```

output
```
a**4*x + 3*a**3*b*x**(4/3) + 6*b*c**3*x**(10/3)/5 + 3*c**4*x**(11/3)/11 +
3*x**(8/3)*(12*a*b*c**2 + 4*b**3*c)/8 + 3*x**(7/3)*(6*a**2*c**2 + 12*a*b**
2*c + b**4)/7 + 3*x**(5/3)*(4*a**3*c + 6*a**2*b**2)/5 + x**3*(4*a*c**3 + 6
*b**2*c**2)/3 + x**2*(12*a**2*b*c + 4*a*b**3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^4\,dx=\frac{3}{11}\,c^4x^{\frac{11}{3}}+\frac{6}{5}\,bc^3x^{\frac{10}{3}}+2\,b^2c^2x^3+\frac{3}{2}\,b^3cx^{\frac{8}{3}}+\frac{3}{7}\,b^4x^{\frac{7}{3}}+a^4x+\frac{3}{5}\left(4\,cx^{\frac{5}{3}}+5\,bx^{\frac{4}{3}}\right)a^3+\frac{6}{35}\left(15\,c^2x^{\frac{7}{3}}+35\,bcx^2+21\,b^2x^{\frac{5}{3}}\right)a^2+\frac{1}{42}\left(56\,c^3x^3+189\,bc^2x^{\frac{8}{3}}+216\,b^2cx^{\frac{7}{3}}+84\,b^3x^2\right)a$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^4,x, algorithm="maxima")
```

output
```
3/11*c^4*x^(11/3) + 6/5*b*c^3*x^(10/3) + 2*b^2*c^2*x^3 + 3/2*b^3*c*x^(8/3)
 + 3/7*b^4*x^(7/3) + a^4*x + 3/5*(4*c*x^(5/3) + 5*b*x^(4/3))*a^3 + 6/35*(1
5*c^2*x^(7/3) + 35*b*c*x^2 + 21*b^2*x^(5/3))*a^2 + 1/42*(56*c^3*x^3 + 189*
b*c^2*x^(8/3) + 216*b^2*c*x^(7/3) + 84*b^3*x^2)*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^4\,dx=\frac{3}{11}\,c^4x^{\frac{11}{3}}+\frac{6}{5}\,bc^3x^{\frac{10}{3}}+2\,b^2c^2x^3+\frac{4}{3}\,ac^3x^3+\frac{3}{2}\,b^3cx^{\frac{8}{3}}+\frac{9}{2}\,abc^2x^{\frac{8}{3}}+\frac{3}{7}\,b^4x^{\frac{7}{3}}+\frac{36}{7}\,ab^2cx^{\frac{7}{3}}+\frac{18}{7}\,a^2c^2x^{\frac{7}{3}}+2\,ab^3x^2+6\,a^2bcx^2+\frac{18}{5}\,a^2b^2x^{\frac{5}{3}}+\frac{12}{5}\,a^3cx^{\frac{5}{3}}+3\,a^3bx^{\frac{4}{3}}+a^4x$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^4,x, algorithm="giac")
```

output
```
3/11*c^4*x^(11/3) + 6/5*b*c^3*x^(10/3) + 2*b^2*c^2*x^3 + 4/3*a*c^3*x^3 + 3
/2*b^3*c*x^(8/3) + 9/2*a*b*c^2*x^(8/3) + 3/7*b^4*x^(7/3) + 36/7*a*b^2*c*x^
(7/3) + 18/7*a^2*c^2*x^(7/3) + 2*a*b^3*x^2 + 6*a^2*b*c*x^2 + 18/5*a^2*b^2*
x^(5/3) + 12/5*a^3*c*x^(5/3) + 3*a^3*b*x^(4/3) + a^4*x
```

Mupad [B] (verification not implemented)

Time = 18.91 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^4\,dx = x^{7/3}\left(\frac{18\,a^2\,c^2}{7}+\frac{36\,a\,b^2\,c}{7}+\frac{3\,b^4}{7}\right)+a^4\,x+\frac{3\,c^4\,x^{11/3}}{11} +x^{5/3}\left(\frac{12\,c\,a^3}{5}+\frac{18\,a^2\,b^2}{5}\right)+x^3\left(2\,b^2\,c^2+\frac{4\,a\,c^3}{3}\right)+3\,a^3\,b\,x^{4/3}+\frac{6\,b\,c^3\,x^{10/3}}{5}+2\,a\,b\,x^2\,\left(b^2+3\,a\,c\right)+\frac{3\,b\,c\,x^{8/3}\,(b^2}{2}$$

input
```
int((a + b*x^(1/3) + c*x^(2/3))^4,x)
```

output
```
x^(7/3)*((3*b^4)/7 + (18*a^2*c^2)/7 + (36*a*b^2*c)/7) + a^4*x + (3*c^4*x^(
11/3))/11 + x^(5/3)*((12*a^3*c)/5 + (18*a^2*b^2)/5) + x^3*((4*a*c^3)/3 + 2
*b^2*c^2) + 3*a^3*b*x^(4/3) + (6*b*c^3*x^(10/3))/5 + 2*a*b*x^2*(3*a*c + b^
2) + (3*b*c*x^(8/3)*(3*a*c + b^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \left(a+b\sqrt[3]{x} +cx^{2/3}\right)^4\,dx = \frac{x\left(5544x^{\frac{2}{3}}a^3c+8316x^{\frac{2}{3}}a^2b^2+10395x^{\frac{5}{3}}ab\,c^2+3465x^{\frac{5}{3}}b^3c+630x^{\frac{8}{3}}c^4+6930x^{\frac{1}{3}}a^3b+5940x^{\frac{4}{3}}\right.}{}$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^4,x)
```

output
```
(x*(5544*x**(2/3)*a**3*c + 8316*x**(2/3)*a**2*b**2 + 10395*x**(2/3)*a*b*c*
*2*x + 3465*x**(2/3)*b**3*c*x + 630*x**(2/3)*c**4*x**2 + 6930*x**(1/3)*a**
3*b + 5940*x**(1/3)*a**2*c**2*x + 11880*x**(1/3)*a*b**2*c*x + 990*x**(1/3)
*b**4*x + 2772*x**(1/3)*b*c**3*x**2 + 2310*a**4 + 13860*a**2*b*c*x + 4620*
a*b**3*x + 3080*a*c**3*x**2 + 4620*b**2*c**2*x**2))/2310
```

3.51 $\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 92

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3 \, dx = a^3x+\frac{9}{4}a^2bx^{4/3} + \frac{9}{5}a\left(b^2+ac\right)x^{5/3}+\frac{1}{2}b\left(b^2+6ac\right)x^2+\frac{9}{7}c\left(b^2+ac\right)x^{7/3}+\frac{9}{8}bc^2x^{8/3}+\frac{c^3x^3}{3}$$

output
```
a^3*x+9/4*a^2*b*x^(4/3)+9/5*a*(a*c+b^2)*x^(5/3)+1/2*b*(6*a*c+b^2)*x^2+9/7*
c*(a*c+b^2)*x^(7/3)+9/8*b*c^2*x^(8/3)+1/3*c^3*x^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3 \, dx = \frac{1}{840}\left(840a^3x+1890a^2bx^{4/3}+1512ab^2x^{5/3}+1512a^2cx^{5/3}+420b^3x^2+2520abcx^2+1080b^2cx^{7/3}+1080ac^2x^{7/3}+945bc^2x^{8/3}+280c^3x^3\right)$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^3,x]
```

output
```
(840*a^3*x + 1890*a^2*b*x^(4/3) + 1512*a*b^2*x^(5/3) + 1512*a^2*c*x^(5/3)
+ 420*b^3*x^2 + 2520*a*b*c*x^2 + 1080*b^2*c*x^(7/3) + 1080*a*c^2*x^(7/3) +
 945*b*c^2*x^(8/3) + 280*c^3*x^3)/840
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1680, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3 dx$$

$$\Big\downarrow \text{1680}$$

$$3\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^3 x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1140}$$

$$3\int \left(x^{2/3}a^3+3bxa^2+3\left(b^2+ac\right)x^{4/3}a+c^3x^{8/3}+3bc^2x^{7/3}+3c\left(b^2+ac\right)x^2+b\left(b^2+6ac\right)x^{5/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^3x}{3}+\frac{3}{4}a^2bx^{4/3}+\frac{3}{5}ax^{5/3}\left(ac+b^2\right)+\frac{3}{7}cx^{7/3}\left(ac+b^2\right)+\frac{1}{6}bx^2\left(6ac+b^2\right)+\frac{3}{8}bc^2x^{8/3}+\frac{c^3x^3}{9}\right)$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^3,x]
```

output
```
3*((a^3*x)/3 + (3*a^2*b*x^(4/3))/4 + (3*a*(b^2 + a*c)*x^(5/3))/5 + (b*(b^2
 + 6*a*c)*x^2)/6 + (3*c*(b^2 + a*c)*x^(7/3))/7 + (3*b*c^2*x^(8/3))/8 + (c^
3*x^3)/9)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

method	result
default	$a^3x+\frac{b^3x^2}{2}+\frac{c^3x^3}{3}+\frac{9b^2a\,x^{\frac{5}{3}}}{5}+\frac{9a\,c^2x^{\frac{7}{3}}}{7}+\frac{9a^2b\,x^{\frac{4}{3}}}{4}+\frac{9a^2c\,x^{\frac{5}{3}}}{5}+\frac{9b\,c^2x^{\frac{8}{3}}}{8}+\frac{9b^2c\,x^{\frac{7}{3}}}{7}+3abc\,x^2$
trager	$\frac{(2c^3x^2+18abcx+3b^3x+2c^3x+6a^3+18abc+3b^3+2c^3)(x-1)}{6}+\frac{9x^{\frac{4}{3}}(4a\,c^2x+4b^2cx+7b\,a^2)}{28}+\frac{9x^{\frac{5}{3}}(5b\,c^2x+8a^2c+8}{40}$
derivativedivides	$\frac{c^3x^3}{3}+\frac{9b\,c^2x^{\frac{8}{3}}}{8}+\frac{3(a\,c^2+2b^2c+c(2ac+b^2))x^{\frac{7}{3}}}{7}+\frac{(4abc+b(2ac+b^2))x^2}{2}+\frac{3(a(2ac+b^2)+2b^2a+a^2c)x^{\frac{5}{3}}}{5}+9a$
orering	Expression too large to display

input
```
int((a+b*x^(1/3)+c*x^(2/3))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+1/2*b^3*x^2+1/3*c^3*x^3+9/5*b^2*a*x^(5/3)+9/7*a*c^2*x^(7/3)+9/4*a^2*
b*x^(4/3)+9/5*a^2*c*x^(5/3)+9/8*b*c^2*x^(8/3)+9/7*b^2*c*x^(7/3)+3*a*b*c*x^
2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3\,dx=\frac{1}{3}\,c^3x^3+a^3x+\frac{1}{2}\,\left(b^3+6\,abc\right)x^2+\frac{9}{40}\,\left(5\,bc^2x^2+8\,\left(ab^2+a^2c\right)x\right)x^{\frac{2}{3}}+\frac{9}{28}\,\left(7\,a^2bx+4\,\left(b^2c+ac^2\right)x^2\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^3,x, algorithm="fricas")
```

output
```
1/3*c^3*x^3 + a^3*x + 1/2*(b^3 + 6*a*b*c)*x^2 + 9/40*(5*b*c^2*x^2 + 8*(a*b
^2 + a^2*c)*x)*x^(2/3) + 9/28*(7*a^2*b*x + 4*(b^2*c + a*c^2)*x^2)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3\,dx=a^3x+\frac{9a^2bx^{\frac{4}{3}}}{4}+\frac{9bc^2x^{\frac{8}{3}}}{8}+\frac{c^3x^3}{3}+\frac{3x^{\frac{7}{3}}\cdot\left(3ac^2+3b^2c\right)}{7}+\frac{3x^{\frac{5}{3}}\cdot\left(3a^2c+3ab^2\right)}{5}+\frac{x^2\cdot\left(6abc+b^3\right)}{2}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**3,x)
```

output
```
a**3*x + 9*a**2*b*x**(4/3)/4 + 9*b*c**2*x**(8/3)/8 + c**3*x**3/3 + 3*x**(7
/3)*(3*a*c**2 + 3*b**2*c)/7 + 3*x**(5/3)*(3*a**2*c + 3*a*b**2)/5 + x**2*(6
*a*b*c + b**3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3\,dx=\frac{1}{3}\,c^3x^3+\frac{9}{8}\,bc^2x^{\frac{8}{3}}+\frac{9}{7}\,b^2cx^{\frac{7}{3}}+\frac{1}{2}\,b^3x^2+a^3x+\frac{9}{20}\left(4\,cx^{\frac{5}{3}}+5\,bx^{\frac{4}{3}}\right)a^2+\frac{3}{35}\left(15\,c^2x^{\frac{7}{3}}+35\,bcx^2+21\,b^2x^{\frac{5}{3}}\right)a$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^3,x, algorithm="maxima")
```

output
```
1/3*c^3*x^3 + 9/8*b*c^2*x^(8/3) + 9/7*b^2*c*x^(7/3) + 1/2*b^3*x^2 + a^3*x
+ 9/20*(4*c*x^(5/3) + 5*b*x^(4/3))*a^2 + 3/35*(15*c^2*x^(7/3) + 35*b*c*x^2
 + 21*b^2*x^(5/3))*a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3\,dx=\frac{1}{3}\,c^3x^3+\frac{9}{8}\,bc^2x^{\frac{8}{3}}+\frac{9}{7}\,b^2cx^{\frac{7}{3}}+\frac{9}{7}\,ac^2x^{\frac{7}{3}}+\frac{1}{2}\,b^3x^2+3\,abcx^2+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{5}\,a^2cx^{\frac{5}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+a^3x$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^3,x, algorithm="giac")
```

output
```
1/3*c^3*x^3 + 9/8*b*c^2*x^(8/3) + 9/7*b^2*c*x^(7/3) + 9/7*a*c^2*x^(7/3) +
1/2*b^3*x^2 + 3*a*b*c*x^2 + 9/5*a*b^2*x^(5/3) + 9/5*a^2*c*x^(5/3) + 9/4*a^
2*b*x^(4/3) + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3\,dx = x^2\left(\frac{b^3}{2}+3\,a\,c\,b\right)+a^3\,x+\frac{c^3\,x^3}{3}+\frac{9\,a^2\,b\,x^{4/3}}{4}+\frac{9\,b\,c^2\,x^{8/3}}{8}+\frac{9\,a\,x^{5/3}\,(b^2+a\,c)}{5}+\frac{9\,c\,x^{7/3}\,(b^2+a\,c)}{7}$$

input
```
int((a + b*x^(1/3) + c*x^(2/3))^3,x)
```

output
```
x^2*(b^3/2 + 3*a*b*c) + a^3*x + (c^3*x^3)/3 + (9*a^2*b*x^(4/3))/4 + (9*b*c
^2*x^(8/3))/8 + (9*a*x^(5/3)*(a*c + b^2))/5 + (9*c*x^(7/3)*(a*c + b^2))/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3\,dx = \frac{x\left(1512x^{\frac{2}{3}}a^2c+1512x^{\frac{2}{3}}a\,b^2+945x^{\frac{5}{3}}b\,c^2+1890x^{\frac{1}{3}}a^2b+1080x^{\frac{4}{3}}a\,c^2+1080x^{\frac{4}{3}}b^2c+840a^3+\right.}{840}$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^3,x)
```

output
```
(x*(1512*x**(2/3)*a**2*c + 1512*x**(2/3)*a*b**2 + 945*x**(2/3)*b*c**2*x +
1890*x**(1/3)*a**2*b + 1080*x**(1/3)*a*c**2*x + 1080*x**(1/3)*b**2*c*x + 8
40*a**3 + 2520*a*b*c*x + 420*b**3*x + 280*c**3*x**2))/840
```

3.52 $\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2\,dx=a^2x+\frac{3}{2}abx^{4/3}+\frac{3}{5}\left(b^2+2ac\right)x^{5/3}+bcx^2+\frac{3}{7}c^2x^{7/3}$$

output
```
a^2*x+3/2*a*b*x^(4/3)+3/5*(2*a*c+b^2)*x^(5/3)+b*c*x^2+3/7*c^2*x^(7/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2\,dx=\frac{1}{70}\left(70a^2x+105abx^{4/3}+42b^2x^{5/3}+84acx^{5/3}+70bcx^2+30c^2x^{7/3}\right)$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^2,x]
```

output
```
(70*a^2*x + 105*a*b*x^(4/3) + 42*b^2*x^(5/3) + 84*a*c*x^(5/3) + 70*b*c*x^2
 + 30*c^2*x^(7/3))/70
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1680, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2 dx$$

$$\Big\downarrow \text{1680}$$

$$3\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^2 x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1140}$$

$$3\int \left(x^{2/3}a^2+2bxa+c^2x^2+2bcx^{5/3}+\left(b^2+2ac\right)x^{4/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^2x}{3}+\frac{1}{5}x^{5/3}\left(2ac+b^2\right)+\frac{1}{2}abx^{4/3}+\frac{1}{3}bcx^2+\frac{1}{7}c^2x^{7/3}\right)$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^2,x]
```

output
```
3*((a^2*x)/3 + (a*b*x^(4/3))/2 + ((b^2 + 2*a*c)*x^(5/3))/5 + (b*c*x^2)/3 +
 (c^2*x^(7/3))/7)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

method	result
derivativedivides	$x\,a^{2}+\frac{3 a b\,x^{\frac{4}{3}}}{2}+\frac{3 \left(2 a c+b^{2}\right) x^{\frac{5}{3}}}{5}+b c\,x^{2}+\frac{3 c^{2} x^{\frac{7}{3}}}{7}$
default	$x\,a^{2}+\frac{3 b^{2} x^{\frac{5}{3}}}{5}+\frac{3 c^{2} x^{\frac{7}{3}}}{7}+\frac{3 a b\,x^{\frac{4}{3}}}{2}+\frac{6 a c\,x^{\frac{5}{3}}}{5}+b c\,x^{2}$
trager	$\left(x -1\right) \left(b c x +a^{2}+b c \right)+\frac{3 x^{\frac{4}{3}} \left(2 c^{2} x +7 b a \right)}{14}+\frac{3 \left(2 a c +b^{2}\right) x^{\frac{5}{3}}}{5}$
orering	$\frac{\left(62 b\,c^{5} x^{3}-280 a^{2} c^{4} x^{2}+133 a\,b^{2} c^{3} x^{2}+42 a^{3} b\,c^{2} x +7 a^{4} b^{2}\right) \left(a +b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{2}}{70 c^{2} \left(b\,x^{2} c^{3}-4 a^{2} c^{2} x +2 a\,b^{2} x c +a^{3} b \right)}+\frac{9 c\,x^{3} \left(-3 b\,c^{2} x +16 a^{2} c -7 b^{2} a \right) \left(a +b\,x^{\frac{1}{3}}\right.}{35 \left(b\,x^{2} c^{3}-4 a^{2} c^{2} x +2 a\,b^{2}\right.}$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^2,x,method=_RETURNVERBOSE)
```

output
```
x*a^2+3/2*a*b*x^(4/3)+3/5*(2*a*c+b^2)*x^(5/3)+b*c*x^2+3/7*c^2*x^(7/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2\,dx = bcx^2+a^2x+\frac{3}{5}\,\left(b^2+2\,ac\right)x^{\frac{5}{3}}+\frac{3}{14}\,\left(2\,c^2x^2+7\,abx\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^2,x, algorithm="fricas")
```

output
```
b*c*x^2 + a^2*x + 3/5*(b^2 + 2*a*c)*x^(5/3) + 3/14*(2*c^2*x^2 + 7*a*b*x)*x
^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2\,dx=a^2x+\frac{3abx^{\frac{4}{3}}}{2}+bcx^2+\frac{3c^2x^{\frac{7}{3}}}{7}+\frac{3x^{\frac{5}{3}}\cdot(2ac+b^2)}{5}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**2,x)
```

output
```
a**2*x + 3*a*b*x**(4/3)/2 + b*c*x**2 + 3*c**2*x**(7/3)/7 + 3*x**(5/3)*(2*a
*c + b**2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2\,dx=\frac{3}{7}\,c^2x^{\frac{7}{3}}+bcx^2+\frac{3}{5}\,b^2x^{\frac{5}{3}}+a^2x+\frac{3}{10}\left(4\,cx^{\frac{5}{3}}+5\,bx^{\frac{4}{3}}\right)a$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^2,x, algorithm="maxima")
```

output
```
3/7*c^2*x^(7/3) + b*c*x^2 + 3/5*b^2*x^(5/3) + a^2*x + 3/10*(4*c*x^(5/3) +
5*b*x^(4/3))*a
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2\,dx=\frac{3}{7}\,c^2x^{\frac{7}{3}}+bcx^2+\frac{3}{5}\,b^2x^{\frac{5}{3}}+\frac{6}{5}\,acx^{\frac{5}{3}}+\frac{3}{2}\,abx^{\frac{4}{3}}+a^2x$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^2,x, algorithm="giac")
```

output
```
3/7*c^2*x^(7/3) + b*c*x^2 + 3/5*b^2*x^(5/3) + 6/5*a*c*x^(5/3) + 3/2*a*b*x^
(4/3) + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2\,dx=a^2\,x+x^{5/3}\left(\frac{3\,b^2}{5}+\frac{6\,a\,c}{5}\right)+\frac{3\,c^2\,x^{7/3}}{7}+\frac{3\,a\,b\,x^{4/3}}{2}+b\,c\,x^2$$

input
```
int((a + b*x^(1/3) + c*x^(2/3))^2,x)
```

output
```
a^2*x + x^(5/3)*((6*a*c)/5 + (3*b^2)/5) + (3*c^2*x^(7/3))/7 + (3*a*b*x^(4/
3))/2 + b*c*x^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2\,dx=\frac{x\left(84x^{\frac{2}{3}}ac+42x^{\frac{2}{3}}b^2+105x^{\frac{1}{3}}ab+30x^{\frac{4}{3}}c^2+70a^2+70bcx\right)}{70}$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^2,x)
```

output
```
(x*(84*x**(2/3)*a*c + 42*x**(2/3)*b**2 + 105*x**(1/3)*a*b + 30*x**(1/3)*c*
*2*x + 70*a**2 + 70*b*c*x))/70
```

3.53 $\int \frac{1}{a+b\sqrt[3]{x}+cx^{2/3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{1}{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \frac{3\sqrt[3]{x}}{c} - \frac{3(b^2-2ac)\,\text{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{3b\log\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}{2c^2}$$

output
```
3*x^(1/3)/c-3*(-2*a*c+b^2)*arctanh((b+2*c*x^(1/3))/(-4*a*c+b^2)^(1/2))/c^2
/(-4*a*c+b^2)^(1/2)-3/2*b*ln(a+b*x^(1/3)+c*x^(2/3))/c^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{1}{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \frac{3\left(2c\sqrt[3]{x}+\frac{2(b^2-2ac)\arctan\left(\frac{b+2c\sqrt[3]{x}}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+b\sqrt[3]{x}+cx^{2/3}\right)\right)}{2c^2}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^(-1),x]
```

output
```
(3*(2*c*x^(1/3) + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^(1/3))/Sqrt[-b^2 + 4*
a*c]])/Sqrt[-b^2 + 4*a*c] - b*Log[a + b*x^(1/3) + c*x^(2/3)]))/(2*c^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1680, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sqrt[3]{x}+cx^{2/3}}dx$$
$$\Big\downarrow \text{1680}$$
$$3\int \frac{x^{2/3}}{a+cx^{2/3}+b\sqrt[3]{x}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{1143}$$
$$3\int \left(\frac{1}{c}-\frac{a+b\sqrt[3]{x}}{c\left(a+cx^{2/3}+b\sqrt[3]{x}\right)}\right)d\sqrt[3]{x}$$
$$\Big\downarrow \text{2009}$$
$$3\left(-\frac{\left(b^2-2ac\right)\text{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}{2c^2}+\frac{\sqrt[3]{x}}{c}\right)$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^(-1),x]
```

output
```
3*(x^(1/3)/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^(1/3))/Sqrt[b^2 - 4*a*c]]
)/(c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^(1/3) + c*x^(2/3)])/(2*c^2))
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{3x^{\frac{1}{3}}}{c}+\frac{-\frac{3b\ln\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)}{2c}+\frac{6\left(-a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{b+2c\,x^{\frac{1}{3}}}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$	83
default	Expression too large to display	1122

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
3*x^(1/3)/c+3/c*(-1/2*b/c*ln(a+b*x^(1/3)+c*x^(2/3))+2*(-a+1/2*b^2/c)/(4*a*
c-b^2)^(1/2)*arctan((b+2*c*x^(1/3))/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.86

$$\int \frac{1}{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \left[-\frac{3\left((b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^4x^2+a^2b^2-2\,a^3c+(b^3c-2\,abc^2)x+\left(b^4-5\,ab^2c+4\,a^2c^2+(2\,c^3\cdots\right.}{\cdots}\right.\right.}{\cdots}\right.$$

$$-\frac{3\left(2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}cx^{\frac{1}{3}}+\sqrt{-b^2+4\,ac}b}{b^2-4\,ac}\right)+(b^3-4\,abc)\log\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)-2\,\cdots\right.}{2\,(b^2c^2-4\,ac^3)}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3)),x, algorithm="fricas")
```

output
```
[-3/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^4*x^2 + a^2*b^2 - 2*a^3*c
+ (b^3*c - 2*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2 + (2*c^3*x + b^3 -
3*a*b*c)*sqrt(b^2 - 4*a*c))*x^(2/3) + (a^2*b + (b^2*c - 2*a*c^2)*x)*sqrt(b
^2 - 4*a*c) - (a*b^3 - 4*a^2*b*c - (b^2*c^2 - 4*a*c^3)*x + (b*c^2*x + a*b^
2 - 2*a^2*c)*sqrt(b^2 - 4*a*c))*x^(1/3))/(c^3*x^2 + a^3 + (b^3 - 3*a*b*c)*
x)) + (b^3 - 4*a*b*c)*log(c*x^(2/3) + b*x^(1/3) + a) - 2*(b^2*c - 4*a*c^2)
*x^(1/3))/(b^2*c^2 - 4*a*c^3), -3/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*ar
ctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^(1/3) + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c
)) + (b^3 - 4*a*b*c)*log(c*x^(2/3) + b*x^(1/3) + a) - 2*(b^2*c - 4*a*c^2)*
x^(1/3))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(82) = 164$.

Time = 1.35 (sec) , antiderivative size = 644, normalized size of antiderivative = 7.58

$$\int \frac{1}{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \begin{cases} \frac{3a^2\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{b^3}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{\frac{2}{3}}}{2b} \\ -\frac{3b^2\log\left(\frac{b}{2c}+\sqrt[3]{x}\right)}{bc^2+2c^3\sqrt[3]{x}}-\frac{3b^2}{bc^2+2c^3\sqrt[3]{x}}-\frac{6bc\sqrt[3]{x}\log\left(\frac{b}{2c}+\sqrt[3]{x}\right)}{bc^2+2c^3\sqrt[3]{x}}+\frac{6c^2x^{\frac{2}{3}}}{bc^2+2c^3\sqrt[3]{x}} \\ -\frac{12abc\log\left(\frac{b}{2c}+\sqrt[3]{x}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}{8ac^3-2b^2c^2}-\frac{12abc\log\left(\frac{b}{2c}+\sqrt[3]{x}+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{8ac^3-2b^2c^2}+\frac{24ac^2\sqrt[3]{x}}{8ac^3-2b^2c^2}+\frac{6ac\sqrt{-4a\cdots}}{\cdots} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3)+c*x**(2/3)),x)
```

output
```
Piecewise((3*a**2*log(a/b + x**(1/3))/b**3 - 3*a*x**(1/3)/b**2 + 3*x**(2/3
)/(2*b), Eq(c, 0)), (-3*b**2*log(b/(2*c) + x**(1/3))/(b*c**2 + 2*c**3*x**(
1/3)) - 3*b**2/(b*c**2 + 2*c**3*x**(1/3)) - 6*b*c*x**(1/3)*log(b/(2*c) + x
**(1/3))/(b*c**2 + 2*c**3*x**(1/3)) + 6*c**2*x**(2/3)/(b*c**2 + 2*c**3*x**
(1/3)), Eq(a, b**2/(4*c))), (-12*a*b*c*log(b/(2*c) + x**(1/3) - sqrt(-4*a*
c + b**2)/(2*c))/(8*a*c**3 - 2*b**2*c**2) - 12*a*b*c*log(b/(2*c) + x**(1/3
) + sqrt(-4*a*c + b**2)/(2*c))/(8*a*c**3 - 2*b**2*c**2) + 24*a*c**2*x**(1/
3)/(8*a*c**3 - 2*b**2*c**2) + 6*a*c*sqrt(-4*a*c + b**2)*log(b/(2*c) + x**(
1/3) - sqrt(-4*a*c + b**2)/(2*c))/(8*a*c**3 - 2*b**2*c**2) - 6*a*c*sqrt(-4
*a*c + b**2)*log(b/(2*c) + x**(1/3) + sqrt(-4*a*c + b**2)/(2*c))/(8*a*c**3
 - 2*b**2*c**2) + 3*b**3*log(b/(2*c) + x**(1/3) - sqrt(-4*a*c + b**2)/(2*c
))/(8*a*c**3 - 2*b**2*c**2) + 3*b**3*log(b/(2*c) + x**(1/3) + sqrt(-4*a*c
+ b**2)/(2*c))/(8*a*c**3 - 2*b**2*c**2) - 6*b**2*c*x**(1/3)/(8*a*c**3 - 2*
b**2*c**2) - 3*b**2*sqrt(-4*a*c + b**2)*log(b/(2*c) + x**(1/3) - sqrt(-4*a
*c + b**2)/(2*c))/(8*a*c**3 - 2*b**2*c**2) + 3*b**2*sqrt(-4*a*c + b**2)*lo
g(b/(2*c) + x**(1/3) + sqrt(-4*a*c + b**2)/(2*c))/(8*a*c**3 - 2*b**2*c**2)
, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = -\frac{3\,b\log\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)}{2\,c^2} + \frac{3\,(b^2-2\,ac)\arctan\left(\frac{2\,cx^{\frac{1}{3}}+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2} + \frac{3\,x^{\frac{1}{3}}}{c}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3)),x, algorithm="giac")
```

output
```
-3/2*b*log(c*x^(2/3) + b*x^(1/3) + a)/c^2 + 3*(b^2 - 2*a*c)*arctan((2*c*x^
(1/3) + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2) + 3*x^(1/3)/c
```

Mupad [B] (verification not implemented)

Time = 18.73 (sec) , antiderivative size = 517, normalized size of antiderivative = 6.08

$$\int \frac{1}{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \frac{3\,x^{1/3}}{c} + \frac{3\,b^3\,\ln\left(9\,a\,b^4+36\,a^3\,c^2+9\,b^5\,x^{1/3}-36\,a^2\,b^2\,c+9\,b^4\,c\,x^{2/3}+36\,a^2\,c^3\,x^{2/3}+36\,a^2\,b\,c^2\,x^{1/3}-36\,a\,b^2\,c^2\,x^{2/3}\right.}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} - \frac{3\,b^2\,\mathrm{atan}\left(\frac{3\,b^3}{\sqrt{4\,a\,c-b^2}\,(6\,a\,c-3\,b^2)}-\frac{12\,a\,c^2\,x^{1/3}}{\sqrt{4\,a\,c-b^2}\,(6\,a\,c-3\,b^2)}+\frac{6\,b^2\,c\,x^{1/3}}{\sqrt{4\,a\,c-b^2}\,(6\,a\,c-3\,b^2)}-\frac{6\,a\,b\,c}{\sqrt{4\,a\,c-b^2}\,(6\,a\,c-3\,b^2)}\right)}{c^2\,\sqrt{4\,a\,c-b^2}} + \frac{6\,a\,\mathrm{atan}\left(\frac{3\,b^3}{\sqrt{4\,a\,c-b^2}\,(6\,a\,c-3\,b^2)}-\frac{12\,a\,c^2\,x^{1/3}}{\sqrt{4\,a\,c-b^2}\,(6\,a\,c-3\,b^2)}+\frac{6\,b^2\,c\,x^{1/3}}{\sqrt{4\,a\,c-b^2}\,(6\,a\,c-3\,b^2)}-\frac{6\,a\,b\,c}{\sqrt{4\,a\,c-b^2}\,(6\,a\,c-3\,b^2)}\right)}{c\,\sqrt{4\,a\,c-b^2}} - \frac{6\,a\,b\,c\,\ln\left(9\,a\,b^4+36\,a^3\,c^2+9\,b^5\,x^{1/3}-36\,a^2\,b^2\,c+9\,b^4\,c\,x^{2/3}+36\,a^2\,c^3\,x^{2/3}+36\,a^2\,b\,c^2\,x^{1/3}-36\,a\,b^2\,c^2\,x\right.}{4\,a\,c^3-b^2\,c^2}$$

input
```
int(1/(a + b*x^(1/3) + c*x^(2/3)),x)
```

output
```
(3*x^(1/3))/c + (3*b^3*log(9*a*b^4 + 36*a^3*c^2 + 9*b^5*x^(1/3) - 36*a^2*b
^2*c + 9*b^4*c*x^(2/3) + 36*a^2*c^3*x^(2/3) + 36*a^2*b*c^2*x^(1/3) - 36*a*
b^2*c^2*x^(2/3) - 36*a*b^3*c*x^(1/3)))/(2*(4*a*c^3 - b^2*c^2)) - (3*b^2*at
an((3*b^3)/((4*a*c - b^2)^(1/2)*(6*a*c - 3*b^2)) - (12*a*c^2*x^(1/3))/((4*
a*c - b^2)^(1/2)*(6*a*c - 3*b^2)) + (6*b^2*c*x^(1/3))/((4*a*c - b^2)^(1/2)
*(6*a*c - 3*b^2)) - (6*a*b*c)/((4*a*c - b^2)^(1/2)*(6*a*c - 3*b^2))))/(c^2
*(4*a*c - b^2)^(1/2)) + (6*a*atan((3*b^3)/((4*a*c - b^2)^(1/2)*(6*a*c - 3*
b^2)) - (12*a*c^2*x^(1/3))/((4*a*c - b^2)^(1/2)*(6*a*c - 3*b^2)) + (6*b^2*
c*x^(1/3))/((4*a*c - b^2)^(1/2)*(6*a*c - 3*b^2)) - (6*a*b*c)/((4*a*c - b^2
)^(1/2)*(6*a*c - 3*b^2))))/(c*(4*a*c - b^2)^(1/2)) - (6*a*b*c*log(9*a*b^4
+ 36*a^3*c^2 + 9*b^5*x^(1/3) - 36*a^2*b^2*c + 9*b^4*c*x^(2/3) + 36*a^2*c^3
*x^(2/3) + 36*a^2*b*c^2*x^(1/3) - 36*a*b^2*c^2*x^(2/3) - 36*a*b^3*c*x^(1/3
)))/(4*a*c^3 - b^2*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.74

$$\int \frac{1}{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \frac{-6\sqrt{4ac-b^2}\,atan\left(\frac{2x^{\frac{1}{3}}c+b}{\sqrt{4ac-b^2}}\right)ac+3\sqrt{4ac-b^2}\,atan\left(\frac{2x^{\frac{1}{3}}c+b}{\sqrt{4ac-b^2}}\right)b^2+12x^{\frac{1}{3}}a\,c^2-3x}{c^2\left(4ac-b^2\right)}$$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3)),x)
```

output
```
(3*( - 4*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a*c - b**2))*a*
c + 2*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a*c - b**2))*b**2
+ 8*x**(1/3)*a*c**2 - 2*x**(1/3)*b**2*c - 4*log(x**(2/3)*c + x**(1/3)*b +
a)*a*b*c + log(x**(2/3)*c + x**(1/3)*b + a)*b**3))/(2*c**2*(4*a*c - b**2))
```

3.54 $\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 91

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}\,dx=-\frac{3\left(ab+\left(b^2-2ac\right)\sqrt[3]{x}\right)}{c\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}+\frac{12a\text{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

output
```
(-3*a*b-3*(-2*a*c+b^2)*x^(1/3))/c/(-4*a*c+b^2)/(a+b*x^(1/3)+c*x^(2/3))+12*
a*arctanh((b+2*c*x^(1/3))/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}\,dx=\frac{3\left(a\left(b-2c\sqrt[3]{x}\right)+b^2\sqrt[3]{x}\right)}{c\left(-b^2+4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}+\frac{12a\arctan\left(\frac{b+2c\sqrt[3]{x}}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^(-2),x]
```

output
```
(3*(a*(b - 2*c*x^(1/3)) + b^2*x^(1/3)))/(c*(-b^2 + 4*a*c)*(a + b*x^(1/3) +
 c*x^(2/3))) + (12*a*ArcTan[(b + 2*c*x^(1/3))/Sqrt[-b^2 + 4*a*c]])/(-b^2 +
 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1680, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}dx$$

$$\Big\downarrow \text{1680}$$

$$3\int \frac{x^{2/3}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1153}$$

$$3\left(\frac{\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}-\frac{2a\int \frac{1}{a+cx^{2/3}+b\sqrt[3]{x}}d\sqrt[3]{x}}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1083}$$

$$3\left(\frac{4a\int \frac{1}{b^2-4ac-x^{2/3}}d\left(b+2c\sqrt[3]{x}\right)}{b^2-4ac}+\frac{\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}\right)$$

$$\Big\downarrow \text{219}$$

$$3\left(\frac{4a\text{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}\right)$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^(-2),x]
```

output
```
3*(((2*a + b*x^(1/3))*x^(1/3))/((b^2 - 4*a*c)*(a + b*x^(1/3) + c*x^(2/3)))
 + (4*a*ArcTanh[(b + 2*c*x^(1/3))/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{-\frac{3\left(2ac-b^{2}\right)x^{\frac{1}{3}}}{c\left(4ac-b^{2}\right)}+\frac{3ab}{c\left(4ac-b^{2}\right)}}{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}+\frac{12a\arctan\left(\frac{b+2c\,x^{\frac{1}{3}}}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}$	104
default	Expression too large to display	8522

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^2,x,method=_RETURNVERBOSE)
```

output
```
3*(-1/c*(2*a*c-b^2)/(4*a*c-b^2)*x^(1/3)+1/c*a*b/(4*a*c-b^2))/(a+b*x^(1/3)+
c*x^(2/3))+12*a/(4*a*c-b^2)^(3/2)*arctan((b+2*c*x^(1/3))/(4*a*c-b^2)^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(79) = 158$.

Time = 0.10 (sec) , antiderivative size = 833, normalized size of antiderivative = 9.15

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^2,x, algorithm="fricas")
```

output
```
[-3*(a^3*b^3 - 4*a^4*b*c + 2*(a*c^4*x^2 + a^4*c + (a*b^3*c - 3*a^2*b*c^2)*
x)*sqrt(b^2 - 4*a*c)*log((2*c^4*x^2 + a^2*b^2 - 2*a^3*c + (b^3*c - 2*a*b*c
^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2 - (2*c^3*x + b^3 - 3*a*b*c)*sqrt(b^2
- 4*a*c))*x^(2/3) - (a^2*b + (b^2*c - 2*a*c^2)*x)*sqrt(b^2 - 4*a*c) - (a*b
^3 - 4*a^2*b*c - (b^2*c^2 - 4*a*c^3)*x - (b*c^2*x + a*b^2 - 2*a^2*c)*sqrt(
b^2 - 4*a*c))*x^(1/3))/(c^3*x^2 + a^3 + (b^3 - 3*a*b*c)*x)) + (b^6 - 8*a*b
^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*x + (a^2*b^3*c - 4*a^3*b*c^2 + (b^4*c^2
 - 6*a*b^2*c^3 + 8*a^2*c^4)*x)*x^(2/3) - (2*a^3*b^2*c - 8*a^4*c^2 + (b^5*c
 - 7*a*b^3*c^2 + 12*a^2*b*c^3)*x)*x^(1/3))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16
*a^5*c^3 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^7*c - 11*a*b^5*c^
2 + 40*a^2*b^3*c^3 - 48*a^3*b*c^4)*x), -3*(a^3*b^3 - 4*a^4*b*c - 4*(a*c^4*
x^2 + a^4*c + (a*b^3*c - 3*a^2*b*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-(2*sqr
t(-b^2 + 4*a*c)*c*x^(1/3) + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^6 -
8*a*b^4*c + 18*a^2*b^2*c^2 - 8*a^3*c^3)*x + (a^2*b^3*c - 4*a^3*b*c^2 + (b^
4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*x)*x^(2/3) - (2*a^3*b^2*c - 8*a^4*c^2 + (
b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*x)*x^(1/3))/(a^3*b^4*c - 8*a^4*b^2*c^2
 + 16*a^5*c^3 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^7*c - 11*a*b
^5*c^2 + 40*a^2*b^3*c^3 - 48*a^3*b*c^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*x**(1/3)+c*x**(2/3))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}\,dx = -\frac{12\,a\arctan\left(\frac{2\,cx^{\frac{1}{3}}+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}} - \frac{3\left(b^2x^{\frac{1}{3}}-2\,acx^{\frac{1}{3}}+ab\right)}{\left(b^2c-4\,ac^2\right)\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^2,x, algorithm="giac")
```

output
```
-12*a*arctan((2*c*x^(1/3) + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^
2 + 4*a*c)) - 3*(b^2*x^(1/3) - 2*a*c*x^(1/3) + a*b)/((b^2*c - 4*a*c^2)*(c*
x^(2/3) + b*x^(1/3) + a))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}\,dx=-\frac{\frac{3\,x^{1/3}\,\left(2\,a\,c-b^2\right)}{c\,\left(4\,a\,c-b^2\right)}-\frac{3\,a\,b}{c\,\left(4\,a\,c-b^2\right)}}{a+b\,x^{1/3}+c\,x^{2/3}}-\frac{12\,a\,\mathrm{atan}\left(\frac{\left(4\,a\,c-b^2\right)\,\left(\frac{6\,a\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{12\,a\,c\,x^{1/3}}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)}{6\,a}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(1/(a + b*x^(1/3) + c*x^(2/3))^2,x)
```

output
```
- ((3*x^(1/3)*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (3*a*b)/(c*(4*a*c - b^2))
)/(a + b*x^(1/3) + c*x^(2/3)) - (12*a*atan(((4*a*c - b^2)*((6*a*(b^3 - 4*a
*b*c))/(4*a*c - b^2)^(5/2) - (12*a*c*x^(1/3))/(4*a*c - b^2)^(3/2)))/(6*a))
)/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.84

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}\,dx=\frac{12x^{\frac{2}{3}}\sqrt{4ac-b^2}\,atan\left(\frac{2x^{\frac{1}{3}}c+b}{\sqrt{4ac-b^2}}\right)abc+12x^{\frac{1}{3}}\sqrt{4ac-b^2}\,atan\left(\frac{2x^{\frac{1}{3}}c+b}{\sqrt{4ac-b^2}}\right)ab^2+12}{b\left(16x^{\frac{2}{3}}a^2c^3-8x^{\frac{2}{3}}ab^2c^2+x^{\frac{2}{3}}b^4c+16x^{\frac{1}{3}}a^2bc\right.}$$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^2,x)
```

output
```
(3*(4*x**(2/3)*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a*c - b**
2))*a*b*c + 4*x**(1/3)*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a
*c - b**2))*a*b**2 + 4*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a
*c - b**2))*a**2*b + 8*x**(2/3)*a**2*c**2 - 6*x**(2/3)*a*b**2*c + x**(2/3)
*b**4 + 8*a**3*c - 2*a**2*b**2))/(b*(16*x**(2/3)*a**2*c**3 - 8*x**(2/3)*a*
b**2*c**2 + x**(2/3)*b**4*c + 16*x**(1/3)*a**2*b*c**2 - 8*x**(1/3)*a*b**3*
c + x**(1/3)*b**5 + 16*a**3*c**2 - 8*a**2*b**2*c + a*b**4))
```

3.55 $\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 153

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3}\,dx=-\frac{3\left(ab+(b^2-2ac)\sqrt[3]{x}\right)}{2c\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}+\frac{3(b^2+2ac)\left(b+2c\sqrt[3]{x}\right)}{2c\left(b^2-4ac\right)^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}-\frac{6(b^2+2ac)\,\text{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

output
```
1/2*(-3*a*b-3*(-2*a*c+b^2)*x^(1/3))/c/(-4*a*c+b^2)/(a+b*x^(1/3)+c*x^(2/3))
^2+3/2*(2*a*c+b^2)*(b+2*c*x^(1/3))/c/(-4*a*c+b^2)^2/(a+b*x^(1/3)+c*x^(2/3)
)-6*(2*a*c+b^2)*arctanh((b+2*c*x^(1/3))/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
5/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3}\,dx=\frac{3\left(a^2\left(6b-4c\sqrt[3]{x}\right)+3b^3x^{2/3}+2b^2cx+2a\left(5b^2\sqrt[3]{x}+3bcx^{2/3}+2c^2x\right)\right)}{2\left(b^2-4ac\right)^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}+\frac{6(b^2+2ac)\arctan\left(\frac{b+2c\sqrt[3]{x}}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^(-3),x]
```

output
```
(3*(a^2*(6*b - 4*c*x^(1/3)) + 3*b^3*x^(2/3) + 2*b^2*c*x + 2*a*(5*b^2*x^(1/
3) + 3*b*c*x^(2/3) + 2*c^2*x)))/(2*(b^2 - 4*a*c)^2*(a + b*x^(1/3) + c*x^(2
/3))^2) + (6*(b^2 + 2*a*c)*ArcTan[(b + 2*c*x^(1/3))/Sqrt[-b^2 + 4*a*c]])/(
-b^2 + 4*a*c)^(5/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1680, 1164, 27, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3}dx$$

$$\Big\downarrow \text{1680}$$

$$3\int \frac{x^{2/3}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1164}$$

$$3\left(\frac{\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{2\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}-\frac{\int\frac{2\left(a-b\sqrt[3]{x}\right)}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^2}d\sqrt[3]{x}}{2\left(b^2-4ac\right)}\right)$$

$\big\downarrow$ 27

$$3\left(\frac{\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{2\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}-\frac{\int\frac{a-b\sqrt[3]{x}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^2}d\sqrt[3]{x}}{b^2-4ac}\right)$$

$\big\downarrow$ 1159

$$3\left(\frac{\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{2\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}-\frac{-\frac{\left(2ac+b^2\right)\int\frac{1}{a+cx^{2/3}+b\sqrt[3]{x}}d\sqrt[3]{x}}{b^2-4ac}-\frac{\sqrt[3]{x}\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}}{b^2-4ac}\right)$$

$\big\downarrow$ 1083

$$3\left(\frac{\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{2\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}-\frac{\frac{2\left(2ac+b^2\right)\int\frac{1}{b^2-4ac-x^{2/3}}d\left(b+2c\sqrt[3]{x}\right)}{b^2-4ac}-\frac{\sqrt[3]{x}\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}}{b^2-4ac}\right)$$

$\big\downarrow$ 219

$$3\left(\frac{\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{2\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^2}-\frac{\frac{2\left(2ac+b^2\right)\text{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{\sqrt[3]{x}\left(2ac+b^2\right)+3ab}{\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}}{b^2-4ac}\right)$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^(-3),x]
```

output
```
3*(((2*a + b*x^(1/3))*x^(1/3))/(2*(b^2 - 4*a*c)*(a + b*x^(1/3) + c*x^(2/3)
)^2) - (-((3*a*b + (b^2 + 2*a*c)*x^(1/3))/((b^2 - 4*a*c)*(a + b*x^(1/3) +
c*x^(2/3)))) + (2*(b^2 + 2*a*c)*ArcTanh[(b + 2*c*x^(1/3))/Sqrt[b^2 - 4*a*c
]])/(b^2 - 4*a*c)^(3/2))/(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{3c\left(2ac+b^2\right)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{9b\left(2ac+b^2\right)x^{\frac{2}{3}}}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3a\left(2ac-5b^2\right)x^{\frac{1}{3}}}{16a^2c^2-8a\,b^2c+b^4}+\frac{9a^2b}{16a^2c^2-8a\,b^2c+b^4}}{\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^2}+\frac{6\left(2ac+b^2\right)\arctan\left(\frac{b+2c\,x^{\frac{1}{3}}}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}$
default	Expression too large to display

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^3,x,method=_RETURNVERBOSE)
```

output
```
3*(c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+3/2*b*(2*a*c+b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^(2/3)-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/3
)+3*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4))/(a+b*x^(1/3)+c*x^(2/3))^2+6*(2*a*c+b
^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((b+2*c*x^(1/3))/(4
*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. 2(131) = 262.

Time = 0.25 (sec) , antiderivative size = 1937, normalized size of antiderivative = 12.66

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^3,x, algorithm="fricas")
```

output
```
[3/2*(6*a^6*b^3 - 24*a^7*b*c + 6*(a*b^4*c^4 - 6*a^2*b^2*c^5 + 8*a^3*c^6)*x
^3 + 3*(b^9 - 10*a*b^7*c + 33*a^2*b^5*c^2 - 34*a^3*b^3*c^3 - 8*a^4*b*c^4)*
x^2 + 2*(a^6*b^2 + 2*a^7*c + (b^2*c^6 + 2*a*c^7)*x^4 + 2*(b^5*c^3 - a*b^3*
c^4 - 6*a^2*b*c^5)*x^3 + (b^8 - 4*a*b^6*c - 3*a^2*b^4*c^2 + 20*a^3*b^2*c^3
 + 4*a^4*c^4)*x^2 + 2*(a^3*b^5 - a^4*b^3*c - 6*a^5*b*c^2)*x)*sqrt(b^2 - 4*
a*c)*log((2*c^4*x^2 + a^2*b^2 - 2*a^3*c + (b^3*c - 2*a*b*c^2)*x + (b^4 - 5
*a*b^2*c + 4*a^2*c^2 - (2*c^3*x + b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c))*x^(2/3
) - (a^2*b + (b^2*c - 2*a*c^2)*x)*sqrt(b^2 - 4*a*c) - (a*b^3 - 4*a^2*b*c -
 (b^2*c^2 - 4*a*c^3)*x - (b*c^2*x + a*b^2 - 2*a^2*c)*sqrt(b^2 - 4*a*c))*x^
(1/3))/(c^3*x^2 + a^3 + (b^3 - 3*a*b*c)*x)) + 6*(2*a^3*b^6 - 15*a^4*b^4*c
+ 30*a^5*b^2*c^2 - 8*a^6*c^3)*x + (a^4*b^5 - 2*a^5*b^3*c - 8*a^6*b*c^2 + 2
*(b^4*c^5 - 2*a*b^2*c^6 - 8*a^2*c^7)*x^3 + (5*b^7*c^2 - 36*a*b^5*c^3 + 66*
a^2*b^3*c^4 - 8*a^3*b*c^5)*x^2 + 2*(2*a*b^8 - 19*a^2*b^6*c + 63*a^3*b^4*c^
2 - 86*a^4*b^2*c^3 + 40*a^5*c^4)*x)*x^(2/3) - (2*a^5*b^4 - 4*a^6*b^2*c - 1
6*a^7*c^2 + (b^5*c^4 - 2*a*b^3*c^5 - 8*a^2*b*c^6)*x^3 + 2*(2*b^8*c - 19*a*
b^6*c^2 + 63*a^2*b^4*c^3 - 86*a^3*b^2*c^4 + 40*a^4*c^5)*x^2 + (5*a^2*b^7 -
 36*a^3*b^5*c + 66*a^4*b^3*c^2 - 8*a^5*b*c^3)*x)*x^(1/3))/(a^6*b^6 - 12*a^
7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3 + (b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b
^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^9*c^3 - 15*a*b^7*c^4 + 84*a^2*b^5*c^5 - 20
8*a^3*b^3*c^6 + 192*a^4*b*c^7)*x^3 + (b^12 - 18*a*b^10*c + 129*a^2*b^8*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*x**(1/3)+c*x**(2/3))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3}\,dx = \frac{6\,(b^2+2\,ac)\arctan\left(\frac{2\,cx^{\frac{1}{3}}+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{3\left(2\,b^2cx+4\,ac^2x+3\,b^3x^{\frac{2}{3}}+6\,abcx^{\frac{2}{3}}+10\,ab^2x^{\frac{1}{3}}-4\,a^2cx^{\frac{1}{3}}+6\,a^2b\right)}{2\,(b^4-8\,ab^2c+16\,a^2c^2)\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^2}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^3,x, algorithm="giac")
```

output
```
6*(b^2 + 2*a*c)*arctan((2*c*x^(1/3) + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b
^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 3/2*(2*b^2*c*x + 4*a*c^2*x + 3*b^
3*x^(2/3) + 6*a*b*c*x^(2/3) + 10*a*b^2*x^(1/3) - 4*a^2*c*x^(1/3) + 6*a^2*b
)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^(2/3) + b*x^(1/3) + a)^2)
```

Mupad [B] (verification not implemented)

Time = 18.97 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.08

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3}\,dx=\frac{\frac{9\,a^2\,b}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{9\,b\,x^{2/3}\,\left(b^2+2\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{3\,a\,x^{1/3}\,\left(2\,a\,c-5\,b^2\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,c\,x\,\left(b^2+2\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^{2/3}\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^{4/3}+2\,b\,c\,x+2\,a\,b\,x^{1/3}}+\frac{6\,\mathrm{atan}\left(\frac{\left(\frac{3\,\left(b^2+2\,a\,c\right)\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{6\,c\,x^{1/3}\,\left(b^2+2\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{3\,b^2+6\,a\,c}\right)\,\left(b^2+2\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

input
```
int(1/(a + b*x^(1/3) + c*x^(2/3))^3,x)
```

output
```
((9*a^2*b)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*x^(2/3)*(2*a*c + b^2))/(2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*a*x^(1/3)*(2*a*c - 5*b^2))/(b^4 + 16
*a^2*c^2 - 8*a*b^2*c) + (3*c*x*(2*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*
c))/(x^(2/3)*(2*a*c + b^2) + a^2 + c^2*x^(4/3) + 2*b*c*x + 2*a*b*x^(1/3))
+ (6*atan((((3*(2*a*c + b^2)*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b
^2)^(5/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (6*c*x^(1/3)*(2*a*c + b^2))/(4
*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*a*c + 3*b^2))*(2*a*c
 + b^2))/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 848, normalized size of antiderivative = 5.54

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^3}\,dx=\text{Too large to display}$$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^3,x)
```

output
```
(3*(16*x**(2/3)*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a*c - b*
*2))*a**2*b*c**2 + 16*x**(2/3)*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/
sqrt(4*a*c - b**2))*a*b**3*c + 4*x**(2/3)*sqrt(4*a*c - b**2)*atan((2*x**(1
/3)*c + b)/sqrt(4*a*c - b**2))*b**5 + 16*x**(1/3)*sqrt(4*a*c - b**2)*atan(
(2*x**(1/3)*c + b)/sqrt(4*a*c - b**2))*a**2*b**2*c + 8*x**(1/3)*sqrt(4*a*c
 - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a*c - b**2))*a*b**4 + 8*x**(1/3)*s
qrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a*c - b**2))*a*b*c**3*x +
 4*x**(1/3)*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a*c - b**2))
*b**3*c**2*x + 8*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a*c - b
**2))*a**3*b*c + 4*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a*c -
 b**2))*a**2*b**3 + 16*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/sqrt(4*a
*c - b**2))*a*b**2*c**2*x + 8*sqrt(4*a*c - b**2)*atan((2*x**(1/3)*c + b)/s
qrt(4*a*c - b**2))*b**4*c*x - 16*x**(2/3)*a**3*c**3 + 12*x**(2/3)*a**2*b**
2*c**2 + 6*x**(2/3)*a*b**4*c - 2*x**(2/3)*b**6 - 32*x**(1/3)*a**3*b*c**2 +
 40*x**(1/3)*a**2*b**3*c - 8*x**(1/3)*a**2*c**4*x - 8*x**(1/3)*a*b**5 - 2*
x**(1/3)*a*b**2*c**3*x + x**(1/3)*b**4*c**2*x - 8*a**4*c**2 + 22*a**3*b**2
*c - 5*a**2*b**4))/(2*b*(128*x**(2/3)*a**4*c**4 - 32*x**(2/3)*a**3*b**2*c*
*3 - 24*x**(2/3)*a**2*b**4*c**2 + 10*x**(2/3)*a*b**6*c - x**(2/3)*b**8 + 1
28*x**(1/3)*a**4*b*c**3 - 96*x**(1/3)*a**3*b**3*c**2 + 64*x**(1/3)*a**3*c*
*5*x + 24*x**(1/3)*a**2*b**5*c - 48*x**(1/3)*a**2*b**2*c**4*x - 2*x**(1...
```

3.56 $\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 268

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}\,dx=\frac{15(b^2-4ac)^2(9b^2-4ac)\left(b+2c\sqrt[3]{x}\right)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{16384c^5}-\frac{5(b^2-4ac)(9b^2-4ac)\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{2048c^4}+\frac{(9b^2-4ac)\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{128c^3}-\frac{3\left(9b-14c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{112c^2}-\frac{15(b^2-4ac)^3(9b^2-4ac)\operatorname{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\right)}{32768c^{11/2}}$$

output
```
15/16384*(-4*a*c+b^2)^2*(-4*a*c+9*b^2)*(b+2*c*x^(1/3))*(a+b*x^(1/3)+c*x^(2
/3))^(1/2)/c^5-5/2048*(-4*a*c+b^2)*(-4*a*c+9*b^2)*(b+2*c*x^(1/3))*(a+b*x^(
1/3)+c*x^(2/3))^(3/2)/c^4+1/128*(-4*a*c+9*b^2)*(b+2*c*x^(1/3))*(a+b*x^(1/3
)+c*x^(2/3))^(5/2)/c^3-3/112*(9*b-14*c*x^(1/3))*(a+b*x^(1/3)+c*x^(2/3))^(7
/2)/c^2-15/32768*(-4*a*c+b^2)^3*(-4*a*c+9*b^2)*arctanh(1/2*(b+2*c*x^(1/3))
/c^(1/2)/(a+b*x^(1/3)+c*x^(2/3))^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.16

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2} dx = \frac{2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\left(945b^7+84b^5c\left(-125a+6cx^{2/3}\right)-630b^6c\sqrt[3]{x}-8b^4\left(-791ac^2\sqrt[3]{x}+\right.\right.}{}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^(5/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + b*x^(1/3) + c*x^(2/3)]*(945*b^7 + 84*b^5*c*(-125*a + 6
*c*x^(2/3)) - 630*b^6*c*x^(1/3) - 8*b^4*(-791*a*c^2*x^(1/3) + 54*c^3*x) +
16*b^3*c^2*(2359*a^2 - 284*a*c*x^(2/3) + 24*c^2*x^(4/3)) + 96*b^2*(-199*a^
2*c^3*x^(1/3) + 36*a*c^4*x + 648*c^5*x^(5/3)) + 64*b*c^3*(-663*a^3 + 174*a
^2*c*x^(2/3) + 2456*a*c^2*x^(4/3) + 1584*c^3*x^2) + 896*(15*a^3*c^4*x^(1/3
) + 118*a^2*c^5*x + 136*a*c^6*x^(5/3) + 48*c^7*x^(7/3))) + 105*(b^2 - 4*a*
c)^3*(9*b^2 - 4*a*c)*Log[b - 2*Sqrt[c]*Sqrt[a + b*x^(1/3) + c*x^(2/3)] + 2
*c*x^(1/3)])/(229376*c^(11/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1680, 1166, 27, 1160, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2} dx$$

$$\Big\downarrow \text{1680}$$

$$3\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{5/2} x^{2/3} d\sqrt[3]{x}$$

$$\Big\downarrow \text{1166}$$

$$3\left(\frac{\int -\frac{1}{2}\left(2a+9b\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{5/2}d\sqrt[3]{x}}{8c}+\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{8c}\right)$$

$\downarrow$ 27

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{8c}-\frac{\int \left(2a+9b\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{5/2}d\sqrt[3]{x}}{16c}\right)$$

$\downarrow$ 1160

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{8c}-\frac{\frac{9b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{7c}-\frac{(9b^2-4ac)\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{5/2}d\sqrt[3]{x}}{2c}}{16c}\right)$$

$\downarrow$ 1087

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{8c}-\frac{\frac{9b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{7c}-\frac{(9b^2-4ac)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\int \left(a+cx^{2/3}+b\sqrt[3]{\cdots}\right.}{24c}\cdots\right.}{2c}}{16c}\cdots\right.$$

$\downarrow$ 1087

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{8c}-\frac{\frac{9b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{7c}-\frac{(9b^2-4ac)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{2c}}{16c}\cdots\right.$$

$\downarrow$ 1087

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{8c}-\frac{9b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{7c}-\frac{(9b^2-4ac)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 1092

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{8c}-\frac{\frac{9b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{7c}-\frac{(9b^2-4ac)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 219

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{8c}-\frac{\frac{9b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}{7c}-\frac{(9b^2-4ac)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{12c}-\frac{5\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(\ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}}{\ldots}\right.$$

input

```
Int[(a + b*x^(1/3) + c*x^(2/3))^(5/2),x]
```

output
```
3*(((a + b*x^(1/3) + c*x^(2/3))^(7/2)*x^(1/3))/(8*c) - ((9*b*(a + b*x^(1/3
) + c*x^(2/3))^(7/2))/(7*c) - ((9*b^2 - 4*a*c)*(((b + 2*c*x^(1/3))*(a + b*
x^(1/3) + c*x^(2/3))^(5/2))/(12*c) - (5*(b^2 - 4*a*c)*(((b + 2*c*x^(1/3))*
(a + b*x^(1/3) + c*x^(2/3))^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x^(
1/3))*Sqrt[a + b*x^(1/3) + c*x^(2/3)])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x^(1/3))/(2*Sqrt[c]*Sqrt[a + b*x^(1/3) + c*x^(2/3)])])/(8*c^(3/2))))/
(16*c)))/(24*c)))/(2*c))/(16*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{3x^{\frac{1}{3}}\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{7}{2}}}{8c} - \frac{27b\left(\frac{\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{7}{2}}}{7c} - \frac{b\left(\frac{\left(b+2c\,x^{\frac{1}{3}}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}{12c} + \frac{5\left(4ac-b^{2}\right)\left(\frac{\left(b+2c\,x^{\frac{1}{3}}\right)\left(a+b\,x^{\frac{1}{3}}\right.}{8c}\right.}{}\right.}{}\right.}{27b\left(\frac{\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{7}{2}}}{7c} - \frac{b\left(\frac{\left(b+2c\,x^{\frac{1}{3}}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}{12c} + \frac{5\left(4ac-b^{2}\right)\left(\frac{\left(b+2c\,x^{\frac{1}{3}}\right)\left(a+b\,x^{\frac{1}{3}}\right.}{8c}\right.}{}\right.}{}\right.}$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
3/8*x^(1/3)*(a+b*x^(1/3)+c*x^(2/3))^(7/2)/c-27/16*b/c*(1/7*(a+b*x^(1/3)+c*
x^(2/3))^(7/2)/c-1/2*b/c*(1/12*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(
5/2)+5/24*(4*a*c-b^2)/c*(1/8*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(3/
2)+3/16*(4*a*c-b^2)/c*(1/4*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(1/2)
+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x^(1/3))/c^(1/2)+(a+b*x^(1/3)+c*x^(2/
3))^(1/2))))))-3/8*a/c*(1/12*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(5/
2)+5/24*(4*a*c-b^2)/c*(1/8*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(3/2)
+3/16*(4*a*c-b^2)/c*(1/4*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(1/2)+1
/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x^(1/3))/c^(1/2)+(a+b*x^(1/3)+c*x^(2/3)
)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 1744, normalized size of antiderivative = 6.51

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**(5/2),x)
```

output
```
3*Piecewise(((-a*(a**3 - 3*a*(3*a**2*c + 3*a*b**2 - 5*a*(17*a*c**2/8 + 243
*b**2*c/224)/(6*c) - 9*b*(237*a*b*c/56 + b**3 - 11*b*(17*a*c**2/8 + 243*b*
*2*c/224)/(12*c))/(10*c))/(4*c) - 5*b*(3*a**2*b - 4*a*(237*a*b*c/56 + b**3
 - 11*b*(17*a*c**2/8 + 243*b**2*c/224)/(12*c))/(5*c) - 7*b*(3*a**2*c + 3*a
*b**2 - 5*a*(17*a*c**2/8 + 243*b**2*c/224)/(6*c) - 9*b*(237*a*b*c/56 + b**
3 - 11*b*(17*a*c**2/8 + 243*b**2*c/224)/(12*c))/(10*c))/(8*c))/(6*c))/(2*c
) - b*(-2*a*(3*a**2*b - 4*a*(237*a*b*c/56 + b**3 - 11*b*(17*a*c**2/8 + 243
*b**2*c/224)/(12*c))/(5*c) - 7*b*(3*a**2*c + 3*a*b**2 - 5*a*(17*a*c**2/8 +
 243*b**2*c/224)/(6*c) - 9*b*(237*a*b*c/56 + b**3 - 11*b*(17*a*c**2/8 + 24
3*b**2*c/224)/(12*c))/(10*c))/(8*c))/(3*c) - 3*b*(a**3 - 3*a*(3*a**2*c + 3
*a*b**2 - 5*a*(17*a*c**2/8 + 243*b**2*c/224)/(6*c) - 9*b*(237*a*b*c/56 + b
**3 - 11*b*(17*a*c**2/8 + 243*b**2*c/224)/(12*c))/(10*c))/(4*c) - 5*b*(3*a
**2*b - 4*a*(237*a*b*c/56 + b**3 - 11*b*(17*a*c**2/8 + 243*b**2*c/224)/(12
*c))/(5*c) - 7*b*(3*a**2*c + 3*a*b**2 - 5*a*(17*a*c**2/8 + 243*b**2*c/224)
/(6*c) - 9*b*(237*a*b*c/56 + b**3 - 11*b*(17*a*c**2/8 + 243*b**2*c/224)/(1
2*c))/(10*c))/(8*c))/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqr
t(a + b*x**(1/3) + c*x**(2/3)) + 2*c*x**(1/3))/sqrt(c), Ne(a - b**2/(4*c),
 0)), ((b/(2*c) + x**(1/3))*log(b/(2*c) + x**(1/3))/sqrt(c*(b/(2*c) + x**(
1/3))**2), True)) + sqrt(a + b*x**(1/3) + c*x**(2/3))*(33*b*c*x**2/112 + c
**2*x**(7/3)/8 + x**(5/3)*(17*a*c**2/8 + 243*b**2*c/224)/(6*c) + x**(4/...
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a + b\sqrt[3]{x} + cx^{2/3}\right)^{5/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage3:=type(sage2):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m
& i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2} dx = \int \left(a+b\,x^{1/3}+c\,x^{2/3}\right)^{5/2} dx$$

input
```
int((a + b*x^(1/3) + c*x^(2/3))^(5/2),x)
```

output
```
int((a + b*x^(1/3) + c*x^(2/3))^(5/2), x)
```

Reduce [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2} dx = \int \left(x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a\right)^{\frac{5}{2}} dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^(5/2),x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^(5/2),x)
```

3.57 $\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2} dx$

Optimal result

Integrand size = 20, antiderivative size = 211

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2} dx = -\frac{3(b^2-4ac)\left(7b^2-4ac\right)\left(b+2c\sqrt[3]{x}\right)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{512c^4} + \frac{\left(7b^2-4ac\right)\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{64c^3} - \frac{\left(7b-10c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{20c^2} + \frac{3(b^2-4ac)^2\left(7b^2-4ac\right)\operatorname{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\right)}{1024c^{9/2}}$$

output
```
-3/512*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(b+2*c*x^(1/3))*(a+b*x^(1/3)+c*x^(2/3))
^(1/2)/c^4+1/64*(-4*a*c+7*b^2)*(b+2*c*x^(1/3))*(a+b*x^(1/3)+c*x^(2/3))^(3/
2)/c^3-1/20*(7*b-10*c*x^(1/3))*(a+b*x^(1/3)+c*x^(2/3))^(5/2)/c^2+3/1024*(-
4*a*c+b^2)^2*(-4*a*c+7*b^2)*arctanh(1/2*(b+2*c*x^(1/3))/c^(1/2)/(a+b*x^(1/
3)+c*x^(2/3))^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}\,dx=\frac{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\left(-105b^5+760ab^3c-1296a^2bc^2+70b^4c\sqrt[3]{x}-432ab^2c^2\sqrt[3]{x}+480a^2c^3\sqrt[3]{\cdots}\right.}{2560c^4}$$
$$+\frac{3(-7b^2+4ac)\left(-b^2+4ac\right)^2\log\left(b-2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}+2c\sqrt[3]{x}\right)}{1024c^{9/2}}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^(3/2),x]
```

output
```
(Sqrt[a + b*x^(1/3) + c*x^(2/3)]*(-105*b^5 + 760*a*b^3*c - 1296*a^2*b*c^2
+ 70*b^4*c*x^(1/3) - 432*a*b^2*c^2*x^(1/3) + 480*a^2*c^3*x^(1/3) - 56*b^3*
c^2*x^(2/3) + 288*a*b*c^3*x^(2/3) + 48*b^2*c^3*x + 2240*a*c^4*x + 1664*b*c
^4*x^(4/3) + 1280*c^5*x^(5/3)))/(2560*c^4) + (3*(-7*b^2 + 4*a*c)*(-b^2 + 4
*a*c)^2*Log[b - 2*Sqrt[c]*Sqrt[a + b*x^(1/3) + c*x^(2/3)] + 2*c*x^(1/3)])/
(1024*c^(9/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1680, 1166, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}\,dx$$

$$\Big\downarrow \ 1680$$

$$3\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{3/2}x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \ 1166$$

$$3\left(\frac{\int -\frac{1}{2}\left(2a+7b\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{3/2}d\sqrt[3]{x}}{6c}+\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{6c}\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{6c}-\frac{\int \left(2a+7b\sqrt[3]{x}\right)\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{3/2}d\sqrt[3]{x}}{12c}\right)$$

$$\Big\downarrow \text{1160}$$

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\int \left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{3/2}d\sqrt[3]{x}}{2c}}{12c}\right)$$

$$\Big\downarrow \text{1087}$$

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{a+cx^{2/3}+b\sqrt[3]{x}}\ldots}{16c}\ldots}{2c}}{12c}\ldots$$

$$\Big\downarrow \text{1087}$$

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\sqrt{\ldots}}{\ldots}\ldots\right.}{\ldots}\ldots\right.}{2c}}{12c}\ldots$$

$$\Big\downarrow \text{1092}$$

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{5c}-\frac{(7b^2-4ac)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\sqrt{}}{}\right.}{2c}\right.}{}}{12c}\right.$$

$\downarrow$ 219

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}{5c}-\frac{(7b^2-4ac)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\sqrt{}}{}\right.}{2c}\right.}{}}{12c}\right.$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^(3/2),x]
```

output

```
3*(((a + b*x^(1/3) + c*x^(2/3))^(5/2)*x^(1/3))/(6*c) - ((7*b*(a + b*x^(1/3
) + c*x^(2/3))^(5/2))/(5*c) - ((7*b^2 - 4*a*c)*(((b + 2*c*x^(1/3))*(a + b*
x^(1/3) + c*x^(2/3))^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x^(1/3))*S
qrt[a + b*x^(1/3) + c*x^(2/3)])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^
(1/3))/(2*Sqrt[c]*Sqrt[a + b*x^(1/3) + c*x^(2/3)])])/(8*c^(3/2))))/(16*c))
)/(2*c))/(12*c))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{x^{\frac{1}{3}}\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}{2c}-\frac{7b\left(\frac{\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{\left(b+2c\,x^{\frac{1}{3}}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^{2}\right)\left(\frac{\left(b+2c\,x^{\frac{1}{3}}\right)\sqrt{a+b\,x^{\frac{1}{3}}\cdots}}{4c}\cdots\right)}{\cdots}\right)}{2c}\cdots\right)}{4c}\cdots$
default	$\frac{x^{\frac{1}{3}}\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}{2c}-\frac{7b\left(\frac{\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{\left(b+2c\,x^{\frac{1}{3}}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^{2}\right)\left(\frac{\left(b+2c\,x^{\frac{1}{3}}\right)\sqrt{a+b\,x^{\frac{1}{3}}\cdots}}{4c}\cdots\right)}{\cdots}\right)}{2c}\cdots\right)}{4c}\cdots$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1/3)*(a+b*x^(1/3)+c*x^(2/3))^(5/2)/c-7/4*b/c*(1/5*(a+b*x^(1/3)+c*x^
(2/3))^(5/2)/c-1/2*b/c*(1/8*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(3/2
)+3/16*(4*a*c-b^2)/c*(1/4*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(1/2)+
1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x^(1/3))/c^(1/2)+(a+b*x^(1/3)+c*x^(2/3
))^(1/2)))))-1/2*a/c*(1/8*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(3/2)+
3/16*(4*a*c-b^2)/c*(1/4*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(1/2)+1/
8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x^(1/3))/c^(1/2)+(a+b*x^(1/3)+c*x^(2/3))
^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.65

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**(3/2),x)
```

output
```
3*Piecewise(((-a*(a**2 - 3*a*(7*a*c/6 + b**2/40)/(4*c) - 5*b*(17*a*b/15 -
7*b*(7*a*c/6 + b**2/40)/(8*c))/(6*c))/(2*c) - b*(-2*a*(17*a*b/15 - 7*b*(7*
a*c/6 + b**2/40)/(8*c))/(3*c) - 3*b*(a**2 - 3*a*(7*a*c/6 + b**2/40)/(4*c)
- 5*b*(17*a*b/15 - 7*b*(7*a*c/6 + b**2/40)/(8*c))/(6*c))/(4*c))/(2*c))*Pie
cewise((log(b + 2*sqrt(c)*sqrt(a + b*x**(1/3) + c*x**(2/3)) + 2*c*x**(1/3)
)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x**(1/3))*log(b/(2*c) + x**
(1/3))/sqrt(c*(b/(2*c) + x**(1/3))**2), True)) + sqrt(a + b*x**(1/3) + c*x
**(2/3))*(13*b*x**(4/3)/60 + c*x**(5/3)/6 + x**(2/3)*(17*a*b/15 - 7*b*(7*a
*c/6 + b**2/40)/(8*c))/(3*c) + x**(1/3)*(a**2 - 3*a*(7*a*c/6 + b**2/40)/(4
*c) - 5*b*(17*a*b/15 - 7*b*(7*a*c/6 + b**2/40)/(8*c))/(6*c))/(2*c) + x*(7*
a*c/6 + b**2/40)/(4*c) + (-2*a*(17*a*b/15 - 7*b*(7*a*c/6 + b**2/40)/(8*c))
/(3*c) - 3*b*(a**2 - 3*a*(7*a*c/6 + b**2/40)/(4*c) - 5*b*(17*a*b/15 - 7*b*
(7*a*c/6 + b**2/40)/(8*c))/(6*c))/(4*c))/c), Ne(c, 0)), (2*(a**2*(a + b*x*
*(1/3))**(5/2)/5 - 2*a*(a + b*x**(1/3))**(7/2)/7 + (a + b*x**(1/3))**(9/2)
/9)/b**3, Ne(b, 0)), (a**(3/2)*x/3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value3*sageVARc*(2*(((((7680*sageVARc^5*1/9
2160/sageVAR
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2} dx = \int \left(a+b\,x^{1/3}+c\,x^{2/3}\right)^{3/2} dx$$

input
```
int((a + b*x^(1/3) + c*x^(2/3))^(3/2),x)
```

output
```
int((a + b*x^(1/3) + c*x^(2/3))^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2} dx = \int \left(x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a\right)^{\frac{3}{2}} dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^(3/2),x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^(3/2),x)
```

3.58 $\int \sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 154

$$\int \sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \frac{3(5b^2-4ac)\left(b+2c\sqrt[3]{x}\right)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{64c^3} - \frac{\left(5b-6c\sqrt[3]{x}\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{8c^2} - \frac{3(b^2-4ac)\,(5b^2-4ac)\,\mathrm{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\right)}{128c^{7/2}}$$

output

```
3/64*(-4*a*c+5*b^2)*(b+2*c*x^(1/3))*(a+b*x^(1/3)+c*x^(2/3))^(1/2)/c^3-1/8*
(5*b-6*c*x^(1/3))*(a+b*x^(1/3)+c*x^(2/3))^(3/2)/c^2-3/128*(-4*a*c+b^2)*(-4
*a*c+5*b^2)*arctanh(1/2*(b+2*c*x^(1/3))/c^(1/2)/(a+b*x^(1/3)+c*x^(2/3))^(1
/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\,dx=\frac{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\left(15b^3-52abc-10b^2c\sqrt[3]{x}+24ac^2\sqrt[3]{x}+8bc^2x^{2/3}+48c^3x\right)}{64c^3}+\frac{3(5b^4-24ab^2c+16a^2c^2)\log\left(b-2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}+2c\sqrt[3]{x}\right)}{128c^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x^(1/3) + c*x^(2/3)],x]
```

output
```
(Sqrt[a + b*x^(1/3) + c*x^(2/3)]*(15*b^3 - 52*a*b*c - 10*b^2*c*x^(1/3) + 2
4*a*c^2*x^(1/3) + 8*b*c^2*x^(2/3) + 48*c^3*x))/(64*c^3) + (3*(5*b^4 - 24*a
*b^2*c + 16*a^2*c^2)*Log[b - 2*Sqrt[c]*Sqrt[a + b*x^(1/3) + c*x^(2/3)] + 2
*c*x^(1/3)])/(128*c^(7/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1680, 1166, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\,dx$$

$$\Big\downarrow \text{1680}$$

$$3\int \sqrt{a+cx^{2/3}+b\sqrt[3]{x}}x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1166}$$

$$3\left(\frac{\int -\frac{1}{2}\left(2a+5b\sqrt[3]{x}\right)\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}d\sqrt[3]{x}}{4c}+\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{4c}-\frac{\int\left(2a+5b\sqrt[3]{x}\right)\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}d\sqrt[3]{x}}{8c}\right)$$

$\big\downarrow$ 1160

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{3c}-\frac{\left(5b^2-4ac\right)\int\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}d\sqrt[3]{x}}{2c}}{8c}\right)$$

$\big\downarrow$ 1087

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{3c}-\frac{\left(5b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}}d\cdots}{8c}\cdots\right.}{2c}\cdots}{8c}\cdots\right.$$

$\big\downarrow$ 1092

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{3c}-\frac{\left(5b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^{2/3}}d\frac{b+2c\cdots}{\sqrt{a+cx^{2/}\cdots}}}{4c}\cdots\right.}{2c}\cdots}{8c}\cdots\right.$$

$\big\downarrow$ 219

$$3\left(\frac{\sqrt[3]{x}\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}{3c}-\frac{\left(5b^2-4ac\right)\left(\frac{\left(b+2c\sqrt[3]{x}\right)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+\cdots}{2\sqrt{c}\sqrt{a+\cdots}}\cdots\right.}{8c^{3/2}}\cdots\right.}{2c}\cdots}{8c}\cdots\right.$$

input
```
Int[Sqrt[a + b*x^(1/3) + c*x^(2/3)],x]
```

output
```
3*(((a + b*x^(1/3) + c*x^(2/3))^(3/2)*x^(1/3))/(4*c) - ((5*b*(a + b*x^(1/3
) + c*x^(2/3))^(3/2))/(3*c) - ((5*b^2 - 4*a*c)*(((b + 2*c*x^(1/3))*Sqrt[a
+ b*x^(1/3) + c*x^(2/3)])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^(1/3))
/(2*Sqrt[c]*Sqrt[a + b*x^(1/3) + c*x^(2/3)])])/(8*c^(3/2))))/(2*c))/(8*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{3x^{\frac{1}{3}}\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}{4c}-\frac{15b\left(\frac{\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{\left(b+2c\,x^{\frac{1}{3}}\right)\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{\frac{1}{3}}}{\sqrt{c}}+\sqrt{a+b\,x^{\frac{1}{3}}+\cdots}\right)}{8c^{\frac{3}{2}}}\cdots\right)}{2c}\cdots\right)}{8c}\cdots$
default	$\frac{3x^{\frac{1}{3}}\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}{4c}-\frac{15b\left(\frac{\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{\left(b+2c\,x^{\frac{1}{3}}\right)\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{\frac{1}{3}}}{\sqrt{c}}+\sqrt{a+b\,x^{\frac{1}{3}}+\cdots}\right)}{8c^{\frac{3}{2}}}\cdots\right)}{2c}\cdots\right)}{8c}\cdots$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/4*x^(1/3)*(a+b*x^(1/3)+c*x^(2/3))^(3/2)/c-15/8*b/c*(1/3*(a+b*x^(1/3)+c*x
^(2/3))^(3/2)/c-1/2*b/c*(1/4*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(1/
2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x^(1/3))/c^(1/2)+(a+b*x^(1/3)+c*x^(
2/3))^(1/2))))-3/4*a/c*(1/4*(b+2*c*x^(1/3))/c*(a+b*x^(1/3)+c*x^(2/3))^(1/2
)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x^(1/3))/c^(1/2)+(a+b*x^(1/3)+c*x^(2
/3))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.83

$$\int \sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = 3\left(\begin{cases} \left(-\frac{a\left(\frac{a}{4}-\frac{5b^2}{48c}\right)}{2c}-\frac{b\left(-\frac{ab}{12c}-\frac{3b\left(\frac{a}{4}-\frac{5b^2}{48c}\right)}{4c}\right)}{2c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{\frac{2}{3}}}+2c\sqrt[3]{x}\right)}{\sqrt{c}} \\ \frac{\left(\frac{b}{2c}+\sqrt[3]{x}\right)\log\left(\frac{b}{2c}+\sqrt[3]{x}\right)}{\sqrt{c\left(\frac{b}{2c}+\sqrt[3]{x}\right)^2}} \end{cases}\right. \\ \frac{2\left(\frac{a^2\left(a+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^{\frac{5}{2}}}{5}+\frac{\left(a+b\sqrt[3]{x}\right)^{\frac{7}{2}}}{7}\right)}{b^3} \\ \frac{\sqrt{a}x}{3} \end{cases}\right.$$

input
```
integrate((a+b*x**(1/3)+c*x**(2/3))**(1/2),x)
```

output
```
3*Piecewise(((-a*(a/4 - 5*b**2/(48*c))/(2*c) - b*(-a*b/(12*c) - 3*b*(a/4 -
 5*b**2/(48*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x**(
1/3) + c*x**(2/3)) + 2*c*x**(1/3))/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2
*c) + x**(1/3))*log(b/(2*c) + x**(1/3))/sqrt(c*(b/(2*c) + x**(1/3))**2), T
rue)) + sqrt(a + b*x**(1/3) + c*x**(2/3))*(b*x**(2/3)/(24*c) + x/4 + x**(1
/3)*(a/4 - 5*b**2/(48*c))/(2*c) + (-a*b/(12*c) - 3*b*(a/4 - 5*b**2/(48*c))
/(4*c))/c), Ne(c, 0)), (2*(a**2*(a + b*x**(1/3))**(3/2)/3 - 2*a*(a + b*x**
(1/3))**(5/2)/5 + (a + b*x**(1/3))**(7/2)/7)/b**3, Ne(b, 0)), (sqrt(a)*x/3
, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.86

$$\int \sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \frac{1}{64}\sqrt{cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a}\left(2\left(4\,x^{\frac{1}{3}}\left(6\,x^{\frac{1}{3}}+\frac{b}{c}\right)-\frac{5\,b^2c-12\,ac^2}{c^3}\right)x^{\frac{1}{3}}+\frac{15\,b^3-52\,ab}{c^3}\right. + \frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\log\left(\left|2\,\sqrt{c}\left(\sqrt{c}x^{\frac{1}{3}}-\sqrt{cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a}\right)+b\right|\right)}{128\,c^{\frac{7}{2}}}$$

input
```
integrate((a+b*x^(1/3)+c*x^(2/3))^(1/2),x, algorithm="giac")
```

output
```
1/64*sqrt(c*x^(2/3) + b*x^(1/3) + a)*(2*(4*x^(1/3)*(6*x^(1/3) + b/c) - (5*
b^2*c - 12*a*c^2)/c^3)*x^(1/3) + (15*b^3 - 52*a*b*c)/c^3) + 3/128*(5*b^4 -
 24*a*b^2*c + 16*a^2*c^2)*log(abs(2*sqrt(c)*(sqrt(c)*x^(1/3) - sqrt(c*x^(2
/3) + b*x^(1/3) + a)) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 19.38 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.25

$$\int \sqrt{a+b\sqrt[3]{x}+cx^{2/3}}\,dx = \frac{3\,x^{1/3}\,\left(a+b\,x^{1/3}+c\,x^{2/3}\right)^{3/2}}{4\,c} - \frac{3\,a\,\left(\left(\frac{b}{4\,c}+\frac{x^{1/3}}{2}\right)\,\sqrt{a+b\,x^{1/3}+c\,x^{2/3}}+\frac{\ln\left(\sqrt{a+b\,x^{1/3}+c\,x^{2/3}}+\frac{\frac{b}{2}+c\,x^{1/3}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c} - \frac{15\,b\,\left(\frac{\left(8\,c\,\left(a+c\,x^{2/3}\right)-3\,b^2+2\,b\,c\,x^{1/3}\right)\,\sqrt{a+b\,x^{1/3}+c\,x^{2/3}}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{a+b\,x^{1/3}+c\,x^{2/3}}+\frac{b+2\,c\,x^{1/3}}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{8\,c}$$

input
```
int((a + b*x^(1/3) + c*x^(2/3))^(1/2),x)
```

output
```
(3*x^(1/3)*(a + b*x^(1/3) + c*x^(2/3))^(3/2))/(4*c) - (3*a*((b/(4*c) + x^(
1/3)/2)*(a + b*x^(1/3) + c*x^(2/3))^(1/2) + (log((a + b*x^(1/3) + c*x^(2/3
))^(1/2) + (b/2 + c*x^(1/3))/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) -
 (15*b*(((8*c*(a + c*x^(2/3)) - 3*b^2 + 2*b*c*x^(1/3))*(a + b*x^(1/3) + c*
x^(2/3))^(1/2))/(24*c^2) + (log(2*(a + b*x^(1/3) + c*x^(2/3))^(1/2) + (b +
 2*c*x^(1/3))/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*c)
```

Reduce [F]

$$\int \sqrt{a + b\sqrt[3]{x} + cx^{2/3}}\, dx = \int \sqrt{x^{\frac{2}{3}}c + x^{\frac{1}{3}}b + a}dx$$

input
```
int((a+b*x^(1/3)+c*x^(2/3))^(1/2),x)
```

output
```
int((a+b*x^(1/3)+c*x^(2/3))^(1/2),x)
```

3.59 $\int \frac{1}{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 99

$$\int \frac{1}{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\,dx = -\frac{3\left(3b-2c\sqrt[3]{x}\right)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{4c^2} + \frac{3(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\right)}{8c^{5/2}}$$

output
```
-3/4*(3*b-2*c*x^(1/3))*(a+b*x^(1/3)+c*x^(2/3))^(1/2)/c^2+3/8*(-4*a*c+3*b^2
)*arctanh(1/2*(b+2*c*x^(1/3))/c^(1/2)/(a+b*x^(1/3)+c*x^(2/3))^(1/2))/c^(5/
2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\,dx = \frac{3\left(-3b+2c\sqrt[3]{x}\right)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{4c^2} + \frac{3(-3b^2+4ac)\log\left(bc^2-2c^{5/2}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}+2c^3\sqrt[3]{x}\right)}{8c^{5/2}}$$

input
```
Integrate[1/Sqrt[a + b*x^(1/3) + c*x^(2/3)],x]
```

output
```
(3*(-3*b + 2*c*x^(1/3))*Sqrt[a + b*x^(1/3) + c*x^(2/3)])/(4*c^2) + (3*(-3*
b^2 + 4*a*c)*Log[b*c^2 - 2*c^(5/2)*Sqrt[a + b*x^(1/3) + c*x^(2/3)] + 2*c^3
*x^(1/3)])/(8*c^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1680, 1166, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}dx$$

$$\Big\downarrow \text{1680}$$

$$3\int \frac{x^{2/3}}{\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1166}$$

$$3\left(\frac{\int -\frac{2a+3b\sqrt[3]{x}}{2\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}}d\sqrt[3]{x}}{2c}+\frac{\sqrt[3]{x}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{2c}\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{\sqrt[3]{x}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{2c}-\frac{\int \frac{2a+3b\sqrt[3]{x}}{\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}}d\sqrt[3]{x}}{4c}\right)$$

$$\Big\downarrow \text{1160}$$

$$3\left(\frac{\sqrt[3]{x}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{2c}-\frac{\frac{3b\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{c}-\frac{(3b^2-4ac)\int\frac{1}{\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}}d\sqrt[3]{x}}{2c}}{4c}\right)$$

$\Big\downarrow$ 1092

$$3\left(\frac{\sqrt[3]{x}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{2c}-\frac{\frac{3b\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{c}-\frac{(3b^2-4ac)\int\frac{1}{4c-x^{2/3}}d\frac{b+2c\sqrt[3]{x}}{\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}}}{c}}{4c}\right)$$

$\Big\downarrow$ 219

$$3\left(\frac{\sqrt[3]{x}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{2c}-\frac{\frac{3b\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{c}-\frac{(3b^2-4ac)\text{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\right)}{2c^{3/2}}}{4c}\right)$$

input
```
Int[1/Sqrt[a + b*x^(1/3) + c*x^(2/3)],x]
```

output
```
3*((Sqrt[a + b*x^(1/3) + c*x^(2/3)]*x^(1/3))/(2*c) - ((3*b*Sqrt[a + b*x^(1
/3) + c*x^(2/3)])/c - ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^(1/3))/(2*Sqrt[c
]*Sqrt[a + b*x^(1/3) + c*x^(2/3)])])/(2*c^(3/2)))/(4*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{3x^{\frac{1}{3}}\sqrt{a+b x^{\frac{1}{3}}+c x^{\frac{2}{3}}}}{2c}-\frac{9b\left(\frac{\sqrt{a+b x^{\frac{1}{3}}+c x^{\frac{2}{3}}}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+c x^{\frac{1}{3}}}{\sqrt{c}}+\sqrt{a+b x^{\frac{1}{3}}+c x^{\frac{2}{3}}}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{3a\ln\left(\frac{\frac{b}{2}+c x^{\frac{1}{3}}}{\sqrt{c}}+\sqrt{a+b x^{\frac{1}{3}}+c x^{\frac{2}{3}}}\right)}{2c^{\frac{3}{2}}}$
default	$\frac{3x^{\frac{1}{3}}\sqrt{a+b x^{\frac{1}{3}}+c x^{\frac{2}{3}}}}{2c}-\frac{9b\left(\frac{\sqrt{a+b x^{\frac{1}{3}}+c x^{\frac{2}{3}}}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+c x^{\frac{1}{3}}}{\sqrt{c}}+\sqrt{a+b x^{\frac{1}{3}}+c x^{\frac{2}{3}}}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{3a\ln\left(\frac{\frac{b}{2}+c x^{\frac{1}{3}}}{\sqrt{c}}+\sqrt{a+b x^{\frac{1}{3}}+c x^{\frac{2}{3}}}\right)}{2c^{\frac{3}{2}}}$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*x^(1/3)/c*(a+b*x^(1/3)+c*x^(2/3))^(1/2)-9/4*b/c*(1/c*(a+b*x^(1/3)+c*x^
(2/3))^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x^(1/3))/c^(1/2)+(a+b*x^(1/3)+c*x^(
2/3))^(1/2)))-3/2*a/c^(3/2)*ln((1/2*b+c*x^(1/3))/c^(1/2)+(a+b*x^(1/3)+c*x^
(2/3))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.01

$$\int \frac{1}{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\,dx = 3\left(\begin{cases} \left(-\frac{a}{2c}+\frac{3b^2}{8c^2}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{\frac{2}{3}}}+2c\sqrt[3]{x}\right)}{\sqrt{c}} & \text{for}\ a-\frac{b^2}{4c}\neq 0 \\ \frac{\left(\frac{b}{2c}+\sqrt[3]{x}\right)\log\left(\frac{b}{2c}+\sqrt[3]{x}\right)}{\sqrt{c\left(\frac{b}{2c}+\sqrt[3]{x}\right)^2}} & \text{otherwise} \end{cases}\right)+\left(- \right. \\ \frac{2\left(a^2\sqrt{a+b\sqrt[3]{x}}-\frac{2a\left(a+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{3}+\frac{\left(a+b\sqrt[3]{x}\right)^{\frac{5}{2}}}{5}\right)}{b^3} \\ \frac{x}{3\sqrt{a}} \end{cases}\right.$$

input
```
integrate(1/(a+b*x**(1/3)+c*x**(2/3))**(1/2),x)
```

output
```
3*Piecewise(((-a/(2*c) + 3*b**2/(8*c**2))*Piecewise((log(b + 2*sqrt(c)*sqr
t(a + b*x**(1/3) + c*x**(2/3)) + 2*c*x**(1/3))/sqrt(c), Ne(a - b**2/(4*c),
 0)), ((b/(2*c) + x**(1/3))*log(b/(2*c) + x**(1/3))/sqrt(c*(b/(2*c) + x**(
1/3))**2), True)) + (-3*b/(4*c**2) + x**(1/3)/(2*c))*sqrt(a + b*x**(1/3) +
 c*x**(2/3)), Ne(c, 0)), (2*(a**2*sqrt(a + b*x**(1/3)) - 2*a*(a + b*x**(1/
3))**(3/2)/3 + (a + b*x**(1/3))**(5/2)/5)/b**3, Ne(b, 0)), (x/(3*sqrt(a)),
 True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\,dx = \frac{3}{4}\sqrt{cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a}\left(\frac{2\,x^{\frac{1}{3}}}{c}-\frac{3\,b}{c^2}\right) - \frac{3\,(3\,b^2-4\,ac)\log\left(\left|2\,\sqrt{c}\left(\sqrt{c}x^{\frac{1}{3}}-\sqrt{cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a}\right)+b\right|\right)}{8\,c^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(1/2),x, algorithm="giac")
```

output
```
3/4*sqrt(c*x^(2/3) + b*x^(1/3) + a)*(2*x^(1/3)/c - 3*b/c^2) - 3/8*(3*b^2 -
 4*a*c)*log(abs(2*sqrt(c)*(sqrt(c)*x^(1/3) - sqrt(c*x^(2/3) + b*x^(1/3) +
a)) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\,dx=\int \frac{1}{\sqrt{a+b\,x^{1/3}+c\,x^{2/3}}}\,dx$$

input
```
int(1/(a + b*x^(1/3) + c*x^(2/3))^(1/2),x)
```

output
```
int(1/(a + b*x^(1/3) + c*x^(2/3))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\,dx=\frac{\frac{3x^{\frac{1}{3}}\sqrt{x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a}\,c^{2}}{2}-\frac{9\sqrt{x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a}\,bc}{4}-\frac{3\sqrt{c}\log\left(\frac{2\sqrt{c}\sqrt{x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a}+2x^{\frac{1}{3}}c+b}{\sqrt{4ac-b^{2}}}\right)ac}{2}+\frac{9\sqrt{c}\log\left(\right.}{}}{c^{3}}$$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(1/2),x)
```

output
```
(3*(4*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*c**2 - 6*sqrt(x**(2/3)*c
+ x**(1/3)*b + a)*b*c - 4*sqrt(c)*log((2*sqrt(c)*sqrt(x**(2/3)*c + x**(1/3
)*b + a) + 2*x**(1/3)*c + b)/sqrt(4*a*c - b**2))*a*c + 3*sqrt(c)*log((2*sq
rt(c)*sqrt(x**(2/3)*c + x**(1/3)*b + a) + 2*x**(1/3)*c + b)/sqrt(4*a*c - b
**2))*b**2))/(8*c**3)
```

3.60 $\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\,dx = -\frac{6\left(ab+(b^2-2ac)\sqrt[3]{x}\right)}{c\,(b^2-4ac)\,\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}} + \frac{3\text{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\right)}{c^{3/2}}$$

output
```
(-6*a*b-6*(-2*a*c+b^2)*x^(1/3))/c/(-4*a*c+b^2)/(a+b*x^(1/3)+c*x^(2/3))^(1/
2)+3*arctanh(1/2*(b+2*c*x^(1/3))/c^(1/2)/(a+b*x^(1/3)+c*x^(2/3))^(1/2))/c^
(3/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\,dx = \frac{6\left(a\left(b-2c\sqrt[3]{x}\right)+b^2\sqrt[3]{x}\right)}{c\,(-b^2+4ac)\,\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}} + \frac{3\text{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\right)}{c^{3/2}}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^(-3/2),x]
```

output
```
(6*(a*(b - 2*c*x^(1/3)) + b^2*x^(1/3)))/(c*(-b^2 + 4*a*c)*Sqrt[a + b*x^(1/
3) + c*x^(2/3)]) + (3*ArcTanh[(b + 2*c*x^(1/3))/(2*Sqrt[c]*Sqrt[a + b*x^(1
/3) + c*x^(2/3)])])/c^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1680, 1164, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1680}$$

$$3\int \frac{x^{2/3}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{3/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1164}$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{\left(b^2-4ac\right)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}-\frac{2\int \frac{2a+b\sqrt[3]{x}}{\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}}d\sqrt[3]{x}}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1160}$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{\left(b^2-4ac\right)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}-\frac{2\left(\frac{b\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}}d\sqrt[3]{x}}{2c}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1092}$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{(b^2-4ac)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}-\frac{2\left(\frac{b\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{c}-\frac{(b^2-4ac)\int\frac{1}{4c-x^{2/3}}d\frac{b+2c\sqrt[3]{x}}{\sqrt{a+cx^{2/3}+b\sqrt[3]{x}}}}{c}\right)}{b^2-4ac}\right)$$

$\Big\downarrow$ 219

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{(b^2-4ac)\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}-\frac{2\left(\frac{b\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}{c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2c\sqrt[3]{x}}{2\sqrt{c}\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}\right)}{2c^{3/2}}\right)}{b^2-4ac}\right)$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^(-3/2),x]
```

output
```
3*((2*(2*a + b*x^(1/3))*x^(1/3))/((b^2 - 4*a*c)*Sqrt[a + b*x^(1/3) + c*x^(
2/3)]) - (2*((b*Sqrt[a + b*x^(1/3) + c*x^(2/3)])/c - ((b^2 - 4*a*c)*ArcTan
h[(b + 2*c*x^(1/3))/(2*Sqrt[c]*Sqrt[a + b*x^(1/3) + c*x^(2/3)])])/(2*c^(3/
2))))/(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.23

method	result
derivativedivides	$-\frac{3x^{\frac{1}{3}}}{c\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}}-\frac{3b\left(-\frac{1}{c\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}}-\frac{b\left(b+2c\,x^{\frac{1}{3}}\right)}{c\left(4ac-b^{2}\right)\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}}\right)}{2c}+\frac{3\ln\left(\frac{\frac{b}{2}+c\,x^{\frac{1}{3}}}{\sqrt{c}}+\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}\right)}{c^{\frac{3}{2}}}$
default	$-\frac{3x^{\frac{1}{3}}}{c\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}}-\frac{3b\left(-\frac{1}{c\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}}-\frac{b\left(b+2c\,x^{\frac{1}{3}}\right)}{c\left(4ac-b^{2}\right)\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}}\right)}{2c}+\frac{3\ln\left(\frac{\frac{b}{2}+c\,x^{\frac{1}{3}}}{\sqrt{c}}+\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}\right)}{c^{\frac{3}{2}}}$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3*x^(1/3)/c/(a+b*x^(1/3)+c*x^(2/3))^(1/2)-3/2*b/c*(-1/c/(a+b*x^(1/3)+c*x^
(2/3))^(1/2)-b/c*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x^(1/3)+c*x^(2/3))^(1/2)
)+3/c^(3/2)*ln((1/2*b+c*x^(1/3))/c^(1/2)+(a+b*x^(1/3)+c*x^(2/3))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x**(1/3)+c*x**(2/3))**(3/2),x)
```

output
```
Integral((a + b*x**(1/3) + c*x**(2/3))**(-3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\,dx = -\frac{6\left(\frac{ab}{b^2c-4\,ac^2}+\frac{(b^2-2\,ac)x^{\frac{1}{3}}}{b^2c-4\,ac^2}\right)}{\sqrt{cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a}} - \frac{3\log\left(\left|2\sqrt{c}\left(\sqrt{c}x^{\frac{1}{3}}-\sqrt{cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a}\right)+b\right|\right)}{c^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(3/2),x, algorithm="giac")
```

output
```
-6*(a*b/(b^2*c - 4*a*c^2) + (b^2 - 2*a*c)*x^(1/3)/(b^2*c - 4*a*c^2))/sqrt(
c*x^(2/3) + b*x^(1/3) + a) - 3*log(abs(2*sqrt(c)*(sqrt(c)*x^(1/3) - sqrt(c
*x^(2/3) + b*x^(1/3) + a)) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 20.60 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\,dx = \frac{3\,\ln\left(\sqrt{a+b\,x^{1/3}+c\,x^{2/3}}+\frac{\frac{b}{2}+c\,x^{1/3}}{\sqrt{c}}\right)}{c^{3/2}} + \frac{3\left(\frac{a\,b}{2}-x^{1/3}\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{a+b\,x^{1/3}+c\,x^{2/3}}}$$

input
```
int(1/(a + b*x^(1/3) + c*x^(2/3))^(3/2),x)
```

output
```
(3*log((a + b*x^(1/3) + c*x^(2/3))^(1/2) + (b/2 + c*x^(1/3))/c^(1/2)))/c^(
3/2) + (3*((a*b)/2 - x^(1/3)*(a*c - b^2/2)))/(c*(a*c - b^2/4)*(a + b*x^(1/
3) + c*x^(2/3))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.68

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\,dx = \frac{-12x^{\frac{1}{3}}\sqrt{x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a}\,a\,c^2+6x^{\frac{1}{3}}\sqrt{x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a}\,b^2c+6\sqrt{x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a}}{}$$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(3/2),x)
```

output
```
(3*( - 4*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*c**2 + 2*x**(1/3)*sq
rt(x**(2/3)*c + x**(1/3)*b + a)*b**2*c + 2*sqrt(x**(2/3)*c + x**(1/3)*b +
a)*a*b*c + 4*x**(2/3)*sqrt(c)*log((2*sqrt(c)*sqrt(x**(2/3)*c + x**(1/3)*b
 + a) + 2*x**(1/3)*c + b)/sqrt(4*a*c - b**2))*a*c**2 - x**(2/3)*sqrt(c)*log
((2*sqrt(c)*sqrt(x**(2/3)*c + x**(1/3)*b + a) + 2*x**(1/3)*c + b)/sqrt(4*a
*c - b**2))*b**2*c - 4*x**(2/3)*sqrt(c)*a*c**2 + 2*x**(2/3)*sqrt(c)*b**2*c
 + 4*x**(1/3)*sqrt(c)*log((2*sqrt(c)*sqrt(x**(2/3)*c + x**(1/3)*b + a) + 2
*x**(1/3)*c + b)/sqrt(4*a*c - b**2))*a*b*c - x**(1/3)*sqrt(c)*log((2*sqrt(
c)*sqrt(x**(2/3)*c + x**(1/3)*b + a) + 2*x**(1/3)*c + b)/sqrt(4*a*c - b**2
))*b**3 - 4*x**(1/3)*sqrt(c)*a*b*c + 2*x**(1/3)*sqrt(c)*b**3 + 4*sqrt(c)*l
og((2*sqrt(c)*sqrt(x**(2/3)*c + x**(1/3)*b + a) + 2*x**(1/3)*c + b)/sqrt(4
*a*c - b**2))*a**2*c - sqrt(c)*log((2*sqrt(c)*sqrt(x**(2/3)*c + x**(1/3)*b
 + a) + 2*x**(1/3)*c + b)/sqrt(4*a*c - b**2))*a*b**2 - 4*sqrt(c)*a**2*c +
2*sqrt(c)*a*b**2))/(c**2*(4*x**(2/3)*a*c**2 - x**(2/3)*b**2*c + 4*x**(1/3)
*a*b*c - x**(1/3)*b**3 + 4*a**2*c - a*b**2))
```

3.61 $\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}\,dx=-\frac{2\left(ab+(b^2-2ac)\sqrt[3]{x}\right)}{c\,(b^2-4ac)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}+\frac{2(b^2+4ac)\left(b+2c\sqrt[3]{x}\right)}{c\,(b^2-4ac)^2\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}$$

output
```
(-2*a*b-2*(-2*a*c+b^2)*x^(1/3))/c/(-4*a*c+b^2)/(a+b*x^(1/3)+c*x^(2/3))^(3/
2)+2*(4*a*c+b^2)*(b+2*c*x^(1/3))/c/(-4*a*c+b^2)^2/(a+b*x^(1/3)+c*x^(2/3))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}\,dx=\frac{2\left(8a^2b+12ab^2\sqrt[3]{x}+3b^3x^{2/3}+12abcx^{2/3}+2b^2cx+8ac^2x\right)}{(b^2-4ac)^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^(-5/2),x]
```

output
```
(2*(8*a^2*b + 12*a*b^2*x^(1/3) + 3*b^3*x^(2/3) + 12*a*b*c*x^(2/3) + 2*b^2*
c*x + 8*a*c^2*x))/((b^2 - 4*a*c)^2*(a + b*x^(1/3) + c*x^(2/3))^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1680, 1156, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1680}$$

$$3\int \frac{x^{2/3}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{5/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1156}$$

$$3\left(\frac{4b\int \frac{\sqrt[3]{x}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{3/2}}d\sqrt[3]{x}}{3\left(b^2-4ac\right)}-\frac{2x^{2/3}\left(b+2c\sqrt[3]{x}\right)}{3\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 1158}$$

$$3\left(\frac{8b\left(2a+b\sqrt[3]{x}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}-\frac{2x^{2/3}\left(b+2c\sqrt[3]{x}\right)}{3\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\right)$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^(-5/2),x]
```

output
```
3*((8*b*(2*a + b*x^(1/3)))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x^(1/3) + c*x^(2/
3)]) - (2*(b + 2*c*x^(1/3))*x^(2/3))/(3*(b^2 - 4*a*c)*(a + b*x^(1/3) + c*x
^(2/3))^(3/2)))
```

Defintions of rubi rules used

rule 1156
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Simp[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c)))   Int[
(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(92) = 184$.

Time $= 0.01$ (sec) , antiderivative size $= 210$, normalized size of antiderivative $= 1.96$

method	result
derivativedivides	$-\frac{3x^{\frac{1}{3}}}{2c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}-\frac{3b\left(-\frac{1}{3c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{2b}{3}+\frac{4c\,x^{\frac{1}{3}}}{3}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}+\frac{16c\left(b+2c\,x^{\frac{1}{3}}\right)}{3\left(4ac-b^{2}\right)^{2}\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}}\right)}{2c}\cdots\right)}{4c}\cdots$
default	$-\frac{3x^{\frac{1}{3}}}{2c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}-\frac{3b\left(-\frac{1}{3c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{2b}{3}+\frac{4c\,x^{\frac{1}{3}}}{3}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}}+\frac{16c\left(b+2c\,x^{\frac{1}{3}}\right)}{3\left(4ac-b^{2}\right)^{2}\sqrt{a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}}}\right)}{2c}\cdots\right)}{4c}\cdots$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-3/2*x^(1/3)/c/(a+b*x^(1/3)+c*x^(2/3))^(3/2)-3/4*b/c*(-1/3/c/(a+b*x^(1/3)+
c*x^(2/3))^(3/2)-1/2*b/c*(2/3*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x^(1/3)+c*x
^(2/3))^(3/2)+16/3*c/(4*a*c-b^2)^2*(b+2*c*x^(1/3))/(a+b*x^(1/3)+c*x^(2/3))
^(1/2)))+3/2*a/c*(2/3*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x^(1/3)+c*x^(2/3))^
(3/2)+16/3*c/(4*a*c-b^2)^2*(b+2*c*x^(1/3))/(a+b*x^(1/3)+c*x^(2/3))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{\frac{2}{3}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*x**(1/3)+c*x**(2/3))**(5/2),x)
```

output
```
Integral((a + b*x**(1/3) + c*x**(2/3))**(-5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.41

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}\,dx = \frac{2\left(\frac{8\,a^2b}{b^4-8\,ab^2c+16\,a^2c^2}+\left(\frac{12\,ab^2}{b^4-8\,ab^2c+16\,a^2c^2}+x^{\frac{1}{3}}\left(\frac{2\,(b^2c+4\,ac^2)x^{\frac{1}{3}}}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\,(b^3+4\,abc)}{b^4-8\,ab^2c+16\,a^2c^2}\right.\right.\right.}{\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(5/2),x, algorithm="giac")
```

output
```
2*(8*a^2*b/(b^4 - 8*a*b^2*c + 16*a^2*c^2) + (12*a*b^2/(b^4 - 8*a*b^2*c + 1
6*a^2*c^2) + x^(1/3)*(2*(b^2*c + 4*a*c^2)*x^(1/3)/(b^4 - 8*a*b^2*c + 16*a^
2*c^2) + 3*(b^3 + 4*a*b*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2)))*x^(1/3))/(c*x^
(2/3) + b*x^(1/3) + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 21.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.49

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}\,dx = \frac{2\,b^3\,\left(a+b\,x^{1/3}+c\,x^{2/3}\right)-2\,a\,b^3-2\,b^4\,x^{1/3}-16\,a^2\,c^2\,x^{1/3}+8\,a^2\,b\,c+16\,a\,c}{\left(16\,a^2\,c^3-\right.}$$

input
```
int(1/(a + b*x^(1/3) + c*x^(2/3))^(5/2),x)
```

output
```
(2*b^3*(a + b*x^(1/3) + c*x^(2/3)) - 2*a*b^3 - 2*b^4*x^(1/3) - 16*a^2*c^2*
x^(1/3) + 8*a^2*b*c + 16*a*c^2*x^(1/3)*(a + b*x^(1/3) + c*x^(2/3)) + 4*b^2
*c*x^(1/3)*(a + b*x^(1/3) + c*x^(2/3)) + 8*a*b*c*(a + b*x^(1/3) + c*x^(2/3
)) + 12*a*b^2*c*x^(1/3))/((b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)*(a + b*x^(1/3
) + c*x^(2/3))^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.77

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}\,dx = \frac{24x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a}\,ab\,c^2+6x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}}c+x^{\frac{1}{3}}b+a}\,b^3c+24x^{\frac{1}{3}}\sqrt{x^{\frac{2}{3}}c+x^{\frac{1}{3}}b-}}{}$$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(5/2),x)
```

output

```
(2*(12*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*b*c**2 + 3*x**(2/3)*sq
rt(x**(2/3)*c + x**(1/3)*b + a)*b**3*c + 12*x**(1/3)*sqrt(x**(2/3)*c + x**
(1/3)*b + a)*a*b**2*c + 8*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**2*b*c + 8*s
qrt(x**(2/3)*c + x**(1/3)*b + a)*a*c**3*x + 2*sqrt(x**(2/3)*c + x**(1/3)*b
 + a)*b**2*c**2*x + 16*x**(2/3)*sqrt(c)*a**2*c**2 - 4*x**(2/3)*sqrt(c)*a*b
**2*c - 6*x**(2/3)*sqrt(c)*b**4 + 16*x**(1/3)*sqrt(c)*a**2*b*c - 12*x**(1/
3)*sqrt(c)*a*b**3 + 8*x**(1/3)*sqrt(c)*a*c**3*x - 6*x**(1/3)*sqrt(c)*b**2*
c**2*x + 8*sqrt(c)*a**3*c - 6*sqrt(c)*a**2*b**2 + 16*sqrt(c)*a*b*c**2*x -
12*sqrt(c)*b**3*c*x))/(c*(32*x**(2/3)*a**3*c**3 - 6*x**(2/3)*a*b**4*c + x*
*(2/3)*b**6 + 32*x**(1/3)*a**3*b*c**2 - 16*x**(1/3)*a**2*b**3*c + 16*x**(1
/3)*a**2*c**4*x + 2*x**(1/3)*a*b**5 - 8*x**(1/3)*a*b**2*c**3*x + x**(1/3)*
b**4*c**2*x + 16*a**4*c**2 - 8*a**3*b**2*c + a**2*b**4 + 32*a**2*b*c**3*x
- 16*a*b**3*c**2*x + 2*b**5*c*x))
```

3.62 $\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 167

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}\,dx = -\frac{6\left(ab+(b^2-2ac)\sqrt[3]{x}\right)}{5c\,(b^2-4ac)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}} + \frac{2(3b^2+4ac)\left(b+2c\sqrt[3]{x}\right)}{5c\,(b^2-4ac)^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}} - \frac{16(3b^2+4ac)\left(b+2c\sqrt[3]{x}\right)}{5\,(b^2-4ac)^3\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}$$

output
```
1/5*(-6*a*b-6*(-2*a*c+b^2)*x^(1/3))/c/(-4*a*c+b^2)/(a+b*x^(1/3)+c*x^(2/3))
^(5/2)+2/5*(4*a*c+3*b^2)*(b+2*c*x^(1/3))/c/(-4*a*c+b^2)^2/(a+b*x^(1/3)+c*x
^(2/3))^(3/2)-16/5*(4*a*c+3*b^2)*(b+2*c*x^(1/3))/(-4*a*c+b^2)^3/(a+b*x^(1/
3)+c*x^(2/3))^(1/2)
```

Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}\,dx = -\frac{2\left(96a^3bc+3b^2x^{2/3}\left(5b^3+30b^2c\sqrt[3]{x}+40bc^2x^{2/3}+16c^3x\right)+8a^2\left(b^3+30b^2c\sqrt[3]{x}+30bc^2x^{2/3}+20c^3x\right)+4\right.}{5\,(b^2-4ac)^3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^(-7/2),x]
```

output
```
(-2*(96*a^3*b*c + 3*b^2*x^(2/3)*(5*b^3 + 30*b^2*c*x^(1/3) + 40*b*c^2*x^(2/
3) + 16*c^3*x) + 8*a^2*(b^3 + 30*b^2*c*x^(1/3) + 30*b*c^2*x^(2/3) + 20*c^3
*x) + 4*a*(5*b^4*x^(1/3) + 50*b^3*c*x^(2/3) + 60*b^2*c^2*x + 40*b*c^3*x^(4
/3) + 16*c^4*x^(5/3))))/(5*(b^2 - 4*a*c)^3*(a + b*x^(1/3) + c*x^(2/3))^(5/
2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1680, 1164, 1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 1680}$$

$$3\int \frac{x^{2/3}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{7/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1164}$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{5\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}-\frac{2\int \frac{2a-3b\sqrt[3]{x}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{5/2}}d\sqrt[3]{x}}{5\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{ 1159}$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{5\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}-\frac{2\left(-\frac{4\left(4ac+3b^2\right)\int\frac{1}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{3/2}}d\sqrt[3]{x}}{3\left(b^2-4ac\right)}-\frac{2\left(\sqrt[3]{x}\left(4ac+3b^2\right)+8ab\right)}{3\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\right)}{5\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{1088}$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{5\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}-\frac{2\left(\frac{8\left(4ac+3b^2\right)\left(b+2c\sqrt[3]{x}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}-\frac{2\left(\sqrt[3]{x}\left(4ac+3b^2\right)+8ab\right)}{3\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\right)}{5\left(b^2-4ac\right)}\right)$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^(-7/2),x]
```

output
```
3*((-2*((-2*(8*a*b + (3*b^2 + 4*a*c)*x^(1/3)))/(3*(b^2 - 4*a*c)*(a + b*x^(
1/3) + c*x^(2/3))^(3/2)) + (8*(3*b^2 + 4*a*c)*(b + 2*c*x^(1/3)))/(3*(b^2 -
 4*a*c)^2*Sqrt[a + b*x^(1/3) + c*x^(2/3)])))/(5*(b^2 - 4*a*c)) + (2*(2*a +
 b*x^(1/3))*x^(1/3))/(5*(b^2 - 4*a*c)*(a + b*x^(1/3) + c*x^(2/3))^(5/2)))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(139) = 278$.

Time = 0.02 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.88

method	result
derivativedivides	$-\frac{3x^{\frac{1}{3}}}{4c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}-\frac{9b\left(-\frac{1}{5c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}-\frac{b\left(\frac{\frac{2b}{5}+\frac{4c\,x^{\frac{1}{3}}}{5}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}+\frac{16c\left(\frac{\frac{2b}{3}+\frac{4c\,x^{\frac{1}{3}}}{3}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right.}\cdots\right.}{5\left(4\cdots\right.}\right)}{2c}\cdots\right)}{8c}\cdots$
default	$-\frac{3x^{\frac{1}{3}}}{4c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}-\frac{9b\left(-\frac{1}{5c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}-\frac{b\left(\frac{\frac{2b}{5}+\frac{4c\,x^{\frac{1}{3}}}{5}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{5}{2}}}+\frac{16c\left(\frac{\frac{2b}{3}+\frac{4c\,x^{\frac{1}{3}}}{3}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right.}\cdots\right.}{5\left(4\cdots\right.}\right)}{2c}\cdots\right)}{8c}\cdots$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-3/4*x^(1/3)/c/(a+b*x^(1/3)+c*x^(2/3))^(5/2)-9/8*b/c*(-1/5/c/(a+b*x^(1/3)+
c*x^(2/3))^(5/2)-1/2*b/c*(2/5*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x^(1/3)+c*x
^(2/3))^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x^(
1/3)+c*x^(2/3))^(3/2)+16/3*c/(4*a*c-b^2)^2*(b+2*c*x^(1/3))/(a+b*x^(1/3)+c*
x^(2/3))^(1/2))))+3/4*a/c*(2/5*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x^(1/3)+c*
x^(2/3))^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x^
(1/3)+c*x^(2/3))^(3/2)+16/3*c/(4*a*c-b^2)^2*(b+2*c*x^(1/3))/(a+b*x^(1/3)+c
*x^(2/3))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}\,dx = \int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{\frac{2}{3}}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*x**(1/3)+c*x**(2/3))**(7/2),x)
```

output
```
Integral((a + b*x**(1/3) + c*x**(2/3))**(-7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(137) = 274$.

Time = 0.17 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.11

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}\,dx = -\frac{2\left(\left(\left(\left(2\left(4\,x^{\frac{1}{3}}\left(\frac{2\,(3\,b^2c^3+4\,ac^4)x^{\frac{1}{3}}}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}+\frac{5\,(3\,b^3c^2+4\,abc^3)}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right)+\frac{5\,(9\,b^4c+24\,ab^2c^2+16\,a^2c^3)}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right)x^{\frac{1}{3}}+\cdots}{5\left(cx^{\frac{2}{3}}+bx^{\frac{1}{3}}+a\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(7/2),x, algorithm="giac")
```

output
```
-2/5*(((2*(4*x^(1/3)*(2*(3*b^2*c^3 + 4*a*c^4)*x^(1/3)/(b^6 - 12*a*b^4*c +
48*a^2*b^2*c^2 - 64*a^3*c^3) + 5*(3*b^3*c^2 + 4*a*b*c^3)/(b^6 - 12*a*b^4*c
 + 48*a^2*b^2*c^2 - 64*a^3*c^3)) + 5*(9*b^4*c + 24*a*b^2*c^2 + 16*a^2*c^3)
/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x^(1/3) + 5*(3*b^5 + 40
*a*b^3*c + 48*a^2*b*c^2)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))
*x^(1/3) + 20*(a*b^4 + 12*a^2*b^2*c)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 -
64*a^3*c^3))*x^(1/3) + 8*(a^2*b^3 + 12*a^3*b*c)/(b^6 - 12*a*b^4*c + 48*a^2
*b^2*c^2 - 64*a^3*c^3))/(c*x^(2/3) + b*x^(1/3) + a)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 22.25 (sec) , antiderivative size = 515, normalized size of antiderivative = 3.08

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}\,dx = \frac{\frac{2\,c^2\,x^{1/3}\,(56\,b^2+32\,a\,c)}{5\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)^2}+\frac{b\,c\,(56\,b^2+32\,a\,c)}{5\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)^2}}{\sqrt{a+b\,x^{1/3}+c\,x^{2/3}}}$$
$$+\frac{x^{1/3}\left(\frac{6\,b^2}{5\,(4\,a\,c^2-b^2\,c)}-\frac{12\,a\,c}{5\,(4\,a\,c^2-b^2\,c)}\right)+\frac{6\,a\,b}{5\,(4\,a\,c^2-b^2\,c)}}{\left(a+b\,x^{1/3}+c\,x^{2/3}\right)^{5/2}}$$
$$+\frac{\frac{16\,c^2\,x^{1/3}}{5\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}+\frac{8\,b\,c}{5\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}}{\sqrt{a+b\,x^{1/3}+c\,x^{2/3}}}-\frac{\frac{4\,x^{1/3}}{5\,(4\,a\,c-b^2)}-\frac{2\,b}{5\,c\,(4\,a\,c-b^2)}}{\left(a+b\,x^{1/3}+c\,x^{2/3}\right)^{3/2}}$$
$$+\frac{x^{1/3}\left(\frac{2\,c\,(8\,b^2+8\,a\,c)}{5\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}+\frac{16\,a\,c^2}{5\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}-\frac{8\,b^2\,c}{5\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}\right)+\frac{b\,(8\,b^2+8\,a\,c)}{5\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}-\frac{8\,a\,b\,c}{5\,(4\,a\,c^2-b^2\,c)\,(\cdots}}{\left(a+b\,x^{1/3}+c\,x^{2/3}\right)^{3/2}}$$

input
```
int(1/(a + b*x^(1/3) + c*x^(2/3))^(7/2),x)
```

output
```
((2*c^2*x^(1/3)*(32*a*c + 56*b^2))/(5*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) +
 (b*c*(32*a*c + 56*b^2))/(5*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x^(
1/3) + c*x^(2/3))^(1/2) + (x^(1/3)*((6*b^2)/(5*(4*a*c^2 - b^2*c)) - (12*a*
c)/(5*(4*a*c^2 - b^2*c))) + (6*a*b)/(5*(4*a*c^2 - b^2*c)))/(a + b*x^(1/3)
+ c*x^(2/3))^(5/2) + ((16*c^2*x^(1/3))/(5*(4*a*c^2 - b^2*c)*(4*a*c - b^2))
 + (8*b*c)/(5*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x^(1/3) + c*x^(2/3)
)^(1/2) - ((4*x^(1/3))/(5*(4*a*c - b^2)) - (2*b)/(5*c*(4*a*c - b^2)))/(a +
 b*x^(1/3) + c*x^(2/3))^(3/2) + (x^(1/3)*((2*c*(8*a*c + 8*b^2))/(5*(4*a*c^
2 - b^2*c)*(4*a*c - b^2)) + (16*a*c^2)/(5*(4*a*c^2 - b^2*c)*(4*a*c - b^2))
 - (8*b^2*c)/(5*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(8*a*c + 8*b^2))/(5
*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*a*b*c)/(5*(4*a*c^2 - b^2*c)*(4*a*c
- b^2)))/(a + b*x^(1/3) + c*x^(2/3))^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 869, normalized size of antiderivative = 5.20

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(7/2),x)
```

output
```
(2*(240*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**2*b*c**2 + 200*x**(2
/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*b**3*c + 64*x**(2/3)*sqrt(x**(2/3)
*c + x**(1/3)*b + a)*a*c**4*x + 15*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b +
 a)*b**5 + 48*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*b**2*c**3*x + 240
*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**2*b**2*c + 20*x**(1/3)*sqrt
(x**(2/3)*c + x**(1/3)*b + a)*a*b**4 + 160*x**(1/3)*sqrt(x**(2/3)*c + x**(
1/3)*b + a)*a*b*c**3*x + 120*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*b*
*3*c**2*x + 96*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**3*b*c + 8*sqrt(x**(2/3
)*c + x**(1/3)*b + a)*a**2*b**3 + 160*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*
*2*c**3*x + 240*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*b**2*c**2*x + 90*sqrt(
x**(2/3)*c + x**(1/3)*b + a)*b**4*c*x - 192*x**(2/3)*sqrt(c)*a**3*c**2 - 3
36*x**(2/3)*sqrt(c)*a**2*b**2*c - 144*x**(2/3)*sqrt(c)*a*b**4 - 192*x**(2/
3)*sqrt(c)*a*b*c**3*x - 144*x**(2/3)*sqrt(c)*b**3*c**2*x - 192*x**(1/3)*sq
rt(c)*a**3*b*c - 144*x**(1/3)*sqrt(c)*a**2*b**3 - 192*x**(1/3)*sqrt(c)*a**
2*c**3*x - 336*x**(1/3)*sqrt(c)*a*b**2*c**2*x - 144*x**(1/3)*sqrt(c)*b**4*
c*x - 64*sqrt(c)*a**4*c - 48*sqrt(c)*a**3*b**2 - 384*sqrt(c)*a**2*b*c**2*x
 - 352*sqrt(c)*a*b**3*c*x - 64*sqrt(c)*a*c**4*x**2 - 48*sqrt(c)*b**5*x - 4
8*sqrt(c)*b**2*c**3*x**2))/(5*(192*x**(2/3)*a**5*c**4 + 48*x**(2/3)*a**4*b
**2*c**3 - 108*x**(2/3)*a**3*b**4*c**2 + 192*x**(2/3)*a**3*b*c**5*x + 33*x
**(2/3)*a**2*b**6*c - 144*x**(2/3)*a**2*b**3*c**4*x - 3*x**(2/3)*a*b**8...
```

3.63 $\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 222

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}\,dx = -\frac{6\left(ab+(b^2-2ac)\sqrt[3]{x}\right)}{7c\,(b^2-4ac)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}} + \frac{6(5b^2+4ac)\left(b+2c\sqrt[3]{x}\right)}{35c\,(b^2-4ac)^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}} - \frac{32(5b^2+4ac)\left(b+2c\sqrt[3]{x}\right)}{35\,(b^2-4ac)^3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}} + \frac{256c(5b^2+4ac)\left(b+2c\sqrt[3]{x}\right)}{35\,(b^2-4ac)^4\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}$$

output
```
1/7*(-6*a*b-6*(-2*a*c+b^2)*x^(1/3))/c/(-4*a*c+b^2)/(a+b*x^(1/3)+c*x^(2/3))
^(7/2)+6/35*(4*a*c+5*b^2)*(b+2*c*x^(1/3))/c/(-4*a*c+b^2)^2/(a+b*x^(1/3)+c*
x^(2/3))^(5/2)-32/35*(4*a*c+5*b^2)*(b+2*c*x^(1/3))/(-4*a*c+b^2)^3/(a+b*x^(
1/3)+c*x^(2/3))^(3/2)+256/35*c*(4*a*c+5*b^2)*(b+2*c*x^(1/3))/(-4*a*c+b^2)^
4/(a+b*x^(1/3)+c*x^(2/3))^(1/2)
```

Mathematica [A] (verified)

Time = 3.50 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}\,dx = \frac{2\left(1920a^4bc^2+320a^3c\left(b^3+21b^2c\sqrt[3]{x}+21bc^2x^{2/3}+14c^3x\right)+8a^2\left(-b^5+14\right.\right.}{}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^(-9/2),x]
```

output
```
(2*(1920*a^4*b*c^2 + 320*a^3*c*(b^3 + 21*b^2*c*x^(1/3) + 21*b*c^2*x^(2/3)
+ 14*c^3*x) + 8*a^2*(-b^5 + 140*b^4*c*x^(1/3) + 1190*b^3*c^2*x^(2/3) + 154
0*b^2*c^3*x + 1120*b*c^4*x^(4/3) + 448*c^5*x^(5/3)) + 5*(-7*b^7*x^(2/3) +
70*b^6*c*x + 560*b^5*c^2*x^(4/3) + 1120*b^4*c^3*x^(5/3) + 896*b^3*c^4*x^2
+ 256*b^2*c^5*x^(7/3)) + 4*a*(-7*b^6*x^(1/3) + 343*b^5*c*x^(2/3) + 2170*b^
4*c^2*x + 3360*b^3*c^3*x^(4/3) + 2240*b^2*c^4*x^(5/3) + 896*b*c^5*x^2 + 25
6*c^6*x^(7/3))))/(35*(b^2 - 4*a*c)^4*(a + b*x^(1/3) + c*x^(2/3))^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1680, 1164, 1159, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 1680}$$

$$3\int \frac{x^{2/3}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{9/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 1164}$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{7\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}-\frac{2\int\frac{2a-5b\sqrt[3]{x}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{7/2}}d\sqrt[3]{x}}{7\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \; 1159$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{7\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}-\frac{2\left(-\frac{8\left(4ac+5b^2\right)\int\frac{1}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{5/2}}d\sqrt[3]{x}}{5\left(b^2-4ac\right)}-\frac{2\left(\sqrt[3]{x}\left(4ac+5b^2\right)+12ab\right)}{5\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}\right)}{7\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \; 1089$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{7\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}-\frac{2\left(-\frac{8\left(4ac+5b^2\right)\left(-\frac{8c\int\frac{1}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{3/2}}d\sqrt[3]{x}}{3\left(b^2-4ac\right)}-\frac{2\left(b+2c\sqrt[3]{x}\right)}{3\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\right)}{5\left(b^2-4ac\right)} \cdots\right.}{7\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \; 1088$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{7\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}-\frac{2\left(-\frac{8\left(4ac+5b^2\right)\left(\frac{16c\left(b+2c\sqrt[3]{x}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}-\frac{2\left(b+2c\sqrt[3]{x}\right)}{3\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\right)}{5\left(b^2-4ac\right)}-\frac{}{5(}\right.}{7\left(b^2-4ac\right)}\right.$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^(-9/2),x]
```

output
```
3*((-2*((-8*(5*b^2 + 4*a*c)*((-2*(b + 2*c*x^(1/3)))/(3*(b^2 - 4*a*c)*(a +
b*x^(1/3) + c*x^(2/3))^(3/2)) + (16*c*(b + 2*c*x^(1/3)))/(3*(b^2 - 4*a*c)^
2*Sqrt[a + b*x^(1/3) + c*x^(2/3)])))/(5*(b^2 - 4*a*c)) - (2*(12*a*b + (5*b
^2 + 4*a*c)*x^(1/3)))/(5*(b^2 - 4*a*c)*(a + b*x^(1/3) + c*x^(2/3))^(5/2)))
)/(7*(b^2 - 4*a*c)) + (2*(2*a + b*x^(1/3))*x^(1/3))/(7*(b^2 - 4*a*c)*(a +
b*x^(1/3) + c*x^(2/3))^(7/2)))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(184) = 368$.

Time = 0.02 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.88

method	result
derivativedivides	$-\frac{x^{\frac{1}{3}}}{2c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{7}{2}}}-\frac{5b\left(-\frac{1}{7c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{7}{2}}}-\frac{b\left(\frac{\frac{2b}{7}+\frac{4c\,x^{\frac{1}{3}}}{7}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{7}{2}}}+\frac{24c\left(\frac{\frac{2b}{5}+\frac{4c\,x^{\frac{1}{3}}}{5}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{4c\cdots}$ $5b\left(-\frac{1}{7c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{7}{2}}}-\frac{b\left(\frac{\frac{2b}{7}+\frac{4c\,x^{\frac{1}{3}}}{7}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{7}{2}}}+\frac{24c\left(\frac{\frac{2b}{5}+\frac{4c\,x^{\frac{1}{3}}}{5}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)\cdots}\right.}{\cdots}\right.}{\cdots}\right.$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(1/3)/c/(a+b*x^(1/3)+c*x^(2/3))^(7/2)-5/4*b/c*(-1/7/c/(a+b*x^(1/3)+
c*x^(2/3))^(7/2)-1/2*b/c*(2/7*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x^(1/3)+c*x
^(2/3))^(7/2)+24/7*c/(4*a*c-b^2)*(2/5*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x^(
1/3)+c*x^(2/3))^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(b+2*c*x^(1/3))/(4*a*c-b^2)/
(a+b*x^(1/3)+c*x^(2/3))^(3/2)+16/3*c/(4*a*c-b^2)^2*(b+2*c*x^(1/3))/(a+b*x^
(1/3)+c*x^(2/3))^(1/2)))))+1/2*a/c*(2/7*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x
^(1/3)+c*x^(2/3))^(7/2)+24/7*c/(4*a*c-b^2)*(2/5*(b+2*c*x^(1/3))/(4*a*c-b^2
)/(a+b*x^(1/3)+c*x^(2/3))^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(b+2*c*x^(1/3))/(4
*a*c-b^2)/(a+b*x^(1/3)+c*x^(2/3))^(3/2)+16/3*c/(4*a*c-b^2)^2*(b+2*c*x^(1/3
))/(a+b*x^(1/3)+c*x^(2/3))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(9/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}\,dx = \int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{\frac{2}{3}}\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(a+b*x**(1/3)+c*x**(2/3))**(9/2),x)
```

output
```
Integral((a + b*x**(1/3) + c*x**(2/3))**(-9/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(182) = 364$.

Time = 0.20 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.78

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}\,dx = \frac{2\left(\left(\left(\left(2\left(8\left(2\left(4\,x^{\frac{1}{3}}\left(\frac{2\left(5\,b^2c^5+4\,ac^6\right)x^{\frac{1}{3}}}{b^8-16\,ab^6c+96\,a^2b^4c^2-256\,a^3b^2c^3+256\,a^4c^4}+\frac{7\left(5\,b^3c^4\cdots\right.}{b^8-16\,ab^6c+96\,a^2b^4c^2\cdots}\right.\right.\right.\right.\right.\right.\right.}{\cdots}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(9/2),x, algorithm="giac")
```

output
```
2/35*(((2*(8*(2*(4*x^(1/3)*(2*(5*b^2*c^5 + 4*a*c^6)*x^(1/3)/(b^8 - 16*a*b^
6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4) + 7*(5*b^3*c^4 + 4*a
*b*c^5)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4
)) + 7*(25*b^4*c^3 + 40*a*b^2*c^4 + 16*a^2*c^5)/(b^8 - 16*a*b^6*c + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x^(1/3) + 35*(5*b^5*c^2 + 24*a*
b^3*c^3 + 16*a^2*b*c^4)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c
^3 + 256*a^4*c^4))*x^(1/3) + 35*(5*b^6*c + 124*a*b^4*c^2 + 176*a^2*b^2*c^3
 + 64*a^3*c^4)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*
a^4*c^4))*x^(1/3) - 7*(5*b^7 - 196*a*b^5*c - 1360*a^2*b^3*c^2 - 960*a^3*b*
c^3)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*
x^(1/3) - 28*(a*b^6 - 40*a^2*b^4*c - 240*a^3*b^2*c^2)/(b^8 - 16*a*b^6*c +
96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x^(1/3) - 8*(a^2*b^5 - 40
*a^3*b^3*c - 240*a^4*b*c^2)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b
^2*c^3 + 256*a^4*c^4))/(c*x^(2/3) + b*x^(1/3) + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 24.54 (sec) , antiderivative size = 942, normalized size of antiderivative = 4.24

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*x^(1/3) + c*x^(2/3))^(9/2),x)
```

output
```
((64*b*c^2)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (128*c^3*x^(1/3))/(35
*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x^(1/3) + c*x^(2/3))^(1/2) - (
(32*b*c^2)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (64*c^3*x^(1/3))/(35*(
4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x^(1/3) + c*x^(2/3))^(1/2) + (x^
(1/3)*((6*b^2)/(7*(4*a*c^2 - b^2*c)) - (12*a*c)/(7*(4*a*c^2 - b^2*c))) + (
6*a*b)/(7*(4*a*c^2 - b^2*c)))/(a + b*x^(1/3) + c*x^(2/3))^(7/2) + ((64*c^3
*x^(1/3)*(28*a*c + 41*b^2))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3) + (32*b
*c^2*(28*a*c + 41*b^2))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3))/(a + b*x^(
1/3) + c*x^(2/3))^(1/2) + (x^(1/3)*((2*c^2*(96*a*c + 160*b^2))/(35*(4*a*c^
2 - b^2*c)*(4*a*c - b^2)^2) - (64*a*c^3)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^
2)^2) + (32*b^2*c^2)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2)) + (b*c*(96*a*
c + 160*b^2))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*a*b*c^2)/(35*(4
*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x^(1/3) + c*x^(2/3))^(3/2) - ((8*
b)/(35*(4*a*c - b^2)^2) - (16*c*x^(1/3))/(35*(4*a*c - b^2)^2))/(a + b*x^(1
/3) + c*x^(2/3))^(3/2) + ((16*c^2*x^(1/3))/(35*(4*a*c^2 - b^2*c)*(4*a*c -
b^2)) + (8*b*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x^(1/3) + c*x
^(2/3))^(3/2) - ((12*x^(1/3))/(35*(4*a*c - b^2)) - (6*b)/(35*c*(4*a*c - b^
2)))/(a + b*x^(1/3) + c*x^(2/3))^(5/2) + (x^(1/3)*((2*c*(24*a*c + 36*b^2))
/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (48*a*c^2)/(35*(4*a*c^2 - b^2*c)*(
4*a*c - b^2)) - (24*b^2*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(...
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1493, normalized size of antiderivative = 6.73

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(9/2),x)
```

output
```
(2*(6720*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**3*b*c**3 + 9520*x**
(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**2*b**3*c**2 + 3584*x**(2/3)*sqr
t(x**(2/3)*c + x**(1/3)*b + a)*a**2*c**5*x + 1372*x**(2/3)*sqrt(x**(2/3)*c
 + x**(1/3)*b + a)*a*b**5*c + 8960*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b +
 a)*a*b**2*c**4*x - 35*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*b**7 + 5
600*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*b**4*c**3*x + 6720*x**(1/3)
*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**3*b**2*c**2 + 1120*x**(1/3)*sqrt(x**
(2/3)*c + x**(1/3)*b + a)*a**2*b**4*c + 8960*x**(1/3)*sqrt(x**(2/3)*c + x*
*(1/3)*b + a)*a**2*b*c**4*x - 28*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a
)*a*b**6 + 13440*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*b**3*c**3*x
+ 1024*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*c**6*x**2 + 2800*x**(1
/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*b**5*c**2*x + 1280*x**(1/3)*sqrt(x**
(2/3)*c + x**(1/3)*b + a)*b**2*c**5*x**2 + 1920*sqrt(x**(2/3)*c + x**(1/3)
*b + a)*a**4*b*c**2 + 320*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**3*b**3*c +
4480*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**3*c**4*x - 8*sqrt(x**(2/3)*c + x
**(1/3)*b + a)*a**2*b**5 + 12320*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**2*b*
*2*c**3*x + 8680*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*b**4*c**2*x + 3584*sq
rt(x**(2/3)*c + x**(1/3)*b + a)*a*b*c**5*x**2 + 350*sqrt(x**(2/3)*c + x**(
1/3)*b + a)*b**6*c*x + 4480*sqrt(x**(2/3)*c + x**(1/3)*b + a)*b**3*c**4*x*
*2 - 4096*x**(2/3)*sqrt(c)*a**4*c**3 - 11264*x**(2/3)*sqrt(c)*a**3*b**2...
```

3.64 $\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{11/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 279

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{11/2}}\,dx = -\frac{2\left(ab+\left(b^2-2ac\right)\sqrt[3]{x}\right)}{3c\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}} + \frac{2(7b^2+4ac)\left(b+2c\sqrt[3]{x}\right)}{21c\left(b^2-4ac\right)^2\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}} - \frac{16(7b^2+4ac)\left(b+2c\sqrt[3]{x}\right)}{35\left(b^2-4ac\right)^3\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}} + \frac{256c(7b^2+4ac)\left(b+2c\sqrt[3]{x}\right)}{105\left(b^2-4ac\right)^4\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}} - \frac{2048c^2(7b^2+4ac)\left(b+2c\sqrt[3]{x}\right)}{105\left(b^2-4ac\right)^5\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}$$

output
```
1/3*(-2*a*b-2*(-2*a*c+b^2)*x^(1/3))/c/(-4*a*c+b^2)/(a+b*x^(1/3)+c*x^(2/3))
^(9/2)+2/21*(4*a*c+7*b^2)*(b+2*c*x^(1/3))/c/(-4*a*c+b^2)^2/(a+b*x^(1/3)+c*
x^(2/3))^(7/2)-16/35*(4*a*c+7*b^2)*(b+2*c*x^(1/3))/(-4*a*c+b^2)^3/(a+b*x^(
1/3)+c*x^(2/3))^(5/2)+256/105*c*(4*a*c+7*b^2)*(b+2*c*x^(1/3))/(-4*a*c+b^2)
^4/(a+b*x^(1/3)+c*x^(2/3))^(3/2)-2048/105*c^2*(4*a*c+7*b^2)*(b+2*c*x^(1/3)
)/(-4*a*c+b^2)^5/(a+b*x^(1/3)+c*x^(2/3))^(1/2)
```

Mathematica [A] (verified)

Time = 6.94 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{11/2}}\,dx =$$

$$-\frac{2\left(17920a^5bc^3+4480a^4c^2\left(b^3+18b^2c\sqrt[3]{x}+18bc^2x^{2/3}+12c^3x\right)+224a^3c\left(-b^5+90b^4c\sqrt[3]{x}+720b^3c^2x^{2/3}+\right.\right.}{}$$

input
```
Integrate[(a + b*x^(1/3) + c*x^(2/3))^(-11/2),x]
```

output
```
(-2*(17920*a^5*b*c^3 + 4480*a^4*c^2*(b^3 + 18*b^2*c*x^(1/3) + 18*b*c^2*x^(
2/3) + 12*c^3*x) + 224*a^3*c*(-b^5 + 90*b^4*c*x^(1/3) + 720*b^3*c^2*x^(2/3
) + 960*b^2*c^3*x + 720*b*c^4*x^(4/3) + 288*c^5*x^(5/3)) + 8*a^2*(b^7 - 12
6*b^6*c*x^(1/3) + 4284*b^5*c^2*x^(2/3) + 27720*b^4*c^3*x + 45360*b^3*c^4*x
^(4/3) + 34272*b^2*c^5*x^(5/3) + 16128*b*c^6*x^2 + 4608*c^7*x^(7/3)) + 7*(
9*b^9*x^(2/3) - 42*b^8*c*x + 504*b^7*c^2*x^(4/3) + 5040*b^6*c^3*x^(5/3) +
13440*b^5*c^4*x^2 + 16128*b^4*c^5*x^(7/3) + 9216*b^3*c^6*x^(8/3) + 2048*b^
2*c^7*x^3) + 4*a*(9*b^8*x^(1/3) - 432*b^7*c*x^(2/3) + 4368*b^6*c^2*x + 357
84*b^5*c^3*x^(4/3) + 75600*b^4*c^4*x^(5/3) + 69888*b^3*c^5*x^2 + 32256*b^2
*c^6*x^(7/3) + 9216*b*c^7*x^(8/3) + 2048*c^8*x^3)))/(105*(b^2 - 4*a*c)^5*(
a + b*x^(1/3) + c*x^(2/3))^(9/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1680, 1164, 1159, 1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{11/2}}dx$$

$$\Big\downarrow \text{1680}$$

$$3\int \frac{x^{2/3}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{11/2}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{1164}$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{9\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}-\frac{2\int \frac{2a-7b\sqrt[3]{x}}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{9/2}}d\sqrt[3]{x}}{9\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{1159}$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{9\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}-\frac{2\left(-\frac{12\left(4ac+7b^2\right)\int \frac{1}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{7/2}}d\sqrt[3]{x}}{7\left(b^2-4ac\right)}-\frac{2\left(\sqrt[3]{x}\left(4ac+7b^2\right)+16ab\right)}{7\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{7/2}}\right)}{9\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{1089}$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{9\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}-\frac{2\left(-\frac{12\left(4ac+7b^2\right)\left(-\frac{16c\int \frac{1}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{5/2}}d\sqrt[3]{x}}{5\left(b^2-4ac\right)}-\frac{2\left(b+2c\sqrt[3]{x}\right)}{5\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{5/2}}\right)}{7\left(b^2-4ac\right)}\right.}{9\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{1089}$$

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{9\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}-\frac{2\left(-\frac{12\left(4ac+7b^2\right)\left(-\frac{16c\left(-\frac{8c\int\frac{1}{\left(a+cx^{2/3}+b\sqrt[3]{x}\right)^{3/2}}d\sqrt[3]{x}}{3\left(b^2-4ac\right)}-\frac{2\left(b+2c\sqrt[3]{x}\right)}{3\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)}\right.}{5\left(b^2-4ac\right)}\right.}{7\left(b^2-4ac\right)}\right.}{9\left(b^2-4a\right.}\right.$$

↓ 1088

$$3\left(\frac{2\sqrt[3]{x}\left(2a+b\sqrt[3]{x}\right)}{9\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{9/2}}-\frac{2\left(-\frac{12(4ac+7b^2)\left(-\frac{16c\left(\frac{16c\left(b+2c\sqrt[3]{x}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+b\sqrt[3]{x}+cx^{2/3}}}-\frac{2\left(b+2c\sqrt[3]{x}\right)}{3\left(b^2-4ac\right)\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{3/2}}\right)}{5\left(b^2-4ac\right)}\right.}{7(b^2-4ac)}\right.}{9\left(b^2-4ac\right)}\right.$$

input
```
Int[(a + b*x^(1/3) + c*x^(2/3))^(-11/2),x]
```

output
```
3*((-2*((-12*(7*b^2 + 4*a*c)*((-16*c*((-2*(b + 2*c*x^(1/3)))/(3*(b^2 - 4*a
*c)*(a + b*x^(1/3) + c*x^(2/3))^(3/2)) + (16*c*(b + 2*c*x^(1/3)))/(3*(b^2
- 4*a*c)^2*Sqrt[a + b*x^(1/3) + c*x^(2/3)])))/(5*(b^2 - 4*a*c)) - (2*(b +
2*c*x^(1/3)))/(5*(b^2 - 4*a*c)*(a + b*x^(1/3) + c*x^(2/3))^(5/2))))/(7*(b^
2 - 4*a*c)) - (2*(16*a*b + (7*b^2 + 4*a*c)*x^(1/3)))/(7*(b^2 - 4*a*c)*(a +
 b*x^(1/3) + c*x^(2/3))^(7/2))))/(9*(b^2 - 4*a*c)) + (2*(2*a + b*x^(1/3))*
x^(1/3))/(9*(b^2 - 4*a*c)*(a + b*x^(1/3) + c*x^(2/3))^(9/2)))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(231) = 462$.

Time = 0.02 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.87

method	result
	$21b\left(-\frac{1}{9c\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{9}{2}}}-\frac{b\left(\frac{\frac{2b}{9}+\frac{4c\,x^{\frac{1}{3}}}{9}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right)^{\frac{9}{2}}}+\frac{32c\left(\frac{\frac{2b}{7}+\frac{4c\,x^{\frac{1}{3}}}{7}}{\left(4ac-b^{2}\right)\left(a+b\,x^{\frac{1}{3}}+c\,x^{\frac{2}{3}}\right.}\right.}{}\right.}{}\right.$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-3/8*x^(1/3)/c/(a+b*x^(1/3)+c*x^(2/3))^(9/2)-21/16*b/c*(-1/9/c/(a+b*x^(1/3
)+c*x^(2/3))^(9/2)-1/2*b/c*(2/9*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x^(1/3)+c
*x^(2/3))^(9/2)+32/9*c/(4*a*c-b^2)*(2/7*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x
^(1/3)+c*x^(2/3))^(7/2)+24/7*c/(4*a*c-b^2)*(2/5*(b+2*c*x^(1/3))/(4*a*c-b^2
)/(a+b*x^(1/3)+c*x^(2/3))^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(b+2*c*x^(1/3))/(4
*a*c-b^2)/(a+b*x^(1/3)+c*x^(2/3))^(3/2)+16/3*c/(4*a*c-b^2)^2*(b+2*c*x^(1/3
))/(a+b*x^(1/3)+c*x^(2/3))^(1/2))))))+3/8*a/c*(2/9*(b+2*c*x^(1/3))/(4*a*c-
b^2)/(a+b*x^(1/3)+c*x^(2/3))^(9/2)+32/9*c/(4*a*c-b^2)*(2/7*(b+2*c*x^(1/3))
/(4*a*c-b^2)/(a+b*x^(1/3)+c*x^(2/3))^(7/2)+24/7*c/(4*a*c-b^2)*(2/5*(b+2*c*
x^(1/3))/(4*a*c-b^2)/(a+b*x^(1/3)+c*x^(2/3))^(5/2)+16/5*c/(4*a*c-b^2)*(2/3
*(b+2*c*x^(1/3))/(4*a*c-b^2)/(a+b*x^(1/3)+c*x^(2/3))^(3/2)+16/3*c/(4*a*c-b
^2)^2*(b+2*c*x^(1/3))/(a+b*x^(1/3)+c*x^(2/3))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(11/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{11/2}}\,dx=\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{\frac{2}{3}}\right)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(a+b*x**(1/3)+c*x**(2/3))**(11/2),x)
```

output
```
Integral((a + b*x**(1/3) + c*x**(2/3))**(-11/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{11/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(11/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. 2(229) = 458.

Time = 0.27 (sec) , antiderivative size = 949, normalized size of antiderivative = 3.40

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x^(1/3)+c*x^(2/3))^(11/2),x, algorithm="giac")
```

output
```
-2/105*(((2*(4*(2*(8*(2*(4*x^(1/3)*(2*(7*b^2*c^7 + 4*a*c^8)*x^(1/3)/(b^10
- 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024
*a^5*c^5) + 9*(7*b^3*c^6 + 4*a*b*c^7)/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2
 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5)) + 9*(49*b^4*c^5 + 5
6*a*b^2*c^6 + 16*a^2*c^7)/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b
^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))*x^(1/3) + 21*(35*b^5*c^4 + 104*
a*b^3*c^5 + 48*a^2*b*c^6)/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b
^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))*x^(1/3) + 63*(35*b^6*c^3 + 300*
a*b^4*c^4 + 272*a^2*b^2*c^5 + 64*a^3*c^6)/(b^10 - 20*a*b^8*c + 160*a^2*b^6
*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))*x^(1/3) + 63*(7
*b^7*c^2 + 284*a*b^5*c^3 + 720*a^2*b^3*c^4 + 320*a^3*b*c^5)/(b^10 - 20*a*b
^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5
))*x^(1/3) - 21*(7*b^8*c - 416*a*b^6*c^2 - 5280*a^2*b^4*c^3 - 5120*a^3*b^2
*c^4 - 1280*a^4*c^5)/(b^10 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^
3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))*x^(1/3) + 9*(7*b^9 - 192*a*b^7*c + 3
808*a^2*b^5*c^2 + 17920*a^3*b^3*c^3 + 8960*a^4*b*c^4)/(b^10 - 20*a*b^8*c +
 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 1024*a^5*c^5))*x^(
1/3) + 36*(a*b^8 - 28*a^2*b^6*c + 560*a^3*b^4*c^2 + 2240*a^4*b^2*c^3)/(b^1
0 - 20*a*b^8*c + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 10
24*a^5*c^5))*x^(1/3) + 8*(a^2*b^7 - 28*a^3*b^5*c + 560*a^4*b^3*c^2 + 22...
```

Mupad [B] (verification not implemented)

Time = 26.86 (sec) , antiderivative size = 1455, normalized size of antiderivative = 5.22

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{11/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*x^(1/3) + c*x^(2/3))^(11/2),x)
```

output
```
((128*b*c^2)/(315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (256*c^3*x^(1/3))/(
315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x^(1/3) + c*x^(2/3))^(3/2)
- ((32*b*c^2)/(315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (64*c^3*x^(1/3))/(
315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x^(1/3) + c*x^(2/3))^(3/2)
+ ((128*b*c^3)/(315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3) + (256*c^4*x^(1/3))
/(315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3))/(a + b*x^(1/3) + c*x^(2/3))^(1/2
) - ((256*b*c^3)/(315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3) + (512*c^4*x^(1/3
))/(315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3))/(a + b*x^(1/3) + c*x^(2/3))^(1
/2) + ((1024*b*c^3)/(315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3) + (2048*c^4*x^
(1/3))/(315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3))/(a + b*x^(1/3) + c*x^(2/3)
)^(1/2) + ((32*b*c)/(315*(4*a*c - b^2)^3) - (64*c^2*x^(1/3))/(315*(4*a*c -
 b^2)^3))/(a + b*x^(1/3) + c*x^(2/3))^(3/2) + (x^(1/3)*((2*b^2)/(3*(4*a*c^
2 - b^2*c)) - (4*a*c)/(3*(4*a*c^2 - b^2*c))) + (2*a*b)/(3*(4*a*c^2 - b^2*c
)))/(a + b*x^(1/3) + c*x^(2/3))^(9/2) + ((256*c^4*x^(1/3)*(164*a*c + 343*b
^2))/(315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^4) + (128*b*c^3*(164*a*c + 343*b
^2))/(315*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^4))/(a + b*x^(1/3) + c*x^(2/3))^
(1/2) + (x^(1/3)*((2*c^2*(160*a*c + 336*b^2))/(105*(4*a*c^2 - b^2*c)*(4*a*
c - b^2)^2) - (64*a*c^3)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*b^2
*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2)) + (b*c*(160*a*c + 336*b^2))
/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*a*b*c^2)/(105*(4*a*c^2 -...
```

Reduce [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 2219, normalized size of antiderivative = 7.95

$$\int \frac{1}{\left(a+b\sqrt[3]{x}+cx^{2/3}\right)^{11/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*x^(1/3)+c*x^(2/3))^(11/2),x)
```

output
```
(2*(80640*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**4*b*c**4 + 161280*
x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**3*b**3*c**3 + 64512*x**(2/3)
*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**3*c**6*x + 34272*x**(2/3)*sqrt(x**(2
/3)*c + x**(1/3)*b + a)*a**2*b**5*c**2 + 274176*x**(2/3)*sqrt(x**(2/3)*c +
 x**(1/3)*b + a)*a**2*b**2*c**5*x - 1728*x**(2/3)*sqrt(x**(2/3)*c + x**(1/
3)*b + a)*a*b**7*c + 302400*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*b
**4*c**4*x + 36864*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*b*c**7*x**
2 + 63*x**(2/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*b**9 + 35280*x**(2/3)*sq
rt(x**(2/3)*c + x**(1/3)*b + a)*b**6*c**3*x + 64512*x**(2/3)*sqrt(x**(2/3)
*c + x**(1/3)*b + a)*b**3*c**6*x**2 + 80640*x**(1/3)*sqrt(x**(2/3)*c + x**
(1/3)*b + a)*a**4*b**2*c**3 + 20160*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b
+ a)*a**3*b**4*c**2 + 161280*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*
*3*b*c**5*x - 1008*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**2*b**6*c
+ 362880*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**2*b**3*c**4*x + 368
64*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**2*c**7*x**2 + 36*x**(1/3)
*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a*b**8 + 143136*x**(1/3)*sqrt(x**(2/3)*
c + x**(1/3)*b + a)*a*b**5*c**3*x + 129024*x**(1/3)*sqrt(x**(2/3)*c + x**(
1/3)*b + a)*a*b**2*c**6*x**2 + 3528*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b
+ a)*b**7*c**2*x + 112896*x**(1/3)*sqrt(x**(2/3)*c + x**(1/3)*b + a)*b**4*
c**5*x**2 + 17920*sqrt(x**(2/3)*c + x**(1/3)*b + a)*a**5*b*c**3 + 4480*...
```

3.65 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 59

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3 \, dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)^7}{7b^3}-\frac{3a\left(a+b\sqrt[3]{x}\right)^8}{4b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^9}{3b^3}$$

output
```
3/7*a^2*(a+b*x^(1/3))^7/b^3-3/4*a*(a+b*x^(1/3))^8/b^3+1/3*(a+b*x^(1/3))^9/
b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3 \, dx=\frac{1}{84}\left(84a^6x+378a^5bx^{4/3}+756a^4b^2x^{5/3}+840a^3b^3x^2+540a^2b^4x^{7/3}+189ab^5x^{8/3}+28b^6x^3\right)$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^3,x]
```

output
```
(84*a^6*x + 378*a^5*b*x^(4/3) + 756*a^4*b^2*x^(5/3) + 840*a^3*b^3*x^2 + 54
0*a^2*b^4*x^(7/3) + 189*a*b^5*x^(8/3) + 28*b^6*x^3)/84
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1379, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3 dx$$

$$\Big\downarrow \text{1379}$$

$$\frac{\int \left(\sqrt[3]{x}b^2+ab\right)^6 dx}{b^6}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\int b^6\left(a+b\sqrt[3]{x}\right)^6 x^{2/3}d\sqrt[3]{x}}{b^6}$$

$$\Big\downarrow \text{27}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^6 x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^8}{b^2}-\frac{2a\left(a+b\sqrt[3]{x}\right)^7}{b^2}+\frac{a^2\left(a+b\sqrt[3]{x}\right)^6}{b^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^2\left(a+b\sqrt[3]{x}\right)^7}{7b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^9}{9b^3}-\frac{a\left(a+b\sqrt[3]{x}\right)^8}{4b^3}\right)$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^3,x]
```

output
```
3*((a^2*(a + b*x^(1/3))^7)/(7*b^3) - (a*(a + b*x^(1/3))^8)/(4*b^3) + (a +
b*x^(1/3))^9/(9*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{b^6x^3}{3}+\frac{9b^5a\,x^{\frac{8}{3}}}{4}+\frac{45b^4a^2x^{\frac{7}{3}}}{7}+10b^3a^3x^2+9a^4b^2x^{\frac{5}{3}}+\frac{9a^5b\,x^{\frac{4}{3}}}{2}+a^6x$
default	$\frac{b^6x^3}{3}+\frac{9b^5a\,x^{\frac{8}{3}}}{4}+\frac{45b^4a^2x^{\frac{7}{3}}}{7}+10b^3a^3x^2+9a^4b^2x^{\frac{5}{3}}+\frac{9a^5b\,x^{\frac{4}{3}}}{2}+a^6x$
trager	$\frac{\left(b^6x^2+30b^3x\,a^3+b^6x+3a^6+30b^3a^3+b^6\right)(x-1)}{3}+\frac{9a^2b\,x^{\frac{4}{3}}\left(10b^3x+7a^3\right)}{14}+\frac{9a\,b^2x^{\frac{5}{3}}\left(b^3x+4a^3\right)}{4}$
orering	$-\frac{\left(-64x^5b^{15}-257x^4b^{12}a^3-388x^3b^9a^6-162x^2b^6a^9-228a^{12}b^3x+9a^{15}\right)\left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^3}{84b^3\left(b^3x+a^3\right)^4}-\frac{9\left(5x^4b^{12}+20x^3b^9\right.}{}$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^6*x^3+9/4*b^5*a*x^(8/3)+45/7*b^4*a^2*x^(7/3)+10*b^3*a^3*x^2+9*a^4*b^
2*x^(5/3)+9/2*a^5*b*x^(4/3)+a^6*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3\,dx=\frac{1}{3}\,b^6x^3+10\,a^3b^3x^2+a^6x+\frac{9}{4}\left(ab^5x^2+4\,a^4b^2x\right)x^{\frac{2}{3}}+\frac{9}{14}\left(10\,a^2b^4x^2+7\,a^5bx\right)x^{\frac{1}{3}}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^3,x, algorithm="fricas")
```

output
```
1/3*b^6*x^3 + 10*a^3*b^3*x^2 + a^6*x + 9/4*(a*b^5*x^2 + 4*a^4*b^2*x)*x^(2/
3) + 9/14*(10*a^2*b^4*x^2 + 7*a^5*b*x)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3\,dx=a^6x+\frac{9a^5bx^{\frac{4}{3}}}{2}+9a^4b^2x^{\frac{5}{3}}+10a^3b^3x^2+\frac{45a^2b^4x^{\frac{7}{3}}}{7}+\frac{9ab^5x^{\frac{8}{3}}}{4}+\frac{b^6x^3}{3}$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**3,x)
```

output
```
a**6*x + 9*a**5*b*x**(4/3)/2 + 9*a**4*b**2*x**(5/3) + 10*a**3*b**3*x**2 +
45*a**2*b**4*x**(7/3)/7 + 9*a*b**5*x**(8/3)/4 + b**6*x**3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(47) = 94$.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3\,dx = \frac{1}{3}\,b^6x^3+\frac{9}{4}\,ab^5x^{\frac{8}{3}}+\frac{36}{7}\,a^2b^4x^{\frac{7}{3}}+4\,a^3b^3x^2+a^6x+\frac{9}{10}\left(2\,b^2x^{\frac{5}{3}}+5\,abx^{\frac{4}{3}}\right)a^4+\frac{3}{35}\left(15\,b^4x^{\frac{7}{3}}+70\,ab^3x^2+84\,a^2b^2x^{\frac{5}{3}}\right)a^2$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^3,x, algorithm="maxima")
```

output
```
1/3*b^6*x^3 + 9/4*a*b^5*x^(8/3) + 36/7*a^2*b^4*x^(7/3) + 4*a^3*b^3*x^2 + a
^6*x + 9/10*(2*b^2*x^(5/3) + 5*a*b*x^(4/3))*a^4 + 3/35*(15*b^4*x^(7/3) + 7
0*a*b^3*x^2 + 84*a^2*b^2*x^(5/3))*a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3\,dx = \frac{1}{3}\,b^6x^3+\frac{9}{4}\,ab^5x^{\frac{8}{3}}+\frac{45}{7}\,a^2b^4x^{\frac{7}{3}}+10\,a^3b^3x^2+9\,a^4b^2x^{\frac{5}{3}}+\frac{9}{2}\,a^5bx^{\frac{4}{3}}+a^6x$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^3,x, algorithm="giac")
```

output
```
1/3*b^6*x^3 + 9/4*a*b^5*x^(8/3) + 45/7*a^2*b^4*x^(7/3) + 10*a^3*b^3*x^2 +
9*a^4*b^2*x^(5/3) + 9/2*a^5*b*x^(4/3) + a^6*x
```

Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3\,dx = a^6\,x+\frac{b^6\,x^3}{3}+\frac{9\,a^5\,b\,x^{4/3}}{2}+\frac{9\,a\,b^5\,x^{8/3}}{4}+10\,a^3\,b^3\,x^2+9\,a^4\,b^2\,x^{5/3}+\frac{45\,a^2\,b^4\,x^{7/3}}{7}$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^3,x)
```

output
```
a^6*x + (b^6*x^3)/3 + (9*a^5*b*x^(4/3))/2 + (9*a*b^5*x^(8/3))/4 + 10*a^3*b
^3*x^2 + 9*a^4*b^2*x^(5/3) + (45*a^2*b^4*x^(7/3))/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3\,dx = \frac{x\left(756x^{\frac{2}{3}}a^4b^2+189x^{\frac{5}{3}}a\,b^5+378x^{\frac{1}{3}}a^5b+540x^{\frac{4}{3}}a^2b^4+84a^6+840a^3b^3x+28b^6x^2\right)}{84}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^3,x)
```

output
```
(x*(756*x**(2/3)*a**4*b**2 + 189*x**(2/3)*a*b**5*x + 378*x**(1/3)*a**5*b +
 540*x**(1/3)*a**2*b**4*x + 84*a**6 + 840*a**3*b**3*x + 28*b**6*x**2))/84
```

3.66 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 57

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2 \, dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)^5}{5b^3}-\frac{a\left(a+b\sqrt[3]{x}\right)^6}{b^3}+\frac{3\left(a+b\sqrt[3]{x}\right)^7}{7b^3}$$

output
```
3/5*a^2*(a+b*x^(1/3))^5/b^3-a*(a+b*x^(1/3))^6/b^3+3/7*(a+b*x^(1/3))^7/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2 \, dx=\frac{1}{35}\left(35a^4x+105a^3bx^{4/3}+126a^2b^2x^{5/3}+70ab^3x^2+15b^4x^{7/3}\right)$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^2,x]
```

output
```
(35*a^4*x + 105*a^3*b*x^(4/3) + 126*a^2*b^2*x^(5/3) + 70*a*b^3*x^2 + 15*b^
4*x^(7/3))/35
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1379, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2 dx$$

$$\Big\downarrow \text{1379}$$

$$\frac{\int \left(\sqrt[3]{x}b^2+ab\right)^4 dx}{b^4}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\int b^4\left(a+b\sqrt[3]{x}\right)^4 x^{2/3}d\sqrt[3]{x}}{b^4}$$

$$\Big\downarrow \text{27}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^4 x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^6}{b^2}-\frac{2a\left(a+b\sqrt[3]{x}\right)^5}{b^2}+\frac{a^2\left(a+b\sqrt[3]{x}\right)^4}{b^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^2\left(a+b\sqrt[3]{x}\right)^5}{5b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^7}{7b^3}-\frac{a\left(a+b\sqrt[3]{x}\right)^6}{3b^3}\right)$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^2,x]
```

output
```
3*((a^2*(a + b*x^(1/3))^5)/(5*b^3) - (a*(a + b*x^(1/3))^6)/(3*b^3) + (a +
b*x^(1/3))^7/(7*b^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{3 b^{4} x^{\frac{7}{3}}}{7}+2 b^{3} a \,x^{2}+\frac{18 a^{2} b^{2} x^{\frac{5}{3}}}{5}+3 a^{3} b \,x^{\frac{4}{3}}+a^{4} x$
default	$\frac{3 b^{4} x^{\frac{7}{3}}}{7}+2 b^{3} a \,x^{2}+\frac{18 a^{2} b^{2} x^{\frac{5}{3}}}{5}+3 a^{3} b \,x^{\frac{4}{3}}+a^{4} x$
trager	$\left(x -1\right) \left(2 b^{3} x +a^{3}+2 b^{3}\right) a +\frac{3 b \,x^{\frac{4}{3}} \left(b^{3} x +7 a^{3}\right)}{7}+\frac{18 a^{2} b^{2} x^{\frac{5}{3}}}{5}$
orering	$\frac{\left(31 b^{9} x^{3}+63 a^{3} b^{6} x^{2}+21 a^{6} b^{3} x +7 a^{9}\right) \left(a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{2}}{35 b^{3} \left(b^{3} x +a^{3}\right)^{2}}-\frac{27 b^{3} x^{3} \left(b^{3} x +2 a^{3}\right) \left(a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right) \left(\frac{2 a b}{3 x^{\frac{2}{3}}}+\frac{2 b^{2}}{3 x^{\frac{1}{3}}}\right)}{35 \left(b^{3} x +a^{3}\right)^{2}}$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^2,x,method=_RETURNVERBOSE)
```

output
```
3/7*b^4*x^(7/3)+2*b^3*a*x^2+18/5*a^2*b^2*x^(5/3)+3*a^3*b*x^(4/3)+a^4*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2\,dx=2\,ab^3x^2+\frac{18}{5}\,a^2b^2x^{\frac{5}{3}}+a^4x+\frac{3}{7}\,\left(b^4x^2+7\,a^3bx\right)x^{\frac{1}{3}}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^2,x, algorithm="fricas")
```

output
```
2*a*b^3*x^2 + 18/5*a^2*b^2*x^(5/3) + a^4*x + 3/7*(b^4*x^2 + 7*a^3*b*x)*x^(
1/3)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2\,dx=a^4x+3a^3bx^{\frac{4}{3}}+\frac{18a^2b^2x^{\frac{5}{3}}}{5}+2ab^3x^2+\frac{3b^4x^{\frac{7}{3}}}{7}$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**2,x)
```

output
```
a**4*x + 3*a**3*b*x**(4/3) + 18*a**2*b**2*x**(5/3)/5 + 2*a*b**3*x**2 + 3*b
**4*x**(7/3)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2\,dx=\frac{3}{7}\,b^4x^{\frac{7}{3}}+2\,ab^3x^2+\frac{12}{5}\,a^2b^2x^{\frac{5}{3}}+a^4x+\frac{3}{5}\left(2\,b^2x^{\frac{5}{3}}+5\,abx^{\frac{4}{3}}\right)a^2$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^2,x, algorithm="maxima")
```

output
```
3/7*b^4*x^(7/3) + 2*a*b^3*x^2 + 12/5*a^2*b^2*x^(5/3) + a^4*x + 3/5*(2*b^2*
x^(5/3) + 5*a*b*x^(4/3))*a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2\,dx=\frac{3}{7}\,b^4x^{\frac{7}{3}}+2\,ab^3x^2+\frac{18}{5}\,a^2b^2x^{\frac{5}{3}}+3\,a^3bx^{\frac{4}{3}}+a^4x$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^2,x, algorithm="giac")
```

output
```
3/7*b^4*x^(7/3) + 2*a*b^3*x^2 + 18/5*a^2*b^2*x^(5/3) + 3*a^3*b*x^(4/3) + a
^4*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2\,dx=a^4\,x+\frac{3\,b^4\,x^{7/3}}{7}+2\,a\,b^3\,x^2+3\,a^3\,b\,x^{4/3}+\frac{18\,a^2\,b^2\,x^{5/3}}{5}$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^2,x)
```

output
```
a^4*x + (3*b^4*x^(7/3))/7 + 2*a*b^3*x^2 + 3*a^3*b*x^(4/3) + (18*a^2*b^2*x^
(5/3))/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2\,dx=\frac{x\left(126x^{\frac{2}{3}}a^2b^2+105x^{\frac{1}{3}}a^3b+15x^{\frac{4}{3}}b^4+35a^4+70a\,b^3x\right)}{35}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^2,x)
```

output
```
(x*(126*x**(2/3)*a**2*b**2 + 105*x**(1/3)*a**3*b + 15*x**(1/3)*b**4*x + 35
*a**4 + 70*a*b**3*x))/35
```

3.67 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)\,dx=a^2x+\frac{3}{2}abx^{4/3}+\frac{3}{5}b^2x^{5/3}$$

output
```
a^2*x+3/2*a*b*x^(4/3)+3/5*b^2*x^(5/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)\,dx=\frac{1}{10}\left(10a^2x+15abx^{4/3}+6b^2x^{5/3}\right)$$

input
```
Integrate[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3),x]
```

output
```
(10*a^2*x + 15*a*b*x^(4/3) + 6*b^2*x^(5/3))/10
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$a^2x+\frac{3}{2}abx^{4/3}+\frac{3}{5}b^2x^{5/3}$$

input
```
Int[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3),x]
```

output
```
a^2*x + (3*a*b*x^(4/3))/2 + (3*b^2*x^(5/3))/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$x\,a^{2}+\frac{3ab\,x^{\frac{4}{3}}}{2}+\frac{3b^{2}x^{\frac{5}{3}}}{5}$	22
default	$x\,a^{2}+\frac{3ab\,x^{\frac{4}{3}}}{2}+\frac{3b^{2}x^{\frac{5}{3}}}{5}$	22
risch	$x\,a^{2}+\frac{3ab\,x^{\frac{4}{3}}}{2}+\frac{3b^{2}x^{\frac{5}{3}}}{5}$	22
parts	$x\,a^{2}+\frac{3ab\,x^{\frac{4}{3}}}{2}+\frac{3b^{2}x^{\frac{5}{3}}}{5}$	22
trager	$a^{2}(x-1)+\frac{3ab\,x^{\frac{4}{3}}}{2}+\frac{3b^{2}x^{\frac{5}{3}}}{5}$	24
orering	$x\left(a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}\right)-\frac{9x^{2}\left(\frac{2ab}{3x^{\frac{2}{3}}}+\frac{2b^{2}}{3x^{\frac{1}{3}}}\right)}{20}+\frac{9x^{3}\left(-\frac{4ab}{9x^{\frac{5}{3}}}-\frac{2b^{2}}{9x^{\frac{4}{3}}}\right)}{20}$	64

input
```
int(a^2+2*a*b*x^(1/3)+b^2*x^(2/3),x,method=_RETURNVERBOSE)
```

output
```
x*a^2+3/2*a*b*x^(4/3)+3/5*b^2*x^(5/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)\,dx=\frac{3}{5}\,b^{2}x^{\frac{5}{3}}+\frac{3}{2}\,abx^{\frac{4}{3}}+a^{2}x$$

input
```
integrate(a^2+2*a*b*x^(1/3)+b^2*x^(2/3),x, algorithm="fricas")
```

output
```
3/5*b^2*x^(5/3) + 3/2*a*b*x^(4/3) + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2 x^{2/3}\right) dx = a^2 x + \frac{3abx^{\frac{4}{3}}}{2} + \frac{3b^2 x^{\frac{5}{3}}}{5}$$

input
```
integrate(a**2+2*a*b*x**(1/3)+b**2*x**(2/3),x)
```

output
```
a**2*x + 3*a*b*x**(4/3)/2 + 3*b**2*x**(5/3)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2 x^{2/3}\right) dx = \frac{3}{5}\, b^2 x^{\frac{5}{3}} + \frac{3}{2}\, abx^{\frac{4}{3}} + a^2 x$$

input
```
integrate(a^2+2*a*b*x^(1/3)+b^2*x^(2/3),x, algorithm="maxima")
```

output
```
3/5*b^2*x^(5/3) + 3/2*a*b*x^(4/3) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2 x^{2/3}\right) dx = \frac{3}{5}\, b^2 x^{\frac{5}{3}} + \frac{3}{2}\, abx^{\frac{4}{3}} + a^2 x$$

input
```
integrate(a^2+2*a*b*x^(1/3)+b^2*x^(2/3),x, algorithm="giac")
```

output
```
3/5*b^2*x^(5/3) + 3/2*a*b*x^(4/3) + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)\,dx=a^2\,x+\frac{3\,b^2\,x^{5/3}}{5}+\frac{3\,a\,b\,x^{4/3}}{2}$$

input
```
int(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3),x)
```

output
```
a^2*x + (3*b^2*x^(5/3))/5 + (3*a*b*x^(4/3))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)\,dx=\frac{x\left(6x^{\frac{2}{3}}b^2+15x^{\frac{1}{3}}ab+10a^2\right)}{10}$$

input
```
int(a^2+2*a*b*x^(1/3)+b^2*x^(2/3),x)
```

output
```
(x*(6*x**(2/3)*b**2 + 15*x**(1/3)*a*b + 10*a**2))/10
```

3.68 $\int \frac{1}{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{1}{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=-\frac{3a^2}{b^3\left(a+b\sqrt[3]{x}\right)}+\frac{3\sqrt[3]{x}}{b^2}-\frac{6a\log\left(a+b\sqrt[3]{x}\right)}{b^3}$$

output
```
-3*a^2/b^3/(a+b*x^(1/3))+3*x^(1/3)/b^2-6*a*ln(a+b*x^(1/3))/b^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{1}{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=\frac{3\left(-a^2+ab\sqrt[3]{x}+b^2x^{2/3}\right)}{b^3\left(a+b\sqrt[3]{x}\right)}-\frac{6a\log\left(a+b\sqrt[3]{x}\right)}{b^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-1),x]
```

output
```
(3*(-a^2 + a*b*x^(1/3) + b^2*x^(2/3)))/(b^3*(a + b*x^(1/3))) - (6*a*Log[a
+ b*x^(1/3)])/b^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1379, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}dx$$

$$\Big\downarrow \text{1379}$$

$$b^2\int \frac{1}{\left(\sqrt[3]{x}b^2+ab\right)^2}dx$$

$$\Big\downarrow \text{774}$$

$$3b^2\int \frac{x^{2/3}}{b^2\left(a+b\sqrt[3]{x}\right)^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{27}$$

$$3\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^2}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)}+\frac{1}{b^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^2}{b^3\left(a+b\sqrt[3]{x}\right)}-\frac{2a\log\left(a+b\sqrt[3]{x}\right)}{b^3}+\frac{\sqrt[3]{x}}{b^2}\right)$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-1),x]
```

output
```
3*(-(a^2/(b^3*(a + b*x^(1/3)))) + x^(1/3)/b^2 - (2*a*Log[a + b*x^(1/3)])/b
^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{3a^2}{b^3\left(a+b\,x^{\frac{1}{3}}\right)}+\frac{3x^{\frac{1}{3}}}{b^2}-\frac{6a\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^3}$
default	$-\frac{3a^4}{b^3(b^3x+a^3)}+b^4\left(\frac{3x^{\frac{1}{3}}}{b^6}+\frac{a\left(\frac{a\,x^{\frac{1}{3}}+\frac{a^2}{b}}{b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2}+\frac{2\ln\left(b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2\right)}{b}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2b^2x^{\frac{1}{3}}-ba\right)\sqrt{3}}{3ba}\right)}{b}\right)}{3b^6}-\right.$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
-3*a^2/b^3/(a+b*x^(1/3))+3*x^(1/3)/b^2-6*a*ln(a+b*x^(1/3))/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \frac{1}{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = -\frac{3\left(a^2b^2x^{\frac{2}{3}}+a^4+2\,(ab^3x+a^4)\log\left(bx^{\frac{1}{3}}+a\right)-(b^4x+2\,a^3b)x^{\frac{1}{3}}\right)}{b^6x+a^3b^3}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3)),x, algorithm="fricas")
```

output
```
-3*(a^2*b^2*x^(2/3) + a^4 + 2*(a*b^3*x + a^4)*log(b*x^(1/3) + a) - (b^4*x
+ 2*a^3*b)*x^(1/3))/(b^6*x + a^3*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(42) = 84$.

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.37

$$\int \frac{1}{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}\,dx = \begin{cases} -\frac{6a^2\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{ab^3+b^4\sqrt[3]{x}} - \frac{6a^2}{ab^3+b^4\sqrt[3]{x}} - \frac{6ab\sqrt[3]{x}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{ab^3+b^4\sqrt[3]{x}} + \frac{3b^2x^{\frac{2}{3}}}{ab^3+b^4\sqrt[3]{x}} & \text{for}\ b \neq 0 \\ \frac{x}{a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3)),x)
```

output
```
Piecewise((-6*a**2*log(a/b + x**(1/3))/(a*b**3 + b**4*x**(1/3)) - 6*a**2/(
a*b**3 + b**4*x**(1/3)) - 6*a*b*x**(1/3)*log(a/b + x**(1/3))/(a*b**3 + b**
4*x**(1/3)) + 3*b**2*x**(2/3)/(a*b**3 + b**4*x**(1/3)), Ne(b, 0)), (x/a**2
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{1}{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}\,dx = -\frac{3\,a^2}{b^4x^{\frac{1}{3}} + ab^3} - \frac{6\,a\log\left(bx^{\frac{1}{3}} + a\right)}{b^3} + \frac{3\,x^{\frac{1}{3}}}{b^2}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3)),x, algorithm="maxima")
```

output
```
-3*a^2/(b^4*x^(1/3) + a*b^3) - 6*a*log(b*x^(1/3) + a)/b^3 + 3*x^(1/3)/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{1}{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=-\frac{6\,a\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3}+\frac{3\,x^{\frac{1}{3}}}{b^2}-\frac{3\,a^2}{\left(bx^{\frac{1}{3}}+a\right)b^3}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3)),x, algorithm="giac")
```

output
```
-6*a*log(abs(b*x^(1/3) + a))/b^3 + 3*x^(1/3)/b^2 - 3*a^2/((b*x^(1/3) + a)*
b^3)
```

Mupad [B] (verification not implemented)

Time = 20.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{1}{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=\frac{3\,x^{1/3}}{b^2}-\frac{3\,a^2}{a\,b^3+b^4\,x^{1/3}}-\frac{6\,a\,\ln\left(a+b\,x^{1/3}\right)}{b^3}$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3)),x)
```

output
```
(3*x^(1/3))/b^2 - (3*a^2)/(a*b^3 + b^4*x^(1/3)) - (6*a*log(a + b*x^(1/3)))
/b^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \frac{1}{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=\frac{3x^{\frac{2}{3}}b^2-6x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+a\right)ab+6x^{\frac{1}{3}}ab-6\log\left(x^{\frac{1}{3}}b+a\right)a^2}{b^3\left(x^{\frac{1}{3}}b+a\right)}$$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3)),x)
```

output
```
(3*(x**(2/3)*b**2 - 2*x**(1/3)*log(x**(1/3)*b + a)*a*b + 2*x**(1/3)*a*b -
2*log(x**(1/3)*b + a)*a**2))/(b**3*(x**(1/3)*b + a))
```

3.69 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 16

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2}\,dx=\frac{x}{a\left(a+b\sqrt[3]{x}\right)^3}$$

output
```
x/a/(a+b*x^(1/3))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. $2(16) = 32$.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.50

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2}\,dx=\frac{-a^2-3ab\sqrt[3]{x}-3b^2x^{2/3}}{b^3\left(a+b\sqrt[3]{x}\right)^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-2),x]
```

output
```
(-a^2 - 3*a*b*x^(1/3) - 3*b^2*x^(2/3))/(b^3*(a + b*x^(1/3))^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1379, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2}dx$$

$$\Big\downarrow \text{ 1379}$$

$$b^4\int \frac{1}{\left(\sqrt[3]{x}b^2+ab\right)^4}dx$$

$$\Big\downarrow \text{ 746}$$

$$\frac{b^3x}{a\left(ab+b^2\sqrt[3]{x}\right)^3}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-2),x]
```

output
```
(b^3*x)/(a*(a*b + b^2*x^(1/3))^3)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.00

method	result
derivativedivides	$-\frac{a^{2}}{b^{3}\left(a+b\,x^{\frac{1}{3}}\right)^{3}}-\frac{3}{b^{3}\left(a+b\,x^{\frac{1}{3}}\right)}+\frac{3a}{b^{3}\left(a+b\,x^{\frac{1}{3}}\right)^{2}}$
trager	$\frac{(x-1)\left(x^{2}b^{6}a^{6}+2a^{3}b^{9}x^{2}+10b^{12}x^{2}-7a^{9}b^{3}x-23a^{6}b^{6}x+2a^{3}b^{9}x+a^{12}-7a^{9}b^{3}+a^{6}b^{6}\right)a^{2}}{\left(b^{3}x+a^{3}\right)^{3}\left(a^{9}+3a^{6}b^{3}+3a^{3}b^{6}+b^{9}\right)}-\frac{3ba\,x^{\frac{4}{3}}\left(-2b^{3}x+a^{3}\right)}{\left(b^{3}x+a^{3}\right)^{3}}+ 3b^{2}$
default	Expression too large to display

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/b^3/(a+b*x^(1/3))^3-3/b^3/(a+b*x^(1/3))+3*a/b^3/(a+b*x^(1/3))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 6.81

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2}\,dx = -\frac{10\,a^2b^6x^2+2\,a^5b^3x+a^8+3\,(b^8x^2-2\,a^3b^5x)x^{\frac{2}{3}}-3\,(2\,ab^7x^2-a^4b^4x)x^{\frac{1}{3}}}{b^{12}x^3+3\,a^3b^9x^2+3\,a^6b^6x+a^9b^3}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^2,x, algorithm="fricas")
```

output
```
-(10*a^2*b^6*x^2 + 2*a^5*b^3*x + a^8 + 3*(b^8*x^2 - 2*a^3*b^5*x)*x^(2/3) -
 3*(2*a*b^7*x^2 - a^4*b^4*x)*x^(1/3))/(b^12*x^3 + 3*a^3*b^9*x^2 + 3*a^6*b^
6*x + a^9*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(12) = 24$.

Time = 0.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 8.81

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2}\,dx = \begin{cases} -\frac{a^2}{a^3b^3+3a^2b^4\sqrt[3]{x}+3ab^5x^{\frac{2}{3}}+b^6x} - \frac{3ab\sqrt[3]{x}}{a^3b^3+3a^2b^4\sqrt[3]{x}+3ab^5x^{\frac{2}{3}}+b^6x} - \frac{3b^2x^{\frac{2}{3}}}{a^3b^3+3a^2b^4\sqrt[3]{x}+3a} \\ \frac{x}{a^4} \end{cases}$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**2,x)
```

output
```
Piecewise((-a**2/(a**3*b**3 + 3*a**2*b**4*x**(1/3) + 3*a*b**5*x**(2/3) + b
**6*x) - 3*a*b*x**(1/3)/(a**3*b**3 + 3*a**2*b**4*x**(1/3) + 3*a*b**5*x**(2
/3) + b**6*x) - 3*b**2*x**(2/3)/(a**3*b**3 + 3*a**2*b**4*x**(1/3) + 3*a*b*
*5*x**(2/3) + b**6*x), Ne(b, 0)), (x/a**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.50

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2}\,dx = -\frac{3\,b^2x^{\frac{2}{3}}+3\,abx^{\frac{1}{3}}+a^2}{b^6x+3\,ab^5x^{\frac{2}{3}}+3\,a^2b^4x^{\frac{1}{3}}+a^3b^3}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^2,x, algorithm="maxima")
```

output
```
-(3*b^2*x^(2/3) + 3*a*b*x^(1/3) + a^2)/(b^6*x + 3*a*b^5*x^(2/3) + 3*a^2*b^
4*x^(1/3) + a^3*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2}\,dx = -\frac{3\,b^2x^{\frac{2}{3}}+3\,abx^{\frac{1}{3}}+a^2}{\left(bx^{\frac{1}{3}}+a\right)^3b^3}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^2,x, algorithm="giac")
```

output
```
-(3*b^2*x^(2/3) + 3*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 20.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.50

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2}\,dx = -\frac{a^2+3\,b^2\,x^{2/3}+3\,a\,b\,x^{1/3}}{b^6\,x+a^3\,b^3+3\,a\,b^5\,x^{2/3}+3\,a^2\,b^4\,x^{1/3}}$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^2,x)
```

output
```
-(a^2 + 3*b^2*x^(2/3) + 3*a*b*x^(1/3))/(b^6*x + a^3*b^3 + 3*a*b^5*x^(2/3)
+ 3*a^2*b^4*x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^2}\,dx = \frac{x}{a\left(3x^{\frac{2}{3}}a\,b^2+3x^{\frac{1}{3}}a^2b+a^3+b^3x\right)}$$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^2,x)
```

output

```
x/(a*(3*x**(2/3)*a*b**2 + 3*x**(1/3)*a**2*b + a**3 + b**3*x))
```

3.70 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 57

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3}\,dx=-\frac{3a^2}{5b^3\left(a+b\sqrt[3]{x}\right)^5}+\frac{3a}{2b^3\left(a+b\sqrt[3]{x}\right)^4}-\frac{1}{b^3\left(a+b\sqrt[3]{x}\right)^3}$$

output
```
-3/5*a^2/b^3/(a+b*x^(1/3))^5+3/2*a/b^3/(a+b*x^(1/3))^4-1/b^3/(a+b*x^(1/3))
^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3}\,dx=\frac{-a^2-5ab\sqrt[3]{x}-10b^2x^{2/3}}{10b^3\left(a+b\sqrt[3]{x}\right)^5}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3),x]
```

output
```
(-a^2 - 5*a*b*x^(1/3) - 10*b^2*x^(2/3))/(10*b^3*(a + b*x^(1/3))^5)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1379, 774, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3}dx$$
$$\Big\downarrow \text{1379}$$
$$b^6\int \frac{1}{\left(\sqrt[3]{x}b^2+ab\right)^6}dx$$
$$\Big\downarrow \text{774}$$
$$3b^6\int \frac{x^{2/3}}{b^6\left(a+b\sqrt[3]{x}\right)^6}d\sqrt[3]{x}$$
$$\Big\downarrow \text{27}$$
$$3\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^6}d\sqrt[3]{x}$$
$$\Big\downarrow \text{53}$$
$$3\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^6}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)^5}+\frac{1}{b^2\left(a+b\sqrt[3]{x}\right)^4}\right)d\sqrt[3]{x}$$
$$\Big\downarrow \text{2009}$$
$$3\left(-\frac{a^2}{5b^3\left(a+b\sqrt[3]{x}\right)^5}+\frac{a}{2b^3\left(a+b\sqrt[3]{x}\right)^4}-\frac{1}{3b^3\left(a+b\sqrt[3]{x}\right)^3}\right)$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3),x]
```

output
```
3*(-1/5*a^2/(b^3*(a + b*x^(1/3))^5) + a/(2*b^3*(a + b*x^(1/3))^4) - 1/(3*b
^3*(a + b*x^(1/3))^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{3a^{2}}{5b^{3}\left(a+b\,x^{\frac{1}{3}}\right)^{5}}+\frac{3a}{2b^{3}\left(a+b\,x^{\frac{1}{3}}\right)^{4}}-\frac{1}{b^{3}\left(a+b\,x^{\frac{1}{3}}\right)^{3}}$
trager	$\frac{(x-1)(a^{12}b^{12}x^{4}-5a^{9}b^{15}x^{4}+240a^{6}b^{18}x^{4}-230a^{3}b^{21}x^{4}+10b^{24}x^{4}-5a^{15}b^{9}x^{3}-74a^{12}b^{12}x^{3}+1095a^{9}b^{15}x^{3}-1010a^{6}b^{18}x^{3}}{}$
default	Expression too large to display

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^3,x,method=_RETURNVERBOSE)
```

output
```
-3/5*a^2/b^3/(a+b*x^(1/3))^5+3/2*a/b^3/(a+b*x^(1/3))^4-1/b^3/(a+b*x^(1/3))
^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(47) = 94$.

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.09

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3}\,dx =$$
$$-\frac{10\,b^{12}x^4-230\,a^3b^9x^3+240\,a^6b^6x^2-5\,a^9b^3x+a^{12}-9\,(5\,ab^{11}x^3-35\,a^4b^8x^2+14\,a^7b^5x)x^{\frac{2}{3}}+9\,(14\,a^2b^{10}}{10\,(b^{18}x^5+5\,a^3b^{15}x^4+10\,a^6b^{12}x^3+10\,a^9b^9x^2+5\,a^{12}b^6x+a^{15}b^3)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^3,x, algorithm="fricas")
```

output
```
-1/10*(10*b^12*x^4 - 230*a^3*b^9*x^3 + 240*a^6*b^6*x^2 - 5*a^9*b^3*x + a^1
2 - 9*(5*a*b^11*x^3 - 35*a^4*b^8*x^2 + 14*a^7*b^5*x)*x^(2/3) + 9*(14*a^2*b
^10*x^3 - 35*a^5*b^7*x^2 + 5*a^8*b^4*x)*x^(1/3))/(b^18*x^5 + 5*a^3*b^15*x^
4 + 10*a^6*b^12*x^3 + 10*a^9*b^9*x^2 + 5*a^12*b^6*x + a^15*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(53) = 106$.

Time = 0.82 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.09

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3}\,dx = \begin{cases} -\frac{a^2}{10a^5b^3+50a^4b^4\sqrt[3]{x}+100a^3b^5x^{\frac{2}{3}}+100a^2b^6x+50ab^7x^{\frac{4}{3}}+10b^8x^{\frac{5}{3}}} - \frac{}{10a^5b^3+50a^4b^4\sqrt[3]{x}+10} \\ \frac{x}{a^6} \end{cases}$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**3,x)
```

output
```
Piecewise((-a**2/(10*a**5*b**3 + 50*a**4*b**4*x**(1/3) + 100*a**3*b**5*x**
(2/3) + 100*a**2*b**6*x + 50*a*b**7*x**(4/3) + 10*b**8*x**(5/3)) - 5*a*b*x
**(1/3)/(10*a**5*b**3 + 50*a**4*b**4*x**(1/3) + 100*a**3*b**5*x**(2/3) + 1
00*a**2*b**6*x + 50*a*b**7*x**(4/3) + 10*b**8*x**(5/3)) - 10*b**2*x**(2/3)
/(10*a**5*b**3 + 50*a**4*b**4*x**(1/3) + 100*a**3*b**5*x**(2/3) + 100*a**2
*b**6*x + 50*a*b**7*x**(4/3) + 10*b**8*x**(5/3)), Ne(b, 0)), (x/a**6, True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3}\,dx = -\frac{10\,b^2x^{\frac{2}{3}}+5\,abx^{\frac{1}{3}}+a^2}{10\left(b^8x^{\frac{5}{3}}+5\,ab^7x^{\frac{4}{3}}+10\,a^2b^6x+10\,a^3b^5x^{\frac{2}{3}}+5\,a^4b^4x^{\frac{1}{3}}+a^5b^3\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^3,x, algorithm="maxima")
```

output
```
-1/10*(10*b^2*x^(2/3) + 5*a*b*x^(1/3) + a^2)/(b^8*x^(5/3) + 5*a*b^7*x^(4/3
) + 10*a^2*b^6*x + 10*a^3*b^5*x^(2/3) + 5*a^4*b^4*x^(1/3) + a^5*b^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3}\,dx = -\frac{10\,b^2x^{\frac{2}{3}}+5\,abx^{\frac{1}{3}}+a^2}{10\left(bx^{\frac{1}{3}}+a\right)^5b^3}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^3,x, algorithm="giac")
```

output
```
-1/10*(10*b^2*x^(2/3) + 5*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^5*b^3)
```

Mupad [B] (verification not implemented)

Time = 21.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3}\,dx = -\frac{a^2+10\,b^2\,x^{2/3}+5\,a\,b\,x^{1/3}}{10\,a^5\,b^3+10\,b^8\,x^{5/3}+100\,a^2\,b^6\,x+50\,a\,b^7\,x^{4/3}+50\,a^4\,b^4\,x^{1/3}+100\,a^3\,b^5\,x^{2/3}}$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^3,x)
```

output
```
-(a^2 + 10*b^2*x^(2/3) + 5*a*b*x^(1/3))/(10*a^5*b^3 + 10*b^8*x^(5/3) + 100
*a^2*b^6*x + 50*a*b^7*x^(4/3) + 50*a^4*b^4*x^(1/3) + 100*a^3*b^5*x^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^3}\,dx = \frac{-10x^{\frac{2}{3}}b^2-5x^{\frac{1}{3}}ab-a^2}{10b^3\left(10x^{\frac{2}{3}}a^3b^2+x^{\frac{5}{3}}b^5+5x^{\frac{1}{3}}a^4b+5x^{\frac{4}{3}}a\,b^4+a^5+10a^2b^3x\right)}$$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^3,x)
```

output
```
( - 10*x**(2/3)*b**2 - 5*x**(1/3)*a*b - a**2)/(10*b**3*(10*x**(2/3)*a**3*b
**2 + x**(2/3)*b**5*x + 5*x**(1/3)*a**4*b + 5*x**(1/3)*a*b**4*x + a**5 + 1
0*a**2*b**3*x))
```

3.71 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^4}\,dx=-\frac{3a^2}{7b^3\left(a+b\sqrt[3]{x}\right)^7}+\frac{a}{b^3\left(a+b\sqrt[3]{x}\right)^6}-\frac{3}{5b^3\left(a+b\sqrt[3]{x}\right)^5}$$

output
```
-3/7*a^2/b^3/(a+b*x^(1/3))^7+a/b^3/(a+b*x^(1/3))^6-3/5/b^3/(a+b*x^(1/3))^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^4}\,dx=\frac{-a^2-7ab\sqrt[3]{x}-21b^2x^{2/3}}{35b^3\left(a+b\sqrt[3]{x}\right)^7}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-4),x]
```

output
```
(-a^2 - 7*a*b*x^(1/3) - 21*b^2*x^(2/3))/(35*b^3*(a + b*x^(1/3))^7)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1379, 774, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^4}dx$$

$$\Big\downarrow \text{1379}$$

$$b^8\int \frac{1}{\left(\sqrt[3]{x}b^2+ab\right)^8}dx$$

$$\Big\downarrow \text{774}$$

$$3b^8\int \frac{x^{2/3}}{b^8\left(a+b\sqrt[3]{x}\right)^8}d\sqrt[3]{x}$$

$$\Big\downarrow \text{27}$$

$$3\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^8}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^8}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)^7}+\frac{1}{b^2\left(a+b\sqrt[3]{x}\right)^6}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^2}{7b^3\left(a+b\sqrt[3]{x}\right)^7}+\frac{a}{3b^3\left(a+b\sqrt[3]{x}\right)^6}-\frac{1}{5b^3\left(a+b\sqrt[3]{x}\right)^5}\right)$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-4),x]
```

output
```
3*(-1/7*a^2/(b^3*(a + b*x^(1/3))^7) + a/(3*b^3*(a + b*x^(1/3))^6) - 1/(5*b
^3*(a + b*x^(1/3))^5))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{3a^2}{7b^3\left(a+b\,x^{\frac{1}{3}}\right)^7}+\frac{a}{b^3\left(a+b\,x^{\frac{1}{3}}\right)^6}-\frac{3}{5b^3\left(a+b\,x^{\frac{1}{3}}\right)^5}$
trager	$\frac{(x-1)(a^{15}b^{18}x^6-28a^{12}b^{21}x^6+1876a^9b^{24}x^6-6020a^6b^{27}x^6+2870a^3b^{30}x^6-140b^{33}x^6+7a^{18}b^{15}x^5-195a^{15}b^{18}x^5+1310}{}$
default	Expression too large to display

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^4,x,method=_RETURNVERBOSE)
```

output
```
-3/7*a^2/b^3/(a+b*x^(1/3))^7+a/b^3/(a+b*x^(1/3))^6-3/5/b^3/(a+b*x^(1/3))^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(46) = 92.

Time = 0.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.36

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^4}\,dx = \frac{140\,ab^{15}x^5-2870\,a^4b^{12}x^4+6020\,a^7b^9x^3-1876\,a^{10}b^6x^2+28\,a^{13}b^3x-a^{16}\cdots}{35\left(b^{24}x^7+7\,a^3b^{2}\cdots\right.}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^4,x, algorithm="fricas")
```

output
```
1/35*(140*a*b^15*x^5 - 2870*a^4*b^12*x^4 + 6020*a^7*b^9*x^3 - 1876*a^10*b^
6*x^2 + 28*a^13*b^3*x - a^16 - 27*(20*a^2*b^14*x^4 - 168*a^5*b^11*x^3 + 18
9*a^8*b^8*x^2 - 28*a^11*b^5*x)*x^(2/3) - 3*(7*b^16*x^5 - 476*a^3*b^13*x^4
+ 1932*a^6*b^10*x^3 - 1160*a^9*b^7*x^2 + 70*a^12*b^4*x)*x^(1/3))/(b^24*x^7
 + 7*a^3*b^21*x^6 + 21*a^6*b^18*x^5 + 35*a^9*b^15*x^4 + 35*a^12*b^12*x^3 +
 21*a^15*b^9*x^2 + 7*a^18*b^6*x + a^21*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(51) = 102.

Time = 1.56 (sec) , antiderivative size = 309, normalized size of antiderivative = 5.52

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^4}\,dx = \begin{cases} -\frac{a^2}{35a^7b^3+245a^6b^4\sqrt[3]{x}+735a^5b^5x^{\frac{2}{3}}+1225a^4b^6x+1225a^3b^7x^{\frac{4}{3}}+735a^2b^8x^{\frac{5}{3}}+245ab^9x^2+35b^{10}x\cdots} \\ \frac{x}{a^8} \end{cases}$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**4,x)
```

output
```
Piecewise((-a**2/(35*a**7*b**3 + 245*a**6*b**4*x**(1/3) + 735*a**5*b**5*x*
*(2/3) + 1225*a**4*b**6*x + 1225*a**3*b**7*x**(4/3) + 735*a**2*b**8*x**(5/
3) + 245*a*b**9*x**2 + 35*b**10*x**(7/3)) - 7*a*b*x**(1/3)/(35*a**7*b**3 +
 245*a**6*b**4*x**(1/3) + 735*a**5*b**5*x**(2/3) + 1225*a**4*b**6*x + 1225
*a**3*b**7*x**(4/3) + 735*a**2*b**8*x**(5/3) + 245*a*b**9*x**2 + 35*b**10*
x**(7/3)) - 21*b**2*x**(2/3)/(35*a**7*b**3 + 245*a**6*b**4*x**(1/3) + 735*
a**5*b**5*x**(2/3) + 1225*a**4*b**6*x + 1225*a**3*b**7*x**(4/3) + 735*a**2
*b**8*x**(5/3) + 245*a*b**9*x**2 + 35*b**10*x**(7/3)), Ne(b, 0)), (x/a**8,
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(46) = 92$.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^4}\,dx =$$
$$-\frac{21\,b^2x^{\frac{2}{3}} + 7\,abx^{\frac{1}{3}} + a^2}{35\left(b^{10}x^{\frac{7}{3}} + 7\,ab^9x^2 + 21\,a^2b^8x^{\frac{5}{3}} + 35\,a^3b^7x^{\frac{4}{3}} + 35\,a^4b^6x + 21\,a^5b^5x^{\frac{2}{3}} + 7\,a^6b^4x^{\frac{1}{3}} + a^7b^3\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^4,x, algorithm="maxima")
```

output
```
-1/35*(21*b^2*x^(2/3) + 7*a*b*x^(1/3) + a^2)/(b^10*x^(7/3) + 7*a*b^9*x^2 +
 21*a^2*b^8*x^(5/3) + 35*a^3*b^7*x^(4/3) + 35*a^4*b^6*x + 21*a^5*b^5*x^(2/
3) + 7*a^6*b^4*x^(1/3) + a^7*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^4}\,dx = -\frac{21\,b^2x^{\frac{2}{3}} + 7\,abx^{\frac{1}{3}} + a^2}{35\left(bx^{\frac{1}{3}} + a\right)^7b^3}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^4,x, algorithm="giac")
```

output
```
-1/35*(21*b^2*x^(2/3) + 7*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^7*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.80

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^4}\,dx =$$
$$-\frac{\frac{a^2}{35\,b^3}+\frac{3\,x^{2/3}}{5\,b}+\frac{a\,x^{1/3}}{5\,b^2}}{a^7+b^7\,x^{7/3}+35\,a^4\,b^3\,x+7\,a\,b^6\,x^2+7\,a^6\,b\,x^{1/3}+21\,a^5\,b^2\,x^{2/3}+35\,a^3\,b^4\,x^{4/3}+21\,a^2\,b^5\,x^{5/3}}$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^4,x)
```

output
```
-(a^2/(35*b^3) + (3*x^(2/3))/(5*b) + (a*x^(1/3))/(5*b^2))/(a^7 + b^7*x^(7/
3) + 35*a^4*b^3*x + 7*a*b^6*x^2 + 7*a^6*b*x^(1/3) + 21*a^5*b^2*x^(2/3) + 3
5*a^3*b^4*x^(4/3) + 21*a^2*b^5*x^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^4}\,dx = \frac{-21x^{\frac{2}{3}}b^2-7x^{\frac{1}{3}}ab-a^2}{35b^3\left(21x^{\frac{2}{3}}a^5b^2+21x^{\frac{5}{3}}a^2b^5+7x^{\frac{1}{3}}a^6b+35x^{\frac{4}{3}}a^3b^4+x^{\frac{7}{3}}b^7+a^7+35a^4b^3x\right.}$$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^4,x)
```

output
```
( - 21*x**(2/3)*b**2 - 7*x**(1/3)*a*b - a**2)/(35*b**3*(21*x**(2/3)*a**5*b
**2 + 21*x**(2/3)*a**2*b**5*x + 7*x**(1/3)*a**6*b + 35*x**(1/3)*a**3*b**4*
x + x**(1/3)*b**7*x**2 + a**7 + 35*a**4*b**3*x + 7*a*b**6*x**2))
```

3.72 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{3b^3}+\frac{3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{10b^3}$$

output
```
3/8*a^2*(a+b*x^(1/3))^7*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-2/3*a*(a
+b*x^(1/3))^8*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+3/10*(a+b*x^(1/3))
^9*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{7/2}\left(120a^7x+630a^6bx^{4/3}+1512a^5b^2x^{5/3}+2100a^4b^3x^2+1800a^3b^4x^{7/3}+9\right.}{120\left(a+b\sqrt[3]{x}\right)^7}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

output
```
(((a + b*x^(1/3))^2)^(7/2)*(120*a^7*x + 630*a^6*b*x^(4/3) + 1512*a^5*b^2*x
^(5/3) + 2100*a^4*b^3*x^2 + 1800*a^3*b^4*x^(7/3) + 945*a^2*b^5*x^(8/3) + 2
80*a*b^6*x^3 + 36*b^7*x^(10/3)))/(120*(a + b*x^(1/3))^7)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\sqrt[3]{x}b^2+ab\right)^7dx}{ab^7+b^8\sqrt[3]{x}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int b^7\left(a+b\sqrt[3]{x}\right)^7x^{2/3}d\sqrt[3]{x}}{ab^7+b^8\sqrt[3]{x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(a+b\sqrt[3]{x}\right)^7x^{2/3}d\sqrt[3]{x}}{ab^7+b^8\sqrt[3]{x}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3b^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^9}{b^2}-\frac{2a\left(a+b\sqrt[3]{x}\right)^8}{b^2}+\frac{a^2\left(a+b\sqrt[3]{x}\right)^7}{b^2}\right)d\sqrt[3]{x}}{ab^7+b^8\sqrt[3]{x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^7\left(\frac{a^2\left(a+b\sqrt[3]{x}\right)^8}{8b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^{10}}{10b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^9}{9b^3}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{ab^7+b^8\sqrt[3]{x}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

output
```
(3*b^7*((a^2*(a + b*x^(1/3))^8)/(8*b^3) - (2*a*(a + b*x^(1/3))^9)/(9*b^3)
+ (a + b*x^(1/3))^10/(10*b^3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(a
*b^7 + b^8*x^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{7}{2}} x\left(36b^{7}x^{\frac{7}{3}}+280a\,b^{6}x^{2}+945a^{2}b^{5}x^{\frac{5}{3}}+1800a^{3}b^{4}x^{\frac{4}{3}}+2100a^{4}b^{3}x+1512a^{5}b^{2}x^{\frac{2}{3}}+630a^{6}b\,x^{\frac{1}{3}}+120a^{7}\right)}{120\left(a+b\,x^{\frac{1}{3}}\right)^{7}}$
default	$\frac{\left(a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}\right)^{\frac{7}{2}}\left(36b^{7}x^{\frac{10}{3}}+945a^{2}b^{5}x^{\frac{8}{3}}+1800a^{3}b^{4}x^{\frac{7}{3}}+1512a^{5}b^{2}x^{\frac{5}{3}}+630a^{6}b\,x^{\frac{4}{3}}+280a\,b^{6}x^{3}+2100a^{4}b^{3}x^{2}+120\right.}{120\left(a+b\,x^{\frac{1}{3}}\right)^{7}}$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*((a+b*x^(1/3))^2)^(7/2)*x*(36*b^7*x^(7/3)+280*a*b^6*x^2+945*a^2*b^5*
x^(5/3)+1800*a^3*b^4*x^(4/3)+2100*a^4*b^3*x+1512*a^5*b^2*x^(2/3)+630*a^6*b
*x^(1/3)+120*a^7)/(a+b*x^(1/3))^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.61

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx = \frac{7}{3}\,ab^6x^3+\frac{35}{2}\,a^4b^3x^2+a^7x + \frac{63}{40}\left(5\,a^2b^5x^2+8\,a^5b^2x\right)x^{\frac{2}{3}}+\frac{3}{20}\left(2\,b^7x^3+100\,a^3b^4x^2+35\,a^6bx\right)x^{\frac{1}{3}}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

output
```
7/3*a*b^6*x^3 + 35/2*a^4*b^3*x^2 + a^7*x + 63/40*(5*a^2*b^5*x^2 + 8*a^5*b^
2*x)*x^(2/3) + 3/20*(2*b^7*x^3 + 100*a^3*b^4*x^2 + 35*a^6*b*x)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.70

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=3\left(\begin{cases}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^9}{360b^3}-\frac{a^8\sqrt[3]{x}}{360b^2}+\frac{a^7x^{\frac{2}{3}}}{360b}+\frac{119a^6x}{360}+\frac{511a^5bx^{\frac{4}{3}}}{360}+\frac{1001a^4b^2x^{\frac{5}{3}}}{360}+\frac{1099a\cdots}{3\cdots}\right. \\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{9}{2}}}{9}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{11}{2}}}{11}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{13}{2}}}{13}}{4a^3b^3} \\ \frac{x\left(a^2\right)^{\frac{7}{2}}}{3}\end{cases}\right.$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

output
```
3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**9/(360*b**3)
- a**8*x**(1/3)/(360*b**2) + a**7*x**(2/3)/(360*b) + 119*a**6*x/360 + 511*
a**5*b*x**(4/3)/360 + 1001*a**4*b**2*x**(5/3)/360 + 1099*a**3*b**3*x**2/36
0 + 701*a**2*b**4*x**(7/3)/360 + 61*a*b**5*x**(8/3)/90 + b**6*x**3/10), Ne
(b**2, 0)), ((a**4*(a**2 + 2*a*b*x**(1/3))**(9/2)/9 - 2*a**2*(a**2 + 2*a*b
*x**(1/3))**(11/2)/11 + (a**2 + 2*a*b*x**(1/3))**(13/2)/13)/(4*a**3*b**3),
 Ne(a*b, 0)), (x*(a**2)**(7/2)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^2x^{\frac{1}{3}}}{8\,b^2}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^3}{8\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{9}{2}}x^{\frac{1}{3}}}{10\,b^2}-\frac{11\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{9}{2}}a}{30\,b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

output
```
3/8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^2*x^(1/3)/b^2 + 3/8*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^3/b^3 + 3/10*(b^2*x^(2/3) + 2*a*b*x^
(1/3) + a^2)^(9/2)*x^(1/3)/b^2 - 11/30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)
^(9/2)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx = \frac{3}{10}\,b^7x^{\frac{10}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{7}{3}\,ab^6x^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + \frac{63}{8}\,a^2b^5x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+15\,a^3b^4x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{35}{2}\,a^4b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right) + \frac{63}{5}\,a^5b^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{21}{4}\,a^6bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^7x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

output
```
3/10*b^7*x^(10/3)*sgn(b*x^(1/3) + a) + 7/3*a*b^6*x^3*sgn(b*x^(1/3) + a) +
63/8*a^2*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15*a^3*b^4*x^(7/3)*sgn(b*x^(1/3)
 + a) + 35/2*a^4*b^3*x^2*sgn(b*x^(1/3) + a) + 63/5*a^5*b^2*x^(5/3)*sgn(b*x
^(1/3) + a) + 21/4*a^6*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^7*x*sgn(b*x^(1/3)
+ a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx = \int {\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)}^{7/2} \,d x$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)
```

output
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.56

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx=\frac{x\left(1512x^{\frac{2}{3}}a^5b^2+945x^{\frac{5}{3}}a^2b^5+630x^{\frac{1}{3}}a^6b+1800x^{\frac{4}{3}}a^3b^4+36x^{\frac{7}{3}}b^7+120a^7+2100a^4b^3x+\right.}{120}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x)
```

output
```
(x*(1512*x**(2/3)*a**5*b**2 + 945*x**(2/3)*a**2*b**5*x + 630*x**(1/3)*a**6
*b + 1800*x**(1/3)*a**3*b**4*x + 36*x**(1/3)*b**7*x**2 + 120*a**7 + 2100*a
**4*b**3*x + 280*a*b**6*x**2))/120
```

3.73 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{a^2\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{7b^3}+\frac{3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3}$$

output
```
1/2*a^2*(a+b*x^(1/3))^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-6/7*a*(a
+b*x^(1/3))^6*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+3/8*(a+b*x^(1/3))^
7*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{5/2}\left(56a^5x+210a^4bx^{4/3}+336a^3b^2x^{5/3}+280a^2b^3x^2+120ab^4x^{7/3}+21b^5x^8\right.}{56\left(a+b\sqrt[3]{x}\right)^5}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

output
```
(((a + b*x^(1/3))^2)^(5/2)*(56*a^5*x + 210*a^4*b*x^(4/3) + 336*a^3*b^2*x^(
5/3) + 280*a^2*b^3*x^2 + 120*a*b^4*x^(7/3) + 21*b^5*x^(8/3)))/(56*(a + b*x
^(1/3))^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\sqrt[3]{x}b^2+ab\right)^5dx}{ab^5+b^6\sqrt[3]{x}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int b^5\left(a+b\sqrt[3]{x}\right)^5x^{2/3}d\sqrt[3]{x}}{ab^5+b^6\sqrt[3]{x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(a+b\sqrt[3]{x}\right)^5x^{2/3}d\sqrt[3]{x}}{ab^5+b^6\sqrt[3]{x}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3b^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^7}{b^2}-\frac{2a\left(a+b\sqrt[3]{x}\right)^6}{b^2}+\frac{a^2\left(a+b\sqrt[3]{x}\right)^5}{b^2}\right)d\sqrt[3]{x}}{ab^5+b^6\sqrt[3]{x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^5\left(\frac{a^2\left(a+b\sqrt[3]{x}\right)^6}{6b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^8}{8b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^7}{7b^3}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{ab^5+b^6\sqrt[3]{x}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

output
```
(3*b^5*((a^2*(a + b*x^(1/3))^6)/(6*b^3) - (2*a*(a + b*x^(1/3))^7)/(7*b^3)
+ (a + b*x^(1/3))^8/(8*b^3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(a*b
^5 + b^6*x^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{5}{2}} x \left(21 b^{5} x^{\frac{5}{3}}+120 a\,b^{4} x^{\frac{4}{3}}+280 a^{2} b^{3} x+336 a^{3} b^{2} x^{\frac{2}{3}}+210 a^{4} b\,x^{\frac{1}{3}}+56 a^{5}\right)}{56 \left(a+b\,x^{\frac{1}{3}}\right)^{5}}$	76
default	$\frac{\left(a^{2}+2 a b\,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{\frac{5}{2}} \left(21 b^{5} x^{\frac{8}{3}}+120 a\,b^{4} x^{\frac{7}{3}}+336 a^{3} b^{2} x^{\frac{5}{3}}+210 a^{4} b\,x^{\frac{4}{3}}+280 a^{2} b^{3} x^{2}+56 x\,a^{5}\right)}{56 \left(a+b\,x^{\frac{1}{3}}\right)^{5}}$	87

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/56*((a+b*x^(1/3))^2)^(5/2)*x*(21*b^5*x^(5/3)+120*a*b^4*x^(4/3)+280*a^2*b
^3*x+336*a^3*b^2*x^(2/3)+210*a^4*b*x^(1/3)+56*a^5)/(a+b*x^(1/3))^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx = 5\,a^2b^3x^2+a^5x + \frac{3}{8}\left(b^5x^2+16\,a^3b^2x\right)x^{\frac{2}{3}}+\frac{15}{28}\left(4\,ab^4x^2+7\,a^4bx\right)x^{\frac{1}{3}}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

output
```
5*a^2*b^3*x^2 + a^5*x + 3/8*(b^5*x^2 + 16*a^3*b^2*x)*x^(2/3) + 15/28*(4*a*
b^4*x^2 + 7*a^4*b*x)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.49

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx = 3\begin{cases}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^7}{168b^3}-\frac{a^6\sqrt[3]{x}}{168b^2}+\frac{a^5x^{\frac{2}{3}}}{168b}+\frac{55a^4x}{168}+\frac{155a^3bx^{\frac{4}{3}}}{168}+\frac{181a^2b^2x^{\frac{5}{3}}}{168}+\frac{33ab^3x^2}{56}\right. \\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{7}{2}}}{7}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{9}{2}}}{9}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{11}{2}}}{11}}{4a^3b^3} \\ \frac{x\left(a^2\right)^{\frac{5}{2}}}{3}\end{cases}$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

output
```
3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**7/(168*b**3)
- a**6*x**(1/3)/(168*b**2) + a**5*x**(2/3)/(168*b) + 55*a**4*x/168 + 155*a
**3*b*x**(4/3)/168 + 181*a**2*b**2*x**(5/3)/168 + 33*a*b**3*x**2/56 + b**4
*x**(7/3)/8), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x**(1/3))**(7/2)/7 - 2*a*
*2*(a**2 + 2*a*b*x**(1/3))**(9/2)/9 + (a**2 + 2*a*b*x**(1/3))**(11/2)/11)/
(4*a**3*b**3), Ne(a*b, 0)), (x*(a**2)**(5/2)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx = \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^2x^{\frac{1}{3}}}{2\,b^2} + \frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^3}{2\,b^3} + \frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}x^{\frac{1}{3}}}{8\,b^2} - \frac{27\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a}{56\,b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

output
```
1/2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^2*x^(1/3)/b^2 + 1/2*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^3/b^3 + 3/8*(b^2*x^(2/3) + 2*a*b*x^(
1/3) + a^2)^(7/2)*x^(1/3)/b^2 - 27/56*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^
(7/2)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^{5/2} dx = \frac{3}{8}b^5x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + \frac{15}{7}ab^4x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + 5a^2b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + 6a^3b^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + \frac{15}{4}a^4bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right) + a^5x\mathrm{sgn}\left(bx^{\frac{1}{3}} + a\right)$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
```

output
```
3/8*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15/7*a*b^4*x^(7/3)*sgn(b*x^(1/3) + a)
 + 5*a^2*b^3*x^2*sgn(b*x^(1/3) + a) + 6*a^3*b^2*x^(5/3)*sgn(b*x^(1/3) + a)
 + 15/4*a^4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^5*x*sgn(b*x^(1/3) + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}\right)^{5/2} dx = \int \left(a^2 + b^2\,x^{2/3} + 2\,a\,b\,x^{1/3}\right)^{5/2} dx$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
```

output
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.40

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx=\frac{x\left(336x^{\frac{2}{3}}a^3b^2+21x^{\frac{5}{3}}b^5+210x^{\frac{1}{3}}a^4b+120x^{\frac{4}{3}}a\,b^4+56a^5+280a^2b^3x\right)}{56}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x)
```

output
```
(x*(336*x**(2/3)*a**3*b**2 + 21*x**(2/3)*b**5*x + 210*x**(1/3)*a**4*b + 12
0*x**(1/3)*a*b**4*x + 56*a**5 + 280*a**2*b**3*x))/56
```

3.74 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{5b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3}$$

output
```
3/4*a^2*(a+b*x^(1/3))^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-6/5*a*(a
+b*x^(1/3))^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+1/2*(a+b*x^(1/3))^
5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.49

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{3/2}\left(20a^3x+45a^2bx^{4/3}+36ab^2x^{5/3}+10b^3x^2\right)}{20\left(a+b\sqrt[3]{x}\right)^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

output
```
(((a + b*x^(1/3))^2)^(3/2)*(20*a^3*x + 45*a^2*b*x^(4/3) + 36*a*b^2*x^(5/3)
 + 10*b^3*x^2))/(20*(a + b*x^(1/3))^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\sqrt[3]{x}b^2+ab\right)^3dx}{ab^3+b^4\sqrt[3]{x}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int b^3\left(a+b\sqrt[3]{x}\right)^3x^{2/3}d\sqrt[3]{x}}{ab^3+b^4\sqrt[3]{x}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(a+b\sqrt[3]{x}\right)^3x^{2/3}d\sqrt[3]{x}}{ab^3+b^4\sqrt[3]{x}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(x^{2/3}a^3+3bxa^2+3b^2x^{4/3}a+b^3x^{5/3}\right)d\sqrt[3]{x}}{ab^3+b^4\sqrt[3]{x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(\frac{a^3x}{3}+\frac{3}{4}a^2bx^{4/3}+\frac{3}{5}ab^2x^{5/3}+\frac{b^3x^2}{6}\right)}{ab^3+b^4\sqrt[3]{x}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

output
```
(3*b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*((a^3*x)/3 + (3*a^2*b*x^(4/
3))/4 + (3*a*b^2*x^(5/3))/5 + (b^3*x^2)/6))/(a*b^3 + b^4*x^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.39

method	result	size
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{3}{2}} x\left(10b^{3}x+36a\,b^{2}x^{\frac{2}{3}}+45a^{2}b\,x^{\frac{1}{3}}+20a^{3}\right)}{20\left(a+b\,x^{\frac{1}{3}}\right)^{3}}$	54
default	$\frac{\left(a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(36b^{2}a\,x^{\frac{5}{3}}+45a^{2}b\,x^{\frac{4}{3}}+10b^{3}x^{2}+20a^{3}x\right)}{20\left(a+b\,x^{\frac{1}{3}}\right)^{3}}$	65

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*((a+b*x^(1/3))^2)^(3/2)*x*(10*b^3*x+36*a*b^2*x^(2/3)+45*a^2*b*x^(1/3)
+20*a^3)/(a+b*x^(1/3))^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.23

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{1}{2}\,b^3x^2+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+a^3x$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

output
```
1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=3\left(\begin{cases}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^5}{60b^3}-\frac{a^4\sqrt[3]{x}}{60b^2}+\frac{a^3x^{\frac{2}{3}}}{60b}+\frac{19a^2x}{60}+\frac{13abx^{\frac{4}{3}}}{30}+\frac{b^2x^{\frac{5}{3}}}{6}\right) & \text{for } b^2\neq 0\\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{5}{2}}}{5}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{7}{2}}}{7}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{9}{2}}}{9}}{4a^3b^3} & \text{for } ab\neq 0\\ \frac{x\left(a^2\right)^{\frac{3}{2}}}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

output
```
3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**5/(60*b**3) -
 a**4*x**(1/3)/(60*b**2) + a**3*x**(2/3)/(60*b) + 19*a**2*x/60 + 13*a*b*x*
*(4/3)/30 + b**2*x**(5/3)/6), Ne(b**2, 0)), ((a**4*(a**2 + 2*a*b*x**(1/3))
**(5/2)/5 - 2*a**2*(a**2 + 2*a*b*x**(1/3))**(7/2)/7 + (a**2 + 2*a*b*x**(1/
3))**(9/2)/9)/(4*a**3*b**3), Ne(a*b, 0)), (x*(a**2)**(3/2)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a^2x^{\frac{1}{3}}}{4\,b^2}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a^3}{4\,b^3}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}x^{\frac{1}{3}}}{2\,b^2}-\frac{7\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a}{10\,b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

output
```
3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^2*x^(1/3)/b^2 + 3/4*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^3/b^3 + 1/2*(b^2*x^(2/3) + 2*a*b*x^(
1/3) + a^2)^(5/2)*x^(1/3)/b^2 - 7/10*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(
5/2)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{1}{2}\,b^3x^2\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{9}{5}\,ab^2x^{\frac{5}{3}}\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{9}{4}\,a^2bx^{\frac{4}{3}}\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^3x\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

output
```
1/2*b^3*x^2*sgn(b*x^(1/3) + a) + 9/5*a*b^2*x^(5/3)*sgn(b*x^(1/3) + a) + 9/
4*a^2*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^3*x*sgn(b*x^(1/3) + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\int {\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)}^{3/2}\,dx$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
```

output
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.24

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx=\frac{x\left(36x^{\frac{2}{3}}a\,b^2+45x^{\frac{1}{3}}a^2b+20a^3+10b^3x\right)}{20}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x)
```

output
```
(x*(36*x**(2/3)*a*b**2 + 45*x**(1/3)*a**2*b + 20*a**3 + 10*b**3*x))/20
```

3.75 $\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=\frac{a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x}{a+b\sqrt[3]{x}}+\frac{3b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x^{4/3}}{4\left(a+b\sqrt[3]{x}\right)}$$

output
```
a*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*x/(a+b*x^(1/3))+3*b*(a^2+2*a*b*x^(
1/3)+b^2*x^(2/3))^(1/2)*x^(4/3)/(4*a+4*b*x^(1/3))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.49

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=\frac{\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\left(4ax+3bx^{4/3}\right)}{4\left(a+b\sqrt[3]{x}\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

output
```
(Sqrt[(a + b*x^(1/3))^2]*(4*a*x + 3*b*x^(4/3)))/(4*(a + b*x^(1/3)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\int \left(\sqrt[3]{x}b^2+ab\right)dx}{ab+b^2\sqrt[3]{x}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(abx+\frac{3}{4}b^2x^{4/3}\right)}{ab+b^2\sqrt[3]{x}}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*(a*b*x + (3*b^2*x^(4/3))/4))/(a*b
 + b^2*x^(1/3))
```

Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

method	result	size
derivativedivides	$\frac{\operatorname{csgn}\left(a+b\,x^{\frac{1}{3}}\right)\left(a+b\,x^{\frac{1}{3}}\right)^{2}\left(3b^{2}x^{\frac{2}{3}}-2ab\,x^{\frac{1}{3}}+a^{2}\right)}{4b^{3}}$	42
default	$\frac{\sqrt{a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}}\,\left(3b\,x^{\frac{4}{3}}+4xa\right)}{4a+4b\,x^{\frac{1}{3}}}$	43

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*csgn(a+b*x^(1/3))*(a+b*x^(1/3))^2*(3*b^2*x^(2/3)-2*a*b*x^(1/3)+a^2)/b^
3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=\frac{3}{4}\,bx^{\frac{4}{3}}+ax$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
```

output
```
3/4*b*x^(4/3) + a*x
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=3\left(\begin{cases}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\left(\frac{a^3}{12b^3}-\frac{a^2\sqrt[3]{x}}{12b^2}+\frac{ax^{\frac{2}{3}}}{12b}+\frac{x}{4}\right) & \text{for } b^2\neq 0\\ \frac{\frac{a^4\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{3}{2}}}{3}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{5}{2}}}{5}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{7}{2}}}{7}}{4a^3b^3} & \text{for } ab\neq 0\\ \frac{x\sqrt{a^2}}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

output
```
3*Piecewise((sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))*(a**3/(12*b**3) -
 a**2*x**(1/3)/(12*b**2) + a*x**(2/3)/(12*b) + x/4), Ne(b**2, 0)), ((a**4*
(a**2 + 2*a*b*x**(1/3))**(3/2)/3 - 2*a**2*(a**2 + 2*a*b*x**(1/3))**(5/2)/5
 + (a**2 + 2*a*b*x**(1/3))**(7/2)/7)/(4*a**3*b**3), Ne(a*b, 0)), (x*sqrt(a
**2)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx=\frac{3\sqrt{b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2}a^2x^{\frac{1}{3}}}{2\,b^2}+\frac{3\sqrt{b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2}a^3}{2\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}x^{\frac{1}{3}}}{4\,b^2}-\frac{5\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a}{4\,b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

output
```
3/2*sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^2*x^(1/3)/b^2 + 3/2*sqrt(b^2
*x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^3/b^3 + 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3)
 + a^2)^(3/2)*x^(1/3)/b^2 - 5/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*
a/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.30

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = \frac{3}{4}\,bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+ax\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
```

output
```
3/4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a*x*sgn(b*x^(1/3) + a)
```

Mupad [B] (verification not implemented)

Time = 21.97 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx = \frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^3-4\,a^2\,b\,x^{1/3}-5\,a\,b^2\,x^{2/3}+3\,b\,x^{1/3}\,\left(a^2+b^2\,x\right.\right.}{4\,b^3}$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

output
```
((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^3 - 4*a^2*b*x^(1/3) - 5*a*b^
2*x^(2/3) + 3*b*x^(1/3)*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))))/(4*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.15

$$\int \sqrt{a^2 + 2ab\sqrt[3]{x} + b^2x^{2/3}}\,dx = \frac{x\left(3x^{\frac{1}{3}}b + 4a\right)}{4}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x)
```

output
```
(x*(3*x**(1/3)*b + 4*a))/4
```

3.76 $\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = -\frac{3a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3\left(a+b\sqrt[3]{x}\right)x^{2/3}}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

output
```
-3*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3/2*(
a+b*x^(1/3))*x^(2/3)/b/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3*a^2*(a+b*x^
(1/3))*ln(a+b*x^(1/3))/b^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \frac{3\left(a+b\sqrt[3]{x}\right)\left(b\left(-2a+b\sqrt[3]{x}\right)\sqrt[3]{x}+2a^2\log\left(a+b\sqrt[3]{x}\right)\right)}{2b^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

output
```
(3*(a + b*x^(1/3))*(b*(-2*a + b*x^(1/3))*x^(1/3) + 2*a^2*Log[a + b*x^(1/3)
]))/(2*b^3*Sqrt[(a + b*x^(1/3))^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab+b^2\sqrt[3]{x}\right)\int \frac{1}{\sqrt[3]{x}b^2+ab}dx}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab+b^2\sqrt[3]{x}\right)\int \frac{x^{2/3}}{b\left(a+b\sqrt[3]{x}\right)}d\sqrt[3]{x}}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab+b^2\sqrt[3]{x}\right)\int \frac{x^{2/3}}{a+b\sqrt[3]{x}}d\sqrt[3]{x}}{b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3\left(ab+b^2\sqrt[3]{x}\right)\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)}-\frac{a}{b^2}+\frac{\sqrt[3]{x}}{b}\right)d\sqrt[3]{x}}{b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(ab+b^2\sqrt[3]{x}\right)\left(\frac{a^2\log\left(a+b\sqrt[3]{x}\right)}{b^3}-\frac{a\sqrt[3]{x}}{b^2}+\frac{x^{2/3}}{2b}\right)}{b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

input
```
Int[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

output
```
(3*(a*b + b^2*x^(1/3))*(-((a*x^(1/3))/b^2) + x^(2/3)/(2*b) + (a^2*Log[a +
b*x^(1/3)])/b^3))/(b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{3\left(a+b\,x^{\frac{1}{3}}\right)\left(b^{2} x^{\frac{2}{3}}+2 a^{2} \ln \left(a+b\,x^{\frac{1}{3}}\right)-2 a b\,x^{\frac{1}{3}}\right)}{2 \sqrt{\left(a+b\,x^{\frac{1}{3}}\right)^{2}}\, b^{3}}$	52
default	$\frac{\left(a+b\,x^{\frac{1}{3}}\right)\left(3 b^{2} x^{\frac{2}{3}}-6 a b\,x^{\frac{1}{3}}+2 a^{2} \ln \left(b^{3} x+a^{3}\right)-2 a^{2} \ln \left(b^{2} x^{\frac{2}{3}}-a b\,x^{\frac{1}{3}}+a^{2}\right)+4 a^{2} \ln \left(a+b\,x^{\frac{1}{3}}\right)\right)}{2 \sqrt{a^{2}+2 a b\,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}}\, b^{3}}$	101

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*(a+b*x^(1/3))*(b^2*x^(2/3)+2*a^2*ln(a+b*x^(1/3))-2*a*b*x^(1/3))/((a+b*
x^(1/3))^2)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \frac{3\left(2\,a^{2}\log\left(bx^{\frac{1}{3}}+a\right)+b^{2}x^{\frac{2}{3}}-2\,abx^{\frac{1}{3}}\right)}{2\,b^{3}}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
```

output
```
3/2*(2*a^2*log(b*x^(1/3) + a) + b^2*x^(2/3) - 2*a*b*x^(1/3))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = 3\left(\begin{cases} \frac{a^2\left(\frac{a}{b}+\sqrt[3]{x}\right)\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{b^2\sqrt{b^2\left(\frac{a}{b}+\sqrt[3]{x}\right)^2}}+\left(-\frac{3a}{2b^3}+\frac{\sqrt[3]{x}}{2b^2}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}} & \text{for } b \\ \frac{a^4\sqrt{a^2+2ab\sqrt[3]{x}}-\frac{2a^2\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{3}{2}}}{3}+\frac{\left(a^2+2ab\sqrt[3]{x}\right)^{\frac{5}{2}}}{5}}{4a^3b^3} & \text{for } a \\ \frac{x}{3\sqrt{a^2}} & \text{othe} \end{cases}\right.$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

output
```
3*Piecewise((a**2*(a/b + x**(1/3))*log(a/b + x**(1/3))/(b**2*sqrt(b**2*(a/
b + x**(1/3))**2)) + (-3*a/(2*b**3) + x**(1/3)/(2*b**2))*sqrt(a**2 + 2*a*b
*x**(1/3) + b**2*x**(2/3)), Ne(b**2, 0)), ((a**4*sqrt(a**2 + 2*a*b*x**(1/3
)) - 2*a**2*(a**2 + 2*a*b*x**(1/3))**(3/2)/3 + (a**2 + 2*a*b*x**(1/3))**(5
/2)/5)/(4*a**3*b**3), Ne(a*b, 0)), (x/(3*sqrt(a**2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.24

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \frac{3\,a^2\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^3}+\frac{3\,x^{\frac{2}{3}}}{2\,b}-\frac{3\,ax^{\frac{1}{3}}}{b^2}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

output
```
3*a^2*log(x^(1/3) + a/b)/b^3 + 3/2*x^(2/3)/b - 3*a*x^(1/3)/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \frac{3\left(bx^{\frac{2}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)-2\,ax^{\frac{1}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)\right)}{2\,b^2} + \frac{3\,a^2\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
```

output
```
3/2*(b*x^(2/3)*sgn(b*x^(1/3) + a) - 2*a*x^(1/3)*sgn(b*x^(1/3) + a))/b^2 +
3*a^2*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \int \frac{1}{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

output
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = \frac{\frac{3x^{\frac{2}{3}}b^2}{2}-3x^{\frac{1}{3}}ab+3\log\left(x^{\frac{1}{3}}b+a\right)a^2}{b^3}$$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x)
```

output
```
(3*(x**(2/3)*b**2 - 2*x**(1/3)*a*b + 2*log(x**(1/3)*b + a)*a**2))/(2*b**3)
```

3.77 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

output
```
6*a/b^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/2*a^2/b^3/(a+b*x^(1/3))/(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3*(a+b*x^(1/3))*ln(a+b*x^(1/3))/b^3/(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=\frac{3a\left(3a+4b\sqrt[3]{x}\right)+6\left(a+b\sqrt[3]{x}\right)^2\log\left(a+b\sqrt[3]{x}\right)}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

output
```
(3*a*(3*a + 4*b*x^(1/3)) + 6*(a + b*x^(1/3))^2*Log[a + b*x^(1/3)])/(2*b^3*
(a + b*x^(1/3))*Sqrt[(a + b*x^(1/3))^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^3+b^4\sqrt[3]{x}\right)\int \frac{1}{\left(\sqrt[3]{x}b^2+ab\right)^3}dx}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab^3+b^4\sqrt[3]{x}\right)\int \frac{x^{2/3}}{b^3\left(a+b\sqrt[3]{x}\right)^3}d\sqrt[3]{x}}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab^3+b^4\sqrt[3]{x}\right)\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^3}d\sqrt[3]{x}}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3\left(ab^3+b^4\sqrt[3]{x}\right)\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^3}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)^2}+\frac{1}{b^2\left(a+b\sqrt[3]{x}\right)}\right)d\sqrt[3]{x}}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(ab^3+b^4\sqrt[3]{x}\right)\left(-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^2}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)}+\frac{\log\left(a+b\sqrt[3]{x}\right)}{b^3}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

output
```
(3*(a*b^3 + b^4*x^(1/3))*(-1/2*a^2/(b^3*(a + b*x^(1/3))^2) + (2*a)/(b^3*(a
 + b*x^(1/3))) + Log[a + b*x^(1/3)]/b^3))/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) +
b^2*x^(2/3)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{3\left(2\ln\left(a+b\,x^{\frac{1}{3}}\right)b^{2}x^{\frac{2}{3}}+4\ln\left(a+b\,x^{\frac{1}{3}}\right)ab\,x^{\frac{1}{3}}+2a^{2}\ln\left(a+b\,x^{\frac{1}{3}}\right)+4ab\,x^{\frac{1}{3}}+3a^{2}\right)\left(a+b\,x^{\frac{1}{3}}\right)}{2b^{3}\left(\left(a+b\,x^{\frac{1}{3}}\right)^{2}\right)^{\frac{3}{2}}}$
default	$\left(-2x^{4}\ln\left(b^{2}x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^{2}\right)b^{12}+24x^{2}\ln\left(a+b\,x^{\frac{1}{3}}\right)a^{6}b^{6}+12x^{2}\ln\left(b^{3}x+a^{3}\right)a^{6}b^{6}-12x^{2}\ln\left(b^{2}x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^{2}\right)a^{6}b^{6}+1\right.$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*(2*ln(a+b*x^(1/3))*b^2*x^(2/3)+4*ln(a+b*x^(1/3))*a*b*x^(1/3)+2*a^2*ln(
a+b*x^(1/3))+4*a*b*x^(1/3)+3*a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3))^2)^(3/2
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx = \frac{3\left(6\,a^3b^3x+3\,a^6+2\,(b^6x^2+2\,a^3b^3x+a^6)\log\left(bx^{\frac{1}{3}}+a\right)+(4\,ab^5x+a\right.}{2\,(b^9x^2+2\,a^3b^6x+a^6b^3)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

output
```
3/2*(6*a^3*b^3*x + 3*a^6 + 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) +
 a) + (4*a*b^5*x + a^4*b^2)*x^(2/3) - (5*a^2*b^4*x + 2*a^5*b)*x^(1/3))/(b^
9*x^2 + 2*a^3*b^6*x + a^6*b^3)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=\frac{3\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^3}+\frac{6\,ax^{\frac{1}{3}}}{b^4\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}+\frac{9\,a^2}{2\,b^5\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

output
```
3*log(x^(1/3) + a/b)/b^3 + 6*a*x^(1/3)/(b^4*(x^(1/3) + a/b)^2) + 9/2*a^2/(
b^5*(x^(1/3) + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.49

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=\frac{3\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}+\frac{3\left(4\,ax^{\frac{1}{3}}+\frac{3\,a^2}{b}\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2b^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

output
```
3*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a)) + 3/2*(4*a*x^(1/3) + 3*
a^2/b)/((b*x^(1/3) + a)^2*b^2*sgn(b*x^(1/3) + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=\int \frac{1}{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{3/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
```

output
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx=\frac{3x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}b+a\right)b^2-3x^{\frac{2}{3}}b^2+6x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+a\right)ab+3\log\left(x^{\frac{1}{3}}b+a\right)a}{b^3\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)}$$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x)
```

output
```
(3*(2*x**(2/3)*log(x**(1/3)*b + a)*b**2 - 2*x**(2/3)*b**2 + 4*x**(1/3)*log
(x**(1/3)*b + a)*a*b + 2*log(x**(1/3)*b + a)*a**2 + a**2))/(2*b**3*(x**(2/
3)*b**2 + 2*x**(1/3)*a*b + a**2))
```

3.78 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 135

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = -\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} + \frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}} - \frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

output
```
-3/4*a^2/b^3/(a+b*x^(1/3))^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+2*a/b^3
/(a+b*x^(1/3))^2/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/2/b^3/(a+b*x^(1/3
))/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.41

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = \frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-4ab\sqrt[3]{x}-6b^2x^{2/3}\right)}{4b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{5/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

output
```
((a + b*x^(1/3))*(-a^2 - 4*a*b*x^(1/3) - 6*b^2*x^(2/3)))/(4*b^3*((a + b*x^
(1/3))^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 774, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^5+b^6\sqrt[3]{x}\right)\int \frac{1}{\left(\sqrt[3]{x}b^2+ab\right)^5}dx}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab^5+b^6\sqrt[3]{x}\right)\int \frac{x^{2/3}}{b^5\left(a+b\sqrt[3]{x}\right)^5}d\sqrt[3]{x}}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab^5+b^6\sqrt[3]{x}\right)\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^5}d\sqrt[3]{x}}{b^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{53}$$

$$\frac{3\left(ab^5+b^6\sqrt[3]{x}\right)\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^5}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)^4}+\frac{1}{b^2\left(a+b\sqrt[3]{x}\right)^3}\right)d\sqrt[3]{x}}{b^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^4}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^3}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^2}\right)\left(ab^5+b^6\sqrt[3]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

output
```
(3*(-1/4*a^2/(b^3*(a + b*x^(1/3))^4) + (2*a)/(3*b^3*(a + b*x^(1/3))^3) - 1
/(2*b^3*(a + b*x^(1/3))^2))*(a*b^5 + b^6*x^(1/3)))/(b^5*Sqrt[a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

method	result
derivativedivides	$-\frac{\left(6b^2x^{\frac{2}{3}}+4ab\,x^{\frac{1}{3}}+a^2\right)\left(a+b\,x^{\frac{1}{3}}\right)}{4b^3\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{5}{2}}}$
default	$-\frac{\left(6x^{\frac{22}{3}}b^{22}+45x^{\frac{20}{3}}a^2b^{20}-36x^{\frac{19}{3}}a^3b^{19}+144x^{\frac{17}{3}}a^5b^{17}-189x^{\frac{16}{3}}a^6b^{16}+126x^{\frac{14}{3}}a^8b^{14}-276x^{\frac{13}{3}}a^9b^{13}-36x^{\frac{11}{3}}a^{11}b^{11}-\right.}{4b^3\left(b^2\right.}$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(6*b^2*x^(2/3)+4*a*b*x^(1/3)+a^2)*(a+b*x^(1/3))/b^3/((a+b*x^(1/3))^2)
^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = \frac{20\,ab^9x^3-60\,a^4b^6x^2-a^{10}-9\,(5\,a^2b^8x^2-4\,a^5b^5x)x^{\frac{2}{3}}-3\,(2\,b^{10}x^3-20}{4\,(b^{15}x^4+4\,a^3b^{12}x^3+6\,a^6b^9x^2+4\,a^9b^6x+a^{12}b^3)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

output
```
1/4*(20*a*b^9*x^3 - 60*a^4*b^6*x^2 - a^10 - 9*(5*a^2*b^8*x^2 - 4*a^5*b^5*x
)*x^(2/3) - 3*(2*b^10*x^3 - 20*a^3*b^7*x^2 + 5*a^6*b^4*x)*x^(1/3))/(b^15*x
^4 + 4*a^3*b^12*x^3 + 6*a^6*b^9*x^2 + 4*a^9*b^6*x + a^12*b^3)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx=\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx=-\frac{3}{2\,b^5\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}+\frac{2\,a}{b^6\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^3}-\frac{3\,a^2}{4\,b^7\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

output
```
-3/2/(b^5*(x^(1/3) + a/b)^2) + 2*a/(b^6*(x^(1/3) + a/b)^3) - 3/4*a^2/(b^7*
(x^(1/3) + a/b)^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx=-\frac{6\,b^2x^{\frac{2}{3}}+4\,abx^{\frac{1}{3}}+a^2}{4\left(bx^{\frac{1}{3}}+a\right)^4b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
```

output
```
-1/4*(6*b^2*x^(2/3) + 4*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^4*b^3*sgn(b*x^
(1/3) + a))
```

Mupad [B] (verification not implemented)

Time = 23.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = -\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+6\,b^2\,x^{2/3}+4\,a\,b\,x^{1/3}\right)}{4\,b^3\,\left(a+b\,x^{1/3}\right)^5}$$

input
```
int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
```

output
```
-((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 6*b^2*x^(2/3) + 4*a*b*x
^(1/3)))/(4*b^3*(a + b*x^(1/3))^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.49

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx = \frac{-6x^{\frac{2}{3}}b^2-4x^{\frac{1}{3}}ab-a^2}{4b^3\left(6x^{\frac{2}{3}}a^2b^2+4x^{\frac{1}{3}}a^3b+x^{\frac{4}{3}}b^4+a^4+4a\,b^3x\right)}$$

input
```
int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x)
```

output
```
( - 6*x**(2/3)*b**2 - 4*x**(1/3)*a*b - a**2)/(4*b**3*(6*x**(2/3)*a**2*b**2
 + 4*x**(1/3)*a**3*b + x**(1/3)*b**4*x + a**4 + 4*a*b**3*x))
```

3.79 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+2p)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+p)}+\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(3+2p)}$$

output
```
3*a^2*(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(1+2*p)-3*a*(a+b
*x^(1/3))^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(p+1)+3*(a+b*x^(1/3))^3*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(a^2-ab(1+2p)\sqrt[3]{x}+b^2\left(1+3p+2p^2\right)x^{2/3}\right)}{b^3(1+p)(1+2p)(3+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

output
```
(3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(a^2 - a*b*(1 + 2*p)*x^(1/3) + b^
2*(1 + 3*p + 2*p^2)*x^(2/3)))/(b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1385, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{774}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\int \left(\frac{a^2\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p}}{b^2}-\frac{2a^2\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+1}}{b^2}+\frac{a^2\left(\frac{\sqrt[3]{x}b}{a}+1\right)^{2p+2}}{b^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2(p+1)}}{b^3(p+1)}+\frac{a^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2p+1}}{b^3(2p+1)}+\frac{a^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2p+3}}{b^3(2p+3)}\right)\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right.$$

input
```
Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

output
```
(3*(-((a^3*(1 + (b*x^(1/3))/a)^(2*(1 + p)))/(b^3*(1 + p))) + (a^3*(1 + (b*
x^(1/3))/a)^(1 + 2*p))/(b^3*(1 + 2*p)) + (a^3*(1 + (b*x^(1/3))/a)^(3 + 2*p
))/(b^3*(3 + 2*p)))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(1 + (b*x^(1/3)
)/a)^(2*p)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(a^2 + 2ab\,x^{\frac{1}{3}} + b^2 x^{\frac{2}{3}}\right)^p dx$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

output
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx = -\frac{3\left(2\,a^2bpx^{\frac{1}{3}}-a^3-(2\,b^3p^2+3\,b^3p+b^3)x-(2\,ab^2p^2+ab^2p)x^{\frac{2}{3}}\right)\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="fricas")
```

output
```
-3*(2*a^2*b*p*x^(1/3) - a^3 - (2*b^3*p^2 + 3*b^3*p + b^3)*x - (2*a*b^2*p^2
 + a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(4*b^3*p^3 + 12
*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Sympy [F]

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx = \int \left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

input
```
integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p,x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.54

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx = \frac{3\left((2\,p^2+3\,p+1)b^3x+(2\,p^2+p)ab^2x^{\frac{2}{3}}-2\,a^2bpx^{\frac{1}{3}}+a^3\right)\left(bx^{\frac{1}{3}}+a\right)^{2\,p}}{(4\,p^3+12\,p^2+11\,p+3)b^3}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="maxima")
```

output
```
3*((2*p^2 + 3*p + 1)*b^3*x + (2*p^2 + p)*a*b^2*x^(2/3) - 2*a^2*b*p*x^(1/3)
 + a^3)*(b*x^(1/3) + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.61

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left(2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pb^3p^2x+2\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^pab^2p^2x^{\frac{2}{3}}+3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+\right.\right.}{}$$

input
```
integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="giac")
```

output
```
3*(2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*p^2*x + 2*(b^2*x^(2/3) + 2*
a*b*x^(1/3) + a^2)^p*a*b^2*p^2*x^(2/3) + 3*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
a^2)^p*b^3*p*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^2*p*x^(2/3) - 2
*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b*p*x^(1/3) + (b^2*x^(2/3) + 2*
a*b*x^(1/3) + a^2)^p*b^3*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3)/(4
*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Mupad [B] (verification not implemented)

Time = 21.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\left(\frac{3\,x\,\left(2\,p^2+3\,p+1\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{3\,a^3}{b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{}{b^2\,\left(4\,p^3\right.}\right.$$

input
```
int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

output
```
(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x*(3*p + 2*p^2 + 1))/(11*p + 12*
p^2 + 4*p^3 + 3) + (3*a^3)/(b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (6*a^2*p*x^
(1/3))/(b^2*(11*p + 12*p^2 + 4*p^3 + 3)) + (3*a*p*x^(2/3)*(2*p + 1))/(b*(1
1*p + 12*p^2 + 4*p^3 + 3)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx=\frac{3\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)^p\left(2x^{\frac{2}{3}}a\,b^2p^2+x^{\frac{2}{3}}a\,b^2p-2x^{\frac{1}{3}}a^2bp+a^3+2b^3p^2x+3b^3px+b^3x\right)}{b^3\left(4p^3+12p^2+11p+3\right)}$$

input
```
int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

output
```
(3*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2)**p*(2*x**(2/3)*a*b**2*p**2 + x*
*(2/3)*a*b**2*p - 2*x**(1/3)*a**2*b*p + a**3 + 2*b**3*p**2*x + 3*b**3*p*x
+ b**3*x))/(b**3*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.80 $\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 176

$$\begin{aligned}\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx =& -\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}} \\ &+ \frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}} \\ &+ \frac{4\left(a+b\sqrt[4]{x}\right)\sqrt[4]{x}}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}} - \frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}\end{aligned}$$

output

```
-12*a^2/b^4/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)+2*a^3/b^4/(a+b*x^(1/4))/
(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)+4*(a+b*x^(1/4))*x^(1/4)/b^3/(a^2+2*a
*b*x^(1/4)+b^2*x^(1/2))^(1/2)-12*a*(a+b*x^(1/4))*ln(a+b*x^(1/4))/b^4/(a^2+
2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\frac{2\left(-5a^3-4a^2b\sqrt[4]{x}+4ab^2\sqrt{x}+2b^3x^{3/4}-6a\left(a+b\sqrt[4]{x}\right)^2\log\left(a+b\sqrt[4]{x}\right)\right)}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{\left(a+b\sqrt[4]{x}\right)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

output
```
(2*(-5*a^3 - 4*a^2*b*x^(1/4) + 4*a*b^2*Sqrt[x] + 2*b^3*x^(3/4) - 6*a*(a +
b*x^(1/4))^2*Log[a + b*x^(1/4)]))/(b^4*(a + b*x^(1/4))*Sqrt[(a + b*x^(1/4)
)^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.64, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\left(ab^3+b^4\sqrt[4]{x}\right)\int \frac{1}{\left(\sqrt[4]{x}b^2+ab\right)^3}dx}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$\Big\downarrow \ 774$$

$$\frac{4\left(ab^3+b^4\sqrt[4]{x}\right)\int \frac{x^{3/4}}{b^3\left(a+b\sqrt[4]{x}\right)^3}d\sqrt[4]{x}}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$\Big\downarrow \ 27$$

$$\frac{4\left(ab^3+b^4\sqrt[4]{x}\right)\int \frac{x^{3/4}}{\left(a+b\sqrt[4]{x}\right)^3}d\sqrt[4]{x}}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{4\left(ab^3+b^4\sqrt[4]{x}\right)\int \left(-\frac{a^3}{b^3\left(a+b\sqrt[4]{x}\right)^3}+\frac{3a^2}{b^3\left(a+b\sqrt[4]{x}\right)^2}-\frac{3a}{b^3\left(a+b\sqrt[4]{x}\right)}+\frac{1}{b^3}\right)d\sqrt[4]{x}}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(ab^3+b^4\sqrt[4]{x}\right)\left(\frac{a^3}{2b^4\left(a+b\sqrt[4]{x}\right)^2}-\frac{3a^2}{b^4\left(a+b\sqrt[4]{x}\right)}-\frac{3a\log\left(a+b\sqrt[4]{x}\right)}{b^4}+\frac{\sqrt[4]{x}}{b^3}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

output
```
(4*(a*b^3 + b^4*x^(1/4))*(a^3/(2*b^4*(a + b*x^(1/4))^2) - (3*a^2)/(b^4*(a
+ b*x^(1/4))) + x^(1/4)/b^3 - (3*a*Log[a + b*x^(1/4)])/b^4))/(b^3*Sqrt[a^2
 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.59

method	result
derivativedivides	$-\frac{2\left(6\ln\left(a+b\,x^{\frac{1}{4}}\right)a\,b^{2}\sqrt{x}-2b^{3}x^{\frac{3}{4}}+12\ln\left(a+b\,x^{\frac{1}{4}}\right)a^{2}b\,x^{\frac{1}{4}}-4a\,b^{2}\sqrt{x}+6\ln\left(a+b\,x^{\frac{1}{4}}\right)a^{3}+4a^{2}b\,x^{\frac{1}{4}}+5a^{3}\right)\left(a+b\,x^{\frac{1}{4}}\right)}{b^{4}\left(\left(a+b\,x^{\frac{1}{4}}\right)^{2}\right)^{\frac{3}{2}}}$
default	Expression too large to display

input
```
int(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(6*ln(a+b*x^(1/4))*a*b^2*x^(1/2)-2*b^3*x^(3/4)+12*ln(a+b*x^(1/4))*a^2*b
*x^(1/4)-4*a*b^2*x^(1/2)+6*ln(a+b*x^(1/4))*a^3+4*a^2*b*x^(1/4)+5*a^3)*(a+b
*x^(1/4))/b^4/((a+b*x^(1/4))^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\frac{2\left(9\,a^5b^4x-5\,a^9-6\left(ab^8x^2-2\,a^5b^4x+a^9\right)\log\left(bx^{\frac{1}{4}}+a\right)-2\left(3\,a^2b^7x\right.\right.}{b^{12}x^2-2\,a^4b^8}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2*(9*a^5*b^4*x - 5*a^9 - 6*(a*b^8*x^2 - 2*a^5*b^4*x + a^9)*log(b*x^(1/4) +
 a) - 2*(3*a^2*b^7*x - a^6*b^3)*x^(3/4) + (7*a^3*b^6*x - 3*a^7*b^2)*sqrt(x
) + 2*(b^9*x^2 - 6*a^4*b^5*x + 3*a^8*b)*x^(1/4))/(b^12*x^2 - 2*a^4*b^8*x +
 a^8*b^4)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**(1/4)+b**2*x**(1/2))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/4) + b**2*sqrt(x))**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx=\frac{4\sqrt{x}}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac{1}{4}}+a^2}b^2}-\frac{12\,a\log\left(x^{\frac{1}{4}}+\frac{a}{b}\right)}{b^4}+\frac{8\,a^2}{\sqrt{b^2\sqrt{x}+2\,abx^{\frac{1}{4}}+a^2}b^4}-\frac{24\,a^2x^{\frac{1}{4}}}{b^5\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^2}-\frac{22\,a^3}{b^6\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^2}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
4*sqrt(x)/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^2) - 12*a*log(x^(1/4)
 + a/b)/b^4 + 8*a^2/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^4) - 24*a^2
*x^(1/4)/(b^5*(x^(1/4) + a/b)^2) - 22*a^3/(b^6*(x^(1/4) + a/b)^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.47

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx = -\frac{12\,a\log\left(\left|bx^{\frac{1}{4}}+a\right|\right)}{b^4\mathrm{sgn}\left(bx^{\frac{1}{4}}+a\right)} + \frac{4\,x^{\frac{1}{4}}}{b^3\mathrm{sgn}\left(bx^{\frac{1}{4}}+a\right)} - \frac{2\left(6\,a^2bx^{\frac{1}{4}}+5\,a^3\right)}{\left(bx^{\frac{1}{4}}+a\right)^2 b^4\mathrm{sgn}\left(bx^{\frac{1}{4}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="giac")
```

output
```
-12*a*log(abs(b*x^(1/4) + a))/(b^4*sgn(b*x^(1/4) + a)) + 4*x^(1/4)/(b^3*sg
n(b*x^(1/4) + a)) - 2*(6*a^2*b*x^(1/4) + 5*a^3)/((b*x^(1/4) + a)^2*b^4*sgn
(b*x^(1/4) + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx = \int \frac{1}{\left(a^2+b^2\,\sqrt{x}+2\,a\,b\,x^{1/4}\right)^{3/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2),x)
```

output
```
int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.52

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx = \frac{4x^{\frac{3}{4}}b^3-24x^{\frac{1}{4}}\log\left(x^{\frac{1}{4}}b+a\right)a^2b-12\sqrt{x}\log\left(x^{\frac{1}{4}}b+a\right)a\,b^2+12\sqrt{x}\,a\,b^2-}{b^4\left(2x^{\frac{1}{4}}ab+\sqrt{x}\,b^2+a^2\right)}$$

input
```
int(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x)
```

output
```
(2*(2*x**(3/4)*b**3 - 12*x**(1/4)*log(x**(1/4)*b + a)*a**2*b - 6*sqrt(x)*l
og(x**(1/4)*b + a)*a*b**2 + 6*sqrt(x)*a*b**2 - 6*log(x**(1/4)*b + a)*a**3
- 3*a**3))/(b**4*(2*x**(1/4)*a*b + sqrt(x)*b**2 + a**2))
```

3.81 $\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = -\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} - \frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} + \frac{6\left(a+b\sqrt[6]{x}\right)\sqrt[6]{x}}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}} - \frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

output
```
-60*a^2/b^6/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)+3/2*a^5/b^6/(a+b*x^(1/6)
)^3/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)-10*a^4/b^6/(a+b*x^(1/6))^2/(a^2+
2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)+30*a^3/b^6/(a+b*x^(1/6))/(a^2+2*a*b*x^(1/
6)+b^2*x^(1/3))^(1/2)+6*(a+b*x^(1/6))*x^(1/6)/b^5/(a^2+2*a*b*x^(1/6)+b^2*x
^(1/3))^(1/2)-30*a*(a+b*x^(1/6))*ln(a+b*x^(1/6))/b^6/(a^2+2*a*b*x^(1/6)+b^
2*x^(1/3))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx=\frac{-77a^5-248a^4b\sqrt[6]{x}-252a^3b^2\sqrt[3]{x}-48a^2b^3\sqrt{x}+48ab^4x^{2/3}+12b^5x^{5/6}-}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{\left(a+b\sqrt[6]{x}\right)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

output
```
(-77*a^5 - 248*a^4*b*x^(1/6) - 252*a^3*b^2*x^(1/3) - 48*a^2*b^3*Sqrt[x] +
48*a*b^4*x^(2/3) + 12*b^5*x^(5/6) - 60*a*(a + b*x^(1/6))^4*Log[a + b*x^(1/
6)])/(2*b^6*(a + b*x^(1/6))^3*Sqrt[(a + b*x^(1/6))^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^5+b^6\sqrt[6]{x}\right)\int \frac{1}{\left(\sqrt[6]{x}b^2+ab\right)^5}dx}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{6\left(ab^5+b^6\sqrt[6]{x}\right)\int \frac{x^{5/6}}{b^5\left(a+b\sqrt[6]{x}\right)^5}d\sqrt[6]{x}}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{6\left(ab^5+b^6\sqrt[6]{x}\right)\int \frac{x^{5/6}}{\left(a+b\sqrt[6]{x}\right)^5}d\sqrt[6]{x}}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{6\left(ab^5+b^6\sqrt[6]{x}\right)\int \left(-\frac{a^5}{b^5\left(a+b\sqrt[6]{x}\right)^5}+\frac{5a^4}{b^5\left(a+b\sqrt[6]{x}\right)^4}-\frac{10a^3}{b^5\left(a+b\sqrt[6]{x}\right)^3}+\frac{10a^2}{b^5\left(a+b\sqrt[6]{x}\right)^2}-\frac{5a}{b^5\left(a+b\sqrt[6]{x}\right)}+\frac{1}{b^5}\right)d\sqrt[6]{x}}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6\left(ab^5+b^6\sqrt[6]{x}\right)\left(\frac{a^5}{4b^6\left(a+b\sqrt[6]{x}\right)^4}-\frac{5a^4}{3b^6\left(a+b\sqrt[6]{x}\right)^3}+\frac{5a^3}{b^6\left(a+b\sqrt[6]{x}\right)^2}-\frac{10a^2}{b^6\left(a+b\sqrt[6]{x}\right)}-\frac{5a\log\left(a+b\sqrt[6]{x}\right)}{b^6}+\frac{\sqrt[6]{x}}{b^5}\right)}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

output
```
(6*(a*b^5 + b^6*x^(1/6))*(a^5/(4*b^6*(a + b*x^(1/6))^4) - (5*a^4)/(3*b^6*(
a + b*x^(1/6))^3) + (5*a^3)/(b^6*(a + b*x^(1/6))^2) - (10*a^2)/(b^6*(a + b
*x^(1/6))) + x^(1/6)/b^5 - (5*a*Log[a + b*x^(1/6)])/b^6))/(b^5*Sqrt[a^2 +
2*a*b*x^(1/6) + b^2*x^(1/3)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.61

method	result
derivativedivides	$-\frac{\left(60\ln\left(a+b\,x^{\frac{1}{6}}\right)a\,b^{4}x^{\frac{2}{3}}-12b^{5}x^{\frac{5}{6}}+240\ln\left(a+b\,x^{\frac{1}{6}}\right)a^{2}b^{3}\sqrt{x}-48a\,b^{4}x^{\frac{2}{3}}+360\ln\left(a+b\,x^{\frac{1}{6}}\right)a^{3}b^{2}x^{\frac{1}{3}}+48a^{2}b^{3}\sqrt{x}+240\,\mathrm{l}}{2b^{6}\left(\left(a+b\,x^{\frac{1}{6}}\right)^{2}\right)^{\frac{5}{2}}}$
default	Expression too large to display

input
```
int(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(60*ln(a+b*x^(1/6))*a*b^4*x^(2/3)-12*b^5*x^(5/6)+240*ln(a+b*x^(1/6))*
a^2*b^3*x^(1/2)-48*a*b^4*x^(2/3)+360*ln(a+b*x^(1/6))*a^3*b^2*x^(1/3)+48*a^
2*b^3*x^(1/2)+240*ln(a+b*x^(1/6))*a^4*b*x^(1/6)+252*a^3*b^2*x^(1/3)+60*ln(
a+b*x^(1/6))*a^5+248*a^4*b*x^(1/6)+77*a^5)*(a+b*x^(1/6))/b^6/((a+b*x^(1/6)
)^2)^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a**2+2*a*b*x**(1/6)+b**2*x**(1/3))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \frac{12\,b^5x^{\frac{5}{6}}+48\,ab^4x^{\frac{2}{3}}-48\,a^2b^3\sqrt{x}-252\,a^3b^2x^{\frac{1}{3}}-248\,a^4bx^{\frac{1}{6}}-77\,a^5}{2\left(b^{10}x^{\frac{2}{3}}+4\,ab^9\sqrt{x}+6\,a^2b^8x^{\frac{1}{3}}+4\,a^3b^7x^{\frac{1}{6}}+a^4b^6\right)} - \frac{30\,a\log\left(bx^{\frac{1}{6}}+a\right)}{b^6}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="maxima")
```

output
```
1/2*(12*b^5*x^(5/6) + 48*a*b^4*x^(2/3) - 48*a^2*b^3*sqrt(x) - 252*a^3*b^2*
x^(1/3) - 248*a^4*b*x^(1/6) - 77*a^5)/(b^10*x^(2/3) + 4*a*b^9*sqrt(x) + 6*
a^2*b^8*x^(1/3) + 4*a^3*b^7*x^(1/6) + a^4*b^6) - 30*a*log(b*x^(1/6) + a)/b
^6
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = -\frac{30\,a\log\left(\left|bx^{\frac{1}{6}}+a\right|\right)}{b^6\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)} + \frac{6\,x^{\frac{1}{6}}}{b^5\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)} - \frac{120\,a^2b^3\sqrt{x}+300\,a^3b^2x^{\frac{1}{3}}+260\,a^4bx^{\frac{1}{6}}+77\,a^5}{2\left(bx^{\frac{1}{6}}+a\right)^4b^6\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="giac")
```

output
```
-30*a*log(abs(b*x^(1/6) + a))/(b^6*sgn(b*x^(1/6) + a)) + 6*x^(1/6)/(b^5*sg
n(b*x^(1/6) + a)) - 1/2*(120*a^2*b^3*sqrt(x) + 300*a^3*b^2*x^(1/3) + 260*a
^4*b*x^(1/6) + 77*a^5)/((b*x^(1/6) + a)^4*b^6*sgn(b*x^(1/6) + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \int \frac{1}{\left(a^2+b^2\,x^{1/3}+2\,a\,b\,x^{1/6}\right)^{5/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2),x)
```

output
```
int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = \frac{12x^{\frac{5}{6}}b^5-240x^{\frac{1}{6}}\log\left(x^{\frac{1}{6}}b+a\right)a^4b-200x^{\frac{1}{6}}a^4b-60x^{\frac{2}{3}}\log\left(x^{\frac{1}{6}}b+a\right)ab^4}{2b^6\left(\right.}$$

input
```
int(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x)
```

output
```
(12*x**(5/6)*b**5 - 240*x**(1/6)*log(x**(1/6)*b + a)*a**4*b - 200*x**(1/6)
*a**4*b - 60*x**(2/3)*log(x**(1/6)*b + a)*a*b**4 + 60*x**(2/3)*a*b**4 - 36
0*x**(1/3)*log(x**(1/6)*b + a)*a**3*b**2 - 180*x**(1/3)*a**3*b**2 - 240*sq
rt(x)*log(x**(1/6)*b + a)*a**2*b**3 - 60*log(x**(1/6)*b + a)*a**5 - 65*a**
5)/(2*b**6*(4*x**(1/6)*a**3*b + x**(2/3)*b**4 + 6*x**(1/3)*a**2*b**2 + 4*s
qrt(x)*a*b**3 + a**4))
```

3.82 $\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 175

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2}\,dx=-\frac{2b^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}}{\left(a+\frac{b}{\sqrt{x}}\right)\sqrt{x}}+\frac{6a^2b\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\sqrt{x}}{a+\frac{b}{\sqrt{x}}}+\frac{a^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}x}{a+\frac{b}{\sqrt{x}}}+\frac{3ab^2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\log(x)}{a+\frac{b}{\sqrt{x}}}$$

output
```
-2*b^3*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/(a+b/x^(1/2))/x^(1/2)+6*a^2*b*(a^2+
b^2/x+2*a*b/x^(1/2))^(1/2)*x^(1/2)/(a+b/x^(1/2))+a^3*(a^2+b^2/x+2*a*b/x^(1
/2))^(1/2)*x/(a+b/x^(1/2))+3*a*b^2*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)*ln(x)/(
a+b/x^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.38

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \frac{\sqrt{\frac{(b+a\sqrt{x})^2}{x}}\left(-2b^3+6a^2bx+a^3x^{3/2}+3ab^2\sqrt{x}\log(x)\right)}{b+a\sqrt{x}}$$

input
```
Integrate[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

output
```
(Sqrt[(b + a*Sqrt[x])^2/x]*(-2*b^3 + 6*a^2*b*x + a^3*x^(3/2) + 3*a*b^2*Sqr
t[x]*Log[x]))/(b + a*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.50, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\int \left(\frac{b^2}{\sqrt{x}}+ab\right)^3 dx}{ab^3+\frac{b^4}{\sqrt{x}}}$$

$$\Big\downarrow \text{ 774}$$

$$\frac{2\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\int b^3\left(a+\frac{b}{\sqrt{x}}\right)^3\sqrt{x}d\sqrt{x}}{ab^3+\frac{b^4}{\sqrt{x}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\int \left(a+\frac{b}{\sqrt{x}}\right)^3\sqrt{x}d\sqrt{x}}{ab^3+\frac{b^4}{\sqrt{x}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\int \frac{(\sqrt{x}a+b)^3}{x}d\sqrt{x}}{ab^3+\frac{b^4}{\sqrt{x}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\int \left(\sqrt{x}a^3+3ba^2+\frac{3b^2a}{\sqrt{x}}+\frac{b^3}{x}\right)d\sqrt{x}}{ab^3+\frac{b^4}{\sqrt{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}\left(\frac{a^3x}{2}+3a^2b\sqrt{x}+3ab^2\log\left(\sqrt{x}\right)-\frac{b^3}{\sqrt{x}}\right)}{ab^3+\frac{b^4}{\sqrt{x}}}$$

input
```
Int[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

output
```
(2*b^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*(-(b^3/Sqrt[x]) + 3*a^2*b*Sqrt[
x] + (a^3*x)/2 + 3*a*b^2*Log[Sqrt[x]]))/(a*b^3 + b^4/Sqrt[x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.37

method	result	size
derivativedivides	$\frac{\left(\frac{x\,a^{2}+b^{2}+2 a b \sqrt{x}}{x}\right)^{\frac{3}{2}} x \left(a^{3} x^{\frac{3}{2}}+3 b^{2} a \ln \left(x \right) \sqrt{x}+6 b\,a^{2} x -2 b^{3}\right)}{\left(\sqrt{x}\,a +b \right)^{3}}$	65
default	$\frac{\left(\frac{a^{2} x^{\frac{3}{2}}+b^{2} \sqrt{x}+2 a b x}{x^{\frac{3}{2}}}\right)^{\frac{3}{2}} \left(x^{\frac{5}{2}} a^{3}+3 \ln \left(x \right) x^{\frac{3}{2}} a\,b^{2}+6 a^{2} b\,x^{2}-2 b^{3} x \right)}{\left(\sqrt{x}\,a +b \right)^{3}}$	71

input
```
int((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((x*a^2+b^2+2*a*b*x^(1/2))/x)^(3/2)*x*(a^3*x^(3/2)+3*b^2*a*ln(x)*x^(1/2)+6
*b*a^2*x-2*b^3)/(x^(1/2)*a+b)^3
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \int \left(a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a**2+b**2/x+2*a*b/x**(1/2))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b/sqrt(x) + b**2/x)**(3/2), x)
```

Maxima [F]

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \int \left(a^2+\frac{2\,ab}{\sqrt{x}}+\frac{b^2}{x}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
a^3*x + 3*a*b^2*integrate(1/x, x) + 6*a^2*b*sqrt(x) - 2*b^3/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.46

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = a^3x\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}(x) + 3\,ab^2\log\left(|x|\right)\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}(x) + 6\,a^2b\sqrt{x}\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}(x) - \frac{2\,b^3\mathrm{sgn}\left(ax+b\sqrt{x}\right)\mathrm{sgn}(x)}{\sqrt{x}}$$

input
```
integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="giac")
```

output
```
a^3*x*sgn(a*x + b*sqrt(x))*sgn(x) + 3*a*b^2*log(abs(x))*sgn(a*x + b*sqrt(x
))*sgn(x) + 6*a^2*b*sqrt(x)*sgn(a*x + b*sqrt(x))*sgn(x) - 2*b^3*sgn(a*x +
b*sqrt(x))*sgn(x)/sqrt(x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \int \left(a^2+\frac{b^2}{x}+\frac{2\,a\,b}{\sqrt{x}}\right)^{3/2} \,dx$$

input
```
int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2),x)
```

output
```
int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.21

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = \frac{6\sqrt{x}\log\left(\sqrt{x}\right) a\, b^2+\sqrt{x}\, a^3x+6a^2bx-2b^3}{\sqrt{x}}$$

input
```
int((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x)
```

output
```
(6*sqrt(x)*log(sqrt(x))*a*b**2 + sqrt(x)*a**3*x + 6*a**2*b*x - 2*b**3)/sqr
t(x)
```

3.83 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 387

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2}dx=-\frac{3b^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{4\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{4/3}}-\frac{7ab^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}-\frac{63a^2b^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}-\frac{105a^3b^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}+\frac{63a^5b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}}+\frac{35a^4b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log(x)}{a+\frac{b}{\sqrt[3]{x}}}$$

output
```
-3/4*b^7*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(4/3)-7*a*b
^6*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x-63/2*a^2*b^5*(a^2
+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(2/3)-105*a^3*b^4*(a^2+b
^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(1/3)+63*a^5*b^2*(a^2+b^2/
x^(2/3)+2*a*b/x^(1/3))^(1/2)*x^(1/3)/(a+b/x^(1/3))+21*a^6*b*(a^2+b^2/x^(2/
3)+2*a*b/x^(1/3))^(1/2)*x^(2/3)/(2*a+2*b/x^(1/3))+a^7*(a^2+b^2/x^(2/3)+2*a
*b/x^(1/3))^(1/2)*x/(a+b/x^(1/3))+35*a^4*b^3*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3
))^(1/2)*ln(x)/(a+b/x^(1/3))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.32

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = \frac{\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\left(-3b^7-28ab^6\sqrt[3]{x}-126a^2b^5x^{2/3}-420a^3b^4x+252a^5b^2x^{5/3}+42a^6bx^2+4a^7x\right.}{4\left(b+a\sqrt[3]{x}\right)x}$$

input
```
Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

output
```
(Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(-3*b^7 - 28*a*b^6*x^(1/3) - 126*a^2*b^5*
x^(2/3) - 420*a^3*b^4*x + 252*a^5*b^2*x^(5/3) + 42*a^6*b*x^2 + 4*a^7*x^(7/
3) + 140*a^4*b^3*x^(4/3)*Log[x]))/(4*(b + a*x^(1/3))*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}\right)^{7/2} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(\frac{b^2}{\sqrt[3]{x}}+ab\right)^7 dx}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{ 774}$$

$$\frac{3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)^7 x^{2/3}d\sqrt[3]{x}}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^7 x^{2/3}d\sqrt[3]{x}}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{ 795}$$

$$\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \frac{\left(\sqrt[3]{x}a+b\right)^7}{x^{5/3}}d\sqrt[3]{x}}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(x^{2/3}a^7+7b\sqrt[3]{x}a^6+21b^2a^5+\frac{35b^3a^4}{\sqrt[3]{x}}+\frac{35b^4a^3}{x^{2/3}}+\frac{21b^5a^2}{x}+\frac{7b^6a}{x^{4/3}}+\frac{b^7}{x^{5/3}}\right)d\sqrt[3]{x}}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(\frac{a^7x}{3}+\frac{7}{2}a^6bx^{2/3}+21a^5b^2\sqrt[3]{x}+35a^4b^3\log\left(\sqrt[3]{x}\right)-\frac{35a^3b^4}{\sqrt[3]{x}}-\frac{21a^2b^5}{2x^{2/3}}-\frac{7ab^6}{3x}-\frac{b^7}{4x^{4/3}}\right)}{ab^7+\frac{b^8}{\sqrt[3]{x}}}$$

input
```
Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

output
```
(3*b^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(-1/4*b^7/x^(4/3) - (7*a*
b^6)/(3*x) - (21*a^2*b^5)/(2*x^(2/3)) - (35*a^3*b^4)/x^(1/3) + 21*a^5*b^2*
x^(1/3) + (7*a^6*b*x^(2/3))/2 + (a^7*x)/3 + 35*a^4*b^3*Log[x^(1/3)]))/(a*b
^7 + b^8/x^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.29

method	result
derivativedivides	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b\, x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} x \left(4 a^{7} x^{\frac{7}{3}}+42 a^{6} b\, x^{2}+140 a^{4} b^{3} \ln \left(x \right) x^{\frac{4}{3}}+252 a^{5} b^{2} x^{\frac{5}{3}}-420 a^{3} b^{4} x -126 a^{2} b^{5} x^{\frac{2}{3}}-28 a\, b^{6} x^{\frac{1}{3}}-3 b^{7}\right)}{4 \left(b +a\, x^{\frac{1}{3}}\right)^{7}}$
default	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b\, x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} \left(42 a^{6} b\, x^{3}+252 x^{\frac{8}{3}} a^{5} b^{2}+140 a^{4} b^{3} \ln \left(x \right) x^{\frac{7}{3}}+4 x^{\frac{10}{3}} a^{7}-28 a\, b^{6} x^{\frac{4}{3}}-420 a^{3} b^{4} x^{2}-126 a^{2} b^{5} x^{\frac{5}{3}}-3 b^{7} x\right.}{4 \left(b +a\, x^{\frac{1}{3}}\right)^{7}}$

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(7/2)*x*(4*a^7*x^(7/3)+42*a^
6*b*x^2+140*a^4*b^3*ln(x)*x^(4/3)+252*a^5*b^2*x^(5/3)-420*a^3*b^4*x-126*a^
2*b^5*x^(2/3)-28*a*b^6*x^(1/3)-3*b^7)/(b+a*x^(1/3))^7
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2}dx=\int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}}dx$$

input
```
integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(7/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2}dx=35\,a^4b^3\log\left(x\right)+\frac{4\,a^7x^{\frac{7}{3}}+42\,a^6bx^2+252\,a^5b^2x^{\frac{5}{3}}-420\,a^3b^4x-126\,a^2b^5x^{\frac{2}{3}}-28\,ab^6x^{\frac{1}{3}}-3\,b^7}{4\,x^{\frac{4}{3}}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="maxima")
```

output
```
35*a^4*b^3*log(x) + 1/4*(4*a^7*x^(7/3) + 42*a^6*b*x^2 + 252*a^5*b^2*x^(5/3
) - 420*a^3*b^4*x - 126*a^2*b^5*x^(2/3) - 28*a*b^6*x^(1/3) - 3*b^7)/x^(4/3
)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.45

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = a^7 x\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+35\,a^4b^3\log\left(|x|\right)\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+\frac{21}{2}\,a^6bx^{\frac{2}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+63\,a^5b^2x^{\frac{1}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)-\frac{420\,a^3b^4x\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+126\,a^2b^5x^{\frac{2}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+28\,ab^6x^{\frac{1}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+\cdots}{4\,x^{\frac{4}{3}}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="giac")
```

output
```
a^7*x*sgn(a*x + b*x^(2/3))*sgn(x) + 35*a^4*b^3*log(abs(x))*sgn(a*x + b*x^(
2/3))*sgn(x) + 21/2*a^6*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 63*a^5*b^2
*x^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 1/4*(420*a^3*b^4*x*sgn(a*x + b*x^(2
/3))*sgn(x) + 126*a^2*b^5*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 28*a*b^6*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 3*b^7*sgn(a*x + b*x^(2/3))*sgn(x))/x^
(4/3)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = \int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{7/2} dx$$

input
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x)
```

output
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.21

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2}dx = \frac{252x^{\frac{5}{3}}a^5b^2-126x^{\frac{2}{3}}a^2b^5+420x^{\frac{4}{3}}\log\left(x^{\frac{1}{3}}\right)a^4b^3+4x^{\frac{7}{3}}a^7-28x^{\frac{1}{3}}a\,b^6+42a^6b\,x^2-420a^3b^4x}{4x^{\frac{4}{3}}}$$

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x)
```

output
```
(252*x**(2/3)*a**5*b**2*x - 126*x**(2/3)*a**2*b**5 + 420*x**(1/3)*log(x**(
1/3))*a**4*b**3*x + 4*x**(1/3)*a**7*x**2 - 28*x**(1/3)*a*b**6 + 42*a**6*b*
x**2 - 420*a**3*b**4*x - 3*b**7)/(4*x**(1/3)*x)
```

3.84 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 287

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = -\frac{3b^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}-\frac{15ab^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}+\frac{30a^3b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}}+\frac{10a^2b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log(x)}{a+\frac{b}{\sqrt[3]{x}}}$$

output
```
-3/2*b^5*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(2/3)-15*a*
b^4*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(1/3)+30*a^3*b^2
*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)*x^(1/3)/(a+b/x^(1/3))+15*a^4*b*(a^2
+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)*x^(2/3)/(2*a+2*b/x^(1/3))+a^5*(a^2+b^2/x
^(2/3)+2*a*b/x^(1/3))^(1/2)*x/(a+b/x^(1/3))+10*a^2*b^3*(a^2+b^2/x^(2/3)+2*
a*b/x^(1/3))^(1/2)*ln(x)/(a+b/x^(1/3))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.34

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = \frac{\left(b+a\sqrt[3]{x}\right)\left(-3b^5-30ab^4\sqrt[3]{x}+60a^3b^2x+15a^4bx^{4/3}+2a^5x^{5/3}+20a^2b^3x^{2/3}\log(x)\right)}{2\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}x}$$

input
```
Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

output
```
((b + a*x^(1/3))*(-3*b^5 - 30*a*b^4*x^(1/3) + 60*a^3*b^2*x + 15*a^4*b*x^(4
/3) + 2*a^5*x^(5/3) + 20*a^2*b^3*x^(2/3)*Log[x]))/(2*Sqrt[(b + a*x^(1/3))^
2/x^(2/3)]*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.42, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}\right)^{5/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(\frac{b^2}{\sqrt[3]{x}}+ab\right)^5 dx}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)^5x^{2/3}d\sqrt[3]{x}}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^5x^{2/3}d\sqrt[3]{x}}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \frac{\left(\sqrt[3]{x}a+b\right)^5}{x}d\sqrt[3]{x}}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(x^{2/3}a^5+5b\sqrt[3]{x}a^4+10b^2a^3+\frac{10b^3a^2}{\sqrt[3]{x}}+\frac{5b^4a}{x^{2/3}}+\frac{b^5}{x}\right)d\sqrt[3]{x}}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(\frac{a^5x}{3}+\frac{5}{2}a^4bx^{2/3}+10a^3b^2\sqrt[3]{x}+10a^2b^3\log\left(\sqrt[3]{x}\right)-\frac{5ab^4}{\sqrt[3]{x}}-\frac{b^5}{2x^{2/3}}\right)}{ab^5+\frac{b^6}{\sqrt[3]{x}}}$$

input
```
Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

output
```
(3*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(-1/2*b^5/x^(2/3) - (5*a*
b^4)/x^(1/3) + 10*a^3*b^2*x^(1/3) + (5*a^4*b*x^(2/3))/2 + (a^5*x)/3 + 10*a
^2*b^3*Log[x^(1/3)]))/(a*b^5 + b^6/x^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.32

method	result	size
derivativedivides	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}} x \left(2 a^{5} x^{\frac{5}{3}}+15 a^{4} b \,x^{\frac{4}{3}}+20 a^{2} b^{3} \ln \left(x \right) x^{\frac{2}{3}}+60 a^{3} b^{2} x -30 a \,b^{4} x^{\frac{1}{3}}-3 b^{5}\right)}{2 \left(b +a \,x^{\frac{1}{3}}\right)^{5}}$	91
default	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}} x \left(2 a^{5} x^{\frac{5}{3}}+15 a^{4} b \,x^{\frac{4}{3}}+20 a^{2} b^{3} \ln \left(x \right) x^{\frac{2}{3}}+60 a^{3} b^{2} x -30 a \,b^{4} x^{\frac{1}{3}}-3 b^{5}\right)}{2 \left(b +a \,x^{\frac{1}{3}}\right)^{5}}$	91

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)*x*(2*a^5*x^(5/3)+15*a^
4*b*x^(4/3)+20*a^2*b^3*ln(x)*x^(2/3)+60*a^3*b^2*x-30*a*b^4*x^(1/3)-3*b^5)/
(b+a*x^(1/3))^5
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = \int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = 10\,a^2b^3\log\left(x\right)+\frac{2\,a^5x^{\frac{5}{3}}+15\,a^4bx^{\frac{4}{3}}+60\,a^3b^2x-30\,ab^4x^{\frac{1}{3}}-3\,b^5}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

output
```
10*a^2*b^3*log(x) + 1/2*(2*a^5*x^(5/3) + 15*a^4*b*x^(4/3) + 60*a^3*b^2*x -
 30*a*b^4*x^(1/3) - 3*b^5)/x^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.45

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = a^5x\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+10\,a^2b^3\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+\frac{15}{2}\,a^4bx^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+30\,a^3b^2x^{\frac{1}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)-\frac{3\left(10\,ab^4x^{\frac{1}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+b^5\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)\right)}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

output
```
a^5*x*sgn(a*x + b*x^(2/3))*sgn(x) + 10*a^2*b^3*log(abs(x))*sgn(a*x + b*x^(
2/3))*sgn(x) + 15/2*a^4*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 30*a^3*b^2
*x^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 3/2*(10*a*b^4*x^(1/3)*sgn(a*x + b*x
^(2/3))*sgn(x) + b^5*sgn(a*x + b*x^(2/3))*sgn(x))/x^(2/3)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx=\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{5/2} dx$$

input
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
```

output
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.21

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx=\frac{20x^{\frac{2}{3}}\log(x)\,a^2b^3+2x^{\frac{5}{3}}a^5+15x^{\frac{4}{3}}a^4b-30x^{\frac{1}{3}}a\,b^4+60a^3b^2x-3b^5}{2x^{\frac{2}{3}}}$$

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x)
```

output
```
(20*x**(2/3)*log(x)*a**2*b**3 + 2*x**(2/3)*a**5*x + 15*x**(1/3)*a**4*b*x -
 30*x**(1/3)*a*b**4 + 60*a**3*b**2*x - 3*b**5)/(2*x**(2/3))
```

3.85 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 184

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \frac{9ab^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[3]{x}}} + \frac{9a^2b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)} + \frac{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}} + \frac{b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\log(x)}{a+\frac{b}{\sqrt[3]{x}}}$$

output
```
9*a*b^2*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)*x^(1/3)/(a+b/x^(1/3))+9*a^2*
b*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)*x^(2/3)/(2*a+2*b/x^(1/3))+a^3*(a^2
+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)*x/(a+b/x^(1/3))+b^3*(a^2+b^2/x^(2/3)+2*a
*b/x^(1/3))^(1/2)*ln(x)/(a+b/x^(1/3))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx=\frac{\left(b+a\sqrt[3]{x}\right)\left(18ab^2\sqrt[3]{x}+9a^2bx^{2/3}+2a^3x+2b^3\log(x)\right)}{2\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\sqrt[3]{x}}$$

input
```
Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

output
```
((b + a*x^(1/3))*(18*a*b^2*x^(1/3) + 9*a^2*b*x^(2/3) + 2*a^3*x + 2*b^3*Log
[x]))/(2*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.49, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(\frac{b^2}{\sqrt[3]{x}}+ab\right)^3 dx}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{ 774}$$

$$\frac{3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^{2/3}d\sqrt[3]{x}}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^{2/3}d\sqrt[3]{x}}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \frac{\left(\sqrt[3]{x}a+b\right)^3}{\sqrt[3]{x}}d\sqrt[3]{x}}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(x^{2/3}a^3+3b\sqrt[3]{x}a^2+3b^2a+\frac{b^3}{\sqrt[3]{x}}\right)d\sqrt[3]{x}}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(\frac{a^3x}{3}+\frac{3}{2}a^2bx^{2/3}+3ab^2\sqrt[3]{x}+b^3\log\left(\sqrt[3]{x}\right)\right)}{ab^3+\frac{b^4}{\sqrt[3]{x}}}$$

input
```
Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

output
```
(3*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(3*a*b^2*x^(1/3) + (3*a^2
*b*x^(2/3))/2 + (a^3*x)/3 + b^3*Log[x^(1/3)]))/(a*b^3 + b^4/x^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] && !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.38

method	result	size
derivativedivides	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b\, x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}} x \left(2 a^{3} x +9 x^{\frac{2}{3}} a^{2} b +2 b^{3} \ln \left(x \right)+18 b^{2} a \,x^{\frac{1}{3}}\right)}{2 \left(b +a \,x^{\frac{1}{3}}\right)^{3}}$	69
default	$\frac{\left(\frac{x^{\frac{2}{3}} a^{2}+2 a b\, x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}} x \left(2 a^{3} x +9 x^{\frac{2}{3}} a^{2} b +2 b^{3} \ln \left(x \right)+18 b^{2} a \,x^{\frac{1}{3}}\right)}{2 \left(b +a \,x^{\frac{1}{3}}\right)^{3}}$	69

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)*x*(2*a^3*x+9*x^(2/3)*a
^2*b+2*b^3*ln(x)+18*b^2*a*x^(1/3))/(b+a*x^(1/3))^3
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \int \left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.16

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = a^3x+b^3\log(x)+\frac{9}{2}a^2bx^{\frac{2}{3}}+9ab^2x^{\frac{1}{3}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

output
```
a^3*x + b^3*log(x) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.43

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = a^3x\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+b^3\log\left(|x|\right)\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+\frac{9}{2}a^2bx^{\frac{2}{3}}\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)+9ab^2x^{\frac{1}{3}}\text{sgn}\left(ax+bx^{\frac{2}{3}}\right)\text{sgn}(x)$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

output
```
a^3*x*sgn(a*x + b*x^(2/3))*sgn(x) + b^3*log(abs(x))*sgn(a*x + b*x^(2/3))*s
gn(x) + 9/2*a^2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 9*a*b^2*x^(1/3)*sg
n(a*x + b*x^(2/3))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{3/2} dx$$

input
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
```

output
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.16

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = \frac{9x^{\frac{2}{3}}a^2b}{2}+9x^{\frac{1}{3}}a\,b^2+\log(x)\,b^3+a^3x$$

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)
```

output
```
(9*x**(2/3)*a**2*b + 18*x**(1/3)*a*b**2 + 2*log(x)*b**3 + 2*a**3*x)/2
```

3.86 $\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=\frac{3b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}x}{a+\frac{b}{\sqrt[3]{x}}}$$

output
```
3*b*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)*x^(2/3)/(2*a+2*b/x^(1/3))+a*(a^2
+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)*x/(a+b/x^(1/3))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=\frac{\left(3b+2a\sqrt[3]{x}\right)\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}x}{2\left(b+a\sqrt[3]{x}\right)}$$

input
```
Integrate[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

output
```
((3*b + 2*a*x^(1/3))*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x)/(2*(b + a*x^(1/3))
)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\int \left(\frac{b^2}{\sqrt[3]{x}}+ab\right)dx}{ab+\frac{b^2}{\sqrt[3]{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(abx+\frac{3}{2}b^2x^{2/3}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}$$

input
```
Int[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

output
```
(Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*((3*b^2*x^(2/3))/2 + a*b*x))/(a
*b + b^2/x^(1/3))
```

Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

method	result	size
derivativedivides	$\frac{\sqrt{\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\, x \left(2 a \,x^{\frac{1}{3}}+3 b \right)}{2 b +2 a \,x^{\frac{1}{3}}}$	47
default	$\frac{\sqrt{\frac{x^{\frac{2}{3}} a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\, x^{\frac{1}{3}} \left(3 x^{\frac{2}{3}} b +2 x a \right)}{2 b +2 a \,x^{\frac{1}{3}}}$	50

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)*x*(2*a*x^(1/3)+3*b)/(b
+a*x^(1/3))
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx=\text{Timed out}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}\, dx = \int \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}}\, dx$$

input
```
integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

output
```
Integral(sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}\, dx = ax + \frac{3}{2}bx^{\frac{2}{3}}$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
```

output
```
a*x + 3/2*b*x^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.39

$$\int \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}\, dx = ax\,\mathrm{sgn}\left(ax + bx^{\frac{2}{3}}\right)\mathrm{sgn}(x) + \frac{3}{2}bx^{\frac{2}{3}}\mathrm{sgn}\left(ax + bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)$$

input
```
integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
```

output
```
a*x*sgn(a*x + b*x^(2/3))*sgn(x) + 3/2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x
)
```

Mupad [B] (verification not implemented)

Time = 20.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}\,dx = \frac{x\left(a + \frac{3b}{2x^{1/3}}\right)\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{x^{1/3}}}}{a + \frac{b}{x^{1/3}}}$$

input
```
int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
```

output
```
(x*(a + (3*b)/(2*x^(1/3)))*(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2))/(a
 + b/x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.11

$$\int \sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}\,dx = \frac{3x^{\frac{2}{3}}b}{2} + ax$$

input
```
int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x)
```

output
```
(3*x**(2/3)*b + 2*a*x)/2
```

3.87 $\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 190

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\frac{3b^2\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{3b\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}{2a^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}+\frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

output
```
3*b^2*(a+b/x^(1/3))*x^(1/3)/a^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-3/2*
b*(a+b/x^(1/3))*x^(2/3)/a^2/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)+(a+b/x^(
1/3))*x/a/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-3*b^3*(a+b/x^(1/3))*ln(b+a
*x^(1/3))/a^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\frac{\left(b+a\sqrt[3]{x}\right)\left(6ab^2\sqrt[3]{x}-3a^2bx^{2/3}+2a^3x-6b^3\log\left(b+a\sqrt[3]{x}\right)\right)}{2a^4\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\sqrt[3]{x}}$$

input
```
Integrate[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

output
```
((b + a*x^(1/3))*(6*a*b^2*x^(1/3) - 3*a^2*b*x^(2/3) + 2*a^3*x - 6*b^3*Log[
b + a*x^(1/3)]))/(2*a^4*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.51, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\int \frac{1}{\frac{b^2}{\sqrt[3]{x}}+ab}dx}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{ 774}$$

$$\frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{b\left(a+\frac{b}{\sqrt[3]{x}}\right)}d\sqrt[3]{x}}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{a+\frac{b}{\sqrt[3]{x}}}d\sqrt[3]{x}}{b\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{ 795}$$

$$\frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\int \frac{x}{\sqrt[3]{x}a+b}d\sqrt[3]{x}}{b\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\int \left(-\frac{b^3}{a^3\left(\sqrt[3]{x}a+b\right)}+\frac{b^2}{a^3}-\frac{\sqrt[3]{x}b}{a^2}+\frac{x^{2/3}}{a}\right)d\sqrt[3]{x}}{b\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\left(-\frac{b^3\log\left(a\sqrt[3]{x}+b\right)}{a^4}+\frac{b^2\sqrt[3]{x}}{a^3}-\frac{bx^{2/3}}{2a^2}+\frac{x}{3a}\right)}{b\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

input
```
Int[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

output
```
(3*(a*b + b^2/x^(1/3))*((b^2*x^(1/3))/a^3 - (b*x^(2/3))/(2*a^2) + x/(3*a)
- (b^3*Log[b + a*x^(1/3)])/a^4))/(b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

method	result	size
derivativedivides	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(-2a^{3}x+3x^{\frac{2}{3}}a^{2}b+6b^{3}\ln\left(b+a\,x^{\frac{1}{3}}\right)-6b^{2}a\,x^{\frac{1}{3}}\right)}{2\sqrt{\frac{x^{\frac{2}{3}}a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\,x^{\frac{1}{3}}a^{4}}$	78
default	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(-2a^{3}x+3x^{\frac{2}{3}}a^{2}b+6b^{3}\ln\left(b+a\,x^{\frac{1}{3}}\right)-6b^{2}a\,x^{\frac{1}{3}}\right)}{2\sqrt{\frac{x^{\frac{2}{3}}a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}}\,x^{\frac{1}{3}}a^{4}}$	78

input
```
int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b+a*x^(1/3))*(-2*a^3*x+3*x^(2/3)*a^2*b+6*b^3*ln(b+a*x^(1/3))-6*b^2*a
*x^(1/3))/((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)/x^(1/3)/a^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \, dx = \int \frac{1}{\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}}} \, dx$$

input
```
integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

output
```
Integral(1/sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \, dx = -\frac{3\,b^3 \log\left(ax^{\frac{1}{3}} + b\right)}{a^4} + \frac{2\,a^2x - 3\,abx^{\frac{2}{3}} + 6\,b^2x^{\frac{1}{3}}}{2\,a^3}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
```

output
```
-3*b^3*log(a*x^(1/3) + b)/a^4 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/
3))/a^3
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}}} \, dx = -\frac{3\,b^3 \log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^4 \mathrm{sgn}\left(ax + bx^{\frac{2}{3}}\right) \mathrm{sgn}\left(x\right)} + \frac{2\,a^2x - 3\,abx^{\frac{2}{3}} + 6\,b^2x^{\frac{1}{3}}}{2\,a^3 \mathrm{sgn}\left(ax + bx^{\frac{2}{3}}\right) \mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
```

output
```
-3*b^3*log(abs(a*x^(1/3) + b))/(a^4*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*
a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3))/(a^3*sgn(a*x + b*x^(2/3))*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}}}\,dx$$

input
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
```

output
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx=\frac{-3x^{\frac{2}{3}}a^2b+6x^{\frac{1}{3}}a\,b^2-6\log\left(x^{\frac{1}{3}}a+b\right)b^3+2a^3x}{2a^4}$$

input
```
int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x)
```

output
```
( - 3*x**(2/3)*a**2*b + 6*x**(1/3)*a*b**2 - 6*log(x**(1/3)*a + b)*b**3 + 2
*a**3*x)/(2*a**4)
```

3.88 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 300

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = \frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2} - \frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)} + \frac{18b^2\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{9b\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}{2a^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} + \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{30b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

output
```
3/2*b^5*(a+b/x^(1/3))/a^6/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(b+a*x^(1/
3))^2-15*b^4*(a+b/x^(1/3))/a^6/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(b+a*
x^(1/3))+18*b^2*(a+b/x^(1/3))*x^(1/3)/a^5/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^
(1/2)-9/2*b*(a+b/x^(1/3))*x^(2/3)/a^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2
)+(a+b/x^(1/3))*x/a^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-30*b^3*(a+b/x^
(1/3))*ln(b+a*x^(1/3))/a^6/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = \frac{\left(b+a\sqrt[3]{x}\right)\left(-27b^5+6ab^4\sqrt[3]{x}+63a^2b^3x^{2/3}+20a^3b^2x-5a^4bx^{4/3}+2a^5x^{5/3}\right.}{2a^6\left(\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}\right)^{3/2}x}$$

input
```
Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

output
```
((b + a*x^(1/3))*(-27*b^5 + 6*a*b^4*x^(1/3) + 63*a^2*b^3*x^(2/3) + 20*a^3*
b^2*x - 5*a^4*b*x^(4/3) + 2*a^5*x^(5/3) - 60*b^3*(b + a*x^(1/3))^2*Log[b +
 a*x^(1/3)]))/(2*a^6*((b + a*x^(1/3))^2/x^(2/3))^(3/2)*x)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.46, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\int \frac{1}{\left(\frac{b^2}{\sqrt[3]{x}}+ab\right)^3}dx}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}d\sqrt[3]{x}}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}d\sqrt[3]{x}}{b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\int \frac{x^{5/3}}{\left(\sqrt[3]{x}a+b\right)^3}d\sqrt[3]{x}}{b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\int \left(-\frac{b^5}{a^5\left(\sqrt[3]{x}a+b\right)^3}+\frac{5b^4}{a^5\left(\sqrt[3]{x}a+b\right)^2}-\frac{10b^3}{a^5\left(\sqrt[3]{x}a+b\right)}+\frac{6b^2}{a^5}-\frac{3\sqrt[3]{x}b}{a^4}+\frac{x^{2/3}}{a^3}\right)d\sqrt[3]{x}}{b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(ab^3+\frac{b^4}{\sqrt[3]{x}}\right)\left(\frac{b^5}{2a^6\left(a\sqrt[3]{x}+b\right)^2}-\frac{5b^4}{a^6\left(a\sqrt[3]{x}+b\right)}-\frac{10b^3\log\left(a\sqrt[3]{x}+b\right)}{a^6}+\frac{6b^2\sqrt[3]{x}}{a^5}-\frac{3bx^{2/3}}{2a^4}+\frac{x}{3a^3}\right)}{b^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

input
```
Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

output
```
(3*(a*b^3 + b^4/x^(1/3))*(b^5/(2*a^6*(b + a*x^(1/3))^2) - (5*b^4)/(a^6*(b
+ a*x^(1/3))) + (6*b^2*x^(1/3))/a^5 - (3*b*x^(2/3))/(2*a^4) + x/(3*a^3) -
(10*b^3*Log[b + a*x^(1/3)])/a^6))/(b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^
(1/3)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.47

method	result
derivativedivides	$-\frac{\left(-2a^{5}x^{\frac{5}{3}}+5a^{4}b\,x^{\frac{4}{3}}+60\ln\left(b+a\,x^{\frac{1}{3}}\right)a^{2}b^{3}x^{\frac{2}{3}}-20a^{3}b^{2}x+120\ln\left(b+a\,x^{\frac{1}{3}}\right)a\,b^{4}x^{\frac{1}{3}}-63b^{3}x^{\frac{2}{3}}a^{2}+60\ln\left(b+a\,x^{\frac{1}{3}}\right)b^{5}-6a \ldots}{2a^{6}x\left(\frac{x^{\frac{2}{3}}a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$
default	$\frac{\left(2a^{5}x^{\frac{5}{3}}-5a^{4}b\,x^{\frac{4}{3}}-60\ln\left(b+a\,x^{\frac{1}{3}}\right)a^{2}b^{3}x^{\frac{2}{3}}+63b^{3}x^{\frac{2}{3}}a^{2}-120\ln\left(b+a\,x^{\frac{1}{3}}\right)a\,b^{4}x^{\frac{1}{3}}+6a\,b^{4}x^{\frac{1}{3}}-60\ln\left(b+a\,x^{\frac{1}{3}}\right)b^{5}+20a^{3}b^{2} \ldots}{2a^{6}x\left(\frac{x^{\frac{2}{3}}a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$

input
```
int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*a^5*x^(5/3)+5*a^4*b*x^(4/3)+60*ln(b+a*x^(1/3))*a^2*b^3*x^(2/3)-20
*a^3*b^2*x+120*ln(b+a*x^(1/3))*a*b^4*x^(1/3)-63*b^3*x^(2/3)*a^2+60*ln(b+a*
x^(1/3))*b^5-6*a*b^4*x^(1/3)+27*b^5)*(b+a*x^(1/3))/a^6/x/((x^(2/3)*a^2+2*a
*b*x^(1/3)+b^2)/x^(2/3))^(3/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = \int \frac{1}{\left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.32

$$\int \frac{1}{\left(a^2 + \frac{b^2}{x^{2/3}} + \frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = \frac{2\,a^5x^{\frac{5}{3}} - 5\,a^4bx^{\frac{4}{3}} + 20\,a^3b^2x + 63\,a^2b^3x^{\frac{2}{3}} + 6\,ab^4x^{\frac{1}{3}} - 27\,b^5}{2\left(a^8x^{\frac{2}{3}} + 2\,a^7bx^{\frac{1}{3}} + a^6b^2\right)} - \frac{30\,b^3\log\left(ax^{\frac{1}{3}} + b\right)}{a^6}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*a^5*x^(5/3) - 5*a^4*b*x^(4/3) + 20*a^3*b^2*x + 63*a^2*b^3*x^(2/3) +
 6*a*b^4*x^(1/3) - 27*b^5)/(a^8*x^(2/3) + 2*a^7*b*x^(1/3) + a^6*b^2) - 30*
b^3*log(a*x^(1/3) + b)/a^6
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = -\frac{30\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^6\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} - \frac{3\left(10\,ab^4x^{\frac{1}{3}}+9\,b^5\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^6\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} + \frac{2\,a^6x-9\,a^5bx^{\frac{2}{3}}+36\,a^4b^2x^{\frac{1}{3}}}{2\,a^9\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

output
```
-30*b^3*log(abs(a*x^(1/3) + b))/(a^6*sgn(a*x + b*x^(2/3))*sgn(x)) - 3/2*(1
0*a*b^4*x^(1/3) + 9*b^5)/((a*x^(1/3) + b)^2*a^6*sgn(a*x + b*x^(2/3))*sgn(x
)) + 1/2*(2*a^6*x - 9*a^5*b*x^(2/3) + 36*a^4*b^2*x^(1/3))/(a^9*sgn(a*x + b
*x^(2/3))*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = \int \frac{1}{{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{3/2}}\,dx$$

input
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
```

output
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx=\frac{-60x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^2b^3+2x^{\frac{5}{3}}a^5+60x^{\frac{2}{3}}a^2b^3-120x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a\,b^4-}{2a^6\left(x^{\frac{2}{3}}a^2+2x^{\frac{1}{3}}ab+b^2\right)}$$

input
```
int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x)
```

output
```
( - 60*x**(2/3)*log(x**(1/3)*a + b)*a**2*b**3 + 2*x**(2/3)*a**5*x + 60*x**
(2/3)*a**2*b**3 - 120*x**(1/3)*log(x**(1/3)*a + b)*a*b**4 - 5*x**(1/3)*a**
4*b*x - 60*log(x**(1/3)*a + b)*b**5 + 20*a**3*b**2*x - 30*b**5)/(2*a**6*(x
**(2/3)*a**2 + 2*x**(1/3)*a*b + b**2))
```

3.89 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 410

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = \frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^4} - \frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^3} + \frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2} - \frac{105b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)} + \frac{45b^2\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{15b\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} + \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{105b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

output
```
3/4*b^7*(a+b/x^(1/3))/a^8/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(b+a*x^(1/
3))^4-7*b^6*(a+b/x^(1/3))/a^8/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(b+a*x
^(1/3))^3+63/2*b^5*(a+b/x^(1/3))/a^8/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
/(b+a*x^(1/3))^2-105*b^4*(a+b/x^(1/3))/a^8/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))
^(1/2)/(b+a*x^(1/3))+45*b^2*(a+b/x^(1/3))*x^(1/3)/a^7/(a^2+b^2/x^(2/3)+2*a
*b/x^(1/3))^(1/2)-15/2*b*(a+b/x^(1/3))*x^(2/3)/a^6/(a^2+b^2/x^(2/3)+2*a*b/
x^(1/3))^(1/2)+(a+b/x^(1/3))*x/a^5/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-1
05*b^3*(a+b/x^(1/3))*ln(b+a*x^(1/3))/a^8/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(
1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.37

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\frac{\left(b+a\sqrt[3]{x}\right)\left(-319b^7-856ab^6\sqrt[3]{x}-444a^2b^5x^{2/3}+544a^3b^4x+556a^4b^3x^{4/3}\right.}{4a^8\left(\frac{\left(b+a\sqrt[3]{x}\right)}{x^{2/3}}\right.}$$

input
```
Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

output
```
((b + a*x^(1/3))*(-319*b^7 - 856*a*b^6*x^(1/3) - 444*a^2*b^5*x^(2/3) + 544
*a^3*b^4*x + 556*a^4*b^3*x^(4/3) + 84*a^5*b^2*x^(5/3) - 14*a^6*b*x^2 + 4*a
^7*x^(7/3) - 420*b^3*(b + a*x^(1/3))^4*Log[b + a*x^(1/3)]))/(4*a^8*((b + a
*x^(1/3))^2/x^(2/3))^(5/2)*x^(5/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.44, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\int \frac{1}{\left(\frac{b^2}{\sqrt[3]{x}}+ab\right)^5}dx}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)^5}d\sqrt[3]{x}}{\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\int \frac{x^{2/3}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^5}d\sqrt[3]{x}}{b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\int \frac{x^{7/3}}{\left(\sqrt[3]{x}a+b\right)^5}d\sqrt[3]{x}}{b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\int \left(-\frac{b^7}{a^7\left(\sqrt[3]{x}a+b\right)^5}+\frac{7b^6}{a^7\left(\sqrt[3]{x}a+b\right)^4}-\frac{21b^5}{a^7\left(\sqrt[3]{x}a+b\right)^3}+\frac{35b^4}{a^7\left(\sqrt[3]{x}a+b\right)^2}-\frac{35b^3}{a^7\left(\sqrt[3]{x}a+b\right)}+\frac{15b^2}{a^7}-\frac{5\sqrt[3]{x}b}{a^6}+\cdots}{b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)\left(\frac{b^7}{4a^8\left(a\sqrt[3]{x}+b\right)^4}-\frac{7b^6}{3a^8\left(a\sqrt[3]{x}+b\right)^3}+\frac{21b^5}{2a^8\left(a\sqrt[3]{x}+b\right)^2}-\frac{35b^4}{a^8\left(a\sqrt[3]{x}+b\right)}-\frac{35b^3\log\left(a\sqrt[3]{x}+b\right)}{a^8}+\frac{15b^2\sqrt[3]{x}}{a^7}-\frac{5bx^{2}\cdots}{2a\cdots}\cdots}{b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

input
```
Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

output
```
(3*(a*b^5 + b^6/x^(1/3))*(b^7/(4*a^8*(b + a*x^(1/3))^4) - (7*b^6)/(3*a^8*(
b + a*x^(1/3))^3) + (21*b^5)/(2*a^8*(b + a*x^(1/3))^2) - (35*b^4)/(a^8*(b
+ a*x^(1/3))) + (15*b^2*x^(1/3))/a^7 - (5*b*x^(2/3))/(2*a^6) + x/(3*a^5) -
 (35*b^3*Log[b + a*x^(1/3)])/a^8))/(b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x
^(1/3)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.49

method	result
derivativedivides	$-\frac{\left(-4a^7x^{\frac{7}{3}}+14a^6bx^2+420\ln\left(b+ax^{\frac{1}{3}}\right)a^4b^3x^{\frac{4}{3}}-84a^5b^2x^{\frac{5}{3}}+1680\ln\left(b+ax^{\frac{1}{3}}\right)a^3b^4x-556a^4b^3x^{\frac{4}{3}}+2520\ln\left(b+ax^{\frac{1}{3}}\right.\right.}{4a^8x^{\frac{5}{3}}\left(\frac{x^{\frac{2}{3}}a^2+2}{}\right.}$
default	$\frac{\left(4a^7x^{\frac{7}{3}}+84a^5b^2x^{\frac{5}{3}}-420\ln\left(b+ax^{\frac{1}{3}}\right)a^4b^3x^{\frac{4}{3}}+556a^4b^3x^{\frac{4}{3}}-2520\ln\left(b+ax^{\frac{1}{3}}\right)a^2b^5x^{\frac{2}{3}}-444a^2b^5x^{\frac{2}{3}}-1680\ln\left(b+ax^{\frac{1}{3}}\right.\right.}{4a^8x^{\frac{5}{3}}\left(\frac{x^{\frac{2}{3}}a^2+2ab}{x^{\frac{2}{3}}}\right.}$

input
```
int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*a^7*x^(7/3)+14*a^6*b*x^2+420*ln(b+a*x^(1/3))*a^4*b^3*x^(4/3)-84*a
^5*b^2*x^(5/3)+1680*ln(b+a*x^(1/3))*a^3*b^4*x-556*a^4*b^3*x^(4/3)+2520*ln(
b+a*x^(1/3))*a^2*b^5*x^(2/3)-544*a^3*b^4*x+1680*ln(b+a*x^(1/3))*a*b^6*x^(1
/3)+444*a^2*b^5*x^(2/3)+420*ln(b+a*x^(1/3))*b^7+856*a*b^6*x^(1/3)+319*b^7)
*(b+a*x^(1/3))/a^8/x^(5/3)/((x^(2/3)*a^2+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\int \frac{1}{\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.34

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx=\frac{4\,a^7x^{\frac{7}{3}}-14\,a^6bx^2+84\,a^5b^2x^{\frac{5}{3}}+556\,a^4b^3x^{\frac{4}{3}}+544\,a^3b^4x-444\,a^2b^5x^{\frac{2}{3}}-85}{4\left(a^{12}x^{\frac{4}{3}}+4\,a^{11}bx+6\,a^{10}b^2x^{\frac{2}{3}}+4\,a^9b^3x^{\frac{1}{3}}+a^8b^4\right)}-\frac{105\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^8}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

output
```
1/4*(4*a^7*x^(7/3) - 14*a^6*b*x^2 + 84*a^5*b^2*x^(5/3) + 556*a^4*b^3*x^(4/
3) + 544*a^3*b^4*x - 444*a^2*b^5*x^(2/3) - 856*a*b^6*x^(1/3) - 319*b^7)/(a
^12*x^(4/3) + 4*a^11*b*x + 6*a^10*b^2*x^(2/3) + 4*a^9*b^3*x^(1/3) + a^8*b^
4) - 105*b^3*log(a*x^(1/3) + b)/a^8
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.36

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = -\frac{105\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^8\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} - \frac{420\,a^3b^4x+1134\,a^2b^5x^{\frac{2}{3}}+1036\,ab^6x^{\frac{1}{3}}+319\,b^7}{4\left(ax^{\frac{1}{3}}+b\right)^4a^8\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} + \frac{2\,a^{10}x-15\,a^9bx^{\frac{2}{3}}+90\,a^8b^2x^{\frac{1}{3}}}{2\,a^{15}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

output
```
-105*b^3*log(abs(a*x^(1/3) + b))/(a^8*sgn(a*x + b*x^(2/3))*sgn(x)) - 1/4*(
420*a^3*b^4*x + 1134*a^2*b^5*x^(2/3) + 1036*a*b^6*x^(1/3) + 319*b^7)/((a*x
^(1/3) + b)^4*a^8*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*a^10*x - 15*a^9*b*
x^(2/3) + 90*a^8*b^2*x^(1/3))/(a^15*sgn(a*x + b*x^(2/3))*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = \int \frac{1}{{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{5/2}}\,dx$$

input
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
```

output
```
int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.48

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = \frac{-2520x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^2b^5+84x^{\frac{5}{3}}a^5b^2-1260x^{\frac{2}{3}}a^2b^5-420x^{\frac{4}{3}}\log\left(x^{\frac{1}{3}}a+\right.}{}$$

input

```
int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x)
```

output

```
( - 2520*x**(2/3)*log(x**(1/3)*a + b)*a**2*b**5 + 84*x**(2/3)*a**5*b**2*x
- 1260*x**(2/3)*a**2*b**5 - 420*x**(1/3)*log(x**(1/3)*a + b)*a**4*b**3*x -
 1680*x**(1/3)*log(x**(1/3)*a + b)*a*b**6 + 4*x**(1/3)*a**7*x**2 + 420*x**
(1/3)*a**4*b**3*x - 1400*x**(1/3)*a*b**6 - 1680*log(x**(1/3)*a + b)*a**3*b
**4*x - 420*log(x**(1/3)*a + b)*b**7 - 14*a**6*b*x**2 - 455*b**7)/(4*a**8*
(6*x**(2/3)*a**2*b**2 + x**(1/3)*a**4*x + 4*x**(1/3)*a*b**3 + 4*a**3*b*x +
 b**4))
```

3.90 $\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 285

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx=-\frac{4b^5\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}}{\left(a+\frac{b}{\sqrt[4]{x}}\right)\sqrt[4]{x}}+\frac{40a^2b^3\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt[4]{x}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^3b^2\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt{x}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^4b\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}x^{3/4}}{3\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{a^5\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}x}{a+\frac{b}{\sqrt[4]{x}}}+\frac{5ab^4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\log(x)}{a+\frac{b}{\sqrt[4]{x}}}$$

output
```
-4*b^5*(a^2+b^2/x^(1/2)+2*a*b/x^(1/4))^(1/2)/(a+b/x^(1/4))/x^(1/4)+40*a^2*
b^3*(a^2+b^2/x^(1/2)+2*a*b/x^(1/4))^(1/2)*x^(1/4)/(a+b/x^(1/4))+20*a^3*b^2
*(a^2+b^2/x^(1/2)+2*a*b/x^(1/4))^(1/2)*x^(1/2)/(a+b/x^(1/4))+20*a^4*b*(a^2
+b^2/x^(1/2)+2*a*b/x^(1/4))^(1/2)*x^(3/4)/(3*a+3*b/x^(1/4))+a^5*(a^2+b^2/x
^(1/2)+2*a*b/x^(1/4))^(1/2)*x/(a+b/x^(1/4))+5*a*b^4*(a^2+b^2/x^(1/2)+2*a*b
/x^(1/4))^(1/2)*ln(x)/(a+b/x^(1/4))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.34

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = \frac{\sqrt{\frac{\left(b+a\sqrt[4]{x}\right)^2}{\sqrt{x}}}\left(-12b^5+120a^2b^3\sqrt{x}+60a^3b^2x^{3/4}+20a^4bx+3a^5x^{5/4}+15ab^4\sqrt[4]{x}\log(x)\right)}{3\left(b+a\sqrt[4]{x}\right)}$$

input
```
Integrate[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

output
```
(Sqrt[(b + a*x^(1/4))^2/Sqrt[x]]*(-12*b^5 + 120*a^2*b^3*Sqrt[x] + 60*a^3*b
^2*x^(3/4) + 20*a^4*b*x + 3*a^5*x^(5/4) + 15*a*b^4*x^(1/4)*Log[x]))/(3*(b
+ a*x^(1/4)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.41, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}\right)^{5/2} dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\int \left(\frac{b^2}{\sqrt[4]{x}}+ab\right)^5 dx}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

$$\Big\downarrow \ 774$$

$$\frac{4\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\int b^5\left(a+\frac{b}{\sqrt[4]{x}}\right)^5 x^{3/4}d\sqrt[4]{x}}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

$$\Big\downarrow \; 27$$

$$\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\int \left(a+\frac{b}{\sqrt[4]{x}}\right)^5 x^{3/4}d\sqrt[4]{x}}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

$$\Big\downarrow \; 795$$

$$\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\int \frac{\left(\sqrt[4]{x}a+b\right)^5}{\sqrt{x}}d\sqrt[4]{x}}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

$$\Big\downarrow \; 49$$

$$\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\int \left(x^{3/4}a^5+5b\sqrt{x}a^4+10b^2\sqrt[4]{x}a^3+10b^3a^2+\frac{5b^4a}{\sqrt[4]{x}}+\frac{b^5}{\sqrt{x}}\right)d\sqrt[4]{x}}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

$$\Big\downarrow \; 2009$$

$$\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}\left(\frac{a^5x}{4}+\frac{5}{3}a^4bx^{3/4}+5a^3b^2\sqrt{x}+10a^2b^3\sqrt[4]{x}+5ab^4\log\left(\sqrt[4]{x}\right)-\frac{b^5}{\sqrt[4]{x}}\right)}{ab^5+\frac{b^6}{\sqrt[4]{x}}}$$

input
```
Int[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

output
```
(4*b^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*(-(b^5/x^(1/4)) + 10*a^2*
b^3*x^(1/4) + 5*a^3*b^2*Sqrt[x] + (5*a^4*b*x^(3/4))/3 + (a^5*x)/4 + 5*a*b^
4*Log[x^(1/4)]))/(a*b^5 + b^6/x^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.32

method	result	size
derivativedivides	$\frac{\left(\frac{a^{2}\sqrt{x}+2ab\,x^{\frac{1}{4}}+b^{2}}{\sqrt{x}}\right)^{\frac{5}{2}} x\left(3a^{5}x^{\frac{5}{4}}+20a^{4}bx+60a^{3}b^{2}x^{\frac{3}{4}}+15a\,b^{4}\ln\left(x\right)x^{\frac{1}{4}}+120a^{2}b^{3}\sqrt{x}-12b^{5}\right)}{3\left(a\,x^{\frac{1}{4}}+b\right)^{5}}$	91
default	$\frac{\left(\frac{a^{2}x^{\frac{3}{4}}+2ab\sqrt{x}+b^{2}x^{\frac{1}{4}}}{x^{\frac{3}{4}}}\right)^{\frac{5}{2}} x\left(3a^{5}x^{\frac{5}{4}}+20a^{4}bx+60a^{3}b^{2}x^{\frac{3}{4}}+15a\,b^{4}\ln\left(x\right)x^{\frac{1}{4}}+120a^{2}b^{3}\sqrt{x}-12b^{5}\right)}{3\left(a\,x^{\frac{1}{4}}+b\right)^{5}}$	95

input
```
int((a^2+b^2/x^(1/2)+2*a*b/x^(1/4))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((a^2*x^(1/2)+2*a*b*x^(1/4)+b^2)/x^(1/2))^(5/2)*x*(3*a^5*x^(5/4)+20*a^
4*b*x+60*a^3*b^2*x^(3/4)+15*a*b^4*ln(x)*x^(1/4)+120*a^2*b^3*x^(1/2)-12*b^5
)/(a*x^(1/4)+b)^5
```

Fricas [F(-1)]

Timed out.

$$\int\left(a^{2}+\frac{b^{2}}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a^2+b^2/x^(1/2)+2*a*b/x^(1/4))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((a**2+b**2/x**(1/2)+2*a*b/x**(1/4))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = 5\,ab^4\log\left(x\right) + \frac{3\,a^5x^{\frac{5}{4}}+20\,a^4bx+60\,a^3b^2x^{\frac{3}{4}}+120\,a^2b^3\sqrt{x}-12\,b^5}{3\,x^{\frac{1}{4}}}$$

input
```
integrate((a^2+b^2/x^(1/2)+2*a*b/x^(1/4))^(5/2),x, algorithm="maxima")
```

output
```
5*a*b^4*log(x) + 1/3*(3*a^5*x^(5/4) + 20*a^4*b*x + 60*a^3*b^2*x^(3/4) + 12
0*a^2*b^3*sqrt(x) - 12*b^5)/x^(1/4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.44

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = a^5x\,\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x) + 5\,ab^4\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x) + \frac{20}{3}\,a^4bx^{\frac{3}{4}}\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x) + 20\,a^3b^2\sqrt{x}\,\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x) + 40\,a^2b^3x^{\frac{1}{4}}\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x) - \frac{4\,b^5\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)}{x^{\frac{1}{4}}}$$

input
```
integrate((a^2+b^2/x^(1/2)+2*a*b/x^(1/4))^(5/2),x, algorithm="giac")
```

output
```
a^5*x*sgn(a*x + b*x^(3/4))*sgn(x) + 5*a*b^4*log(abs(x))*sgn(a*x + b*x^(3/4
))*sgn(x) + 20/3*a^4*b*x^(3/4)*sgn(a*x + b*x^(3/4))*sgn(x) + 20*a^3*b^2*sq
rt(x)*sgn(a*x + b*x^(3/4))*sgn(x) + 40*a^2*b^3*x^(1/4)*sgn(a*x + b*x^(3/4)
)*sgn(x) - 4*b^5*sgn(a*x + b*x^(3/4))*sgn(x)/x^(1/4)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = \int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2\,a\,b}{x^{1/4}}\right)^{5/2} dx$$

input
```
int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2),x)
```

output
```
int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.21

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = \frac{60x^{\frac{3}{4}}a^3b^2+120x^{\frac{1}{4}}\log\left(x^{\frac{1}{8}}\right)a\,b^4+3x^{\frac{5}{4}}a^5+120\sqrt{x}\,a^2b^3+20a^4bx-12b^5}{3x^{\frac{1}{4}}}$$

input
```
int((a^2+b^2/x^(1/2)+2*a*b/x^(1/4))^(5/2),x)
```

output
```
(60*x**(3/4)*a**3*b**2 + 120*x**(1/4)*log(x**(1/8))*a*b**4 + 3*x**(1/4)*a*
*5*x + 120*sqrt(x)*a**2*b**3 + 20*a**4*b*x - 12*b**5)/(3*x**(1/4))
```

3.91 $\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 286

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \frac{25ab^4\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\sqrt[5]{x}}{a+\frac{b}{\sqrt[5]{x}}} + \frac{25a^2b^3\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{2/5}}{a+\frac{b}{\sqrt[5]{x}}} + \frac{50a^3b^2\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{3/5}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)} + \frac{25a^4b\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{4/5}}{4\left(a+\frac{b}{\sqrt[5]{x}}\right)} + \frac{a^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x}{a+\frac{b}{\sqrt[5]{x}}} + \frac{b^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\log(x)}{a+\frac{b}{\sqrt[5]{x}}}$$

output
```
25*a*b^4*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)*x^(1/5)/(a+b/x^(1/5))+25*a^
2*b^3*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)*x^(2/5)/(a+b/x^(1/5))+50*a^3*b
^2*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)*x^(3/5)/(3*a+3*b/x^(1/5))+25*a^4*
b*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)*x^(4/5)/(4*a+4*b/x^(1/5))+a^5*(a^2
+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)*x/(a+b/x^(1/5))+b^5*(a^2+b^2/x^(2/5)+2*a
*b/x^(1/5))^(1/2)*ln(x)/(a+b/x^(1/5))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.35

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \frac{\left(b+a\sqrt[5]{x}\right)\left(300ab^4\sqrt[5]{x}+300a^2b^3x^{2/5}+200a^3b^2x^{3/5}+75a^4bx^{4/5}+12a^5x+12b^5\log(x)\right)}{12\sqrt{\frac{\left(b+a\sqrt[5]{x}\right)^2}{x^{2/5}}}\sqrt[5]{x}}$$

input
```
Integrate[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

output
```
((b + a*x^(1/5))*(300*a*b^4*x^(1/5) + 300*a^2*b^3*x^(2/5) + 200*a^3*b^2*x^
(3/5) + 75*a^4*b*x^(4/5) + 12*a^5*x + 12*b^5*Log[x]))/(12*Sqrt[(b + a*x^(1
/5))^2/x^(2/5)]*x^(1/5))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.42, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}\right)^{5/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\int \left(\frac{b^2}{\sqrt[5]{x}}+ab\right)^5dx}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{5\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\int b^5\left(a+\frac{b}{\sqrt[5]{x}}\right)^5x^{4/5}d\sqrt[5]{x}}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5b^5\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\int \left(a+\frac{b}{\sqrt[5]{x}}\right)^5x^{4/5}d\sqrt[5]{x}}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

$$\Big\downarrow \text{795}$$

$$\frac{5b^5\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\int \frac{\left(\sqrt[5]{x}a+b\right)^5}{\sqrt[5]{x}}d\sqrt[5]{x}}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{5b^5\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\int \left(x^{4/5}a^5+5bx^{3/5}a^4+10b^2x^{2/5}a^3+10b^3\sqrt[5]{x}a^2+5b^4a+\frac{b^5}{\sqrt[5]{x}}\right)d\sqrt[5]{x}}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5b^5\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}\left(\frac{a^5x}{5}+\frac{5}{4}a^4bx^{4/5}+\frac{10}{3}a^3b^2x^{3/5}+5a^2b^3x^{2/5}+5ab^4\sqrt[5]{x}+b^5\log\left(\sqrt[5]{x}\right)\right)}{ab^5+\frac{b^6}{\sqrt[5]{x}}}$$

input
```
Int[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

output
```
(5*b^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*(5*a*b^4*x^(1/5) + 5*a^2*
b^3*x^(2/5) + (10*a^3*b^2*x^(3/5))/3 + (5*a^4*b*x^(4/5))/4 + (a^5*x)/5 + b
^5*Log[x^(1/5)]))/(a*b^5 + b^6/x^(1/5))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.32

method	result	size
derivativedivides	$\frac{\left(\frac{a^{2} x^{\frac{2}{5}}+2 a b \,x^{\frac{1}{5}}+b^{2}}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}} x \left(12 x \,a^{5}+75 a^{4} b \,x^{\frac{4}{5}}+200 a^{3} b^{2} x^{\frac{3}{5}}+300 a^{2} b^{3} x^{\frac{2}{5}}+12 b^{5} \ln \left(x \right)+300 a \,b^{4} x^{\frac{1}{5}}\right)}{12 \left(a \,x^{\frac{1}{5}}+b \right)^{5}}$	91
default	$\frac{\left(\frac{a^{2} x^{\frac{2}{5}}+2 a b \,x^{\frac{1}{5}}+b^{2}}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}} x \left(12 x \,a^{5}+75 a^{4} b \,x^{\frac{4}{5}}+200 a^{3} b^{2} x^{\frac{3}{5}}+300 a^{2} b^{3} x^{\frac{2}{5}}+12 b^{5} \ln \left(x \right)+300 a \,b^{4} x^{\frac{1}{5}}\right)}{12 \left(a \,x^{\frac{1}{5}}+b \right)^{5}}$	91

input
```
int((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*((a^2*x^(2/5)+2*a*b*x^(1/5)+b^2)/x^(2/5))^(5/2)*x*(12*x*a^5+75*a^4*b*
x^(4/5)+200*a^3*b^2*x^(3/5)+300*a^2*b^3*x^(2/5)+12*b^5*ln(x)+300*a*b^4*x^(
1/5))/(a*x^(1/5)+b)^5
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2}dx=\int \left(a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}}dx$$

input
```
integrate((a**2+b**2/x**(2/5)+2*a*b/x**(1/5))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.18

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2}dx=a^5x+b^5\log(x)+\frac{25}{4}a^4bx^{\frac{4}{5}}+\frac{50}{3}a^3b^2x^{\frac{3}{5}}+25\,a^2b^3x^{\frac{2}{5}}+25\,ab^4x^{\frac{1}{5}}$$

input
```
integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="maxima")
```

output
```
a^5*x + b^5*log(x) + 25/4*a^4*b*x^(4/5) + 50/3*a^3*b^2*x^(3/5) + 25*a^2*b^
3*x^(2/5) + 25*a*b^4*x^(1/5)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.44

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2}dx=a^5x\mathrm{sgn}\left(ax+bx^{\frac{4}{5}}\right)\mathrm{sgn}(x)+b^5\log(|x|)\,\mathrm{sgn}\left(ax+bx^{\frac{4}{5}}\right)\mathrm{sgn}(x)+\frac{25}{4}a^4bx^{\frac{4}{5}}\mathrm{sgn}\left(ax+bx^{\frac{4}{5}}\right)\mathrm{sgn}(x)+\frac{50}{3}a^3b^2x^{\frac{3}{5}}\mathrm{sgn}\left(ax+bx^{\frac{4}{5}}\right)\mathrm{sgn}(x)+25\,a^2b^3x^{\frac{2}{5}}\mathrm{sgn}\left(ax+bx^{\frac{4}{5}}\right)\mathrm{sgn}(x)+25\,ab^4x^{\frac{1}{5}}\mathrm{sgn}\left(ax+bx^{\frac{4}{5}}\right)\mathrm{sgn}(x)$$

input
```
integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="giac")
```

output
```
a^5*x*sgn(a*x + b*x^(4/5))*sgn(x) + b^5*log(abs(x))*sgn(a*x + b*x^(4/5))*s
gn(x) + 25/4*a^4*b*x^(4/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 50/3*a^3*b^2*x^(3
/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 25*a^2*b^3*x^(2/5)*sgn(a*x + b*x^(4/5))*
sgn(x) + 25*a*b^4*x^(1/5)*sgn(a*x + b*x^(4/5))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2\,a\,b}{x^{1/5}}\right)^{5/2} dx$$

input
```
int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x)
```

output
```
int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.18

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = \frac{25x^{\frac{4}{5}}a^4b}{4}+\frac{50x^{\frac{3}{5}}a^3b^2}{3} + 25x^{\frac{2}{5}}a^2b^3+25x^{\frac{1}{5}}a\,b^4+\log(x)\,b^5+a^5x$$

input
```
int((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x)
```

output
```
(75*x**(4/5)*a**4*b + 200*x**(3/5)*a**3*b**2 + 300*x**(2/5)*a**2*b**3 + 30
0*x**(1/5)*a*b**4 + 12*log(x)*b**5 + 12*a**5*x)/12
```

3.92 $\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 222

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} - \frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} + \frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} - \frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} + \frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

output
```
20*a/b^5/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)-5/4*a^4/b^5/(a+b*x^(1/5))^3
/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)+20/3*a^3/b^5/(a+b*x^(1/5))^2/(a^2+2
*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)-15*a^2/b^5/(a+b*x^(1/5))/(a^2+2*a*b*x^(1/5
)+b^2*x^(2/5))^(1/2)+5*(a+b*x^(1/5))*ln(a+b*x^(1/5))/b^5/(a^2+2*a*b*x^(1/5
)+b^2*x^(2/5))^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx=\frac{5a\left(25a^3+88a^2b\sqrt[5]{x}+108ab^2x^{2/5}+48b^3x^{3/5}\right)+60\left(a+b\sqrt[5]{x}\right)^4\log\left(a+\right.}{12b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{\left(a+b\sqrt[5]{x}\right)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

output
```
(5*a*(25*a^3 + 88*a^2*b*x^(1/5) + 108*a*b^2*x^(2/5) + 48*b^3*x^(3/5)) + 60
*(a + b*x^(1/5))^4*Log[a + b*x^(1/5)])/(12*b^5*(a + b*x^(1/5))^3*Sqrt[(a +
 b*x^(1/5))^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 774, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\left(ab^5+b^6\sqrt[5]{x}\right)\int \frac{1}{\left(\sqrt[5]{x}b^2+ab\right)^5}dx}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$\Big\downarrow \text{774}$$

$$\frac{5\left(ab^5+b^6\sqrt[5]{x}\right)\int \frac{x^{4/5}}{b^5\left(a+b\sqrt[5]{x}\right)^5}d\sqrt[5]{x}}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(ab^5+b^6\sqrt[5]{x}\right)\int \frac{x^{4/5}}{\left(a+b\sqrt[5]{x}\right)^5}d\sqrt[5]{x}}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$\Big\downarrow \text{49}$$

$$\frac{5\left(ab^5+b^6\sqrt[5]{x}\right)\int \left(\frac{a^4}{b^4\left(a+b\sqrt[5]{x}\right)^5}-\frac{4a^3}{b^4\left(a+b\sqrt[5]{x}\right)^4}+\frac{6a^2}{b^4\left(a+b\sqrt[5]{x}\right)^3}-\frac{4a}{b^4\left(a+b\sqrt[5]{x}\right)^2}+\frac{1}{b^4\left(a+b\sqrt[5]{x}\right)}\right)d\sqrt[5]{x}}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\left(ab^5+b^6\sqrt[5]{x}\right)\left(-\frac{a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^4}+\frac{4a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^3}-\frac{3a^2}{b^5\left(a+b\sqrt[5]{x}\right)^2}+\frac{4a}{b^5\left(a+b\sqrt[5]{x}\right)}+\frac{\log\left(a+b\sqrt[5]{x}\right)}{b^5}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

input
```
Int[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

output
```
(5*(a*b^5 + b^6*x^(1/5))*(-1/4*a^4/(b^5*(a + b*x^(1/5))^4) + (4*a^3)/(3*b^
5*(a + b*x^(1/5))^3) - (3*a^2)/(b^5*(a + b*x^(1/5))^2) + (4*a)/(b^5*(a + b
*x^(1/5))) + Log[a + b*x^(1/5)]/b^5))/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*
x^(2/5)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{5\left(12\ln\left(a+b\,x^{\frac{1}{5}}\right)b^{4}x^{\frac{4}{5}}+48\ln\left(a+b\,x^{\frac{1}{5}}\right)a\,b^{3}x^{\frac{3}{5}}+72\ln\left(a+b\,x^{\frac{1}{5}}\right)a^{2}b^{2}x^{\frac{2}{5}}+48a\,b^{3}x^{\frac{3}{5}}+48\ln\left(a+b\,x^{\frac{1}{5}}\right)a^{3}b\,x^{\frac{1}{5}}+108a^{2}b^{2}\cdots\right.}{12b^{5}\left(\left(a+b\,x^{\frac{1}{5}}\right)^{2}\right)^{\frac{5}{2}}}$
default	Expression too large to display

input
```
int(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
5/12*(12*ln(a+b*x^(1/5))*b^4*x^(4/5)+48*ln(a+b*x^(1/5))*a*b^3*x^(3/5)+72*l
n(a+b*x^(1/5))*a^2*b^2*x^(2/5)+48*a*b^3*x^(3/5)+48*ln(a+b*x^(1/5))*a^3*b*x
^(1/5)+108*a^2*b^2*x^(2/5)+12*ln(a+b*x^(1/5))*a^4+88*a^3*b*x^(1/5)+25*a^4)
*(a+b*x^(1/5))/b^5/((a+b*x^(1/5))^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \frac{5\left(300\,a^5b^{15}x^3+100\,a^{15}b^5x+25\,a^{20}+12\left(b^{20}x^4+4\,a^5b^{15}x^3+6\,a^{10}b^{10}x\cdots\right.\right.}{\cdots}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="fricas")
```

output
```
5/12*(300*a^5*b^15*x^3 + 100*a^15*b^5*x + 25*a^20 + 12*(b^20*x^4 + 4*a^5*b
^15*x^3 + 6*a^10*b^10*x^2 + 4*a^15*b^5*x + a^20)*log(b*x^(1/5) + a) + (48*
a*b^19*x^3 - 226*a^6*b^14*x^2 + 104*a^11*b^9*x + 3*a^16*b^4)*x^(4/5) - (84
*a^2*b^18*x^3 - 228*a^7*b^13*x^2 + 67*a^12*b^8*x + 4*a^17*b^3)*x^(3/5) + (
136*a^3*b^17*x^3 - 197*a^8*b^12*x^2 + 48*a^13*b^7*x + 6*a^18*b^2)*x^(2/5)
- (207*a^4*b^16*x^3 - 124*a^9*b^11*x^2 + 56*a^14*b^6*x + 12*a^19*b)*x^(1/5
))/(b^25*x^4 + 4*a^5*b^20*x^3 + 6*a^10*b^15*x^2 + 4*a^15*b^10*x + a^20*b^5
)
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx=\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{\frac{2}{5}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*b*x**(1/5)+b**2*x**(2/5))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**(1/5) + b**2*x**(2/5))**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx=\frac{5\left(48\,ab^3x^{\frac{3}{5}}+108\,a^2b^2x^{\frac{2}{5}}+88\,a^3bx^{\frac{1}{5}}+25\,a^4\right)}{12\left(b^9x^{\frac{4}{5}}+4\,ab^8x^{\frac{3}{5}}+6\,a^2b^7x^{\frac{2}{5}}+4\,a^3b^6x^{\frac{1}{5}}+a^4b^5\right)}+\frac{5\,\log\left(bx^{\frac{1}{5}}+a\right)}{b^5}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="maxima")
```

output
```
5/12*(48*a*b^3*x^(3/5) + 108*a^2*b^2*x^(2/5) + 88*a^3*b*x^(1/5) + 25*a^4)/
(b^9*x^(4/5) + 4*a*b^8*x^(3/5) + 6*a^2*b^7*x^(2/5) + 4*a^3*b^6*x^(1/5) + a
^4*b^5) + 5*log(b*x^(1/5) + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \frac{5\log\left(\left|bx^{\frac{1}{5}}+a\right|\right)}{b^5\mathrm{sgn}\left(bx^{\frac{1}{5}}+a\right)} + \frac{5\left(48\,ab^2x^{\frac{3}{5}}+108\,a^2bx^{\frac{2}{5}}+88\,a^3x^{\frac{1}{5}}+\frac{25\,a^4}{b}\right)}{12\left(bx^{\frac{1}{5}}+a\right)^4b^4\mathrm{sgn}\left(bx^{\frac{1}{5}}+a\right)}$$

input
```
integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="giac")
```

output
```
5*log(abs(b*x^(1/5) + a))/(b^5*sgn(b*x^(1/5) + a)) + 5/12*(48*a*b^2*x^(3/5
) + 108*a^2*b*x^(2/5) + 88*a^3*x^(1/5) + 25*a^4/b)/((b*x^(1/5) + a)^4*b^4*
sgn(b*x^(1/5) + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \int \frac{1}{\left(a^2+b^2\,x^{2/5}+2\,a\,b\,x^{1/5}\right)^{5/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2),x)
```

output
```
int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = \frac{5x^{\frac{4}{5}}\log\left(x^{\frac{1}{5}}b+a\right)b^4-5x^{\frac{4}{5}}b^4+20x^{\frac{3}{5}}\log\left(x^{\frac{1}{5}}b+a\right)a\,b^3+30x^{\frac{2}{5}}\log\left(x^{\frac{1}{5}}b\right.}{b^5\left(x^{\frac{4}{5}}b^4+4x^{\frac{3}{5}}a\right.}$$

input
```
int(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x)
```

output
```
(5*(12*x**(4/5)*log(x**(1/5)*b + a)*b**4 - 12*x**(4/5)*b**4 + 48*x**(3/5)*
log(x**(1/5)*b + a)*a*b**3 + 72*x**(2/5)*log(x**(1/5)*b + a)*a**2*b**2 + 3
6*x**(2/5)*a**2*b**2 + 48*x**(1/5)*log(x**(1/5)*b + a)*a**3*b + 40*x**(1/5
)*a**3*b + 12*log(x**(1/5)*b + a)*a**4 + 13*a**4))/(12*b**5*(x**(4/5)*b**4
 + 4*x**(3/5)*a*b**3 + 6*x**(2/5)*a**2*b**2 + 4*x**(1/5)*a**3*b + a**4))
```

3.93 $\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 387

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = -\frac{6b^7\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}}{\left(a+\frac{b}{\sqrt[6]{x}}\right)\sqrt[6]{x}}$$
$$+\frac{126a^2b^5\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[6]{x}}{a+\frac{b}{\sqrt[6]{x}}}$$
$$+\frac{105a^3b^4\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{70a^4b^3\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt{x}}{a+\frac{b}{\sqrt[6]{x}}}$$
$$+\frac{63a^5b^2\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x^{2/3}}{2\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{42a^6b\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x^{5/6}}{5\left(a+\frac{b}{\sqrt[6]{x}}\right)}$$
$$+\frac{a^7\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}x}{a+\frac{b}{\sqrt[6]{x}}}+\frac{7ab^6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\log(x)}{a+\frac{b}{\sqrt[6]{x}}}$$

output
```
-6*b^7*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))/x^(1/6)+126*a^2
*b^5*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)*x^(1/6)/(a+b/x^(1/6))+105*a^3*b
^4*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)*x^(1/3)/(a+b/x^(1/6))+70*a^4*b^3*
(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)*x^(1/2)/(a+b/x^(1/6))+63*a^5*b^2*(a^
2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)*x^(2/3)/(2*a+2*b/x^(1/6))+42*a^6*b*(a^2
+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)*x^(5/6)/(5*a+5*b/x^(1/6))+a^7*(a^2+b^2/x
^(1/3)+2*a*b/x^(1/6))^(1/2)*x/(a+b/x^(1/6))+7*a*b^6*(a^2+b^2/x^(1/3)+2*a*b
/x^(1/6))^(1/2)*ln(x)/(a+b/x^(1/6))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.32

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = \frac{\sqrt{\frac{\left(b+a\sqrt[6]{x}\right)^2}{\sqrt[3]{x}}}\left(-60b^7+1260a^2b^5\sqrt[3]{x}+1050a^3b^4\sqrt{x}+700a^4b^3x^{2/3}+315a^5b^2x^{5/6}+84a^6bx\right.}{10\left(b+a\sqrt[6]{x}\right)}$$

input
```
Integrate[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

output
```
(Sqrt[(b + a*x^(1/6))^2/x^(1/3)]*(-60*b^7 + 1260*a^2*b^5*x^(1/3) + 1050*a^
3*b^4*Sqrt[x] + 700*a^4*b^3*x^(2/3) + 315*a^5*b^2*x^(5/6) + 84*a^6*b*x + 1
0*a^7*x^(7/6) + 70*a*b^6*x^(1/6)*Log[x]))/(10*(b + a*x^(1/6)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.39, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1384, 774, 27, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}\right)^{7/2}dx$$

$\Big\downarrow$ 1384

$$\frac{\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\int \left(\frac{b^2}{\sqrt[6]{x}}+ab\right)^7dx}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

$\Big\downarrow$ 774

$$\frac{6\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\int b^7\left(a+\frac{b}{\sqrt[6]{x}}\right)^7x^{5/6}d\sqrt[6]{x}}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

$\Big\downarrow$ 27

$$\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\int \left(a+\frac{b}{\sqrt[6]{x}}\right)^7x^{5/6}d\sqrt[6]{x}}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

$\Big\downarrow$ 795

$$\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\int \frac{\left(\sqrt[6]{x}a+b\right)^7}{\sqrt[3]{x}}d\sqrt[6]{x}}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

$\Big\downarrow$ 49

$$\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\int \left(x^{5/6}a^7+7bx^{2/3}a^6+21b^2\sqrt{x}a^5+35b^3\sqrt[3]{x}a^4+35b^4\sqrt[6]{x}a^3+21b^5a^2+\frac{7b^6a}{\sqrt[6]{x}}+\frac{b^7}{\sqrt[3]{x}}\right)d\sqrt[6]{x}}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

$\Big\downarrow$ 2009

$$\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}\left(\frac{a^7x}{6}+\frac{7}{5}a^6bx^{5/6}+\frac{21}{4}a^5b^2x^{2/3}+\frac{35}{3}a^4b^3\sqrt{x}+\frac{35}{2}a^3b^4\sqrt[3]{x}+21a^2b^5\sqrt[6]{x}+7ab^6\log\left(\sqrt[6]{x}\right)-\right.}{ab^7+\frac{b^8}{\sqrt[6]{x}}}$$

input
```
Int[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

output
```
(6*b^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*(-(b^7/x^(1/6)) + 21*a^2*
b^5*x^(1/6) + (35*a^3*b^4*x^(1/3))/2 + (35*a^4*b^3*Sqrt[x])/3 + (21*a^5*b^
2*x^(2/3))/4 + (7*a^6*b*x^(5/6))/5 + (a^7*x)/6 + 7*a*b^6*Log[x^(1/6)]))/(a
*b^7 + b^8/x^(1/6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.30

$$\frac{\left(\frac{a^{2} \sqrt{x}+2 a b \,x^{\frac{1}{3}}+x^{\frac{1}{6}} b^{2}}{\sqrt{x}}\right)^{\frac{7}{2}} x \left(10 a^{7} x^{\frac{7}{6}}+84 a^{6} b x +315 a^{5} b^{2} x^{\frac{5}{6}}+700 a^{4} b^{3} x^{\frac{2}{3}}+1050 a^{3} b^{4} \sqrt{x}+70 a \,b^{6} \ln \left(x \right) x^{\frac{1}{6}}+12\right.}{10 \left(a \,x^{\frac{1}{6}}+b \right)^{7}}$$

input
```
int((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x)
```

output
```
1/10*((a^2*x^(1/2)+2*a*b*x^(1/3)+x^(1/6)*b^2)/x^(1/2))^(7/2)*x*(10*a^7*x^(
7/6)+84*a^6*b*x+315*a^5*b^2*x^(5/6)+700*a^4*b^3*x^(2/3)+1050*a^3*b^4*x^(1/
2)+70*a*b^6*ln(x)*x^(1/6)+1260*a^2*b^5*x^(1/3)-60*b^7)/(a*x^(1/6)+b)^7
```

Fricas [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = \text{Timed out}$$

input
```
integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = \text{Timed out}$$

input
```
integrate((a**2+b**2/x**(1/3)+2*a*b/x**(1/6))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.20

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = 7\,ab^6\log(x) + \frac{10\,a^7x^{\frac{7}{6}} + 84\,a^6bx + 315\,a^5b^2x^{\frac{5}{6}} + 700\,a^4b^3x^{\frac{2}{3}} + 1050\,a^3b^4\sqrt{x} + 1260\,a^2b^5x^{\frac{1}{3}} - 60\,b^7}{10\,x^{\frac{1}{6}}}$$

input
```
integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="maxima")
```

output
```
7*a*b^6*log(x) + 1/10*(10*a^7*x^(7/6) + 84*a^6*b*x + 315*a^5*b^2*x^(5/6) +
 700*a^4*b^3*x^(2/3) + 1050*a^3*b^4*sqrt(x) + 1260*a^2*b^5*x^(1/3) - 60*b^
7)/x^(1/6)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.44

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2} dx = a^7x\,\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + 7\,ab^6\log(|x|)\,\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + \frac{42}{5}\,a^6bx^{\frac{5}{6}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + \frac{63}{2}\,a^5b^2x^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + 70\,a^4b^3\sqrt{x}\,\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + 105\,a^3b^4x^{\frac{1}{3}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) + 126\,a^2b^5x^{\frac{1}{6}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x) - \frac{6\,b^7\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)}{x^{\frac{1}{6}}}$$

input
```
integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="giac")
```

output
```
a^7*x*sgn(a*x + b*x^(5/6))*sgn(x) + 7*a*b^6*log(abs(x))*sgn(a*x + b*x^(5/6
))*sgn(x) + 42/5*a^6*b*x^(5/6)*sgn(a*x + b*x^(5/6))*sgn(x) + 63/2*a^5*b^2*
x^(2/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 70*a^4*b^3*sqrt(x)*sgn(a*x + b*x^(5/
6))*sgn(x) + 105*a^3*b^4*x^(1/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 126*a^2*b^5
*x^(1/6)*sgn(a*x + b*x^(5/6))*sgn(x) - 6*b^7*sgn(a*x + b*x^(5/6))*sgn(x)/x
^(1/6)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}\,dx=\int \left(a^2+\frac{b^2}{x^{1/3}}+\frac{2\,a\,b}{x^{1/6}}\right)^{7/2}\,dx$$

input
```
int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x)
```

output
```
int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.21

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}\,dx=\frac{315x^{\frac{5}{6}}a^5b^2+420x^{\frac{1}{6}}\log\left(x^{\frac{1}{6}}\right)a\,b^6+10x^{\frac{7}{6}}a^7+700x^{\frac{2}{3}}a^4b^3+1260x^{\frac{1}{3}}a^2b^5+1050\sqrt{x}\,a^3b^4+84}{10x^{\frac{1}{6}}}$$

input
```
int((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x)
```

output
```
(315*x**(5/6)*a**5*b**2 + 420*x**(1/6)*log(x**(1/6))*a*b**6 + 10*x**(1/6)*
a**7*x + 700*x**(2/3)*a**4*b**3 + 1260*x**(1/3)*a**2*b**5 + 1050*sqrt(x)*a
**3*b**4 + 84*a**6*b*x - 60*b**7)/(10*x**(1/6))
```

3.94 $\int \left(a+bx^n+cx^{2n}\right)^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int \left(a+bx^n+cx^{2n}\right)^3\,dx=a^3x+\frac{3a^2bx^{1+n}}{1+n}+\frac{3a(b^2+ac)\,x^{1+2n}}{1+2n}+\frac{b(b^2+6ac)\,x^{1+3n}}{1+3n}+\frac{3c(b^2+ac)\,x^{1+4n}}{1+4n}+\frac{3bc^2x^{1+5n}}{1+5n}+\frac{c^3x^{1+6n}}{1+6n}$$

output
```
a^3*x+3*a^2*b*x^(1+n)/(1+n)+3*a*(a*c+b^2)*x^(1+2*n)/(1+2*n)+b*(6*a*c+b^2)*
x^(1+3*n)/(1+3*n)+3*c*(a*c+b^2)*x^(1+4*n)/(1+4*n)+3*b*c^2*x^(1+5*n)/(1+5*n
)+c^3*x^(1+6*n)/(1+6*n)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

$$\int \left(a+bx^n+cx^{2n}\right)^3\,dx=x\left(a^3+\frac{3a^2bx^n}{1+n}+\frac{3a(b^2+ac)\,x^{2n}}{1+2n}+\frac{b(b^2+6ac)\,x^{3n}}{1+3n}+\frac{3c(b^2+ac)\,x^{4n}}{1+4n}+\frac{3bc^2x^{5n}}{1+5n}+\frac{c^3x^{6n}}{1+6n}\right)$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
x*(a^3 + (3*a^2*b*x^n)/(1 + n) + (3*a*(b^2 + a*c)*x^(2*n))/(1 + 2*n) + (b*
(b^2 + 6*a*c)*x^(3*n))/(1 + 3*n) + (3*c*(b^2 + a*c)*x^(4*n))/(1 + 4*n) + (
3*b*c^2*x^(5*n))/(1 + 5*n) + (c^3*x^(6*n))/(1 + 6*n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1682, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^n+cx^{2n}\right)^3 \, dx$$

$$\Big\downarrow \text{1682}$$

$$\int \left(a^3+3a^2bx^n+3ab^2x^{2n}\left(\frac{ac}{b^2}+1\right)+3b^2cx^{4n}\left(\frac{ac}{b^2}+1\right)+b^3x^{3n}\left(\frac{6ac}{b^2}+1\right)+3bc^2x^{5n}+c^3x^{6n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3x+\frac{3a^2bx^{n+1}}{n+1}+\frac{3ax^{2n+1}\left(ac+b^2\right)}{2n+1}+\frac{bx^{3n+1}\left(6ac+b^2\right)}{3n+1}+\frac{3cx^{4n+1}\left(ac+b^2\right)}{4n+1}+\frac{3bc^2x^{5n+1}}{5n+1}+\frac{c^3x^{6n+1}}{6n+1}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^3,x]
```

output
```
a^3*x + (3*a^2*b*x^(1 + n))/(1 + n) + (3*a*(b^2 + a*c)*x^(1 + 2*n))/(1 + 2
*n) + (b*(b^2 + 6*a*c)*x^(1 + 3*n))/(1 + 3*n) + (3*c*(b^2 + a*c)*x^(1 + 4*
n))/(1 + 4*n) + (3*b*c^2*x^(1 + 5*n))/(1 + 5*n) + (c^3*x^(1 + 6*n))/(1 + 6
*n)
```

Defintions of rubi rules used

rule 1682
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Exp
andIntegrand[(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, n}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96

method	result
risch	$a^3x+\frac{c^3x\,x^{6n}}{1+6n}+\frac{b(6ac+b^2)x\,x^{3n}}{1+3n}+\frac{3a(ac+b^2)x\,x^{2n}}{1+2n}+\frac{3b\,a^2x\,x^n}{1+n}+\frac{3b\,c^2x\,x^{5n}}{1+5n}+\frac{3c(ac+b^2)x\,x^{4n}}{1+4n}$
norman	$a^3x+\frac{c^3x\,e^{6n\ln(x)}}{1+6n}+\frac{b(6ac+b^2)x\,e^{3n\ln(x)}}{1+3n}+\frac{3a(ac+b^2)x\,e^{2n\ln(x)}}{1+2n}+\frac{3b\,a^2x\,e^{n\ln(x)}}{1+n}+\frac{3b\,c^2x\,e^{5n\ln(x)}}{1+5n}+\frac{3c(ac+b^2}{1}$
parallelrisch	$\frac{1383x\,x^{2n}a^2c\,n^3+465x\,x^na^2b\,n^2+411x\,x^{2n}a^2c\,n^2+60x\,x^na^2bn+57x\,x^{2n}a^2cn+321x\left(x^{2n}\right)^2b^2c\,n^2+285x\,x^nx^{4n}b\,c^2n^2+51x}{}$
orering	Expression too large to display

input
```
int((a+b*x^n+c*x^(2*n))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+c^3/(1+6*n)*x*(x^n)^6+b*(6*a*c+b^2)/(1+3*n)*x*(x^n)^3+3*a*(a*c+b^2)/
(1+2*n)*x*(x^n)^2+3*b*a^2/(1+n)*x*x^n+3*b*c^2/(1+5*n)*x*(x^n)^5+3*c*(a*c+b
^2)/(1+4*n)*x*(x^n)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(132) = 264.

Time = 0.09 (sec) , antiderivative size = 525, normalized size of antiderivative = 3.98

$$\int \left(a+bx^n+cx^{2n}\right)^3\,dx$$
$$=\frac{(120\,c^3n^5+274\,c^3n^4+225\,c^3n^3+85\,c^3n^2+15\,c^3n+c^3)xx^{6\,n}+3\,(144\,bc^2n^5+324\,bc^2n^4+260\,bc^2n^3+}{}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
((120*c^3*n^5 + 274*c^3*n^4 + 225*c^3*n^3 + 85*c^3*n^2 + 15*c^3*n + c^3)*x
*x^(6*n) + 3*(144*b*c^2*n^5 + 324*b*c^2*n^4 + 260*b*c^2*n^3 + 95*b*c^2*n^2
 + 16*b*c^2*n + b*c^2)*x*x^(5*n) + 3*(180*(b^2*c + a*c^2)*n^5 + 396*(b^2*c
 + a*c^2)*n^4 + 307*(b^2*c + a*c^2)*n^3 + b^2*c + a*c^2 + 107*(b^2*c + a*c
^2)*n^2 + 17*(b^2*c + a*c^2)*n)*x*x^(4*n) + (240*(b^3 + 6*a*b*c)*n^5 + 508
*(b^3 + 6*a*b*c)*n^4 + 372*(b^3 + 6*a*b*c)*n^3 + b^3 + 6*a*b*c + 121*(b^3
+ 6*a*b*c)*n^2 + 18*(b^3 + 6*a*b*c)*n)*x*x^(3*n) + 3*(360*(a*b^2 + a^2*c)*
n^5 + 702*(a*b^2 + a^2*c)*n^4 + 461*(a*b^2 + a^2*c)*n^3 + a*b^2 + a^2*c +
137*(a*b^2 + a^2*c)*n^2 + 19*(a*b^2 + a^2*c)*n)*x*x^(2*n) + 3*(720*a^2*b*n
^5 + 1044*a^2*b*n^4 + 580*a^2*b*n^3 + 155*a^2*b*n^2 + 20*a^2*b*n + a^2*b)*
x*x^n + (720*a^3*n^6 + 1764*a^3*n^5 + 1624*a^3*n^4 + 735*a^3*n^3 + 175*a^3
*n^2 + 21*a^3*n + a^3)*x)/(720*n^6 + 1764*n^5 + 1624*n^4 + 735*n^3 + 175*n
^2 + 21*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3337 vs. $2(119) = 238$.

Time = 12.26 (sec) , antiderivative size = 3337, normalized size of antiderivative = 25.28

$$\int \left(a + bx^n + cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*log(x) - 3*a**2*c/x - 3*a*b**2/x - 3*a*b*c/x*
*2 - a*c**2/x**3 - b**3/(2*x**2) - b**2*c/x**3 - 3*b*c**2/(4*x**4) - c**3/
(5*x**5), Eq(n, -1)), (a**3*x + 6*a**2*b*sqrt(x) + 3*a**2*c*log(x) + 3*a*b
**2*log(x) - 12*a*b*c/sqrt(x) - 3*a*c**2/x - 2*b**3/sqrt(x) - 3*b**2*c/x -
 2*b*c**2/x**(3/2) - c**3/(2*x**2), Eq(n, -1/2)), (a**3*x + 9*a**2*b*x**(2
/3)/2 + 9*a**2*c*x**(1/3) + 9*a*b**2*x**(1/3) + 6*a*b*c*log(x) - 9*a*c**2/
x**(1/3) + b**3*log(x) - 9*b**2*c/x**(1/3) - 9*b*c**2/(2*x**(2/3)) - c**3/
x, Eq(n, -1/3)), (a**3*x + 4*a**2*b*x**(3/4) + 6*a*sqrt(x)*(a*c + b**2) -
12*b*c**2/x**(1/4) + 4*b*x**(1/4)*(6*a*c + b**2) - 2*c**3/sqrt(x) + 12*c*(
a*c + b**2)*log(x**(1/4)), Eq(n, -1/4)), (a**3*x + 15*a**2*b*x**(4/5)/4 +
5*a*x**(3/5)*(a*c + b**2) + 15*b*c**2*log(x**(1/5)) + 5*b*x**(2/5)*(6*a*c
+ b**2)/2 - 5*c**3/x**(1/5) + 15*c*x**(1/5)*(a*c + b**2), Eq(n, -1/5)), (a
**3*x + 18*a**2*b*x**(5/6)/5 + 9*a*x**(2/3)*(a*c + b**2)/2 + 18*b*c**2*x**
(1/6) + 2*b*sqrt(x)*(6*a*c + b**2) + 6*c**3*log(x**(1/6)) + 9*c*x**(1/3)*(
a*c + b**2), Eq(n, -1/6)), (720*a**3*n**6*x/(720*n**6 + 1764*n**5 + 1624*n
**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1764*a**3*n**5*x/(720*n**6 + 1764*
n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1624*a**3*n**4*x/(720
*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 735*a**3
*n**3*x/(720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1
) + 175*a**3*n**2*x/(720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.32

$$\int \left(a + bx^n + cx^{2n}\right)^3 \, dx = a^3x + \frac{c^3x^{6\,n+1}}{6\,n+1} + \frac{3\,bc^2x^{5\,n+1}}{5\,n+1} + \frac{3\,b^2cx^{4\,n+1}}{4\,n+1} + \frac{b^3x^{3\,n+1}}{3\,n+1} + 3\,a^2\left(\frac{cx^{2\,n+1}}{2\,n+1} + \frac{bx^{n+1}}{n+1}\right) + 3\left(\frac{c^2x^{4\,n+1}}{4\,n+1} + \frac{2\,bcx^{3\,n+1}}{3\,n+1} + \frac{b^2x^{2\,n+1}}{2\,n+1}\right)a$$

input
```
integrate((a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
a^3*x + c^3*x^(6*n + 1)/(6*n + 1) + 3*b*c^2*x^(5*n + 1)/(5*n + 1) + 3*b^2*
c*x^(4*n + 1)/(4*n + 1) + b^3*x^(3*n + 1)/(3*n + 1) + 3*a^2*(c*x^(2*n + 1)
/(2*n + 1) + b*x^(n + 1)/(n + 1)) + 3*(c^2*x^(4*n + 1)/(4*n + 1) + 2*b*c*x
^(3*n + 1)/(3*n + 1) + b^2*x^(2*n + 1)/(2*n + 1))*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(132) = 264$.

Time = 0.14 (sec) , antiderivative size = 824, normalized size of antiderivative = 6.24

$$\int \left(a + bx^n + cx^{2n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
(720*a^3*n^6*x + 120*c^3*n^5*x*x^(6*n) + 432*b*c^2*n^5*x*x^(5*n) + 540*b^2
*c*n^5*x*x^(4*n) + 540*a*c^2*n^5*x*x^(4*n) + 240*b^3*n^5*x*x^(3*n) + 1440*
a*b*c*n^5*x*x^(3*n) + 1080*a*b^2*n^5*x*x^(2*n) + 1080*a^2*c*n^5*x*x^(2*n)
+ 2160*a^2*b*n^5*x*x^n + 1764*a^3*n^5*x + 274*c^3*n^4*x*x^(6*n) + 972*b*c^
2*n^4*x*x^(5*n) + 1188*b^2*c*n^4*x*x^(4*n) + 1188*a*c^2*n^4*x*x^(4*n) + 50
8*b^3*n^4*x*x^(3*n) + 3048*a*b*c*n^4*x*x^(3*n) + 2106*a*b^2*n^4*x*x^(2*n)
+ 2106*a^2*c*n^4*x*x^(2*n) + 3132*a^2*b*n^4*x*x^n + 1624*a^3*n^4*x + 225*c
^3*n^3*x*x^(6*n) + 780*b*c^2*n^3*x*x^(5*n) + 921*b^2*c*n^3*x*x^(4*n) + 921
*a*c^2*n^3*x*x^(4*n) + 372*b^3*n^3*x*x^(3*n) + 2232*a*b*c*n^3*x*x^(3*n) +
1383*a*b^2*n^3*x*x^(2*n) + 1383*a^2*c*n^3*x*x^(2*n) + 1740*a^2*b*n^3*x*x^n
 + 735*a^3*n^3*x + 85*c^3*n^2*x*x^(6*n) + 285*b*c^2*n^2*x*x^(5*n) + 321*b^
2*c*n^2*x*x^(4*n) + 321*a*c^2*n^2*x*x^(4*n) + 121*b^3*n^2*x*x^(3*n) + 726*
a*b*c*n^2*x*x^(3*n) + 411*a*b^2*n^2*x*x^(2*n) + 411*a^2*c*n^2*x*x^(2*n) +
465*a^2*b*n^2*x*x^n + 175*a^3*n^2*x + 15*c^3*n*x*x^(6*n) + 48*b*c^2*n*x*x^
(5*n) + 51*b^2*c*n*x*x^(4*n) + 51*a*c^2*n*x*x^(4*n) + 18*b^3*n*x*x^(3*n) +
 108*a*b*c*n*x*x^(3*n) + 57*a*b^2*n*x*x^(2*n) + 57*a^2*c*n*x*x^(2*n) + 60*
a^2*b*n*x*x^n + 21*a^3*n*x + c^3*x*x^(6*n) + 3*b*c^2*x*x^(5*n) + 3*b^2*c*x
*x^(4*n) + 3*a*c^2*x*x^(4*n) + b^3*x*x^(3*n) + 6*a*b*c*x*x^(3*n) + 3*a*b^2
*x*x^(2*n) + 3*a^2*c*x*x^(2*n) + 3*a^2*b*x*x^n + a^3*x)/(720*n^6 + 1764*n^
5 + 1624*n^4 + 735*n^3 + 175*n^2 + 21*n + 1)
```

Mupad [B] (verification not implemented)

Time = 20.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \left(a + bx^n + cx^{2n}\right)^3 \, dx = a^3 \, x + \frac{x \, x^{3\,n} \, (b^3 + 6\,a\,c\,b)}{3\,n + 1} + \frac{c^3 \, x \, x^{6\,n}}{6\,n + 1} + \frac{3\,b\,c^2 \, x \, x^{5\,n}}{5\,n + 1} + \frac{3\,a^2 \, b \, x \, x^n}{n + 1} + \frac{3\,a\,x\,x^{2\,n} \, (b^2 + a\,c)}{2\,n + 1} + \frac{3\,c\,x\,x^{4\,n} \, (b^2 + a\,c)}{4\,n + 1}$$

input
```
int((a + b*x^n + c*x^(2*n))^3,x)
```

output
```
a^3*x + (x*x^(3*n)*(b^3 + 6*a*b*c))/(3*n + 1) + (c^3*x*x^(6*n))/(6*n + 1)
+ (3*b*c^2*x*x^(5*n))/(5*n + 1) + (3*a^2*b*x*x^n)/(n + 1) + (3*a*x*x^(2*n)
*(a*c + b^2))/(2*n + 1) + (3*c*x*x^(4*n)*(a*c + b^2))/(4*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.78

$$\int \left(a + bx^n + cx^{2n}\right)^3 \, dx = \frac{x(432x^{5n}b\,c^2n^5 + 972x^{5n}b\,c^2n^4 + 780x^{5n}b\,c^2n^3 + 285x^{5n}b\,c^2n^2 + 48x^{5n}b\,c^2n + 540x^{4n}a\,c^2n^5 + 1188x^{4n}a\,c}{}$$

input
```
int((a+b*x^n+c*x^(2*n))^3,x)
```

output

```
(x*(120*x**(6*n)*c**3*n**5 + 274*x**(6*n)*c**3*n**4 + 225*x**(6*n)*c**3*n*
*3 + 85*x**(6*n)*c**3*n**2 + 15*x**(6*n)*c**3*n + x**(6*n)*c**3 + 432*x**(
5*n)*b*c**2*n**5 + 972*x**(5*n)*b*c**2*n**4 + 780*x**(5*n)*b*c**2*n**3 + 2
85*x**(5*n)*b*c**2*n**2 + 48*x**(5*n)*b*c**2*n + 3*x**(5*n)*b*c**2 + 540*x
**(4*n)*a*c**2*n**5 + 1188*x**(4*n)*a*c**2*n**4 + 921*x**(4*n)*a*c**2*n**3
 + 321*x**(4*n)*a*c**2*n**2 + 51*x**(4*n)*a*c**2*n + 3*x**(4*n)*a*c**2 + 5
40*x**(4*n)*b**2*c*n**5 + 1188*x**(4*n)*b**2*c*n**4 + 921*x**(4*n)*b**2*c*
n**3 + 321*x**(4*n)*b**2*c*n**2 + 51*x**(4*n)*b**2*c*n + 3*x**(4*n)*b**2*c
 + 1440*x**(3*n)*a*b*c*n**5 + 3048*x**(3*n)*a*b*c*n**4 + 2232*x**(3*n)*a*b
*c*n**3 + 726*x**(3*n)*a*b*c*n**2 + 108*x**(3*n)*a*b*c*n + 6*x**(3*n)*a*b*
c + 240*x**(3*n)*b**3*n**5 + 508*x**(3*n)*b**3*n**4 + 372*x**(3*n)*b**3*n*
*3 + 121*x**(3*n)*b**3*n**2 + 18*x**(3*n)*b**3*n + x**(3*n)*b**3 + 1080*x*
*(2*n)*a**2*c*n**5 + 2106*x**(2*n)*a**2*c*n**4 + 1383*x**(2*n)*a**2*c*n**3
 + 411*x**(2*n)*a**2*c*n**2 + 57*x**(2*n)*a**2*c*n + 3*x**(2*n)*a**2*c + 1
080*x**(2*n)*a*b**2*n**5 + 2106*x**(2*n)*a*b**2*n**4 + 1383*x**(2*n)*a*b**
2*n**3 + 411*x**(2*n)*a*b**2*n**2 + 57*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b*
*2 + 2160*x**n*a**2*b*n**5 + 3132*x**n*a**2*b*n**4 + 1740*x**n*a**2*b*n**3
 + 465*x**n*a**2*b*n**2 + 60*x**n*a**2*b*n + 3*x**n*a**2*b + 720*a**3*n**6
 + 1764*a**3*n**5 + 1624*a**3*n**4 + 735*a**3*n**3 + 175*a**3*n**2 + 21*a*
*3*n + a**3))/(720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 +...
```

3.95 $\int (a+bx^n+cx^{2n})^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int (a+bx^n+cx^{2n})^2\,dx = a^2x+\frac{2abx^{1+n}}{1+n}+\frac{(b^2+2ac)\,x^{1+2n}}{1+2n}+\frac{2bcx^{1+3n}}{1+3n}+\frac{c^2x^{1+4n}}{1+4n}$$

output
```
a^2*x+2*a*b*x^(1+n)/(1+n)+(2*a*c+b^2)*x^(1+2*n)/(1+2*n)+2*b*c*x^(1+3*n)/(1
+3*n)+c^2*x^(1+4*n)/(1+4*n)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int (a+bx^n+cx^{2n})^2\,dx = x\left(a^2+\frac{2abx^n}{1+n}+\frac{(b^2+2ac)\,x^{2n}}{1+2n}+\frac{2bcx^{3n}}{1+3n}+\frac{c^2x^{4n}}{1+4n}\right)$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
x*(a^2 + (2*a*b*x^n)/(1 + n) + ((b^2 + 2*a*c)*x^(2*n))/(1 + 2*n) + (2*b*c*
x^(3*n))/(1 + 3*n) + (c^2*x^(4*n))/(1 + 4*n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1682, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^n+cx^{2n}\right)^2 \, dx$$

$$\Big\downarrow \text{ 1682}$$

$$\int \left(a^2+b^2x^{2n}\left(\frac{2ac}{b^2}+1\right)+2abx^n+2bcx^{3n}+c^2x^{4n}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2x+\frac{x^{2n+1}\left(2ac+b^2\right)}{2n+1}+\frac{2abx^{n+1}}{n+1}+\frac{2bcx^{3n+1}}{3n+1}+\frac{c^2x^{4n+1}}{4n+1}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^2,x]
```

output
```
a^2*x + (2*a*b*x^(1 + n))/(1 + n) + ((b^2 + 2*a*c)*x^(1 + 2*n))/(1 + 2*n)
+ (2*b*c*x^(1 + 3*n))/(1 + 3*n) + (c^2*x^(1 + 4*n))/(1 + 4*n)
```

Defintions of rubi rules used

rule 1682
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Exp
andIntegrand[(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, n}, x] &&
 EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

method	result
risch	$x\,a^{2}+\frac{c^{2} x\,x^{4 n}}{1+4 n}+\frac{\left(2 a c +b^{2}\right) x\,x^{2 n}}{1+2 n}+\frac{2 b a x\,x^{n}}{1+n}+\frac{2 b c x\,x^{3 n}}{1+3 n}$
norman	$x\,a^{2}+\frac{c^{2} x\,{\mathrm e}^{4 n \ln \left(x \right)}}{1+4 n}+\frac{\left(2 a c +b^{2}\right) x\,{\mathrm e}^{2 n \ln \left(x \right)}}{1+2 n}+\frac{2 b a x\,{\mathrm e}^{n \ln \left(x \right)}}{1+n}+\frac{2 b c x\,{\mathrm e}^{3 n \ln \left(x \right)}}{1+3 n}$
parallelrisch	$12x\,x^{2n}b^{2}n^{3}+6x\,x^{4n}c^{2}n^{3}+19x\,x^{2n}b^{2}n^{2}+11x\,x^{4n}c^{2}n^{2}+8x\,x^{2n}b^{2}n+6x\,x^{4n}c^{2}n+16x\,x^{n}x^{2n}bc\,n^{3}+28x\,x^{n}x^{2n}bc\,n^{2}+14x\,x^{n}x$
orering	$x\left(a+b\,x^{n}+c\,x^{2 n}\right)^{2}-\frac{20 x^{2} n \left(5 n^{2}+1\right) \left(a+b\,x^{n}+c\,x^{2 n}\right) \left(\frac{b\,x^{n} n}{x}+\frac{2 c\,x^{2 n} n}{x}\right)}{\left(12 n^{3}+19 n^{2}+8 n +1\right) \left(1+2 n \right)}+\frac{5 x^{3} \left(7 n^{2}-4 n +1\right) \left(2 \left(\frac{b\,x^{n} n}{x}+\frac{2 c\,x^{2 n} n}{x}\right)\right.}{(12}$

input
```
int((a+b*x^n+c*x^(2*n))^2,x,method=_RETURNVERBOSE)
```

output
```
x*a^2+c^2/(1+4*n)*x*(x^n)^4+(2*a*c+b^2)/(1+2*n)*x*(x^n)^2+2*b*a/(1+n)*x*x^
n+2*b*c/(1+3*n)*x*(x^n)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(79) = 158.

Time = 0.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.59

$$\int \left(a+bx^n+cx^{2n}\right)^2\,dx$$
$$=\frac{(6\,c^2n^3+11\,c^2n^2+6\,c^2n+c^2)xx^{4\,n}+2\,(8\,bcn^3+14\,bcn^2+7\,bcn+bc)xx^{3\,n}+(12\,(b^2+2\,ac)n^3+19\,(b^2}{2}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
((6*c^2*n^3 + 11*c^2*n^2 + 6*c^2*n + c^2)*x*x^(4*n) + 2*(8*b*c*n^3 + 14*b*
c*n^2 + 7*b*c*n + b*c)*x*x^(3*n) + (12*(b^2 + 2*a*c)*n^3 + 19*(b^2 + 2*a*c
)*n^2 + b^2 + 2*a*c + 8*(b^2 + 2*a*c)*n)*x*x^(2*n) + 2*(24*a*b*n^3 + 26*a*
b*n^2 + 9*a*b*n + a*b)*x*x^n + (24*a^2*n^4 + 50*a^2*n^3 + 35*a^2*n^2 + 10*
a^2*n + a^2)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(70) = 140$.

Time = 2.28 (sec) , antiderivative size = 1027, normalized size of antiderivative = 13.00

$$\int \left(a + bx^n + cx^{2n}\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*log(x) - 2*a*c/x - b**2/x - b*c/x**2 - c**2/(3*x
**3), Eq(n, -1)), (a**2*x + 4*a*b*sqrt(x) + 2*a*c*log(x) + b**2*log(x) - 4
*b*c/sqrt(x) - c**2/x, Eq(n, -1/2)), (a**2*x + 3*a*b*x**(2/3) + 6*a*c*x**(
1/3) + 3*b**2*x**(1/3) + 2*b*c*log(x) - 3*c**2/x**(1/3), Eq(n, -1/3)), (a*
*2*x + 8*a*b*x**(3/4)/3 + 8*b*c*x**(1/4) + 4*c**2*log(x**(1/4)) - 2*sqrt(x
)*(-2*a*c - b**2), Eq(n, -1/4)), (24*a**2*n**4*x/(24*n**4 + 50*n**3 + 35*n
**2 + 10*n + 1) + 50*a**2*n**3*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
+ 35*a**2*n**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 10*a**2*n*x/(2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a**2*x/(24*n**4 + 50*n**3 + 35*n*
*2 + 10*n + 1) + 48*a*b*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
1) + 52*a*b*n**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a*b*
n*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*b*x*x**n/(24*n**4
+ 50*n**3 + 35*n**2 + 10*n + 1) + 24*a*c*n**3*x*x**(2*n)/(24*n**4 + 50*n**
3 + 35*n**2 + 10*n + 1) + 38*a*c*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n
**2 + 10*n + 1) + 16*a*c*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n
+ 1) + 2*a*c*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 12*b**2
*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 19*b**2*n**2*x
*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*b**2*n*x*x**(2*n)/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b**2*x*x**(2*n)/(24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1) + 16*b*c*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + ...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int \left(a+bx^n+cx^{2n}\right)^2\,dx = a^2x+\frac{c^2x^{4n+1}}{4n+1}+\frac{2\,bcx^{3n+1}}{3n+1}+\frac{b^2x^{2n+1}}{2n+1}+2\,a\left(\frac{cx^{2n+1}}{2n+1}+\frac{bx^{n+1}}{n+1}\right)$$

input
```
integrate((a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
a^2*x + c^2*x^(4*n + 1)/(4*n + 1) + 2*b*c*x^(3*n + 1)/(3*n + 1) + b^2*x^(2
*n + 1)/(2*n + 1) + 2*a*(c*x^(2*n + 1)/(2*n + 1) + b*x^(n + 1)/(n + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(79) = 158.

Time = 0.12 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.75

$$\int \left(a+bx^n+cx^{2n}\right)^2\,dx$$
$$= \frac{24\,a^2n^4x+6\,c^2n^3xx^{4\,n}+16\,bcn^3xx^{3\,n}+12\,b^2n^3xx^{2\,n}+24\,acn^3xx^{2\,n}+48\,abn^3xx^n+50\,a^2n^3x+11\,c^2n^2}{}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
(24*a^2*n^4*x + 6*c^2*n^3*x*x^(4*n) + 16*b*c*n^3*x*x^(3*n) + 12*b^2*n^3*x*
x^(2*n) + 24*a*c*n^3*x*x^(2*n) + 48*a*b*n^3*x*x^n + 50*a^2*n^3*x + 11*c^2*
n^2*x*x^(4*n) + 28*b*c*n^2*x*x^(3*n) + 19*b^2*n^2*x*x^(2*n) + 38*a*c*n^2*x
*x^(2*n) + 52*a*b*n^2*x*x^n + 35*a^2*n^2*x + 6*c^2*n*x*x^(4*n) + 14*b*c*n*
x*x^(3*n) + 8*b^2*n*x*x^(2*n) + 16*a*c*n*x*x^(2*n) + 18*a*b*n*x*x^n + 10*a
^2*n*x + c^2*x*x^(4*n) + 2*b*c*x*x^(3*n) + b^2*x*x^(2*n) + 2*a*c*x*x^(2*n)
 + 2*a*b*x*x^n + a^2*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Mupad [B] (verification not implemented)

Time = 20.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int (a+bx^n+cx^{2n})^2\,dx = a^2\,x+\frac{x\,x^{2\,n}\,(b^2+2\,a\,c)}{2\,n+1}+\frac{c^2\,x\,x^{4\,n}}{4\,n+1}+\frac{2\,a\,b\,x\,x^n}{n+1}+\frac{2\,b\,c\,x\,x^{3\,n}}{3\,n+1}$$

input
```
int((a + b*x^n + c*x^(2*n))^2,x)
```

output
```
a^2*x + (x*x^(2*n)*(2*a*c + b^2))/(2*n + 1) + (c^2*x*x^(4*n))/(4*n + 1) +
(2*a*b*x*x^n)/(n + 1) + (2*b*c*x*x^(3*n))/(3*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.43

$$\int (a+bx^n+cx^{2n})^2\,dx$$
$$= \frac{x(16x^{3n}bc\,n^3+28x^{3n}bc\,n^2+14x^{3n}bcn+24x^{2n}ac\,n^3+38x^{2n}ac\,n^2+16x^{2n}acn+48x^nab\,n^3+52x^nab\,n^2}{}$$

input
```
int((a+b*x^n+c*x^(2*n))^2,x)
```

output
```
(x*(6*x**(4*n)*c**2*n**3 + 11*x**(4*n)*c**2*n**2 + 6*x**(4*n)*c**2*n + x**
(4*n)*c**2 + 16*x**(3*n)*b*c*n**3 + 28*x**(3*n)*b*c*n**2 + 14*x**(3*n)*b*c
*n + 2*x**(3*n)*b*c + 24*x**(2*n)*a*c*n**3 + 38*x**(2*n)*a*c*n**2 + 16*x**
(2*n)*a*c*n + 2*x**(2*n)*a*c + 12*x**(2*n)*b**2*n**3 + 19*x**(2*n)*b**2*n*
*2 + 8*x**(2*n)*b**2*n + x**(2*n)*b**2 + 48*x**n*a*b*n**3 + 52*x**n*a*b*n*
*2 + 18*x**n*a*b*n + 2*x**n*a*b + 24*a**2*n**4 + 50*a**2*n**3 + 35*a**2*n*
*2 + 10*a**2*n + a**2))/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
```

3.96 $\int (a + bx^n + cx^{2n}) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int (a + bx^n + cx^{2n}) \, dx = ax + \frac{bx^{1+n}}{1+n} + \frac{cx^{1+2n}}{1+2n}$$

output
```
a*x+b*x^(1+n)/(1+n)+c*x^(1+2*n)/(1+2*n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (a + bx^n + cx^{2n}) \, dx = ax + \frac{bx^{1+n}}{1+n} + \frac{cx^{1+2n}}{1+2n}$$

input
```
Integrate[a + b*x^n + c*x^(2*n),x]
```

output
```
a*x + (b*x^(1 + n))/(1 + n) + (c*x^(1 + 2*n))/(1 + 2*n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^n+cx^{2n}\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{bx^{n+1}}{n+1}+\frac{cx^{2n+1}}{2n+1}$$

input
```
Int[a + b*x^n + c*x^(2*n),x]
```

output
```
a*x + (b*x^(1 + n))/(1 + n) + (c*x^(1 + 2*n))/(1 + 2*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
risch	$xa+\frac{bx\,x^{n}}{1+n}+\frac{cx\,x^{2n}}{1+2n}$	31
default	$xa+\frac{b\,x^{1+n}}{1+n}+\frac{c\,x^{1+2n}}{1+2n}$	33
parts	$xa+\frac{b\,x^{1+n}}{1+n}+\frac{c\,x^{1+2n}}{1+2n}$	33
norman	$xa+\frac{bx\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{cx\,\mathrm{e}^{2n\ln(x)}}{1+2n}$	35
parallelrisch	$\frac{2b\,x^{n}xn+c\,x^{2n}xn+b\,x^{n}x+c\,x^{2n}x}{(1+n)(1+2n)}+xa$	50
orering	$\left(a+b\,x^{n}+c\,x^{2n}\right)x-\frac{3n\,x^{2}\left(\frac{b\,x^{n}n}{x}+\frac{2c\,x^{2n}n}{x}\right)}{2n^{2}+3n+1}+\frac{x^{3}\left(\frac{b\,x^{n}n^{2}}{x^{2}}-\frac{b\,x^{n}n}{x^{2}}+\frac{4c\,x^{2n}n^{2}}{x^{2}}-\frac{2c\,x^{2n}n}{x^{2}}\right)}{2n^{2}+3n+1}$	122

input
```
int(a+b*x^n+c*x^(2*n),x,method=_RETURNVERBOSE)
```

output
```
x*a+b/(1+n)*x*x^n+c/(1+2*n)*x*x^(2*n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int \left(a+bx^n+cx^{2n}\right)\,dx=\frac{(cn+c)xx^{2\,n}+(2\,bn+b)xx^{n}+(2\,an^{2}+3\,an+a)x}{2\,n^{2}+3\,n+1}$$

input
```
integrate(a+b*x^n+c*x^(2*n),x, algorithm="fricas")
```

output
```
((c*n + c)*x*x^(2*n) + (2*b*n + b)*x*x^n + (2*a*n^2 + 3*a*n + a)*x)/(2*n^2
 + 3*n + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \left(a+bx^n+cx^{2n}\right)\,dx = ax + b\left(\begin{cases} \frac{x^{n+1}}{n+1} & \text{for}\ n \neq -1 \\ \log\left(x\right) & \text{otherwise} \end{cases}\right) + c\left(\begin{cases} \frac{x^{2n+1}}{2n+1} & \text{for}\ n \neq -\frac{1}{2} \\ \log\left(x\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*x**n+c*x**(2*n),x)
```

output
```
a*x + b*Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True)) + c*Pie
cewise((x**(2*n + 1)/(2*n + 1), Ne(n, -1/2)), (log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \left(a+bx^n+cx^{2n}\right)\,dx = ax + \frac{cx^{2\,n+1}}{2\,n+1} + \frac{bx^{n+1}}{n+1}$$

input
```
integrate(a+b*x^n+c*x^(2*n),x, algorithm="maxima")
```

output
```
a*x + c*x^(2*n + 1)/(2*n + 1) + b*x^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \left(a+bx^n+cx^{2n}\right)\,dx = ax + \frac{cx^{2\,n+1}}{2\,n+1} + \frac{bx^{n+1}}{n+1}$$

input
```
integrate(a+b*x^n+c*x^(2*n),x, algorithm="giac")
```

output
```
a*x + c*x^(2*n + 1)/(2*n + 1) + b*x^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 20.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \left(a + bx^n + cx^{2n}\right) dx = a\,x + \frac{c\,x\,x^{2\,n}}{2\,n+1} + \frac{b\,x\,x^n}{n+1}$$

input
```
int(a + b*x^n + c*x^(2*n),x)
```

output
```
a*x + (c*x*x^(2*n))/(2*n + 1) + (b*x*x^n)/(n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \left(a + bx^n + cx^{2n}\right) dx = \frac{x(x^{2n}cn + x^{2n}c + 2x^n bn + x^n b + 2a\,n^2 + 3an + a)}{2n^2 + 3n + 1}$$

input
```
int(a+b*x^n+c*x^(2*n),x)
```

output
```
(x*(x**(2*n)*c*n + x**(2*n)*c + 2*x**n*b*n + x**n*b + 2*a*n**2 + 3*a*n + a
))/(2*n**2 + 3*n + 1)
```

3.97 $\int \frac{1}{a+bx^n+cx^{2n}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int \frac{1}{a+bx^n+cx^{2n}} \, dx = -\frac{2cx \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

output
```
-2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*
a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4
*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 261 vs. $2(124) = 248$.

Time = 0.74 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.10

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx = -2cx\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-1/n}\text{Hypergeometric2F1}\left(-\frac{1}{n},-\frac{1}{n},\frac{-1+n}{n},\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} + \frac{1-2^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}\text{Hypergeometric2F1}\left(-\frac{1}{n},-\frac{1}{n},\frac{-1+n}{n},\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(-1),x]
```

output
```
-2*c*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 -
4*a*c])/c + x^n))^n^(-1))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hyper
geometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)]/(2^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n))^n^(-1)))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1685, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1685}$$

$$\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^n+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{778}$$

$$\frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(-1),x]
```

output
```
(2*c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])) - (2*c*x*Hypergeome
tric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[
b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{1}{a + b\,x^{n} + c\,x^{2n}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{a + bx^n + cx^{2n}}\,dx = \int \frac{1}{cx^{2\,n} + bx^n + a}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]

$$\int \frac{1}{a + bx^n + cx^{2n}}\,dx = \int \frac{1}{a + bx^n + cx^{2n}}\,dx$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n)),x)
```

output
```
Integral(1/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{1}{a + bx^n + cx^{2n}}\,dx = \int \frac{1}{cx^{2\,n} + bx^n + a}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{1}{a + bx^n + cx^{2n}}\,dx = \int \frac{1}{cx^{2\,n} + bx^n + a}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + bx^n + cx^{2n}}\,dx = \int \frac{1}{a + b\,x^n + c\,x^{2\,n}}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n)),x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n)), x)
```

Reduce [F]

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx=\int \frac{1}{x^{2n}c+x^nb+a}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n)),x)
```

output
```
int(1/(x**(2*n)*c + x**n*b + a),x)
```

3.98 $\int \frac{1}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 283

$$\int \frac{1}{(a+bx^n+cx^{2n})^2}\,dx = \frac{x(b^2-2ac+bcx^n)}{a(b^2-4ac)\,n\,(a+bx^n+cx^{2n})} - \frac{c\left(4ac(1-2n)-b^2(1-n)-b\sqrt{b^2-4ac}(1-n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{a\,(b^2-4ac)\left(b^2-4ac-b\sqrt{b^2-4ac}\right)n} - \frac{c\left(4ac(1-2n)-b^2(1-n)+b\sqrt{b^2-4ac}(1-n)\right)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\,(b^2-4ac)\left(b^2-4ac+b\sqrt{b^2-4ac}\right)n}$$

output
```
x*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))-c*(4*a*c*(1-2*n
)-b^2*(1-n)-b*(-4*a*c+b^2)^(1/2)*(1-n))*x*hypergeom([1, 1/n],[1+1/n],-2*c*
x^n/(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2)
)/n-c*(4*a*c*(1-2*n)-b^2*(1-n)+b*(-4*a*c+b^2)^(1/2)*(1-n))*x*hypergeom([1,
 1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)/(b*(-4*a*c+b
^2)^(1/2)-4*a*c+b^2)/n
```

Mathematica [A] (warning: unable to verify)

Time = 4.56 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+bx^n+cx^{2n})^2}\,dx = -\frac{x\left(\frac{4a^2cn-b^2(-1+n)x^n(b+cx^n)+a\left(-b^2n+bc(-3+4n)x^n+2c^2(-1+2n)x^{2n}\right)}{a+x^n(b+cx^n)}+\frac{2^{-1/n}ac\left(4ac\sqrt{b^2-4ac}(1-2n)+b^3(-1+n)-4abc(-1+n \ldots\right.}{\ldots}\right.}{\ldots}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(-2),x]
```

output
```
-((x*((4*a^2*c*n - b^2*(-1 + n)*x^n*(b + c*x^n) + a*(-(b^2*n) + b*c*(-3 +
4*n)*x^n + 2*c^2*(-1 + 2*n)*x^(2*n)))/(a + x^n*(b + c*x^n)) + (a*c*(4*a*c*
Sqrt[b^2 - 4*a*c]*(1 - 2*n) + b^3*(-1 + n) - 4*a*b*c*(-1 + n) + b^2*Sqrt[b
^2 - 4*a*c]*(-1 + n))*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b -
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*Sqrt[b^2
 - 4*a*c]*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*((c*x^n)/(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n))^n^(-1)) + (a*c*(-(b^2*(-1 + n)) + b*Sqrt[b^2 - 4*a*c]*(-1
 + n) + 4*a*c*(-1 + 2*n))*Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n,
(b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^n^(-1)*Sqrt
[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n))^n^(-1))))/(a^2*(b^2 - 4*a*c)*n))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1683, 25, 1752, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n+cx^{2n})^2}\,dx$$

$$\Big\downarrow \text{ 1683}$$

$$\frac{x\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}-\frac{\int -\frac{-bc(1-n)x^n+2ac(1-2n)-b^2(1-n)}{bx^n+cx^{2n}+a}dx}{an\left(b^2-4ac\right)}$$

$\big\downarrow$ 25

$$\frac{\int \frac{-bc(1-n)x^n+2ac(1-2n)-b^2(1-n)}{bx^n+cx^{2n}+a}dx}{an\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$\big\downarrow$ 1752

$$\frac{\frac{c\left(-b(1-n)\sqrt{b^2-4ac}+4ac(1-2n)-\left(b^2(1-n)\right)\right)\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{2\sqrt{b^2-4ac}}-\frac{c\left(b(1-n)\sqrt{b^2-4ac}+4ac(1-2n)-\left(b^2(1-n)\right)\right)\int \frac{1}{cx^n+\frac{1}{2}\left(b+\sqrt{b^2-4a}\right.}}{2\sqrt{b^2-4ac}}}{an\left(b^2-4ac\right)}$$
$$\frac{x\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

$\big\downarrow$ 778

$$\frac{\frac{cx\left(-b(1-n)\sqrt{b^2-4ac}+4ac(1-2n)-\left(b^2(1-n)\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx\left(b(1-n)\sqrt{b^2-4ac}+4ac(1-2n)-\left(b^2(1-\right.\right.}{\sqrt{b^2-4c}}}{an\left(b^2-4ac\right)}$$
$$\frac{x\left(-2ac+b^2+bcx^n\right)}{an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(-2),x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))) +
((c*(4*a*c*(1 - 2*n) - b^2*(1 - n) - b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hyperg
eometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(S
qrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])) - (c*(4*a*c*(1 - 2*n) - b^2*(1 -
 n) + b*Sqrt[b^2 - 4*a*c]*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-
1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])))/(a*(b^2 - 4*a*c)*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1683
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-
x)*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c +
 n*(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n
))^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && ILtQ[p, -1]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}+c\,x^{2n}\right)^{2}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{1}{(a + bx^n + cx^{2n})^2} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)^2} \, dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(1/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x^n + a*c
)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + bx^n + cx^{2n})^2} \, dx = \text{Timed out}$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a + bx^n + cx^{2n})^2} \, dx = \int \frac{1}{(cx^{2n} + bx^n + a)^2} \, dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

output
```
(b*c*x*x^n + (b^2 - 2*a*c)*x)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*
c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) - integrate(-(b*c*(n - 1)*x^
n - 2*a*c*(2*n - 1) + b^2*(n - 1))/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4
*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^2}\,dx=\int \frac{1}{(cx^{2n}+bx^n+a)^2}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n+cx^{2n})^2}\,dx=\int \frac{1}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n))^2,x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n))^2, x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^2}\,dx=\int \frac{1}{x^{4n}c^2+2x^{3n}bc+2x^{2n}ac+x^{2n}b^2+2x^nab+a^2}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^2,x)
```

output
```
int(1/(x**(4*n)*c**2 + 2*x**(3*n)*b*c + 2*x**(2*n)*a*c + x**(2*n)*b**2 + 2
*x**n*a*b + a**2),x)
```

3.99 $\int \frac{1}{(a+bx^n+cx^{2n})^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 489

$$\int \frac{1}{(a+bx^n+cx^{2n})^3}\,dx = \frac{x(b^2-2ac+bcx^n)}{2a(b^2-4ac)\,n\,(a+bx^n+cx^{2n})^2}$$
$$-\frac{x(4a^2c^2(1-4n)-5ab^2c(1-3n)+b^4(1-2n)-bc(2ac(2-7n)-b^2(1-2n))\,x^n)}{2a^2(b^2-4ac)^2n^2(a+bx^n+cx^{2n})}$$
$$+\frac{c\left(b\sqrt{b^2-4ac}(2ac(2-7n)-b^2(1-2n))\,(1-n)-b^4(1-3n+2n^2)+6ab^2c(1-4n+3n^2)-8a^2c^2(\right.}{2a^2(b^2-4ac)^2\left(b^2-4ac-b\sqrt{b^2-4ac}\right)}$$
$$-\frac{c\left(b\sqrt{b^2-4ac}(2ac(2-7n)-b^2(1-2n))\,(1-n)+b^4(1-3n+2n^2)-6ab^2c(1-4n+3n^2)+8a^2c^2(\right.}{2a^2(b^2-4ac)^2\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}$$

output
```
1/2*x*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))^2-1/2*x*(4*
a^2*c^2*(1-4*n)-5*a*b^2*c*(1-3*n)+b^4*(1-2*n)-b*c*(2*a*c*(2-7*n)-b^2*(1-2*
n))*x^n)/a^2/(-4*a*c+b^2)^2/n^2/(a+b*x^n+c*x^(2*n))+1/2*c*(b*(-4*a*c+b^2)^
(1/2)*(2*a*c*(2-7*n)-b^2*(1-2*n))*(1-n)-b^4*(2*n^2-3*n+1)+6*a*b^2*c*(3*n^2
-4*n+1)-8*a^2*c^2*(8*n^2-6*n+1))*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-
(-4*a*c+b^2)^(1/2)))/a^2/(-4*a*c+b^2)^2/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))/n
^2-1/2*c*(b*(-4*a*c+b^2)^(1/2)*(2*a*c*(2-7*n)-b^2*(1-2*n))*(1-n)+b^4*(2*n^
2-3*n+1)-6*a*b^2*c*(3*n^2-4*n+1)+8*a^2*c^2*(8*n^2-6*n+1))*x*hypergeom([1,
1/n],[1+1/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/a^2/(-4*a*c+b^2)^2/(b*(-4*a*
c+b^2)^(1/2)-4*a*c+b^2)/n^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5445 vs. $2(489) = 978$.

Time = 6.31 (sec) , antiderivative size = 5445, normalized size of antiderivative = 11.13

$$\int \frac{1}{(a+bx^n+cx^{2n})^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(-3),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1683, 25, 1760, 25, 1752, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n+cx^{2n})^3}\,dx$$

$$\Big\downarrow \text{ 1683}$$

$$\frac{x(-2ac+b^2+bcx^n)}{2an(b^2-4ac)(a+bx^n+cx^{2n})^2}-\frac{\int -\frac{-bc(1-3n)x^n+2ac(1-4n)-b^2(1-2n)}{(bx^n+cx^{2n}+a)^2}dx}{2an(b^2-4ac)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-bc(1-3n)x^n+2ac(1-4n)-b^2(1-2n)}{(bx^n+cx^{2n}+a)^2}dx}{2an(b^2-4ac)}+\frac{x(-2ac+b^2+bcx^n)}{2an(b^2-4ac)(a+bx^n+cx^{2n})^2}$$

$$\Big\downarrow \text{ 1760}$$

$$-\frac{\frac{\int -\frac{-bc\left(2ac(2-7n)-b^2(1-2n)\right)(1-n)x^n+b^4\left(2n^2-3n+1\right)+4a^2c^2\left(8n^2-6n+1\right)-ab^2c\left(16n^2-21n+5\right)}{bx^n+cx^{2n}+a}dx}{an(b^2-4ac)}-\frac{x\left(4a^2c^2(1-4n)-bcx^n\left(2ac(2-7n)-b^2(1-\ldots\right.\right.}{an(b^2-4ac)(a+bx\ldots}}{2an\left(b^2-4ac\right)}$$

$$\frac{x\left(-2ac+b^2+bcx^n\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{-bc\left(2ac(2-7n)-b^2(1-2n)\right)(1-n)x^n+b^4\left(2n^2-3n+1\right)+4a^2c^2\left(8n^2-6n+1\right)-ab^2c\left(16n^2-21n+5\right)}{bx^n+cx^{2n}+a}dx}{an(b^2-4ac)}-\frac{x\left(4a^2c^2(1-4n)-bcx^n\left(2ac(2-7n)-b^2(1-2n)\right)\ldots\right.}{an(b^2-4ac)(a+bx^n+c\ldots}}{2an\left(b^2-4ac\right)}$$

$$\frac{x\left(-2ac+b^2+bcx^n\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$\Big\downarrow$ 1752

$$\frac{\frac{c\left(8a^2c^2\left(8n^2-6n+1\right)-6ab^2c\left(3n^2-4n+1\right)+b\sqrt{b^2-4ac}\left(b^2\left(2n^2-3n+1\right)-2ac\left(7n^2-9n+2\right)\right)+b^4\left(2n^2-3n+1\right)\right)\int \frac{1}{cx^n+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{2\sqrt{b^2-4ac}}-\frac{c\left(8a^2c^2\left(8n^2-6n\ldots\right.\right.}{\ldots}}{an(b^2-4ac)}\ldots$$

$$\frac{x\left(-2ac+b^2+bcx^n\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

$\Big\downarrow$ 778

$$\frac{\frac{cx\left(8a^2c^2\left(8n^2-6n+1\right)-6ab^2c\left(3n^2-4n+1\right)+b\sqrt{b^2-4ac}\left(b^2\left(2n^2-3n+1\right)-2ac\left(7n^2-9n+2\right)\right)+b^4\left(2n^2-3n+1\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\ldots\right.}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}\ldots}{an(\ldots}$$

$$\frac{x\left(-2ac+b^2+bcx^n\right)}{2an\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)^2}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(-3),x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^n))/(2*a*(b^2 - 4*a*c)*n*(a + b*x^n + c*x^(2*n))^2
) + (-((x*(4*a^2*c^2*(1 - 4*n) - 5*a*b^2*c*(1 - 3*n) + b^4*(1 - 2*n) - b*c
*(2*a*c*(2 - 7*n) - b^2*(1 - 2*n))*x^n))/(a*(b^2 - 4*a*c)*n*(a + b*x^n + c
*x^(2*n)))) + ((c*(b^4*(1 - 3*n + 2*n^2) - 6*a*b^2*c*(1 - 4*n + 3*n^2) + 8
*a^2*c^2*(1 - 6*n + 8*n^2) + b*Sqrt[b^2 - 4*a*c]*(b^2*(1 - 3*n + 2*n^2) -
2*a*c*(2 - 9*n + 7*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])
) - (c*(b^4*(1 - 3*n + 2*n^2) - 6*a*b^2*c*(1 - 4*n + 3*n^2) + 8*a^2*c^2*(1
 - 6*n + 8*n^2) - b*Sqrt[b^2 - 4*a*c]*(b^2*(1 - 3*n + 2*n^2) - 2*a*c*(2 -
9*n + 7*n^2)))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])))/(a*(b^2
- 4*a*c)*n))/(2*a*(b^2 - 4*a*c)*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1683
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-
x)*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c +
 n*(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n
))^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && ILtQ[p, -1]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1760
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/
(a*n*(p + 1)*(b^2 - 4*a*c))   Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*
d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n
+ c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n
] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Maple [F]

$$\int \frac{1}{(a+b\,x^{n}+c\,x^{2n})^{3}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^3,x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^3}\,dx=\int \frac{1}{(cx^{2\,n}+bx^n+a)^3}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral(1/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^
3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n + a^2*c
)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n+cx^{2n})^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^3}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

output
```
-1/2*((2*a*b*c^3*(7*n - 2) - b^3*c^2*(2*n - 1))*x*x^(3*n) + (a*b^2*c^2*(29
*n - 9) - 4*a^2*c^3*(4*n - 1) - 2*b^4*c*(2*n - 1))*x*x^(2*n) + (4*a*b^3*c*
(3*n - 1) - b^5*(2*n - 1) + 2*a^2*b*c^2*n)*x*x^n + (a^2*b^2*c*(21*n - 5) -
 4*a^3*c^2*(6*n - 1) - a*b^4*(3*n - 1))*x)/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2
+ 16*a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*
x^(4*n) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n)
 + (a^2*b^6*n^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n
^2 - 8*a^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) - integrate(-1/2*((2*n^2 - 3
*n + 1)*b^4 - (16*n^2 - 21*n + 5)*a*b^2*c + 4*(8*n^2 - 6*n + 1)*a^2*c^2 +
((2*n^2 - 3*n + 1)*b^3*c - 2*(7*n^2 - 9*n + 2)*a*b*c^2)*x^n)/(a^3*b^4*n^2
- 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 +
16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^
2)*x^n), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^3}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{1}{(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n))^3,x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n))^3, x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^3}\,dx$$
$$= \int \frac{1}{x^{6n}c^3+3x^{5n}b\,c^2+3x^{4n}a\,c^2+3x^{4n}b^2c+6x^{3n}abc+x^{3n}b^3+3x^{2n}a^2c+3x^{2n}a\,b^2+3x^na^2b+a^3}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^3,x)
```

output
```
int(1/(x**(6*n)*c**3 + 3*x**(5*n)*b*c**2 + 3*x**(4*n)*a*c**2 + 3*x**(4*n)*
b**2*c + 6*x**(3*n)*a*b*c + x**(3*n)*b**3 + 3*x**(2*n)*a**2*c + 3*x**(2*n)
*a*b**2 + 3*x**n*a**2*b + a**3),x)
```

3.100 $\int \left(a+bx^n+cx^{2n}\right)^{5/2} dx$

Optimal result

Integrand size = 18, antiderivative size = 142

$$\int \left(a+bx^n+cx^{2n}\right)^{5/2} dx = \frac{a^2x\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{5}{2},-\frac{5}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
a^2*x*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(1/n,-5/2,-5/2,1+1/n,-2*c*x^n/(b-(
-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^n/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 706 vs. 2(142) = 284.

Time = 3.83 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.97

$$\int \left(a+bx^n+cx^{2n}\right)^{5/2} dx = \frac{x\left(15bn^3(b^4(4+8n+3n^2)+48a^2c^2(1+5n+5n^2)-4ab^2c(7+23n+10n^2))\,x^n\sqrt{\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2}}}\right.}{}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(5/2),x]
```

output
```
(x*(15*b*n^3*(b^4*(4 + 8*n + 3*n^2) + 48*a^2*c^2*(1 + 5*n + 5*n^2) - 4*a*b
^2*c*(7 + 23*n + 10*n^2))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 + n)*((a + x^n*(
b + c*x^n))*(-15*b^4*n^3*(2 + 3*n) + 16*a^2*c^2*(1 + 15*n + 85*n^2 + 210*n
^3 + 184*n^4) + 30*b^3*c*n^3*(1 + n)*x^n + 4*b^2*c^2*(4 + 50*n + 215*n^2 +
 355*n^3 + 186*n^4)*x^(2*n) + 8*b*c^3*(4 + 45*n + 170*n^2 + 255*n^3 + 126*
n^4)*x^(3*n) + 16*c^4*(1 + 10*n + 35*n^2 + 50*n^3 + 24*n^4)*x^(4*n) + 4*a*
c*(45*b^2*n^3*(1 + 3*n) + 2*b*c*(4 + 55*n + 270*n^2 + 530*n^3 + 311*n^4)*x
^n + 4*c^2*(2 + 25*n + 105*n^2 + 170*n^3 + 88*n^4)*x^(2*n))) + 15*a*n^3*(-
12*a*b^2*c*(1 + 3*n) + b^4*(2 + 3*n) + 16*a^2*c^2*(1 + 6*n + 8*n^2))*Sqrt[
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2
, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])])))/(32*c^2*(1 + n)^2*(1 + 2*n)*(1 + 3*n)*(1 + 4*n)*(1 + 5*n)*S
qrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^n+cx^{2n}\right)^{5/2} dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{a^2\sqrt{a+bx^n+cx^{2n}} \int \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{5/2} \left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{5/2} dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{a^2x\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{5}{2},-\frac{5}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(5/2),x]
```

output
```
(a^2*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -5/2, -5/2, 1 + n^(-1)
, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
/(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{5}{2}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(5/2),x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a+bx^n+cx^{2n}\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{5/2} dx = \int \left(a+bx^n+cx^{2n}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(5/2),x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(5/2), x)
```

Maxima [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{5/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{5/2}\,dx=\int \left(cx^{2\,n}+bx^n+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^n+cx^{2n}\right)^{5/2}\,dx=\int {\left(a+b\,x^n+c\,x^{2\,n}\right)}^{5/2}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(5/2),x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(5/2), x)
```

Reduce [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{5/2}\,dx=\text{too large to display}$$

input
```
int((a+b*x^n+c*x^(2*n))^(5/2),x)
```

output

```
(384*x**(4*n)*sqrt(x**(2*n)*c + x**n*b + a)*c**3*n**4*x + 1184*x**(4*n)*sq
rt(x**(2*n)*c + x**n*b + a)*c**3*n**3*x + 976*x**(4*n)*sqrt(x**(2*n)*c + x
**n*b + a)*c**3*n**2*x + 304*x**(4*n)*sqrt(x**(2*n)*c + x**n*b + a)*c**3*n
*x + 32*x**(4*n)*sqrt(x**(2*n)*c + x**n*b + a)*c**3*x + 1008*x**(3*n)*sqrt
(x**(2*n)*c + x**n*b + a)*b*c**2*n**4*x + 3048*x**(3*n)*sqrt(x**(2*n)*c +
x**n*b + a)*b*c**2*n**3*x + 2392*x**(3*n)*sqrt(x**(2*n)*c + x**n*b + a)*b*
c**2*n**2*x + 688*x**(3*n)*sqrt(x**(2*n)*c + x**n*b + a)*b*c**2*n*x + 64*x
**(3*n)*sqrt(x**(2*n)*c + x**n*b + a)*b*c**2*x + 1408*x**(2*n)*sqrt(x**(2*
n)*c + x**n*b + a)*a*c**2*n**4*x + 4128*x**(2*n)*sqrt(x**(2*n)*c + x**n*b
+ a)*a*c**2*n**3*x + 2992*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*a*c**2*n*
*2*x + 768*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*a*c**2*n*x + 64*x**(2*n)
*sqrt(x**(2*n)*c + x**n*b + a)*a*c**2*x + 744*x**(2*n)*sqrt(x**(2*n)*c + x
**n*b + a)*b**2*c*n**4*x + 2164*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*b**
2*c*n**3*x + 1536*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*b**2*c*n**2*x + 3
84*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*b**2*c*n*x + 32*x**(2*n)*sqrt(x*
*(2*n)*c + x**n*b + a)*b**2*c*x + 2488*x**n*sqrt(x**(2*n)*c + x**n*b + a)*
a*b*c*n**4*x + 6728*x**n*sqrt(x**(2*n)*c + x**n*b + a)*a*b*c*n**3*x + 3912
*x**n*sqrt(x**(2*n)*c + x**n*b + a)*a*b*c*n**2*x + 848*x**n*sqrt(x**(2*n)*
c + x**n*b + a)*a*b*c*n*x + 64*x**n*sqrt(x**(2*n)*c + x**n*b + a)*a*b*c*x
+ 30*x**n*sqrt(x**(2*n)*c + x**n*b + a)*b**3*n**4*x + 60*x**n*sqrt(x**(...
```

3.101 $\int (a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\int (a+bx^n+cx^{2n})^{3/2}\,dx=\frac{ax\sqrt{a+bx^n+cx^{2n}}\operatorname{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
a*x*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(1/n,-3/2,-3/2,1+1/n,-2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^n/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 466 vs. 2(140) = 280.

Time = 1.72 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.33

$$\int (a+bx^n+cx^{2n})^{3/2}\,dx=\frac{x\left(-3bn^2(b^2(2+n)-4ac(2+3n))\,x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+\frac{1}{n},\right.\right.}{}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(x*(-3*b*n^2*(b^2*(2 + n) - 4*a*c*(2 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-
2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*
(1 + n)*((3*b^2*n^2 + 4*a*c*(1 + 6*n + 8*n^2) + 2*b*c*(2 + 9*n + 7*n^2)*x^
n + 4*c^2*(1 + 3*n + 2*n^2)*x^(2*n))*(a + x^n*(b + c*x^n)) - 3*a*n^2*(b^2
- 4*a*c*(1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*A
ppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + n)^2*(1 + 2*n)*(1 + 3*n)*
Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1686}$$

$$\frac{a\sqrt{a+bx^n+cx^{2n}}\int \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{ax\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{3}{2},-\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

output
```
(a*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2, 1 + n^(-1),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(
Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \int \left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(3/2), x)
```

Maxima [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(cx^{2n}+bx^n+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \int \left(a+b\,x^n+c\,x^{2n}\right)^{3/2} dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \text{too large to display}$$

input
```
int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
(8*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*n**2*x + 20*x**(2*n)*sqrt(x**(
2*n)*c + x**n*b + a)*c*n*x + 8*x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a)*c*x
+ 14*x**n*sqrt(x**(2*n)*c + x**n*b + a)*b*n**2*x + 32*x**n*sqrt(x**(2*n)*c
 + x**n*b + a)*b*n*x + 8*x**n*sqrt(x**(2*n)*c + x**n*b + a)*b*x + 68*sqrt(
x**(2*n)*c + x**n*b + a)*a*n**2*x + 44*sqrt(x**(2*n)*c + x**n*b + a)*a*n*x
 + 8*sqrt(x**(2*n)*c + x**n*b + a)*a*x + 144*int(sqrt(x**(2*n)*c + x**n*b
+ a)/(6*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 11*x**(2*n)*c*n + 2*x**(2*n
)*c + 6*x**n*b*n**3 + 17*x**n*b*n**2 + 11*x**n*b*n + 2*x**n*b + 6*a*n**3 +
 17*a*n**2 + 11*a*n + 2*a),x)*a**2*n**6 + 408*int(sqrt(x**(2*n)*c + x**n*b
 + a)/(6*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 11*x**(2*n)*c*n + 2*x**(2*
n)*c + 6*x**n*b*n**3 + 17*x**n*b*n**2 + 11*x**n*b*n + 2*x**n*b + 6*a*n**3
+ 17*a*n**2 + 11*a*n + 2*a),x)*a**2*n**5 + 264*int(sqrt(x**(2*n)*c + x**n*
b + a)/(6*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 11*x**(2*n)*c*n + 2*x**(2
*n)*c + 6*x**n*b*n**3 + 17*x**n*b*n**2 + 11*x**n*b*n + 2*x**n*b + 6*a*n**3
 + 17*a*n**2 + 11*a*n + 2*a),x)*a**2*n**4 + 48*int(sqrt(x**(2*n)*c + x**n*
b + a)/(6*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 11*x**(2*n)*c*n + 2*x**(2
*n)*c + 6*x**n*b*n**3 + 17*x**n*b*n**2 + 11*x**n*b*n + 2*x**n*b + 6*a*n**3
 + 17*a*n**2 + 11*a*n + 2*a),x)*a**2*n**3 - 216*int((x**(2*n)*sqrt(x**(2*n
)*c + x**n*b + a))/(6*x**(2*n)*c*n**3 + 17*x**(2*n)*c*n**2 + 11*x**(2*n)*c
*n + 2*x**(2*n)*c + 6*x**n*b*n**3 + 17*x**n*b*n**2 + 11*x**n*b*n + 2*x*...
```

3.102 $\int \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

output
```
x*(a+b*x^n+c*x^(2*n))^(1/2)*AppellF1(1/n,-1/2,-1/2,1+1/n,-2*c*x^n/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(139) = 278.

Time = 0.78 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.53

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \frac{x\left(bnx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+2(1+\right.}{2(1+n)^2}$$

input
```
Integrate[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*(b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1
[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 + n)*(a + x^n*(b + c*x^n) + a*n*Sq
rt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2,
1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])))/(2*(1 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{\sqrt{a+bx^n+cx^{2n}}\int \sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{a+bx^n+cx^{2n}}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},-\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 + n^(-1), (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{a+b\,x^{n}+c\,x^{2n}}dx$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \int \sqrt{a+bx^n+cx^{2n}}\,dx$$

input
```
integrate((a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx = \int \sqrt{cx^{2n}+bx^n+a}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{cx^{2\,n}+bx^n+a}\,dx$$

input
```
integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx=\int \sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

input
```
int((a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int((a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^n+cx^{2n}}\,dx$$
$$=\frac{2\sqrt{x^{2n}c+x^nb+a}\,x+\left(\int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{2n}cn+2x^{2n}c+x^nbn+2x^nb+an+2a}dx\right)a\,n^2+2\left(\int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{2n}cn+2x^{2n}c+x^nbn+2x^nb+an+2a}dx\right)an}{n+2}$$

input
```
int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

output

```
(2*sqrt(x**(2*n)*c + x**n*b + a)*x + int(sqrt(x**(2*n)*c + x**n*b + a)/(x*
*(2*n)*c*n + 2*x**(2*n)*c + x**n*b*n + 2*x**n*b + a*n + 2*a),x)*a*n**2 + 2
*int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(2*n)*c*n + 2*x**(2*n)*c + x**n*b*n
 + 2*x**n*b + a*n + 2*a),x)*a*n - int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b +
 a))/(x**(2*n)*c*n + 2*x**(2*n)*c + x**n*b*n + 2*x**n*b + a*n + 2*a),x)*c*
n**2 - 2*int((x**(2*n)*sqrt(x**(2*n)*c + x**n*b + a))/(x**(2*n)*c*n + 2*x*
*(2*n)*c + x**n*b*n + 2*x**n*b + a*n + 2*a),x)*c*n)/(n + 2)
```

3.103 $\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 139

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

output
```
x*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)*AppellF1(1/n,1/2,1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+x^n(b+cx^n)}}$$

input
```
Integrate[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1),
1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b +
Sqrt[b^2 - 4*a*c])])/Sqrt[a + x^n*(b + c*x^n)]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}dx$$
$$\Big\downarrow \text{ 1686}$$
$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^n+cx^{2n}}}$$
$$\Big\downarrow \text{ 936}$$
$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},\frac{1}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

output
```
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^n + c*
x^(2*n)]
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\,x^{n}+c\,x^{2n}}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + bx^n + cx^{2n}}}\,dx = \int \frac{1}{\sqrt{a + bx^n + cx^{2n}}}\,dx$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + bx^n + cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + bx^n + cx^{2n}}}\,dx = \int \frac{1}{\sqrt{cx^{2n} + bx^n + a}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{1}{\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \int \frac{\sqrt{x^{2n}c+x^nb+a}}{x^{2n}c+x^nb+a}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

output
```
int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(2*n)*c + x**n*b + a),x)
```

3.104 $\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 142

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2c}{b+\sqrt{b}}\right.}{a\sqrt{a+bx^n+cx^{2n}}}$$

output
```
x*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)*AppellF1(1/n,3/2,3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 384 vs. 2(142) = 284.

Time = 1.21 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.70

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{x\left(2bcx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},\frac{1}{2},2+\frac{1}{n},-\frac{2}{b+\sqrt{}}\right.\right.}{}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

output
```
(x*(2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c
])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appell
F1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (1 + n)*(2*(b^2 - 2*a*c + b*c*x^n) +
(b^2*(-2 + n) - 4*a*c*(-1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*n*(1
 + n)*Sqrt[a + x^n*(b + c*x^n)])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},\frac{3}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

output
```
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^n +
 c*x^(2*n)])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(cx^{2n}+bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{1}{(a+b\,x^n+c\,x^{2\,n})^{3/2}}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \int \frac{\sqrt{x^{2n}c+x^n b+a}}{x^{4n}c^2+2x^{3n}bc+2x^{2n}ac+x^{2n}b^2+2x^n ab+a^2}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

output
```
int(sqrt(x**(2*n)*c + x***n*b + a)/(x**(4*n)*c**2 + 2*x***(3*n)*b*c + 2*x**(
2*n)*a*c + x**(2*n)*b**2 + 2*x***n*a*b + a**2),x)
```

3.105 $\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 142

$$\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{n},\frac{5}{2},\frac{5}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2c}{b+\sqrt{b}}\right.}{a^2\sqrt{a+bx^n+cx^{2n}}}$$

output
```
x*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2
)))^(1/2)*AppellF1(1/n,5/2,5/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))/a^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 564 vs. 2(142) = 284.

Time = 5.32 (sec) , antiderivative size = 564, normalized size of antiderivative = 3.97

$$\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \frac{x\left(-2bc(4ac(2-5n)+b^2(-2+3n))\,x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}(a+\right.}{}$$

input
```
Integrate[(a + b*x^n + c*x^(2*n))^(-5/2),x]
```

output
```
(x*(-2*b*c*(4*a*c*(2 - 5*n) + b^2*(-2 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]*(a + x^n*(b + c*x^n))*AppellF1[1 + n^(-1), 1
/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + S
qrt[b^2 - 4*a*c])] + (1 + n)*(2*(8*a^3*c^2*(-1 + 4*n) + b^3*(-2 + 3*n)*x^n
*(b + c*x^n)^2 + 2*a^2*c*(b^2*(5 - 14*n) + 4*c^2*(-1 + 3*n)*x^(2*n)) + 2*a
*b*(b + c*x^n)*(b^2*(-1 + 2*n) - b*c*(-5 + 11*n)*x^n - 2*c^2*(-2 + 5*n)*x^
(2*n))) + (b^4*(4 - 8*n + 3*n^2) + 16*a^2*c^2*(1 - 4*n + 3*n^2) - 4*a*b^2*
c*(5 - 14*n + 6*n^2))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
]*(a + x^n*(b + c*x^n))*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(3*a^2*(b^
2 - 4*a*c)^2*n^2*(1 + n)*(a + x^n*(b + c*x^n))^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1686, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}dx$$

$$\Big\downarrow \text{1686}$$

$$\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{5/2}\left(\frac{2cx^n}{b+\sqrt{b^2-4ac}}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n+cx^{2n}}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{5}{2},\frac{5}{2},1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a^2\sqrt{a+bx^n+cx^{2n}}}$$

input
```
Int[(a + b*x^n + c*x^(2*n))^(-5/2),x]
```

output
```
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[n^(-1), 5/2, 5/2, 1 + n^(-1), (-2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a^2*Sqrt[a + b*x^n
 + c*x^(2*n)])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1686
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p]))   Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - S
qrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{(a+b\,x^n+c\,x^{2n})^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(5/2),x)
```

output
```
int(1/(a+b*x^n+c*x^(2*n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \int \frac{1}{(a+bx^n+cx^{2n})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*x**n+c*x**(2*n))**(5/2),x)
```

output
```
Integral((a + b*x**n + c*x**(2*n))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \int \frac{1}{(cx^{2\,n}+bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n+c*x^(2*n))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^(2*n) + b*x^n + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \int \frac{1}{(a+b\,x^n+c\,x^{2\,n})^{5/2}}\,dx$$

input
```
int(1/(a + b*x^n + c*x^(2*n))^(5/2),x)
```

output
```
int(1/(a + b*x^n + c*x^(2*n))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n+cx^{2n})^{5/2}}\,dx = \int \frac{\sqrt{x^{2n}c+x^n b+a}}{x^{6n}c^3+3x^{5n}b\,c^2+3x^{4n}a\,c^2+3x^{4n}b^2c+6x^{3n}abc+x^{3n}b^3+3x^{2n}a^2c+3x^{2n}}$$

input
```
int(1/(a+b*x^n+c*x^(2*n))^(5/2),x)
```

output
```
int(sqrt(x**(2*n)*c + x**n*b + a)/(x**(6*n)*c**3 + 3*x**(5*n)*b*c**2 + 3*x
**(4*n)*a*c**2 + 3*x**(4*n)*b**2*c + 6*x**(3*n)*a*b*c + x**(3*n)*b**3 + 3*
x**(2*n)*a**2*c + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b + a**3),x)
```

3.106 $\int (a^2 + 2acx^n + c^2x^{2n})^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int (a^2 + 2acx^n + c^2x^{2n})^3 \, dx = a^6x + \frac{6a^5cx^{1+n}}{1+n} + \frac{15a^4c^2x^{1+2n}}{1+2n} + \frac{20a^3c^3x^{1+3n}}{1+3n} + \frac{15a^2c^4x^{1+4n}}{1+4n} + \frac{6ac^5x^{1+5n}}{1+5n} + \frac{c^6x^{1+6n}}{1+6n}$$

output
```
a^6*x+6*a^5*c*x^(1+n)/(1+n)+15*a^4*c^2*x^(1+2*n)/(1+2*n)+20*a^3*c^3*x^(1+3
*n)/(1+3*n)+15*a^2*c^4*x^(1+4*n)/(1+4*n)+6*a*c^5*x^(1+5*n)/(1+5*n)+c^6*x^(
1+6*n)/(1+6*n)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int (a^2 + 2acx^n + c^2x^{2n})^3 \, dx = x\left(a^6 + \frac{6a^5cx^n}{1+n} + \frac{15a^4c^2x^{2n}}{1+2n} + \frac{20a^3c^3x^{3n}}{1+3n} + \frac{15a^2c^4x^{4n}}{1+4n} + \frac{6ac^5x^{5n}}{1+5n} + \frac{c^6x^{6n}}{1+6n}\right)$$

input
```
Integrate[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^3,x]
```

output
```
x*(a^6 + (6*a^5*c*x^n)/(1 + n) + (15*a^4*c^2*x^(2*n))/(1 + 2*n) + (20*a^3*
c^3*x^(3*n))/(1 + 3*n) + (15*a^2*c^4*x^(4*n))/(1 + 4*n) + (6*a*c^5*x^(5*n)
)/(1 + 5*n) + (c^6*x^(6*n))/(1 + 6*n))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1379, 775, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^3 dx$$

$$\Big\downarrow \text{1379}$$

$$\frac{\int \left(c^2x^n+ac\right)^6 dx}{c^6}$$

$$\Big\downarrow \text{775}$$

$$\frac{\int \left(6a^5c^7x^n+15a^4c^8x^{2n}+20a^3c^9x^{3n}+15a^2c^{10}x^{4n}+6ac^{11}x^{5n}+c^{12}x^{6n}+a^6c^6\right) dx}{c^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6c^6x+\frac{6a^5c^7x^{n+1}}{n+1}+\frac{15a^4c^8x^{2n+1}}{2n+1}+\frac{20a^3c^9x^{3n+1}}{3n+1}+\frac{15a^2c^{10}x^{4n+1}}{4n+1}+\frac{6ac^{11}x^{5n+1}}{5n+1}+\frac{c^{12}x^{6n+1}}{6n+1}}{c^6}$$

input
```
Int[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^3,x]
```

output
```
(a^6*c^6*x + (6*a^5*c^7*x^(1 + n))/(1 + n) + (15*a^4*c^8*x^(1 + 2*n))/(1 +
 2*n) + (20*a^3*c^9*x^(1 + 3*n))/(1 + 3*n) + (15*a^2*c^10*x^(1 + 4*n))/(1
+ 4*n) + (6*a*c^11*x^(1 + 5*n))/(1 + 5*n) + (c^12*x^(1 + 6*n))/(1 + 6*n))/
c^6
```

Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

method	result
risch	$a^6x+\frac{c^6x\,x^{6n}}{1+6n}+\frac{6a\,c^5x\,x^{5n}}{1+5n}+\frac{15a^2c^4x\,x^{4n}}{1+4n}+\frac{20a^3c^3x\,x^{3n}}{1+3n}+\frac{15a^4c^2x\,x^{2n}}{1+2n}+\frac{6a^5cx\,x^n}{1+n}$
norman	$a^6x+\frac{c^6x\,e^{6n\ln(x)}}{1+6n}+\frac{6a\,c^5x\,e^{5n\ln(x)}}{1+5n}+\frac{15a^2c^4x\,e^{4n\ln(x)}}{1+4n}+\frac{20a^3c^3x\,e^{3n\ln(x)}}{1+3n}+\frac{15a^4c^2x\,e^{2n\ln(x)}}{1+2n}+\frac{6a^5cx\,e^{n\ln(x)}}{1+n}$
parallelrisch	Expression too large to display
orering	Expression too large to display

input
```
int((a^2+2*a*c*x^n+c^2*x^(2*n))^3,x,method=_RETURNVERBOSE)
```

output
```
a^6*x+c^6/(1+6*n)*x*(x^n)^6+6*a*c^5/(1+5*n)*x*(x^n)^5+15*a^2*c^4/(1+4*n)*x
*(x^n)^4+20*a^3*c^3/(1+3*n)*x*(x^n)^3+15*a^4*c^2/(1+2*n)*x*(x^n)^2+6*a^5*c
/(1+n)*x*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(126) = 252$.

Time = 0.09 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.60

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^3\,dx$$
$$= \frac{(120\,c^6n^5+274\,c^6n^4+225\,c^6n^3+85\,c^6n^2+15\,c^6n+c^6)xx^{6\,n}+6\,(144\,ac^5n^5+324\,ac^5n^4+260\,ac^5n^3+}{}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^3,x, algorithm="fricas")
```

output
```
((120*c^6*n^5 + 274*c^6*n^4 + 225*c^6*n^3 + 85*c^6*n^2 + 15*c^6*n + c^6)*x
*x^(6*n) + 6*(144*a*c^5*n^5 + 324*a*c^5*n^4 + 260*a*c^5*n^3 + 95*a*c^5*n^2
 + 16*a*c^5*n + a*c^5)*x*x^(5*n) + 15*(180*a^2*c^4*n^5 + 396*a^2*c^4*n^4 +
 307*a^2*c^4*n^3 + 107*a^2*c^4*n^2 + 17*a^2*c^4*n + a^2*c^4)*x*x^(4*n) + 2
0*(240*a^3*c^3*n^5 + 508*a^3*c^3*n^4 + 372*a^3*c^3*n^3 + 121*a^3*c^3*n^2 +
 18*a^3*c^3*n + a^3*c^3)*x*x^(3*n) + 15*(360*a^4*c^2*n^5 + 702*a^4*c^2*n^4
 + 461*a^4*c^2*n^3 + 137*a^4*c^2*n^2 + 19*a^4*c^2*n + a^4*c^2)*x*x^(2*n) +
 6*(720*a^5*c*n^5 + 1044*a^5*c*n^4 + 580*a^5*c*n^3 + 155*a^5*c*n^2 + 20*a^
5*c*n + a^5*c)*x*x^n + (720*a^6*n^6 + 1764*a^6*n^5 + 1624*a^6*n^4 + 735*a^
6*n^3 + 175*a^6*n^2 + 21*a^6*n + a^6)*x)/(720*n^6 + 1764*n^5 + 1624*n^4 +
735*n^3 + 175*n^2 + 21*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2445 vs. $2(114) = 228$.

Time = 6.24 (sec) , antiderivative size = 2445, normalized size of antiderivative = 19.40

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((a**2+2*a*c*x**n+c**2*x**(2*n))**3,x)
```

output
```
Piecewise((a**6*x + 6*a**5*c*log(x) - 15*a**4*c**2/x - 10*a**3*c**3/x**2 -
 5*a**2*c**4/x**3 - 3*a*c**5/(2*x**4) - c**6/(5*x**5), Eq(n, -1)), (a**6*x
 + 12*a**5*c*sqrt(x) + 15*a**4*c**2*log(x) - 40*a**3*c**3/sqrt(x) - 15*a**
2*c**4/x - 4*a*c**5/x**(3/2) - c**6/(2*x**2), Eq(n, -1/2)), (a**6*x + 9*a*
*5*c*x**(2/3) + 45*a**4*c**2*x**(1/3) + 20*a**3*c**3*log(x) - 45*a**2*c**4
/x**(1/3) - 9*a*c**5/x**(2/3) - c**6/x, Eq(n, -1/3)), (a**6*x + 8*a**5*c*x
**(3/4) + 30*a**4*c**2*sqrt(x) + 80*a**3*c**3*x**(1/4) + 60*a**2*c**4*log(
x**(1/4)) - 24*a*c**5/x**(1/4) - 2*c**6/sqrt(x), Eq(n, -1/4)), (a**6*x + 1
5*a**5*c*x**(4/5)/2 + 25*a**4*c**2*x**(3/5) + 50*a**3*c**3*x**(2/5) + 75*a
**2*c**4*x**(1/5) + 30*a*c**5*log(x**(1/5)) - 5*c**6/x**(1/5), Eq(n, -1/5)
), (a**6*x + 36*a**5*c*x**(5/6)/5 + 45*a**4*c**2*x**(2/3)/2 + 40*a**3*c**3
*sqrt(x) + 45*a**2*c**4*x**(1/3) + 36*a*c**5*x**(1/6) + 6*c**6*log(x**(1/6
)), Eq(n, -1/6)), (720*a**6*n**6*x/(720*n**6 + 1764*n**5 + 1624*n**4 + 735
*n**3 + 175*n**2 + 21*n + 1) + 1764*a**6*n**5*x/(720*n**6 + 1764*n**5 + 16
24*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 1624*a**6*n**4*x/(720*n**6 + 1
764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 735*a**6*n**3*x/(
720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n + 1) + 175*a
**6*n**2*x/(720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 + 21*n
+ 1) + 21*a**6*n*x/(720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2
 + 21*n + 1) + a**6*x/(720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 17...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.52

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^3\,dx = a^6x+\frac{c^6x^{6\,n+1}}{6\,n+1}+\frac{6\,ac^5x^{5\,n+1}}{5\,n+1}+\frac{12\,a^2c^4x^{4\,n+1}}{4\,n+1}+\frac{8\,a^3c^3x^{3\,n+1}}{3\,n+1}+3\left(\frac{c^2x^{2\,n+1}}{2\,n+1}+\frac{2\,acx^{n+1}}{n+1}\right)a^4+3\left(\frac{c^4x^{4\,n+1}}{4\,n+1}+\frac{4\,ac^3x^{3\,n+1}}{3\,n+1}+\frac{4\,a^2c^2x^{2\,n+1}}{2\,n+1}\right)a^2$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^3,x, algorithm="maxima")
```

output
```
a^6*x + c^6*x^(6*n + 1)/(6*n + 1) + 6*a*c^5*x^(5*n + 1)/(5*n + 1) + 12*a^2
*c^4*x^(4*n + 1)/(4*n + 1) + 8*a^3*c^3*x^(3*n + 1)/(3*n + 1) + 3*(c^2*x^(2
*n + 1)/(2*n + 1) + 2*a*c*x^(n + 1)/(n + 1))*a^4 + 3*(c^4*x^(4*n + 1)/(4*n
 + 1) + 4*a*c^3*x^(3*n + 1)/(3*n + 1) + 4*a^2*c^2*x^(2*n + 1)/(2*n + 1))*a
^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(126) = 252$.

Time = 0.13 (sec) , antiderivative size = 618, normalized size of antiderivative = 4.90

$$\int \left(a^2 + 2acx^n + c^2x^{2n}\right)^3 dx$$
$$= \frac{720\, a^6 n^6 x + 120\, c^6 n^5 x x^{6\,n} + 864\, a c^5 n^5 x x^{5\,n} + 2700\, a^2 c^4 n^5 x x^{4\,n} + 4800\, a^3 c^3 n^5 x x^{3\,n} + 5400\, a^4 c^2 n^5 x x^{2\,n} + \cdots}{\cdots}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^3,x, algorithm="giac")
```

output
```
(720*a^6*n^6*x + 120*c^6*n^5*x*x^(6*n) + 864*a*c^5*n^5*x*x^(5*n) + 2700*a^
2*c^4*n^5*x*x^(4*n) + 4800*a^3*c^3*n^5*x*x^(3*n) + 5400*a^4*c^2*n^5*x*x^(2
*n) + 4320*a^5*c*n^5*x*x^n + 1764*a^6*n^5*x + 274*c^6*n^4*x*x^(6*n) + 1944
*a*c^5*n^4*x*x^(5*n) + 5940*a^2*c^4*n^4*x*x^(4*n) + 10160*a^3*c^3*n^4*x*x^
(3*n) + 10530*a^4*c^2*n^4*x*x^(2*n) + 6264*a^5*c*n^4*x*x^n + 1624*a^6*n^4*
x + 225*c^6*n^3*x*x^(6*n) + 1560*a*c^5*n^3*x*x^(5*n) + 4605*a^2*c^4*n^3*x*
x^(4*n) + 7440*a^3*c^3*n^3*x*x^(3*n) + 6915*a^4*c^2*n^3*x*x^(2*n) + 3480*a
^5*c*n^3*x*x^n + 735*a^6*n^3*x + 85*c^6*n^2*x*x^(6*n) + 570*a*c^5*n^2*x*x^
(5*n) + 1605*a^2*c^4*n^2*x*x^(4*n) + 2420*a^3*c^3*n^2*x*x^(3*n) + 2055*a^4
*c^2*n^2*x*x^(2*n) + 930*a^5*c*n^2*x*x^n + 175*a^6*n^2*x + 15*c^6*n*x*x^(6
*n) + 96*a*c^5*n*x*x^(5*n) + 255*a^2*c^4*n*x*x^(4*n) + 360*a^3*c^3*n*x*x^(
3*n) + 285*a^4*c^2*n*x*x^(2*n) + 120*a^5*c*n*x*x^n + 21*a^6*n*x + c^6*x*x^
(6*n) + 6*a*c^5*x*x^(5*n) + 15*a^2*c^4*x*x^(4*n) + 20*a^3*c^3*x*x^(3*n) +
15*a^4*c^2*x*x^(2*n) + 6*a^5*c*x*x^n + a^6*x)/(720*n^6 + 1764*n^5 + 1624*n
^4 + 735*n^3 + 175*n^2 + 21*n + 1)
```

Mupad [B] (verification not implemented)

Time = 20.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^3 dx = a^6 x + \frac{c^6 x x^{6n}}{6n+1} + \frac{6 a c^5 x x^{5n}}{5n+1} + \frac{6 a^5 c x x^n}{n+1} + \frac{15 a^4 c^2 x x^{2n}}{2n+1} + \frac{20 a^3 c^3 x x^{3n}}{3n+1} + \frac{15 a^2 c^4 x x^{4n}}{4n+1}$$

input
```
int((a^2 + c^2*x^(2*n) + 2*a*c*x^n)^3,x)
```

output
```
a^6*x + (c^6*x*x^(6*n))/(6*n + 1) + (6*a*c^5*x*x^(5*n))/(5*n + 1) + (6*a^5
*c*x*x^n)/(n + 1) + (15*a^4*c^2*x*x^(2*n))/(2*n + 1) + (20*a^3*c^3*x*x^(3*
n))/(3*n + 1) + (15*a^2*c^4*x*x^(4*n))/(4*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 575, normalized size of antiderivative = 4.56

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^3 dx = \frac{x(864x^{5n}a c^5n^5 + 1944x^{5n}a c^5n^4 + 1560x^{5n}a c^5n^3 + 570x^{5n}a c^5n^2 + 96x^{5n}a c^5n + 2700x^{4n}a^2c^4n^5 + 5940}{}$$

input
```
int((a^2+2*a*c*x^n+c^2*x^(2*n))^3,x)
```

output

```
(x*(120*x**(6*n)*c**6*n**5 + 274*x**(6*n)*c**6*n**4 + 225*x**(6*n)*c**6*n*
*3 + 85*x**(6*n)*c**6*n**2 + 15*x**(6*n)*c**6*n + x**(6*n)*c**6 + 864*x**(
5*n)*a*c**5*n**5 + 1944*x**(5*n)*a*c**5*n**4 + 1560*x**(5*n)*a*c**5*n**3 +
 570*x**(5*n)*a*c**5*n**2 + 96*x**(5*n)*a*c**5*n + 6*x**(5*n)*a*c**5 + 270
0*x**(4*n)*a**2*c**4*n**5 + 5940*x**(4*n)*a**2*c**4*n**4 + 4605*x**(4*n)*a
**2*c**4*n**3 + 1605*x**(4*n)*a**2*c**4*n**2 + 255*x**(4*n)*a**2*c**4*n +
15*x**(4*n)*a**2*c**4 + 4800*x**(3*n)*a**3*c**3*n**5 + 10160*x**(3*n)*a**3
*c**3*n**4 + 7440*x**(3*n)*a**3*c**3*n**3 + 2420*x**(3*n)*a**3*c**3*n**2 +
 360*x**(3*n)*a**3*c**3*n + 20*x**(3*n)*a**3*c**3 + 5400*x**(2*n)*a**4*c**
2*n**5 + 10530*x**(2*n)*a**4*c**2*n**4 + 6915*x**(2*n)*a**4*c**2*n**3 + 20
55*x**(2*n)*a**4*c**2*n**2 + 285*x**(2*n)*a**4*c**2*n + 15*x**(2*n)*a**4*c
**2 + 4320*x**n*a**5*c*n**5 + 6264*x**n*a**5*c*n**4 + 3480*x**n*a**5*c*n**
3 + 930*x**n*a**5*c*n**2 + 120*x**n*a**5*c*n + 6*x**n*a**5*c + 720*a**6*n*
*6 + 1764*a**6*n**5 + 1624*a**6*n**4 + 735*a**6*n**3 + 175*a**6*n**2 + 21*
a**6*n + a**6))/(720*n**6 + 1764*n**5 + 1624*n**4 + 735*n**3 + 175*n**2 +
21*n + 1)
```

3.107 $\int (a^2 + 2acx^n + c^2x^{2n})^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^2 \, dx = a^4x+\frac{4a^3cx^{1+n}}{1+n}+\frac{6a^2c^2x^{1+2n}}{1+2n}+\frac{4ac^3x^{1+3n}}{1+3n}+\frac{c^4x^{1+4n}}{1+4n}$$

output
```
a^4*x+4*a^3*c*x^(1+n)/(1+n)+6*a^2*c^2*x^(1+2*n)/(1+2*n)+4*a*c^3*x^(1+3*n)/
(1+3*n)+c^4*x^(1+4*n)/(1+4*n)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^2 \, dx = x\left(a^4+\frac{4a^3cx^n}{1+n}+\frac{6a^2c^2x^{2n}}{1+2n}+\frac{4ac^3x^{3n}}{1+3n}+\frac{c^4x^{4n}}{1+4n}\right)$$

input
```
Integrate[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^2,x]
```

output
```
x*(a^4 + (4*a^3*c*x^n)/(1 + n) + (6*a^2*c^2*x^(2*n))/(1 + 2*n) + (4*a*c^3*
x^(3*n))/(1 + 3*n) + (c^4*x^(4*n))/(1 + 4*n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1379, 775, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^2 dx$$

$$\Big\downarrow \text{1379}$$

$$\frac{\int \left(c^2x^n+ac\right)^4dx}{c^4}$$

$$\Big\downarrow \text{775}$$

$$\frac{\int \left(4a^3c^5x^n+6a^2c^6x^{2n}+4ac^7x^{3n}+c^8x^{4n}+a^4c^4\right)dx}{c^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4c^4x+\frac{4a^3c^5x^{n+1}}{n+1}+\frac{6a^2c^6x^{2n+1}}{2n+1}+\frac{4ac^7x^{3n+1}}{3n+1}+\frac{c^8x^{4n+1}}{4n+1}}{c^4}$$

input
```
Int[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^2,x]
```

output
```
(a^4*c^4*x + (4*a^3*c^5*x^(1 + n))/(1 + n) + (6*a^2*c^6*x^(1 + 2*n))/(1 +
2*n) + (4*a*c^7*x^(1 + 3*n))/(1 + 3*n) + (c^8*x^(1 + 4*n))/(1 + 4*n))/c^4
```

Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result
risch	$a^4x+\frac{c^4 x\, x^{4n}}{1+4n}+\frac{4a\,c^3 x\, x^{3n}}{1+3n}+\frac{6a^2c^2x\, x^{2n}}{1+2n}+\frac{4a^3cx\, x^{n}}{1+n}$
norman	$a^4x+\frac{c^4x\, e^{4n\ln(x)}}{1+4n}+\frac{4a\,c^3x\, e^{3n\ln(x)}}{1+3n}+\frac{6a^2c^2x\, e^{2n\ln(x)}}{1+2n}+\frac{4a^3cx\, e^{n\ln(x)}}{1+n}$
parallelrisch	$\frac{32x\, x^n x^{2n} a\, c^3 n^3+56x\, x^n x^{2n} a\, c^3 n^2+28x\, x^n x^{2n} a\, c^3 n+4x\, x^n a^3 c+6x\, x^{2n} a^2 c^2+96x\, x^n a^3 c\, n^3+72x\, x^{2n} a^2 c^2 n^3+104x\, x^n a^3 c\, n}{}$
orering	$x(a^2+2ac\,x^n+c^2x^{2n})^2-\frac{20x^2n(5n^2+1)(a^2+2ac\,x^n+c^2x^{2n})\left(\frac{2ac\,x^n n}{x}+\frac{2c^2x^{2n}n}{x}\right)}{(12n^3+19n^2+8n+1)(1+2n)}+\frac{5x^3(7n^2-4n+1)\left(2\left(\frac{2ac}{}\right.\right.}{}$

input
```
int((a^2+2*a*c*x^n+c^2*x^(2*n))^2,x,method=_RETURNVERBOSE)
```

output
```
a^4*x+c^4/(1+4*n)*x*(x^n)^4+4*a*c^3/(1+3*n)*x*(x^n)^3+6*a^2*c^2/(1+2*n)*x*
(x^n)^2+4*a^3*c/(1+n)*x*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(82) = 164.

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.63

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^2\,dx$$
$$=\frac{(6\,c^4n^3+11\,c^4n^2+6\,c^4n+c^4)xx^{4\,n}+4\,(8\,ac^3n^3+14\,ac^3n^2+7\,ac^3n+ac^3)xx^{3\,n}+6\,(12\,a^2c^2n^3+19\,a^2c}{24\,n^4+}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^2,x, algorithm="fricas")
```

output
```
((6*c^4*n^3 + 11*c^4*n^2 + 6*c^4*n + c^4)*x*x^(4*n) + 4*(8*a*c^3*n^3 + 14*
a*c^3*n^2 + 7*a*c^3*n + a*c^3)*x*x^(3*n) + 6*(12*a^2*c^2*n^3 + 19*a^2*c^2*
n^2 + 8*a^2*c^2*n + a^2*c^2)*x*x^(2*n) + 4*(24*a^3*c*n^3 + 26*a^3*c*n^2 +
9*a^3*c*n + a^3*c)*x*x^n + (24*a^4*n^4 + 50*a^4*n^3 + 35*a^4*n^2 + 10*a^4*
n + a^4)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(73) = 146$.

Time = 1.79 (sec) , antiderivative size = 918, normalized size of antiderivative = 11.20

$$\int \left(a^2 + 2acx^n + c^2x^{2n}\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((a**2+2*a*c*x**n+c**2*x**(2*n))**2,x)
```

output
```
Piecewise((a**4*x + 4*a**3*c*log(x) - 6*a**2*c**2/x - 2*a*c**3/x**2 - c**4
/(3*x**3), Eq(n, -1)), (a**4*x + 8*a**3*c*sqrt(x) + 6*a**2*c**2*log(x) - 8
*a*c**3/sqrt(x) - c**4/x, Eq(n, -1/2)), (a**4*x + 6*a**3*c*x**(2/3) + 18*a
**2*c**2*x**(1/3) + 4*a*c**3*log(x) - 3*c**4/x**(1/3), Eq(n, -1/3)), (a**4
*x + 16*a**3*c*x**(3/4)/3 + 12*a**2*c**2*sqrt(x) + 16*a*c**3*x**(1/4) + 4*
c**4*log(x**(1/4)), Eq(n, -1/4)), (24*a**4*n**4*x/(24*n**4 + 50*n**3 + 35*
n**2 + 10*n + 1) + 50*a**4*n**3*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + 35*a**4*n**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 10*a**4*n*x/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a**4*x/(24*n**4 + 50*n**3 + 35*n
**2 + 10*n + 1) + 96*a**3*c*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*
n + 1) + 104*a**3*c*n**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) +
 36*a**3*c*n*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 4*a**3*c*x*
x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 72*a**2*c**2*n**3*x*x**(2*
n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 114*a**2*c**2*n**2*x*x**(2*n
)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a**2*c**2*n*x*x**(2*n)/(24
*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*a**2*c**2*x*x**(2*n)/(24*n**4 +
50*n**3 + 35*n**2 + 10*n + 1) + 32*a*c**3*n**3*x*x**(3*n)/(24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1) + 56*a*c**3*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 +
35*n**2 + 10*n + 1) + 28*a*c**3*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2
+ 10*n + 1) + 4*a*c**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^2 dx = a^4x+\frac{c^4x^{4n+1}}{4n+1}+\frac{4ac^3x^{3n+1}}{3n+1}+\frac{4a^2c^2x^{2n+1}}{2n+1}+2\left(\frac{c^2x^{2n+1}}{2n+1}+\frac{2acx^{n+1}}{n+1}\right)a^2$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^2,x, algorithm="maxima")
```

output
```
a^4*x + c^4*x^(4*n + 1)/(4*n + 1) + 4*a*c^3*x^(3*n + 1)/(3*n + 1) + 4*a^2*
c^2*x^(2*n + 1)/(2*n + 1) + 2*(c^2*x^(2*n + 1)/(2*n + 1) + 2*a*c*x^(n + 1)
/(n + 1))*a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(82) = 164.

Time = 0.13 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.39

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^2 dx$$
$$= \frac{24a^4n^4x+6c^4n^3xx^{4n}+32ac^3n^3xx^{3n}+72a^2c^2n^3xx^{2n}+96a^3cn^3xx^n+50a^4n^3x+11c^4n^2xx^{4n}+56}{}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^2,x, algorithm="giac")
```

output
```
(24*a^4*n^4*x + 6*c^4*n^3*x*x^(4*n) + 32*a*c^3*n^3*x*x^(3*n) + 72*a^2*c^2*
n^3*x*x^(2*n) + 96*a^3*c*n^3*x*x^n + 50*a^4*n^3*x + 11*c^4*n^2*x*x^(4*n) +
 56*a*c^3*n^2*x*x^(3*n) + 114*a^2*c^2*n^2*x*x^(2*n) + 104*a^3*c*n^2*x*x^n
+ 35*a^4*n^2*x + 6*c^4*n*x*x^(4*n) + 28*a*c^3*n*x*x^(3*n) + 48*a^2*c^2*n*x
*x^(2*n) + 36*a^3*c*n*x*x^n + 10*a^4*n*x + c^4*x*x^(4*n) + 4*a*c^3*x*x^(3*
n) + 6*a^2*c^2*x*x^(2*n) + 4*a^3*c*x*x^n + a^4*x)/(24*n^4 + 50*n^3 + 35*n^
2 + 10*n + 1)
```

Mupad [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \left(a^2 + 2acx^n + c^2x^{2n}\right)^2 \, dx = a^4\,x + \frac{c^4\,x\,x^{4\,n}}{4\,n+1} + \frac{4\,a\,c^3\,x\,x^{3\,n}}{3\,n+1} + \frac{4\,a^3\,c\,x\,x^n}{n+1} + \frac{6\,a^2\,c^2\,x\,x^{2\,n}}{2\,n+1}$$

input
```
int((a^2 + c^2*x^(2*n) + 2*a*c*x^n)^2,x)
```

output
```
a^4*x + (c^4*x*x^(4*n))/(4*n + 1) + (4*a*c^3*x*x^(3*n))/(3*n + 1) + (4*a^3
*c*x*x^n)/(n + 1) + (6*a^2*c^2*x*x^(2*n))/(2*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.13

$$\int \left(a^2 + 2acx^n + c^2x^{2n}\right)^2 \, dx$$
$$= \frac{x(6x^{4n}c^4n^3 + 11x^{4n}c^4n^2 + 6x^{4n}c^4n + x^{4n}c^4 + 32x^{3n}a\,c^3n^3 + 56x^{3n}a\,c^3n^2 + 28x^{3n}a\,c^3n + 4x^{3n}a\,c^3 + 72x}{}$$

input
```
int((a^2+2*a*c*x^n+c^2*x^(2*n))^2,x)
```

output
```
(x*(6*x**(4*n)*c**4*n**3 + 11*x**(4*n)*c**4*n**2 + 6*x**(4*n)*c**4*n + x**
(4*n)*c**4 + 32*x**(3*n)*a*c**3*n**3 + 56*x**(3*n)*a*c**3*n**2 + 28*x**(3*
n)*a*c**3*n + 4*x**(3*n)*a*c**3 + 72*x**(2*n)*a**2*c**2*n**3 + 114*x**(2*n
)*a**2*c**2*n**2 + 48*x**(2*n)*a**2*c**2*n + 6*x**(2*n)*a**2*c**2 + 96*x**
n*a**3*c*n**3 + 104*x**n*a**3*c*n**2 + 36*x**n*a**3*c*n + 4*x**n*a**3*c +
24*a**4*n**4 + 50*a**4*n**3 + 35*a**4*n**2 + 10*a**4*n + a**4))/(24*n**4 +
 50*n**3 + 35*n**2 + 10*n + 1)
```

3.108 $\int (a^2 + 2acx^n + c^2x^{2n}) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int (a^2 + 2acx^n + c^2x^{2n}) \, dx = a^2x + \frac{2acx^{1+n}}{1+n} + \frac{c^2x^{1+2n}}{1+2n}$$

output
```
a^2*x+2*a*c*x^(1+n)/(1+n)+c^2*x^(1+2*n)/(1+2*n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int (a^2 + 2acx^n + c^2x^{2n}) \, dx = x\left(a^2 + \frac{2acx^n}{1+n} + \frac{c^2x^{2n}}{1+2n}\right)$$

input
```
Integrate[a^2 + 2*a*c*x^n + c^2*x^(2*n),x]
```

output
```
x*(a^2 + (2*a*c*x^n)/(1 + n) + (c^2*x^(2*n))/(1 + 2*n))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2acx^n+c^2x^{2n}\right) dx$$

$$\Big\downarrow \; 2009$$

$$a^2x+\frac{2acx^{n+1}}{n+1}+\frac{c^2x^{2n+1}}{2n+1}$$

input
```
Int[a^2 + 2*a*c*x^n + c^2*x^(2*n),x]
```

output
```
a^2*x + (2*a*c*x^(1 + n))/(1 + n) + (c^2*x^(1 + 2*n))/(1 + 2*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
risch	$x\,a^{2}+\frac{2 a c x\,x^{n}}{1+n}+\frac{c^{2} x\,x^{2 n}}{1+2 n}$	37
default	$x\,a^{2}+\frac{2 a c\,x^{1+n}}{1+n}+\frac{c^{2} x^{1+2 n}}{1+2 n}$	39
parts	$x\,a^{2}+\frac{2 a c\,x^{1+n}}{1+n}+\frac{c^{2} x^{1+2 n}}{1+2 n}$	39
norman	$x\,a^{2}+\frac{2 a c x\,{\mathrm e}^{n \ln \left(x \right)}}{1+n}+\frac{c^{2} x\,{\mathrm e}^{2 n \ln \left(x \right)}}{1+2 n}$	41
parallelrisch	$\frac{4 a c\,x^{n} x n +c^{2} x^{2 n} x n +2 a c\,x^{n} x +c^{2} x^{2 n} x}{\left(1+n \right) \left(1+2 n \right)}+x\,a^{2}$	59
orering	$x \left(a^{2}+2 a c\,x^{n}+c^{2} x^{2 n}\right)-\frac{3 n\,x^{2} \left(\frac{2 a c\,x^{n} n}{x}+\frac{2 c^{2} x^{2 n} n}{x}\right)}{2 n^{2}+3 n +1}+\frac{x^{3} \left(\frac{2 a c\,x^{n} n^{2}}{x^{2}}-\frac{2 a c\,x^{n} n}{x^{2}}+\frac{4 c^{2} x^{2 n} n^{2}}{x^{2}}-\frac{2 c^{2} x^{2 n} n}{x^{2}}\right)}{2 n^{2}+3 n +1}$	139

input
```
int(a^2+2*a*c*x^n+c^2*x^(2*n),x,method=_RETURNVERBOSE)
```

output
```
x*a^2+2*a*c/(1+n)*x*x^n+c^2/(1+2*n)*x*x^(2*n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)\,dx = \frac{(c^2n+c^2)xx^{2\,n}+2\,(2\,acn+ac)xx^n+(2\,a^2n^2+3\,a^2n+a^2)x}{2\,n^2+3\,n+1}$$

input
```
integrate(a^2+2*a*c*x^n+c^2*x^(2*n),x, algorithm="fricas")
```

output
```
((c^2*n + c^2)*x*x^(2*n) + 2*(2*a*c*n + a*c)*x*x^n + (2*a^2*n^2 + 3*a^2*n
+ a^2)*x)/(2*n^2 + 3*n + 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \left(a^2 + 2acx^n + c^2x^{2n}\right) dx = a^2x + 2ac\left(\begin{cases} \frac{x^{n+1}}{n+1} & \text{for}\ n \neq -1 \\ \log\left(x\right) & \text{otherwise} \end{cases}\right) + c^2\left(\begin{cases} \frac{x^{2n+1}}{2n+1} & \text{for}\ n \neq -\frac{1}{2} \\ \log\left(x\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a**2+2*a*c*x**n+c**2*x**(2*n),x)
```

output
```
a**2*x + 2*a*c*Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
+ c**2*Piecewise((x**(2*n + 1)/(2*n + 1), Ne(n, -1/2)), (log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \left(a^2 + 2acx^n + c^2x^{2n}\right) dx = a^2x + \frac{c^2x^{2n+1}}{2n+1} + \frac{2acx^{n+1}}{n+1}$$

input
```
integrate(a^2+2*a*c*x^n+c^2*x^(2*n),x, algorithm="maxima")
```

output
```
a^2*x + c^2*x^(2*n + 1)/(2*n + 1) + 2*a*c*x^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \left(a^2 + 2acx^n + c^2x^{2n}\right) dx = a^2x + \frac{c^2x^{2n+1}}{2n+1} + \frac{2acx^{n+1}}{n+1}$$

input
```
integrate(a^2+2*a*c*x^n+c^2*x^(2*n),x, algorithm="giac")
```

output
```
a^2*x + c^2*x^(2*n + 1)/(2*n + 1) + 2*a*c*x^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 19.90 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)\,dx=a^2\,x+\frac{c^2\,x\,x^{2\,n}}{2\,n+1}+\frac{2\,a\,c\,x\,x^n}{n+1}$$

input
```
int(a^2 + c^2*x^(2*n) + 2*a*c*x^n,x)
```

output
```
a^2*x + (c^2*x*x^(2*n))/(2*n + 1) + (2*a*c*x*x^n)/(n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)\,dx=\frac{x(x^{2n}c^2n+x^{2n}c^2+4x^nacn+2x^nac+2a^2n^2+3a^2n+a^2)}{2n^2+3n+1}$$

input
```
int(a^2+2*a*c*x^n+c^2*x^(2*n),x)
```

output
```
(x*(x**(2*n)*c**2*n + x**(2*n)*c**2 + 4*x**n*a*c*n + 2*x**n*a*c + 2*a**2*n
**2 + 3*a**2*n + a**2))/(2*n**2 + 3*n + 1)
```

3.109 $\int \frac{1}{a^2+2acx^n+c^2x^{2n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{1}{a^2+2acx^n+c^2x^{2n}}\,dx=\frac{x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^2}$$

output
```
x*hypergeom([2, 1/n],[1+1/n],-c*x^n/a)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+2acx^n+c^2x^{2n}}\,dx=\frac{x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^2}$$

input
```
Integrate[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(-1),x]
```

output
```
(x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((c*x^n)/a)])/a^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1379, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2 + 2acx^n + c^2x^{2n}} \, dx$$

$$\Big\downarrow \text{1379}$$

$$c^2 \int \frac{1}{(c^2x^n + ac)^2} dx$$

$$\Big\downarrow \text{778}$$

$$\frac{x \text{ Hypergeometric2F1}\left(2, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{cx^n}{a}\right)}{a^2}$$

input
```
Int[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(-1),x]
```

output
```
(x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((c*x^n)/a)])/a^2
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

Maple [F]

$$\int \frac{1}{a^2 + 2ac\,x^n + c^2x^{2n}}dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n)),x)
```

output
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{1}{a^2 + 2acx^n + c^2x^{2n}}\,dx = \int \frac{1}{c^2x^{2\,n} + 2\,acx^n + a^2}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(1/(c^2*x^(2*n) + 2*a*c*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{1}{a^2 + 2acx^n + c^2x^{2n}}\,dx = \int \frac{1}{a^2 + 2acx^n + c^2x^{2n}}\,dx$$

input
```
integrate(1/(a**2+2*a*c*x**n+c**2*x**(2*n)),x)
```

output
```
Integral(1/(a**2 + 2*a*c*x**n + c**2*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{1}{a^2 + 2acx^n + c^2x^{2n}}\,dx = \int \frac{1}{c^2x^{2\,n} + 2\,acx^n + a^2}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n)),x, algorithm="maxima")
```

output
```
(n - 1)*integrate(1/(a*c*n*x^n + a^2*n), x) + x/(a*c*n*x^n + a^2*n)
```

Giac [F]

$$\int \frac{1}{a^2 + 2acx^n + c^2x^{2n}}\,dx = \int \frac{1}{c^2x^{2\,n} + 2\,acx^n + a^2}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(1/(c^2*x^(2*n) + 2*a*c*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a^2 + 2acx^n + c^2x^{2n}}\,dx = \int \frac{1}{a^2 + c^2\,x^{2\,n} + 2\,a\,c\,x^n}\,dx$$

input
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n),x)
```

output
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n), x)
```

Reduce [F]

$$\int \frac{1}{a^2 + 2acx^n + c^2x^{2n}}\,dx = \int \frac{1}{x^{2n}c^2 + 2x^n ac + a^2}dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n)),x)
```

output
```
int(1/(x**(2*n)*c**2 + 2*x**n*a*c + a**2),x)
```

3.110 $\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^2}\,dx = \frac{x\,\text{Hypergeometric2F1}\left(4,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^4}$$

output
```
x*hypergeom([4, 1/n],[1+1/n],-c*x^n/a)/a^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^2}\,dx = \frac{x\,\text{Hypergeometric2F1}\left(4,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^4}$$

input
```
Integrate[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(-2),x]
```

output
```
(x*Hypergeometric2F1[4, n^(-1), 1 + n^(-1), -((c*x^n)/a)])/a^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1379, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^2}\,dx$$

$$\Big\downarrow \; 1379$$

$$c^4\int \frac{1}{(c^2x^n+ac)^4}dx$$

$$\Big\downarrow \; 778$$

$$\frac{x\,\text{Hypergeometric2F1}\left(4,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^4}$$

input
```
Int[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(-2),x]
```

output
```
(x*Hypergeometric2F1[4, n^(-1), 1 + n^(-1), -((c*x^n)/a)])/a^4
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

Maple [F]

$$\int \frac{1}{(a^2 + 2ac\,x^n + c^2x^{2n})^2} dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^2,x)
```

output
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^2} dx = \int \frac{1}{(c^2x^{2\,n} + 2\,acx^n + a^2)^2} dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(1/(c^4*x^(4*n) + 4*a^2*c^2*x^(2*n) + 4*a^3*c*x^n + a^4 + 2*(2*a*c
^3*x^n + a^2*c^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^2} dx = \int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^2} dx$$

input
```
integrate(1/(a**2+2*a*c*x**n+c**2*x**(2*n))**2,x)
```

output
```
Integral((a**2 + 2*a*c*x**n + c**2*x**(2*n))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^2}\,dx = \int \frac{1}{(c^2x^{2n} + 2\,acx^n + a^2)^2}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^2,x, algorithm="maxima")
```

output
```
(6*n^3 - 11*n^2 + 6*n - 1)*integrate(1/6/(a^3*c*n^3*x^n + a^4*n^3), x) + 1
/6*((6*n^2 - 5*n + 1)*c^2*x*x^(2*n) + (15*n^2 - 11*n + 2)*a*c*x*x^n + (11*
n^2 - 6*n + 1)*a^2*x)/(a^3*c^3*n^3*x^(3*n) + 3*a^4*c^2*n^3*x^(2*n) + 3*a^5
*c*n^3*x^n + a^6*n^3)
```

Giac [F]

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^2}\,dx = \int \frac{1}{(c^2x^{2n} + 2\,acx^n + a^2)^2}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate((c^2*x^(2*n) + 2*a*c*x^n + a^2)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^2}\,dx = \int \frac{1}{(a^2 + c^2\,x^{2\,n} + 2\,a\,c\,x^n)^2}\,dx$$

input
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n)^2,x)
```

output
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n)^2, x)
```

Reduce [F]

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^2} \, dx = \int \frac{1}{x^{4n}c^4 + 4x^{3n}a\,c^3 + 6x^{2n}a^2c^2 + 4x^na^3c + a^4} dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^2,x)
```

output
```
int(1/(x**(4*n)*c**4 + 4*x**(3*n)*a*c**3 + 6*x**(2*n)*a**2*c**2 + 4*x**n*a
**3*c + a**4),x)
```

3.111 $\int \frac{1}{\left(a^2+2acx^n+c^2x^{2n}\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{1}{\left(a^2+2acx^n+c^2x^{2n}\right)^3}\,dx=\frac{x\,\text{Hypergeometric2F1}\left(6,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^6}$$

output
```
x*hypergeom([6, 1/n],[1+1/n],-c*x^n/a)/a^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a^2+2acx^n+c^2x^{2n}\right)^3}\,dx=\frac{x\,\text{Hypergeometric2F1}\left(6,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^6}$$

input
```
Integrate[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(-3),x]
```

output
```
(x*Hypergeometric2F1[6, n^(-1), 1 + n^(-1), -((c*x^n)/a)])/a^6
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1379, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^3}dx$$

$$\Big\downarrow \text{1379}$$

$$c^6\int \frac{1}{(c^2x^n+ac)^6}dx$$

$$\Big\downarrow \text{778}$$

$$\frac{x\,\text{Hypergeometric2F1}\left(6,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^6}$$

input
```
Int[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(-3),x]
```

output
```
(x*Hypergeometric2F1[6, n^(-1), 1 + n^(-1), -((c*x^n)/a)])/a^6
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

Maple [F]

$$\int \frac{1}{(a^2 + 2ac\,x^n + c^2x^{2n})^3}dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^3,x)
```

output
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^3}\,dx = \int \frac{1}{(c^2x^{2\,n} + 2\,acx^n + a^2)^3}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^3,x, algorithm="fricas")
```

output
```
integral(1/(c^6*x^(6*n) + 8*a^3*c^3*x^(3*n) + 12*a^4*c^2*x^(2*n) + 6*a^5*c
*x^n + a^6 + 3*(2*a*c^5*x^n + a^2*c^4)*x^(4*n) + 3*(4*a^2*c^4*x^(2*n) + 4*
a^3*c^3*x^n + a^4*c^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^3}\,dx = \int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^3}\,dx$$

input
```
integrate(1/(a**2+2*a*c*x**n+c**2*x**(2*n))**3,x)
```

output
```
Integral((a**2 + 2*a*c*x**n + c**2*x**(2*n))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^3}\,dx = \int \frac{1}{(c^2x^{2\,n}+2\,acx^n+a^2)^3}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^3,x, algorithm="maxima")
```

output
```
(120*n^5 - 274*n^4 + 225*n^3 - 85*n^2 + 15*n - 1)*integrate(1/120/(a^5*c*n
^5*x^n + a^6*n^5), x) + 1/120*((120*n^4 - 154*n^3 + 71*n^2 - 14*n + 1)*c^4
*x*x^(4*n) + (540*n^4 - 663*n^3 + 296*n^2 - 57*n + 4)*a*c^3*x*x^(3*n) + (9
40*n^4 - 1083*n^3 + 464*n^2 - 87*n + 6)*a^2*c^2*x*x^(2*n) + (770*n^4 - 799
*n^3 + 324*n^2 - 59*n + 4)*a^3*c*x*x^n + (274*n^4 - 225*n^3 + 85*n^2 - 15*
n + 1)*a^4*x)/(a^5*c^5*n^5*x^(5*n) + 5*a^6*c^4*n^5*x^(4*n) + 10*a^7*c^3*n^
5*x^(3*n) + 10*a^8*c^2*n^5*x^(2*n) + 5*a^9*c*n^5*x^n + a^10*n^5)
```

Giac [F]

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^3}\,dx = \int \frac{1}{(c^2x^{2\,n}+2\,acx^n+a^2)^3}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^3,x, algorithm="giac")
```

output
```
integrate((c^2*x^(2*n) + 2*a*c*x^n + a^2)^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^3}\,dx = \int \frac{1}{(a^2+c^2\,x^{2\,n}+2\,a\,c\,x^n)^3}\,dx$$

input
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n)^3,x)
```

output
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n)^3, x)
```

Reduce [F]

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^3} \, dx$$
$$= \int \frac{1}{x^{6n}c^6 + 6x^{5n}a\,c^5 + 15x^{4n}a^2c^4 + 20x^{3n}a^3c^3 + 15x^{2n}a^4c^2 + 6x^n a^5c + a^6} dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^3,x)
```

output
```
int(1/(x**(6*n)*c**6 + 6*x**(5*n)*a*c**5 + 15*x**(4*n)*a**2*c**4 + 20*x**(
3*n)*a**3*c**3 + 15*x**(2*n)*a**4*c**2 + 6*x**n*a**5*c + a**6),x)
```

3.112 $\int (a^2+2acx^n+c^2x^{2n})^{5/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 324

$$\int (a^2+2acx^n+c^2x^{2n})^{5/2} \, dx = \frac{a^5x\sqrt{a^2+2acx^n+c^2x^{2n}}}{a+cx^n} + \frac{5a^4c^2x^{1+n}\sqrt{a^2+2acx^n+c^2x^{2n}}}{(1+n)(ac+c^2x^n)} + \frac{10a^3c^3x^{1+2n}\sqrt{a^2+2acx^n+c^2x^{2n}}}{(1+2n)(ac+c^2x^n)} + \frac{10a^2c^4x^{1+3n}\sqrt{a^2+2acx^n+c^2x^{2n}}}{(1+3n)(ac+c^2x^n)} + \frac{5ac^5x^{1+4n}\sqrt{a^2+2acx^n+c^2x^{2n}}}{(1+4n)(ac+c^2x^n)} + \frac{c^6x^{1+5n}\sqrt{a^2+2acx^n+c^2x^{2n}}}{(1+5n)(ac+c^2x^n)}$$

output
```
a^5*x*(a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2)/(a+c*x^n)+5*a^4*c^2*x^(1+n)*(a^2+2
*a*c*x^n+c^2*x^(2*n))^(1/2)/(1+n)/(a*c+c^2*x^n)+10*a^3*c^3*x^(1+2*n)*(a^2+
2*a*c*x^n+c^2*x^(2*n))^(1/2)/(1+2*n)/(a*c+c^2*x^n)+10*a^2*c^4*x^(1+3*n)*(a
^2+2*a*c*x^n+c^2*x^(2*n))^(1/2)/(1+3*n)/(a*c+c^2*x^n)+5*a*c^5*x^(1+4*n)*(a
^2+2*a*c*x^n+c^2*x^(2*n))^(1/2)/(1+4*n)/(a*c+c^2*x^n)+c^6*x^(1+5*n)*(a^2+2
*a*c*x^n+c^2*x^(2*n))^(1/2)/(1+5*n)/(a*c+c^2*x^n)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.36

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{5/2}\,dx=\frac{x\left((a+cx^n)^2\right)^{5/2}\left(a^5+\frac{5a^4cx^n}{1+n}+\frac{10a^3c^2x^{2n}}{1+2n}+\frac{10a^2c^3x^{3n}}{1+3n}+\frac{5ac^4x^{4n}}{1+4n}+\frac{c^5x^{5n}}{1+5n}\right)}{(a+cx^n)^5}$$

input
```
Integrate[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(5/2),x]
```

output
```
(x*((a + c*x^n)^2)^(5/2)*(a^5 + (5*a^4*c*x^n)/(1 + n) + (10*a^3*c^2*x^(2*n
))/(1 + 2*n) + (10*a^2*c^3*x^(3*n))/(1 + 3*n) + (5*a*c^4*x^(4*n))/(1 + 4*n
) + (c^5*x^(5*n))/(1 + 5*n)))/(a + c*x^n)^5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.46, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1384, 775, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2acx^n+c^2x^{2n}}\int \left(c^2x^n+ac\right)^5dx}{ac^5+c^6x^n}$$

$$\Big\downarrow \text{775}$$

$$\frac{\sqrt{a^2+2acx^n+c^2x^{2n}}\int \left(5a^4c^6x^n+10a^3c^7x^{2n}+10a^2c^8x^{3n}+5ac^9x^{4n}+c^{10}x^{5n}+a^5c^5\right)dx}{ac^5+c^6x^n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2acx^n+c^2x^{2n}}\left(a^5c^5x+\frac{5a^4c^6x^{n+1}}{n+1}+\frac{10a^3c^7x^{2n+1}}{2n+1}+\frac{10a^2c^8x^{3n+1}}{3n+1}+\frac{5ac^9x^{4n+1}}{4n+1}+\frac{c^{10}x^{5n+1}}{5n+1}\right)}{ac^5+c^6x^n}$$

input
```
Int[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*c*x^n + c^2*x^(2*n)]*(a^5*c^5*x + (5*a^4*c^6*x^(1 + n))/(1
 + n) + (10*a^3*c^7*x^(1 + 2*n))/(1 + 2*n) + (10*a^2*c^8*x^(1 + 3*n))/(1 +
 3*n) + (5*a*c^9*x^(1 + 4*n))/(1 + 4*n) + (c^10*x^(1 + 5*n))/(1 + 5*n)))/(
a*c^5 + c^6*x^n)
```

Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.68

method	result
risch	$\frac{\sqrt{\left(a+c\,x^{n}\right)^{2}}\,x\,a^{5}}{a+c\,x^{n}}+\frac{\sqrt{\left(a+c\,x^{n}\right)^{2}}\,c^{5}x\,x^{5n}}{\left(a+c\,x^{n}\right)\left(1+5n\right)}+\frac{5\sqrt{\left(a+c\,x^{n}\right)^{2}}\,a\,c^{4}x\,x^{4n}}{\left(a+c\,x^{n}\right)\left(1+4n\right)}+\frac{10\sqrt{\left(a+c\,x^{n}\right)^{2}}\,a^{2}c^{3}x\,x^{3n}}{\left(a+c\,x^{n}\right)\left(1+3n\right)}+\frac{10\sqrt{\left(a+c\,x^{n}\right)^{2}}\,c^{2}a^{3}x\,x^{2n}}{\left(a+c\,x^{n}\right)\left(1+2n\right)}$

input
```
int((a^2+2*a*c*x^n+c^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((a+c*x^n)^2)^(1/2)/(a+c*x^n)*x*a^5+((a+c*x^n)^2)^(1/2)/(a+c*x^n)*c^5/(1+5
*n)*x*(x^n)^5+5*((a+c*x^n)^2)^(1/2)/(a+c*x^n)*a*c^4/(1+4*n)*x*(x^n)^4+10*(
(a+c*x^n)^2)^(1/2)/(a+c*x^n)*a^2*c^3/(1+3*n)*x*(x^n)^3+10*((a+c*x^n)^2)^(1
/2)/(a+c*x^n)*c^2*a^3/(1+2*n)*x*(x^n)^2+5*((a+c*x^n)^2)^(1/2)/(a+c*x^n)*c*
a^4/(1+n)*x*x^n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00

$$\int \left(a^2 + 2acx^n + c^2x^{2n}\right)^{5/2} dx = \frac{(24\,c^5n^4 + 50\,c^5n^3 + 35\,c^5n^2 + 10\,c^5n + c^5)xx^{5\,n} + 5\,(30\,ac^4n^4 + 61\,ac^4n^3 + 41\,ac^4n^2 + 11}{}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^(5/2),x, algorithm="fricas")
```

output
```
((24*c^5*n^4 + 50*c^5*n^3 + 35*c^5*n^2 + 10*c^5*n + c^5)*x*x^(5*n) + 5*(30
*a*c^4*n^4 + 61*a*c^4*n^3 + 41*a*c^4*n^2 + 11*a*c^4*n + a*c^4)*x*x^(4*n) +
 10*(40*a^2*c^3*n^4 + 78*a^2*c^3*n^3 + 49*a^2*c^3*n^2 + 12*a^2*c^3*n + a^2
*c^3)*x*x^(3*n) + 10*(60*a^3*c^2*n^4 + 107*a^3*c^2*n^3 + 59*a^3*c^2*n^2 +
13*a^3*c^2*n + a^3*c^2)*x*x^(2*n) + 5*(120*a^4*c*n^4 + 154*a^4*c*n^3 + 71*
a^4*c*n^2 + 14*a^4*c*n + a^4*c)*x*x^n + (120*a^5*n^5 + 274*a^5*n^4 + 225*a
^5*n^3 + 85*a^5*n^2 + 15*a^5*n + a^5)*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85
*n^2 + 15*n + 1)
```

Sympy [F]

$$\int \left(a^2 + 2acx^n + c^2x^{2n}\right)^{5/2} dx = \int \left(a^2 + 2acx^n + c^2x^{2n}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a**2+2*a*c*x**n+c**2*x**(2*n))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*c*x**n + c**2*x**(2*n))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.68

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{5/2}\,dx = \frac{(24\,n^4+50\,n^3+35\,n^2+10\,n+1)c^5xx^{5\,n}+5\,(30\,n^4+61\,n^3+41\,n^2+11\,n+1)ac^4xx^{4\,n}}{}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^(5/2),x, algorithm="maxima")
```

output
```
((24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)*c^5*x*x^(5*n) + 5*(30*n^4 + 61*n^3
+ 41*n^2 + 11*n + 1)*a*c^4*x*x^(4*n) + 10*(40*n^4 + 78*n^3 + 49*n^2 + 12*n
 + 1)*a^2*c^3*x*x^(3*n) + 10*(60*n^4 + 107*n^3 + 59*n^2 + 13*n + 1)*a^3*c^
2*x*x^(2*n) + 5*(120*n^4 + 154*n^3 + 71*n^2 + 14*n + 1)*a^4*c*x*x^n + (120
*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)*a^5*x)/(120*n^5 + 274*n^4 +
225*n^3 + 85*n^2 + 15*n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. 2(312) = 624.

Time = 0.19 (sec) , antiderivative size = 679, normalized size of antiderivative = 2.10

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^(5/2),x, algorithm="giac")
```

output
```
(120*a^5*n^5*x*sgn(c*x^n + a) + 24*c^5*n^4*x*x^(5*n)*sgn(c*x^n + a) + 150*
a*c^4*n^4*x*x^(4*n)*sgn(c*x^n + a) + 400*a^2*c^3*n^4*x*x^(3*n)*sgn(c*x^n +
 a) + 600*a^3*c^2*n^4*x*x^(2*n)*sgn(c*x^n + a) + 600*a^4*c*n^4*x*x^n*sgn(c
*x^n + a) + 274*a^5*n^4*x*sgn(c*x^n + a) + 50*c^5*n^3*x*x^(5*n)*sgn(c*x^n
+ a) + 305*a*c^4*n^3*x*x^(4*n)*sgn(c*x^n + a) + 780*a^2*c^3*n^3*x*x^(3*n)*
sgn(c*x^n + a) + 1070*a^3*c^2*n^3*x*x^(2*n)*sgn(c*x^n + a) + 770*a^4*c*n^3
*x*x^n*sgn(c*x^n + a) + 225*a^5*n^3*x*sgn(c*x^n + a) + 35*c^5*n^2*x*x^(5*n
)*sgn(c*x^n + a) + 205*a*c^4*n^2*x*x^(4*n)*sgn(c*x^n + a) + 490*a^2*c^3*n^
2*x*x^(3*n)*sgn(c*x^n + a) + 590*a^3*c^2*n^2*x*x^(2*n)*sgn(c*x^n + a) + 35
5*a^4*c*n^2*x*x^n*sgn(c*x^n + a) + 85*a^5*n^2*x*sgn(c*x^n + a) + 10*c^5*n*
x*x^(5*n)*sgn(c*x^n + a) + 55*a*c^4*n*x*x^(4*n)*sgn(c*x^n + a) + 120*a^2*c
^3*n*x*x^(3*n)*sgn(c*x^n + a) + 130*a^3*c^2*n*x*x^(2*n)*sgn(c*x^n + a) + 7
0*a^4*c*n*x*x^n*sgn(c*x^n + a) + 15*a^5*n*x*sgn(c*x^n + a) + c^5*x*x^(5*n)
*sgn(c*x^n + a) + 5*a*c^4*x*x^(4*n)*sgn(c*x^n + a) + 10*a^2*c^3*x*x^(3*n)*
sgn(c*x^n + a) + 10*a^3*c^2*x*x^(2*n)*sgn(c*x^n + a) + 5*a^4*c*x*x^n*sgn(c
*x^n + a) + a^5*x*sgn(c*x^n + a))/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 +
15*n + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{5/2}\,dx=\int \left(a^2+c^2\,x^{2\,n}+2\,a\,c\,x^n\right)^{5/2}\,dx$$

input
```
int((a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(5/2),x)
```

output
```
int((a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.23

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{5/2}\,dx=\frac{x(a^5+24x^{5n}c^5n^4+50x^{5n}c^5n^3+35x^{5n}c^5n^2+10x^{5n}c^5n+5x^{4n}a\,c^4+10x^{3n}a^2c^3+10x^{2n}a^3}{}$$

input

```
int((a^2+2*a*c*x^n+c^2*x^(2*n))^(5/2),x)
```

output

```
(x*(24*x**(5*n)*c**5*n**4 + 50*x**(5*n)*c**5*n**3 + 35*x**(5*n)*c**5*n**2
+ 10*x**(5*n)*c**5*n + x**(5*n)*c**5 + 150*x**(4*n)*a*c**4*n**4 + 305*x**(
4*n)*a*c**4*n**3 + 205*x**(4*n)*a*c**4*n**2 + 55*x**(4*n)*a*c**4*n + 5*x**
(4*n)*a*c**4 + 400*x**(3*n)*a**2*c**3*n**4 + 780*x**(3*n)*a**2*c**3*n**3 +
 490*x**(3*n)*a**2*c**3*n**2 + 120*x**(3*n)*a**2*c**3*n + 10*x**(3*n)*a**2
*c**3 + 600*x**(2*n)*a**3*c**2*n**4 + 1070*x**(2*n)*a**3*c**2*n**3 + 590*x
**(2*n)*a**3*c**2*n**2 + 130*x**(2*n)*a**3*c**2*n + 10*x**(2*n)*a**3*c**2
+ 600*x**n*a**4*c*n**4 + 770*x**n*a**4*c*n**3 + 355*x**n*a**4*c*n**2 + 70*
x**n*a**4*c*n + 5*x**n*a**4*c + 120*a**5*n**5 + 274*a**5*n**4 + 225*a**5*n
**3 + 85*a**5*n**2 + 15*a**5*n + a**5))/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1)
```

3.113 $\int \left(a^2+2acx^n+c^2x^{2n}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{3/2} dx = \frac{a^3x\sqrt{a^2+2acx^n+c^2x^{2n}}}{a+cx^n} + \frac{3a^2c^2x^{1+n}\sqrt{a^2+2acx^n+c^2x^{2n}}}{(1+n)\left(ac+c^2x^n\right)} + \frac{3ac^3x^{1+2n}\sqrt{a^2+2acx^n+c^2x^{2n}}}{(1+2n)\left(ac+c^2x^n\right)} + \frac{c^4x^{1+3n}\sqrt{a^2+2acx^n+c^2x^{2n}}}{(1+3n)\left(ac+c^2x^n\right)}$$

output
```
a^3*x*(a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2)/(a+c*x^n)+3*a^2*c^2*x^(1+n)*(a^2+2
*a*c*x^n+c^2*x^(2*n))^(1/2)/(1+n)/(a*c+c^2*x^n)+3*a*c^3*x^(1+2*n)*(a^2+2*a
*c*x^n+c^2*x^(2*n))^(1/2)/(1+2*n)/(a*c+c^2*x^n)+c^4*x^(1+3*n)*(a^2+2*a*c*x
^n+c^2*x^(2*n))^(1/2)/(1+3*n)/(a*c+c^2*x^n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.59

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{3/2}\,dx=\frac{x\sqrt{(a+cx^n)^2}\left(a^3(1+6n+11n^2+6n^3)+3a^2c(1+5n+6n^2)\,x^n+3ac^2(1+4n+3n^2)\,x\right.}{(1+n)(1+2n)(1+3n)\,(a+cx^n)}$$

input
```
Integrate[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(3/2),x]
```

output
```
(x*Sqrt[(a + c*x^n)^2]*(a^3*(1 + 6*n + 11*n^2 + 6*n^3) + 3*a^2*c*(1 + 5*n
+ 6*n^2)*x^n + 3*a*c^2*(1 + 4*n + 3*n^2)*x^(2*n) + c^3*(1 + 3*n + 2*n^2)*x
^(3*n)))/((1 + n)*(1 + 2*n)*(1 + 3*n)*(a + c*x^n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.51, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {1384, 775, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2acx^n+c^2x^{2n}}\int \left(c^2x^n+ac\right)^3dx}{ac^3+c^4x^n}$$

$$\Big\downarrow \text{ 775}$$

$$\frac{\sqrt{a^2+2acx^n+c^2x^{2n}}\int \left(3a^2c^4x^n+3ac^5x^{2n}+c^6x^{3n}+a^3c^3\right)dx}{ac^3+c^4x^n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2acx^n+c^2x^{2n}}\left(a^3c^3x+\frac{3a^2c^4x^{n+1}}{n+1}+\frac{3ac^5x^{2n+1}}{2n+1}+\frac{c^6x^{3n+1}}{3n+1}\right)}{ac^3+c^4x^n}$$

input
```
Int[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*c*x^n + c^2*x^(2*n)]*(a^3*c^3*x + (3*a^2*c^4*x^(1 + n))/(1
 + n) + (3*a*c^5*x^(1 + 2*n))/(1 + 2*n) + (c^6*x^(1 + 3*n))/(1 + 3*n)))/(a
*c^3 + c^4*x^n)
```

Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{\sqrt{(a+c\,x^n)^2}\,a^3x}{a+c\,x^n}+\frac{\sqrt{(a+c\,x^n)^2}\,c^3x\,x^{3n}}{(a+c\,x^n)(1+3n)}+\frac{3\sqrt{(a+c\,x^n)^2}\,a\,c^2x\,x^{2n}}{(a+c\,x^n)(1+2n)}+\frac{3\sqrt{(a+c\,x^n)^2}\,a^2cx\,x^n}{(a+c\,x^n)(1+n)}$	138

input
```
int((a^2+2*a*c*x^n+c^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((a+c*x^n)^2)^(1/2)/(a+c*x^n)*a^3*x+((a+c*x^n)^2)^(1/2)/(a+c*x^n)*c^3/(1+3
*n)*x*(x^n)^3+3*((a+c*x^n)^2)^(1/2)/(a+c*x^n)*a*c^2/(1+2*n)*x*(x^n)^2+3*((
a+c*x^n)^2)^(1/2)/(a+c*x^n)*a^2*c/(1+n)*x*x^n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int (a^2+2acx^n +c^2x^{2n})^{3/2}\,dx = \frac{(2\,c^3n^2+3\,c^3n+c^3)xx^{3\,n}+3\,(3\,ac^2n^2+4\,ac^2n+ac^2)xx^{2\,n}+3\,(6\,a^2cn^2+5\,a^2cn+a^2c)x}{6\,n^3+11\,n^2+6\,n+1}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
((2*c^3*n^2 + 3*c^3*n + c^3)*x*x^(3*n) + 3*(3*a*c^2*n^2 + 4*a*c^2*n + a*c^
2)*x*x^(2*n) + 3*(6*a^2*c*n^2 + 5*a^2*c*n + a^2*c)*x*x^n + (6*a^3*n^3 + 11
*a^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [F]

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{3/2}\,dx = \int \left(a^2+2acx^n+c^2x^{2n}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a**2+2*a*c*x**n+c**2*x**(2*n))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*c*x**n + c**2*x**(2*n))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.49

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{3/2}\,dx=\frac{(2\,n^2+3\,n+1)c^3xx^{3\,n}+3\,(3\,n^2+4\,n+1)ac^2xx^{2\,n}+3\,(6\,n^2+5\,n+1)a^2cxx^n+(6\,n^3+}{6\,n^3+11\,n^2+6\,n+1}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
((2*n^2 + 3*n + 1)*c^3*x*x^(3*n) + 3*(3*n^2 + 4*n + 1)*a*c^2*x*x^(2*n) + 3
*(6*n^2 + 5*n + 1)*a^2*c*x*x^n + (6*n^3 + 11*n^2 + 6*n + 1)*a^3*x)/(6*n^3
+ 11*n^2 + 6*n + 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.28

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{3/2}\,dx=\frac{6\,a^3n^3x\mathrm{sgn}(cx^n+a)+2\,c^3n^2xx^{3\,n}\mathrm{sgn}(cx^n+a)+9\,ac^2n^2xx^{2\,n}\mathrm{sgn}(cx^n+a)+18\,a^2cn^2xx^n}{}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
(6*a^3*n^3*x*sgn(c*x^n + a) + 2*c^3*n^2*x*x^(3*n)*sgn(c*x^n + a) + 9*a*c^2
*n^2*x*x^(2*n)*sgn(c*x^n + a) + 18*a^2*c*n^2*x*x^n*sgn(c*x^n + a) + 11*a^3
*n^2*x*sgn(c*x^n + a) + 3*c^3*n*x*x^(3*n)*sgn(c*x^n + a) + 12*a*c^2*n*x*x^
(2*n)*sgn(c*x^n + a) + 15*a^2*c*n*x*x^n*sgn(c*x^n + a) + 6*a^3*n*x*sgn(c*x
^n + a) + c^3*x*x^(3*n)*sgn(c*x^n + a) + 3*a*c^2*x*x^(2*n)*sgn(c*x^n + a)
+ 3*a^2*c*x*x^n*sgn(c*x^n + a) + a^3*x*sgn(c*x^n + a))/(6*n^3 + 11*n^2 + 6
*n + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{3/2}\,dx=\int \left(a^2+c^2\,x^{2\,n}+2\,a\,c\,x^n\right)^{3/2}\,dx$$

input
```
int((a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(3/2),x)
```

output
```
int((a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.71

$$\int \left(a^2+2acx^n+c^2x^{2n}\right)^{3/2}\,dx=\frac{x(2x^{3n}c^3n^2+3x^{3n}c^3n+x^{3n}c^3+9x^{2n}a\,c^2n^2+12x^{2n}a\,c^2n+3x^{2n}a\,c^2+18x^na^2c\,n^2+15x^n}{6n^3+11n^2+6n+1}$$

input
```
int((a^2+2*a*c*x^n+c^2*x^(2*n))^(3/2),x)
```

output
```
(x*(2*x**(3*n)*c**3*n**2 + 3*x**(3*n)*c**3*n + x**(3*n)*c**3 + 9*x**(2*n)*
a*c**2*n**2 + 12*x**(2*n)*a*c**2*n + 3*x**(2*n)*a*c**2 + 18*x**n*a**2*c*n*
*2 + 15*x**n*a**2*c*n + 3*x**n*a**2*c + 6*a**3*n**3 + 11*a**3*n**2 + 6*a**
3*n + a**3))/(6*n**3 + 11*n**2 + 6*n + 1)
```

3.114 $\int \sqrt{a^2+2acx^n+c^2x^{2n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \sqrt{a^2+2acx^n+c^2x^{2n}}\,dx=\frac{ax\sqrt{a^2+2acx^n+c^2x^{2n}}}{a+cx^n}+\frac{c^2x^{1+n}\sqrt{a^2+2acx^n+c^2x^{2n}}}{(1+n)\,(ac+c^2x^n)}$$

output
```
a*x*(a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2)/(a+c*x^n)+c^2*x^(1+n)*(a^2+2*a*c*x^n
+c^2*x^(2*n))^(1/2)/(1+n)/(a*c+c^2*x^n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \sqrt{a^2+2acx^n+c^2x^{2n}}\,dx=\frac{x\sqrt{(a+cx^n)^2}(a+an+cx^n)}{(1+n)\,(a+cx^n)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*c*x^n + c^2*x^(2*n)],x]
```

output
```
(x*Sqrt[(a + c*x^n)^2]*(a + a*n + c*x^n))/((1 + n)*(a + c*x^n))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2acx^n+c^2x^{2n}}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2acx^n+c^2x^{2n}}\int \left(c^2x^n+ac\right)dx}{ac+c^2x^n}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2acx^n+c^2x^{2n}}\left(acx+\frac{c^2x^{n+1}}{n+1}\right)}{ac+c^2x^n}$$

input
```
Int[Sqrt[a^2 + 2*a*c*x^n + c^2*x^(2*n)],x]
```

output
```
(Sqrt[a^2 + 2*a*c*x^n + c^2*x^(2*n)]*(a*c*x + (c^2*x^(1 + n))/(1 + n)))/(a
*c + c^2*x^n)
```

Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{\sqrt{(a+c\,x^n)^2}\,xa}{a+c\,x^n}+\frac{\sqrt{(a+c\,x^n)^2}\,cx\,x^n}{(a+c\,x^n)(1+n)}$	56

input
```
int((a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+c*x^n)^2)^(1/2)/(a+c*x^n)*x*a+((a+c*x^n)^2)^(1/2)/(a+c*x^n)*c/(1+n)*x*
x^n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.23

$$\int \sqrt{a^2+2acx^n+c^2x^{2n}}\,dx=\frac{cxx^n+(an+a)x}{n+1}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
(c*x*x^n + (a*n + a)*x)/(n + 1)
```

Sympy [F]

$$\int \sqrt{a^2+2acx^n+c^2x^{2n}}\,dx=\int \sqrt{a^2+2acx^n+c^2x^{2n}}\,dx$$

input
```
integrate((a**2+2*a*c*x**n+c**2*x**(2*n))**(1/2),x)
```

output
```
Integral(sqrt(a**2 + 2*a*c*x**n + c**2*x**(2*n)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.22

$$\int \sqrt{a^2 + 2acx^n + c^2x^{2n}}\,dx = \frac{a(n+1)x + cxx^n}{n+1}$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
(a*(n + 1)*x + c*x*x^n)/(n + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.28

$$\int \sqrt{a^2 + 2acx^n + c^2x^{2n}}\,dx = \left(ax + \frac{cx^{n+1}}{n+1}\right)\mathrm{sgn}(cx^n + a)$$

input
```
integrate((a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
(a*x + c*x^(n + 1)/(n + 1))*sgn(c*x^n + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2 + 2acx^n + c^2x^{2n}}\,dx = \int \sqrt{a^2 + c^2\,x^{2\,n} + 2\,a\,c\,x^n}\,dx$$

input
```
int((a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(1/2),x)
```

output
```
int((a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.19

$$\int \sqrt{a^2 + 2acx^n + c^2x^{2n}}\, dx = \frac{x(x^n c + an + a)}{n + 1}$$

input
```
int((a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2),x)
```

output
```
(x*(x**n*c + a*n + a))/(n + 1)
```

3.115 $\int \frac{1}{\sqrt{a^2+2acx^n+c^2x^{2n}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{1}{\sqrt{a^2+2acx^n+c^2x^{2n}}}\,dx=\frac{x(a+cx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a\sqrt{a^2+2acx^n+c^2x^{2n}}}$$

output
```
x*(a+c*x^n)*hypergeom([1, 1/n],[1+1/n],-c*x^n/a)/a/(a^2+2*a*c*x^n+c^2*x^(2
*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a^2+2acx^n+c^2x^{2n}}}\,dx=\frac{x(a+cx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a\sqrt{(a+cx^n)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*c*x^n + c^2*x^(2*n)],x]
```

output
```
(x*(a + c*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((c*x^n)/a)])/(a*
Sqrt[(a + c*x^n)^2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1384, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2acx^n+c^2x^{2n}}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{(ac+c^2x^n)\int \frac{1}{c^2x^n+ac}dx}{\sqrt{a^2+2acx^n+c^2x^{2n}}}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x(ac+c^2x^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{ac\sqrt{a^2+2acx^n+c^2x^{2n}}}$$

input
```
Int[1/Sqrt[a^2 + 2*a*c*x^n + c^2*x^(2*n)],x]
```

output
```
(x*(a*c + c^2*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((c*x^n)/a)])
/(a*c*Sqrt[a^2 + 2*a*c*x^n + c^2*x^(2*n)])
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] && !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{1}{\sqrt{a^2 + 2ac\,x^n + c^2x^{2n}}}dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2),x)
```

output
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a^2 + 2acx^n + c^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{c^2x^{2\,n} + 2\,acx^n + a^2}}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(c^2*x^(2*n) + 2*a*c*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a^2 + 2acx^n + c^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{a^2 + 2acx^n + c^2x^{2n}}}\,dx$$

input
```
integrate(1/(a**2+2*a*c*x**n+c**2*x**(2*n))**(1/2),x)
```

output
```
Integral(1/sqrt(a**2 + 2*a*c*x**n + c**2*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a^2 + 2acx^n + c^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{c^2x^{2\,n} + 2\,acx^n + a^2}}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c^2*x^(2*n) + 2*a*c*x^n + a^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a^2 + 2acx^n + c^2x^{2n}}}\,dx = \int \frac{1}{\sqrt{c^2x^{2\,n} + 2\,acx^n + a^2}}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c^2*x^(2*n) + 2*a*c*x^n + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2+2acx^n+c^2x^{2n}}}\,dx=\int \frac{1}{\sqrt{a^2+c^2\,x^{2\,n}+2\,a\,c\,x^n}}\,dx$$

input
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(1/2),x)
```

output
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a^2+2acx^n+c^2x^{2n}}}\,dx=\int \frac{1}{x^nc+a}dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(1/2),x)
```

output
```
int(1/(x**n*c + a),x)
```

3.116 $\int \frac{1}{\left(a^2+2acx^n+c^2x^{2n}\right)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{3/2}} dx = \frac{x(a+cx^n)\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^3\sqrt{a^2+2acx^n+c^2x^{2n}}}$$

output
```
x*(a+c*x^n)*hypergeom([3, 1/n],[1+1/n],-c*x^n/a)/a^3/(a^2+2*a*c*x^n+c^2*x^
(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{3/2}} dx = \frac{x(a+cx^n)^3\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^3\left((a+cx^n)^2\right)^{3/2}}$$

input
```
Integrate[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(-3/2),x]
```

output
```
(x*(a + c*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((c*x^n)/a)])/(
a^3*((a + c*x^n)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1384, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{3/2}}\,dx$$
$$\Big\downarrow \text{ 1384}$$
$$\frac{(ac^3+c^4x^n)\int \frac{1}{(c^2x^n+ac)^3}dx}{\sqrt{a^2+2acx^n+c^2x^{2n}}}$$
$$\Big\downarrow \text{ 778}$$
$$\frac{x(ac^3+c^4x^n)\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^3c^3\sqrt{a^2+2acx^n+c^2x^{2n}}}$$

input
```
Int[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(-3/2),x]
```

output
```
(x*(a*c^3 + c^4*x^n)*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((c*x^n)/a)
])/(a^3*c^3*Sqrt[a^2 + 2*a*c*x^n + c^2*x^(2*n)])
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{1}{\left(a^{2}+2 a c\,x^{n}+c^{2} x^{2 n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(3/2),x)
```

output
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{3/2}}\,dx=\int \frac{1}{\left(c^{2} x^{2\,n}+2\,a c x^{n}+a^{2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^(2*n) + 2*a*c*x^n + a^2)/(c^4*x^(4*n) + 4*a^2*c^2*x^(2
*n) + 4*a^3*c*x^n + a^4 + 2*(2*a*c^3*x^n + a^2*c^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{3/2}}\,dx=\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*c*x**n+c**2*x**(2*n))**(3/2),x)
```

output
```
Integral((a**2 + 2*a*c*x**n + c**2*x**(2*n))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{3/2}}\,dx=\int \frac{1}{(c^2x^{2\,n}+2\,acx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

output
```
(2*n^2 - 3*n + 1)*integrate(1/2/(a^2*c*n^2*x^n + a^3*n^2), x) + 1/2*(c*(2*
n - 1)*x*x^n + a*(3*n - 1)*x)/(a^2*c^2*n^2*x^(2*n) + 2*a^3*c*n^2*x^n + a^4
*n^2)
```

Giac [F]

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{3/2}}\,dx=\int \frac{1}{(c^2x^{2\,n}+2\,acx^n+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(3/2),x, algorithm="giac")
```

output
```
integrate((c^2*x^(2*n) + 2*a*c*x^n + a^2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{3/2}}\,dx=\int \frac{1}{\left(a^2+c^2\,x^{2\,n}+2\,a\,c\,x^n\right)^{3/2}}\,dx$$

input
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(3/2),x)
```

output
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{3/2}}\,dx=\int \frac{1}{x^{3n}c^3+3x^{2n}a\,c^2+3x^na^2c+a^3}dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(3/2),x)
```

output
```
int(1/(x**(3*n)*c**3 + 3*x**(2*n)*a*c**2 + 3*x**n*a**2*c + a**3),x)
```

3.117 $\int \frac{1}{\left(a^2+2acx^n+c^2x^{2n}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{1}{\left(a^2+2acx^n+c^2x^{2n}\right)^{5/2}}\,dx=\frac{x(a+cx^n)\,\text{Hypergeometric2F1}\left(5,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^5\sqrt{a^2+2acx^n+c^2x^{2n}}}$$

output
```
x*(a+c*x^n)*hypergeom([5, 1/n],[1+1/n],-c*x^n/a)/a^5/(a^2+2*a*c*x^n+c^2*x^
(2*n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a^2+2acx^n+c^2x^{2n}\right)^{5/2}}\,dx=\frac{x(a+cx^n)^5\,\text{Hypergeometric2F1}\left(5,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^5\left((a+cx^n)^2\right)^{5/2}}$$

input
```
Integrate[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(-5/2),x]
```

output
```
(x*(a + c*x^n)^5*Hypergeometric2F1[5, n^(-1), 1 + n^(-1), -((c*x^n)/a)])/(
a^5*((a + c*x^n)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1384, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{5/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{(ac^5+c^6x^n)\int \frac{1}{(c^2x^n+ac)^5}dx}{\sqrt{a^2+2acx^n+c^2x^{2n}}}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x(ac^5+c^6x^n)\,\text{Hypergeometric2F1}\left(5,\frac{1}{n},1+\frac{1}{n},-\frac{cx^n}{a}\right)}{a^5c^5\sqrt{a^2+2acx^n+c^2x^{2n}}}$$

input
```
Int[(a^2 + 2*a*c*x^n + c^2*x^(2*n))^(-5/2),x]
```

output
```
(x*(a*c^5 + c^6*x^n)*Hypergeometric2F1[5, n^(-1), 1 + n^(-1), -((c*x^n)/a)
])/(a^5*c^5*Sqrt[a^2 + 2*a*c*x^n + c^2*x^(2*n)])
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{1}{(a^2 + 2ac\,x^n + c^2x^{2n})^{\frac{5}{2}}}dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(5/2),x)
```

output
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^{5/2}}\,dx = \int \frac{1}{(c^2x^{2\,n} + 2\,acx^n + a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c^2*x^(2*n) + 2*a*c*x^n + a^2)/(c^6*x^(6*n) + 8*a^3*c^3*x^(3
*n) + 12*a^4*c^2*x^(2*n) + 6*a^5*c*x^n + a^6 + 3*(2*a*c^5*x^n + a^2*c^4)*x
^(4*n) + 3*(4*a^2*c^4*x^(2*n) + 4*a^3*c^3*x^n + a^4*c^2)*x^(2*n)), x)
```

Sympy [F]

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{5/2}}\,dx = \int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a**2+2*a*c*x**n+c**2*x**(2*n))**(5/2),x)
```

output
```
Integral((a**2 + 2*a*c*x**n + c**2*x**(2*n))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{5/2}}\,dx = \int \frac{1}{(c^2x^{2n}+2\,acx^n+a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(5/2),x, algorithm="maxima")
```

output
```
(24*n^4 - 50*n^3 + 35*n^2 - 10*n + 1)*integrate(1/24/(a^4*c*n^4*x^n + a^5*
n^4), x) + 1/24*((24*n^3 - 26*n^2 + 9*n - 1)*c^3*x*x^(3*n) + (84*n^3 - 85*
n^2 + 28*n - 3)*a*c^2*x*x^(2*n) + (104*n^3 - 94*n^2 + 29*n - 3)*a^2*c*x*x^
n + (50*n^3 - 35*n^2 + 10*n - 1)*a^3*x)/(a^4*c^4*n^4*x^(4*n) + 4*a^5*c^3*n
^4*x^(3*n) + 6*a^6*c^2*n^4*x^(2*n) + 4*a^7*c*n^4*x^n + a^8*n^4)
```

Giac [F]

$$\int \frac{1}{(a^2+2acx^n+c^2x^{2n})^{5/2}}\,dx = \int \frac{1}{(c^2x^{2n}+2\,acx^n+a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(5/2),x, algorithm="giac")
```

output
```
integrate((c^2*x^(2*n) + 2*a*c*x^n + a^2)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^{5/2}}\,dx = \int \frac{1}{\left(a^2 + c^2\,x^{2\,n} + 2\,a\,c\,x^n\right)^{5/2}}\,dx$$

input
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(5/2),x)
```

output
```
int(1/(a^2 + c^2*x^(2*n) + 2*a*c*x^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a^2 + 2acx^n + c^2x^{2n})^{5/2}}\,dx = \int \frac{1}{x^{5n}c^5 + 5x^{4n}a\,c^4 + 10x^{3n}a^2c^3 + 10x^{2n}a^3c^2 + 5x^n a^4 c + a^5}dx$$

input
```
int(1/(a^2+2*a*c*x^n+c^2*x^(2*n))^(5/2),x)
```

output
```
int(1/(x**(5*n)*c**5 + 5*x**(4*n)*a*c**4 + 10*x**(3*n)*a**2*c**3 + 10*x**(
2*n)*a**3*c**2 + 5*x**n*a**4*c + a**5),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                   ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
         return True
      else:
         return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file